GRADE 8 ADV

EOT 1

MATH COVERAGE

Module 1,2,3 & 4

Name: .....

Class: .....

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Module 1: Exponents and Scientific Notation

M1L1: Powers and Exponents

Remember this

$\underbrace{a \times a \times a \times a \times a}_{\text{4 times}} = a^5$ (Base: a, Exponent: 5)

Expanded Form = *Power*

$\underbrace{2 \times 2 \times 2 \times 2}_{\text{4 times}} = 2^4$ (Base: 2, Exponent: 4)

Expanded Form = *Power*

Example 4 Evaluate Algebraic Expressions

Evaluate $a^2 + b^4$ if $a = 3$ and $b = \frac{1}{2}$.

Example 5 Evaluate Algebraic Expressions

Evaluate $d^3 + (c^2 - 2)$ if $c = -4$ and $d = \frac{2}{5}$.

Write each expression using exponents. (Examples 1 and 2)



1. $(-7) \cdot (-7) \cdot 5 \cdot 5 \cdot 5 \cdot 5 =$ _____

2. $n \cdot n \cdot p \cdot p \cdot r \cdot r \cdot r =$ _____

Evaluate each numerical expression. (Example 3)

3. $3^4 - (-4)^2 =$ _____

4. $6 + 2^6 =$ _____

5. Evaluate $x^3 - y^2$ if $x = 2$ and $y = \frac{3}{4}$. (Example 4)

6. Evaluate $(g + h)^3$ if $g = 2$ and $h = -3$. (Example 5)

7. Replace ☐ with $<$, $>$, or $=$ to make a true statement: $(-3)^4$ ☐ $(-4)^3$.

Test Practice

8. A scientist estimates that, after a certain amount of time, there would be $2^5 \cdot 3^3 \cdot 10^5$ bacteria in a Petri dish. How many bacteria is this?

9. **Multiselect** Select all of the expressions that evaluate to negative rational numbers.

- ☐ $(-9)^4$
- ☐ $\left(-\frac{4}{5}\right)^3$
- ☐ $3^5 - 10^4$
- ☐ $(9.8)^2 - 10^2$
- ☐ $\left(-\frac{3}{8}\right)^2$

Module 1: Exponents and Scientific Notation

M1L2: Multiply and Divide Monomials

Remember this

Monomials

A monomial is a number, a variable, or a product of a number and one or more variables.

Monomial	Not Monomial
$6y^2$	$x^2 - y^2$

Product of Powers Property

$$a^n \times a^p = a^{n+p}$$

Quotient of Powers Property

$$\frac{a^n}{a^p} = a^{n-p}$$

Simplify each expression. (Examples 1–3)

MCQ

1. $3^8 \cdot 3 =$ _____

2. $m^5 \cdot m^2 =$ _____

3. $3m^3n^2 \cdot 8mn^3 =$ _____

4. $9p^4 \cdot (-8p^2) =$ _____

5. Simplify $\frac{b^{12}}{b^5}$. (Example 4)

6. Simplify $\frac{5^5 \cdot 6^3 \cdot 8^{10}}{5^3 \cdot 6 \cdot 8^9}$. (Example 6)

7. A publisher sells 10^6 copies of a new science fiction book and 10^3 copies of a new mystery book. How many times as many science fiction books were sold than mystery books? (Example 5)

Hint:
The phrase 'how many times as many' indicates division

Test Practice

8. Simplify $\frac{45x^{15}}{9x^{10}}$. (Example 7)

9. Equation Editor Simplify $\frac{a^4c^6}{a^2c}$.

Module 1: Exponents and Scientific Notation

M1L3: Powers of Monomials

Remember this

Power of a Power

$$(a^m)^n = a^{m \times n} \qquad (5^2)^3 = 5^{2 \times 3} = 5^6$$



Simplify each expression. (Examples 1–4)

1. $(7^2)^3 =$ _____

2. $(8^3)^3 =$ _____

3. $(d^7)^6 =$ _____

4. $(z^7)^3 =$ _____

5. $(2m^5)^6 =$ _____

6. $(7a^5b^6)^4 =$ _____

7. $(-3w^3z^8)^5 =$ _____

8. $(-5r^4s^{10})^4 =$ _____

9. Which is greater: 1,000 or $(6^2)^3$? Explain.

Test Practice

10. Multiselect Select all of the expressions that simplify to the same expression.

- ☐ $(x^3y^4)^2$
- ☐ $(x^2y)^2$
- ☐ $(x^3)^2y^6$
- ☐ $x^6(y^4)^2$
- ☐ $(x^3)^2(y^2)^4$

Module 1: Exponents and Scientific Notation

M1L4: Zero And Negative Exponents

Remember this

Exponent of Zero

Any nonzero number to the zero power is equivalent to 1.

$x^0 = 1, x \neq 0$ $\quad 5^0 = 1$

Negative Exponents

Any nonzero number to the negative n power is the multiplicative inverse of its n^{th} power.

$x^{-n} = \frac{1}{x^n}, x \neq 0$ $\quad 7^{-3} = \frac{1}{7^3}$

Simplify each expression. (Example 1)

1. $46^0 =$ _____

2. w^0, where $w \neq 0$

_____

Express each using a positive exponent. (Example 2)

3. $8^{-4} =$ _____

4. $y^{-9} =$ _____



Express each fraction using a negative exponent. (Example 3)

5. $\frac{1}{d^6} =$ _____

6. $\frac{1}{10^5} =$ _____

Simplify each expression. (Examples 4 and 5)

7. $9^4 \cdot 9^{-6} =$ _____

8. $y^{-9} \cdot y^3 =$ _____

9. $\frac{x^{-8}}{x^{-12}} =$ _____

10. $\frac{d^{-13}}{d^{-2}} =$ _____

Test Practice

11. Simplify $8^{-7} \cdot 8^7 \cdot 10^4 \cdot 10^{-4}$.

12. **Multiselect** Select all of the expressions that are simplified.

- ☐ n^4
- ☐ $\frac{1}{n^{-5}}$
- ☐ $n^6 \cdot n^{-8}$
- ☐ $n^7 \cdot p^8$
- ☐ $\frac{1}{n^3}$

Module 1: Exponents and Scientific Notation

M1L5: Scientific Notation

Remember this

Scientific Notation

$a \times 10^b$

The "coefficient" will be at least 1 and less than 10

If b is positive = bigger number
If b is negative = small number

A way to write BIG and smaller numbers

Scientific Notation ⇒ Standard Form

Ex 1: 4.5×10^3 ⇒ 4,500 ⇒ 4,500

Ex 2: 2.8×10^{-4} ⇒ 0002.8 ⇒ .00028

Did You Notice

If b is positive you move the decimal to the right to get a bigger number
If b is negative you move the decimal to the left to get a smaller number

Write each number in standard form. (Examples 1 and 2)

1. $1.6 \times 10^3 =$ _____

2. $1.49 \times 10^{-7} =$ _____

MCQ ? A. B. C. D.

Write each number in scientific notation. (Examples 4 and 5)

5. 2,204,000,000 = _____

6. 0.0000000642 = _____

Standard From ⇒ Scientific Notation

Ex 1: 536,000 ⇒ 5.36000 ⇒ 5.36000 ⇒ 5.36×10^5

New decimal must go here. Decimal is 5 spots away from original position

Ex 2: .00407 ⇒ 004.07 ⇒ 004.07 ⇒ 4.07×10^{-3}

New decimal must go here. Decimal is 3 spots away from original position

7. A common race is a 5K race, where runners travel 5 kilometers. Is it more appropriate to report the distance as 5 kilometers or 5×10^6 millimeters? Explain your reasoning. (Example 6)

8. The population of Florida was recently recorded as 20,612,439 people. Write an estimation in scientific notation for the population. (Example 7)

Test Practice

9. The diameter of a grain of sand is 0.0024 inch. Write an estimation in scientific notation for the diameter. (Example 7)

10. **Equation Editor** The mass of planet Earth is about 5.98×10^{24} kilograms. When this number is written in standard notation, how many zeros are in the number?

Module 1: Exponents and Scientific Notation

M1L6: Compute with Scientific Notation

Remember this

Multiplying Numbers in Scientific Notation

$$(a \times 10^b)(c \times 10^d) = ac \times 10^{b+d}$$

Dividing Numbers in Scientific Notation

$$\frac{a \times 10^b}{c \times 10^d} = \frac{a}{c} \times 10^{b-d}$$

1. There are about 3×10^{11} stars in our galaxy and about 100 billion galaxies in the observable universe. Suppose every galaxy has as many stars as ours. How many stars are in the observable universe? Write in scientific notation. (Example 1)

MCQ

2. Humpback whales are known to weigh as much as 80,000 pounds. The tiny krill they eat weigh only 2.1875×10^{-3} pound. About how many times greater is the weight of a humpback whale? (Example 2)

Adding & Subtracting in Scientific Notation

Must be same number

$$(a \times 10^b) \pm (c \times 10^b) = (a \pm c) \times 10^b$$

Evaluate. Express each result in scientific notation. (Example 3)

3. $(1.28 \times 10^5) + (1.13 \times 10^3) =$

4. $(7.26 \times 10^6) - (1.3 \times 10^4) =$

To increase the exponent -> Move to the Left

To decrease the exponent -> Move to the Right

Test Practice

5. The speed of light is about 1.86×10^5 miles per second. The star Sirius is about 5.062×10^{13} miles from Earth. About how many seconds does it take light to travel from Sirius to Earth? Write in scientific notation, rounded to the nearest hundredth.

6. Table Item The table shows the amount of money raised by each region. The four regions raised a total of $\$(5.38 \times 10^4)$. How much did the West raise?

Region	Amount Raised ($)
East	1.46×10^4
North	2.38×10^4
South	6.75×10^3
West	

Module 2: Real Numbers

M2L1: Terminating and Repeating Decimals

Remember this

Terminating decimals are decimals with a final digit.

Example:
37.2
150.9000...

Repeating decimals are decimals with one or more digits that repeat infinitely.

Repeating decimals are often written with a bar over the digit or digits that repeat.

Example:
7.414141...

$5.2\overline{2} = 5.222...$

Fractions To Repeating Decimal

$$\begin{array}{r} 0.444\ldots \\ 9\overline{)\,4.000} \\ -36\downarrow \\ \hline 40 \\ -36\downarrow \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

$\frac{4}{9}$ → Dividend
$\frac{4}{9}$ → Divisor

MCQ ? A. B. C. D.

Write each rational number in decimal form. Then determine whether the decimal is a terminating decimal. (Examples 1 and 2)

1. $-\frac{11}{16} =$ _____

2. $\frac{5}{33} =$ _____

3. $4\frac{3}{8} =$ _____

4. $-9\frac{11}{30} =$ _____

Module 2: Real Numbers

M2L1: Terminating and Repeating Decimals

Remember this

Repeating Decimals to Fractions

1 repeated digit

EXAMPLE

$-0.\overline{7}$

Let $N = 0.\overline{7}$ **Step 1**

$10 \times N = 0.77777\ldots \times 10$ **Step 2**

$10N = 7.77777\ldots$

$-N = -0.77777\ldots$

Subtracting step (1) from step (2)

$9N = 7$

$N = \frac{7}{9}$

So $-0.\overline{7} = -\frac{7}{9}$

2 repeated digits

EXAMPLE

$0.\overline{13}$

Let $N = 0.\overline{13}$ **Step 1**

$100 \times N = 0.131313\ldots \times 100$ **Step 2**

$100N = 13.131313\ldots$

$-N = -0.131313\ldots\ldots$

Subtracting step (1) from step (2)

$99N = 13$

$N = \frac{13}{99}$

So $0.\overline{13} = \frac{13}{99}$

Example 3 Write Repeating Decimals as Fractions

Write $0.\overline{5}$ as a fraction in simplest form.

Example 4 Write Repeating Decimals as Mixed Numbers

Write $2.\overline{18}$ as a mixed number in simplest form.

Write each decimal as a fraction or mixed number in simplest form. (Examples 3 and 4)

5. $0.\overline{8} =$ _____

6. $-0.\overline{18} =$ _____



7. $-1.\overline{5} =$ _____

8. $4.\overline{45} =$ _____

Module 2: Real Numbers

M2L2: Roots

Squares and Cubes 1 to 20 Chart

$1^2 = 1$	$11^2 = 121$	$1^3 = 1$	$11^3 = 1331$
$2^2 = 4$	$12^2 = 144$	$2^3 = 8$	$12^3 = 1728$
$3^2 = 9$	$13^2 = 169$	$3^3 = 27$	$13^3 = 2197$
$4^2 = 16$	$14^2 = 196$	$4^3 = 64$	$14^3 = 2744$
$5^2 = 25$	$15^2 = 225$	$5^3 = 125$	$15^3 = 3375$
$6^2 = 36$	$16^2 = 256$	$6^3 = 216$	$16^3 = 4096$
$7^2 = 49$	$17^2 = 289$	$7^3 = 343$	$17^3 = 4913$
$8^2 = 64$	$18^2 = 324$	$8^3 = 512$	$18^3 = 5832$
$9^2 = 81$	$19^2 = 361$	$9^3 = 729$	$19^3 = 6859$
$10^2 = 100$	$20^2 = 400$	$10^3 = 1000$	$20^3 = 8000$



Square Root If $y = x^2$ then $\sqrt{y} = x$

Since $25 = 5^2$,

$\sqrt{25} = 5$,

$-\sqrt{25} = -5$,

$\pm\sqrt{25} = \pm 5$,

$\sqrt{-25}$ does not exist

Cube Root If $y = x^3$ then $\sqrt[3]{y} = x$

Since $27 = 3^3$

$\sqrt[3]{27} = 3$

$-\sqrt{27} = -3$

$\pm\sqrt{27} = \pm 3$

$\sqrt{-27} = -3$

Simplify using rational numbers. If the expression cannot be simplified, explain why. (Examples 1–4)

1. $\sqrt{361} =$ _____

2. $\pm\sqrt{1.96} =$ _____



3. $-\sqrt{\frac{9}{16}} =$ _____

4. $\sqrt{-441} =$ _____



5. Solve $m^2 = 0.04$. (Example 5) _____

Simplify using rational numbers. (Examples 6 and 7)

6. $\sqrt[3]{343} =$ _____

7. $\sqrt[3]{-512} =$ _____

Module 2: Real Numbers

M2L2: Roots



8. A basin of a water fountain is cube shaped and has a volume of 91.125 cubic feet. Solve $s^3 = 91.125$ to find the length s of one side of the basin. (Example 8)

Test Practice

9. Moesha has 196 pepper plants that she wants to plant in a square formation. How many pepper plants should she plant in each row?

10. Equation Editor What is the value of p in the equation shown?

$p^3 = -0.027$

Apply

11. A cement path consists of six equal-sized cement squares arranged in a row to form a rectangle. If the total area of the path is 96 square feet, what is the length in feet of the path?

Step 1: Find the area of one cement square
= Total area ÷ number of squares

Step 2: Find the side length of one cement square
s^2 = area of one cement square

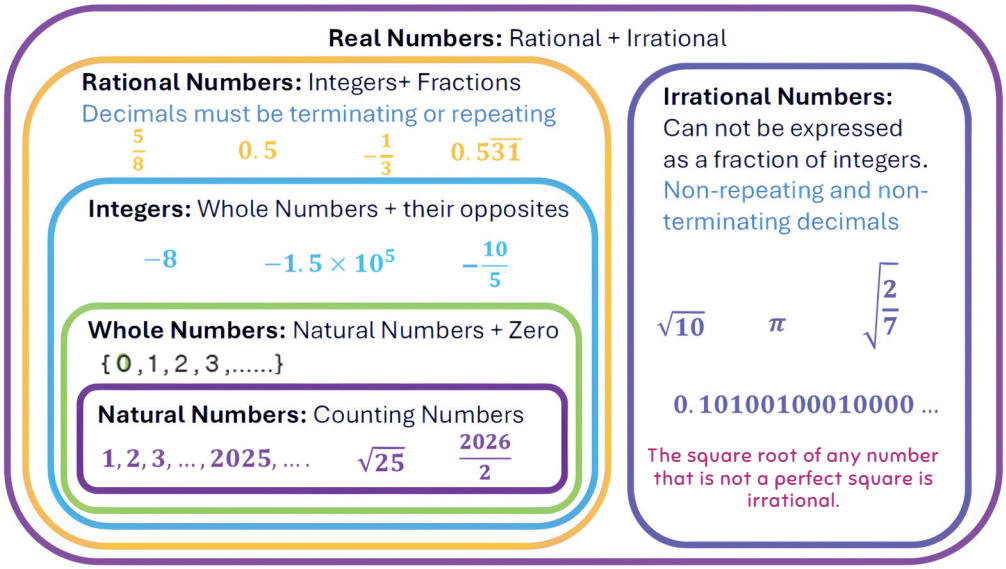
Step 3: Find the total length of the path
Side length x number of squares

12. A photo collage consists of seven equal-sized square photos arranged in a row to form a rectangle. If the total area of the collage is 567 square inches, what is the length of the collage?

Module 2: Real Numbers

M2L3: Real Numbers

Remember this



Identify whether each number is *rational* or *irrational*. (Example 1)

MCQ

1. $-\sqrt{10}$

2. $-\frac{3}{11}$

3. $0.\overline{3}$

4. $\sqrt{81}$

5. 0

6. $-\frac{\sqrt{2}}{2}$

7. $\sqrt{7}$

8. $\frac{\sqrt{2}}{\sqrt{2}}$

Select all the sets of numbers to which each real number belongs. (Examples 2–

9. $\sqrt[3]{343}$

(A) Rational
(B) Irrational
(C) Integer
(D) Whole
(E) Natural

10. $\frac{7}{\sqrt{2}}$

(A) Rational
(B) Irrational
(C) Integer
(D) Whole
(E) Natural

11. $-\frac{7}{1}$

(A) Rational
(B) Irrational
(C) Integer
(D) Whole
(E) Natural

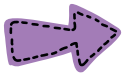
Module 2: Real Numbers

M2L4: Estimate Rational Numbers

To approximate $\sqrt{x}$ to the nearest integer, follow the steps:

Example: Approximate $\sqrt{74}$ to the nearest integer





Step 1: Find the two consecutive perfect squares that the x is between, and write the inequality:
$a^2 < x < b^2$

$64 < 74 < 81$

Step 2: Take the square root of each number:
$\sqrt{a^2} < \sqrt{x} < \sqrt{b^2}$

$\sqrt{64} < \sqrt{74} < \sqrt{81}$

Step 3: Simplify the square roots of perfect squares:
$a < \sqrt{x} < b$, then, $\sqrt{x}$ is between a and b.

$8 < \sqrt{74} < 9$

Step 4: To find a better estimate, to which perfect square a^2 or b^2 is x closer and conclude to which integer a or b is $\sqrt{x}$ is closer.

64 — 10 — 74 — 7 — 81

Since 74 is closer to 81 than 64,
$\sqrt{74}$ is closer to 9 than 8.

So, $\sqrt{74} \approx 9$.

Example 3 Estimate Cube Roots to the Nearest Integer

Estimate $\sqrt[3]{320}$ to the nearest integer.



Estimate each square root or cube root to the nearest integer. (Examples 1 and 3)

1. $\sqrt{125} \approx$ _____

2. $\sqrt{55} \approx$ _____

3. $\sqrt[3]{70} \approx$ _____

4. $\sqrt[3]{923} \approx$ _____

Module 2: Real Numbers

M2L5: Compare and Order Real Numbers

To approximate $\sqrt{x}$ to the nearest tenth, follow the steps:

Example: Approximate $\sqrt{74}$ to the nearest tenth

Step 1:

To find the whole part, approximate $\sqrt{x}$ to the nearest integer by following the previous steps. $\sqrt{x} \approx a$ (but take the **smallest integer**, not the closest).

$8 < \sqrt{74} < 9$ → So, $\sqrt{74} \approx 8$.

Step 2:

For the decimal part, start testing for intervals with **decimals such** that when you multiply each by itself, the **product is close to x,** but not greater than it.

8.1	8.2	8.3	8.4	8.5	8.6	8.7
× 8.1	× 8.2	× 8.3	× 8.4	× 8.5	× 8.6	× 8.7
65.61	67.24	68.89	70.56	72.25	73.96	75.69

$\sqrt{74}$ is closer to 8.6 than 8.7. So, $\sqrt{74} = 8.6$

Hint:
First, rewrite each fraction, square root, or expression as a decimal number. Then, compare the decimal numbers.

Complete each statement using <, >, or =. Then graph the numbers on the number line. (Examples 1 and 2)

1. $\sqrt{11}$ _____ $3\frac{2}{3}$

3.2 3.3 3.4 3.5 3.6 3.7

2. $\sqrt{3}$ _____ $\frac{\sqrt{10}}{2}$

1.4 1.5 1.6 1.7 1.8 1.9

3. $-\pi^2$ _____ $-\sqrt{93}$

−10 −9.9 −9.8 −9.7 −9.6 −9.5

4. $-\sqrt{12}$ _____ -320%

−3.6 −3.5 −3.4 −3.3 −3.2 −3.1

REMEMBER! 3.14

Module 2: Real Numbers

M2L5: Compare and Order Real Numbers



To approximate $\sqrt{x}$ to the nearest tenth, follow the steps:

Example: Approximate $\sqrt{74}$ to the nearest tenth

Step 1:

To find the whole part, approximate $\sqrt{x}$ to the nearest integer by following the previous steps. $\sqrt{x} \approx a$ (but take the **smallest integer**, not the closest).

$8 < \sqrt{74} < 9$ → **So, $\sqrt{74} \approx 8$.**

Step 2:

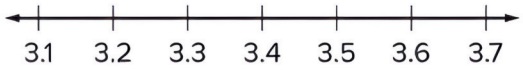
For the decimal part, start testing for intervals with **decimals such** that when you multiply each by itself, the **product is close to x,** but not greater than it.

8.1	8.2	8.3	8.4	8.5	8.6	8.7
× 8.1	× 8.2	× 8.3	× 8.4	× 8.5	× 8.6	× 8.7
65.61	67.24	68.89	70.56	72.25	73.96	75.69

$\sqrt{74}$ is closer to 8.6 than 8.7. So, $\sqrt{74} = 8.6$

5. Order the set $\left\{3\frac{1}{2}, \frac{10}{3}, \pi, \sqrt{13}\right\}$ from least to greatest. Then graph the set on the number line. (Example 3)

Hint:
First, rewrite each fraction, square root, or expression as a decimal number. Then, compare the decimal numbers.



6. The table shows the foul-shot statistics for three players in a recent basketball game. Which player had the greatest foul-shot statistic? (Example 4)

Player	Foul-Shot Statistic
1	$\frac{7}{9}$
2	72%
3	8 out of 10

Test Practice

7. **Multiple Choice** Select the symbol that makes the sentence true.

$\sqrt{27} \_\_ \frac{\sqrt{95}}{2}$

(A) $<$

(B) $>$

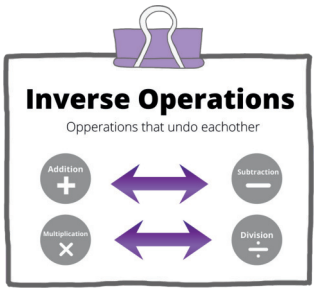
(C) $=$

(D) $\leq$

Module 3: Solve Equations with Variables on Each Side

M3L1: Solve Equations with Variables on Each Side

Remember this



How to solve the equation?

Step 1: Rearrange the equation

To rearrange an equation, move all variables to one side and all constants to the other.

Remember to change the sign when moving a term across the equals sign.

$$5 - 6 + x = 4x + 10$$

Step 2: Collect like terms

$$5 - 6 - 10 = 4x - x$$

$$-11 = 3x$$

$$\frac{-11}{3} = \frac{3x}{3}$$

$$x = -\frac{11}{3}$$

Step 3: Undo each operation (find the inverse)

Whatever you do to one side, you must do to the other side to keep it balanced.

How to check your solution?

Step 1: Write the original equation and substitute x

$$5 - 6 + x = 4x + 10$$

$$5 - 6 + (-11/3) = 4(-11/3) + 10$$

Step 2: Evaluate each side

P	E	M	D	A	S
Parentheses	Exponents	Multiplication	Division	Addition	Subtraction
(....)	a^2	X	÷	+	-

left side:

$$-1 + \left(-\frac{11}{3}\right)$$

$$-\frac{3}{3} + \left(-\frac{11}{3}\right)$$

$$-\frac{3+11}{3} = -\frac{14}{3}$$

Right side:

$$-\frac{44}{3} + 10.$$

$$-\frac{44}{3} + \frac{30}{3}$$

$$\frac{-44+30}{3} = \frac{-14}{3}$$

Step 3: Compare both sides

They are equal, so the substitution checks: $x = -\frac{11}{3}$ is indeed a correct solution.

Solve each equation. Check your solution. (Examples 1–3)

MCQ

1. $-2a - 9 = 6a + 15$

2. $14 + 3n = 5n - 6$

Module 3: Solve Equations with Variables on Each Side

M3L1: Solve Equations with Variables on Each Side



3. $\frac{1}{2}x - 5 = 10 - \frac{3}{4}x$

4. $\frac{2}{3}y + 1 = \frac{1}{6}y + 8$

5. $5.4p + 13.1 = -2.6p + 3.5$

6. $0.15w + 0.35 = 0.22w - 0.14$

Test Practice

7. Twelve more than seven times a number equals the number less six. Solve the equation $7x + 12 = x - 6$ to find the number, x.

8. Equation Editor Solve the equation shown for x.

$3x - 15 = 17 - x$

Module 3: Solve Equations with Variables on Each Side

M3L2: Write and solve Equations with Variables on Each Side

Remember this

How to Translate a Word Problem into an Equation ?

1. Understand the Situation
2. Define the Variable
3. Write an Expression for Each Side
4. Write the Equation

Clue Words for Mathematical Operations

=	Equals – the same as – equivalent – is.
+	Sum – more than – increased by – total – gains – alltogether – added to – add.
–	Difference – less than – decreased by – mines – fewer –
×	Product – each – every – per – product – multiplied by – twice – times – half – third – quarter
÷	Quotient – half – split – per – shared equally – divided by.

EXAMPLE

An aquarium holds 50 gallons of water and is being drained at a rate of 1/2 gallon per second. Another aquarium holds 70 gallons of water and is being filled at a rate of 3/4 gallon per second. After how many seconds will the two aquariums have the same amount of water?

Aquarium	Starting amount	Rate	Change per second
A	50 gallons	½ gallon drained per second	losing water (−)
B	70 gallons	¾ gallon filled per second	gaining water (+)

Let x=number of seconds

$$\text{Water in A} = \text{Water in B}$$

$$50 - \frac{1}{2}x = 70 + \frac{3}{4}x$$

Write and solve an equation for each exercise. Check your solution.

(Examples 1 and 2)

1. Marko has 45 comic books in his collection, and Tamara has 61 comic books. Marko buys 4 new comic books each month and Tamara buys 2 comic books each month. After how many months will Marko and Tamara have the same number of comic books?

MCQ A. B. C. D.

2. A fish tank has 150 gallons of water and is being drained at a rate of $\frac{1}{2}$ gallon each second. A second fish tank has 120 gallons of water and is being filled at a rate of $\frac{1}{4}$ gallon each second. After how many seconds will the two fish tanks have the same amount of water?

Module 3: Solve Equations with Variables on Each Side

M3L2: Write and solve Equations with Variables on Each Side



Write and solve an equation for each exercise. Check your solution.
(Examples 1 and 2)

3. Shipping Company A charges $14 plus $2.25 a pound to ship overnight packages. Shipping Company B charges $20 plus $1.50 a pound to ship an overnight package. For what weight is the charge the same for the two companies?

4. A bicycle rental company charges a $20 fee plus $5.50 per hour to rent a bicycle. Another bicycle rental company charges a $15 fee plus $6.50 per hour to rent a bicycle. For what number of hours is the cost for the rental the same?

Test Practice

5. Open Response Deanna and Lulu are playing games at the arcade. Deanna starts with $15, and the machine she is playing costs $0.75 per game. Lulu starts with $13, and her machine costs $0.50 per game. After how many games will the two friends have the same amount of money remaining? Let g represent the number of games.

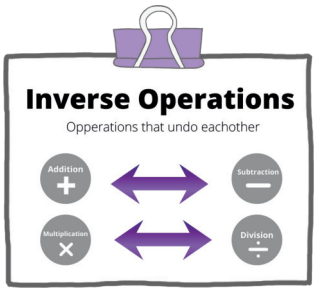
Equation:

Number of Games:

Module 3: Solve Equations with Variables on Each Side

M3L3: Solve Multi-Step Equations

Remember this



How to solve the equation?

Step 1: Use distributive property

$5-2(3-x)=4x+10$

Step 2: Rearrange the equation

To rearrange an equation, move all variables to one side and all constants to the other.

Remember to change the sign when moving a term across the equals sign.

$5 - 6 + 2x = 4x + 10$

Step 2: Collect like terms

$5 - 6 - 10 = 4x - 2x$

$-11 = 2x$

Step 3: Undo each operation (find the inverse)

Whatever you do to one side, you must do to the other side to keep it balanced.

$\frac{-11}{2} = \frac{2x}{2}$

$x = -\frac{11}{2} \rightarrow x = -5.5$

How to check your solution?

Step 1: Write the original equation and substitute x

$5 - 2(3-x) = 4x + 10$

$5 - 6 + (3-(-5.5)) = 4(-5.5) + 10$

Step 2: Evaluate each side

P	E	M	D	A	S
Parentheses	Exponents	Multiplication	Division	Addition	Subtraction
(....)	a^2	X	÷	+	-

Left side:

$5-2(3+5.5)$

$5-2(8.5)$

$5-17$

$= -12$

Right side:

$4(-5.5)+10$

$-22+10$

$=-12$

Step 3: Compare both sides

Conclusion: Left side = -12 and right side = -12.

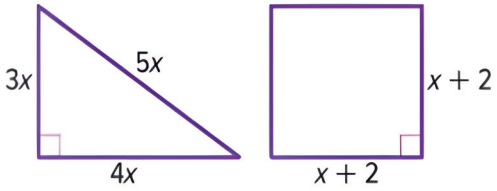
Apply

8. At a health club, 96 members participate in the morning workout, and this number has been increasing by 2 people per week. In the afternoon workout, there are 80 members, and this number has been decreasing by 3 people per week. Solve the equation $96 + 2w = 2(80 - 3w)$ to find in how many weeks w the number of people working out in the morning will be double the number of people working out in the afternoon.



M3L3: Solve Multi-Step Equations

9. The triangle and the square shown have the same perimeter. Solve the equation $3x + 4x + 5x = 4(x + 2)$ to find the value of x. Then find the length of one side of the square.



10. **Find the Error** A student solved the equation $3(-4 + x) - 5x = 7x + 15$. Find her mistake and correct it.

$$
\begin{aligned}
3(-4 + x) - 5x &= 7x + 15 \\
-12 + 3x - 15x &= 7x + 15 \\
-12 - 12x &= 7x + 15 \\
-12 - 12x + 12x &= 7x + 12x + 15 \\
-12 &= 19x + 15 \\
-12 - 15 &= 19x + 15 - 15 \\
-27 &= 19x \\
-1\tfrac{8}{19} &= x
\end{aligned}
$$

11. **Identify Structure** Describe the role of the Distributive Property when solving multi-step equations that contain expressions with grouping symbols.

12. Suppose your friend says he can solve the equation $15 - (2x + 6) = \frac{1}{2}(7x - 4)$ by first using the Multiplication Property of Equality to multiply each side of the equation by 2. Is he correct? Justify your response.

Module 3: Solve Equations with Variables on Each Side

M3L4: Write and Solve Multi-Step Equations

Remember this

How to Translate a Word Problem into an Equation ?

1. Understand the Situation
2. Define the Variable
3. Write an Expression for Each Side
4. Write the Equation

Clue Words for Mathematical Operations

=	Equals – the same as – equivalent – is.
+	Sum – more than – increased by – total – gains – alltogether – added to – add.
–	Difference – less than – decreased by – mines – fewer –
×	Product – each – every – per – product – multiplied by – twice – times – half – third – quarter
÷	Quotient – half – split – per – shared equally – divided by.

EXAMPLE

A rectangular television's length is 3 inches shorter than twice its width. The perimeter of the television is 144 inches. What is the width of the television?

The phrase "8 less than 12" or "8 shorter than 12" means you subtract 8 from 12, so it's 12 – 8 (not 8 – 12).

The perimeter of a rectangle is:

$P=2l+2w$

1) Translate the sentence "length is 3 inches shorter than twice its width"

$L=2w-3$

2) We are told the perimeter P is 144 inches, so:

$P=2l+2w$

$144 = 2(2w-3)+2w$

☑ This is the **equation** that represents the problem.

Then Solve it step-by-step to find w

Write and solve an equation for each exercise. Check your solution.

(Examples 1 and 2)



1. Mr. Reed is drawing a blueprint of a rectangular patio. The width of the patio is $40\frac{3}{4}$ feet shorter than twice its length. The perimeter of the patio is $86\frac{1}{2}$ feet. What is the length of the patio?

2. The Yearbook Club is going to an amusement park, and each of their 12 members will pay for admission and will also help pay for parking. The Robotics Club is going to a waterpark, and each of their 14 members will pay for admission and will also purchase a meal ticket. Admission to the amusement park is 1.5 times that of the waterpark's admission, as shown in the table. If the total cost is the same at both the amusement park and the waterpark, what is the admission per student to the waterpark?

Amusement Park	Waterpark
Admission: $1.5x per student	Admission: $x per student
Parking: $2 per student	Meal Ticket: $10.50 per student

Module 3: Solve Equations with Variables on Each Side

M3L4: Write and Solve Multi-Step Equations



Test Practice

3. Open Response Arjun purchased 5 tickets to a play, each with the same price. He was also charged an online service fee of $3.50 per ticket. Emilia purchased 3 tickets to the same play and paid twice as much for her tickets as Arjun. Emilia was also charged a service fee of $2.75 per ticket. If they spent the same amount, what is the cost of each of the tickets Arjun purchased? Let t represent the cost of each of Arjun's tickets.

Equation:

Cost of a Ticket:

Apply

4. Four siblings have a dog walking business. The table shows the hours worked by each sibling. Each sibling earns $25.50 per hour and the total number of hours worked is represented by $10h + 15$, where h represents the number of hours Michael worked. What was the total amount the siblings earned?

Sibling	Hours Worked
Martin	$2.5h + 3$
Emilio	$4(h - 2)$
Michael	h
Mario	31

5. Create Write a real-world problem that can be solved using a multi-step equation. Then write and solve an equation for your problem.

6. Persevere with Problems Elijah put $2x + 3$ dollars in the bank the first week. The following week he doubled the first week's savings and put that amount in the bank. The next week, he doubled what was in the bank and put that amount in the bank. He now has $477 in the bank. Write and solve an equation to find how much money he put in the bank the first week.

7. Find the Error A student wrote the equation $4(x + 5.5) = 3(1.5x + 4.75)$ to represent the problem shown. Find her mistake and correct it.

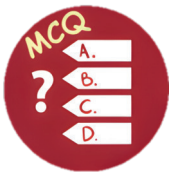
Petra and her 4 friends went to the movies. They each bought a ticket for x dollars and spent $5.50 each on snacks. Valentina and her 3 friends went to the movies and paid 1.5 times as much for each of their tickets as Petra. Everyone in Valentina's group also spent $4.75 each on snacks. If both groups spent the same amount, what is the cost of a ticket for Petra's group?

Module 3: Solve Equations with Variables on Each Side

M3L5: Determine the Number of Solutions

Remember this

	No Solution	One Solution	Infinite Solutions
Symbols	$a = b$	$x = a$	$a = a$
Examples	$3x + 4 = 3x$ $4 = 0$ Since $4 \neq 0$, there is no solution.	$2x = 20$ $x = 10$ Since $x = 10$, there is one solution.	$4x + 2 = 4x + 2$ $2 = 2$ Since $2 = 2$, all of the values of x are solutions.



Solve each equation. Determine whether the equation has one solution, no solution, or infinitely many solutions. (Examples 1 and 2)

1. $4(x - 8) + 12 = 2(2x - 9)$

2. $3(2k - 5) = 6(k - 4) + 9$

3. $-4y - 3 = \frac{1}{3}(12y - 9) - 8y$

4. $6(3 - 5w) = 5(4 - 2w) - 20w$

Complete each equation so that it has infinitely many solutions. (Example 3)

5. $2x - 7(x + 10) = \square x - \square$

6. $12x - x + 8 + 3x = \square x + \square$

Infinite Solutions
Left side is equal to Right side
$14x + 5 = 14x + 5$
Same Same

Complete each equation so that it has no solution. (Example 4)

7. $-15x + 4x + 2 - x = \square x + \square$

8. $9(x - 4) - 5x = \square x - \square$

No Solution
Left side is not equal to Right
$14x + 5 = 14x + 7$
Same Different

Test Practice

9. Multiple Choice Which of the following explains why $\frac{2}{3}(x + 3) = \frac{2}{3}(x - 6)$ has no solution?

(A) The coefficients are different, and the constants are different.

(B) The coefficients are the same, and the constants are the same.

(C) The coefficients are different, and the constants are the same.

(D) The coefficients are the same, and the constants are different.

Module 4: Linear Relationships and Slope

M4L1: Proportional Relationships and Slope

Different ways to find the Slope :

1) Using words:

Unit Rate $= \frac{y}{x}$

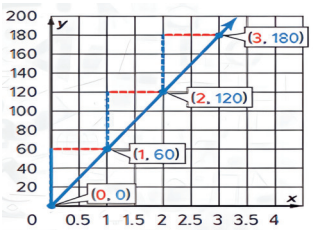
2) Table :

Time (minutes), x	Number of Songs, y
0	0
1	2
2	4
3	6

(+1 → +2 for each step)

Rate of change $= \frac{change\ in\ y}{change\ in\ x}$

3) Graph OR 2 points :



Formula $= \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{\text{Rise}}{\text{Run}}$

4) Equation:

Direct variation:
(Proportional Relationships)

$y = mx$

m = Slope

Slope-intercept form:
(Non-Proportional Relationships)

$y = mx + b$

m = Slope, b = y-intercept

EXAMPLE

Time (minutes), x	Number of Songs, y
0	0
1	2
2	4
3	6

(+1 → +2 for each step)

Slope:

Rate of change $= \frac{change\ in\ y}{change\ in\ x} = \frac{2}{1} = 2$



Example 5 Compare Proportional Relationships

The cost y for computer repairs at Computer Access for x hours is shown in the table. Macro Repair charges $23.50 per hour for computer repairs. Assume that the cost is proportional to the number of hours.

Which company has the lower repair cost? Explain.

Computer Access	
Number of Hours, x	Cost ($), y
2	50
3	75
4	100
5	125

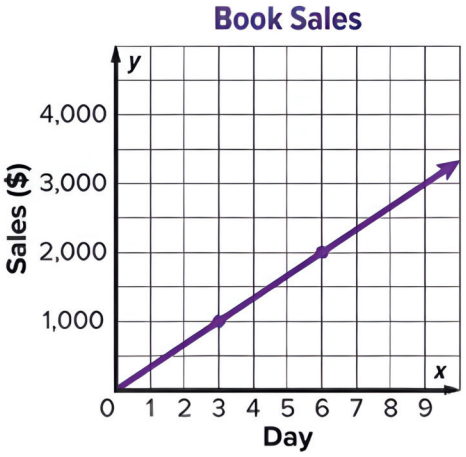
(+1 → +25 for each step)

Module 4: Linear Relationships and Slope

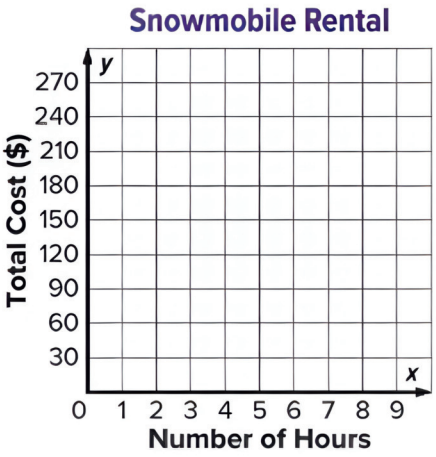
M4L1: Proportional Relationships and Slope



1. The graph shows the amount of book sales over several days. Find and interpret the slope. Then find the unit rate and compare it to the slope. (Example 1)



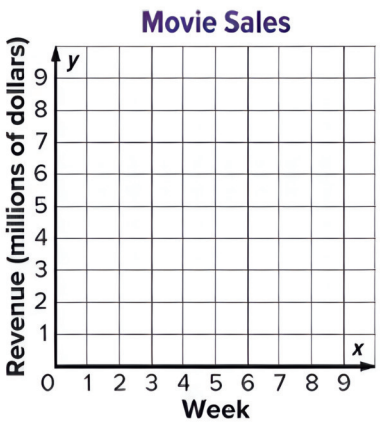
2. The cost y of renting a snowmobile for x hours is a proportional relationship. This can be represented by the equation $y = 33.75x$. Graph the equation. Then find and interpret the slope. (Example 2)



"**Remember:** Make a table first (using the equation) to graph the points."

x		y
0		
1		
2		

3. By the end of its fourth week, a movie had grossed $9.2 million. Assume the revenue y in millions of dollars is proportional to the week x. Graph this relationship on the coordinate plane. Then find and interpret the slope. (Example 3)



Step 1: Find the slope:

Unit Rate = $\frac{y}{x}$

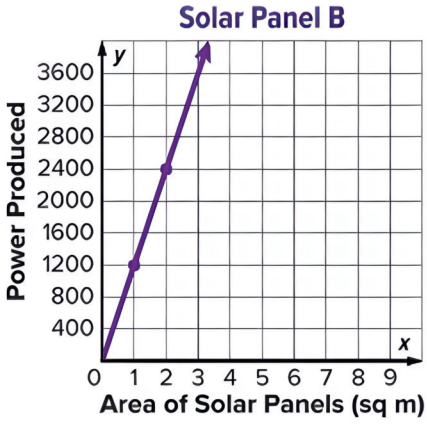
Step 2: Write the equation

y = (slope)x

Step 3: Construct a table to graph the points

x		y
0		
1		
2		

4. The amount of power y solar panel A can produce with an area of x square meters can be represented by the equation $y = 1{,}020x$. The amount of power a solar panel B can produce is shown on the graph. Which solar panel can produce more power? Explain. (Example 4)



Module 4: Linear Relationships and Slope

M4L2: Slope of a Line

Different ways to find the Slope :

1) Using words:

Unit Rate $= \frac{y}{x}$

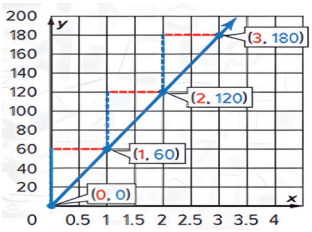
2) Table :

Time (minutes), x	Number of Songs, y
0	0
1	2
2	4
3	6

(+1 → +2 for each step)

Rate of change $= \frac{change\ in\ y}{change\ in\ x}$

3) Graph OR 2 points :



Formula $= \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{\text{Rise}}{\text{Run}}$

4) Equation:

Direct variation:
(Proportional Relationships)

$y = mx$

m: Slope

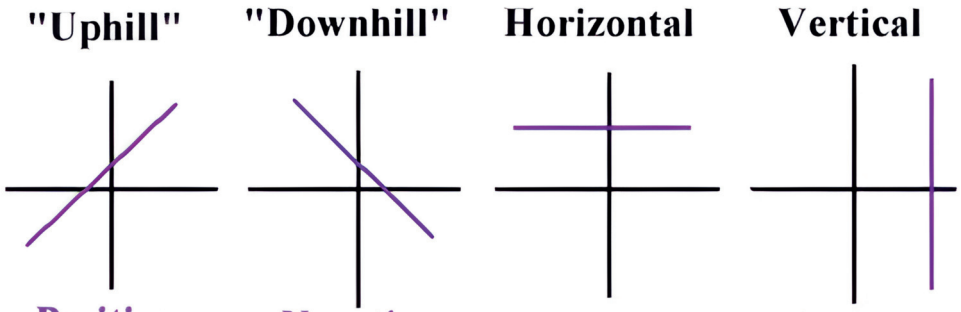
Slope-intercept form:
(Non-Proportional Relationships)

$y = mx + b$

m: Slope, b: y-intercept

The images below summarize the slopes of different types of lines:

Remember this



Slope $= \frac{\text{Zero}}{\text{Number}}$

Slope $= \frac{\text{Number}}{\text{Zero}}$

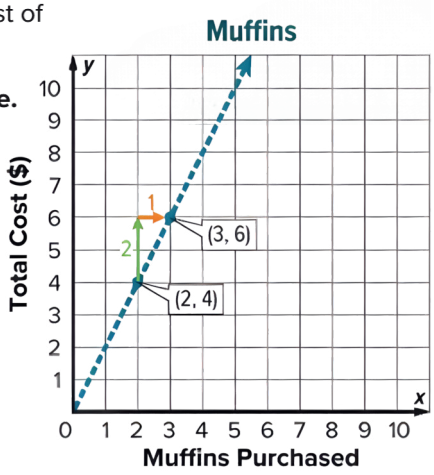
Module 4: Linear Relationships and Slope

M4L2: Slope of a Line

Example 1 Find Slope from a Graph

The graph shows the cost of muffins at a bake sale.

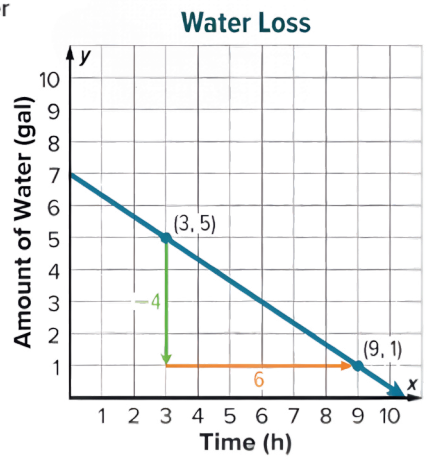
Find the slope of the line.



Example 2 Find Slope from a Graph

The graph shows the amount of water in a leaking bucket over time.

Find the slope of the line.



Example 3 Find Slope from a Table

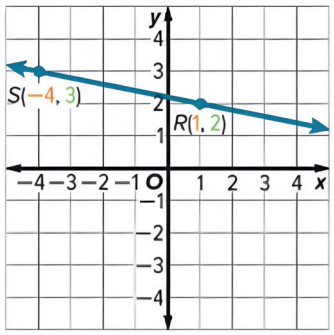
The points given in the table lie on a line.

Find the slope of the line. Check your solution.

x	−6	−2	2	6
y	−2	−1	0	1

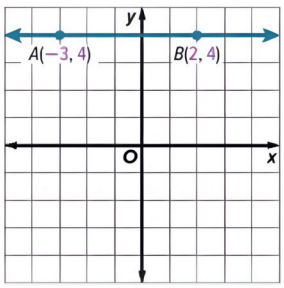
Example 4 Find Slope Using the Slope Formula

Find the slope of the line that passes through *R*(1, 2), *S*(−4, 3). Check your solution.



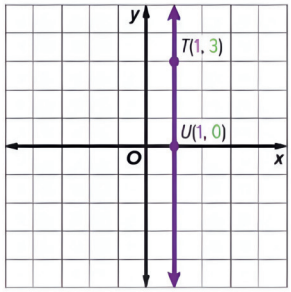
Example 5 Zero Slope

Find the slope of the line that passes through *A*(−3, 4), *B*(2, 4).



Example 6 Undefined Slope

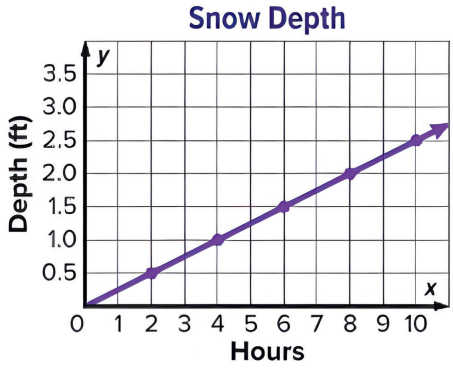
Find the slope of the line that passes through *T*(1, 3), *U*(1, 0).



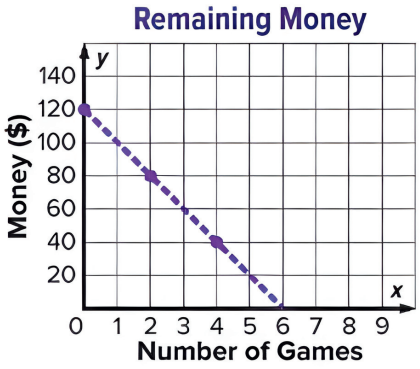
Module 4: Linear Relationships and Slope

M4L2: Slope of a Line

1. The graph shows the depth in feet of snow after each two-hour period during a snowstorm. Find the slope of the line. (Example 1)



2. The graph shows the amount of money left after buying video games. Find the slope of the line. (Example 2)



3. The points given in the table lie on a line. Find the slope of the line. (Example 3)

x	−1	2	5	8
y	3	−1	−5	−9

Find the slope of the line that passes through each pair of points. (Examples 4–6)

4. $M(3, 5)$, $N(2, 6)$

5. $G(-3, 2)$, $H(7, 2)$

6. $E(6, 8)$, $F(6, -2)$

Test Practice

7. Multiple Choice The points given in each table lie on lines. Which table, when graphed, would show a negative slope?

(A)

x	−2	3	8	13
y	−2	−1	0	1

(B)

x	−1	1	3	5
y	−2	0	2	4

(C)

x	3	5	6	8
y	8	0	−4	−12

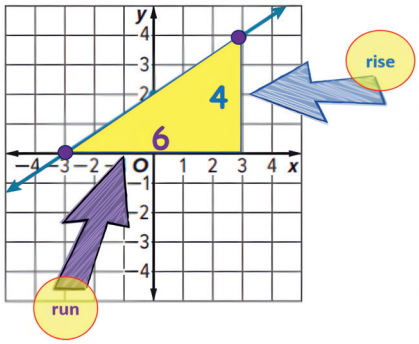
(D)

x	−13	−9	−5	−1
y	−4	−2	0	2

Module 4: Linear Relationships and Slope

M4L3: Similar Triangles and Slope

Remember this

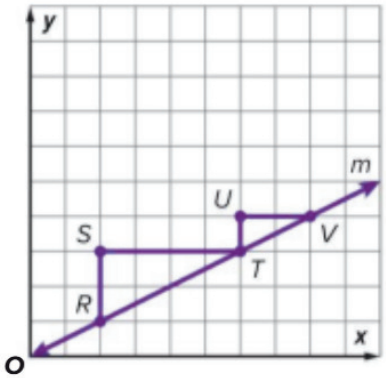


Slope:

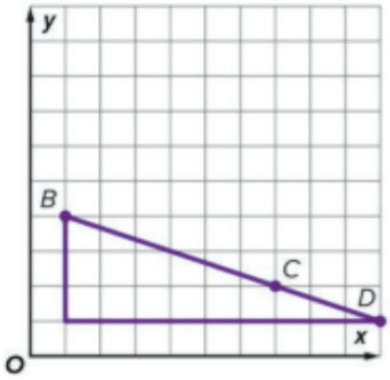
$$\frac{\text{Rise}}{\text{Run}} = \frac{4}{6} = \frac{2}{3}$$

MCQ

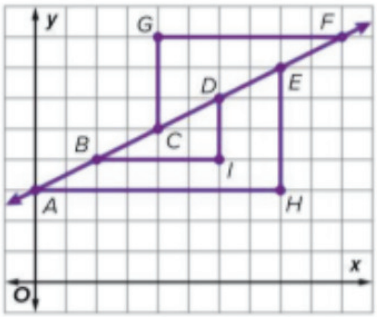
1. The graph of line m is shown. Use the similar slope triangles to compare the slope of segment RT and TV. (Example 1)



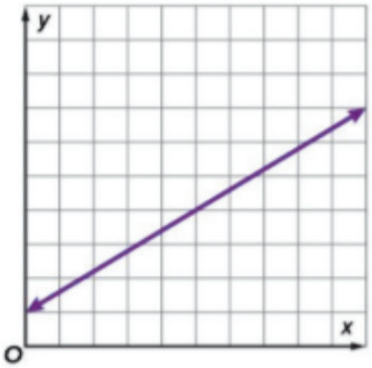
2. The plans for a zipline are shown. Use two points to determine the slope of the zipline. Then verify that the slope is the same by choosing a different set of points. (Example 2)



3. Name the slope triangles shown in the graph. What is the slope of the line?

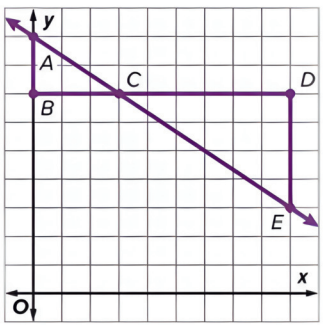


4. Draw two slope triangles on the line. Determine the slope of the line.



Test Practice

5. Multiselect The graph shows similar slope triangles on a line. Select all of the statements that are true.



- ☐ The slope of the line is negative.
- ☐ The slopes of each triangle are the same because they lie on the same line.
- ☐ Triangle CDE has a greater slope because the triangle is larger.
- ☐ The slope of each triangle is $\frac{2}{3}$.
- ☐ The slope of the line is positive.

Module 4: Linear Relationships and Slope

M4L4: Direct Variation

Different ways to find the Slope :

1) Using words:

Unit Rate = $\frac{y}{x}$

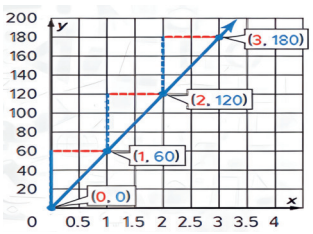
2) Table :

Time (minutes), x	Number of Songs, y
0	0
1	2
2	4
3	6

(+1 → +2 for each step)

Rate of change = $\frac{change\ in\ y}{change\ in\ x}$

3) Graph OR 2 points :



Formula = $\frac{y_2 - y_1}{x_2 - x_1}$ = $\frac{\text{Rise}}{\text{Run}}$

4) Equation:

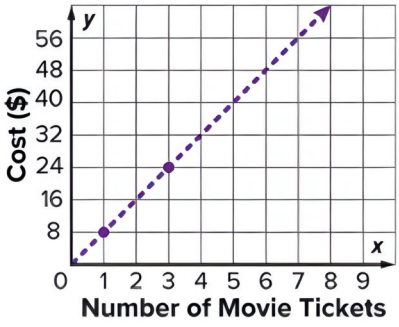
Direct variation:
(Proportional Relationships)

$y = mx$ (m = Slope)

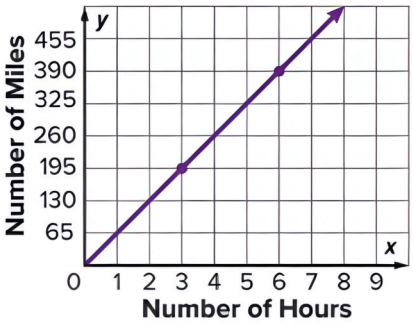
Slope-intercept form:
(Non-Proportional Relationships)

$y = mx + b$ (m = Slope, b = y-intercept)

1. The cost y of movie tickets varies directly with the number of tickets x as shown in the graph. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 1)



2. The number of miles y varies directly with the number of hours x as shown in the graph. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 1)



3. The cost of paper varies directly with the number of reams bought. Suppose two reams cost $5.10. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 2)

4. The amount of flour needed for a recipe varies directly with the number of servings planned. Three servings require $4\frac{1}{2}$ cups of flour. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 2)

Test Practice

5. Open Response The distance a bus travels varies directly with time as shown in the table. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 3)

Time (h), x	Distance (mi), y
1.5	93.75
3	187.5
4.5	281.25
6	375

Module 4: Linear Relationships and Slope

M4L5: Slope-Intercept Form



Different ways to find the Slope :

1) Using words:

Unit Rate = $\frac{y}{x}$

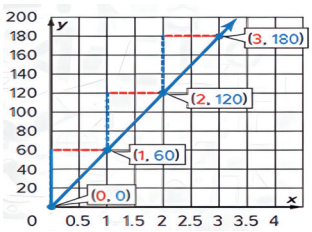
2) Table :

Time (minutes), x	Number of Songs, y
0	0
1	2
2	4
3	6

+1, +1, +1 / +2, +2, +2

Rate of change = $\frac{change\ in\ y}{change\ in\ x}$

3) Graph OR 2 points :



Formula = $\frac{y_2 - y_1}{x_2 - x_1}$ = $\frac{Rise}{Run}$

4) Equation:

Direct variation:
(Proportional Relationships)

$y = mx$

m: Slope

Slope-intercept form:
(Non-Proportional Relationships)

$y = mx + b$

m: Slope, b: y-intercept

Proportional VS Non-proportional Relationships

1) Table :

Proportional

Time (min.)	Distance (ft.)
0	0
2	6
4	12
6	18

$\frac{2}{6} = \frac{6}{18}$

Ratios are equivalent.

Include both the unit rate and the rate of change.

Non-Proportional

Time (min.)	Distance (ft.)
0	4
2	10
4	16
6	22

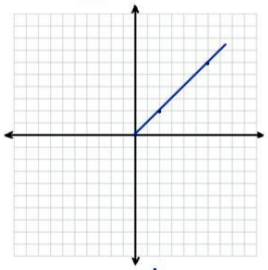
$\frac{1}{5} \frac{2}{10} = \frac{6}{22} \frac{3}{11}$

Ratios are not equivalent.

Include only the rate of change

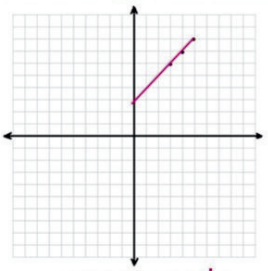
2) Graph :

Proportional



y = kx
crosses origin (0,0)
linear

Non-Proportional



y = mx + b
does NOT cross origin
linear

How to find the y-intercept (b) ?

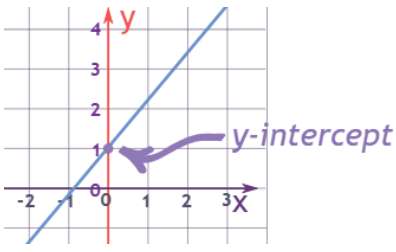
1) Table :

x	y
-5	-16
-2	-7
0	-1
3	8

look for the row where the **x-value is (0).** The corresponding y-value is the y-intercept.

b= -1

2) Graph :



locate the point where the line **crosses the vertical y-axis.**

b= 1

Module 4: Linear Relationships and Slope

M4L5: Slope-Intercept Form

Identify the slope and *y*-intercept of the graph of each equation. (Example 1)



1. $y = \frac{1}{2}x - 5$

2. $y = 3x - 1$

Write the equation of a line in slope-intercept form with each slope and *y*-intercept. (Example 2)

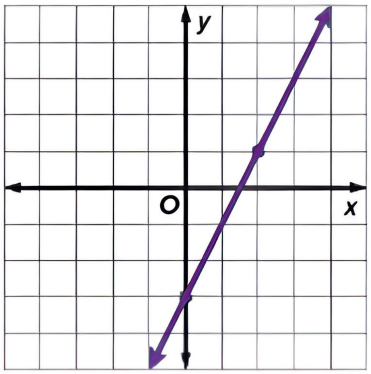
3. slope $= -\frac{1}{3}$, *y*-intercept $= 4$

4. slope $= \frac{3}{2}$, *y*-intercept $= -3$

5. slope $= 4$, *y*-intercept $= -2$

6. slope $= -1$, *y*-intercept $= 6$

7. Write an equation in slope-intercept form for the graph shown. (Example 3)



8. The Augello family is driving from Columbus to St. Louis at a constant rate of 65 miles per hour. The distance between the two cities is 420 miles. Write an equation in slope-intercept form to represent the distance *y* in miles remaining after driving *x* hours. (Example 4)

9. The table shows the costs for art show participants, including the $30 registration fee. Write an equation in slope-intercept form that represents the data in the table. (Example 5)

Number of Pieces of Art	Cost ($)
0	30
2	90
4	150
6	210
8	270

Test Practice

10. Multiselect Select all of the statements that are true about the equation $y = -\frac{3}{5}x + 8$.

- ☐ The slope of the line is negative.
- ☐ The slope of the line is 8.
- ☐ The *y*-intercept of the line is 8.
- ☐ The *y*-intercept of the line is $-\frac{3}{5}$.
- ☐ The slope of the line is $-\frac{3}{5}$.

Module 4: Linear Relationships and Slope

M4L5: Graph Linear Equations

How to graph linear functions?

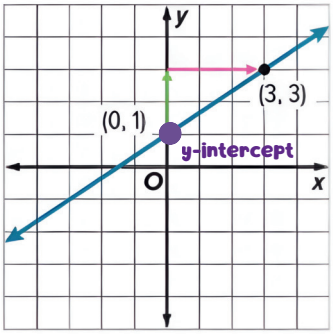
Step 1 Find the slope and *y*-intercept. $m = \frac{2}{3}$, $b = 1$

Step 2 Graph the *y*-intercept.

Step 3 Use the slope to locate a second point on the line. $\frac{\text{Rise}}{\text{Run}} = \frac{2}{3}$

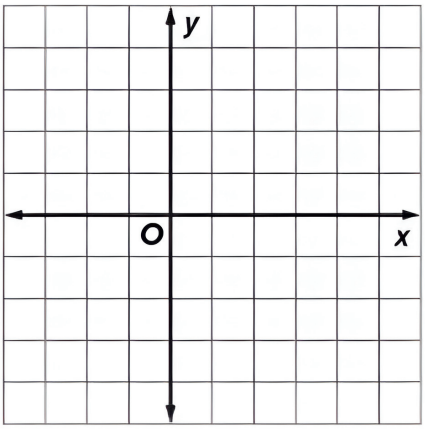
Step 4 Draw a line through the points.

$y = \frac{2}{3}x + 1$

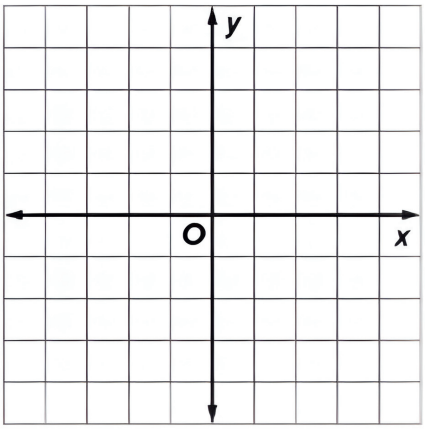


Graph each equation using the slope and *y*-intercept. (Example 1)

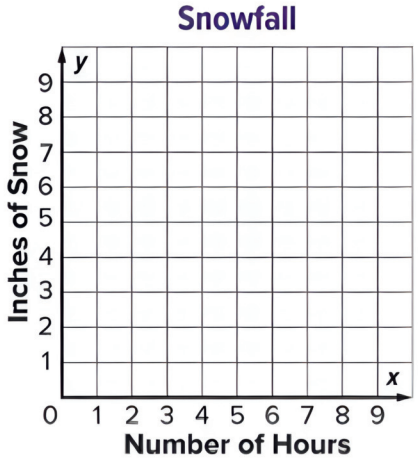
1. $y = \frac{3}{5}x - 3$



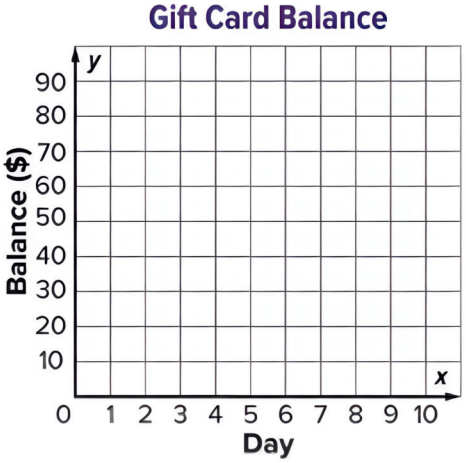
2. $y = -\frac{2}{3}x - 2$



3. The equation $y = \frac{1}{5}x + 3.5$ can be used to find the amount of accumulated snow *y* in inches *x* hours after 5 P.M. on a certain day. Graph this equation. (Example 2)



4. Alliyah's gift card balance can be represented by the equation $y = -5x + 50$, where *y* represents the gift card balance after *x* number of days. Graph this equation. (Example 2)

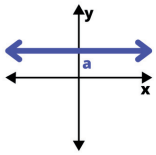


Module 4: Linear Relationships and Slope

M4L6: Graph Linear Equations

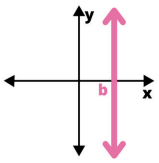
Equation of a Horizontal Line

$y=a$



Equation of a Vertical Line

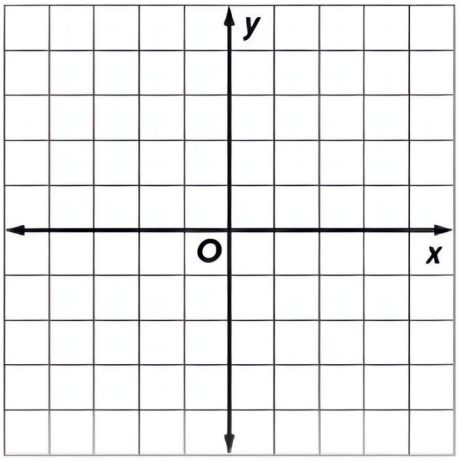
$x=b$





Graph each equation. (Examples 3 and 4)

5. $y = -3$



6. $x = 2$

