5 Current and Resistance



FIGURE 5.1 A current flowing through a wire makes this light bulb shine.

Electric lighting is so commonplace that you don't even think about it. You walk into a dark room and simply flick on a switch, bringing the room to nearly daytime brightness (Figure 5.1). However, what happens when the switch is flicked depends ultimately on principles of physics and engineering devices that took decades to develop and refine.

This chapter is the first to focus on electric charges in motion. It presents some of the fundamental concepts that we'll use in Chapter 6 to analyze basic electric circuits, which are integral to all electronics applications. These chapters concentrate on the electrical effects of moving charges, but you should be aware that moving charges also give rise to other effects, which we'll start to examine in Chapter 7 on magnetism.

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WHAT WE WILL LEARN

- Electric current at a point in a circuit is the rate at which net charge moves past that point.
- Direct current is current flowing in a direction that does not change with time. The direction of current is defined as the direction in which positive charge would be moving.
- The current density passing a given point in a conductor is the current per cross-sectional area.
- The conductivity of a material characterizes the ability of that material to conduct current. Its inverse is resistivity.
- The resistance of a device depends on its geometry and on the material of which it is made.
- The resistivity of a conductor increases approximately linearly with temperature.
- Electromotive force (usually referred to as emf) is a potential difference in an electric circuit.
- Ohm's Law states that the potential drop across a device is equal to the current flowing through the device times the resistance of the device.
- A simple circuit consists of a source of emf and resistors connected in series or in parallel.
- In a circuit diagram, an equivalent resistance can replace resistors connected in series or in parallel.
- The power in a circuit is the product of the current and the voltage drop.
- A diode conducts current in one direction but not in the opposite direction.



5.1 Electric Current

Up to this point, our study of electricity has focused on electrostatics, which deals with the properties of stationary electric charges and fields. If electrostatics were all there was to electricity, it would not be nearly as important to modern society as it is. The world-changing impact of electricity is due to the properties of charges in motion, or electric current. All electrical devices rely on some kind of current for their operation.

Let's start by looking at a few very simple experiments. When you were a kid, you very likely had some toys that ran on batteries, and probably some of them contained small light bulbs. Consider a very simple circuit consisting only of a battery, a switch, and a light bulb (see Figure 5.2). If the switch is open, as in Figure 5.2a, the light bulb does not shine. If the switch is closed, as in Figure 5.2b, the light bulb turns on. We all know why this happens—because a current flows through the closed circuit. In this chapter, we'll examine what it means for the current to flow, what the physical basis of current is, and how it is related to the potential difference provided by the battery. We'll see that the light bulb acts as a resistor in the circuit, and we'll examine how resistors behave.

Let's begin by considering the simple experiment performed in Figure 5.2c, where the battery's orientation is the reverse of what it is in Figure 5.2b. The light bulb shines just

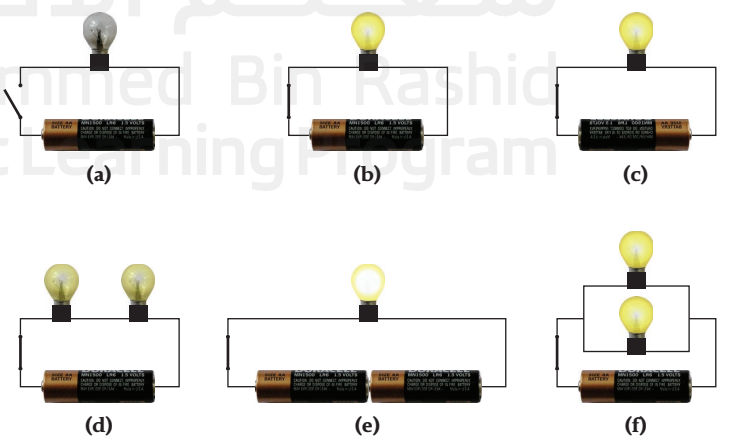


FIGURE 5.2 Experiments with batteries and light bulbs.

the same, despite the fact that the sign of the potential difference provided by the battery is reversed. (The positive terminal of the battery is on the end with the copper color.) In Figure 5.2d, two light bulbs are in the circuit, one behind the other. (Section 5.5 will focus on this arrangement of resistors, which is a connection *in series*.) Each of the two light bulbs shines with significantly less intensity than the single bulb in Figure 5.2c, so the current in the circuit may be smaller than before. On the other hand, two batteries in series, as in Figure 5.2e, double the potential difference in the circuit, and the bulb shines significantly brighter. Finally, if separate wires are used to connect the two light bulbs to a single battery, as shown in Figure 5.2f, the light bulbs shine with about the same intensity as in Figure 5.2b or Figure 5.2c. This way of wiring resistors in a circuit is called a *parallel connection* and will be explored in Section 5.6.

Quantitatively, the **electric current,** i, is the net charge passing a given point in a given time, divided by that time. The random motion of electrons in a conductor is not a current, in spite of the fact that large amounts of charge are moving past a given point, because no *net* charge flows. If net charge dq passes a point during time dt, the current at that point is, by definition,

$$i = \frac{dq}{dt}. \tag{5.1}$$

The net amount of charge passing a given point in time t is the integral of the current with respect to time:

$$q = \int dq = \int_0^t i\, dt'. \tag{5.2}$$

Total charge is conserved, implying that charge flowing in a conductor is never lost. Therefore, the same amount of charge flows into one end of a conductor as emerges from the other end.

The unit of current, coulombs per second, was given the name **ampere** (abbreviated A, or sometimes amp), after the French physicist André Ampère (1775–1836):

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}.$$

Some typical currents are 1 A for a light bulb, 200 A for the starter in your car, 1 mA = $1 \cdot 10^{-3}$A to power your MP3 player, 1 nA for the currents in your brain's neurons and synaptic connections, and 10,000 A = 10^4 A in a lightning strike (for a short time). The smallest currents that can be measured are those from individual electrons tunneling in scanning tunneling microscopes and are on the order of 10 pA. The largest current in the Solar System is the solar wind, which is in the GA range. Other examples of the broad range of currents are shown in Figure 5.3.

You should remember this handy safety rule related to the orders of magnitudes of currents: 1-10-100. That is, 1 mA of current flowing through a human body can be felt (as

Hohlraums

Hohlraums are devices for inertial confinement fusion.

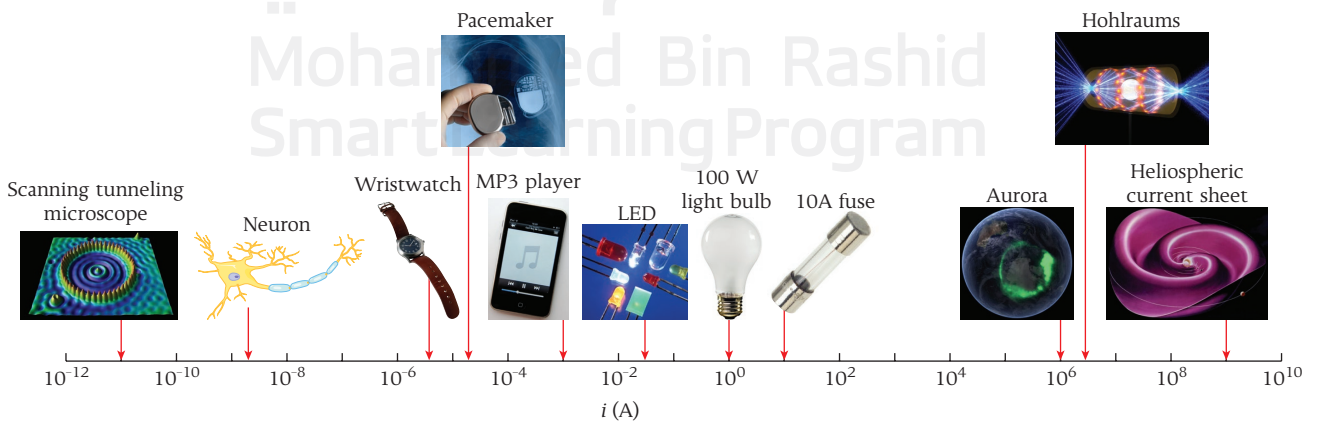


FIGURE 5.3 Examples of electrical currents ranging from 1 pA to 10 GA.

a tingle, usually), 10 mA of current makes muscles contract to the point that the person cannot let go of the wire carrying the current, and 100 mA is sufficient to stop the heart.

A current that flows in only one direction, which does not change with time, is called **direct current.** In this chapter, the direction of the current flowing in a conductor is indicated by an arrow.

Physically, the charge carriers in a conductor are electrons, which are negatively charged. However, by convention, positive current is defined as flowing from the positive to the negative terminal. The reason for this counterintuitive definition of current direction is that the definition originated in the second half of the 19th century, when it was not known that electrons are the charge carriers responsible for current. And so the current direction was simply defined as the direction in which the positive charges would flow.

Self-Test Opportunity 5.1

A typical rechargeable AA battery is rated at 700 mAh. How long can this battery provide a current of 100 μA ?

EXAMPLE 5.1 Iontophoresis

There are three ways to administer anti-inflammatory medication. The painless way is the oral one–simply swallowing the drug. However, this method typically leads to a small amount of the drug in the affected tissue, on the order of 1 μg. The second way is to have the drug injected locally with a needle. This hurts but can deposit on the order of 10 mg of the drug in the affected tissue–four orders of magnitude more than with the oral method. However, since the 1990s, a third method has been available, which is also painless and can deposit on the order of 100 μg of a drug in the area where it is needed. This method, called *iontophoresis*, uses (very weak) electrical currents that are sent through the patient's tissue (Figure 5.4). The iontophoresis device consists of a battery and two electrodes (plus other electronic circuitry that allows the nurse to control the strength of the current applied). The anti-inflammatory drug, usually dexamethasone, is applied to the underside of the negatively charged electrode. A current flows through the patient's skin and deposits the drug in the tissue to a depth of up to 1.7 cm.

PROBLEM

A nurse wants to administer 80 μg of dexamethasone to the heel of an injured soccer player. If she uses an iontophoresis device that applies a current of 0.14 mA, as shown in Figure 5.4, how long does the administration of the dose take? Assume that the instrument has an application rate of 650 μg/C and that the current flows at a constant rate.

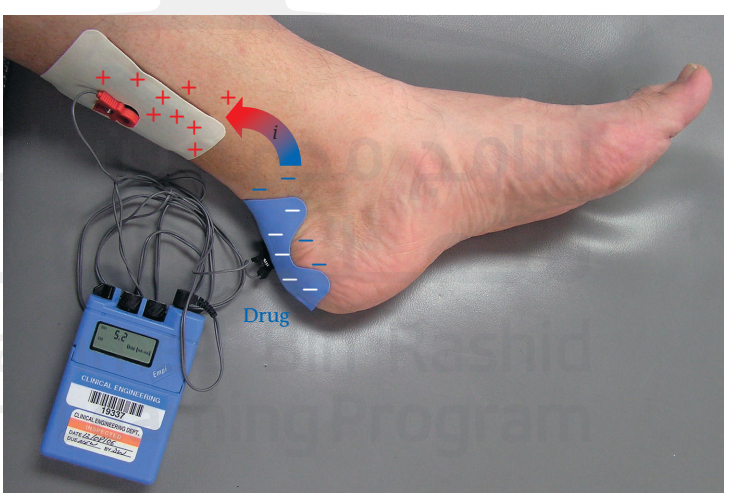


FIGURE 5.4 Iontophoresis is the application of medication under the skin with the aid of electrical current.

SOLUTION

If the drug application rate is 650 μg/C, to apply 80 μg requires a total charge of

$$q = \frac{80\ \mu g}{650\ \mu g/C} = 0.123\ C.$$

The current flows at a constant rate, so the integral in equation 5.2 is simply

$$q = \int_0^t i\,dt' = it.$$

Solving for t and inserting the numbers, we find

$$q = it \Rightarrow t = \frac{q}{i} = \frac{0.123\ \text{C}}{0.14 \times 10^{-3}\ \text{A}} = 880\ \text{s}.$$

The iontophoresis treatment of the athlete will take approximately 15 min.

5.2 Current Density

Consider a current flowing in a conductor. For a perpendicular plane through the conductor, the current per unit area flowing through the conductor at that point (the cross-sectional area A in Figure 5.5) is the **current density,** $\vec{J}$. The direction of $\vec{J}$ is defined as the direction of the velocity of the positive charges (or opposite to the direction of negative charges) crossing the plane. The current flowing through the plane is

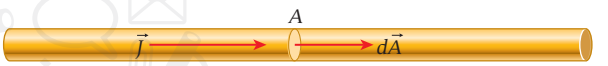


FIGURE 5.5 Segment of a conductor (wire), with a perpendicular plane intersecting it and forming a cross-sectional area A.

$$i = \int \vec{J} \cdot d\vec{A}, \tag{5.3}$$

where $d\vec{A}$ is the differential area element of the perpendicular plane, as indicated in Figure 5.5. If the current is uniform and perpendicular to the plane, then $i = JA$, and the magnitude of the current density can be expressed as

$$J = \frac{i}{A}. \tag{5.4}$$

In a conductor that is not carrying current, the conduction electrons move randomly. When current flows through the conductor, electrons still move randomly but also have an additional **drift velocity,** $\vec{v}_d$, in the direction opposite to that of the electric field driving the current. The magnitude of the velocity of random motion is on the order of 10^6 m/s, while the magnitude of the drift velocity is on the order of 10^{-4} m/s or even less. With such a slow drift velocity, you might wonder why a light comes on almost immediately after you turn on a switch. The answer is that the switch establishes an electric field almost immediately throughout the circuit (with a speed on the order of $3 \cdot 10^8$ m/s), causing the free electrons in the entire circuit (including in the light bulb) to move almost instantly.

The current density is related to the drift velocity of the moving electrons. Consider a conductor with cross-sectional area A and electric field $\vec{E}$ applied to it. Suppose the conductor has n conduction electrons per unit volume, and assume that all the electrons have the same drift velocity and that the current density is uniform. The negatively charged electrons will drift in a direction opposite to that of the electric field. In a time interval dt, each electron moves a net distance $v_d\,dt$. The volume of electrons passing a cross section of the conductor in time dt is then $Av_d\,dt$, and the number of electrons in this volume is $nAv_d\,dt$. Each electron has charge $-e$, so the charge dq that flows through the area in time dt is

$$dq = -nev_dA\,dt. \tag{5.5}$$

Therefore, the current is

$$i = \frac{dq}{dt} = -nev_dA. \tag{5.6}$$

The resulting current density is

$$J = \frac{i}{A} = -nev_d. \tag{5.7}$$

Equation 5.7 was derived in one spatial dimension, as is appropriate for a wire. However, it can be readily generalized to arbitrary directions in three-dimensional space:

$$\vec{J} = -(ne)\vec{v}_d.$$

You can see that the drift velocity vector is antiparallel to the current density vector, as stated before.

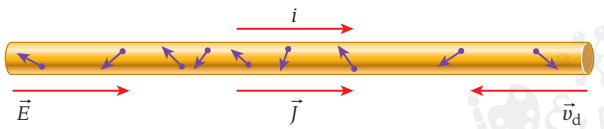


FIGURE 5.6 Electrons moving in a wire from right to left, causing a current in the direction from left to right.

Figure 5.6 shows a schematic drawing of a wire carrying a current. The physical current carriers are negatively charged electrons. In Figure 5.6, these electrons are moving to the left with drift velocity $\vec{v}_d$. However, the electric field, the current density, and the current are all directed to the right because of the convention that these quantities refer to positive charges. You may find this convention somewhat confusing, but you'll need to keep it in mind.

SOLVED PROBLEM 5.1 Drift Velocity of Electrons in a Copper Wire

PROBLEM

You are playing "Galactic Destroyer" on your video game console. Your game controller operates at 12 V and is connected to the main box with an 18 gauge copper wire of length 1.5 m. As you fly your spaceship into battle, you hold the joystick in the forward position for 5.3 s, sending a current of 0.78 mA to the console. How far have the electrons in the wire moved during those few seconds, while on the screen your spaceship crossed half of a star system?

SOLUTION

THINK To find out how far electrons in a wire move during a given time interval, we need to calculate their drift velocity. To determine the drift velocity for electrons in a copper wire carrying a current, we need to find the density of charge-carrying electrons in copper. Then, we can apply the definition of the charge density to calculate the drift velocity.

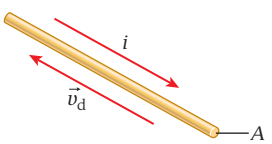


FIGURE 5.7 A copper wire with cross-sectional area A carrying a current, i.

SKETCH A copper wire with cross-sectional area A carrying a current, i, is shown in Figure 5.7, which also shows that, by convention, the electrons drift in the direction opposite to the direction of the current.

RESEARCH We obtain the distance x traveled by the electrons during time t from

$$x = v_d t,$$

where v_d is the magnitude of the drift velocity of the electrons. The drift velocity is related to the current density via equation 5.7:

$$\frac{i}{A} = -nev_d \qquad \text{(i)}$$

where i is the current, A is the cross-sectional area (0.823 mm^2 for an 18 gauge wire), n is the density of electrons, and $-e$ is the charge of an electron. The density of electrons is defined as

$$n = \frac{\text{number of conduction electrons}}{\text{volume}}.$$

We can calculate the density of electrons by assuming there is one conduction electron per copper atom. The density of copper is

$$\rho_{Cu} = 8.96 \text{ g/cm}^3 = 8960 \text{ kg/m}^3.$$

One mole of copper has a mass of 63.5 g and contains 6.02×10^{23} atoms. Thus, the density of electrons is

$$n = \left(\frac{1 \text{ electron}}{1 \text{ atom}}\right)\left(\frac{6.02 \times 10^{23} \text{ atoms}}{63.5 \text{ g}}\right)\left(\frac{8.96 \text{ g}}{1 \text{ cm}^3}\right)\left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3}\right) = 8.49 \times 10^{28} \frac{\text{electrons}}{\text{m}^3}.$$

SIMPLIFY We solve equation (i) for the magnitude of the drift velocity:

$$v_d = \frac{i}{neA}.$$

Thus, the distance traveled by the electrons is

$$x = v_d t = \frac{i}{neA} t.$$

CALCULATE Putting in the numerical values, we get

$$x = v_d t = \frac{it}{neA} = \frac{\left(0.78 \times 10^{-3}\ \text{A}\right)(5.3\ \text{s})}{\left(8.49 \times 10^{28}\ \text{m}^{-3}\right)\left(1.602 \times 10^{-19}\ \text{C}\right)\left(0.823\ \text{mm}^2\right)}$$

$$= \left(6.96826 \times 10^{-8}\ \text{m/s}\right)(5.3\ \text{s})$$

$$= 3.69318 \times 10^{-7}\ \text{m}.$$

ROUND We report our results to two significant figures:

$$v_d = 7.0 \times 10^{-8}\ \text{m/s},$$

and

$$x = 3.7 \times 10^{-7}\ \text{m} = 0.37\ \mu\ \text{m}.$$

DOUBLE-CHECK Our result for the magnitude of the drift velocity turns out to be a stunningly small number. Earlier, it was stated that typical drift velocities are on the order of 10^{-4} m/s or smaller. Since the current is proportional to the drift velocity, a relatively small current implies a relatively small drift velocity. An 18 gauge wire can carry a current of several amperes, so the current specified in the problem statement is less than 1% of the maximum current. Therefore, the fact that our calculated drift velocity is less than 1% of 10^{-4} m/s, a typical drift velocity for high currents, is reasonable.

The distance we calculated for the movement of the electrons is less than 0.001 of the thickness of a fingernail, a very small distance compared to the length of the wire. This result provides a valuable reminder that the electromagnetic field moves with nearly the speed of light (in vacuum) inside a conductor and causes all conduction electrons to drift basically at the same time. Therefore, the signal from your game controller arrives almost instantaneously at the console, despite the incredibly slow pace of the individual electrons.

5.3 Resistivity and Resistance



Some materials conduct electricity better than others. Applying a given potential difference across a good conductor results in a relatively large current; applying the same potential difference across an insulator produces little current. The **resistivity,** ρ, is a measure of how strongly a material opposes the flow of electric current. The **resistance,** R, is a material's opposition to the flow of electric current.

If a known electric potential difference, ΔV, is applied across a conductor (some physical device or material that conducts current) and the resulting current, i, is measured, the resistance of that conductor is given by

$$R = \frac{\Delta V}{i}. \tag{5.8}$$

The units of resistance are volts per ampere, a combination that was given the name **ohm** and the symbol Ω (the capital Greek letter omega), in honor of the German physicist Georg Simon Ohm (1789–1854):

$$1\ \Omega = \frac{1\ \text{V}}{1\ \text{A}}.$$

Rearrangement of equation 5.8 results in

$$i = \frac{\Delta V}{R}, \tag{5.9}$$

which states that for a given potential difference, ΔV, the current, i, is inversely proportional to the resistance, R. This equation is commonly referred to as **Ohm's Law.** A rearrangement of equation 5.9, $\Delta V = iR$, is sometimes also referred to as Ohm's Law.

Sometimes devices are described in terms of the **conductance,** G, defined as

$$G = \frac{i}{\Delta V} = \frac{1}{R}.$$

Conductance has the SI derived unit of siemens (S), in honor of German inventor and industrialist Ernst Werner von Siemens (1816–1892):

$$1\ \text{S} = \frac{1\ \text{A}}{1\ \text{V}} = \frac{1}{1\ \Omega}.$$

In some conductors, the resistivity depends on the direction in which the current is flowing. This chapter assumes that the resistivity of a material is uniform for all directions of the current.

The resistance of a device depends on the material of which the device is made as well as its geometry. As stated earlier, the resistivity of a material characterizes how much it opposes the flow of current. The resistivity is defined in terms of the magnitude of the applied electric field, E, and the magnitude of the resulting current density, J:

$$\rho = \frac{E}{J}. \tag{5.10}$$

The units of resistivity are

$$[\rho] = \frac{[E]}{[J]} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V m}}{\text{A}} = \Omega\ \text{m}.$$

Table 5.1 lists the resistivities of some representative conductors at 20 °C. As you can see, typical values for the resistivity of metal conductors used in wires are on the order of $10^{-8}\ \Omega$ m. For example, copper has a resistivity of about $2 \cdot 10^{-8}\ \Omega$ m. Several metal alloys listed in Table 5.1 have useful properties. For example, wire made from Nichrome (80% nickel and 20% chromium) is often used as a heating element in devices such as toasters. The next time you are toasting an English muffin, look inside the toaster. The glowing elements are probably Nichrome wires. The resistivity of Nichrome ($108 \cdot 10^{-8}\ \Omega$ m) is about 50 times that of copper. Thus, when current is run through the Nichrome wires of the toaster, the wires dissipate power and heat up until they glow with a dull red color, while the copper wires of the electrical cord that connects the toaster to the wall remain cool.

Table 5.1 The Resistivity and the Temperature Coefficient of the Resistivity for Some Representative Conductors

Material	Resistivity, ρ, at 20 °C ($10^{-8}\ \Omega$ m)	Temperature Coefficient, α ($10^{-3}\ \text{K}^{-1}$)
Silver	1.62	3.8
Copper	1.72	3.9
Gold	2.44	3.4
Aluminum	2.82	3.9
Brass	3.9	2
Tungsten	5.51	4.5
Nickel	7	5.9
Iron	9.7	5
Steel	11	5
Tantalum	13	3.1
Lead	22	4.3
Constantan	49	0.01
Stainless steel	70	1
Mercury	95.8	0.89
Nichrome	108	0.4

The values for steel and stainless steel depend strongly on the type of steel.

Sometimes materials are specified in terms of their **conductivity,** σ, rather than their resistivity, ρ; conductivity is defined as

$$\sigma = \frac{1}{\rho}.$$

The units of conductivity are $(\Omega\ \text{m})^{-1}$.

The resistance of a conductor can be found from its resistivity and its geometry. For a homogeneous conductor of length L and constant cross-sectional area A, the equation $\Delta V = -\int \vec{E} \cdot d\vec{s}$ can be used to relate the electric field, E, and the electric potential difference, ΔV, across the conductor:

$$E = \frac{\Delta V}{L}.$$

Note that in contrast to electrostatics, where the surface of any conductor is an equipotential surface and has no electric field inside and no current flowing through it, the conductor in this situation has $\Delta V \neq 0$ and $\vec{E} \neq 0$, causing a current to flow. The magnitude of the current density is the current divided by the cross-sectional area:

$$J = \frac{i}{A}.$$

From the definition of resistivity (equation 5.10) and using $J = i/A$ and Ohm's Law (equation 5.8), we obtain

$$\rho = \frac{E}{J} = \frac{\Delta V/L}{i/A} = \frac{\Delta V}{i}\frac{A}{L} = \frac{iR}{i}\frac{A}{L} = R\frac{A}{L}.$$

Rearranging terms yields an expression for the resistance of a conductor in terms of the resistivity of its constituent material, the length, and the cross-sectional area:

$$R = \rho \frac{L}{A}. \tag{5.11}$$

Note that in the limit $A \to \infty$, equation 5.11 implies that $R \to 0$. Therefore, the equation is only an approximation, because the conduction electrons will still encounter some resistance in traveling through some length L of a material of density ρ. However, for all wires in actual use, equation 5.11 is an excellent approximation.

Size Convention for Wires

The American Wire Gauge (AWG) size convention for wires specifies diameters and thus cross-sectional areas on a logarithmic scale. The AWG size convention is shown in Table 5.2. The wire gauge is related to the diameter: The higher the gauge number, the thinner the wire. For large-diameter wires, gauge numbers consist of one or more zeros, as shown in Table 5.2. A 00 gauge wire is equivalent to a –1 gauge, a 000 gauge wire is equivalent to a –2 gauge, and so on. By definition, a 36 gauge wire has a diameter of exactly 0.005 in, and a 0000 gauge wire has a diameter of exactly 0.46 in. (These sizes appear in red in Table 5.2.) There are 39 gauge values from 0000-gauge to 36 gauge, and the gauge number is a logarithmic representation of the wire diameter. Therefore, the formula to convert from the AWG gauge to the wire diameter, in inches, is $d = (0.005)92^{(36-n)/39}$, where n is the gauge number. Typical residential wiring uses 12 gauge to 10 gauge wires. An important rule of thumb is that a reduction by 3 gauges doubles the cross-sectional area of the wire. Examining equation 5.11, you can see that to cut the resistance of a given length of wire in half, you have to reduce the gauge number by 3.

Table 5.2 Wire Diameters and Cross-Sectional Areas as Defined by the American Wire Gauge Convention

AWG	d (in)	d (mm)	A (mm^2)
000000	0.5800	14.733	170.49
00000	0.5165	13.120	135.20
0000	0.46	11.684	107.22
000	0.4096	10.405	85.029
00	0.3648	9.2658	67.431
0	0.3249	8.2515	53.475
1	0.2893	7.3481	42.408
	. . .		
8	0.1285	3.2636	8.3656
9	0.1144	2.9064	6.6342
10	0.1019	2.5882	5.2612
11	0.0907	2.3048	4.1723
12	0.0808	2.0525	3.3088
13	0.0720	1.8278	2.6240
14	0.0641	1.6277	2.0809
15	0.0571	1.4495	1.6502
16	0.0508	1.2908	1.3087
17	0.0453	1.1495	1.0378
18	0.0403	1.0237	0.8230
	. . .		
35	0.0056	0.1426	0.0160
36	0.005	0.1270	0.0127
37	0.0045	0.1131	0.0100
	. . .		

EXAMPLE 5.2 Resistance of a Copper Wire

Standard wires that electricians put into residential housing have fairly low resistance.

PROBLEM

What is the resistance of the 100.0 m standard 12 gauge copper wire that is typically used in wiring household electrical outlets?

SOLUTION

A 12 gauge copper wire has a diameter of 2.053 mm (see Table 5.2). Its cross-sectional area is then

$$A = 3.31 \text{ mm}^2.$$

Using the value for the resistivity of copper from Table 5.1 and equation 5.11, we find

$$R = \rho \frac{L}{A} = (1.72 \times 10^{-8}\ \Omega\ \text{m}) \frac{100.0 \text{ m}}{3.31 \times 10^{-6}\ \text{m}^2} = 0.520\ \Omega.$$

Concept Check 5.1

If the diameter of the wire in Example 5.2 is doubled, its resistance will

a) increase by a factor of 4.

b) increase by a factor of 2.

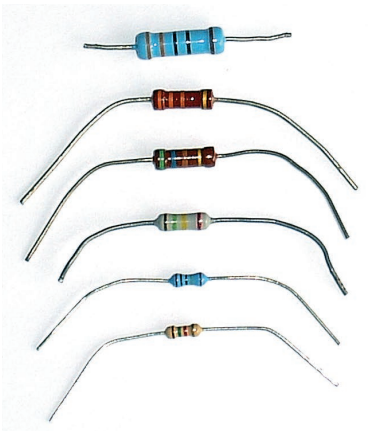
c) stay the same.

d) decrease by a factor of 2.

e) decrease by a factor of 4.

Resistor Codes

In many applications, circuit design calls for a range of resistances in various parts of a circuit. Commercially available resistors, such as those shown in Figure 5.8a, have a wide range of resistances. Resistors are commonly made of carbon enclosed in a plastic cover that looks like a medicine capsule, with wires sticking out at the ends for electrical connection. The value of the resistance is indicated by three or four color bands on the plastic covering. The first two bands indicate numbers for the mantissa, the third represents a power of 10, and the fourth indicates a tolerance for the range of values. For the mantissa and power of 10, the numbers associated with the colors are black = 0, brown = 1, red = 2, orange = 3,



(a)



(b)

FIGURE 5.8 (a) Selection of resistors with various resistances. (b) Color-coding of a 150 Ω resistor.

Concept Check 5.2

If the temperature of a copper wire with a resistance of 100 Ω is increased by 25 K, the resistance will

a) increase by approximately 10 Ω.

b) increase by approximately 4 m Ω.

c) decrease by approximately 4 m Ω.

d) decrease by approximately 10 Ω.

e) stay the same.

yellow = 4, green = 5, blue = 6, purple = 7, gray = 8, and white = 9. For the tolerance, brown means 1%, red means 2%, gold means 5%, silver means 10%, and no band at all means 20%. For example, the single resistor shown in Figure 5.8b has the colors (left to right) brown, green, brown, and gold. From the code, the resistance of this resistor is 15×10^1 Ω = 150 Ω, with a tolerance of 5%.

Temperature Dependence and Superconductivity

The values of resistivity and resistance vary with temperature. For metals, this dependence on temperature is linear over a broad range of temperatures. An empirical relationship for the temperature dependence of the resistivity of a metal is

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0), \tag{5.12}$$

where ρ is the resistivity at temperature T, ρ_0 is the resistivity at temperature T_0, and α is the **temperature coefficient of electric resistivity** for the particular conductor.

In everyday applications, the temperature dependence of the resistance is often important. Equation 5.11 states that the resistance of a device depends on its length and cross-sectional area. Thus, the temperature dependence of the resistance of a conductor can be approximated as

$$R - R_0 = R_0 \alpha (T - T_0). \tag{5.13}$$

Note that equations 5.12 and 5.13 deal with temperature differences, so the temperatures can be expressed in degrees Celsius or kelvins (but not in degrees Fahrenheit!). Also note that T_0 does not necessarily have to be room temperature.

Values of α for representative conductors are listed in Table 5.1. Note that common metal conductors such as copper have a temperature coefficient of electric resistivity on the order of 4×10^{-3} K^{-1}. However, one metal alloy, constantan (60% copper and 40% nickel), has the special characteristic that its temperature coefficient of electric resistivity is very small: $\alpha = 1 \times 10^{-5}$ K^{-1}. The name of this alloy comes from shortening the phrase "constant resistance." The small temperature coefficient of constantan combined with its relatively high resistivity of 4.9×10^{-7} Ω m makes it useful for precision resistors whose resistances have little dependence on temperature. Note also that Nichrome has a relative small temperature coefficient, 4×10^{-4} K^{-1}, which makes it suitable for the construction of heating elements, as noted earlier.

According to equation 5.12, most materials have a resistivity that varies linearly with temperature under ordinary circumstances. However, some materials do not follow this rule at low temperatures. At very low temperatures, the resistivity of some materials goes to exactly zero. These materials are called **superconductors.** Superconductors have applications in the construction of magnets for devices such as magnetic resonance imagers (MRI). Magnets constructed with superconductors use less power and can produce stronger magnetic fields than magnets constructed with conventional resistive conductors. A more extensive discussion of superconductivity will be presented in Chapter 8.

The resistance of some semiconducting materials actually decreases as the temperature increases, which implies a negative temperature coefficient of electric resistivity. These materials are often employed in high-resolution detectors for optical measurements or in particle detectors. Such devices must be kept cold to keep their resistance high, which is accomplished with refrigerators or liquid nitrogen.

A *thermistor* is a semiconductor whose resistance depends strongly on temperature. Thermistors are used to measure temperature. The temperature dependence of the resistance of a typical thermistor is shown in Figure 5.9a. Here you can see that the resistance of a thermistor falls with increasing temperature. This drop is in contrast to the increase in resistance of a copper wire over the same temperature range, shown in Figure 5.9b.

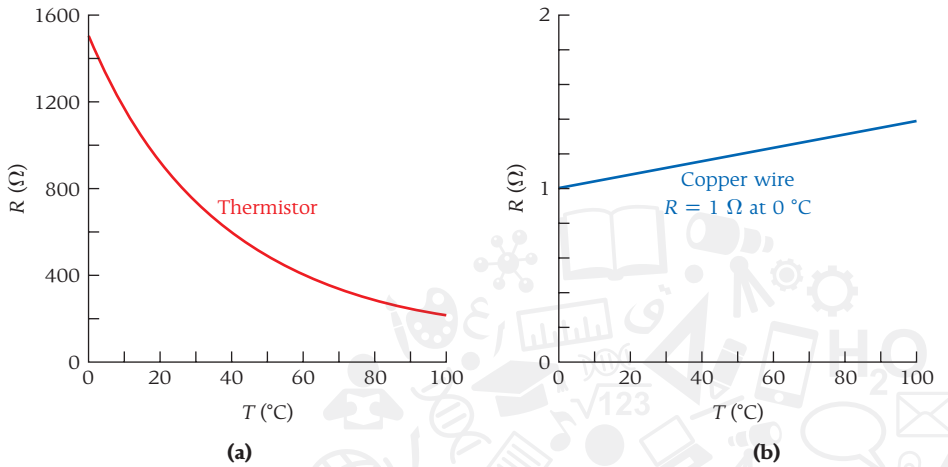


FIGURE 5.9 (a) The temperature dependence of the resistance of a thermistor. (b) The temperature dependence of the resistance of a copper wire that has a resistance of 1 Ω at $T = 0$ °C.

Microscopic Basis of Conduction in Solids

Conduction of current through solids results from the motion of electrons. In a metal conductor such as copper, the atoms of the metal form a regular array called a *crystal lattice.* The outermost electrons of each atom are essentially free to move randomly in this lattice. When an electric field is applied, the electrons drift in the direction opposite to that of the electric field. Resistance to drift occurs when electrons interact with the metal atoms in the lattice. When the temperature of the metal is increased, the motion of the atoms in the lattice increases. This, in turn, increases the probability that electrons will interact with the atoms, effectively increasing the resistance of the metal. A detailed discussion of the interaction between the lattice atoms and the conduction electrons is beyond the scope of this book. The approximate relations given by equations 5.12 and 5.13, which result from this interaction, are sufficient for our purposes.

The atoms of a semiconductor are also arranged in a crystal lattice. However, the outermost electrons of the atoms of the semiconductor are not free to move about within the lattice. To move about, the electrons must be given enough energy to attain an energy state where they can move freely. Thus, a typical semiconductor has a higher resistance than a metal conductor because it has many fewer conduction electrons. In addition, when a semiconductor is heated, many more electrons gain enough energy to move freely; thus, the resistance of the semiconductor decreases as its temperature increases.

5.4 Electromotive Force and Ohm's Law



For current to flow through a resistor, a potential difference must be established across the resistor. This potential difference, supplied by a battery or other device, is termed an **electromotive force,** abbreviated **emf.** (Electromotive force is not a force at all, but rather a potential difference. The term is still in widespread use but mainly in the form of its abbreviation, pronounced "ee-em-eff.") A device that maintains a potential difference is called an *emf device* and does work on the charge carriers. The potential difference created by the emf device is represented as V_{emf}. This text assumes that emf devices have terminals to which a circuit can be connected. The emf device is assumed to maintain a constant potential difference, V_{emf}, between these terminals.

Solar cells convert light energy from the Sun to electric energy. If you examine a battery, you will find its potential difference (sometimes colloquially called "voltage") written on it. This "voltage" is the potential difference (emf) that the battery can provide to a circuit.

(Note that a battery is a source of constant emf, it does *not* supply constant current to a circuit.) Rechargeable batteries also display a rating in mAh (milliampere-hour), which provides information on the total charge the battery can deliver when fully charged. The mAh is another unit of charge:

$$1 \text{ mAh} = (10^{-3} \text{ A})(3600 \text{ s}) = 3.6 \text{ As} = 3.6 \text{ C}.$$

Electrical components in a circuit can be sources of emf, capacitors, resistors, or other electrical devices. These components are connected with conducting wires. At least one component must be a source of emf because the potential difference created by the emf device is what drives the current through the circuit. You can think of an emf device as the pump in a water pipeline; without the pump, the water sits in the pipe and doesn't move. Once the pump is turned on, the water moves through the pipe in a continuous flow.

An electric circuit starts and ends at an emf device. Since the emf device maintains a constant potential difference, V_{emf}, between its terminals, positive current leaves the device at the higher potential of its positive terminal and enters its negative terminal at a lower potential. This lower potential is conventionally set to zero.

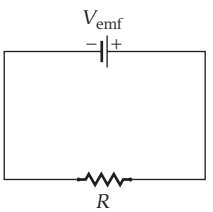


FIGURE 5.10 Simple circuit containing a source of emf and a resistor.

Consider a simple circuit of the form shown in Figure 5.10, where a source of emf provides a potential difference, V_{emf}, across a resistor with resistance R. Note an important convention for circuit diagrams: A resistor is always symbolized by a zigzag line, and it is assumed that all of the resistance, R, is concentrated there. The wires connecting the different circuit elements are represented by straight lines; it is implied that they do not have a resistance. Physical wires do, of course, have some resistance, but it is assumed to be negligible for the purpose of the diagram.

For a circuit like the one shown in Figure 5.10, the emf device provides the potential difference that creates the current flowing through the resistor. Therefore, in this case, Ohm's Law (equation 5.9) can be written in terms of the external emf as

$$V_{emf} = iR. \tag{5.14}$$

Note that, unlike Newton's law of gravitation or the law of conservation of energy, Ohm's Law is not a law of nature. It is not even obeyed by all resistors. For many resistors, called *ohmic resistors*, the current is directly proportional to the potential difference across the resistor over a wide range of temperatures and a wide range of applied potential differences. For other resistors, called *non-ohmic resistors*, current and potential difference are not directly proportional at all. Non-ohmic resistors include many kinds of transistors, which means that many modern electronic devices do not obey Ohm's Law. We'll take a closer look at one of these devices, the diode, in Section 5.8. Nevertheless, a large class of materials and devices (such as conventional wires, for example) do obey Ohm's Law, and thus it is worth devoting attention to its consequences. The remainder of this chapter (with the exception of Section 5.8) treats resistors as ohmic devices; that is, devices that obey Ohm's Law.

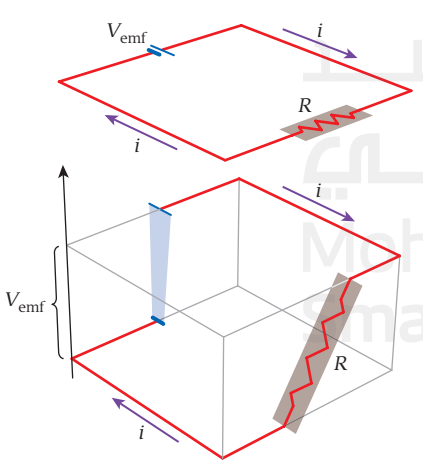


FIGURE 5.11 (a) Conventional representation of a simple circuit with a resistor and a source of emf. (b) Three-dimensional representation of the same circuit, displaying the potential at each point in the circuit. The current in the circuit is shown for both views.

The current, i, that flows through the resistor in Figure 5.10 also flows through the source of emf and the wires connecting the components. Because the wires are assumed to have zero resistance (as noted above), the change in potential of the current must occur in the resistor, according to Ohm's Law. This change is referred to as the **potential drop** across the resistor. Thus, the circuit shown in Figure 5.10 can be represented in a different way, making it clearer where the potential drop happens and showing which parts of the circuit are at which potential. Figure 5.11a shows the circuit in Figure 5.10. Figure 5.11b shows the same circuit but with the vertical dimension representing the value of the electric potential at different points around the circuit. The potential difference is supplied by the source of emf, and the entire potential drop occurs across the single resistor. (Remember, the convention is that the lines connecting the circuit elements in a circuit diagram represent wires with no resistance. Therefore, these connecting wires are represented in Figure 5.11b by horizontal lines, signifying that the entire wire is at exactly the same potential.) Ohm's Law

applies for the potential drop across the resistor, and the current in the circuit can be calculated using equation 5.9.

Figure 5.11 illustrates an important point about circuits. Sources of emf add potential difference to a circuit, and potential drops through resistors reduce potential in the circuit. However, the total potential difference on any closed path around the complete circuit must be zero. This is a straightforward consequence of the law of conservation of energy. An analogy with gravity may help: You can gain and lose potential energy by moving up and down in a gravitational field (for example, by climbing up and down hills), but if you arrive back at the same point from which you started, the net energy gained or lost is exactly zero. The same holds for current flowing in a circuit: It does not matter how many potential drops or sources of emf are encountered on any closed loop; a given point always has the same value of the electric potential. The current can flow through the loop in either direction with the same result.

Resistance of the Human Body

This short introduction of resistance and Ohm's Law leads to a point about electrical safety. It was mentioned earlier that currents above 100 mA can be deadly if they flow through human heart muscle. Ohm's Law makes it clear that the resistance of the human body determines whether a given potential difference—say, from a car battery—can be dangerous. Since we usually handle tools with our hands, the most relevant measure for the human body's resistance, R_{body}, is the resistance along a path from the fingertips of one hand to the fingertips of the other hand. (Note that the heart is pretty much in the middle of this path!) For most people this resistance is in the range $500\ \text{k}\Omega < R_{body} < 2\ \text{M}\Omega$. Most of this resistance comes from the skin, in particular, the layers of dead skin on the outside. However, if the skin is wet, its conductivity is drastically increased, and consequently, the body's resistance is drastically lowered. For a given potential difference, Ohm's Law implies that the current then drastically increases. Handling electrical devices in wet environments or touching them with your tongue is thus a very bad idea.

Wires in a circuit can have sharp points where they are cut. If these points penetrate the skin at the fingertips, the resistance of the skin is eliminated, and the fingertip-to-fingertip resistance is very drastically lowered. If a wire penetrates a blood vessel, the human body's resistance decreases even further, because blood has a high salinity and is thus a good conductor. In this case, even relatively small potential differences from batteries can have a deadly effect.

Self-Test Opportunity 5.2

A resistor with $R = 10.0\ \Omega$ is connected across a source of emf with potential difference $V_{emf} = 1.50$ V. What is the current flowing though the circuit?

5.5 Resistors in Series



A circuit can contain more than one resistor and/or more than one source of emf. The analysis of circuits with multiple resistors requires different techniques. Let's first examine resistors connected in series.

Two resistors, R_1 and R_2, are connected in series with one source of emf with potential difference V_{emf} in the circuit shown in Figure 5.12. The potential drop across resistor R_1 is denoted by ΔV_1, and the potential drop across resistor R_2 by ΔV_2. The two potential drops must sum to the potential difference supplied by the source of emf:

$$V_{emf} = \Delta V_1 + \Delta V_2.$$

The crucial insight is that the same current must flow through all the elements of the circuit. How do we know this? Remember, at the beginning of this chapter, current was defined as the rate of change of the charge in time: $i = dq/dt$. Current has to be the same everywhere along a wire, and also in a resistor, because charge is conserved everywhere. No charge is lost or gained along the wire, and so the current is the same everywhere around the loop in Figure 5.12.

To clear up a common misconception, note that *there is no such thing as current getting "used up" in a resistor*. No matter how many resistors are connected in series, the current that flows into the first one is the same current that flows out of the last one. An analogy to water flowing in a pipe may help: No matter how long the pipe is and how many bends it may have, all water that flows into one end has to come out the other end.

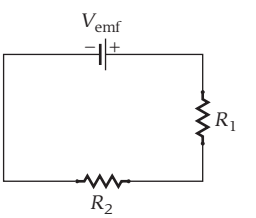


FIGURE 5.12 Circuit with two resistors in series with one source of emf.

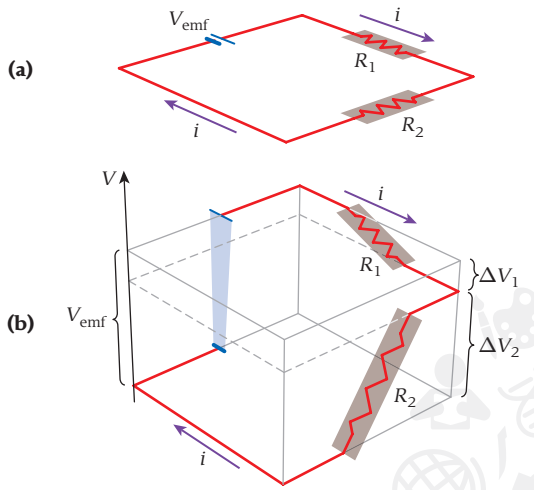


FIGURE 5.13 (a) Conventional representation of a simple circuit with two resistors in series and a source of emf. (b) Three-dimensional representation of the same circuit, displaying the potential at each point in the circuit. The current in the circuit is shown for both views.

Thus, the current flowing through each resistor in Figure 5.12 is the same. For each resistor, we can apply Ohm's Law and get

$$V_{emf} = iR_1 + iR_2.$$

An equivalent resistance, R_{eq}, can replace the two individual resistances:

$$V_{emf} = iR_1 + iR_2 = iR_{eq},$$

where

$$R_{eq} = R_1 + R_2.$$

Thus, two resistors in series can be replaced with an equivalent resistance equal to the sum of the two resistances. Figure 5.13 illustrates the potential drops in the series circuit of Figure 5.12, using a three-dimensional view.

The expression for the equivalent resistance of two resistors in series can be generalized to a circuit with n resistors in series:

$$R_{eq} = \sum_{i=1}^{n} R_i \quad \text{(for resistors in series).} \tag{5.15}$$

That is, if resistors are connected in a single path so that the same current flows through all of them, their total resistance is just the sum of their individual resistances.

Concept Check 5.3

What are the relative magnitudes of the two resistances in Figure 5.13?

a) $R_1 < R_2$

b) $R_1 = R_2$

c) $R_1 > R_2$

d) Not enough information is given in the figure to compare the resistances.

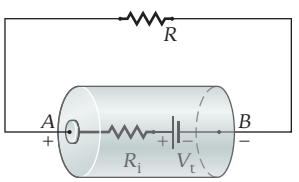


FIGURE 5.14 Battery (gray cylinder) with internal resistance R_i connected to an external resistor, R.

EXAMPLE 5.3 Internal Resistance of a Battery

When a battery is not connected in a circuit, the potential difference across its terminals is V_t. When the battery is connected in series with a resistor with resistance R, current i flows through the circuit. When the current is flowing, the potential difference, V_{emf}, across the terminals of the battery is less than V_t. This drop occurs because the battery has an internal resistance, R_i, which can be thought of as being in series with the external resistor (Figure 5.14). That is,

$$V_t = iR_{eq} = i(R + R_i).$$

The battery is depicted by the gray cylinder in Figure 5.14. The terminals of the battery are represented by points A and B.

PROBLEM

Consider a battery that has $V_t = 12.0$ V when it is not connected to a circuit. When a 10.0-Ω resistor is connected with the battery, the potential difference across the battery's terminals drops to 10.9 V. What is the internal resistance of the battery?

SOLUTION

The current flowing through the external resistor is given by

$$i = \frac{\Delta V}{R} = \frac{10.9\ \text{V}}{10.0\ \Omega} = 1.09\ \text{A}.$$

The current flowing in the complete circuit, including the battery, must be the same as the current flowing in the external resistor. Thus, we have

$$V_t = iR_{eq} = i(R + R_i)$$

$$(R + R_i) = \frac{V_t}{i}$$

$$R_i = \frac{V_t}{i} - R = \frac{12.0\ \text{V}}{1.09\ \text{A}} - 10.0\ \Omega = 1.00\ \Omega.$$

The internal resistance of the battery is 1.00 Ω. Batteries with internal resistance are said to be *nonideal*. Unless otherwise specified, batteries in circuits will be assumed to have zero internal resistance. Such batteries are said to be *ideal*. An ideal battery maintains a constant potential difference between its terminals, independent of the current flowing.

Whether a battery can still provide energy cannot be determined by simply measuring the potential difference across the terminals. Instead, you must place a resistance on the battery

and then measure the potential difference. If the battery is no longer functional, it may still provide its rated potential difference when not connected, but its potential difference may drop to zero when connected to an external resistance. Some brands of batteries have built-in devices to measure the functioning potential difference simply by pressing on a particular spot on the battery and observing an indicator.

Resistor with a Nonconstant Cross Section

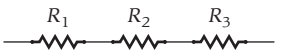
Up to now the discussion has assumed that a resistor has the same cross-sectional area, A, and the same resistivity, ρ, everywhere along its length (this was the implicit assumption in the derivation leading to equation 5.11). This is, of course, not always the case. How do we handle the analysis of a resistor whose cross-sectional area is a function of the position x along the resistor, $A(x)$, and/or whose resistivity can change as a function of position, $\rho(x)$? We simply divide the resistor into many very short pieces of length Δx and sum over all of them, since equation 5.15 says that the total resistance is the sum of all of the resistances of the individual short pieces; then we take the limit $\Delta x \to 0$. If this sounds like an integration to you, you are right. The general formula for computing the resistance of a resistor of length L with a nonuniform cross-sectional area, $A(x)$, is

$$R = \int_0^L \frac{\rho(x)}{A(x)}\,dx. \tag{5.16}$$

A concrete example will help clarify this equation.

Concept Check 5.4

Three identical resistors, R_1, R_2, and R_3, are wired together as shown in the figure. An electric current is flowing through the three resistors. The current through R_2



a) is the same as the current through R_1 and R_3.

b) is a third of the current through R_1 and R_3.

c) is twice the sum of the current through R_1 and R_3.

d) is three times the current through R_1 and R_3.

e) cannot be determined.

SOLVED PROBLEM 5.2 Brain Probe

Reasearchers use a technique known as electrocorticography (ECoG) to measure the electric field generated by neurons in the brain. Some of these measurements can only be done by inserting very thin wires into the brain in order to probe directly into neurons. These wires are insulated, with only a very short tip exposed, which is pulled into a very fine conical tip. ECoG is being used to treat an epilepsy patient in Figure 5.15.

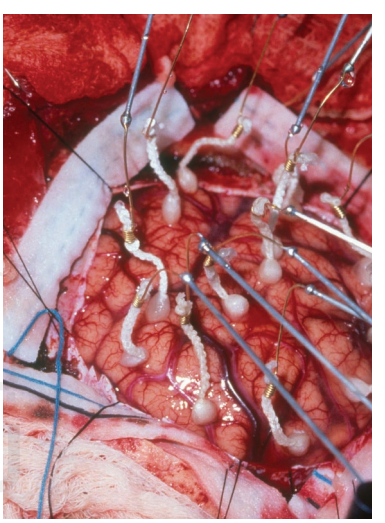


FIGURE 5.15 Electrocorticography performed with electrode grids on the cerebral cortex of an epilepsy patient.

PROBLEM

If the wire used for ECoG is made of tungsten and has a diameter of 0.74 mm and the tip has a length of 2.0 mm and is sharpened to a diameter of 2.4 μm at the end, what is the resistance of the tip? (The resistivity of tungsten is listed in Table 5.1 as $5.51 \times 10^{-8}\ \Omega$ m.)

SOLUTION

THINK First, why might one want to know the resistance? To measure electrical fields or potential differences in neurons, probes with a large resistance, say, on the order of kilo-ohms, cannot be used because the fields or differences will not be detectable. However, since resistance is inversely proportional to the cross-sectional area, a very small area means a relatively large resistance. The problem statement says that the probe has a very fine tip, much pointier than any sewing needle. Hence the need to find out the resistance of the probe before inserting it into the brain!

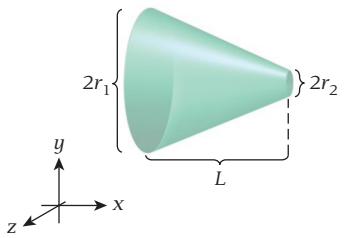
Clearly, we are dealing with a case of nonconstant cross-sectional area, and so we will need to perform the integration of equation 5.16. However, since the tip is entirely made of tungsten, the resistivity is constant throughout its volume, which will simplify the task.

SKETCH Figure 5.16a shows a three-dimensional view of the tip, and Figure 5.16b presents a cut through its symmetry plane and the integration path.

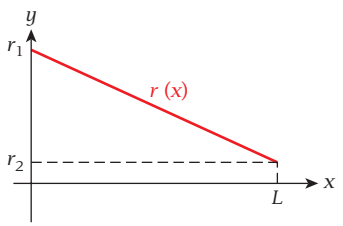
RESEARCH The research part is quite simple for this problem, because we already know which equation we need to use. However, equation 5.16 needs to be altered to reflect the fact that the resistivity is constant throughout the tip:

$$R = \rho \int_0^L \frac{1}{A(x)}\,dx, \tag{i}$$

– Continued



(a)



(b)

FIGURE 5.16 (a) Shape of the tip of the probe. (b) Coordinate system for the integration.

where $A(x)$ is the area of a circle, $A(x) = \pi[r(x)]^2$. The radius of the circle falls linearly from r_1 to r_2 (see Figure 5.16b):

$$r(x) = r_1 + \frac{(r_2 - r_1)x}{L}. \qquad \text{(ii)}$$

SIMPLIFY We substitute the expression for the radius from equation (ii) into the formula for the area and then substitute the resulting expression for $A(x)$ into equation (i). We arrive at

$$R = \rho \int_0^L \frac{1}{\pi\left(r_1 + (r_2 - r_1)x/L\right)^2}\, dx.$$

This integral may look daunting at first sight, but except for x all the other quantities are constants. We consult an integration table or software and find

$$R = -\left.\frac{\rho L}{\pi(r_2 - r_1)\left(r_1 + (r_2 - r_1)x/L\right)}\right|_0^L = \frac{\rho L}{\pi r_1 r_2}.$$

CALCULATE Putting in the numerical values, we get

$$R = \frac{\left(5.51\times10^{-8}\ \Omega\ \text{m}\right)\left(2.0\times10^{-3}\ \text{m}\right)}{\pi(0.37\times10^{-3}\ \text{m})(1.2\times10^{-6}\ \text{m})} = 7.90039\times10^{-2}\Omega\ .$$

ROUND We report our result to the two significant figures to which the geometric properties of the tip were given:

$$R = 7.9\times10^{-2}\Omega\ .$$

DOUBLE-CHECK The value of 79 mΩ seems a very small resistance to a current that has to pass through a tip that is sharpened to a diameter of 2.4 μm. On the other hand, the tip has a very small length, which argues for a small resistance. An additional confidence builder is the fact that the units worked out properly.

However, there are also a few tests we can perform to convince ourselves that at least the asymptotic limits of the solution to $R = \rho L/(\pi r_1 r_2)$ are reasonable. First, as the length approaches zero, so does the resistance, as expected. Second, as the radius of either end of the tip approaches zero, the formula predicts an infinite resistance, which is also expected.



5.6 Resistors in Parallel

Instead of being connected in series, which causes all the current to pass through both resistors, two resistors can be connected in parallel, which divides the current between them, as shown in Figure 5.17. Again, to better illustrate the potential drops, Figure 5.18 shows the same circuit in a three-dimensional view.

In this case, the potential drop across each resistor is equal to the potential difference provided by the source of emf. Using Ohm's Law (equation 5.14) for the current i_1 in R_1 and the current i_2 in R_2, we have

$$i_1 = \frac{V_{\text{emf}}}{R_1}$$

and

$$i_2 = \frac{V_{\text{emf}}}{R_2}.$$

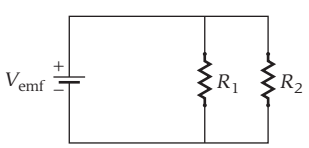


FIGURE 5.17 Circuit with two resistors connected in parallel and a single source of emf.

The total current from the source of emf, i, must be

$$i = i_1 + i_2.$$

Inserting the expressions for i_1 and i_2, we obtain

$$i = i_1 + i_2 = \frac{V_{\text{emf}}}{R_1} + \frac{V_{\text{emf}}}{R_2} = V_{\text{emf}}\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

Ohm's Law (equation 5.14) can be rewritten as

$$i = V_{\text{emf}}\left(\frac{1}{R_{\text{eq}}}\right).$$

Thus, two resistors connected in parallel can be replaced with an equivalent resistance given by

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

In general, the equivalent resistance for n resistors connected in parallel is given by

$$\frac{1}{R_{\text{eq}}} = \sum_{i=1}^{n} \frac{1}{R_i} \quad \text{(resistors in parallel).} \qquad (5.17)$$

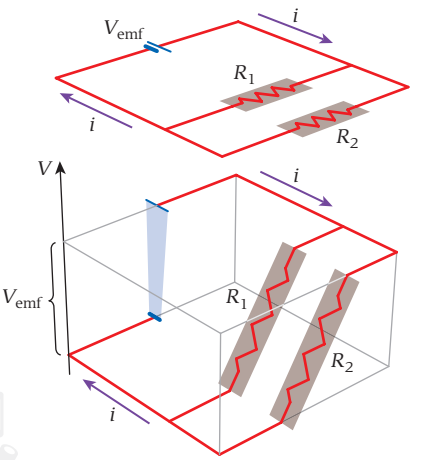


FIGURE 5.18 (a) Conventional representation of a simple circuit with two resistors in parallel and a source of emf. (b) Three-dimensional representation of the same circuit, displaying the potential at each point in the circuit.

Concept Check 5.5

Three identical resistors, R_1, R_2, and R_3, are wired together as shown in the figure. An electric current is flowing from point A to point B. The current flowing through R_2

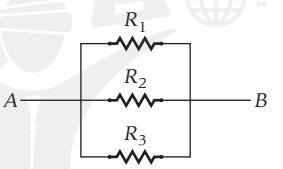
a) is the same as the current through R_1 and R_3.

b) is a third of the current through R_1 and R_3.

c) is twice the sum of the current through R_1 and R_3.

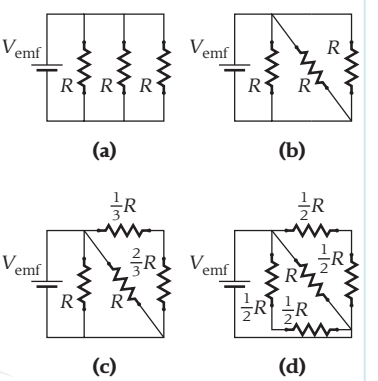
d) is three times the current through R_1 and R_3.

e) cannot be determined.



Concept Check 5.6

Which combination of resistors has the highest equivalent resistance?



a) combination (a)

b) combination (b)

c) combination (c)

d) combination (d)

e) The equivalent resistance is the same for all four.

EXAMPLE 5.4 Equivalent Resistance in a Circuit with Six Resistors

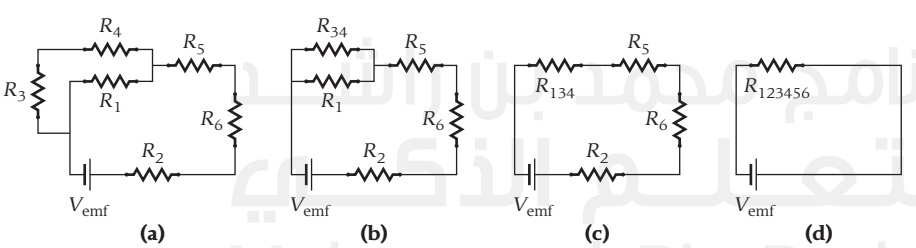


FIGURE 5.19 (a) Circuit with six resistors. (b)–(d) Steps in combining these resistors to determine the equivalent resistance.

PROBLEM

Figure 5.19a shows a circuit with six resistors, R_1 through R_6. What is the current flowing through resistors R_2 and R_3 in terms of V_{emf} and R_1 through R_6?

SOLUTION

We begin by identifying parts of the circuit that are clearly wired in parallel or in series. The current flowing through R_2 is the current flowing from the source of emf. We note that R_3 and R_4 are in series. Thus, we can write

$$R_{34} = R_3 + R_4. \qquad \text{(i)}$$

– Continued

This substitution is made in Figure 5.19b. This figure shows us that R_{34} and R_1 are in parallel. We can then write

$$\frac{1}{R_{134}} = \frac{1}{R_1} + \frac{1}{R_{34}},$$

or

$$R_{134} = \frac{R_1 R_{34}}{R_1 + R_{34}}. \quad \text{(ii)}$$

This substitution is depicted in Figure 5.19c. From this figure, we can see that R_2, R_5, R_6, and R_{134} are in series. Thus, we can write

$$R_{123456} = R_2 + R_5 + R_6 + R_{134}. \quad \text{(iii)}$$

This substitution is shown in Figure 5.19d. We substitute for R_{34} and R_{134} from equations (i) and (ii) into equation (iii):

$$R_{123456} = R_2 + R_5 + R_6 + \frac{R_1 R_{34}}{R_1 + R_{34}} = R_2 + R_5 + R_6 + \frac{R_1(R_3 + R_4)}{R_1 + R_3 + R_4}.$$

Thus, i_2, the current flowing through R_2, is given by

$$i_2 = \frac{V_{\text{emf}}}{R_{123456}}.$$

Now we turn to the determination of the current flowing through R_3. Current i_2 is also flowing through the equivalent resistance R_{134} that contains R_3 (see Figure 5.19c). Thus, we can write

$$V_{134} = i_2 R_{134},$$

where V_{134} is the potential drop across the equivalent resistance R_{134}. The resistor R_1 and the equivalent resistance R_{34} are in parallel. Thus, V_{34}, the potential drop across R_{34}, is the same as the potential drop across R_{134} which is V_{134}. The resistors R_3 and R_4 are in series, and thus, i_3, the current flowing through R_3, is the same as i_{34}, the current flowing through R_{34}. We can thus write

$$V_{34} = V_{134} = i_{34} R_{34} = i_3 R_{34}.$$

Now we can express i_3 in terms of V and R_1 through R_6:

$$i_3 = \frac{V_{134}}{R_{34}} = \frac{i_2 R_{134}}{R_{34}} = \frac{\left(\frac{V_{\text{emf}}}{R_{123456}}\right) R_{134}}{R_{34}} = \frac{V_{\text{emf}} R_{134}}{R_{34} R_{123456}} = \frac{V_{\text{emf}} \left(\frac{R_1 R_{34}}{R_1 + R_{34}}\right)}{R_{34} R_{123456}} = \frac{V_{\text{emf}} R_1}{R_{123456}(R_1 + R_{34})}$$

or

$$i_3 = \frac{V_{\text{emf}} R_1}{\left(R_2 + R_5 + R_6 + \frac{R_1(R_3 + R_4)}{R_1 + R_3 + R_4}\right)(R_1 + R_3 + R_4)} = \frac{V_{\text{emf}} R_1}{(R_2 + R_5 + R_6)(R_1 + R_3 + R_4) + R_1(R_3 + R_4)}.$$

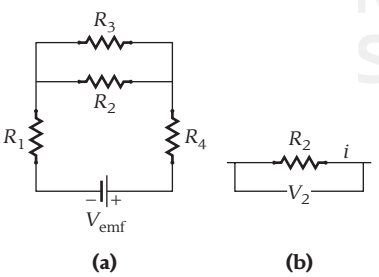


FIGURE 5.20 (a) A circuit with four resistors and a battery. (b) Potential drop across resistor R_2.

SOLVED PROBLEM 5.3 Potential Drop across a Resistor in a Circuit

PROBLEM

The circuit shown in Figure 5.20a has four resistors and a battery with $V_{\text{emf}} = 149$ V. The values of the four resistors are $R_1 = 17.0\ \Omega$, $R_2 = 51.0\ \Omega$, $R_3 = 114.0\ \Omega$, and $R_4 = 55.0\ \Omega$. What is the magnitude of the potential drop across R_2?

SOLUTION

THINK The resistors R_2 and R_3 are in parallel and can be replaced with an equivalent resistance, R_{23}. The resistors R_1 and R_4 are in series with R_{23}. The current flowing through R_1, R_4, and R_{23} is the same because they are in series. We can obtain the current in the circuit by calculating the equivalent resistance for R_1, R_4, and R_{23} and using Ohm's Law. The potential drop across R_{23} is equal to the current flowing in the circuit times R_{23}. The potential drop across R_2 is the same as the potential drop across R_{23} because R_2 and R_3 are in parallel.

SKETCH The potential drop across resistor R_2 is illustrated in Figure 5.20b.

RESEARCH The equivalent resistance for R_2 and R_3 can be calculated using equation 5.17:

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}. \quad \text{(i)}$$

The equivalent resistance of the three resistors in series can be found using equation 5.15:

$$R_{eq} = \sum_{i=1}^{n} R_i = R_1 + R_{23} + R_4.$$

Finally, we obtain the current in the circuit using Ohm's Law:

$$V_{emf} = iR_{eq} = i(R_1 + R_{23} + R_4).$$

SIMPLIFY The potential drop, V_2, across R_2 is equal to the potential drop, V_{23}, across the equivalent resistance R_{23}:

$$V_2 = V_{23} = iR_{23} = \frac{V_{emf}}{R_1 + R_{23} + R_4} R_{23} = \frac{R_{23} V_{emf}}{R_1 + R_{23} + R_4}. \quad \text{(ii)}$$

We can solve equation (i) for R_{23} to obtain

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3}.$$

We can then use equation (ii) to determine the potential drop V_2 as

$$V_2 = \frac{\left(\frac{R_2 R_3}{R_2 + R_3}\right) V_{emf}}{R_1 + \left(\frac{R_2 R_3}{R_2 + R_3}\right) + R_4} = \frac{R_2 R_3 V_{emf}}{R_1 (R_2 + R_3) + R_2 R_3 + R_4 (R_2 + R_3)},$$

which we can rewrite as

$$V_2 = \frac{R_2 R_3 V_{emf}}{(R_1 + R_4)(R_2 + R_3) + R_2 R_3}.$$

CALCULATE Putting in the numerical values, we get

$$\begin{aligned} V_2 &= \frac{R_2 R_3 V_{emf}}{(R_1 + R_4)(R_2 + R_3) + R_2 R_3} \\ &= \frac{(51.0\ \Omega)(114.0\ \Omega)(149\ \text{V})}{(17.0\ \Omega + 55.0\ \Omega)(51.0\ \Omega + 114.0\ \Omega) + (51.0\ \Omega)(114.0\ \Omega)} \\ &= 48.9593\ \text{V}. \end{aligned}$$

ROUND We report our result to three significant figures:

$$V = 49.0\ \text{V}.$$

DOUBLE-CHECK You may be tempted to avoid completing the analytic solution as we've done here. Instead, you may want to insert numbers earlier, for example, into the expression for R_{23}. So, to double-check our result, let's calculate the current in the circuit explicitly and then calculate the potential drop across R_{23} using that current. The equivalent resistance for R_2 and R_3 in parallel is

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(51.0\ \Omega)(114.0\ \Omega)}{51.0\ \Omega + 114.0\ \Omega} = 35.2\ \Omega.$$

The current in the circuit is then

$$i = \frac{V_{emf}}{R_1 + R_{23} + R_4} = \frac{149.0\ \text{V}}{17.0\ \Omega + 35.2\ \Omega + 55.0\ \Omega} = 1.39\ \text{A}.$$

The potential drop across R_2 is then

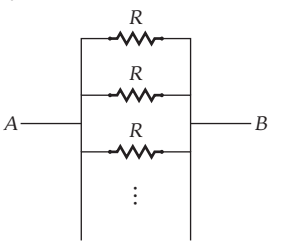
$$V_2 = iR_{23} = (1.39\ \text{A})(35.2\ \Omega) = 48.9\ \text{V},$$

which agrees with our result within rounding error. It is reassuring that both methods lead to the same answer.

We can also check that the potential drops across R_1, R_{23}, and R_4 sum to V_{emf}, as they should since R_1, R_{23}, and R_4 are in series. The potential drop across R_1 is $V_1 = iR_1 = (1.39\ \text{A})(17.0\ \Omega) = 23.6\ \text{V}$. The potential drop across R_4 is $V_4 = iR_4 = (1.39\ \text{A})(55.0\ \Omega) = 76.5\ \text{V}$. So the total potential drop is $V_{total} = V_1 + V_{23} + V_4 = (23.6\ \text{V}) + (48.9\ \text{V}) + (76.5\ \text{V}) = 149\ \text{V}$, which is equal to V_{emf}. Thus, our answer is consistent.

Concept Check 5.7

As more identical resistors, *R*, are added to the circuit shown in the figure, the resistance between points *A* and *B* will



a) increase.

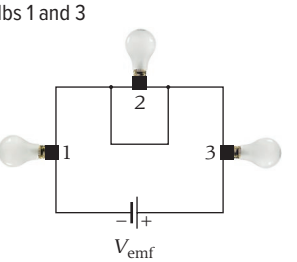
b) stay the same.

c) decrease.

d) change in an unpredictable manner.

Concept Check 5.8

Three light bulbs are connected in series with a battery that delivers a constant potential difference, V_{emf}. When a wire is connected across light bulb 2 as shown in the figure, light bulbs 1 and 3



a) burn just as brightly as they did before the wire was connected.

b) burn more brightly than they did before the wire was connected.

c) burn less brightly than they did before the wire was connected.

d) go out.

5.7 Energy and Power in Electric Circuits

Consider a simple circuit in which a source of emf with potential difference ΔV causes a current, i, to flow. The work required from the emf device to move a differential amount of charge, dq, from the negative terminal to the positive terminal (within the emf device) is equal to the increase in electric potential energy of that charge, dU:

$$dU = dq\,\Delta V.$$

Remembering that current is defined as $i = dq/dt$, we can rewrite the differential electric potential energy as

$$dU = i\,dt\,\Delta V.$$

Using the definition of power, $P = dU/dt$, and substituting into it the expression for the differential potential energy, we obtain

$$P = \frac{dU}{dt} = \frac{i\,dt\,\Delta V}{dt} = i\Delta V.$$

Thus, the product of the current times the potential difference gives the power supplied by the source of emf. By conservation of energy, this power is equal to the power dissipated in a circuit containing one resistor. In a more complicated circuit, each resistor will dissipate power at the rate given by this equation, where i and ΔV refer to the current through and potential difference across that resistor. Ohm's Law (equation 5.9) leads to different formulations of the power:

$$P = i\Delta V = i^2R = \frac{(\Delta V)^2}{R}. \tag{5.18}$$

The unit of power is the watt (W). Electrical devices, such as light bulbs, are rated in terms of how much power they consume. Your electric bill depends on how much electrical energy your appliances consume, and this energy is measured in kilowatt-hours (kWh).

Qualitatively, much or most of the energy dissipated in resistors is converted into heat. This phenomenon is employed in incandescent lighting, where heating a metal filament to a very high temperature causes it to emit light. The heat dissipated in electrical circuits is a huge problem for large-scale computer systems and server farms for the biggest Internet databases. These computer systems use thousands of processors for computing applications that can be parallelized. All of these processors emit heat, and very expensive cooling has to be provided to offset it. It turns out that the cost of cooling is one of the most stringent boundary conditions limiting the maximum size of these supercomputers.

Some of the power dissipated in circuits can be converted into mechanical energy by motors. The functioning of electric motors requires an understanding of magnetism and will be covered later.

EXAMPLE 5.5 Temperature Dependence of a Light Bulb's Resistance

A 100-W light bulb is connected in series to a source of emf with $V_{\text{emf}} = 100$ V. When the light bulb is lit, the temperature of its tungsten filament is 2520 °C.

PROBLEM

What is the resistance of the light bulb's tungsten filament at room temperature (20 °C)?

SOLUTION

The resistance of the filament when the light bulb is lit can be obtained using equation 5.18:

$$P = \frac{V_{\text{emf}}^2}{R}.$$

We rearrange this equation and substitute the numerical values to get the resistance of the filament:

$$R = \frac{V_{\text{emf}}^2}{P} = \frac{(100\ \text{V})^2}{100\ \text{W}} = 100\ \Omega.$$

The temperature dependence of the filament's resistance is given by equation 5.13:

$$R - R_0 = R_0\alpha(T - T_0).$$

We solve for the resistance at room temperature, R_0:

$$R = R_0 + R_0\alpha(T - T_0) = R_0\left[1 + \alpha(T - T_0)\right]$$

$$R_0 = \frac{R}{1 + \alpha(T - T_0)}.$$

Using the temperature coefficient of resistivity for tungsten from Table 5.1, we get

$$R_0 = \frac{R}{1 + \alpha(T - T_0)} = \frac{100\ \Omega}{1 + \left(4.5 \cdot 10^{-3}\ ^\circ\text{C}^{-1}\right)(2520\ ^\circ\text{C} - 20\ ^\circ\text{C})} = 8.2\ \Omega.$$

High-Voltage Direct Current Power Transmission

The transmission of electrical power from power-generating stations to users of electricity is of great practical interest. Often, electrical power-generating stations are located in remote areas, and thus the power must be transmitted long distances. This is particularly true for clean power sources, such as hydroelectric dams and large solar farms in deserts.

The power, P, transmitted to users is the product of the current, i, and the potential difference, ΔV, in the power line: $P = i\Delta V$. Thus, the current required for a given power is $i = P/\Delta V$, and a higher potential difference means a lower current in the power line. Equation 5.18 indicates that the power dissipated in an electrical power transmission line, P_{loss}, is given by $P_{\text{loss}} = i^2R$. The resistance, R, of the power line is fixed; thus, decreasing the power lost during transmission means reducing the current carried in the transmission line. This reduction is accomplished by transmitting the power using a very high potential difference and a very low current. Looking at equation 5.18, you might argue that we could also write $P_{\text{loss}} = (\Delta V)^2/R$ and that a high potential difference means a large power loss instead of a small power loss. However, ΔV in this equation is the potential drop across the power line, not the potential difference at which the electrical power is being transmitted. The potential drop across the power line is $V_{\text{drop}} = iR$, which is much lower than the high potential difference used to transmit the electrical power. The expressions for the transmitted power and the dissipated power can be combined to give $P_{\text{loss}} = (P/\Delta V)^2R = P^2R/(\Delta V)^2$, which means that for a given amount of power, the dissipated power decreases as the square of the potential difference used to transmit the power.

However, alternating currents have the inherent disadvantage of high power losses. High-voltage direct current (HVDC) transmission lines do not have this problem and suffer only power losses due to the resistance of the power line. However, HVDC transmission lines have the extra requirement that alternating current must be converted to direct current for transmission and the direct current must be converted back to alternating current at the destination.

Part of the power produced by the hydroelectric plant shown in Figure 5.21 is transmitted via the world's largest HVDC transmission line a distance of about 800 km from the Itaipú Dam to São Paulo, Brazil, one of the ten largest metropolitan areas in the world. The transmission line carries 6300 MW of electrical power using direct current with a potential difference of ±600 kV. The station at the Itaipú Dam that converts the alternating current to direct current is shown in Figure 5.21. The station in São Paulo that converts the transmitted direct current back to alternating current is shown in Figure 5.21.



FIGURE 5.21 The station that converts alternating current to direct current at the Itaipú Dam on the Paraná River in Brazil and Paraguay.

Self-Test Opportunity 5.3

Consider a battery with internal resistance R_i. What external resistance, R, will undergo the maximum heating when connected to this battery?

Future applications of HVDC power transmission include the transmission of power from solar power stations located in remote areas in the southwest United States to densely populated areas, such as large cities in California and Texas. Some of the most promising research on power transmission is focusing on the use of superconducting wires for this purpose. As mentioned earlier, once a material becomes superconducting, its resistance approaches zero. Therefore, it can transmit electrical power without significant losses. The most promising materials for superconducting power transmission wires are *high-temperature superconductors*, discovered only in 1986. These wires have to be cooled with liquid nitrogen, but we are approaching the point where their widespread use in long-distance power transmission can become a reality.



FIGURE 5.22 HVDC power transmission line.

SOLVED PROBLEM 5.4 Size of Wire for a Power Line

PROBLEM

Imagine you are designing the HVDC power line from the Itaipú Dam on the Paraná River in Brazil and Paraguay to the city of São Paulo in Brazil. The power line is 800 km long and transmits 6300 MW of power at a potential difference of 1.20 MV. (Figure 5.22 shows an HVDC line.)

The electric company requires that no more than 25% of the power be lost in transmission. If the line consists of one wire made out of copper and having a circular cross section, what is the minimum diameter of the wire?

SOLUTION

THINK Knowing the power transmitted and the potential difference with which it is transmitted, we can calculate the current carried in the line. We can then express the power lost in terms of the resistance of the transmission line. With the current and the resistance of the wire, we can write an expression for the power lost during transmission. The resistance of the wire is a function of the diameter of the wire, the length of the wire, and the resistivity of copper. We can then solve for the diameter of the wire that will keep the power loss within the specified limit.

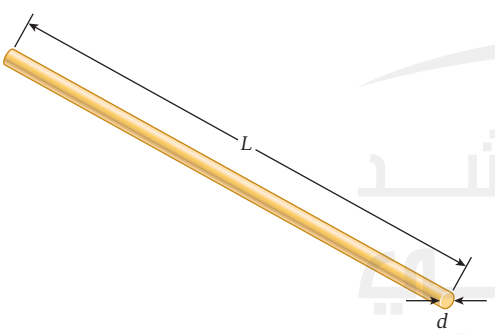


FIGURE 5.23 An HVDC transmission line consisting of a copper conductor (not to scale).

SKETCH A sketch of a copper wire of length L and diameter d is shown in Figure 5.23.

RESEARCH The power, P, carried in the line is related to the current, i, and the potential difference, ΔV: $P = i\Delta V$. The power lost in transmission, P_{lost}, can be related (see equation 5.18) to the current in the wire and the resistance, R, of the wire:

$$P_{\text{lost}} = i^2 R. \tag{i}$$

The resistance of the wire is given by equation 5.11:

$$R = \rho_{\text{Cu}} \frac{L}{A}, \tag{ii}$$

where ρ_{Cu} is the resistivity of copper, L is the length of the wire, and A is the cross-sectional area of the wire.

The cross-sectional area of the wire is the area of a circle:

$$A = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4},$$

where d is the diameter of the wire. Thus, with the area of a circle substituted for A, equation (ii) becomes

$$R = \rho_{\text{Cu}} \frac{L}{\pi d^2/4}. \tag{iii}$$

SIMPLIFY We can solve $P = i\Delta V$ for the current in the wire:

$$i = \frac{P}{\Delta V}.$$

Substituting this expression for the current and that for the resistance from equation (iii) into equation (i) for the lost power gives

$$P_{\text{lost}} = \left(\frac{P}{\Delta V}\right)^2 \left(\rho_{\text{Cu}} \frac{L}{\pi d^2/4}\right) = \frac{4P^2 \rho_{\text{Cu}} L}{\pi (\Delta V)^2 d^2}.$$

The fraction of lost power relative to total power, f, is

$$\frac{P_{\text{lost}}}{P} = \frac{\left(\frac{4P^2 \rho_{\text{Cu}} L}{\pi (\Delta V)^2 d^2}\right)}{P} = \frac{4P \rho_{\text{Cu}} L}{\pi (\Delta V)^2 d^2} = f.$$

Solving this equation for the diameter of the wire gives

$$d = \sqrt{\frac{4P \rho_{\text{Cu}} L}{f \pi (\Delta V)^2}}.$$

CALCULATE Putting in the numerical values gives us

$$d = \sqrt{\frac{4\left(6300 \times 10^6 \text{ W}\right)\left(1.72 \times 10^{-8} \ \Omega \text{ m}\right)\left(800 \times 10^3 \text{ m}\right)}{(0.25)\pi\left(1.20 \times 10^6 \text{ V}\right)^2}} = 0.0175099 \text{ m}.$$

ROUND Rounding to three significant figures gives us the minimum diameter of the copper wire:

$$d = 1.75 \text{ cm}.$$

DOUBLE-CHECK To double-check our result, let's calculate the resistance of this transmission line. Using our calculated value for the diameter, we can find the cross-sectional area and then, using equation 5.11, obtain

$$R = \rho_{\text{Cu}} \frac{L}{\pi d^2/4} = \frac{4\rho_{\text{Cu}} L}{\pi d^2} = \frac{4\left(1.72 \times 10^{-8} \Omega \text{ m}\right)\left(800 \times 10^3 \text{ m}\right)}{\pi\left(1.75 \times 10^{-2} \text{ m}\right)^2} = 57.2 \ \Omega.$$

The current transmitted is

$$i = \frac{P}{V} = \frac{6300 \times 10^6 \text{ W}}{1.20 \times 10^6 \text{ V}} = 5250 \text{ A}.$$

The power lost is then

$$P = i^2 R = (5250 \text{ A})^2 (57.2 \ \Omega) = 1580 \text{ MW},$$

which is close (within rounding error) to 25% of the total power of 6300 MW. Thus, our result seems reasonable.

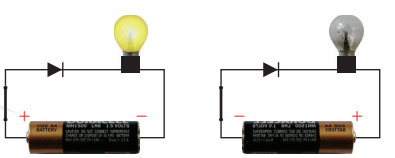


FIGURE 5.24 (a) The circuit of Figure 5.2c, but with a diode included. (b) Reversing the potential difference from the battery causes the current to stop flowing and the light bulb to stop shining.

5.8 Diodes: One-Way Streets in Circuits

Section 5.4 stated that many resistors obey Ohm's Law. It was noted, however, that there are also non-ohmic resistors that do not obey Ohm's Law. A very common and extremely useful example is a diode. A *diode* is an electronic device that is designed to conduct current in one direction and not in the other direction. Remember that Figure 5.2c showed that a light bulb was still shining with the same intensity when the battery it was connected to was reversed. If a diode (represented by the symbol ⟶|⟵) is added to the same circuit, the diode prevents the current from flowing when the potential difference delivered by the battery is reversed; see Figure 5.24. The diode acts like a one-way street for the current.

Figure 5.25 shows current versus potential difference for a 3-Ω ohmic resistor and a silicon diode. The resistor obeys Ohm's Law, with the current flowing in the opposite direction when the potential difference is negative. The plot of current versus potential difference for the resistor is a straight line with a slope of $\frac{1}{3}$ Ω. The silicon diode is wired so that it will not conduct

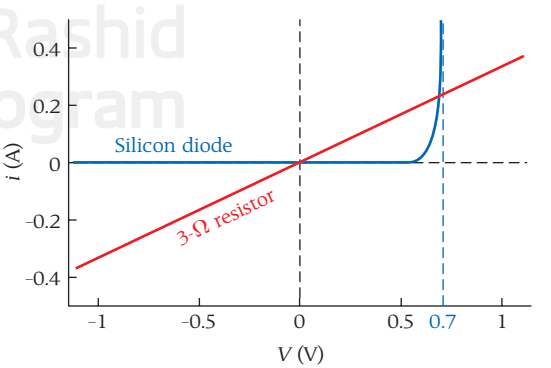


FIGURE 5.25 Current as a function of potential difference for a resistor (red) and a diode (blue).

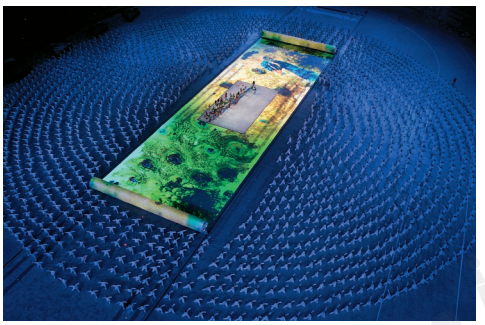


FIGURE 5.26 Giant LED screen used during the opening ceremony of the 2008 Olympic Games in Beijing.

Self-Test Opportunity 5.4

Suppose the battery in Figure 5.24 has a potential difference of 1.5 V across its terminals and the diode is a silicon diode like the one in Figure 5.25. What are the potential drops across the diode and the light bulb in parts (a) and (b) of Figure 5.24?

any current when there is a negative potential difference. This silicon diode, like most, will conduct current if the potential difference is above 0.7 V. For potential differences above this threshold, the diode is essentially a conductor; below this threshold, the diode will not conduct current. The turn-on of the diode above the threshold potential difference increases exponentially; it can be close to instantaneous, as is visible in Figure 5.25.

The fundamental physics principles that underlie the functioning of diodes require an understanding of quantum mechanics.

One particularly useful kind of diode is the light-emitting diode (LED), which not only regulates current in a circuit but also emits light of a single wavelength in a very controlled way. LEDs that emit light of many different wavelengths have been manufactured, and they emit light much more efficiently than conventional incandescent bulbs do. Light intensity is measured in lumens (lm). Light sources can be compared in terms of how many lumens they produce per watt of electrical power. During the last decade, intensive research into LED technology has resulted in huge increases in LED output efficiency, which has reached values of 130 to 170 lm/W. This compares very favorably with conventional incandescent lights (which are in the range from 5 to 20 lm/W), halogen lights (20 to 30 lm/W), and even fluorescent high-efficiency lights (30 to 95 lm/W). Prices for LEDs (in particular, "white" LEDs) are still comparatively high but are expected to decrease significantly. The United States uses over 100 billion kWh of electrical energy for lighting alone each year, which is approximately 10% of the total U.S. energy consumption. Universal use of LED lighting could save 70% to 90% of those 100 billion kWh, approximately the annual energy output of 10 nuclear power plants (~1 GW power each).

LEDs are also used in large display screens, where high light output is desirable. Perhaps the most impressive of these was showcased during the opening ceremony of the 2008 Beijing Olympics (Figure 5.26). It used 44,000 individual LEDs and measured an astounding 147 m by 22 m.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- Current, i, is defined as the rate at which charge, q, flows past a particular point: $i = \frac{dq}{dt}$.
- The magnitude of the average current density, J, at a given cross-sectional area, A, in a conductor is given by $J = \frac{i}{A}$.
- The magnitude of the current density, J, is related to the magnitude of the drift velocity, v_d, of the current-carrying charges, $-e$, by $J = \frac{i}{A} = -nev_d$, where n is the number of charge carriers per unit volume.
- The resistivity, ρ, of a material is defined in terms of the magnitudes of the electric field applied across the material, E, and the resulting current density, J: $\rho = \frac{E}{J}$.
- The resistance, R, of a specific device having resistivity ρ, length L, and constant cross-sectional area A, is $R = \rho\frac{L}{A}$.
- The temperature dependence of the resistivity of a material is given by $\rho - \rho_0 = \rho_0\alpha(T - T_0)$, where ρ is the final resistivity, ρ_0 is the initial resistivity, α is the temperature coefficient of electric resistivity, T is the final temperature, and T_0 is the initial temperature.
- The electromotive force, or emf, is a potential difference created by a device that drives current through a circuit.
- Ohm's Law states that when a potential difference, ΔV, appears across a resistor, R, the current, i, flowing through the resistor is $i = \frac{\Delta V}{R}$.
- Resistors connected in series can be replaced with an equivalent resistance, R_{eq}, given by the sum of the resistances of the resistors: $R_{eq} = \sum_{i=1}^{n} R_i$.
- Resistors connected in parallel can be replaced with an equivalent resistance, R_{eq}, given by $\frac{1}{R_{eq}} = \sum_{i=1}^{n}\frac{1}{R_i}$.
- The power, P, dissipated by a resistor, R, through which a current, i, flows is given by $P = i\Delta V = i^2R = \frac{(\Delta V)^2}{R}$, where ΔV is the potential drop across the resistor.

ANSWERS TO SELF-TEST OPPORTUNITIES

5.1 $\dfrac{700\ \text{mAh}}{0.1\ \text{mA}} = 7000\ \text{h} \approx 292\ \text{days}.$

5.2 $\Delta V = iR \Rightarrow i = \dfrac{\Delta V}{R} = \dfrac{1.50\ \text{V}}{10.0\ \Omega} = 0.150\ \text{A}.$

5.3 The maximum heating of the external resistance occurs when the external resistance is equal to the internal resistance.

$$V_t = V_{emf} + iR_i = i(R + R_i)$$

$$P_{heat} = i^2R = \frac{V_t^2 R}{(R + R_i)^2}$$

$$\frac{dP_{heat}}{dR} = -\frac{2V_t^2 R}{(R + R_i)^3} + \frac{V_t^2}{(R + R_i)^2} = 0 \text{ at extremum.}$$

From this, it follows that $R = R_i$.

You can check that this extremum is a maximum by taking the second derivative at $R = R_i$:

$$\left.\frac{d^2P_{heat}}{dR^2}\right|_{R=R_i} = V_t^2\left(\frac{6}{16R^3} - \frac{5}{8R^3}\right) = -\frac{V_t^2}{R^3}\left(\frac{1}{2}\right) < 0.$$

5.4 The sum of the two potential drops, across the diode and across the light bulb, has to equal 1.5 V, the potential difference supplied by the battery. In part (b), the diode prevents any current from flowing; therefore, the potential drop across the diode is 1.5 V and that across the light bulb is zero. In part (a), the potential drop across the diode is 0.7 V (see Figure 5.25), and therefore that across the light bulb is 1.5 V − 0.7 V = 0.8 V.

PROBLEM-SOLVING GUIDELINES

1. If a circuit diagram is not given as part of the problem statement, draw one yourself and label all the given values and unknown components. Indicate the direction of the current, starting from the emf source. (Don't worry about getting the direction of the current wrong in your diagram; if you guess wrong, your final answer for the current will be a negative number.)

2. Sources of emf supply potential to a circuit, and resistors reduce potential in the circuit. However, be careful to check the direction of the potential of the emf source relative to that of the current; a current flowing in the direction opposite to the potential of an emf device picks up a negative potential difference.

3. The sum of the potential drops across resistors in a circuit equals the net amount of emf supplied to the circuit. (This is a consequence of the law of conservation of energy.)

4. In any given wire segment, the current is the same everywhere. (This is a consequence of the law of conservation of charge.)

MULTIPLE-CHOICE QUESTIONS

5.1 If the current through a resistor is increased by a factor of 2, how does this affect the power that is dissipated?

a) It decreases by a factor of 4.
b) It increases by a factor of 2.
c) It decreases by a factor of 8.
d) It increases by a factor of 4.

5.2 You make a parallel connection between two resistors, resistor A having a very large resistance and resistor B having a very small resistance. The equivalent resistance for this combination will be

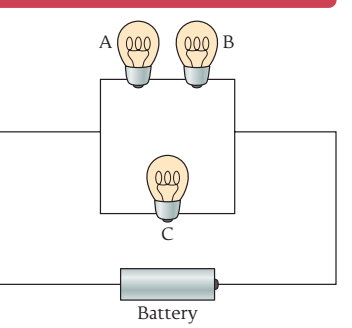
a) slightly greater than the resistance of resistor A.
b) slightly less than the resistance of resistor A.
c) slightly greater than the resistance of resistor B.
d) slightly less than the resistance of resistor B.

5.3 Two cylindrical wires, 1 and 2, made of the same material, have the same resistance. If the length of wire 2 is twice that of wire 1, what is the ratio of their cross-sectional areas, A_1 and A_2?

a) $A_1/A_2 = 2$
b) $A_1/A_2 = 4$
c) $A_1/A_2 = 0.5$
d) $A_1/A_2 = 0.25$

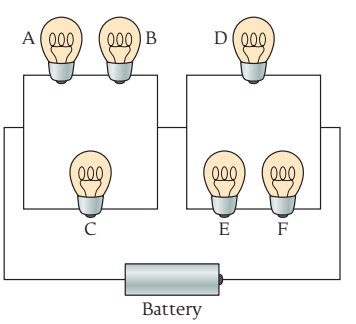
5.4 All three light bulbs in the circuit shown in the figure are identical. Which of the three shines the brightest?

a) A
b) B
c) C
d) A and B
e) All three are equally bright.



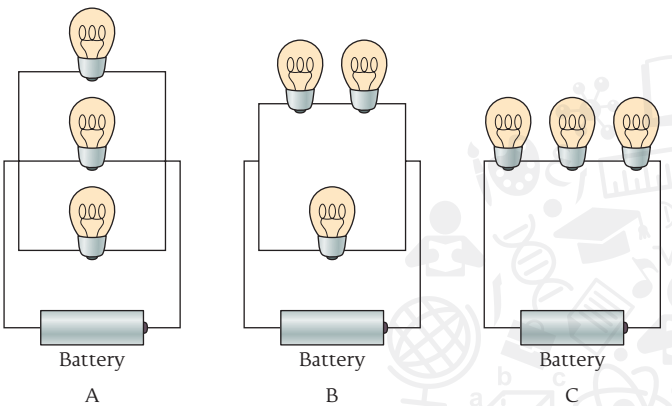
5.5 All of the six light bulbs in the circuit shown in the figure are identical. Which ordering correctly expresses the relative brightness of the bulbs? (*Hint:* The more current flowing through a light bulb, the brighter it is!)

a) A = B > C = D > E = F
b) A = B = E = F > C = D
c) C = D > A = B = E = F
d) A = B = C = D = E = F



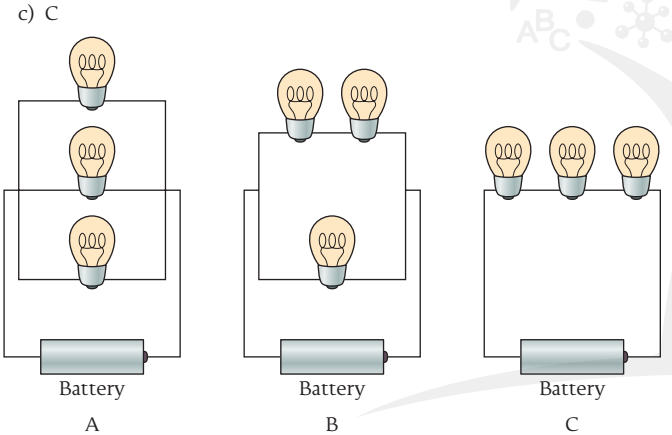
5.6 Which of the arrangements of three identical light bulbs shown in the figure draws the most current from the battery?

a) A
b) B
c) C
d) All three draw equal current.
e) A and C are tied for drawing the most current.

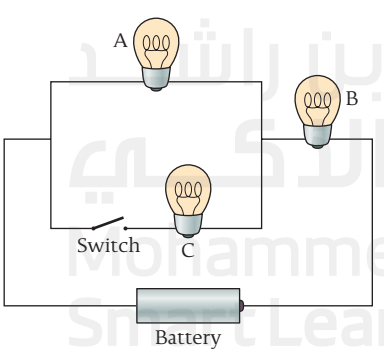


5.7 Which of the arrangements of three identical light bulbs shown in the figure has the highest resistance?

a) A
b) B
c) C
d) All three have equal resistance.
e) A and C are tied for having the highest resistance.



5.8 Three identical light bulbs are connected as shown in the figure. Initially the switch is closed. When the switch is opened (as shown in the figure), bulb C goes off. What happens to bulbs A and B?



a) Bulb A gets brighter, and bulb B gets dimmer.
b) Both bulbs A and B get brighter.
c) Both bulbs A and B get dimmer.
d) Bulb A gets dimmer, and bulb B gets brighter.

5.9 Which of the following wires has the largest current flowing through it?

a) a 1 m long copper wire of diameter 1 mm connected to a 10-V battery
b) a 0.5-m long copper wire of diameter 0.5 mm connected to a 5-V battery
c) a 2-m long copper wire of diameter 2 mm connected to a 20-V battery
d) a 1 m long copper wire of diameter 0.5 mm connected to a 5-V battery
e) All of the wires have the same current flowing through them.

5.10 Ohm's Law states that the potential difference across a device is equal to

a) the current flowing through the device times the resistance of the device.
b) the current flowing through the device divided by the resistance of the device.
c) the resistance of the device divided by the current flowing through the device.
d) the current flowing through the device times the cross-sectional area of the device.
e) the current flowing through the device times the length of the device.

5.11 A constant electric field is maintained inside a semiconductor. As the temperature is lowered, the magnitude of the current density inside the semiconductor

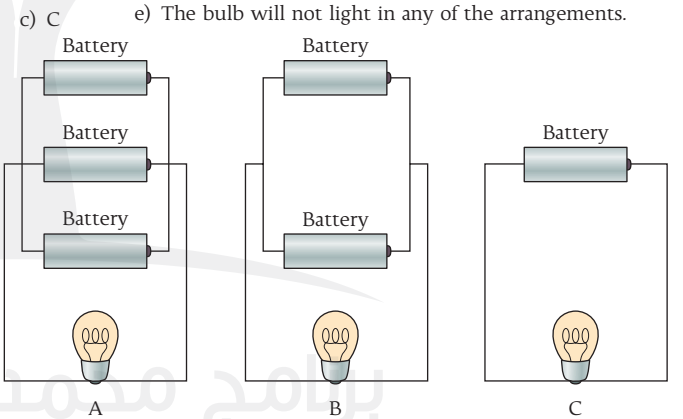
a) increases.
b) stays the same.
c) decreases.
d) may increase or decrease.

5.12 Which of the following is an incorrect statement?

a) The currents through electronic devices connected in series are equal.
b) The potential drops across electronic devices connected in parallel are equal.
c) More current flows across the smaller resistance when two resistors are connected in parallel.
d) More current flows across the smaller resistance when two resistors are connected in series.

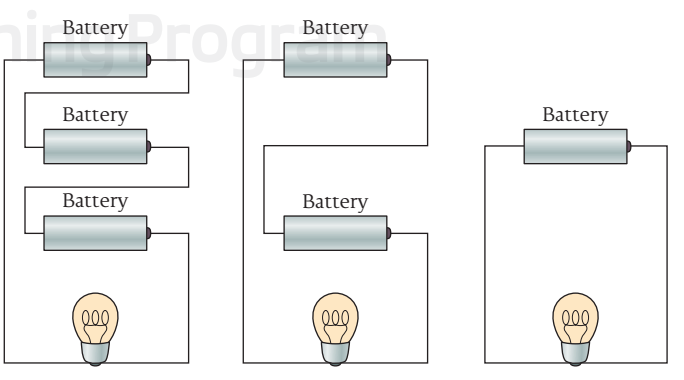
5.13 Identical batteries are connected in three different arrangements to the same light bulb as shown in the figure. Assume that the batteries have no internal resistance. In which arrangement will the light bulb shine the brightest?

a) A
b) B
c) C
d) The bulb will have the same brightness in all three arrangements.
e) The bulb will not light in any of the arrangements.



5.14 Identical batteries are connected in three different arrangements to the same light bulb as shown in the figure. Assume that the batteries have no internal resistance. In which arrangement will the light bulb shine the brightest?

a) A
b) B
c) C
d) The bulb will have the same brightness in all three arrangements.
e) The bulb will not light in any of the arrangements.



CONCEPTUAL QUESTIONS

5.15 What would happen to the drift velocity of electrons in a wire if the resistance due to collisions between the electrons and the atoms in the crystal lattice of the metal disappeared?

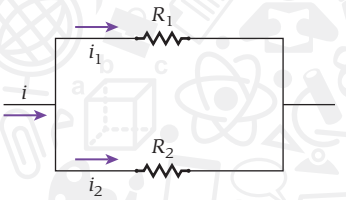
5.16 Why do light bulbs typically burn out just as they are turned on rather than while they are lit?

5.17 Two identical light bulbs are connected to a battery. Will the light bulbs be brighter if they are connected in series or in parallel?

5.18 Two resistors with resistances R_1 and R_2 are connected in parallel. Demonstrate that, no matter what the actual values of R_1 and R_2 are, the equivalent resistance is always less than the smaller of the two resistances.

5.19 Show that for resistors connected in series, it is always the highest resistance that dissipates the most power, while for resistors connected in parallel, it is always the lowest resistance that dissipates the most power.

5.20 For the connections shown in the figure, determine the current i_1 in terms of the total current, i, and R_1 and R_2.



5.21 An infinite number of resistors are connected in parallel. If $R_1 = 10\ \Omega$, $R_2 = 10^2\ \Omega$, $R_3 = 10^3\ \Omega$, and so on, show that $R_{eq} = 9\ \Omega$.

5.22 You are given two identical batteries and two pieces of wire. The red wire has a higher resistance than the black wire. You place the red wire across the terminals of one battery and the black wire across the terminals of the other battery. Which wire gets hotter?

5.23 Should light bulbs (ordinary incandescent bulbs with tungsten filaments) be considered ohmic resistors? Why or why not? How would this be determined experimentally?

5.24 A charged-particle beam is used to inject a charge, Q_0, into a small, irregularly shaped region (not a cavity, just some region within the solid block) in the interior of a block of ohmic material with conductivity σ and permittivity ε at time $t = 0$. Eventually, all the injected charge will move to the outer surface of the block, but how quickly?

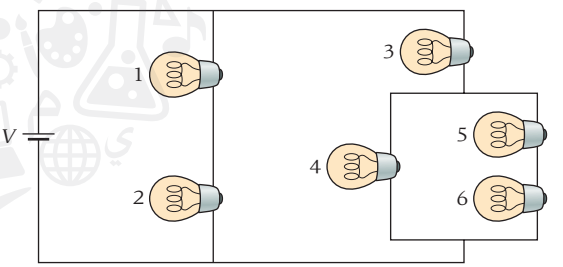
a) Derive a differential equation for the charge, $Q(t)$, in the injection region as a function of time.

b) Solve the equation from part (a) to find $Q(t)$ for all $t \geq 0$.

c) For copper, a good conductor, and for quartz (crystalline SiO_2), an insulator, calculate the time for the charge in the injection region to decrease by half. Look up the necessary values. Assume that the effective "dielectric constant" of copper is 1.00000.

5.25 Show that the drift speed of free electrons in a wire does not depend on the cross-sectional area of the wire.

5.26 Rank the brightness of the six identical light bulbs in the circuit in the figure. Each light bulb may be treated as an identical resistor with resistance R.



5.27 Two conductors of the same length and radius are connected to the same emf device. If the resistance of one is twice that of the other, to which conductor is more power delivered?

EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

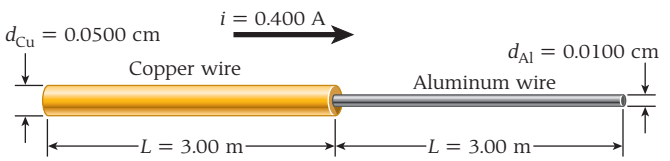
Sections 5.1 and 5.2

5.28 How many protons are in the beam traveling close to the speed of light in the Tevatron at Fermilab, which is carrying 11 mA of current around the 6.3 km circumference of the main Tevatron ring?

5.29 What is the current density in an aluminum wire having a radius of 1.00 mm and carrying a current of 1.00 mA? What is the drift speed of the electrons carrying this current? The density of aluminum is 2.70×10^3 kg/m^3, and 1 mole of aluminum has a mass of 26.98 g. There is one conduction electron per atom in aluminum.

•5.30 A copper wire has a diameter $d_{Cu} = 0.0500$ cm, is 3.00 m long, and has a charge-carrier density of 8.50×10^{28} electrons/m^3. As shown in the figure, the copper wire is attached to an equal length of aluminum wire with a diameter $d_{Al} = 0.0100$ cm and a charge-carrier density of 6.02×10^{28} electrons/m^3. A current of 0.400 A flows through the copper wire.

a) What is the ratio of the current densities in the two wires, J_{Cu}/J_{Al}?



b) What is the ratio of the drift velocities in the two wires, v_{d-Cu}/v_{d-Al}?

•5.31 A current of 0.123 mA flows in a silver wire whose cross-sectional area is 0.923 mm^2.

a) Find the density of electrons in the wire, assuming that there is one conduction electron per silver atom.

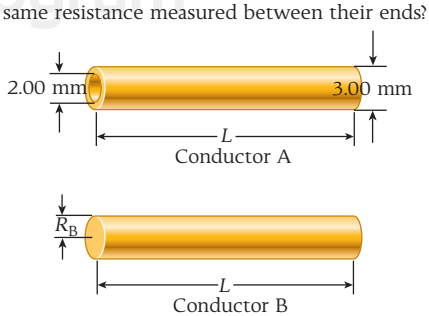
b) Find the current density in the wire assuming that the current is uniform.

c) Find the electrons' drift speed.

Section 5.3

5.32 What is the resistance of a copper wire of length $l = 10.9$ m and diameter $d = 1.30$ mm? The resistivity of copper is $1.72 \times 10^{-8}\ \Omega$ m.

5.33 Two conductors are made of the same material and have the same length L. Conductor A is a hollow tube with inside diameter 2.00 mm and outside diameter 3.00 mm; conductor B is a solid wire with radius R_B. What value of R_B is required for the two conductors to have the same resistance measured between their ends?



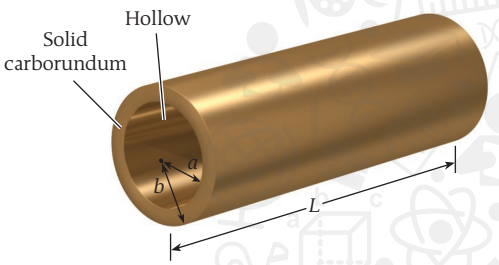
5.34 A copper coil has a resistance of 0.100 Ω at room temperature (20.0 °C). What is its resistance when it is cooled to −100. °C?

5.35 What gauge of aluminum wire will have the same resistance per unit length as 12 gauge copper wire?

5.36 A rectangular wafer of pure silicon, with resistivity $\rho = 2300\ \Omega\,\text{m}$, measures 2.00 cm by 3.00 cm by 0.0100 cm. Find the maximum resistance of this rectangular wafer between any two faces.

•5.37 A copper wire that is 1.00 m long and has a radius of 0.500 mm is stretched to a length of 2.00 m. What is the fractional change in resistance, $\Delta R/R$, as the wire is stretched? What is $\Delta R/R$ for a wire of the same initial dimensions made out of aluminum?

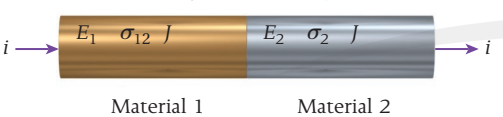
•5.38 The most common material used for sandpaper, silicon carbide, is also widely used in electrical applications. One common device is a tubular resistor made of a special grade of silicon carbide called *carborundum*. A particular carborundum resistor (see the figure) consists of a thick-walled cylindrical shell (a pipe) of inner radius $a = 1.50$ cm, outer radius $b = 2.50$ cm, and length $L = 60.0$ cm. The resistance of this carborundum resistor at 20.0 °C is 1.00 Ω.



a) Calculate the resistivity of carborundum at room temperature. Compare this to the resistivities of the most commonly used conductors (copper, aluminum, and silver).

b) Carborundum has a high temperature coefficient of resistivity: $\alpha = 2.14 \times 10^{-3}\ \text{K}^{-1}$. If, in a particular application, the carborundum resistor heats up to 300. °C, what is the percentage change in its resistance between room temperature (20.0 °C) and this operating temperature?

••5.39 As illustrated in the figure, a current, i, flows through the junction of two materials with the same cross-sectional area and with conductivities σ_1 and σ_2. Show that the total amount of charge at the junction is $\varepsilon_0 i(1/\sigma_2 - 1/\sigma_1)$.



Section 5.4

5.40 A potential difference of 12.0 V is applied across a wire of cross-sectional area 4.50 mm^2 and length 1000. km. The current passing through the wire is 3.20×10^{-3} A.

a) What is the resistance of the wire?

b) What type of wire is this?

5.41 One brand of 12.0 V automotive battery used to be advertised as providing "600 cold-cranking amps." Assuming that this is the current the battery supplies if its terminals are shorted, that is, connected to negligible resistance, determine the internal resistance of the battery. (*IMPORTANT*: **Do not** attempt such a connection as it could be lethal!)

5.42 A copper wire has radius $r = 0.0250$ cm, is 3.00 m long, has resistivity $\rho = 1.72 \times 10^{-8}\ \Omega\,\text{m}$, and carries a current of 0.400 A. The wire has a charge-carrier density of $8.50 \cdot 10^{28}$ electrons/m^3.

a) What is the resistance, R, of the wire?

b) What is the electric potential difference, ΔV, across the wire?

c) What is the electric field, E, in the wire?

•5.43 A 34 gauge copper wire ($A = 0.0201$ mm^2), with a constant potential difference of 0.100 V applied across its 1.00 m length at room temperature (20.0 °C), is cooled to liquid nitrogen temperature (77 K = −196 °C).

a) Determine the percentage change in the wire's resistance during the drop in temperature.

b) Determine the percentage change in current flowing in the wire.

c) Compare the drift speeds of the electrons at the two temperatures.

Section 5.5

5.44 A resistor of unknown resistance and a 35.0-Ω resistor are connected across a 120. V emf device in such a way that an 11.0 A current flows. What is the value of the unknown resistance?

5.45 A battery has a potential difference of 14.50 V when it is not connected in a circuit. When a 17.91 Ω resistor is connected across the battery, the potential difference of the battery drops to 12.68 V. What is the internal resistance of the battery?

5.46 When a battery is connected to a 100. Ω resistor, the current is 4.00 A. When the same battery is connected to a 400. Ω resistor, the current is 1.01 A. Find the emf supplied by the battery and the internal resistance of the battery.

•5.47 A light bulb is connected to a source of emf. There is a 6.20 V drop across the light bulb and a current of 4.10 A flowing through the light bulb.

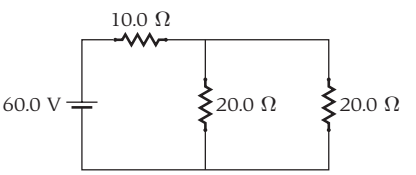
a) What is the resistance of the light bulb?

b) A second light bulb, identical to the first, is connected in series with the first bulb. The potential drop across the bulbs is now 6.29 V, and the current through the bulbs is 2.90 A. Calculate the resistance of each light bulb.

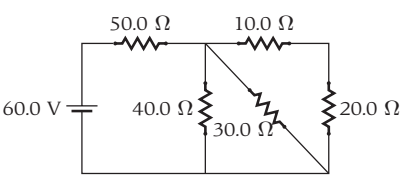
c) Why are your answers to parts (a) and (b) not the same?

Section 5.6

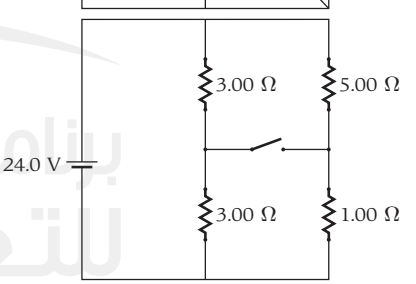
5.48 What is the current in the 10.0 Ω resistor in the circuit in the figure?



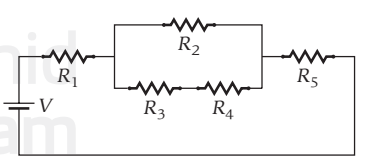
5.49 What is the equivalent resistance of the five resistors in the circuit in the figure?



•5.50 What is the current in the circuit shown in the figure when the switch is (a) open and (b) closed?



5.51 For the circuit shown in the figure, $R_1 = 6.00\ \Omega$, $R_2 = 6.00\ \Omega$, $R_3 = 2.00\ \Omega$, $R_4 = 4.00\ \Omega$, $R_5 = 3.00\ \Omega$, and the potential difference is 12.0 V.

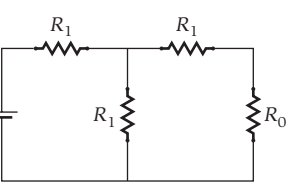


a) What is the equivalent resistance for the circuit?

b) What is the current through R_5?

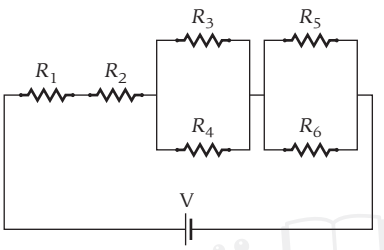
c) What is the potential drop across R_3?

5.52 Four resistors are connected in a circuit as shown in the figure. What value of R_1, expressed as a multiple of R_0, will make the equivalent resistance for the circuit equal to R_0?



•5.53 As shown in the figure, a circuit consists of an emf source with $V = 20.0$ V and six resistors. Resistors $R_1 = 5.00\ \Omega$ and $R_2 = 10.00\ \Omega$ are connected in series. Resistors

$R_3 = 5.00\ \Omega$ and $R_4 = 5.00\ \Omega$ are connected in parallel and are in series with R_1 and R_2. Resistors $R_5 = 2.00\ \Omega$ and $R_6 = 2.00\ \Omega$ are connected in parallel and are also in series with R_1 and R_2.



a) What is the potential drop across each resistor?

b) How much current flows through each resistor?

•**5.54** When a 40.0 V emf device is placed across two resistors in series, a current of 10.0 A flows through each of the resistors. When the same emf device is placed across the same two resistors in parallel, the current through the emf device is 50.0 A. What is the magnitude of the larger of the two resistances?

Section 5.7

5.55 A voltage spike causes the line voltage in a home to jump rapidly from 110. V to 150. V. What is the percentage increase in the power output of a 100.-W tungsten-filament incandescent light bulb during this spike, assuming that the bulb's resistance remains constant?

5.56 A thundercloud similar to the one described in Example 24.3 produces a lightning bolt that strikes a radio tower. If the lightning bolt transfers 5.00 C of charge in about 0.100 ms and the potential remains constant at 70.0 MV, find (a) the average current, (b) the average power, (c) the total energy, and (d) the effective resistance of the air during the lightning strike.

5.57 A hair dryer consumes 1600. W of power and operates at 110. V. (Assume that the current is DC. In fact, these are root-mean-square values of AC quantities, but the calculation is not affected.

a) Will the hair dryer trip a circuit breaker designed to interrupt the circuit if the current exceeds 15.0 A?

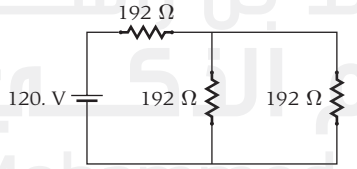
b) What is the resistance of the hair dryer when it is operating?

5.58 How much money will a homeowner owe an electric company if he turns on a 100.00 W incandescent light bulb and leaves it on for an entire year? (Assume that the cost of electricity is \$0.12000/kWh and that the light bulb lasts that long.) The same amount of light can be provided by a 26.000 W compact fluorescent light bulb. What would it cost the homeowner to leave one of those on for a year?

5.59 Three resistors are connected across a battery as shown in the figure.

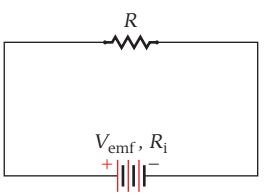
a) How much power is dissipated across the three resistors?

b) Determine the potential drop across each resistor.



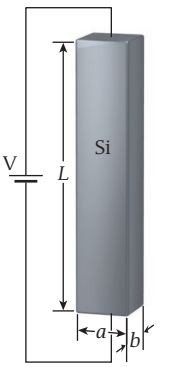
5.60 Suppose an AAA battery is able to supply 625 mAh before its potential drops below 1.50 V. How long will it be able to supply power to a 5.00 W bulb before the potential drops below 1.50 V?

•**5.61** Show that the power supplied to the circuit in the figure by the battery with internal resistance R_i is maximum when the resistance of the resistor in the circuit, R, is equal to R_i. Determine the power supplied to R. For practice, calculate the power dissipated by a 12.0 V battery with an internal resistance of 2.00 Ω when $R = 1.00\ \Omega$, $R = 2.00\ \Omega$, and $R = 3.00\ \Omega$.



•**5.62** A water heater consisting of a metal coil that is connected across the terminals of a 15.0 V power supply is able to heat 250 mL of water from room temperature to boiling point in 45.0 s. What is the resistance of the coil?

•**5.63** A potential difference of $V = 0.500$ V is applied across a block of silicon with resistivity $8.70 \times 10^{-4}\ \Omega$ m. As indicated in the figure, the dimensions of the silicon block are width $a = 2.00$ mm and length $L = 15.0$ cm. The resistance of the silicon block is 50.0 Ω, and the density of charge carriers is $1.23 \times 10^{23}\ \text{m}^{-3}$. Assume that the current density in the block is uniform and that current flows in silicon according to Ohm's Law. The total length of 0.500 mm-diameter copper wire in the circuit is 75.0 cm, and the resistivity of copper is $1.69 \times 10^{-8}\ \Omega$ m.



a) What is the resistance, R_w, of the copper wire?

b) What are the direction and the magnitude of the electric current, i, in the block?

c) What is the thickness, b, of the block?

d) On average, how long does it take an electron to pass from one end of the block to the other?

e) How much power, P, is dissipated by the block?

f) As what form of energy does this dissipated power appear?

Additional Exercises

5.64 In an emergency, you need to run a radio that uses 30.0 W of power when attached to a 10.0-V power supply. The only power supply you have access to provides 25.0 kV, but you do have a large number of 25.0-Ω resistors. If you want the power to the radio to be as close as possible to 30.0 W, how many resistors should you use, and how should they be connected (in series or in parallel)?

5.65 A certain brand of hot dog cooker applies a potential difference of 120. V to opposite ends of the hot dog and cooks it by means of the heat produced. If 48.0 kJ is needed to cook each hot dog, what current is needed to cook three hot dogs simultaneously in 2.00 min? Assume a parallel connection.

5.66 A circuit consists of a copper wire of length 10.0 m and radius 1.00 mm connected to a 10.0-V battery. An aluminum wire of length 5.00 m is connected to the same battery and dissipates the same amount of power. What is the radius of the aluminum wire?

5.67 The resistivity of a conductor is $\rho = 1.00 \times 10^{-5}\ \Omega$ m. If a cylindrical wire is made of this conductor, with a cross-sectional area of $1.00 \times 10^{-6}\ \text{m}^2$, what should the length of the wire be for its resistance to be 10.0 Ω?

5.68 Two cylindrical wires of identical length are made of copper and aluminum. If they carry the same current and have the same potential difference across their length, what is the ratio of their radii?

5.69 Two resistors with resistances 200. Ω and 400. Ω are connected (a) in series and (b) in parallel with an ideal 9.00-V battery. Compare the power delivered to the 200.-Ω resistor.

5.70 What are (a) the conductance and (b) the radius of a 3.50-m-long iron heating element for a 110.-V, 1500.-W heater?

5.71 A 100.-W, 240.-V European light bulb is used in an American household, where the electricity is delivered at 120. V. What power will it consume?

5.72 A modern house is wired for 115 V, and the current is limited by circuit breakers to a maximum of 200. A. (For the purpose of this problem, treat these as DC quantities.)

a) Calculate the minimum total resistance the circuitry in the house can have at any time.

b) Calculate the maximum electrical power the house can consume.

•**5.73** A 12.0 V battery with an internal resistance $R_i = 4.00\ \Omega$ is attached across an external resistor of resistance R. Find the maximum power that can be delivered to the resistor.

•**5.74** A multiclad wire consists of a zinc core of radius 1.00 mm surrounded by a copper sheath of thickness 1.00 mm. The resistivity of zinc is $\rho = 5.964 \times 10^{-8}\ \Omega$ m. What is the resistance of a 10.0 m long strand of this wire?

•**5.75** The Stanford Linear Accelerator accelerated a beam consisting of 2.0×10^{14} electrons per second through a potential difference of 2.0×10^{10} V.

a) Calculate the current in the beam.

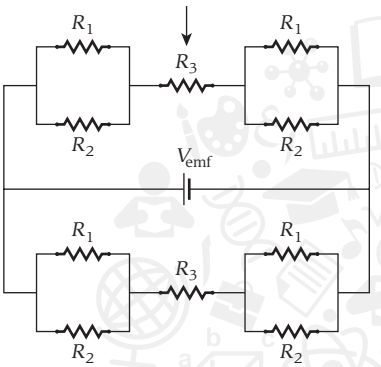
b) Calculate the power of the beam.

c) Calculate the effective ohmic resistance of the accelerator.

•**5.76** In the circuit shown in the figure, $R_1 = 3.00\ \Omega$, $R_2 = 6.00\ \Omega$, $R_3 = 20.0\ \Omega$, and $V_{emf} = 12.0$ V.

a) Determine a value for the equivalent resistance.

b) Calculate the magnitude of the current flowing through R_3 on the top branch of the circuit (marked with a vertical arrow).

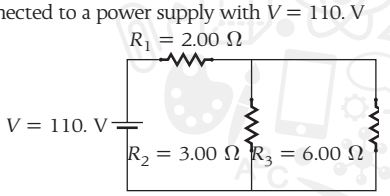


•**5.77** Three resistors are connected to a power supply with $V = 110$. V as shown in the figure.

a) Find the potential drop across R_3.

b) Find the current in R_1.

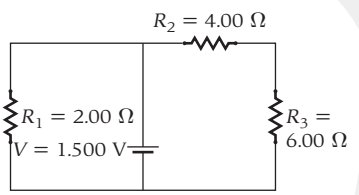
c) Find the rate at which thermal energy is dissipated from R_2.



•**5.78** A battery with $V = 1.500$ V is connected to three resistors as shown in the figure.

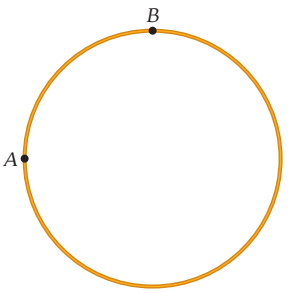
a) Find the potential drop across each resistor.

b) Find the current in each resistor.



•**5.79** A 2.50 m long copper cable is connected across the terminals of a 12.0 V car battery. Assuming that it is completely insulated from its environment, how long after the connection is made will the copper start to melt? (Useful information: copper has a mass density of 8960 kg/m^3, a melting point of 1359 K, and a specific heat of 386 J/kg/K.)

•**5.80** A piece of copper wire is used to form a circular loop of radius 10.0 cm. The wire has a cross-sectional area of 10.0 mm^2. Points A and B are 90.0° apart, as shown in the figure. Find the resistance between points A and B.



•**5.81** Two conducting wires have identical lengths $L_1 = L_2 = L = 10.0$ km and identical circular cross sections of radius $r_1 = r_2 = r = 1.00$ mm. One wire is made of steel (with resistivity $\rho_{steel} = 40.0 \times 10^{-8}\ \Omega$ m); the other is made of copper (with resistivity $\rho_{copper} = 1.68 \times 10^{-8}\ \Omega$ m).

a) Calculate the ratio of the power dissipated by the two wires, P_{copper}/P_{steel}, when they are connected in parallel and a potential difference of $V = 100$. V is applied to them.

b) Based on this result, how do you explain the fact that conductors for power transmission are made of copper and not steel?

•**5.82** Before bendable tungsten filaments were developed, Thomas Edison used carbon filaments in his light bulbs. Though carbon has a very high melting temperature (3599 °C), its sublimation rate is high at high temperatures. So carbon-filament bulbs were kept at lower temperatures, thereby rendering them dimmer than later tungsten-based bulbs.

A typical carbon-filament bulb requires an average power of 40. W, when 110 volts is applied across it, and has a filament temperature of 1800 °C. Carbon, unlike copper, has a negative temperature coefficient of resistivity: $\alpha = -0.00050\ °C^{-1}$. Calculate the resistance at room temperature (20. °C) of this carbon filament.

••**5.83** A material is said to be *ohmic* if an electric field, $\vec{E}$, in the material gives rise to current density $\vec{J} = \sigma\vec{E}$, where the conductivity, σ, is a constant independent of $\vec{E}$ or $\vec{J}$. (This is the precise form of Ohm's Law.) Suppose in some material an electric field, $\vec{E}$, produces current density, $\vec{J}$, not necessarily related by Ohm's Law; that is, the material may or may not be ohmic.

a) Calculate the rate of energy dissipation (sometimes called *ohmic heating* or *joule heating*) per unit volume in this material, in terms of $\vec{E}$ and $\vec{J}$.

b) Express the result of part (a) in terms of $\vec{E}$ alone and $\vec{J}$ alone, for $\vec{E}$ and $\vec{J}$ related via Ohm's Law, that is, in an ohmic material with conductivity σ or resistivity ρ.

MULTI-VERSION EXERCISES

5.84 A high-voltage direct current (HVDC) transmission line carries electrical power a distance of 643.1 km. The line transmits 7935 MW of power at a potential difference of 1.177 MV. If the HVDC line consists of one copper wire of diameter 2.353 cm, what fraction of the power is lost in transmission?

5.85 A high-voltage direct current (HVDC) transmission line carries 5319 MW of electrical power a distance of 411.7 km. The HVDC line consists of one copper wire of diameter 2.125 cm. The fraction of the power lost in transmission is 7.538×10^{-2}. What is the potential difference in the line?

5.86 A high-voltage direct current (HVDC) transmission line carries 5703 MW of electrical power at a potential difference of 1.197 MV. The HVDC line consists of one copper wire of diameter 1.895 cm. The fraction of the power lost in transmission is 1.166×10^{-1}. How long is the line?

5.87 The reserve capacity (RC) of a car battery is defined as the number of minutes the battery can provide 25.0 A of current at a potential difference of 10.5 V. Thus, the RC indicates how long the battery can power a car whose charging system has failed. How much energy is stored in a car battery with an RC of 110.0?

5.88 The reserve capacity (RC) of a car battery is defined as the number of minutes the battery can provide 25.0 A of current at a potential difference of 10.5 V. Thus, the RC indicates how long the battery can power a car whose charging system has failed. If a car battery stores $1.843 \cdot 10^6$ J of energy, what is its RC?

5.89 A flashlight bulb with a tungsten filament draws 374.3 mA of current when a potential difference of 3.907 V is applied. When the bulb is at room temperature (20.00 °C) and is not lit, its resistance is 1.347 Ω. What is the temperature of the tungsten filament when the bulb is lit?

5.90 A flashlight bulb with a tungsten filament draws 420.1 mA when a potential difference of 3.949 V is applied. The temperature of the tungsten filament when the bulb is lit is 1291 °C. What is the resistance of the bulb when it is at room temperature (20.00 °C) and is not lit?

5.91 When a flashlight bulb with a tungsten filament is lit, the applied potential difference is 3.991 V and the temperature of the tungsten filament is 1.110×10^3 °C. The resistance of the bulb when it is at room temperature (20.00 °C) and is not lit is 1.451 Ω. What current does the bulb draw when it is lit?