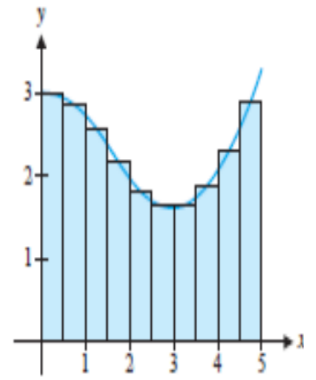


## (6-1) Area Between Curves

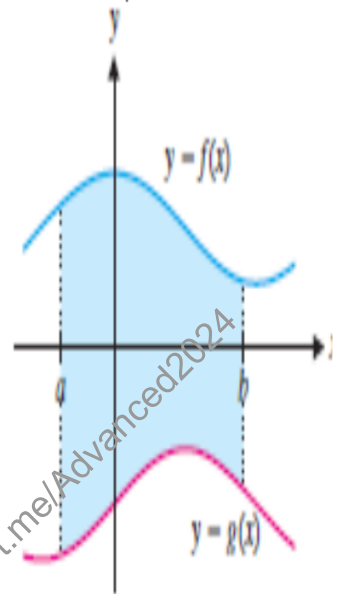
We have learned before that if  $f(x) > 0$  then the area between the curve of  $f(x)$  and the  $x$ -axis in the interval  $[a, b]$  is given by

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x = \int_a^b f(x) dx$$



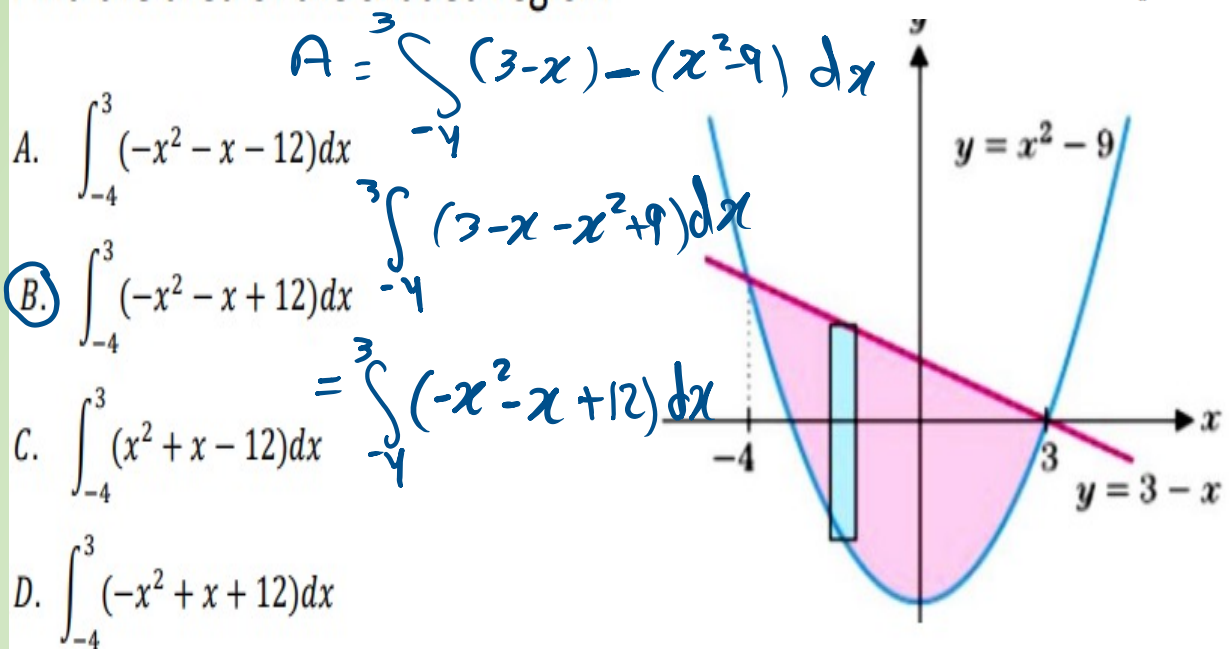
The accurate area could be found by

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n [f(c_i) - g(c_i)] \Delta x = \int_a^b [f(x) - g(x)] dx$$



Find the area of the shaded region:

المنطقة المظللة في الشكل تعطى بالتكامل :



Exercise page 414: find the area between the curves on the given interval.

Q1)  $y = x^3, y = x^2 - 1, 1 \leq x \leq 3$

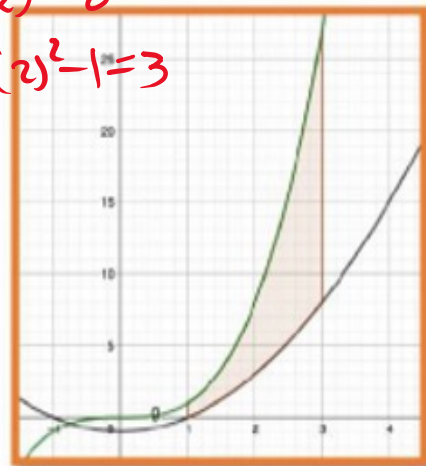
$y = (2)^3 = 8$

$y = (2)^2 - 1 = 3$

$$A = \int_1^3 (x^3) - (x^2 - 1) dx$$

$$A = \int_1^3 (x^3 - x^2 + 1) dx$$

$$A = \left[ \frac{x^4}{4} - \frac{x^3}{3} + x \right]_1^3 = \left( \frac{3^4}{4} - \frac{3^3}{3} + 3 \right) - \left( \frac{1^4}{4} - \frac{1^3}{3} + 1 \right) = \checkmark$$



Q2)  $y = \cos x, y = x^2 + 2, 0 \leq x \leq 2$

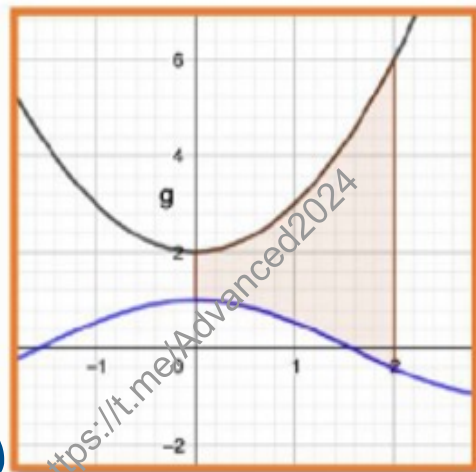
$y = \cos(1) \} y = (1)^2 + 2$   
 $= 0.54 \} y = 3$

$$A = \int_0^2 (x^2 + 2) - (\cos x) dx$$

$$= \left[ \frac{x^3}{3} + 2x - \sin x \right]_0^2$$

$$= \left( \frac{2^3}{3} + 2(2) - \sin 2 \right) - (0 + 2(0) - \sin 0)$$

$$= \left[ \frac{20}{3} - \sin 2 \right]$$



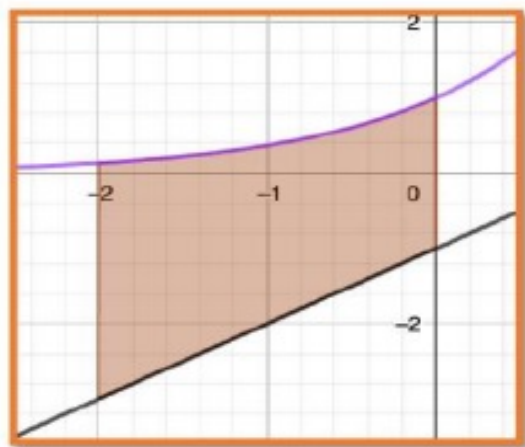
Q3)  $y = e^x, y = x - 1, -2 \leq x \leq 0$

$y = e^{-1}, y = (-1) - 1 = -2$

$$A = \int_{-2}^0 (e^x) - (x - 1) dx$$

$$= \left[ e^x - \frac{x^2}{2} + x \right]_{-2}^0$$

$$(e^0 - 0 + 0) - \left( e^{-2} - \frac{(-2)^2}{2} + (-2) \right) = \checkmark$$

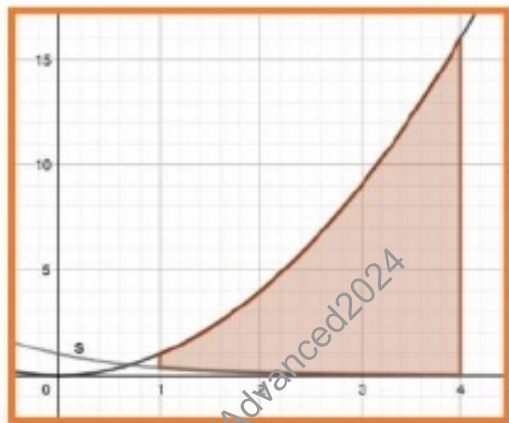


Q4)  $y = e^{-x}, y = x^2, 1 \leq x \leq 4$

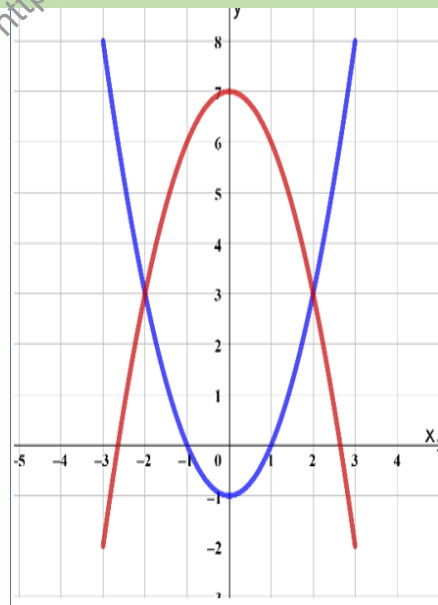
$y = e^{-2} = 0.1 \} y = (2)^2 = 4$

$$A = \int_1^4 (x^2 - e^{-x}) dx$$

$$= \left[ \frac{x^3}{3} - \frac{e^{-x}}{-1} \right]_1^4 = \checkmark$$



$$y = x^2 - 1, y = 7 - x^2, \dots \leq x \leq \dots$$



$$y = x^3, y = 3x + 2, \dots -1 \leq x \leq 2$$

$$x^3 = 3x + 2$$

$$x = 2$$

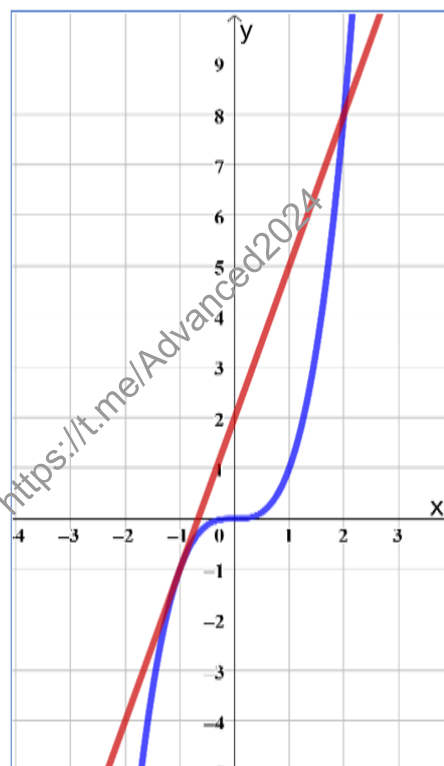
$$x^3 - 3x - 2 = 0$$

$$x = -1$$

$$A = \int_{-1}^2 (3x + 2) - (x^3) dx$$

$$= \left[ \frac{3x^2}{2} + 2x - \frac{x^4}{4} \right]_{-1}^2$$

$$= \left( \frac{3(2)^2}{2} + 2(2) - \frac{(2)^4}{4} \right) - \left( \frac{3(-1)^2}{2} + 2(-1) - \frac{(-1)^4}{4} \right)$$



$$y = \sqrt{x}, y = x^2, \dots 0 \leq x \leq 1$$

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x^4 - x = 0$$

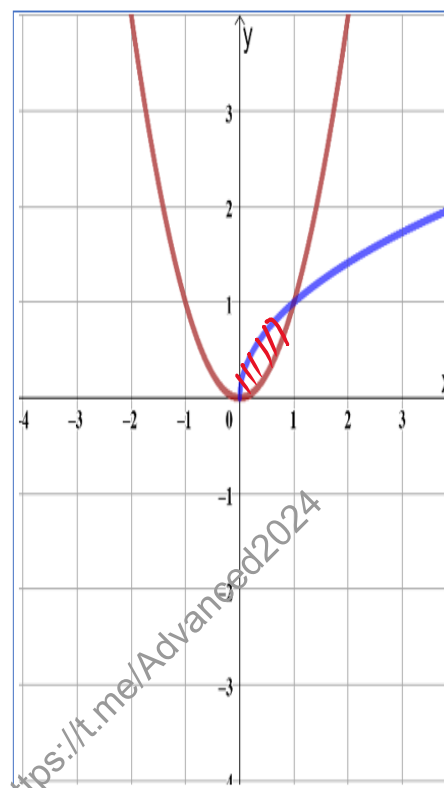
$$x(x^3 - 1) = 0$$

$$\boxed{x=0} \left\{ \begin{array}{l} x^3 - 1 = 0 \\ x^3 = 1 \\ \boxed{x=1} \end{array} \right.$$

$$A = \int_0^1 (\sqrt{x} - x^2) dx$$

$$= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3} \right]_0^1$$

$$=$$



The area between the curves  
 $y = x^2 + 1$ ,  $y = 3 - x$  is given by?

مساحة المحصورة بين المنحنيين  
 $y = x^2 + 1$ ,  $y = 3 - x$  تعطى بالتكامل؟

$$x^2 + 1 = 3 - x$$

$$x^2 + 1 - 3 + x = 0$$

$$A = \int_{-2}^1 (-x^2 + x - 2) dx$$

$$x^2 + x - 2 = 0$$

$$x = -2, x = 1$$

$$A = \int_{-2}^1 (3 - x) - (x^2 + 1) dx$$

$$B. A = \int_{-2}^1 (-x^2 - x + 2) dx$$

$$\int_{-2}^1 (-x^2 - x + 2) dx$$

$$C. A = \int_{-2}^1 (x^2 + x - 3) dx$$

$$D. A = \int_{-2}^1 (x^2 + x + 2) dx$$

Find the area of the region bounded among  $y = e^x$ ,  $y = 4e^{-x}$ ,  $x = 0$

$$A. \int_0^{e^2} (4e^{-x} - e^x) dx$$

$$B. \int_0^{e^2} (e^x - 4e^{-x}) dx$$

$$C. \int_0^{\ln 2} (4e^{-x} + e^x) dx$$

$$D. \int_0^{\ln 2} (4e^{-x} - e^x) dx$$

بالتكامل  $y = e^x$ ,  $y = 4e^{-x}$ ,  $x = 0$  تعطى المساحة بين المنحنيات

$$e^x = 4e^{-x}$$

$$e^x = \frac{4}{e^x}$$

$$(e^x)^2 = 4$$

$$e^x = 2, e^x = -2 \times$$

$$x = \ln 2$$

$$x = 0$$

$$\int_0^{\ln 2} (4e^{-x} - e^x) dx$$

$$y = 4xe^{-x^2}, y = |x|$$

$$4xe^{-x^2} = x$$

$$4xe^{-x^2} - x = 0$$

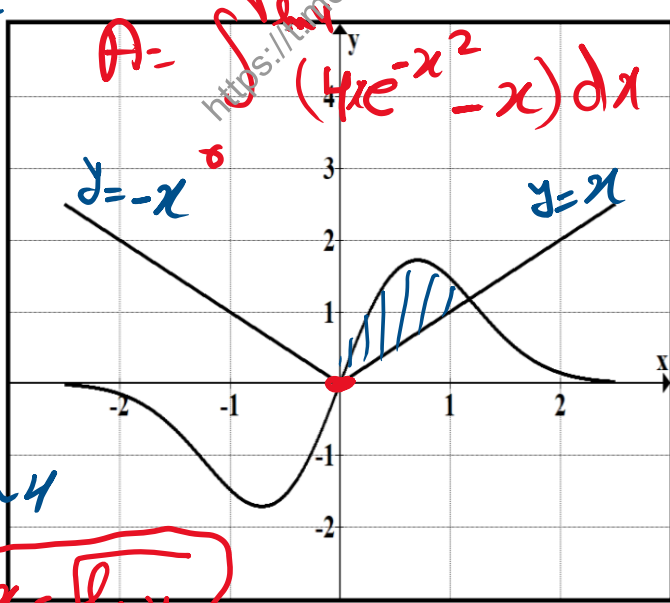
$$x(4e^{-x^2} - 1) = 0$$

$$x = 0 \quad \left\{ \begin{array}{l} 4e^{-x^2} - 1 = 0 \\ 4e^{-x^2} = 1 \\ e^{-x^2} = \frac{1}{4} \end{array} \right.$$

$$-x^2 = \ln \frac{1}{4} = \ln 1 - \ln 4$$

$$-x^2 = -\ln 4$$

$$x^2 = \ln 4 \rightarrow x = \sqrt{\ln 4}$$

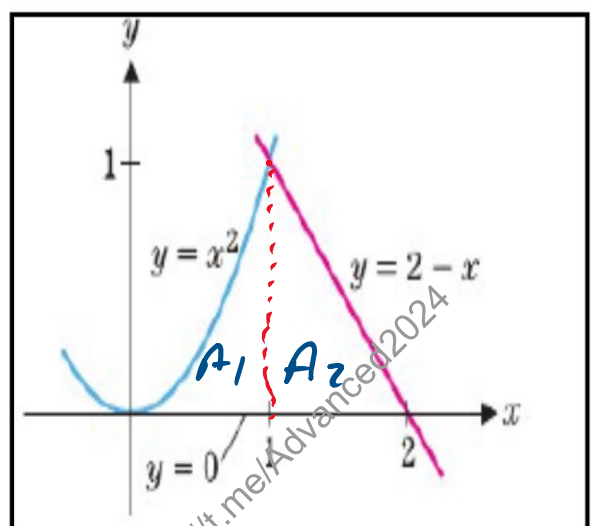


Find the area between the curves  $y = x^2$ ,  $y = 2 - x$  and  $y = 0$

$$A_1 = \int_0^1 x^2 dx, A_2 = \int_1^2 (2 - x) dx$$

$$A = \int_0^1 x^2 dx + \int_1^2 (2 - x) dx$$

$$= \left[ \frac{x^3}{3} \right]_0^1 + \left[ 2x - \frac{x^2}{2} \right]_1^2$$



Find the area of the shaded region:

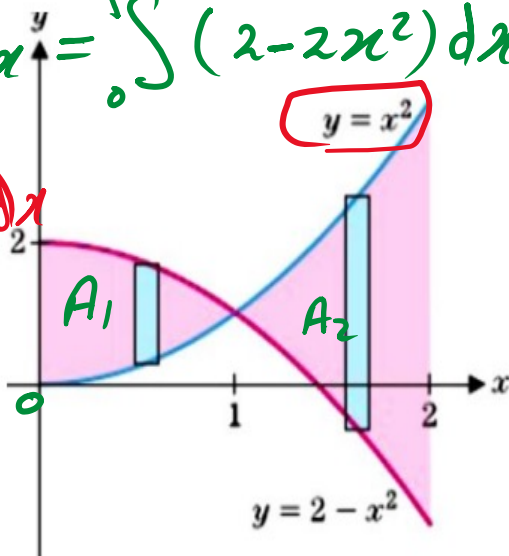
$$A_1 = \int_0^1 (2 - x^2) - x^2 dx = \int_0^1 (2 - 2x^2) dx$$

$$A_2 = \int_1^2 (x^2 - (2 - x^2)) dx$$

$$= \int_1^2 (2x^2 - 2) dx$$

C.  $\int_0^1 (2 - 2x^2) dx + \int_1^2 (2x^2 - 2) dx$

D.  $\int_0^1 (2x^2 - 2) dx + \int_1^2 (2 - 2x^2) dx$



The area of the region bounded between the curves  $y = \cos x$ ,  $y = \sin x$  على  $[0, \pi]$

A.  $\int_0^{\pi} (\cos x - \sin x) dx$

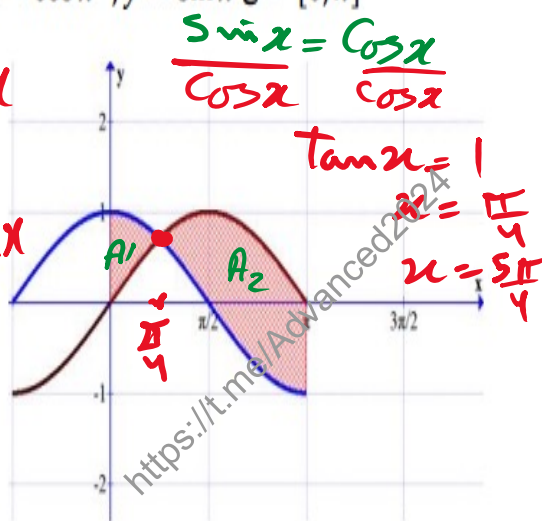
B.  $\int_0^{\pi} (\sin x - \cos x) dx$

C.  $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$

D.  $\int_0^{\pi/4} (\sin x - \cos x) dx + \int_{\pi/4}^{\pi} (\cos x - \sin x) dx$

$$A_1 = \int_0^{\pi/4} (\cos x - \sin x) dx$$

$$A_2 = \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$$



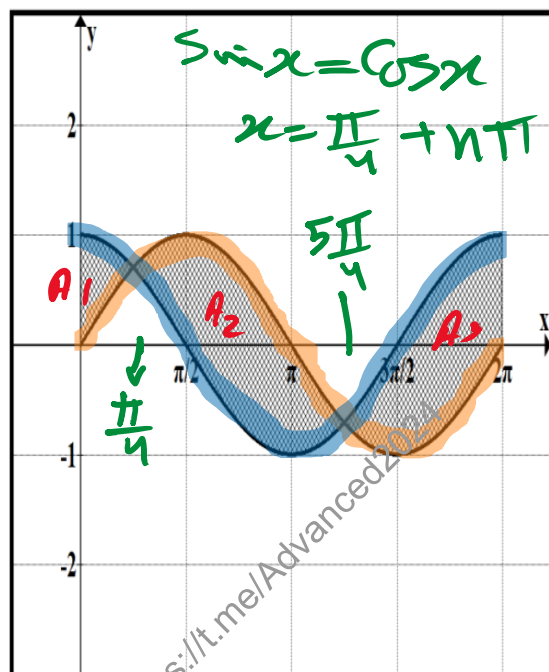
**Ex4:** Find the area between the curves  $y = \sin x$  and  $y = \cos x$  on the interval  $[0, 2\pi]$  (hint: graph the curves or solve the equations together to find the intersection points)

$$A_1 = \int_0^{\pi/4} (\cos x - \sin x) dx$$

$$A_2 = \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$$

$$A_3 = \int_{5\pi/4}^{2\pi} (\cos x - \sin x) dx$$

$$A = A_1 + A_2 + A_3$$



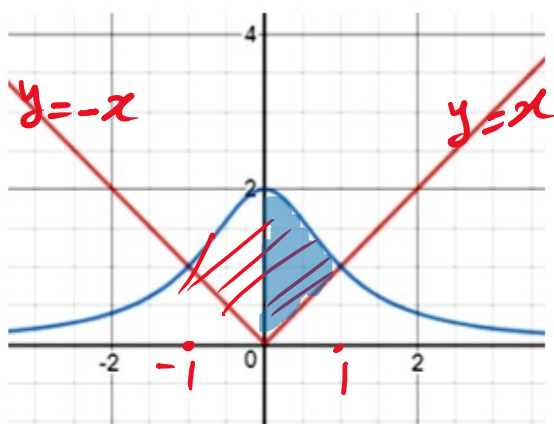
The area of the region bounded between the curves  $y = \frac{2}{1+x^2}$ ,  $y = |x|$  is given by

A.  $\int_0^2 (|x| - \frac{2}{1+x^2}) dx$

B.  $\int_0^2 (\frac{2}{1+x^2} - |x|) dx$

C.  $\int_{-1}^1 (\frac{2}{1+x^2} - |x|) dx$  ✓ (1) ✓

D.  $\int_{-1}^1 (|x| - \frac{2}{1+x^2}) dx$  (2) ✓



(3) ✓  $2 \int_0^1 (\frac{2}{1+x^2} - x) dx = 2 \int_{-1}^0 (\frac{2}{1+x^2} - (-x)) dx$

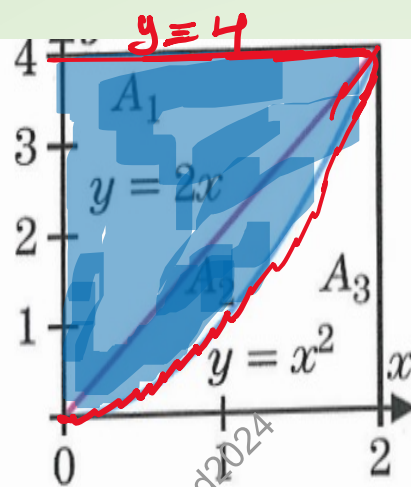
(4) ✓  $\int_{-1}^0 (\frac{2}{1+x^2} - (-x)) dx + \int_0^1 (\frac{2}{1+x^2} - x) dx$

$= 2 \tan^{-1} x + \frac{x^2}{2} \Big|_{-1}^0 + 2 \tan^{-1} x - \frac{x^2}{2} \Big|_0^1$

or  $2 [2 \tan^{-1} x + \frac{x^2}{2}]_{-1}^0$

$2 [(2 \tan^{-1}(0) + 0^2/2) - (2 \tan^{-1}(-1) + \frac{(-1)^2}{2})]$   
 $= 2 (-2(-\frac{\pi}{4}) + \frac{1}{2}) = \boxed{\pi + 1}$

Find the area that given by  $\int_0^2 (4 - x^2) dx$  using A1, A2, A3



A

A1

(B) ✓

A1 + A2

C

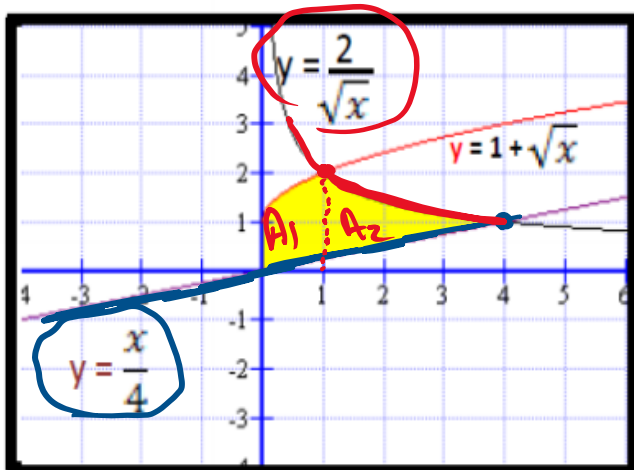
A2

D

A3

2) Find The area of the shaded region?

2) أوجد مساحة المنطقة المظللة ؟



$$A_1 = \int_0^1 ((1 + \sqrt{x}) - (\frac{x}{4})) dx$$

$$A_2 = \int_1^4 (\frac{2}{\sqrt{x}} - \frac{x}{4}) dx$$

A)  $\frac{11}{3}$  ✓✓

B)  $\frac{3}{11}$

C)  $\frac{17}{8}$

D)  $\frac{139}{24}$

Ministry question

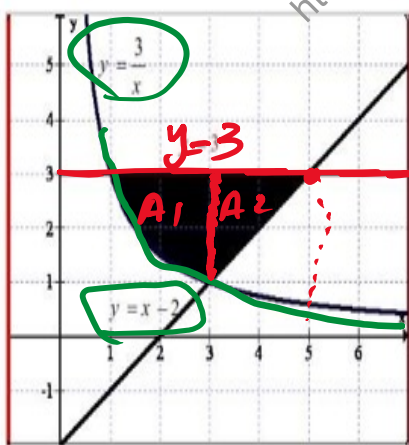
Find the area in the first quadrant and bounded by the curve

$f(x) = \frac{3}{x}$  and the lines  $y = x - 2$ ,  $y = 3$

$$A_1 = \int_1^3 (3 - \frac{3}{x}) dx$$

$$A_2 = \int_3^5 (3 - (x - 2)) dx$$

$$A = A_1 + A_2$$



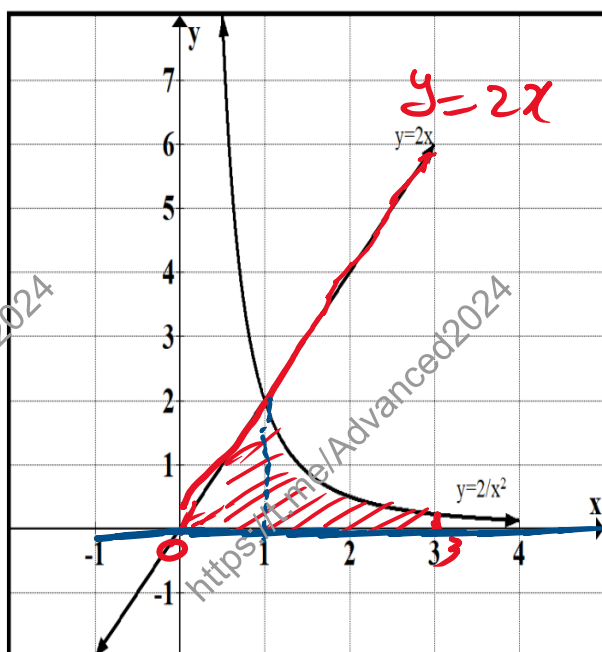
**Ex10:** Find the area bounded by the curves  $f(x) = \frac{2}{x^2}$ ,  $f(x) = 2x$  and the x-axis in the interval  $[0, 3]$

$$A_1 = \int_0^1 (2x) dx = 2 \frac{x^2}{2} \Big|_0^1 = 1$$

$$A_2 = \int_1^3 (\frac{2}{x^2}) dx$$

$$\int_1^3 (2x^{-2}) dx = 2 \frac{x^{-1}}{-1} \Big|_1^3 = -2 \left( \frac{1}{3} - 1 \right) = \frac{8}{3}$$

$$A = A_1 + A_2$$



أوجد مساحة المنطقة المحصورة بين المنحنيين  $y = x$  ،  $y = x^2$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x = 0$$

$$x = 1$$

a)  $\frac{1}{6}$  ✓✓

b)  $\frac{1}{4}$

c)  $\frac{8}{3}$

d)  $\frac{2}{3}$

$$A = \int_0^1 (x - x^2) dx$$

$$= \frac{1}{6}$$

أوجد مساحة المنطقة المحصورة بين المنحنيين  $g(x) = x - 1$  ،  $f(x) = 4 + 5x - x^2$

$g(0) = -1$  ،  $f(0) = 4$

$$4 + 5x - x^2 = x - 1$$

$$-x^2 + 5x + 4 - x + 1 = 0$$

$$-x^2 + 4x + 5 = 0$$

$$x = 5 , x = -1$$

a)  $\frac{80}{3}$

b)  $\frac{200}{3}$

c) 60

d) 36

$$A = \int_{-1}^5 ((4 + 5x - x^2) - (x - 1)) dx$$

أوجد مساحة المنطقة المحصورة بين المنحنيين  $y = \sqrt{x}$  ،  $y = x$

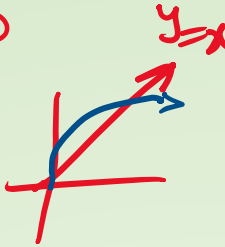
$$x = \sqrt{x}$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x = 0$$

$$x = 1$$



a)  $\frac{1}{6}$

b)  $\frac{7}{6}$

c)  $\frac{2}{5}$

d)  $\frac{2}{3}$

$$A = \int_0^1 (\sqrt{x} - x) dx$$

أوجد مساحة المنطقة المحدودة بالمنحنيات  $y = 0$  و  $y = 2 - x$  و  $y = x^2$

$$A = \int_0^1 x^2 dx + \int_1^2 (2 - x) dx$$

a)  $\frac{2}{3}$

b)  $\frac{7}{6}$

c)  $\frac{5}{6}$

d)  $\frac{11}{6}$

