نماذج أجابة أسئله أمتحان تقييمى أول

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(1)

$$\int (x+2)\sqrt[3]{x^2+4x-1}\,dx$$

الحل :

بفرض $u = x^2 + 4x - 1$

$du = (2x+4)dx \quad , \quad \frac{1}{2}du = (x+2)dx$

$$\int (x+2)\sqrt[3]{x^2+4x-1}\,dx = \int u^{\frac{1}{3}}\left(\frac{1}{2}\,du\right)$$

$$= \frac{1}{2}\int u^{\frac{1}{3}}\,du$$

$$= \frac{1}{2}\left[\frac{3}{4}\,u^{\frac{4}{3}}\right] + C$$

$$\therefore \int (x+2)\sqrt[3]{x^2+4x-1}\,dx = \frac{3}{8}(x^2+4x-1)^{\frac{4}{3}} + C$$

(2)

أوجد :

$$\int \frac{5}{\sqrt{x}\,(\sqrt{x}+2)^3}\,dx$$

$$\int \frac{5}{x^{\frac{1}{2}}(x^{\frac{1}{2}}+2)^3}\,dx =$$

$$= \int 5\,x^{\frac{-1}{2}}\,(x^{\frac{1}{2}}+2)^{-3}\,dx = 5\int (x^{\frac{1}{2}}+2)^{-3}\,x^{\frac{-1}{2}}\,dx$$

نفرض أن $u = x^{\frac{1}{2}} + 2$ تفاضل $du = \frac{1}{2}x^{\frac{-1}{2}}\,dx$

$$2\,du = x^{\frac{-1}{2}}\,dx$$

$$5\int (x^{\frac{1}{2}}+2)^{-3}\,x^{\frac{-1}{2}}\,dx = 5\int u^{-3}.2\,du = 10\int u^{-3}\,du$$

$$= 10.\frac{u^{-2}}{-2} + C = \frac{-5}{u^2} + C = \frac{-5}{(\sqrt{x}+2)^2} + C$$

(3)

$$\int x(2x-1)^3\,dx$$

الحل:

$$u=2x-1 \Rightarrow du=2dx \Rightarrow \frac{du}{2}=dx$$

$$u=2x-1 \Rightarrow 2x=u+1 \Rightarrow x=\frac{u+1}{2}$$

$$\int x(2x-1)^3\,dx=\int\left(\frac{u+1}{2}\right)u^3\frac{du}{2}=\frac{1}{4}\int(u^4+u^3)du$$

$$=\frac{1}{4}\left(\frac{u^5}{5}+\frac{u^4}{4}+C\right)=\frac{1}{20}(2x-1)^5+\frac{1}{16}(2x-1)^4+C$$

(4)

$$\int x^5\sqrt{3+x^2}\,dx$$

الحل:

$$u=3+x^2 \Rightarrow du=2xdx \Rightarrow \frac{du}{2}=xdx$$

$$\int x^5\sqrt{3+x^2}\,dx=\int \sqrt{3+x^2}\,(x^4)(xdx)$$

$$u=3+x^2 \Rightarrow x^2=u-3 \Rightarrow x^4=(u-3)^2$$

$$\int x^5\sqrt{3+x^2}\,dx=\int \sqrt{3+x^2}\,(x^4)(xdx)$$

$$=\int \sqrt{u}\,(u-3)^2\frac{du}{2}=\frac{1}{2}\int u^{\frac{1}{2}}(u^2-6u+9)du=\frac{1}{2}\int\left(u^{\frac{5}{2}}-6u^{\frac{3}{2}}+9u^{\frac{1}{2}}\right)du$$

$$=\frac{1}{2}\left(\frac{u^{\frac{7}{2}}}{\frac{7}{2}}-\frac{6u^{\frac{5}{2}}}{\frac{5}{2}}+\frac{9u^{\frac{3}{2}}}{\frac{3}{2}}+C\right)=\frac{1}{7}u^{\frac{7}{2}}-\frac{6}{5}u^{\frac{5}{2}}+3u^{\frac{3}{2}}+C$$

$$=\frac{1}{7}(3+x^2)^{\frac{7}{2}}-\frac{6}{5}(3+x^2)^{\frac{5}{2}}+3(3+x^2)^{\frac{3}{2}}+C$$

(5)

$$\int x\sec^2(x^2+2)\,dx$$

الحل:

$$u = x^2+2 \Longrightarrow du = 2xdx \Longrightarrow \frac{1}{2}du = xdx$$

$$\int x sec^2(x^2+2)dx = \int sec^2 u\left(\frac{1}{2}du\right)$$

$$= \frac{1}{2}\int sec^2 u\,du = \frac{1}{2}tan\,u + C = \frac{1}{2}tan\,(x^2+2) + C$$

(6)

$$\int \csc^5 x\ \cot x\,dx$$

الحل:

$$u = \csc x \Rightarrow du = -\csc x\cot x\,dx \Rightarrow -du = \csc x\cot x\,dx$$

$$\int \csc^5 x.\cot x\,dx = \int \csc^4 x\,.\csc x\,.\cot x\,dx = \int u^4(-du)$$

$$= -\int u^4\,du = -\frac{u^5}{5} + C = -\frac{1}{5}\csc^5 x + C$$

(7)

$$\int \cot x\, dx$$

الحل:

$$\int \cot x\, dx = \int \frac{\cos x}{\sin x} dx$$

$$u = \sin x \Longrightarrow du = \cos x\, dx$$

$$\int \cot x\, dx = \int \frac{du}{u} = \ln|u| + C = \ln|\sin x| + C$$

(8)

اوجد قيمة التكامل :

$$\int (3+\sin 2x)^5 \cos 2x dx$$

الحل

$$u = (3+\sin 2x)$$

$$du = 2\cos 2x du$$

$$\frac{1}{2} du = \cos 2x du$$

$$\int (3+\sin 2x)^5 \cos 2x dx$$

$$= \int \frac{1}{2} u^5 du$$

$$= \frac{u^6}{12} + c$$

$$= \frac{(3+\sin 2x)^6}{12} + c$$

(9)

$$\int \frac{dx}{(\sin^2 x)\sqrt{1+\cot x}}\, dx$$

$$= \int \frac{1}{\sqrt{1+\cot x}} \cdot \frac{1}{\sin^2 x}\, dx$$

$$= \int (1+\cot x)^{-1/2} \cdot \csc^2 x\, dx$$

نفرض أن

$$u = 1 + \cot x$$

$$du = -\csc^2 x\, dx$$

$$\therefore -\int (1+\cot x)^{-1/2} \cdot -\csc^2 x\, dx$$

بالتعويض

$$= -\int u^{-1/2}\, du$$

$$= -\frac{u^{1/2}}{\frac{1}{2}} + c$$

$$= -2\sqrt{u} + c$$

$$= -2\sqrt{1+\cot x} + c$$

(10)

$$\int \frac{3t^2 - 6t}{t^3 - 3t^2 + 8} dt$$

نفرض أن $u = t^3 - 3t^2 + 8 \longrightarrow du = (3t^2 - 6t)\ dt$

$$\int \frac{3t^2 - 6t}{t^3 - 3t^2 + 8} dx = \int \frac{1}{u} du = ln\,|u| + C$$

$$= ln\left|t^3 - 3t^2 + 8\right| + C$$

....

$$\int \frac{x^3 + 4}{x} dx$$

$$= \int (\frac{x^3}{x} + \frac{4}{x}) dx$$

$$= \int (x^2 + \frac{4}{x})\ dx$$

$$= \frac{x^3}{3} + 4\ ln\ |x| + C$$

(11)

$$\int \frac{x+1}{\sqrt[3]{x}+1}dx = \int \frac{\left(\sqrt[3]{x}+1\right)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)dx}{\left(\sqrt[3]{x}+1\right)}$$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$$= \int \left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)dx$$

$$= \int \left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)dx$$

قاعدة الجمع والطرح

$$= \frac{3}{5}x^{\frac{5}{3}}-\frac{3}{4}x^{\frac{4}{3}}+x+C$$

$$= \frac{3}{5}x\sqrt[3]{x^2}-\frac{3}{4}x\sqrt[3]{x}+x+C$$

(12)

أوجد: $\int (x^2-2)e^{x^3-6x}\,dx$

الحل

$$I = \int e^{x^3-6x}\cdot(x^2-2)\,dx$$

$$= \int e^{u}\cdot \frac{1}{3}\,du$$

$$= \frac{1}{3}e^{u}+c$$

$$= \frac{1}{3}e^{x^3-6x}+c$$

$u = x^3-6x$

$du = (3x^2-6)dx$

$du = 3(x^2-2)dx$

$\frac{1}{3}du = (x^2-2)\,dx$

(13)

أوجد: $\int \frac{1}{x^2} e^{\frac{1}{x}} dx$

الحل

$I = \int e^{\frac{1}{x}} \cdot \frac{1}{x^2} dx$

$= -\int e^{u} \cdot du$

$= -e^{u} + c = -e^{\frac{1}{x}} + c$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2} dx$

$-du = \frac{1}{x^2} dx$

(14)

أوجد: $\int \frac{e^x}{e^x + 1} dx$

الحل

$I = \int \frac{1}{u} du$

$= \ln|u| + c$

$= \ln|e^x + 1| + c$

$u = e^x + 1$

$du = e^x dx$

(15)

$$\int (2\tan x - \csc^2 x)\, dx$$

$$\int tanx\, dx = \int \frac{sin\, x}{cos\, x} dx \qquad \left| \begin{array}{l} u = \cos x \\ du = -\sin x\, dx \end{array} \right.$$

$$= -\int \frac{du}{u}$$

$$= -\,ln\,|u| + C$$

$$= -\,ln\,|\ cos\, x\,| + C$$

$$\therefore \mathrm{I} = -2\ln|\cos x| + \cot x + C$$