7 Magnetism



FIGURE 7.1 The Shanghai Maglev Train in the Pudong Airport station. The inset is a display inside the train showing the maximum speed of 430 km/h (267 mph) attained during the 7-minute 20-second trip from the airport to downtown Shanghai.

While much of our knowledge of magnetism has been known for two centuries and some of it since ancient times, new and exciting technical applications of this phenomenon continue to be developed today. Many of the devices we use every day make use of magnetism. Cars, computers, power generators, almost anything that uses an electric motor, and almost any information storage technology are just a few examples. One particularly impressive example is Shanghai's Maglev Train, shown in Figure 7.1, which floats on artificially generated magnetic fields and is thus able to reach very high speeds. In order to appreciate these innovations, however, we need to start with the basics of magnetism.

This chapter is the first to consider magnetism, describing magnetic fields and magnetic forces and their effects on charged particles and currents. We'll continue to study magnetism in the next few chapters, describing the causes of magnetic fields and their connection to electric fields. You will see that electricity and magnetism are really parts of the same universal force, called the *electromagnetic force*; their connection is one of the most spectacular successes in physical theory.

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WHAT WE WILL LEARN

- Permanent magnets exist in nature. A magnet always has a north pole and a south pole. A single magnetic north pole or south pole cannot be isolated–magnetic poles always come in pairs.
- Opposite magnetic poles attract, and like poles repel.
- Breaking a bar magnet in half results in two new magnets, each with a north and a south pole.
- A magnetic field exerts a force on a moving charged particle.
- The force exerted on a charged particle moving in a magnetic field is perpendicular to both the magnetic field and the velocity of the particle.
- The torque on a current-carrying loop can be expressed in terms of the vector product of the magnetic dipole moment of the loop and the magnetic field.
- The Hall effect can be used to measure magnetic fields.



7.1 Permanent Magnets

In the region of Magnesia (in central Greece), the ancient Greeks found several types of naturally occurring minerals that attract and repel each other and attract certain kinds of metal, such as iron. They also, if floating freely, line up with the North and South Poles of the Earth. These minerals are various forms of iron oxide and are called **permanent magnets.** Other examples of permanent magnets include refrigerator magnets and magnetic door latches, which are made of compounds of iron, nickel, or cobalt. If you touch an iron bar to a piece of the mineral lodestone (magnetic magnetite), the iron bar will be magnetized. If you float this iron bar in water, it will align with the Earth's magnetic poles. The end of the magnet that points north is called the **north magnetic pole,** and the other end is called the **south magnetic pole.**

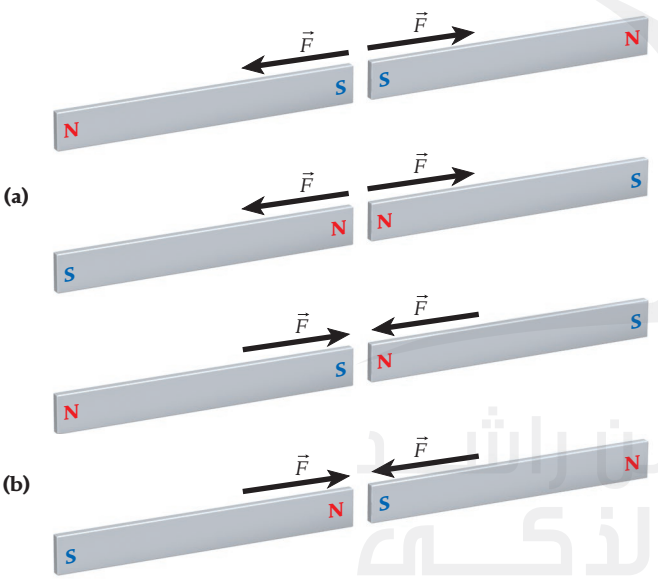


FIGURE 7.2 (a) Like magnetic poles repel; (b) unlike magnetic poles attract.

If two permanent magnets are brought close together with the two north poles or two south poles almost touching, the magnets repel each other (Figure 7.2a). If a north pole and a south pole are brought close together, the magnets attract each other (Figure 7.2b). What is called the North Pole of Earth is actually a magnetic south pole, which is why it attracts the north pole of permanent magnets.

Breaking a permanent magnet in half does not yield one north pole and one south pole. Instead, two new magnets, each with its own north and south pole result (Figure 7.3). Unlike electric charge, which exists as separate positive (proton) and negative (electron) charges, no separate magnetic monopoles (isolated north and south poles) exist. Scientists have carried out extensive searches for magnetic monopoles, and none has been found. The discussion of the source of magnetism in this chapter will help you understand why there are no magnetic monopoles.

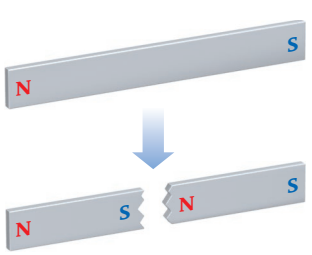
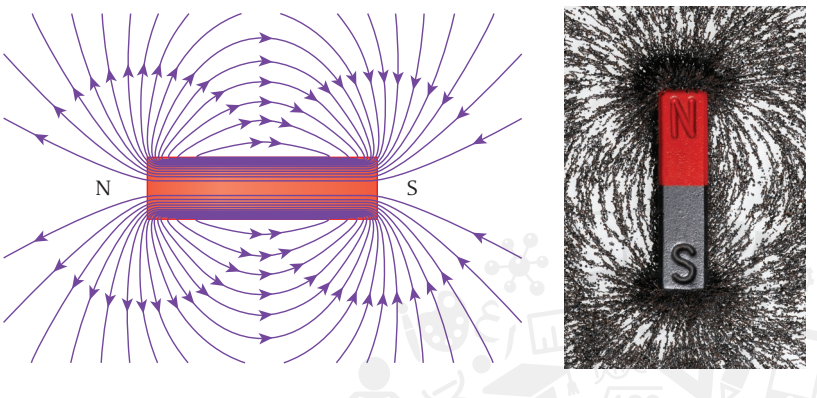


FIGURE 7.3 Breaking a bar magnet in half yields two magnets, each with its own north and south pole.

Magnetic Field Lines

Permanent magnets interact with each other at some distance, without touching. In analogy with the gravitational field and the electric field, the concept of a **magnetic field** is used to describe the magnetic force. The vector $\vec{B}(\vec{r})$ denotes the magnetic field vector at any given point in space.

Like an electric field, a magnetic field is represented using field lines. The magnetic field vector is always tangent to the **magnetic field lines.** The magnetic field lines from a permanent bar magnet are shown in Figure 7.4a. As with electric field lines, closer spacing between lines indicates higher field strength. In an electric field, the electric force on a positive test charge points in the same direction as the electric field vector.



(a) (b)

FIGURE 7.4 (a) Computer-generated magnetic field lines from a permanent bar magnet. (b) Iron filings align themselves with the magnetic field lines and make them visible.

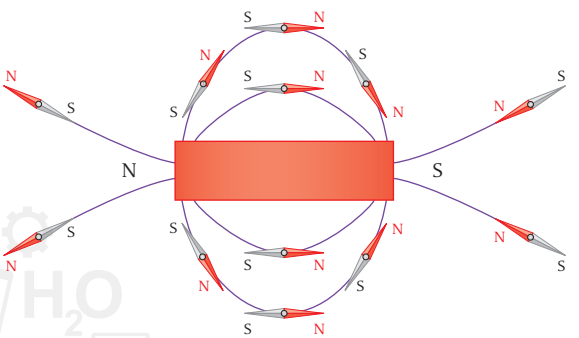


FIGURE 7.5 Using a compass needle to determine the direction of the magnetic field from a bar magnet.

However, because no magnetic monopole exists, the magnetic force cannot be described in an analogous way.

The direction of the magnetic field is established in terms of the direction in which a compass needle points. A compass needle, with a north pole and a south pole, will orient itself so that its north pole points in the direction of the magnetic field. Thus, the direction of the field can be determined at any point by noting the direction in which a compass needle placed at that point points, as illustrated in Figure 7.5 for a bar magnet.

Externally, magnetic field lines appear to originate on north poles and terminate on south poles, but these field lines are actually closed loops that penetrate the magnet itself. This formation of loops is an important difference between electric and magnetic field lines (for static fields–this statement does not apply to time-dependent fields, as we'll see in subsequent chapters). Recall that electric field lines start at positive charges and end on negative charges. However, because no magnetic monopoles exist, magnetic field lines cannot start or stop at particular points. Instead, they form closed loops that do not start or stop anywhere. We'll see later that this difference is important in describing the interaction of electric and magnetic fields. If you see a field pattern and don't know at first if it is an electric field or a magnetic field, check for closed loops. If you find some, it is a magnetic field; if the field lines do not form loops, it is an electric field.

Earth's Magnetic Field

Earth itself is a magnet, with a magnetic field similar to the magnetic field of a bar magnet (Figure 7.4). This magnetic field is important because it protects us from high-energy radiation from space called *cosmic rays*. These cosmic rays consist mostly of charged particles that are deflected away from Earth's surface by its magnetic field. The poles of Earth's magnetic field do not coincide with the geographic poles, defined as the points where Earth's rotation axis intersects its surface.

Figure 7.6 shows a cross section of Earth's magnetic field lines. The field lines are close together, forming a surface that wraps around Earth like a doughnut. Earth's magnetic field is distorted by the solar wind, a flow of ionized particles, mainly protons, emitted by the Sun and moving outward from the Sun at approximately 400 km/s. Two bands of charged particles captured from the solar wind circle the Earth. These are called the **Van Allen radiation belts** (Figure 7.6), after James A. Van Allen (1914–2006),

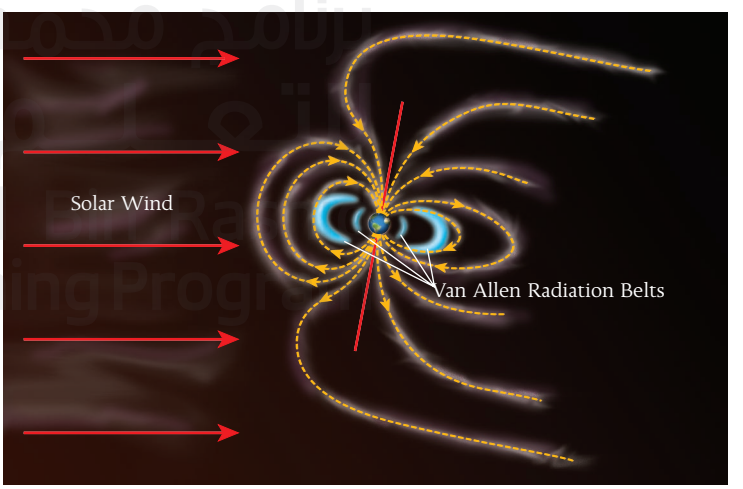


FIGURE 7.6 Cross section through Earth's magnetic field. The dashed lines represent the magnetic field lines. The axis defined by the north and south magnetic poles (red line) currently forms an angle of approximately 11° with the rotation axis.



FIGURE 7.7 Aurora borealis over Finland, photographed from the International Space Station.

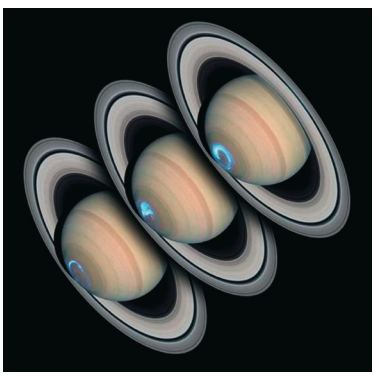


FIGURE 7.8 Aurora on Saturn, photographed by the Hubble Space Telescope.

who discovered them in the early days of space flight by putting radiation counters on satellites. The Van Allen radiation belts come closest to Earth near the north and south magnetic poles, where the charged particles trapped within the belts often collide with atoms in the atmosphere, exciting them. These excited atoms emit light of different colors as they collide and lose energy; the results are the fabulous **aurora borealis** (northern lights) in the northern latitudes (see Figure 7.7) and the **aurora australis** (southern lights) in the southern latitudes. Aurorae are not unique to Earth; they have been seen on other planets with strong magnetic fields, such as Jupiter and Saturn (shown in Figure 7.8).

Earth's magnetic poles move at a current rate of up to 40 km in a single year. Right now, the magnetic north pole is located approximately 2800 km away from the geographic South Pole, at the edge of Antarctica, and is moving toward Australia. The magnetic south pole is located in the Canadian Arctic and, if its present rate of motion continues, will reach Siberia in 2050. Earth's magnetic field has decreased steadily at a rate of about 7% per century since it was first measured accurately around 1840. At that rate, Earth's magnetic field will disappear in a few thousand years. However, some geological evidence indicates that the magnetic field of Earth has reversed itself approximately 170 times in the past 100 million years. The last reversal occurred about 770,000 years ago. Thus, rather than disappear, Earth's magnetic field may reverse its direction. What is the cause of Earth's magnetic field? Surprisingly, the answer to this question is not known exactly and is under intense current research. Most likely, it is caused by strong electrical currents inside the Earth, caused by the spinning liquid iron-nickel core. This spinning is often referred to as the *dynamo effect*. (We'll see how currents create magnetic fields in Chapter 8.)

Because the geographic North Pole and the magnetic north pole are not in the same location, a compass needle generally does not point exactly to the geographic North Pole. This difference is called the **magnetic declination.** The magnetic declination is taken to be positive when magnetic north is east of true north and negative when magnetic north is west of true north. The magnetic north pole currently lies on a line that passes through southeastern Missouri, western Illinois, eastern Iowa, and western Wisconsin. Along this line, the magnetic declination is zero. West of this line, the magnetic declination is positive and reaches 18° in Seattle. East of this line, the declination is negative, up to −18° in Maine. A map showing the magnetic declinations in the United States as of 2004 is presented in Figure 7.9.

Because the positions of Earth's magnetic poles move with time, the magnetic declinations for all locations on Earth's surface also change with time. For example, Figure 7.10 shows the estimated magnetic declination for Lansing, Michigan, for the period 1900−2004. A similar graph can be drawn for any location on Earth.

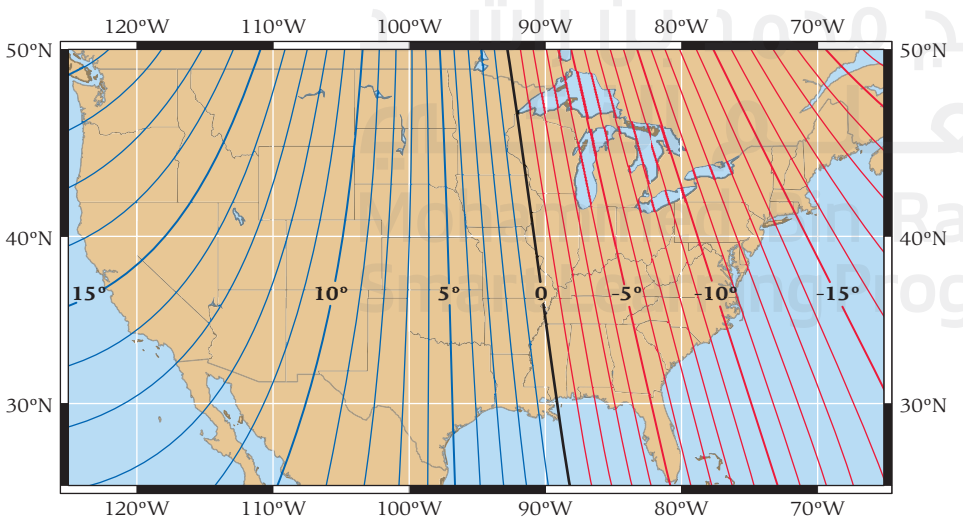


FIGURE 7.9 Magnetic declinations in the United States in 2004. Red lines represent negative magnetic declinations, and blue lines signify positive magnetic declinations. Lines of magnetic declination are separated by 1 degree.

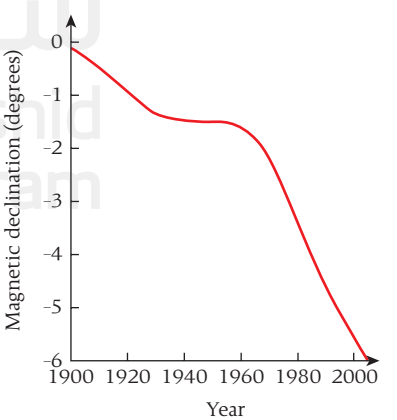


FIGURE 7.10 The magnetic declination at Lansing, Michigan, from 1900 to 2004.

Superposition of Magnetic Fields

If several sources of magnetic field, such as several permanent magnets, are close together, the magnetic field at any given point in space is given by the superposition of the magnetic fields from all the sources. The superposition principle for the total magnetic field, $\vec{B}_{\text{total}}(\vec{r})$, due to n magnetic field sources can be stated as

$$\vec{B}_{\text{total}}(\vec{r}) = \vec{B}_1(\vec{r}) + \vec{B}_2(\vec{r}) + \cdots + \vec{B}_n(\vec{r}). \tag{7.1}$$

7.2 Magnetic Force

The qualitative discussion in the preceding section pointed out that a magnetic field has a direction, along the magnetic field lines. The magnitude of a magnetic field is determined by examining its effect on a moving charged particle. We'll start with a constant magnetic field and study its effect on a single charge. As a reminder, the electric field exerts a force on a charge given by $\vec{F}_E = q\vec{E}$. Experiments such as the one shown in Figure 7.11 show that a magnetic field does not exert a force on a charge at rest but only on a *moving* charge.

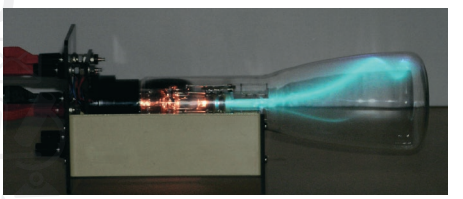


FIGURE 7.11 A beam of electrons (bluish green), made visible by a small amount of gas in an evacuated tube, is bent by a magnet (at the right edge of picture).

A magnetic field is defined in terms of the force exerted by the field on a moving charged particle. The magnetic force exerted by a magnetic field on a moving charged particle with charge q moving with velocity $\vec{v}$ is given by

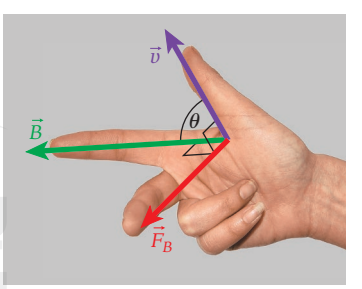
$$\vec{F}_B = q\vec{v} \times \vec{B}. \tag{7.2}$$



The direction of the force is perpendicular to both the velocity of the moving charged particle and the magnetic field (Figure 7.12). This statement is right-hand rule 1. The right-hand rule gives the force direction on a positive charge given the known velocity and field directions. However, for a negative charge the force will be in the opposite direction.

The magnitude of the magnetic force on a moving charged particle is

$$F_B = |q| vB \sin\theta, \tag{7.3}$$



where θ is the angle between the velocity of the charged particle and the magnetic field. (The angle θ is always between 0° and 180°, and therefore, $\sin\theta \geq 0$.) You can see that no magnetic force acts on a charged particle moving parallel to a magnetic field because in that case $\theta = 0°$. If a charged particle is moving perpendicularly to the magnetic field, $\theta = 90°$ and (for fixed values of v and B) the magnitude of the magnetic force has its maximum value of

$$F_B = |q| vB \quad \left(\text{for } \vec{v} \perp \vec{B}\right). \tag{7.4}$$

FIGURE 7.12 Right-hand rule 1 for the force exerted by a magnetic field, on a particle with charge q moving with velocity To find the direction of the magnetic force, point your thumb in the direction of the velocity of the moving charged particle and your index finger in the direction of the magnetic field, and your middle finger then gives you the direction of the magnetic force.

Magnetic Force and Work

Equation 7.2 established that the magnetic force is the vector product of the velocity vector and magnetic field vector and thus is perpendicular to both vectors. This implies that $\vec{F}_B \cdot \vec{v} = 0$ and, since the force is the product of mass and acceleration, also that $\vec{a} \cdot \vec{v} = 0$. We saw that this condition means that the direction of the velocity vector can change but the magnitude of the velocity vector, the speed, remains the same. Therefore, the kinetic energy, $\frac{1}{2}mv^2$, remains constant for a particle subjected to a magnetic force, and the magnetic force *does no work* on the moving particle.

This is a profound result: A constant magnetic field cannot be used to do work on a particle. The kinetic energy of a particle moving in a constant magnetic field remains constant, even though the direction of the particle's velocity vector can change as a function of time while the particle is moving through the magnetic field. An electric field, on the other hand, can easily be used to do work on a particle.

Units of Magnetic Field Strength

To discuss the motion of charges in magnetic fields, we need to know what units are used to measure the magnetic field strength. Solving equation 7.4 for the field strength and inserting the units of the other quantities gives

$$[F_B] = [q][v][B] \Rightarrow [B] = \frac{[F_B]}{[q][v]} = \frac{\text{N s}}{\text{C m}}.$$

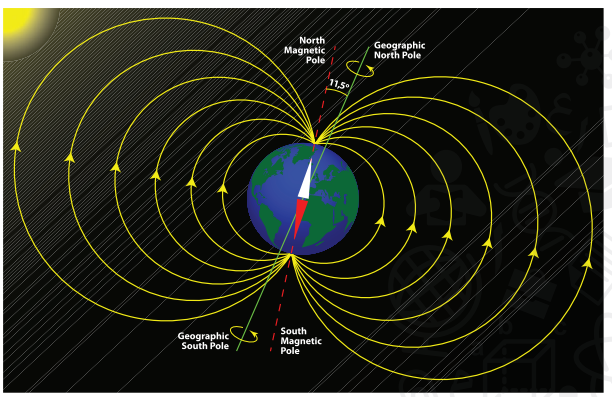


FIGURE 7.13 Global map of the strength of Earth's magnetic field.

Because the ampere (A) is defined as 1 C/s, (N s)/(C m) = N/(A m). The unit of magneticfield strength has been named the **tesla** (T), in honor of Croatian-born American physicist and inventor Nikola Tesla (1856–1943):

$$1\ \text{T} = 1\ \frac{\text{N s}}{\text{C m}} = 1\ \frac{\text{N}}{\text{A m}}.$$

A tesla is a rather large amount of magnetic field strength. Sometimes magnetic field strength is given in gauss (G), which is not an SI unit:

$$1\ \text{G} = 10^{-4}\ \text{T}.$$

For example, the strength of Earth's magnetic field at Earth's surface is on the order of 0.5 G (5×10^{-5} T). It varies with location from 0.2 G to 0.6 G, as illustrated in Figure 7.13.

SOLVED PROBLEM 7.1 Cathode Ray Tube

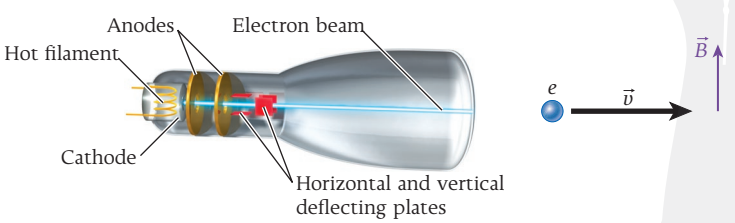


FIGURE 7.14 (a) A cathode ray tube. (b) Electrons moving with velocity $\vec{v}$ enter a constant magnetic field.

PROBLEM

Consider a cathode ray tube similar to the one shown in Figure 7.11. In this tube, a potential difference of $\Delta V = 111$ V accelerates electrons horizontally (starting essentially from rest) in an electron gun, as shown in Figure 7.14a. The electron gun has a specially coated filament that emits electrons when heated. A negatively charged cathode controls the number of electrons emitted. Positively charged anodes focus and accelerate the electrons into a beam. Downstream from the anodes are horizontal and vertical deflecting plates. Beyond the electron gun is a constant magnetic field with magnitude $B = 3.40 \times 10^{-4}$ T. The direction of the magnetic field is upward, perpendicular to the initial velocity of the electrons. What is the magnitude of the acceleration of the electrons due to the magnetic field? (The mass of an electron is 9.11×10^{-31} kg.)

SOLUTION

THINK The electrons gain kinetic energy in the electron gun of the cathode ray tube. The gain in kinetic energy of each electron is equal to the charge of the electron times the potential difference. The speed of the electrons can be found from the definition of kinetic energy. The magnetic force on an electron can be found from the electron charge, the electron velocity, and the strength of the magnetic field, and it is equal to the mass of the electron times its acceleration.

SKETCH Figure 7.14b shows an electron, moving with velocity $\vec{v}$, entering a constant magnetic field that is perpendicular to the electron path.

RESEARCH The change in kinetic energy, ΔK, of the electrons plus the change in potential energy of the electrons is equal to zero:

$$\Delta K + \Delta U = \tfrac{1}{2}mv^2 + q\Delta V = 0.$$

Since, in this case, $q = -e$, we see that

$$e\Delta V = \tfrac{1}{2}mv^2, \qquad \text{(i)}$$

where ΔV is the magnitude of the potential difference across which the electrons were accelerated and m is the mass of an electron. We can solve equation (i) for the speed of the electrons:

$$v = \sqrt{\frac{2e\Delta V}{m}}. \qquad \text{(ii)}$$

The magnitude of the force exerted by the magnetic field on the electrons is given by equation 7.3:

$$F_B = evB\sin 90° = evB,$$

where $-e$ is the charge of an electron and B is the magnitude of the magnetic field. According to Newton's Second Law, $F_{net} = ma$. Since the only force present is the magnetic one, we have

$$F_B = ma = evB, \qquad \text{(iii)}$$

where a is the magnitude of the acceleration of the electrons.

SIMPLIFY We can rearrange equation (iii) and substitute the expression for the speed of the electrons from equation (ii) to obtain the acceleration of the electrons:

$$a = \frac{evB}{m} = \frac{eB\sqrt{\frac{2e\Delta V}{m}}}{m} = B\sqrt{2\Delta V\frac{e^3}{m^3}}.$$

CALCULATE Putting in the numerical values gives us

$$a = \left(3.40\times10^{-4}\ \text{T}\right)\sqrt{2(111\ \text{V})\frac{\left(1.602\times10^{-19}\ \text{C}\right)^3}{\left(9.11\times10^{-31}\ \text{kg}\right)^3}} = 3.7357\times10^{14}\ \text{m/s}^2.$$

ROUND We report our result to three significant figures:

$$a = 3.74\times10^{14}\ \text{m/s}^2.$$

DOUBLE-CHECK The calculated acceleration is tremendously large, almost 40 trillion times the Earth's gravitational acceleration. So we certainly want to double-check. We first calculate the speed of the electrons:

$$v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2\left(1.602\times10^{-19}\ \text{C}\right)(111\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 6.25\times10^{6}\ \text{m/s}.$$

A speed of 6250 km/s may seem large, but it is reasonable for electrons because it is only 2% of the speed of light. The magnetic force on each electron is then

$$F_B = evB = \left(1.602\times10^{-19}\ \text{C}\right)\left(6.25\times10^{6}\ \text{m/s}\right)\left(3.40\times10^{-4}\ \text{T}\right) = 3.40\times10^{-16}\ \text{N}.$$

The acceleration is very large because the mass of an electron is very small.

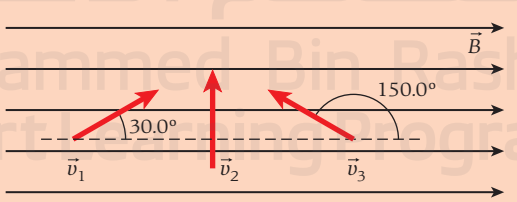
Concept Check 7.1

In what direction will the electron in Figure 7.14b be deflected as it enters the constant magnetic field?

a) into the page
b) out of the page
c) upward
d) downward
e) no deflection

Self-Test Opportunity 7.1

Three particles, each with charge $q = 6.15\ \mu\text{C}$ and speed $v = 465$ m/s, enter a uniform magnetic field with magnitude $B = 0.165$ T (see the figure). What is the magnitude of the magnetic force on each of the particles?



7.3 Motion of Charged Particles in a Magnetic Field

The fact that the force due to a magnetic field acting on a moving charged particle is perpendicular to both the field and the particle's velocity makes this force different from any we've considered so far. However, the tools we use to analyze this force—Newton's laws and the laws of conservation of energy, momentum, and angular momentum—are the same.

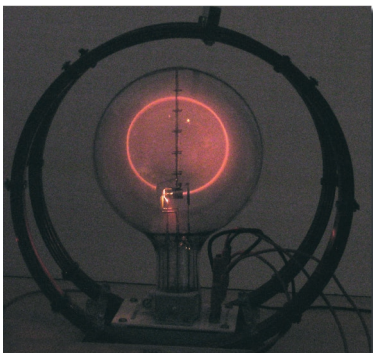


FIGURE 7.15 Electron beam bent into a circular path by the magnetic field generated by two coils.

Paths of Moving Charged Particles in a Constant Magnetic Field

Suppose you drive your car at constant speed around a circular track. The friction between the tires and the road provides the centripetal force that keeps the car moving in a circle. This force always points toward the center of the circle and creates a centripetal acceleration. A similar physical situation occurs when a particle with charge q and mass m moves with velocity $\vec{v}$ perpendicular to a uniform magnetic field, $\vec{B}$, as illustrated in Figure 7.15.

In this situation, the particle moves in a circle with constant speed v and the magnetic force of magnitude $F_B = |q|vB$ supplies the centripetal force that keeps the particle moving in a circle. Particles with opposite charges and the same mass will orbit in opposite directions at the same orbital radius. For example, electrons and positrons are elementary particles with the same mass; the electron has a negative charge, and the positron has a positive charge. Figure 7.16 is a bubble chamber photograph showing two electron-positron pairs. A bubble chamber is a device that can track charged particles moving in a constant magnetic field. Pair 1 has an electron and a positron that have the same relatively low speed. The particles initially travel in a circle. However, as they move through the bubble chamber, they slow down. (This slowdown is not due to the magnetic force but to collisions of the particles with the molecules of the gas in the bubble chamber.) Thus, the radius of the circle gets smaller and smaller, creating a spiral. The electron and positron in pair 2 have a much higher speed. Their tracks are curved but do not form a complete circle before the particles exit the bubble chamber.

If the velocity of a charged particle is parallel (or antiparallel) to the magnetic field, the particle experiences no magnetic force and continues to travel in a straight line.

For motion perpendicular to a magnetic field, as in Figure 7.15, the force required to keep a particle moving with speed v in a circle with radius r is the centripetal force:

$$F = \frac{mv^2}{r}.$$

Setting this expression for the centripetal force equal to that for the magnetic force, we obtain

$$vB|q| = \frac{mv^2}{r}.$$

Rearranging gives an expression for the radius of the circle in which the particle is traveling:

$$r = \frac{mv}{|q|B}. \tag{7.5}$$

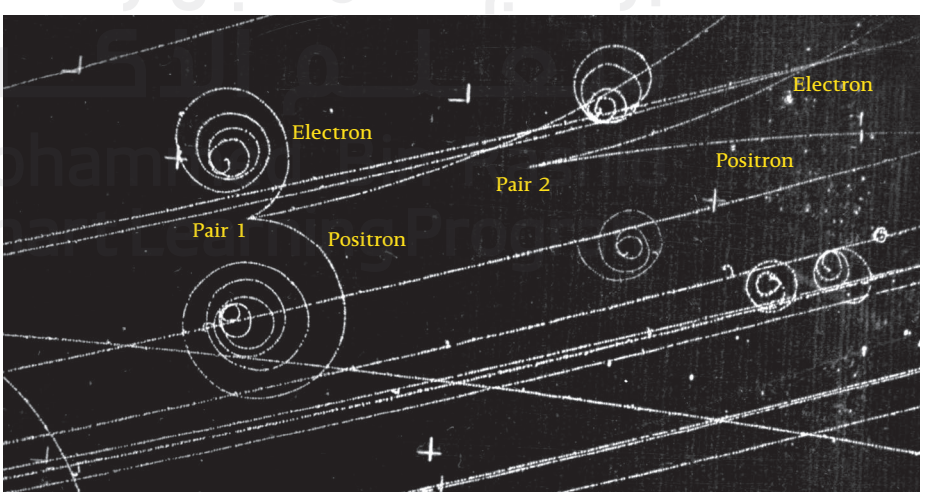


FIGURE 7.16 Bubble chamber photograph showing two electron-positron pairs. The bubble chamber is located in a constant magnetic field pointing directly out of the page.

A common way to express this relationship is in terms of the magnitude of the momentum of the particle:

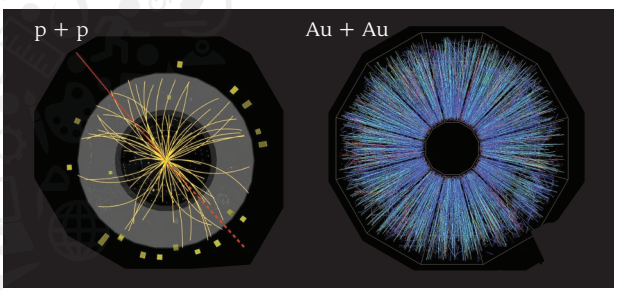
$$Br = \frac{p}{|q|}. \tag{7.6}$$

If the velocity $\vec{v}$ is not perpendicular to $\vec{B}$, then the velocity component perpendicular to $\vec{B}$ causes circular motion while the parallel velocity component is unaffected by $\vec{B}$ and drags this orbit into a helical shape.

Time Projection Chamber

Particle physicists create new elementary particles by colliding larger particles at the highest energies. In these collisions, many particles stream away from the interaction point at high speeds. A simple particle detector is not sufficient to identify these particles. A device that can help physicists study these collisions is a time projection chamber (TPC). The STAR TPC was described in Example 2.4.

Figure 7.17 shows collisions of two protons and two gold nuclei. The proton-proton collision creates dozens of particles; the gold-gold collision creates thousands of particles. Each charged particle leaves a track in the chamber. The color assigned by a computer to the track represents the ionization density of the track as particles pass through the gas of the chamber. As they pass through the gas, the particles ionize the atoms of the gas, releasing free electrons. The gas allows the free electrons to drift without recombining with positive ions. Electric fields applied between the center of the chamber and the end caps of the cylinder exert an electric force on the free electrons, making them drift toward the end caps, where they are recorded electronically. Using the drift time and the recording positions, computer software reconstructs the trajectories that the particles took through the chamber. The particles produced in the collisions have a velocity component that is perpendicular to the chamber's magnetic field and thus have circular trajectories.



(a) (b)

FIGURE 7.17 Curved tracks left by the motion of charged particles produced in collisions of (a) two protons, each with kinetic energy of 100 GeV, and (b) two gold nuclei, each with kinetic energy of 100 GeV.

EXAMPLE 7.1 Transverse Momentum of a Particle Seen by the ATLAS

One track of a moving charged particle from Figure 7.17a is shown in Figure 7.18. The radius of the circular trajectory this particle is following is $r = 2.300$ m. The magnitude of the magnetic field in the TPC is $B = 0.5000$ T. We can assume that the particle has charge $|q| = 1.602177 \times 10^{-19}$ C.

PROBLEM

What is the component of the particle's momentum that is perpendicular to the magnetic field?

SOLUTION

We'll call this component the transverse momentum of the particle, p_t. We use equation 7.6, replacing p with p_t, because the magnetic force depends only on p_t, and not on the component of the momentum that is parallel to $\vec{B}$:

$$Br = \frac{p_t}{|q|}.$$

We can express the magnitude of the transverse momentum of the particle in terms of the magnitude of the TPC's magnetic field and the absolute value of the charge of the particle:

$$p_t = |q|Br = \left(1.602177 \times 10^{-19}\ \text{C}\right)(0.5000\ \text{T})(2.300\ \text{m}) = 1.843 \times 10^{-19}\ \text{kg m/s}.$$

– Continued

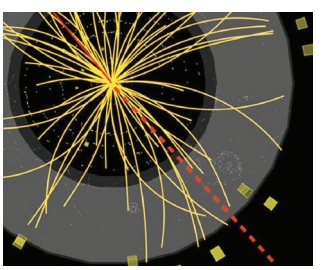


FIGURE 7.18 Circle fitted to the trajectory of one of the charged particles produced from the proton-proton collision in the ATLAS detector at CERN shown in Figure 7.17a.

Instead of these SI units for momentum, particle physicists often use MeV/c (recall Example 7.5). Since 1 MeV $= 1.602 \times 10^{-13}$ J, the conversion between kg m/s and MeV/c is

$$1 \text{ MeV}/c = \frac{1.60218 \times 10^{-13} \text{J}}{2.99792 \times 10^{8} \text{m/s}} = 5.3443 \times 10^{-22} \text{kg m/s} \Leftrightarrow 1 \text{ kg m/s} = 1.87115 \times 10^{21} \text{MeV}/c.$$

Therefore, our result for the transverse momentum is

$$p_t = 344.8 \text{ MeV}/c.$$

The analysis of a particle's transverse momentum carried out here can be done by automated computer algorithms on up to approximately 5000 charged particles created in a single collision of two gold nuclei. This very complex task takes about 30 seconds for a computer (3-GHz processor) to finish. In contrast, the TPC can record up to 1000 events per second, which corresponds to 1 millisecond per event.

EXAMPLE 7.2 The Solar Wind and Earth's Magnetic Field

Section 7.1 discussed the Van Allen radiation belts that trap particles emitted from the Sun. The Sun throws approximately 1 million tons of matter into space every second. This matter is mostly protons traveling at a speed of around 400. km/s.

PROBLEM

If protons from the Sun are incident perpendicular to Earth's magnetic field (which has a magnitude of 50.0 μT at the Equator), what is the radius of the orbit of the protons? The mass of a proton is 1.67×10^{-27} kg.

SOLUTION

Equation 7.5 relates the magnitude of the magnetic field, B, the radius of a circular orbit, r, and the speed, v, of a particle with mass m and charge q traveling perpendicular to a magnetic field:

$$r = \frac{mv}{|q|B}.$$

Putting in the numerical values, we get

$$r = \frac{\left(1.67 \times 10^{-27} \text{ kg}\right)\left(400. \times 10^{3} \text{ m/s}\right)}{\left(1.602 \times 10^{-19} \text{ C}\right)\left(50.0 \times 10^{-6} \text{ T}\right)} = 83.5 \text{ m}.$$

Thus, the protons of the solar wind orbit around the Earth's magnetic field lines at the Equator in circles of radius 83.5 m. Protons that are incident on the Earth's magnetic field away from the Equator are not traveling perpendicular to the magnetic field, so their orbital radius is larger. However, the magnetic field lines are closer together, meaning that the field is stronger toward the poles. The protons thus spiral along the field lines as they approach the poles. The shape of the Earth's magnetic field forces these protons traveling toward the poles to reverse and travel back toward the Equator, trapping the protons in the Van Allen radiation belts. Thus, the Earth's magnetic field completely blocks the solar wind from reaching the Earth's surface. This is vital, because the blocked cosmic radiation would otherwise make it impossible for higher organisms to live on Earth by ionizing (removing electrons from) atoms and destroying large molecules, for example, DNA.

Cyclotron Frequency

If a particle performs a complete circular orbit inside a uniform magnetic field–for example, like the electrons in the beam shown in Figure 7.15–then the period of revolution, T, of the particle is the circumference of the circle divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}. \tag{7.7}$$

The frequency, f, of the motion of the charged particle is the inverse of the period:

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}. \tag{7.8}$$

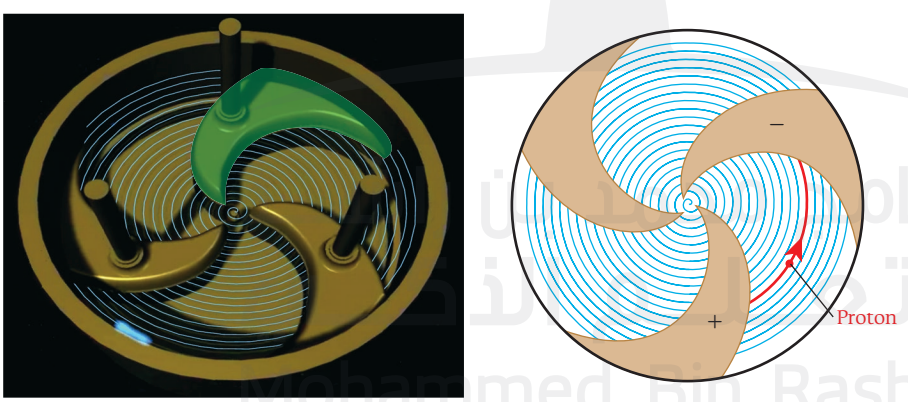
The angular speed, ω, of the motion is

$$\omega = 2\pi f = \frac{|q|B}{m}. \tag{7.9}$$

Thus, the frequency and the angular speed of the particle's motion are independent of the particle's speed and thus independent of the particle's kinetic energy. This fact is used in cyclotrons, which is why ω as given by equation 7.9 is referred to as the **cyclotron frequency.** In a cyclotron, particles are accelerated to higher and higher kinetic energies, and the fact that the cyclotron frequency is independent of the kinetic energy makes designing a cyclotron much easier.

EXAMPLE 7.3 Energy of a Cyclotron

A cyclotron is a particle accelerator (Figure 7.19). The golden horn-shaped pieces of metal shown in the figure (historically called *dees*) have alternating electric potentials applied to them, so a positively charged particle always has a negatively charged dee ahead when it emerges from under any dee, which is now positively charged. The resulting electric field accelerates the particle. Because the cyclotron sits in a strong magnetic field, the particle's trajectory is curved. The radius of the trajectory is proportional to the magnitude of the particle's momentum, according to equation 7.6, so the accelerated particle spirals outward until it reaches the edge of the magnetic field (where its path is no longer bent by the field) and is extracted. According to equation 7.9, the angular frequency is independent of the particle's momentum or energy, so the frequency with which the polarity of the dees is changed does not have to be adjusted as the particle is accelerated. (This holds true only as long as the speed of the accelerated particles does not approach a sizable fraction of the speed of light. To compensate for relativistic effects, the magnetic field of a cyclotron increases with the orbital radius of the accelerated particles.)



(a) (b)

FIGURE 7.19 (a) Computer-generated drawing of the central section of the K500 superconducting cyclotron at the National Superconducting Cyclotron Laboratory at Michigan State University, with the spiral trajectory of an accelerated particle superimposed. One of the three dees of the cyclotron is highlighted in green. (b) Top view of the K500, showing a proton being accelerated between two dees.

PROBLEM

What is the kinetic energy, in mega-electron-volts (MeV), of a proton extracted from a cyclotron with radius $r = 1.81$ m, if the magnetic field of the cyclotron is uniform and has magnitude $B = 0.851$ T? The mass of a proton is 1.67×10^{-27} kg.

SOLUTION

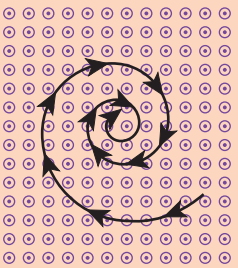
We can solve equation 7.5 for the speed, v, of the proton:

$$v = \frac{r|q|B}{m}.$$

– Continued

Self-Test Opportunity 7.2

A uniform magnetic field is directed out of the page (the standard notation of a dot within a circle represents the tip of the arrowhead of a field line). A charged particle is traveling in the plane of the page, as shown by the arrows in the figure.



a) Is the charge of the particle positive or negative?

b) Is the particle slowing down, speeding up, or moving at constant speed?

c) Is the magnetic field doing work on the particle?

Concept Check 7.2

Cosmic rays would continuously bombard Earth's surface if most of them were not deflected by Earth's magnetic field. Given that Earth is approximately a magnetic dipole (see Figure 7.6), the intensity of cosmic rays incident on its surface is greatest at the

a) North and South Poles.

b) Equator.

c) middle latitudes.

We substitute this expression for v into the equation for kinetic energy:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{r|q|B}{m}\right)^2 = \frac{r^2q^2B^2}{2m}.$$

Putting in the given numbers, we get the kinetic energy in joules:

$$K = \frac{(1.81\ \text{m})^2\left(1.602\times10^{-19}\ \text{C}\right)^2(0.851\ \text{T})^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)} = 1.82\times10^{-11}\ \text{J}.$$

Since 1 eV = 1.602×10^{-19} J and 1 MeV = 10^6 eV, we have

$$K = 1.82\times10^{-11}\ \text{J}\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{MeV}}{10^6\ \text{eV}}\right) = 114\ \text{MeV}.$$

Mass Spectrometer

One application of the motion of charged particles in a magnetic field is a **mass spectrometer,** which allows precise determination of atomic and molecular masses and can be useful for carbon dating and the analysis of unknown chemical compounds. A mass spectrometer operates by ionizing the atoms or molecules to be studied and accelerating them through an electric potential. The ions are then passed through a velocity selector (described further in Solved Problem 7.2), which allows only ions with a given velocity to pass through and blocks the remaining ions. The ions then enter a region of constant magnetic field. In the magnetic field, the radius of curvature of the orbit of each ion is given by equation 7.5: $r = mv/|q|B$. Assuming that all the atoms or molecules are singly ionized (have a charge of +1 or −1), the radius of curvature is proportional to the mass of the ion. A schematic diagram of a mass spectrometer is shown in Figure 7.20.

Ions with different masses will have orbits with different radii in the constant magnetic field. For example, in Figure 7.20, ions with orbital radius r_1 have a smaller mass than ions with orbital radius r_2. The particle detector measures the distances from the entrance point, d_1 and d_2, which can be related to the orbital radii and thus the mass of the ions.

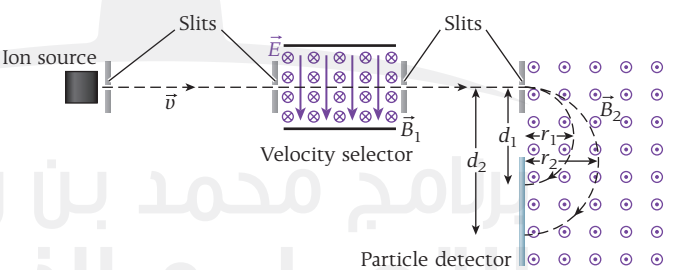


FIGURE 7.20 Schematic diagram of a mass spectrometer showing an ion source, a velocity selector consisting of crossed electric and magnetic fields (see Solved Problem 7.2), a region of constant magnetic field, and a particle detector.

SOLVED PROBLEM 7.2 Velocity Selector

Protons are accelerated from rest through an electric potential difference of $\Delta V = 14.0$ kV. The protons enter a velocity selector, consisting of a parallel plate capacitor in a constant magnetic field, directed perpendicularly into the plane of the page in Figure 7.21a. The electric field between the plates of the parallel plate capacitor is $\vec{E} = 4.30 \times 10^5$ V/m, directed along the plane of the page and downward in Figure 7.21a. This arrangement of perpendicular electric and magnetic fields is referred to as *crossed fields*.

PROBLEM

What magnetic field is required for the protons to move through the velocity selector without being deflected?

SOLUTION

THINK For a proton to move on a straight line without deflection requires that the net force on the proton be zero. Since the proton has a certain velocity and since the magnetic force depends on the velocity, it is plausible that this condition of zero net force cannot be realized for arbitrary speeds of the proton–hence the name *velocity selector*.

SKETCH Figure 7.21b shows the electric and magnetic forces on the protons as they pass through the velocity selector. Note that the two forces point in opposite directions.

RESEARCH The change in kinetic energy of the protons plus the change in electric potential energy is equal to zero, which can be expressed as

$$\Delta K = -\Delta U = \tfrac{1}{2}mv^2 = e\Delta V,$$

where m is the mass of a proton, v is the speed of the proton after acceleration, e is the charge of the proton, and ΔV is the electric potential difference across which the protons were accelerated. The speed of the protons after acceleration is

$$v = \sqrt{\frac{2e\Delta V}{m}}. \quad \text{(i)}$$

When the protons enter the velocity selector, the direction of the electric force is in the direction of the electric field, which is downward (negative y-direction). The magnitude of the electric force is

$$F_E = eE, \quad \text{(ii)}$$

where E is the magnitude of the electric field in the velocity selector. Right-hand rule 1 gives the direction of the magnetic force: With your thumb in the direction of the velocity of the protons (positive x-direction) and your index finger in the direction of the magnetic field (into the page), your middle finger points up (positive y-direction). Thus, the direction of the magnetic force on the protons is upward. The magnitude of the magnetic force is given by

$$F_B = evB, \quad \text{(iii)}$$

where B is the magnitude of the magnetic field in the velocity selector.

SIMPLIFY The condition that allows the protons to pass though the velocity selector without being deflected is that the electric force balances the magnetic force, or $F_E = F_B$. Using equations (ii) and (iii), we can express this condition as

$$eE = evB.$$

Solving for the magnetic field, B, and substituting for v from equation (i), we obtain

$$B = \frac{E}{\sqrt{\frac{2e\Delta V}{m}}} = E\sqrt{\frac{m}{2e\Delta V}}.$$

CALCULATE Putting in the numerical values gives us

$$B = \left(4.30\times10^{5}\ \text{V/m}\right)\sqrt{\frac{1.67\times10^{-27}\ \text{kg}}{2\left(1.602\times10^{-19}\ \text{C}\right)\left(14.0\times10^{3}\ \text{V}\right)}} = 0.262371\ \text{T}.$$

ROUND We report our result to three significant figures:

$$B = 0.262\ \text{T}.$$

DOUBLE-CHECK We verify that the electric force is equal to the magnetic force. The electric force is

$$F_E = eE = \left(1.602\times10^{-19}\ \text{C}\right)\left(4.30\times10^{5}\ \text{V/m}\right) = 6.89\times10^{-14}\ \text{N}.$$

To calculate the magnitude of the magnetic force, we need to find the speed of the protons:

$$v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2\left(1.602\times10^{-19}\ \text{C}\right)\left(14.0\times10^{3}\ \text{V}\right)}{1.67\times10^{-27}\ \text{kg}}} = 1.64\times10^{6}\ \text{m/s}.$$

– Continued

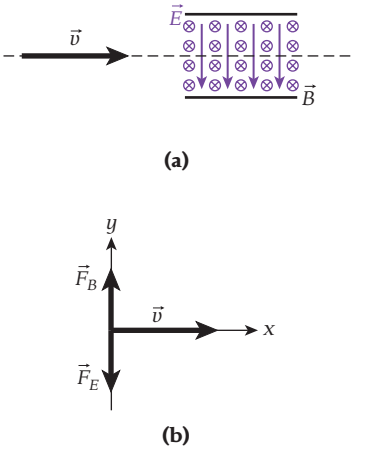


FIGURE 7.21 (a) A proton entering a velocity selector, consisting of crossed electric and magnetic fields. (b) Electric and magnetic forces on a proton passing through the combined fields.

This speed is 0.55% of the speed of light, which is not totally impossible. The magnitude of the magnetic force is then

$$F_B = evB = \left(1.602\times10^{-19}\ \text{C}\right)\left(1.64\times10^{6}\ \text{m/s}\right)(0.262\ \text{T}) = 6.88\times10^{-14}\ \text{N},$$

which agrees with the value of the electric force within rounding error. Thus, our result seems reasonable.

Magnetic Levitation

An interesting application of magnetic force is **magnetic levitation,** a situation in which an upward magnetic force on an object balances the downward gravitational force, achieving static equilibrium with no need for direct contact of surfaces. But if you try to balance a magnet over another magnet by orienting the north poles (or south poles) toward each other, you'll see right away that this is not possible. Instead, one of the magnets will simply flip over, and then the opposite poles will point toward each other, and the attractive force between them will cause the two magnets to snap together. A stable equilibrium requires a local minimum of the potential energy, which does not exist for the pure repulsive interaction of two like magnetic poles.

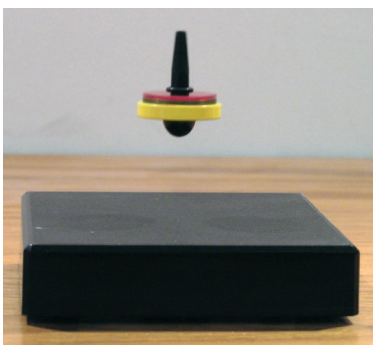


FIGURE 7.22 The Levitron, a toy demonstrating the magnetic levitation of a spinning magnet above a base magnet.

Figure 7.22 shows a toy called the Levitron demonstrating the principle of magnetic levitation. The magnetic top is spun on a plate and then lifted to the proper height and released. The top can remain suspended for several minutes. How does this toy work, considering the requirement for stable equilibrium just mentioned? The answer is that the rapid rotation of the top provides a sufficiently large angular momentum and creates a potential energy barrier that prevents the magnet from flipping over.

Of course, there are other ways to create stable magnetic levitation systems, all of them involving multiple magnets attached rigidly to each other. Magnetic levitation has real-world applications in magnetic levitation (maglev) trains. These trains have several advantages over trains that run on normal steel rails: There are no moving parts to wear out, there is less vibration, and reduced friction means that high speeds are possible. Several maglev trains are already in service around the world and more are being planned. One example is the Shanghai Maglev Train (Figure 7.1), which operates between the Shanghai Pudong Airport and downtown Shanghai and reaches speeds of up to 430 km/h (267 mph).

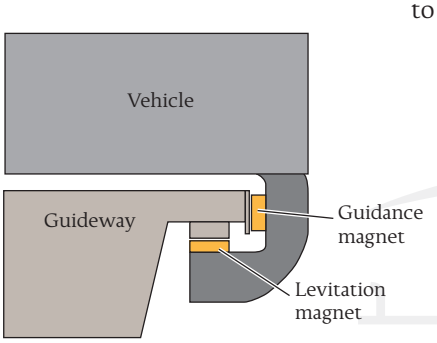


FIGURE 7.23 Cross section of one side of a train car of the Shanghai Maglev Train. The levitation magnets lift the cars 15 cm off the guideway, and the guidance magnets keep the cars centered on the guideway. The magnets are all mounted on the moving vehicle.

The Shanghai Maglev Train operates using magnets attached to the cars (Figure 7.23). These are normal, non-superconducting magnetic coils with electronic feedback to produce stable levitation and guidance. The train cars are held 15 cm above the guideway to allow clearance of any objects that may be on the guideway. The levitation and guidance magnets are held at a distance of 10 mm from the guideway, which is constructed of a magnetic material. The propulsion of the train is provided by magnetic fields built into the guideway. The train propulsion system operates like an electric motor (see Section 7.5) whose circular loops have been unwrapped to a linear configuration.

Maglev trains that use superconducting magnets have been tested, but some technical problems have yet to be resolved, including the maintenance of the superconducting coils and the exposure of the passengers to high magnetic fields.



7.4 Magnetic Force on a Current-Carrying Wire

Consider a wire carrying a current, i, in a constant magnetic field, $\vec{B}$ (Figure 7.24a). The magnetic field exerts a force on the moving charges in the wire. The charge, q, flowing past a point in the wire in a given time, t, is $q = ti$. During this time, the charge occupies a length, L, of wire given by $L = v_d t$, where v_d is the drift speed (the magnitude of the drift velocity) of the charge carriers in the wire. Thus, we obtain

$$q = ti = \frac{L}{v_d} i. \tag{7.10}$$

The magnitude of the magnetic force is then

$$F_B = qv_d B \sin\theta = \left(\frac{L}{v_d} i\right) v_d B \sin\theta = iLB \sin\theta, \qquad (7.11)$$

where θ is the angle between the direction of the current flow and the direction of the magnetic field. The direction of the force is perpendicular to both the current and the magnetic field and is given by a variant of right-hand rule 1, with the current in the direction of the velocity of a charged particle, as illustrated in Figure 7.24b. This variant of right-hand rule 1 takes advantage of the fact that current can be thought of as charges in motion.

Equation 7.11 can be expressed as a vector product:

$$\vec{F}_B = i\vec{L} \times \vec{B}, \qquad (7.12)$$

where the notation $i\vec{L}$ represents the current in a length of wire. Equation 7.12 is simply a reformulation of equation 7.2 for the case in which the moving charges make up a current flowing in a wire. Since physical situations involving currents are far more common than those involving the motion of an isolated charged particle, equation 7.12 is the most useful form for determining the magnetic force in practical applications.

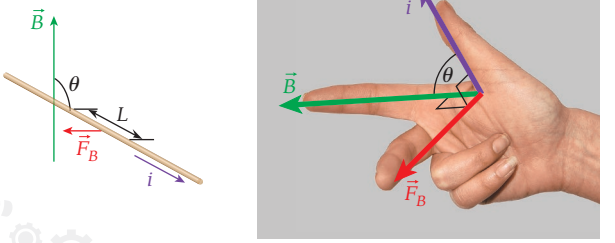


FIGURE 7.24 (a) Magnetic force on a current-carrying wire. (b) A variant of right-hand rule 1 giving the direction of the magnetic force on a current-carrying wire. To determine the direction of the force on a current-carrying wire using your right hand, point your thumb in the direction of the current and your index finger in the direction of the magnetic field; then your middle finger will point in the direction of the force.

EXAMPLE 7.4 Force on the Voice Coil of a Loudspeaker

A loudspeaker produces sound by exerting a magnetic force on a voice coil in a magnetic field, as shown in Figure 7.25. The movable voice coil is connected to a speaker cone that actually produces the sounds. The magnetic field is produced by the two permanent magnets as shown. The magnitude of the magnetic field is $B = 1.50$ T. The voice coil is composed of $n = 100$ turns of wire carrying a current, $i = 1.00$ mA. The diameter of the voice coil is $d = 2.50$ cm.

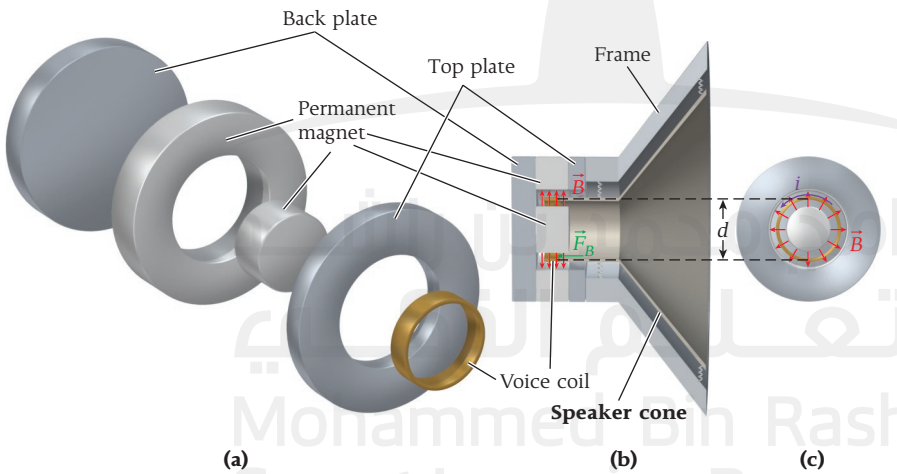


FIGURE 7.25 Schematic diagram of a loudspeaker: (a) an exploded three-dimensional view of the driver end of the loudspeaker; (b) a cross-sectional side view of the loudspeaker; (c) a front view of the driver end of the loudspeaker.

PROBLEM

What is the magnetic force exerted by the magnetic field on the loudspeaker's voice coil?

SOLUTION

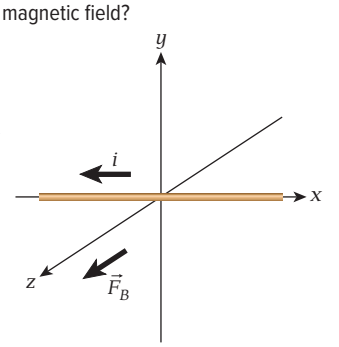
The magnitude of the magnetic force on the voice coil is given by equation 7.11:

$$F = iLB \sin\theta,$$

– Continued

Concept Check 7.3

The figure shows a wire lying along the x-axis with a current, i, flowing in the negative x-direction. The wire is in a uniform magnetic field. The magnetic force, $\vec{F}_B$, acts on the wire in the positive z-direction. The magnetic field is oriented so that the force is maximum. What is the direction of the magnetic field?



a) positive y-direction
b) negative x-direction
c) negative y-direction
d) positive z-direction
e) negative z-direction

where L is the length of wire carrying current i in the magnetic field with magnitude B. The wire makes an angle θ with the magnetic field. In this case, the wire is always perpendicular to the magnetic field, so $\theta = 90°$. The length of wire in the voice coil is given by the number of turns, n, times the circumference, πd, of each turn

$$L = n\pi d.$$

Thus, the force on the voice coil is

$$F = i(n\pi d)B(\sin 90°) = n\pi idB.$$

Putting in the numerical values, we get

$$F = n\pi idB = (100)(\pi)\left(1.00\times10^{-3}\text{ A}\right)\left(2.50\times10^{-2}\text{ m}\right)(1.50\text{ T}) = 0.0118\text{ N}.$$

From the right-hand rule 1 illustrated in Figure 7.24b, the direction of the force exerted by the magnetic field on the voice coil is toward the left in Figure 7.25b and perpendicularly into the page in Figure 7.25c.

If the current in the voice coil is reversed, the force will be in the opposite direction. If the current is proportional to the amplitude of a sound wave, sound waves can be reproduced in the cone of the loudspeaker. This basic idea is used in most speakers and headphones.

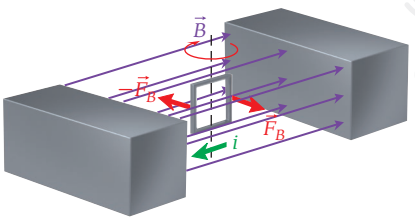


FIGURE 7.26 A primitive element of an electric motor consisting of a current-carrying loop in a magnetic field.

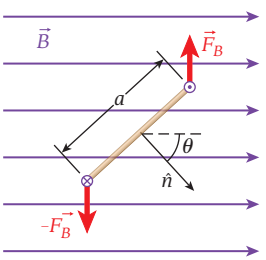


FIGURE 7.27 Bottom view of a current-carrying loop in a magnetic field, showing the forces acting on the loop.

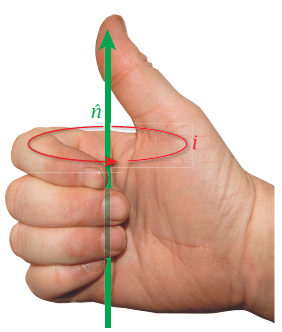


FIGURE 7.28 Right-hand rule 2 gives the direction of the unit normal vector for a current-carrying loop. According to the rule, if you curl the fingers of your right hand in the direction of the current in the loop, your thumb points in the direction of the unit normal vector.

7.5 Torque on a Current-Carrying Loop

Electric motors rely on the magnetic force exerted on a current-carrying wire. This force is used to create a torque that turns a shaft. Let's consider a simple electric motor, consisting of a single square loop carrying a current, i, in a constant magnetic field, $\vec{B}$. The loop is oriented so that its horizontal sections are parallel to the magnetic field and its vertical sections are perpendicular to the magnetic field, as shown in Figure 7.26. The magnitude of the magnetic force on the two vertical sections of the loop is given by equation 7.11 with $\theta = 90°$:

$$F = iLB.$$

The direction of the magnetic force is given by the variant of right-hand rule 1 illustrated in Figure 7.24b. The two magnetic forces, $\vec{F}_B$ and $-\vec{F}_B$, shown in Figure 7.26 have equal magnitudes and opposite directions. These forces create a torque that tends to rotate the loop around a vertical axis of rotation. These two forces sum to zero. The two horizontal sections of the loop are parallel to the magnetic field and thus experience no magnetic force. Thus, no net force acts on the coil, even though a torque is produced.

Now we consider the case where the loop rotates about its center. As the loop turns in the magnetic field, the forces on the vertical sides of the loop, perpendicular to the direction of the field, do not change. The forces on the square loop, with side length a, are illustrated in Figure 7.27, which shows a bottom view of the coil. In Figure 7.27, θ is the angle between a unit vector, $\hat{n}$, normal to the plane of the coil, and the magnetic field, $\vec{B}$. The unit normal vector is perpendicular to the plane of the wire loop and points in a direction given by right-hand rule 2 (Figure 7.28), based on the current flowing around the loop.

In Figure 7.27, the current is flowing out of the page in the right side of the loop, indicated by the dot in a circle (representing the tip of an arrowhead) and flowing into the page in the left side of the loop, indicated by the cross in a circle (representing the tail of an arrow). The magnitude of the force on each of these vertical segments is

$$F = iaB.$$

The forces on the two horizontal segments of the loop are parallel or antiparallel to the axis of rotation and do not cause a torque, and these two forces sum to zero. Therefore, there is no net force on the loop.

The sum of the torques on the two vertical segments of the square loop gives the net torque exerted on the loop about its center:

$$\tau_1 = (iaB)\left(\frac{a}{2}\right)\sin\theta + (iaB)\left(\frac{a}{2}\right)\sin\theta = ia^2B\sin\theta = iAB\sin\theta, \qquad (7.13)$$

where the index 1 on τ_1 indicates that it is the torque on a single loop and $A = a^2$ is the area of the loop. The reason that the loop continues to rotate and doesn't stop at $\theta = 0°$ is that it is connected to a device called a **commutator,** which causes the current to change directions as the coil rotates. This commutator consists of a split ring, with one end of the loop connected to each half of the loop, as shown in Figure 7.29. The current in the loop switches direction two times for every complete rotation of the loop.

If the single loop is replaced with a coil consisting of many loops wound closely together, the torque on the coil is found by multiplying the torque on a loop, τ_1 from equation 7.13, by the number of windings (loops in the coil), N:

$$\tau = N\tau_1 = NiAB\sin\theta. \tag{7.14}$$

Does this expression for the torque hold for other shapes with area A, other than squares? The answer is yes, though we will not prove it here.

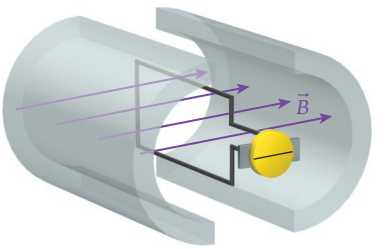


FIGURE 7.29 A wire loop connected to a source of current through a commutator ring.



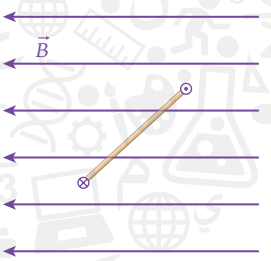
Concept Check 7.4

The top view of a current-carrying loop in a constant magnetic field is shown in the figure. The torque on the loop will cause it to rotate

a) clockwise.

b) counterclockwise.

c) not at all.



7.6 Magnetic Dipole Moment

A current-carrying coil can be described with one parameter, which contains information about a key characteristic of the coil in a magnetic field. The magnitude of the **magnetic dipole moment,** $\vec{\mu}$, of a current-carrying coil of wire is defined to be

$$\mu = NiA, \tag{7.15}$$

where N is the number of windings, i is the current through the wire, and A is the area of the loops. The direction of the magnetic dipole moment is given by right-hand rule 2 and is the direction of the unit normal vector, $\hat{n}$. Using equation 7.15, we can rewrite equation 7.14 as

$$\tau = (NiA)B\sin\theta = \mu B\sin\theta. \tag{7.16}$$

The torque on a magnetic dipole is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \tag{7.17}$$

That is, the torque on a current-carrying coil is the vector product of the magnetic dipole moment of the coil and the magnetic field.

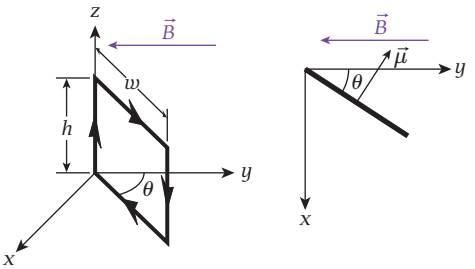
SOLVED PROBLEM 7.3 Torque on a Rectangular Current-Carrying Loop

A rectangular loop with height $h = 6.50$ cm and width $w = 4.50$ cm is in a uniform magnetic field of magnitude $B = 0.250$ T, which points in the negative y-direction (Figure 7.30a). The loop makes an angle of $\theta = 33.0°$ with the y-axis, as shown in the figure. The loop carries a current of magnitude $i = 9.00$ A in the direction indicated by the arrows.

PROBLEM

What is the magnitude of the torque on the loop around the z-axis?

– Continued



(a) (b)

FIGURE 7.30 (a) A rectangular loop carrying a current in a magnetic field. (b) View of the rectangular loop looking down on the *xy*-plane. The magnetic dipole moment is perpendicular to the plane of the loop, with a direction determined by right-hand rule 2.

SOLUTION

THINK The torque on the loop is equal to the vector cross product of the magnetic dipole moment and the magnetic field. The magnetic dipole moment is perpendicular to the plane of the loop, with the direction given by right-hand rule 2.

SKETCH Figure 7.30b is a view of the loop looking down on the *xy*-plane.

RESEARCH The magnitude of the magnetic dipole moment of the loop is

$$\mu = NiA = iwh. \tag{i}$$

The magnitude of the torque on the loop is

$$\tau = \mu B \sin\theta_{\mu B}, \tag{ii}$$

where $\theta_{\mu B}$ is the angle between the magnetic dipole moment and the magnetic field. From Figure 7.30b, we can see that

$$\theta_{\mu B} = \theta + 90°. \tag{iii}$$

SIMPLIFY We can combine equations (i), (ii), and (iii) to obtain

$$\tau = iwhB\sin(\theta + 90°).$$

CALCULATE Putting in the numerical values, we get

$$\tau = (9.00\text{ A})\left(4.50\times10^{-2}\text{ m}\right)\left(6.50\times10^{-2}\text{ m}\right)(0.250\text{ T})\left[\sin(33.0° + 90°)\right]$$
$$= 0.0055195\text{ N m}.$$

ROUND We report our result to three significant figures:

$$\tau = 5.52\times10^{-3}\text{ N m}.$$

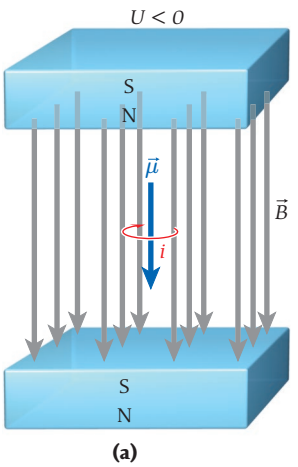
DOUBLE-CHECK The magnitude of the force on each of the vertical segments of the loop is

$$F_B = ihB = (9.00\text{ A})\left(6.50\times10^{-2}\text{ m}\right)(0.250\text{ T}) = 0.146\text{ N}.$$

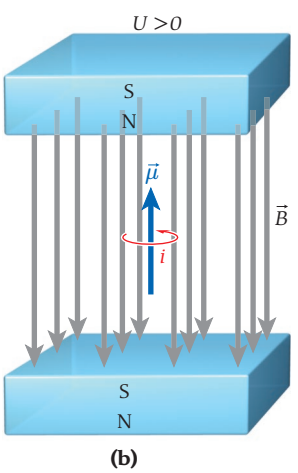
The magnitude of the torque is then the magnitude of the force on the vertical segment that is not along the *z*-axis times the moment arm (which is *w*) times the sine of the angle between the force and the moment arm:

$$\tau = Fw\sin(33.0° + 90°) = 0.146\text{ N}\left(4.50\times10^{-2}\text{ m}\right)\left[\sin(33.0° + 90°)\right] = 5.52\times10^{-3}\text{ N m}.$$

This is the same as the result calculated above.



(a)



(b)

FIGURE 7.31 Magnetic dipole moment vector in an external magnetic field: (a) magnetic dipole and external magnetic field are parallel, resulting in a negative potential energy; (b) magnetic dipole and external magnetic field are antiparallel, resulting in a positive potential energy.

A magnetic dipole has potential energy in an external magnetic field. If the dipole moment is aligned with the magnetic field, the dipole has its minimum potential energy. If the dipole moment is oriented in a direction opposite to the external field, the dipole has its maximum potential energy. The work done by a torque is

$$W = \int_{\theta_0}^{\theta} \tau(\theta')\, d\theta'. \tag{7.18}$$

Using the work-energy theorem and equation 7.16 and setting $\theta_0 = 90°$, we can express the magnetic potential energy, *U*, of a magnetic dipole in an external magnetic field, $\vec{B}$, as

$$W = \int_{\theta_0}^{\theta} \tau(\theta')\, d\theta' = \int_{\theta_0}^{\theta} \mu B\sin\theta'\, d\theta' = -\mu B\cos\theta'\Big|_{\theta_0}^{\theta} = U(\theta) - U(90°),$$

or

$$U(\theta) = -\mu B\cos\theta = -\vec{\mu}\cdot\vec{B}, \tag{7.19}$$

where θ is the angle between the magnetic dipole moment and the external magnetic field.

The lowest value, $-\mu B$, of the potential energy of a magnetic dipole in an external magnetic field is achieved when the dipole's magnetic moment vector is parallel to the external magnetic field vector, and the highest value, $+\mu B$, results when the two vectors are antiparallel (see Figure 7.31). This dependence of potential energy on orientation occurs

in diverse physical situations, for which magnetic dipoles in external magnetic fields are a simple model. So far, the only magnetic dipoles we have discussed are current-carrying loops. However, other types of magnetic dipoles exist, including bar magnets and even the Earth. In addition, elementary charged particles such as protons have intrinsic magnetic dipole moments.

Self-Test Opportunity 7.3

What is the maximum difference in magnetic potential energy between two orientations of a loop with area 0.100 m^2 carrying a current of 2.00 A in a constant magnetic field of magnitude 0.500 T?

7.7 Hall Effect

Consider a conductor carrying a current, i, flowing in a direction perpendicular to a magnetic field, $\vec{B}$ (Figure 7.32a). The electrons in the conductor are moving with velocity $\vec{v}_d$ in the direction opposite to the current. The moving electrons experience a force perpendicular to their velocity, causing them to move toward one edge of the conductor. After some time, many electrons have moved to one edge of the conductor, creating a net negative charge on that edge and leaving a net positive charge on the opposite edge of the conductor. This charge distribution creates an electric field, $\vec{E}$, which exerts a force on the electrons in a direction opposite to that exerted by the magnetic field. When the magnitude of the force exerted on the electrons by the electric field is equal to the magnitude of the force exerted on them by the magnetic field, the net number of electrons on the edges of the conductor no longer changes with time. This result is called the **Hall effect.** The potential difference, ΔV_H, between the edges of the conductor when equilibrium is reached is termed the **Hall potential difference,** given by

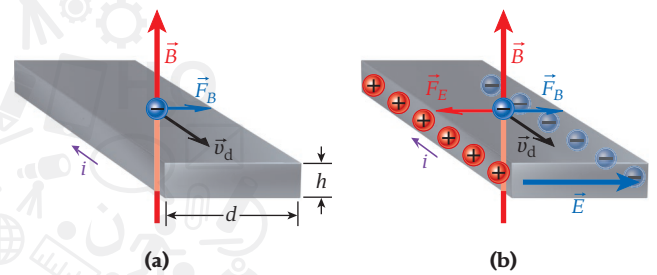


FIGURE 7.32 (a) A conductor carrying a current in a magnetic field. The charge carriers are electrons. (b) The electrons have drifted to one side of the conductor, leaving a net positive charge on the opposite side. This distribution of charges creates an electric field. The potential difference across the conductor is the Hall potential difference.

$$\Delta V_H = Ed, \tag{7.20}$$

where d is the width of the conductor and E is the magnitude of the created electric field.

The Hall effect can be used to demonstrate that the charge carriers in metals are negatively charged. If the charge carriers in a metal were positive and moving in the direction of the current shown in Figure 7.32a, those positive charges would collect on the same edge of the conductor as the electrons in Figure 7.32b, giving an electric field with the opposite sign. Thus, the charge carriers in conductors are negatively charged and must be electrons. The Hall effect also establishes that in some semiconductors the charge carriers are electron holes (missing electrons), which appear to be positively charged carriers.

The Hall effect can also be used to determine a magnetic field by measuring the current flowing through the conductor and the resulting potential difference across the conductor. To obtain the formula for the magnetic field, we start with the equilibrium condition of the Hall effect, that the magnitudes of the magnetic and electric forces are equal:

$$F_E = F_B \Rightarrow eE = v_d Be \Rightarrow B = \frac{E}{v_d} = \frac{\Delta V_H}{v_d d}, \tag{7.21}$$

where substitution for E from equation 7.20 is used in the last step. In Chapter 5, we saw that the drift speed, v_d, of an electron in a conductor can be related to the magnitude of the current density, J, in the conductor:

$$J = \frac{i}{A} = nev_d,$$

where A is the cross-sectional area of the conductor and n is the number of electrons per unit volume in the conductor. As shown in Figure 7.32a, the cross-sectional area is given by $A = dh$, where d is the width and h is the height of the conductor. Solving $i/A = nev_d$ for the drift speed and substituting hd for A gives

$$v_d = \frac{i}{Ane} = \frac{i}{hdne}.$$

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Substituting this expression for v_d into equation 7.21, we have

$$B = \frac{\Delta V_H}{v_d d} = \frac{\Delta V_H dhne}{id} = \frac{\Delta V_H hne}{i}. \tag{7.22}$$

Thus, equation 7.22 gives the magnetic field strength (magnitude) from a measured value of the Hall potential difference, ΔV_H, and the known height, h, and density of charge carriers, n, of the conductor. Equivalently, a rearranged form of equation 7.22 can be used to find the Hall voltage if the magnetic field strength is known:

$$\Delta V_H = \frac{iB}{neh}. \tag{7.23}$$

EXAMPLE 7.5 Hall Effect

Suppose we use a Hall probe to measure the magnitude of a constant magnetic field. The Hall probe is a strip of copper with a height, h, of 2.00 mm. We measure a voltage of 0.250 μV across the probe when we run a current of 1.25 A through it.

PROBLEM

What is the magnitude of the magnetic field?

SOLUTION

The magnetic field is given by equation 7.22:

$$B = \frac{\Delta V_H hne}{i}.$$

We have been given the values of V_H, h, and i, and we know e. The density of the electrons, n, is defined as the number of electrons per unit volume:

$$n = \frac{\text{number of electrons}}{\text{volume}}.$$

The density of copper is $\rho_{Cu} = 8.96\ \text{g/cm}^3 = 8960\ \text{kg/m}^3$, and 1 mole of copper has a mass of 63.5 g and $6.02 \cdot 10^{23}$ atoms. Each copper atom has one conduction electron. Thus, the density of the electrons is

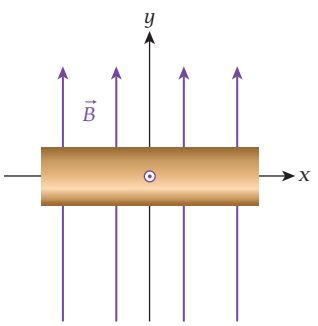
$$n = \left(\frac{1\ \text{electron}}{1\ \text{atom}}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{63.5\ \text{g}}\right)\left(\frac{8.96\ \text{g}}{1\ \text{cm}^3}\right)\left(\frac{1.0\times10^{6}\ \text{cm}^3}{1\ \text{m}^3}\right) = 8.49\times10^{28}\ \frac{\text{electrons}}{\text{m}^3}$$

We can now calculate the magnitude of the magnetic field:

$$B = \frac{\left(0.250\times10^{-6}\ \text{V}\right)(0.002\ \text{m})\left(8.49\times10^{28}\ \frac{\text{electrons}}{\text{m}^3}\right)\left(1.602\times10^{-19}\ \text{C}\right)}{1.25\ \text{A}} = 5.44\ \text{T}.$$

Concept Check 7.5

A conductor is carrying a current in a constant magnetic field, as shown in the figure. The resulting electric field due to the Hall effect is



a) in the positive x-direction.

b) in the negative x-direction.

c) in the positive y-direction.

d) in the negative y-direction.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- Magnetic field lines indicate the direction of a magnetic field in space. Magnetic field lines do not end on magnetic poles, but form closed loops instead.
- The magnetic force on a particle with charge q moving with velocity $\vec{v}$ in a magnetic field, $\vec{B}$, is given by $\vec{F} = q\vec{v} \times \vec{B}$ Right-hand rule 1 gives the direction of the force.
- For a particle with charge q moving with speed v perpendicular to a magnetic field of magnitude B, the magnitude of the magnetic force on the moving charged particle is $F = |q|vB$.
- The unit of magnetic field is the tesla, abbreviated T.
- The average magnitude of the Earth's magnetic field at the surface is approximately 0.5×10^{-4} T.
- A particle with mass m and charge q moving with speed v perpendicular to a magnetic field with magnitude B has a trajectory that is a circle with radius $r = mv / |q|B$.
- The cyclotron frequency, ω, of a particle with charge q and mass m moving in a circular orbit in a constant magnetic field of magnitude B is given by $\omega = |q|B/m$.

- The force exerted by a magnetic field, $\vec{B}$, on a length of wire, $\vec{L}$, carrying a current, i, is given by $\vec{F} = i\vec{L} \times \vec{B}$. The magnitude of this force is $F = iLB \sin\theta$, where θ is the angle between the direction of the current and the direction of the magnetic field.
- The magnitude of the torque on a loop carrying a current, i, in a magnetic field with magnitude B is $\tau = iAB \sin\theta$, where A is the area of the loop and θ is the angle between a unit vector normal to the loop and the direction of the magnetic field. Right-hand rule 2 gives the direction of the unit normal vector to the loop.
- The magnitude of the magnetic dipole moment of a coil carrying a current, i, is given by $\mu = NiA$, where N is the number of loops (windings) and A is the area of a loop. The direction of the dipole moment is given by right-hand rule 2 and is the direction in which the unit normal vector points.
- The Hall effect results when a current, i, flowing through a conductor with height h in a magnetic field of magnitude B produces a potential difference across the conductor (the Hall potential difference), given by $\Delta VH = iB/neh$, where n is the number of electrons per unit volume and e is the magnitude of an electron's charge.

ANSWERS TO SELF-TEST OPPORTUNITIES

7.1 Particle 1: $F_B = qvB \sin\theta = (6.15 \times 10^{-6}\ \text{C})(465\ \text{m/s})(0.165\ \text{T})(\sin 30.0^\circ) = 2.36 \times 10^{-4}\ \text{N}$.

Particle 2: $F_B = qvB \sin\theta = (6.15 \times 10^{-6}\ \text{C})(465\ \text{m/s})(0.165\ \text{T})(\sin 90.0^\circ) = 4.72 \times 10^{-4}\ \text{N}$.

Particle 3: $F_B = qvB \sin\theta = (6.15 \times 10^{-6}\ \text{C})(465\ \text{m/s})(0.165\ \text{T})(\sin 150.0^\circ) = 2.36 \times 10^{-4}\ \text{N}$.

7.2 a) positive

b) slowing down

c) no (Therefore, another force must be acting on the particle to slow it down.)

7.3 $\Delta U = U_{max} - U_{min} = 2\mu B = 2iAB = 2(2.00\ \text{A})(0.100\ \text{m}^2)(0.500\ \text{T}) = 0.200\ \text{J}$.

PROBLEM-SOLVING GUIDELINES

1. When working with magnetic fields and forces, you need to sketch a clear diagram of the problem situation in three dimensions. Often, a separate sketch of the velocity and magnetic field vectors (or of the length of wire and the field vectors) is useful to visualize the plane in which they lie, since the magnetic force will be perpendicular to that plane.

2. Remember that the right-hand rules apply for positive charges and currents. If a charge or a current is negative, you can use the right-hand rule, but the force will then be in the opposite direction.

3. A particle in both an electric and a magnetic field experiences an electric force, $\vec{F}_E = q\vec{E}$, and a magnetic force, $\vec{F}_B = q\vec{v} \times \vec{B}$. Be sure you take the vector sum of the individual forces.

MULTIPLE-CHOICE QUESTIONS

7.1 A magnetic field is oriented in a certain direction in a horizontal plane. An electron moves in a certain direction in the horizontal plane. For this situation, there

a) is one possible direction for the magnetic force on the electron.

b) are two possible directions for the magnetic force on the electron.

c) are infinite possible directions for the magnetic force on the electron.

7.2 A particle with charge q is at rest when a magnetic field is suddenly turned on. The field points in the z-direction. What is the direction of the net force acting on the charged particle?

a) in the x-direction

b) in the y-direction

c) The net force is zero.

d) in the z-direction

7.3 Which of the following has the largest cyclotron frequency?

a) an electron with speed v in a magnetic field with magnitude B

b) an electron with speed $2v$ in a magnetic field with magnitude B

c) an electron with speed $v/2$ in a magnetic field with magnitude B

d) an electron with speed $2v$ in a magnetic field with magnitude $B/2$

e) an electron with speed $v/2$ in a magnetic field with magnitude $2B$

7.4 In the Hall effect, a potential difference produced across a conductor of finite thickness in a magnetic field by a current flowing through the conductor is given by

a) the product of the density of electrons, the charge of an electron, and the conductor's thickness divided by the product of the magnitudes of the current and the magnetic field.

b) the reciprocal of the expression described in part (a).

c) the product of the charge on an electron and the conductor's thickness divided by the product of the density of electrons and the magnitudes of the current and the magnetic field.

d) the reciprocal of the expression described in (c).

e) none of the above.

7.5 An electron (with charge $-e$ and mass m_e) moving in the positive x-direction enters a velocity selector. The velocity selector consists of crossed electric and magnetic fields: $\vec{E}$ is directed in the positive y-direction, and $\vec{B}$ is directed in the positive z-direction. For a velocity v (in the positive x-direction), the net force on the electron is zero, and the electron moves straight through the velocity selector. With what velocity

will a proton (with charge $+e$ and mass $m_p = 1836\ m_e$) move straight through the velocity selector?

a) v c) $v/1836$
b) $-v$ d) $-v/1836$

7.6 In which direction does a magnetic force act on an electron that is moving in the positive x-direction in a magnetic field pointing in the positive z-direction?

a) the positive y-direction c) the negative x-direction
b) the negative y-direction d) any direction in the xy-plane

7.7 A charged particle is moving in a constant magnetic field. Which of the following statements concerning the magnetic force exerted on the particle is (are) true? (Assume that the magnetic field is not parallel or antiparallel to the velocity.)

a) It does no work on the particle.
b) It may increase the speed of the particle.
c) It may change the velocity of the particle.
d) It can act only on the particle while the particle is in motion.
e) It does not change the kinetic energy of the particle.

7.8 An electron moves in a circular trajectory with radius r_i in a constant magnetic field. What is the final radius of the trajectory when the magnetic field is doubled?

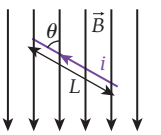
a) $\frac{r_i}{4}$ c) r_i
d) $2r_i$
b) $\frac{r_i}{2}$ e) $4r_i$

7.9 Protons in the solar wind reach Earth's magnetic field with a speed of 400 km/s. If the magnitude of this field is $5.0 \cdot 10^{-5}$ T and the velocity of the protons is perpendicular to it, what is the cyclotron frequency of the protons after entering the field?

a) 122 Hz c) 321 Hz e) 763 Hz
b) 233 Hz d) 432 Hz

7.10 An isolated segment of wire of length $L = 4.50$ m carries a current of magnitude $i = 35.0$ A at an angle $\theta = 50.3°$ with respect to a constant magnetic field of magnitude $B = 6.70 \times 10^{-2}$ T (see the figure). What is the magnitude of the magnetic force on the wire?

a) 2.66 N
b) 3.86 N
c) 5.60 N
d) 8.12 N
e) 11.8 N

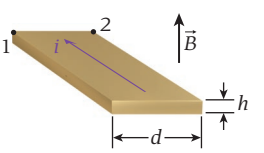


7.11 A coil is composed of circular loops of radius $r = 5.13$ cm and has $N = 47$ windings. A current, $i = 1.27$ A, flows through the coil, which is inside a homogeneous magnetic field of magnitude 0.911 T. What is the maximum torque on the coil due to the magnetic field?

a) 0.148 N m
b) 0.211 N m
c) 0.350 N m
d) 0.450 N m
e) 0.622 N m

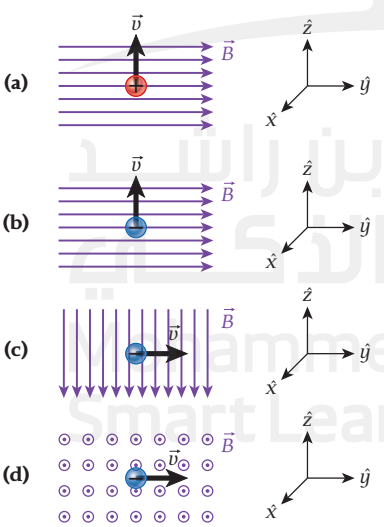
7.12 A copper conductor has a current, $i = 1.41$ A, flowing through it, perpendicular to a constant magnetic field, $B = 4.94$ T, as shown in the figure. The conductor is $d = 0.100$ m wide and $h = 2.00$ mm high. What is the electric potential difference between points 1 and 2?

a) 2.56×10^{-9} V
b) 5.12×10^{-9} V
c) 7.50×10^{-8} V
d) 2.56×10^{-7} V
e) 9.66×10^{-7} V



CONCEPTUAL QUESTIONS

7.13 Draw on the xyz-coordinate system and specify (in terms of the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$) the direction of the magnetic force on each of the moving particles shown in the figures. *Note:* The positive y-axis is toward the right, the positive z-axis is toward the top of the page, and the positive x-axis is directed out of the page.



7.14 A particle with mass m, charge q, and velocity v enters a magnetic field of magnitude B and with direction perpendicular to the initial velocity of the particle. What is the work done by the magnetic field on the particle? How does this affect the particle's motion?

7.15 An electron is moving with a constant velocity. When it enters an electric field that is perpendicular to its velocity, the electron will follow a _____ trajectory. When the electron enters a magnetic field that is perpendicular to its velocity, it will follow a _____ trajectory.

7.16 A proton, moving in negative y-direction in a magnetic field, experiences a force of magnitude F, acting in the negative x-direction.

a) What is the direction of the magnetic field producing this force?

b) Does your answer change if the word "proton" in the statement is replaced by "electron"?

7.17 It would be mathematically possible, for a region with zero current density, to define a scalar magnetic potential analogous to the electrostatic potential: $V_B(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{B} \cdot d\vec{s}$, or $\vec{B}(\vec{r}) = -\nabla V_B(\vec{r})$. However, this has not been done. Explain why not.

7.18 A current-carrying wire is positioned within a large, uniform magnetic field, $\vec{B}$. However, the wire experiences no force. Explain how this might be possible.

7.19 A charged particle moves under the influence of an electric field only. Is it possible for the particle to move with a constant speed? What if the electric field is replaced with a magnetic field?

7.20 A charged particle travels with speed v, at an angle θ with respect to the z-axis. It enters at time $t = 0$ a region of space where there is a magnetic field of magnitude B in the positive z-direction. When does it emerge from this region of space?

7.21 An electron is traveling horizontally from the northwest toward the southeast in a region of space where the Earth's magnetic field is directed horizontally toward the north. What is the direction of the magnetic force on the electron?

7.22 At the Earth's surface, there is an electric field that points approximately straight down and has magnitude 150 V/m. Suppose you had a tuneable electron gun (you can release electrons with whatever kinetic energy you like) and a detector to determine the direction of motion of the electrons when they leave the gun. Explain how you could use the gun to find the direction toward the north magnetic pole. Specifically, what kinetic energy would the electrons need to have? (*Hint:* It might be easier to think about finding which direction is east or west.)

7.23 The work done by the magnetic field on a charged particle in motion in a cyclotron is zero. How, then, can a cyclotron be used as a particle accelerator, and what essential feature of the particle's motion makes it possible?

EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Section 7.2

7.24 A proton moving with a speed of 4.00×10^5 m/s in the positive y-direction enters a uniform magnetic field of 0.400 T pointing in the positive x-direction. Calculate the magnitude of the force on the proton.

7.25 The magnitude of the magnetic force on a particle with charge $-2e$ moving with speed $v = 1.00\times 10^5$ m/s is 3.00×10^{-18} N. What is the magnitude of the magnetic field component perpendicular to the direction of motion of the particle?

•7.26 A particle with a charge of +10.0 μC is moving at 300. m/s in the positive z-direction.

a) Find the minimum magnetic field required to keep it moving in a straight line at constant speed if there is a uniform electric field of magnitude 100. V/m pointing in the positive y-direction.

b) Find the minimum magnetic field required to keep the particle moving in a straight line at constant speed if there is a uniform electric field of magnitude 100. V/m pointing in the positive z-direction.

•7.27 A particle with a charge of 20.0 μC moves along the x-axis with a speed of 50.0 m/s. It enters a magnetic field given by $\vec{B} = 0.300\hat{y} + 0.700\hat{z}$ in teslas. Determine the magnitude and the direction of the magnetic force on the particle.

••7.28 The magnetic field in a region in space (where $x > 0$ and $y > 0$) is given by $\vec{B} = (x - az)\hat{y} + (xy - b)$ where a and b are positive constants. An electron moving with a constant velocity, $\vec{v} = v_0\hat{x}$, enters this region. What are the coordinates of the points at which the net force acting on the electron is zero?

Section 7.3

7.29 A proton is accelerated from rest by a potential difference of 400. V. The proton enters a uniform magnetic field and follows a circular path of radius 20.0 cm. Determine the magnitude of the magnetic field.

7.30 An electron with a speed of 4.00×10^5 m/s enters a uniform magnetic field of magnitude 0.0400 T at an angle of 35.0° to the magnetic field lines. The electron will follow a helical path.

a) Determine the radius of the helical path.

b) How far forward will the electron have moved after completing one circle?

7.31 A particle with mass m and charge q is moving within both an electric field and a magnetic field, $\vec{E}$ and $\vec{B}$. The particle has velocity $\vec{v}$, momentum $\vec{p}$, and kinetic energy, K. Find general expressions for $d\vec{p}/dt$ and dK/dt, in terms of these seven quantities.

7.32 The Earth is showered with particles from space known as *muons*. They have a charge identical to that of an electron but are many times heavier ($m = 1.88\times 10^{-28}$ kg). Suppose a strong magnetic field is established in a lab ($B = 0.500$ T) and a muon enters this field with a velocity of 3.00×10^6 m/s at a right angle to the field. What will be the radius of the muon's resulting orbit?

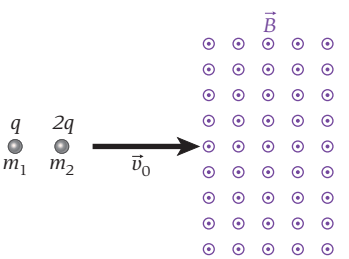
7.33 An electron in a magnetic field moves counterclockwise on a circle in the xy-plane, with a cyclotron frequency of $\omega = 1.20\times 10^{12}$ Hz. What is the magnetic field, $\vec{B}$?

7.34 An electron with energy equal to 4.00×10^2 eV and an electron with energy equal to 2.00×10^2 eV are trapped in a uniform magnetic field and move in circular paths in a plane perpendicular to the magnetic field. What is the ratio of the radii of their orbits?

•7.35 A proton with an initial velocity given by $(1.00\hat{x} + 2.00\hat{y} + 3.00\hat{z})(10^5$ m/s) enters a magnetic field given by $(0.500$ T$)\hat{z}$. Describe the motion of the proton.

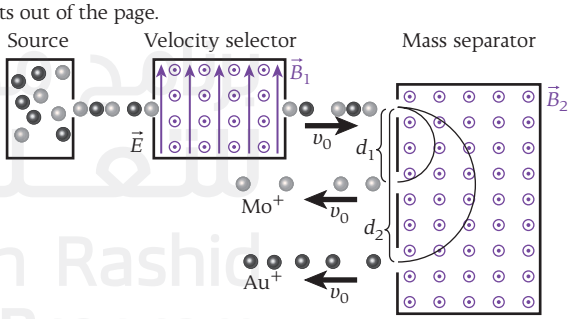
•7.36 Initially at rest, a small copper sphere with a mass of 3.00×10^{-6} kg and a charge of 5.00×10^{-4} C is accelerated through a 7000. V potential difference before entering a magnetic field of magnitude 4.00 T, directed perpendicular to its velocity. What is the radius of curvature of the sphere's motion in the magnetic field?

•7.37 Two particles with masses m_1 and m_2 and charges q and $2q$ travel with the same velocity, v, and enter a magnetic field of strength B at the same point, as shown in the figure. In the magnetic field, they move in semicircles with radii R and $2R$. What is the ratio of their masses? Is it possible to apply an electric field that would cause the particles to move in a straight line in the magnetic field? If yes, what would be the magnitude and the direction of the field?



•7.38 The figure shows a schematic diagram of a simple mass spectrometer, consisting of a velocity selector and a particle detector and being used to separate singly ionized atoms ($q = +e = 1.602\times 10^{-19}$ C) of gold (Au) and molybdenum (Mo). The electric field inside the velocity selector has magnitude $E = 1.789\times 10^4$ V/m and points toward the top of the page, and the magnetic field has magnitude $B_1 = 1.000$ T and points out of the page.

a)



Draw the electric force vector, $\vec{F}_E$, and the magnetic force vector, $\vec{F}_B$, acting on the ions inside the velocity selector.

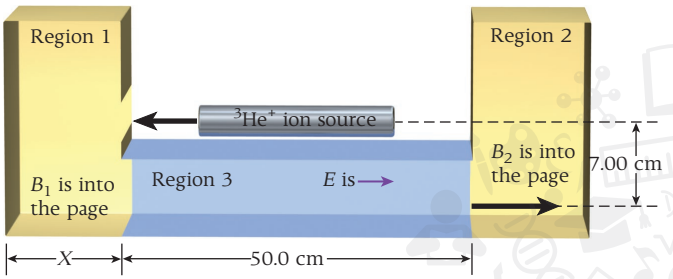
b) Calculate the velocity, v_0, of the ions that make it through the velocity selector (those that travel in a straight line). Does v_0 depend on the type of ion (gold versus molybdenum), or is it the same for both types of ions?

c) Write the equation for the radius of the semicircular path of an ion in the particle detector: $R = R(m, v_0, q, B_2)$.

d) The gold ions (represented by the dark gray spheres) exit the particle detector at a distance $d_2 = 40.00$ cm from the entrance slit, while the molybdenum ions (represented by the light gray spheres) exit the particle detector at a distance $d_1 = 19.81$ cm from the entrance slit. The mass of a gold ion is $m_{gold} = 3.271\times 10^{-25}$ kg. Calculate the mass of a molybdenum ion.

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••**7.39** A small particle accelerator for accelerating $^3He^+$ ions is shown in the figure. The $^3He^+$ ions exit the ion source with a kinetic energy of 4.00 keV. Regions 1 and 2 contain magnetic fields directed into the page, and region 3 contains an electric field directed from left to right. The $^3He^+$ ion beam exits the accelerator from a hole on the right that is 7.00 cm below the ion source, as shown in the figure.



a) If $B_1 = 1.00$ T and region 3 is 50.0 cm long with $E = 60.0$ kV/m, what value should B_2 have to cause the ions to move straight through the exit hole after being accelerated twice in region 3?

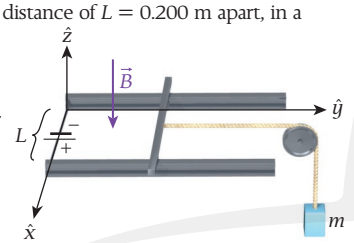
b) What minimum width X should region 1 have?

c) What is the velocity of the ions when they leave the accelerator?

Section 7.4

7.40 A straight wire of length 2.00 m carries a current of 24.0 A. It is placed on a horizontal tabletop in a uniform horizontal magnetic field. The wire makes an angle of 30.0° with the magnetic field lines. If the magnitude of the force on the wire is 0.500 N, what is the magnitude of the magnetic field?

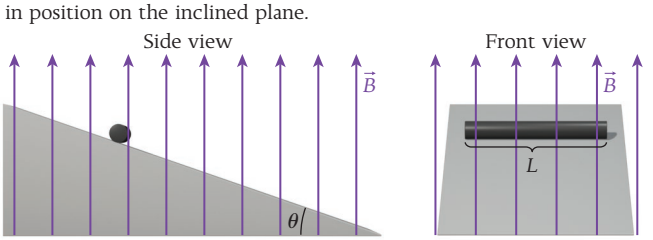
7.41 As shown in the figure, a straight conductor parallel to the x-axis can slide without friction on top of two horizontal conducting rails that are parallel to the y-axis and a distance of $L = 0.200$ m apart, in a vertical magnetic field of 1.00 T. A 20.0-A current is maintained through the conductor. If a string is connected exactly at the center of the conductor and passes over a frictionless pulley, what mass m suspended from the string allows the conductor to be at rest?



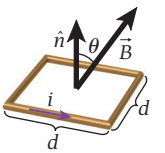
7.42 A copper wire of radius 0.500 mm is carrying a current at the Earth's Equator. Assuming that the magnetic field of the Earth has magnitude 0.500 G at the Equator and is parallel to the surface of the Earth and that the current in the wire flows toward the east, what current is required to allow the wire to levitate?

•**7.43** A copper sheet with length 1.00 m, width 0.500 m, and thickness 1.00 mm is oriented so that its largest surface area is perpendicular to a magnetic field of strength 5.00 T. The sheet carries a current of 3.00 A across its length. What is the magnitude of the force on this sheet? How does this magnitude compare to that of the force on a thin copper wire carrying the same current and oriented perpendicularly to the same magnetic field?

•**7.44** A conducting rod of length L slides freely down an inclined plane, as shown in the figure. The plane is inclined at an angle θ from the horizontal. A uniform magnetic field of strength B acts in the positive y-direction. Determine the magnitude and the direction of the current that would have to be passed through the rod to hold it in position on the inclined plane.



•**7.45** A square loop of wire, with side length $d = 8.00$ cm, carries a current of magnitude $i = 0.150$ A and is free to rotate. It is placed between the poles of an electromagnet that produce a uniform magnetic field of 1.00 T. The loop is initially placed so that its normal vector, $\hat{n}$, is at a 35.0° angle relative to the direction of the magnetic field vector, with the angle θ defined as shown in the figure. The wire is copper (with a density of $\rho = 8960$ kg/m^3), and its diameter is 0.500 mm. What is the magnitude of the initial angular acceleration of the loop when it is released?



•**7.46** A rail gun accelerates a projectile from rest by using the magnetic force on a current-carrying wire. The wire has radius $r = 5.10\times10^{-4}$ m and is made of copper having a density of $\rho = 8960$ kg/m^3. The gun consists of rails of length $L = 1.00$ m in a constant magnetic field of magnitude $B = 2.00$ T, oriented perpendicular to the plane defined by the rails. The wire forms an electrical connection across the rails at one end of the rails. When triggered, a current of 1.00×10^4 A flows through the wire, which accelerates the wire along the rails. Calculate the final speed of the wire as it leaves the rails. (Neglect friction.)

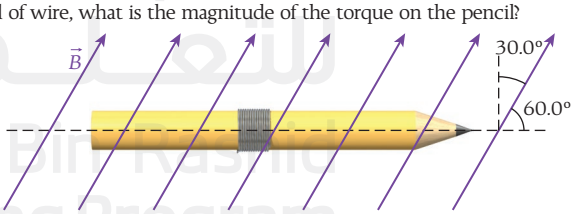
Sections 7.5 and 7.6

•**7.47** A square loop of wire of side length ℓ lies in the xy-plane, with its center at the origin and its sides parallel to the x- and y-axes. It carries a current, i, flowing in the counterclockwise direction, as viewed looking down the z-axis from the positive direction. The loop is in a magnetic field given by $\vec{B} = (B_0/a)(z\hat{x} + x\hat{z})$, where B_0 is a constant field strength, a is a constant with the dimension of length, and $\hat{x}$ and $\hat{z}$ are unit vectors in the positive x-direction and positive z-direction. Calculate the net force on the loop.

7.48 A rectangular coil with 20 windings carries a current of 2.00 mA flowing in the counterclockwise direction. It has two sides that are parallel to the y-axis and have length 8.00 cm and two sides that are parallel to the x-axis and have length 6.00 cm. A uniform magnetic field of 50.0 μT acts in the positive x-direction. What torque must be applied to the loop to hold it steady?

7.49 A coil consists of 120. circular loops of wire of radius 4.80 cm. A current of 0.490 A runs through the coil, which is oriented vertically and is free to rotate about a vertical axis (parallel to the z-axis). It experiences a uniform horizontal magnetic field in the positive x-direction. When the coil is oriented parallel to the x-axis, a force of 1.20 N applied to the edge of the coil in the positive y-direction can keep it from rotating. Calculate the strength of the magnetic field.

7.50 Twenty loops of wire are tightly wound around a round pencil that has a diameter of 6.00 mm. The pencil is then placed in a uniform 5.00 T magnetic field, as shown in the figure. If a 3.00 A current is present in the coil of wire, what is the magnitude of the torque on the pencil?

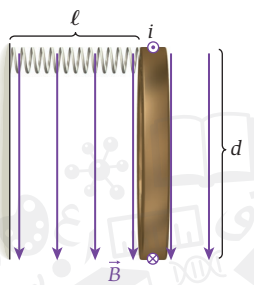


•**7.51** A copper wire with density $\rho = 8960$ kg/m^3 is formed into a circular loop of radius 50.0 cm. The cross-sectional area of the wire is 1.00×10^{-5} m^2, and a potential difference of 0.0120 V is applied to the wire. What is the maximum angular acceleration of the loop when it is placed in a magnetic field of magnitude 0.250 T? The loop rotates about an axis in the plane of the loop that corresponds to a diameter.

7.52 A simple galvanometer is made from a coil that consists of N loops of wire of area A. The coil is attached to a mass, M, by a light rigid rod of length L. With no current in the coil, the mass hangs straight down, and the coil lies in a horizontal plane. The coil is in a uniform magnetic field of magnitude B that is oriented horizontally. Calculate the angle from the vertical of the rigid rod as a function of the current, i, in the coil.

7.53 Show that the magnetic dipole moment of an electron orbiting in a hydrogen atom is proportional to its angular momentum, L: $\mu = -eL/(2m)$, where $-e$ is the charge of the electron and m is its mass.

•7.54 The figure shows a top view of a current-carrying ring of wire having a diameter $d = 8.00$ cm, which is suspended from the ceiling via a thin string. A 1.00A current flows in the ring in the direction indicated in the figure. The ring is connected to one end of a spring with a spring constant of 100. N/m. When the ring is in the position shown in the figure, the spring is at its equilibrium length, ℓ. Determine the extension of the spring when a magnetic field of magnitude $B = 2.00$ T is applied parallel to the plane of the ring, as shown in the figure.



•7.55 A coil of wire consisting of 40 rectangular loops, with width 16.0 cm and height 30.0 cm, is placed in a constant magnetic field given by $\vec{B} = 0.0650\text{T}\hat{x} + 0.250\text{T}\hat{z}$. The coil is hinged to a fixed thin rod along the y-axis (along segment da in the figure) and is originally located in the xy-plane. A current of 0.200 A runs through the wire.

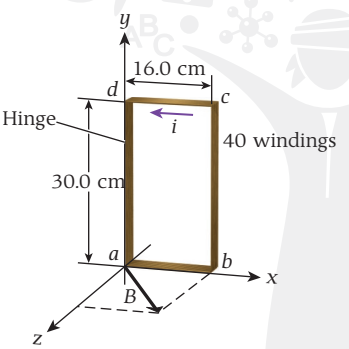
a) What are the magnitude and the direction of the force, $\vec{F}_{ab}$, that $\vec{B}$ exerts on segment ab of the coil?

b) What are the magnitude and the direction of force, $\vec{F}_{bc}$, that $\vec{B}$ exerts on segment bc of the coil?

c) What is the magnitude of the net force, F_{net}, that $\vec{B}$ exerts on the coil?

d) What are the magnitude and the direction of the torque, $\vec{\tau}$, that $\vec{B}$ exerts on the coil?

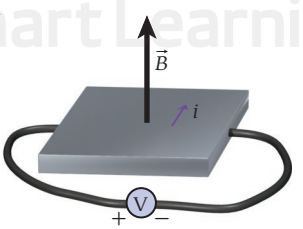
e) In what direction, if any, will the coil rotate about the y-axis (viewed from above and looking down that axis)?



Section 7.7

7.56 A high electron mobility transistor (HEMT) controls large currents by applying a small voltage to a thin sheet of electrons. The density and mobility of the electrons in the sheet are critical for the operation of the HEMT. HEMTs consisting of AlGaN/GaN/Si are being studied because they promise better performance at higher currents, temperatures, and frequencies than conventional silicon HEMTs can achieve. In one study, the Hall effect was used to measure the density of electrons in one of these new HEMTs. When a current of 10.0 μA flows through the length of the electron sheet, which is 1.00 mm long, 0.300 mm wide, and 10.0 nm thick, a magnetic field of 1.00 T perpendicular to the sheet produces a voltage of 0.680 mV across the width of the sheet. What is the density of electrons in the sheet?

•7.57 The figure shows schematically a setup for a Hall effect measurement using a thin film of zinc oxide of thickness 1.50 μm. The current, i, across the thin film is 12.3 mA, and the Hall potential, V_{H}, is −20.1 mV when a magnetic field of magnitude $B = 0.900$ T is applied perpendicular to the current flow.



a) What are the charge carriers in the thin film? [*Hint:* They can be either electrons with charge $-e$ or electron holes (missing electrons) with charge $+e$.]

b) Calculate the density of charge carriers in the thin film.

Additional Exercises

7.58 A cyclotron in a magnetic field of 9.00 T is used to accelerate protons to 50.0% of the speed of light. What is the cyclotron frequency of these protons? What is the radius of their trajectory in the cyclotron? What are the cyclotron frequency and the trajectory radius of the same protons in the Earth's magnetic field? Assume that the Earth's magnetic field is 0.500 G.

7.59 A straight wire carrying a current of 3.41 A is placed at an angle of 10.0° to the horizontal between the pole tips of a magnet producing a field of 0.220 T upward. The poles' tips each have a 10.0 cm diameter. The magnetic force causes the wire to move out of the space between the poles. What is the magnitude of that force?

7.60 An electron is moving at $v = 6.00 \cdot 10^7$ m/s perpendicular to the Earth's magnetic field. If the field strength is $0.500 \cdot 10^{-4}$ T, what is the radius of the electron's circular path?

7.61 A straight wire with a constant current running through it is in Earth's magnetic field, at a location where the magnitude is 0.430 G. What is the minimum current that must flow through the wire for a 10.0-cm length of it to experience a force of 1.00 N?

7.62 A circular coil with a radius of 10.0 cm has 100 turns of wire and carries a current, $i = 100.$ mA. It is free to rotate in a region with a constant horizontal magnetic field given by $\vec{B} = (0.0100\text{ T})\hat{x}$. If the unit normal vector to the plane of the coil makes an angle of 30.0° with the horizontal, what is the magnitude of the net torque acting on the coil?

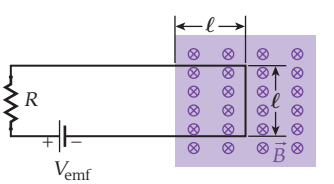
7.63 At $t = 0$ an electron crosses the positive y-axis (so $x = 0$) at 60.0 cm from the origin with velocity $2.00 \cdot 10^5$ m/s in the positive x-direction. It is in a uniform magnetic field.

a) Find the magnitude and the direction of the magnetic field that will cause the electron to cross the x-axis at $x = 60.0$ cm.

b) What work is done on the electron during this motion?

c) How long will the trip from y-axis to x-axis take?

•7.64 A 12.0V battery is connected to a 3.00Ω resistor in a rigid rectangular loop of wire measuring 3.00 m by 1.00 m. As shown in the figure, a length $\ell = 1.00$ m of wire at the end of the loop extends into a 2.00 m by 2.00 m region with a magnetic field of magnitude 5.00 T, directed into the page. What is the net force on the loop?



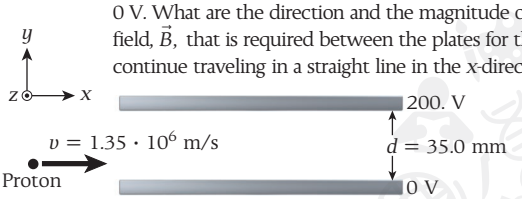
7.65 An alpha particle ($m = 6.64 \times 10^{-27}$ kg, $q = +2e$) is accelerated by a potential difference of 2700. V and moves in a plane perpendicular to a constant magnetic field of magnitude 0.340 T, which curves the trajectory of the alpha particle. Determine the radius of curvature and the period of revolution.

•7.66 In a certain area, the electric field near the surface of the Earth is given by $\vec{E} = (-150.\text{ N/C})\hat{z}$, and the Earth's magnetic field is given by $\vec{B} = (50.0\ \mu\text{T})\hat{r}_{\text{N}} - (20.0\ \mu\text{T})\hat{z}$, where $\hat{z}$ is a unit vector pointing vertically upward and $\hat{r}_{\text{N}}$ is a horizontal unit vector pointing due north. What velocity, $\vec{v}$, will allow an electron in this region to move in a straight line at constant speed?

•7.67 A helium leak detector uses a mass spectrometer to detect tiny leaks in a vacuum chamber. The chamber is evacuated with a vacuum pump and then sprayed with helium gas on the outside. If there is any leak, the helium molecules pass through the leak and into the chamber, whose volume is sampled by the leak detector. In the spectrometer, helium ions are accelerated and released into a tube, where their motion is perpendicular to an applied magnetic field, $\vec{B}$, and they follow a circular path of radius r and then hit a detector. Estimate the velocity required if the radius of the ions' circular path is to be no more than 5.00 cm, the magnetic field is 0.150 T, and the mass of a helium-4 atom is about 6.64×10^{-27} kg. Assume that each ion is singly ionized (has one electron less than the neutral atom). By what factor does the required velocity change if helium-3 atoms, which have about $\frac{3}{4}$ as much mass as helium-4 atoms, are used?

•**7.68** In your laboratory, you set up an experiment with an electron gun that emits electrons with energy of 7.50 keV toward an atomic target. What deflection (magnitude and direction) will Earth's magnetic field (0.300 G) produce in the beam of electrons if the beam is initially directed due east and covers a distance of 1.00 m from the gun to the target? (*Hint:* First calculate the radius of curvature, and then determine how far away from a straight line the electron beam has deviated after 1.00 m.)

•**7.69** A proton enters the region between the two plates shown in the figure moving in the *x*-direction with a speed $v = 1.35 \cdot 10^6$ m/s. The potential of the top plate is 200. V, and the potential of the bottom plate is 0 V. What are the direction and the magnitude of the magnetic field, $\vec{B}$, that is required between the plates for the proton to continue traveling in a straight line in the *x*-direction?



•**7.70** An electron moving at a constant velocity, $\vec{v} = v_0\hat{x}$, enters a region in space where a magnetic field is present. The magnetic field, $\vec{B}$, is constant and points in the *z*-direction. What are the magnitude and the direction of the magnetic force acting on the electron? If the width of the region where the magnetic field is present is *d*, what is the minimum velocity the electron must have in order to escape this region?

•**7.71** A 30-turn square coil with a mass of 0.250 kg and a side length of 0.200 m is hinged along a horizontal side and carries a 5.00 A current. It is placed in a magnetic field pointing vertically downward and having a magnitude of 0.00500 T. Determine the angle that the plane of the coil makes with the vertical when the coil is in equilibrium. Use $g = 9.81$ m/s^2.

•**7.72** A semicircular loop of wire of radius *R* is in the *xy*-plane, centered about the origin. The wire carries a current, *i*, counterclockwise around the semicircle, from $x = -R$ to $x = +R$ on the *x*-axis. A magnetic field, $\vec{B}$, is pointing out of the plane, in the positive *z*-direction. Calculate the net force on the semicircular loop.

•**7.73** A proton moving at speed $v = 1.00 \cdot 10^6$ m/s enters a region in space where a magnetic field given by $\vec{B} = (-0.500\text{ T})\hat{z}$ exists. The velocity vector of the proton is at an angle $\theta = 60.0°$ with respect to the positive *z*-axis.

a) Analyze the motion of the proton and describe its trajectory (in qualitative terms only).

b) Calculate the radius, *r*, of the trajectory projected onto a plane perpendicular to the magnetic field (in the *xy*-plane).

c) Calculate the period, *T*, and frequency, *f*, of the motion in that plane.

d) Calculate the pitch of the motion (the distance traveled by the proton in the direction of the magnetic field in 1 period).

MULTI-VERSION EXERCISES

7.74 A small aluminum ball with a mass of 5.063 g and a charge of 11.03 C is moving northward at 3079 m/s. You want the ball to travel in a horizontal circle with a radius of 2.137 m and in a clockwise direction when viewed from above. Ignoring gravity, what is the magnitude of the magnetic field that must be applied to the aluminum ball to cause it to move in this way?

7.75 A small aluminum ball with a charge of 11.17 C is moving northward at 3131 m/s. You want the ball to travel in a horizontal circle with a radius of 2.015 m and in a clockwise direction when viewed from above. Ignoring gravity, the magnitude of the magnetic field that must be applied to the aluminum ball to cause it to move in this way is $B = 0.8000$ T. What is the mass of the ball?

7.76 A small charged aluminum ball with a mass of 3.435 g is moving northward at 3183 m/s. You want the ball to travel in a horizontal circle with a radius of 1.893 m and in a clockwise direction when viewed from above. Ignoring gravity, the magnitude of the magnetic field that must be applied to the aluminum ball to cause it to move in this way is $B = 0.5107$ T. What is the charge on the ball?

7.77 A velocity selector is used in a mass spectrometer to produce a beam of charged particles with uniform velocity. Suppose the fields in a selector are given by $\vec{E} = (1.749 \times 10^4\text{ V/m})\hat{x}$ and $\vec{B} = (46.23\text{ mT})\hat{y}$. Find the speed with which a charged particle can travel through the selector in the *z*-direction without being deflected.

7.78 A velocity selector is used in a mass spectrometer to produce a beam of charged particles with uniform velocity. Suppose the fields in a selector are given by $\vec{E} = (2.207 \times 10^4\text{ V/m})\hat{x}$ and $\vec{B} = B_y\hat{y}$. The speed with which a charged particle can travel through the selector in the *z*-direction without being deflected is 4.713×10^5 m/s. What is the value of B_y?

7.79 A velocity selector is used in a mass spectrometer to produce a beam of charged particles of uniform velocity. Suppose the fields in a selector are given by $\vec{E} = E_x\hat{x}$ and $\vec{B} = (47.45\text{ mT})\hat{y}$. The speed with which a charged particle can travel through the selector in the *z*-direction without being deflected is 5.616×10^5 m/s. What is the value of E_x?

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