8-1 Introduction to Vectors

State whether each quantity described is a *vector* quantity or a *scalar* quantity.

1. a box being pushed at a force of 125 newtons

   SOLUTION:

   This quantity has a magnitude of 125 newtons, but no direction is given. This is a scalar quantity.

   ANSWER:

   scalar

2. wind blowing at 20 knots

   SOLUTION:

   This quantity has a magnitude of 20 knots, but no direction is given. This is a scalar quantity.

   ANSWER:

   scalar

3. a deer running 15 meters per second due west

   SOLUTION:

   This quantity has a magnitude of 15 meters per second and a direction of due west. This is a vector quantity.

   ANSWER:

   vector

4. a baseball thrown with a speed of 85 miles per hour

   SOLUTION:

   This quantity has a magnitude of 85 miles per hour, but no direction is given. This is a scalar quantity.

   ANSWER:

   scalar

5. a 15-pound tire hanging from a rope

   SOLUTION:

   Weight is a vector quantity that is calculated using the mass of the tire and the downward pull due to gravity.

   ANSWER:

   vector

6. a rock thrown straight up at a velocity of 50 feet per second

   SOLUTION:

   This quantity has a magnitude of 50 feet per second and a direction of straight up. This is a vector quantity.

   ANSWER:

   vector

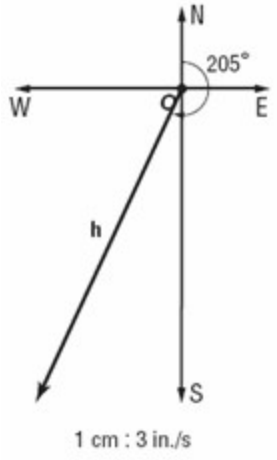
8-1 Introduction to Vectors

Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

7. **h** = 13 inches per second at a bearing of 205°

SOLUTION:

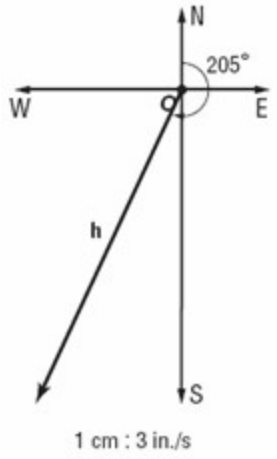
Sample answer: Using a scale of 1 cm : 3 in./s, draw and label a 13 ÷ 3 or about 4.33-centimeter arrow at an angle of 205° clockwise from the north.



Drawing may not be to scale.

ANSWER:

Sample answer:

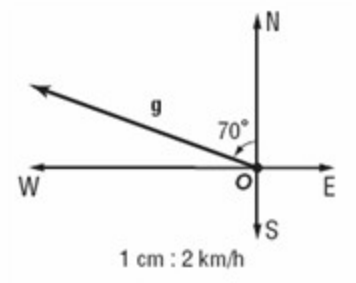


Drawing may not be to scale.

8. **g** = 6 kilometers per hour at a bearing of N70°W

SOLUTION:

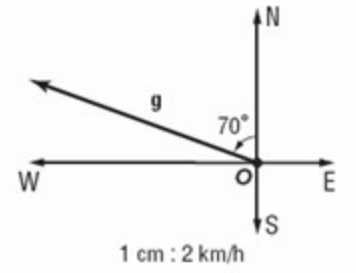
Sample answer: Using a scale of 1 cm : 2 km/h, draw and label a 3-centimeter arrow at an angle of 70° west of north.



Drawing may not be to scale.

ANSWER:

Sample answer:

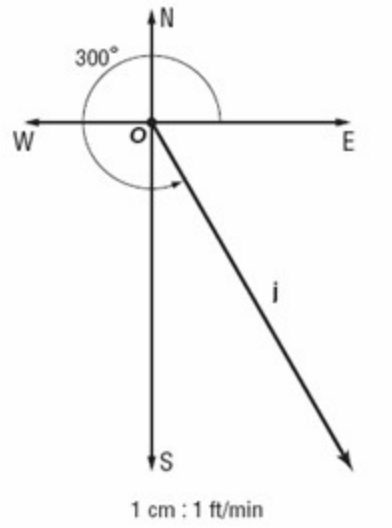


Drawing may not be to scale.

9. **j** = 5 feet per minute at 300° to the horizontal

SOLUTION:

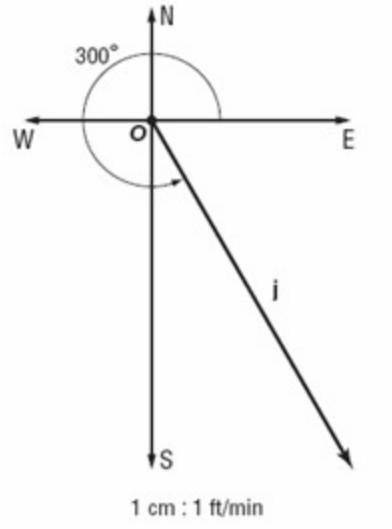
Sample answer: Using a scale of 1 cm : 1 ft/min, draw and label a 5-centimeter arrow at an angle of 300° to the *x*-axis.



Drawing may not be to scale.

ANSWER:

Sample answer:

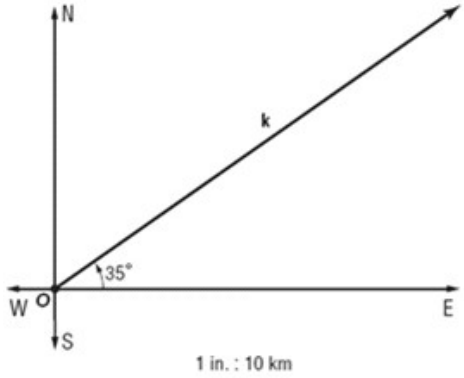


Drawing may not be to scale.

10. **k** = 28 kilometers at 35° to the horizontal

SOLUTION:

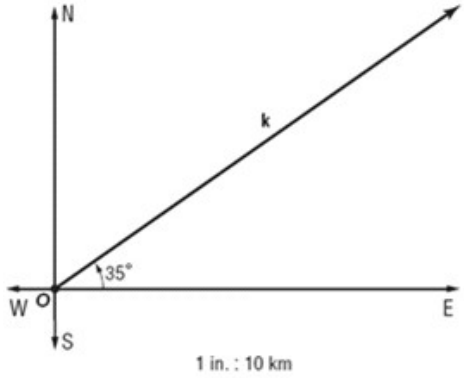
Sample answer: Using a scale of 1 in : 10 km, draw and label a 2.8-inch arrow at an angle of 35° to the *x*-axis.



Drawing may not be to scale.

ANSWER:

Sample answer:

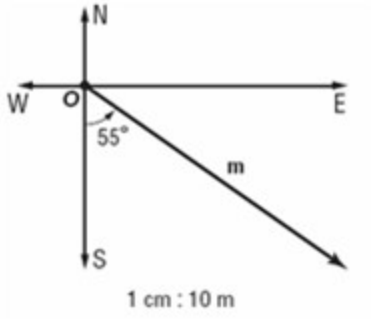


Drawing may not be to scale.

11. **m** = 40 meters at a bearing of S55°E

SOLUTION:

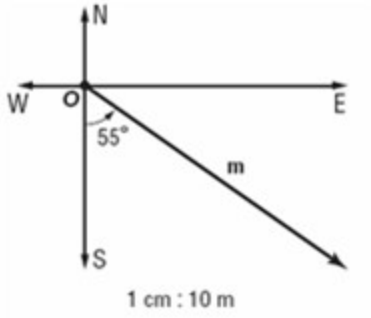
Sample answer: Using a scale of 1 cm : 10 m, draw and label a 4-centimeter arrow at an angle of 55° east of south.



Drawing may not be to scale.

ANSWER:

Sample answer:

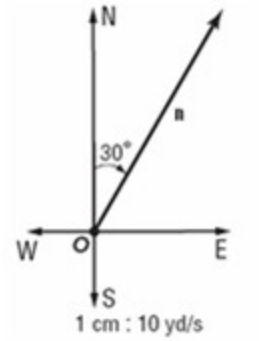


Drawing may not be to scale.

12. **n** = 32 yards per second at a bearing of 030°

SOLUTION:

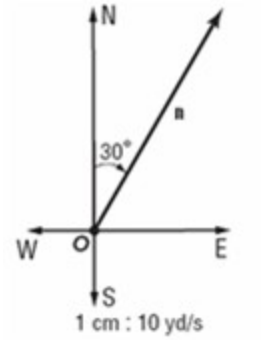
Sample answer: Using a scale of 1 cm : 10 yd/sec, draw and label a 3.2-centimeter arrow at an angle of 30° clockwise from the north.



Drawing may not be to scale.

ANSWER:

Sample answer:

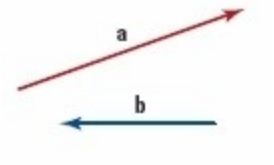


Drawing may not be to scale.

8-1 Introduction to Vectors

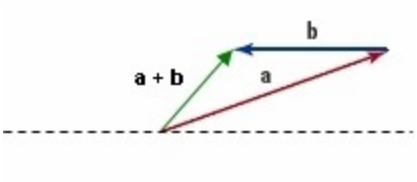
Find the resultant of each pair of vectors using either the triangle or parallelogram method. State the magnitude of the resultant to the nearest tenth of a centimeter and its direction relative to the horizontal.

13.



SOLUTION:

Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b** as shown. Draw the horizontal.



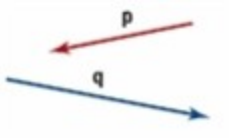
Drawing may not be to scale.

Measure the length of **a** + **b** and then measure the angle this vector makes with the horizontal.
The vector has a length of approximately 1.4 centimeters and is at an approximate angle of 50° with the horizontal.

ANSWER:

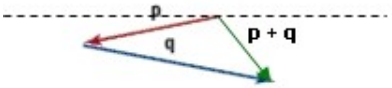
1.4 cm, 50^{o}

14.



SOLUTION:

Translate **q** so that its tail touches the tip of **p**. Then draw the resultant vector **p** + **q** as shown. Draw the horizontal.



Drawing may not be to scale.

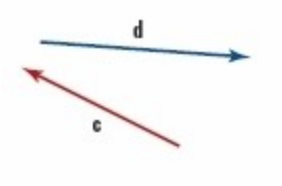
Measure the length of **p** + **q** and then measure the angle this vector makes with the horizontal.
The vector has a length of approximately 1.1 centimeters and is at an approximate angle of 310° with the horizontal.

ANSWER:

1.1 cm, 310^{o}

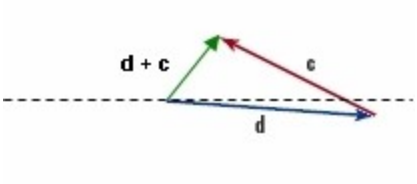
8-1 Introduction to Vectors

15.



SOLUTION:

Translate **c** so that its tail touches the tip of **d**. Then draw the resultant vector **d** + **c** as shown. Draw the horizontal.



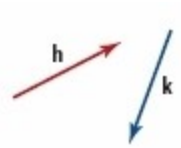
Drawing may not be to scale.

Measure the length of **d** + **c** and then measure the angle this vector makes with the horizontal.
The vector has a length of approximately 1.0 centimeter and is at an approximate angle of 46° with the horizontal.

ANSWER:

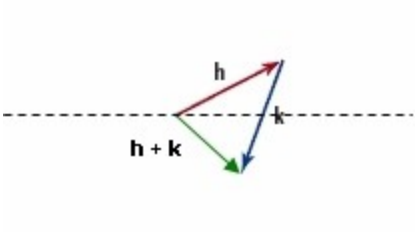
1.0 cm, 46^{o}

16.



SOLUTION:

Translate **k** so that its tail touches the tip of **h**. Then draw the resultant vector **h** + **k** as shown. Draw the horizontal.



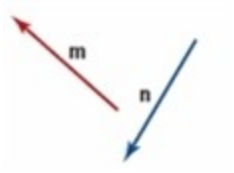
Drawing may not be to scale.

Measure the length of **h** + **k** and then measure the angle this vector makes with the horizontal.
The vector has a length of approximately 1.1 centimeters and is at an approximate angle of 320° with the horizontal.

ANSWER:

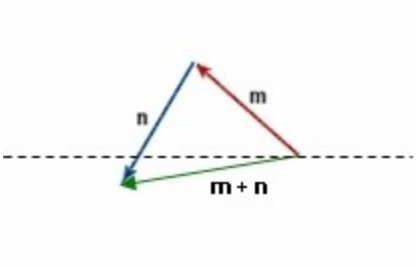
1.1 cm, 320^{o}

8-1 Introduction to Vectors

17. 

SOLUTION:

Translate **n** so that its tail touches the tip of **m**. Then draw the resultant vector **m** + **n** as shown. Draw the horizontal.

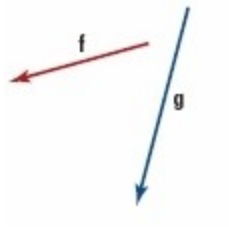


Drawing may not be to scale.

Measure the length of **m** + **n** and then measure the angle this vector makes with the horizontal.
The vector has a length of approximately 2.3 centimeters and is at an approximate angle of 188° with the horizontal.

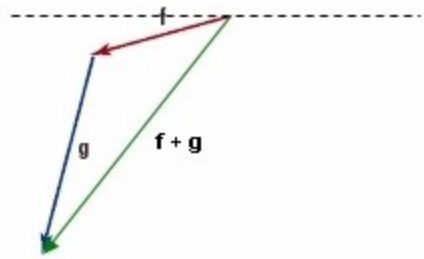
ANSWER:

2.3 cm, 188°

18. 

SOLUTION:

Translate **g** so that its tail touches the tip of **f**. Then draw the resultant vector **f** + **g** as shown. Draw the horizontal.



Drawing may not be to scale.

Measure the length of **f** + **g** and then measure the angle this vector makes with the horizontal.
The vector has a length of approximately 3.8 centimeters and is at an approximate angle of 231° with the horizontal.

ANSWER:

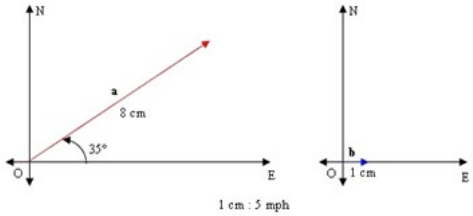
3.8 cm, 231°

19. **GOLFING** While playing a golf video game, Ana hits a ball 35° above the horizontal at a speed of 40 miles per hour with a 5 miles per hour wind blowing, as shown. Find the resulting speed and direction of the ball.

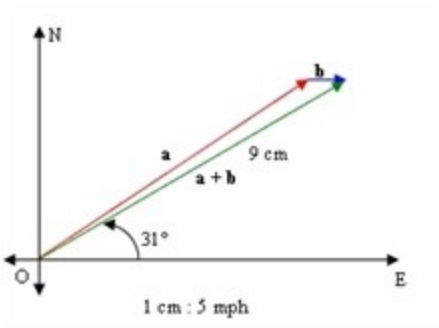


SOLUTION:

Let **a** = hitting a ball 40 miles per hour at an angle of 35° above the horizontal and **b** = a 5 mph wind blowing due east. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 5 mph.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a + b**.



Drawings may not be to scale.

Measure the length of **a + b** and then measure the angle this vector makes with the horizontal. The length of the vector is approximately 9 centimeters, which is 9 × 5 or 45 miles per hour. Therefore, Ana's ball is traveling approximately 45 miles per hour at an angle of 31° with the horizontal.

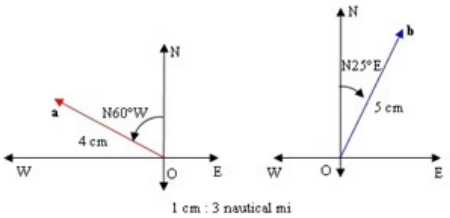
ANSWER:

45 mph, 31°

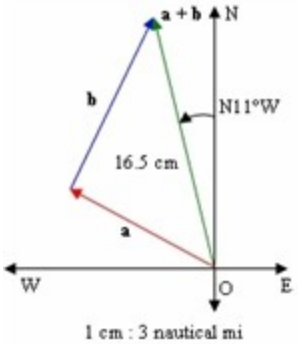
20. **BOATING** A charter boat leaves port on a heading of N60°W for 12 nautical miles. The captain changes course to a bearing of N25°E for the next 15 nautical miles. Determine the ship's distance and direction from port to its current location.

SOLUTION:

Let **a** = the boat leaving port on a heading of N60°W for 12 nautical miles and **b** = the new course of N25°E for 15 nautical miles. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 3 mi/h.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a + b**.



Drawings may not be to scale.

Measure the length of **a + b** and then measure the angle this vector makes with the horizontal. The length of the vector is approximately 6.5 centimeters, which is 6.5 × 3 or 19.5 nautical miles. Therefore, the ship traveled approximately 19.5 nautical miles at a bearing of N11°W.

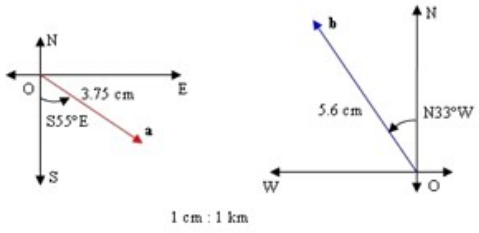
ANSWER:

19.5 nautical mi, N11°W

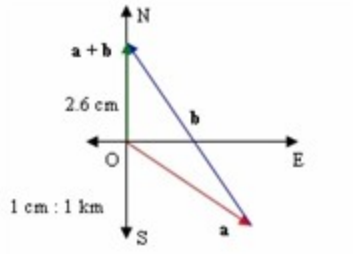
21. **HIKING** Nick and Lauren hiked 3.75 kilometers to a lake 55° east of south from their campsite. Then they hiked 33° west of north to the nature center 5.6 kilometers from the lake. Where is the nature center in relation to their campsite?

SOLUTION:

Let **a** = Nick and Lauren hiking 3.75 kilometers 55° east of south to the lake and **b** = Nick and Lauren hiking 5.6 kilometers 33° west of north to the nature center. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 1 km.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Measure the length of **a** + **b** and then measure the angle this vector makes with the horizontal. The length of the vector is approximately 2.6 centimeters, which is 2.6 kilometers. Therefore, the nature center is approximately 2.6 kilometers due north of the campsite.
Drawings may not be to scale.

ANSWER:

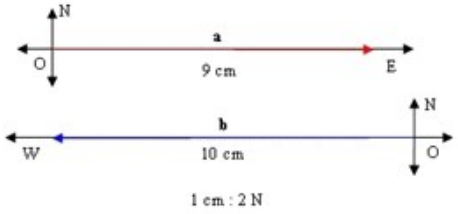
2.6 km due north

Determine the magnitude and direction of the resultant of each vector sum.

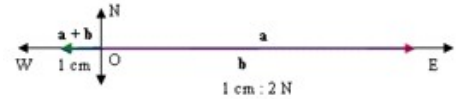
22. 18 newtons directly forward and then 20 newtons directly backward

SOLUTION:

Let **a** = 18 newtons directly forward and **b** = 20 newtons directly backward. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 2 N. Since **a** is going forward and **b** is going backward, draw **a** so that it is headed due east and draw **b** so that it is headed due west.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Drawings may not be to scale.

Measure the length of **a** + **b**. The length of the vector is approximately 1.0 centimeter, which is 1.0 × 2 or 2 newtons. **a** + **b** is in the direction of **b**. Since the direction of **b** is backwards, the resultant vector is 2 newtons backwards.

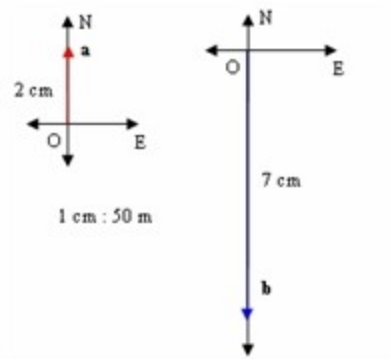
ANSWER:

2 N backward

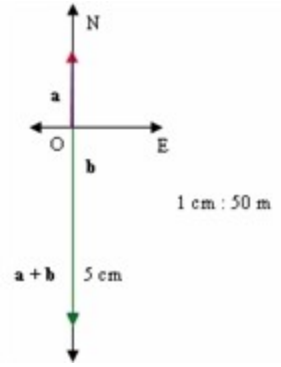
23. 100 meters due north and then 350 meters due south

SOLUTION:

Let **a** = 100 meters due north and **b** =350 meters due south. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 50 m.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Drawings may not be to scale.

Measure the length of **a** + **b**. The length of the vector is approximately 5.0 centimeters, which is 5.0 × 50 or 250 meters. **a** + **b** is in the direction of **b**. Since the direction of **b** is due south, the resultant vector is 250 meters due south.

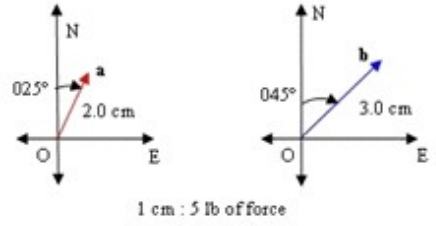
ANSWER:

250 m due south

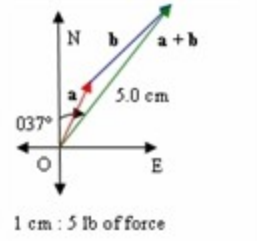
24. 10 pounds of force at a bearing of 025° and then 15 pounds of force at a bearing of 045°

SOLUTION:

Let **a** = 10 pounds of force at a bearing of 025° and **b** = 15 pounds of force at a bearing of 045°. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 5 lb of force.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Drawings may not be to scale.

Measure the length of **a** + **b** and then measure the angle this vector makes with the horizontal. The length of the vector is approximately 5 centimeters, which is 5 × 5 or 25 pounds of force. Therefore, the resultant is about 25 pounds of force at a bearing of 037°.

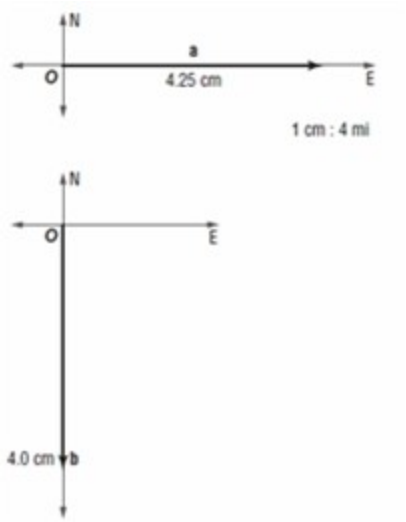
ANSWER:

25 lb of force at a bearing of 037°

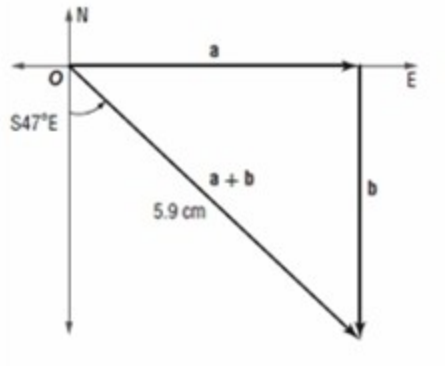
25. 17 miles east and then 16 miles south

SOLUTION:

Let **a** = 17 miles east and **b** = 16 miles south. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 4 mi.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Measure the length of **a** + **b** and then measure the angle this vector makes with the horizontal. The length of the vector is approximately 5.9 centimeters, which is 5.9 × 4 or 23.6 miles. Therefore, the resultant is about 23.6 miles at a bearing of S47°E. Drawings may not be to scale.

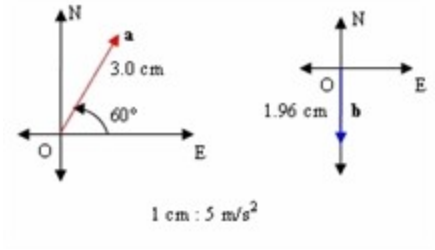
ANSWER:

23.6 mi at a bearing of S47°E

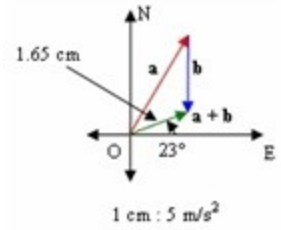
26. 15 meters per second squared at a 60° angle to the horizontal and then 9.8 meters per second squared downward

SOLUTION:

Let **a** = 15 meters per second squared at a 60° angle to the horizontal and **b** = 9.8 meters per second squared downward. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 5 m/s^2.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Drawings may not be to scale.

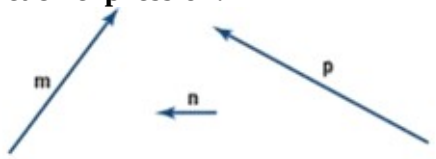
Measure the length of **a** + **b** and then measure the angle this vector makes with the horizontal. The length of the vector is approximately 1.65 centimeters, which is 1.65 × 5 or 8.25 meters per second squared. Therefore, the resultant is about 8.25 meters per second squared at an angle of 23° to the horizontal.

ANSWER:

8.25 m/s^2 at 23° to the horizontal

8-1 Introduction to Vectors

Use the set of vectors to draw a vector diagram of each expression.



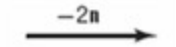
27. $\mathbf{m} - 2\mathbf{n}$

SOLUTION:

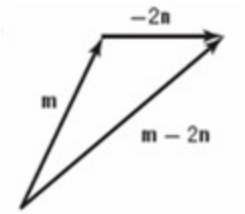
Rewrite the expression as the addition of two vectors: $\mathbf{m} - 2\mathbf{n} = \mathbf{m} + (-2\mathbf{n})$. Draw $\mathbf{m}$.



To represent $-2\mathbf{n}$, draw a vector 2 times as long as $\mathbf{n}$ in the opposite direction from $\mathbf{n}$.

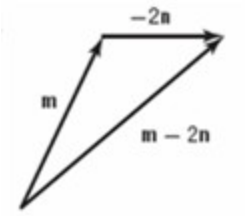


Then use the triangle method to draw the resultant vector.



Drawings may not be to scale.

ANSWER:



Drawing may not be to scale.

28. $\mathbf{n} - \frac{3}{4}\mathbf{m}$

SOLUTION:

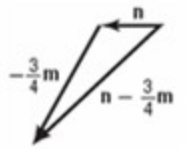
Rewrite the expression as the addition of two vectors: $\mathbf{n} - \frac{3}{4}\mathbf{m} = \mathbf{n} + \left(-\frac{3}{4}\mathbf{m}\right)$. Draw $\mathbf{n}$.



To represent $-\frac{3}{4}\mathbf{m}$, draw a vector $\frac{3}{4}$ the length of $\mathbf{m}$ in the opposite direction from $\mathbf{m}$.

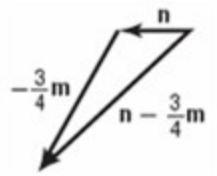


Then use the triangle method to draw the resultant vector.



Drawings may not be to scale.

ANSWER:



Drawing may not be to scale.

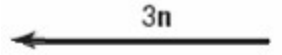
29. $\frac{1}{2}\mathbf{p} + 3\mathbf{n}$

SOLUTION:

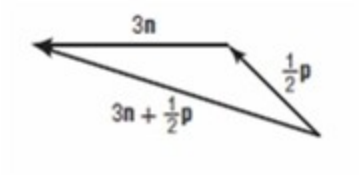
The expression is the addition of two vectors: $\frac{1}{2}\mathbf{p} + 3\mathbf{n}$. To represent $\frac{1}{2}\mathbf{p}$, draw a vector $\frac{1}{2}$ the length of **p** in the same direction as **p**.



To represent 3**n**, draw a vector 3 times as long as **n** in the same direction as **n**.

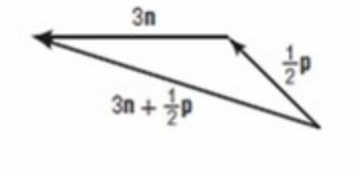


Then use the triangle method to draw the resultant vector.



Drawings may not be to scale.

ANSWER:

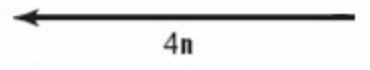


Drawing may not be to scale.

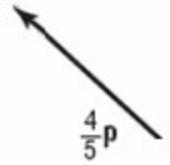
30. $4\mathbf{n} + \frac{4}{5}\mathbf{p}$

SOLUTION:

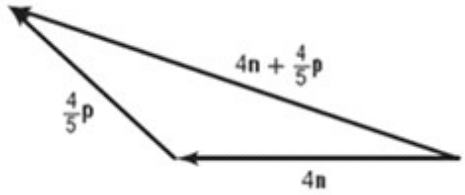
The expression is the addition of two vectors: $4\mathbf{n} + \frac{4}{5}\mathbf{p}$. To represent 4**n**, draw a vector 4 times as long as **n** in the same direction as **n**.



To represent $\frac{4}{5}\mathbf{p}$, draw a vector $\frac{4}{5}$ the length of **p** in the same direction as **p**.

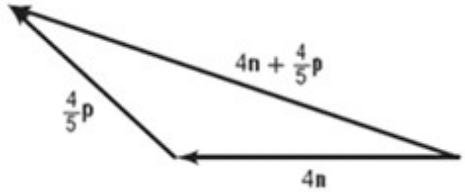


Then use the triangle method to draw the resultant vector.



Drawings may not be to scale.

ANSWER:

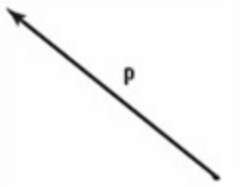


Drawing may not be to scale.

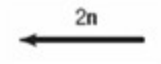
31. $\mathbf{p} + 2\mathbf{n} - \mathbf{m}$

SOLUTION:

Rewrite the expression as the addition of three vectors: $\mathbf{p} + 2\mathbf{n} - \mathbf{m} = \mathbf{p} + 2\mathbf{n} + (-\mathbf{m})$. Draw $\mathbf{p}$.



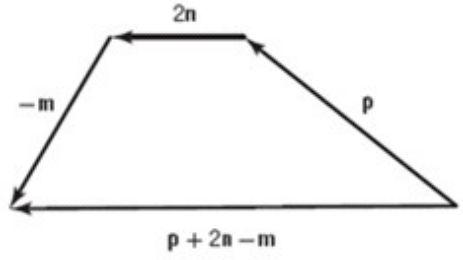
To represent $2\mathbf{n}$, draw a vector 2 times as long as $\mathbf{n}$ in the same direction as $\mathbf{n}$.



To represent $-\mathbf{m}$, draw a vector the same length as $\mathbf{m}$ in the opposite direction from $\mathbf{m}$.

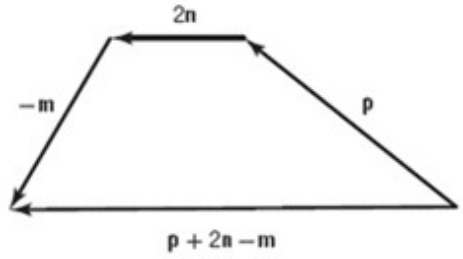


Translate $2\mathbf{n}$ so that its tail touches the tip of $\mathbf{p}$. Then, translate $-\mathbf{m}$ so that its tail touches the tip of $2\mathbf{n}$. Finally, draw the resultant vector $\mathbf{p} + 2\mathbf{n} - \mathbf{m}$.



Drawings may not be to scale.

ANSWER:



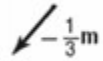
Drawing may not be to scale.

32. $-\frac{1}{3}\mathbf{m} + \mathbf{p} - 2\mathbf{n}$

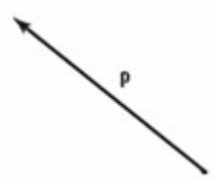
SOLUTION:

Rewrite the expression as the addition of three vectors: $-\frac{1}{3}\mathbf{m} + \mathbf{p} - 2\mathbf{n} = -\frac{1}{3}\mathbf{m} + \mathbf{p} + (-2\mathbf{n})$.

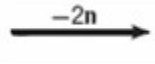
To represent $-\frac{1}{3}\mathbf{m}$, draw a vector $\frac{1}{3}$ the length of $\mathbf{m}$ in the opposite direction from $\mathbf{m}$.



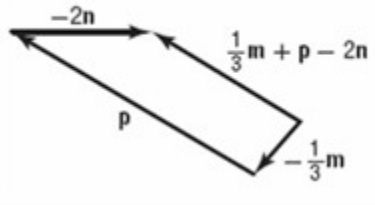
Draw $\mathbf{p}$.



To represent $-2\mathbf{n}$, draw a vector 2 times as long as $\mathbf{n}$ in the opposite direction from $\mathbf{n}$.

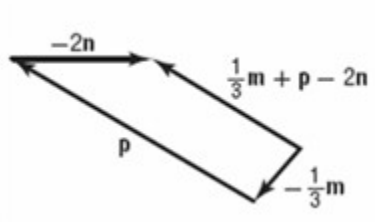


Translate $\mathbf{p}$ so that its tail touches the tip of $-\frac{1}{3}\mathbf{m}$. Then, translate $-2\mathbf{n}$ so that its tail touches the tip of $\mathbf{p}$. Finally, draw the resultant vector $-\frac{1}{3}\mathbf{m} + \mathbf{p} - 2\mathbf{n}$.



Drawings may not be to scale.

ANSWER:



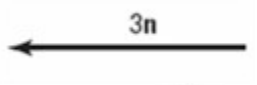
Drawing may not be to scale.

33. $3\mathbf{n} - \frac{1}{2}\mathbf{p} + \mathbf{m}$

SOLUTION:

Rewrite the expression as the addition of three vectors: $3\mathbf{n} - \frac{1}{2}\mathbf{p} + \mathbf{m} = 3\mathbf{n} + \left(-\frac{1}{2}\mathbf{p}\right) + \mathbf{m}$.

To represent 3**n**, draw a vector 3 times as long as **n** in the same direction as **n**.



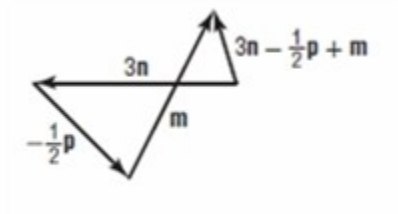
To represent $-\frac{1}{2}\mathbf{p}$, draw a vector $\frac{1}{2}$ the length of **p** in the opposite direction from **p**.



Draw **m**.

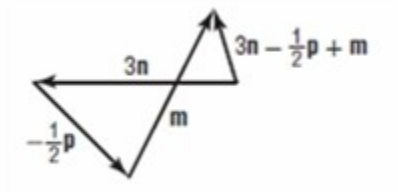


Translate $-\frac{1}{2}\mathbf{p}$ so that its tail touches the tip of 3**n**. Then, translate **m** so that its tail touches the tip of $-\frac{1}{2}\mathbf{p}$. Finally, draw the resultant vector $3\mathbf{n} - \frac{1}{2}\mathbf{p} + \mathbf{m}$.



Drawings may not be to scale.

ANSWER:



Drawing may not be to scale.

34. $\mathbf{m} - 3\mathbf{n} + \frac{1}{4}\mathbf{p}$

SOLUTION:

Rewrite the expression as the addition of three vectors: $\mathbf{m} - 3\mathbf{n} + \frac{1}{4}\mathbf{p} = \mathbf{m} + (-3\mathbf{n}) + \frac{1}{4}\mathbf{p}$. Draw **m**.



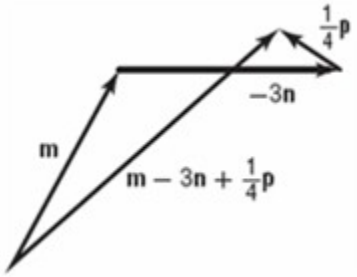
To represent −3**n**, draw a vector 3 times as long as **n** in the opposite direction from **n**.



To represent $\frac{1}{4}\mathbf{p}$, draw a vector $\frac{1}{4}$ the length of **p** in the same direction as **p**.

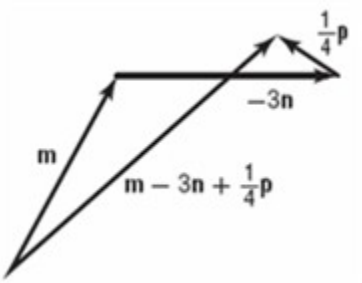


Translate −3**n** so that its tail touches the tip of **m**. Then, translate $\frac{1}{4}\mathbf{p}$ so that its tail touches the tip of −3**n**. Finally, draw the resultant vector $\mathbf{m} - 3\mathbf{n} + \frac{1}{4}\mathbf{p}$.



Drawings may not be to scale.

ANSWER:

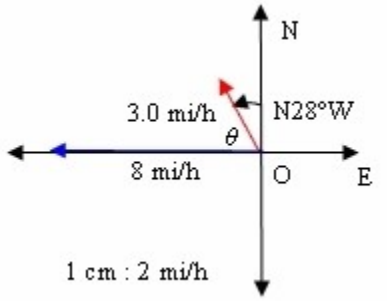


Drawing may not be to scale.

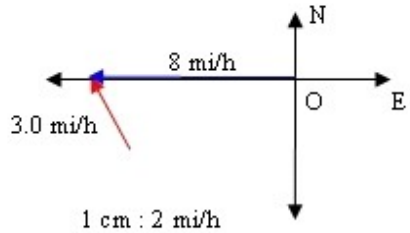
35. **RUNNING** A runner's resultant velocity is 8 miles per hour due west running with a wind of 3 miles per hour N28°W. What is the runner's speed, to the nearest mile per hour, without the effect of the wind?

SOLUTION:

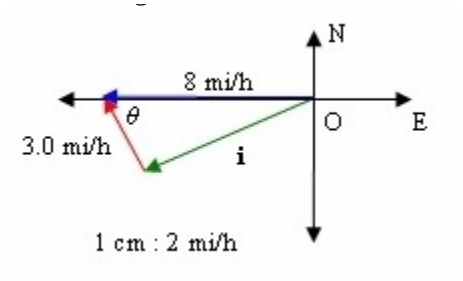
Draw a diagram to represent the runner's resultant velocity and the wind.



The compliment θ to the angle created by the wind blowing at N28°W measures 90 − 28 or 62°.
The vector representing the runner's resultant velocity is the sum of the vector representing the wind and a vector **i**, the runner's speed and direction without the effect of the wind. Translate the wind vector as shown.



Draw the vector **i**, the runner's speed and direction without the effect of the wind. Using the Alternate Interior Angles Theorem, we can label θ as shown.



Drawings may not be to scale.

Use the Law of Cosines to find $|\mathbf{i}|$, the runner's speed without the effect of the wind.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\mathbf{i}|^2 = 8^2 + 3^2 - 2(8)(3)\cos 62^\circ$$

$$|\mathbf{i}| = \sqrt{8^2 + 3^2 - 2(8)(3)\cos 62^\circ}$$

$$|\mathbf{i}| \approx 7.1039$$

The runner's speed, to the nearest mile per hour, without the effect of the wind is 7 miles per hour.

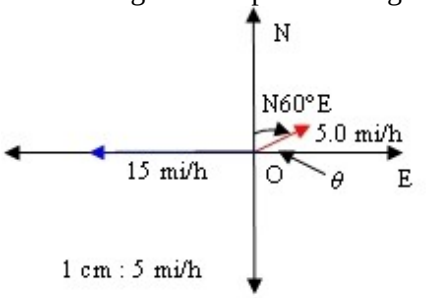
ANSWER:

7 mi/h

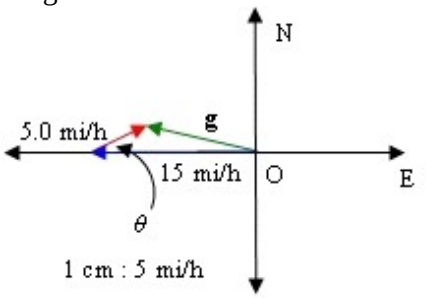
36. **GLIDING** A glider is traveling at an air speed of 15 miles per hour due west. If the wind is blowing at 5 miles per hour in the direction N60°E, what is the resulting ground speed of the glider?

SOLUTION:

Draw a diagram to represent the glider and the wind.



The compliment θ to the angle created by the wind blowing at N60°E measures 90 − 60 or 30°.
Translate the wind vector as shown and draw the resultant vector **g** representing the ground speed of the glider.



Drawings may not be to scale.

Use the Law of Cosines to find $|\mathbf{g}|$, the ground speed of the glider.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\mathbf{g}|^2 = 15^2 + 5^2 - 2(15)(5)\cos 30^\circ$$

$$|\mathbf{g}| = \sqrt{15^2 + 5^2 - 2(15)(5)\cos 30^\circ}$$

$$|\mathbf{g}| \approx 10.9588$$

The ground speed of the glider is approximately 11.0 mi/h.

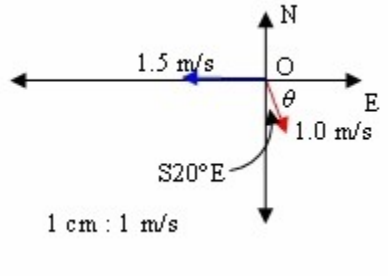
ANSWER:

about 11.0 mi/h

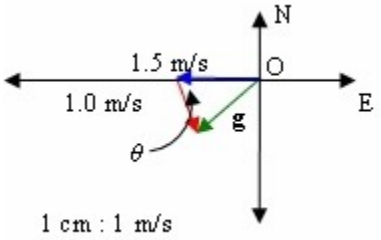
37. **CURRENT** Kaya is swimming due west at a rate of 1.5 meters per second. A strong current is flowing S20°E at a rate of 1 meter per second. Find Kaya's resulting speed and direction.

SOLUTION:

Draw a diagram to represent Kaya and the current.



The compliment θ to the angle created by the current at S20°E measures 90 − 20 or 70°.
Translate the vector representing the current as shown and draw the resultant vector **g** representing Kaya's resulting speed and direction.



Use the Law of Cosines to find $|\mathbf{g}|$, the ground speed of the glider.

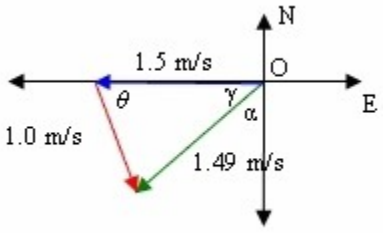
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\mathbf{g}|^2 = 1.5^2 + 1.0^2 - 2(1.5)(1.0)\cos 70^\circ$$

$$|\mathbf{g}| = \sqrt{1.5^2 + 1.0^2 - 2(1.5)(1.0)\cos 70^\circ}$$

$$\approx 1.4913$$

Kaya's resulting speed is about 1.49 meters per second.
The heading of the resultant **g** is represented by angle α, as shown. To find α, first calculate γ using the Law of Sines.



Drawings may not be to scale.

$$\frac{\sin\gamma}{a}=\frac{\sin\theta}{b}$$

$$\frac{\sin\gamma}{1.0}=\frac{\sin 70^\circ}{1.49}$$

$$\sin\gamma=\frac{1.0\cdot\sin 70^\circ}{1.49}$$

$$\gamma=\sin^{-1}\frac{1.0\cdot\sin 70^\circ}{1.49}$$

$$\gamma\approx 39.059$$

The measure of α is 90° − γ, which is about 50.94. Therefore, the speed of Kaya is about 1.49 meters per second at a bearing of S51°W.

ANSWER:

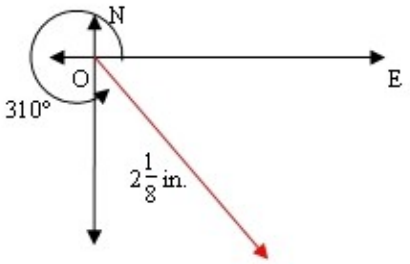
about 1.49 m/s at a bearing of S51°W

Draw a diagram that shows the resolution of each vector into its rectangular components. Then find the magnitudes of the vector's horizontal and vertical components.

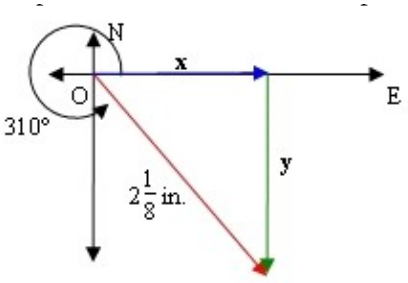
38. $2\frac{1}{8}$ inches at 310° to the horizontal

SOLUTION:

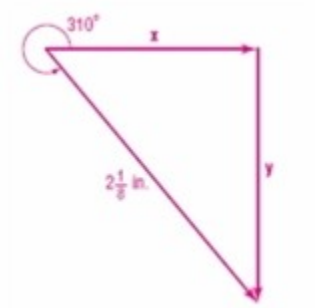
Draw a vector to represent $2\frac{1}{8}$ inches at 310° to the horizontal.



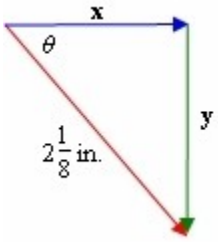
The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.



Remove the axes.



The horizontal and vertical components of the vector form a right triangle. The angle θ is 360° − 310° or 50°. Use the sine or cosine ratios to find the magnitude of each component.



Drawings may not be to scale.

$$\cos 50^\circ=\frac{|\mathbf{x}|}{2\frac{1}{8}}$$

$$|\mathbf{x}|=2\frac{1}{8}\cos 50^\circ$$

$$|\mathbf{x}|\approx 1.3659$$

$$\sin 50^\circ=\frac{|\mathbf{y}|}{2\frac{1}{8}}$$

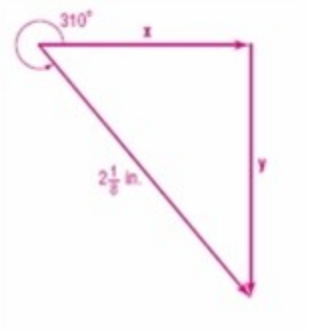
$$|\mathbf{y}|=2\frac{1}{8}\sin 50^\circ$$

$$|\mathbf{y}|\approx 1.6278$$

The magnitude of the horizontal component is about 1.37 inches and the magnitude of the vertical component is about 1.63 inches.

ANSWER:

about 1.37 in.; about 1.63 in.

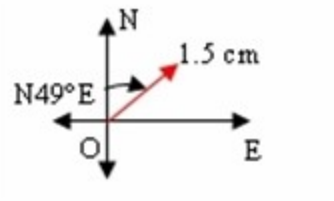


Drawing may not be to scale.

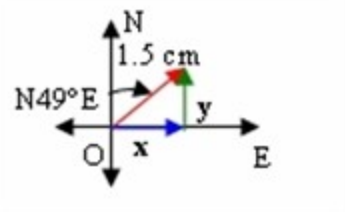
39. 1.5 centimeters at a bearing of N49°E

SOLUTION:

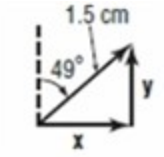
Draw a vector to represent 1.5 centimeters at a bearing of N49°E.



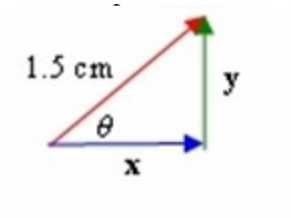
The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.



Remove the axes.



The horizontal and vertical components of the vector form a right triangle. The angle θ is 90° − 49° or 41°. Use the sine or cosine ratios to find the magnitude of each component.



Drawings may not be to scale.

$$\cos 41^\circ = \frac{|x|}{1.5} \qquad \sin 41^\circ = \frac{|y|}{1.5}$$

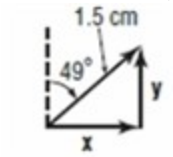
$$|x| = 1.5\cos 41^\circ \qquad |y| = 1.5\sin 41^\circ$$

$$|x| \approx 1.1321 \qquad |y| \approx 0.9841$$

The magnitude of the horizontal component is about 1.13 centimeters and the magnitude of the vertical component is about 0.98 centimeter.

ANSWER:

about 1.13 cm; about 0.98 cm

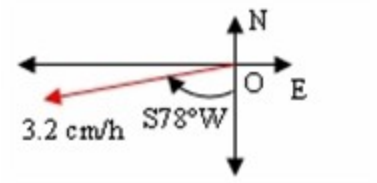


Drawing may not be to scale.

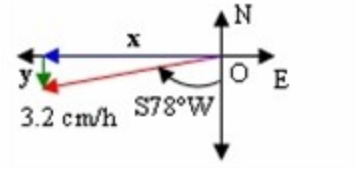
40. 3.2 centimeters per hour at a bearing of S78°W

SOLUTION:

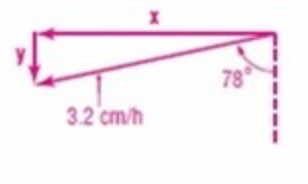
Draw a vector to represent 3.2 centimeters per hour at a bearing of S78°W.



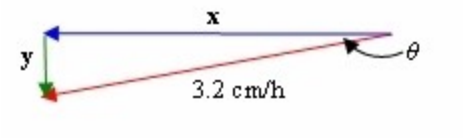
The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.



Remove the axes.



The horizontal and vertical components of the vector form a right triangle. The angle θ is 90° − 78° or 12°. Use the sine or cosine ratios to find the magnitude of each component.



Drawings may not be to scale.

$$\cos 12^\circ = \frac{|\mathbf{x}|}{3.2} \qquad \sin 12^\circ = \frac{|\mathbf{y}|}{3.2}$$

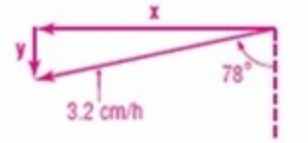
$$|\mathbf{x}| = 3.2\cos 12^\circ \qquad |\mathbf{y}| = 3.2\sin 12^\circ$$

$$|\mathbf{x}| \approx 3.1301 \qquad |\mathbf{y}| \approx 0.6653$$

The magnitude of the horizontal component is about 3.13 centimeters per hour and the magnitude of the vertical component is about 0.67 centimeter per hour.

ANSWER:

about 3.13 cm/h; about 0.67 cm/h

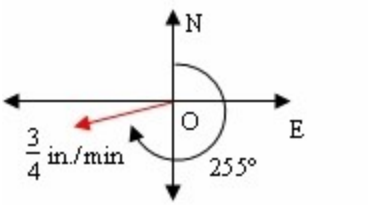


Drawing may not be to scale.

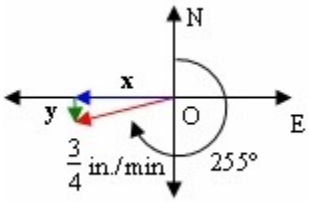
41. $\frac{3}{4}$ inch per minute at a bearing of 255°

SOLUTION:

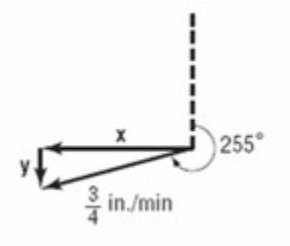
Draw a vector to represent $\frac{3}{4}$ inch per minute at a bearing of 255°.



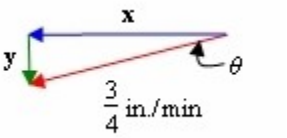
The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.



Remove the axes.



The horizontal and vertical components of the vector form a right triangle. The angle θ is 270° − 255° or 15°. Use the sine or cosine ratios to find the magnitude of each component.



Drawings may not be to scale.

$$\cos 15^\circ = \frac{|\mathbf{x}|}{\frac{3}{4}} \qquad \sin 15^\circ = \frac{|\mathbf{y}|}{\frac{3}{4}}$$

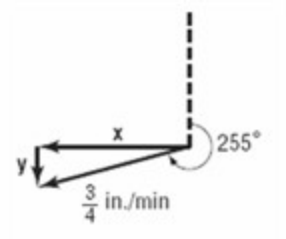
$$|\mathbf{x}| = \frac{3}{4}\cos 15^\circ \qquad |\mathbf{y}| = \frac{3}{4}\sin 15^\circ$$

$$|\mathbf{x}| \approx 0.7244 \qquad |\mathbf{y}| \approx 0.1941$$

The magnitude of the horizontal component is about 0.72 inch per minute and the magnitude of the vertical component is about 0.19 inch per minute.

ANSWER:

about 0.72 in./min; about 0.19 in./min



Drawing may not be to scale.

42. **CLEANING** Aiko is pushing the handle of a push broom with a force of 190 newtons at an angle of 33° with the ground.

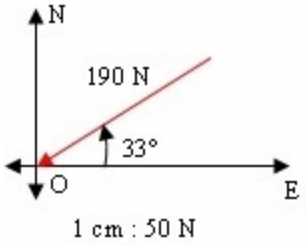


a. Draw a diagram that shows the resolution of this force into its rectangular components.

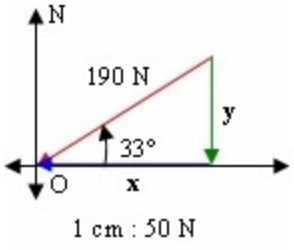
b. Find the magnitudes of the horizontal and vertical components.

SOLUTION:

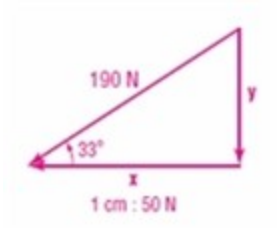
a. Aiko is pushing the handle of the push broom down with a force of 190 newtons at an angle of 33° with the ground. Draw a vector to represent the push broom.



The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.

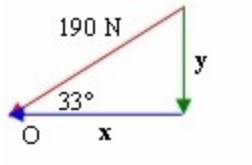


Remove the axes.



Drawings may not be to scale.

b. The horizontal and vertical components of the vector form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.



Drawings may not be to scale.

$$\cos 33^\circ = \frac{|x|}{190} \qquad \sin 33^\circ = \frac{|y|}{190}$$

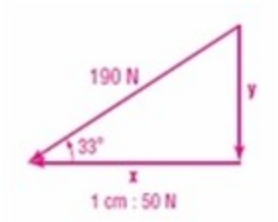
$$|x| = 190\cos 33^\circ \qquad |y| = 190\sin 33^\circ$$

$$|x| \approx 159.347 \qquad |y| \approx 103.481$$

The magnitude of the horizontal component is about 159.3 newtons and the magnitude of the vertical component is about 103.5 newtons.

ANSWER:

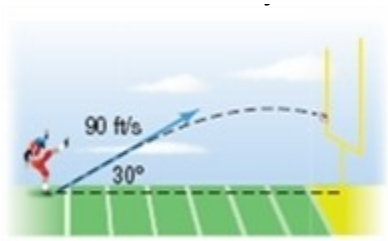
a.



Drawing may not be to scale.

b. about 159.3 N; about 103.5 N

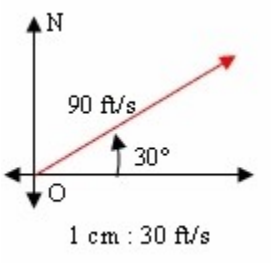
43. **FOOTBALL** For a field goal attempt, a football is kicked with the velocity shown in the diagram below.



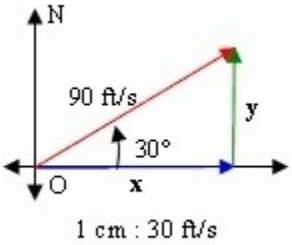
a. Draw a diagram that shows the resolution of this force into its rectangular components.
b. Find the magnitudes of the horizontal and vertical components.

SOLUTION:

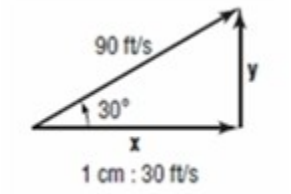
a. The football is kicked 90 feet per second at 30° to the horizontal. Draw a vector to represent the football.



The vector can be resolved into a horizontal component **x** and a vertical component **y** as shown.

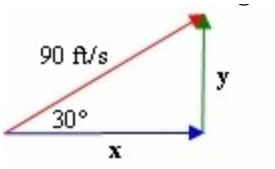


Remove the axes.



Drawings may not be to scale.

b. The horizontal and vertical components of the vector form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.



Drawings may not be to scale.

$$\cos 30^\circ = \frac{|x|}{90} \qquad \sin 30^\circ = \frac{|y|}{90}$$

$$|x| = 90\cos 30^\circ \qquad |y| = 90\sin 30^\circ$$

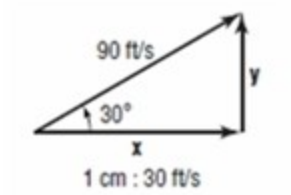
$$|x| = 90\left(\frac{\sqrt{3}}{2}\right) \qquad |y| = 90\left(\frac{1}{2}\right)$$

$$|x| = 45\sqrt{3} \qquad |y| = 45$$

The magnitude of the horizontal component is $45\sqrt{3}$ or about 77.9 feet per second and the magnitude of the vertical component is 45 feet per second.

ANSWER:

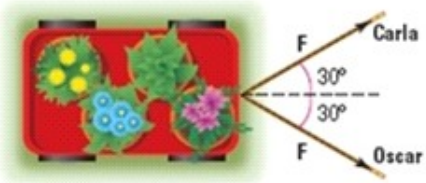
a.



Drawing may not be to scale.

b. $45\sqrt{3}$ ft/s or about 77.9 ft/s; 45 ft/s

44. **GARDENING** Carla and Oscar are pulling a wago pulls on the wagon with equal force at an angle of 30 wagon. The resultant force is 120 newtons.

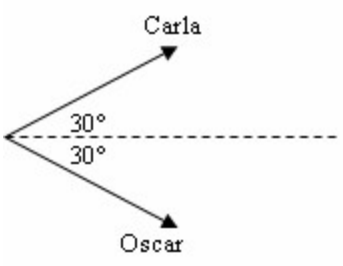


a. How much force is each person exerting?
b. If each person exerts a force of 75 newtons, wha
c. How will the resultant force be affected if Carla together?

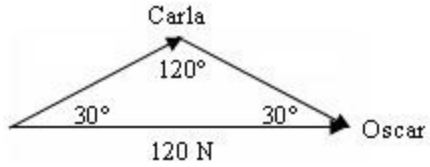
SOLUTION:

a. Draw vectors to represent Carla and Oscar pullin

8-1 Introduction to Vectors



Translate the vector representing Oscar so that its ta
vector representing Carla. Then draw the resultant v
has a force of 120 N. The two angles formed by the
forces exerted by Carla and Oscar are both 30°. The
and Oscar's forces is 120°.



Drawings may not be to scale.

Use the Law of Sines to find the magnitude of Oscar

$$\frac{\sin 30^\circ}{x} = \frac{\sin 120^\circ}{120}$$

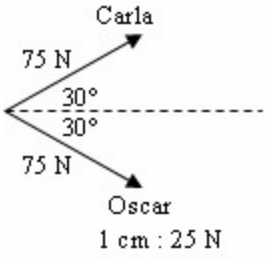
$$120 \cdot \sin 30^\circ = x \sin 120^\circ$$

$$\frac{120 \cdot \sin 30^\circ}{\sin 120^\circ} = x$$

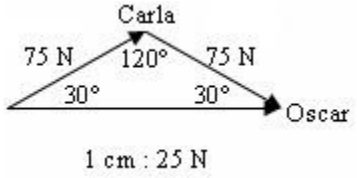
$$69.282 \approx x$$

Since Carla and Oscar are pulling on the wagon with
pulling with a force of about 69 newtons.

b. Draw vectors to represent Carla and Oscar pullin
force of 75 newtons.



Translate the vector representing Oscar so that its ta
vector representing Carla. Then draw the resultant v
formed by the axis of the wagon and the forces exer
both 30°. The angle that joins Carla's and Oscar's fo



Drawings may not be to scale.

Use the Law of Cosines to find the magnitude of the

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

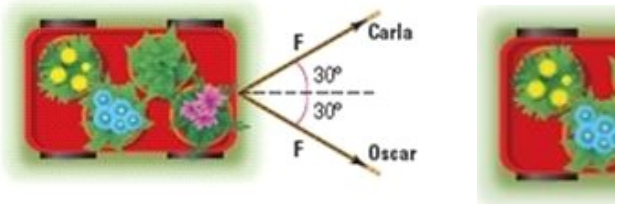
$$c^2 = 75^2 + 75^2 - 2(75)(75)\cos 120^\circ$$

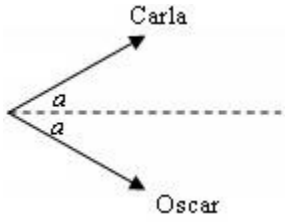
$$c = \sqrt{75^2 + 75^2 - 2(75)(75)\cos 120^\circ}$$

$$c \approx 129.904$$

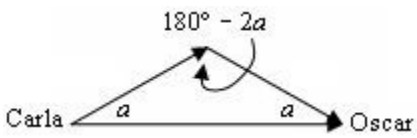
The resultant force is about 130 newtons.

c. Let a be the angles created by the axis of the wag
and Oscar.





Translate the vector representing Oscar so that its ta
vector representing Carla. Then draw the resultant v
formed by the axis of the wagon and the forces exer
both a. The angle that joins Carla's and Oscar's forc



Drawings may not be to scale.

As Oscar and Carla move closer together, a decreas
third angle of the triangle, $180° - 2a$, increases. Due
in triangles, as one angle in a triangle increases, the s
also increase. Thus, if Carla and Oscar move closer
would be greater.

ANSWER:

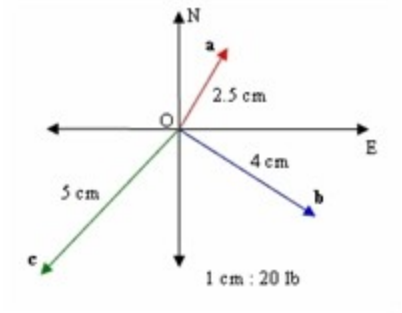
a. about 69 N
b. about 130 N
c. It would be greater.

The magnitude and true bearings of three forces acting on an object are given. Find the magnitude and direction of the resultant of these forces.

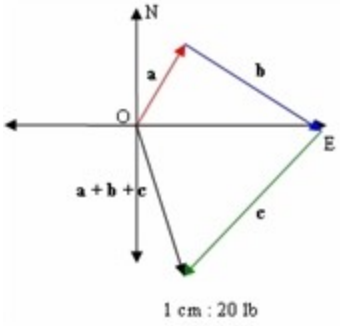
45. 50 lb at 30°, 80 lb at 125°, and 100 lb at 220°

SOLUTION:

Let **a** = 50 lb at 30°, **b** = 80 lb at 125°, and **c** = 100 lb at 220°. Draw a diagram to represent **a**, **b**, and **c** using a scale of 1 cm : 20 lb.



Translate **b** so that its tail touches the tip of **a**. Then, translate **c** so that its tail touches the tip of **b**. Finally, draw the resultant vector **a** + **b** + **c**.



Drawings may not be to scale.

Measure the length of **a** + **b** + **c** and then measure the angle this vector makes from north.

The length of the vector is approximately 4.2 centimeters, which is 4.2 × 20 or 84 pounds. Therefore, the resultant is about 84 pounds at a bearing of 162°.

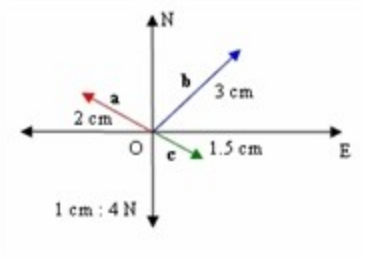
ANSWER:

84 lb at 162°

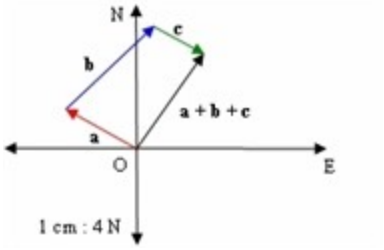
46. 8 newtons at 300°, 12 newtons at 45°, and 6 newtons at 120°

SOLUTION:

Let **a** = 8 N at 300°, **b** = 12 N at 45°, and **c** = 6 N at 120°. Draw a diagram to represent **a**, **b**, and **c** using a scale of 1 cm : 4 N.



Translate **b** so that its tail touches the tip of **a**. Then, translate **c** so that its tail touches the tip of **b**. Finally, draw the resultant vector **a** + **b** + **c**.



Drawings may not be to scale.

Measure the length of **a** + **b** + **c** and then measure the angle this vector makes from north.
The length of the vector is approximately 2.9 centimeters, which is 2.9 × 4 or 11.6 newtons. Therefore, the resultant is about 11.6 newtons at a bearing of 35°.

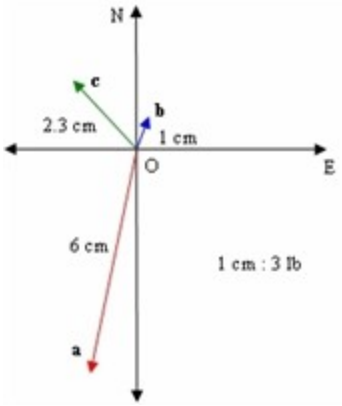
ANSWER:

11.6 N at 35°

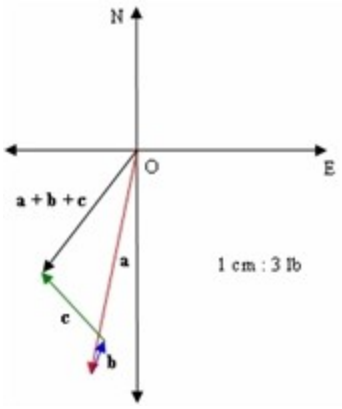
47. 18 lb at 190°, 3 lb at 20°, and 7 lb at 320°

SOLUTION:

Let **a** = 18 lb at 190°, **b** = 3 lb at 20°, and **c** = 7 lb at 320°. Draw a diagram to represent **a**, **b**, and **c** using a scale of 1 cm : 3 lb.



Translate **b** so that its tail touches the tip of **a**. Then, translate **c** so that its tail touches the tip of **b**. Finally, draw the resultant vector **a** + **b** + **c**.



Drawings may not be to scale.

Measure the length of **a** + **b** + **c** and then measure the angle this vector makes from north.
The length of the vector is approximately 3.9 centimeters, which is 3.9 × 3 or 11.7 pounds. Therefore, the resultant is about 11. 7 pounds at a bearing of 215°.

ANSWER:

11.7 lb at 215°

48. **DRIVING** Carrie's school is on a direct path three miles from her house. She drives on two different streets on her way to school. She travels at an angle of 20.9° with the path on the first street and then turns 45.4° onto the second street.

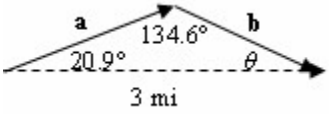
8-1 Introduction to Vectors



a. How far does Carrie drive on the first street?
b. How far does she drive on the second street?
c. If it takes her 10 minutes to get to school, and she averages 25 miles per hour on the first street, what speed does Carrie average after she turns onto the second street?

SOLUTION:

a. The direct path and the streets that Carrie uses to arrive at school form a triangle.



The remaining angle θ is 24.5°. Use the Law of Sines to find the magnitude of **a**.

$$\frac{\sin 24.5^\circ}{\mathbf{a}} = \frac{\sin 134.6^\circ}{3}$$

$$3 \cdot \sin 24.5^\circ = \mathbf{a} \sin 134.6^\circ$$

$$\frac{3 \cdot \sin 24.5^\circ}{\sin 134.6^\circ} = \mathbf{a}$$

$$1.7472 \approx \mathbf{a}$$

Carrie drives about 1.75 miles on the first street.

b. Use the Law of Sines to find the magnitude of **b**.

$$\frac{\sin 20.9^\circ}{\mathbf{b}} = \frac{\sin 134.6^\circ}{3}$$

$$3 \cdot \sin 20.9^\circ = \mathbf{b} \sin 134.6^\circ$$

$$\frac{3 \cdot \sin 20.9^\circ}{\sin 134.6^\circ} = \mathbf{b}$$

$$1.5031 \approx \mathbf{b}$$

Carrie drives about 1.5 miles on the second street.

c. On the first street, Carrie drives about 1.75 miles at an average rate of 25 miles per hour. Use $d = rt$ to find the time t that Carrie took to drive on the first street.

$$d = rt$$

$$1.75 = 25t$$

$$0.07 = t$$

Carrie traveled on the first street for about 0.07 hour. This means that Carrie traveled 0.07 × 60 or 4.2 minutes on the first street. It also means that Carrie traveled 10 − 4.2 or 5.8 minutes on the second street. Since the rate that is desired is miles per hour, convert 5.8 minutes to hours by using $t = \frac{5.8}{60}$. Substitute $t = \frac{5.8}{60}$ and $d = 1.5$ into $d = rt$ and solve for r.

$$d = rt$$

$$1.5 = r\left(\frac{5.8}{60}\right)$$

$$15.517 \approx r$$

Carrie averages a speed of about 15.5 miles per hour on the second street.

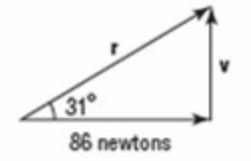
ANSWER:

a. about 1.75 mi
b. about 1.5 mi
c. about 15.5 mi/h

49. **SLEDDING** Irwin is pulling his sister on a sled. The direction of his resultant force is 31°, and the horizontal component of the force is 86 newtons.
a. What is the vertical component of the force?
b. What is the magnitude of the resultant force?

SOLUTION:

a. Let **v** represent the vertical component of the force and **r** represent the magnitude of the resultant force.



Use the tangent ratio to find **v**.

$$\tan 31^\circ = \frac{|\mathbf{v}|}{86}$$

$$|\mathbf{v}| = 86 \tan 31^\circ$$

$$|\mathbf{v}| \approx 51.674$$

The vertical component of the force is about 52 newtons.

b. Use the cosine ratio to find **r**.

$$\cos 31^\circ = \frac{86}{|\mathbf{r}|}$$

$$|\mathbf{r}| \cos 31^\circ = 86$$

$$|\mathbf{r}| = \frac{86}{\cos 31^\circ}$$

$$|\mathbf{r}| \approx 100.33$$

The magnitude of the resultant force is about 100 newtons.

ANSWER:

a. about 52 N
b. about 100 N

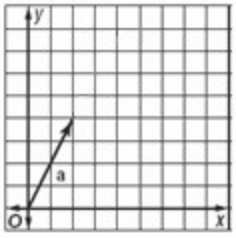
50. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate multiplication of a vector by a scalar.
a. GRAPHICAL On a coordinate plane, draw a vector **a** so that the tail is located at the origin. Choose a value for a scalar k. Then draw the vector that results if you multiply the original vector by k on the same coordinate plane. Repeat the process for four additional vectors **b**, **c**, **d**, and **e**. Use the same value for k each time.
b. TABULAR Copy and complete the table below for each vector you drew in part **a**.

Vector	Terminal Point of Vector	Terminal Point of Vector × k
a		
b		
c		
d		
e		

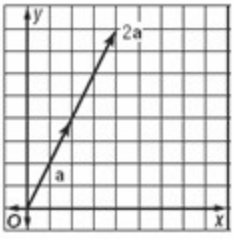
c. ANALYTICAL If the terminal point of a vector **a** is located at the point (a, b), what is the location of the terminal point of the vector k**a**?

SOLUTION:

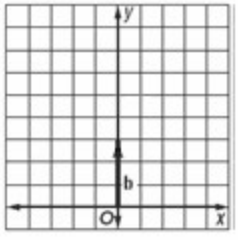
a. Sample answer: Draw vector **a** so that its tail is located at the origin and its terminal point is located at (2, 4).



Let $k = 2$. Multiply **a** by k. To represent 2**a**, draw a vector 2 times as long as **a** in the same direction as **a**. Graph 2**a** on the same coordinate plane as **a**.

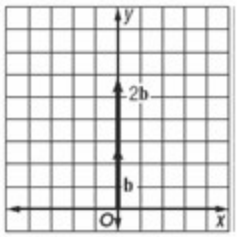


Draw vector **b** so that its tail is located at the origin and its terminal point is located at (0, 3).

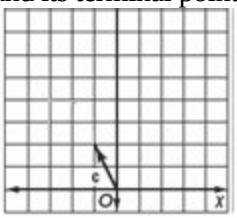


Let $k = 2$. Multiply **b** by k. To represent 2**b**, draw a vector 2 times as long as **b** in the same direction as **b**. Graph 2**b** on the same coordinate plane as **b**.

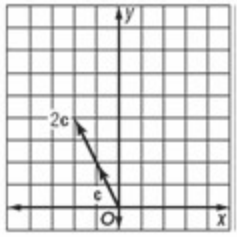
8-1 Introduction to Vectors



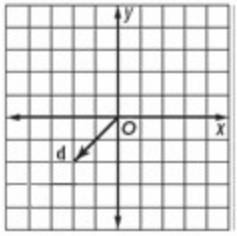
Draw vector **c** so that its tail is located at the origin and its terminal point is located at (−1, 2).



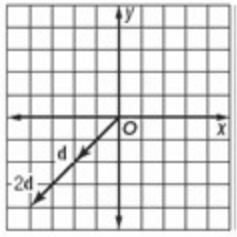
Let $k = 2$. Multiply **c** by k. To represent 2**c**, draw a vector 2 times as long as **c** in the same direction as **c**. Graph 2**c** on the same coordinate plane as **c**.



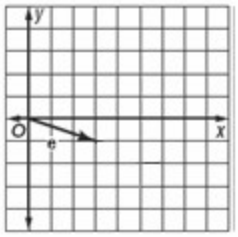
Draw vector **d** so that its tail is located at the origin and its terminal point is located at (−2, −2).



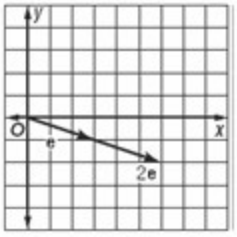
Let $k = 2$. Multiply **d** by k. To represent 2**d**, draw a vector 2 times as long as **d** in the same direction as **d**. Graph 2**d** on the same coordinate plane as **d**.



Draw vector **e** so that its tail is located at the origin and its terminal point is located at (3, −1).



Let $k = 2$. Multiply **e** by k. To represent 2**e**, draw a vector 2 times as long as **e** in the same direction as **e**. Graph 2**e** on the same coordinate plane as **e**.



b. Sample answers: Analyze the graphs to find the terminal points of the vectors. The terminal point of **a** is (2, 4) and the terminal point of 2**a** is (4, 8).

The terminal point of **b** is (0, 3) and the terminal point of 2**b** is (0, 6).
The terminal point of **c** is (−1, 2) and the terminal point of 2**c** is (−2, 4).
The terminal point of **d** is (−2, −2) and the terminal point of 2**d** is (−4, −4).
The terminal point of **e** is (3, −1) and the terminal point of 2**e** is (6, −2).

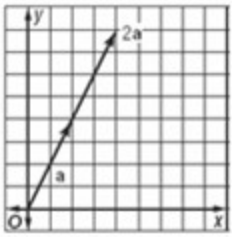
Vector	Terminal Point of Vector	Terminal Point of Vector × k
a	(2, 4)	(4, 8)
b	(0, 3)	(0, 6)
c	(−1, 2)	(−2, 4)
d	(−2, −2)	(−4, −4)
e	(3, −1)	(6, −2)

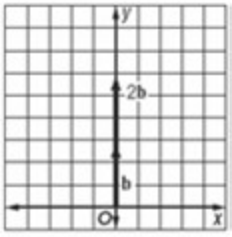
c. The terminal point of 2**a** is (4, 8) or (2 · 2, 2 · 4). The terminal point of 2**b** is (0, 6) or (2 · 0, 2 · 3). The terminal point of 2**c** is (−2, 4) or (2 · −1, 2 · 2). The terminal point of 2**d** is (−4, −4) or (2 · −2, 2 · −2). The terminal point of 2**e** is (6, −2) or (2 · 3, 2 · −1). To find the terminal point of the new vector, we can multiply the coordinates of the terminal point of the original vector by 2 or k. Thus, if the terminal point of vector **a** is located at the point (a, b), the location

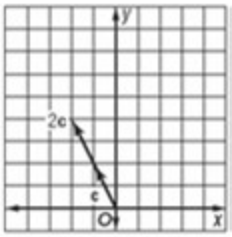
of the terminal point of the vector $k\mathbf{a}$ is (ka, kb).

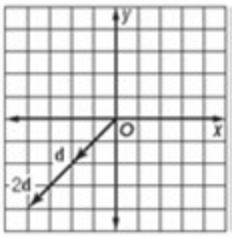
ANSWER:

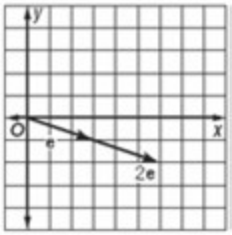
a. Sample answers: $k = 2$









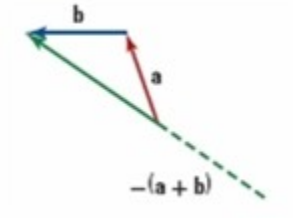


b. Sample answers are given.

Vector	Terminal Point of Vector	Terminal Point of Vector × k
a	(2, 4)	(4, 8)
b	(0, 3)	(0, 6)
c	(−1, 2)	(−2, 4)
d	(−2, −2)	(−4, −4)
e	(3, −1)	(6, −2)

c. (ka, kb)

An *equilibrant* vector is the opposite of a resultant vector. It balances a combination of vectors such that the sum of the vectors and the equilibrant is the zero vector. The equilibrant vector of a + b is −(a + b).

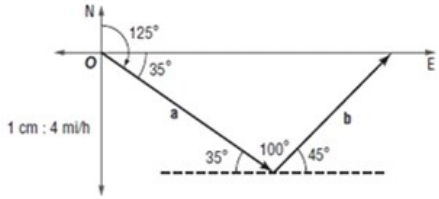


Find the magnitude and direction of the equilibrant vector for each set of vectors.

51. **a** = 15 miles per hour at a bearing of 125°
b = 12 miles per hour at a bearing of 045°

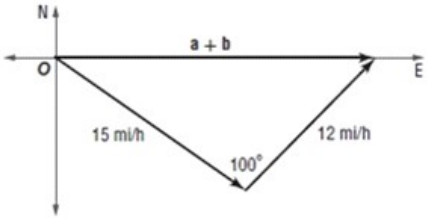
SOLUTION:

Draw a diagram to represent **a** and **b** using a scale of 1 cm : 4 mi/h.



The angle created by **a** and the x-axis is 35°. Draw a horizontal where the tip of **a** and the tail of **b** meet, as shown. **b** makes a 45° angle and **a** makes a 35° angle with the horizontal. Thus, the angle created by **a** and **b** is 100°.

Draw the resultant **a** + **b**. The three vectors form a triangle.



Drawings may not be to scale

Use the Law of Cosines to find the magnitude of **a** + **b**.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\mathbf{a}+\mathbf{b}|^2 = 15^2 + 12^2 - 2(15)(12)\cos 100^\circ$$

$$|\mathbf{a}+\mathbf{b}| = \sqrt{15^2 + 12^2 - 2(15)(12)\cos 100^\circ}$$

$$\approx 20.77$$

Use the Law of Sines to find the angle opposite of **b**.

$$\frac{\sin 100^\circ}{20.77} = \frac{\sin B}{12}$$

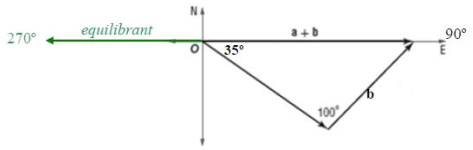
$$\frac{12\sin 100^\circ}{20.77} = \sin B$$

$$\sin^{-1}\frac{12\sin 100^\circ}{20.77} = B$$

$$34.68 \approx B$$

The angle opposite **b** is about 35°. (Note that is is not exactly 35°, so **a** + **b** is actually slightly below the *x*-axis.)

To find the bearing of **a** + **b**, subtract 35° from 125°. Thus, the direction of **a** + **b** is a bearing of 90°. Since the equilibrant vector is the opposite of the resultant vector, it will have a magnitude of about 20.77 mi/h at a bearing of about 270°.



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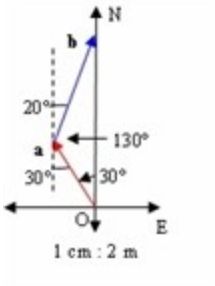
ANSWER:

about 20.77 mi/h at a bearing of 270°

52. **a** = 4 meters at a bearing of N30W°
b = 6 meters at a bearing of N20E°

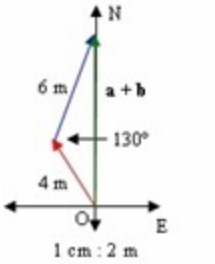
SOLUTION:

Draw a diagram to represent **a** and **b** using a scale of 1 cm : 2 m.



The angle created by **a** and the *y*-axis is 30°. Draw a vertical line where the tip of **a** and the tail of **b** meet, as shown. **b** makes a 20° angle and **a** makes a 30° angle with the vertical line. Thus, the angle created by **a** and **b** is 130°.

Draw the resultant **a** + **b**. The three vectors form a triangle.



Drawings may not be to scale.

Use the Law of Cosines to find the magnitude of **a** + **b**.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\mathbf{a}+\mathbf{b}|^2 = 4^2 + 6^2 - 2(4)(6)\cos 130^\circ$$

$$|\mathbf{a}+\mathbf{b}| = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 130^\circ}$$

$$\approx 9.10$$

Use the Law of Sines to find the angle opposite of **b**.

$$\frac{\sin 130^\circ}{9.10} = \frac{\sin B}{6}$$

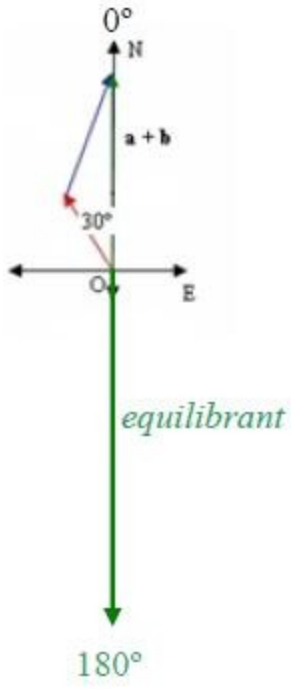
$$\frac{6\sin 130^\circ}{9.10} = \sin B$$

$$\sin^{-1}\frac{6\sin 130^\circ}{9.10} = B$$

$$30.34 \approx B$$

The angle opposite **b** is about 30°. To find the

bearing of **a** + **b**, subtract 30° from 30°. Thus, the direction of **a** + **b** is a bearing of 0° or due north. Since the equilibrant vector is the opposite of the resultant vector, it will have a magnitude of about 9.1 m at a bearing of about 180°.



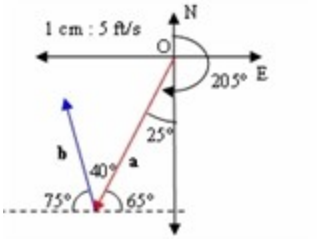
ANSWER:

about 9.1 m at a bearing of 180°

53. **a** = 23 feet per second at a bearing of 205°
b = 16 feet per second at a bearing of 345°

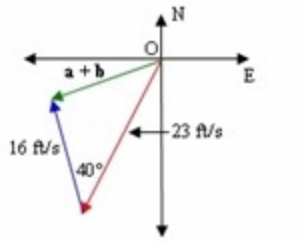
SOLUTION:

Draw a diagram to represent **a** and **b** using a scale of 1 cm : 5 ft/s.



The angle created by **a** and the *y*-axis is 205° − 180° or 25°. Draw a horizontal where the tip of **a** and the tail of **b** meet, as shown. Since **a** forms a right triangle with the horizontal and the *y*-axis, the angle created by **a** and the horizontal is 180° − 90° − 25° or 65°. **b** is at a bearing of 345°, so the angle created by **b** and the horizontal is 345° − 270° or 75°. Thus, the angle created by **a** and **b** is 180° − 75° − 65° or 40°.

Draw the resultant **a** + **b**. The three vectors form a triangle.



Drawings may not be to scale.

Use the Law of Cosines to find the magnitude of **a** + **b**.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\mathbf{a}+\mathbf{b}|^2 = 23^2 + 16^2 - 2(23)(16)\cos 40^\circ$$

$$|\mathbf{a}+\mathbf{b}| = \sqrt{23^2 + 16^2 - 2(23)(16)\cos 40^\circ}$$

$$\approx 14.87$$

Use the Law of Sines to find the angle opposite of **b**.

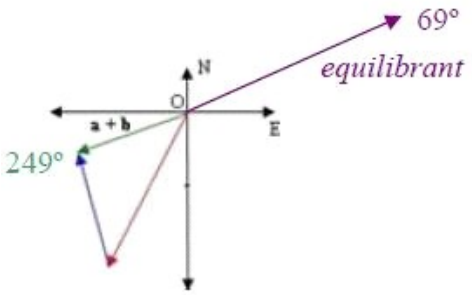
$$\frac{\sin 40^\circ}{14.87} = \frac{\sin B}{16}$$

$$\frac{16\sin 40^\circ}{14.87} = \sin B$$

$$\sin^{-1}\frac{16\sin 40^\circ}{14.87} = B$$

$$43.76 \approx B$$

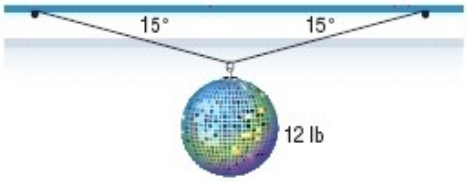
The angle opposite **b** is about 44°. To find the bearing of **a** + **b**, add 44° to 205°. Thus, the direction of **a** + **b** is a bearing of 249°. Since the equilibrant vector is the opposite of the resultant vector, it will have a magnitude of about 14.87 ft/s at a bearing of about 249° − 180° or 69°.



ANSWER:

about 14.87 ft/s at a bearing of 69°

54. **PARTY PLANNING** A disco ball is suspended above a dance floor by two wires of equal length as shown.



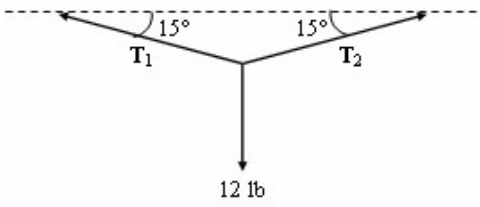
a. Draw a vector diagram of the situation that indicates that two tension vectors $\mathbf{T_1}$ and $\mathbf{T_2}$ with equal magnitude are keeping the disco ball stationary or at equilibrium.

b. Redraw the diagram using the triangle method to find $\mathbf{T_1} + \mathbf{T_2}$.

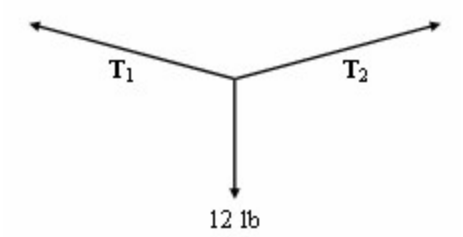
c. Use your diagram from part **b** and the fact that the equilibrant of the resultant $\mathbf{T}_1 + \mathbf{T}_2$ and the vector representing the weight of the disco ball are equivalent vectors to calculate the magnitudes of $\mathbf{T}_1$ and $\mathbf{T}_2$.

SOLUTION:

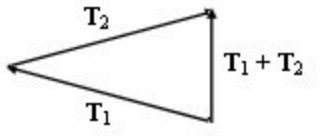
a. Draw a diagram to represent $\mathbf{T_1}$, $\mathbf{T_2}$, and the disco ball.



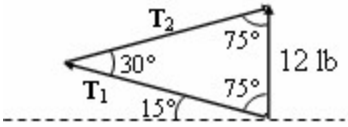
The wires are exerting an equal force away from the disco ball at 15° angles with the horizontal, as shown. The disco ball is exerting a force of 12 pounds downward.



b. Translate $\mathbf{T_2}$ so that its tail touches the tip of $\mathbf{T_1}$. Then draw the resultant vector $\mathbf{T_1} + \mathbf{T_2}$, as shown.



c. The three vectors, $\mathbf{T_1}$, $\mathbf{T_2}$, and $\mathbf{T_1} + \mathbf{T_2}$, form a triangle. Draw a horizontal that touches the tail of $\mathbf{T_1}$.



Since the equilibrant of $\mathbf{T_1} + \mathbf{T_2}$ and the vector representing the weight of the disco ball are equivalent vectors, $\mathbf{T_1} + \mathbf{T_2}$ is exerting a force of 12 lb upward. The angle created by $\mathbf{T}_1$ and the horizontal is 15°. Since the direction of $\mathbf{T_1} + \mathbf{T_2}$ is upward, $\mathbf{T_1} + \mathbf{T_2}$ creates a right angle with the horizontal. Thus, the angle created by $\mathbf{T_1}$ and $\mathbf{T_1} + \mathbf{T_2}$ is 90° − 15° or 75°. Since $\mathbf{T_1} = \mathbf{T_2}$, the triangle is isosceles and the angle created by $\mathbf{T_2}$ and $\mathbf{T_1} + \mathbf{T_2}$ is also 75°. The remaining angle of the triangle is 180° − 75° − 75° or 30°.

Use the Law of Sines to find the magnitude of $\mathbf{T_1}$.

$$\frac{\sin 75^\circ}{T_1} = \frac{\sin 30^\circ}{12}$$

$$12 \sin 75^\circ = T_1 \sin 30^\circ$$

$$\frac{12 \sin 75^\circ}{\sin 30^\circ} = T_1$$

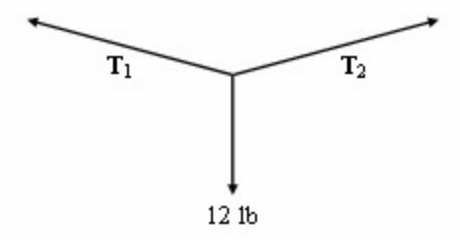
$$23.1822 \approx T_1$$

Since $\mathbf{T_1}$ and $\mathbf{T_2}$ are equal in length, they both have

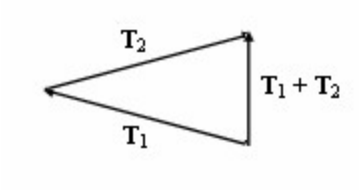
the same magnitude. Thus, $\mathbf{T_1} \approx 23.2$ lb and $\mathbf{T_2} \approx 23.2$ lb.

ANSWER:

a.

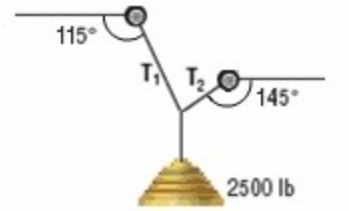


b.



c. $\mathbf{T_1} \approx 23.2$ lb, $\mathbf{T_2} \approx 23.2$ lb

55. **CABLE SUPPORT** Two cables with tensions $\mathbf{T_1}$ and $\mathbf{T_2}$ are tied together to support a 2500-pound load at equilibrium.



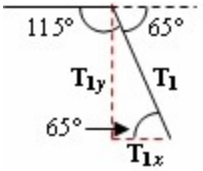
a. Write expressions to represent the horizontal and vertical components of $\mathbf{T_1}$ and $\mathbf{T_2}$.

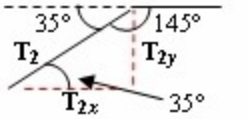
b. Given that the equilibrant of the resultant $\mathbf{T_1} + \mathbf{T_2}$ and the vector representing the weight of the load are equivalent vectors, calculate the magnitudes of $\mathbf{T_1}$ and $\mathbf{T_2}$ to the nearest tenth of a pound.

c. Use your answers from parts **a** and **b** to find the magnitudes of the horizontal and vertical components of $\mathbf{T_1}$ and $\mathbf{T_2}$ to the nearest tenth of a pound.

SOLUTION:

a. Sample answers: Diagram each cable with its vertical and horizontal components. Label the components and the angles as shown.





Use the sine or cosine ratios to find expressions to represent the vertical and horizontal components of $\mathbf{T_1}$.

$$\cos 65^\circ = \frac{T_{1x}}{T_1} \qquad \sin 65^\circ = \frac{T_{1y}}{T_1}$$

$$T_1 \cos 65^\circ = T_{1x} \qquad T_1 \sin 65^\circ = T_{1y}$$

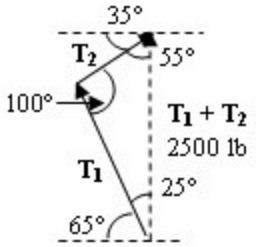
Use the sine or cosine ratios to find expressions to represent the vertical and horizontal components of $\mathbf{T_2}$.

$$\cos 35^\circ = \frac{T_{2x}}{T_2} \qquad \sin 35^\circ = \frac{T_{2y}}{T_2}$$

$$T_2 \cos 35^\circ = T_{2x} \qquad T_2 \sin 35^\circ = T_{2y}$$

$\mathbf{T_{1x}} = \mathbf{T_1} \cos 65°$; $\mathbf{T_{1y}} = \mathbf{T_1} \sin 65°$; $\mathbf{T_{2x}} = \mathbf{T_2} \cos 35°$; $\mathbf{T_{2y}} = \mathbf{T_2} \sin 35°$

b. Since the equilibrant of $\mathbf{T_1} + \mathbf{T_2}$ and the vector representing the weight of the load are equivalent vectors, $\mathbf{T_1} + \mathbf{T_2}$ is exerting a force of 2500 lb upward. Draw the resultant $\mathbf{T_1} + \mathbf{T_2}$ by translating $\mathbf{T_2}$ so that its tail touches the tip of $\mathbf{T_1}$. Label the angles as shown.



Use the Law of Sines to find the magnitude of $\mathbf{T_1}$ and $\mathbf{T_2}$.

$$\frac{\sin 55^\circ}{T_1} = \frac{\sin 100^\circ}{2500} \qquad \frac{\sin 25^\circ}{T_2} = \frac{\sin 100^\circ}{2500}$$

$$2500 \sin 55^\circ = T_1 \sin 100^\circ \qquad 2500 \sin 25^\circ = T_2 \sin 100^\circ$$

$$\frac{2500 \sin 55^\circ}{\sin 100^\circ} = T_1 \qquad \frac{2500 \sin 25^\circ}{\sin 100^\circ} = T_2$$

$$2079.5 \approx T_1 \qquad 1072.8 \approx T_2$$

$\mathbf{T_1}$ is about 2079.5 pounds and $\mathbf{T_2}$ is about 1072.8 pounds.

c. Substitute $\mathbf{T_1} = 2079.5$ and $\mathbf{T_2} = 1072.8$ into the equations found in part **a**.

$$T_{1x} = T_1 \cos 65^\circ \qquad T_{1y} = T_1 \sin 65^\circ$$

$$T_{1x} = 2079.5 \cos 65^\circ \qquad T_{1y} = 2079.5 \sin 65^\circ$$

$$T_{1x} \approx 878.8 \qquad T_{1y} \approx 1884.7$$

$$T_{2x} = T_2 \cos 35^\circ \qquad T_{2y} = T_2 \sin 35^\circ$$

$$T_{2x} = 1072.8 \cos 35^\circ \qquad T_{2y} = 1072.8 \sin 35^\circ$$

$$T_{2x} \approx 878.8 \qquad T_{2y} \approx 615.3$$

$\mathbf{T_{1x}}$ is about 878.8 pounds, $\mathbf{T_{1y}}$ is about 1884.7 pounds, $\mathbf{T_{2x}}$ is about 878.8 pounds, and $\mathbf{T_{2y}}$ is about 615.3 pounds.

ANSWER:

a. Sample answers: $\mathbf{T_{1x}} = \mathbf{T_1} \cos 65°$; $\mathbf{T_{1y}} = \mathbf{T_1} \sin 65°$; $\mathbf{T_{2x}} = \mathbf{T_2} \cos 35°$; $\mathbf{T_{2y}} = \mathbf{T_2} \sin 35°$

b. $\mathbf{T_1} \approx 2079.5$ lb; $\mathbf{T_2} \approx 1072.8$ lb

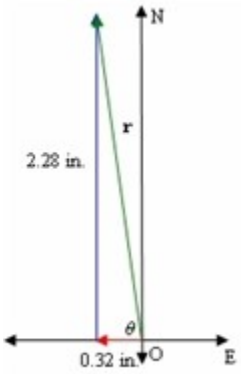
c. $\mathbf{T_{1x}} \approx 878.8$ lb ; $\mathbf{T_{1y}} \approx 1884.7$ lb; $\mathbf{T_{2x}} \approx 878.8$ lb; $\mathbf{T_{2y}} \approx 615.3$ lb

Find the magnitude and direction of each vector given its vertical and horizontal components and the range of values for the angle of direction θ to the horizontal.

56. horizontal: 0.32 in., vertical: 2.28 in., $90° < \theta < 180°$

SOLUTION:

Since $90° < \theta < 180°$, the vector **r** will be in the second quadrant. Thus, the horizontal component must go left and the vertical component must go up. Diagram the components and the vector **r**.



Drawing may not be to scale.

To find the magnitude of **r**, use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$0.32^2 + 2.28^2 = |\mathbf{r}|^2$$

$$\sqrt{0.32^2 + 2.28^2} = |\mathbf{r}|$$

$$2.3 \approx |\mathbf{r}|$$

Use the tangent ratio to find θ.

$$\tan\theta = \frac{2.28}{0.32}$$

$$\theta = \tan^{-1}\frac{2.28}{0.32}$$

$$\theta \approx 82.01$$

θ is about 82°. Thus, the vector is about 2.3 inches at 180° − 82° or 98° to the horizontal.

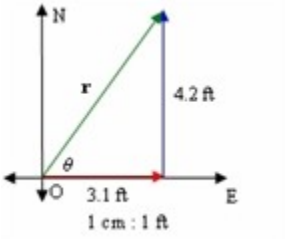
ANSWER:

about 2.3 in. at 98°

57. horizontal: 3.1 ft, vertical: 4.2 ft, $0° < \theta < 90°$

SOLUTION:

Since $0° < \theta < 90°$, the vector **r** will be in the first quadrant. Thus, the horizontal component must go right and the vertical component must go up. Diagram the components and the vector **r**.



Drawing may not be to scale.

To find the magnitude of **r**, use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$3.1^2 + 4.2^2 = |\mathbf{r}|^2$$

$$\sqrt{3.1^2 + 4.2^2} = |\mathbf{r}|$$

$$5.2 \approx |\mathbf{r}|$$

Use the tangent ratio to find θ.

$$\tan\theta = \frac{4.2}{3.1}$$

$$\theta = \tan^{-1}\frac{4.2}{3.1}$$

$$\theta \approx 53.57$$

θ is about 54°. Thus, the vector is about 5.2 feet at 54° to the horizontal.

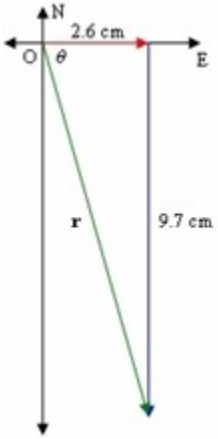
ANSWER:

about 5.2 ft at 54°

58. horizontal: 2.6 cm, vertical: 9.7 cm, $270° < \theta < 360°$

SOLUTION:

Since $270° < \theta < 360°$, the vector **r** will be in the fourth quadrant. Thus, the horizontal component must go right and the vertical component must go down. Diagram the components and the vector **r**.



Drawing may not be to scale.

To find the magnitude of **r**, use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$2.6^2 + 9.7^2 = |\mathbf{r}|^2$$

$$\sqrt{2.6^2 + 9.7^2} = |\mathbf{r}|$$

$$10.0 \approx |\mathbf{r}|$$

Use the tangent ratio to find θ.

$$\tan\theta = \frac{9.7}{2.6}$$

$$\theta = \tan^{-1}\frac{9.7}{2.6}$$

$$\theta \approx 75.00$$

θ is about 75°. Thus, the vector is about 10 centimeters at $360° - 75°$ or 285° to the horizontal.

ANSWER:

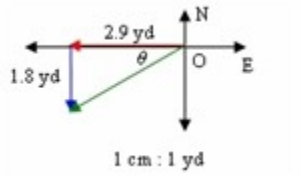
about 10 cm at 285°

8-1 Introduction to Vectors

59. horizontal: 2.9 yd, vertical: 1.8 yd, $180° < \theta < 270°$

SOLUTION:

Since $180° < \theta < 270°$, the vector **r** will be in the third quadrant. Thus, the horizontal component must go left and the vertical component must go down. Diagram the components and the vector **r**.



Drawing may not be to scale.

To find the magnitude of **r**, use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$2.9^2 + 1.8^2 = |\mathbf{r}|^2$$

$$\sqrt{2.9^2 + 1.8^2} = |\mathbf{r}|$$

$$3.4 \approx |\mathbf{r}|$$

Use the tangent ratio to find θ.

$$\tan\theta = \frac{1.8}{2.9}$$

$$\theta = \tan^{-1}\frac{1.8}{2.9}$$

$$\theta \approx 31.83$$

θ is about 32°. Thus, the vector is about 3.4 yards at 180° + 32° or 212° to the horizontal.

ANSWER:

about 3.4 yd at 212°

Draw any three vectors a, b, and c. Show geometrically that each of the following vector properties holds using these vectors.

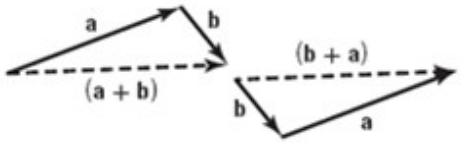
60. Commutative Property: $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

SOLUTION:

Sample answer: Draw two vectors, **a** and **b**.



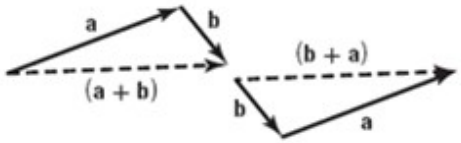
For (**a** + **b**), translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector (**a** + **b**). For (**b** + **a**), translate **a** so that its tail touches the tip of **b**. Then draw the resultant vector (**b** + **a**).



The resultants are equivalent vectors. Thus, the Commutative Property holds.

ANSWER:

Sample answer:



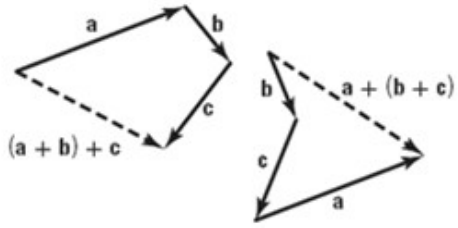
61. Associative Property: (**a** + **b**) + **c** = **a** + (**b** + **c**)

SOLUTION:

Sample answer: Draw three vectors, **a**, **b**, and **c**.



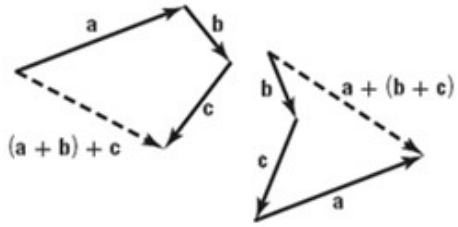
For (**a** + **b**) + **c**, translate **b** so that its tail touches the tip of **a**. Then translate **c** so that its tail touches the tip of **b**. Finally, draw the resultant vector (**a** + **b**) + **c**. For **a** + (**b** + **c**), translate **c** so that its tail touches the tip of **b**. Then translate **a** so that its tail touches the tip of **c**. Finally, draw the resultant vector **a** + (**b** + **c**).



The resultants are equivalent vectors. Thus, the Associative Property holds.

ANSWER:

Sample answer:



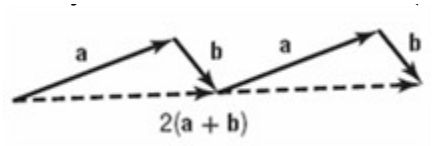
62. Distributive Property: $k(\mathbf{a} + \mathbf{b}) = k\mathbf{a} + k\mathbf{b}$, for k = 2, 0.5, and −2

SOLUTION:

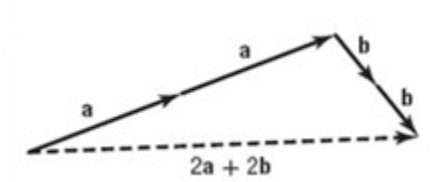
Sample answers: Draw two vectors, **a** and **b**.



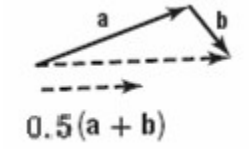
Let k = 2. Show that 2(**a** + **b**) = 2**a** + 2**b**.
For 2(**a** + **b**), start by finding **a** + **b** by translating **b** so that its tail touches the tip of **a**. Draw the resultant vector (**a** + **b**). Repeat the process starting by translating **a** so that its tail touches the tip of (**a** + **b**). Then translate **b** so that its tail touches the tip of **a**. Finally, draw the resultant vector 2(**a** + **b**).



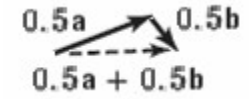
For 2**a** + 2**b**, start with two **a** vectors and two **b** vectors. Translate one **a** so that its tail touches the tip of the other **a**. Then take one **b** and translate it so its tail touches the tip of the second **a**. Take the second **b** and translate it so its tail touches the tip of the first **b**. Finally, draw the resultant vector 2**a** + 2**b**.



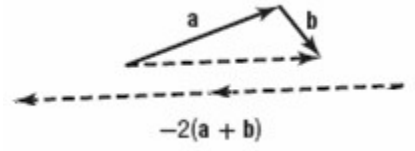
Let k =0.5. Show that 0.5(**a** + **b**) = 0.5**a** + 0.5**b**.
For 0.5(**a** + **b**), start by finding **a** + **b** by translating **b** so that its tail touches the tip of **a**. Draw the resultant vector (**a** + **b**). Then, draw a vector 0.5 the length of (**a** + **b**) in the same direction as (**a** + **b**).



For 0.5**a** + 0.5**b**, draw a vector 0.5 the length of **a** in the same direction as **a**. Repeat the process for **b**. Then, translate the new **b** so that its tail touches the tip of the new **a**. Finally, draw the resultant vector 0.5**a** + 0.5**b**.

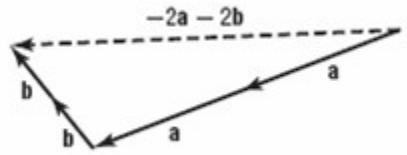


Let k = −2. Show that −2(**a** + **b**) = −2**a** + (−2)**b**.
For −2(**a** + **b**), start by finding **a** + **b** by translating **b** so that its tail touches the tip of **a**. Draw the resultant vector (**a** + **b**). Then, draw a vector 2 times as long as (**a** + **b**) in the opposite direction from (**a** + **b**).



For −2**a** + (−2)**b** or −2**a** − 2**b**, start with two **a** vectors that are drawn in the opposite direction from **a** and two **b** vectors that are drawn in the opposite direction from **b**. Translate one **a** so that its tail touches the tip of the other **a**. Then take one **b** and

translate it so its tail touches the tip of the second **a**. Take the second **b** and translate it so its tail touches the tip of the first **b**. Finally, draw the resultant vector −2**a** − 2**b**.

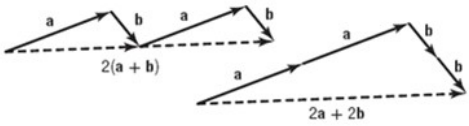


The resultants are equivalent vectors. Thus, the Distributive Property holds.

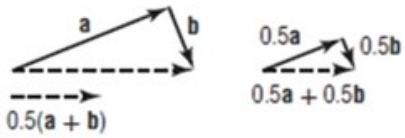
ANSWER:

Sample answers:

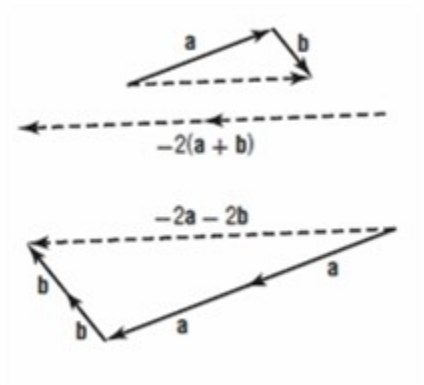
$k = 2$



$k = 0.5$



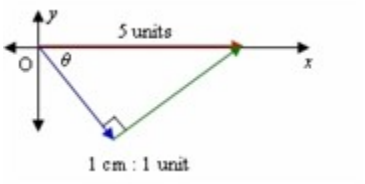
$k = -2$



63. **OPEN ENDED** Consider a vector of 5 units directed along the positive *x*-axis. Resolve the vector into two perpendicular components in which no component is horizontal or vertical.

SOLUTION:

Sample answer: Start by drawing a vector of 5 units along the positive *x*-axis. Sketch two components of the vector that are neither horizontal nor vertical and are joined by a right angle.



The triangle formed by the three vectors is a right triangle with a hypotenuse of length 5 units. Sketching the components so that the blue component is of length 3 units allows for the length of the green component to be found using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

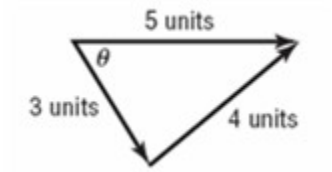
$$3^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$b^2 = 16$$

$$b = \pm 4$$

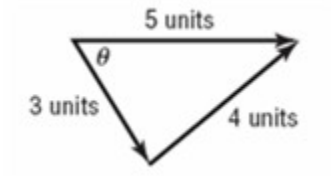
Since the component cannot have a negative length, the length of the green component is 4 units. Sketch the vector with its two perpendicular components without the axes.



Drawing may not be to scale.

ANSWER:

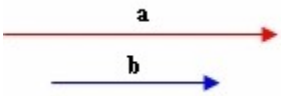
Sample answer:



64. **REASONING** Is it *sometimes*, *always*, or *never* possible to find the sum of two parallel vectors using the parallelogram method? Explain your reasoning.

SOLUTION:

Start by drawing two parallel vectors **a** and **b**.



Translate **b** so that its tail touches the tail of **a**.



Never; sample answer: If two vectors are parallel, then they share the same direction. If you place the two vectors so that their initial points coincide, they would be superimposed and there would be no angle between them. Thus, it would be impossible to complete the parallelogram.

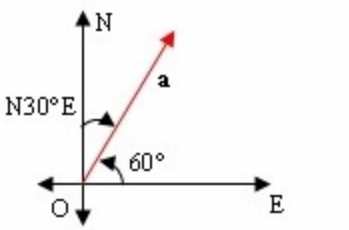
ANSWER:

Never; sample answer: If two vectors are parallel, then they share the same direction. If you place the two vectors so that their initial points coincide, they would be superimposed and there would be no angle between them. Thus, it would be impossible to complete the parallelogram.

65. **REASONING** Why is it important to establish a common reference for measuring the direction of a vector, for example, from the positive *x*-axis?

SOLUTION:

Sample answer: In order for the direction to have a consistent meaning, it must be measured using a common reference. Lack of a common reference would cause ambiguity in the reporting of the direction of the vector. For example, the direction of vector **a** can be measured as N30°E or 60° to the horizontal depending on the reference established for measurement.



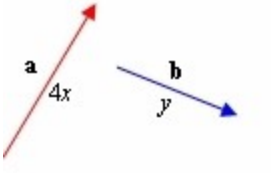
ANSWER:

Sample answer: In order for the direction to have a consistent meaning, it must be measured using a common reference. Lack of a common reference would cause ambiguity in the reporting of the direction of the vector.

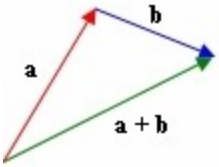
66. **CHALLENGE** The resultant of **a** + **b** is equal to the resultant of **a** – **b**. If the magnitude of **a** is 4*x*, what is the magnitude of **b**?

SOLUTION:

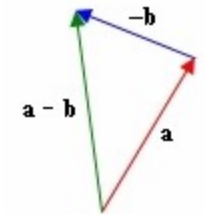
Draw vector **a** with a magnitude of 4*x* and vector **b** with a magnitude of *y*.



Draw **a** + **b** by translating **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a** + **b**.



Draw **a** − **b**. To represent −**b**, draw a vector the same length as **b** in the opposite direction from **b**. Then translate −**b** so that its tail touches the tip of **a**. Finally, draw the resultant vector **a** − **b**.



In order for **a** + **b** to equal **a** − **b**, **b** would have to be equal to −**b**. This only occurs when **b** is the zero vector. Therefore, the magnitude of **b** is 0.

ANSWER:

0

67. **REASONING** Consider the statement | **a** | + | **b** | ≥ | **a** + **b** |.

a. Express this statement using words.

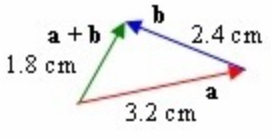
b. Is this statement true or false? Justify your answer.

SOLUTION:

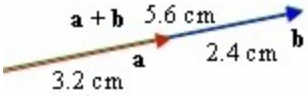
a. The magnitude of **a** added to the magnitude of **b** is greater than or equal to the magnitude of the vector created by **a** + **b**.

b. True; sample answer: The vector created by **a** + **b** has to account for the direction of both vectors. This can result in a very small magnitude, | **a** + **b** |, if **a** and **b** have opposite directions. Calculating the sum of the magnitudes, | **a** | + | **b** |, will result in the greatest possible value because direction is not being considered. This value can only be achieved by | **a** + **b** | if **a** and **b** are parallel vectors with the same direction.

For example, draw two vectors **a** and **b** that have different directions and find | **a** + **b** |.



| **a** + **b** | = 1.8 cm. However, | **a** | + | **b** | is 2.4 + 3.2 or 5.6. So, | **a** | + | **b** | ≥ | **a** + **b** | holds true. To further test the inequality, draw **a** and **b** so that they have the same direction and find | **a** + **b** |.



Drawings may not be drawn to scale.

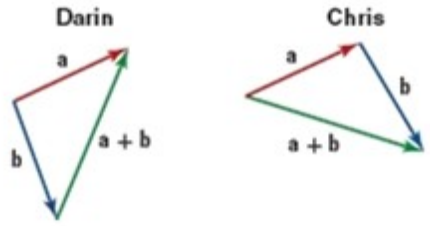
Here, | **a** + **b** | = 5.6 cm. The inequality | **a** | + | **b** | ≥ | **a** + **b** | still holds true.

ANSWER:

a. The magnitude of **a** added to the magnitude of **b** is greater than or equal to the magnitude of the vector created by **a** + **b**.

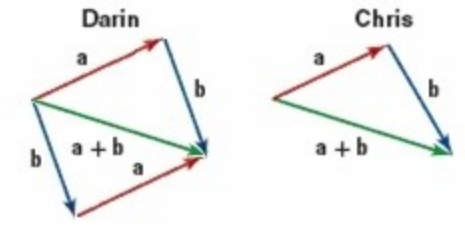
b. True; sample answer: The vector created by **a** + **b** has to account for the direction of both vectors. This can result in a very small magnitude, | **a** + **b** |, if **a** and **b** have opposite directions. Calculating the sum of the magnitudes, | **a** | + | **b** |, will result in the greatest possible value because direction is not being considered. This value can only be achieved by | **a** + **b** | if **a** and **b** are parallel vectors with the same direction.

68. **ERROR ANALYSIS** Darin and Cris are finding the resultant of vectors **a** and **b**. Is either of them correct? Explain your reasoning.



SOLUTION:

Cris is correct. Cris placed the initial point of the second vector on the terminal point of the first vector and then drew the resultant from the initial point of the first vector to the terminal point of the second vector, which is the correct way to use the triangle method. Darin placed the initial points of the two vectors together, which is the first step in using the parallelogram method, but then he did not complete the parallelogram. For Darin to also be correct, he would have to complete the parallelogram and then draw the resultant vector, which is the diagonal of the parallelogram.



ANSWER:

Cris; sample answer: Cris placed the initial point of the second vector on the terminal point of the first vector and then drew the resultant from the initial point of the first vector to the terminal point of the second vector, which is the correct way to use the triangle method. Darin placed the initial points of the two vectors together, which is the first step in using the parallelogram method, but then he did not complete the parallelogram.

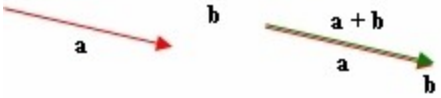
69. **REASONING** Is it possible for the sum of two vectors to equal one of the vectors? Explain.

SOLUTION:

Yes; sample answer: It is possible for the sum of two vectors to be equal to one of the components only when one of the vectors is the zero vector. For example, draw two vectors **a** and **b**, where neither vector is the zero vector and **a** and **b** have the same direction. Find **a** + **b**.



In this instance, (**a**) + (**b**) = (**a** + **b**). However, if we let **b** equal to the zero vector, then (**a**) + (**b**) = (**a** + **b**) = (**a**), as shown.



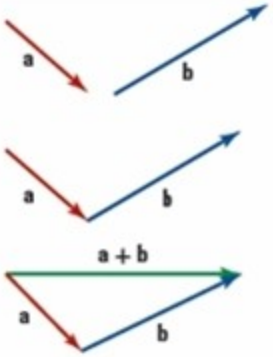
ANSWER:

Yes; sample answer: It is possible for the sum of two vectors to be equal to one of the components only when one of the vectors is the zero vector.

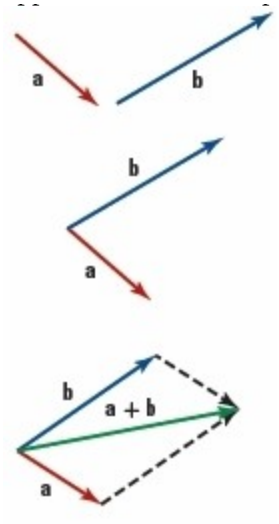
70. ***Writing in Math*** Compare and contrast the parallelogram and triangle methods of finding the resultant of two or more vectors.

SOLUTION:

Sample answer: Using the triangle method, you place the initial point of subsequent vectors at the terminal point of previous vectors and then draw the resultant from the initial point of the first vector to the terminal point of the last vector.



Using the parallelogram method, you place the initial points of the two vectors at the same point, then complete the parallelogram and draw the resultant from the initial points of the two vectors to the opposite vertex of the parallelogram.



Both the triangle and parallelogram methods can be used to find the resultant of two or more vectors.

ANSWER:

Sample answer: Using the triangle method, you place the initial point of subsequent vectors at the terminal point of previous vectors and then draw the resultant from the initial point of the first vector to the terminal point of the last vector. Using the parallelogram method, you place the initial points of the two vectors at the same point, then complete the parallelogram and draw the resultant from the initial points of the two vectors to the opposite vertex of the parallelogram. Both the triangle and parallelogram methods can be used to find the resultant of two or more vectors.

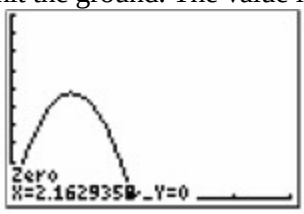
71. **KICKBALL** Suppose a kickball player kicks a ball at a 32° angle to the horizontal with an initial speed of 20 meters per second. How far away will the ball land?

SOLUTION:

To determine the distance the ball travels, you need the horizontal distance that the ball has traveled when the ball hits the ground or when the height of the ball is 0. First, write a parametric equation for the vertical position of the ball.

$$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$$
$$= t(20)\sin 32 - \frac{1}{2}(9.8)t^2 + 0$$

Graph the equation for the vertical position. Use the **CALC** menu to find the time t for when the ball will hit the ground. The value is about 2.163 seconds.



[0,5] scl: 1 by [0,10] scl: 1

Determine the horizontal position of the ball at 2.163 seconds.

$$x = tv_0 \cos\theta$$
$$= 2.163(20)\cos 32$$
$$\approx 36.687$$

The ball will travel a distance of about 36.7 meters.

ANSWER:

36.7 m

72. Graph $(x')^2 + y' - 5 = 1$ if it has been rotated 45° from its position in the xy-plane.

SOLUTION:

Use the rotation formulas for x' and y' to find the equation of the rotated conic in the xy–plane.

$$x' = x\cos\theta + y\sin\theta$$
$$x' = x\cos 45° + y\sin 45°$$
$$x' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y$$

$$y' = y\cos\theta - x\sin\theta$$
$$y' = y\cos 45° - x\sin 45°$$

$$y' = \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}x$$

Substitute these values into the original equation.

$$(x')^2 + y' - 5 = 1$$

$$\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)^2 + \left(\frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}x\right) - 5 = 1$$

$$\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}x - 5 = 1$$

$$x^2 + 2xy + y^2 + \sqrt{2}y - \sqrt{2}x - 10 = 2$$

$$x^2 + 2xy + y^2 + \sqrt{2}y - \sqrt{2}x - 12 = 0$$

Graph the equation by solving for y.

$y^2 + (2x + \sqrt{2})y + x^2 - \sqrt{2}x - 12 = 0$

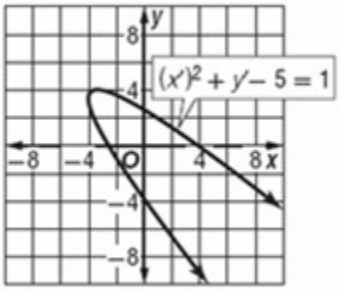
Use the quadratic formula.

$$y = \frac{-(2x+\sqrt{2}) \pm \sqrt{(2x+\sqrt{2})^2 - 4(1)(x^2 - \sqrt{2}x - 12)}}{2(1)}$$

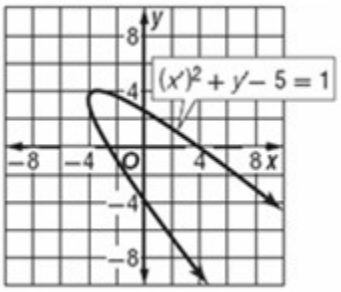
$$y = \frac{-2x - \sqrt{2} \pm \sqrt{4x^2 + 4\sqrt{2}x + 2 - 4x^2 + 4\sqrt{2}x + 48}}{2}$$

$$y = \frac{-2x - \sqrt{2} \pm \sqrt{8\sqrt{2}x + 50}}{2}$$

Use a graphing calculator to assist in graphing the conic.



ANSWER:



Write an equation for a circle that satisfies each set of conditions. Then graph the circle.

73. center at (4, 5), radius 4

SOLUTION:

The center is located at (4, 5), so $h = 4$ and $k = 5$.

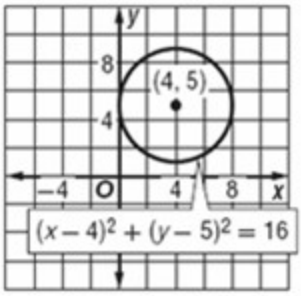
The radius is 4, so $r^2 = 16$.

Use the values of h, k, and r to write the equation of the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

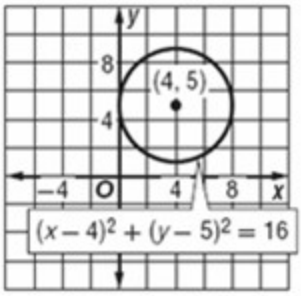
$$(x-4)^2 + (y-5)^2 = 16$$

Graph a circle that has a center at (4, 5) and a radius of 4.



ANSWER:

$(x - 4)^2 + (y - 5)^2 = 16$



74. center at (1, –4), diameter 7

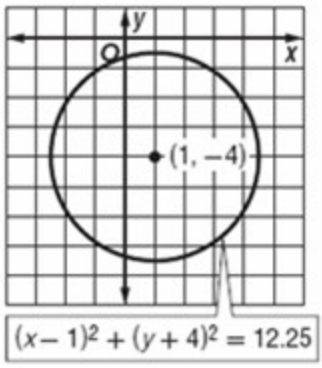
SOLUTION:

The center is located at (1, −4), so $h = 1$ and $k = -4$. The diameter is 7, so $r = 3.5$, and $r^2 = 12.25$.

Use the values of h, k, and r to write the equation of the circle.

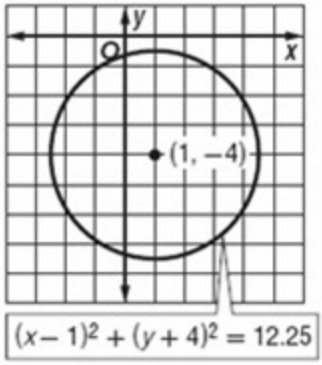
$$(x-h)^2+(y-k)^2=r^2$$
$$(x-1)^2+(y+4)^2=12.25$$

Graph a circle that has a center at (1, −4) and a radius of 3.5.



ANSWER:

$(x - 1)^2 + (y + 4)^2 = 12.25$



Determine the equation of and graph a parabola with the given focus *F* and vertex *V*.

75. $F(2, 4)$, $V(2, 3)$

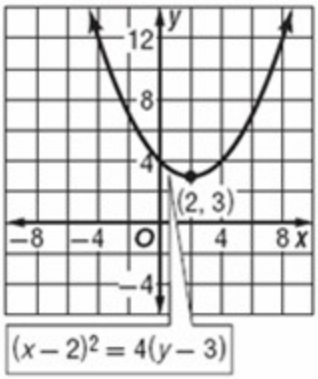
SOLUTION:

Because the focus and vertex share the same x–coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $4 - 3$ or 1. Because p is positive, the graph opens up.

Write the equation for the parabola in standard form using the values of h, p, and k.

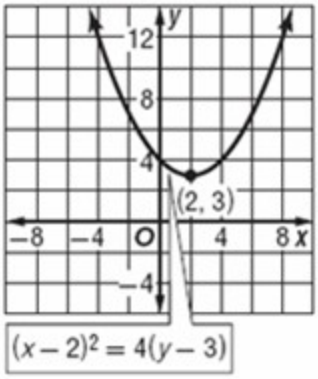
$$4p(y-k)=(x-h)^2$$
$$4(1)(y-3)=(x-2)^2$$
$$4(y-3)=(x-2)^2$$

The standard form of the equation is $(x - 2)^2 = 4(y - 3)$. Graph the vertex and focus. Then make a table of values to graph the parabola.



ANSWER:

$(x - 2)^2 = 4(y - 3)$



76. $F(1, 5)$, $V(-7, 5)$

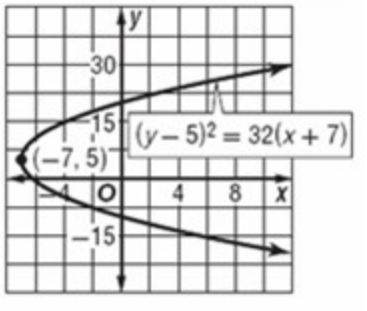
SOLUTION:

Because the focus and vertex share the same y–coordinate, the graph is horizontal. The focus is $(h + p, k)$, so the value of p is $1 - (-7)$ or 8. Because p is positive, the graph opens to the right.

Write the equation for the parabola in standard form using the values of h, p, and k.

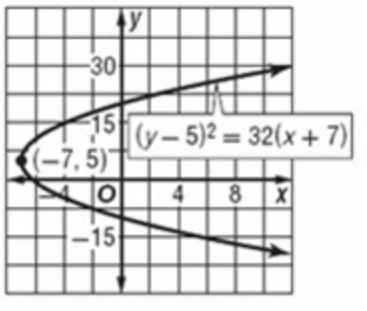
$$4p(x-h) = (y-k)^2$$
$$4(8)[x-(-7)] = (y-5)^2$$
$$32(x+7) = (y-5)^2$$

The standard form of the equation $(y - 5)^2 = 32(x + 7)$. Graph the vertex and focus. Then make a table of values to graph the parabola.



ANSWER:

$(y - 5)^2 = 32(x + 7)$



77. **CRAFTS** Sanjay is selling wood carvings. He sells large statues for \$60, clocks for \$40, dollhouse furniture for \$25, and chess pieces for \$5. He takes the following number of items to the fair: 12 large statues, 25 clocks, 45 pieces of dollhouse furniture, and 50 chess pieces.

a. Write an inventory matrix for the number of each item and a cost matrix for the price of each item.

b. Find Sanjay's total income if he sells all of the items.

SOLUTION:

a. Sample answer: The number of each item can be written in a 1×4 inventory matrix.

$$\begin{bmatrix} 12 & 25 & 45 & 50 \end{bmatrix}$$

The price of each item can be written in a 4×1 cost matrix.

$$\begin{bmatrix} 60 \\ 40 \\ 25 \\ 5 \end{bmatrix}$$

b. Find the product of the inventory and cost matrix found in part **a**.

$$\begin{bmatrix} 12 & 25 & 45 & 50 \end{bmatrix} \cdot \begin{bmatrix} 60 \\ 40 \\ 25 \\ 5 \end{bmatrix} = [12(60) + 25(40) + 45(25) + 50(5)]$$
$$= [3095]$$

If Stanley sells all of the items, his total income will be \$3095.

ANSWER:

a. Sample answer: $\begin{bmatrix} 12 & 25 & 45 & 50 \end{bmatrix}, \begin{bmatrix} 60 \\ 40 \\ 25 \\ 5 \end{bmatrix}$

b. \$3095

Solve each equation for all values of x.

78. $4 \sin x \cos x - 2 \sin x = 0$

SOLUTION:

$$4\sin x\cos x - 2\sin x = 0$$
$$(2\sin x)(2\cos x - 1) = 0$$

$$2\sin x = 0$$
$$\sin x = 0$$
$$x = 0 \text{ or } \pi$$

$$2\cos x - 1 = 0$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}$$

The period of sine and cosine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are 0, π, $\frac{\pi}{3}$, and $\frac{5\pi}{3}$. The solutions on the interval $(-\infty,\infty)$ are then found by adding integer multiples of 2π. The solutions $x = 0 + 2n\pi$ and $x = \pi + 2n\pi$ can be combined to $x = n\pi$. Therefore, the general form of the solutions is $x = n\pi$, $x = \frac{\pi}{3} + 2n\pi$, and $x = \frac{5\pi}{3} + 2n\pi$, where n is an integer.

ANSWER:

$$n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \text{ where } n \text{ is an integer}$$

79. $\sin x - 2 \cos^2 x = -1$

SOLUTION:

$$\sin x - 2\cos^2 x = -1$$
$$\sin x - 2(1 - \sin^2 x) = -1$$
$$\sin x - 2 + 2\sin^2 x = -1$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$\sin x + 1 = 0$$
$$\sin x = -1$$
$$x = \frac{3\pi}{2}$$

The period of sine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions on this interval are $\frac{\pi}{6}$, $\frac{5\pi}{6}$, and $\frac{3\pi}{2}$. The solutions on the interval $(-\infty,\infty)$ are then found by adding integer multiples of 2π. Therefore, the general form of the solutions is $x = \frac{\pi}{6} + 2n\pi$, $x = \frac{5\pi}{6} + 2n\pi$, and $x = \frac{3\pi}{2} + 2n\pi$, where n is an integer.

ANSWER:

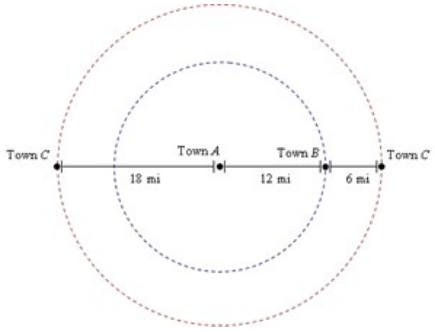
$$\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi, \text{ where } n \text{ is an integer}$$

80. **SAT/ACT** If town *A* is 12 miles from town *B* and town *C* is 18 miles from town *A*, then which of the following cannot be the distance from town *B* to town *C*?

A 5 miles
B 7 miles
C 10 miles
D 12 miles
E 18 miles

SOLUTION:

Sketch the possible locations of the towns. The possible locations can be represented by two concentric circles, one with a radius of 12 miles and the other with a radius of 18 miles, with town *A* at the center.

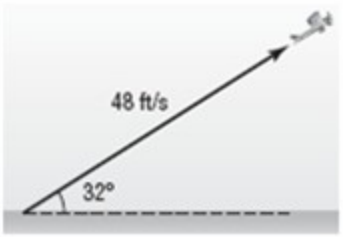


Town *B* and town *C* will be at their closest distance to each other when they are on the same side of town *A*. As shown in the drawing, town *B* and town *C* can achieve a minimal distance of 6 miles from each other. Town *B* and town *C* will be at their farthest from each other when they are on exact opposite sides of town *A*. As also shown in the drawing, town *B* and town *C* can achieve a maximum distance of 30 miles from each other. Thus, the distance *d* that town *B* and town *C* can be from each other is $6 \le d \le 30$. The towns cannot be 5 miles apart.
The correct answer is A.

ANSWER:

A

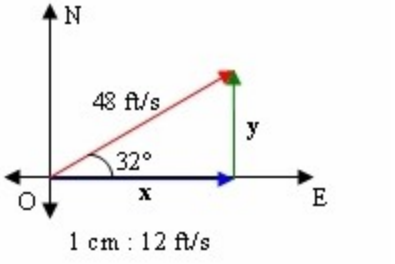
81. A remote control airplane flew along an initial path of 32° to the horizontal at a velocity of 48 feet per second as shown. Which of the following represent the magnitudes of the horizontal and vertical components of the velocity?



F 25.4 ft/s, 40.7 ft/s
G 40.7 ft/s, 25.4 ft/s
H 56.6 ft/s, 90.6 ft/s
J 90.6 ft/s, 56.6 ft/s

SOLUTION:

The remote control airplane flew along an initial path of 32° to the horizontal at a velocity of 48 feet per second. Draw a vector to represent the airplane. Also include the horizontal component **x** and a vertical component **y**.



The horizontal and vertical components of the vector form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.

$$\cos 32^\circ = \frac{|\mathbf{x}|}{48} \qquad \sin 32^\circ = \frac{|\mathbf{y}|}{48}$$

$$|\mathbf{x}| = 48\cos 32^\circ \qquad |\mathbf{y}| = 48\sin 32^\circ$$

$$|\mathbf{x}| \approx 40.7 \qquad |\mathbf{y}| \approx 25.4$$

Drawings may not be to scale.

The magnitude of the horizontal component is about 40.7 feet per second and the magnitude of the vertical component is about 25.4 feet per second.
The correct answer is G.

ANSWER:

G

82. **REVIEW** Triangle *ABC* has vertices *A*(−4, 2), *B*(−4, −3), and *C*(3, −3). After a dilation, triangle *A′B′*C' has vertices *A′*(−12, 6), *B′*(−12, −9), and *C′*(9, −9).

How many times as great is the area of $\Delta A'B'C'$ than the area of ΔABC?

A $\frac{1}{9}$

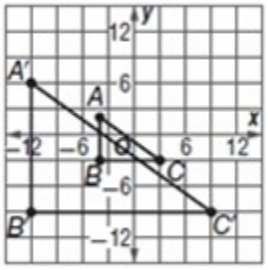
B $\frac{1}{3}$

C 3

D 9

SOLUTION:

Graph ΔABC and $\Delta A'B'C'$.



To find the area of ΔABC, find the height and the length of the base. The length of the base is the length of $\overline{BC}$, which is 7. The height of ΔABC is the length of $\overline{AB}$, which is 5. Substitute these values into the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(7)(5)$$
$$= 17.5$$

The area of ΔABC is 17.5 units.

To find the area of $\Delta A'B'C'$, find the height and the length of the base. The length of the base is the length of $\overline{B'C'}$, which is 21. The height of $\Delta A'B'C'$ is the length of $\overline{A'B'}$, which is 15. Substitute these values into the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(21)(15)$$
$$= 157.5$$

The area of $\Delta A'B'C'$ is 157.5 units. To find how many times greater the area of $\Delta A'B'C'$ is than the area of ΔABC, divide 157.5 by 17.5. So, the area of $\Delta A'B'C'$ is 9 times greater than the area of ΔABC.

The correct answer is D.

ANSWER:

D

83. **REVIEW** Holly is drawing a map of her neighborhood. Her house is represented by quadrilateral $ABCD$ with vertices $A(2, 2)$, $B(6, 2)$, $C(6, 6)$, and $D(2, 6)$. She wants to use the same coordinate system to make another map that is one half the size of the original map. What could be the new vertices of Holly's house?

F $A'(0, 0)$, $B'(2, 1)$, $C'(3, 3)$, $D'(0, 3)$

G $A'(0, 0)$, $B'(3, 1)$, $C'(2, 3)$, $D'(0, 2)$

H $A'(1, 1)$, $B'(3, 1)$, $C'(3, 3)$, $D'(1, 3)$

J $A'(1, 2)$, $B'(3, 0)$, $C'(2, 2)$, $D'(2, 3)$

SOLUTION:

To make another map that is one half the size of the original map, Holly can perform a dilation on the original map using a scale factor of $\frac{1}{2}$. To find the new coordinates, she can multiply the original coordinates by the scale factor.

$A = \frac{1}{2}(2, 2)$ or $(1, 1)$

$B = \frac{1}{2}(6, 2)$ or $(3, 1)$

$C = \frac{1}{2}(6, 6)$ or $(3, 3)$

$D = \frac{1}{2}(2, 6)$ or $(1, 3)$

The correct answer is H.

ANSWER:

H

8-2 Vectors in the Coordinate Plane

Find the component form and magnitude of $\overline{AB}$ with the given initial and terminal points.

1. $A(-3, 1)$, $B(4, 5)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 4 - (-3), 5 - 1 \rangle \\ &= \langle 7, 4 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = 7$ and $y_2 - y_1 = 4$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overline{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{7^2 + 4^2} \\ &= \sqrt{65} \text{ or about } 8.1\end{aligned}$$

ANSWER:

$\langle 7, 4 \rangle$; $\sqrt{65} \approx 8.1$

2. $A(2, -7)$, $B(-6, 9)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -6 - 2, 9 - (-7) \rangle \\ &= \langle -8, 16 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = -8$ and $y_2 - y_1 = 16$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overline{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-8)^2 + 16^2} \\ &= \sqrt{320} \\ &= 8\sqrt{5} \text{ or about } 17.9\end{aligned}$$

ANSWER:

$\langle -8, 16 \rangle$; $8\sqrt{5} \approx 17.9$

3. $A(10, -2)$, $B(3, -5)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 3 - 10, -5 - (-2) \rangle \\ &= \langle -7, -3 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = -7$ and $y_2 - y_1 = -3$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overline{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-7)^2 + (-3)^2} \\ &= \sqrt{58} \text{ or about } 7.6\end{aligned}$$

ANSWER:

$\langle -7, -3 \rangle$; $\sqrt{58} \approx 7.6$

4. $A(-2, 7)$, $B(-9, -1)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -9 - (-2), -1 - 7 \rangle \\ &= \langle -7, -8 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = -7$ and $y_2 - y_1 = -8$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overline{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-7)^2 + (-8)^2} \\ &= \sqrt{113} \text{ or about } 10.6\end{aligned}$$

ANSWER:

$\langle -7, -8 \rangle$; $\sqrt{113} \approx 10.6$

5. $A(-5, -4)$, $B(8, -2)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 8-(-5), -2-(-4) \rangle \\ &= \langle 13, 2 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = 13$ and $y_2 - y_1 = 2$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{13^2 + 2^2} \\ &= \sqrt{173} \text{ or about } 13.2\end{aligned}$$

ANSWER:

$\langle 13, 2 \rangle$; $\sqrt{173} \approx 13.2$

6. $A(-2, 6)$, $B(1, 10)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1-(-2), 10-6 \rangle \\ &= \langle 3, 4 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = 3$ and $y_2 - y_1 = 4$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{3^2 + 4^2} \\ &= 5\end{aligned}$$

ANSWER:

$\langle 3, 4 \rangle$; 5

7. $A(2.5, -3)$, $B(-4, 1.5)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -4-2.5, 1.5-(-3) \rangle \\ &= \langle -6.5, 4.5 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = -6.5$ and $y_2 - y_1 = 4.5$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-6.5)^2 + 4.5^2} \\ &= \sqrt{62.5} \text{ or about } 7.9\end{aligned}$$

ANSWER:

$\langle -6.5, 4.5 \rangle$; $\sqrt{62.5} \approx 7.9$

8. $A(-4.3, 1.8)$, $B(9.4, -6.2)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 9.4-(-4.3), -6.2-1.8 \rangle \\ &= \langle 13.7, -8 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = 13.7$ and $y_2 - y_1 = -8$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(13.7)^2 + (-8)^2} \\ &= \sqrt{251.69} \text{ or about } 15.9\end{aligned}$$

ANSWER:

$\langle 13.7, -8 \rangle$; $\sqrt{251.69} \approx 15.9$

9. $A\left(\frac{1}{2}, -9\right)$, $B\left(6, \frac{5}{2}\right)$

SOLUTION:

First, find the component form.

$$\overline{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \left\langle 6 - \left(\frac{1}{2}\right), \frac{5}{2} - (-9) \right\rangle$$
$$= \left\langle \frac{11}{2}, \frac{23}{2} \right\rangle$$

Next, find the magnitude. Substitute $x_2 - x_1 = \frac{11}{2}$ and $y_2 - y_1 = \frac{23}{2}$ into the formula for the magnitude of a vector in the coordinate plane.

$$\left|\overline{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(\frac{11}{2}\right)^2 + \left(\frac{23}{2}\right)^2}$$
$$= \sqrt{\frac{325}{2}} \text{ or about } 12.7$$

ANSWER:

$\left\langle \frac{11}{2}, \frac{23}{2} \right\rangle; \sqrt{\frac{325}{2}} \approx 12.7$

10. $A\left(\frac{3}{5}, -\frac{2}{5}\right)$, $B(-1, 7)$

SOLUTION:

First, find the component form.

$$\overline{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \left\langle -1 - \left(\frac{3}{5}\right), 7 - \left(-\frac{2}{5}\right) \right\rangle$$
$$= \left\langle -\frac{8}{5}, \frac{37}{5} \right\rangle$$

Next, find the magnitude. Substitute $x_2 - x_1 = -\frac{8}{5}$ and $y_2 - y_1 = \frac{37}{5}$ into the formula for the magnitude of a vector in the coordinate plane.

$$\left|\overline{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(-\frac{8}{5}\right)^2 + \left(\frac{37}{5}\right)^2}$$
$$= \sqrt{\frac{1433}{25}} \text{ or about } 7.6$$

ANSWER:

$\left\langle -\frac{8}{5}, \frac{37}{5} \right\rangle; \sqrt{\frac{1433}{25}} \approx 7.6$

Find each of the following for $\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$.

11. $4\mathbf{h} - \mathbf{g}$

SOLUTION:

$$4\mathbf{h} - \mathbf{g} = (4)\langle -6, 2 \rangle - \langle -3, -5 \rangle$$
$$= \langle -24, 8 \rangle - \langle -3, -5 \rangle$$
$$= \langle -21, 13 \rangle$$

ANSWER:

$\langle -21, 13 \rangle$

12. $\mathbf{f} + 2\mathbf{h}$

SOLUTION:

$$\begin{aligned}\mathbf{f} + 2\mathbf{h} &= \langle 8,0\rangle + (2)\langle -6,2\rangle \\ &= \langle 8,0\rangle + \langle -12,4\rangle \\ &= \langle -4,4\rangle\end{aligned}$$

ANSWER:

$\langle -4, 4\rangle$

13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$

SOLUTION:

$$\begin{aligned}3\mathbf{g} - 5\mathbf{f} + \mathbf{h} &= 3\mathbf{g} + (-5)\mathbf{f} + \mathbf{h} \\ &= (3)\langle -3,-5\rangle + (-5)\langle 8,0\rangle + \langle -6,2\rangle \\ &= \langle -9,-15\rangle + \langle -40,0\rangle + \langle -6,2\rangle \\ &= \langle -55,-13\rangle\end{aligned}$$

ANSWER:

$\langle -55, -13\rangle$

14. $2\mathbf{f} + \mathbf{g} - 3\mathbf{h}$

SOLUTION:

$$\begin{aligned}2\mathbf{f} + \mathbf{g} - 3\mathbf{h} &= 2\mathbf{f} + \mathbf{g} + (-3)\mathbf{h} \\ &= (2)\langle 8,0\rangle + \langle -3,-5\rangle + (-3)\langle -6,2\rangle \\ &= \langle 16,0\rangle + \langle -3,-5\rangle + \langle 18,-6\rangle \\ &= \langle 31,-11\rangle\end{aligned}$$

ANSWER:

$\langle 31, -11\rangle$

15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$

SOLUTION:

$$\begin{aligned}\mathbf{f} - 2\mathbf{g} - 2\mathbf{h} &= \mathbf{f} + (-2)\mathbf{g} + (-2)\mathbf{h} \\ &= \langle 8,0\rangle + (-2)\langle -3,-5\rangle + (-2)\langle -6,2\rangle \\ &= \langle 8,0\rangle + \langle 6,10\rangle + \langle 12,-4\rangle \\ &= \langle 26,6\rangle\end{aligned}$$

ANSWER:

$\langle 26, 6\rangle$

16. $\mathbf{h} - 4\mathbf{f} + 5\mathbf{g}$

SOLUTION:

$$\begin{aligned}\mathbf{h} - 4\mathbf{f} + 5\mathbf{g} &= \mathbf{h} + (-4)\mathbf{f} + 5\mathbf{g} \\ &= \langle -6,2\rangle + (-4)\langle 8,0\rangle + (5)\langle -3,-5\rangle \\ &= \langle -6,2\rangle + \langle -32,0\rangle + \langle -15,-25\rangle \\ &= \langle -53,-23\rangle\end{aligned}$$

ANSWER:

$\langle -53, -23\rangle$

17. $4\mathbf{g} - 3\mathbf{f} + \mathbf{h}$

SOLUTION:

$$\begin{aligned}4\mathbf{g} - 3\mathbf{f} + \mathbf{h} &= 4\mathbf{g} + (-3)\mathbf{f} + \mathbf{h} \\ &= (4)\langle -3,-5\rangle + (-3)\langle 8,0\rangle + \langle -6,2\rangle \\ &= \langle -12,-20\rangle + \langle -24,0\rangle + \langle -6,2\rangle \\ &= \langle -42,-18\rangle\end{aligned}$$

ANSWER:

$\langle -42, -18\rangle$

18. $6\mathbf{h} + 5\mathbf{f} - 10\mathbf{g}$

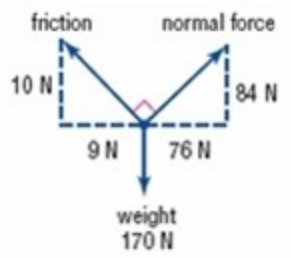
SOLUTION:

$$\begin{aligned}6\mathbf{h} + 5\mathbf{f} - 10\mathbf{g} &= 6\mathbf{h} + 5\mathbf{f} + (-10)\mathbf{g} \\ &= (6)\langle -6,2\rangle + (5)\langle 8,0\rangle + (-10)\langle -3,-5\rangle \\ &= \langle -36,12\rangle + \langle 40,0\rangle + \langle 30,50\rangle \\ &= \langle 34,62\rangle\end{aligned}$$

ANSWER:

$\langle 34, 62\rangle$

19. **PHYSICS** In physics, force diagrams are used to show the effects of all the different forces acting upon an object. The following force diagram could represent the forces acting upon a child sliding down a slide.



a. Using the blue dot representing the child as the origin, express each force as a vector in component form.

b. Find the component form of the resultant vector representing the force that causes the child to move down the slide.

SOLUTION:

a. Since the blue dot represents the child as the origin, the rectangular components for a force will be the horizontal and vertical components of the force.

The force of friction is applying 9 newtons of force to the left parallel to the *x*-axis and 10 newtons of force up parallel to the *y*-axis. Thus, $\mathbf{f} = \langle -9, 10 \rangle$.

Normal force is applying 76 newtons of force to the right parallel to the *x*-axis and 84 newtons of force up parallel to the *y*-axis. Thus, $\mathbf{n} = \langle 76, 84 \rangle$.

Weight is applying 170 newtons of force down parallel to the *y*-axis. Thus, $\mathbf{w} = \langle 0, -170 \rangle$.

b. To find the component form of the resultant vector representing the force that causes the child to move down the slide, find the sum of the three force vectors, or $\mathbf{f} + \mathbf{n} + \mathbf{w}$.

$$\begin{aligned}\mathbf{f} + \mathbf{n} + \mathbf{w} &= \langle -9, 10 \rangle + \langle 76, 84 \rangle + \langle 0, -170 \rangle \\ &= \langle 67, -76 \rangle\end{aligned}$$

The component form of the resultant vector representing the force is $\langle 67, -76 \rangle$.

ANSWER:

a. $\mathbf{f} = \langle -9, 10 \rangle$, $\mathbf{n} = \langle 76, 84 \rangle$, $\mathbf{w} = \langle 0, -170 \rangle$

b. $\langle 67, -76 \rangle$

Find a unit vector u with the same direction as v.

20. $\mathbf{v} = \langle -2, 7 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} &= \frac{1}{|\mathbf{v}|}\mathbf{v} \\ &= \frac{1}{|\langle -2, 7 \rangle|}\langle -2, 7 \rangle \\ &= \frac{1}{\sqrt{(-2)^2 + 7^2}}\langle -2, 7 \rangle \\ &= \frac{1}{\sqrt{53}}\langle -2, 7 \rangle \\ &= \left\langle -\frac{2}{\sqrt{53}}, \frac{7}{\sqrt{53}} \right\rangle \\ &= \left\langle -\frac{2\sqrt{53}}{53}, \frac{7\sqrt{53}}{53} \right\rangle\end{aligned}$$

ANSWER:

$\mathbf{u} = \left\langle -\frac{2\sqrt{53}}{53}, \frac{7\sqrt{53}}{53} \right\rangle$

21. $\mathbf{v} = \langle 9, -3 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle 9,-3\rangle|}\langle 9,-3\rangle$$

$$= \frac{1}{\sqrt{9^2+(-3)^2}}\langle 9,-3\rangle$$

$$= \frac{1}{\sqrt{90}}\langle 9,-3\rangle$$

$$= \left\langle \frac{9}{\sqrt{90}}, -\frac{3}{\sqrt{90}} \right\rangle$$

$$= \left\langle \frac{9\sqrt{90}}{90}, -\frac{3\sqrt{90}}{90} \right\rangle$$

$$= \left\langle \frac{3\sqrt{10}}{10}, -\frac{\sqrt{10}}{10} \right\rangle$$

ANSWER:

$$\mathbf{u} = \left\langle \frac{3\sqrt{10}}{10}, -\frac{\sqrt{10}}{10} \right\rangle$$

22. $\mathbf{v} = \langle -8, -5 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle -8,-5\rangle|}\langle -8,-5\rangle$$

$$= \frac{1}{\sqrt{(-8)^2+(-5)^2}}\langle -8,-5\rangle$$

$$= \frac{1}{\sqrt{89}}\langle -8,-5\rangle$$

$$= \left\langle -\frac{8}{\sqrt{89}}, -\frac{5}{\sqrt{89}} \right\rangle$$

$$= \left\langle -\frac{8\sqrt{89}}{89}, -\frac{5\sqrt{89}}{89} \right\rangle$$

ANSWER:

$$\mathbf{u} = \left\langle -\frac{8\sqrt{89}}{89}, -\frac{5\sqrt{89}}{89} \right\rangle$$

23. $\mathbf{v} = \langle 6, 3 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle 6,3 \rangle|}\langle 6,3 \rangle$$

$$= \frac{1}{\sqrt{6^2 + 3^2}}\langle 6,3 \rangle$$

$$= \frac{1}{\sqrt{45}}\langle 6,3 \rangle$$

$$= \left\langle \frac{6}{\sqrt{45}}, \frac{3}{\sqrt{45}} \right\rangle$$

$$= \left\langle \frac{6\sqrt{45}}{45}, \frac{3\sqrt{45}}{45} \right\rangle$$

$$= \left\langle \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$$

ANSWER:

$\mathbf{u} = \left\langle \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$

24. $\mathbf{v} = \langle -2, 9 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle -2,9 \rangle|}\langle -2,9 \rangle$$

$$= \frac{1}{\sqrt{(-2)^2 + 9^2}}\langle -2,9 \rangle$$

$$= \frac{1}{\sqrt{85}}\langle -2,9 \rangle$$

$$= \left\langle -\frac{2}{\sqrt{85}}, \frac{9}{\sqrt{85}} \right\rangle$$

$$= \left\langle -\frac{2\sqrt{85}}{85}, \frac{9\sqrt{85}}{85} \right\rangle$$

ANSWER:

$\mathbf{u} = \left\langle -\frac{2\sqrt{85}}{85}, \frac{9\sqrt{85}}{85} \right\rangle$

25. $\mathbf{v} = \langle -1, -5 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle -1, -5 \rangle|}\langle -1, -5 \rangle$$

$$= \frac{1}{\sqrt{(-1)^2 + (-5)^2}}\langle -1, -5 \rangle$$

$$= \frac{1}{\sqrt{26}}\langle -1, -5 \rangle$$

$$= \left\langle -\frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}} \right\rangle$$

$$= \left\langle -\frac{\sqrt{26}}{26}, -\frac{5\sqrt{26}}{26} \right\rangle$$

ANSWER:

$$\mathbf{u} = \left\langle -\frac{\sqrt{26}}{26}, -\frac{5\sqrt{26}}{26} \right\rangle$$

26. $\mathbf{v} = \langle 1, 7 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle 1, 7 \rangle|}\langle 1, 7 \rangle$$

$$= \frac{1}{\sqrt{1^2 + 7^2}}\langle 1, 7 \rangle$$

$$= \frac{1}{\sqrt{50}}\langle 1, 7 \rangle$$

$$= \left\langle \frac{1}{\sqrt{50}}, \frac{7}{\sqrt{50}} \right\rangle$$

$$= \left\langle \frac{\sqrt{50}}{50}, \frac{7\sqrt{50}}{50} \right\rangle$$

$$= \left\langle \frac{\sqrt{2}}{10}, \frac{7\sqrt{2}}{10} \right\rangle$$

ANSWER:

$$\mathbf{u} = \left\langle \frac{\sqrt{2}}{10}, \frac{7\sqrt{2}}{10} \right\rangle$$

27. $\mathbf{v} = \langle 3, -4 \rangle$

SOLUTION:

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

$$= \frac{1}{|\langle 3,-4 \rangle|}\langle 3,-4 \rangle$$

$$= \frac{1}{\sqrt{3^2+(-4)^2}}\langle 3,-4 \rangle$$

$$= \frac{1}{\sqrt{25}}\langle 3,-4 \rangle$$

$$= \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$$

ANSWER:

$\mathbf{u} = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

Let $\overline{DE}$ be the vector with the given initial and terminal points. Write $\overline{DE}$ as a linear combination of the vectors i and j.

28. $D(4, -1)$, $E(5, -7)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$= \langle 5-4, -7-(-1) \rangle$$

$$= \langle 1, -6 \rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle 1, -6 \rangle$$

$$= \mathbf{i} - 6\mathbf{j}$$

ANSWER:

$\mathbf{i} - 6\mathbf{j}$

29. $D(9, -6)$, $E(-7, 2)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$= \langle -7-9, 2-(-6) \rangle$$

$$= \langle -16, 8 \rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle -16, 8 \rangle$$

$$= -16\mathbf{i} + 8\mathbf{j}$$

ANSWER:

$-16\mathbf{i} + 8\mathbf{j}$

30. $D(3, 11)$, $E(-2, -8)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$= \langle -2-3, -8-11 \rangle$$

$$= \langle -5, -19 \rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle -5, -19 \rangle$$

$$= -5\mathbf{i} - 19\mathbf{j}$$

ANSWER:

$-5\mathbf{i} - 19\mathbf{j}$

31. $D(9.5, 1)$, $E(0, -7.3)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle 0 - 9.5, -7.3 - 1 \rangle$$
$$= \langle -9.5, -8.3 \rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle -9.5, -8.3 \rangle$$
$$= -9.5\mathbf{i} - 8.3\mathbf{j}$$

ANSWER:

$-9.5\mathbf{i} - 8.3\mathbf{j}$

32. $D(-3, -5.7)$, $E(6, -8.1)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle 6 - (-3), -8.1 - (-5.7) \rangle$$
$$= \langle 9, -2.4 \rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle 9, -2.4 \rangle$$
$$= 9\mathbf{i} - 2.4\mathbf{j}$$

ANSWER:

$9\mathbf{i} - 2.4\mathbf{j}$

33. $D(-4, -6)$, $E(9, 5)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle 9 - (-4), 5 - (-6) \rangle$$
$$= \langle 13, 11 \rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \langle 13, 11 \rangle$$
$$= 13\mathbf{i} + 11\mathbf{j}$$

ANSWER:

$13\mathbf{i} + 11\mathbf{j}$

34. $D\left(\frac{1}{8}, 3\right), E\left(-4, \frac{2}{7}\right)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\overline{DE} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \left\langle -4 - \left(\frac{1}{8}\right), \frac{2}{7} - 3 \right\rangle$$
$$= \left\langle -\frac{33}{8}, -\frac{19}{7} \right\rangle$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\overline{DE} = \left\langle -\frac{33}{8}, -\frac{19}{7} \right\rangle$$
$$= -\frac{33}{8}\mathbf{i} - \frac{19}{7}\mathbf{j}$$

ANSWER:

$-\frac{33}{8}\mathbf{i} - \frac{19}{7}\mathbf{j}$

35. $D(-3, 1.5)$, $E(-3, 1.5)$

SOLUTION:

First, find the component form of $\overline{DE}$.

$$\begin{aligned}\overline{DE} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -3 - (-3), 1.5 - 1.5 \rangle \\ &= \langle 0, 0 \rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\begin{aligned}\overline{DE} &= \langle 0, 0 \rangle \\ &= 0\mathbf{i} + 0\mathbf{j} \\ &= 0\end{aligned}$$

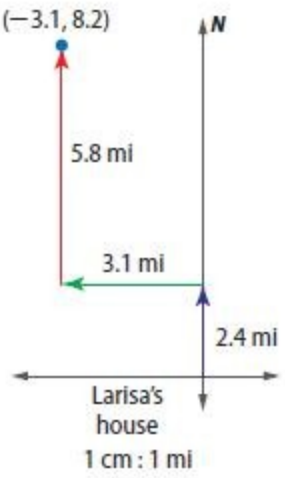
ANSWER:

0

36. **COMMUTE** To commute to school, Larisa leaves her house and drives north on Pepper Lane for 2.4 miles. She turns left on Cinnamon Drive for 3.1 miles and then turns right on Maple Street for 5.8 miles. Express Larisa's commute as a linear combination of unit vectors **i** and **j**.

SOLUTION:

Let Larisa's house be located at the origin, (0, 0). Make a drawing to represent Larisa's commute and find the coordinates of her destination.



Drawing may not be to scale.

The initial and terminal points of Larisa's commute are (0, 0) and (−3.1, 8.2), respectively. First, find the component form of the vector representing Larisa's commute.

$$\begin{aligned}\langle x_2 - x_1, y_2 - y_1 \rangle &= \langle -3.1 - 0, 8.2 - 0 \rangle \\ &= \langle -3.1, 8.2 \rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\langle -3.1, 8.2 \rangle = -3.1\mathbf{i} + 8.2\mathbf{j}$$

Larisa's commute expressed as a linear combination is $-3.1\mathbf{i} + 8.2\mathbf{j}$.

ANSWER:

$-3.1\mathbf{i} + 8.2\mathbf{j}$

37. **ROWING** Nadia is rowing across a river at a speed of 5 miles per hour perpendicular to the shore.

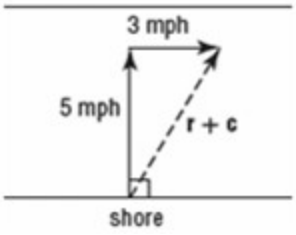
The river has a current of 3 miles per hour heading downstream.

a. At what speed is she traveling?

b. At what angle is she traveling with respect to the shore?

SOLUTION:

a. Nadia's rowing can be represented by the vector $\mathbf{r} = \langle 0,5 \rangle$ and the current can be represented by the vector $\mathbf{c} = \langle 3,0 \rangle$.



Add the vectors representing **r** and **c** to find the resultant vector, **v**.

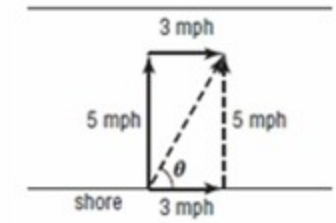
$$\mathbf{v} = \mathbf{r} + \mathbf{c}$$
$$= \langle 0,5 \rangle + \langle 3,0 \rangle \text{ or } \langle 3,5 \rangle$$

The speed at which Nadia is traveling is the magnitude of **v**.

$$|\mathbf{v}| = \sqrt{3^2 + 5^2}$$
$$= \sqrt{34} \text{ or about } 5.8$$

Thus, she is traveling at about 5.8 miles per hour.

b. Find θ by constructing a right triangle with the given vectors and using the tangent ratio.



$$\tan\theta = \frac{5}{3}$$
$$\theta = \tan^{-1}\frac{5}{3}$$
$$\theta \approx 59^\circ$$

Thus, Nadia is traveling at an angle of about 59° with respect to the shore.

ANSWER:

a. about 5.8 mph

b. about 59º

Find the component form of v with the given magnitude and direction angle.

38. $|\mathbf{v}| = 12$, $\theta = 60°$

SOLUTION:

$$\mathbf{v} = \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle$$
$$= \langle 12\cos 60^\circ, 12\sin 60^\circ \rangle$$
$$= \left\langle 12\left(\frac{1}{2}\right), 12\left(\frac{\sqrt{3}}{2}\right) \right\rangle$$
$$= \langle 6, 6\sqrt{3} \rangle$$

ANSWER:

$\langle 6, 6\sqrt{3} \rangle$

39. $|\mathbf{v}| = 4$, $\theta = 135°$

SOLUTION:

$$\mathbf{v} = \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle$$
$$= \langle 4\cos 135^\circ, 4\sin 135^\circ \rangle$$
$$= \left\langle 4\left(-\frac{\sqrt{2}}{2}\right), 4\left(\frac{\sqrt{2}}{2}\right) \right\rangle$$
$$= \langle -2\sqrt{2}, 2\sqrt{2} \rangle$$

ANSWER:

$\langle -2\sqrt{2}, 2\sqrt{2} \rangle$

40. $|\mathbf{v}| = 6$, $\theta = 240°$

SOLUTION:

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 6\cos 240°, 6\sin 240° \rangle \\ &= \left\langle 6\left(-\frac{1}{2}\right), 6\left(-\frac{\sqrt{3}}{2}\right) \right\rangle \\ &= \langle -3, -3\sqrt{3} \rangle\end{aligned}$$

ANSWER:

$\langle -3, -3\sqrt{3} \rangle$

41. $|\mathbf{v}| = 16$, $\theta = 330°$

SOLUTION:

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 16\cos 330°, 16\sin 330° \rangle \\ &= \left\langle 16\left(\frac{\sqrt{3}}{2}\right), 16\left(-\frac{1}{2}\right) \right\rangle \\ &= \langle 8\sqrt{3}, -8 \rangle\end{aligned}$$

ANSWER:

$\langle 8\sqrt{3}, -8 \rangle$

42. $|\mathbf{v}| = 28$, $\theta = 273°$

SOLUTION:

$$\begin{aligned}v &= \langle |v|\cos\theta, |v|\sin\theta \rangle \\ &= \langle 28\cos 273°, 28\sin 273° \rangle \\ &= \langle 28(0.05234), 28(-0.9986) \rangle \\ &= \langle 1.47, -27.96 \rangle\end{aligned}$$

ANSWER:

$\langle 1.47, -27.96 \rangle$

43. $|\mathbf{v}| = 15$, $\theta = 125°$

SOLUTION:

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 15\cos 125°, 15\sin 125° \rangle \\ &= \langle 15(-0.5736), 15(0.8192) \rangle \\ &= \langle -8.60, 12.29 \rangle\end{aligned}$$

ANSWER:

$\langle -8.6, 12.29 \rangle$

Find the direction angle of each vector to the nearest tenth of a degree.

44. $3\mathbf{i} + 6\mathbf{j}$

SOLUTION:

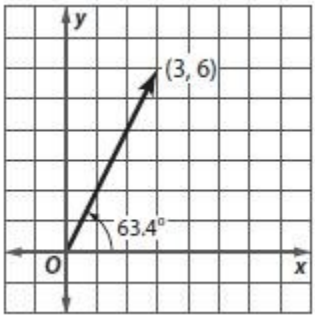
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{6}{3}$$

$$\theta = \tan^{-1} 2$$

$$\theta \approx 63.4°$$

So, the direction angle of the vector is about 63.4°.



ANSWER:

63.4°

45. $-2\mathbf{i} + 5\mathbf{j}$

SOLUTION:

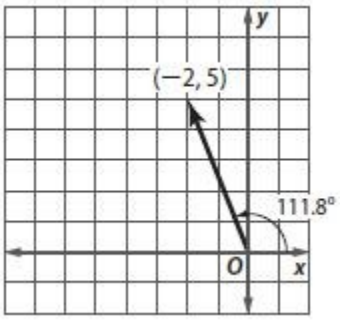
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{5}{-2}$$

$$\theta = \tan^{-1}\left(-\frac{5}{2}\right)$$

$$\theta \approx -68.2^\circ$$

Since the vector lies in Quadrant II, $\theta = 180° + (-68.2°)$ or about 111.8°.



ANSWER:

111.8º

46. $8\mathbf{i} - 2\mathbf{j}$

SOLUTION:

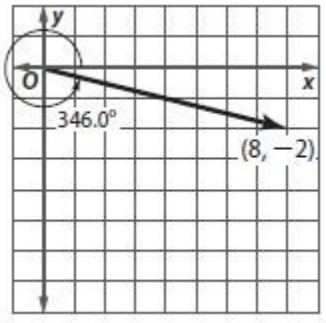
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{-2}{8}$$

$$\theta = \tan^{-1}\left(-\frac{1}{4}\right)$$

$$\theta \approx -14.0^\circ$$

Since the vector lies in Quadrant IV, $\theta = 360° + (-14.0°)$ or about 346.0°.



ANSWER:

346.0º

47. $-4\mathbf{i} - 3\mathbf{j}$

SOLUTION:

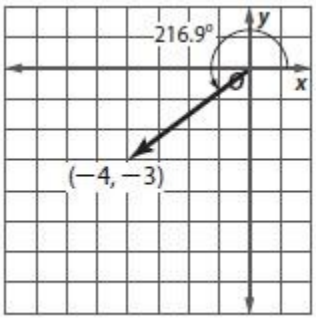
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{-3}{-4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta \approx 36.9^\circ$$

Since the vector lies in Quadrant III, $\theta = 180° + 36.9°$ or about 216.9°.



ANSWER:

216.9º

48. $\langle -5, 9\rangle$

SOLUTION:

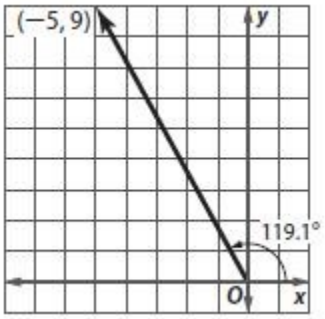
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{9}{-5}$$

$$\theta = \tan^{-1}\left(-\frac{9}{5}\right)$$

$$\theta \approx -60.9^\circ$$

Since the vector lies in Quadrant II, $\theta = 180° + (-60.9)°$ or about 119.1 °.



ANSWER:

119.1º

49. $\langle 7, 7 \rangle$

SOLUTION:

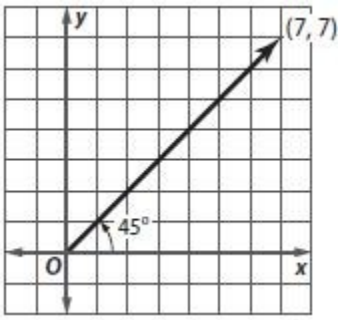
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{7}{7}$$

$$\theta = \tan^{-1} 1$$

$$\theta \approx 45^\circ$$

So, the direction angle of the vector is 45°.



ANSWER:

45°

50. $\langle -6, -4 \rangle$

SOLUTION:

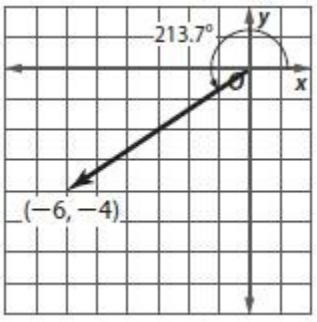
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{-4}{-6}$$

$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$

$$\theta \approx 33.7^\circ$$

Since the vector lies in Quadrant III, θ = 180° + 33.7° or about 213.7°.



ANSWER:

213.7°

51. $\langle 3, -8 \rangle$

SOLUTION:

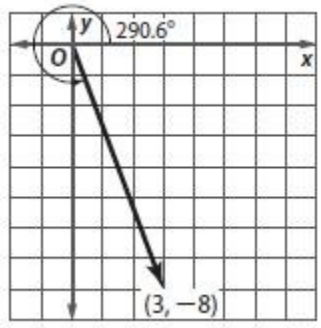
$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{-8}{3}$$

$$\theta = \tan^{-1}\left(-\frac{8}{3}\right)$$

$$\theta \approx -69.4^\circ$$

Since the vector lies in Quadrant IV, $\theta = 360° + (-69.4)°$ or about 290.6°.



ANSWER:

290.6º

52. **SLEDDING** Maggie is pulling a sled with a force of 275 newtons by holding its rope at a 58º angle. Her brother is going to help by pushing the sled with a force of 320 N. Determine the magnitude and direction of the total resultant force on the sled.



SOLUTION:

Use the magnitude and direction of the force that Maggie is using to pull the sled to write the force in component form as $\mathbf{f_1}$. Let $\theta = 122°$ since θ is the direction angle that $\mathbf{f_1}$ makes with the positive x-axis.

$$\mathbf{f_1} = \langle |\mathbf{f_1}|\cos\theta, |\mathbf{f_1}|\sin\theta \rangle$$

$$= \langle 275\cos 122^\circ, 275\sin 122^\circ \rangle$$

$$= \langle -145.7, 233.2 \rangle$$

Since Maggie's brother is pushing the sled to the left, the component form of his force $\mathbf{f_2}$ is $\langle -320, 0 \rangle$. Add the algebraic vectors representing $\mathbf{f_1}$ and $\mathbf{f_2}$ to find the resultant force, vector $\mathbf{v}$.

$$\mathbf{v} = \mathbf{f_1} + \mathbf{f_2}$$

$$= \langle -145.7, 233.2 \rangle + \langle -320, 0 \rangle$$

$$= \langle -465.7, 233.2 \rangle$$

Find the magnitude of the resultant.

$$|\mathbf{v}| = \sqrt{(-465.7)^2 + 233.2^2}$$

$$= \sqrt{271,258.73} \text{ or about } 520.8$$

Find the resultant direction angle θ.

$$\tan\theta = \frac{233.2}{-465.7}$$

$$\theta = \tan^{-1}\frac{233.2}{-465.7}$$

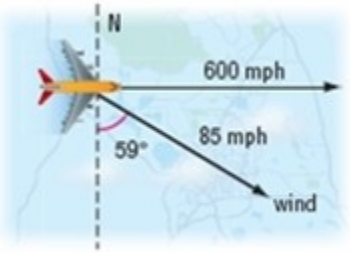
$$\theta \approx -26.6$$

Since $\mathbf{v}$ lies in Quadrant II, $\theta = 180 + (-26.6)$ or 153.4°. Therefore, the resultant force on the sled is about 520.8 newtons at an angle of about 153.4° with the horizontal.

ANSWER:

about 520.8 N; about 153.4°

53. **NAVIGATION** An airplane is traveling due east with a speed of 600 miles per hour. The wind blows at 85 miles per hour at an angle of S59°E.



a. Determine the speed of the airplane's flight.
b. Determine the angle of the airplane's flight.

SOLUTION:

a. Since the airplane is traveling due east with a speed of 600 miles per hour, the component form of the speed $\mathbf{v_1}$ is $\langle 600, 0 \rangle$. Use the magnitude and the direction of the wind $\mathbf{v_2}$ to write this vector in component form. Let $\theta = -31°$ since θ is the

direction angle that $\mathbf{v_2}$ makes with the positive *x*-axis.

$$\begin{aligned}\mathbf{v}_2 &= \langle |\mathbf{v}_2|\cos\theta, |\mathbf{v}_2|\sin\theta\rangle \\ &= \langle 85\cos(-31^\circ), 85\sin(-31^\circ)\rangle \\ &= \langle 72.9, -43.8\rangle\end{aligned}$$

Add the algebraic vectors representing $\mathbf{v_1}$ and $\mathbf{v_2}$ to find the resultant velocity, vector $\mathbf{r}$.

$$\begin{aligned}\mathbf{r} &= \mathbf{v}_1 + \mathbf{v}_2 \\ &= \langle 600, 0\rangle + \langle 72.9, -43.8\rangle \\ &= \langle 672.9, -43.8\rangle\end{aligned}$$

Find the magnitude of the resultant.

$$\begin{aligned}|\mathbf{r}| &= \sqrt{672.9^2 + (-43.8)^2} \\ &= \sqrt{454,712.85} \text{ or about } 674.3\end{aligned}$$

The speed of the airplane's flight is about 674.3 miles per hour.

b. Find the resultant direction angle θ.

$$\begin{aligned}\tan\theta &= \frac{-43.8}{672.9} \\ \theta &= \tan^{-1}\frac{-43.8}{672.9} \\ \theta &\approx -3.7^\circ\end{aligned}$$

Since $\mathbf{r}$ lies in Quadrant IV, $\theta = 360 + (-3.7)$ or 356.3°. Therefore, the angle of the airplane's flight is about S86°E.

ANSWER:

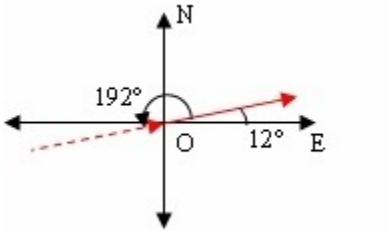
a. about 674.3 mph
b. about S86°E

54. **HEADING** A pilot needs to plot a course that will result in a velocity of 500 miles per hour in a direction of due west. If the wind is blowing 100 miles per hour from the directed angle of 192°, find the direction and the speed the pilot should set to achieve this resultant.

SOLUTION:

The pilot's final heading of 500 miles per hour in a direction of due west can be represented by the vector $\mathbf{r} = \langle -500, 0\rangle$. The final heading is the sum of the vector representing the initial course $\mathbf{v_1}$ plotted by the pilot and the vector representing the wind $\mathbf{v_2}$, or $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$.

Since the wind is blowing from the directed angle of 192°, it is blowing at an angle of 12° with the horizontal.



Use the magnitude and direction of the wind to write $\mathbf{v_2}$ in component form.

$$\begin{aligned}\mathbf{v}_2 &= \langle |\mathbf{v}_2|\cos\theta, |\mathbf{v}_2|\sin\theta\rangle \\ &= \langle 100\cos 12^\circ, 100\sin 12^\circ\rangle \\ &= \langle 97.81, 20.79\rangle\end{aligned}$$

Substitute the component forms of $\mathbf{r}$ and $\mathbf{v_2}$ into $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$ and solve for $\mathbf{v_1}$.

$$\begin{aligned}\mathbf{r} &= \mathbf{v}_1 + \mathbf{v}_2 \\ \langle -500, 0\rangle &= \mathbf{v}_1 + \langle 97.81, 20.79\rangle \\ \langle -500, 0\rangle - \langle 97.81, 20.79\rangle &= \mathbf{v}_1 \\ \langle -597.81, -20.79\rangle &= \mathbf{v}_1\end{aligned}$$

The component form of the initial course is $\mathbf{v}_1 = \langle -597.81, -20.79\rangle$.

Find the magnitude of $\mathbf{v_1}$.

$$\begin{aligned}|\mathbf{v}_1| &= \sqrt{(-597.81)^2 + (-20.79)^2} \\ &= \sqrt{357,797.06} \\ &\approx 598.2\end{aligned}$$

Find the resultant direction angle θ.

$$\begin{aligned}\tan\theta &= \frac{-20.79}{-597.81} \\ \theta &= \tan^{-1}\frac{20.79}{597.81} \\ \theta &\approx 2.0\end{aligned}$$

Since $\mathbf{v_1}$ lies in Quadrant III, $\theta = 180 + 2.0$ or about 182°. Thus, the pilot needs to plot a course of about 598.2 miles per hour at a directed angle of 182°.

ANSWER:

about 598.2 mph at a directed angle of 182°

Determine whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ with the initial and terminal points given are equivalent. If so, prove that $\overrightarrow{AB} = \overrightarrow{CD}$. If not, explain why not.

55. $A(3, 5)$, $B(6, 9)$, $C(–4, –4)$, $D(–2, 0)$

SOLUTION:

Find the magnitude and direction of each vector.

For $\overrightarrow{AB}$, find the component form.

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle 6 - 3, 9 - 5 \rangle$$
$$= \langle 3, 4 \rangle$$

Use the component form of the vector to find the magnitude. Substitute $x_2 - x_1 = 3$ and $y_2 - y_1 = 4$ into the formula for the magnitude of a vector in the coordinate plane.

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= 5$$

Next, find the direction angle of $\overrightarrow{AB}$.

$$\tan\theta = \frac{b}{a}$$
$$\tan\theta = \frac{4}{3}$$
$$\theta = \tan^{-1}\frac{4}{3}$$
$$\theta \approx 53.1^\circ$$

For $\overrightarrow{CD}$, find the component form.

$$\overrightarrow{CD} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle -2 - (-4), 0 - (-4) \rangle$$
$$= \langle 2, 4 \rangle$$

Use the component form of the vector to find the magnitude. Substitute $x_2 - x_1 = 2$ and $y_2 - y_1 = 4$ into the formula for the magnitude of a vector in the coordinate plane.

$$|\overrightarrow{CD}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{2^2 + 4^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$
$$\approx 4.5$$

Next, find the direction angle of $\overrightarrow{CD}$.

$$\tan\theta = \frac{b}{a}$$
$$\tan\theta = \frac{4}{2}$$
$$\theta = \tan^{-1} 2$$
$$\theta \approx 63.4^\circ$$

No; $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are not equivalent. The magnitude and direction are not the same for both vectors, so they are not equivalent.

ANSWER:

No; sample answer: The magnitude and direction are not the same for both vectors, so they are not equivalent.

56. $A(–4, –5)$, $B(–8, 1)$, $C(3, –3)$, $D(1, 0)$

SOLUTION:

Find the magnitude and direction of each vector.

For $\overrightarrow{AB}$, find the component form.

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle -8 - (-4), 1 - (-5) \rangle$$
$$= \langle -4, 6 \rangle$$

Use the component form of the vector to find the magnitude. Substitute $x_2 - x_1 = -4$ and $y_2 - y_1 = 6$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4)^2 + 6^2} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \\ &\approx 7.2\end{aligned}$$

Next, find the direction angle of $\overrightarrow{AB}$.

$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{6}{-4}$$

$$\theta = \tan^{-1}\left(-\frac{3}{2}\right)$$

$$\theta \approx -56.3^\circ$$

For $\overrightarrow{CD}$, find the component form.

$$\begin{aligned}\overrightarrow{CD} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1 - 3, 0 - (-3) \rangle \\ &= \langle -2, 3 \rangle\end{aligned}$$

Use the component form of the vector to find the magnitude. Substitute $x_2 - x_1 = -2$ and $y_2 - y_1 = 3$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{CD}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2)^2 + 3^2} \\ &= \sqrt{13} \\ &\approx 3.6\end{aligned}$$

Next, find the direction angle of $\overrightarrow{CD}$.

$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{3}{-2}$$

$$\theta = \tan^{-1}\left(-\frac{3}{2}\right)$$

$$\theta \approx -56.3^\circ$$

No; $\overrightarrow{AB}$ nd $\overrightarrow{CD}$ are not equivalent: The direction of the two vectors is the same, but the magnitude is different, so they are not equivalent.

ANSWER:

No; sample answer: The direction of the two vectors is the same, but the magnitude is different, so they are not equivalent.

57. $A(1, -3)$, $B(0, -10)$, $C(11, 8)$, $D(10, 1)$

SOLUTION:

Find the magnitude and direction of each vector.

For $\overrightarrow{AB}$, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 0 - 1, -10 - (-3) \rangle \\ &= \langle -1, -7 \rangle\end{aligned}$$

Use the component form of the vector to find the magnitude. Substitute $x_2 - x_1 = -1$ and $y_2 - y_1 = -7$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1)^2 + (-7)^2} \\ &= \sqrt{50} \\ &= 2\sqrt{5}\end{aligned}$$

Next, find the direction angle of $\overrightarrow{AB}$.

$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{-7}{-1}$$

$$\theta = \tan^{-1} 7$$

$$\theta \approx 81.9^\circ$$

For $\overrightarrow{CD}$, find the component form.

$$\begin{aligned}\overrightarrow{CD} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 10 - 11, 1 - 8 \rangle \\ &= \langle -1, -7 \rangle\end{aligned}$$

Use the component form of the vector to find the magnitude. Substitute $x_2 - x_1 = -1$ and $y_2 - y_1 = -7$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{CD}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1)^2 + (-7)^2} \\ &= \sqrt{50} \\ &= 2\sqrt{5}\end{aligned}$$

Next, find the direction angle of $\overrightarrow{CD}$.

$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{-7}{-1}$$

$$\theta = \tan^{-1} 7$$

$$\theta \approx 81.9^\circ$$

Yes; $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent. The magnitude and direction are the same for both vectors, so they are equivalent.

ANSWER:

Yes; sample answer: The magnitude and direction are the same for both vectors, so they are equivalent.

58. **RAFTING** The Soto family is rafting across a river. Suppose that they are on a stretch of the river that is 150 meters wide, flowing south at a rate of 1.0 meter per second. In still water, their raft travels 5.0 meters per second.

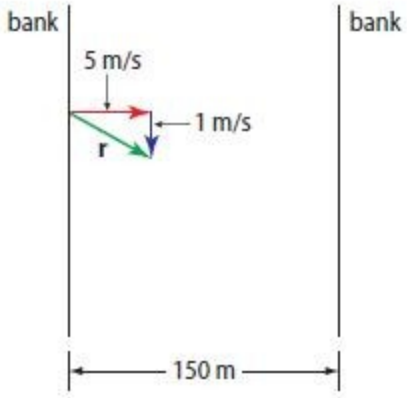
a. What is the speed of the raft?

b. How far downriver will the raft land?

c. How long does it take them to travel from one bank to the other if they head directly across the river?

SOLUTION:

a. Diagram the situation.



The speed of the raft is represented by the magnitude of the resultant vector **r**. The resultant vector **r** is the sum of the vector representing the path of the raft $\mathbf{v_1}$ and the vector representing the current of the river $\mathbf{v_2}$, or $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$. Writing in component form, $\mathbf{v_1} = \langle 5, 0 \rangle$ and $\mathbf{v_2} = \langle -1, 0 \rangle$. Substitute these vectors into $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$ and solve for **r**.

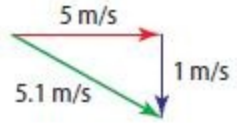
$$\begin{aligned}\mathbf{r} &= \mathbf{v_1} + \mathbf{v_2} \\ &= \langle 5, 0 \rangle + \langle 0, -1 \rangle \\ &= \langle 5, -1 \rangle\end{aligned}$$

Find the magnitude of **r**.

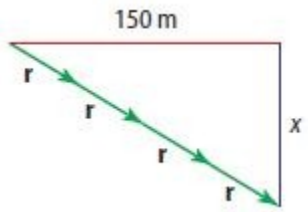
$$\begin{aligned}|r| &= \sqrt{5^2 + (-1)^2} \\ &= \sqrt{26} \\ &\approx 5.1\end{aligned}$$

So, the speed of the raft is about 5.1 meters per second.

b. The vectors representing the resultant, the path of the raft, and the current of the river form a right triangle.



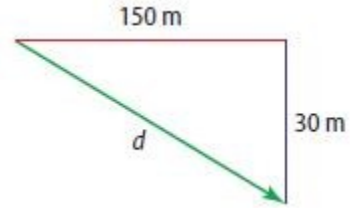
Assuming the raft continues on the path represented by **r**, a similar triangle can be formed using **r** and the width of the river.



Drawing is not to scale.

Using properties of similar triangles, $\frac{5}{150} = \frac{1}{x}$ or $x = 30$. So, the raft will land 30 meters downriver.

c. First, find the total distance d traveled by the raft using the answer from part **b** and the Pythagorean Theorem.



$$d^2 = 150^2 + 30^2$$
$$d = \sqrt{150^2 + 30^2}$$
$$d = \sqrt{23,400}$$
$$d \approx 153$$

The total distance traveled by the raft is about 153 meters. If the raft traveled at a speed of 5.1 meters per second, the time that it took the family to cross the river is $\frac{153}{5.1} = 30$ seconds.

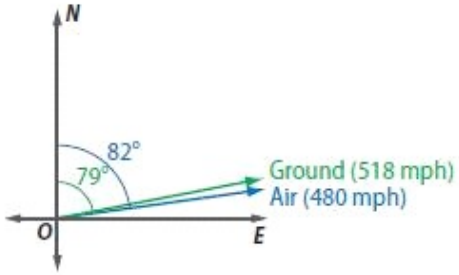
ANSWER:

a. about 5.1 m/s
b. 30 m
c. 30 s

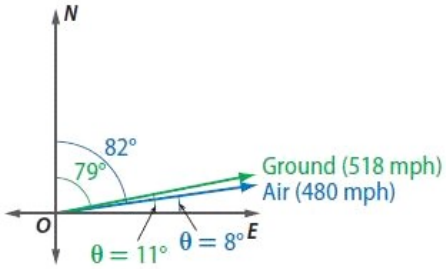
59. **NAVIGATION** A jet is flying with an air speed of 480 miles per hour at a bearing of N82°E. Because of the wind, the ground speed of the plane is 518 miles per hour at a bearing of N79°E.
a. Draw a diagram to represent the situation.
b. What are the speed and direction of the wind?
c. If the pilot increased the air speed of the plane to 500 miles per hour, what would be the resulting ground speed and direction of the plane?

SOLUTION:

a. Sample answer: Draw a diagram to represent the situation. If the initial bearing of the jet is N82°E but is flying at a bearing of N79°E because of the wind, the angle created by the two vectors is 3°.



b. The resultant vector **r** for the ground speed of the jet is the sum of the vector representing the air speed of the jet $\mathbf{v_1}$ and the vector representing the wind $\mathbf{v_2}$, or $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$.



Use the air speed and bearing of the jet to write $\mathbf{v_1}$ in component form. Let $\theta = 8°$ since the bearing is N82°E.

$$\mathbf{v_1} = \langle |\mathbf{v_1}|\cos\theta, |\mathbf{v_1}|\sin\theta \rangle$$
$$= \langle 480\cos 8^\circ, 480\sin 8^\circ \rangle$$
$$= \langle 475.3, 66.8 \rangle$$

Use the ground speed and bearing of the jet to write **r** in component form. Let $\theta = 11°$ since the bearing is N79°E.

$$\mathbf{r} = \langle |\mathbf{r}|\cos\theta, |\mathbf{r}|\sin\theta \rangle$$
$$= \langle 518\cos 11^\circ, 518\sin 11^\circ \rangle$$
$$= \langle 508.5, 98.8 \rangle$$

Substitute the component forms of **r** and $\mathbf{v_1}$ into $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$ and solve for $\mathbf{v_2}$.

$$r = v_1 + v_2$$
$$\langle 508.5, 98.8\rangle = \langle 475.3, 66.8\rangle + v_2$$
$$\langle 508.5, 98.8\rangle - \langle 475.3, 66.8\rangle = v_2$$
$$\langle 33.2, 32\rangle = v_2$$

The component form of the wind is $\mathbf{v_2} = \langle 33.2, 32\rangle$.

Find the magnitude of $\mathbf{v_2}$.

$$|\mathbf{v_2}| = \sqrt{33.2^2 + 32^2}$$
$$= \sqrt{2126.24} \text{ or about } 46.1$$

Find the resultant direction angle θ.

$$\tan\theta = \frac{32}{33.2}$$
$$\theta = \tan^{-1}\frac{32}{33.2}$$
$$\theta \approx 43.9$$

The directed angle of the wind is about 44°. Thus, the wind is blowing about 46.1 miles per hour at a bearing of about N46°E.

c. Use the air speed and bearing of the jet to write $\mathbf{v_1}$ in component form. Let θ = 8° since the bearing is N82°E.

$$\mathbf{v_1} = \langle |\mathbf{v_1}|\cos\theta, |\mathbf{v_1}|\sin\theta\rangle$$
$$= \langle 500\cos 8^\circ, 500\sin 8^\circ\rangle$$
$$= \langle 495.1, 69.6\rangle$$

Substitute the component forms of $\mathbf{v_1}$ and $\mathbf{v_2}$ into $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$ and solve for $\mathbf{r}$.

$$\mathbf{r} = \mathbf{v_1} + \mathbf{v_2}$$
$$= \langle 495.1, 69.6\rangle + \langle 33.2, 32\rangle$$
$$= \langle 528.3, 101.6\rangle$$

Find the magnitude of $\mathbf{r}$.

$$|\mathbf{r}| = \sqrt{528.3^2 + 101.6^2}$$
$$= \sqrt{289{,}423.45} \text{ or about } 538.0$$

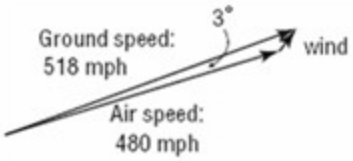
Find the resultant direction angle θ.

$$\tan\theta = \frac{101.6}{528.3}$$
$$\theta = \tan^{-1}\frac{101.6}{528.3}$$
$$\theta \approx 10.9$$

The directed angle of the jet is about 11°. Thus, the jet is traveling at a resulting ground speed of about 538 miles per hour at a directed angle of about N79° E.

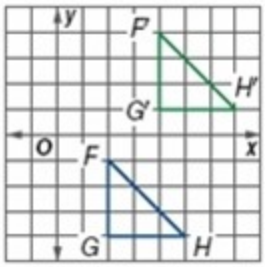
ANSWER:

a. Sample answer:



b. about 46.1 mph; about N46°E
c. about 538 mph; about N79°E

60. **TRANSLATIONS** You can translate a figure along a translation vector $\langle a, b\rangle$ by adding a to each x-coordinate and b to each y-coordinate. Consider the triangles shown below.



a. Describe the translation from ΔFGH to $\Delta F'G'H'$ using a translation vector.
b. Graph $\Delta F'G'H'$ and its translated image $\Delta F''G''H''$ along $\langle -3, -6\rangle$.
c. Describe the translation from ΔFGH to $\Delta F''G''H''$ using a translation vector.

SOLUTION:

a. Find the coordinates of F and F'. $F(2, -1)$ and $F'(4, 4)$. Since a translation vector $\langle a, b\rangle$ is added to each vertex to obtain its image, write F' as the sum of F and the translation vector. Then solve for a and b.

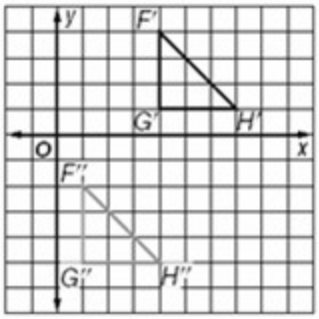
$$F + \langle a,b \rangle = F'$$
$$(2,-1) + \langle a,b \rangle = (4,4)$$
$$(2+a,-1+b) = (4,4)$$

So, 2 + a = 4 and −1 + b = 4. Thus, a = 2 and b = 5. The translation vector is $\langle 2,5 \rangle$.

b. Find the coordinates of F', G', and H'. $F'(4, 4)$, $G'(4, 1)$, and $H'(7, 1)$. Add the translation vector to each coordinate.

$$F' + \langle a,b \rangle = F''$$
$$(4,4) + \langle -3,-6 \rangle =$$
$$[4+(-3),4+(-6)] =$$
$$(1,-2) =$$
$$G' + \langle a,b \rangle = G''$$
$$(4,1) + \langle -3,-6 \rangle =$$
$$[4+(-3),1+(-6)] =$$
$$(1,-5) =$$
$$H' + \langle a,b \rangle = H''$$
$$(7,1) + \langle -3,-6 \rangle =$$
$$[7+(-3),1+(-6)] =$$
$$(4,-5) =$$

Graph triangle $\triangle F'G'H'$ and $\triangle F''G''H''$.



c. Find the coordinates F and F''. $F(2, -1)$ and $F''(1, -2)$. Since a translation vector $\langle a,b \rangle$ is added to each vertex to obtain its image, write F'' as the sum of F and the translation vector. Then solve for a and b.

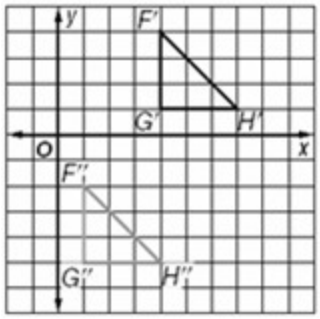
$$F + \langle a,b \rangle = F''$$
$$(2,-1) + \langle a,b \rangle = (1,-2)$$
$$(2+a,-1+b) = (1,-2)$$

So, 2 + a = 1 and −1 + b = −2. Thus, a = −1 and b = −1. The translation vector is $\langle -1,-1 \rangle$.

ANSWER:

a. $\langle 2, 5 \rangle$

b.



c. $\langle -1, -1 \rangle$

Given the initial point and magnitude of each vector, determine a possible terminal point of the vector.

61. (–1, 4); $\sqrt{37}$

SOLUTION:

Sample answer: Substitute (−1, 4) and $\sqrt{37}$ into the formula for the magnitude of a vector and solve for (x_2, y_2).

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\sqrt{37} = \sqrt{[x_2 - (-1)]^2 + (y_2 - 4)^2}$$
$$\sqrt{37} = \sqrt{(x_2 + 1)^2 + (y_2 - 4)^2}$$
$$37 = (x_2 + 1)^2 + (y_2 - 4)^2$$

Let x = 0.

$$37 = (0+1)^2 + (y_2 - 4)^2$$
$$37 = 1 + (y_2 - 4)^2$$
$$36 = (y_2 - 4)^2$$
$$\pm 6 = y_2 - 4$$

Solving $-6 = y_2 - 4$, $y_2 = -2$. So, a possible terminal point of the vector is (0, −2).

ANSWER:

Sample answer: (0, –2)

62. (–3, –7); 10

SOLUTION:

Sample answer: Substitute (−3, −7) and 10 into the formula for the magnitude of a vector and solve for (x_2, y_2).

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{[x_2 - (-3)]^2 + [y_2 - (-7)]^2}$$

$$10 = \sqrt{(x_2 + 3)^2 + (y_2 + 7)^2}$$

$$100 = (x_2 + 3)^2 + (y_2 + 7)^2$$

Let $x = 5$.

$$100 = (5 + 3)^2 + (y_2 + 7)^2$$

$$100 = 64 + (y_2 + 7)^2$$

$$36 = (y_2 + 7)^2$$

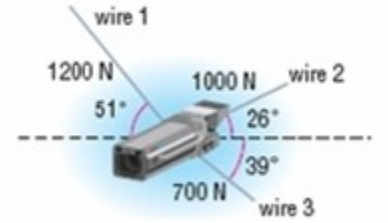
$$\pm 6 = y_2 + 7$$

Solving $6 = y_2 + 7$, $y_2 = -1$. So, a possible terminal point of the vector is (5, −1).

ANSWER:

Sample answer: (5, –1)

63. **CAMERA** A video camera that follows the action at a sporting event is supported by three wires. The tension in each wire can be modeled by a vector.



a. Find the component form of each vector.

b. Find the component form of the resultant vector acting on the camera.

c. Find the magnitude and direction of the resulting force.

SOLUTION:

a. Use the tension and directed angle of each wire to write the vectors representing each of the wires in component form. For wire 1, the directed angle is 180 − 51 or 129°, and for wire 3, the directed angle is 360 −39 or 321°.

wire 1:

$$\langle |\mathbf{v}_1|\cos\theta, |\mathbf{v}_1|\sin\theta \rangle = \langle 1200\cos 129^\circ, 1200\sin 129^\circ \rangle$$
$$= \langle -755.2, 932.6 \rangle$$

wire 2:

$$\langle |\mathbf{v}_2|\cos\theta, |\mathbf{v}_2|\sin\theta \rangle = \langle 1000\cos 26^\circ, 1000\sin 26^\circ \rangle$$
$$= \langle 898.8, 438.4 \rangle$$

wire 3:

$$\langle |\mathbf{v}_3|\cos\theta, |\mathbf{v}_3|\sin\theta \rangle = \langle 700\cos 321^\circ, 700\sin 321^\circ \rangle$$
$$= \langle 544.0, -440.5 \rangle$$

b. The resultant vector **r** is the sum of wire 1 $\mathbf{v_1}$, wire 2 $\mathbf{v_2}$, and wire 3 $\mathbf{v_3}$. Substitute the values found in part **a** into $\mathbf{r} = \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}$ and solve for **r**.

$$\mathbf{r} = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$$
$$= \langle -755.2, 932.6 \rangle + \langle 898.8, 438.4 \rangle + \langle 544.0, -440.5 \rangle$$
$$= \langle 687.6, 930.5 \rangle$$

c. Find the magnitude of **r**.

$$|r| = \sqrt{687.6^2 + 930.5^2}$$
$$= \sqrt{1,338,624.01}$$
$$\approx 1157.0$$

Find the resultant direction angle θ.

$$\tan\theta = \frac{930.5}{687.6}$$

$$\theta = \tan^{-1}\frac{930.5}{687.6}$$

$$\theta \approx 53.5$$

The resultant force is about 1157 newtons at a directed angle of about 53.5°.

ANSWER:

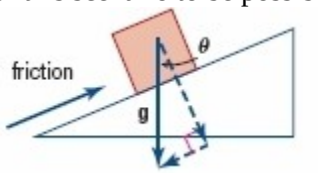
a. wire 1: $\langle -755.2, 932.6 \rangle$; wire 2: $\langle 898.8, 438.4 \rangle$; wire 3: $\langle 544.0, -440.5 \rangle$

b. $\langle 687.6, 930.5 \rangle$

c. about 1157 N; about 53.5°

8-2 Vectors in the Coordinate Plane

64. **FORCE** A box is stationary on a ramp. Both gravity **g** and friction are being exerted on the box. The components of gravity are shown in the diagram. What must be true of the force of friction for this scenario to be possible?



SOLUTION:

The friction force cannot be less than the component of gravity that is parallel to the ramp. If the friction force is less than the component of gravity that is parallel to the ramp, the box would slide down the ramp.

ANSWER:

The friction force cannot be less than the component of gravity that is parallel to the ramp.

65. **REASONING** If vectors **a** and **b** are parallel, write a vector equation relating **a** and **b**.

SOLUTION:

Sample answer: Let $\mathbf{a} = \langle x_1, y_1 \rangle$ and $\mathbf{b} = \langle x_2, y_2 \rangle$.

Find the directed angle for each vector.

For **a**: $\tan\theta = \frac{y_1}{x_1}$, $\theta = \tan^{-1}\frac{y_1}{x_1}$

For **b**: $\tan\theta = \frac{y_2}{x_2}$, $\theta = \tan^{-1}\frac{y_2}{x_2}$

If **a** and **b** are parallel, then their directed angles must be equal. Thus,

$$\tan^{-1}\frac{y_1}{x_1} = \tan^{-1}\frac{y_2}{x_2}$$

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}.$$

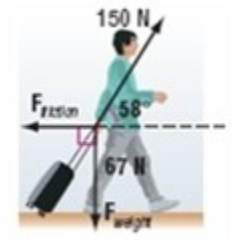
If $\frac{y_1}{x_1} = \frac{y_2}{x_2}$, then $\frac{y_1}{x_1} = \frac{ky_2}{kx_2}$ for any real number k.

Thus, **a** = k**b**, where k is any real number scalar.

ANSWER:

Sample answer: **a** = k**b**, where k is any real number scalar.

66. **CHALLENGE** To pull luggage, Greg exerts a force of 150 newtons at an angle of 58° with the horizontal. If the resultant force on the luggage is 72 newtons at an angle of 56.7° with the horizontal, what is the magnitude of the resultant of $\mathbf{F}_{friction}$ and $\mathbf{F}_{weight}$?



SOLUTION:

To find the magnitude of the resultant of $\mathbf{F}_{friction}$ and $\mathbf{F}_{weight}$, find $\mathbf{F}_{friction}$. The resultant force on the luggage **r** is the sum of the vector representing the force exerted by Greg **v** and the vectors representing the friction $\mathbf{F}_{friction}$ and the weight of the luggage $\mathbf{F}_{weight}$, or $\mathbf{r} = \mathbf{v} + \mathbf{F}_{friction} + \mathbf{F}_{weight}$.

Use the magnitude and direction of the resultant force on the luggage to write **r** in component form.

$$\begin{aligned}\mathbf{r} &= \langle |\mathbf{r}|\cos\theta, |\mathbf{r}|\sin\theta \rangle \\ &= \langle 72\cos 56.7^\circ, 72\sin 56.7^\circ \rangle \\ &= \langle 39.5, 60.2 \rangle\end{aligned}$$

Use the magnitude and direction of the force exerted by Greg to write **v** in component form.

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 150\cos 58^\circ, 150\sin 58^\circ \rangle \\ &= \langle 79.5, 127.2 \rangle\end{aligned}$$

From the diagram, $\mathbf{F}_{weight}$ can be written as $\mathbf{F}_{weight} = \langle 0, -67 \rangle$.

Substitute the component forms of **r**, **v**, and $\mathbf{F}_{weight}$ into $\mathbf{r} = \mathbf{v} + \mathbf{F}_{friction} + \mathbf{F}_{weight}$ and solve for $\mathbf{F}_{friction}$.

$$\begin{aligned}\mathbf{r} &= \mathbf{v} + \mathbf{F}_{friction} + \mathbf{F}_{weight} \\ \langle 39.5, 60.2 \rangle &= \langle 79.5, 127.2 \rangle + \mathbf{F}_{friction} + \langle 0, -67 \rangle \\ \langle 39.5, 60.2 \rangle &= \mathbf{F}_{friction} + \langle 79.5, 60.2 \rangle \\ \langle 39.5, 60.2 \rangle - \langle 79.5, 60.2 \rangle &= \mathbf{F}_{friction} \\ \langle -40, 0 \rangle &= \mathbf{F}_{friction}\end{aligned}$$

The resultant of $\mathbf{F}_{friction}$ and $\mathbf{F}_{weight}$ is $\langle -40, 0 \rangle + \langle 0, -67 \rangle = \langle -40, -67 \rangle$.

Find the magnitude of $\langle -40, -67 \rangle$.

$$\sqrt{a^2+b^2} = \sqrt{(-40)^2+(-67)^2}$$
$$= \sqrt{6089} \text{ or about } 78.0$$

The magnitude of the resultant of $\mathbf{F}_{friction}$ and $\mathbf{F}_{weight}$ is about 78 newtons.

ANSWER:

about 78 N

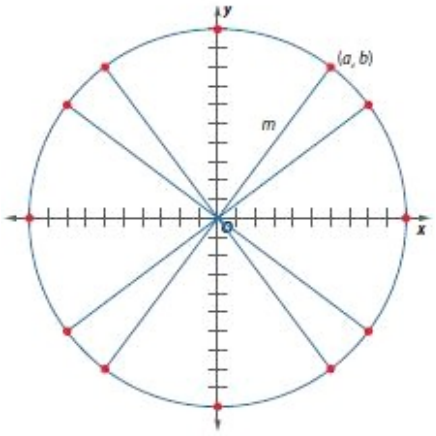
67. **REASONING** If given the initial point of a vector and its magnitude, describe the locus of points that represent possible locations for the terminal point.

SOLUTION:

Sample answer: If the initial point of a vector is (a, b) and the vector has magnitude m, then the locus of terminal points (x, y) are the points that satisfy the equation $\sqrt{(x-a)^2+(y-b)^2} = m$. The locus of terminal points (x, y) are the points that lie on the circle that has a center at (a, b) and a radius of m.

Consider a vector with an initial point at the origin and a magnitude of m.

The circle below is formed by the terminal points of many vectors with this initial point and magnitude.



ANSWER:

Sample answer: If the initial point of a vector is (a, b) and the vector has magnitude m, then the locus of terminal points (x, y) are the points that satisfy the equation $\sqrt{(x-a)^2+(y-b)^2} = m$.

68. ***Writing in Math*** Explain how to find the direction angle of a vector in the fourth quadrant.

SOLUTION:

Sample answer: To find the direction angle of a vector in the fourth quadrant, use the vertical and horizontal components of the vector and the trigonometric equation $\tan\theta = \frac{b}{a}$ to determine the angle that the vector makes with the positive x-axis. Since the horizontal component will be positive and the vertical component will be negative in the fourth quadrant, using the trigonometric equation will produce a negative value for θ. Add this value to 360° to find the direction angle of the vector in the fourth quadrant.

ANSWER:

Sample answer: To find the direction angle of a vector in the fourth quadrant, use the vertical and horizontal components of the vector and the trigonometric equation $\tan\theta = \frac{b}{a}$ to determine the angle that the vector makes with the positive x-axis. Then add this value to 360° to find the direction angle of the vector in the fourth quadrant.

69. **CHALLENGE** The direction angle of $\langle x, y \rangle$ is $(4y)°$. Find x in terms of y.

SOLUTION:

$$\tan\theta = \frac{b}{a}$$
$$\tan 4y = \frac{y}{x}$$
$$x\tan 4y = y$$
$$x = \frac{y}{\tan 4y}$$

ANSWER:

$$x = \frac{y}{\tan 4y}$$

PROOF Prove each vector property. Let $\mathbf{a} = \langle x_1, y_1 \rangle$, $\mathbf{b} = \langle x_2, y_2 \rangle$, and $\mathbf{c} = \langle x_3, y_3 \rangle$.

70. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

SOLUTION:

$\mathbf{a}+\mathbf{b} = \langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle$	Substitute.
$= \langle x_1 + x_2, y_1 + y_2 \rangle$	Vector addition
$= \langle x_2 + x_1, y_2 + y_1 \rangle$	Commutative Property
$= \langle x_2, y_2 \rangle + \langle x_1, y_1 \rangle$	Vector addition
$= \mathbf{b} + \mathbf{a}$	Substitute.

ANSWER:

$$\begin{aligned}\mathbf{a}+\mathbf{b} &= \langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle \\ &= \langle x_1 + x_2, y_1 + y_2 \rangle \\ &= \langle x_2 + x_1, y_2 + y_1 \rangle \\ &= \langle x_2, y_2 \rangle + \langle x_1, y_1 \rangle \\ &= \mathbf{b} + \mathbf{a}\end{aligned}$$

71. $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$

SOLUTION:

$(\mathbf{a}+\mathbf{b})+\mathbf{c} = (\langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle) + \langle x_3, y_3 \rangle$	Substitute.
$= \langle x_1 + x_2, y_1 + y_2 \rangle + \langle x_3, y_3 \rangle$	Vector addition
$= \langle x_1 + x_2 + x_3, y_1 + y_2 + y_3 \rangle$	Vector addition
$= \langle x_1, y_1 \rangle + \langle x_2 + x_3, y_2 + y_3 \rangle$	Remove $\langle x_1, y_1 \rangle$.
$= \langle x_1, y_1 \rangle + (\langle x_2, y_2 \rangle + \langle x_3, y_3 \rangle)$	Vector addition
$= \mathbf{a} + (\mathbf{b} + \mathbf{c})$	Substitute.

ANSWER:

$$\begin{aligned}(\mathbf{a}+\mathbf{b})+\mathbf{c} &= (\langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle) + \langle x_3, y_3 \rangle \\ &= \langle x_1 + x_2, y_1 + y_2 \rangle + \langle x_3, y_3 \rangle \\ &= \langle x_1 + x_2 + x_3, y_1 + y_2 + y_3 \rangle \\ &= \langle x_1, y_1 \rangle + \langle x_2 + x_3, y_2 + y_3 \rangle \\ &= \langle x_1, y_1 \rangle + (\langle x_2, y_2 \rangle + \langle x_3, y_3 \rangle) \\ &= \mathbf{a} + (\mathbf{b} + \mathbf{c})\end{aligned}$$

72. $k(\mathbf{a} + \mathbf{b}) = k\mathbf{a} + k\mathbf{b}$, where k is a scalar

SOLUTION:

$k(\mathbf{a}+\mathbf{b}) = k(\langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle)$	Substitute.
$= k\langle x_1 + x_2, y_1 + y_2 \rangle$	Vector addition
$= \langle k(x_1 + x_2), k(y_1 + y_2) \rangle$	Distribute.
$= \langle kx_1 + kx_2, ky_1 + ky_2 \rangle$	Distribute.
$= \langle kx_1, ky_1 \rangle + \langle kx_2, ky_2 \rangle$	Vector addition
$= k\langle x_1, y_1 \rangle + k\langle x_2, y_2 \rangle$	Factor k.
$= k\mathbf{a} + k\mathbf{b}$	Substitute.

ANSWER:

$$\begin{aligned}k(\mathbf{a}+\mathbf{b}) &= k(\langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle) \\ &= k\langle x_1 + x_2, y_1 + y_2 \rangle \\ &= \langle k(x_1 + x_2), k(y_1 + y_2) \rangle \\ &= \langle kx_1 + kx_2, ky_1 + ky_2 \rangle \\ &= \langle kx_1, ky_1 \rangle + \langle kx_2, ky_2 \rangle \\ &= k\langle x_1, y_1 \rangle + k\langle x_2, y_2 \rangle \\ &= k\mathbf{a} + k\mathbf{b}\end{aligned}$$

73. $|k\mathbf{a}| = |k|\,|\mathbf{a}|$, where k is a scalar

SOLUTION:

$\lvert k\mathbf{a}\rvert = \lvert k\langle x_1, y_1 \rangle\rvert$	Substitute.
$= \lvert\langle kx_1, ky_1 \rangle\rvert$	Distribute.
$= \sqrt{(kx_1)^2 + (ky_1)^2}$	Magnitude of a Vector
$= \sqrt{k^2x_1^2 + k^2y_1^2}$	Distribute.
$= \sqrt{k^2(x_1^2 + y_1^2)}$	Factor.
$= \sqrt{k^2}\sqrt{x_1^2 + y_1^2}$	Distribute square root.
$= \lvert k\rvert\lvert\langle x_1, y_1 \rangle\rvert$	Simplify and Definition of Magnitude
$= \lvert k\rvert\lvert a\rvert$	Substitute.

ANSWER:

$$\begin{aligned}|k\mathbf{a}| &= |k\langle x_1, y_1 \rangle| \\ &= |\langle kx_1, ky_1 \rangle| \\ &= \sqrt{(kx_1)^2 + (ky_1)^2} \\ &= \sqrt{k^2x_1^2 + k^2y_1^2} \\ &= \sqrt{k^2(x_1^2 + y_1^2)} \\ &= \sqrt{k^2}\sqrt{x_1^2 + y_1^2} \\ &= |k||\langle x_1, y_1 \rangle| \\ &= |k||\mathbf{a}|\end{aligned}$$

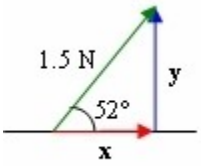
74. **TOYS** Roman is pulling a toy by exerting a force of 1.5 newtons on a string attached to the toy.

a. The string makes an angle of 52° with the floor. Find the horizontal and vertical components of the force.

b. If Roman raises the string so that it makes a 78° angle with the floor, what are the magnitudes of the horizontal and vertical components of the force?

SOLUTION:

a. Diagram the situation.



The horizontal and vertical components of the force form a right triangle. Use the sine or cosine ratios to find the magnitude of each force.

$$\cos 52^\circ = \frac{|x|}{1.5} \qquad \sin 52^\circ = \frac{|y|}{1.5}$$

$$|x| = 1.5\cos 52^\circ \qquad |y| = 1.5\sin 52^\circ$$

$$|x| \approx 0.92 \qquad |y| \approx 1.18$$

The horizontal component is about 0.92 newton and the vertical component is about 1.18 newtons.

b. Repeat the process used in part **a**, but use an angle of 78° instead of 52°.

$$\cos 78^\circ = \frac{|x|}{1.5} \qquad \sin 78^\circ = \frac{|y|}{1.5}$$

$$|x| = 1.5\cos 78^\circ \qquad |y| = 1.5\sin 78^\circ$$

$$|x| \approx 0.31 \qquad |y| \approx 1.47$$

The horizontal component is about 0.31 newton and the vertical component is about 1.47 newtons.

ANSWER:

a. ≈0.92 N, ≈1.18 N

b. ≈0.31 N, ≈1.47 N

Write each pair of parametric equations in rectangular form.

75. $y = t^2 + 2,\ x = 3t - 6$

SOLUTION:

Solve for t.

$$x = 3t - 6$$

$$x + 6 = 3t$$

$$\frac{x+6}{3} = t$$

Substitute for t.

$$y = t^2 + 2$$

$$y = \left(\frac{x+6}{3}\right)^2 + 2$$

$$y = \frac{x^2 + 12x + 36}{9} + 2$$

$$y = \frac{x^2}{9} + \frac{4x}{3} + 6$$

ANSWER:

$$y = \frac{x^2}{9} + \frac{4x}{3} + 6$$

76. $y = t^2 - 5,\ x = 2t + 8$

SOLUTION:

Solve for t.

$$x = 2t + 8$$

$$x - 8 = 2t$$

$$\frac{x-8}{2} = t$$

Substitute for t.

$$y = t^2 - 5$$

$$y = \left(\frac{x-8}{2}\right)^2 - 5$$

$$y = \frac{x^2 - 16x + 64}{4} - 5$$

$$y = \frac{x^2}{4} - 4x + 11$$

ANSWER:

$$y = \frac{x^2}{4} - 4x + 11$$

77. $y = 7t,\ x = t^2 - 1$

SOLUTION:

Solve for t.

$$x = t^2 - 1$$
$$x + 1 = t^2$$
$$\pm\sqrt{x+1} = t$$

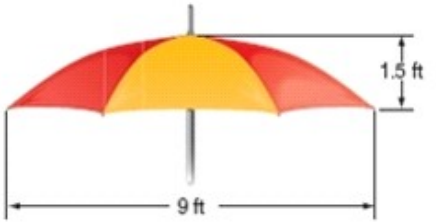
Substitute for t.

$$y = 7t$$
$$y = 7\left(\pm\sqrt{x+1}\right)$$
$$y = \pm 7\sqrt{x+1}$$

ANSWER:

$y = \pm 7\sqrt{x+1}$

78. **UMBRELLAS** A beach umbrella has an arch in the shape of a parabola. Write an equation to model the arch, assuming that the origin and the vertex are at the point where the pole and the canopy of the umbrella meet. Express y in terms of x.



SOLUTION:

The standard form for a parabola that opens vertically is $(x - h)^2 = 4p(y - k)$. If the vertex is at the origin, $h = 0$ and $k = 0$. The equation becomes $x^2 = 4py$. If the vertex of the umbrella is at the origin, then the two tips of the umbrella occur at the points $(-4.5, -1.5)$ and $(4.5, 1.5)$. Substitute one of these points into the equation and solve for p.

$$x^2 = 4py$$
$$4.5^2 = 4p(-1.5)$$
$$20.25 = -6p$$
$$-\frac{27}{8} = p$$
$$-3.375 = p$$

To write an equation to model the arch in terms of y, substitute $p = -\frac{27}{8}$ into $x^2 = 4py$ and solve for y.

$$x^2 = 4py$$
$$x^2 = 4\left(-\frac{27}{8}\right)y$$
$$x^2 = -\frac{27}{2}y$$
$$-\frac{27}{2}x^2 = y$$

ANSWER:

$$y = -\frac{2}{27}x^2$$

Decompose each expression into partial fractions.

79. $\dfrac{5z - 11}{2z^2 + z - 6}$

SOLUTION:

Rewrite the expression as partial fractions with constant numerators, *A* and *B*, and denominators that are the linear factors of the original denominator.

$$\frac{5z-11}{2z^2+z-6}=\frac{A}{z+2}+\frac{B}{2z-3}$$

Multiply each side by the LCD, $2z^2+z-6$.

$$5z-11=A(2z-3)+B(z+2)$$

Group the like terms.

$$5z-11=A(2z-3)+B(z+2)$$
$$5z-11=2Az-3A+Bz+2B$$
$$5z-11=2Az+Bz-3A+2B$$
$$5z-11=(2A+B)z-3A+2B$$

Equate the coefficients on the left and right side of the equation to form two equations with 2 variables. In other words, the coefficients of the *z*-terms on the left side of the equation must equal the coefficients of the *z*-terms on the right side.

$$2A+B=5$$
$$-3A+2B=-11$$

Use any method to solve the new system.

$$\begin{aligned}2A+B&=5\\B&=5-2A\\-3A+2B&=-11\\-3A+2(5-2A)&=-11\\-3A+10-4A&=-11\\10-7A&=-11\\-7A&=-21\\7A&=21\\A&=3\\B&=5-2(3)\\B&=-1\end{aligned}$$

Replace *A* and *B* with 3 and −1 in the partial fraction decomposition.

$$\frac{5z-11}{2z^2+z-6}=\frac{A}{z+2}+\frac{B}{2z-3}$$
$$\frac{5z-11}{2z^2+z-6}=\frac{3}{z+2}-\frac{1}{2z-3}$$

ANSWER:

$$\frac{3}{z+2}-\frac{1}{2z-3}$$

80. $$\frac{7x^2+18x-1}{(x^2-1)(x+2)}$$

SOLUTION:

Rewrite the expression as partial fractions with constant numerators, *A* and *B*, and denominators that are the linear factors of the original denominator.

$$\frac{7x^2+18x-1}{(x^2-1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x-1}$$

Multiply each side by the LCD, $(x^2-1)(x+2)$.

$$7x^2+18x-1=A(x+2)(x-1)+B(x+1)(x-1)+C(x+1)(x+2)$$

Group the like terms.

$$7x^2+18x-1=A(x^2+x-2)+B(x^2-1)$$
$$+C(x^2+3x+2)$$
$$7x^2+18x-1=Ax^2+Ax-2A+Bx^2-B$$
$$+Cx^2+3Cx+2C$$
$$7x^2+18x-1=Ax^2+Bx^2+Cx^2+Ax$$
$$+3Cx-2A-B+2C$$
$$7x^2+18x-1=(A+B+C)x^2+(A+3C)x$$
$$-2A-B+2C$$

Equate the coefficients on the left and right side of the equation to form three equations with 3 variables. In other words, the coefficients of the *x*-terms on the left side of the equation must equal the coefficients of the *x*-terms on the right side.

$$A+B+C=7$$
$$A+3C=18$$
$$-2A-B+2C=-1$$

Use any method to solve the new system.

$$\begin{aligned}A+B+C&=7\\(+)\ -2A-B+2C&=-1\\-A+3C&=6\\(+)A+3C&=18\\6C&=24\\C&=4\end{aligned}$$

$$\begin{aligned} C &= 4 \\ A + 3(4) &= 18 \\ A + 12 &= 18 \\ A &= 6 \\ 6 + B + 4 &= 7 \\ B + 10 &= 7 \\ B &= -3 \end{aligned}$$

Replace A, B, and C with 6, –3 and 4 in the partial fraction decomposition.

$$\frac{7x^2+18x-1}{(x^2-1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$\frac{7x^2+18x-1}{(x^2-1)(x+2)} = \frac{6}{x+1} - \frac{3}{x+2} + \frac{4}{x-1}$$

ANSWER:

$$\frac{6}{x+1} - \frac{3}{x+2} + \frac{4}{x-1}$$

81. $\dfrac{9-9x}{x^2-9}$

SOLUTION:

Rewrite the expression as partial fractions with constant numerators, A and B, and denominators that are the linear factors of the original denominator.

$$\frac{9-9x}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$$

Multiply each side by the LCD, $x^2 - 9$.

$$9 - 9x = A(x-3) + B(x+3)$$

Group the like terms.

$$\begin{aligned} 9 - 9x &= A(x-3) + B(x+3) \\ -9x + 9 &= Ax - 3A + Bx + 3B \\ -9x + 9 &= Ax + Bx - 3A + 3B \\ -9x + 9 &= (A+B)x - 3A + 3B \end{aligned}$$

Equate the coefficients on the left and right side of the equation to form two equations with 2 variables. In other words, the coefficients of the z-terms on the left side of the equation must equal the coefficients of the z-terms on the right side.

$$\begin{aligned} A + B &= -9 \\ -3A + 3B &= 9 \end{aligned}$$

Use any method to solve the new system.

$$\begin{aligned} A + B &= -9 \\ B &= -9 - A \\ -3A + 3B &= 9 \\ -3A + 3(-9 - A) &= 9 \\ -3A - 27 - 3A &= 9 \\ -27 - 6A &= 9 \\ -6A &= 36 \\ A &= -6 \\ B &= -9 - (-6) \\ B &= -3 \end{aligned}$$

Replace A and B with -6 and -3 in the partial fraction decomposition.

$$\frac{9-9x}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$$

$$\frac{9-9x}{x^2-9} = \frac{-6}{x+3} + \frac{-3}{x-3}$$

ANSWER:

$$\frac{-6}{x+3} + \frac{-3}{x-3}$$

Verify each identity.

82. $\sin(\theta + 180°) = -\sin\theta$

SOLUTION:

$$\begin{aligned} \sin(\theta + 180^\circ) &= \sin\theta\cos 180^\circ + \cos\theta\sin 180^\circ \\ &= \sin\theta(-1) + \cos\theta(0) \\ &= -\sin\theta \end{aligned}$$

ANSWER:

$$\begin{aligned} \sin(\theta + 180^\circ) &= \sin\theta\cos 180^\circ + \cos\theta\sin 180^\circ \\ &= \sin\theta(-1) + \cos\theta(0) \\ &= -\sin\theta \end{aligned}$$

83. $\sin(60° + \theta) + \sin(60° - \theta) = \sqrt{3}\cos\theta$

SOLUTION:

$$\begin{aligned}&\sin(60^\circ+\theta)+\sin(60^\circ-\theta)\\&=\sin60^\circ\cos\theta+\cos60^\circ\sin\theta+\sin60^\circ\cos\theta-\cos60^\circ\sin\theta\\&=\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\\&=\frac{2\sqrt{3}}{2}\cos\theta\\&=\sqrt{3}\cos\theta\end{aligned}$$

ANSWER:

$$\begin{aligned}&\sin(60^\circ+\theta)+\sin(60^\circ-\theta)\\&=\sin60^\circ\cos\theta+\cos60^\circ\sin\theta+\sin60^\circ\cos\theta-\cos60^\circ\sin\theta\\&=\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\\&=\frac{2\sqrt{3}}{2}\cos\theta\\&=\sqrt{3}\cos\theta\end{aligned}$$

Express each logarithm in terms of ln 3 and ln 7.

84. ln 189

SOLUTION:

$$\begin{aligned}\ln 189&=\ln(27\cdot 7)\\&=\ln 27+\ln 7\\&=\ln 3^3+\ln 7\\&=3\ln 3+\ln 7\end{aligned}$$

ANSWER:

3 ln 3 + ln 7

85. $\ln 5.\overline{4}$

SOLUTION:

$$\begin{aligned}\ln 5.\overline{4}&=\ln\left(5+\frac{4}{9}\right)\\&=\ln\frac{49}{9}\\&=\ln 49-\ln 9\\&=\ln 7^2-\ln 3^2\\&=2\ln 7-2\ln 3\end{aligned}$$

ANSWER:

2 ln 7 – 2 ln 3

86. ln 441

SOLUTION:

$$\begin{aligned}\ln 441&=\ln(9\cdot 49)\\&=\ln 9+\ln 49\\&=\ln 3^2+\ln 7^2\\&=2\ln 3+2\ln 7\end{aligned}$$

ANSWER:

2 ln 3 + 2 ln 7

87. $\ln\frac{9}{343}$

SOLUTION:

$$\begin{aligned}\ln\frac{9}{343}&=\ln 9-\ln 343\\&=\ln 3^2-\ln 7^3\\&=2\ln 3-3\ln 7\end{aligned}$$

ANSWER:

2 ln 3 – 3 ln 7

Find each $f(c)$ using synthetic substitution.

88. $f(x) = 6x^6 - 9x^4 + 12x^3 - 16x^2 + 8x + 24$; $c = 6$

SOLUTION:

6	6	0	–9	12	–16	8	24
		36	216	1242	7524	45048	270336
	6	36	207	1254	7508	45056	270360

The remainder is 270,360. Therefore, $f(6) = 270{,}360$.

ANSWER:

270,360

89. $f(x) = 8x^5 - 12x^4 + 18x^3 - 24x^2 + 36x - 48$, $c = 4$

SOLUTION:

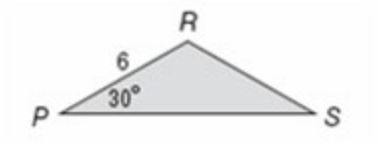
4	8	–12	18	–24	36	–48
		32	80	392	1472	6032
	8	20	98	368	1508	5984

The remainder is 5,984. Therefore, $f(4) = 5{,}984$.

ANSWER:

5984

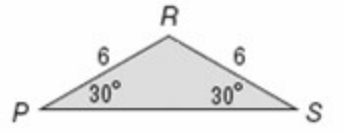
90. **SAT/ACT** If *PR* = *RS*, what is the area of triangle *PRS*?



A $9\sqrt{2}$
B $9\sqrt{3}$
C $18\sqrt{2}$
D $18\sqrt{3}$
E $36\sqrt{3}$

SOLUTION:

If *PR* = *RS*, then *RS* = 6 and $\angle S = 30°$ as shown.



Therefore, $\angle R = 120°$. Using $\angle R$, *PR*, and *RS*, the area of the triangle can now be found.

$$\text{Area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2}(6)(6)\sin 120°$$
$$= 18\left(\frac{\sqrt{3}}{2}\right)$$
$$= 9\sqrt{3}$$

The correct answer is B.

ANSWER:

B

91. **REVIEW** Dalton has made a game for his younger sister's birthday party. The playing board is a circle divided evenly into 8 sectors. If the circle has a radius of 18 inches, what is the approximate area of one of the sectors?

F 4 in^2
G 32 in^2
H 127 in^2
J 254 in^2

SOLUTION:

Find the area of the circular board by substituting $r = 18$ into the formula for the area of a circle, $A = \pi r^2$.

$$A = \pi r^2$$
$$= \pi(18)^2$$
$$= 324\pi \text{ or about } 1017.9$$

The area of one of the 8 sectors is $\frac{324\pi}{8} \approx 127.2$ square inches.
The correct answer is H.

ANSWER:

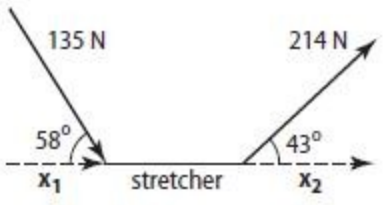
H

92. Paramedics Lydia Gonzalez and Theo Howard are moving a person on a stretcher. Ms. Gonzalez is pushing the stretcher from behind with a force of 135 newtons at 58° with the horizontal, while Mr. Howard is pulling the stretcher from the front with a force of 214 newtons at 43° with the horizontal. What is the magnitude of the horizontal force exerted on the stretcher?

A 228 N
B 260 N
C 299 N
D 346 N

SOLUTION:

Diagram the situation.



Since Ms. Gonzalez and Mr. Howard are pushing and pulling the stretcher in the same direction, the horizontal forces will both be positive. Calculate the magnitudes of both horizontal forces $\mathbf{x_1}$ and $\mathbf{x_2}$.

$$\mathbf{x}_1 = 135\cos 58^\circ \approx 71.5 \qquad \mathbf{x}_2 = 214\cos 43^\circ \approx 156.5$$

The magnitude of the horizontal force exerted on the stretcher is about 71.5 + 156.5 or 228 newtons.
The correct answer is A.

ANSWER:

A

93. **REVIEW** Find the center and radius of the circle with equation $(x - 4)^2 + y^2 - 16 = 0$.

F $C(-4, 0)$; $r = 4$ units
G $C(-4, 0)$; $r = 16$ units
H $C(4, 0)$; $r = 4$ units
J $C(4, 0)$; $r = 16$ units

SOLUTION:

Write the equation in standard form as $(x - 4)^2 + y^2 = 16$. The circle has a center at (4, 0) and a radius of 4.
The correct answer is H.

ANSWER:

H

8-3 Dot Products and Vector Projections

Find the dot product of u and v. Then determine if u and v are orthogonal.

1. $\mathbf{u} = \langle 3, -5 \rangle$, $\mathbf{v} = \langle 6, 2 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 3, -5 \rangle \cdot \langle 6, 2 \rangle \\ &= 3(6) + (-5)(2) \\ &= 8\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

8; not orthogonal

2. $\mathbf{u} = \langle -10, -16 \rangle$, $\mathbf{v} = \langle -8, 5 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle -10, -16 \rangle \cdot \langle -8, 5 \rangle \\ &= -10(-8) + (-16)(5) \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

3. $\mathbf{u} = \langle 9, -3 \rangle$, $\mathbf{v} = \langle 1, 3 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 9, -3 \rangle \cdot \langle 1, 3 \rangle \\ &= 9(1) + (-3)(3) \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

4. $\mathbf{u} = \langle 4, -4 \rangle$, $\mathbf{v} = \langle 7, 5 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 4, -4 \rangle \cdot \langle 7, 5 \rangle \\ &= 4(7) + (-4)(5) \\ &= 8\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

8; not orthogonal

5. $\mathbf{u} = \langle 1, -4 \rangle$, $\mathbf{v} = \langle 2, 8 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 1, -4 \rangle \cdot \langle 2, 8 \rangle \\ &= 1(2) + (-4)(8) \\ &= -30\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−30; not orthogonal

6. $\mathbf{u} = 11\mathbf{i} + 7\mathbf{j}$; $\mathbf{v} = -7\mathbf{i} + 11\mathbf{j}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle 11, 7 \rangle$ and $\mathbf{v} = \langle -7, 11 \rangle$.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 11, 7 \rangle \cdot \langle -7, 11 \rangle \\ &= 11(-7) + 7(11) \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

7. $\mathbf{u} = \langle -4, 6 \rangle$, $\mathbf{v} = \langle -5, -2 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle -4, 6 \rangle \cdot \langle -5, -2 \rangle \\ &= -4(-5) + 6(-2) \\ &= 8\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

8; not orthogonal

8. $\mathbf{u} = 8\mathbf{i} + 6\mathbf{j}$; $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle 8, 6 \rangle$ and $\mathbf{v} = \langle -1, 2 \rangle$.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 8, 6 \rangle \cdot \langle -1, 2 \rangle \\ &= 8(-1) + 6(2) \\ &= 4\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

4; not orthogonal

9. **SPORTING GOODS** The vector $\mathbf{u} = \langle 406, 297 \rangle$ gives the numbers of men's basketballs and women's basketballs, respectively, in stock at a sporting goods store. The vector $\mathbf{v} = \langle 27.5, 15 \rangle$ gives the prices in dollars of the two types of basketballs, respectively.
   a. Find the dot product $\mathbf{u} \cdot \mathbf{v}$.
   b. Interpret the result in the context of the problem.

SOLUTION:

a.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 406, 297 \rangle \cdot \langle 27.5, 15 \rangle \\ &= 406(27.5) + 297(15) \\ &= 15,620\end{aligned}$$

b. The product of the number of men's basketballs and the price of one men's basketball is 406 · 27.5 or \$11,165. This is the revenue that can be made by selling all of the men's basketballs. The product of the number of women's basketballs and the price of one women's basketball is 297 · 15 or \$4455. This is the revenue that can be made by selling all of the women's basketballs. The dot product represents the sum of these two numbers. The total revenue that can be made by selling all of the basketballs is \$15,620.

ANSWER:

a. 15,620
b. The total revenue that can be made by selling all of the basketballs is \$15,620.

Use the dot product to find the magnitude of the given vector.

10. $\mathbf{m} = \langle -3, 11 \rangle$

SOLUTION:

Since $|\mathbf{m}|^2 = \mathbf{m} \cdot \mathbf{m}$, then $|\mathbf{m}| = \sqrt{\mathbf{m} \cdot \mathbf{m}}$.

$$\begin{aligned}|\mathbf{m}| &= \sqrt{\mathbf{m} \cdot \mathbf{m}} \\ |\langle -3,11 \rangle| &= \sqrt{\langle -3,11 \rangle \cdot \langle -3,11 \rangle} \\ &= \sqrt{(-3)^2 + 11^2} \\ &= \sqrt{130} \text{ or about } 11.4\end{aligned}$$

ANSWER:

$\sqrt{130} \approx 11.4$

11. $\mathbf{r} = \langle -9, -4 \rangle$

SOLUTION:

Since $|\mathbf{r}|^2 = \mathbf{r} \cdot \mathbf{r}$, then $|\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$.

$$\begin{aligned}|\mathbf{r}| &= \sqrt{\mathbf{r} \cdot \mathbf{r}} \\ |\langle -9,4 \rangle| &= \sqrt{\langle -9,4 \rangle \cdot \langle -9,4 \rangle} \\ &= \sqrt{(-9)^2 + 4^2} \\ &= \sqrt{97} \text{ or about } 9.8\end{aligned}$$

ANSWER:

$\sqrt{97} \approx 9.8$

12. $\mathbf{n} = \langle 6, 12 \rangle$

SOLUTION:

Since $|\mathbf{n}|^2 = \mathbf{n} \cdot \mathbf{n}$, then $|\mathbf{n}| = \sqrt{\mathbf{n} \cdot \mathbf{n}}$.

$$\begin{aligned}|\mathbf{n}| &= \sqrt{\mathbf{n} \cdot \mathbf{n}} \\ |\langle 6,12 \rangle| &= \sqrt{\langle 6,12 \rangle \cdot \langle 6,12 \rangle} \\ &= \sqrt{6^2 + 12^2} \\ &= \sqrt{180} \\ &= 6\sqrt{5} \text{ or about } 13.4\end{aligned}$$

ANSWER:

$6\sqrt{5} \approx 13.4$

13. $\mathbf{v} = \langle 1, -18 \rangle$

SOLUTION:

Since $|\mathbf{v}|^2 = \mathbf{v} \cdot \mathbf{v}$, then $|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$.

$$\begin{aligned}|\mathbf{v}| &= \sqrt{\mathbf{v} \cdot \mathbf{v}} \\ |\langle 1,-18 \rangle| &= \sqrt{\langle 1,-18 \rangle \cdot \langle 1,-18 \rangle} \\ &= \sqrt{1^2 + (-18)^2} \\ &= \sqrt{325} \\ &= 5\sqrt{13} \text{ or about } 18.0\end{aligned}$$

ANSWER:

$5\sqrt{13} \approx 18.0$

14. $\mathbf{p} = \langle -7, -2 \rangle$

SOLUTION:

Since $|\mathbf{p}|^2 = \mathbf{p} \cdot \mathbf{p}$, then $|\mathbf{p}| = \sqrt{\mathbf{p} \cdot \mathbf{p}}$.

$$|\mathbf{p}| = \sqrt{\mathbf{p} \cdot \mathbf{p}}$$
$$|\langle -7, -2 \rangle| = \sqrt{\langle -7, -2 \rangle \cdot \langle -7, -2 \rangle}$$
$$= \sqrt{(-7)^2 + (-2)^2}$$
$$= \sqrt{53} \text{ or about } 7.3$$

ANSWER:

$\sqrt{53} \approx 7.3$

15. $\mathbf{t} = \langle 23, -16 \rangle$

SOLUTION:

Since $|\mathbf{t}|^2 = \mathbf{t} \cdot \mathbf{t}$, then $|\mathbf{t}| = \sqrt{\mathbf{t} \cdot \mathbf{t}}$.

$$|\mathbf{t}| = \sqrt{\mathbf{t} \cdot \mathbf{t}}$$
$$|\langle 23, -16 \rangle| = \sqrt{\langle 23, -16 \rangle \cdot \langle 23, -16 \rangle}$$
$$= \sqrt{23^2 + (-16)^2}$$
$$= \sqrt{785} \text{ or about } 28.0$$

ANSWER:

$\sqrt{785} \approx 28.0$

Find the angle θ between u and v to the nearest tenth of a degree.

16. $\mathbf{u} = \langle 0, -5 \rangle$, $\mathbf{v} = \langle 1, -4 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle 0, -5 \rangle \cdot \langle 1, -4 \rangle}{|\langle 0, -5 \rangle||\langle 1, -4 \rangle|}$$
$$\cos\theta = \frac{0(1) + (-5)(-4)}{\sqrt{0^2 + (-5)^2}\sqrt{1^2 + (-4)^2}}$$
$$\cos\theta = \frac{0 + 20}{\sqrt{0 + 25}\sqrt{1 + 16}}$$
$$\cos\theta = \frac{20}{\sqrt{25}\sqrt{17}}$$
$$\cos\theta = \frac{20}{5\sqrt{17}}$$
$$\cos\theta = \frac{4}{\sqrt{17}}$$
$$\theta = \cos^{-1}\frac{4}{\sqrt{17}}$$
$$\theta \approx 14.0^\circ$$

ANSWER:

14.0°

17. $\mathbf{u} = \langle 7, 10 \rangle$, $\mathbf{v} = \langle 4, -4 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 7, 10 \rangle \cdot \langle 4, -4 \rangle}{|\langle 7, 10 \rangle||\langle 4, -4 \rangle|}$$

$$\cos\theta = \frac{7 \cdot 4 + 10(-4)}{\sqrt{7^2 + 10^2}\sqrt{4^2 + (-4)^2}}$$

$$\cos\theta = \frac{28 + (-40)}{\sqrt{49 + 100}\sqrt{16 + 16}}$$

$$\cos\theta = \frac{-12}{\sqrt{149}\sqrt{32}}$$

$$\cos\theta = \frac{-12}{\sqrt{149} \cdot 4\sqrt{2}}$$

$$\cos\theta = \frac{-12}{4\sqrt{298}}$$

$$\cos\theta = \frac{-3}{\sqrt{298}}$$

$$\theta = \cos^{-1}\frac{-3}{\sqrt{298}}$$

$$\theta \approx 100.0^\circ$$

ANSWER:

100.0°

18. $\mathbf{u} = \langle -2, 4 \rangle$, $\mathbf{v} = \langle 2, -10 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle -2, 4 \rangle \cdot \langle 2, -10 \rangle}{|\langle -2, 4 \rangle||\langle 2, -10 \rangle|}$$

$$\cos\theta = \frac{(-2)2 + 4(-10)}{\sqrt{(-2)^2 + 4^2}\sqrt{2^2 + (-10)^2}}$$

$$\cos\theta = \frac{-4 + (-40)}{\sqrt{4 + 16}\sqrt{4 + 100}}$$

$$\cos\theta = \frac{-44}{\sqrt{20}\sqrt{104}}$$

$$\cos\theta = \frac{-44}{4\sqrt{130}}$$

$$\cos\theta = \frac{-11}{\sqrt{130}}$$

$$\theta = \cos^{-1}\frac{-11}{\sqrt{130}}$$

$$\theta \approx 164.7^\circ$$

ANSWER:

164.7°

19. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle -2, 3 \rangle$ and $\mathbf{v} = \langle -4, -2 \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle -2, 3 \rangle \cdot \langle -4, -2 \rangle}{|\langle -2, 3 \rangle||\langle -4, -2 \rangle|}$$

$$\cos\theta = \frac{(-2)(-4) + 3(-2)}{\sqrt{(-2)^2 + 3^2}\sqrt{(-4)^2 + (-2)^2}}$$

$$\cos\theta = \frac{8 + (-6)}{\sqrt{4+9}\sqrt{16+4}}$$

$$\cos\theta = \frac{2}{\sqrt{13}\sqrt{20}}$$

$$\cos\theta = \frac{2}{\sqrt{13} \cdot 2\sqrt{5}}$$

$$\cos\theta = \frac{2}{2\sqrt{65}}$$

$$\cos\theta = \frac{1}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{65}}$$

$$\theta \approx 82.9^\circ$$

ANSWER:

82.9°

20. $\mathbf{u} = \langle -9, 0 \rangle$, $\mathbf{v} = \langle -1, -1 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle -9, 0 \rangle \cdot \langle -1, -1 \rangle}{|\langle -9, 0 \rangle||\langle -1, -1 \rangle|}$$

$$\cos\theta = \frac{(-9)(-1) + 0(-1)}{\sqrt{(-9)^2 + 0^2}\sqrt{(-1)^1 + (-1)^2}}$$

$$\cos\theta = \frac{9 + 0}{\sqrt{81+0}\sqrt{1+1}}$$

$$\cos\theta = \frac{9}{\sqrt{81}\sqrt{2}}$$

$$\cos\theta = \frac{9}{9\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{2}}$$

$$\theta = 45.0^\circ$$

ANSWER:

45.0°

21. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle -1, -3 \rangle$ and $\mathbf{v} = \langle -7, -3 \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle -1, -3 \rangle \cdot \langle -7, -3 \rangle}{|\langle -1, -3 \rangle||\langle -7, -3 \rangle|}$$

$$\cos\theta = \frac{(-1)(-7) + (-3)(-3)}{\sqrt{\left(-1^2 + (-3)\right)^2}\sqrt{\left(-7\right)^2 + (-3)^3}}$$

$$\cos\theta = \frac{7 + 9}{\sqrt{1+9}\sqrt{49+9}}$$

$$\cos\theta = \frac{7 + 9}{\sqrt{10}\sqrt{58}}$$

$$\cos\theta = \frac{16}{2\sqrt{145}}$$

$$\cos\theta = \frac{8}{\sqrt{145}}$$

$$\theta = \cos^{-1}\frac{8}{\sqrt{145}}$$

$$\theta \approx 48.4^\circ$$

ANSWER:

48.4°

22. $\mathbf{u} = \langle 6, 0 \rangle$, $\mathbf{v} = \langle -10, 8 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 6, 0 \rangle \cdot \langle -10, 8 \rangle}{|\langle 6, 0 \rangle||\langle -10, 8 \rangle|}$$

$$\cos\theta = \frac{6(-10) + 0(8)}{\sqrt{6^2 + 0^2}\sqrt{\left(-10\right)^2 + 8^2}}$$

$$\cos\theta = \frac{-60 + 0}{\sqrt{36 + 0}\sqrt{100 + 64}}$$

$$\cos\theta = \frac{-60}{\sqrt{36}\sqrt{164}}$$

$$\cos\theta = \frac{-60}{12\sqrt{41}}$$

$$\cos\theta = \frac{-5}{\sqrt{41}}$$

$$\theta = \cos^{-1}\frac{-5}{\sqrt{41}}$$

$$\theta \approx 141.3^\circ$$

ANSWER:

141.3°

23. $\mathbf{u} = -10\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle -10, 1 \rangle$ and $\mathbf{v} = \langle 10, -5 \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle -10, 1 \rangle \cdot \langle 10, -5 \rangle}{|\langle -10, 1 \rangle||\langle 10, -5 \rangle|}$$

$$\cos\theta = \frac{(-10)10 + 1(-5)}{\sqrt{(-10)^2 + 1^2}\sqrt{10^2 + (-5)^2}}$$

$$\cos\theta = \frac{-100 + (-5)}{\sqrt{100 + 1}\sqrt{100 + 25}}$$

$$\cos\theta = \frac{-100 + (-5)}{\sqrt{101}\sqrt{125}}$$

$$\cos\theta = \frac{-105}{5\sqrt{505}}$$

$$\cos\theta = \frac{-21}{\sqrt{505}}$$

$$\theta = \cos^{-1}\frac{-21}{\sqrt{505}}$$

$$\theta \approx 159.1^\circ$$

ANSWER:

159.1°

24. **CAMPING** Regina and Luis set off from their campsite to search for firewood. The path that Regina takes can be represented by $\mathbf{u} = \langle 3, -5 \rangle$. The path that Luis takes can be represented by $\mathbf{v} = \langle -7, 6 \rangle$. Find the angle between the pair of vectors.

SOLUTION:

Use the formula for finding the angle between two vectors.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 3, -5 \rangle \cdot \langle -7, 6 \rangle}{|\langle 3, -5 \rangle||\langle -7, 6 \rangle|}$$

$$\cos\theta = \frac{-21 + (-30)}{\sqrt{34}\sqrt{85}}$$

$$\cos\theta = \frac{-51}{17\sqrt{10}}$$

$$\theta = \cos^{-1}\frac{-51}{17\sqrt{10}} \text{ or about } 161.6^\circ$$

ANSWER:

about 161.6°

Find the projection of u onto v. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.

25. $\mathbf{u} = 3\mathbf{i} + 6\mathbf{j}$, $\mathbf{v} = -5\mathbf{i} + 2\mathbf{j}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle 3, 6 \rangle$ and $\mathbf{v} = \langle -5, 2 \rangle$.

Find the projection of **u** onto **v**.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}\\ &= \frac{\langle 3,6\rangle\cdot\langle -5,2\rangle}{|\langle -5,2\rangle|^2}\langle -5,2\rangle\\ &= \frac{-15+12}{\left(\sqrt{29}\right)^2}\langle -5,2\rangle\\ &= \frac{-3}{29}\langle -5,2\rangle\\ &= \left\langle \frac{15}{29}, -\frac{6}{29}\right\rangle\end{aligned}$$

To write **u** as the sum of two orthogonal vectors, start by writing **u** as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of **u** onto **v**, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w}_1 + \mathbf{w}_2\\ \mathbf{u} - \mathbf{w}_1 &= \mathbf{w}_2\\ \langle 3, 6\rangle - \left\langle \frac{15}{29}, -\frac{6}{29}\right\rangle &= \mathbf{w}_2\\ \left\langle \frac{87}{29}, \frac{174}{29}\right\rangle - \left\langle \frac{15}{29}, -\frac{6}{29}\right\rangle &= \mathbf{w}_2\\ \left\langle \frac{72}{29}, \frac{180}{29}\right\rangle &= \mathbf{w}_2\end{aligned}$$

Thus, $\mathbf{u} = \left\langle \frac{15}{29}, -\frac{6}{29}\right\rangle + \left\langle \frac{72}{29}, \frac{180}{29}\right\rangle$.

ANSWER:

$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{15}{29}, -\frac{6}{29}\right\rangle; \mathbf{u} = \left\langle \frac{15}{29}, -\frac{6}{29}\right\rangle + \left\langle \frac{72}{29}, \frac{180}{29}\right\rangle$

26. $\mathbf{u} = \langle 5, 7\rangle$, $\mathbf{v} = \langle -4, 4\rangle$

SOLUTION:

Find the projection of **u** onto **v**.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}\\ &= \frac{\langle 5,7\rangle\cdot\langle -4,4\rangle}{|\langle -4,4\rangle|^2}\langle -4,4\rangle\\ &= \frac{-20+28}{\left(\sqrt{32}\right)^2}\langle -4,4\rangle\\ &= \frac{8}{32}\langle -4,4\rangle\\ &= \frac{1}{4}\langle -4,4\rangle\\ &= \langle -1,1\rangle\end{aligned}$$

To write **u** as the sum of two orthogonal vectors, start by writing **u** as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of **u** onto **v**, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w}_1 + \mathbf{w}_2\\ \mathbf{u} - \mathbf{w}_1 &= \mathbf{w}_2\\ \langle 5, 7\rangle - \langle -1, 1\rangle &= \mathbf{w}_2\\ \langle 6, 6\rangle &= \mathbf{w}_2\end{aligned}$$

Thus, $\mathbf{u} = \langle -1,1\rangle + \langle 6,6\rangle$.

ANSWER:

$\text{proj}_{\mathbf{v}}\mathbf{u} = \langle -1, 1\rangle; \mathbf{u} = \langle -1, 1\rangle + \langle 6, 6\rangle$

27. $\mathbf{u} = \langle 8, 2 \rangle$, $\mathbf{v} = \langle -4, 1 \rangle$

SOLUTION:

Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle 8,2\rangle\cdot\langle -4,1\rangle}{|\langle -4,1\rangle|^2}\langle -4,1\rangle \\ &= \frac{-32+2}{\left(\sqrt{17}\right)^2}\langle -4,1\rangle \\ &= \frac{-30}{17}\langle -4,1\rangle \\ &= \left\langle \frac{120}{17}, -\frac{30}{17}\right\rangle\end{aligned}$$

To write $\mathbf{u}$ as the sum of two orthogonal vectors, start by writing $\mathbf{u}$ as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of $\mathbf{u}$ onto $\mathbf{v}$, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle 8, 2\rangle - \left\langle \frac{120}{17}, -\frac{30}{17}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{136}{17}, \frac{34}{17}\right\rangle - \left\langle \frac{120}{17}, -\frac{30}{17}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{16}{17}, \frac{64}{17}\right\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \left\langle \frac{120}{17}, -\frac{30}{17}\right\rangle + \left\langle \frac{16}{17}, \frac{64}{17}\right\rangle$.

ANSWER:

$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{120}{17}, -\frac{30}{17}\right\rangle; \mathbf{u} = \left\langle \frac{120}{17}, -\frac{30}{17}\right\rangle + \left\langle \frac{16}{17}, \frac{64}{17}\right\rangle$

28. $\mathbf{u} = 6\mathbf{i} + \mathbf{j}$, $\mathbf{v} = -3\mathbf{i} + 9\mathbf{j}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle 6,1\rangle$ and $\mathbf{v} = \langle -3,9\rangle$.

Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle 6,1\rangle\cdot\langle -3,9\rangle}{|\langle -3,9\rangle|^2}\langle -3,9\rangle \\ &= \frac{-18+9}{\left(\sqrt{90}\right)^2}\langle -3,9\rangle \\ &= \frac{-9}{90}\langle -3,9\rangle \\ &= -\frac{1}{10}\langle -3,9\rangle \\ &= \left\langle \frac{3}{10}, -\frac{9}{10}\right\rangle\end{aligned}$$

To write $\mathbf{u}$ as the sum of two orthogonal vectors, start by writing $\mathbf{u}$ as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of $\mathbf{u}$ onto $\mathbf{v}$, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle 6, 1\rangle - \left\langle \frac{3}{10}, -\frac{9}{10}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{60}{10}, \frac{10}{10}\right\rangle - \left\langle \frac{3}{10}, -\frac{9}{10}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{57}{10}, \frac{19}{10}\right\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \left\langle \frac{3}{10}, -\frac{9}{10}\right\rangle + \left\langle \frac{57}{10}, \frac{19}{10}\right\rangle$.

ANSWER:

$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{3}{10}, -\frac{9}{10}\right\rangle; \mathbf{u} = \left\langle \frac{3}{10}, -\frac{9}{10}\right\rangle + \left\langle \frac{57}{10}, \frac{19}{10}\right\rangle$

29. $\mathbf{u} = \langle 2, 4 \rangle$, $\mathbf{v} = \langle -3, 8 \rangle$

SOLUTION:

Find the projection of **u** onto **v**.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle 2,4\rangle\cdot\langle -3,8\rangle}{|\langle -3,8\rangle|^2}\langle -3,8\rangle \\ &= \frac{-6+32}{\left(\sqrt{73}\right)^2}\langle -3,8\rangle \\ &= \frac{26}{73}\langle -3,8\rangle \\ &= \left\langle -\frac{78}{73}, \frac{208}{73}\right\rangle\end{aligned}$$

To write **u** as the sum of two orthogonal vectors, start by writing **u** as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of **u** onto **v**, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle 2,4\rangle - \left\langle -\frac{78}{73}, \frac{208}{73}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{146}{73}, \frac{292}{73}\right\rangle - \left\langle -\frac{78}{73}, \frac{208}{73}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{224}{73}, \frac{84}{73}\right\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \left\langle -\frac{78}{73}, \frac{208}{73}\right\rangle + \left\langle \frac{224}{73}, \frac{84}{73}\right\rangle$.

ANSWER:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -\frac{78}{73}, \frac{208}{73}\right\rangle; \mathbf{u} = \left\langle -\frac{78}{73}, \frac{208}{73}\right\rangle + \left\langle \frac{224}{73}, \frac{84}{73}\right\rangle$$

30. $\mathbf{u} = \langle -5, 9 \rangle$, $\mathbf{v} = \langle 6, 4 \rangle$

SOLUTION:

Find the projection of **u** onto **v**.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle -5,9\rangle\cdot\langle 6,4\rangle}{|\langle 6,4\rangle|^2}\langle 6,4\rangle \\ &= \frac{-30+36}{\left(\sqrt{52}\right)^2}\langle 6,4\rangle \\ &= \frac{6}{52}\langle 6,4\rangle \\ &= \frac{3}{26}\langle 6,4\rangle \\ &= \left\langle \frac{9}{13}, \frac{6}{13}\right\rangle\end{aligned}$$

To write **u** as the sum of two orthogonal vectors, start by writing **u** as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of **u** onto **v**, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle -5,9\rangle - \left\langle \frac{9}{13}, \frac{6}{13}\right\rangle &= \mathbf{w_2} \\ \left\langle -\frac{65}{13}, \frac{117}{13}\right\rangle - \left\langle \frac{9}{13}, \frac{6}{13}\right\rangle &= \mathbf{w_2} \\ \left\langle -\frac{74}{13}, \frac{111}{13}\right\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \left\langle \frac{9}{13}, \frac{6}{13}\right\rangle + \left\langle -\frac{74}{13}, \frac{111}{13}\right\rangle$.

ANSWER:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{9}{13}, \frac{6}{13}\right\rangle; \mathbf{u} = \left\langle \frac{9}{13}, \frac{6}{13}\right\rangle + \left\langle -\frac{74}{13}, \frac{111}{13}\right\rangle$$

31. $\mathbf{u} = 5\mathbf{i} - 8\mathbf{j}$, $\mathbf{v} = 6\mathbf{i} - 4\mathbf{j}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle 5, -8 \rangle$ and $\mathbf{v} = \langle 6, -4 \rangle$.

Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle 5,-8\rangle\cdot\langle 6,-4\rangle}{|\langle 6,-4\rangle|^2}\langle 6,-4\rangle \\ &= \frac{30+32}{\left(\sqrt{52}\right)^2}\langle 6,-4\rangle \\ &= \frac{62}{52}\langle 6,-4\rangle \\ &= \frac{31}{26}\langle 6,-4\rangle \\ &= \left\langle \frac{93}{13}, -\frac{62}{13}\right\rangle\end{aligned}$$

To write $\mathbf{u}$ as the sum of two orthogonal vectors, start by writing $\mathbf{u}$ as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of $\mathbf{u}$ onto $\mathbf{v}$, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle 5, -8\rangle - \left\langle \frac{93}{13}, -\frac{62}{13}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{65}{13}, -\frac{104}{13}\right\rangle - \left\langle \frac{93}{13}, -\frac{62}{13}\right\rangle &= \mathbf{w_2} \\ \left\langle -\frac{28}{13}, -\frac{42}{13}\right\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \left\langle \frac{93}{13}, -\frac{62}{13}\right\rangle + \left\langle -\frac{28}{13}, -\frac{42}{13}\right\rangle$.

ANSWER:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{93}{13}, -\frac{62}{13}\right\rangle; \mathbf{u} = \left\langle \frac{93}{13}, -\frac{62}{13}\right\rangle + \left\langle -\frac{28}{13}, -\frac{42}{13}\right\rangle$$

32. $\mathbf{u} = -2\mathbf{i} - 5\mathbf{j}$, $\mathbf{v} = 9\mathbf{i} + 7\mathbf{j}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle -2, -5 \rangle$ and $\mathbf{v} = \langle 9, 7 \rangle$.

Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle -2,-5\rangle\cdot\langle 9,7\rangle}{|\langle 9,7\rangle|^2}\langle 9,7\rangle \\ &= \frac{-18-35}{\left(\sqrt{130}\right)^2}\langle 9,7\rangle \\ &= -\frac{53}{130}\langle 9,7\rangle \\ &= \left\langle -\frac{477}{130}, -\frac{371}{130}\right\rangle\end{aligned}$$

To write $\mathbf{u}$ as the sum of two orthogonal vectors, start by writing $\mathbf{u}$ as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of $\mathbf{u}$ onto $\mathbf{v}$, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle -2, -5\rangle - \left\langle -\frac{477}{130}, -\frac{371}{130}\right\rangle &= \mathbf{w_2} \\ \left\langle -\frac{260}{130}, -\frac{650}{130}\right\rangle - \left\langle -\frac{477}{130}, -\frac{371}{130}\right\rangle &= \mathbf{w_2} \\ \left\langle \frac{217}{130}, -\frac{279}{130}\right\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \left\langle -\frac{477}{130}, -\frac{371}{130}\right\rangle + \left\langle \frac{217}{130}, -\frac{279}{130}\right\rangle$.

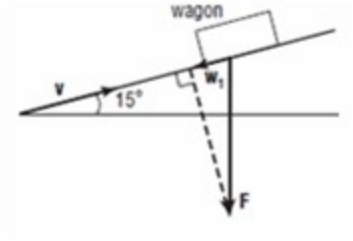
ANSWER:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -\frac{477}{130}, -\frac{371}{130}\right\rangle; \mathbf{u} = \left\langle -\frac{477}{130}, -\frac{371}{130}\right\rangle + \left\langle \frac{217}{130}, -\frac{279}{130}\right\rangle$$

33. **WAGON** Malcolm is pulling his sister in a wagon up a small slope at an incline of 15°. If the combined weight of Malcolm's sister and the wagon is 78 pounds, what force is required to keep her from rolling down the slope?

SOLUTION:

The combined weight of Malcolm's sister and the wagon is the force exerted due to gravity, $\mathbf{F} = \langle 0, -78 \rangle$. To find the force $-\mathbf{w_1}$ required to keep her from sliding down the slope, project $\mathbf{F}$ onto a unit vector $\mathbf{v}$ in the direction of the side of the hill.



Find a unit vector $\mathbf{v}$ in the direction of the side of the hill.

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 1(\cos 15^\circ), 1(\sin 15^\circ) \rangle \\ &= \langle 0.966, 0.259 \rangle\end{aligned}$$

Find $\mathbf{w_1}$, the projection of $\mathbf{F}$ onto the unit vector $\mathbf{v}$, $\text{proj}_{\mathbf{v}}\mathbf{F}$.

$$\begin{aligned}\mathbf{w_1} &= \text{proj}_{\mathbf{v}}\mathbf{F} \\ &= \left(\frac{\mathbf{F}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= (\mathbf{F}\cdot\mathbf{v})\mathbf{v} \\ &= (\langle 0, -78\rangle \cdot \langle 0.966, 0.259\rangle)\mathbf{v} \\ &\approx -20.2\mathbf{v}\end{aligned}$$

Since $\mathbf{w_1}$ points down the hill, the force required is $-\mathbf{w_1} = -(-20.2\mathbf{v})$ or $20.2\mathbf{v}$. Since $\mathbf{v}$ is a unit vector, 20.2 pounds represents the magnitude of the force required to keep Malcolm's sister from sliding down the hill.

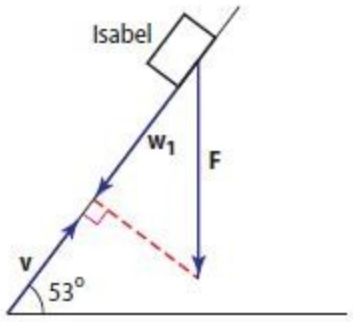
ANSWER:

about 20.2 lb

34. **SLIDE** Isabel is going down a slide but stops herself when she notices that another student is lying hurt at the bottom of the slide. What force is required to keep her from sliding down the slide if the incline is 53° and she weighs 62 pounds?

SOLUTION:

Isabel's weight is the force exerted due to gravity, $\mathbf{F} = \langle 0, -62 \rangle$. To find the force $-\mathbf{w_1}$ required to keep her from sliding down the slope, project $\mathbf{F}$ onto a unit vector $\mathbf{v}$ in the direction of the side of the hill.



Find a unit vector $\mathbf{v}$ in the direction of the side of the hill.

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 1(\cos 53^\circ), 1(\sin 53^\circ) \rangle \\ &= \langle 0.602, 0.799 \rangle\end{aligned}$$

Find $\mathbf{w_1}$, the projection of $\mathbf{F}$ onto the unit vector $\mathbf{v}$, $\text{proj}_{\mathbf{v}}\mathbf{F}$.

$$\begin{aligned}\mathbf{w_1} &= \text{proj}_{\mathbf{v}}\mathbf{F} \\ &= \left(\frac{\mathbf{F}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= (\mathbf{F}\cdot\mathbf{v})\mathbf{v} \\ &= (\langle 0, -62\rangle \cdot \langle 0.602, 0.799\rangle)\mathbf{v} \\ &\approx -49.5\mathbf{v}\end{aligned}$$

Since $\mathbf{w_1}$ points down the hill, the force required is $-\mathbf{w_1} = -(-49.5\mathbf{v})$ or $49.5\mathbf{v}$. Since $\mathbf{v}$ is a unit vector, 49.5 pounds represents the magnitude of the force required to keep Isabel from sliding down the slide.

ANSWER:

about 49.5 lb

35. **PHYSICS** Alexa is pushing a construction barrel up a ramp 1.5 meters long into the back of a truck. She is using a force of 534 newtons and the ramp is 25° from the horizontal. How much work in joules is Alexa doing?



SOLUTION:

Use the projection formula for work. Since the force vector **F** that represents Alexa pushing the construction barrel up the ramp is parallel to $\overrightarrow{AB}$, **F** does not need to be projected on $\overrightarrow{AB}$. So, $\mathbf{F} = \text{proj}_{\overrightarrow{AB}}\mathbf{F}$. The magnitude of the directed distance $\overrightarrow{AB}$ is 1.5.

$$\begin{aligned} W &= \left|\text{proj}_{\overrightarrow{AB}}\mathbf{F}\right|\left|\overrightarrow{AB}\right| \\ &= |\mathbf{F}|\left|\overrightarrow{AB}\right| \\ &= (534)(1.5) \\ &= 801 \end{aligned}$$

Verify the result using the dot product formula for work. The component form of the force vector **F** in terms of magnitude and direction angle given is $\langle 534\cos 25^\circ, 534\sin 25^\circ\rangle$. The component form of the directed distance the barrel is moved in terms of magnitude and direction angle given is $\langle 1.5\cos 25^\circ, 1.5\sin 25^\circ\rangle$.

$$\begin{aligned} W &= \mathbf{F}\cdot\overrightarrow{AB} \\ &= \langle 534\cos 25^\circ, 534\sin 25^\circ\rangle \cdot \langle 1.5\cos 25^\circ, 1.5\sin 25^\circ\rangle \\ &= (534\cos 25^\circ)(1.5\cos 25^\circ) + (534\sin 25^\circ)(1.5\sin 25^\circ) \\ &= 801 \end{aligned}$$

The work that Alexa is doing is 801 joules.

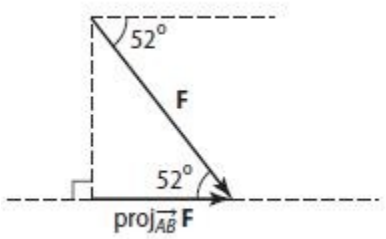
ANSWER:

801 joules

36. **SHOPPING** Sophia is pushing a shopping cart with a force of 125 newtons at a downward angle, or angle of depression, of 52°. How much work in joules would Sophia do if she pushed the shopping cart 200 meters?

SOLUTION:

Diagram the situation.



Use the projection formula for work. The magnitude of the projection of **F** onto $\overrightarrow{AB}$ is $|\mathbf{F}|\cos\theta = 152\cos 52^\circ$. The magnitude of the directed distance $\overrightarrow{AB}$ is 200.

$$\begin{aligned} W &= \left|\text{proj}_{\overrightarrow{AB}}\mathbf{F}\right|\left|\overrightarrow{AB}\right| \\ &= (125\cos 52^\circ)(200) \\ &\approx 15{,}391.5 \end{aligned}$$

Therefore, Sophia does about 15,391.5 joules of work pushing the shopping cart.

ANSWER:

about 15,391.5 joules

Find a vector orthogonal to each vector.

37. $\langle -2, -8 \rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle -2, -8 \rangle$ and $\mathbf{b} = \langle x, y \rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \langle -2, -8 \rangle \cdot \langle x, y \rangle \\ &= -2x + (-8)y \\ &= -2x - 8y\end{aligned}$$

If **a** and **b** are orthogonal, then $-2x - 8y = 0$. Solve for y.

$$\begin{aligned}-2x - 8y &= 0 \\ -2x &= 8y \\ -\frac{x}{4} &= y\end{aligned}$$

Substitute a value for x and solve for y. A value of x that is divisible by 4 will produce an integer value for y. Let $x = -12$.

$$\begin{aligned}-\frac{x}{4} &= y \\ -\frac{(-12)}{4} &= y \\ 3 &= y\end{aligned}$$

A vector orthogonal to $\langle -2, -8 \rangle$ is $\langle -12, 3 \rangle$.

ANSWER:

Sample answer: $\langle -12, 3 \rangle$

38. $\langle 3, 5 \rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle 3, 5 \rangle$ and $\mathbf{b} = \langle x, y \rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \langle 3, 5 \rangle \cdot \langle x, y \rangle \\ &= 3x + 5y\end{aligned}$$

If **a** and **b** are orthogonal, then $3x + 5y = 0$. Solve for y.

$$\begin{aligned}3x + 5y &= 0 \\ 3x &= -5y \\ -\frac{3x}{5} &= y\end{aligned}$$

Substitute a value for x and solve for y. A value of x that results in a value for $3x$ that is divisible by 5 will produce an integer value for y. Let $x = 10$.

$$\begin{aligned}-\frac{3x}{5} &= y \\ -\frac{3(10)}{5} &= y \\ -6 &= y\end{aligned}$$

A vector orthogonal to $\langle 3, 5 \rangle$ is $\langle 10, -6 \rangle$.

ANSWER:

Sample answer: $\langle 10, -6 \rangle$

39. $\langle 7, -4 \rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle 7, -4 \rangle$ and $\mathbf{b} = \langle x, y \rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \langle 7, -4 \rangle \cdot \langle x, y \rangle \\ &= 7x + (-4)y \\ &= 7x - 4y\end{aligned}$$

If **a** and **b** are orthogonal, then $7x - 4y = 0$. Solve for y.

$$\begin{aligned}7x - 4y &= 0 \\ 7x &= 4y \\ \frac{7x}{4} &= y\end{aligned}$$

Substitute a value for x and solve for y. A value of x that results in a value for $7x$ that is divisible by 4 will produce an integer value for y. Let $x = 8$.

$$\begin{aligned}\frac{7x}{4} &= y \\ \frac{7(8)}{4} &= y \\ 14 &= y\end{aligned}$$

A vector orthogonal to $\langle 7, -4 \rangle$ is $\langle 8, 14 \rangle$.

ANSWER:

Sample answer: $\langle 8, 14 \rangle$

40. $\langle -1, 6 \rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle -1, 6 \rangle$ and $\mathbf{b} = \langle x, y \rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \langle -1, 6 \rangle \cdot \langle x, y \rangle \\ &= (-1)x + 6y \\ &= -x + 6y\end{aligned}$$

If **a** and **b** are orthogonal, then $-x + 6y = 0$. Solve for y.

$$\begin{aligned}-x + 6y &= 0 \\ -x &= -6y \\ \frac{x}{6} &= y\end{aligned}$$

Substitute a value for x and solve for y. A value of x that is divisible by 6 will produce an integer value for y. Let $x = 6$.

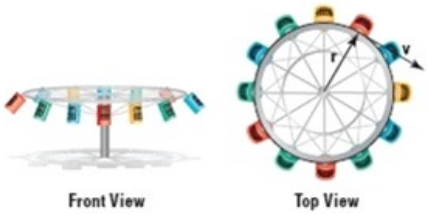
$$\begin{aligned}\frac{x}{6} &= y \\ \frac{(6)}{6} &= y \\ 1 &= y\end{aligned}$$

A vector orthogonal to $\langle -1, 6 \rangle$ is $\langle 6, 1 \rangle$.

ANSWER:

Sample answer: $\langle 6, 1 \rangle$

41. **RIDES** For a circular amusement park ride, the position vector **r** is perpendicular to the tangent velocity vector **v** at any point on the circle, as shown below.

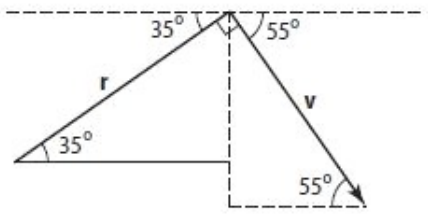


a. If the radius of the ride is 20 feet and the speed of the ride is constant at 40 feet per second, write the component forms of the position vector **r** and the tangent velocity vector **v** when **r** is at a directed angle of 35°.

b. What method can be used to prove that the position vector and the velocity vector that you developed in part **a** are perpendicular? Show that the two vectors are perpendicular.

SOLUTION:

a. Diagram the situation.



Drawing is not to scale.

The component form of **r** can be found using its magnitude and directed angle.

$$\begin{aligned}\mathbf{r} &= \langle |\mathbf{r}|\cos\theta, |\mathbf{r}|\sin\theta \rangle \\ &= \langle 20\cos 35^\circ, 20\sin 35^\circ \rangle \\ &= \langle 16.38, 11.47 \rangle\end{aligned}$$

The component form of **v** can be found using its magnitude and the 55° angle. Since the direction of the vector is pointing down, the vertical component will be negative.

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 40\cos 55^\circ, -40\sin 55^\circ \rangle \\ &= \langle 22.94, -32.77 \rangle\end{aligned}$$

So, $\mathbf{r} = \langle 16.38, 11.47 \rangle$ and $\mathbf{v} = \langle 22.94, -32.77 \rangle$.

b. If the dot product of the two vectors is equal to 1, then the two vectors are orthogonal or perpendicular. To avoid rounding error, use the exact expression for the components of the vectors found in part **a.**

$$\begin{aligned}\mathbf{r} \cdot \mathbf{v} &= \langle 20\cos 35^\circ, 20\sin 35^\circ \rangle \cdot \langle 40\cos 55^\circ, -40\sin 55^\circ \rangle \\ &= (20\cos 35^\circ)(40\cos 55^\circ) + (20\sin 35^\circ)(-40\sin 55^\circ) \\ &= 0\end{aligned}$$

ANSWER:

a. $\langle 16.38, 11.47 \rangle, \langle 22.94, -32.77 \rangle$

b. dot product;

$(20\cos 35^\circ)(40\cos 55^\circ) + (20\sin 35^\circ)(-40\sin 55^\circ) = 0$

Given v and u · v, find u.

42. $\mathbf{v} = \langle 3, -6 \rangle$, $\mathbf{u} \cdot \mathbf{v} = 33$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle x, y \rangle \cdot \langle 3, -6 \rangle \\ 33 &= x(3) + y(-6) \\ 33 &= 3x - 6y\end{aligned}$$

Solve for y.

$$\begin{aligned}33 &= 3x - 6y \\ 33 - 3x &= -6y \\ \frac{11 - x}{-2} &= y\end{aligned}$$

Substitute a value for x and solve for y. A value of x that results in a value for $11 - x$ that is divisible by -2 will produce an integer value for y. Let $x = 5$.

$$\begin{aligned}\frac{11 - x}{-2} &= y \\ \frac{11 - 5}{-2} &= y \\ -3 &= y\end{aligned}$$

Therefore, $\mathbf{u} = \langle 5, -3 \rangle$.

ANSWER:

Sample answer: $\mathbf{u} = \langle 5, -3 \rangle$

43. $\mathbf{v} = \langle 4, 6 \rangle$, $\mathbf{u} \cdot \mathbf{v} = 38$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\mathbf{u} \cdot \mathbf{v} = \langle x, y \rangle \cdot \langle 4, 6 \rangle$$
$$38 = x(4) + y(6)$$
$$38 = 4x + 6y$$

Solve for y.

$$38 = 4x + 6y$$
$$38 - 4x = 6y$$
$$\frac{19 - 2x}{3} = y$$

Substitute a value for x and solve for y. A value of x that results in a value for $19 - 2x$ that is divisible by 3 will produce an integer value for y. Let $x = -1$.

$$\frac{19 - 2x}{3} = y$$
$$\frac{19 - 2(-1)}{3} = y$$
$$\frac{21}{3} = y$$
$$7 = y$$

Therefore, $\mathbf{u} = \langle -1, 7 \rangle$.

ANSWER:

Sample answer: $\mathbf{u} = \langle -1, 7 \rangle$

44. $\mathbf{v} = \langle -5, -1 \rangle$, $\mathbf{u} \cdot \mathbf{v} = -8$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\mathbf{u} \cdot \mathbf{v} = \langle x, y \rangle \cdot \langle -5, -1 \rangle$$
$$-8 = x(-5) + y(-1)$$
$$-8 = -5x - y$$

Solve for y.

$$-8 = -5x - y$$
$$-8 + 5x = -y$$
$$8 - 5x = y$$

Substitute a value for x and solve for y. Let $x = 1$.

$$8 - 5x = y$$
$$8 - 5(1) = y$$
$$3 = y$$

Therefore, $\mathbf{u} = \langle 1, 3 \rangle$.

ANSWER:

Sample answer: $\mathbf{u} = \langle 1, 3 \rangle$

45. $\mathbf{v} = \langle -2, 7 \rangle$, $\mathbf{u} \cdot \mathbf{v} = -43$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\mathbf{u} \cdot \mathbf{v} = \langle x, y \rangle \cdot \langle -2, 7 \rangle$$
$$-43 = x(-2) + y(7)$$
$$-43 = -2x + 7y$$

Solve for y.

$$-43 = -2x + 7y$$
$$-43 + 2x = 7y$$
$$\frac{-43 + 2x}{7} = y$$

Substitute a value for x and solve for y. A value of x that results in a value for $-43 + 2x$ that is divisible by 7 will produce an integer value for y. Let $x = 4$.

$$\frac{-43 + 2x}{7} = y$$
$$\frac{-43 + 2(4)}{7} = y$$
$$\frac{-35}{7} = y$$
$$-5 = y$$

Therefore, $\mathbf{u} = \langle 4, -5 \rangle$.

ANSWER:

Sample answer: $\mathbf{u} = \langle 4, -5 \rangle$

46. **SCHOOL** A student rolls her backpack from her Chemistry classroom to her English classroom using a force of 175 newtons.



a. If she exerts 3060 joules to pull her backpack 31 meters, what is the angle of her force?

b. If she exerts 1315 joules at an angle of 60°, how far did she pull her backpack?

SOLUTION:

a. Use the projection formula for work. The magnitude of the projection of **F** onto $\overrightarrow{AB}$ is $|\mathbf{F}|\cos\theta = 175\cos\theta$. The magnitude of the directed distance $\overrightarrow{AB}$ is 31 and the work exerted is 3060 joules. Solve for θ.

$$W = |\text{proj}_{\overrightarrow{AB}}\mathbf{F}||\overrightarrow{AB}|$$
$$3060 = (175\cos\theta)(31)$$
$$0.564 \approx \cos\theta$$
$$\cos^{-1} 0.564 \approx \theta$$
$$55.7^\circ \approx \theta$$

Therefore, the angle of the student's force is about 55.7°.

b. Use the projection formula for work. The magnitude of the projection of **F** onto $\overrightarrow{AB}$ is $|\mathbf{F}|\cos\theta = 175\cos 60^\circ$. The work exerted is 1315 joules. Solve for $\overrightarrow{AB}$.

$$W = |\text{proj}_{\overrightarrow{AB}}\mathbf{F}||\overrightarrow{AB}|$$
$$1315 = (175\cos 60^\circ)|\overrightarrow{AB}|$$
$$\frac{1315}{(175\cos 60^\circ)} = |\overrightarrow{AB}|$$
$$15.0 \approx |\overrightarrow{AB}|$$

Therefore, the student pulled her backpack about 15 meters.

ANSWER:

a. about 55.7°

b. about 15 m

Determine whether each pair of vectors are *parallel*, *perpendicular*, or *neither*. Explain your reasoning.

47. $\mathbf{u} = \left\langle -\frac{2}{3}, \frac{3}{4} \right\rangle$, $\mathbf{v} = \langle 9, 8 \rangle$

SOLUTION:

Find the angle between **u** and **v**.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\left\langle -\frac{2}{3}, \frac{3}{4} \right\rangle \cdot \langle 9, 8 \rangle}{\left|\left\langle -\frac{2}{3}, \frac{3}{4} \right\rangle\right| |\langle 9, 8 \rangle|}$$

$$\cos\theta = \frac{-6 + 6}{\sqrt{\frac{145}{144}}\sqrt{145}}$$

$$\cos\theta = \frac{0}{\sqrt{\frac{145}{144}}\sqrt{145}}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the two vectors are orthogonal (perpendicular). The equation can be further worked out to verify this conclusion.

$$\cos\theta = \frac{0}{\sqrt{\frac{145}{144}}\sqrt{145}}$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

Perpendicular; sample answer: Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are perpendicular.

ANSWER:

Perpendicular; sample answer: Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are perpendicular.

48. $\mathbf{u} = \langle -1, -4 \rangle$, $\mathbf{v} = \langle 3, 6 \rangle$

SOLUTION:

Find the angle between **u** and **v**.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle -1, -4 \rangle \cdot \langle 3, 6 \rangle}{|\langle -1, -4 \rangle||\langle 3, 6 \rangle|}$$

$$\cos\theta = \frac{-3 + (-24)}{\sqrt{17}\sqrt{45}}$$

$$\cos\theta = \frac{-27}{3\sqrt{85}}$$

$$\theta = \cos^{-1}\frac{-27}{3\sqrt{85}} \text{ or about } 167.5^\circ$$

The pair of vectors are neither parallel nor perpendicular. Using the formula for the angle between two vectors, $\theta \approx 167.5°$.

ANSWER:

Neither; sample answer: Using the formula for the angle between two vectors, $\theta \approx 167.5°$.

49. $\mathbf{u} = \langle 5, 7 \rangle$, $\mathbf{v} = \langle -15, -21 \rangle$

SOLUTION:

Find the angle between **u** and **v**.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 5, 7 \rangle \cdot \langle -15, -21 \rangle}{|\langle 5, 7 \rangle||\langle -15, -21 \rangle|}$$

$$\cos\theta = \frac{-75 + (-147)}{\sqrt{74}\sqrt{666}}$$

$$\cos\theta = \frac{-222}{222}$$

$$\cos\theta = -1$$

$$\theta = \cos^{-1}(-1) \text{ or } 180^\circ$$

The vectors are parallel. The angle between the two vectors is 180°. They are headed in opposite directions.

ANSWER:

Parallel; sample answer: Using the formula for the angle between two vectors, $\cos\theta = -1$. The angle between the two vectors is 180°. They are headed in opposite directions.

50. $\mathbf{u} = \langle \sec\theta, \csc\theta \rangle$, $\mathbf{v} = \langle \csc\theta, -\sec\theta \rangle$

SOLUTION:

Find the angle between **u** and **v**.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle \sec\theta, \csc\theta \rangle \cdot \langle \csc\theta, -\sec\theta \rangle}{|\langle \sec\theta, \csc\theta \rangle||\langle \csc\theta, -\sec\theta \rangle|}$$

$$\cos\theta = \frac{\sec\theta\csc\theta + [\csc\theta(-\sec\theta)]}{|\langle \sec\theta, \csc\theta \rangle||\langle \csc\theta, -\sec\theta \rangle|}$$

$$\cos\theta = \frac{\sec\theta\csc\theta - \sec\theta\csc\theta}{|\langle \sec\theta, \csc\theta \rangle||\langle \csc\theta, -\sec\theta \rangle|}$$

$$\cos\theta = \frac{0}{|\langle \sec\theta, \csc\theta \rangle||\langle \csc\theta, -\sec\theta \rangle|}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the two vectors are orthogonal (perpendicular). The equation can be further worked out to verify this conclusion.

$$\cos\theta = \frac{0}{|\langle \sec\theta, \csc\theta \rangle||\langle \csc\theta, -\sec\theta \rangle|}$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

Perpendicular; sample answer: Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are perpendicular.

ANSWER:

Perpendicular; sample answer: Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are perpendicular.

Find the angle between the two vectors in radians.

51. $\mathbf{u} = \cos\left(\frac{\pi}{3}\right)\mathbf{i} + \sin\left(\frac{\pi}{3}\right)\mathbf{j}$, $\mathbf{v} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)\mathbf{j}$

SOLUTION:

Simplify each vector.

$$\mathbf{u} = \cos\left(\frac{\pi}{3}\right)\mathbf{i} + \sin\left(\frac{\pi}{3}\right)\mathbf{j} \qquad \mathbf{v} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)$$

$$= \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j} \qquad = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

Find the angle between **u** and **v**.

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$\cos\theta = \frac{\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \cdot \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle}{\left|\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle\right|\left|\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle\right|}$$

$$\cos\theta = \frac{-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{\sqrt{\frac{1}{4} + \frac{3}{4}}\sqrt{\frac{2}{4} + \frac{2}{4}}}$$

$$\cos\theta = \frac{-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{1 \cdot 1}$$

$$\cos\theta = -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\right)$$

$$\theta = 75$$

Thus, the angle between the two vectors is $\frac{75 \cdot \pi}{180} = \frac{5\pi}{12}$.

ANSWER:

$\frac{5\pi}{12}$

52. $\mathbf{u} = \cos\left(\frac{7\pi}{6}\right)\mathbf{i} + \sin\left(\frac{7\pi}{6}\right)\mathbf{j}$, $\mathbf{v} = \cos\left(\frac{5\pi}{4}\right)\mathbf{i} + \sin\left(\frac{5\pi}{4}\right)$

SOLUTION:

Simplify each vector.

$$\mathbf{u} = \cos\left(\frac{7\pi}{6}\right)\mathbf{i} + \sin\left(\frac{7\pi}{6}\right)\mathbf{j}$$
$$= -\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$$
$$\mathbf{v} = \cos\left(\frac{5\pi}{4}\right)\mathbf{i} + \sin\left(\frac{5\pi}{4}\right)\mathbf{j}$$
$$= -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$$

Find the angle between **u** and **v**.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle \cdot \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle}{\left\| \left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle \right\| \left\| \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle \right\|}$$
$$\cos\theta = \frac{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{\sqrt{\frac{3}{4} + \frac{1}{4}}\sqrt{\frac{2}{4} + \frac{2}{4}}}$$
$$\cos\theta = \frac{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{1 \cdot 1}$$
$$\cos\theta = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$\theta = \cos^{-1}\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)$$
$$\theta = 15$$

Thus, the angle between the two vectors is $\frac{15 \cdot \pi}{180} = \frac{\pi}{12}$.

ANSWER:

$\frac{\pi}{12}$

53. $\mathbf{u} = \cos\left(\frac{\pi}{6}\right)\mathbf{i} + \sin\left(\frac{\pi}{6}\right)\mathbf{j}$, $\mathbf{v} = \cos\left(\frac{2\pi}{3}\right)\mathbf{i} + \sin\left(\frac{2\pi}{3}\right)\mathbf{j}$

SOLUTION:

Simplify each vector.

$$\mathbf{u} = \cos\left(\frac{\pi}{6}\right)\mathbf{i} + \sin\left(\frac{\pi}{6}\right)\mathbf{j}$$
$$= \frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$$
$$\mathbf{v} = \cos\left(\frac{2\pi}{3}\right)\mathbf{i} + \sin\left(\frac{2\pi}{3}\right)\mathbf{j}$$
$$= -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$$

Find the angle between **u** and **v**.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \cdot \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle}{\left\| \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \right\| \left\| \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \right\|}$$
$$\cos\theta = \frac{-\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}}{\left\| \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \right\| \left\| \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \right\|}$$
$$\cos\theta = \frac{0}{\left\| \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \right\| \left\| \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \right\|}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the two vectors are orthogonal (perpendicular). The equation can be further worked out to verify this conclusion.

$$\cos\theta = \frac{0}{\left\| \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \right\| \left\| \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \right\|}$$
$$\cos\theta = 0$$
$$\theta = 90^\circ$$

Thus, the angle between the two vectors is $\frac{90 \cdot \pi}{180} = \frac{\pi}{2}$.

ANSWER:

$\frac{\pi}{2}$

54. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j}$, $\mathbf{v} = \cos\left(\frac{5\pi}{6}\right)\mathbf{i} + \sin\left(\frac{5\pi}{6}\right)\mathbf{j}$

SOLUTION:

Simplify each vector.

$$\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

$$\mathbf{v} = \cos\left(\frac{5\pi}{6}\right)\mathbf{i} + \sin\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$$

Find the angle between **u** and **v**.

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$\cos\theta = \frac{\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \cdot \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle}{\left\| \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \right\| \left\| \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \right\|}$$

$$\cos\theta = \frac{-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{\sqrt{\frac{2}{4} + \frac{2}{4}}\sqrt{\frac{3}{4} + \frac{1}{4}}}$$

$$\cos\theta = \frac{-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{1 \cdot 1}$$

$$\cos\theta = -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)$$

$$\theta = 105$$

Thus, the angle between the two vectors is $\frac{105 \cdot \pi}{180} = \frac{7\pi}{12}$.

ANSWER:

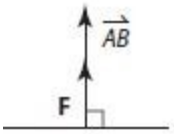
$\frac{7\pi}{12}$

55. **WORK** Tommy lifts his nephew, who weighs 16 kilograms, a distance of 0.9 meter. The force of weight in newtons can be calculated using $F = mg$, where m is the mass in kilograms and g is 9.8 meters per second squared. How much work did Tommy do to lift his nephew?

SOLUTION:

The force that Tommy is using to lift his nephew is $F = 16 \cdot 9.8$ or 156.8 newtons.

Diagram the situation.



Use the projection formula for work. Since the force vector **F** that represents Tommy lifting his nephew is parallel to $\overline{AB}$, **F** does not need to be projected on $\overline{AB}$. So, $\mathbf{F} = \text{proj}_{\overline{AB}}\mathbf{F}$. The magnitude of the directed distance $\overline{AB}$ is 0.9.

$$W = \left|\text{proj}_{\overline{AB}}\mathbf{F}\right|\left|\overline{AB}\right| = |\mathbf{F}|\left|\overline{AB}\right| = (156.8)(0.9) = 141.12$$

The work that Tommy did to lift his nephew was about 141.1 joules.

ANSWER:

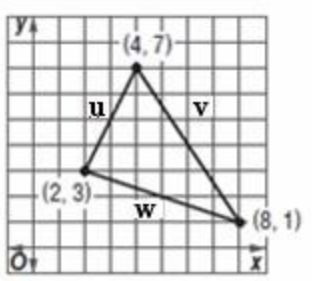
about 141.1 joules

The vertices of a triangle on the coordinate plane are given. Find the measures of the angles of each triangle using vectors. Round to the nearest tenth of a degree.

56. (2, 3), (4, 7), (8, 1)

SOLUTION:

Graph the triangle. Label **u**, **v**, and **w**.



The vectors can go in either direction, so find the vectors in component form for each initial point.

$\mathbf{u} = \langle 2, 4 \rangle$ or $\mathbf{u} = \langle -2, -4 \rangle$
$\mathbf{v} = \langle 4, -6 \rangle$ or $\mathbf{v} = \langle -4, 6 \rangle$
$\mathbf{w} = \langle -6, 2 \rangle$ or $\mathbf{w} = \langle 6, -2 \rangle$

Find the angle between **u** and **v**. The vertex (4, 7) must be the initial point of both vectors. Thus, use $\mathbf{u} = \langle -2, -4 \rangle$ and $\mathbf{v} = \langle 4, -6 \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -2, -4 \rangle \cdot \langle 4, -6 \rangle}{|\langle -2, -4 \rangle||\langle 4, -6 \rangle|}$$
$$\cos\theta = \frac{-8 + 24}{\sqrt{20}\sqrt{52}}$$
$$\cos\theta = \frac{16}{2\sqrt{5} \cdot 2\sqrt{13}}$$
$$\cos\theta = \frac{4}{\sqrt{65}}$$
$$\theta = \cos^{-1}\frac{4}{\sqrt{65}}$$
$$\theta \approx 60.3$$

Find the angle between **v** and **w**. The vertex (8, 1) must be the initial point of both vectors. Thus, use $\mathbf{v} = \langle -4, 6 \rangle$ and $\mathbf{w} = \langle -6, 2 \rangle$.

$$\cos\theta = \frac{v \cdot w}{|v||w|}$$
$$\cos\theta = \frac{\langle -4, 6 \rangle \cdot \langle -6, 2 \rangle}{|\langle -4, 6 \rangle||\langle -6, 2 \rangle|}$$
$$\cos\theta = \frac{24 + 12}{\sqrt{52}\sqrt{40}}$$
$$\cos\theta = \frac{36}{2\sqrt{13} \cdot 2\sqrt{10}}$$
$$\cos\theta = \frac{9}{\sqrt{130}}$$
$$\theta = \cos^{-1}\frac{9}{\sqrt{130}}$$
$$\theta \approx 37.9^\circ$$

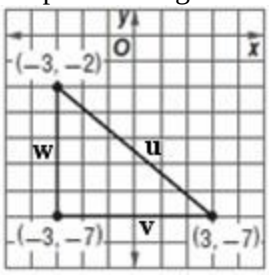
180 – (60.3 + 37.9) = 81.8

The three angles are about 37.9°, 60.3°, and 81.8°.

ANSWER:
37.9°, 60.3°, 81.8°

57. (–3, –2), (–3, –7), (3, –7)

SOLUTION:
Graph the triangle. Label **u**, **v**, and **w**.



The vectors can go either directions, so find the vectors in component form for each initial point.

$\mathbf{u} = \langle 6, -5 \rangle$ or $\mathbf{u} = \langle -6, 5 \rangle$
$\mathbf{v} = \langle -6, 0 \rangle$ or $\mathbf{v} = \langle 6, 0 \rangle$
$\mathbf{w} = \langle 0, 5 \rangle$ or $\mathbf{w} = \langle 0, -5 \rangle$

Find the angle between **u** and **v**. The vertex (3, –7) must be the initial point of both vectors. Thus, use $\mathbf{u} = \langle -6, 5 \rangle$ and $\mathbf{v} = \langle -6, 0 \rangle$.

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$
$$\cos\theta = \frac{\langle -6, 5\rangle \cdot \langle -6, 0\rangle}{|\langle -6, 5\rangle||\langle -6, 0\rangle|}$$
$$\cos\theta = \frac{(-6)(-6) + 5(0)}{\sqrt{(-6)^2 + 5^2} \cdot \sqrt{(-6)^2 + 0^2}}$$
$$\cos\theta = \frac{36 + 0}{\sqrt{61} \cdot 6}$$
$$\cos\theta = \frac{6}{\sqrt{61}}$$
$$\theta = \cos^{-1}\frac{6}{\sqrt{61}}$$
$$\theta \approx 39.8^\circ$$

Find the angle between **v** and **w**. The vertex (−3, −7) must be the initial point of both vectors. Thus, use $\mathbf{v} = \langle 6, 0\rangle$ and $\mathbf{w} = \langle 0, 5\rangle$.

$$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}||\mathbf{w}|}$$
$$\cos\theta = \frac{\langle 6, 0\rangle \cdot \langle 0, 5\rangle}{|\langle 6, 0\rangle||\langle 0, 5\rangle|}$$
$$\cos\theta = \frac{6 \cdot 0 + 0 \cdot 5}{\sqrt{6^2} \cdot \sqrt{5^2}}$$
$$\cos\theta = \frac{0}{6 \cdot 5}$$
$$\cos\theta = 0$$

Since **v** · **w** = 0, the two vectors are perpendicular. The equation can be further worked out to verify this conclusion.

$$\cos\theta = \frac{0}{6 \cdot 5}$$
$$\cos\theta = 0$$
$$\theta = 90^\circ$$

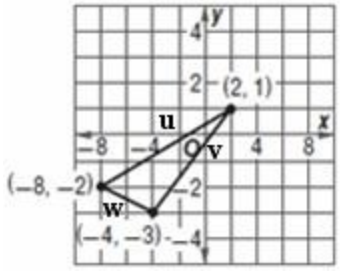
Find the angle between **u** and **w**. The vertex (−3, −2) must be the initial point of both vectors. Thus, use $\mathbf{u} = \langle 6, -5\rangle$ and $\mathbf{w} = \langle 0, -5\rangle$.

180 – (90 + 50.2) = 39.8
The three angles are about 39.8°, 50.2°, and 90°.

ANSWER:
39.8°, 50.2°, 90°

58. (−4, −3), (−8, −2), (2, 1)

SOLUTION:
Graph the triangle. Label **u**, **v**, and **w**.



The vectors can go in either direction, so find the vectors in component form for each initial point.
$\mathbf{u} = \langle 10, 3\rangle$ or $\mathbf{u} = \langle -10, -3\rangle$
$\mathbf{v} = \langle 6, 4\rangle$ or $\mathbf{v} = \langle -6, -4\rangle$
$\mathbf{w} = \langle -4, 1\rangle$ or $\mathbf{w} = \langle 4, -1\rangle$

Find the angle between **u** and **v**. The vertex (2, 1) must be the initial point of both vectors. Thus, use $\mathbf{u} = \langle -10, -3\rangle$ and $\mathbf{v} = \langle -6, -4\rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -10, -3\rangle \cdot \langle -6, -4\rangle}{|\langle 10, 3\rangle||\langle -6, -4\rangle|}$$
$$\cos\theta = \frac{(-10)(-6) + (-3)(-4)}{\sqrt{10^2 + 3^2}\sqrt{(-6)^2 + (-4)^2}}$$
$$\cos\theta = \frac{60 + 12}{\sqrt{109}\sqrt{52}}$$
$$\cos\theta = \frac{36}{\sqrt{1417}}$$
$$\theta = \cos^{-1}\frac{36}{\sqrt{1417}}$$
$$\theta \approx 17.0^\circ$$

Find the angle between **v** and **w**. The vertex (−4, −3) must be the initial point of both vectors. Thus, use $\mathbf{v} = \langle 6, 4\rangle$ and $\mathbf{w} = \langle -4, 1\rangle$.

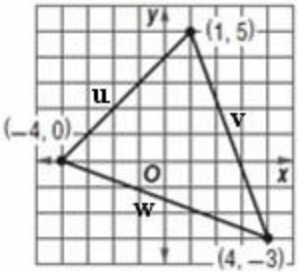
$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{|\mathbf{v}||\mathbf{w}|}$$
$$\cos\theta = \frac{\langle 6,4\rangle\cdot\langle -4,1\rangle}{|\langle 6,4\rangle||\langle -4,1\rangle|}$$
$$\cos\theta = \frac{6(-4)+4(1)}{\sqrt{6^2+4^2}\sqrt{4^2+1^2}}$$
$$\cos\theta = \frac{-24+4}{\sqrt{52}\sqrt{17}}$$
$$\cos\theta = \frac{-10}{\sqrt{221}}$$
$$\theta = \cos^{-1}\frac{-10}{\sqrt{221}}$$
$$\theta \approx 132.3^\circ$$

180 – (17.0 + 132.3) = 30.7
Thus, the three angles are about 17.0°, 30.7°, and 132.3°.

ANSWER:
17.0°, 30.7°, 132.3°

59. (1, 5), (4, –3), (–4, 0)

SOLUTION:
Graph the triangle. Label **u**, **v**, and **w**.



The vectors can go in either direction, so find the vectors in component form for each initial point.
$\mathbf{u} = \langle 5,5\rangle$ or $\mathbf{u} = \langle -5,-5\rangle$
$\mathbf{v} = \langle 3,-8\rangle$ or $\mathbf{v} = \langle -3,8\rangle$
$\mathbf{w} = \langle 8,-3\rangle$ or $\mathbf{w} = \langle -8,3\rangle$

Find the angle between **u** and **v**. The vertex (−4, 0) must be the initial point of both vectors. Thus, use $\mathbf{u} = \langle -5,-5\rangle$ and $\mathbf{v} = \langle 3,-8\rangle$.

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -5,-5\rangle\cdot\langle 3,-8\rangle}{|\langle -5,-5\rangle||\langle 3,-8\rangle|}$$
$$\cos\theta = \frac{(-5)3+(-5)(-8)}{\sqrt{(-5)^2+(-5)^2}\sqrt{3^2+(-8)^2}}$$
$$\cos\theta = \frac{-15+40}{\sqrt{50}\sqrt{73}}$$
$$\cos\theta = \frac{5}{\sqrt{146}}$$
$$\theta = \cos^{-1}\frac{5}{\sqrt{146}}$$
$$\theta \approx 65.6^\circ$$

Find the angle between **v** and **w**. The vertex (4, −3) must be the initial point of both vectors. Thus, use $\mathbf{v} = \langle -3,8\rangle$ and $\mathbf{w} = \langle -8,3\rangle$.

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{|\mathbf{v}||\mathbf{w}|}$$
$$\cos\theta = \frac{\langle -3,8\rangle\cdot\langle -8,3\rangle}{|\langle -3,8\rangle||\langle -8,3\rangle|}$$
$$\cos\theta = \frac{(-3)(-8)+8(3)}{\sqrt{(-3)^2+8^2}\sqrt{(-8)^2+3^2}}$$
$$\cos\theta = \frac{24+24}{\sqrt{73}\sqrt{73}}$$
$$\cos\theta = \frac{48}{73}$$
$$\theta = \cos^{-1}\frac{48}{73}$$
$$\theta \approx 48.9^\circ$$

180 – (65.6 + 48.9) = 65.6
The three angles are about 48.9°, 65.6°, and 65.6°.

ANSWER:
48.9°, 65.6°, 65.6°

Given u, |v| , and θ, the angle between u and v, find possible values of v. Round to the nearest hundredth.

60. $\mathbf{u} = \langle 4, -2\rangle$, $|\mathbf{v}| = 10$, 45°

SOLUTION:
Let $\mathbf{v} = \langle x, y\rangle$. Substitute the given values into the

equation for the angle between to vectors..

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos 45^\circ = \frac{\langle 4,-2\rangle\cdot\langle x,y\rangle}{|\langle 4,-2\rangle|\cdot 10}$$
$$\frac{\sqrt{2}}{2} = \frac{4x-2y}{\sqrt{20}\cdot 10}$$
$$\frac{\sqrt{2}}{2} = \frac{4x-2y}{20\sqrt{5}}$$
$$20\sqrt{5}\cdot\frac{\sqrt{2}}{2} = 4x-2y$$
$$10\sqrt{10} = 4x-2y$$

Solve $10\sqrt{10} = 4x - 2y$ for y.

$$10\sqrt{10} = 4x-2y$$
$$10\sqrt{10}-4x = -2y$$
$$\frac{10\sqrt{10}-4x}{-2} = y$$
$$-5\sqrt{10}+2x = y$$

Since $|\mathbf{v}| = 10$, $x^2 + y^2 = 100$. Substitute $y = -5\sqrt{10} + 2x$ into this equation and solve for x.

$$x^2+y^2 = 100$$
$$x^2+(-5\sqrt{10}+2x)^2 = 100$$
$$x^2+250-20\sqrt{10}x+4x^2 = 100$$
$$5x^2-20\sqrt{10}x+150 = 0$$

Use the Pythagorean Theorem.

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-20\sqrt{10})\pm\sqrt{(-20\sqrt{10})^2-4(5)(150)}}{2(5)}$$
$$= \frac{20\sqrt{10}\pm\sqrt{1000}}{10}$$
$$= \frac{20\sqrt{10}\pm 10\sqrt{10}}{10}$$
$$= 2\sqrt{10}\pm 1\sqrt{10}$$
$$\approx 9.49 \text{ or } 3.16$$

Find y when x = 9.49.

$$-5\sqrt{10}+2x = y$$
$$-5\sqrt{10}+2(9.49) = y$$
$$3.17 \approx y$$

Find y when x = 3.16.

$$-5\sqrt{10}+2x = y$$
$$-5\sqrt{10}+2(3.16) = y$$
$$-9.49 \approx y$$

$\langle 9.49, 3.17\rangle$ or $\langle 3.16, -9.49\rangle$

ANSWER:
$\langle 9.49, 3.17\rangle$ or $\langle 3.16, -9.49\rangle$

61. $\mathbf{u} = \langle 3, 4\rangle$, $|\mathbf{v}| = \sqrt{29}$, 121°

SOLUTION:

Let $\mathbf{v} = \langle x, y\rangle$. Substitute the given values into the equation for the angle between to vectors..

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos 121^\circ = \frac{\langle 3,4\rangle\cdot\langle x,y\rangle}{|\langle 3,4\rangle|\cdot\sqrt{29}}$$
$$\cos 121^\circ = \frac{3x+4y}{5\cdot\sqrt{29}}$$
$$5\sqrt{29}\cdot\cos 121^\circ = 3x+4y$$
$$-13.87 = 3x+4y$$
$$-13.87-3x = 4y$$
$$\frac{-13.87-3x}{4} = y$$

Since $|\mathbf{v}| = \sqrt{29}$, $x^2 + y^2 = 29$. Substitute $y = \frac{-13.87-3x}{4}$ into this equation and solve for x.

$$x^2+y^2 = 29$$
$$x^2+\left(\frac{-13.87-3x}{4}\right)^2 = 29$$
$$x^2+\frac{192.38+83.22x+9x^2}{16} = 29$$
$$16x^2+192.38+83.22x+9x^2 = 464$$
$$25x^2+83.22x-271.62 = 0$$

Use the Pythagorean Theorem.

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(83.22)\pm\sqrt{(83.22)^2-4(25)(-271.62)}}{2(25)}$$
$$=\frac{-83.22\pm\sqrt{34,087.57}}{50}$$
$$\approx 2.03 \text{ or } -5.36$$

Find y when x = 2.03.

$$\frac{-13.87-3x}{4}=y$$
$$\frac{-13.87-3(2.03)}{4}=y$$
$$-4.99\approx y$$

Find y when x = −5.36.

$$\frac{-13.87-3x}{4}=y$$
$$\frac{-13.87-3(-5.36)}{4}=y$$
$$0.55\approx y$$

$\langle -5.36, 0.55\rangle$ or $\langle 2.03. -4.99\rangle$

ANSWER:

$\langle -5.36, 0.55\rangle$ or $\langle 2.03, -4.99\rangle$

62. $\mathbf{u}=\langle -1, -6\rangle$, $|\mathbf{v}|=7$, 96°

SOLUTION:

Let $\mathbf{v}=\langle x, y\rangle$. Substitute the given values into the equation for the angle between to vectors..

$$\cos\theta=\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos 96^\circ=\frac{\langle -1,-6\rangle\cdot\langle x,y\rangle}{|\langle -1,-6\rangle|\cdot 7}$$
$$\cos 96^\circ=\frac{-x-6y}{\sqrt{37}\cdot 7}$$
$$7\sqrt{37}\cdot\cos 96^\circ=-x-6y$$
$$-4.45=-x-6y$$
$$-4.45+x=-6y$$
$$\frac{-4.45+x}{-6}=y$$

Since $|\mathbf{v}|=7$, $x^2+y^2=49$. Substitute $y=\frac{-4.45+x}{-6}$ into this equation and solve for x.

$$x^2+y^2=49$$
$$x^2+\left(\frac{-4.45+x}{-6}\right)^2=49$$
$$x^2+\frac{19.8-8.9x+x^2}{36}=49$$
$$36x^2+19.8-8.9x+x^2=1764$$
$$37x^2-8.9x-1744.2=0$$

Use the Pythagorean Theorem.

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-8.9)\pm\sqrt{(-8.9)^2-4(37)(-1744.2)}}{2(37)}$$
$$=\frac{8.9\pm\sqrt{258,220.81}}{74}$$
$$\approx 6.99 \text{ or } -6.75$$

Find y when x = 6.99.

$$\frac{-4.45+x}{-6}=y$$
$$\frac{-4.45+6.99}{-6}=y$$
$$-0.42\approx y$$

Find y when x = −6.75.

$$\frac{-4.45+x}{-6}=y$$
$$\frac{-4.45+(-6.75)}{-6}=y$$
$$1.87\approx y$$

$-\langle 6.75, 1.87\rangle$ or $\langle 6.99, -0.42\rangle$

ANSWER:

$\langle -6.75, 1.87\rangle$ or $\langle 6.99, -0.42\rangle$

63. $\mathbf{u}=\langle -2, 5\rangle$, $|\mathbf{v}|=12$, 27°

SOLUTION:

Let $\mathbf{v}=\langle x, y\rangle$. Substitute the given values into the

equation for the angle between to vectors..

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos 27^\circ = \frac{\langle -2,5\rangle \cdot \langle x,y\rangle}{|\langle -2,5\rangle| \cdot 12}$$

$$\cos 27^\circ = \frac{-2x+5y}{\sqrt{29}\cdot 12}$$

$$12\sqrt{29}\cdot\cos 27^\circ = -2x+5y$$

$$57.58 = -2x+5y$$

$$57.58+2x = 5y$$

$$\frac{57.58+2x}{5} = y$$

Since $|\mathbf{v}| = 12$, $x^2 + y^2 = 144$. Substitute $y = \frac{57.58+2x}{5}$ into this equation and solve for x.

$$x^2 + y^2 = 144$$

$$x^2 + \left(\frac{57.58+2x}{5}\right)^2 = 144$$

$$x^2 + \frac{3315.46+230.32x+4x^2}{25} = 144$$

$$25x^2 + 3315.46 + 230.32x + 4x^2 = 3600$$

$$29x^2 + 230.32x - 284.54 = 0$$

Use the Pythagorean Theorem.

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(230.32)\pm\sqrt{(230.32)^2-4(29)(-284.54)}}{2(29)}$$

$$= \frac{-230.32\pm\sqrt{86,053.94}}{58}$$

$$\approx 1.09 \text{ or } -9.03$$

Find y when $x = 1.09$.

$$\frac{57.58+2x}{5} = y$$

$$\frac{57.58+2(1.09)}{5} = y$$

$$11.95 \approx y$$

Find y when $x = -9.03$.

$$\frac{57.58+2x}{5} = y$$

$$\frac{57.58+2(-9.03)}{5} = y$$

$$7.90 \approx y$$

$\langle -9.03, 7.90\rangle$ or $\langle 1.09, 11.95\rangle$

ANSWER:

$\langle -9.03, 7.90\rangle$ or $\langle 1.09, 11.95\rangle$

64. **CARS** A car is stationary on a 9° incline. Assuming that the only forces acting on the car are gravity and the 275 Newton force applied by the brakes, about how much does the car weigh?



SOLUTION:

The car's weight is the force exerted due to gravity and can be represented by the force vector $\mathbf{F} = \langle 0, y \rangle$. The force required to keep the car from rolling is 275 newtons. Project **F** onto a unit vector **v** in the direction of the side of the incline to solve for the weight of the car.

Find a unit vector **v** in the direction of the side of the incline.

$$\begin{aligned}\mathbf{v} &= \langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \rangle \\ &= \langle 1(\cos 9^\circ), 1(\sin 9^\circ) \rangle \\ &= \langle \cos 9^\circ, \sin 9^\circ \rangle\end{aligned}$$

Find the projection of **F** onto the unit vector **v**, $\text{proj}_{\mathbf{v}}\mathbf{F}$.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{F} &= \left(\frac{\mathbf{F}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= (\mathbf{F}\cdot\mathbf{v})\mathbf{v} \\ &= \left(\langle 0, y \rangle \cdot \langle \cos 9^\circ, \sin 9^\circ \rangle\right)\mathbf{v} \\ &= (y \sin 9^\circ)\mathbf{v}\end{aligned}$$

The force required is (y sin 9°)**v**. Since **v** is a unit vector, the magnitude of the force required to keep the car stationary is (y sin 9°). Solve for y.

$$\begin{aligned}y \sin 9^\circ &= 275 \\ y &= \frac{275}{\sin 9^\circ} \\ y &\approx 1757.9\end{aligned}$$

Since the force is given in units, the car's weight will be given in kilograms. Thus, the car weighs about 1757.9 kilograms.

ANSWER:

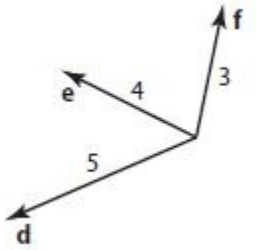
about 1757.9 kg

65. **REASONING** Determine whether the statement below is *true* or *false*. Explain.
If |**d**|, |**e**|, and |**f**| form a Pythagorean triple, and the angles between **d** and **e** and between **e** and **f** are acute, then the angle between **d** and **f** must be a right angle. Explain your reasoning.

SOLUTION:

The statement is false. All three vectors may originate at the same point and not form a triangle at all. If so, the angle between **d** and **f** may be acute, right, or obtuse.

For example:



Drawing is not to scale.

|**d**|, |**e**|, and |**f**| form a Pythagorean triple, and the angles between **d** and **e** and between **e** and **f** are acute, but the angle between **d** and **f** is obtuse.

ANSWER:

False; sample answer: All three vectors may originate at the same point and not form a triangle at all. If so, the angle between **d** and **f** may be acute, right, or obtuse.

66. **ERROR ANALYSIS** Beng and Ethan are studying the properties of the dot product. Beng concludes that the dot product is associative because it is commutative; that is $(\mathbf{u}\cdot\mathbf{v})\cdot\mathbf{w} = \mathbf{u}\cdot(\mathbf{v}\cdot\mathbf{w})$. Ethan disagrees. Is either of them correct? Explain your reasoning.

SOLUTION:

Ethan is correct. The dot product of $\mathbf{u}\cdot\mathbf{v}$ will result in some value x. Since x is a scalar and not a vector, $x\cdot\mathbf{w}$ is undefined. So, $(\mathbf{u}\cdot\mathbf{v})\cdot\mathbf{w}$ is undefined.

ANSWER:

Ethan; sample answer: **u** · **v** is a scalar, so (**u** · **v**) · **w** is undefined.

67. **REASONING** Determine whether the statement below is *true* or *false*.
If **a** and **b** are both orthogonal to **v** in the plane, then **a** and **b** are parallel. Explain your reasoning.

SOLUTION:

The statement is false. If either **a** or **b** is a zero vector, it would be orthogonal to **v**, but could not be parallel to the other vector. Recall that the terms perpendicular and orthogonal have essentially the same meaning, except when **a** or **b** is the zero vector. The zero vector is orthogonal to any vector, but since it has no magnitude or direction, it cannot be perpendicular to a vector.

ANSWER:

False; sample answer: If either **a** or **b** is a zero vector, it would be orthogonal to **v**, but could not be parallel to the other vector.

68. **CHALLENGE** If **u** and **v** are perpendicular, what is the projection of **u** onto **v**?

SOLUTION:

The equation for the projection of **u** onto **v** is $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$. If **u** and **v** are perpendicular, then they are orthogonal. So, $\mathbf{u}\cdot\mathbf{v} = 0$. Substitute $\mathbf{u}\cdot\mathbf{v} = 0$ into the equation for the projection of **u** onto **v**.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{0}{|\mathbf{v}|^2}\right)\mathbf{v}$$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = 0$$

So, the projection of **u** onto **v** is 0.

ANSWER:

0

69. **PROOF** Show that if the angle between vectors **u** and **v** is 90°, $\mathbf{u}\cdot\mathbf{v} = 0$ using the formula for the angle between two nonzero vectors.

SOLUTION:

$$\cos 90° = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|} \quad \text{Angle between } \mathbf{u} \text{ and } \mathbf{v} \text{ where } \theta = 90°.$$

$$0 = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|} \quad \cos 90° = 0.$$

$$0 = \mathbf{u}\cdot\mathbf{v} \quad \text{Multiply each side by } |\mathbf{u}|\cdot|\mathbf{v}|.$$

ANSWER:

$$\cos 90° = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|} \quad \text{Angle between } \mathbf{u} \text{ and } \mathbf{v} \text{ where } \theta = 90°.$$

$$0 = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|} \quad \cos 90° = 0.$$

$$0 = \mathbf{u}\cdot\mathbf{v} \quad \text{Multiply each side by } |\mathbf{u}|\cdot|\mathbf{v}|.$$

PROOF Prove each dot product property. Let $\mathbf{u} = \langle u_1, u_2 \rangle$, $\mathbf{v} = \langle v_1, v_2 \rangle$, and $\mathbf{w} = \langle w_1, w_2 \rangle$.

70. $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$

SOLUTION:

$$\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$$

$$\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle \stackrel{?}{=} \langle v_1, v_2 \rangle \cdot \langle u_1, u_2 \rangle$$

$$u_1v_1 + u_2v_2 = u_1v_1 + u_2v_2$$

ANSWER:

$$\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$$

$$\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle \stackrel{?}{=} \langle v_1, v_2 \rangle \cdot \langle u_1, u_2 \rangle$$

$$u_1v_1 + u_2v_2 = u_1v_1 + u_2v_2$$

71. $\mathbf{u}\cdot(\mathbf{v}+\mathbf{w}) = \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}$

SOLUTION:

$$\mathbf{u}\cdot(\mathbf{v}+\mathbf{w}) = \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}$$

$$\langle u_1, u_2 \rangle \cdot (\langle v_1, v_2 \rangle + \langle w_1, w_2 \rangle) \stackrel{?}{=} \langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle \cdot \langle w_1, w_2 \rangle$$

$$\langle u_1, u_2 \rangle \cdot \langle v_1 + w_1, v_2 + w_2 \rangle \stackrel{?}{=} (u_1v_1 + u_2v_2) + (u_1w_1 + u_2w_2)$$

$$u_1(v_1 + w_1) + u_2(v_2 + w_2) \stackrel{?}{=} u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2$$

$$u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2 = u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2$$

ANSWER:

$$\mathbf{u}\cdot(\mathbf{v}+\mathbf{w}) = \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{w}$$

$$\langle u_1, u_2 \rangle \cdot (\langle v_1, v_2 \rangle + \langle w_1, w_2 \rangle) \stackrel{?}{=} \langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle \cdot \langle w_1, w_2 \rangle$$

$$\langle u_1, u_2 \rangle \cdot \langle v_1 + w_1, v_2 + w_2 \rangle \stackrel{?}{=} (u_1v_1 + u_2v_2) + (u_1w_1 + u_2w_2)$$

$$u_1(v_1 + w_1) + u_2(v_2 + w_2) \stackrel{?}{=} u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2$$

$$u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2 = u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2$$

72. $k(\mathbf{u} \cdot \mathbf{v}) = k\mathbf{u} \cdot \mathbf{v} = \mathbf{u}k \cdot \mathbf{v}$

SOLUTION:

$$\begin{array}{ll} k(\mathbf{u} \cdot \mathbf{v}) = k\mathbf{u} \cdot \mathbf{v} & = \mathbf{u} \cdot k\mathbf{v} \\ k(\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle) \stackrel{?}{=} k\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle & \stackrel{?}{=} \langle u_1, u_2\rangle \cdot k\langle v_1, v_2\rangle \\ k(u_1v_1 + u_2v_2) \stackrel{?}{=} \langle ku_1, ku_2\rangle \cdot \langle v_1, v_2\rangle & \stackrel{?}{=} \langle u_1, u_2\rangle \cdot \langle kv_1, kv_2\rangle \\ ku_1v_1 + ku_2v_2 = ku_1v_1 + ku_2v_2 & = ku_1v_1 + ku_2v_2 \end{array}$$

ANSWER:

$$\begin{array}{ll} k(\mathbf{u} \cdot \mathbf{v}) = k\mathbf{u} \cdot \mathbf{v} & = \mathbf{u} \cdot k\mathbf{v} \\ k(\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle) \stackrel{?}{=} k\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle & \stackrel{?}{=} \langle u_1, u_2\rangle \cdot k\langle v_1, v_2\rangle \\ k(u_1v_1 + u_2v_2) \stackrel{?}{=} \langle ku_1, ku_2\rangle \cdot \langle v_1, v_2\rangle & \stackrel{?}{=} \langle u_1, u_2\rangle \cdot \langle kv_1, kv_2\rangle \\ ku_1v_1 + ku_2v_2 = ku_1v_1 + ku_2v_2 & = ku_1v_1 + ku_2v_2 \end{array}$$

73. ***Writing in Math*** Explain how to find the dot product of two nonzero vectors.

SOLUTION:

Sample answer: For two nonzero vectors $\langle a, b\rangle$ and $\langle c, d\rangle$, the dot product $\langle a,b\rangle \cdot \langle c,d\rangle$ is the sum of the products of the *x*-coordinates and the *y*-coordinates, or $ac + bd$.

ANSWER:

Sample answer: For two nonzero vectors $\langle a, b\rangle$ and $\langle c, d\rangle$, the dot product $\langle a, b\rangle \cdot \langle c, d\rangle$ is the sum of the products of the *x*-coordinates and the *y*-coordinates, or $ac + bd$.

Find each of the following for $a = \langle 10, 1\rangle$, $b = \langle -5, 2.8\rangle$, and $c = \left\langle \frac{3}{4}, -9 \right\rangle$.

74. **b** – **a** + 4**c**

SOLUTION:

$$\begin{aligned} \mathbf{b} - \mathbf{a} + 4\mathbf{c} &= \langle -5, 2.8\rangle - \langle 10, 1\rangle + (4)\left\langle \frac{3}{4}, -9 \right\rangle \\ &= \langle -5, 2.8\rangle - \langle 10, 1\rangle + \langle 3, -36\rangle \\ &= \langle -15, 1.8\rangle + \langle 3, -36\rangle \\ &= \langle -12, -34.2\rangle \end{aligned}$$

ANSWER:

$\langle -12, -34.2\rangle$

75. **c** – 3**a** + **b**

SOLUTION:

$$\begin{aligned} \mathbf{c} - 3\mathbf{a} + \mathbf{b} &= \left\langle \frac{3}{4}, -9 \right\rangle - (3)\langle 10, 1\rangle + \langle -5, 2.8\rangle \\ &= \left\langle \frac{3}{4}, -9 \right\rangle - \langle 30, 3\rangle + \langle -5, 2.8\rangle \\ &= \left\langle -\frac{117}{4}, -12 \right\rangle + \langle -5, 2.8\rangle \\ &= \left\langle -\frac{137}{4}, -9.2 \right\rangle \end{aligned}$$

ANSWER:

$\left\langle -\frac{137}{4}, -9.2 \right\rangle$

76. 2**a** – 4**b** + **c**

SOLUTION:

$$\begin{aligned} 2\mathbf{a} - 4\mathbf{b} + \mathbf{c} &= (2)\langle 10, 1\rangle - (4)\langle -5, 2.8\rangle + \left\langle \frac{3}{4}, -9 \right\rangle \\ &= \langle 20, 2\rangle - \langle -20, 11.2\rangle + \left\langle \frac{3}{4}, -9 \right\rangle \\ &= \langle 40, -9.2\rangle + \left\langle \frac{3}{4}, -9 \right\rangle \\ &= \left\langle \frac{163}{4}, -18.2 \right\rangle \end{aligned}$$

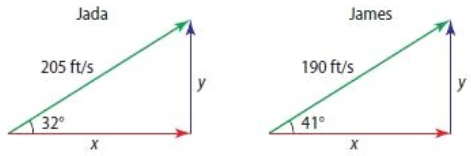
ANSWER:

$\left\langle \frac{163}{4}, -18.2 \right\rangle$

77. **GOLF** Jada drives a golf ball with a velocity of 205 feet per second at an angle of 32° with the ground. On the same hole, James drives a golf ball with a velocity of 190 feet per second at an angle of 41°. Find the magnitudes of the horizontal and vertical components for each force.

SOLUTION:

Draw a diagram that shows the resolution of the force that Jada and James exert on the golf ball into its rectangular components.



The horizontal and vertical components of each force form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.

For Jada:

$$\cos 32^\circ = \frac{x}{205} \qquad \sin 32^\circ = \frac{y}{205}$$
$$205\cos 32^\circ = x \qquad 205\sin 32^\circ = y$$
$$173.8 \approx x \qquad 108.6 \approx y$$

For James:

$$\cos 41^\circ = \frac{x}{190} \qquad \sin 41^\circ = \frac{y}{190}$$
$$190\cos 41^\circ = x \qquad 190\sin 41^\circ = y$$
$$143.4 \approx x \qquad 124.7 \approx y$$

So, the horizontal and vertical components for Jada are about 173.8 feet per second and about 108.6 feet per second. The horizontal and vertical components for James are about 143.4 feet per second and about 124.7 feet per second.

ANSWER:

Jada: about 173.8 ft/s and about 108.6 ft/s; James: about 143.4 ft/s and about 124.7 ft/s

Graph the hyperbola given by each equation.

78. $\frac{(x-5)^2}{48} - \frac{y^2}{5} = 1$

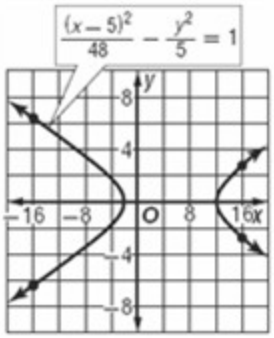
SOLUTION:

The equation is in standard form, where $h = 5$, $k = 0$, $a = \sqrt{48}$ or about 6.93, $b = \sqrt{5}$ or about 2.24, and $c = \sqrt{48+5}$ or about 7.28. In the standard form of the equation, the y−term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.

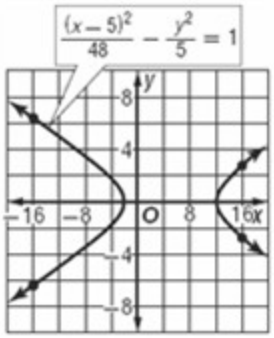
center: $(h, k) = (5, 0)$
vertices: $(h \pm a, k) = (11.93, 0)$ and $(-1.93, 0)$
foci: $(h \pm c, k) = (12.28, 0)$ and $(-2.28, 0)$
asymptotes:

$$y - k = \pm\frac{b}{a}(x-h)$$
$$y - 0 = \pm\frac{\sqrt{5}}{4\sqrt{3}}(x-5)$$
$$y = \pm\frac{\sqrt{15}}{12}(x-5)$$
$$y = \pm\left[\frac{\sqrt{15}}{12}x + \frac{5\sqrt{15}}{12}\right]$$
$$y = \frac{\sqrt{15}}{12}x + \frac{5\sqrt{15}}{12} \quad OR$$
$$y = -\frac{\sqrt{15}}{12}x - \frac{5\sqrt{15}}{12}$$

Use a table of values to sketch the hyperbola.



ANSWER:



79. $\frac{x^2}{81} - \frac{y^2}{49} = 1$

SOLUTION:

The equation is in standard form, where $h = 0$, $k = 0$, $a = 9$, $b = 7$, and $c = \sqrt{81+49}$ or about 11.4. In the standard form of the equation, the y-term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.

center: $(h, k) = (0, 0)$

vertices: $(h \pm a, k) = (9, 0)$ and $(-9, 0)$

foci: $(h \pm c, k) = (11.4, 0)$ and $(-11.4, 0)$

asymptotes:

$$y - k = \pm\frac{b}{a}(x - h)$$

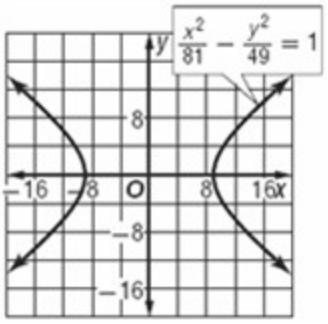
$$y - 0 = \pm\frac{7}{9}(x - 0)$$

$$y = \pm\frac{7}{9}x$$

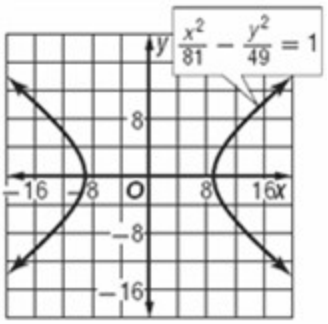
$$y = \frac{7}{9}x \quad OR$$

$$y = -\frac{7}{9}x$$

Use a table of values to sketch the hyperbola.



ANSWER:



80. $\frac{y^2}{36} - \frac{x^2}{4} = 1$

SOLUTION:

The equation is in standard form, where $h = 0$, $k = 0$, $a = 6$, $b = 2$, and $c = \sqrt{36+4}$ or about 6.32. In the standard form of the equation, the x-term is being subtracted. Therefore, the orientation of the hyperbola is vertical.

center: $(h, k) = (0, 0)$

vertices: $(h, k \pm a) = (0, 6)$ and $(0, -6)$

foci: $(h, k \pm c) = (0, 6.32)$ and $(0, -6.32)$

asymptotes:

$$y - k = \pm\frac{a}{b}(x - h)$$

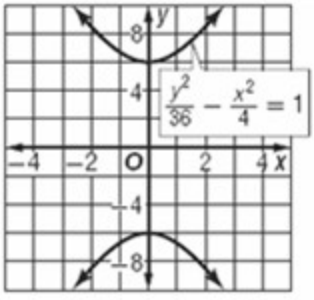
$$y - 0 = \pm\frac{6}{2}(x - 9)$$

$$y = \pm 3x$$

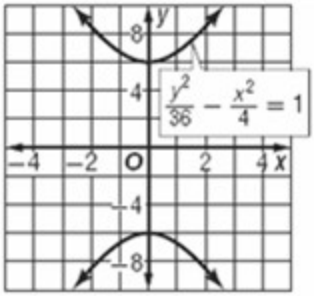
$$y = 3x \quad OR$$

$$y = -3x$$

Use a table of values to sketch the hyperbola.



ANSWER:



Find the exact value of each expression, if it exists.

81. $\arcsin\left(\sin\frac{\pi}{6}\right)$

SOLUTION:

First, find $\sin\frac{\pi}{6}$. On the unit circle, $\frac{\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$. So, $\sin\frac{\pi}{6}=\frac{1}{2}$.

Next, find arcsin.

The inverse property applies, because $\frac{1}{2}$ is on the interval [–1, 1]. Therefore, $\arcsin\frac{1}{2}=\frac{\pi}{6}$, and $\arcsin\left(\sin\frac{\pi}{6}\right)=\frac{\pi}{6}$.

ANSWER:

$\frac{\pi}{6}$

82. $\arctan\left(\tan\frac{1}{2}\right)$

SOLUTION:

The inverse property applies, because $\frac{1}{2}$ lies on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Therefore, $\arctan\left(\tan\frac{1}{2}\right)=\frac{1}{2}$.

ANSWER:

$\frac{1}{2}$

83. $\sin\left(\cos^{-1}\frac{3}{4}\right)$

SOLUTION:

To simplify the expression, let $u=\cos^{-1}\left(\frac{3}{4}\right)$ so $\cos u=\frac{3}{4}$. Because the cosine function is positive in Quadrants I and IV, and the domain of the inverse cosine function is restricted to Quadrants I and II, u must lie in Quadrant I. Using the Pythagorean Theorem, you can find that the length of the opposite side is $\sqrt{7}$. $\sin u=\frac{\text{opp}}{\text{hyp}}=\frac{\sqrt{7}}{4}$. So, $\sin\left(\cos^{-1}\frac{3}{4}\right)=\frac{\sqrt{7}}{4}$.

ANSWER:

$\frac{\sqrt{7}}{4}$

Solve each equation.

84. $\log_{12}(x^3+2)=\log_{12}127$

SOLUTION:

$$\log_{12}(x^3+2)=\log_{12}127$$
$$x^3+2=127$$
$$x^3=125$$
$$x=5$$

ANSWER:

5

85. $\log_2 x = \log_2 6 + \log_2 (x - 5)$

SOLUTION:

$$\log_2 x = \log_2 6 + \log_2 (x - 5)$$
$$\log_2 x = \log_2 6(x - 5)$$
$$\log_2 x = \log_2 (6x - 30)$$
$$x = 6x - 30$$
$$-5x = -30$$
$$x = 6$$

ANSWER:

6

86. $e^{5x-4} = 70$

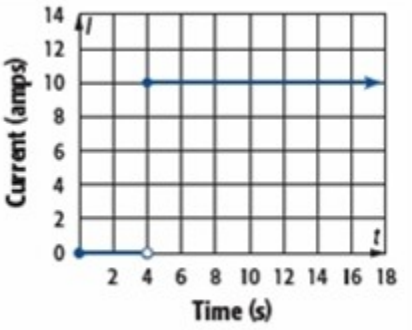
SOLUTION:

$$e^{5x-4} = 70$$
$$\ln e^{5x-4} = \ln 70$$
$$5x - 4 = \ln 70$$
$$5x = \ln 70 + 4$$
$$x = \frac{\ln 70 + 4}{5}$$
$$x \approx 1.65$$

ANSWER:

$\frac{\ln 70 + 4}{5} \approx 1.65$

87. **ELECTRICITY** A simple electric circuit contains only a power supply and a resistor. When the power supply is off, there is no current in the circuit. When the power supply is turned on, the current almost instantly becomes a constant value. This situation can be modeled by a graph like the one shown. I represents current in amps, and t represents time in seconds.



a. At what t-value is this function discontinuous?

b. When was the power supply turned on?

c. If the person who turned on the power supply left and came back hours later, what would he or she measure the current in the circuit to be?

SOLUTION:

a. There appears to be a jump discontinuity at $t = 4$. Therefore, the graph is discontinuous at $t = 4$.

b. The current is 0 amps for $0 \leq t < 4$ and 10 amps for $4 \leq t$. Therefore, it appears that the power supply was turned on at $t = 4$.

c. From the graph, it appears $\lim_{t \to \infty} I(t) = 10$.

Therefore, the current in the circuit would be 10 amps.

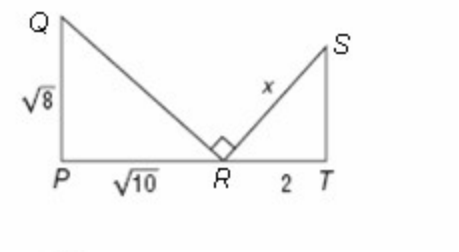
ANSWER:

a. $t = 4$

b. when $t = 4$

c. 10 amps

88. **SAT/ACT** In the figure below, $\Delta PQR \sim \Delta TRS$. What is the value of x?



A $\sqrt{2}$
B $\sqrt{5}$
C 3
D $3\sqrt{2}$
E 6

SOLUTION:

Since $\Delta PQR \sim \Delta TRS$, then $\frac{PQ}{TR} = \frac{QR}{RS}$. Use the Pythagorean Theorem to solve for QR.

$$(PR)^2 + (PQ)^2 = (QR)^2$$
$$\left(\sqrt{10}\right)^2 + \left(\sqrt{8}\right)^2 = (QR)^2$$
$$18 = (QR)^2$$
$$3\sqrt{2} = QR$$

Substitute PQ, TR, and QR into $\frac{PQ}{TR} = \frac{QR}{RS}$ and solve for RS.

$$\frac{\sqrt{8}}{2} = \frac{3\sqrt{2}}{RS}$$
$$RS\left(\sqrt{8}\right) = 2\left(3\sqrt{2}\right)$$
$$RS = \frac{2\left(3\sqrt{2}\right)}{\left(\sqrt{8}\right)}$$
$$RS = 3$$

So, $x = 3$.
The correct answer is C.

ANSWER:
C

89. **REVIEW** Consider $C(-9, 2)$ and $D(-4, -3)$. Which of the following is the component form and magnitude of $\overrightarrow{CD}$?

F $\langle 5, -5 \rangle, 5\sqrt{2}$
G $\langle 5, -5 \rangle, 6\sqrt{2}$
H $\langle 6, -5 \rangle, 5\sqrt{2}$
J $\langle 6, -6 \rangle, 6\sqrt{2}$

SOLUTION:

First, find the component form.

$$\overrightarrow{CD} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle -4 - (-9), -3 - 2 \rangle$$
$$= \langle 5, -5 \rangle$$

Next, find the magnitude. Substitute $x_2 - x_1 = 5$ and $y_2 - y_1 = -5$ into the formula for the magnitude of a vector in the coordinate plane.

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{5^2 + (-5)^2}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

The correct answer is F.

ANSWER:
F

90. A snow sled is pulled by exerting a force of 25 pounds on a rope that makes a 20° angle with the horizontal, as shown in the figure. What is the approximate work done in pulling the sled 50 feet?



A 428 foot-pounds
B 1093 foot-pounds
C 1175 foot-pounds
D 1250 foot-pounds

SOLUTION:

Use the projection formula for work. The magnitude of the projection of **F** onto $\overrightarrow{AB}$ is $|\mathbf{F}|\cos\theta = 25\cos 20^\circ$. The magnitude of the directed distance $\overrightarrow{AB}$ is 50.

$$\begin{aligned} W &= \left|\text{proj}_{\overrightarrow{AB}}\mathbf{F}\right|\left|\overrightarrow{AB}\right| \\ &= (25\cos 20^\circ)(50) \\ &\approx 1174.6 \end{aligned}$$

Therefore, the approximate work done in pulling the sled is about 1174.6 foot-pounds.
The correct answer is C.

ANSWER:

C

91. **REVIEW** If $\mathbf{s} = \langle 4, -3\rangle$, $\mathbf{t} = \langle -6, 2\rangle$, which of the following represents $\mathbf{t} - 2\mathbf{s}$?

F $\langle 14, 8\rangle$
G $\langle 14, 6\rangle$
H $\langle -14, 8\rangle$
J $\langle -14, -8\rangle$

SOLUTION:

$$\begin{aligned} \mathbf{t} - 2\mathbf{s} &= \mathbf{t} + (-2)\mathbf{s} \\ &= \langle -6, 2\rangle + (-2)\langle 4, -3\rangle \\ &= \langle -6, 2\rangle + \langle -8, 6\rangle \\ &= \langle -14, 8\rangle \end{aligned}$$

The correct answer is H.

ANSWER:

H

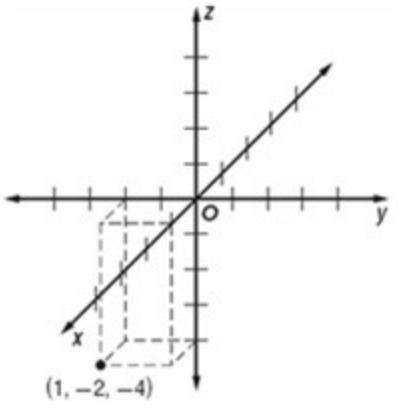
8-4 Vectors in Three-Dimensional Space

Plot each point in a three-dimensional coordinate system.

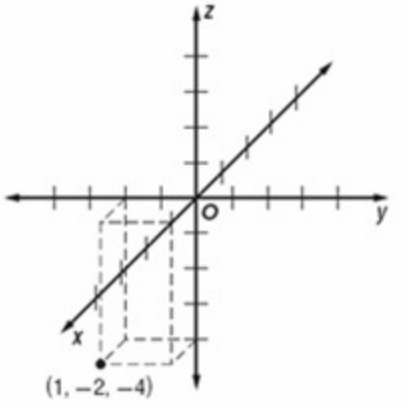
1. (1, −2, −4)

SOLUTION:

Starting at the origin, move 1 unit forward along the *x*-axis, 2 units to the left parallel to the *y*-axis, and then 4 units down parallel to the *z*-axis. Plot the point (1, −2, −4).



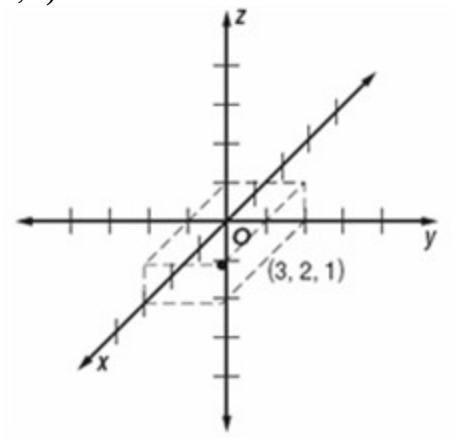
ANSWER:



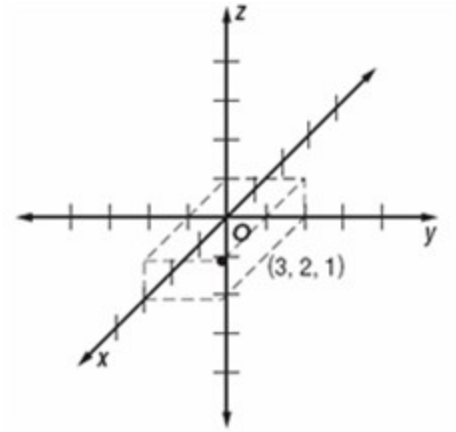
2. (3, 2, 1)

SOLUTION:

Starting at the origin, move 3 units forward along the *x*-axis, 2 units to the right parallel to the *y*-axis, and then 1 unit up parallel to the *z*-axis. Plot the point (3, 2, 1).



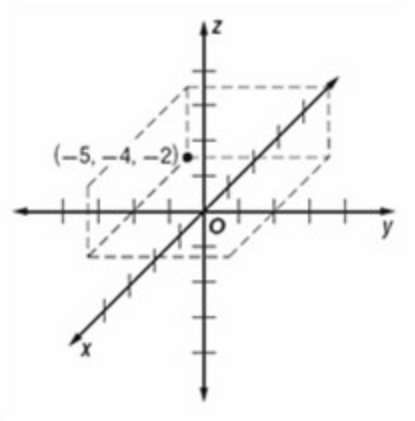
ANSWER:



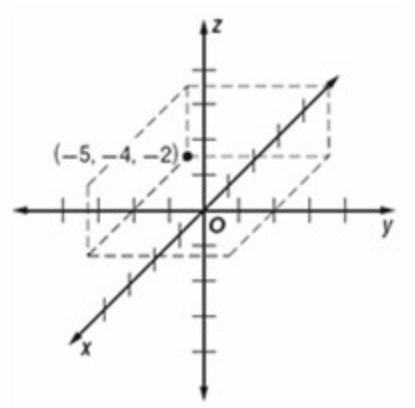
3. (−5, −4, −2)

SOLUTION:

Starting at the origin, move 5 units backwards along the x-axis, 4 units to the left parallel to the y-axis, and then 2 units down parallel to the z-axis. Plot the point
(−5, −4, −2).



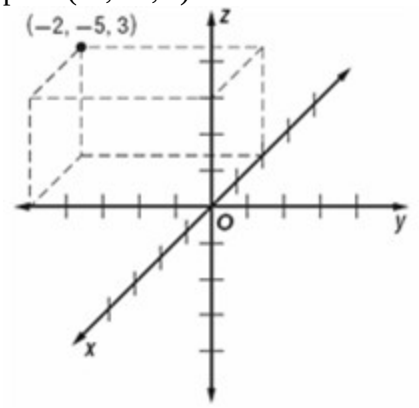
ANSWER:



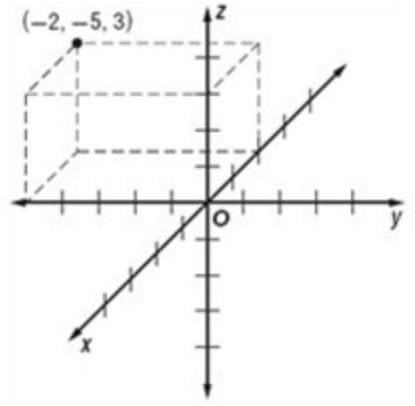
4. (−2, −5, 3)

SOLUTION:

Starting at the origin, move 2 units backwards along the x-axis, 5 units to the left parallel to the y-axis, and then 3 units up parallel to the z-axis. Plot the point (−2, −5, 3).



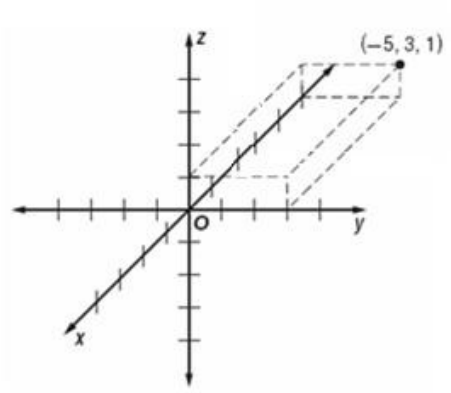
ANSWER:



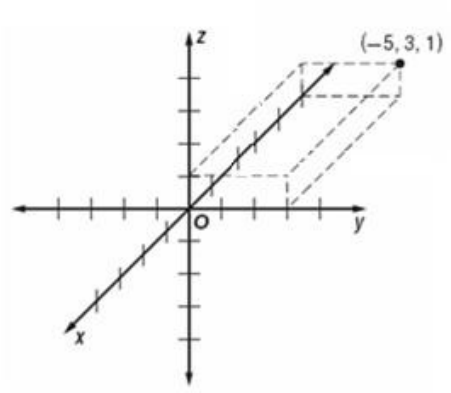
5. (−5, 3, 1)

SOLUTION:

Starting at the origin, move 5 units backwards along the x-axis, 3 units to the right parallel to the y-axis, and then 1 unit up parallel to the z-axis. Plot the point (−5, 3, 1).



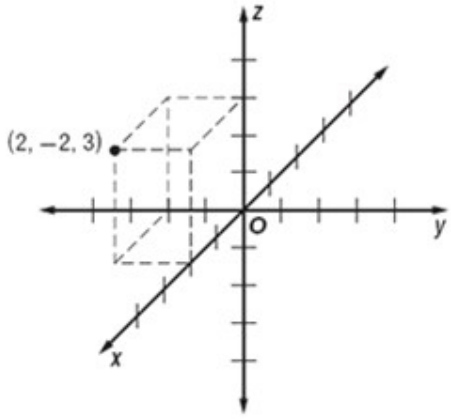
ANSWER:



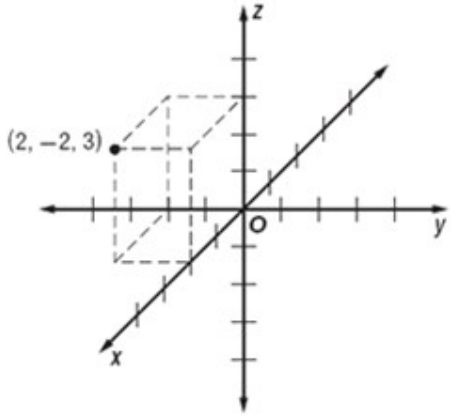
6. (2, −2, 3)

SOLUTION:

Starting at the origin, move 2 units forward along the x-axis, 2 units to the left parallel to the y-axis, and then 3 units up parallel to the z-axis. Plot the point (2, −2, 3).



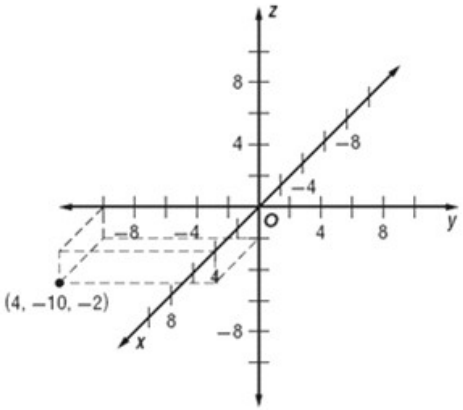
ANSWER:



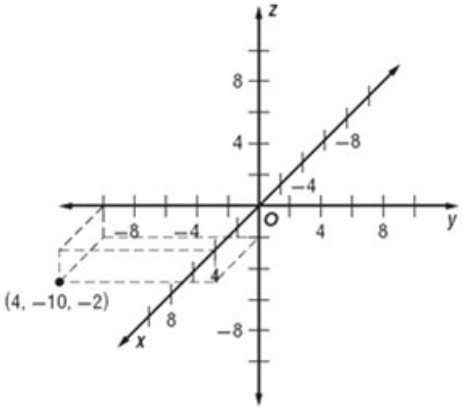
7. (4, −10, −2)

SOLUTION:

Starting at the origin, move 4 units forward along the x-axis, 10 units to the left parallel to the y-axis, and then 2 units down parallel to the z-axis. Plot the point (4, −10, −2).



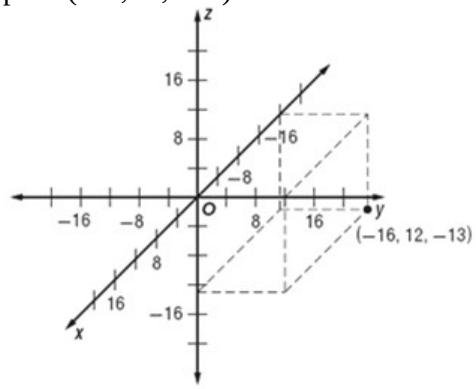
ANSWER:



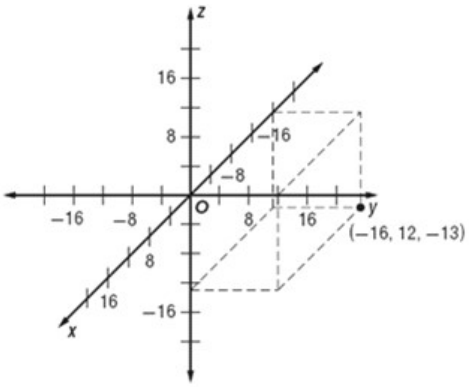
8. (−16, 12, −13)

SOLUTION:

Starting at the origin, move 16 units backwards along the x-axis, 12 units to the right parallel to the y-axis, and then 13 units down parallel to the z-axis. Plot the point (−16, 12, −13).



ANSWER:



Find the length and midpoint of the segment with the given endpoints.

9. (−4, 10, 4), (1, 0, 9)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{[1 - (-4)]^2 + (0 - 10)^2 + (9 - 4)^2} \\ &= \sqrt{25 + 100 + 25} \\ &= \sqrt{150} \\ &= 5\sqrt{6} \\ &\approx 12.2 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \left(\frac{-4 + 1}{2}, \frac{10 + 0}{2}, \frac{4 + 9}{2}\right) = \left(-\frac{3}{2}, 5, \frac{13}{2}\right)$$

ANSWER:

$$5\sqrt{6} \approx 12.2; \left(-\frac{3}{2}, 5, \frac{13}{2}\right)$$

10. (−6, 6, 3), (−9, −2, −2)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{[-9 - (-6)]^2 + (-2 - 6)^2 + (-2 - 3)^2} \\ &= \sqrt{9 + 64 + 25} \\ &= \sqrt{98} \\ &= 7\sqrt{2} \\ &\approx 9.9 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \left(\frac{-6 + (-9)}{2}, \frac{6 + (-2)}{2}, \frac{3 + (-2)}{2}\right) = \left(-\frac{15}{2}, 2, \frac{1}{2}\right)$$

ANSWER:

$$7\sqrt{2} \approx 9.9; \left(-\frac{15}{2}, 2, \frac{1}{2}\right)$$

11. (6, 1, 10), (−9, −10, −4)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{[6 - (-9)]^2 + [(1 - (-10)]^2 + [(10 - (-4)]^2} \\ &= \sqrt{225 + 121 + 196} \\ &= \sqrt{542} \\ &\approx 23.3 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \left(\frac{6 + (-9)}{2}, \frac{1 + (-10)}{2}, \frac{10 + (-4)}{2}\right) = \left(-\frac{3}{2}, -\frac{9}{2}, 3\right)$$

ANSWER:

$$\sqrt{542} \approx 23.3; \left(-\frac{3}{2}, -\frac{9}{2}, 3\right)$$

12. (8, 3, 4), (−4, −7, 5)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{[8 - (-4)]^2 + [(3 - (-7)]^2 + (4 - 5)^2} \\ &= \sqrt{144 + 100 + 1} \\ &= \sqrt{245} \\ &= 7\sqrt{5} \\ &\approx 15.7 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) &= \left(\frac{8 + (-4)}{2}, \frac{3 + (-7)}{2}, \frac{4 + 5}{2}\right) \\ &= \left(2, -2, \frac{9}{2}\right) \end{aligned}$$

ANSWER:

$7\sqrt{5} \approx 15.7; \left(2, -2, \frac{9}{2}\right)$

13. (−3, 2, 8), (9, 6, 0)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(-3 - 9)^2 + (2 - 6)^2 + (8 - 0)^2} \\ &= \sqrt{144 + 16 + 64} \\ &= \sqrt{224} \\ &= 4\sqrt{14} \\ &\approx 15.0 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) &= \left(\frac{-3 + 9}{2}, \frac{2 + 6}{2}, \frac{8 + 0}{2}\right) \\ &= (3, 4, 4) \end{aligned}$$

ANSWER:

$4\sqrt{14} \approx 15.0$; (3, 4, 4)

14. (−7, 2, −5), (−2, −5, −8)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{[-7 - (-2)]^2 + [(2 - (-5)]^2 + [(-5 - (-8)]^2} \\ &= \sqrt{25 + 49 + 9} \\ &= \sqrt{83} \\ &\approx 9.1 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) &= \left(\frac{-7 + (-2)}{2}, \frac{2 + (-5)}{2}, \frac{-5 + (-8)}{2}\right) \\ &= \left(-\frac{9}{2}, -\frac{3}{2}, -\frac{13}{2}\right) \end{aligned}$$

ANSWER:

$\sqrt{83} \approx 9.1; \left(-\frac{9}{2}, -\frac{3}{2}, -\frac{13}{2}\right)$

15. **VACATION** A family from Wichita, Kansas, is using a GPS device to plan a vacation to Castle Rock, Colorado. According to the device, the coordinates for the family's home are (37.7º, 97.2º, 433 m), and the coordinates to Castle Rock are (39.4º, 104.8º, 1981 m). Determine the longitude, latitude, and altitude of the halfway point between Wichita and Castle Rock.

SOLUTION:

Use the Midpoint Formula for points in space to find the halfway point between Wichita and Castle Rock.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = \left(\frac{37.7+39.4}{2}, \frac{97.2+104.8}{2}, \frac{433+1981}{2}\right)$$
$$= (38.55, 101, 1207)$$

So, the halfway point between Wichita and Castle Rock is located at longitude: 38.55º, latitude: 101º, and altitude: 1207 m

ANSWER:

longitude: 38.55º, latitude: 101º, altitude: 1207 m

16. **FIGHTER PILOTS** During a training session, the location of two F-18 fighter jets are represented by the coordinates (675, −121, 19,300) and (−289, 715, 16,100), where the coordinates are given in feet.

a. Determine the distance between the two jets.

b. To what location would one of the fighter pilots have to fly the F-18 in order to reduce the distance between the two jets by half?

SOLUTION:

a. Use the Distance Formula for points in space to find the distance between the two jets.

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$
$$= \sqrt{(-289-675)^2 + [715-(-121)]^2 + (16,100-19,300)^2}$$
$$= \sqrt{929,296 + 698,896 + 10,240,000}$$
$$= \sqrt{11,868,192}$$
$$\approx 3445.0$$

The distance between the two jets is about 3445 feet.

b. To reduce the distance between the two jets by half, one of the pilots can fly to the midpoint between the pilots' current positions. Use the Midpoint Formula for points in space to find the midpoint between the two jets.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = \left(\frac{675+(-289)}{2}, \frac{-121+715}{2}, \frac{19,300+16,100}{2}\right)$$
$$= (193,\ 297,\ 17,700)$$

Sample answer: (193, 297, 17,700)

ANSWER:

a. about 3445 ft

b. Sample answer: (193, 297, 17,700)

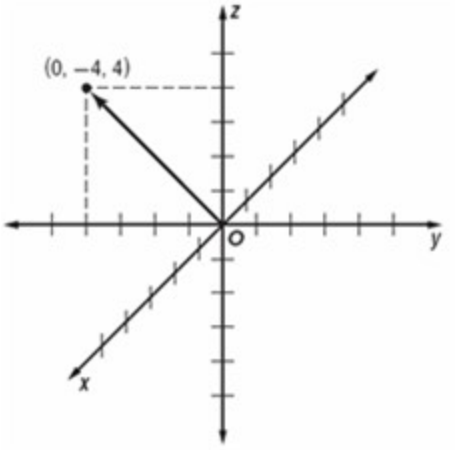
8-4 Vectors in Three-Dimensional Space

Locate and graph each vector in space.

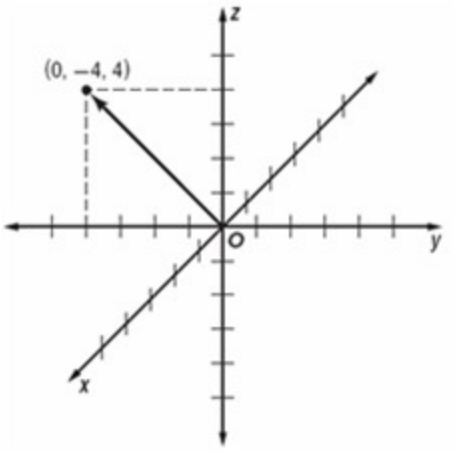
17. **a** = $\langle 0, -4, 4 \rangle$

SOLUTION:

Start by plotting the point (0, −4, 4). From the origin, move 4 units to the left along the *y*-axis, and then 4 units up parallel to the *z*-axis. Draw **a** with an initial point at the origin and terminal point (0, −4, 4).



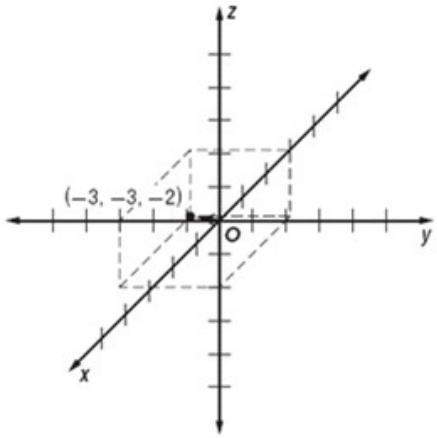
ANSWER:



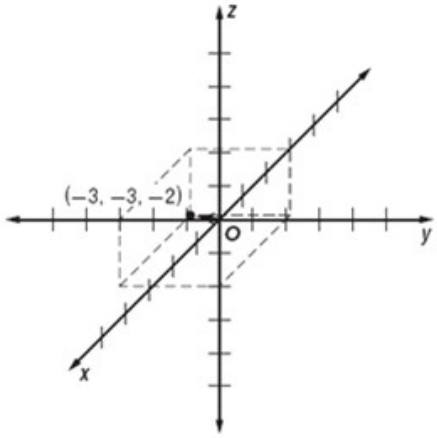
18. **b** = $\langle -3, -3, -2 \rangle$

SOLUTION:

Start by plotting the point (−3, −3, −2). From the origin, move 3 units backwards along the *x*-axis, 3 units to the left parallel to the *y*-axis, and then 2 units down parallel to the *z*-axis. Draw **b** with an initial point at the origin and terminal point (−3, −3, −2).



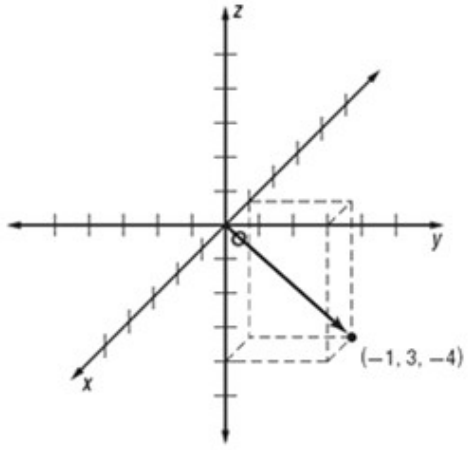
ANSWER:



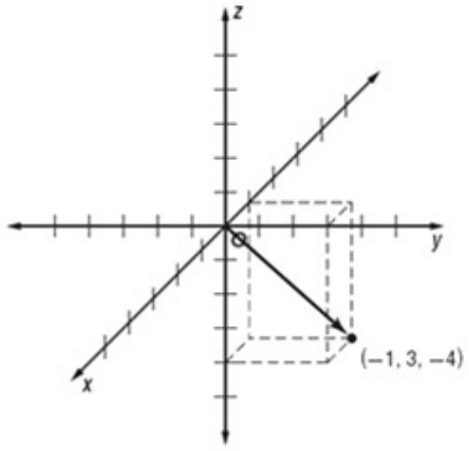
19. $\mathbf{c} = \langle -1, 3, -4 \rangle$

SOLUTION:

Start by plotting the point (−1, 3, −4). From the origin, move 1 unit backwards along the *x*-axis, 3 units to the right parallel to the *y*-axis, and then 4 units down parallel to the *z*-axis. Draw **c** with an initial point at the origin and terminal point (−1, 3, −4).



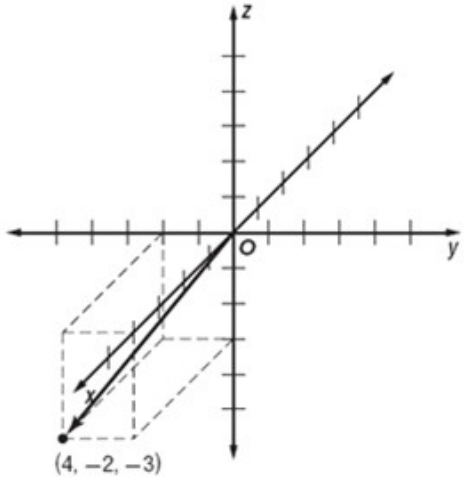
ANSWER:



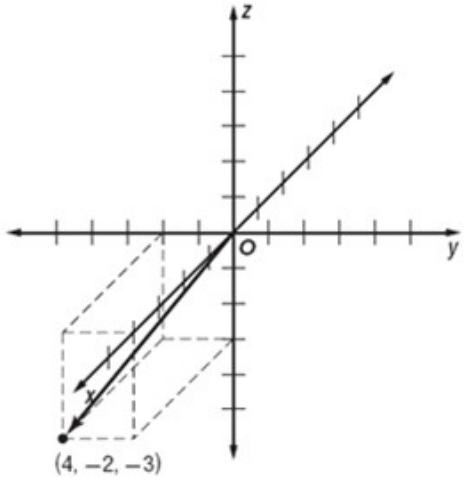
20. $\mathbf{d} = \langle 4, -2, -3 \rangle$

SOLUTION:

Start by plotting the point (4, −2, −3). From the origin, move 4 units forward along the *x*-axis, 2 units to the left parallel to the *y*-axis, and then 3 units down parallel to the *z*-axis. Draw **d** with an initial point at the origin and terminal point (4, −2, −3).



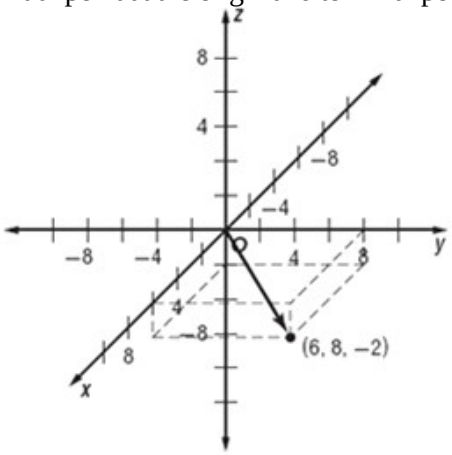
ANSWER:



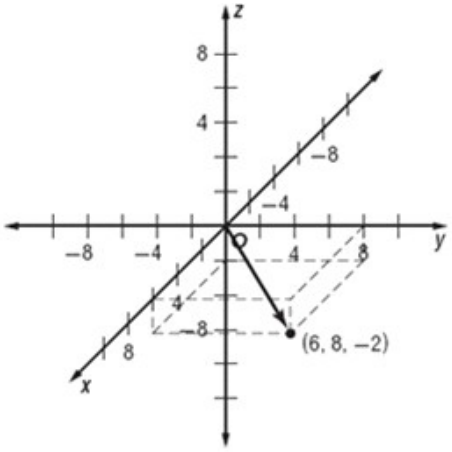
21. $\mathbf{v} = 6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$

SOLUTION:

Start by writing **v** in component form as $\mathbf{v} = \langle 6, 8, -2 \rangle$. Next plot the point (6, 8, −2). From the origin, move 6 units forward along the *x*-axis, 8 units to the right parallel to the *y*-axis, and then 2 units down parallel to the *z*-axis. Draw **v** with an initial point at the origin and terminal point (6, 8, −2).



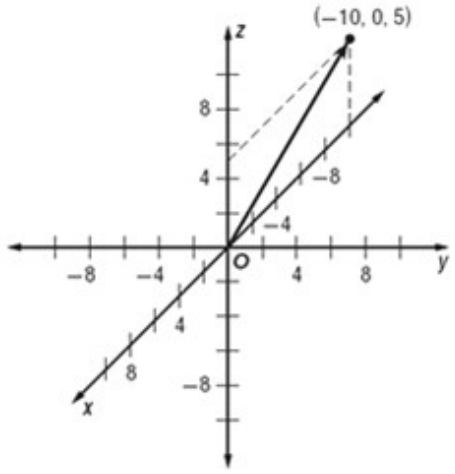
ANSWER:



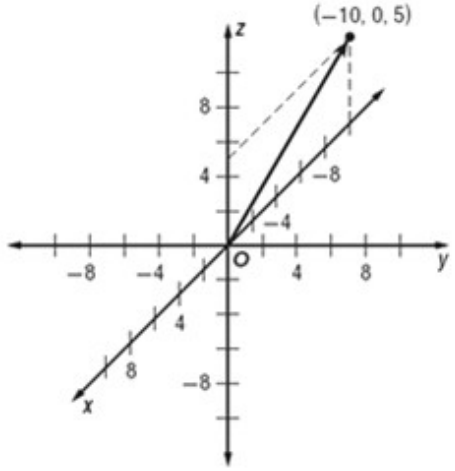
22. $\mathbf{w} = -10\mathbf{i} + 5\mathbf{k}$

SOLUTION:

Start by writing **w** in component form as $\mathbf{w} = \langle -10, 0, 5 \rangle$. Next plot the point (−10, 0, 5). From the origin, move 10 units backwards along the *x*-axis, then 5 units up parallel to the *z*-axis. Draw **w** with an initial point at the origin and terminal point (−10, 0, 5).



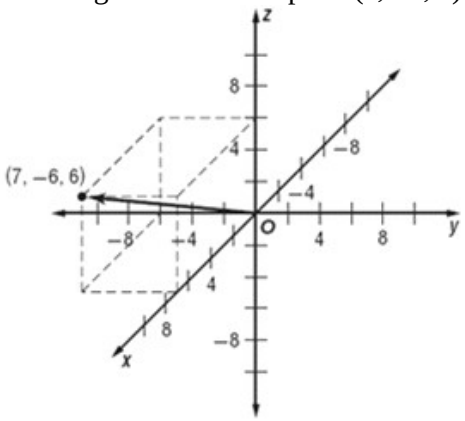
ANSWER:



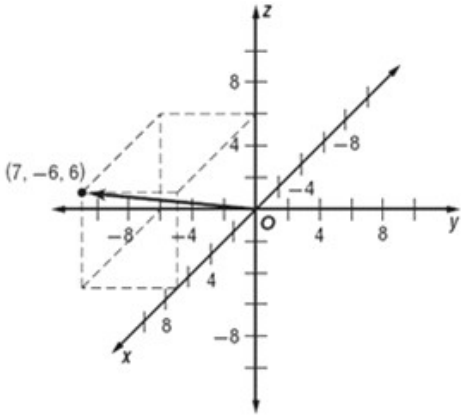
23. $\mathbf{m} = 7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$

SOLUTION:

Start by writing **m** in component form as $\mathbf{m} = \langle 7, -6, 6 \rangle$. Next plot the point (7, −6, 6). From the origin, move 7 units forward along the *x*-axis, 6 units to the left parallel to the *y*-axis, and then 6 units up parallel to the *z*-axis. Draw **m** with an initial point at the origin and terminal point (7, −6, 6).



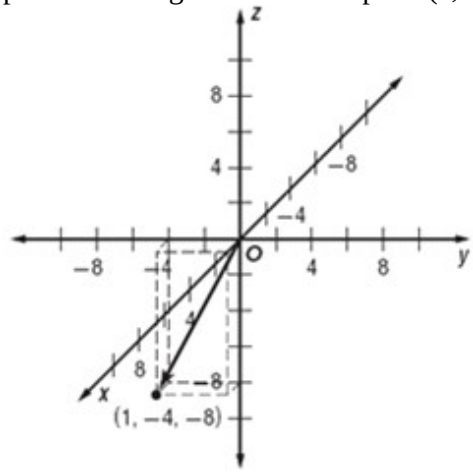
ANSWER:



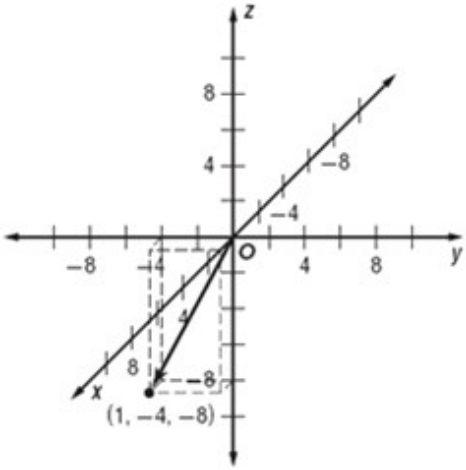
24. $\mathbf{n} = \mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

SOLUTION:

Start by writing **n** in component form as $\mathbf{n} = \langle 1, -4, -8 \rangle$. Next plot the point (1, −4, −8). From the origin, move 1 unit forward along the *x*-axis, 4 units to the left parallel to the *y*-axis, and then 8 units down parallel to the *z*-axis. Draw **n** with an initial point at the origin and terminal point (1, −4, −8).



ANSWER:



Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. Then find a unit vector in the direction of $\overrightarrow{AB}$.

25. $A(-5, -5, -9)$, $B(11, -3, -1)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 11-(-5), -3-(-5), -1-(-9) \rangle \\ &= \langle 16, 2, 8 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{16^2 + 2^2 + 8^2} \\ &= \sqrt{324} \\ &= 18\end{aligned}$$

Using this magnitude and component form, find a unit vector $\mathbf{u}$ in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle 16, 2, 8 \rangle}{18} \\ &= \left\langle \frac{16}{18}, \frac{2}{18}, \frac{8}{18} \right\rangle \\ &= \left\langle \frac{8}{9}, \frac{1}{9}, \frac{4}{9} \right\rangle\end{aligned}$$

ANSWER:

$\langle 16, 2, 8 \rangle$; 18; $\left\langle \frac{8}{9}, \frac{1}{9}, \frac{4}{9} \right\rangle$

26. $A(-4, 0, -3)$, $B(-4, -8, 9)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle -4-(-4), -8-0, 9-(-3) \rangle \\ &= \langle 0, -8, 12 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{0^2 + (-8)^2 + 12^2} \\ &= \sqrt{208} \\ &= 4\sqrt{13}\end{aligned}$$

Using this magnitude and component form, find a unit vector $\mathbf{u}$ in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle 0, -8, 12 \rangle}{4\sqrt{13}} \\ &= \left\langle \frac{0}{4\sqrt{13}}, \frac{-8}{4\sqrt{13}}, \frac{12}{4\sqrt{13}} \right\rangle \\ &= \left\langle 0, \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle \\ &= \left\langle 0, -\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right\rangle\end{aligned}$$

ANSWER:

$\langle 0, -8, 12 \rangle$; $4\sqrt{13}$; $\left\langle 0, -\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right\rangle$

27. $A(3, 5, 1)$, $B(0, 0, -9)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 0-3, 0-5, -9-1 \rangle \\ &= \langle -3, -5, -10 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(-3)^2 + (-5)^2 + (-10)^2} \\ &= \sqrt{134}\end{aligned}$$

Using this magnitude and component form, find a unit vector **u** in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle -3, -5, -10 \rangle}{\sqrt{134}} \\ &= \left\langle \frac{-3}{\sqrt{134}}, \frac{-5}{\sqrt{134}}, \frac{-10}{\sqrt{134}} \right\rangle \\ &= \left\langle -\frac{3\sqrt{134}}{134}, -\frac{5\sqrt{134}}{134}, -\frac{5\sqrt{134}}{67} \right\rangle\end{aligned}$$

ANSWER:

$\langle -3, -5, -10 \rangle$; $\sqrt{134}$; $\left\langle -\frac{3\sqrt{134}}{134}, -\frac{5\sqrt{134}}{134}, -\frac{5\sqrt{134}}{67} \right\rangle$

28. $A(-3, -7, -12)$, $B(-7, 1, 8)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle -7-(-3), 1-(-7), 8-(-12) \rangle \\ &= \langle -4, 8, 20 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(-4)^2 + 8^2 + 20^2} \\ &= \sqrt{480} \\ &= 4\sqrt{30}\end{aligned}$$

Using this magnitude and component form, find a unit vector **u** in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle -4, 8, 20 \rangle}{4\sqrt{30}} \\ &= \left\langle -\frac{4}{4\sqrt{30}}, \frac{8}{4\sqrt{30}}, \frac{20}{4\sqrt{30}} \right\rangle \\ &= \left\langle -\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right\rangle \\ &= \left\langle -\frac{\sqrt{30}}{30}, \frac{\sqrt{30}}{15}, \frac{\sqrt{30}}{6} \right\rangle\end{aligned}$$

ANSWER:

$\langle -4, 8, 20 \rangle$; $4\sqrt{30}$; $\left\langle -\frac{\sqrt{30}}{30}, \frac{\sqrt{30}}{15}, \frac{\sqrt{30}}{6} \right\rangle$

29. $A(2, -5, 4)$, $B(1, 3, -6)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 1-2, 3-(-5), -6-4 \rangle \\ &= \langle -1, 8, -10 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(-1)^2 + 8^2 + (-10)^2} \\ &= \sqrt{165}\end{aligned}$$

Using this magnitude and component form, find a unit vector **u** in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle -1, 8, -10 \rangle}{\sqrt{165}} \\ &= \left\langle -\frac{1}{\sqrt{165}}, \frac{8}{\sqrt{165}}, -\frac{10}{\sqrt{165}} \right\rangle \\ &= \left\langle -\frac{\sqrt{165}}{165}, \frac{8\sqrt{165}}{165}, -\frac{2\sqrt{165}}{33} \right\rangle\end{aligned}$$

ANSWER:

$$\langle -1, 8, -10 \rangle; \sqrt{165}; \left\langle -\frac{\sqrt{165}}{165}, \frac{8\sqrt{165}}{165}, -\frac{2\sqrt{165}}{33} \right\rangle$$

30. $A(8, 12, 7)$, $B(2, -3, 11)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 2-8, -3-12, 11-7 \rangle \\ &= \langle -6, -15, 4 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(-6)^2 + (-15)^2 + 4^2} \\ &= \sqrt{277}\end{aligned}$$

Using this magnitude and component form, find a unit vector **u** in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle -6, -15, 4 \rangle}{\sqrt{277}} \\ &= \left\langle -\frac{6}{\sqrt{277}}, -\frac{15}{\sqrt{277}}, \frac{4}{\sqrt{277}} \right\rangle \\ &= \left\langle -\frac{6\sqrt{277}}{277}, -\frac{15\sqrt{277}}{277}, \frac{4\sqrt{277}}{277} \right\rangle\end{aligned}$$

ANSWER:

$$\langle -6, -15, 4 \rangle; \sqrt{277}; \left\langle -\frac{6\sqrt{277}}{277}, -\frac{15\sqrt{277}}{277}, \frac{4\sqrt{277}}{277} \right\rangle$$

31. $A(3, 14, -5)$, $B(7, -1, 0)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 7-3, -1-14, 0-(-5) \rangle \\ &= \langle 4, -15, 5 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{4^2 + (-15)^2 + 5^2} \\ &= \sqrt{266}\end{aligned}$$

Using this magnitude and component form, find a unit vector **u** in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle 4, -15, 5 \rangle}{\sqrt{266}} \\ &= \left\langle \frac{4}{\sqrt{266}}, -\frac{15}{\sqrt{266}}, \frac{5}{\sqrt{266}} \right\rangle \\ &= \left\langle \frac{2\sqrt{266}}{133}, -\frac{15\sqrt{266}}{266}, \frac{5\sqrt{266}}{266} \right\rangle\end{aligned}$$

ANSWER:

$$\langle 4, -15, 5 \rangle;\ \sqrt{266};\ \left\langle \frac{2\sqrt{266}}{133}, -\frac{15\sqrt{266}}{266}, \frac{5\sqrt{266}}{266} \right\rangle$$

32. $A(1, -18, -13)$, $B(21, 14, 29)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 21-1, 14-(-18), 29-(-13) \rangle \\ &= \langle 20, 32, 42 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{20^2 + 32^2 + 42^2} \\ &= \sqrt{3188} \\ &= 2\sqrt{797}\end{aligned}$$

Using this magnitude and component form, find a unit vector **u** in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle 20, 32, 42 \rangle}{2\sqrt{797}} \\ &= \left\langle \frac{20}{2\sqrt{797}}, \frac{32}{2\sqrt{797}}, \frac{42}{2\sqrt{797}} \right\rangle \\ &= \left\langle \frac{10\sqrt{797}}{797}, \frac{16\sqrt{797}}{797}, \frac{21\sqrt{797}}{797} \right\rangle\end{aligned}$$

ANSWER:

$$\langle 20, 32, 42 \rangle;\ 2\sqrt{797};\ \left\langle \frac{10\sqrt{797}}{797}, \frac{16\sqrt{797}}{797}, \frac{21\sqrt{797}}{797} \right\rangle$$

33. $A(-5, 12, 17)$, $B(6, -11, 4)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 6-(-5), -11-12, 4-17 \rangle \\ &= \langle 11, -23, -13 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{11^2 + (-23)^2 + (-13)^2} \\ &= \sqrt{819} \\ &= 3\sqrt{91}\end{aligned}$$

Using this magnitude and component form, find a unit vector $\mathbf{u}$ in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle 11, -23, -13 \rangle}{3\sqrt{91}} \\ &= \left\langle \frac{11}{3\sqrt{91}}, -\frac{23}{3\sqrt{91}}, -\frac{13}{3\sqrt{91}} \right\rangle \\ &= \left\langle \frac{11\sqrt{91}}{273}, -\frac{23\sqrt{91}}{273}, -\frac{\sqrt{91}}{21} \right\rangle\end{aligned}$$

ANSWER:

$$\langle 11, -23, -13 \rangle; 3\sqrt{91}; \left\langle \frac{11\sqrt{91}}{273}, -\frac{23\sqrt{91}}{273}, -\frac{\sqrt{91}}{21} \right\rangle$$

34. $A(9, 3, 7)$, $B(-5, -7, 2)$

SOLUTION:

Find the component form of $\overrightarrow{AB}$.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle -5-9, -7-3, 2-7 \rangle \\ &= \langle -14, -10, -5 \rangle\end{aligned}$$

Use the component form to find the magnitude of $\overrightarrow{AB}$.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(-14)^2 + (-10)^2 + (-5)^2} \\ &= \sqrt{321}\end{aligned}$$

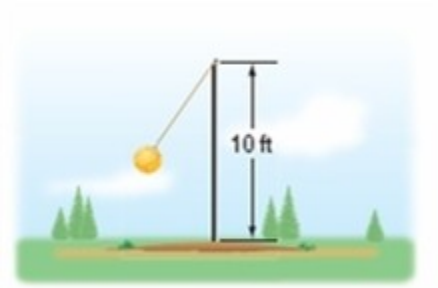
Using this magnitude and component form, find a unit vector $\mathbf{u}$ in the direction of $\overrightarrow{AB}$.

$$\begin{aligned}\mathbf{u} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \\ &= \frac{\langle -14, -10, -5 \rangle}{\sqrt{321}} \\ &= \left\langle -\frac{14}{\sqrt{321}}, -\frac{10}{\sqrt{321}}, -\frac{5}{\sqrt{321}} \right\rangle \\ &= \left\langle -\frac{14\sqrt{321}}{321}, -\frac{10\sqrt{321}}{321}, -\frac{5\sqrt{321}}{321} \right\rangle\end{aligned}$$

ANSWER:

$$\langle -14, -10, -5 \rangle; \sqrt{321}; \left\langle -\frac{14\sqrt{321}}{321}, -\frac{10\sqrt{321}}{321}, -\frac{5\sqrt{321}}{321} \right\rangle$$

35. **TETHERBALL** In the game of tetherball, a ball is attached to a 10-foot pole by a length of rope. Two players hit the ball in opposing directions in attempt to wind the entire length of rope around the pole. To serve, a certain player holds the ball so that its coordinates are (5, 3.6, 4.7) and the coordinates of the end of the rope connected to the pole are (0, 0, 9.8), where the coordinates are given in feet. Determine the magnitude of the vector representing the length of the rope.



SOLUTION:

Let P represent the point at which the rope and the pole are connected and let B represent the position of the ball as it is being served. Then $\overrightarrow{PB}$ represents the vector from the pole to the ball.

$$\begin{aligned}\overrightarrow{PB} &= \langle 0-5, 0-3.6, 9.8-4.7\rangle \\ &= \langle -5, -3.6, 5.1\rangle\end{aligned}$$

The magnitude of $\overrightarrow{PB}$ is

$$\sqrt{(-5)^2+(-3.6)^2+(5.1)^2} \approx 8.0$$

Thus, the rope is about 8 feet long.

ANSWER:

about 8 ft

Find each of the following for $\mathbf{a} = \langle -5, -4, 3\rangle$, $\mathbf{b} = \langle 6, -2, -7\rangle$, and $\mathbf{c} = \langle -2, 2, 4\rangle$.

36. $6\mathbf{a} - 7\mathbf{b} + 8\mathbf{c}$

SOLUTION:

$$\begin{aligned}6\mathbf{a}-7\mathbf{b}+8\mathbf{c} &= 6\mathbf{a}+(-7)\mathbf{b}+8\mathbf{c} \\ &= 6\langle -5,-4,3\rangle + (-7)\langle 6,-2,-7\rangle + 8\langle -2,2,4\rangle \\ &= \langle -30,-24,18\rangle + \langle -42,14,49\rangle + \langle -16,16,32\rangle \\ &= \langle -88,6,99\rangle\end{aligned}$$

ANSWER:

$\langle -88, 6, 99\rangle$

37. $7\mathbf{a} - 5\mathbf{b}$

SOLUTION:

$$\begin{aligned}7\mathbf{a}-5\mathbf{b} &= 7\mathbf{a}+(-5)\mathbf{b} \\ &= 7\langle -5,-4,3\rangle + (-5)\langle 6,-2,-7\rangle \\ &= \langle -35,-28,21\rangle + \langle -30,10,35\rangle \\ &= \langle -65,-18,56\rangle\end{aligned}$$

ANSWER:

$\langle -65, -18, 56\rangle$

38. $2\mathbf{a} + 5\mathbf{b} - 9\mathbf{c}$

SOLUTION:

$$\begin{aligned}2\mathbf{a}+5\mathbf{b}-9\mathbf{c} &= 2\mathbf{a}+5\mathbf{b}+(-9)\mathbf{c} \\ &= 2\langle -5,-4,3\rangle + 5\langle 6,-2,-7\rangle + (-9)\langle -2,2,4\rangle \\ &= \langle -10,-8,6\rangle + \langle 30,-10,-35\rangle + \langle 18,-18,-36\rangle \\ &= \langle 38,-36,-65\rangle\end{aligned}$$

ANSWER:

$\langle 38, -36, -65\rangle$

39. $6\mathbf{b} + 4\mathbf{c} - 4\mathbf{a}$

SOLUTION:

$$\begin{aligned}6\mathbf{b}+4\mathbf{c}-4\mathbf{a} &= 6\mathbf{b}+4\mathbf{c}+(-4)\mathbf{a} \\ &= 6\langle 6,-2,-7\rangle + 4\langle -2,2,4\rangle + (-4)\langle -5,-4,3\rangle \\ &= \langle 36,-12,-42\rangle + \langle -8,8,16\rangle + \langle 20,16,-12\rangle \\ &= \langle 48,12,-38\rangle\end{aligned}$$

ANSWER:

$\langle 48, 12, -38\rangle$

40. $8\mathbf{a} - 5\mathbf{b} - \mathbf{c}$

SOLUTION:

$$\begin{aligned}8\mathbf{a}-5\mathbf{b}-\mathbf{c} &= 8\mathbf{a}+(-5)\mathbf{b}+(-1)\mathbf{c} \\ &= 8\langle -5,-4,3\rangle + (-5)\langle 6,-2,-7\rangle + (-1)\langle -2,2,4\rangle \\ &= \langle -40,-32,24\rangle + \langle -30,10,35\rangle + \langle 2,-2,-4\rangle \\ &= \langle -68,-24,55\rangle\end{aligned}$$

ANSWER:

$\langle -68, -24, 55\rangle$

41. $-6\mathbf{a} + \mathbf{b} + 7\mathbf{c}$

SOLUTION:

$$\begin{aligned}-6\mathbf{a}+\mathbf{b}+7\mathbf{c} &= (-6)\mathbf{a}+\mathbf{b}+7\mathbf{c}\\ &= (-6)\langle -5,-4,3\rangle + \langle 6,-2,-7\rangle + 7\langle -2,2,4\rangle\\ &= \langle 30,24,-18\rangle + \langle 6,-2,-7\rangle + \langle -14,14,28\rangle\\ &= \langle 22,36,3\rangle\end{aligned}$$

ANSWER:

$\langle 22, 36, 3\rangle$

Find each of the following for $\mathbf{x} = -9\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{y} = 6\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$, and $\mathbf{z} = -2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$.

42. $7\mathbf{x} + 6\mathbf{y}$

SOLUTION:

Write $\mathbf{x}$ and $\mathbf{y}$ in component form as

$$\mathbf{x} = \langle -9,4,3\rangle \text{ and } \mathbf{y} = \langle 6,-2,-7\rangle.$$

$$\begin{aligned}7\mathbf{x}+6\mathbf{y} &= 7\langle -9,4,3\rangle + 6\langle 6,-2,-7\rangle\\ &= \langle -63,28,21\rangle + \langle 36,-12,-42\rangle\\ &= \langle -27,16,-21\rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\langle -27,16,-21\rangle = -27\mathbf{i} + 16\mathbf{j} - 21\mathbf{k}$$

ANSWER:

$-27\mathbf{i} + 16\mathbf{j} - 21\mathbf{k}$

43. $3\mathbf{x} - 5\mathbf{y} + 3\mathbf{z}$

SOLUTION:

Write $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ in component form as

$$\mathbf{x} = \langle -9,4,3\rangle,\ \mathbf{y} = \langle 6,-2,-7\rangle \text{ and } \mathbf{z} = \langle -2,2,4\rangle.$$

$$\begin{aligned}3\mathbf{x}-5\mathbf{y}+3\mathbf{z} &= 3\langle -9,4,3\rangle + (-5)\langle 6,-2,-7\rangle + 3\langle -2,2,4\rangle\\ &= \langle -27,12,9\rangle + \langle -30,10,35\rangle + \langle -6,6,12\rangle\\ &= \langle -63,28,56\rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\langle -63,28,56\rangle = -63\mathbf{i} + 28\mathbf{j} + 56\mathbf{k}$$

ANSWER:

$-63\mathbf{i} + 28\mathbf{j} + 56\mathbf{k}$

44. $4\mathbf{x} + 3\mathbf{y} + 2\mathbf{z}$

SOLUTION:

Write $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ in component form as

$$\mathbf{x} = \langle -9,4,3\rangle,\ \mathbf{y} = \langle 6,-2,-7\rangle \text{ and } \mathbf{z} = \langle -2,2,4\rangle.$$

$$\begin{aligned}4\mathbf{x}+3\mathbf{y}+2\mathbf{z} &= 4\langle -9,4,3\rangle + 3\langle 6,-2,-7\rangle + 2\langle -2,2,4\rangle\\ &= \langle -36,16,12\rangle + \langle 18,-6,-21\rangle + \langle -4,4,8\rangle\\ &= \langle -22,14,-1\rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\langle -22,14,-1\rangle = -22\mathbf{i} + 14\mathbf{j} - \mathbf{k}$$

ANSWER:

$-22\mathbf{i} + 14\mathbf{j} - \mathbf{k}$

45. $-8\mathbf{x} - 2\mathbf{y} + 5\mathbf{z}$

SOLUTION:

Write $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ in component form as

$$\mathbf{x} = \langle -9,4,3\rangle,\ \mathbf{y} = \langle 6,-2,-7\rangle \text{ and } \mathbf{z} = \langle -2,2,4\rangle.$$

$$\begin{aligned}-8\mathbf{x}-2\mathbf{y}+5\mathbf{z} &= (-8)\langle -9,4,3\rangle + (-2)\langle 6,-2,-7\rangle + 5\langle -2,2,4\rangle\\ &= \langle 72,-32,-24\rangle + \langle -12,4,14\rangle + \langle -10,10,20\rangle\\ &= \langle 50,-18,10\rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\langle 50,-18,10\rangle = 50\mathbf{i} - 18\mathbf{j} + 10\mathbf{k}$$

ANSWER:

$50\mathbf{i} - 18\mathbf{j} + 10\mathbf{k}$

46. $-6\mathbf{y} - 9\mathbf{z}$

SOLUTION:

Write $\mathbf{y}$ and $\mathbf{z}$ in component form as

$$\mathbf{y} = \langle 6,-2,-7\rangle \text{ and } \mathbf{z} = \langle -2,2,4\rangle.$$

$$\begin{aligned}-6\mathbf{y}-9\mathbf{z} &= (-6)\langle 6,-2,-7\rangle + (-9)\langle -2,2,4\rangle\\ &= \langle -36,12,42\rangle + \langle 18,-18,-36\rangle\\ &= \langle -18,-6,6\rangle\end{aligned}$$

Then rewrite the vector as a linear combination of the standard unit vectors.

$$\langle -18,-6,6\rangle = -18\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$$

ANSWER:

$-18\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$

47. $-\mathbf{x} - 4\mathbf{y} - \mathbf{z}$

SOLUTION:

Write **x**, **y**, and **z** in component form as

$\mathbf{x} = \langle -9, 4, 3 \rangle$, $\mathbf{y} = \langle 6, -2, -7 \rangle$ and $\mathbf{z} = \langle -2, 2, 4 \rangle$.

$$\begin{aligned} -\mathbf{x} - 4\mathbf{y} - \mathbf{z} &= (-1)\langle -9, 4, 3 \rangle + (-4)\langle 6, -2, -7 \rangle + (-1)\langle -2, 2, 4 \rangle \\ &= \langle 9, -4, -3 \rangle + \langle -24, 8, 28 \rangle + \langle 2, -2, -4 \rangle \\ &= \langle -13, 2, 21 \rangle \end{aligned}$$

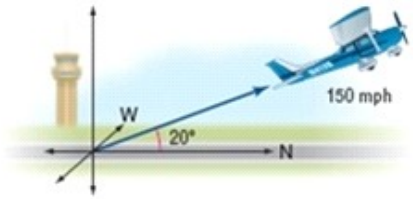
Then rewrite the vector as a linear combination of the standard unit vectors.

$\langle -13, 2, 21 \rangle = -13\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}$

ANSWER:

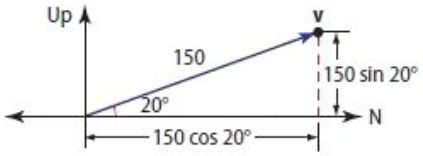
$-13\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}$

48. **AIRPLANES** An airplane is taking off headed due north with an air speed of 150 miles per hour at an angle of 20° relative to the horizontal. The wind is blowing with a velocity of 8 miles per hour from the southwest. Find a vector that represents the resultant velocity of the plane relative to the point of takeoff. Let **i** point east, **j** point north, and **k** point up.



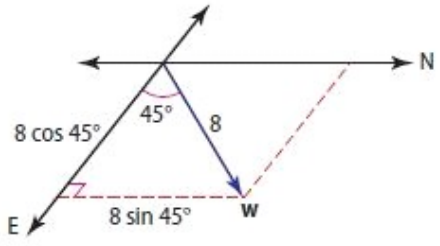
SOLUTION:

Let **v** be a vector representing the airplane's velocity and **w** be a vector representing the wind's velocity. Since **v** has a magnitude of 150 and a direction angle of 20°, find the component form of **v** using trigonometric ratios as shown.



$v = \langle 0, 150\cos 20^\circ, 150\sin 20^\circ \rangle \approx \langle 0, 141.0, 51.3 \rangle$

Since the wind is blowing from the southwest, **w** points toward the northeast and will have a direction angle of 45° with east as the positive *x*-axis. Find the component form of **w** using trigonometric ratios as shown.



$w = \langle 8\cos 45^\circ, 8\sin 45^\circ, 0 \rangle \approx \langle 5.7, 5.7, 0 \rangle$

The resultant velocity of the airplane is **v** + **w**.

$$\begin{aligned} v + w &= \langle 0, 141.0, 51.3 \rangle + \langle 5.7, 5.7, 0 \rangle \\ &= \langle 5.7, 146.7, 51.3 \rangle \\ &= 5.7i + 146.7j + 51.3k \end{aligned}$$

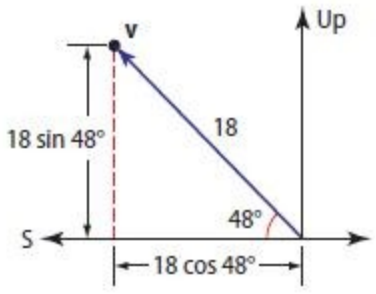
ANSWER:

$5.7\mathbf{i} + 146.7\mathbf{j} + 51.3\mathbf{k}$

49. **TRACK AND FIELD** Lena throws a javelin due south at a speed of 18 miles per hour and at an angle of 48° relative to the horizontal. If the wind is blowing with a velocity of 12 miles per hour at an angle of S15°E, find a vector that represents the resultant velocity of the javelin. Let **i** point east, **j** point north, and **k** point up.

SOLUTION:

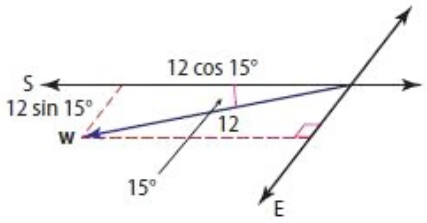
Let **v** be a vector representing the javelin's velocity and **w** be a vector representing the wind's velocity. Since **v** has a magnitude of 18 and a direction angle of 48°, find the component form of **v** using trigonometric ratios as shown. Since the direction of the javelin is due south, which is the negative *y*-axis, the *y*-component will be negative.



$v = \langle 0, -18\cos 48^\circ, 18\sin 48^\circ \rangle \approx \langle 0, -12.0, 13.4 \rangle$

Since the wind is blowing with a velocity of 12 miles per hour at an angle of S15°E, find the component

form of **w** using trigonometric ratios as shown. Since the direction of the wind's velocity is S15°E, the *y*-component will be negative.



$w = \langle 12\sin 15^\circ, -12\cos 15^\circ, 0\rangle \approx \langle 3.1, -11.6, 0\rangle$

The resultant velocity of the javelin is **v** + **w**.

$$\begin{aligned}\mathbf{v}+\mathbf{w} &= \langle 0, -12.0, 13.4\rangle + \langle 3.1, -11.6, 0\rangle \\ &= \langle 3.1, -23.6, 13.4\rangle \\ &= 3.1\mathbf{i} - 23.6\mathbf{j} + 13.4\mathbf{k}\end{aligned}$$

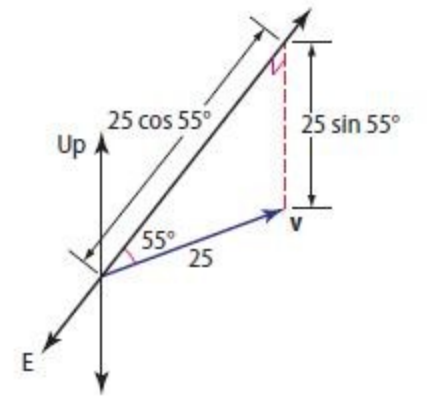
ANSWER:

3.1**i** − 23.6**j** + 13.4**k**

50. **SUBMARINE** A submarine heading due west dives at a speed of 25 knots and an angle of decline of 55°. The current is moving with a velocity of 4 knots at an angle of S20°W. Find a vector that represents the resultant velocity of the submarine relative to the initial point of the dive. Let **i** point east, **j** point north, and **k** point up.

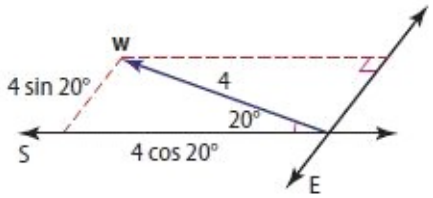
SOLUTION:

Let **v** be a vector representing the submarine's velocity and **w** be a vector representing the current's velocity. Since **v** has a magnitude of 25 and a direction angle of 55°, find the component form of **v** using trigonometric ratios as shown. Since the submarine is diving at a direction of due west, both the *x*- and *z*-components will be negative.



$\mathbf{v} = \langle -25\cos 55^\circ, 0, -25\sin 55^\circ\rangle \approx \langle -14.3, 0, -20.5\rangle$

Since the current is moving with a velocity of 4 knots at an angle of S20°W, find the component form of **w** using trigonometric ratios as shown. Since the direction of the current's velocity is S20°W, both the *x*- and *y*-components will be negative.



$\mathbf{w} = \langle -4\sin 20^\circ, -4\cos 20^\circ, 0\rangle \approx \langle -1.4, -3.8, 0\rangle$

The resultant velocity of the javelin is **v** + **w**.

$$\begin{aligned}\mathbf{v}+\mathbf{w} &= \langle -14.3, 0, -20.5\rangle + \langle -1.4, -3.8, 0\rangle \\ &= \langle -15.7, -3.8, -20.5\rangle \\ &= -15.7\mathbf{i} - 3.8\mathbf{j} - 20.5\mathbf{k}\end{aligned}$$

ANSWER:

−15.7**i** − 3.8**j** − 20.5**k**

If *N* is the midpoint of $\overline{MP}$, find *P*.

51. $M(3, 4, 5)$; $N\left(\frac{7}{2}, 1, 2\right)$

SOLUTION:

Let $P = (x_2, y_2, z_2)$. Use the Midpoint Formula and the points M and N to solve for P.

$$N = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$$\left(\frac{7}{2}, 1, 2\right) = \left(\frac{3+x_2}{2}, \frac{4+y_2}{2}, \frac{5+z_2}{2}\right)$$

Solve for x_2, y_2, and z_2.

$$\frac{3+x_2}{2} = \frac{7}{2} \qquad \frac{4+y_2}{2} = 1 \qquad \frac{5+z_2}{2} = 2$$

$$3 + x_2 = 7 \qquad 4 + y_2 = 2 \qquad 5 + z_2 = 4$$

$$x_2 = 4 \qquad y_2 = -2 \qquad z_2 = -1$$

So, $P = (4, -2, -1)$.

ANSWER:

(4, –2, –1)

52. $M(-1, -4, -9)$; $N(-2, 1, -5)$

SOLUTION:

Let $P = (x_2, y_2, z_2)$. Use the Midpoint Formula and the points M and N to solve for P.

$$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$(-2, 1, -5) = \left(\frac{-1 + x_2}{2}, \frac{-4 + y_2}{2}, \frac{-9 + z_2}{2}\right)$$

Solve for x_2, y_2, and z_2.

$$\frac{-1 + x_2}{2} = -2 \qquad \frac{-4 + y_2}{2} = 1 \qquad \frac{-9 + z_2}{2} = -5$$

$$-1 + x_2 = -4 \qquad -4 + y_2 = 2 \qquad -9 + z_2 = -10$$

$$x_2 = -3 \qquad y_2 = 6 \qquad z_2 = -1$$

So, $P = (-3, 6, -1)$.

ANSWER:

$(-3, 6, -1)$

53. $M(7, 1, 5)$; $N\left(5, -\frac{1}{2}, 6\right)$

SOLUTION:

Let $P = (x_2, y_2, z_2)$. Use the Midpoint Formula and the points M and N to solve for P.

$$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$\left(5, -\frac{1}{2}, 6\right) = \left(\frac{7 + x_2}{2}, \frac{1 + y_2}{2}, \frac{5 + z_2}{2}\right)$$

Solve for x_2, y_2, and z_2.

$$\frac{7 + x_2}{2} = 5 \qquad \frac{1 + y_2}{2} = -\frac{1}{2} \qquad \frac{5 + z_2}{2} = 6$$

$$7 + x_2 = 10 \qquad 1 + y_2 = -1 \qquad 5 + z_2 = 12$$

$$x_2 = 3 \qquad y_2 = -2 \qquad z_2 = 7$$

So, $P = (3, -2, 7)$.

ANSWER:

$(3, -2, 7)$

54. $M\left(\frac{3}{2}, -5, 9\right)$; $N\left(-2, -\frac{13}{2}, \frac{11}{2}\right)$

SOLUTION:

Let $P = (x_2, y_2, z_2)$. Use the Midpoint Formula and the points M and N to solve for P.

$$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$\left(-2, -\frac{13}{2}, \frac{11}{2}\right) = \left(\frac{\frac{3}{2} + x_2}{2}, \frac{-5 + y_2}{2}, \frac{9 + z_2}{2}\right)$$

Solve for x_2, y_2, and z_2.

$$\frac{\frac{3}{2} + x_2}{2} = -2 \qquad \frac{-5 + y_2}{2} = -\frac{13}{2} \qquad \frac{9 + z_2}{2} = \frac{11}{2}$$

$$\frac{3}{2} + x_2 = -4 \qquad -5 + y_2 = -13 \qquad 9 + z_2 = 11$$

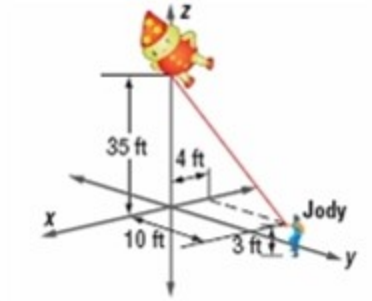
$$x_2 = -\frac{11}{2} \qquad y_2 = -8 \qquad z_2 = 2$$

So, $P = \left(-\frac{11}{2}, -8, 2\right)$.

ANSWER:

$\left(-\frac{11}{2}, -8, 2\right)$

55. **VOLUNTEERING** Jody is volunteering to help guide a balloon in a parade. If the balloon is 35 feet high and she is holding the tether three feet above the ground as shown, how long is the tether to the nearest foot?



SOLUTION:

According to the graph, the coordinates of the balloon are (0, 0, 35) and the coordinates of the end of the tether that Jody is holding are (−4, 10, 3). Use the Distance Formula for points in space to find the length of the tether.

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(-4 - 0)^2 + (10 - 0)^2 + (3 - 35)^2}$$
$$\approx 33.8$$

The length of the tether to the nearest foot is 34 feet.

ANSWER:

34 ft

Determine whether the triangle with the given vertices is *isosceles* or *scalene*.

56. $A(3, 1, 2)$, $B(5, -1, 1)$, $C(1, 3, 1)$

SOLUTION:

Use the Distance Formula for points in space to find the length of each segment.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(5 - 3)^2 + (-1 - 1)^2 + (1 - 2)^2}$$
$$= 3$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(1 - 5)^2 + [3 - (-1)]^2 + (1 - 1)^2}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(1 - 3)^2 + (3 - 1)^2 + (1 - 2)^2}$$
$$= 3$$

The triangle is isosceles.

ANSWER:

isosceles

57. $A(4, 3, 4)$, $B(4, 6, 4)$, $C(4, 3, 6)$

SOLUTION:

Use the Distance Formula for points in space to find the length of each segment.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(4-4)^2 + (6-3)^2 + (4-4)^2}$$
$$= 3$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(4-4)^2 + (3-6)^2 + (6-4)^2}$$
$$= \sqrt{13}$$

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(4-4)^2 + (3-3)^2 + (6-4)^2}$$
$$= 2$$

The triangle is scalene.

ANSWER:

scalene

58. $A(-1, 4, 3)$, $B(2, 5, 1)$, $C(0, -6, 6)$

SOLUTION:

Use the Distance Formula for points in space to find the length of each segment.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{[2-(-1)]^2 + (5-4)^2 + (1-3)^2}$$
$$= \sqrt{14}$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(0-2)^2 + (-6-5)^2 + (6-1)^2}$$
$$= \sqrt{150}$$
$$= 5\sqrt{6}$$

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{[0-(-1)]^2 + (-6-4)^2 + (6-3)^2}$$
$$= \sqrt{110}$$

The triangle is scalene.

ANSWER:

scalene

59. $A(-2.2, 4.3, 5.6)$, $B(0.7, 9.3, 15.6)$, $C(3.6, 14.3, 5.6)$

SOLUTION:

Use the Distance Formula for points in space to find the length of each segment.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{[0.7-(-2.2)]^2 + (9.3-4.3)^2 + (15.6-5.6)^2}$$
$$= \sqrt{2.9^2 + 5^2 + 10^2}$$
$$= \sqrt{8.41 + 25 + 100}$$
$$= \sqrt{133.41}$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(3.6-0.7)^2 + (14.3-9.3)^2 + (5.6-15.6)^2}$$
$$= \sqrt{2.9^2 + 5^2 + (-10)^2}$$
$$= \sqrt{8.41 + 25 + 100}$$
$$= \sqrt{133.41}$$

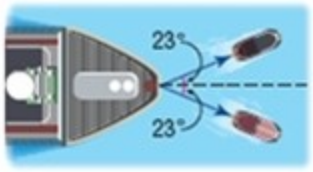
$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{[3.6-(-2.2)]^2 + (14.3-4.3)^2 + (5.6-5.6)^2}$$
$$= \sqrt{5.8^2 + 10^2 + 0^2}$$
$$= \sqrt{33.64 + 100}$$
$$= \sqrt{133.64}$$

The triangle is isosceles.

ANSWER:

isosceles

60. **TUGBOATS** Two tugboats are pulling a disabled s tow lines makes an angle 23° west of north and the o 23° east of north. Each tug exerts a constant force o depressed 15° below the point where the lines attach They pull the supertanker two miles due north.



a. Write a three-dimensional vector to describe the f tugboat.

b. Find the vector that describes the total force on t

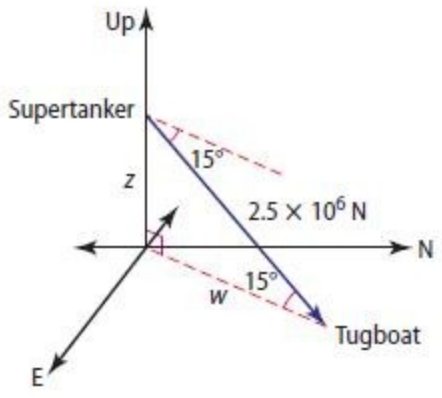
c. If each tow line is 300 feet long, about how far ap

SOLUTION:

a. The tugboats exert a constant force of 2.5×10^6 n below the point where the lines attach to the superta

Diagram this situation for one of the tugboats.

8-4 Vectors in Three-Dimensional Space

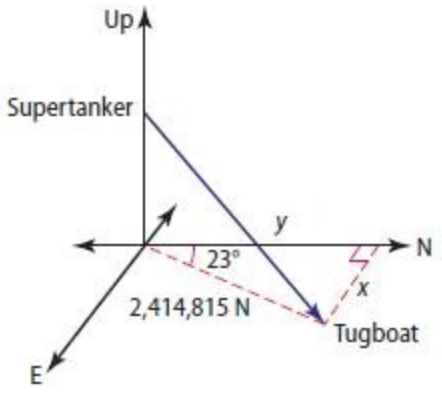


Use trigonometric ratios to solve for z and w.

$$\sin 15^\circ = \frac{z}{2.5\times10^6}$$
$$2.5\times10^6 \cdot \sin 15^\circ = z$$
$$647{,}048 \approx z$$
$$6.5\times10^5 \approx z$$

$$\cos 15^\circ = \frac{w}{2.5\times10^6}$$
$$2.5\times10^6 \cdot \cos 15^\circ = w$$
$$2{,}414{,}815 \approx w$$
$$2.4\times10^6 \approx w$$

w represents the force on the xy-plane that the tugbo supertanker. Use w and the angle that the tugboat is the x and y components.



$$\sin 23^\circ = \frac{x}{2{,}414{,}815}$$
$$2{,}414{,}815 \cdot \sin 23^\circ = x$$
$$943{,}543 \approx x$$
$$9.4\times10^5 \approx x$$

$$\cos 23^\circ = \frac{y}{2{,}414{,}815}$$
$$2{,}414{,}815 \cdot \cos 23^\circ = y$$
$$2{,}222{,}849 \approx y$$
$$2.2\times10^6 \approx y$$

So, $x \approx 9.4 \times 10^5$, $y \approx 2.2 \times 10^6$, and $z \approx 6.5 \times 10^5$. Si by the tugboats is pulling down from the supertanker, Also, x will be positive for the tugboat that is located negative for the tugboat located at N23°W. The thre that describe the force from each tugboat are $\langle 9.4\times10^5, 2.2\times10^6, -6.5\times10^5\rangle$, $\langle -9.4\times10^5, 2.2\times10^6, -6.5\times10^5\rangle$.

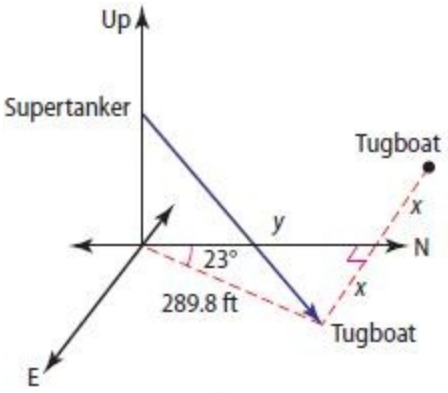
b. Add the two vectors found in part **a**.

$\langle 9.4\times10^5, 2.2\times10^6, -6.5\times10^5\rangle +$
$\langle -9.4\times10^5, 2.2\times10^6, -6.5\times10^5\rangle = \langle 0, 4.4\times1$

c. First, find w, the distance on the xy-plane that the each of the tugboats by using trigonometric ratios. Di one of the tugboats.

$$\cos 15^\circ = \frac{w}{300}$$
$$300 \cdot \cos 15^\circ = w$$
$$289.8 \approx w$$

Use w and the angle that the tugboat is from north to



$$\sin 23^\circ = \frac{x}{289.8}$$
$$289.8 \cdot \sin 23^\circ = x$$
$$113.23 \approx x$$

If $x \approx 113.23$, then the tugboats are $2 \cdot 113.23$ or abo each other.

ANSWER:

a. $\langle 9.4 \times 10^5, 2.2 \times 10^6, -6.5 \times 10^5 \rangle$
$\langle -9.4 \times 10^5, 2.2 \times 10^6, -6.5 \times 10^5 \rangle$

b. $\langle 0, 4.4 \times 10^6, -1.3 \times 10^6 \rangle$

c. about 226.5 ft

61. **SPHERES** Use the distance formula for two points in space to prove that the standard form of the equation of a sphere with center (h, k, ℓ) and radius r is $r^2 = (x - h)^2 + (y - k)^2 + (z - \ell)^2$.

SOLUTION:

Let the center of a sphere with radius r be located at (h, k, ℓ) and the point (x, y, z) be a point on the sphere. Substitute these points into the distance formula for two points in space. Since the distance between the center of a sphere and a point on the sphere is equal to the radius, let $d = r$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$r = \sqrt{(x-h)^2 + (y-k)^2 + (z-\ell)^2}$$
$$r^2 = (x-h)^2 + (y-k)^2 + (z-\ell)^2$$

ANSWER:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$r = \sqrt{(x-h)^2 + (y-k)^2 + (z-\ell)^2}$$
$$r^2 = (x-h)^2 + (y-k)^2 + (z-\ell)^2$$

Use the formula $r^2 = (x - h)^2 + (y - k)^2 + (z - \ell)^2$ to write an equation for the sphere with the given center and radius.

62. center = (–4, –2, 3), radius = 4

SOLUTION:

The formula developed in Exercise 61 for the standard form of the equation of a sphere with a center (h, k, ℓ) and radius r is $(x - h)^2 + (y - k)^2 + (z - \ell)^2 = r^2$. Substitute $h = -4$, $k = -2$, $\ell = 3$, and $r = 4$ into the formula.

$$(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$$
$$[x-(-4)]^2 + [y-(-2)]^2 + (z-3)^2 = 4^2$$
$$(x+4)^2 + (y+2)^2 + (z-3)^2 = 16$$

ANSWER:

$(x + 4)^2 + (y + 2)^2 + (z - 3)^2 = 16$

63. center = (6, 0, –1), radius = $\frac{1}{2}$

SOLUTION:

The formula developed in Exercise 61 for the standard form of the equation of a sphere with a center (h, k, ℓ) and radius r is $(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$. Substitute $h = 6$, $k = 0$, $\ell = -1$, and $r = \frac{1}{2}$ into the formula.

$$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$$
$$(x-6)^2+(y-0)^2+[z-(-1)]^2=\left(\frac{1}{2}\right)^2$$
$$(x-6)^2+y^2+(z+1)^2=\frac{1}{4}$$

ANSWER:

$$(x-6)^2+y^2+(z+1)^2=\frac{1}{4}$$

64. center = (5, –3, 4), radius = $\sqrt{3}$

SOLUTION:

The formula developed in Exercise 61 for the standard form of the equation of a sphere with a center (h, k, ℓ) and radius r is $(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$. Substitute $h = 5$, $k = -3$, $\ell = 4$, and $r = \sqrt{3}$ into the formula.

$$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$$
$$(x-5)^2+[y-(-3)]^2+(z-4)^2=\left(\sqrt{3}\right)^2$$
$$(x-5)^2+(y+3)^2+(z-4)^2=3$$

ANSWER:

$(x-5)^2 + (y+3)^2 + (z-4)^2 = 3$

65. center = (0, 7, –1), radius = 12

SOLUTION:

The formula developed in Exercise 61 for the standard form of the equation of a sphere with a center (h, k, ℓ) and radius r is $(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$. Substitute $h = 0$, $k = 7$, $\ell = -1$, and $r = 12$ into the formula.

$$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$$
$$(x-0)^2+(y-7)^2+[z-(-1)]^2=12^2$$
$$x^2+(y-7)^2+(z+1)^2=144$$

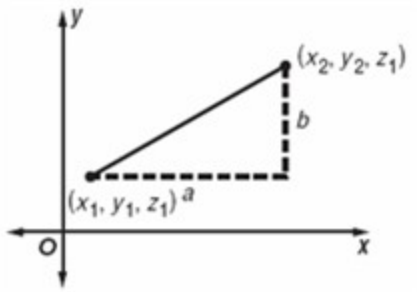
ANSWER:

$x^2 + (y-7)^2 + (z+1)^2 = 144$

66. **REASONING** Prove the Distance Formula in Space. (*Hint*: Use the Pythagorean Theorem twice.)

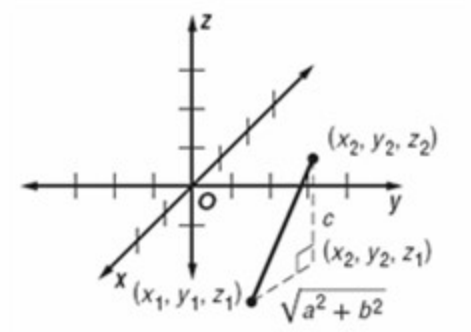
SOLUTION:

Let the two points be (x_1, y_1, z_1) and (x_2, y_2, z_2). The point in the *xy*-plane directly below the second point is (x_2, y_2, z_1). Use the Pythagorean Theorem to find the distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_1). Let the *x*-component of the distance be *a* and the *y*-component of the distance be *b*.



Therefore, the distance from (x_1, y_1, z_1) to (x_2, y_2, z_1) is $\sqrt{a^2+b^2}$.

Now, use the Pythagorean Theorem again to find the distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2). The *xy*-component of the distance is $\sqrt{a^2+b^2}$. Let the *z*-component of the distance be *c*.

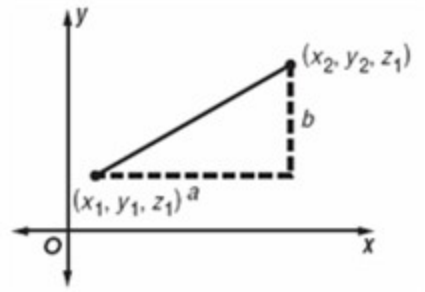


Therefore, the distance from (x_1, y_1, z_1) to (x_2, y_2, z_2) is $\sqrt{\left(\sqrt{a^2+b^2}\right)^2+c^2} = \sqrt{a^2+b^2+c^2}$.

Since $a = x_2 - x_1$, $b = y_2 - y_1$, and $c = z_2 - z_1$, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

ANSWER:

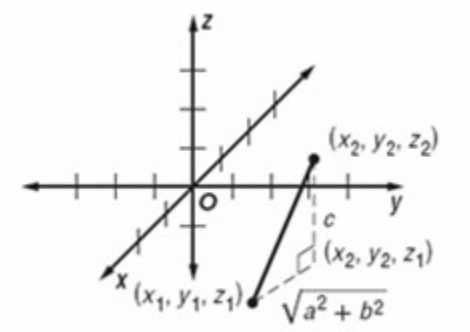
Sample answer: Let the two points be (x_1, y_1, z_1) and (x_2, y_2, z_2). The point in the xy-plane directly below the second point is (x_2, y_2, z_1). Use the Pythagorean Theorem to find the distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_1). Let the x-component of the distance be a and the y-component of the distance be b.



Therefore, the distance from (x_1, y_1, z_1) to (x_2, y_2, z_1) is $\sqrt{a^2+b^2}$.

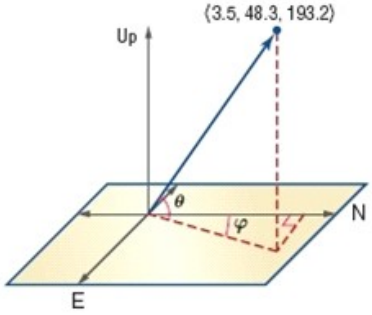
Now, use the Pythagorean Theorem again to find the distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2).

The xy-component of the distance is $\sqrt{a^2+b^2}$. Let the z-component of the distance be c.



Therefore, the distance from (x_1, y_1, z_1) to (x_2, y_2, z_2) is $\sqrt{\left(\sqrt{a^2+b^2}\right)^2+c^2}$ or $\sqrt{a^2+b^2+c^2}$. Since $a = x_2 - x_1$, $b = y_2 - y_1$, and $c = z_2 - z_1$, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

67. **CHALLENGE** Refer to Example 6.



a. Calculate the resultant speed of the rocket.
b. Find the quadrant bearing φ of the rocket.
c. Calculate the resultant angle of incline θ of the rocket relative to the horizontal.

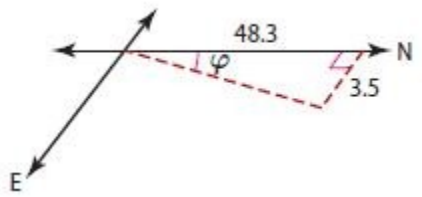
SOLUTION:

a. The resultant speed of the rocket is the magnitude of the vector representing the resultant velocity of the rocket. Find the magnitude of $\langle 3.5, 48.3, 193.2 \rangle$.

$$\sqrt{3.5^2 + 48.3^2 + 193.2^2} = \sqrt{39,671.38}$$
$$\approx 199.2$$

The resultant speed of the rocket is about 199.2 miles per hour.

b. The quadrant bearing of the rocket is the angle in the xy-plane that the rocket makes with the positive y-axis, or north. Use the graph and trigonometric ratios to solve for φ.



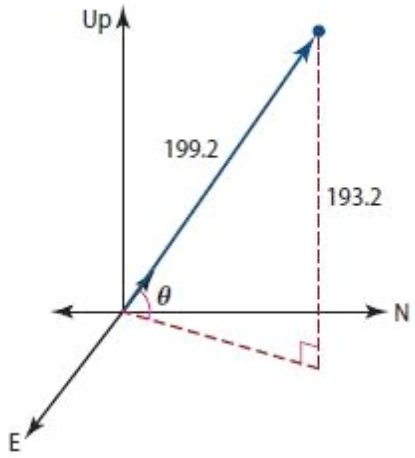
$$\tan\varphi = \frac{3.5}{48.3}$$

$$\varphi = \tan^{-1}\frac{3.5}{48.3}$$

$$\varphi \approx 4.1^\circ$$

The quadrant bearing of the rocket is N4.1°E.

c. The vector representing the velocity of the rocket forms a right triangle with its vertical component. The magnitude of the vector was found in part **a** to be 199.2. Use trigonometric ratios to solve for θ.



$$\sin\theta = \frac{193.2}{199.2}$$

$$\theta = \sin^{-1}\frac{193.2}{199.2}$$

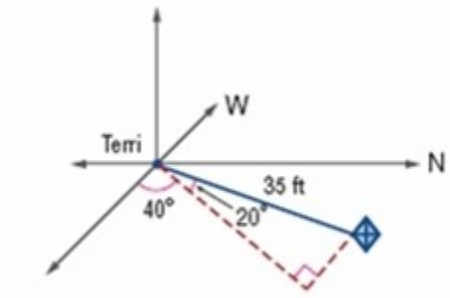
$$\theta \approx 75.9^\circ$$

The angle of incline θ of the rocket is about 75.9°.

ANSWER:

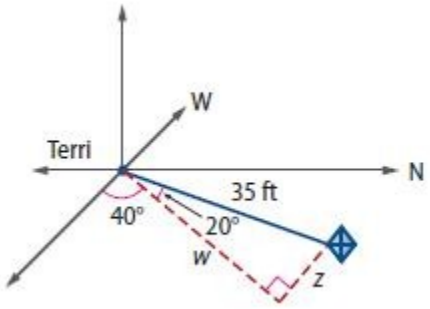
a. about 199.2 mi/h
b. N4.1°E
c. about 75.9°

68. **CHALLENGE** Terri is standing in an open field facing N50°E. She is holding a kite on a 35-foot string that is flying at a 20° angle with the field. Find the components of the vector from Terri to the kite. (*Hint:* Use trigonometric ratios and two right triangles to find **x**, **y**, and **z**.)



SOLUTION:

Use trigonometric ratios to solve for w and z.

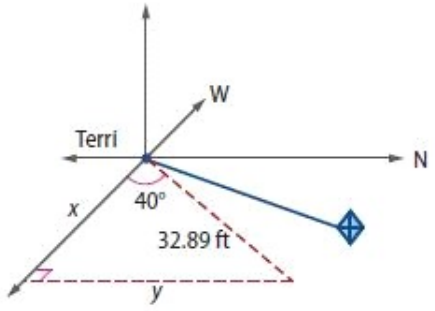


$$\sin 20^\circ = \frac{z}{35} \qquad \cos 20^\circ = \frac{w}{35}$$

$$35\sin 20^\circ = z \qquad 35\cos 20^\circ = w$$

$$11.97 \approx z \qquad 32.89 \approx w$$

w represents the distance on the xy-plane that Terri is from the kite. Use w, the angle given, and trigonometric ratios to solve for x and y.



$$\cos 40^\circ = \frac{x}{32.89} \qquad \sin 40^\circ = \frac{y}{32.89}$$

$$32.89\cos 40^\circ = x \qquad 32.89\sin 40^\circ = y$$

$$25.20 \approx x \qquad 21.14 \approx y$$

The components of the vector from Terri to the kite are $\langle 25.20, 21.14, 11.97 \rangle$.

ANSWER:
$\langle 25.20, 21.14, 11.97 \rangle$

69. **WRITING IN MATH** Describe a situation where it is more reasonable to use a two-dimensional coordinate system and one where it is more reasonable to use a three-dimensional coordinate system.

SOLUTION:

Sample answer: It is more reasonable to use two dimensions when describing position on a map since the map itself is two-dimensional. When the description of the position of an object requires the added dimension of altitude, such as the position of an airplane, it is more reasonable to use three dimensions.

ANSWER:

Sample answer: It is more reasonable to use two dimensions when describing position on a map since the map itself is two-dimensional. When the description of the position of an object requires the added dimension of altitude, such as the position of an airplane, it is more reasonable to use three dimensions.

Find the projection of u onto v. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.

70. $\mathbf{u} = \langle 6, 8 \rangle$, $\mathbf{v} = \langle 2, -1 \rangle$

SOLUTION:

Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}\\ &= \frac{\langle 6,8\rangle\cdot\langle 2,-1\rangle}{|\langle 2,-1\rangle|^2}\langle 2,-1\rangle\\ &= \frac{12-8}{(\sqrt{5})^2}\langle 2,-1\rangle\\ &= \frac{4}{5}\langle 2,-1\rangle\\ &= \left\langle \frac{8}{5}, -\frac{4}{5}\right\rangle\\ &= \langle 1.6, -0.8\rangle\end{aligned}$$

To write $\mathbf{u}$ as the sum of two orthogonal vectors, start by writing $\mathbf{u}$ as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of $\mathbf{u}$ onto $\mathbf{v}$, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2}\\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2}\\ \langle 6, 8\rangle - \langle 1.6, -0.8\rangle &= \mathbf{w_2}\\ \langle 4.4, 8.8\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \langle 1.6, -0.8\rangle + \langle 4.4, 8.8\rangle$.

ANSWER:
$\langle 1.6, -0.8\rangle$; $\mathbf{u} = \langle 1.6, -0.8\rangle + \langle 4.4, 8.8\rangle$

71. $\mathbf{u} = \langle -1, 4 \rangle$, $\mathbf{v} = \langle 5, 1 \rangle$

SOLUTION:

Find the projection of **u** onto **v**.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle -1,4\rangle\cdot\langle 5,1\rangle}{|\langle 5,1\rangle|^2}\langle 5,1\rangle \\ &= \frac{-5+4}{\left(\sqrt{26}\right)^2}\langle 5,1\rangle \\ &= -\frac{1}{26}\langle 5,1\rangle \\ &= \left\langle -\frac{5}{26}, -\frac{1}{26}\right\rangle \\ &= \langle -0.19, -0.04\rangle\end{aligned}$$

To write **u** as the sum of two orthogonal vectors, start by writing **u** as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of **u** onto **v**, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle -1, 4\rangle - \langle -0.19, -0.04\rangle &= \mathbf{w_2} \\ \langle -0.81, 4.04\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \langle -0.19, -0.04\rangle + \langle -0.81, 4.04\rangle$.

ANSWER:

$\langle -0.19, -0.04\rangle$; $\mathbf{u} = \langle -0.19, -0.04\rangle + \langle -0.81, 4.04\rangle$

72. $\mathbf{u} = \langle 5, 4 \rangle$, $\mathbf{v} = \langle 4, -2 \rangle$

SOLUTION:

Find the projection of **u** onto **v**.

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} \\ &= \frac{\langle 5,4\rangle\cdot\langle 4,-2\rangle}{|\langle 4,-2\rangle|^2}\langle 4,-2\rangle \\ &= \frac{20-8}{\left(\sqrt{20}\right)^2}\langle 4,-2\rangle \\ &= \frac{12}{20}\langle 4,-2\rangle \\ &= \left\langle \frac{48}{20}, -\frac{24}{20}\right\rangle \\ &= \langle 2.4, -1.2\rangle\end{aligned}$$

To write **u** as the sum of two orthogonal vectors, start by writing **u** as the sum of two vectors $\mathbf{w_1}$ and $\mathbf{w_2}$, or $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$. Since one of the vectors is the projection of **u** onto **v**, let $\mathbf{w_1} = \text{proj}_{\mathbf{v}}\mathbf{u}$ and solve for $\mathbf{w_2}$.

$$\begin{aligned}\mathbf{u} &= \mathbf{w_1} + \mathbf{w_2} \\ \mathbf{u} - \mathbf{w_1} &= \mathbf{w_2} \\ \langle 5, 4\rangle - \langle 2.4, -1.2\rangle &= \mathbf{w_2} \\ \langle 2.6, 5.2\rangle &= \mathbf{w_2}\end{aligned}$$

Thus, $\mathbf{u} = \langle 2.4, -1.2\rangle + \langle 2.6, 5.2\rangle$.

ANSWER:

$\langle 2.4, -1.2\rangle$; $\mathbf{u} = \langle 2.4, -1.2\rangle + \langle 2.6, 5.2\rangle$

Find the component form and magnitude of $\overline{AB}$ with the given initial and terminal points.

73. $A(6, -4)$, $B(-7, -7)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -7 - (6), -7 - (-4) \rangle \\ &= \langle -13, -3 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = -13$ and $y_2 - y_1 = -3$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}|\overline{AB}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-13)^2 + (-3)^2} \\ &= \sqrt{169 + 9} \\ &= \sqrt{178} \text{ or about } 13.3\end{aligned}$$

ANSWER:

$\langle -13, -3 \rangle$, $\sqrt{178} \approx 13.3$

74. $A(-4, -8)$, $B(1, 6)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1 - (-4), 6 - (-8) \rangle \\ &= \langle 5, 14 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = 5$ and $y_2 - y_1 = 14$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}|\overline{AB}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{5^2 + 14^2} \\ &= \sqrt{221} \text{ or about } 14.9\end{aligned}$$

ANSWER:

$\langle 5, 14 \rangle$, $\sqrt{221} \approx 14.9$

75. $A(-5, -12)$, $B(1, 6)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1 - (-5), 6 - (-12) \rangle \\ &= \langle 6, 18 \rangle\end{aligned}$$

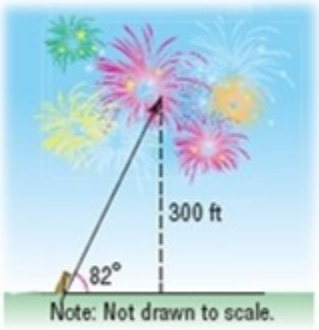
Next, find the magnitude. Substitute $x_2 - x_1 = 6$ and $y_2 - y_1 = 18$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}|\overline{AB}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{6^2 + 18^2} \\ &= \sqrt{360} \text{ or about } 19.0\end{aligned}$$

ANSWER:

$\langle 6, 18 \rangle$, $\sqrt{360} \approx 19.0$

76. **ENTERTAINMENT** The Independence Day fireworks at Memorial Park are fired at an angle of 82° with the horizontal. The technician firing the shells expects them to explode about 300 feet in the air 4.8 seconds after they are fired.



a. Find the initial speed of a shell fired from ground level.

b. Safety barriers will be placed around the launch area to protect spectators. If the barriers are placed 100 yards from the point directly below the explosion of the shells, how far should the barriers be from the point where the fireworks are launched?

SOLUTION:

a. Use the equation for the vertical position of an object, $y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$, and solve for v_0.

Let $t = 4.8$, $h_0 = 0$, $y = 300$, $\theta = 82°$, and $g = 32$ since units is given in feet.

$$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$$

$$300 = (4.8)v_0 \sin 82° - \frac{1}{2}(32)(4.8)^2 + 0$$

$$300 + \frac{1}{2}(32)(4.8)^2 = (4.8)v_0 \sin 82°$$

$$\frac{300 + \frac{1}{2}(32)(4.8)^2}{4.8 \cdot \sin 82°} = v_0$$

$$140.7 \approx v_0$$

The initial speed of the firework is about 140.7 feet per second.

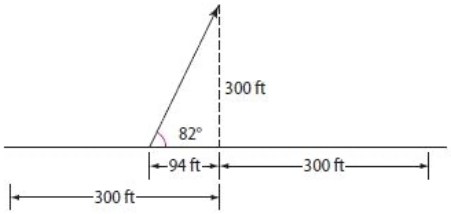
b. Use the equation for the horizontal distance of an object, $x = tv_0 \cos\theta$, and solve for x. Let $t = 4.8$, $v_0 = 140.7$, and $\theta = 82°$.

$$x = tv_0 \cos\theta$$

$$x = (4.8)(140.7)\cos 82°$$

$$x \approx 94.0$$

The horizontal distance from the launch area to the explosion is about 94 feet. The safety barriers will be placed 100 yards, or 300 feet, from the point directly below the explosion as shown.



Drawing is not to scale.

So, the barriers should be about 300 + 94 or 394 feet from the launch site in the direction of the launch and about 300 − 94 or 206 feet from the launch site in the direction away from the launch.

ANSWER:

a. 140.7 ft/s

b. The barriers should be about 394 feet from the launch site in the direction of the launch and about 206 feet from the launch site in the direction away from the launch.

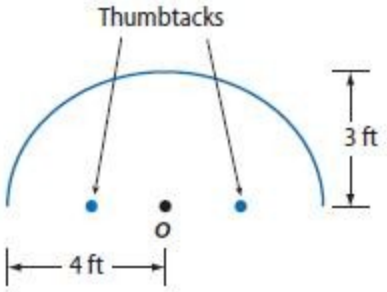
77. **CONSTRUCTION** A stone fireplace that was designed as an arch in the shape of a semi-ellipse will have an opening with a height of 3 feet at the center and a width of 8 feet along the base. To sketch an outline of the fireplace, a contractor uses a string tied to two thumbtacks.

a. At what locations should the thumbtacks be placed?

b. What length of string needs to be used? Explain your reasoning.

SOLUTION:

a. The thumbtacks should be placed at the foci of the ellipse. Diagram the situation.



The distance from the center of the ellipse to the thumbtacks, or foci c, can be solved using the relationship $c^2 = a^2 - b^2$, where $a = 4$ and $b = 3$.

$$c^2 = a^2 - b^2$$

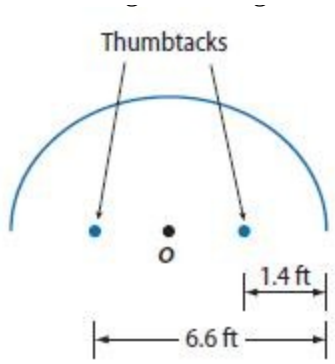
$$c^2 = 4^2 - 3^2$$

$$c^2 = 7$$

$$c = \pm\sqrt{7} \text{ or about } \pm 2.6$$

The thumbtacks should be placed about 2.6 feet to the left and to the right of the center of the arch.

b. 8 ft; Sample answer: With the string anchored by thumbtacks at the foci of the arch and held taut by a pencil, the sum of the distances from each thumbtack to the pencil will remain constant. When the pencil is at one of the bottom corners of the arch, exactly 4 feet from the center, the distance from the far thumbtack to the bottom corner will be 4 + 2.6 = 6.6 feet and the distance from the other thumbtack to the bottom corner will be 4 − 2.6 = 1.4 feet. Thus, the total length of string needed is 8 feet.



ANSWER:

a. about 2.6 feet to the left and to the right of the center of the arch

b. 8 ft; Sample answer: With the string anchored by thumbtacks at the foci of the arch and held taut by a pencil, the sum of the distances from each thumbtack to the pencil will remain constant. When the pencil is at one of the bottom corners of the arch, exactly 4 feet from the center, the distance from the far thumbtack to the bottom corner will be 4 + 2.6 or about 6.6 feet and the distance from the other thumbtack to the bottom corner will be 4 − 2.6 or about 1.4 feet. Thus, the total length of string needed is 6.6 + 1.4 or 8 feet.

Solve each equation for all values of θ.

78. $\csc\theta + 2\cot\theta = 0$

SOLUTION:

$$\csc\theta + 2\cot\theta = 0$$
$$\frac{1}{\sin\theta} + \frac{2\cos\theta}{\sin\theta} = 0$$
$$\frac{1+2\cos\theta}{\sin\theta} = 0$$
$$1+2\cos\theta = 0$$
$$2\cos\theta = -1$$
$$\cos\theta = -\frac{1}{2}$$

The period of cosine is $[0, 2\pi)$. The solutions on this interval are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$, where n is an integer.

ANSWER:

$\frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}$

79. $\sec^2 \theta - 9 = 0$

SOLUTION:

$$\sec^2 \theta - 9 = 0$$
$$\sec^2 \theta = 9$$
$$\frac{1}{\cos^2 \theta} = 9$$
$$\cos^2 \theta = \frac{1}{9}$$
$$\cos \theta = \sqrt{\frac{1}{9}}$$
$$\cos \theta = \pm \frac{1}{3}$$

The period of cosine is $[0, 2\pi)$. The solutions on this interval are $\cos^{-1} \frac{1}{3}$ and $\cos^{-1}\left(-\frac{1}{3}\right)$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π. Therefore, the general form of the solutions is $\cos^{-1} \frac{1}{3} + n\pi, \cos^{-1}\left(-\frac{1}{3}\right) + n\pi$, where n is an integer.

ANSWER:

$\cos^{-1} \frac{1}{3} + n\pi, \cos^{-1}\left(-\frac{1}{3}\right) + n\pi, n \in \mathbb{Z}$

80. $2 \csc \theta - 3 = 5 \sin \theta$

SOLUTION:

$$2 \csc \theta - 3 = 5 \sin \theta$$
$$\frac{2}{\sin \theta} - 3 = 5 \sin \theta$$
$$\sin\theta \cdot \left(\frac{2}{\sin\theta} - 3\right) = \sin\theta \cdot 5 \sin \theta$$
$$2 - 3\sin\theta = 5\sin^2\theta$$
$$-5\sin^2\theta - 3\sin\theta + 2 = 0$$
$$5\sin^2\theta + 3\sin\theta - 2 = 0$$
$$(\sin\theta + 1)(5\sin\theta - 2) = 0$$

$$\sin\theta + 1 = 0$$
$$\sin\theta = -1$$

$$5\sin\theta - 2 = 0$$
$$5\sin\theta = 2$$
$$\sin\theta = \frac{2}{5}$$

The period of sine is $[0, 2\pi)$. The solutions on this interval are $\frac{3\pi}{2}$ and $\sin^{-1} \frac{2}{5}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{3\pi}{2} + 2n\pi, \sin^{-1} \frac{2}{5} + 2n\pi, 2\pi - \sin^{-1} \frac{2}{5} + 2n\pi$, where n is an integer.

ANSWER:

$\frac{3\pi}{2} + 2n\pi, \sin^{-1} \frac{2}{5} + 2n\pi, 2\pi - \sin^{-1} \frac{2}{5} + 2n\pi, n \in \mathbb{Z}$

Sketch the graph of each function.

81. $y = \cos^{-1}(x - 2)$

SOLUTION:

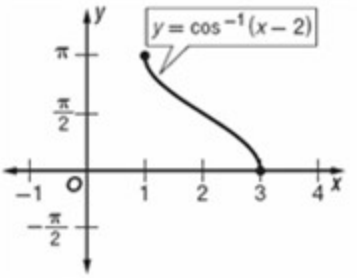
First, rewrite $y = \cos^{-1}(x - 3)$ in the form $\cos y = x$.

$$y = \cos^{-1}(x-2)$$
$$\cos y = (x-2)$$
$$\cos y + 2 = x$$

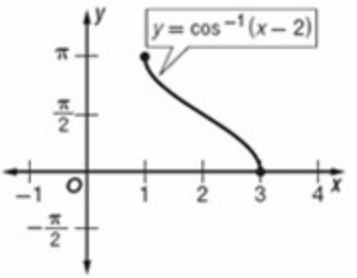
Next, assign values to y on the interval $[0, \pi]$ to make a table of values.

y	$x = \cos y + 2$
0	3
$\frac{\pi}{4}$	2.71
$\frac{\pi}{2}$	2
$\frac{3\pi}{4}$	1.29
π	1

Plot the points and connect them with a smooth curve.



ANSWER:



82. $y = \arccos x + 3$

SOLUTION:

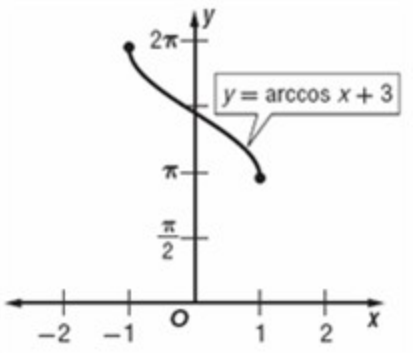
First, rewrite $y = \arccos x + 3$ in the form $\cos y = x$.

$$y = \arccos x + 3$$
$$y - 3 = \arccos x$$
$$\cos(y-3) = x$$

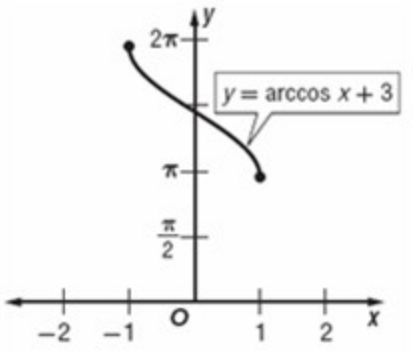
Next, assign values to y on the interval $[\pi, 2\pi]$ to make a table of values.

y	$x = \cos(y - 3)$
π	0.99
$\frac{5\pi}{4}$	0.60
$\frac{3\pi}{2}$	−0.14
$\frac{7\pi}{4}$	−0.80
2π	−0.99

Plot the points and connect them with a smooth curve.



ANSWER:



83. $y = \sin^{-1} 3x$

SOLUTION:

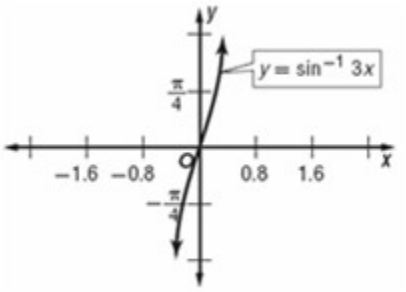
First, rewrite $y = \sin^{-1} 3x$ in the form $\sin y = x$.

$$y = \sin^{-1} 3x$$
$$\sin y = 3x$$
$$\frac{1}{3}\sin y = x$$

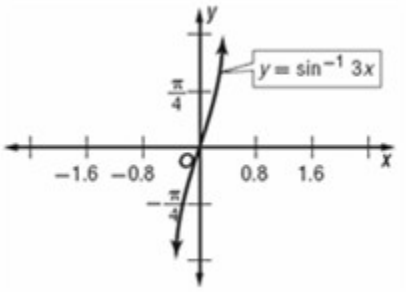
Next, assign values to y on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ to make a table of values.

y	$x = \frac{1}{3}\sin y$
$-\frac{\pi}{2}$	$-\frac{1}{3}$
$-\frac{\pi}{6}$	$-\frac{1}{6}$
0	0
$\frac{\pi}{6}$	$\frac{1}{6}$
$\frac{\pi}{2}$	$\frac{1}{3}$

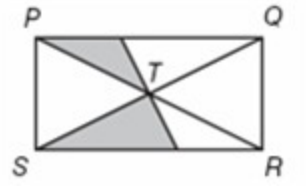
Plot the points and connect them with a smooth curve.



ANSWER:



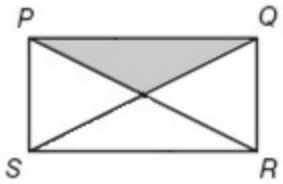
84. **SAT/ACT** What percent of the area of rectangle *PQRS* is shaded?



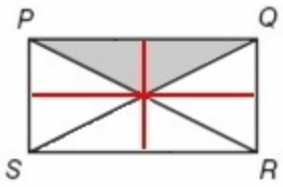
A 22%
B 25%
C 30%
D $33\frac{1}{3}\%$
E 35%

SOLUTION:

By rotating one of the shaded areas, we can see that one of the triangles created by the intersection of the two diagonals is completely shaded.



We now have to determine what portion the shaded triangle is of the rectangle. Draw two additional segments that divide the rectangle into 8 equal parts as shown.



Since 2 of the 8 pieces are shaded, exactly $\frac{1}{4}$ or 25% of the rectangle is shaded.
The correct answer is B.

ANSWER:

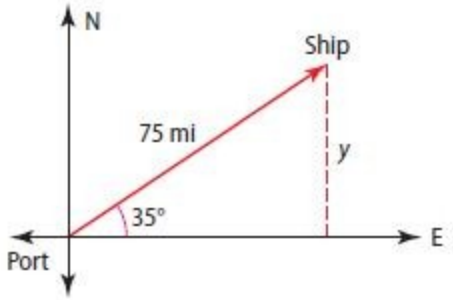
B

85. **REVIEW** A ship leaving port sails for 75 miles in a direction 35° north of east. At that point, how far north of its starting point is the ship?

F 43 miles
G 55 miles
H 61 miles
J 72 miles

SOLUTION:

Diagram the situation.



Use trigonometric ratios to solve for y, the distance north the ship is from its starting point.

$$\sin 35^\circ = \frac{y}{75}$$
$$75 \sin 35^\circ = y$$
$$43.0 \approx y$$

The correct answer is F.

ANSWER:

F

86. During a storm, the force of the wind blowing against a skyscraper can be expressed by the vector $\langle 132, 3454, -76 \rangle$, where the force of the wind is measured in newtons. What is the approximate magnitude of this force?

A 3457 N
B 3568 N
C 3692 N
D 3717 N

SOLUTION:

Use the component form of the vector representing the wind to find the magnitude.

$$\sqrt{132^2 + 3454^2 + (-76)^2} = \sqrt{11{,}953{,}316}$$
$$\approx 3457.4$$

The wind is blowing with a magnitude of about 3457.4 newtons.
The correct answer is A.

ANSWER:

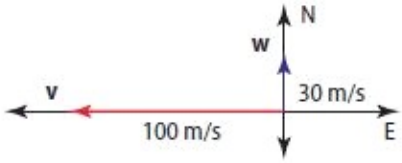
A

87. **REVIEW** An airplane is flying due west at a velocity of 100 meters a second. The wind is blowing from the south at 30 meters a second. What is the approximate magnitude of the airplane's resultant velocity?

F 4 m/s
G 95.4 m/s
H 100 m/s
J 104.4 m/s

SOLUTION:

Diagram the situation.



A vector that represents the velocity of the airplane in component form is $\mathbf{v} = \langle 100, 0 \rangle$. A vector that represents the velocity of the wind in component form is $\mathbf{w} = \langle 0, 30 \rangle$. The airplane's resultant velocity **r** is **v** + **w**.

$$\begin{aligned}\mathbf{r} &= \mathbf{v} + \mathbf{w} \\ &= \langle 100, 0 \rangle + \langle 0, 30 \rangle \\ &= \langle 100, 30 \rangle\end{aligned}$$

Use the component form of **r** to find the magnitude of the airplane's resultant velocity.

$$\begin{aligned}|\mathbf{r}| &= \sqrt{100^2 + 30^2} \\ &= \sqrt{10{,}900} \\ &\approx 104.4\end{aligned}$$

The airplane's resultant velocity is about 104.4 meters per second.
The correct answer is J.

ANSWER:

J

8-5 Dot and Cross Products of Vectors in Space

Find the dot product of u and v. Then determine if u and v are orthogonal.

1. $\mathbf{u} = \langle 3, -9, 6 \rangle, \mathbf{v} = \langle -8, 2, 7 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 3, -9, 6 \rangle \cdot \langle -8, 2, 7 \rangle \\ &= 3(-8) + (-9)(2) + 6(7) \\ &= -24 - 18 + 42 \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

2. $\mathbf{u} = \langle 5, 0, -4 \rangle, \mathbf{v} = \langle 6, -1, 4 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 5, 0, -4 \rangle \cdot \langle 6, -1, 4 \rangle \\ &= 5(6) + 0(-1) + (-4)(4) \\ &= 30 + 0 - 16 \\ &= 14\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

14; not orthogonal

3. $\mathbf{u} = \langle 2, -8, -7 \rangle, \mathbf{v} = \langle 5, 9, -7 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 2, -8, -7 \rangle \cdot \langle 5, 9, -7 \rangle \\ &= 2(5) + (-8)(9) + (-7)(-7) \\ &= 10 - 72 + 49 \\ &= -13\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−13; not orthogonal

4. $\mathbf{u} = \langle -7, -3, 1 \rangle, \mathbf{v} = \langle -4, 5, -13 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle -7, -3, 1 \rangle \cdot \langle -4, 5, -13 \rangle \\ &= (-7)(-4) + (-3)(5) + 1(-13) \\ &= 28 - 15 - 13 \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

5. $\mathbf{u} = \langle 11, 4, -2 \rangle, \mathbf{v} = \langle -1, 3, 8 \rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 11, 4, -2 \rangle \cdot \langle -1, 3, 8 \rangle \\ &= 11(-1) + 4(3) + (-2)(8) \\ &= -11 + 12 + -16 \\ &= -15\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−15; not orthogonal

6. $\mathbf{u} = 6\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}, \mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle 6, -2, -5 \rangle$ and $\mathbf{v} = \langle 3, -2, 6 \rangle$.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 6, -2, -5 \rangle \cdot \langle 3, -2, 6 \rangle \\ &= 6(3) + (-2)(-2) + (-5)(6) \\ &= 18 + 4 + -30 \\ &= -8\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−8; not orthogonal

7. $\mathbf{u} = 3\mathbf{i} - 10\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle 3, -10, 1 \rangle$ and $\mathbf{v} = \langle 7, 2, -1 \rangle$.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 3, -10, 1 \rangle \cdot \langle 7, 2, -1 \rangle \\ &= 3(7) + (-10)(2) + 1(-1) \\ &= 21 - 20 - 1 \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

8. $\mathbf{u} = 9\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$, $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle 9, -9, 6 \rangle$ and $\mathbf{v} = \langle 6, 4, -3 \rangle$.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 9, -9, 6 \rangle \cdot \langle 6, 4, -3 \rangle \\ &= 9(6) + (-9)(4) + 6(-3) \\ &= 54 - 36 - 18 \\ &= 0\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

ANSWER:

0; orthogonal

9. **CHEMISTRY** A water molecule, in which the oxygen atom is centered at the origin, has one hydrogen atom centered at $\langle 55.5, 55.5, -55.5 \rangle$ and the second hydrogen atom centered at $\langle -55.5, -55.5, -55.5 \rangle$. Determine the bond angle between the vectors formed by the oxygen-hydrogen bonds.

SOLUTION:

Let $\mathbf{u} = \langle 55.5, 55.5, -55.5 \rangle$ and $\mathbf{v} = \langle -55.5, -55.5, -55.5 \rangle$. Use the formula for the angle between two vectors.

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$\cos\theta = \frac{\langle 55.5, 55.5, -55.5 \rangle \cdot \langle -55.5, -55.5, -55.5 \rangle}{|\langle 55.5, 55.5, -55.5 \rangle||\langle -55.5, -55.5, -55.5 \rangle|}$$

$$\cos\theta = \frac{-3080.25 + (-3080.25) + 3080.25}{\sqrt{9240.75}\sqrt{9240.75}}$$

$$\cos\theta = \frac{-3080.25}{9240.75}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

$$\theta \approx 109.5°$$

The angle between the vectors formed by the oxygen-hydrogen bonds is about 109.5°.

ANSWER:

about 109.5°

Find the angle θ between vectors u and v to the nearest tenth of a degree.

10. $\mathbf{u} = \langle 3, -2, 2 \rangle$, $\mathbf{v} = \langle 1, 4, -7 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

$$\cos\theta = \frac{\langle 3, -2, 2 \rangle \cdot \langle 1, 4, -7 \rangle}{|\langle 3, -2, 2 \rangle||\langle 1, 4, -7 \rangle|}$$

$$\cos\theta = \frac{3(1) + (-2)4 + 2(-7)}{\sqrt{3^2 + (-2)^2 + 2^2}\sqrt{1^2 + 4^2 + (-7)^2}}$$

$$\cos\theta = \frac{3 + (-8) + (-14)}{\sqrt{9 + 4 + 4}\sqrt{1 + 16 + 49}}$$

$$\cos\theta = \frac{-19}{\sqrt{17}\sqrt{66}}$$

$$\cos\theta = \frac{-19}{\sqrt{1122}}$$

$$\theta \approx 124.6°$$

ANSWER:

124.6°

11. $\mathbf{u} = \langle 6, -5, 1 \rangle, \mathbf{v} = \langle -8, -9, 5 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle 6, -5, 1 \rangle \cdot \langle -8, -9, 5 \rangle}{|\langle 6, -5, 1 \rangle||\langle -8, -9, 5 \rangle|}$$
$$\cos\theta = \frac{6(-8) + (-5)(-9) + 1(5)}{\sqrt{6^2 + (-5)^2 + 1^2}\sqrt{(-8)^2 + (-9)^2 + 5^2}}$$
$$\cos\theta = \frac{-48 + 45 + 5}{\sqrt{36 + 25 + 1}\sqrt{64 + 81 + 25}}$$
$$\cos\theta = \frac{2}{\sqrt{62}\sqrt{170}}$$
$$\cos\theta = \frac{2}{2\sqrt{2635}}$$
$$\cos\theta = \frac{1}{\sqrt{2635}}$$
$$\theta \approx 88.9°$$

ANSWER:

88.9°

12. $\mathbf{u} = \langle -8, 1, 12 \rangle, \mathbf{v} = \langle -6, 4, 2 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -8, 1, 12 \rangle \cdot \langle -6, 4, 2 \rangle}{|\langle -8, 1, 12 \rangle||\langle -6, 4, 2 \rangle|}$$
$$\cos\theta = \frac{(-8)(-6) + 1(4) + 12(2)}{\sqrt{(-8)^2 + 1^2 + 12^2}\sqrt{(-6)^2 + 4^2 + 2^2}}$$
$$\cos\theta = \frac{48 + 4 + 24}{\sqrt{64 + 1 + 144}\sqrt{36 + 16 + 4}}$$
$$\cos\theta = \frac{76}{\sqrt{209}\sqrt{56}}$$
$$\cos\theta = \frac{76}{2\sqrt{2926}}$$
$$\cos\theta = \frac{38}{\sqrt{2926}}$$
$$\theta \approx 45.4°$$

ANSWER:

45.4°

13. $\mathbf{u} = \langle 10, 0, -8 \rangle, \mathbf{v} = \langle 3, -1, -12 \rangle$

SOLUTION:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle 10, 0, -8 \rangle \cdot \langle 3, -1, -12 \rangle}{|\langle 10, 0, -8 \rangle||\langle 3, -1, -12 \rangle|}$$
$$\cos\theta = \frac{10(3) + 0(-1) + (-8)(-12)}{\sqrt{10^2 + 0^2 + (-8)^2}\sqrt{3^2 + (-1)^2 + (-12)^2}}$$
$$\cos\theta = \frac{30 + 0 + 96}{\sqrt{100 + 64}\sqrt{9 + 1 + 144}}$$
$$\cos\theta = \frac{126}{\sqrt{164}\sqrt{154}}$$
$$\cos\theta = \frac{126}{2\sqrt{6314}}$$
$$\cos\theta = \frac{63}{\sqrt{6314}}$$
$$\theta \approx 37.5°$$

ANSWER:

37.5°

14. $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$, $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle -3, 2, 9 \rangle$ and $\mathbf{v} = \langle 4, 3, -10 \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -3, 2, 9 \rangle \cdot \langle 4, 3, -10 \rangle}{|\langle -3, 2, 9 \rangle||\langle 4, 3, -10 \rangle|}$$
$$\cos\theta = \frac{(-3)4 + 2(3) + 9(-10)}{\sqrt{(-3)^2 + 2^2 + 9^2}\sqrt{4^2 + 3^2 + (-10)^2}}$$
$$\cos\theta = \frac{-12 + 6 + (-90)}{\sqrt{9 + 4 + 81}\sqrt{16 + 9 + 100}}$$
$$\cos\theta = \frac{-96}{\sqrt{94}\sqrt{125}}$$
$$\cos\theta = \frac{-96}{5\sqrt{470}}$$
$$\theta \approx 152.3°$$

ANSWER:

152.3°

15. $\mathbf{u} = -6\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = -4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle -6, 3, 5 \rangle$ and $\mathbf{v} = \langle -4, 2, 6 \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$
$$\cos\theta = \frac{\langle -6, 3, 5 \rangle \cdot \langle -4, 2, 6 \rangle}{|\langle -6, 3, 5 \rangle||\langle -4, 2, 6 \rangle|}$$
$$\cos\theta = \frac{(-6)(-4) + 3(2) + 5(6)}{\sqrt{(-6)^2 + 3^2 + 5^2}\sqrt{(-4)^2 + 2^2 + 6^2}}$$
$$\cos\theta = \frac{24 + 6 + 30}{\sqrt{36 + 9 + 25}\sqrt{16 + 4 + 36}}$$
$$\cos\theta = \frac{60}{\sqrt{70}\sqrt{56}}$$
$$\cos\theta = \frac{60}{28\sqrt{5}}$$
$$\cos\theta = \frac{15}{7\sqrt{5}}$$
$$\theta \approx 16.6°$$

ANSWER:

16.6°

Find the cross product of u and v. Then show that u × v is orthogonal to both u and v.

16. $\mathbf{u} = \langle -1, 3, 5 \rangle, \mathbf{v} = \langle 2, -6, -3 \rangle$

SOLUTION:

Find the cross product of **u** and **v**.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 5 \\ 2 & -6 & -3 \end{vmatrix}$$
$$= \begin{vmatrix} 3 & 5 \\ -6 & -3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -1 & 5 \\ 2 & -3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -1 & 3 \\ 2 & -6 \end{vmatrix}\mathbf{k}$$
$$= [-9 - (-30)]\mathbf{i} - (3 - 10)\mathbf{j} + (6 - 6)\mathbf{k}$$
$$= 21\mathbf{i} + 7\mathbf{j} + 0\mathbf{k}$$
$$= \langle 21, 7, 0 \rangle$$

To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both **u** and **v**, find the dot product of $\mathbf{u} \times \mathbf{v}$ with **u** and $\mathbf{u} \times \mathbf{v}$ with **v**.

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 21, 7, 0 \rangle \cdot \langle -1, 3, 5 \rangle$$
$$= 21(-1) + 7(3) + 0(5)$$
$$= -21 + 21 + 0$$
$$= 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 21, 7, 0 \rangle \cdot \langle 2, -6, -3 \rangle$$
$$= 21(2) + 7(-6) + 0(-3)$$
$$= 42 - 42 + 0$$
$$= 0$$

Because both dot products are zero, the vectors are orthogonal.

ANSWER:

$\mathbf{u} \times \mathbf{v} = \langle 21, 7, 0 \rangle$
$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$
$= \langle 21, 7, 0 \rangle \cdot \langle -1, 3, 5 \rangle$
$= 21(-1) + 7(3) + 0(5)$
$= 0$
$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$
$= \langle 21, 7, 0 \rangle \cdot \langle 2, -6, -3 \rangle$
$= 21(2) + 7(-6) + 0(-3)$
$= 0$

17. $\mathbf{u} = \langle 4, 7, -2 \rangle, \mathbf{v} = \langle -5, 9, 1 \rangle$

SOLUTION:

Find the cross product of **u** and **v**.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 7 & -2 \\ -5 & 9 & 1 \end{vmatrix}$$
$$= \begin{vmatrix} 7 & -2 \\ 9 & 1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 4 & -2 \\ -5 & 1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 4 & 7 \\ -5 & 9 \end{vmatrix}\mathbf{k}$$
$$= [7 - (-18)]\mathbf{i} - (4 - 10)\mathbf{j} + [36 - (-35)]\mathbf{k}$$
$$= 25\mathbf{i} + 6\mathbf{j} + 71\mathbf{k}$$
$$= \langle 25, 6, 71 \rangle$$

To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both **u** and **v**, find the dot product of $\mathbf{u} \times \mathbf{v}$ with **u** and $\mathbf{u} \times \mathbf{v}$ with **v**.

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 25, 6, 71 \rangle \cdot \langle 4, 7, -2 \rangle$$
$$= 25(4) + 6(7) + 71(-2)$$
$$= 100 + 42 - 142$$
$$= 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 25, 6, 71 \rangle \cdot \langle -5, 9, 1 \rangle$$
$$= 25(-5) + 6(9) + 71(1)$$
$$= -125 + 54 + 71$$
$$= 0$$

Because both dot products are zero, the vectors are orthogonal.

ANSWER:

$\mathbf{u} \times \mathbf{v} = \langle 25, 6, 71 \rangle$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$
$= \langle 25, 6, 71 \rangle \cdot \langle 4, 7, -2 \rangle$
$= 25(4) + 6(7) + 71(-2)$
$= 0$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$
$= \langle 25, 6, 71 \rangle \cdot \langle -5, 9, 1 \rangle$
$= 25(-5) + 6(9) + 71(1)$
$= 0$

18. $\mathbf{u} = \langle 3, -6, 2 \rangle, \mathbf{v} = \langle 1, 5, -8 \rangle$

SOLUTION:

Find the cross product of **u** and **v**.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 2 \\ 1 & 5 & -8 \end{vmatrix}$$
$$= \begin{vmatrix} -6 & 2 \\ 5 & -8 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & 2 \\ 1 & -8 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & -6 \\ 1 & 5 \end{vmatrix}\mathbf{k}$$
$$= (48 - 10)\mathbf{i} - (-24 - 2)\mathbf{j} + [15 - (-6)]\mathbf{k}$$
$$= 38\mathbf{i} + 26\mathbf{j} + 21\mathbf{k}$$
$$= \langle 38, 26, 21 \rangle$$

To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both **u** and **v**, find the dot product of $\mathbf{u} \times \mathbf{v}$ with **u** and $\mathbf{u} \times \mathbf{v}$ with **v**.

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 38, 26, 21 \rangle \cdot \langle 3, -6, 2 \rangle$$
$$= 38(3) + 26(-6) + 21(2)$$
$$= 114 - 156 + 42$$
$$= 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 38, 26, 21 \rangle \cdot \langle 1, 5, -8 \rangle$$
$$= 38(1) + 26(5) + 21(-8)$$
$$= 38 + 130 - 168$$
$$= 0$$

Because both dot products are zero, the vectors are orthogonal.

ANSWER:

$\mathbf{u} \times \mathbf{v} = \langle 38, 26, 21 \rangle$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$
$= \langle 38, 26, 21 \rangle \cdot \langle 3, -6, 2 \rangle$
$= 38(3) + 26(-6) + 21(2)$
$= 0$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$
$= \langle 38, 26, 21 \rangle \cdot \langle 1, 5, -8 \rangle$
$= 38(1) + 26(5) + 21(-8)$
$= 0$

19. $\mathbf{u} = \langle 5, -8, 0 \rangle, \mathbf{v} = \langle -4, -2, 7 \rangle$

SOLUTION:

Find the cross product of **u** and **v**.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -8 & 0 \\ -4 & -2 & 7 \end{vmatrix}$$
$$= \begin{vmatrix} -8 & 0 \\ -2 & 7 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 5 & 0 \\ -4 & 7 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 5 & -8 \\ -4 & -2 \end{vmatrix}\mathbf{k}$$
$$= (-56 - 0)\mathbf{i} - (35 - 0)\mathbf{j} + (-10 - 32)\mathbf{k}$$
$$= -56\mathbf{i} - 35\mathbf{j} - 42\mathbf{k}$$
$$= \langle -56, -35, -42 \rangle$$

To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both **u** and **v**, find the dot product of $\mathbf{u} \times \mathbf{v}$ with **u** and $\mathbf{u} \times \mathbf{v}$ with **v**.

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle -56, -35, -42 \rangle \cdot \langle 5, -8, 0 \rangle$$
$$= (-56)(5) + (-35)(-8) + (-42)(0)$$
$$= -280 + 280 + 0$$
$$= 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle -56, -35, -42 \rangle \cdot \langle -4, -2, 7 \rangle$$
$$= (-56)(-4) + (-35)(-2) + (-42)(7)$$
$$= 224 + 70 - 294$$
$$= 0$$

Because both dot products are zero, the vectors are orthogonal.

ANSWER:

$\mathbf{u} \times \mathbf{v} = \langle -56, -35, -42 \rangle$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle -56, -35, -42 \rangle \cdot \langle 5, -8, 0 \rangle$$
$$= (-56)(5) + (-35)(-8) + (-42)(0)$$
$$= 0$$
$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle -56, -35, -42 \rangle \cdot \langle -4, -2, 7 \rangle$$
$$= (-56)(-4) + (-35)(-2) + (-42)(7)$$
$$= 0$$

20. $\mathbf{u} = -2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}, \mathbf{v} = 7\mathbf{i} + \mathbf{j} - 6\mathbf{k}$

SOLUTION:

Write **u** and **v** in component form as $\mathbf{u} = \langle -2, -2, 5 \rangle$ and $\mathbf{v} = \langle 7, 1, -6 \rangle$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -2 & 5 \\ 7 & 1 & -6 \end{vmatrix}$$
$$= \begin{vmatrix} -2 & 5 \\ 1 & -6 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -2 & 5 \\ 7 & -6 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -2 & -2 \\ 7 & 1 \end{vmatrix}\mathbf{k}$$
$$= (12 - 5)\mathbf{i} - (12 - 35)\mathbf{j} + [-2 - (-14)]\mathbf{k}$$
$$= 7\mathbf{i} + 23\mathbf{j} + 12\mathbf{k}$$
$$= \langle 7, 23, 12 \rangle$$

To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both **u** and **v**, find the dot product of $\mathbf{u} \times \mathbf{v}$ with **u** and $\mathbf{u} \times \mathbf{v}$ with **v**.

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 7, 23, 12 \rangle \cdot \langle -2, -2, 5 \rangle$$
$$= 7(-2) + 23(-2) + 12(5)$$
$$= -14 - 46 + 60$$
$$= 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 7, 23, 12 \rangle \cdot \langle 7, 1, -6 \rangle$$
$$= 7(7) + 23(1) + 12(-6)$$
$$= 49 + 23 - 72$$
$$= 0$$

Because both dot products are zero, the vectors are orthogonal.

ANSWER:

$\mathbf{u} \times \mathbf{v} = \langle 7, 23, 12 \rangle$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 7, 23, 12 \rangle \cdot \langle -2, -2, 5 \rangle$$
$$= 7(-2) + 23(-2) + 12(5)$$
$$= 0$$
$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 7, 23, 12 \rangle \cdot \langle 7, 1, -6 \rangle$$
$$= 7(7) + 23(1) + 12(-6)$$
$$= 0$$

21. $\mathbf{u} = -4\mathbf{i} + \mathbf{j} + 8\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$

SOLUTION:

Write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle -4, 1, 8 \rangle$ and $\mathbf{v} = \langle 3, -4, -3 \rangle$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 8 \\ 3 & -4 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 8 \\ -4 & -3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -4 & 8 \\ 3 & -3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -4 & 1 \\ 3 & -4 \end{vmatrix}\mathbf{k}$$

$$= [-3 - (-32)]\mathbf{i} - (12 - 24)\mathbf{j} + (16 - 3)\mathbf{k}$$

$$= 29\mathbf{i} + 12\mathbf{j} + 13\mathbf{k}$$

$$= \langle 29, 12, 13 \rangle$$

To show that $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, find the dot product of $\mathbf{u} \times \mathbf{v}$ with $\mathbf{u}$ and $\mathbf{u} \times \mathbf{v}$ with $\mathbf{v}$.

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 29, 12, 13 \rangle \cdot \langle -4, 1, 8 \rangle$$
$$= 29(-4) + 12(1) + 13(8)$$
$$= -116 + 12 + 104$$
$$= 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 29, 12, 13 \rangle \cdot \langle 3, -4, -3 \rangle$$
$$= 29(3) + 12(-4) + 13(-3)$$
$$= 87 - 48 - 39$$
$$= 0$$

Because both dot products are zero, the vectors are orthogonal.

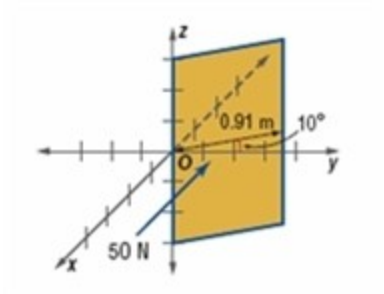
ANSWER:

$\mathbf{u} \times \mathbf{v} = \langle 29, 12, 13 \rangle$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u}$$
$$= \langle 29, 12, 13 \rangle \cdot \langle -4, 1, 8 \rangle$$
$$= 29(-4) + 12(1) + 13(8)$$
$$= 0$$

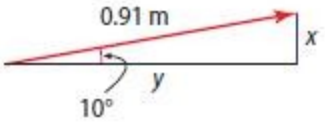
$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}$$
$$= \langle 29, 12, 13 \rangle \cdot \langle 3, -4, -3 \rangle$$
$$= 29(3) + 12(-4) + 13(-3)$$
$$= 0$$

22. **RESTAURANTS** A restaurant server applies 50 newtons of force to open a door. Find the magnitude and direction of the torque about the door's hinge.



SOLUTION:

The component form of the vector representing the directed distance from the axis of rotation to the end of the door can be found using the triangle shown and trigonometric ratios.



Vector $\mathbf{r}$ is therefore $\langle -0.91\sin 10°, 0.91\cos 10°, 0 \rangle$ or about $\langle -0.16, 0.90, 0 \rangle$. Notice that the x-component is negative since the door is resting 10° from the y-axis in the direction of the negative x-axis. The vector representing the force applied to the end of the door is 50 newtons in the direction of the negative x-axis, so $\mathbf{F} = \langle -50, 0, 0 \rangle$.

Use the cross product of these vectors to find the vector representing the torque about the hinge.

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.16 & 0.90 & 0 \\ -50 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0.90 & 0 \\ 0 & 0 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -0.16 & 0 \\ -50 & 0 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -0.16 & 0.90 \\ -50 & 0 \end{vmatrix}\mathbf{k}$$

$$= 0\mathbf{i} - 0\mathbf{j} - 45\mathbf{k}$$

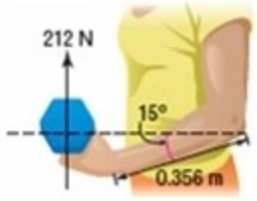
$$= \langle 0, 0, -45 \rangle$$

The component form of the torque vector $\langle 0, 0, -45 \rangle$ tells us that the magnitude of the vector is about 45 newton-meters parallel to the negative z-axis.

ANSWER:

about 45 N· m parallel to the negative z-axis

23. **WEIGHTLIFTING** A weightlifter doing bicep curls applies 212 newtons of force to lift the dumbbell. The weightlifter's forearm is 0.356 meters long and she begins the bicep curl with her elbow bent at a 15° angle below the horizontal, in the direction of the positive *x*-axis.

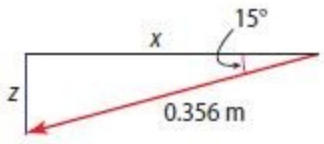


a. Find the vector representing the torque about the weightlifter's elbow in component form.

b. Find the magnitude and direction of the torque.

SOLUTION:

a. The component form of the vector representing the directed distance from the axis of rotation to the end of the weightlifter's forearm can be found using the triangle shown and trigonometric ratios.



Vector **r** is therefore $\langle 0.356\cos 15^\circ, 0, 0.356\sin 15^\circ \rangle$ or about $\langle 0.34, 0, -0.09 \rangle$. Notice that the *z*-component is negative since the weightlifter's arm is at a 15° angle below the horizontal. The vector representing the force applied to the weight is 212 newtons up, so $\mathbf{F} = \langle 0, 0, 212 \rangle$.

Use the cross product of these vectors to find the vector representing the torque about the hinge.

$$\begin{aligned}\mathbf{T} &= \mathbf{r} \times \mathbf{F}\\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.34 & 0 & -0.09 \\ 0 & 0 & 212 \end{vmatrix}\\ &= \begin{vmatrix} 0 & -0.09 \\ 0 & 212 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0.34 & -0.09 \\ 0 & 212 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0.34 & 0 \\ 0 & 0 \end{vmatrix}\mathbf{k}\\ &= 0\mathbf{i} - 72.08\mathbf{j} - 0\mathbf{k}\\ &= \langle 0, -72.08, 0 \rangle\end{aligned}$$

The component form of the torque vector is $\langle 0, -72.08, 0 \rangle$.

b. The component form of the torque vector $\langle 0, -72.08, 0 \rangle$ tells us that the magnitude of the vector is about 72.08 newton-meters parallel to the negative *y*-axis.

ANSWER:

a. $\langle 0, -72.08, 0 \rangle$

b. about 72.08 N · m parallel to the negative *y*-axis

Find the area of the parallelogram with adjacent sides u and v.

24. $\mathbf{u} = \langle 2, -5, 3 \rangle, \mathbf{v} = \langle 4, 6, -1 \rangle$

SOLUTION:

First, find $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 3 \\ 4 & 6 & -1 \end{vmatrix}\\ &= \begin{vmatrix} -5 & 3 \\ 6 & -1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 2 & -5 \\ 4 & 6 \end{vmatrix}\mathbf{k}\\ &= (5-18)\mathbf{i} - (-2-12)\mathbf{j} + [12-(-20)]\mathbf{k}\\ &= -13\mathbf{i} + 14\mathbf{j} + 32\mathbf{k}\\ &= \langle -13, 14, 32 \rangle\end{aligned}$$

Then, find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}|\mathbf{u} \times \mathbf{v}| &= \sqrt{(-13)^2 + 14^2 + 32^2}\\ &= \sqrt{169 + 196 + 1024}\\ &= \sqrt{1389}\\ &\approx 37.3\end{aligned}$$

The area of the parallelogram is about 37.3 square units.

ANSWER:

$\sqrt{1389} \approx 37.3$ units^2

25. $\mathbf{u} = \langle -9, 1, 2 \rangle, \mathbf{v} = \langle 6, -5, 3 \rangle$

SOLUTION:

First, find $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -9 & 1 & 2 \\ 6 & -5 & 3 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 2 \\ -5 & 3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -9 & 2 \\ 6 & 3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -9 & 1 \\ 6 & -5 \end{vmatrix}\mathbf{k} \\ &= [3-(-10)]\mathbf{i} - (-27-12)\mathbf{j} + (45-6)\mathbf{k} \\ &= 13\mathbf{i} + 39\mathbf{j} + 39\mathbf{k} \\ &= \langle 13, 39, 39 \rangle\end{aligned}$$

Then, find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}|\mathbf{u} \times \mathbf{v}| &= \sqrt{13^2 + 39^2 + 39^2} \\ &= \sqrt{169 + 1521 + 1521} \\ &= \sqrt{3211} \\ &= 13\sqrt{19} \\ &\approx 56.7\end{aligned}$$

The area of the parallelogram is about 56.7 square units.

ANSWER:

$13\sqrt{19} \approx 56.7$ units^2

26. $\mathbf{u} = \langle 4, 3, -1 \rangle, \mathbf{v} = \langle 7, 2, -2 \rangle$

SOLUTION:

First, find $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & -1 \\ 7 & 2 & -2 \end{vmatrix} \\ &= \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 4 & -1 \\ 7 & -2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 4 & 3 \\ 7 & 2 \end{vmatrix}\mathbf{k} \\ &= [-6-(-2)]\mathbf{i} - [-8-(-7)]\mathbf{j} + (8-21)\mathbf{k} \\ &= -4\mathbf{i} - 1\mathbf{j} - 13\mathbf{k} \\ &= \langle -4, -1, -13 \rangle\end{aligned}$$

Then, find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}|\mathbf{u} \times \mathbf{v}| &= \sqrt{(-4)^2 + (-1)^2 + (-13)^2} \\ &= \sqrt{16 + 1 + 169} \\ &= \sqrt{186} \\ &\approx 13.6\end{aligned}$$

The area of the parallelogram is about 13.6 square units.

ANSWER:

$\sqrt{186} \approx 13.6$ units^2

27. $\mathbf{u} = 6\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = 5\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

SOLUTION:

First, write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle 6, -2, 5 \rangle$ and $\mathbf{v} = \langle 5, -4, -8 \rangle$. Next, find $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -2 & 5 \\ 5 & -4 & -8 \end{vmatrix} \\ &= \begin{vmatrix} -2 & 5 \\ -4 & -8 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 6 & 5 \\ 5 & -8 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 6 & -2 \\ 5 & -4 \end{vmatrix}\mathbf{k} \\ &= [16 - (-20)]\mathbf{i} - (-48 - 25)\mathbf{j} + [-24 - (-10)]\mathbf{k} \\ &= 36\mathbf{i} + 73\mathbf{j} - 14\mathbf{k} \\ &= \langle 36, 73, -14 \rangle\end{aligned}$$

Then, find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}|u \times v| &= \sqrt{36^2 + 73^2 + (-14)^2} \\ &= \sqrt{1296 + 5329 + 196} \\ &= \sqrt{6821} \\ &\approx 82.6\end{aligned}$$

The area of the parallelogram is about 82.6 square units.

ANSWER:

$\sqrt{6821} \approx 82.6$ units^2

28. $\mathbf{u} = \mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

SOLUTION:

First, write $\mathbf{u}$ and $\mathbf{v}$ in component form as $\mathbf{u} = \langle 1, 4, -8 \rangle$ and $\mathbf{v} = \langle -2, 3, -7 \rangle$. Next, find $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -8 \\ -2 & 3 & -7 \end{vmatrix} \\ &= \begin{vmatrix} 4 & -8 \\ 3 & -7 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & -8 \\ -2 & -7 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 4 \\ -2 & 3 \end{vmatrix}\mathbf{k} \\ &= [-28 - (-24)]\mathbf{i} - (-7 - 16)\mathbf{j} + [3 - (-8)]\mathbf{k} \\ &= -4\mathbf{i} + 23\mathbf{j} + 11\mathbf{k} \\ &= \langle -4, 23, 11 \rangle\end{aligned}$$

Then, find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}|u \times v| &= \sqrt{(-4)^2 + 23^2 + 11^2} \\ &= \sqrt{16 + 529 + 121} \\ &= \sqrt{666} \\ &= 3\sqrt{74} \\ &\approx 25.8\end{aligned}$$

The area of the parallelogram is about 25.8 square units.

ANSWER:

$3\sqrt{74} \approx 25.8$ units^2

29. $\mathbf{u} = -3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = 4\mathbf{i} - \mathbf{j} + 6\mathbf{k}$

SOLUTION:

First, write **u** and **v** in component form as $\mathbf{u} = \langle -3, -5, 3 \rangle$ and $\mathbf{v} = \langle 4, -1, 6 \rangle$. Next, find $\mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -5 & 3 \\ 4 & -1 & 6 \end{vmatrix}$$
$$= \begin{vmatrix} -5 & 3 \\ -1 & 6 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -3 & 3 \\ 4 & 6 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -3 & -5 \\ 4 & -1 \end{vmatrix}\mathbf{k}$$
$$= [-30 - (-3)]\mathbf{i} - (-18 - 12)\mathbf{j} + [3 - (-20)]\mathbf{k}$$
$$= -27\mathbf{i} + 30\mathbf{j} + 23\mathbf{k}$$
$$= \langle -27, 30, 23 \rangle$$

Then, find the magnitude of $\mathbf{u} \times \mathbf{v}$.

$$|u \times v| = \sqrt{(-27)^2 + 30^2 + 23^2}$$
$$= \sqrt{729 + 900 + 529}$$
$$= \sqrt{2158}$$
$$\approx 46.5$$

The area of the parallelogram is about 46.5 square units.

ANSWER:

$\sqrt{2158} \approx 46.5$ units^2

Find the volume of the parallelepiped having t, u, and v as adjacent edges.

30. $\mathbf{t} = \langle -1, -9, 2 \rangle, \mathbf{u} = \langle 4, -7, -5 \rangle, \mathbf{v} = \langle 3, -2, 6 \rangle$

SOLUTION:

$$\mathbf{t} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} -1 & -9 & 2 \\ 4 & -7 & -5 \\ 3 & -2 & 6 \end{vmatrix}$$
$$= \begin{vmatrix} -7 & -5 \\ -2 & 6 \end{vmatrix}(-1) - \begin{vmatrix} 4 & -5 \\ 3 & 6 \end{vmatrix}(-9) + \begin{vmatrix} 4 & -7 \\ 3 & -2 \end{vmatrix}(2)$$
$$= (-42 - 10)(-1) - [24 - (-15)](-9) + [-8 - (-21)](2)$$
$$= 52 + 351 + 26$$
$$= 429$$

The volume of the parallelepiped is 429 cubic units.

ANSWER:

429 units^3

31. $\mathbf{t} = \langle -6, 4, -8 \rangle, \mathbf{u} = \langle -3, -1, 6 \rangle, \mathbf{v} = \langle 2, 5, -7 \rangle$

SOLUTION:

$$\mathbf{t} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} -6 & 4 & -8 \\ -3 & -1 & 6 \\ 2 & 5 & -7 \end{vmatrix}$$
$$= \begin{vmatrix} -1 & 6 \\ 5 & -7 \end{vmatrix}(-6) - \begin{vmatrix} -3 & 6 \\ 2 & -7 \end{vmatrix}(4) + \begin{vmatrix} -3 & -1 \\ 2 & 5 \end{vmatrix}(-8)$$
$$= (7 - 30)(-6) - (21 - 12)(4) + [-15 - (-2)](-8)$$
$$= 138 - 36 + 104$$
$$= 206$$

The volume of the parallelepiped is 206 cubic units.

ANSWER:

206 units^3

32. $\mathbf{t} = \langle 2, -3, -1 \rangle, \mathbf{u} = \langle 4, -6, 3 \rangle, \mathbf{v} = \langle -9, 5, -4 \rangle$

SOLUTION:

$$\mathbf{t} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} 2 & -3 & -1 \\ 4 & -6 & 3 \\ -9 & 5 & -4 \end{vmatrix}$$
$$= \begin{vmatrix} -6 & 3 \\ 5 & -4 \end{vmatrix}(2) - \begin{vmatrix} 4 & 3 \\ -9 & -4 \end{vmatrix}(-3) + \begin{vmatrix} 4 & -6 \\ -9 & 5 \end{vmatrix}(-1)$$
$$= (24 - 15)(2) - [-16 - (-27)](-3) + (20 - 54)(-1)$$
$$= 18 + 33 + 34$$
$$= 85$$

The volume of the parallelepiped is 85 cubic units.

ANSWER:

85 units^3

33. $\mathbf{t} = -4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{u} = 5\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$

SOLUTION:

Write **t**, **u**, and **v** in component form as $\mathbf{t} = \langle -4, 1, 3 \rangle, \mathbf{u} = \langle 5, 7, -6 \rangle$, and $\mathbf{v} = \langle 3, -2, -5 \rangle$.

$$\mathbf{t} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} -4 & 1 & 3 \\ 5 & 7 & -6 \\ 3 & -2 & -5 \end{vmatrix}$$
$$= \begin{vmatrix} 7 & -6 \\ -2 & -5 \end{vmatrix}(-4) - \begin{vmatrix} 5 & -6 \\ 3 & -5 \end{vmatrix}(1) + \begin{vmatrix} 5 & 7 \\ 3 & -2 \end{vmatrix}(3)$$
$$= (-35 - 12)(-4) - [-25 - (-18)](1) + (-10 - 21)(3)$$
$$= 188 + 7 - 93$$
$$= 102$$

The volume of the parallelepiped is 102 cubic units.

ANSWER:

102 units^3

34. $\mathbf{t} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - 6\mathbf{j} + 8\mathbf{k}$

SOLUTION:

Write **t**, **u**, and **v** in component form as $\mathbf{t} = \langle 1,1,-4 \rangle$, $\mathbf{u} = \langle -3,2,7 \rangle$, and $\mathbf{v} = \langle 2,-6,8 \rangle$.

$$\mathbf{t} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} 1 & 1 & -4 \\ -3 & 2 & 7 \\ 2 & -6 & 8 \end{vmatrix}$$
$$= \begin{vmatrix} 2 & 7 \\ -6 & 8 \end{vmatrix}(1) - \begin{vmatrix} -3 & 7 \\ 2 & 8 \end{vmatrix}(1) + \begin{vmatrix} -3 & 2 \\ 2 & -6 \end{vmatrix}(-4)$$
$$= [16 - (-42)](1) - (-24 - 14)(1) + (18 - 4)(-4)$$
$$= 58 + 38 - 56$$
$$= 40$$

The volume of the parallelepiped is 40 cubic units.

ANSWER:

40 units^3

35. $\mathbf{t} = 5\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $\mathbf{u} = 3\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$, $\mathbf{v} = 8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

SOLUTION:

Write **t**, **u**, and **v** in component form as $\mathbf{t} = \langle 5,-2,6 \rangle$, $\mathbf{u} = \langle 3,-5,7 \rangle$, and $\mathbf{v} = \langle 8,-1,4 \rangle$.

$$\mathbf{t} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} 5 & -2 & 6 \\ 3 & -5 & 7 \\ 8 & -1 & 4 \end{vmatrix}$$
$$= \begin{vmatrix} -5 & 7 \\ -1 & 4 \end{vmatrix}(5) - \begin{vmatrix} 3 & 7 \\ 8 & 4 \end{vmatrix}(-2) + \begin{vmatrix} 3 & -5 \\ 8 & -1 \end{vmatrix}(6)$$
$$= [-20 - (-7)](5) - (12 - 56)(-2) + [-3 - (-40)](6)$$
$$= -65 - 88 + 222$$
$$= 69$$

The volume of the parallelepiped is 69 cubic units.

ANSWER:

69 units^3

Find a vector that is orthogonal to each vector.

36. $\langle 3, -8, 4 \rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle 3,-8,4 \rangle$ and $\mathbf{b} = \langle x,y,z \rangle$. Find the dot product of **a** and **b**.

$$\mathbf{a} \cdot \mathbf{b} = \langle 3,-8,4 \rangle \cdot \langle x,y,z \rangle$$
$$= 3x + (-8)y + 4z$$
$$= 3x - 8y + 4z$$

If **a** and **b** are orthogonal, then $3x - 8y + 4z = 0$. Substitute values for x and y and solve for z. Let $x = 4$ and $y = 3$.

$$3x - 8y + 4z = 0$$
$$3(4) - 8(3) + 4z = 0$$
$$-12 + 4z = 0$$
$$4z = 12$$
$$z = 3$$

A vector orthogonal to $\langle 3,-8,4 \rangle$ is $\langle 4,3,3 \rangle$.

ANSWER:

Sample answer: $\langle 4, 3, 3 \rangle$

37. $\langle -1, -2, 5\rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle -1, -2, 5\rangle$ and $\mathbf{b} = \langle x, y, z\rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \langle -1, -2, 5\rangle \cdot \langle x, y, z\rangle \\ &= -1x + (-2)y + 5z \\ &= -x - 2y + 5z\end{aligned}$$

If **a** and **b** are orthogonal, then $-x - 2y + 5z = 0$. Substitute values for x and y and solve for z. Let $x = 5$ and $y = 5$.

$$\begin{aligned}-x - 2y + 5z &= 0 \\ -(5) - 2(5) + 5z &= 0 \\ -15 + 5z &= 0 \\ 5z &= 15 \\ z &= 3\end{aligned}$$

A vector orthogonal to $\langle -1, -2, 5\rangle$ is $\langle 5, 5, 3\rangle$.

ANSWER:

Sample answer: $\langle 5, 5, 3\rangle$

38. $\left\langle 6, -\frac{1}{3}, -3\right\rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \left\langle 6, -\frac{1}{3}, -3\right\rangle$ and $\mathbf{b} = \langle x, y, z\rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \left\langle 6, -\frac{1}{3}, -3\right\rangle \cdot \langle x, y, z\rangle \\ &= 6x + \left(-\frac{1}{3}\right)y + (-3)z \\ &= 6x - \frac{1}{3}y - 3z\end{aligned}$$

If **a** and **b** are orthogonal, then $6x - \frac{1}{3}y - 3z = 0$.

Substitute values for x and y and solve for z. Let $x = 1$ and $y = 9$.

$$\begin{aligned}6x - \frac{1}{3}y - 3z &= 0 \\ 6(1) - \frac{1}{3}(9) - 3z &= 0 \\ 3 - 3z &= 0 \\ -3z &= -3 \\ z &= 1\end{aligned}$$

A vector orthogonal to $\left\langle 6, -\frac{1}{3}, -3\right\rangle$ is $\langle 1, 9, 1\rangle$.

ANSWER:

Sample answer: $\langle 1, 9, 1\rangle$

39. $\langle 7, 0, 8 \rangle$

SOLUTION:

Sample answer: Two vectors are orthogonal if and only if their dot product is equal to 0. Let $\mathbf{a} = \langle 7,0,8 \rangle$ and $\mathbf{b} = \langle x, y, z \rangle$. Find the dot product of **a** and **b**.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= \langle 7,0,8 \rangle \cdot \langle x, y, z \rangle \\ &= 7x + 0y + 8z \\ &= 7x + 8z\end{aligned}$$

If **a** and **b** are orthogonal, then $7x + 8z = 0$. Substitute a value for x and solve for z. Let $x = -8$.

$$\begin{aligned}7x + 8z &= 0 \\ 7(-8) + 8z &= 0 \\ -56 + 8z &= 0 \\ 8z &= 56 \\ z &= 7\end{aligned}$$

A vector orthogonal to $\langle 7,0,8 \rangle$ is $\langle -8,0,7 \rangle$.

ANSWER:

Sample answer: $\langle -8, 0, 7 \rangle$

Given v and u · v, find u.

40. $v = \langle 2, -4, -6 \rangle, \mathbf{u} \cdot \mathbf{v} = -22$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y, z \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle x, y, z \rangle \cdot \langle 2, -4, -6 \rangle \\ -22 &= x(2) + y(-4) + z(-6) \\ -22 &= 2x - 4y - 6z\end{aligned}$$

Substitute values for x and y and solve for z. Let $x = -6$ and $y = -2$.

$$\begin{aligned}-22 &= 2x - 4y - 6z \\ -22 &= 2(-6) - 4(-2) - 6z \\ -22 &= -4 - 6z \\ -18 &= -6z \\ 3 &= z\end{aligned}$$

Therefore, $\mathbf{u} = \langle -6, -2, 3 \rangle$.

ANSWER:

Sample answer: $\langle -6, -2, 3 \rangle$

41. $\mathbf{v} = \left\langle \frac{1}{2}, 0, 4 \right\rangle, \mathbf{u} \cdot \mathbf{v} = \frac{31}{2}$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y, z \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle x, y, z \rangle \cdot \left\langle \frac{1}{2}, 0, 4 \right\rangle \\ \frac{31}{2} &= x\left(\frac{1}{2}\right) + y(0) + z(4) \\ \frac{31}{2} &= \frac{1}{2}x + 4z\end{aligned}$$

Substitute a value for x and solve for z. Let $x = -1$.

$$\begin{aligned}\frac{31}{2} &= \frac{1}{2}x + 4z \\ \frac{31}{2} &= \frac{1}{2}(-1) + 4z \\ \frac{31}{2} &= -\frac{1}{2} + 4z \\ 16 &= 4z \\ 4 &= z\end{aligned}$$

Therefore, $\mathbf{u} = \langle -1, 0, 4 \rangle$.

ANSWER:

Sample answer: $\langle -1, 0, 4 \rangle$

42. $\mathbf{v} = \langle -2, -6, -5 \rangle$, $\mathbf{u} \cdot \mathbf{v} = 35$

SOLUTION:

Sample answer: Let $\mathbf{u} = \langle x, y, z \rangle$. Substitute **u**, **v**, and $\mathbf{u} \cdot \mathbf{v}$ into the equation for a dot product.

$$\mathbf{u} \cdot \mathbf{v} = \langle x, y, z \rangle \cdot \langle -2, -6, -5 \rangle$$
$$35 = x(-2) + y(-6) + z(-5)$$
$$35 = -2x - 6y - 5z$$

Substitute values for x and y and solve for z. Let $x = -3$ and $y = 1$.

$$35 = -2x - 6y - 5z$$
$$35 = -2(-3) - 6(1) - 5z$$
$$35 = -5z$$
$$-7 = z$$

Therefore, $\mathbf{u} = \langle -3, 1, -7 \rangle$.

ANSWER:

Sample answer: $\langle -3, 1, -7 \rangle$

Determine whether the points are collinear.

43. $(-1, 7, 7), (-3, 9, 11), (-5, 11, 13)$

SOLUTION:

Let $a = (-1, 7, 7)$, $b = (-3, 9, 11)$, and $c = (-5, 11, 13)$. Form two vectors, $\overrightarrow{ba}$ and $\overrightarrow{bc}$. If the three points are collinear, then the angle between $\overrightarrow{ba}$ and $\overrightarrow{bc}$ will be 0° or 180°.

Find the component form of each vector.

$$\overrightarrow{ba} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$
$$= \langle -1 - (-3), 7 - 9, 7 - 11 \rangle$$
$$= \langle 2, -2, -4 \rangle$$
$$\overrightarrow{bc} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$
$$= \langle -5 - (-3), 11 - 9, 13 - 11 \rangle$$
$$= \langle -2, 2, 2 \rangle$$

Use the formula for the angle between two vectors.

$$\cos\theta = \frac{\overrightarrow{ba} \cdot \overrightarrow{bc}}{\left|\overrightarrow{ba}\right| \left|\overrightarrow{bc}\right|}$$
$$\cos\theta = \frac{\langle 2, -2, -4 \rangle \cdot \langle -2, 2, 2 \rangle}{|\langle 2, -2, -4 \rangle| |\langle -2, 2, 2 \rangle|}$$
$$\cos\theta = \frac{-4 + (-4) - 8}{\sqrt{24}\sqrt{12}}$$
$$\cos\theta = \frac{-16}{12\sqrt{2}}$$
$$\theta \approx 160.5°$$

The angle between the vectors is not 180° nor 0°. Therefore, the points are not collinear.

ANSWER:

not collinear

44. (11, 8, –1), (17, 5, –7), (8, 11, 5)

SOLUTION:

Let a = (11, 8, −1), b = (17, 5, −7), and c = (8, 11, 5). Form two vectors, $\overrightarrow{ba}$ and $\overrightarrow{bc}$. If the three points are collinear, then the angle between $\overrightarrow{ba}$ and $\overrightarrow{bc}$ will be 0° or 180°.

Find the component form of each vector.

$$\overrightarrow{ba} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$
$$= \langle 11-17, 8-5, -1-(-7) \rangle$$
$$= \langle -6, 3, 6 \rangle$$
$$\overrightarrow{bc} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$
$$= \langle 8-17, 11-5, 5-(-7) \rangle$$
$$= \langle -9, 6, 12 \rangle$$

Use the formula for the angle between two vectors.

$$\cos\theta = \frac{\overrightarrow{ba} \cdot \overrightarrow{bc}}{\left|\overrightarrow{ba}\right|\left|\overrightarrow{bc}\right|}$$
$$\cos\theta = \frac{\langle -6, 3, 6\rangle \cdot \langle -9, 6, 12\rangle}{|\langle -6, 3, 6\rangle||\langle -9, 6, 12\rangle|}$$
$$\cos\theta = \frac{54 + 18 + 72}{\sqrt{36+9+36}\sqrt{81+36+144}}$$
$$\cos\theta = \frac{144}{\sqrt{81}\sqrt{261}}$$
$$\cos\theta = \frac{144}{9\sqrt{261}}$$
$$\theta = \cos^{-1}\frac{16}{\sqrt{261}}$$
$$\theta \approx 8$$

The angle between the vectors is about 8°. Therefore, the points are not collinear.

ANSWER:

not collinear

Determine whether each pair of vectors are parallel.

45. $\mathbf{m} = \langle 2, -10, 6\rangle, \mathbf{n} = \langle 3, -15, 9\rangle$

SOLUTION:

If two vectors are parallel, then the angle between the vectors will be 0 or 180°. Use the formula for the angle between two vectors.

$$\cos\theta = \frac{m \cdot n}{|m||n|}$$
$$\cos\theta = \frac{\langle 2, -10, 6\rangle \cdot \langle 3, -15, 9\rangle}{|\langle 2, -10, 6\rangle||\langle 3, -15, 9\rangle|}$$
$$\cos\theta = \frac{6 + 150 + 54}{\sqrt{4+100+36}\sqrt{9+225+81}}$$
$$\cos\theta = \frac{210}{\sqrt{140}\sqrt{315}}$$
$$\cos\theta = \frac{210}{6\sqrt{35}\sqrt{35}}$$
$$\cos\theta = \frac{210}{210}$$
$$\theta = \cos^{-1}1$$
$$\theta = 0$$

The angle between the vectors is 0°. Therefore, the vectors are parallel.

ANSWER:

parallel

46. $\mathbf{a} = \langle 6, 3, -7 \rangle, \mathbf{b} = \langle -4, -2, 3 \rangle$

SOLUTION:

If two vectors are parallel, then the angle between the vectors will be 0 or 180°. Use the formula for the angle between two vectors.

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\cos\theta = \frac{\langle 6, 3, -7 \rangle \cdot \langle -4, -2, 3 \rangle}{|\langle 6, 3, -7 \rangle||\langle -4, -2, 3 \rangle|}$$

$$\cos\theta = \frac{-24 + (-6) + (-21)}{\sqrt{36+9+49}\sqrt{16+4+9}}$$

$$\cos\theta = \frac{-51}{\sqrt{94}\sqrt{29}}$$

$$\cos\theta = \frac{-51}{\sqrt{94}\sqrt{29}}$$

$$\theta = \cos^{-1}\frac{-51}{\sqrt{94}\sqrt{29}}$$

$$\theta \approx 167.6°$$

The angle between the vectors is about 167.6°. Therefore, the vectors are not parallel.

ANSWER:

not parallel

47. $\mathbf{w} = \left\langle -\frac{3}{2}, \frac{3}{4}, -\frac{9}{8} \right\rangle, \mathbf{z} = \langle -4, 2, -3 \rangle$

SOLUTION:

If two vectors are parallel, then the angle between the vectors will be 0 or 180°. Use the formula for the angle between two vectors.

$$\cos\theta = \frac{w \cdot z}{|w||z|}$$

$$\cos\theta = \frac{\left\langle -\frac{3}{2}, \frac{3}{4}, -\frac{9}{8} \right\rangle \cdot \langle -4, 2, -3 \rangle}{\left|\left\langle -\frac{3}{2}, \frac{3}{4}, -\frac{9}{8} \right\rangle\right||\langle -4, 2, -3 \rangle|}$$

$$\cos\theta = \frac{6 + \frac{3}{2} + \frac{27}{8}}{\sqrt{\frac{9}{4} + \frac{9}{16} + \frac{81}{64}}\sqrt{16+4+9}}$$

$$\cos\theta = \frac{\frac{87}{8}}{\sqrt{\frac{261}{64}}\sqrt{29}}$$

$$\cos\theta = \frac{\frac{87}{8}}{\frac{\sqrt{7569}}{8}}$$

$$\cos\theta = \frac{\frac{87}{8}}{\frac{87}{8}}$$

$$\theta = \cos^{-1}1$$

$$\theta = 0$$

The angle between the vectors is 0°. Therefore, the vectors are parallel.

ANSWER:

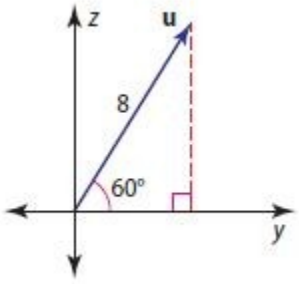
parallel

Write the component form of each vector.

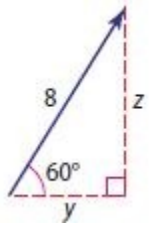
48. **u** lies in the *yz*-plane, has a magnitude of 8, and makes a 60° angle above the positive *y*-axis.

SOLUTION:

If **u** lies in the *yz* plane, then the *x*-component is 0. Sketch the vector in the *yz* plane.



The vector can be represented by a triangle as shown.



Use trigonometric ratios to find *y* and *z*.

$$\cos 60^\circ = \frac{y}{8} \qquad \sin 60^\circ = \frac{z}{8}$$
$$8\cos 60^\circ = y \qquad 8\sin 60^\circ = z$$
$$8\left(\frac{1}{2}\right) = y \qquad 8\left(\frac{\sqrt{3}}{2}\right) = z$$
$$4 = y \qquad 4\sqrt{3} = y$$

So, $\mathbf{u} = \langle 0, 4, 4\sqrt{3} \rangle$.

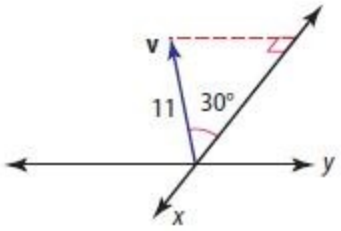
ANSWER:

$\langle 0, 4, 4\sqrt{3} \rangle$

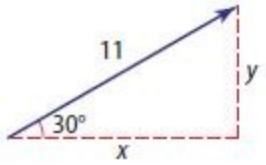
49. **v** lies in the *xy*-plane, has a magnitude of 11, and makes a 30° angle to the left of the negative *x*-axis.

SOLUTION:

If **v** lies in the *xy* plane, then the *z*-component is 0. Sketch the vector in the *xy* plane.



The vector can be represented by a triangle as shown.



Use trigonometric ratios to find *y* and *x*.

$$\cos 30^\circ = \frac{x}{11} \qquad \sin 30^\circ = \frac{y}{11}$$
$$11\cos 30^\circ = y \qquad 11\sin 30^\circ = y$$
$$11\left(\frac{\sqrt{3}}{2}\right) = x \qquad 11\left(\frac{1}{2}\right) = y$$
$$\frac{11\sqrt{3}}{2} = x \qquad \frac{11}{2} = y$$

Since **v** is to the left of the negative *x*-axis, *x* and *y* are negative.

So, $\mathbf{v} = \left\langle -\frac{11\sqrt{3}}{2}, -\frac{11}{2}, 0 \right\rangle$.

ANSWER:

$\left\langle -\frac{11\sqrt{3}}{2}, -\frac{11}{2}, 0 \right\rangle$

Given the four vertices, determine whether quadrilateral *ABCD* is a parallelogram. If it is, find its area, and determine whether it is a rectangle.

50. *A*(3, 0, –2), *B*(0, 4, –1), *C*(0, 2, 5), *D*(3, 2, 4)

SOLUTION:

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Use the Distance Formula for points in space to find the length of each side.

$$\begin{aligned} AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(0-3)^2+(4-0)^2+[-1-(-2)]^2} \\ &= \sqrt{26} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(0-0)^2+(2-4)^2+[5-(-1)]^2} \\ &= \sqrt{40} \\ &= 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(3-0)^2+(2-2)^2+(4-5)^2} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(3-3)^2+(0-2)^2+(-2-4)^2} \\ &= \sqrt{40} \\ &= 2\sqrt{10} \end{aligned}$$

Since both pairs of opposite sides are not congruent, *ABCD* is not a parallelogram.

ANSWER:

not a parallelogram

51. *A*(7, 5, 5), *B*(4, 4, 4), *C*(4, 6, 2), *D*(7, 7, 3)

SOLUTION:

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Use the Distance Formula for points in space to find the length of each side.

$$\begin{aligned} AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(4-7)^2+(4-5)^2+(4-5)^2} \\ &= \sqrt{11} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(4-4)^2+(6-4)^2+(2-4)^2} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(7-4)^2+(7-6)^2+(3-2)^2} \\ &= \sqrt{11} \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \\ &= \sqrt{(7-7)^2+(5-7)^2+(5-3)^2} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

Since both pairs of opposite sides are congruent, *ABCD* is a parallelogram. The area of a parallelogram is $A = bh$.

$$\begin{aligned} A &= bh \\ &= \left(\sqrt{11}\right)\left(2\sqrt{2}\right) \\ &= 2\sqrt{22} \\ &\approx 9.4 \end{aligned}$$

The area of the parallelogram is about 9.4 units^2.

If a parallelogram has one right angle, then it has four right angles and is a rectangle. Form two vectors, $\overrightarrow{ba}$ and $\overrightarrow{bc}$. If $\overrightarrow{ba} \cdot \overrightarrow{bc} = 0$, then $\overrightarrow{ba}$ and $\overrightarrow{bc}$ are orthogonal and perpendicular. If $\overrightarrow{ba}$ and $\overrightarrow{bc}$ are perpendicular, then *ABCD* is a rectangle.

Find the component form of each vector.

$$\begin{aligned} \overrightarrow{ba} &= \langle x_2-x_1, y_2-y_1, z_2-z_1 \rangle \\ &= \langle 7-4, 5-4, 5-4 \rangle \\ &= \langle 3,1,1 \rangle \end{aligned}$$

$$\begin{aligned} \overrightarrow{bc} &= \langle x_2-x_1, y_2-y_1, z_2-z_1 \rangle \\ &= \langle 4-4, 6-4, 2-4 \rangle \\ &= \langle 0,2,-2 \rangle \end{aligned}$$

$$\begin{aligned} \overrightarrow{ba} \cdot \overrightarrow{bc} &= \langle 3,1,1 \rangle \cdot \langle 0,2,-2 \rangle \\ &= 3(0)+1(2)+1(-2) \\ &= 0 \end{aligned}$$

Since $\overrightarrow{ba} \cdot \overrightarrow{bc} = 0$, $\overrightarrow{ba}$ and $\overrightarrow{bc}$ are orthogonal. Thus, *ABCD* is a rectangle.

ANSWER:

parallelogram; about 9.4 units^2; rectangle

52. **AIR SHOWS** In an air show, two airplanes take off simultaneously. The first plane starts at the position (0, –2, 0) and is at the position (6, –10, 15) after three seconds. The second plane starts at the position (0, 2, 0) and is at the position (6, 10, 15) after three seconds. Are the paths of the two planes parallel? Explain.

SOLUTION:

Let $\mathbf{f_1}$ be the vector that represents the path of the first plane and let $\mathbf{f_2}$ be the vector that represents the path of the second plane. Write $\mathbf{f_1}$ and $\mathbf{f_2}$ in component form.

$$\begin{aligned}\mathbf{f_1} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 6-0, -10-(-2), 15-0 \rangle \\ &= \langle 6, -8, 15 \rangle\end{aligned}$$

$$\begin{aligned}\mathbf{f_2} &= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \\ &= \langle 6-0, 10-2, 15-0 \rangle \\ &= \langle 6, 8, 15 \rangle\end{aligned}$$

If two vectors are parallel, then the angle between the vectors will be 0 or 180°. Use the formula for the angle between two vectors.

$$\cos\theta = \frac{f_1 \cdot f_2}{|f_1||f_2|}$$

$$\cos\theta = \frac{\langle 6, -8, 15\rangle \cdot \langle 6, 8, 15\rangle}{|\langle 6, -8, 15\rangle||\langle 6, 8, 15\rangle|}$$

$$\cos\theta = \frac{36 + (-64) + 225}{\sqrt{36+64+225}\sqrt{36+64+225}}$$

$$\cos\theta = \frac{197}{\sqrt{325}\sqrt{325}}$$

$$\cos\theta = \frac{197}{325}$$

$$\theta = \cos^{-1}\frac{197}{325}$$

$$\theta \approx 52.7°$$

The angle between the vectors is about 52.7°. Therefore, the paths of the two planes are not parallel.

(Note that for this problem we are also assuming that the planes fly in a straight line.)

ANSWER:

No; sample answer: Using vectors in component form to represent the path of each airplane, it can be determined that the angle between the two vectors is 52.7°. Therefore, the paths of the two planes are not parallel.

For $\mathbf{u} = \langle 3, 2, -2 \rangle$ and $\mathbf{v} = \langle -4, 4, 5 \rangle$, find each of the following, if possible.

53. $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$

SOLUTION:

Find $\mathbf{u} \times \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -2 \\ -4 & 4 & 5 \end{vmatrix} \\ &= \begin{vmatrix} 2 & -2 \\ 4 & 5 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & -2 \\ -4 & 5 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & 2 \\ -4 & 4 \end{vmatrix}\mathbf{k} \\ &= [10-(-8)]\mathbf{i} - (15-8)\mathbf{j} + [12-(-8)]\mathbf{k} \\ &= 18\mathbf{i} - 7\mathbf{j} + 20\mathbf{k} \\ &= \langle 18, -7, 20 \rangle\end{aligned}$$

Find $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$.

$$\begin{aligned}\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) &= \langle 3, 2, -2 \rangle \cdot \langle 18, -7, 20 \rangle \\ &= 3(18) + 2(-7) + (-2)(20) \\ &= 0\end{aligned}$$

ANSWER:

0

54. $\mathbf{v} \times (\mathbf{u} \cdot \mathbf{v})$

SOLUTION:

Find $\mathbf{u} \cdot \mathbf{v}$.

$$\begin{aligned}\mathbf{u} \cdot \mathbf{v} &= \langle 3, 2, -2 \rangle \cdot \langle -4, 4, 5 \rangle \\ &= 3(-4) + 2(4) + (-2)(5) \\ &= -14\end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v}$ is the scalar −14, $\mathbf{v} \times (\mathbf{u} \cdot \mathbf{v})$ is not possible because the cross product definition only applies to vectors in three-dimensional space.

ANSWER:

not possible

55. $\mathbf{u} \times \mathbf{u} \times \mathbf{v}$

SOLUTION:

Find $\mathbf{u} \times \mathbf{u}$.

$$\mathbf{u} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -2 \\ 3 & 2 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -2 \\ 2 & -2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & -2 \\ 3 & -2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}\mathbf{k}$$

$$= [-4-(-4)]\mathbf{i} - [-6-(-6)]\mathbf{j} + (6-6)\mathbf{k}$$

$$= 0\mathbf{i} - 0\mathbf{j} + 0\mathbf{k}$$

$$= \langle 0, 0, 0 \rangle$$

Find $\mathbf{u} \times \mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{u} \times \mathbf{v} = \langle 0, 0, 0 \rangle \times \mathbf{v}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0 \\ -4 & 4 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 \\ 4 & 5 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0 & 0 \\ -4 & 5 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0 & 0 \\ -4 & 4 \end{vmatrix}\mathbf{k}$$

$$= (0-0)\mathbf{i} - (0-0)\mathbf{j} + (0-0)\mathbf{k}$$

$$= 0\mathbf{i} - 0\mathbf{j} + 0\mathbf{k}$$

$$= \langle 0, 0, 0 \rangle$$

ANSWER:

$\langle 0, 0, 0 \rangle$

56. $\mathbf{v} \cdot \mathbf{v} \cdot \mathbf{u}$

SOLUTION:

Find $\mathbf{v} \cdot \mathbf{v}$.

$$\mathbf{v} \cdot \mathbf{v} = \langle -4, 4, 5 \rangle \cdot \langle -4, 4, 5 \rangle$$

$$= (-4)(-4) + 4(4) + 5(5)$$

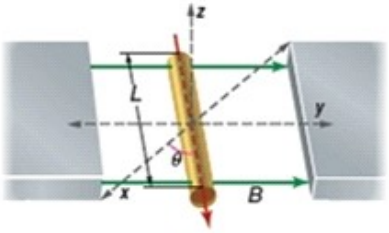
$$= 57$$

Since $\mathbf{v} \cdot \mathbf{v}$ is the scalar 57, $\mathbf{v} \cdot \mathbf{v} \cdot \mathbf{u}$ is not possible because the dot product definition only applies to vectors.

ANSWER:

not possible

57. **ELECTRICITY** When a wire carrying an electric current is placed in a magnetic field, the force on the wire in newtons is given by $\overline{F} = I\overline{L} \times \overline{B}$, where I represents the current flowing through the wire in amps, $\overline{L}$ represents the vector length of the wire pointing in the direction of the current in meters, and $\overline{B}$ is the force of the magnetic field in teslas. In the figure below, the wire is rotated through an angle θ in the xy-plane.

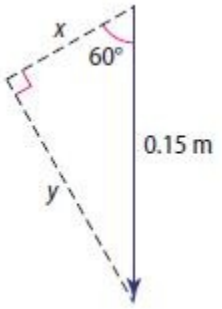


a. If the force of a magnetic field is 1.1 teslas, find the magnitude of the force on a wire in the xy-plane that is 0.15 meter in length carrying a current of 25 amps at an angle of 60°.

b. If the force on the wire is $\overline{F} = \langle 0, 0, -0.63 \rangle$, what is the angle of the wire?

SOLUTION:

a. Since the force of the magnetic field is moving parallel to the y-axis, it can be written in component form as $\overline{B} = \langle 0, 1.1, 0 \rangle$. Use the length and direction of the wire to write $\overline{L}$ in component form.



The angle between the wire and the x-axis is 60°. Since the wire is located in the xy-plane, the z-component is 0. We can use trigonometric ratios to solve for the x- and y-components.

$$\cos 60^\circ = \frac{x}{0.15}$$

$$x = 0.15 \cos 60^\circ$$

$$x = 0.15\left(\frac{1}{2}\right)$$

$$x = 0.075$$

$$\sin 60^\circ = \frac{y}{0.15}$$

$$y = 0.15 \sin 60^\circ$$

$$y = 0.15\left(\frac{\sqrt{3}}{2}\right)$$

$$y \approx 0.130$$

Thus, $\overline{L} = \langle 0.075, 0.130, 0 \rangle$. Substitute $\overline{L}$, $\overline{B}$, and I into the equation $\overline{F} = I\overline{L} \times \overline{B}$ and find $\overline{F}$.

$$\begin{aligned}\vec{F} &= I\vec{L}\times\vec{B}\\ &= 25\langle 0.075, 0.130, 0\rangle \times \langle 0, 1.1, 0\rangle\\ &= \langle 1.875, 3.25, 0\rangle \times \langle 0, 1.1, 0\rangle\\ &= \begin{vmatrix} 3.25 & 0\\ 1.1 & 0\end{vmatrix}\mathbf{i} - \begin{vmatrix} 1.875 & 0\\ 0 & 0\end{vmatrix}\mathbf{j} + \begin{vmatrix} 1.875 & 3.25\\ 0 & 1.1\end{vmatrix}\mathbf{k}\\ &= 0\mathbf{i} - 0\mathbf{j} + 2.06\mathbf{k}\\ &= \langle 0, 0, 2.06\rangle\end{aligned}$$

Therefore, $\vec{F} = \langle 0, 0, 2.06\rangle$. The magnitude of the force on the wire is about 2.1 newtons.

b.

Since $\vec{F} = \langle 0, 0, -0.63\rangle$, $z = -0.63$. Substitute $z = -0.63$ into the equation and solve for θ.

$$\begin{aligned}F &= IL\times B\\ -0.63 &= 25(0.15\cos\theta)(11)\\ \frac{-0.63}{1.1(0.15)(25)} &= \cos\theta\\ \cos^{-1}\left(\frac{-0.63}{1.1(0.15)(25)}\right) &= \theta\\ 98.8 &\approx \theta\end{aligned}$$

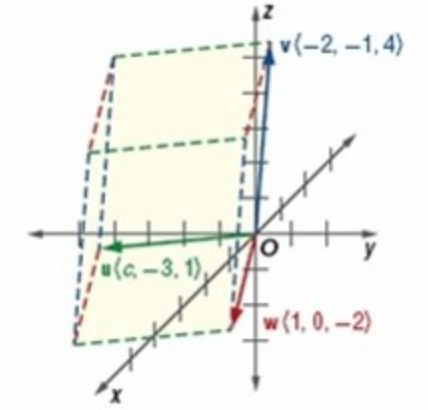
Therefore, the angle of the wire is about 98.8°.

ANSWER:

a. about 2.1 N
b. about 98.8º

Given v, w, and the volume of the parallelepiped having adjacent edges u, v, and w, find *c*.

58. $\mathbf{v} = \langle -2, -1, 4\rangle, \mathbf{w} = \langle 1, 0, -2\rangle, \mathbf{u} = \langle c, -3, 1\rangle$ and $V = 7$ cubic units



SOLUTION:

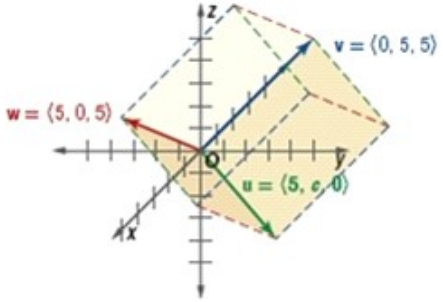
The volume of the parallelepiped can be found by calculating $V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|$. Substitute **v**, **u**, **w**, and V and solve for c.

$$\begin{aligned}V &= |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|\\ 7 &= \begin{vmatrix} c & -3 & 1\\ -2 & -1 & 4\\ 1 & 0 & -2\end{vmatrix}\\ 7 &= \begin{vmatrix} -1 & 4\\ 0 & -2\end{vmatrix}(c) - \begin{vmatrix} -2 & 4\\ 1 & -2\end{vmatrix}(-3) + \begin{vmatrix} -2 & -1\\ 1 & 0\end{vmatrix}(1)\\ 7 &= (2-0)(c) - (4-4)(-3) + [0-(-1)](1)\\ 7 &= 2c + 1\\ 6 &= 2c\\ 3 &= c\end{aligned}$$

ANSWER:

3

59. $\mathbf{v} = \langle 0, 5, 5\rangle, \mathbf{w} = \langle 5, 0, 5\rangle, \mathbf{u} = \langle 5, c, 0\rangle$ and V = 250 cubic units



SOLUTION:

The volume of the parallelepiped can be found by calculating $V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$. Substitute **v**, **u**, **w**, and V and solve for c.

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$$

$$250 = \begin{vmatrix} 5 & c & 0 \\ 0 & 5 & 5 \\ 5 & 0 & 5 \end{vmatrix}$$

$$250 = \begin{vmatrix} 5 & 5 \\ 0 & 5 \end{vmatrix}(5) - \begin{vmatrix} 0 & 5 \\ 5 & 5 \end{vmatrix}(c) + \begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix}(0)$$

$$250 = (25 - 0)(5) - (0 - 25)(c) + (0 - 25)(0)$$

$$250 = 125 + 25c$$

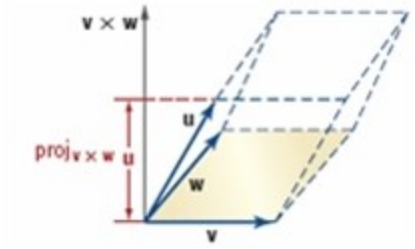
$$125 = 25c$$

$$5 = c$$

ANSWER:

5

60. **PROOF** Verify the formula for the volume of a parallelepiped. (*Hint*: Use the projection of **u** onto **v** × **w**.)



SOLUTION:

Sample answer: The base of the parallelepiped is a parallelogram with area $|\mathbf{v} \times \mathbf{w}|$. The height is $|\text{proj}_{\mathbf{w} \times \mathbf{v}} \mathbf{u}|$. So, the volume of the parallelepiped is $V = |\mathbf{v} \times \mathbf{w}|\,|\text{proj}_{\mathbf{w} \times \mathbf{v}} \mathbf{u}|$.

$$\begin{aligned} V &= |\mathbf{v} \times \mathbf{w}| \cdot |\text{proj}_{\mathbf{w} \times \mathbf{v}} \mathbf{u}| && \text{Volume formula} \\ &= |\mathbf{v} \times \mathbf{w}| \cdot \left| \frac{\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})}{|(\mathbf{v} \times \mathbf{w})|^2} (\mathbf{v} \times \mathbf{w}) \right| && \text{Projection of } \mathbf{u} \text{ onto } \mathbf{v} \times \mathbf{w} \\ &= |\mathbf{v} \times \mathbf{w}| \cdot \frac{|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|}{|(\mathbf{v} \times \mathbf{w})|^2} |\mathbf{v} \times \mathbf{w}| && \text{Rule of Absolute Values} \\ &= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| && \text{Simplify.} \end{aligned}$$

ANSWER:

Sample answer: The base of the parallelepiped is a parallelogram with area $|\mathbf{v} \times \mathbf{w}|$. The height is $|\text{proj}_{\mathbf{w} \times \mathbf{v}} \mathbf{u}|$. So, the volume of the parallelepiped is $V = |\mathbf{v} \times \mathbf{w}|\,|\text{proj}_{\mathbf{w} \times \mathbf{v}} \mathbf{u}|$.

$$\begin{aligned} V &= |\mathbf{v} \times \mathbf{w}| \cdot |\text{proj}_{\mathbf{w} \times \mathbf{v}} \mathbf{u}| \\ &= |\mathbf{v} \times \mathbf{w}| \cdot \left| \frac{u \cdot (\mathbf{v} \times \mathbf{w})}{|(\mathbf{v} \times \mathbf{w})|^2} (\mathbf{v} \times \mathbf{w}) \right| \\ &= |\mathbf{v} \times \mathbf{w}| \cdot \frac{|u \cdot (\mathbf{v} \times \mathbf{w})|}{|(\mathbf{v} \times \mathbf{w})|^2} |\mathbf{v} \times \mathbf{w}| \text{ or } |u \cdot (\mathbf{v} \times \mathbf{w})| \end{aligned}$$

61. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain.
For any two nonzero, nonparallel vectors in space, there is a vector that is perpendicular to both.

SOLUTION:

Always; sample answer: The cross product of two three-dimensional vectors results in a vector that is perpendicular to both original vectors.

ANSWER:

Always; sample answer: The cross product of two three-dimensional vectors results in a vector that is perpendicular to both original vectors.

62. **REASONING** If **u** and **v** are parallel in space, then how many vectors are perpendicular to both? Explain.

SOLUTION:

Sample answer: There are infinitely many vectors that are perpendicular to a pair of parallel vectors. If two vectors are parallel, then they are coplanar. Due to the Perpendicular Transversal Theorem, if a vector is perpendicular to one of the two parallel vectors, then it is perpendicular to the other. Since a vector can have infinitely many perpendicular vectors, there are infinitely many vectors that are perpendicular to the pair of parallel vectors.

ANSWER:

Sample answer: There are infinitely many vectors that are perpendicular to a pair of parallel vectors. If two vectors are parallel, then they are coplanar. Due to the Perpendicular Transversal Theorem, if a vector is perpendicular to one of the two parallel vectors, then it is perpendicular to the other. Since a vector can have infinitely many perpendicular vectors, there are infinitely many vectors that are perpendicular to the pair of parallel vectors.

63. **CHALLENGE** Given $\mathbf{u} = \langle 4, 6, c \rangle$ and $\mathbf{v} = \langle -3, -2, 5 \rangle$, find the value of c for which $\mathbf{u} \times \mathbf{v} = 34\mathbf{i} - 26\mathbf{j} + 10\mathbf{k}$.

SOLUTION:

Find $\mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & c \\ -3 & -2 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & c \\ -2 & 5 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 4 & c \\ -3 & 5 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 4 & 6 \\ -3 & -2 \end{vmatrix}\mathbf{k}$$

$$= [30 - (-2c)]\mathbf{i} - [20 - (-3c)]\mathbf{j} + [-8 - (-18)]\mathbf{k}$$

$$= (30 + 2c)\mathbf{i} - (20 + 3c)\mathbf{j} + 10\mathbf{k}$$

Since $30 + 2c = 34$, solve for c.

$$30 + 2c = 34$$
$$2c = 4$$
$$c = 2$$

Substitute $c = 2$ into $-(20 + 3c) = -26$ to verify the solution.

$$-(20 + 3c) = -26$$
$$-[20 + 3(2)] = -26$$
$$-26 = -26$$

Thus, $c = 2$.

ANSWER:

2

64. **REASONING** Explain why the cross product is not defined for vectors in the two-dimensional coordinate system.

SOLUTION:

Sample answer: The definition of the cross product of vectors **a** and **b** is the vector perpendicular to the plane that contains both **a** and **b**. To have a vector that is perpendicular to a two-dimensional plane, a third direction is needed.

ANSWER:

Sample answer: The definition of the cross product of vectors **a** and **b** is the vector perpendicular to the plane that contains both **a** and **b**. To have a vector that is perpendicular to a two-dimensional plane, a third direction is needed.

65. ***Writing in Math*** Compare and contrast the methods for determining whether vectors in space are parallel or perpendicular.

SOLUTION:

Sample answer: To determine if two vectors are parallel or perpendicular, you can use the formula for the angle between two vectors. If the angle measures 0 or 180, they are parallel. If the angle measures 90, they are perpendicular. You can also find the component form of the two vectors and use the ratios of the coordinates to determine if two vectors are parallel. If the ratios of all three coordinates of the component vectors are the same, then the vectors are parallel. This method does not work for determining if two vectors are perpendicular. To determine if two vectors are perpendicular, you can find the dot product of the vectors. If the dot product is 0, then the vectors are perpendicular. This method does not work for determining if two vectors are parallel.

ANSWER:

Sample answer: To determine if two vectors are parallel or perpendicular, you can use the formula for the angle between two vectors. If the angle is measures 0 or 180, they are parallel. If the angle measures 90, they are perpendicular. You can also find the component form of the two vectors and use the ratios of the coordinates to determine if two vectors are parallel. If the ratios of all three coordinates of the component vectors are the same, then the vectors are parallel. To determine if two vectors are perpendicular, you can find the dot product of the vectors. If the dot product is 0, then the vectors are perpendicular.

Find the length and midpoint of the segment with the given endpoints.

66. (1, 10, 13), (−2, 22, −6)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(-2-1)^2 + (22-10)^2 + (-6-13)^2} \\ &= \sqrt{9 + 144 + 361} \\ &= \sqrt{514} \\ &\approx 22.7 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) &= \left(\frac{1 + (-2)}{2}, \frac{10 + 22}{2}, \frac{13 + (-6)}{2}\right) \\ &= \left(-\frac{1}{2}, 16, \frac{7}{2}\right) \end{aligned}$$

ANSWER:

$$\sqrt{514} \approx 22.7; \left(-\frac{1}{2}, 16, \frac{7}{2}\right)$$

67. (12, −1, −14), (21, 19, −23)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(21-12)^2 + [19 - (-1)]^2 + [-23 - (-14)]^2} \\ &= \sqrt{81 + 400 + 81} \\ &= \sqrt{562} \\ &\approx 23.7 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) &= \left(\frac{12 + 21}{2}, \frac{-1 + 19}{2}, \frac{-14 + (-23)}{2}\right) \\ &= \left(\frac{33}{2}, 9, -\frac{37}{2}\right) \end{aligned}$$

ANSWER:

$$\sqrt{562} \approx 23.7; \left(\frac{33}{2}, 9, -\frac{37}{2}\right)$$

68. (−22, 24, −9), (10, 10, 2)

SOLUTION:

Use the Distance Formula for points in space to find the length of the segment.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{[10 - (-22)]^2 + (10 - 24)^2 + [2 - (-9)]^2} \\ &= \sqrt{1024 + 196 + 121} \\ &= \sqrt{1341} \\ &= 3\sqrt{149} \\ &\approx 36.6 \end{aligned}$$

Use the Midpoint Formula for points in space to find the midpoint of the segment.

$$\begin{aligned} &\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) \\ &= \left(\frac{-22 + 10}{2}, \frac{24 + 10}{2}, \frac{-9 + 2}{2}\right) \\ &= \left(-6, 17, -\frac{7}{2}\right) \end{aligned}$$

ANSWER:

$3\sqrt{149} \approx 36.6; \left(-6, 17, -\frac{7}{2}\right)$

Find the dot product of u and v. Then determine if u and v are orthogonal.

69. $\langle -8, -7 \rangle \cdot \langle 1, 2 \rangle$

SOLUTION:

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= \langle -8, -7 \rangle \cdot \langle 1, 2 \rangle \\ &= (-8)(1) + (-7)(2) \\ &= -22 \end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−22; not orthogonal

70. $\langle -4, -6 \rangle \cdot \langle 7, 5 \rangle$

SOLUTION:

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= \langle -4, -6 \rangle \cdot \langle 7, 5 \rangle \\ &= (-4)(7) + (-6)(5) \\ &= -58 \end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−58; not orthogonal

71. $\langle 6, -3 \rangle \cdot \langle -3, 5 \rangle$

SOLUTION:

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= \langle 6, -3 \rangle \cdot \langle -3, 5 \rangle \\ &= (6)(-3) + (-3)(5) \\ &= -33 \end{aligned}$$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

ANSWER:

−33; not orthogonal

72. **BAKERY** Hector's bakery has racks that can hold up to 900 bagels and muffins. Due to costs, the number of bagels produced must be less than or equal to 300 plus twice the number of muffins produced. The demand for bagels is at least three times that of muffins. Hector makes a profit of \$3 per muffin sold and \$1.25 per bagel sold. How many of each item should he make to maximize profit?

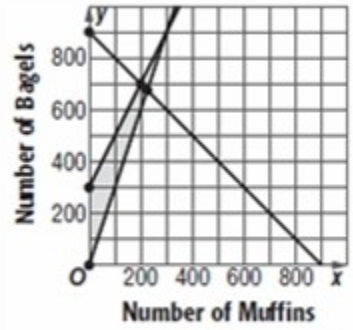
SOLUTION:

Let x represent the number of muffins produced and y the number of bagels produced. The objective function is then given by $f(x, y) = 3x + 1.25y$.

The constraints are given by the following.

$x + y \leq 900$ Bakery capacity constraint
$y \leq 300 + 2x$ Production constraint
$y \geq 3x$ Market demand constraint

Because x and y cannot be negative, additional constraints are $x \geq 0$ and $y \geq 0$.
Sketch a graph of the region determined by the constraints to find how many of each product the bakery should produce to maximize profit.



The shaded polygonal region has four vertex points at (0, 0), (0, 300), (200, 700), and (225, 675). Find the value of $f(x, y) = 3x + 1.25y$ at each of the four vertices.

$f(0, 0) = 3(0) + 1.25(0) = 0$
$f(0, 300) = 3(0) + 1.25(300) = 375$
$f(200, 700) = 3(200) + 1.25(700) = 1475$
$f(225, 675) = 3(225) + 1.25(675) = 1518.75$

Because f is greatest at (225, 675), the bakery should produce 225 muffins and 675 bagels.

ANSWER:

225 muffins and 675 bagels

73. Decompose $\frac{2m+16}{m^2-16}$ into partial fractions.

SOLUTION:

Rewrite the expression as partial fractions with constant numerators, A and B, and denominators that are the linear factors of the original denominator.

$$\frac{2m+16}{m^2-16} = \frac{A}{m-4} + \frac{B}{m+4}$$

Multiply each side by the LCD, $m^2 - 16$.

$$2m+16 = A(m+4) + B(m-4)$$

Group the like terms.

$$2m+16 = A(m+4) + B(m-4)$$
$$2m+16 = Am + 4A + Bm - 4B$$
$$2m+16 = Am + Bm + 4A - 4B$$
$$2m+16 = (A+B)m + 4A - 4B$$

Equate the coefficients on the left and right side of the equation to form two equations with 2 variables. In other words, the coefficients of the m-terms on the left side of the equation must equal the coefficients of the m-terms on the right side.

$$A + B = 2$$
$$4A - 4B = 16$$

Use any method to solve the new system.

$$A + B = 2$$
$$B = 2 - A$$
$$4A - 4B = 16$$
$$4A - 4(2 - A) = 16$$
$$4A - 8 + 4A = 16$$
$$-8 + 8A = 16$$
$$8A = 24$$
$$A = 3$$
$$B = 2 - (3)$$
$$B = -1$$

Replace A and B with 3 and -1 in the partial fraction decomposition.

$$\frac{2m+16}{m^2-16} = \frac{A}{m-4} + \frac{B}{m+4}$$
$$\frac{2m+16}{m^2-16} = \frac{3}{m-4} + \frac{-1}{m+4}$$

ANSWER:

$$\frac{3}{m-4}+\frac{-1}{m+4}$$

Verify each identity.

74. $\tan^2\theta+\cos^2\theta+\sin^2\theta=\sec^2\theta$

SOLUTION:

$$\begin{aligned}&\tan^2\theta+\cos^2\theta+\sin^2\theta\\&=\tan^2\theta+(\cos^2\theta+\sin^2\theta)\\&=\tan^2\theta+1 && \text{Pythagorean Identity}\\&=\sec^2\theta && \text{Pythagorean Identity}\end{aligned}$$

ANSWER:

$$\begin{aligned}\tan^2\theta+\cos^2\theta+\sin^2\theta &= \tan^2\theta+(\cos^2\theta+\sin^2\theta)\\&=\tan^2\theta+1\\&=\sec^2\theta\end{aligned}$$

75. $\sec^2\theta\cot^2\theta-\cot^2\theta=1$

SOLUTION:

$$\begin{aligned}&\sec^2\theta\cot^2\theta-\cot^2\theta\\&=\cot^2\theta(\sec^2\theta-1) && \text{Factor}\\&=\cot^2\theta\tan^2\theta && \text{Pythagorean Identity}\\&=1 && \text{Reciprocal Identity}\end{aligned}$$

ANSWER:

$$\begin{aligned}\sec^2\theta\cot^2\theta-\cot^2\theta &= \cot^2\theta(\sec^2\theta-1)\\&=\cot^2\theta\tan^2\theta\\&=1\end{aligned}$$

76. $\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$

SOLUTION:

$$\begin{aligned}&\tan^2\theta-\sin^2\theta\\&=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta && \text{Quotient Identity}\\&=\sin^2\theta\left(\frac{1}{\cos^2\theta}-1\right) && \text{Factor}\\&=\sin^2\theta\left(\sec^2\theta-1\right) && \text{Reciprocal Identity}\\&=\sin^2\theta\tan^2\theta && \text{Pythagorean Identity}\end{aligned}$$

ANSWER:

$$\begin{aligned}\tan^2\theta-\sin^2\theta &= \frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta\\&=\sin^2\theta\left(\frac{1}{\cos^2\theta}-1\right)\\&=\sin^2\theta(\sec^2\theta-1)\\&=\sin^2\theta\tan^2\theta\end{aligned}$$

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

77. $a = 20$, $c = 24$, $B = 47°$

SOLUTION:

Use the Law of Cosines to find the missing side measure.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 20^2 + 24^2 - 2(20)(24)\cos 47°$$
$$b^2 = 400 + 576 - (960)\cos 47°$$
$$b^2 = 321.28$$
$$b \approx 17.9$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin A}{20} = \frac{\sin 47°}{17.9}$$
$$\sin A = \frac{20\sin 47°}{17.9}$$
$$A = \sin^{-1}\left(\frac{20\sin 47°}{17.9}\right)$$
$$A \approx 55°$$

Find the measure of the remaining angle.

$$C \approx 180° - (47° + 55°)$$
$$\approx 78°$$

Therefore, $A \approx 55°$, $C \approx 78°$ and $b \approx 17.9$.

ANSWER:

$A \approx 55°$, $C \approx 78°$, $b \approx 17.9$

78. $A = 25°$, $B = 78°$, $a = 13.7$

SOLUTION:

Find the measure of the remaining angle.

$$C = 180° - (25° + 78°)$$
$$= 77°$$

Use the Law of Sines to find missing side measure b.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 25°}{13.7} = \frac{\sin 78°}{b}$$
$$b\sin 25° = 13.7\sin 78°$$
$$b = \frac{13.7\sin 78°}{\sin 25°}$$
$$b \approx 31.7$$

Use the Law of Sines to find missing side measure c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 25°}{13.7} = \frac{\sin 77°}{c}$$
$$c\sin 25° = 13.7\sin 77°$$
$$c = \frac{13.7\sin 77°}{\sin 25°}$$
$$c \approx 31.6$$

Therefore, $C = 77°$, $b \approx 31.7$, $c \approx 31.6$.

ANSWER:

$C = 77°$, $b \approx 31.7$, $c \approx 31.6$

79. $a = 21.5$, $b = 16.7$, $c = 10.3$

SOLUTION:

Use the Law of Cosines to find missing angle measure A.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 21.5^2 &= 16.7^2 + 10.3^2 - 2(16.7)(10.3)\cos A \\ 462.25 &= 384.98 - (344.02)\cos A \\ 77.27 &= -344.02\cos A \\ \frac{77.27}{-344.02} &= \cos A \\ A &= \cos^{-1}\frac{77.27}{-344.02} \\ A &\approx 103^\circ \end{aligned}$$

Use the Law of Sines to find missing angle measure B.

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 103^\circ}{21.5} &= \frac{\sin B}{16.7} \\ 21.5\sin B &= 16.7\sin 103^\circ \\ \sin B &= \frac{16.7\sin 103^\circ}{21.5} \\ B &= \sin^{-1}\frac{16.7\sin 103^\circ}{21.5} \\ B &\approx 49^\circ \end{aligned}$$

Find the measure of the remaining angle.

$$\begin{aligned} C &\approx 180^\circ - (103^\circ + 49^\circ) \\ &\approx 28^\circ \end{aligned}$$

Therefore, $A \approx 103°$, $B \approx 49°$ and $C \approx 28°$.

ANSWER:

$A \approx 103°$, $B \approx 49°$, $C \approx 28°$

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

80. $-72.775°$

SOLUTION:

First, convert 0.775° into minutes and seconds.

$$\begin{aligned} -72.775^\circ &= -72^\circ + 0.775^\circ\left(\frac{60'}{1^\circ}\right) \\ &= -72^\circ + 46.5' \end{aligned}$$

Next, convert 0.5' into seconds.

$$\begin{aligned} -72.775^\circ &= -72^\circ + 46' + 0.5'\left(\frac{60''}{1'}\right) \\ &= 72^\circ + 46' + 30'' \end{aligned}$$

Therefore, $-72.775°$ can be written as $-72° 46' 30''$.

ANSWER:

$-72° 46′ 30″$

81. 29° 6′ 6″

SOLUTION:

Each minute is $\frac{1}{60}$ of a degree and each second is $\frac{1}{60}$ of a minute, so each second is $\frac{1}{3600}$ of a degree.

$$\begin{aligned} 29^\circ\,6'6'' &= 29^\circ + 6'\left(\frac{1^\circ}{60'}\right) + 6''\left(\frac{1^\circ}{3600''}\right) \\ &\approx 29^\circ + 0.1^\circ + 0.002 \\ &\approx 29.102^\circ \end{aligned}$$

Therefore, 29° 6′ 6″ can be written as about 29.102°.

ANSWER:

29.102°

82. 132° 18′ 31″

SOLUTION:

Each minute is $\frac{1}{60}$ of a degree and each second is $\frac{1}{60}$ of a minute, so each second is $\frac{1}{3600}$ of a degree.

$$\begin{aligned} 132°\ 18'31" &= 132° + 18'\left(\frac{1°}{60'}\right) + 31"\left(\frac{1°}{3600"}\right) \\ &\approx 132° + 0.3° + 0.009 \\ &\approx 132.309° \end{aligned}$$

Therefore, 132° 18′ 31″ can be written as about $132.309°$.

ANSWER:

132.309°

83. **SAT/ACT** The graph represents the set of all possible solutions to which of the following statements?



A $|x - 1| > 1$
B $|x - 1| < 1$
C $|x + 1| < 1$
D $|x + 1| > 1$

SOLUTION:

The graph represents the solution set $x < -2$ or $x > 0$. Solve each inequality to find the inequality that has this solution set.

For choice A:

$x - 1$ is positive
$x - 1 > 1$
$x > 2$

$x - 1$ is negative
$-(x - 1) > 1$
$x - 1 < -1$
$x < 0$

For choice B:

$x - 1$ is positive
$x - 1 < 1$
$x < 2$

$x - 1$ is negative
$-(x - 1) < 1$
$x - 1 > -1$
$x > 0$

For choice C:

$x + 1$ is positive
$x + 1 < 1$
$x < 0$

$x + 1$ is negative
$-(x + 1) < 1$
$x + 1 > -1$
$x > -2$

For choice D:

$x + 1$ is positive
$x + 1 > 1$
$x > 0$

$x + 1$ is negative
$-(x + 1) > 1$
$x + 1 < -1$
$x < -2$

The correct answer is D.

ANSWER:

D

84. What is the cross product of $\mathbf{u} = \langle 3, 8, 0 \rangle$ and $\mathbf{v} = \langle -4, 2, 6 \rangle$?

F $48\mathbf{i} - 18\mathbf{j} + 38\mathbf{k}$
G $48\mathbf{i} - 22\mathbf{j} + 38\mathbf{k}$
H $46\mathbf{i} - 22\mathbf{j} + 38\mathbf{k}$
J $46\mathbf{i} - 18\mathbf{j} + 38\mathbf{k}$

SOLUTION:

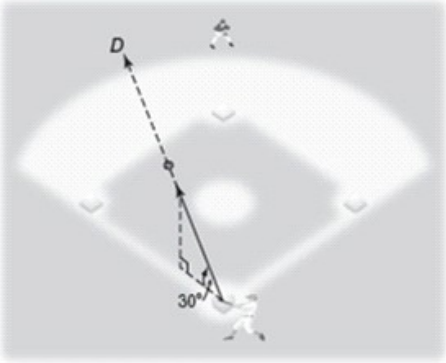
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 8 & 0 \\ -4 & 2 & 6 \end{vmatrix}$$
$$= \begin{vmatrix} 8 & 0 \\ 2 & 6 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & 0 \\ -4 & 6 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & 8 \\ -4 & 2 \end{vmatrix}\mathbf{k}$$
$$= (48 - 0)\mathbf{i} - (18 - 0)\mathbf{j} + [6 - (-32)]\mathbf{k}$$
$$= 48\mathbf{i} - 18\mathbf{j} + 38\mathbf{k}$$

The correct answer is F.

ANSWER:

F

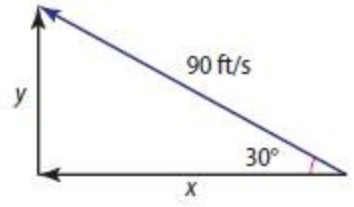
85. **FREE RESPONSE** A batter hits a ball at a 30° angle with the ground at an initial speed of 90 feet per second.



a. Find the magnitude of the horizontal and vertical components of the velocity.
b. Are the values in part **a** vectors or scalars?
c. Assume that the ball is not caught and the player hit it one yard off the ground. How far will it travel in the air?
d. Assume that home plate is at the origin and second base lies due north. If the ball is hit at a bearing of N20°W and lands at point *D*, find the component form of $\overrightarrow{CD}$.
e. Determine the unit vector of $\overrightarrow{CD}$.
f. The fielder is standing at (0, 150) when the ball is hit. At what quadrant bearing should the fielder run in order to meet the ball where it will hit the ground?

SOLUTION:

a. The force exerted on the baseball can be resolved into horizontal and vertical components as shown.



Use the sine or cosine ratios to find the magnitude of each component.

$$\cos 30^\circ = \frac{|\mathbf{x}|}{90} \qquad \sin 30^\circ = \frac{|\mathbf{y}|}{90}$$
$$|\mathbf{x}| = 90\cos 30^\circ \qquad |\mathbf{y}| = 90\sin 30^\circ$$
$$|\mathbf{x}| = 90\left(\frac{\sqrt{3}}{2}\right) \qquad |\mathbf{y}| = 90\left(\frac{1}{2}\right)$$
$$|\mathbf{x}| = 45\sqrt{3} \qquad |\mathbf{y}| = 45$$

The magnitude of the horizontal component is about $45\sqrt{3}$ feet per second and the magnitude of the vertical component 45 feet per second.

b. Since the vertical and horizontal components lack a defined direction, they are both scalars.

c. To determine the distance the ball travels, you need the horizontal distance that the ball has traveled when the ball hits the ground or when the height of the ball is 0. First, write a parametric equation for the vertical position of the ball. Let $y = 0$, $v_0 = 90$, $g = 32$, and $h_0 = 3$ and solve for t.

$$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$$
$$0 = t(90)\sin 30^\circ - \frac{1}{2}(32)t^2 + 3$$
$$0 = 16t^2 - 45t - 3$$

Use the Pythagorean Theorem to solve for t.

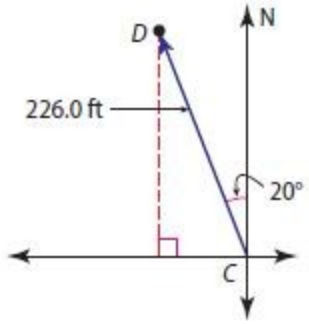
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-45) \pm \sqrt{(-45)^2 - 4(16)(-3)}}{2(16)}$$
$$= \frac{45 \pm \sqrt{2217}}{32}$$
$$\approx 2.9 \text{ or } -0.07$$

Since time cannot be negative, $t = 2.9$. Determine the horizontal position of the ball at 2.9 seconds.

$$x = tv_0 \cos\theta$$
$$= (2.9)(90)\cos 30^\circ$$
$$\approx 226.0$$

The ball will travel about 226.0 feet in the air.

d. Diagram the situation.



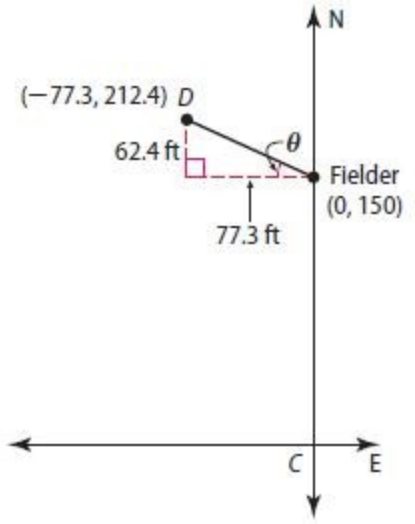
The direction angle that $\overline{CD}$ makes with the positive x-axis is 90° + 20° or 110°. Use the direction angle and the magnitude of $\overline{CD}$ to write $\overline{CD}$ in component form.

$$\overline{CD} = \left\langle \left|\overline{CD}\right|\cos\theta, \left|\overline{CD}\right|\sin\theta \right\rangle$$
$$= \left\langle 226\cos 110^\circ, 226\sin 110^\circ \right\rangle$$
$$= \left\langle -77.3, 212.4 \right\rangle$$

e.

$$\mathbf{u} = \frac{1}{\left|\overline{CD}\right|}\overline{CD}$$
$$= \frac{1}{226.0}\left\langle -77.3, 212.4 \right\rangle$$
$$= \left\langle -\frac{77.3}{226.0}, \frac{212.4}{226.0} \right\rangle$$
$$= \left\langle -0.342, 0.940 \right\rangle$$

f. Diagram the situation.



Use the tangent ratio to solve for θ.

$$\tan\theta = \frac{62.4}{77.3}$$
$$\theta = \tan^{-1}\frac{62.4}{77.3}$$
$$\theta \approx 38.9^\circ$$

Thus, the fielder will have to take an angle of about N51.1°W in order to meet the ball where it will hit the ground.

ANSWER:

a. horizontal: $45\sqrt{3}$ ft/s; vertical: 45 ft/s
b. scalars
c. about 226.0 ft
d. $\langle -77.3, 212.4 \rangle$
e. $\langle -0.342, 0.940 \rangle$
f. about N51.1°W