



Al Maarifah 2 Girls School
Fuelling Minds with Knowledge

2024 - 2023

Notebook - Math Reveal

TERM 3

Students Name :.....

Grade:

**Grade 9 Advanced
Student Edition**



Done by :T. Amal Alzeyoudi

MODULE 12

LESSON 12.1

Lesson 12-1

Today's Goals

- Write and analyze conjectures by using inductive reasoning.
- Disprove conjectures by using counterexamples.

Conjectures and Counterexamples

Check

Write a conjecture that describes the pattern in the sequence. Then use your conjecture to find the next term in the sequence.

$\frac{1}{2}, 1, 2, 4, \dots$

The next number in the sequence is ____?____ the preceding number.

The next number in the sequence is ____?____.

Check

Make a conjecture about the sum of two odd numbers.

The sum of two odd numbers is always a(n) ____?____ number.

Check

Make a conjecture about the relationships between AD and AB , if C is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{AC}$.

AD is ____?____ of AB .

.....

.....

.....

MODULE 12

LESSON 12.1

Check

Find a counterexample to show that each conjecture is false.

- a. If n is a real number, then $\frac{1}{n} < n$. Select all that apply.
- A. $n = -3$ B. $n = \frac{1}{4}$ C. $n = 1$ D. $n = 5$ E. $n = 100$
- b. If a line intersects a segment at its midpoint, then the line is perpendicular to the segment. Draw a diagram to represent the counterexample.

.....

.....

3. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

4. $6, \frac{11}{2}, 5, \frac{9}{2}$

.....

.....

.....

Practice

Example 1

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next term in the sequence.

1. 4, 8, 12, 16, 20

2. 2, 22, 222, 2222

.....

.....

.....

MODULE 12

LESSON 12.1

5. Arrival times: 3:00 P.M., 12:30 P.M., 10:00 A.M., . . .

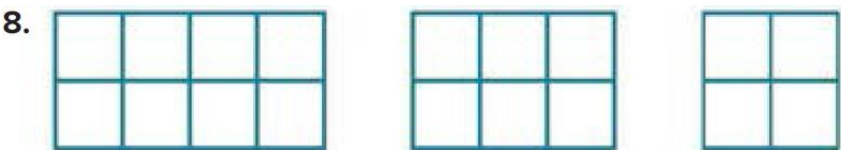
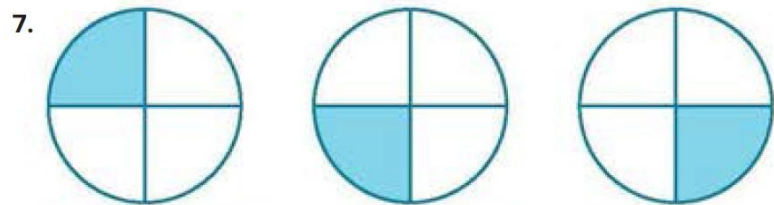
.....

.....

6. Percent humidity: 100%, 93%, 86%, . . .

.....

.....



Examples 2 and 3

Make a conjecture about each value or geometric relationship.

9. the product of two odd numbers The product of two odd numbers is odd.

10. the product of two and a number, plus one The result is odd.

11. the relationship between a and c if $ab = bc$, $b \neq 0$ The result is $a = c$.

MODULE 12

LESSON 12.1

- 12. the relationship between a and b if $ab = 1$ They are reciprocals.
- 13. the relationship between two intersecting lines that form four congruent angles
The lines are perpendicular.
- 14. the relationship between the angles of a triangle with all sides congruent
All the angles are congruent.
- 15. the relationship between NP and PQ if point P is midpoint of NQ P .
- 16. the relationship between the volume of a prism and a pyramid with the same base and equal heights The volume of the prism is three times the volume of the pyramid.

.....

.....

.....

.....

.....

MODULE 12

LESSON 12.2

Lesson 12-2

Statements, Conditionals, and Biconditionals

TRUTH TABLE for Conjunction ,Disjunction and Negation

Conjunction:		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation:	
p	$\sim p$
T	F
F	T

Example 1 Truth Values of Conjunctions

Use the statements to write a compound statement for each conjunction. Then find the truth values. Explain your reasoning.

p : The figure is a trapezoid.

q : The figure has four congruent sides.

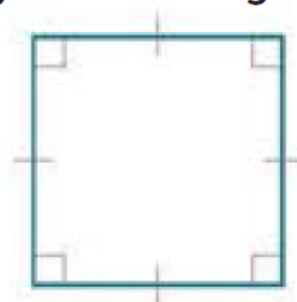
r : The figure has four right angles.

a. p and r

p and r : The figure is a trapezoid, and the figure has four right angles. Although r is true, p is false. So, p and r is false.

b. $\sim p \wedge q$

$\sim p \wedge q$: The figure is not a trapezoid, and the figure has four congruent sides. Both $\sim p$ and q are true, so $\sim p \wedge q$ is true.



MODULE12

LESSON 12.2

Check

Identify the hypothesis and conclusion of each conditional statement.

- a. If the forecast is rain, then I will take an umbrella.

Hypothesis: ? _____

Conclusion: ? _____

- b. A number is divisible by 10 if its last digit is a 0.

Hypothesis: ? _____

Conclusion: ? _____

Check

Identify the hypothesis and conclusion of the conditional statement ***A polygon with two sets of parallel sides is a parallelogram.*** Then write the statement in if-then form.

Hypothesis: ? _____

Conclusion: ? _____

? _____

Biconditional Statement

Words

A **biconditional statement** is the conjunction of a conditional and its converse.

Symbols

$(p \rightarrow q) \wedge (q \rightarrow p) \rightarrow (p \leftrightarrow q)$, read *p if and only if q*

So, the biconditional statement for the example above is *Aarón will be accepted into culinary school if and only if he earns a 3.5 GPA or higher this semester.*

Practice

Examples 1 and 2

Use the statements to write a compound statement for each conjunction or disjunction. Then find the truth values. Explain your reasoning.

$p: -3 - 2 = -5$

q : Vertical angles are congruent.

$r: 2 + 8 > 10$

1. p and q

2. $p \wedge r$

3. $q \vee \sim r$

4. $r \vee q$

5. $\sim p \wedge \sim q$

6. $\sim r \vee \sim p$

MODULE 12

LESSON 12.6

Lesson 12-6

Proving Angle Relationships

Learn Angle Addition

A protractor is used to measure angles. The Protractor Postulate illustrates the relationship between angle measures and real numbers. You will use these theorems and postulates to find angle measures.

Postulate 12.10: Protractor Postulate

The measure of any angle has a measure that is between 0 and 180.

Postulate 12.11: Angle Addition Postulate

D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.

Theorem 12.3: Supplement Theorem

If two angles form a linear pair, then they are supplementary angles.

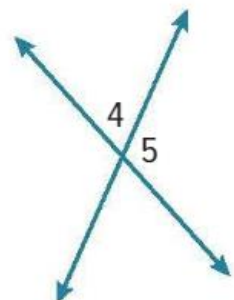
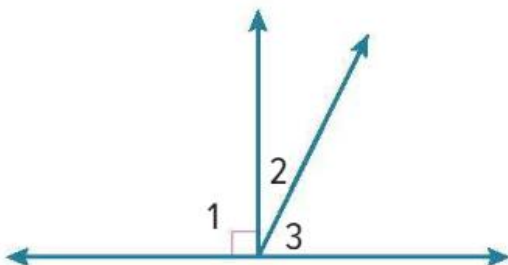
Theorem 12.4: Complement Theorem

If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Check

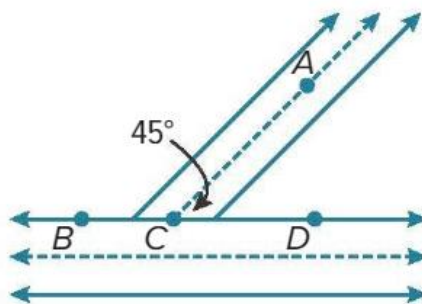
What is $m\angle 3$ if $m\angle 2 = 26^\circ$
_____?

What is $m\angle 4$ if $m\angle 5 = (2x^\circ)$ and
 $m\angle 4 = (x + 9)^\circ$ _____?



LESSON 12.6

CITY PLANNING A city planner is designing an entrance ramp for a freeway. In the diagram, $m\angle ACD = 45^\circ$. What is $m\angle BCA$? Copy and complete the calculations and justify each step.



$$m\angle BCA + m\angle ACD = \underline{\hspace{2cm}}?$$

$$m\angle BCA + \underline{\quad ? \quad} = \underline{\quad ? \quad}$$

$$m\angle BCA = \underline{\hspace{1cm}}?$$

The properties of algebra that apply to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

Reflexive Property of Congruence

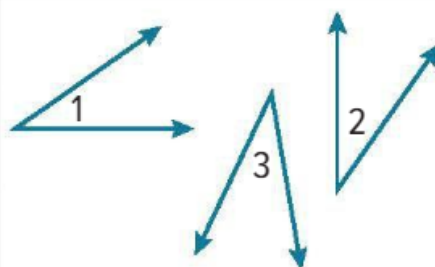
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.



Given: $\angle J \cong \angle K$

Prove: $\angle K \cong \angle J$

Paragraph Proof:

We are given that $\angle J \cong \angle K$. By the definition of congruent angles, $m\angle J = m\angle K$. Using the Symmetric Property of Equality, $m\angle K = m\angle J$. Thus, $\angle K \cong \angle J$ by the definition of congruent angles.

MODULE 12

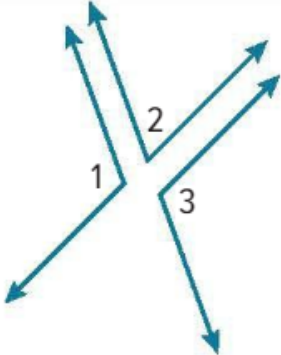
LESSON 12.6

Theorems

Theorem 12.6: Congruent Supplements Theorem

Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation $\angle$ s suppl. to same $\angle$ or $\cong \angle$ s are $\cong$.

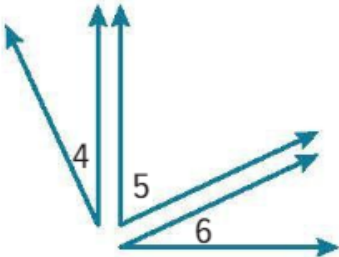


If $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 2 + m\angle 3 = 180^\circ$, then $\angle 1 \cong \angle 3$.

Theorem 12.7: Congruent Complements Theorem

Angles complementary to the same angle or to congruent angles are congruent.

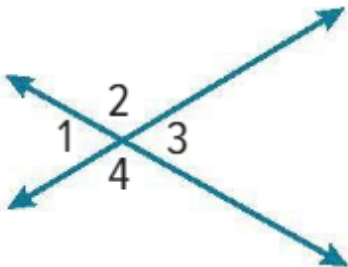
Abbreviation $\angle$ s compl. to same $\angle$ or $\cong \angle$ s are $\cong$.



If $m\angle 4 + m\angle 5 = 90^\circ$ and $m\angle 5 + m\angle 6 = 90^\circ$, then $\angle 4 \cong \angle 6$.

Theorem 12.8: Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.



$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

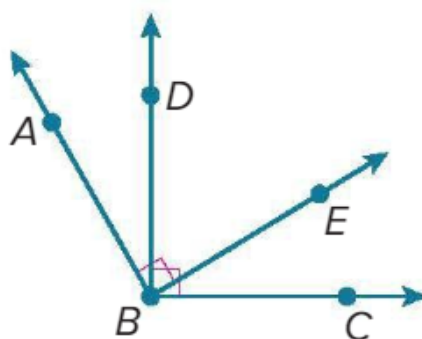
Example 3 Congruent Supplements and Complements

In the figure, $\angle ABE$ and $\angle DBC$ are right angles.

Select from the reasons provided to complete the proof.

Given: $\angle ABE$ and $\angle DBC$ are right angles.

Prove: $\angle ABD \cong \angle EBC$



Proof:

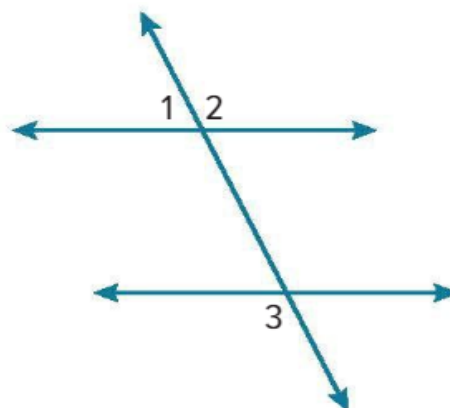
Statements	Reasons
1. $\angle ABE$ and $\angle DBC$ are right angles.	1. Given
2. $m\angle ABE = 90^\circ$; $m\angle DBC = 90^\circ$	2. Definition of right angle
3. $\angle ABD$ and $\angle DBE$ are complementary; $\angle DBE$ and $\angle EBC$ are complementary.	3. Complement Theorem
4. $\angle ABD \cong \angle EBC$	4. Congruent Complements Theorem

Check

Copy and complete the proof.

Given: $\angle 1$ and $\angle 3$ are supplementary.

Prove: $\angle 2 \cong \angle 3$



Proof:

It is given that $\angle 1$ and $\angle 3$ are _____. By the definition of linear pair, $\angle 1$ and $\angle 2$ are a linear pair. So, by the _____, $\angle 1$ and $\angle 2$ are supplementary. Thus, $\angle 2 \cong \angle 3$ by the _____.

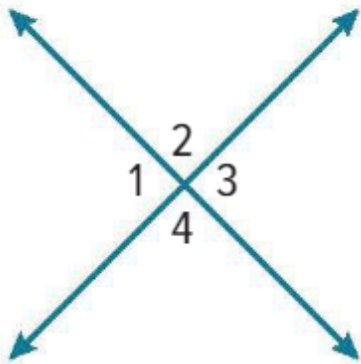
Example 4 Vertical Angles

Complete the proof.

Choose from the statements and reasons provided.

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1 \cong \angle 3$; $\angle 2 \cong \angle 4$	2. Vertical Angles Theorem
3. $\angle 3 \cong \angle 1$	3. Symmetric Property of Congruence
4. $\angle 3 \cong \angle 2$	4. Transitive Property of Congruence
5. $\angle 3 \cong \angle 4$	5. Transitive Property of Congruence

Check

Copy and complete the proof. Choose from the reasons provided.

Given: $\angle 4 \cong \angle 7$

Prove: $\angle 5 \cong \angle 6$



Proof:

Statements	Reasons
1. $\angle 4 \cong \angle 7$	1. Given
2. $\angle 5 \cong \angle 4$ and $\angle 7 \cong \angle 6$	2. _____ ?
3. $\angle 5 \cong \angle 7$	3. _____ ?
4. $\angle 5 \cong \angle 6$	4. _____ ?

Learn Right Angle Theorems

You can prove the following theorems about right angles using what you already know about angle measures.

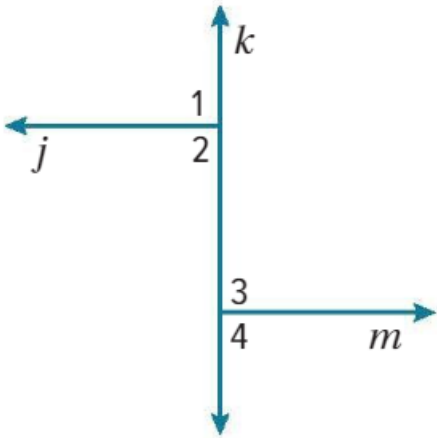
Theorem 12.9	Perpendicular lines intersect to form four right angles.
Theorem 12.10	All right angles are congruent.
Theorem 12.11	Perpendicular lines form congruent adjacent angles.
Theorem 12.12	If two angles are congruent and supplementary, then each angle is a right angle.
Theorem 12.13	If two congruent angles form a linear pair, then they are right angles.

Check

Copy and complete the proof. Choose from the reasons provided.

Given: $\angle 1 \cong \angle 4$
Lines j and k are perpendicular.

Prove: $\angle 2 \cong \angle 4$



Proof:

Statements	Reasons
1. Lines j and k are perpendicular.	1. Given
2. $\angle 2 \cong \angle 1$	2. _____ ?
3. $\angle 1 \cong \angle 4$	3. Given
4. $\angle 2 \cong \angle 4$	4. Transitive Property of Congruence

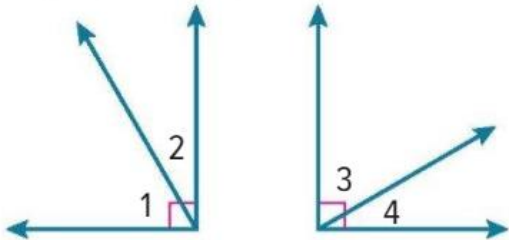
MODULE 12

LESSON 12.6

Example 3

PROOF Write a two-column proof.

6. Given: $\angle 2 \cong \angle 4$
Prove: $\angle 1 \cong \angle 3$



.....

.....

.....

.....

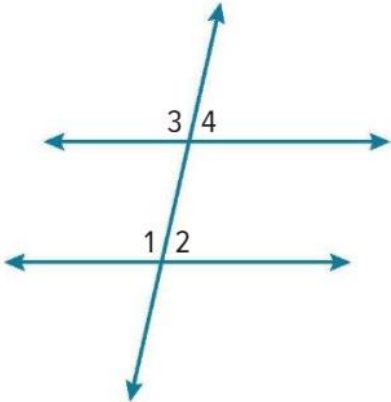
.....

.....

.....

.....

7. Given: $\angle 1 \cong \angle 3$
Prove: $\angle 2 \cong \angle 4$



.....

.....

.....

.....

.....

.....

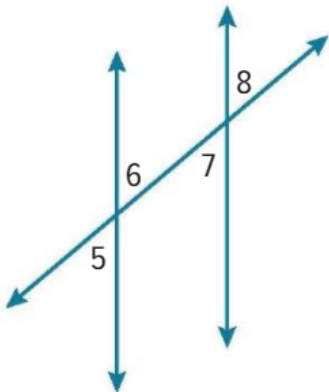
.....

.....

Example 4

PROOF Write a two-column proof.

8. Given: $\angle 5 \cong \angle 7$
Prove: $\angle 5 \cong \angle 8$



.....

.....

.....

.....

.....

.....

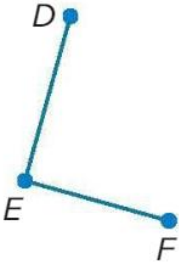
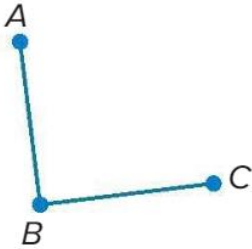
.....

.....

Example 5

PROOF Write a two-column proof.

9. Given: $m\angle ABC = m\angle DEF$
 $\angle ABC$ and $\angle DEF$ are supplementary.
Prove: $\angle ABC$ and $\angle DEF$ are right angles.



.....

.....

.....

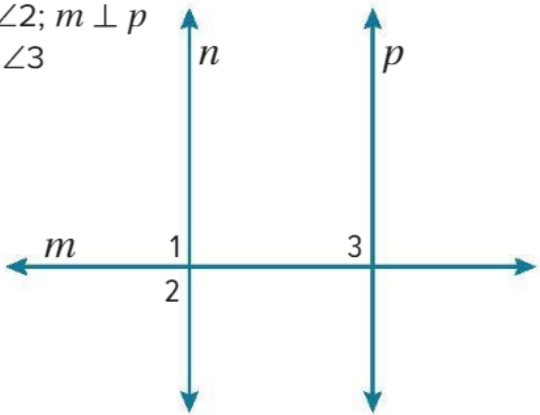
.....

.....

.....

.....

10. Given: $\angle 1 \cong \angle 2$; $m \perp p$
Prove: $\angle 2 \cong \angle 3$



.....

.....

.....

.....

.....

.....

Parallel Lines and Transversals

Learn Parallel Lines and Transversals

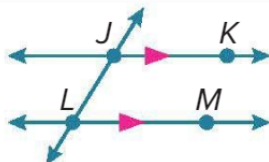
If two lines do not intersect, then they are either parallel or skew.

Parallel and Skew

Parallel Lines

Parallel lines are coplanar lines that do not intersect.

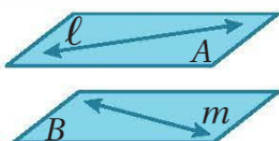
Example $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$



Skew Lines

Skew lines are lines that do not intersect and are not coplanar.

Example Lines ℓ and m are skew.



Parallel Planes

Parallel planes are planes that do not intersect.

Example Planes A and B are parallel.



If segments or rays are contained within lines that are parallel or skew, then the segments or rays are parallel or skew.

Example 1 Identify Parallel and Skew Relationships

Identify each of the following using the cube shown. Assume lines and planes that appear to be parallel or perpendicular are parallel or perpendicular, respectively.

a. all lines skew to $\overleftrightarrow{BC}$

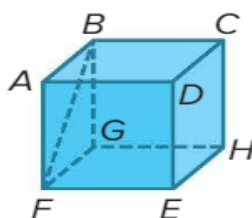
$\overleftrightarrow{AF}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{FG}$, and $\overleftrightarrow{HE}$

b. all lines parallel to $\overleftrightarrow{EH}$

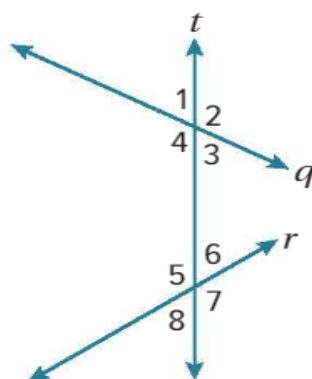
$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{FG}$

c. all planes parallel to plane DCH

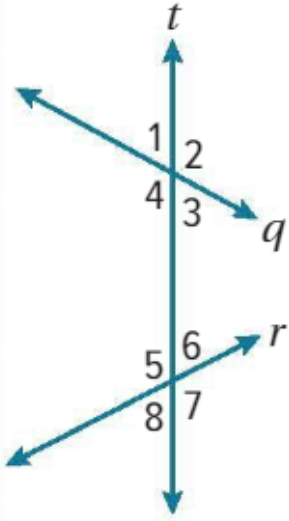
Plane ABG is the only plane parallel to plane DCH .



A line that intersects two or more lines in a plane at different points is called a **transversal**. In the diagram, line t is a transversal of lines q and r . Notice that line t forms a total of eight angles with lines q and r . These angles and specific pairings of these angles are given special names.



Transversal Angle Pair Relationships

Four interior angles lie in the region between lines q and r .	$\angle 3, \angle 4, \angle 5, \angle 6$	
Four exterior angles lie in the two regions that are not between lines q and r .	$\angle 1, \angle 2, \angle 7, \angle 8$	
Consecutive interior angles are interior angles that lie on the same side of transversal t .	$\angle 4$ and $\angle 5, \angle 3$ and $\angle 6$	
Alternate interior angles are nonadjacent interior angles that lie on opposite sides of transversal t .	$\angle 3$ and $\angle 5, \angle 4$ and $\angle 6$	
Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of transversal t .	$\angle 1$ and $\angle 7, \angle 2$ and $\angle 8$	
Corresponding angles lie on the same side of transversal t and on the same side of lines q and r .	$\angle 1$ and $\angle 5, \angle 2$ and $\angle 6, \angle 3$ and $\angle 7, \angle 4$ and $\angle 8$	

Theorem 12.15: Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 2 \cong \angle 6, \angle 3 \cong \angle 7$
---	--

Theorem 12.16: Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$\angle 1$ and $\angle 3, \angle 6$ and $\angle 7$
---	--

Theorem 12.17: Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 5, \angle 4 \cong \angle 8$
---	--

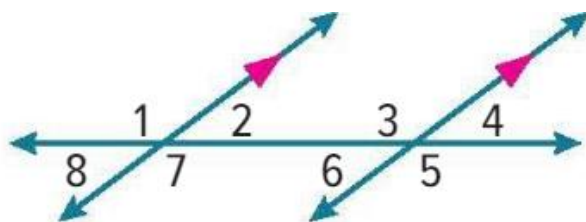
A special relationship also exists when the transversal of two parallel lines is a perpendicular line.

Theorem 12.18: Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

MODULE 12

LESSON 12.7



Theorem 12.15: Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

$$\angle 2 \cong \angle 6,$$

$$\angle 3 \cong \angle 7$$

Theorem 12.16: Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

$$\angle 1 \text{ and } \angle 3,$$

$$\angle 6 \text{ and } \angle 7$$

Theorem 12.17: Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

$$\angle 1 \cong \angle 5,$$

$$\angle 4 \cong \angle 8$$

A special relationship also exists when the transversal of two parallel lines is a perpendicular line.

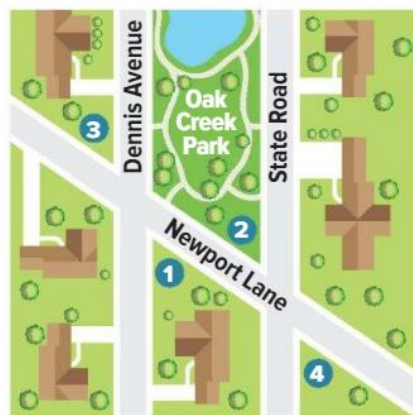
Theorem 12.18: Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

Check

COMMUNITY PLANNING Dennis Avenue and State Road are parallel streets that intersect Newport Lane along the south side of Oak Creek Park. If $m\angle 3 = 62^\circ$, find $m\angle 4$

_____?



MODULE 12

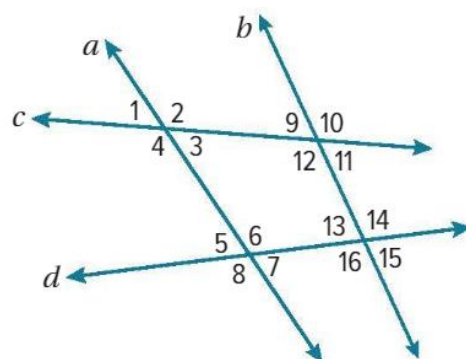
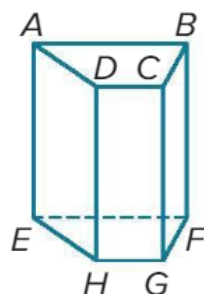
LESSON 12.7

Practice

Example 1

Identify each of the following using the figure shown. Assume lines and planes that appear to be parallel or perpendicular are parallel or perpendicular, respectively.

1. three segments parallel to $\overline{AE}$
2. a segment skew to $\overline{AB}$
3. a pair of parallel planes
4. a segment parallel to $\overline{AD}$
5. three segments parallel to $\overline{HG}$
6. five segments skew to $\overline{BC}$
7. How could you characterize the relationship between faces $ABCD$ and $DCGH$? Explain.



Examples 2 and 3

Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

8. $\angle 4$ and $\angle 5$
9. $\angle 5$ and $\angle 15$
10. $\angle 12$ and $\angle 14$
11. $\angle 7$ and $\angle 15$
12. $\angle 2$ and $\angle 12$
13. $\angle 3$ and $\angle 6$
14. $\angle 1$ and $\angle 9$
15. $\angle 3$ and $\angle 9$
16. $\angle 10$ and $\angle 16$
17. $\angle 5$ and $\angle 13$

Lesson 12-8

Slope and Equations of Lines

Postulate 12.12: Slope Criteria for Parallel and Perpendicular Lines

Slopes of Parallel Lines

Two distinct nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

Slopes of Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1 . Vertical and horizontal lines are perpendicular.

Check

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for $A(14, 13)$, $B(-11, 0)$, $C(-3, 7)$, and $D(-4, -5)$. Graph each line to verify your answer.

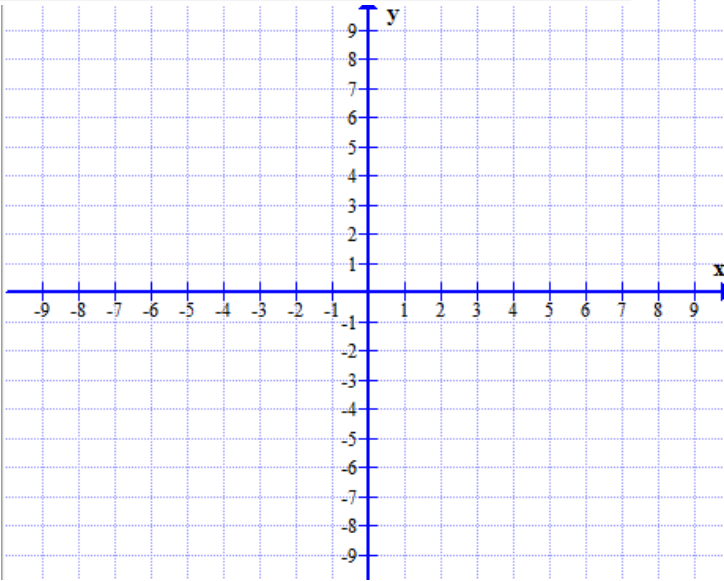
.....

.....

.....

.....

.....



Check

Determine whether the pair of lines is *parallel*, *perpendicular*, or *neither*.

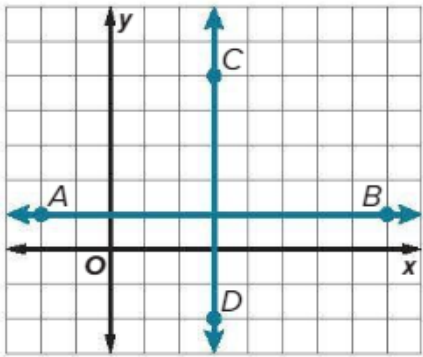
.....

.....

.....

.....

.....



Learn Equations of Lines

An equation of a nonvertical line can be written in different but equivalent forms.

Key Concept • Nonvertical Line Equations

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y -intercept.

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is any point on the line and m is the slope of the line.

$y = mx + b$ $y = 3x + 8$

slope y-intercept

point (3, 5)

$y - 5 = -2(x - 3)$

slope

The equations of horizontal and vertical lines involve only one variable.

Key Concept • Horizontal and Vertical Line Equations

The equation of a horizontal line is $y = b$, where b is the y -intercept of the line.

The equation of a vertical line is $x = a$, where a is the x -intercept of the line.

When given the equations of two lines, you can compare the equations to determine the relationship between the lines.

Check

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

- a. $y = 3x - 9$; $y = -\frac{1}{3}x + 2$

.....
- b. $y = \frac{9}{7}x - \frac{19}{7}$; $y - 1 = \frac{9}{7}(x + 3)$

.....
- c. $x = -3$; $x = 4$

.....

.....

.....

Check

Write an equation in slope-intercept form for the line parallel to

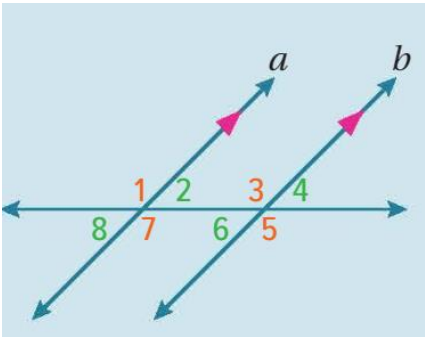
$y = \frac{1}{2}x + \frac{5}{2}$ containing $(\frac{3}{2}, 1)$.

.....

.....

.....

Proving Lines Parallel



Theorem 12.20: Alternate Exterior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the lines are parallel.	If $\angle 1 \cong \angle 5$, then $a \parallel b$.
---	---

Theorem 12.21: Consecutive Interior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.	If $m\angle 7 + m\angle 6 = 180^\circ$, then $a \parallel b$.
---	---

Theorem 12.22: Alternate Interior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.	If $\angle 7 \cong \angle 3$, then $a \parallel b$.
---	---

Theorem 12.23: Perpendicular Transversal Converse

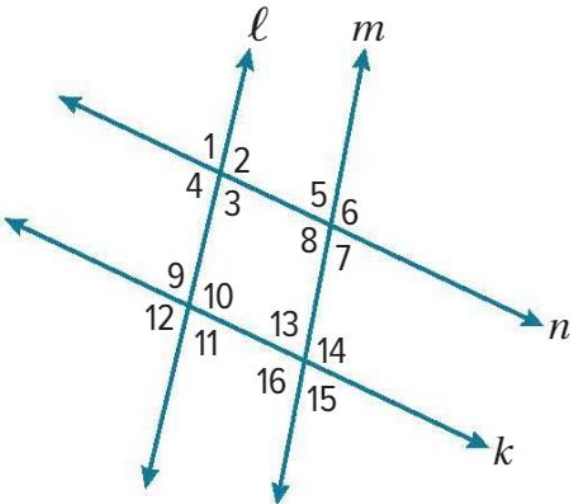
If two lines in a plane are perpendicular to the same line, then the lines are parallel.
--

MODULE 12

LESSON 12.9

Check

Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



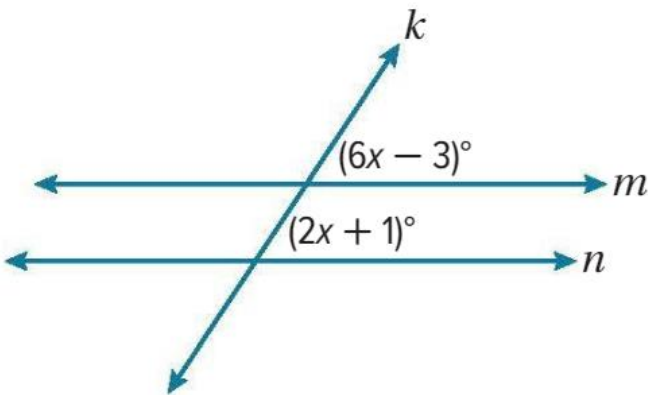
- a. $\angle 1 \cong \angle 15$
- A. $\ell \parallel m$; Alternate Exterior Angles Converse
 - B. $n \parallel k$; Alternate Exterior Angles Converse
 - C. $\ell \parallel m$; Converse of Corresponding Angles Theorem
 - D. It is not possible to determine whether the lines are parallel.

b. $m\angle 3 + m\angle 10 = 180$

- c. $\angle 3 \cong \angle 5$
- A. $\ell \parallel m$; Alternate Interior Angles Converse
 - B. $\ell \parallel m$; Consecutive Interior Angles Converse
 - C. $n \parallel k$; Alternate Interior Angles Converse
 - D. It is not possible to determine whether the lines are parallel.

Check

- a. Find the value of x so that $m \parallel n$. Identify the postulate or theorem you used.

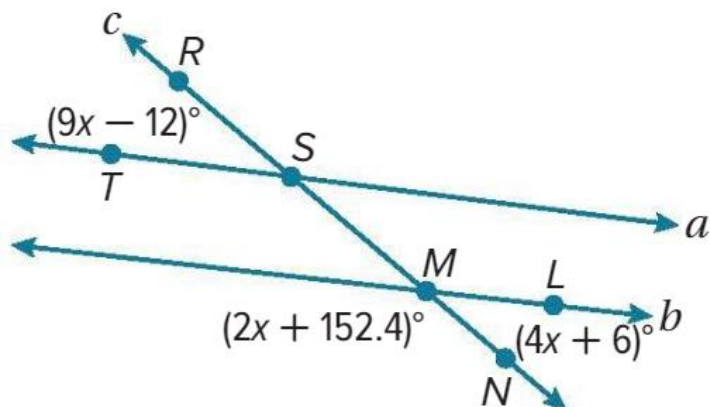


$x = \underline{\hspace{1cm}} ? \hspace{1cm}; \hspace{1cm} \underline{\hspace{1cm}} ? \hspace{1cm}$

MODULE 12

LESSON 12.9

b. Find $m\angle LMN$ so that $a \parallel b$.



A. 3.6°

B. 20.4°

C. 100°

D. 159.6°

Check

ANTENNAS Is it possible to prove that the support poles of the antenna complex are parallel? Justify your answer.



- A. No; because the consecutive interior angles are not supplementary, the support poles cannot be parallel.
- B. No; because the alternate interior angles are not supplementary, the support poles cannot be parallel.
- C. Yes; because the alternate interior angles are supplementary, the support poles are parallel.
- D. Yes; because the consecutive interior angles are congruent, the support poles are parallel.

Practice

Example 1

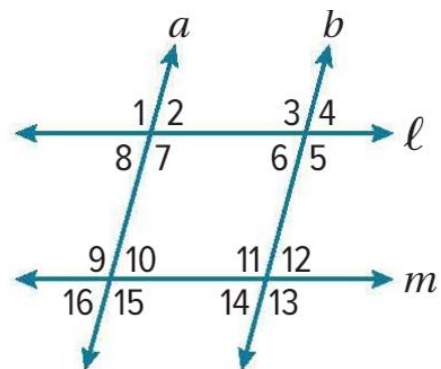
Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. $\angle 3 \cong \angle 7$

2. $\angle 9 \cong \angle 11$

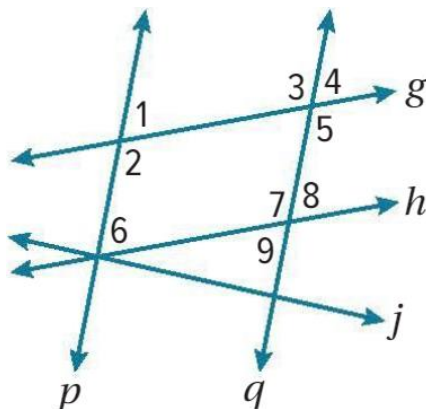
3. $\angle 2 \cong \angle 16$

4. $m\angle 5 + m\angle 12 = 180^\circ$



Given the following information, determine which lines, if any, are parallel. State the theorem that justifies your answer.

5. $\angle 1 \cong \angle 6$



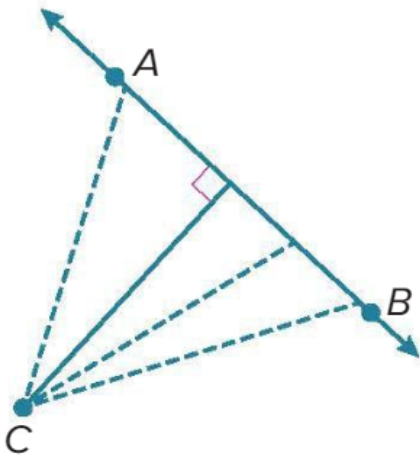
6. $m\angle 7 + m\angle 6 = 180^\circ$

Perpendiculars and Distance

Key Concept • Distance Between a Point and a Line

The distance between a line and a point not on the line is the length of the segment perpendicular to the line from the point.

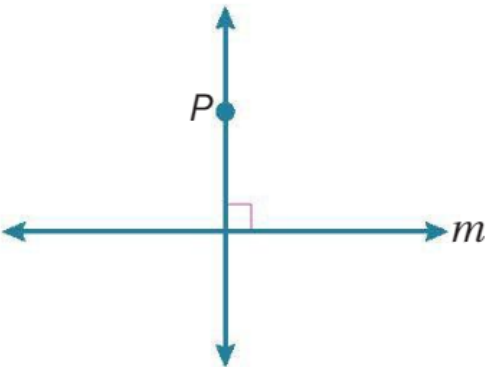
Given $\overleftrightarrow{AB}$ and point C not on the line, there are an infinite number of lines that pass through the point and intersect the line. The shortest distance between the point and the line is the length of the segment that is perpendicular to the line through the point. So, the distance between C and $\overleftrightarrow{AB}$ is CD .



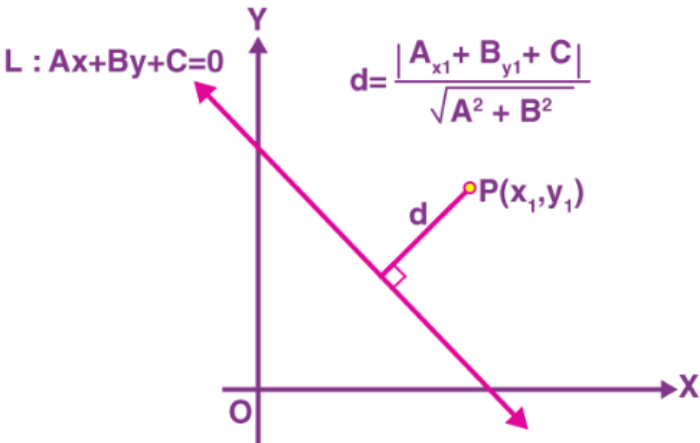
Just as there is one shortest distance from C to $\overleftrightarrow{AB}$, there is exactly one line that passes through C and is perpendicular to $\overleftrightarrow{AB}$.

Postulate 12.14: Perpendicular Postulate

If given a line and a point not on the line, then there exists exactly one line through the point that is perpendicular to the given line.



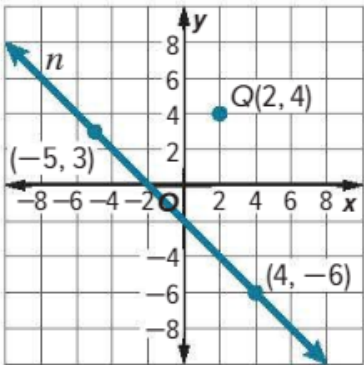
.....
.....
.....
.....
.....



Check

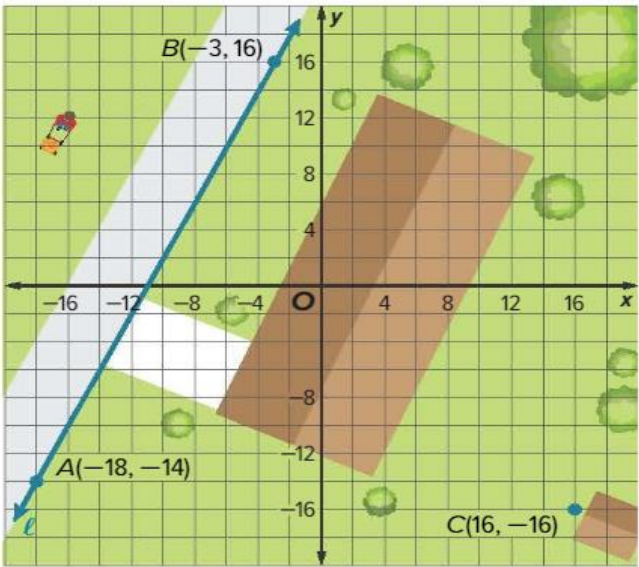
Line n contains points $(-5, 3)$ and $(4, -6)$. Find the distance between line n and point $Q(2, 4)$. Round to the nearest tenth, if necessary.

_____?



Check

ZONING Javier wants to build a shed on his property. According to zoning laws, the shed must be at least 20 feet from his property line. Javier knows that points A and B fall on his property line. If Javier plans to build the shed behind his house at point C , will he satisfy the zoning laws? If yes, how far away will the shed be from Javier's property line? Each unit on the coordinate plane represents 1 foot.



- A. no **B. yes; 31.3 ft** C. yes; 34.1 ft D. yes; 37.2 ft

Learn Distance Between Parallel Lines

By definition, parallel lines do not intersect. An alternate definition states that two lines in a plane are parallel if they are always **equidistant**. Two lines are equidistant from each other if the distance between the two lines, measured along a perpendicular line or segment to the two lines, is always the same.

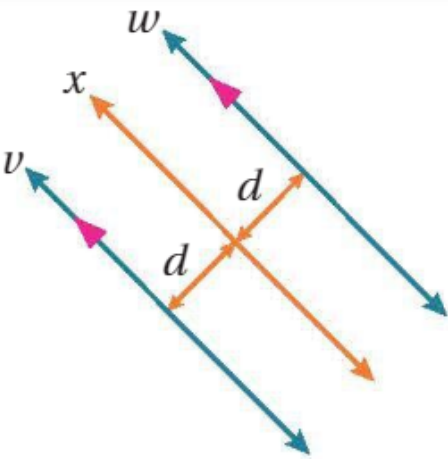
Key Concept • Distance Between Parallel Lines

The distance between two parallel lines is the perpendicular distance between one of the lines and any point on the other line.

Theorem 12.24: Two Lines Equidistant from a Third

In a plane, if two lines are each equidistant from a third line, then the two lines are parallel to each other.

Example If line w and line v are equidistant from line x , then line w and line v are parallel.



Example 3 Distance Between Parallel Lines

Find the distance between the parallel lines r and t with equations $y = -3x - 5$ and $y = -3x + 6$, respectively.

You need to solve a system of equations to find the endpoints of a segment perpendicular to lines r and t . Lines r and t have slope -3 .

.....

.....

.....

.....

.....

Check

Find the distance between parallel lines a and b with equations $x + 3y = 6$ and $x + 3y = -14$, respectively. Round to the nearest hundredth, if necessary.

_____?

.....

.....

.....

.....

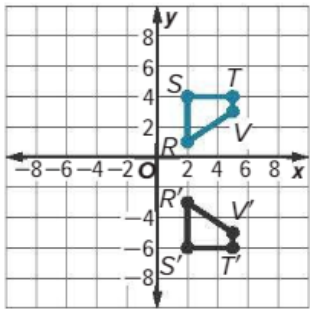
.....

Reflections

Example 1 Reflection in a Horizontal or Vertical Line

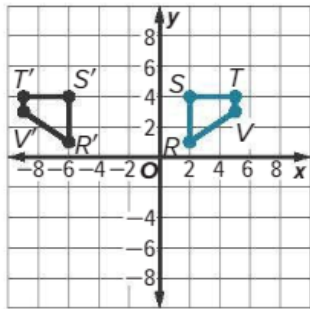
Consider quadrilateral $RSTV$ with vertices at $R(2, 1)$, $S(2, 4)$, $T(5, 4)$, and $V(5, 3)$. Graph the image of quadrilateral $RSTV$ under each reflection. Determine the coordinates of the image.

a. in the line $y = -1$



$R'(2, -3)$, $S'(2, -6)$,
 $T'(5, -6)$, $V'(5, -5)$

b. in the line $x = -2$



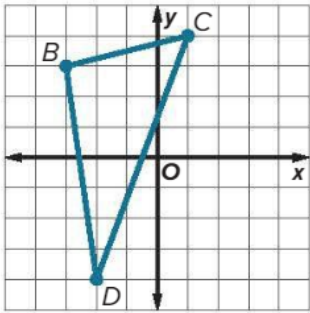
$R'(-6, 1)$, $S'(-6, 4)$,
 $T'(-9, 4)$, $V'(-9, 3)$

Check

Triangle BCD has coordinates $B(-3, 3)$, $C(1, 4)$, and $D(-2, -4)$.

Select the coordinates of the vertices of the image after a reflection in the line $x = 3$.

- A $B'(3, 3)$, $C'(-1, 4)$, $D'(2, -4)$
- B $B'(-3, -3)$, $C'(1, -4)$, $D'(-2, 4)$
- C $B'(9, 3)$, $C'(5, 4)$, $D'(8, -4)$
- D $B'(-3, -3)$, $C'(1, 2)$, $D'(-2, 10)$



.....

.....

.....

.....

.....

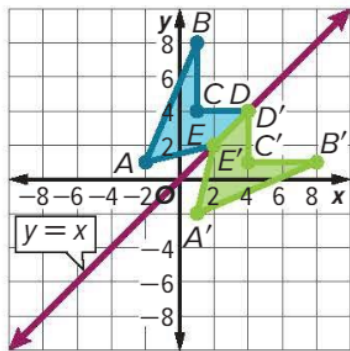
MODULE 13

LESSON 13.1

 Example 2 Reflection in the Line $y = x$

DESIGN Winona is designing a logo for her blog header. She graphs a figure on the coordinate plane and wants to reflect it in the line $y = x$ to complete the basic shape for her logo design. What are the coordinates of the vertices of the image after the reflection?

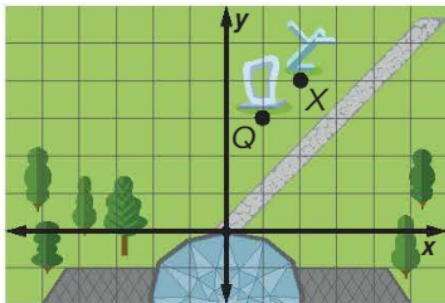
- $(x, y) \rightarrow (y, x)$
- $A(-2, 1) \rightarrow A'(-2, -2)$
- $B(1, 8) \rightarrow B'(8, 1)$
- $C(1, 4) \rightarrow C'(4, 1)$
- $D(4, 4) \rightarrow D'(4, 4)$
- $E(2, 2) \rightarrow E'(2, 2)$



Check

LANDSCAPE Tomas is designing a sculpture garden for an art museum. There is a sidewalk connecting the center of the museum entrance to the edge of the lawn. Tomas has a set of 4 sculptures in a series that he wants to be equidistant from this sidewalk. He plotted positions for Q and X on the graph. If pieces Q and R are a pair and X and Y are a pair, where will pieces R and Y be placed in the garden?

- A** $R(3, 1), Y(4, 2)$
- B** $R(3, -1), Y(4, -2)$
- C** $R(-3, 1), Y(-4, 2)$
- D** $R(-3, -1), Y(-4, -2)$



Museum Entrance

MODULE 13

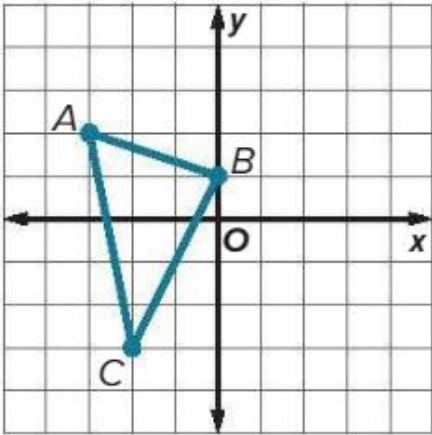
LESSON 13.1

Practice

Examples 1 and 2

Graph the image of each figure under the given reflection. Determine the coordinates of the image.

1. $\triangle ABC$ in the line $y = x$



.....

.....

.....

.....

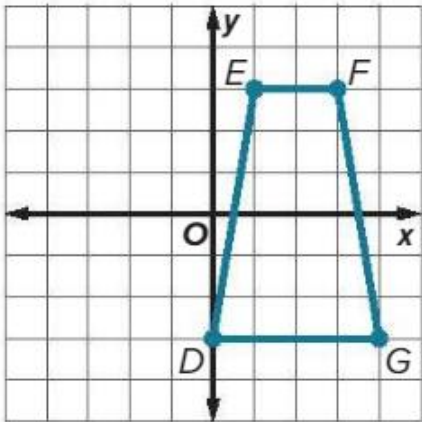
.....

.....

.....

.....

2. trapezoid $DEFG$ in the line $x = -1$



.....

.....

.....

.....

.....

.....

.....

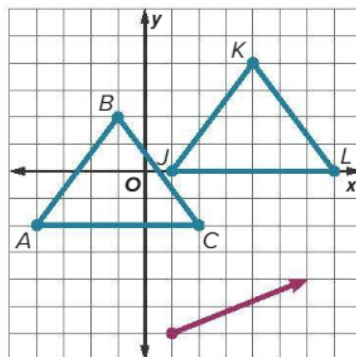
.....

Translations

Learn Translations

You've learned that a translation is a function in which all of the points of a figure move the same distance in the same direction as described by a translation vector.

When a translation has been applied to a figure:



1. The distance between each pair of corresponding vertices is the same.
2. The segments that connect each pair of corresponding vertices are parallel.

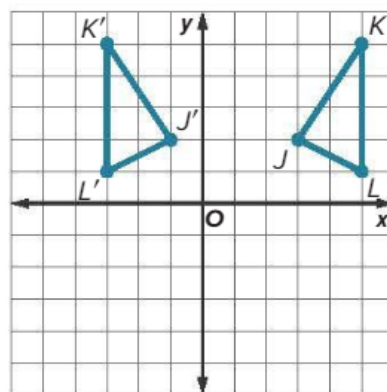
Recall that a translation vector describes the magnitude and direction of the translation. The **magnitude** of a vector is its length from the initial point to the terminal point.

Example 1 Determine a Translation Vector

Determine whether a translation maps $\triangle JKL$ onto $\triangle J'K'L'$. If so, find the translation vector. If not, explain why.

$\triangle J'K'L'$ is not a translation of $\triangle JKL$.

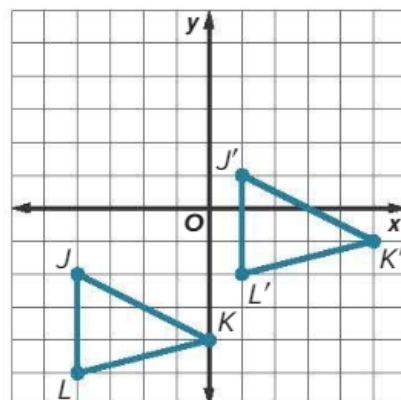
The distances between corresponding vertices are not equal.



Check

Determine whether a translation maps $\triangle JKL$ onto $\triangle J'K'L'$. If so, find the translation vector. If not, explain why.

- A** yes; $\langle 5, 3 \rangle$
B yes; $\langle 3, 5 \rangle$
C No; this is a reflection.
D No; the triangles are not congruent.

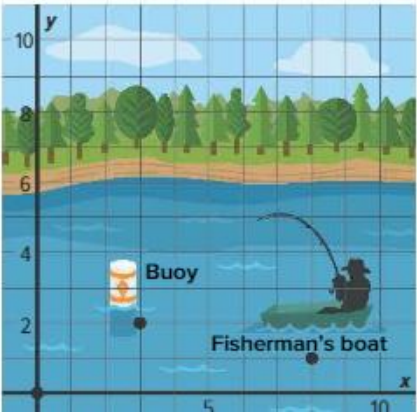


MODULE 13

LESSON 13.2

 Apply Example 2 Translations on the Coordinate Plane

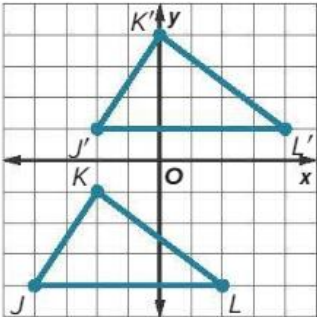
SCAVENGER HUNT Travis is on a scavenger hunt at the lake. He needs to swim to the nearest buoy, pick up a card, and then swim until he reaches a fisherman who will give him the next clue in exchange for the card. Describe the translation from the buoy to the fisherman's boat by using a translation vector.



Practice

Examples 1 and 2

- 1. Determine whether a translation maps $\triangle JKL$ onto $\triangle J'K'L'$. If so, find the translation vector. If not, explain why.



.....

.....

.....

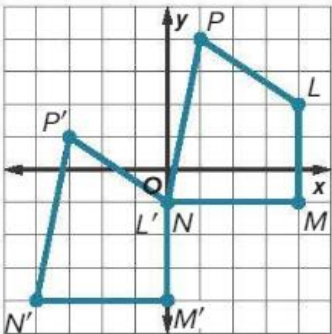
.....

.....

.....

.....

- 2. Determine whether a translation maps quadrilateral $LMNP$ onto quadrilateral $L'M'N'P'$. If so, find the translation vector. If not, explain why.



.....

.....

.....

.....

.....

.....

.....

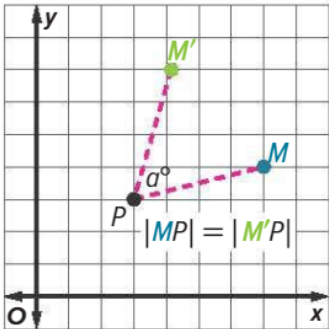
Rotations

Learn Rotations About Points that Are Not the Origin

Key Concept • Rotation

A rotation about a fixed point P through an angle of α° is a function that maps point M to point M' such that:

- point P does not move,
- $m\angle MPM'$ is α° , and
- $|MP| = |M'P|$



When a point is rotated 90° , 180° , or 270° counterclockwise about the origin, you can use the following rules to determine the coordinates of an image. A rotation of 360° will map an image onto the preimage.

Rotations on the Coordinate Plane (About the Origin)

90° Rotation $(x, y) \rightarrow (-y, x)$

180° Rotation $(x, y) \rightarrow (-x, -y)$

270° Rotation $(x, y) \rightarrow (y, -x)$

When combined with translations, these rules can also be used to rotate figures about points that are not the origin.

Example 1 Rotation About a Point That Is Not the Origin

Triangle ABC has vertices $A(-8, 5)$, $B(-6, 9)$, and $C(-3, 6)$. Graph $\triangle ABC$ and its image after a rotation of 180° about $(-5, 3)$.

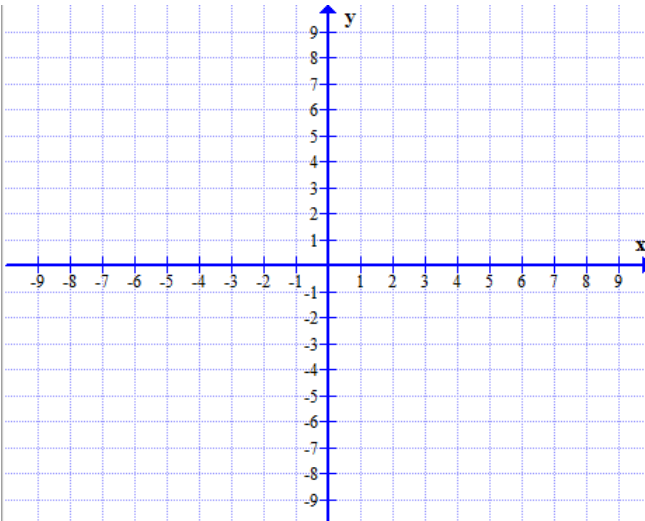
.....

.....

.....

.....

.....

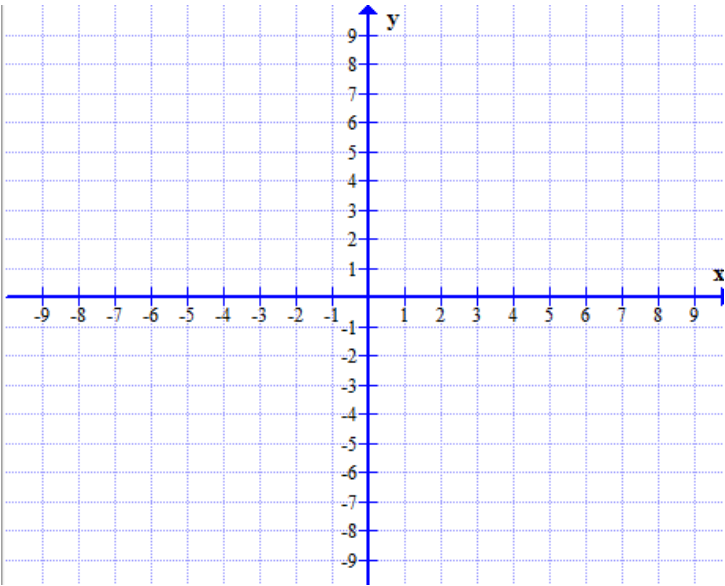


MODULE 13

LESSON 13.3

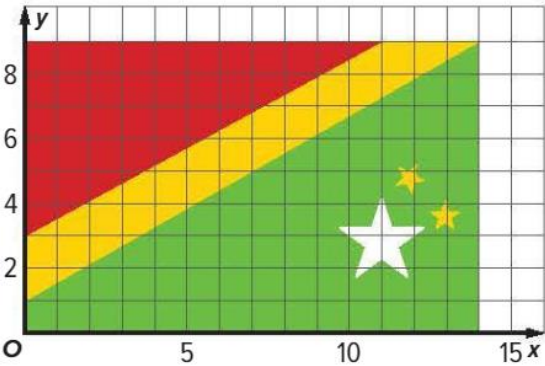
Check

Triangle PQR has vertices $P(2, 1)$, $Q(2, 4)$, and $R(5, 1)$. Graph $\triangle PQR$ and its image after a rotation 270° counterclockwise about $(7, 5)$.



 Example 2 Describe the Effect of a Rotation

FLAGS Kendrick is working with a team in his social studies class to create a new country and its government. Kendrick is responsible for creating the country's flag. He is using geometry software to design the flag on the coordinate plane. Describe how the two yellow stars would be affected if they were rotated 90° counterclockwise about the center of the white star.



If the stars were rotated, they would curve around the top-left sides of the white star. Together the preimage and the image would create a semicircle of yellow stars above the white star.

.....

.....

.....

.....

.....

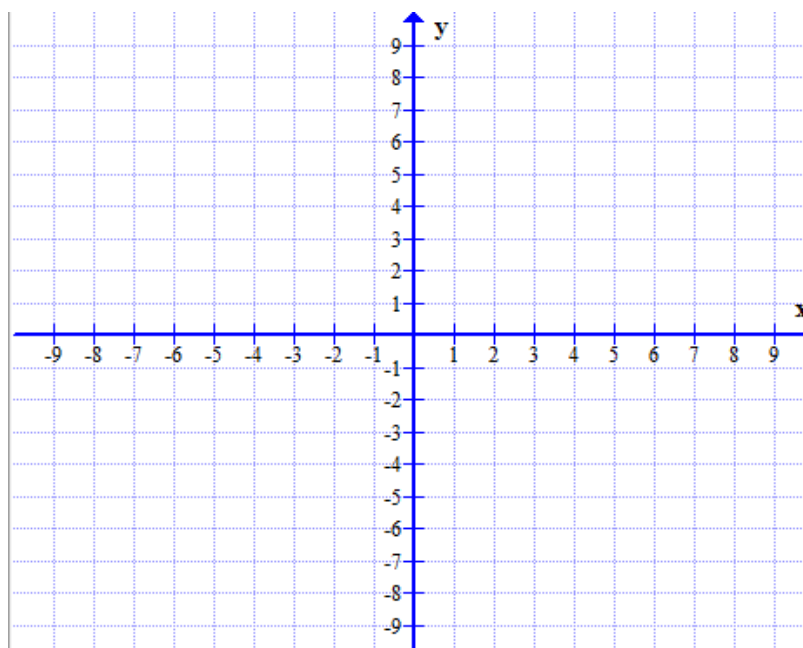
MODULE 13

LESSON 13.3

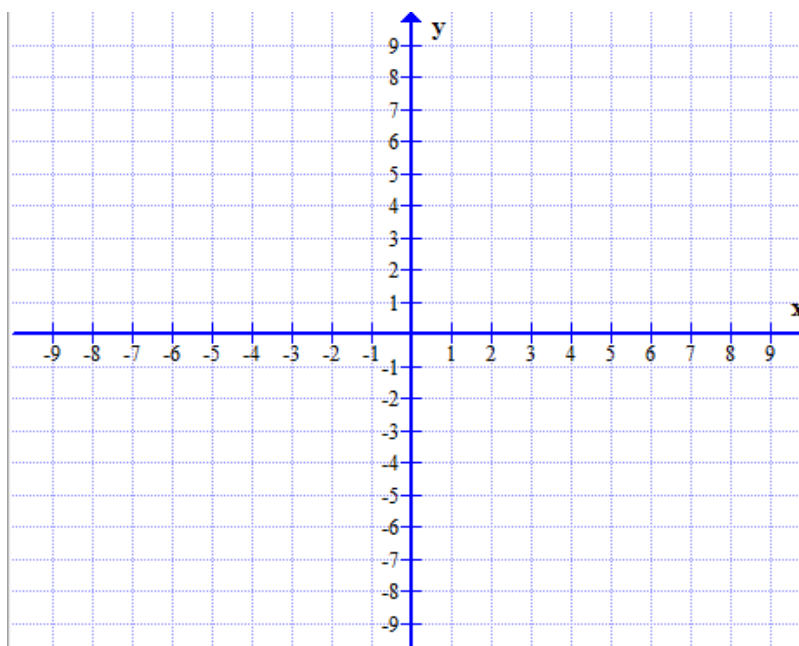
Practice

Examples 1 and 2

1. Triangle XYZ has vertices $X(0, 2)$, $Y(4, 4)$, and $Z(3, -1)$. Graph $\triangle XYZ$ and its image after a rotation of 180° about $(2, -3)$.



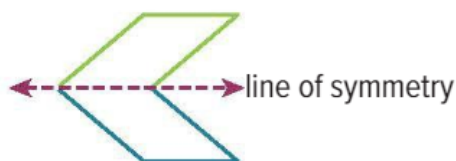
2. Triangle ABC has vertices $A(1, 7)$, $B(3, 2)$, and $C(-2, -2)$. Graph $\triangle ABC$ and its image after a rotation of 270° counterclockwise about $(-4, 2)$.



Learn Line Symmetry

A figure has **symmetry** if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself. Figures that have symmetry are self-congruent. One type of symmetry is *line symmetry*.

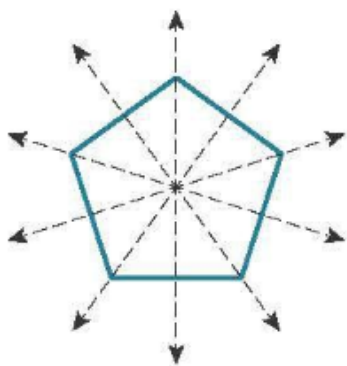
A figure in the plane has **line symmetry** (or *reflectional symmetry*) if each half of the figure matches the other side exactly. When a figure has line symmetry, the figure can be mapped onto itself by a reflection in a line, called the **line of symmetry** (or *axis of symmetry*).



Example 1 Identify Line Symmetry

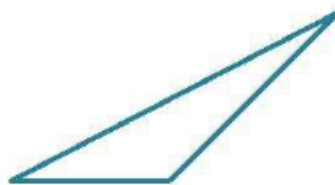
Determine whether each figure has a line of symmetry. If so, draw the lines of symmetry and state how many lines of symmetry it has.

a.



5 lines of symmetry

b.



0 lines of symmetry

MODULE 13

LESSON 13.6

Check

Determine whether each figure has a line of symmetry. If so, copy the figure and draw the lines of symmetry and state how many lines of symmetry it has.



This figure has ? line(s) of symmetry.



This figure has ? line(s) of symmetry.



This figure has ? line(s) of symmetry.

MODULE 13

LESSON 13.6

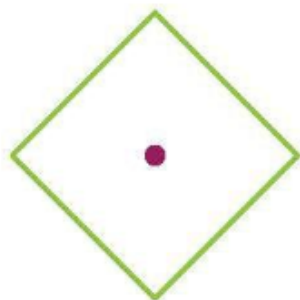
Learn Rotational Symmetry

A figure in the plane has **rotational symmetry** (or *radial symmetry*) if the figure can be mapped onto itself by being rotated less than 360° about the center of the figure so the image and the preimage are indistinguishable. The point in which a figure can be rotated onto itself is called the **center of symmetry** (or *point of symmetry*).

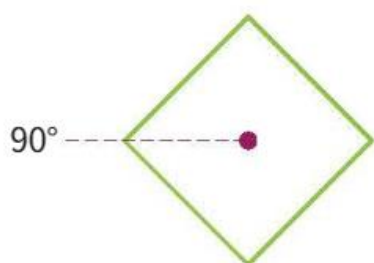
This figure has rotational symmetry because a rotation of 90° , 180° , or 270° maps the figure onto itself.

The number of times that a figure maps onto itself as it rotates from 0° to 360° is called the **order of symmetry**. The **magnitude of symmetry** (or *angle of rotation*) is the smallest angle through which a figure can be rotated so it maps onto itself. The order and magnitude of a rotation are related by the following equation.

$$\text{magnitude} = 360^\circ \div \text{order}$$



This figure has order 4 and magnitude 90° .



MODULE 13

LESSON 13.6

Key Concept • Point Symmetry

A figure has **point symmetry** if it can be mapped onto itself by a rotation of 180° . If a figure has point symmetry, then the center of symmetry in the figure is called the **point of symmetry**.

Example

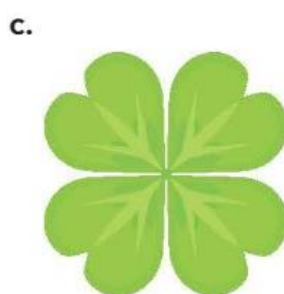
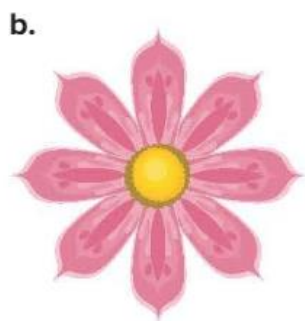
A rhombus has point symmetry because it looks the same right-side up as upside down.



Example 2 Identify Rotational Symmetry

NATURE Objects found in nature often have rotational symmetry.

Determine whether each figure has rotational symmetry. Explain.



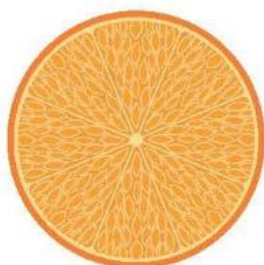
No; no rotation less than 360° maps the leaf onto itself.

Yes; the flower can map onto itself with a rotation that is less than 360° .

Yes; the clover can map onto itself with a rotation that is less than 360° .

Check

HOUSEHOLD Below are several objects that you might find around your house. Determine whether each figure has rotational symmetry. Explain.



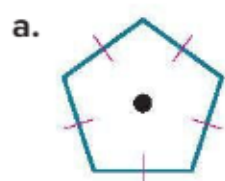
MODULE 13

LESSON 13.6

Example 3 Determine Order and Magnitude of Symmetry

Part A State the order and magnitude of symmetry.

Determine whether each figure has rotational symmetry. If so, locate the center of symmetry and state the order and magnitude of symmetry.



rotational
symmetry: yes

order = 5
magnitude = 72°



rotational
symmetry: no

order = none
magnitude = none



rotational
symmetry: yes

order = 2
magnitude = 180°

Check

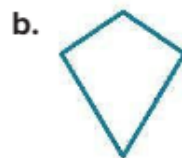
Determine whether each figure has rotational symmetry. If so, copy the figure and locate the center of symmetry and state the order and magnitude of symmetry.



rotational
symmetry: ?

order = ?

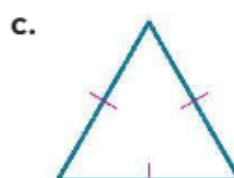
magnitude = ?



rotational
symmetry: ?

order = ?

magnitude = ?



rotational
symmetry: ?

order = ?

magnitude = ?

Which figure(s) has point symmetry? Justify your answer.

MODULE 13

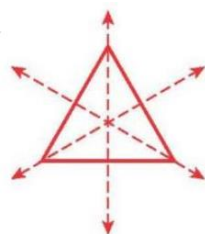
LESSON 13.6

Practice

Example 1

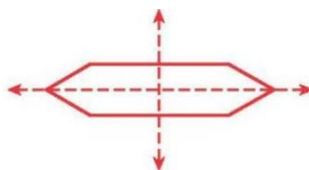
Determine whether each figure has a line of symmetry. If so, draw the lines of symmetry and state how many lines of symmetry it has.

1.



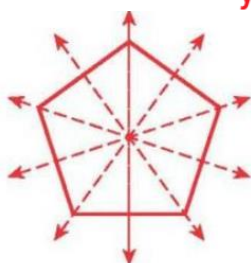
yes; 3

2.



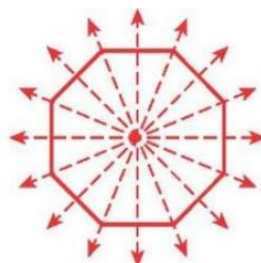
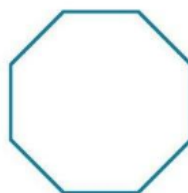
yes; 2

5.



yes; 5

6.



yes; 8

Example 2

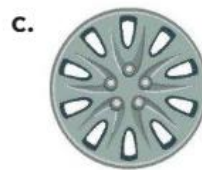
7. **CARS** Steve found the hubcaps shown below at his local junkyard. Determine whether each hubcap has rotational symmetry. Explain.



Yes; the hubcap can map onto itself with a rotation that is less than 360° .



Yes; the hubcap can map onto itself with a rotation that is less than 360° .

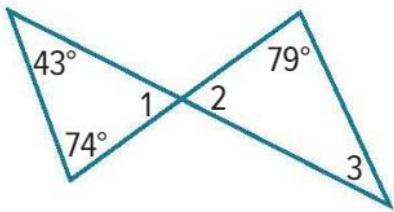


Yes; the hubcap can map onto itself with a rotation that is less than 360° .

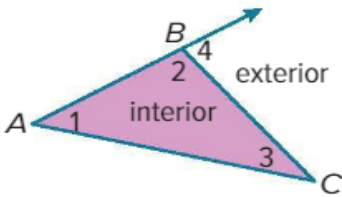
Angles of Triangles

Check

Find the measure of each numbered angle.



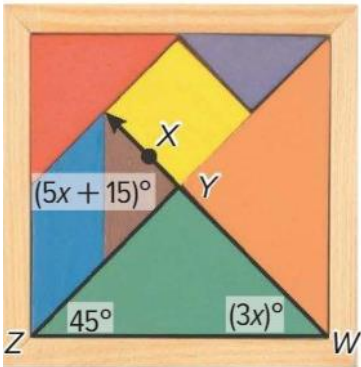
Learn Exterior Angles of Triangles



interior angles	The sum of the measures of the interior angles of a triangle is 180° .
exterior angles	An exterior angle of a triangle is formed by one side of the triangle and the extension of an adjacent side. A triangle has three exterior angles.
remote interior angles	Each exterior angle of a triangle has two remote interior angles that are not adjacent to the exterior angle.

Check

PUZZLES Find the measure of $\angle XYZ$ created by the triangle.



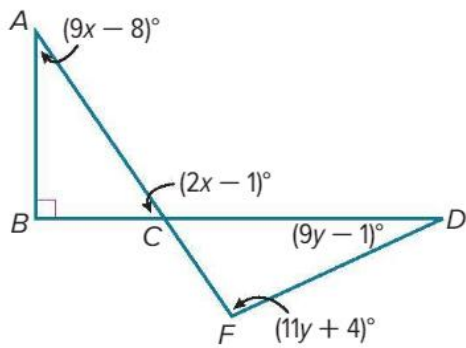
MODULE 14

LESSON 14.1

Check

Find each measure.

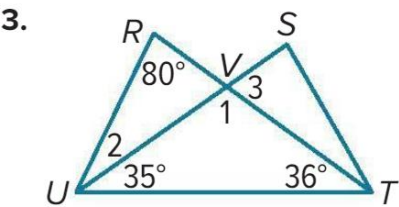
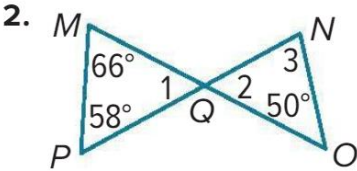
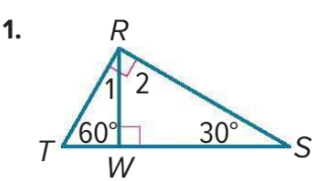
- $m\angle BAC = \underline{\hspace{1cm}}?$
- $m\angle BCA = \underline{\hspace{1cm}}?$
- $m\angle DCF = \underline{\hspace{1cm}}?$
- $m\angle CDF = \underline{\hspace{1cm}}?$
- $m\angle CFD = \underline{\hspace{1cm}}?$



Practice

Example 1

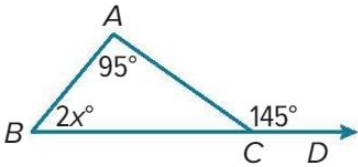
Find the measure of each numbered angle.



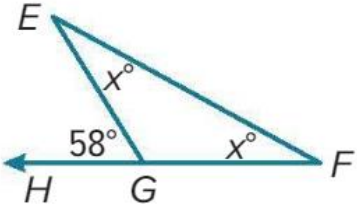
Example 2

Find each measure.

5. $m\angle ABC$



6. $m\angle F$



Congruent Triangles

Check

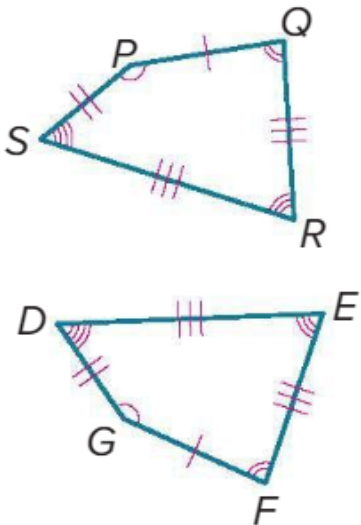
Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.

$\angle P \cong \underline{\hspace{1cm}} ; \angle Q \cong \underline{\hspace{1cm}} ; \angle R \cong \underline{\hspace{1cm}} ; \angle S \cong \underline{\hspace{1cm}}$

$\overline{PQ} \cong \underline{\hspace{1cm}} ; \overline{QR} \cong \underline{\hspace{1cm}} ; \overline{RS} \cong \underline{\hspace{1cm}} ; \overline{SP} \cong \underline{\hspace{1cm}}$

Complete the congruence statement.

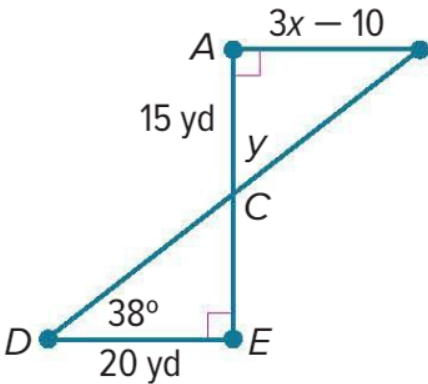
Polygon $PQRS \cong$ Polygon $\underline{\hspace{1cm}} ? \underline{\hspace{1cm}}$



Check

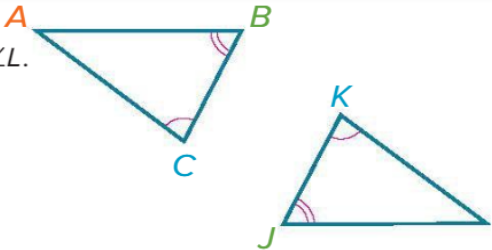
In the diagram, $\triangle ABC \cong \triangle EDC$. Find the values of x and y .

$x = \underline{\hspace{1cm}} ; y = \underline{\hspace{1cm}}$



Learn Third Angles Theorem and Triangle Congruence

Theorem 14.3: Third Angles Theorem	
Words	If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.
Example	If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.



You will prove Theorem 14.3 in Exercise 25.

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

Theorem 14.4: Properties of Triangle Congruence

Reflexive Property of Triangle Congruence

$$\triangle ABC \cong \triangle ABC$$

Symmetric Property of Triangle Congruence

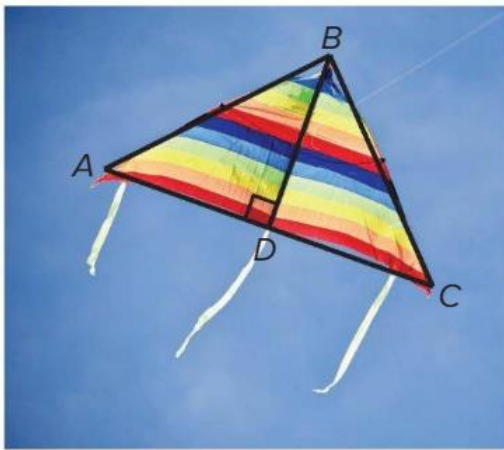
If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Check

KITES The kite shown is made of two congruent triangles. If $m\angle BAD = m\angle BCD = 45^\circ$, find $m\angle ABD$. $m\angle ABD = \underline{\hspace{1cm}}$



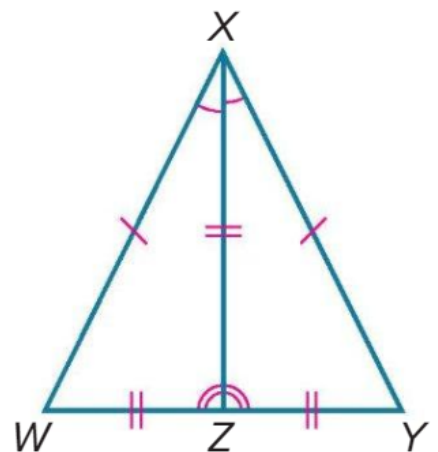
Check

Write a paragraph proof.

Given: $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$,
 $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$

Prove: $\triangle WXZ \cong \triangle YXZ$

It is given that $\overline{WX} \cong \overline{YX}$ and $\overline{WZ} \cong \overline{YZ}$. By the SSS Property, $\overline{XZ} \cong \overline{XZ}$. It is also given that $\angle WXZ \cong \angle YXZ$ and $\angle XZW \cong \angle XZY$. So, by ASA the Theorem, $\angle W \cong \angle Y$. By the definition of congruent triangles, $\triangle WXZ \cong \triangle YXZ$.



MODULE 14 Practice

LESSON 14.2 Example 1

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.

1.

.....

.....

.....

.....

2.

.....

.....

.....

.....

3.

.....

.....

.....

.....

4.

.....

.....

.....

.....

5.

.....

.....

.....

.....

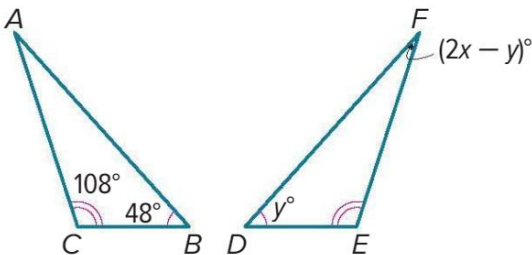
MODULE 14

LESSON 14.2

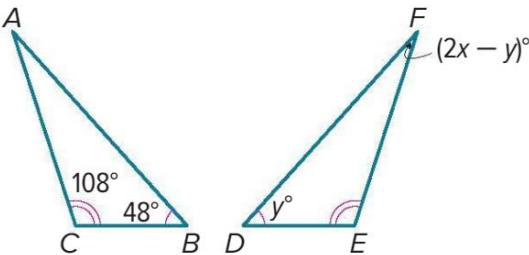
Example 2

In the diagram, $\triangle ABC \cong \triangle FDE$.

6. Find the value of x .

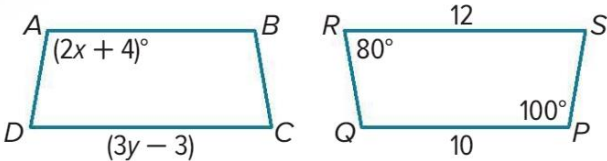


7. Find the value of y .

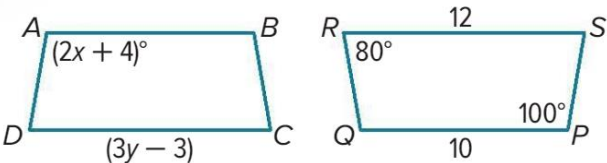


In the diagram, polygon $ABCD \cong$ polygon $PQRS$.

8. Find the value of x .

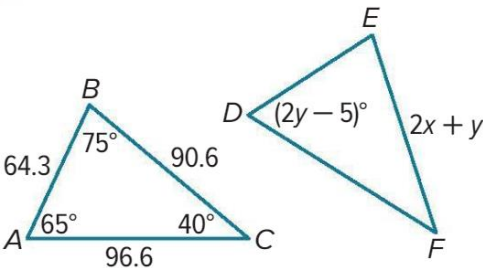


9. Find the value of y .



In the diagram, $\triangle ABC \cong \triangle DEF$.

10. Find the value of x .

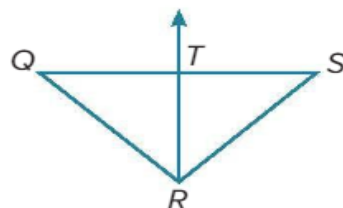


Example 1 Use SSS to Prove Triangles CongruentWrite a flow proof to show that $\triangle QRT \cong \triangle SRT$.

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$. $\overrightarrow{RT}$ bisects $\overline{QS}$ at point T .

Prove: $\triangle QRT \cong \triangle SRT$

Proof:



$\triangle QRS$ is isosceles
with $\overline{QR} \cong \overline{SR}$.

Given

$\overrightarrow{RT}$ bisects $\overline{QS}$
at point T .

Given

$\overline{RT} \cong \overline{RT}$

Reflexive Property
of Congruence

$\overline{QT} \cong \overline{ST}$

Definition of
segment bisector

$\triangle QRT \cong \triangle SRT$

SSS

Check

Triangle ABC has vertices $A(1, 1)$, $B(0, 3)$, and $C(2, 5)$. Triangle EFG has vertices $E(1, -1)$, $F(2, -5)$, and $G(4, -4)$. Is $\triangle ABC \cong \triangle EFG$?

Part AGraph $\triangle ABC$ and $\triangle EFG$ on the same coordinate plane.**Part B**

Find the side lengths of each triangle.

$AB = \underline{\hspace{1cm}}; BC = \underline{\hspace{1cm}}; AC = \underline{\hspace{1cm}}; EF = \underline{\hspace{1cm}}; FG = \underline{\hspace{1cm}}; EG = \underline{\hspace{1cm}}$

Part CIs triangle ABC congruent to triangle EFG ? Justify your argument.

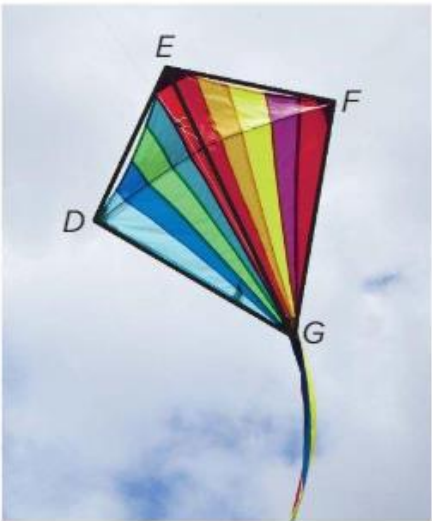
- A. No; $AC \neq FG$, so SSS congruence is not met.
- B. No; $BC \neq FG$, so SSS congruence is not met.
- C. Yes; all corresponding sides have the same measure, so SSS congruence is met.
- D. Yes; all corresponding sides have the same measure, so by the definition of congruent figures, $\triangle ABC \cong \triangle EFG$.

MODULE 14

LESSON 14.3

Check

KITES The kite shown appears to be made up of congruent triangles. If $\overline{DE} \cong \overline{FE}$ and $\overline{EG}$ bisects $\angle DEF$, prove that $\triangle DEG \cong \triangle FEG$. Copy and complete the two-column proof by selecting the correct statements and reasons.



Given: $\overline{DE} \cong \overline{FE}$, $\overline{EG}$ bisects $\angle DEF$.
Prove: $\triangle DEG \cong \triangle FEG$
Proof:

Statements	Reasons
1. <u> ? </u>	1. Given
2. $\overline{EG}$ bisects $\angle DEF$.	2. <u> ? </u>
3. <u> ? </u>	3. Definition of angle bisector
4. <u> ? </u>	4. <u> ? </u>
5. $\triangle DEG \cong \triangle FEG$	5. <u> ? </u>

Practice

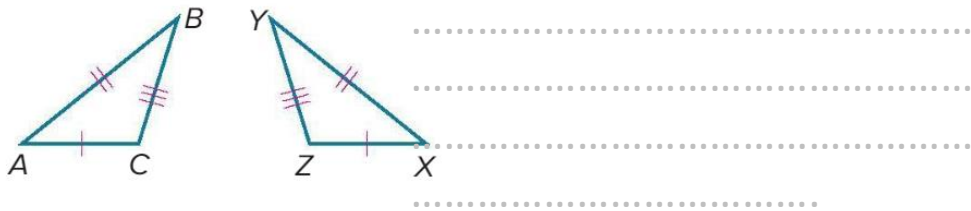
Example 1

PROOF Write the specified type of proof.

1. two-column proof

Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

Prove: $\triangle ABC \cong \triangle XYZ$



2. flow proof

Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

Prove: $\triangle RST \cong \triangle UTS$



3. two-column proof

Given: $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$.

Prove: $\triangle ABD \cong \triangle CBD$



3. Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$. (Given)

2. $\overline{AD} \cong \overline{DC}$ (Definition of midpoint)

3. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Congruence)

4. $\triangle ABD \cong \triangle CBD$ (SSS)

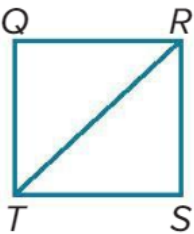
MODULE 14

LESSON 14.3

5. paragraph proof

Given: $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$

Prove: $\triangle QRT \cong \triangle SRT$



.....
.....
.....
.....
.....

5. Proof: We know that $\overline{QR} \cong \overline{SR}$ and $\overline{ST} \cong \overline{QT}$. $\overline{RT} \cong \overline{RT}$ by the Reflexive Property. Because $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$, and $\overline{RT} \cong \overline{RT}$, $\triangle QRT \cong \triangle SRT$ by SSS.

.....
.....
.....

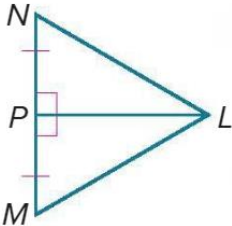
Example 3

PROOF Write the specified type of proof.

11. two-column proof

Given: $NP = PM$, $\overline{NP} \perp \overline{PL}$

Prove: $\triangle NPL \cong \triangle MPL$

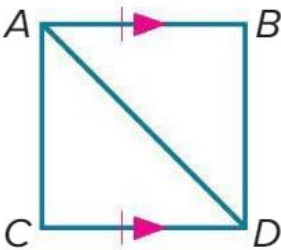


.....
.....
.....
.....

12. two-column proof

Given: $AB = CD$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle DBA$

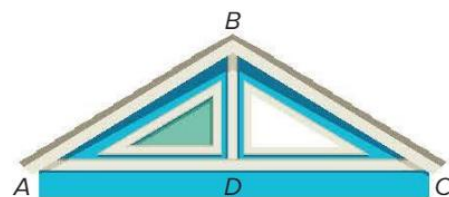


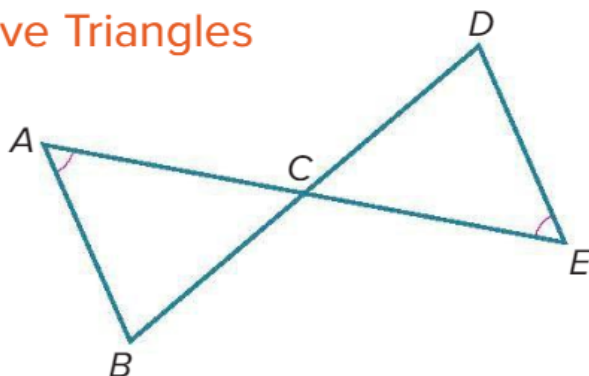
.....
.....
.....
.....

MODULE 14

LESSON 14.3

15. **GAMING** Devontae is building a house in a simulation video game. He wants the roof of the house and the main support beam to create congruent triangles. If $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ bisects $\overline{AC}$, write a two-column proof to prove $\triangle ABD \cong \triangle CBD$.

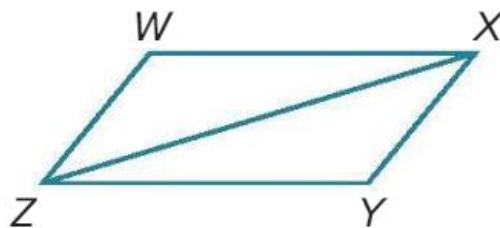


Example 1 Use ASA to Prove Triangles Congruent**Complete the two-column proof.****Given:** $\angle BAC \cong \angle DEC$;
 $\overline{BD}$ bisects $\overline{AE}$.**Prove:** $\triangle ACB \cong \triangle ECD$ 

Statements	Reasons
1. $\angle BAC \cong \angle DEC$	1. Given
2. $\overline{BD}$ bisects $\overline{AE}$	2. Given
3. $\overline{AC} \cong \overline{EC}$	3. Definition of segment bisector
4. $\angle ACB \cong \angle ECD$	4. Vertical Angles Theorem
5. $\triangle ACB \cong \triangle ECD$	5. ASA

Check

Complete the two-column proof.

Given: $\overline{WX} \parallel \overline{YZ}$ and $\overline{WZ} \parallel \overline{YX}$ **Prove:** $\triangle WXZ \cong \triangle YZX$ 

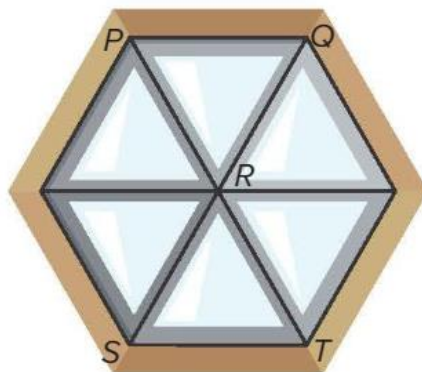
Statements	Reasons
1. <u> ?</u>	1. Given
2. $\overline{WZ} \parallel \overline{YX}$	2. <u> ?</u>
3. $\angle WXZ \cong \angle YZX$	3. <u> ?</u>
4. <u> ?</u>	4. Alternate Interior Angles Theorem
5. <u> ?</u>	5. Reflexive Property of Congruence
6. $\triangle WXZ \cong \triangle YZX$	6. <u> ?</u>

MODULE 14

LESSON 14.4

Example 2 Apply ASA Congruence

PRODUCTION A company that manufactures windows needs to determine the amount of glass required to make the hexagonal window shown. $\overline{PQ} \parallel \overline{TS}$, R is the midpoint of $\overline{PT}$, and ST is 12 inches.



Part A Determine whether $\triangle PRQ$ is congruent to $\triangle TRS$.

Because $\overline{PQ}$ is parallel to $\overline{TS}$, $\angle RPQ \cong \angle RTS$ by the Alternate Interior Angles Theorem.

Because point R is the midpoint of $\overline{PT}$, $\overline{TR} \cong \overline{PR}$ by the Midpoint Theorem.

$\angle TRS$ and $\angle PRQ$ are vertical angles, so they are congruent by the Vertical Angles Theorem.

Therefore, by ASA, $\triangle PRQ \cong \triangle TRS$.

Part B Find the area of the window.

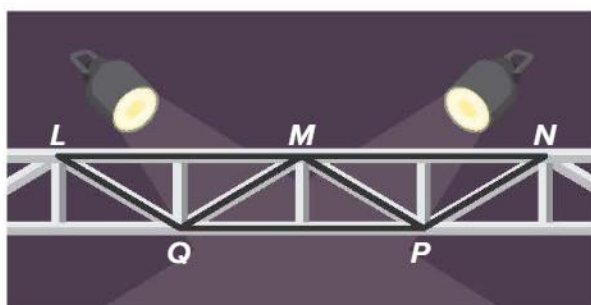
If the six triangles that form the window are congruent and the height of $\triangle TRS$ is about 10.39 inches, how much glass is required to manufacture the window?

$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area of a triangle} \\ &\approx \frac{1}{2}(12)(10.39) && b = 12 \text{ and } h = 10.39 \\ &\approx 62.34 && \text{Simplify.} \end{aligned}$$

The area of the window is approximately 62.34(6) or about 374.04 square inches.

Check

LIGHTING A theater uses scaffolding to hang stage lighting. The stage manager needs to determine how much electrical wire is needed to hang lights across the scaffolding from point L to N . $\overline{LN} \parallel \overline{QP}$, $\overline{NP} \parallel \overline{MQ}$, and $\overline{LQ} \parallel \overline{MP}$. If MN is 4 feet, how many feet of electrical wire is needed to display lights across the scaffolding?



Learn Proving Triangles Congruent: AAS

The congruence of two angles and a nonincluded side is also sufficient to prove two triangles congruent. This congruence relationship is a theorem because it can be proved using the Third Angles Theorem.

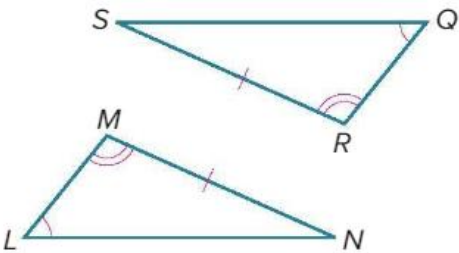
Theorem 14.5: Angle-Angle-Side (AAS) Congruence

Words If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and nonincluded side of a second triangle, then the two triangles are congruent.

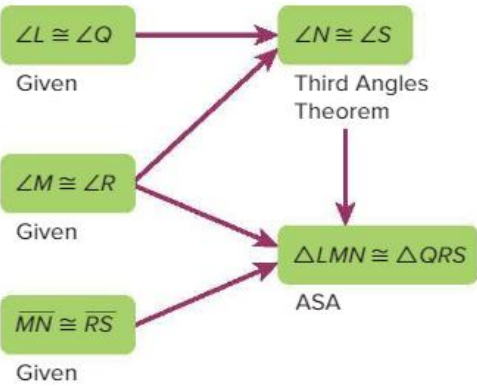
Example If $\angle A \cong \angle D$,
 $\angle B \cong \angle E$, and
 $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

The proof of the AAS Congruence Theorem is below.

Given: $\angle L \cong \angle Q$
 $\angle M \cong \angle R$
 $\overline{MN} \cong \overline{RS}$
Prove: $\triangle LMN \cong \triangle QRS$



Proof:



MODULE 14

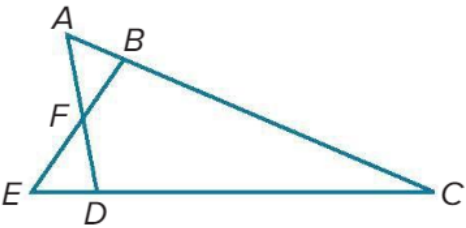
LESSON 14.4

Check

Choose the correct statements and reasons to complete the two-column proof.

Given: $\angle DAC \cong \angle BEC$ and $\overline{DC} \cong \overline{BC}$

Prove: $\triangle ACD \cong \triangle ECB$



Statements	Reasons
1. <u> ?</u>	1. Given
2. $\overline{DC} \cong \overline{BC}$	2. <u> ?</u>
3. <u> ?</u>	3. <u> ?</u>
4. $\triangle ACD \cong \triangle ECB$	4. <u> ?</u>

Practice

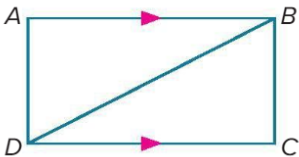
Example 1

PROOF Write the specified type of proof.

1. two-column proof

Given: $\overline{AB} \parallel \overline{CD}$, $\angle CBD \cong \angle ADB$

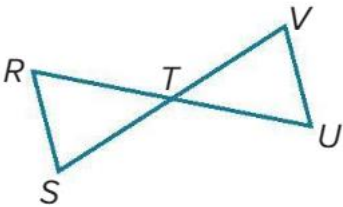
Prove: $\triangle ABD \cong \triangle CDB$



2. two-column proof

Given: $\angle S \cong \angle V$, and T is the midpoint of $\overline{SV}$.

Prove: $\triangle RTS \cong \triangle UTV$



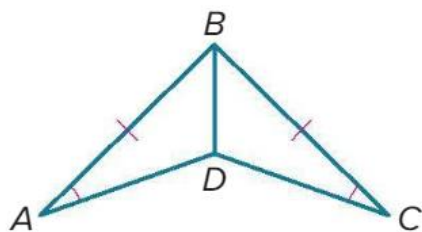
MODULE 14

LESSON 14.4

3. flow proof

Given: $\overline{AB} \cong \overline{CB}$, $\angle A \cong \angle C$, and
 $\overline{DB}$ bisects $\angle ABC$.

Prove: $\overline{AD} \cong \overline{CD}$



.....

.....

.....

.....

.....

.....

.....

.....

Proving Right Triangles Congruent

Learn Right Triangle Congruence

Theorem 14.6: Leg-Leg (LL) Congruence

Words	If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.
Example	Given right $\triangle ABC$ and right $\triangle DEF$, $\overline{AB} \cong \overline{DE}$ and $\overline{CB} \cong \overline{FE}$. So, $\triangle ABC \cong \triangle DEF$ by LL.

Theorem 14.7: Hypotenuse-Angle (HA) Congruence

Words	If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and the corresponding acute angle of another right triangle, then the triangles are congruent.
Example	Given right $\triangle ABC$ and right $\triangle DEF$, $\overline{AC} \cong \overline{DF}$ and $\angle C \cong \angle F$. So, $\triangle ABC \cong \triangle DEF$ by HA.

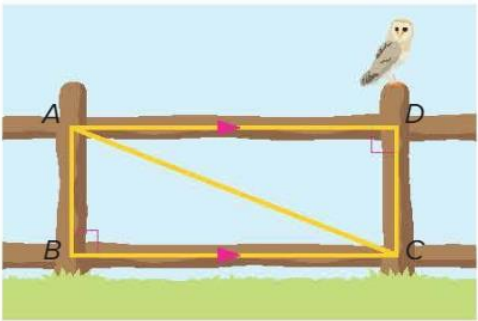
Check

FENCES The fence has parallel supports and a crossbar that forms two triangles. Complete the proof to show that the triangles are congruent.

Given: $\angle B$ and $\angle D$ are right angles. $\overline{AD} \parallel \overline{BC}$.

Prove: $\triangle ABC \cong \triangle CDA$

Proof:



Statements	Reasons
1. $\angle B$ and $\angle D$ are right angles.	1. <u> ?</u>
2. $\triangle ABC$ and $\triangle CDA$ are right triangles.	2. <u> ?</u>
3. $\overline{AD} \parallel \overline{BC}$	3. <u> ?</u>
4. $\angle DAC \cong \angle BCA$	4. <u> ?</u>
5. $\overline{AC} \cong \overline{AC}$	5. <u> ?</u>
6. $\triangle ABC \cong \triangle CDA$	6. <u> ?</u>

MODULE 14

LESSON 14.5

Practice

Example 1

- 1. CAMPING** In the diagram of the pup tent, the support pole is perpendicular to the ground. The base of the support pole is located at the midpoint of the segment connecting the bottom of the sides of the tent. Write a two-column proof to show that the triangles formed by the support pole are congruent.

Given: $\overline{XZ} \perp \overline{WY}$; Z is the midpoint of $\overline{WY}$.

Prove: $\triangle WXZ \cong \triangle YXZ$

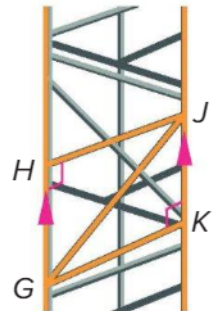


- 2. TOWERS** The cell phone tower has parallel poles and diagonal support beams that form two triangles. Write a two-column proof to show that the triangles are congruent.

Given: $\angle H$ and $\angle K$ are right angles.

$$\overline{GH} \parallel \overline{KJ}$$

Prove: $\triangle GKJ \cong \triangle JHG$



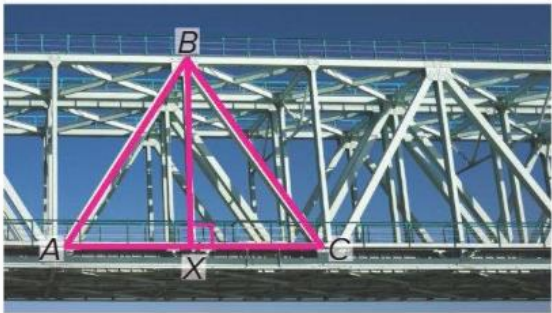
MODULE 14

LESSON 14.5

3. **BRIDGES** In the diagram, the vertical support beam, $\overline{BX}$, is perpendicular to the deck of the bridge. The two diagonals, $\overline{AB}$ and $\overline{CB}$, are equal in length. Write a two-column proof to show that the triangles formed by the vertical support beam are congruent.

Given: $\overline{BX} \perp \overline{AC}$; $AB = CB$

Prove: $\triangle AXB \cong \triangle CXB$



.....

.....

.....

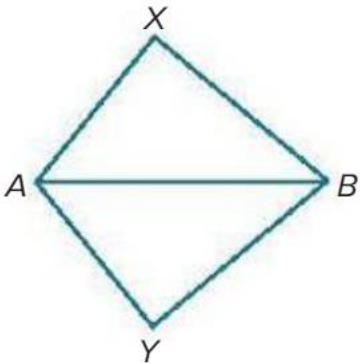
.....

.....

11. **PROOF** Write a two-column proof.

Given: $\overline{BX} \perp \overline{XA}$, $\overline{BY} \perp \overline{YA}$, and $\overline{XA} \cong \overline{YA}$

Prove: $\triangle BXA \cong \triangle BYA$



.....

.....

.....

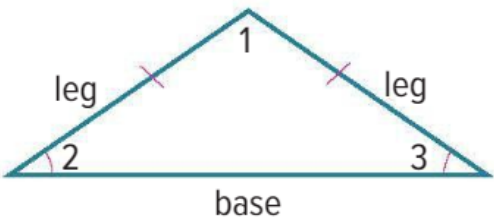
.....

.....

Isosceles and Equilateral Triangles

Learn Isosceles Triangles

An **isosceles triangle** is a triangle with at least two sides congruent. The two congruent sides are called the **legs of an isosceles triangle**. The angle between the sides that are the legs is called the **vertex angle of an isosceles triangle**. $\angle 1$ is the vertex angle of the triangle. The side of the triangle opposite the vertex angle is called the **base**. The two angles formed by the base and the congruent sides are called the **base angles of an isosceles triangle**. $\angle 2$ and $\angle 3$ are the base angles.



Theorem 14.10: Isosceles Triangle Theorem

Words If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.

A diagram of triangle ABC. Vertex A is on the left, vertex B is on the right, and vertex C is at the bottom. Side AC is labeled with angle 1 at vertex A and has a single tick mark. Side BC is labeled with angle 2 at vertex B and has a single tick mark. The base is AB.

Theorem 14.11: Converse of the Isosceles Triangle Theorem

Words If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.

A diagram of triangle DEF. Vertex D is at the top, vertex F is at the bottom left, and vertex E is at the bottom right. Angle 1 is at vertex D, and angle 2 is at vertex F. Both angles are marked with single arcs. The sides opposite these angles are FE and DE.

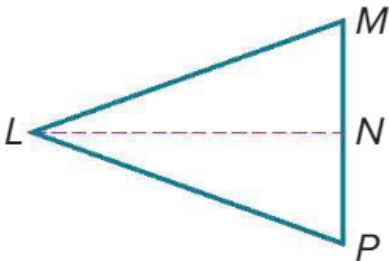
Example 1 Prove the Isosceles Triangle Theorem

Prove the Isosceles Triangle Theorem.

To prove the Isosceles Triangle Theorem, draw an *auxiliary line* and use the two triangles that are formed. An **auxiliary line** is an extra line or segment drawn in a figure to help analyze geometric relationships.

Given: $\triangle LMP$, $\overline{LM} \cong \overline{LP}$

Prove: $\angle M \cong \angle P$



Proof:	Statements	Reasons
	1. Let N be the midpoint of $\overline{MP}$.	1. Every segment has exactly one midpoint.
	2. Draw an auxiliary segment $\overline{LN}$.	2. Two points determine a line.
	3. $\overline{MN} \cong \overline{PN}$	3. Midpoint Theorem
	4. $\overline{LN} \cong \overline{LN}$	4. Reflexive Property of Congruence
	5. $\overline{LM} \cong \overline{LP}$	5. Given
	6. $\triangle LMN \cong \triangle LPN$	6. SSS
	7. $\angle M \cong \angle P$	7. CPCTC

Example 2 Find Missing Measures in Isosceles Triangles

Find $m\angle B$ and $m\angle C$.

Part A Determine side relationships.

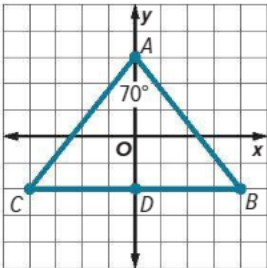
Use the Distance Formula to determine the measures of the sides of $\triangle ABC$. The coordinates of $\triangle ABC$ are $A(0, 3)$, $B(-4, -2)$, and $C(4, -2)$.

$AB = \sqrt{[0 - (-4)]^2 + [3 - (-2)]^2}$ or $\sqrt{41}$ units

$AC = \sqrt{[0 - 4]^2 + [3 - (-2)]^2}$ or $\sqrt{41}$ units

$CB = \sqrt{[4 - (-4)]^2 + [-2 - (-2)]^2}$ or 8 units

So, $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{AC}$.



MODULE 14

LESSON 14.6

Part B Determine the angle measures.

Because $\overline{AB} \cong \overline{AC}$, we know that $\angle C \cong \angle B$ by the Isosceles Triangle Theorem.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$m\angle A + 2m\angle B = 180^\circ$$

$$70^\circ + 2m\angle B = 180^\circ$$

$$m\angle B = m\angle C = 55^\circ$$

Triangle Angle-Sum Theorem

Definition of congruent

Substitute.

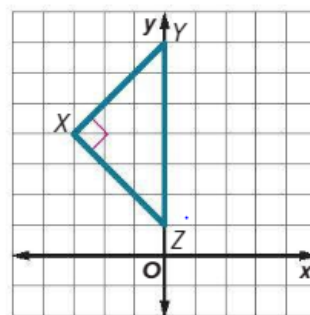
Solve.

Check

Find $m\angle XYZ$ and $m\angle YZX$.

$$m\angle XYZ = \underline{\hspace{1cm}} ? \underline{\hspace{1cm}}$$

$$m\angle YZX = \underline{\hspace{1cm}} ? \underline{\hspace{1cm}}$$



Learn Equilateral Triangles

The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

Corollary 14.3

A triangle is equilateral if and only if it is equiangular.

Corollary 14.4

Each angle of an equilateral triangle measures 60° .

Example 3 Find Missing Measures in Equilateral Triangles

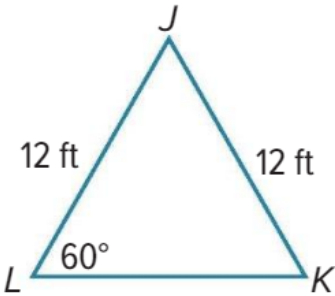
Find $m\angle J$.

Because $JL = JK$, $\overline{JL} \cong \overline{JK}$. By the Isosceles Triangle Theorem, base angles L and K are congruent, so $m\angle L = m\angle K$.

Use the Triangle Angle-Sum Theorem to write and solve an equation to find $m\angle J$.

$$m\angle J + m\angle K + m\angle L = 180^\circ$$
$$m\angle J + 60^\circ + 60^\circ = 180^\circ$$
$$m\angle J = 60^\circ$$

Triangle Angle-Sum Theorem
Isosceles Triangle Theorem
Solve.

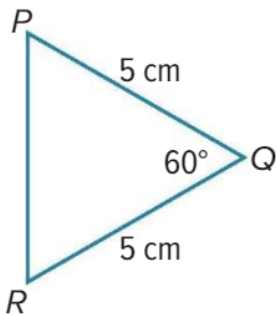


Check

Find $m\angle R$ and PR .

$m\angle R = \underline{\hspace{1cm}}$

$PR = \underline{\hspace{1cm}} \text{ cm}$



.....

.....

.....

.....

.....

 Example 4 Find Missing Values

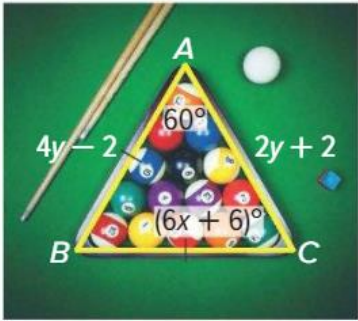
BILLIARDS Find the value of each variable.

Because $\overline{AB} \cong \overline{BC}$, $\angle ACB \cong \angle BAC$ by the Isosceles Triangle Theorem.

$(6x + 6)^\circ = 60^\circ$ Isosceles Triangle Theorem
 $x = 9$ Solve.

Because each angle of the triangle measures 60° by the Triangle Angle-Sum Theorem, the triangle is an equilateral triangle by Corollary 5.3.

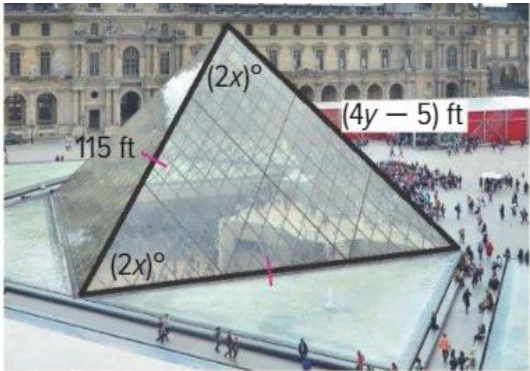
$4y - 2 = 2y + 2$ Corollary 14.3; definition of equilateral $\triangle$
 $y = 2$ Solve.



Check

ARCHITECTURE The main entrance to the Louvre Museum is a unique metal and glass pyramid. Find the value of each variable.

$x = \underline{\hspace{1cm}}?$ and $y = \underline{\hspace{1cm}}?$



.....

.....

.....

.....

.....

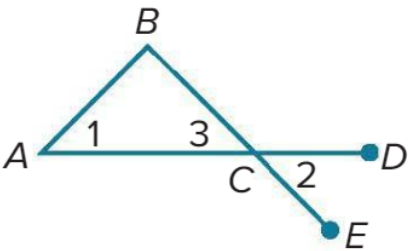
Practice

Example 1

1. **PROOF** Write a two-column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{CB}$



.....

.....

.....

.....

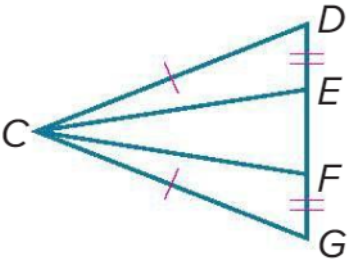
.....

2. **PROOF** Write a two-column proof.

Given: $\overline{CD} \cong \overline{CG}$

$\overline{DE} \cong \overline{GF}$

Prove: $\overline{CE} \cong \overline{CF}$



.....

.....

.....

.....

.....

MODULE 14

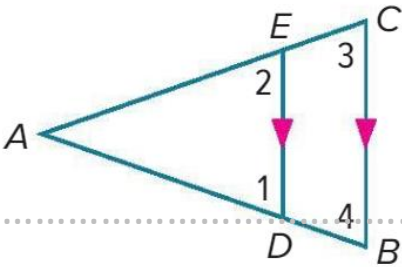
LESSON 14.6

3. **PROOF** Write a two-column proof.

Given: $\overline{DE} \parallel \overline{BC}$

$\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{AC}$



4. **ROOFS** In the picture, $\overline{BD} \perp \overline{AC}$ and $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$. Write a two-column proof to prove that $\overline{BD}$ bisects the angle formed by the sloped sides of the roof, $\angle ABC$.



Example 2

5. Refer to the figure.

- a. Find the measures of the sides of $\triangle ABC$. Show your work.
- b. Find $m\angle A$. Show your work.

