

# Formulae of Differentiation

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	$f(x)$	$f'(x)$	$f(u)$	$f'(u)$
	<b>Non-Chain Rule</b>			<b>Chain Rule</b>
1	$x^n$	$n \cdot x^{n-1}$	$u^n$	$n \cdot u^{n-1} \cdot u'$
2	$\sin x$	$\cos x$	$\sin u$	$\cos u \cdot u'$
3	$\cos x$	$-\sin x$	$\cos u$	$-\sin u \cdot u'$
4	$\tan x$	$\sec^2 x$	$\tan u$	$\sec^2 u \cdot u'$
5	$\cot x$	$-\csc^2 x$	$\cot u$	$-\csc^2 u \cdot u'$
6	$\sec x$	$\sec x \cdot \tan x$	$\sec u$	$\sec u \cdot \tan u \cdot u'$
7	$\csc x$	$-\csc x \cdot \cot x$	$\csc u$	$-\csc u \cdot \cot u \cdot u'$
	<b>Inverse Trigonometric Functions</b>			
8	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} u$	$\frac{u'}{\sqrt{1-u^2}}$
9	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1} u$	$-\frac{u'}{\sqrt{1-u^2}}$
10	$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\tan^{-1} u$	$\frac{u'}{1+u^2}$
11	$\cot^{-1} x$	$-\frac{1}{1+x^2}$	$\cot^{-1} u$	$-\frac{u'}{1+u^2}$
12	$\sec^{-1} x$	$\frac{1}{ x \sqrt{x^2-1}}$	$\sec^{-1} u$	$\frac{u'}{ u \sqrt{u^2-1}}$
13	$\csc^{-1} x$	$-\frac{1}{ x \sqrt{x^2-1}}$	$\csc^{-1} u$	$-\frac{u'}{ u \sqrt{u^2-1}}$
	<b>Exponential and Logarithmic Functions</b>			
14	$e^x$	$e^x$	$e^u$	$e^u \cdot u'$
15	$a^x$	$a^x \cdot \ln a$	$a^u$	$a^u \cdot u' \cdot \ln a$
16	$\ln x$	$\frac{1}{x}$	$\ln u$	$\frac{u'}{u}$
	<b>Product Rule</b>		<b>Quotient Rule</b>	
17	$u \cdot v$	$u \cdot v' + v \cdot u'$	$\frac{u}{v}$	$\frac{v \cdot u' - u \cdot v'}{v^2}$

## Formulae of Integration

	Rule القاعدة	General بشكل عام
1	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$ $\int (ax+b)^{-1} dx = \frac{1}{a} \ln ax+b  + c$
2	$\int \sin x dx = -\cos x + c$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c, a \in R$
3	$\int \cos x dx = \sin x + c$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
4	$\int \sec^2 x dx = \tan x + c$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
5	$\int \csc^2 x dx = -\cot x + c$	$\int \csc^2(ax) dx = -\frac{1}{a} \cot(ax) + c$
6	$\int \sec x \cdot \tan x dx = \sec x + c$	$\int \sec(ax) \cdot \tan(ax) dx = \frac{1}{a} \sec(ax) + c$
7	$\int \csc x \cdot \cot x dx = -\csc x + c$	$\int \csc(ax) \cdot \cot(ax) dx = -\frac{1}{a} \csc(ax) + c$
8	$\int \tan x dx = -\ln \cos x  + c = \ln \sec x  + c$	$\int \tan(ax) dx = -\frac{1}{a} \ln \cos(ax)  + c = \frac{1}{a} \ln \sec(ax)  + c$
9	$\int \cot x dx = \ln \sin x  + c$	$\int \cot(ax) dx = \frac{1}{a} \ln \sin(ax)  + c$
10	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
11	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
12	$\int \frac{1}{ x \sqrt{x^2-1}} dx = \sec^{-1} x + c$	$\int \frac{1}{ x \sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + c$
13	$\int \frac{-1}{\sqrt{1-x^2}} dx = -\sin^{-1} x + c = \cos^{-1} x + C$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = -\sin^{-1}\left(\frac{x}{a}\right) + c = \cos^{-1}\left(\frac{x}{a}\right) + C$
14	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
15	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
<b>قواعد الدالة الأسية</b>		
16	$\int e^x dx = e^x + c$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
17	$\int a^x dx = \frac{a^x}{\ln a } + c, a \in R - \{0\}$	$\int f'(x) \cdot e^{f(x)} dx = e^{f(x)} + c$
<b>قواعد الدالة اللوغاريتمية</b>		
18	$\int \frac{1}{x} dx = \ln x  + c$	$\int \frac{k}{ax+b} dx = \frac{k}{a} \ln ax+b  + c$
19		$\int \frac{f'(x)}{f(x)} dx = \ln f(x)  + c$

Washer Formula	قاعدة الحلقات	Shell Formula	قاعدة الاصداف
$V = \pi \int_a^b (R^2 - r^2) dx$		$V = 2\pi \int_a^b h(x).r(x) dx$	
Arc Length طول المنحنى		Integration By Parts التكامل بالأجزاء	
$s = \int \sqrt{1 + [f'(x)]^2} dx$		$\int u. dv = uv - \int v. du$	
Integration by Substitution التكامل بالتعويض			
$\int g(x). (f(x))^n. dx$ where $f(x)$ is not linear function, Let $u = f(x)$		$\int g(x). \sin(f(x)). dx$ where $f(x)$ is not linear function, Let $u = f(x)$	
Integration by Parts التكامل بالأجزاء			
$\int g(x). (f(x))^n. dx$ where $f(x)$ is linear function i.e. $f(x) = ax + b$ , Let $u = g(x)$ , $dv = (f(x))^n$		$\int g(x). \sin(f(x)). dx$ where $f(x)$ is linear function, Let $u = g(x)$ , $dv = \sin(f(x))$	
Trigonometrical Formulae متطابقات هامة			
1) $\sin^2 x + \cos^2 x = 1$	$\sin^2 x = 1 - \cos^2 x$	2) $\sin 2x = 2 \sin x \cdot \cos x$	
	$\cos^2 x = 1 - \sin^2 x$	$\sin 4x = 2 \sin 2x \cdot \cos 2x$	
$\tan^2 x + 1 = \sec^2 x$ and $\sec^2 x - \tan^2 x = 1$		$\sin 10x = 2 \sin 5x \cdot \cos 5x$	
$\cot^2 x + 1 = \csc^2 x$ and $\csc^2 x - \cot^2 x = 1$		$\cos 2x = 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$ $= \cos^2 x - \sin^2 x$	
	$\text{Sinh } x = \frac{e^x - e^{-x}}{2}$	$\text{Cosh } x = \frac{e^x + e^{-x}}{2}$	
$\int \sin^2 x dx$		$\int \cos^2 x dx$	
use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$		use $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$	
$\int \tan^2 x dx$		$\int \cot^2 x dx$	
use $\tan^2 x = \sec^2 x - 1$		use $\cot^2 x = \csc^2 x - 1$	
$\int \sin^3 x dx$ , use $\sin^3 x = \sin x \cdot \sin^2 x$		$\int \cos^3 x dx$ , use $\cos^3 x = \cos x \cdot \cos^2 x$	
use $\sin^2 x = 1 - \cos^2 x$		use $\cos^2 x = 1 - \sin^2 x$	

## Trigonometrical Formulae متطابقات هامة

$$\sin A \cdot \cos B = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$$

$$\cos A \cdot \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \cdot \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

Integral of type	Substitution	Figure
$\int \sqrt{a^2 - x^2} dx$	$x = a \sin \theta$	
$\int \sqrt{a^2 + x^2} dx$	$x = a \tan \theta$	
$\int \sqrt{x^2 - a^2} dx$	$x = a \sec \theta$	

## التكامل بالكسور الجزئية Integration using Partial Fractions

تذكرة في الكسور الجزئية

$$1) \frac{1}{x^2 - 3x + 2} = \frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$2) \frac{4x^2 + 3x + 1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{D}{x+1}$$

$$3) \frac{3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+D}{x^2+1}$$

$$4) \frac{x^2+x+2}{(x-1)^3(x^2+x+1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{D}{(x-1)^3} + \frac{Ex+F}{x^2+x+1} + \frac{Gx+H}{(x^2+x+1)^2}$$