التقعر واختبار المشتقة الثانية

Concavity and second derivative test

طريقة سهلة لمعرفة فترات التقعر لأعلى او أسفل

12 متقدم

Definition 5.1

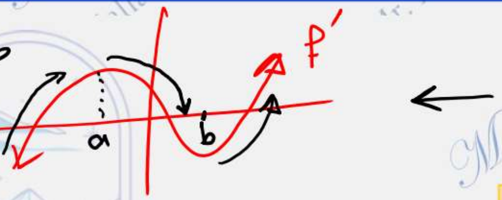
For a function f that is differentiable on an interval I, the graph of f is

تقعر لأعلى (i) **concave up** on I if f' is increasing on I or

تقعر لأسفل (ii) **concave down** on I if f'' is decreasing on I.

$(-\infty, a)$ (b, ∞) → $f(x)$ Concave up

(a, b) $f(x)$ Concave down

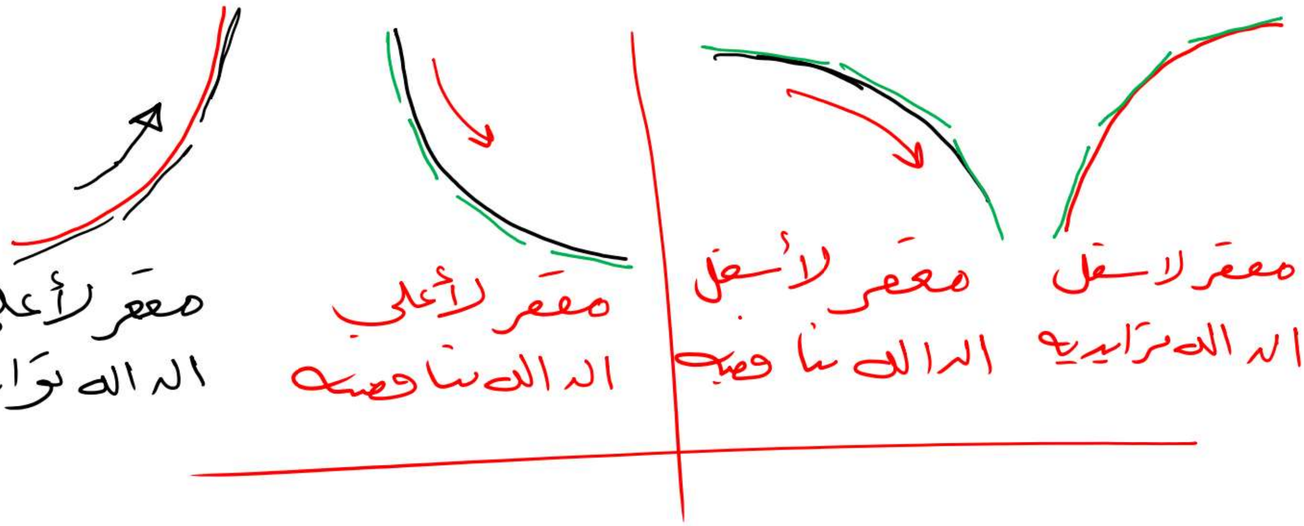


Notes

- The graph of the function $y = f(x)$ is concave up on open interval I if the curve is above all its tangents.
- The graph of the function $y = f(x)$ is concave down on open interval I if the curve is below all its tangents.

ملاحظات

- الرسم البياني للدالة $y = f(x)$ يكون مقعرا لأعلى على فترة مفتوحة I إذا كان منحنى الدالة يقع فوق كل مماساته.
- الرسم البياني للدالة $y = f(x)$ يكون مقعرا لأسفل على فترة مفتوحة I إذا كان منحنى الدالة يقع أسفل كل مماساته.



مقعر لأعلى الدالة تزايدية

مقعر لأعلى الدالة تناقصية

مقعر لأسفل الدالة تناقصية

مقعر لأسفل الدالة تزايدية

Theorem 5.1

$f''(x)$

Suppose that f'' exists on an interval I.

(i) If $f''(x) > 0$ on I, then the graph of f is **concave up** on I.

(ii) If $f''(x) < 0$ on I, then the graph of f is **concave down** on I.

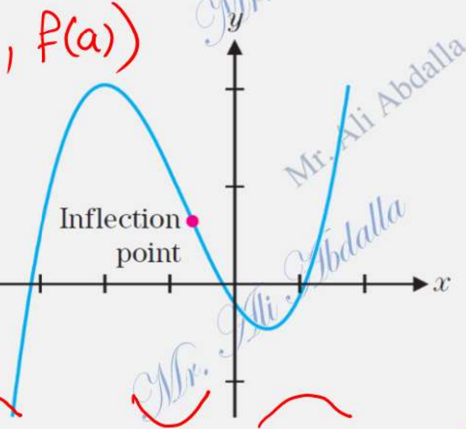
f'' — c_1 — c_2

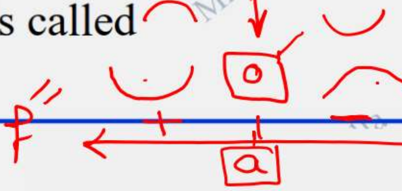
$f''(x) = 0$ ← $f''(x)$ undefined

Definition 5.2

Suppose that f is continuous on the interval (a, b) and that the graph changes concavity at a point $c \in (a, b)$ (i.e., the graph is concave down on one side of c and concave up on the other). Then, the point $(c, f(c))$ is called an **inflection point** of f.

$(a, f(a))$





$x = a$

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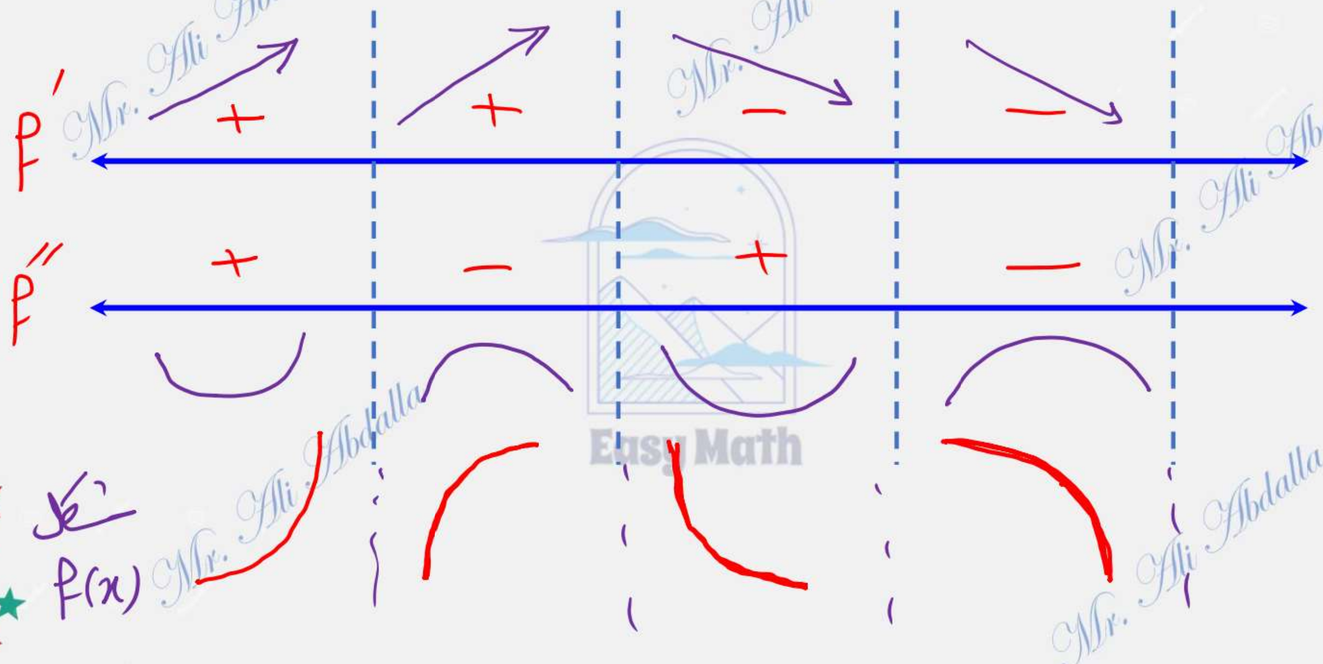


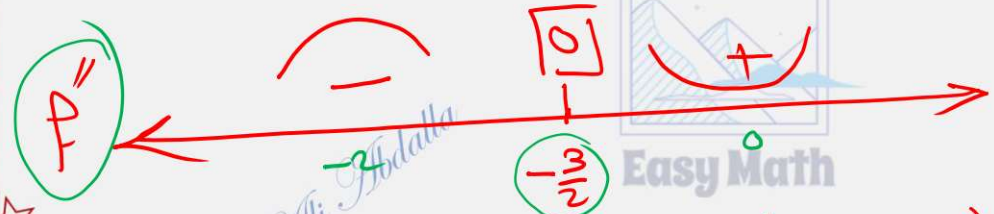
Determine where the graph of $f(x) = 2x^3 + 9x^2 - 24x - 10$ is concave up and concave down

$-24+18$

$f'(x) = 6x^2 + 18x - 24 \Rightarrow f''(x) = 12x + 18$

$f''(x) = 0 \qquad 12x + 18 = 0 \Rightarrow 12x = -18$

$\boxed{x = \frac{-3}{2}}$



f(x) Concave up $(-\frac{3}{2}, \infty)$

Concave down $(-\infty, -\frac{3}{2})$

نقط انعطاف

$x = \frac{-3}{2}$

$f(-\frac{3}{2}) =$

$(-\frac{3}{2}, f(-\frac{3}{2}))$

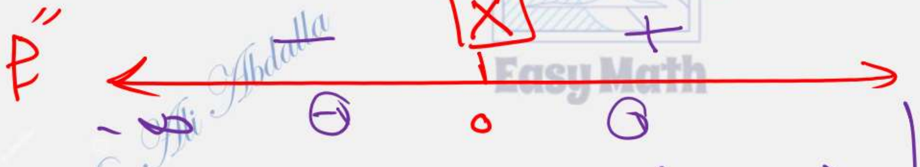
Determine the intervals where the graph of the given function is concave up and concave down and identify inflection points $f(x) = x + \frac{25}{x}$ $x \neq 0$

$f(x) = x + 25x^{-1}$

$f'(x) = 1 - 25x^{-2} \Rightarrow f''(x) = 50x^{-3} = \boxed{\frac{\boxed{50}}{x^3}}$

$f''(x) \neq 0$ $f''(x)$ undefined

$x^3 = 0 \Rightarrow \boxed{x = 0} \notin$ Domai



Concave down $(-\infty, 0)$

Concave up $(0, \infty)$

No inflection

$x = 0 \notin$ Domain

ابدا

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If $f(x) = 2x^3 - ax^2 - 2$ has inflection point at $x = \frac{1}{2}$, find the value of a?

انعطاف

$f''\left(\frac{1}{2}\right) = 0$

$f'(x) = 6x^2 - 2ax$

$f''(x) = 12x - 2a$

$12\left(\frac{1}{2}\right) - 2a = 0$

$6 - 2a = 0$

$2a = 6 \Rightarrow \boxed{a = 3}$

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If $f(x) = ax^3 + bx^2 - 5$ has inflection point at (2, 11), find the value of a and b?

$f''(2) = 0$ ✓ | $\boxed{f(2) = 11}$

$a(2)^3 + b(2)^2 - 5 = 11$

$8a + 4b = 16$

$\boxed{2a + b = 4}$

$f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$

$6a(2) + 2b = 0$

$12a + 2b = 0$

$\boxed{6a + b = 0}$

$\boxed{b = -6a}$

$2a + (-6a) = 4$

$-4a = 4$

$\boxed{a = -1}$

$b = -6(-1)$

$= 6$

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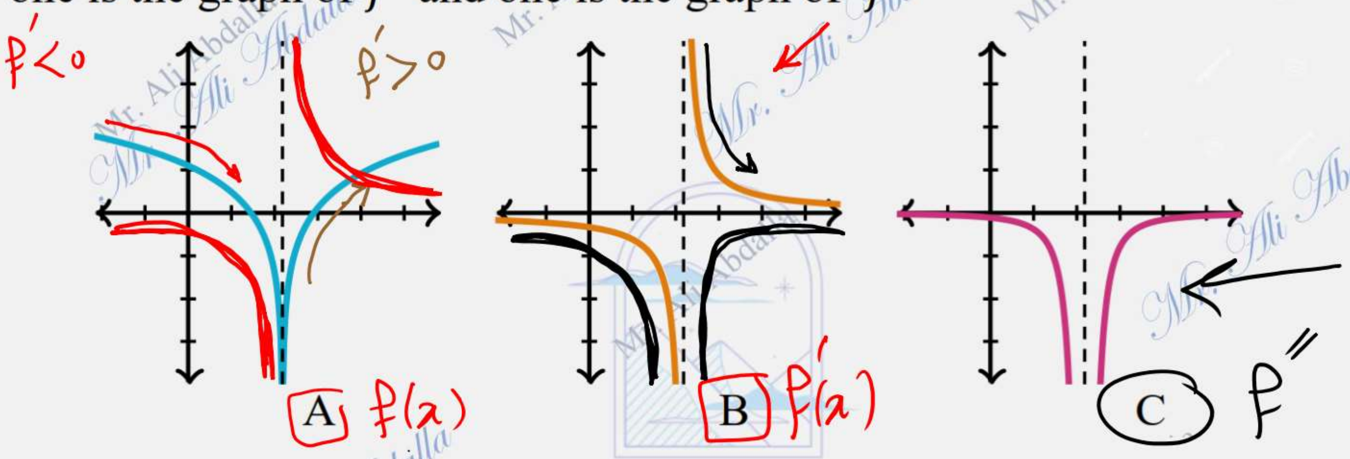
Three graphs labeled A, B, C are shown below. One is the graph of f, one is the graph of f' and one is the graph of f''



Which of the following is correct identifies each graph?

(A)

f	f'	f''
B	A	C

[✓ **(B)**]

f	f'	f''
A	B	C

(C)

f	f'	f''
A	C	B

(D)

f	f'	f''
B	C	A

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If the graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection at $\boxed{x = -1}$ then the value of k is

A) -1 B) 0 C) 1 ✓ D) 2

$$\boxed{f''(-1) = 0}$$

$$f(x) = 2x^2 + kx^{-1}$$

$$f'(x) = 4x - kx^{-2}$$

$$f''(x) = 4 + 2kx^{-3}$$

$$= 4 + \frac{2k}{x^3}$$

$$4 + \frac{2k}{(-1)^3} = 0$$

$$4 - 2k = 0$$

$$2k = 4$$

$$\Rightarrow \boxed{k=2}$$

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Let f be the function given by $\boxed{f(x) = 2xe^x}$. The graph of f is concave down when

A) $x < -2$ B) $x > -2$ C) $x < -1$ D) $x > -1$

$$f'(x) = 2\cdot e^x + 2x\cdot e^x$$

$$= (2+2x)\,e^x$$

$$f''(x) = 2e^x + (2+2x)\cdot e^x$$

$$= (2+2+2x)e^x = (4+2x)e^x$$

$$\boxed{f''(x) = 0}$$

$$4+2x=0 \Rightarrow \boxed{x=-2}$$

$4+(-2)$ ②

f'': $-$ $\boxed{0}$ $+$

-3 -2 -1

Concave down $(-\infty, -2)$

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$(\frac{1}{\sqrt[4]{3}}, \frac{\pi}{6})$ $f(\frac{1}{\sqrt[4]{3}}) = \tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$

Determine the intervals where the graph of the given function is concave up and concave down and identify inflection points.

$$f(x) = \tan^{-1} x^2$$

$(-\infty,\infty)$ Domain

$$f'(x) = \frac{1}{1+(x^2)^2}\frac{d}{dx}(x^2) = \frac{2x}{1+x^4}$$

$$f''(x) = \frac{2(1+x^4) - 2x^{①}(4x^{③})}{(1+x^4)^2} = \frac{2+2x^4-8x^4}{(1+x^4)^2}$$

$$f''(x) = \frac{2-6x^4}{(1+x^4)^2}$$

موجب

$f''(x) = 0 \Rightarrow 2-6x^4=0$

$6x^4 = 2$

$x^4 = \frac{1}{3}$

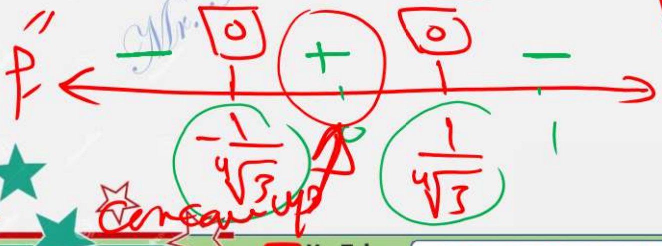
$$\boxed{x = \pm\frac{1}{\sqrt[4]{3}}}$$

f''(x) undefined

$1+x^4 \neq 0$

Concave down

$(-\infty, -\frac{1}{\sqrt[4]{3}}), (\frac{1}{\sqrt[4]{3}}, \infty)$



The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The following graph represents the graph of $f''(x)$.

$f''(x) = 0$ التقاطع مع محور x ،

تتغير الـ ∪ ∩

For what values of x does the graph of f have a point of inflection?

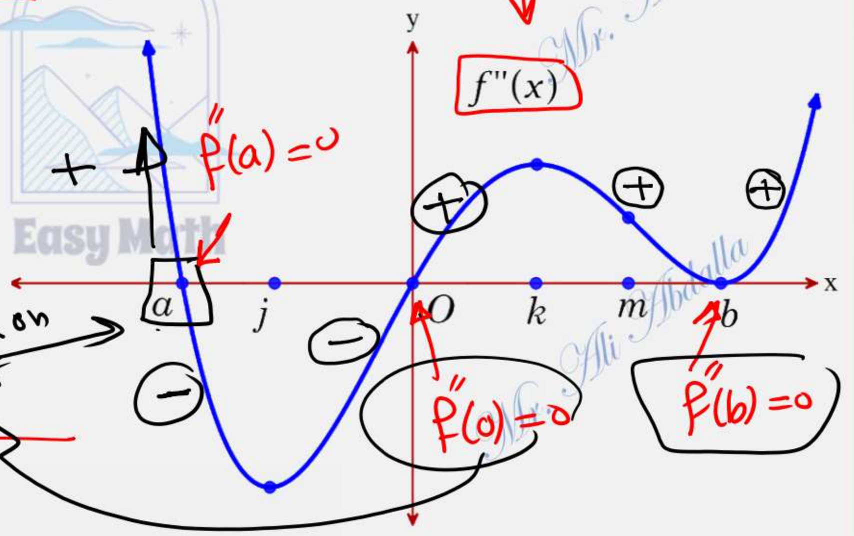
[✓ A) 0 and a only]

B) 0 and m only

C) b and j only

D) 0, a, and b

E) b, j, and k



Theorem 5.2 (Second Derivative Test)

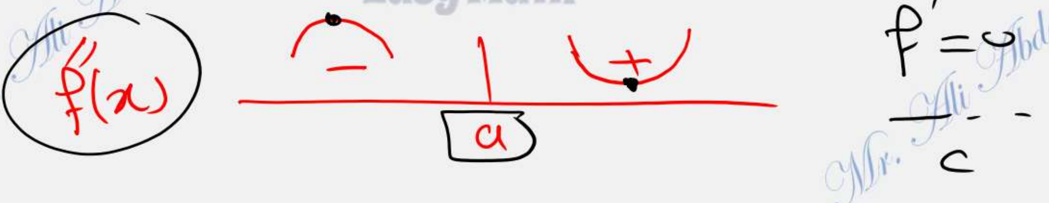
Suppose that f'' is continuous on the interval (a, b) and $f'(c) = 0$, for some number $c \in (a, b)$.

(i) If $f''(c) < 0$, then $f(c)$ is a **local maximum**.

(ii) If $f''(c) > 0$, then $f(c)$ is a **local minimum**.

Notes:

1) If $f'(c)$ not exist, then can not use "***Second Derivative Test***".
2) Second Derivative Test fail if $f''(c) = 0$ then we must use "First Derivative Test".



To use second derevative test to find the local extrema	لاستخدام اختبار المشتقة الثانية لمعرفة القيم القصوى
1- Find the first derivative test and use it to find the critiacl numbers.	1- نوجد المشتقة الأولى ومنها نوجد الأعداد الحرجة للدالة
2- Find second dreivative.	2- نوجد المشتقة الثانية.
3- Substitute by the critical numbers in the second derivative.	3- نعوض باستخدام الاعداد الحرجة في المشتقة الثانية.
4- After Substitution:	4- بعد التعويض
A) If $f''(c) < 0$ (the value of $f''(x)$ is negative) the we have a local maximum.	أ) إذا كانت قيمة المشتقة الثانية سالبة $f''(c) < 0$ (تكون قيمة عظمى محلية)
B) If $f''(c) > 0$ (the value of $f''(x)$ is positive) the we have a local minimum.	ب) إذا كانت قيمة المشتقة الثانية موجبة $f''(c) > 0$ (تكون قيمة صغرى محلية)

What is the value of x at which the maximum value of $y = \boxed{\ln x} - 2x^2$ occurs? $x>0$

A) $x = 0$

[✓ B) $x = \frac{1}{2}$]

C) $x = \frac{e}{2}$

D) There is no maximum

$y' = \frac{1}{\boxed{x}} - 4x = x^{-1} - 4x$

y' undefined

~~$\boxed{x=0} \notin$ Domain~~

$\frac{1}{x} - 4x = 0 \Rightarrow \frac{1}{x} = \frac{4x}{1}$

$4x^2 = 1 \Rightarrow x^2 = \frac{1}{4}$

$\boxed{x = \pm\frac{1}{2}} \leftarrow$

$f''(x) = -x^{-2} - 4$

$f''(x) = -\frac{1}{x^2} - 4$

$x = \frac{1}{2}$ $f''(\frac{1}{2}) < 0$ local Max

$x = -\frac{1}{2}$ $f''(-\frac{1}{2}) < 0$ local Max

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Use the Second Derivative Test to find the local extrema of:

$$f(x) = x^4 - 8x^2 + 10 \quad (-\infty, \infty)$$

$f'(x) = 4x^3 - 16x \Rightarrow f'(x) = 0 \Rightarrow 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$4x = 0$ $x^2 - 4 = 0$

$\boxed{x = 0}$ $\boxed{x = \pm 2}$

$f''(x) = 12x^2 - 16$

$f''(0) = -16 < 0$ local Max

$f''(2) = 32 > 0$ local Min

$f''(-2) = 32 > 0$ local Min

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Find all critical numbers and use the Second Derivative Test to determine all local extrema. $f(x) = x \ln x$ $\boxed{x>0}$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$\boxed{f'(x) = \ln x + 1}$

$f'(x) = 0 \Rightarrow \ln x + 1 = 0$

$e^{\cancel{\ln} x} = e^{-1}$

$\boxed{x = e^{-1}} = \frac{1}{e}$

$f''(x) = \frac{1}{x}$

$f''(\frac{1}{e}) = \frac{1}{1/e} = e > 0$ local Min

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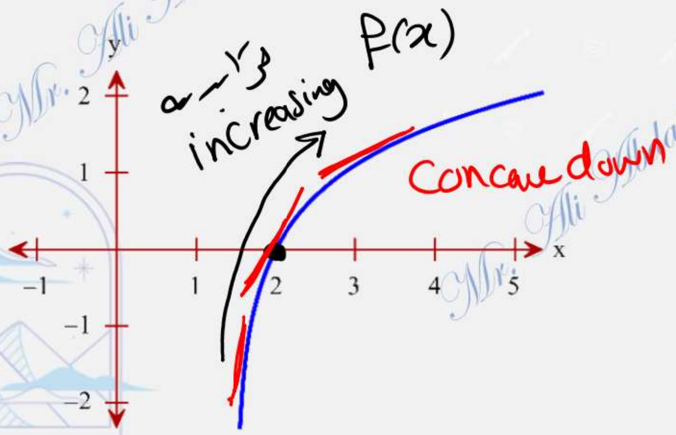
The graph of the twice differentiable function $f(x)$ is shown. Which of the following is true

A) $f(2) < f'(2) < f''(2)$

B) $f'(2) < f(2) < f''(2)$

C) $f''(2) < f'(2) < f(2)$

✓ D) $f''(2) < f(2) < f'(2)$



من الأصغر الى الأكبر

$f''(2) < f(2) < f'(2)$

$f(2) = 0$

$f'(2) > 0$ ⊕

$f''(2) < 0$ ⊖

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Complete the following using the following graph.

1) $f' = 0$ and $f'' < 0$ at the point(s) ..A..

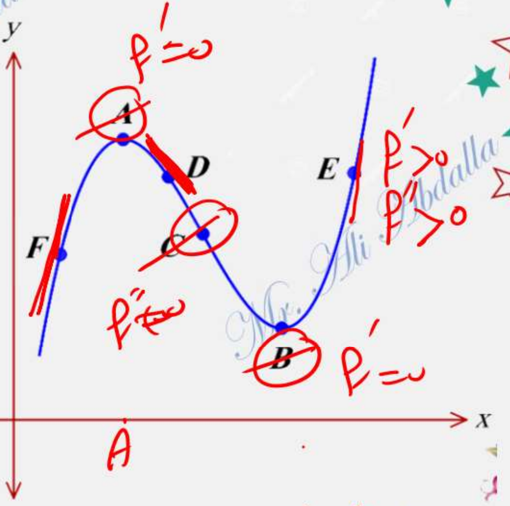
2) $f' = 0$ and $f'' > 0$ at the point(s) ..B...

3) $f'' = 0$ at the point(s) C.....

4) $f' \times f'' > 0$ at the point(s) ..D.., E

5) $f' \times f'' < 0$ at the point(s) ..F.....

6) $f' \times f'' = 0$ at the point(s) .A, B, C



f $f' > 0$ $f'' < 0$

⊕ ⊖ -

at A $f' = 0$, $f'' < 0$

at B $f' = 0$ $f'' > 0$

D $f' < 0$ $f'' < 0$

⊖ ⊖ ⊕

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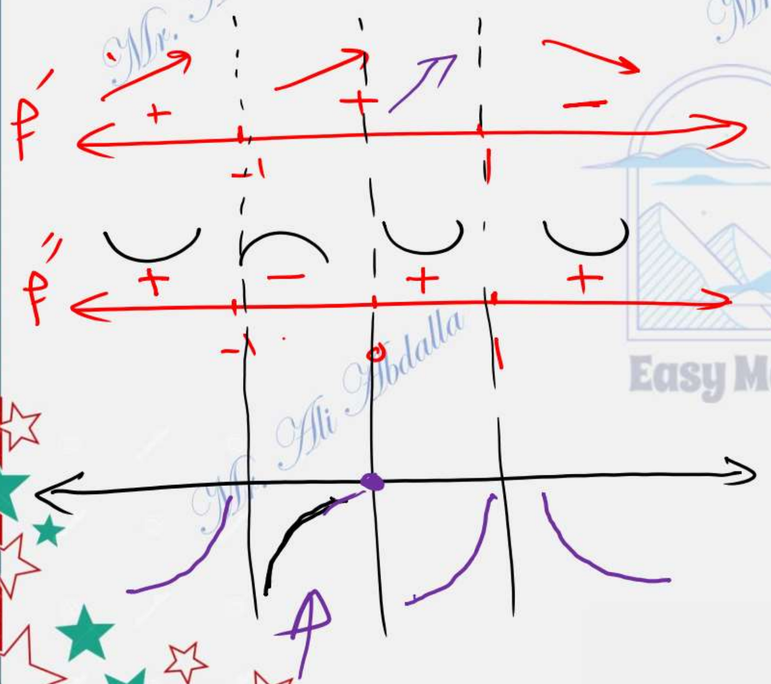
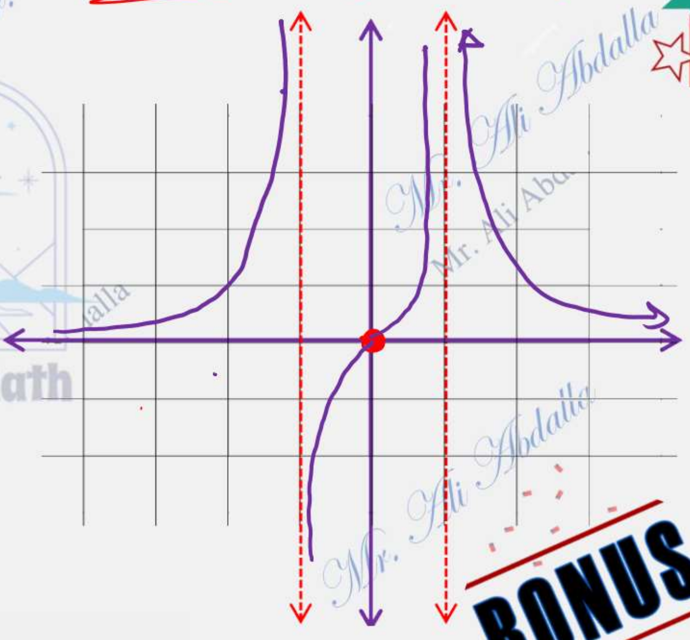
Sketch a graph with the given properties:

$f(0) = 0$, $f'(x) > 0$ for $x < -1$ and $-1 < x < 1$, $f'(x) < 0$ for $x > 1$, $f''(x) > 0$ for $x < -1$, $0 < x < 1$ and $x > 1$, $f''(x) < 0$ for $-1 < x < 0$

⊕ ⊖ +





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