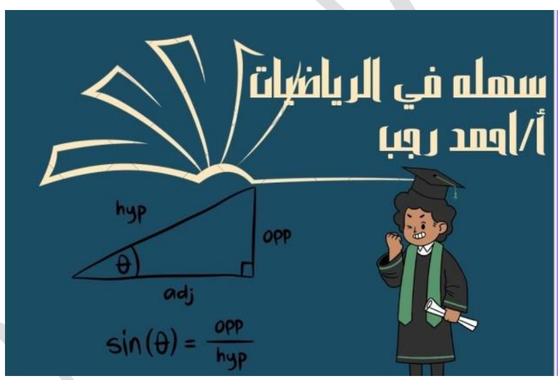
@MATHE1111

التقويمي الثاني لماده رياضيات

الصف الحادي عشر علمي

الفصل الدراسي الثاني2024/2023

اعداد الاستاذ / احمد رجب



أضغط هنا

أضغط هنا

أضغط هنا
لارسال رساله

بند (9-2)

اثبت صحه المتطابقه $\frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta} = \tan^2\theta$

الحل

قواعد هامه

$1-\cos^2\theta = \sin^2\theta$:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

الطرف الايسر

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \tan^2\theta$$

18/17دور ثاني

اثبت صحه المتطابقه :

$$\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$$

الحل

قواعد هامه

$1-\sin^2 x = \cos^2 x$

Lhs: $\frac{\cos x}{1-\sin x} \times \frac{1+\sin x}{1+\sin x} = \frac{\cos x(1+\sin x)}{1-\sin^2 x}$

$$\frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x}$$

اثبت صحه المتطابقه: $\frac{1-\cos x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}=(\csc x-\cot x)^2$

الحل

الطرف الايسر :

$1-\cos^2\theta=\sin^2\theta$

$\frac{1}{\sin x}=\csc x$

$\frac{\cos x}{\sin x}=\cot x$

$$\frac{1-\cos x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}$$

$$=\frac{(1-\cos x)^2}{1-\cos^2 x}$$

$$=\frac{(1-\cos x)^2}{\sin^2\theta}$$

$$=(\frac{1-\cos x}{\sin x})^2$$

$$=(\frac{1}{\sin x}-\frac{\cos x}{\sin x})^2=(\csc x-\cot x)^2$$

اثبت صحه المتطابقه :

$\tan x+\cot x=\sec x.\csc x$

الحل

$\sin^2\theta+\cos^2\theta=1$

$\frac{1}{\sin x}=\csc x$

$\frac{1}{\cos x}=\sec x$

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\cos x.\sin x}$$

$$=\frac{1}{\cos x.\sin x}=\frac{1}{\cos x}.\frac{1}{\sin x}$$

$$=\sec x.\csc x$$

اثبت صحه المتطابقه :

19/18

$$\frac{1}{1-\cos x}+\frac{1}{1+\cos x}=2\csc^2 x$$

الحل

$$\frac{1}{1-\cos x}+\frac{1}{1+\cos x}=\frac{1+\cos x+1-\cos x}{(1-\cos x)(1+\cos x)}$$

$$=\frac{2}{1-cos^2x}$$

$$\frac{2}{1-cos^2x}=\frac{2}{sin^2x}=2\csc^2 x$$

$$1-cos^2x=sin^2x$$

$$\frac{1}{sin^2x}=\csc^2 x$$

اثبت صحه المتطابقه : $2cotx\ cscx\ =\frac{1}{1-\cos x}+\frac{1}{1+\cos x}$

اثبت صحه المتطابقه $(1-tanx)^2=sec^2x-2tanx$

اثبت صحه المتطابقه: $\frac{1+\cos x}{1-\cos x}-\frac{1-\cos x}{1+\cos x}=4tanx\,.\,secx$

بند (3-9)

حل المعادله : $\sqrt{2}\, cos\ x = 1$

الحل

$$\cos x = \frac{1}{\sqrt{2}}$$

نفرض ان $\propto$ هي زاويه الاسناد

$$\cos \alpha = |\cos x| \rightarrow \alpha = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$\cos \alpha > 0$ تقع في الربع الاول الرابع

الاول $x = \frac{\pi}{4} + 2k\pi$

الرابع $x = \left(2\pi - \frac{\pi}{4}\right) + 2k\pi$

حل المعادله :

$x = \frac{\pi}{4} + 2k\pi$ او $x = \left(\frac{7\pi}{4}\right) + 2k\pi$

حل المعادله : $3\sin\theta + 1 = \sin\theta$

حل المعادلة : $\cos^2 x + 3\cos x + 2 = 0$

الحل

$(\cos x + 1)(\cos x + 2) = 0$

$\cos x + 2 = 0$

$\cos x = -2 \notin [-1,1]$

مرفوضه

$\cos x + 1 = 0$

$\cos x = -1$

زاويه ربعيه

$x = \pi + 2k\pi$

حل المعادله :

$x = \pi + 2k\pi$

حل المعادله :

$2\sin^2 x - \sin x - 2 = 0$

الحل

19/18

$(2\sin x + 1)(\sin x - 2) = 0$

$2\sin x + 1 = 0 \rightarrow \sin x = \frac{-1}{2}$

$(\sin x - 2) = 0$

$\sin x = 2$

$2 \notin [-1,1]$

مرفوضه

نفرض ان ∝ هي زاويه الاسناد

$\sin\alpha = |\sin x| \rightarrow \alpha = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\sin\alpha < 0$ تقع في الربع الثالث الرابع

$x = (\pi + \frac{\pi}{6}) + 2k\pi$ الثالث $\rightarrow x = \frac{7\pi}{6} + 2k\pi$

$x = \left(2\pi - \frac{\pi}{6}\right) + 2k\pi$ الرابع $\rightarrow x = \left(\frac{11\pi}{6}\right) + 2k\pi$

حل المعادله : $x = \left(\frac{11\pi}{6}\right) + 2k\pi$, $x = \frac{7\pi}{6} + 2k\pi$

حل المعادلة : $\mathbf{\sin\theta\cos\theta - \cos\theta = 0}$

الحل

$$\cos\theta\,(\sin\theta - 1) = 0$$

$\sin\theta - 1 = 0$

$\sin\theta = 1$

زاويه ربعيه

$\theta = \frac{\pi}{2} + 2k\pi$,

$\cos\theta = 0$

زاويه ربعيه

$\theta = \frac{\pi}{2} + 2k\pi$, $\theta = \frac{3\pi}{4} + 2k\pi$

حل المعاداله :

$$\theta = \frac{\pi}{2} + 2k\pi \;,\; \theta = \frac{3\pi}{4} + 2k\pi$$

حل المعادلة : $\mathbf{2\cos x \,.\, \sin x - \cos\theta = 0}$

الحل

حل المعادلة : $\tan x = \sqrt{3}$

الحل

$$tan\, x = \sqrt{3}$$

نفرض ان α هي زاويه الاسناد

$$\tan\alpha = |\tan x| \qquad \rightarrow \alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

تقع في الربع الاول , الثالث $\tan x > 0$

$$x = \frac{\pi}{3} + k\pi$$

حل المعادله: $x = \frac{\pi}{3} + k\pi$

حل المعادلة : $\sqrt{3}\tan x = 1$

18/17دور ثاني

بند (3-9) و(4-9)

اذا كان $0 < \alpha < \frac{\pi}{2}$, $\sin\alpha = \frac{4}{5}$,

$\frac{3\pi}{2} > \alpha > \pi$, $\cos\beta = \frac{-12}{13}$ اوجد مما يلي :

$$(1)\sin(\alpha+\beta) \qquad (2)\tan 2\beta$$

الحل

$\sin^2 + \cos^2 = 1$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$(\frac{4}{5})^2 + \cos^2\alpha = 1 \quad \rightarrow \quad \cos^2\alpha = 1 - (\frac{4}{5})^2$$

$$\cos\alpha = \pm\frac{3}{5} \quad \rightarrow \quad 0 < \alpha < \frac{\pi}{2} \quad \rightarrow \quad \cos\alpha = \frac{3}{5}$$

$$\sin^2\beta + \cos^2\beta = 1$$

$$\sin^2\beta + (\frac{-12}{13})^2 = 1 \quad \rightarrow \quad \sin^2\beta = 1 - (\frac{-12}{13})^2$$

$$\sin\beta = \pm\frac{5}{13} \quad \rightarrow \pi < \alpha < \frac{3\pi}{2} \quad \rightarrow \quad \sin\beta = -\frac{5}{13}$$

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{-\frac{5}{13}}{\frac{-12}{13}} = \frac{5}{12}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \frac{4}{5}\times\frac{-12}{13} + \frac{3}{5}\times -\frac{5}{13} = \frac{-63}{65}$$

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\times\frac{5}{12}}{1-(\frac{5}{12})^2} = \frac{120}{119}$$

19/18دور تاني

اذا كان $\frac{3\pi}{2} < \alpha < 2\pi$, $\sin\theta = \frac{-12}{13}$, اوجد $\sin 2\theta$

الحل

$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin^2\theta + \cos^2\theta = 1$

$(\frac{-12}{13})^2 + \cos^2\theta = 1 \quad \rightarrow \quad \cos^2\theta = 1 - (\frac{-12}{13})^2$

$\cos\theta = \pm\frac{5}{13} \quad \rightarrow \quad \frac{3\pi}{2} < \alpha < 2\pi \quad \rightarrow \quad \cos\theta = \frac{5}{13}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2 \times \frac{-12}{13} \times \frac{5}{13} = \frac{-120}{169}$

18/17

$\pi < \alpha < \frac{3\pi}{2}$, $\sin\theta = \frac{-3}{5}$, اوجد

$\sin\frac{\theta}{2}$(1) $\quad$ $\tan 2\theta$(2)

الحل

$\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$(\frac{-3}{5})^2 + \cos^2\theta = 1 \quad \rightarrow \quad \cos^2\theta = 1 - (\frac{-3}{5})^2$

$\cos\theta = \pm\frac{4}{5} \quad \rightarrow \quad \pi < \alpha < \frac{3\pi}{2} \quad \rightarrow \quad \cos\theta = \frac{-4}{5}$

$\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos\theta}{2}} = \pm\sqrt{\frac{1-(\frac{-4}{5})}{2}}$

$\sin(\frac{\theta}{2}) = \pm\frac{3\sqrt{10}}{10} \quad \rightarrow \frac{\pi}{2} < \alpha < \frac{3\pi}{4} \rightarrow \sin(\frac{\theta}{2}) = \frac{3\sqrt{10}}{10}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-3}{5}}{\frac{-4}{5}} = \frac{3}{4} \quad \rightarrow \quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{3}{4}}{1-(\frac{3}{4})^2} = \frac{24}{7}$

17/16

اذا كان $\sin\alpha = \frac{3}{5}$, $\cos\beta = \frac{24}{25}$ زاويتين حادتين اوجد :

$\cos(\alpha-\beta)$, $\sin(\frac{\pi}{2}-\beta)$

الحل

$\sin^2\alpha + \cos^2\alpha = 1$

$(\frac{3}{5})^2 + \cos^2\alpha = 1 \quad \rightarrow \quad \cos^2\alpha = 1 - (\frac{3}{5})^2$

$\cos\alpha = \pm\frac{4}{5} \quad \rightarrow$ زاويه حاده $\rightarrow \quad \cos\alpha = \frac{4}{5}$

$\sin^2\beta + \cos^2\beta = 1$

$\sin^2\beta + (\frac{24}{25})^2 = 1 \quad \rightarrow \quad \sin^2\beta = 1 - (\frac{24}{25})^2$

$\sin\beta = \pm\frac{7}{25} \quad \rightarrow$ زاويه حاده $\rightarrow \quad \sin\beta = \frac{7}{25}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \frac{4}{5}\times\frac{24}{25} + \frac{3}{5}\times\frac{7}{25} = \frac{117}{125}$

$\sin(\frac{\pi}{2}-\beta) = \cos\beta = \frac{24}{25}$

$\sin^2 + \cos^2 = 1$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\frac{\pi}{2}-\beta) = \cos\beta$

قوانين هامه

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 - \tan\alpha\times\tan\beta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$