Math EOT
Term 3
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Grade 10 ADV

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2.1

Polynomial Functions

Example 1

Describe the end behavior of each function using the leading coefficient and degree, and state the domain and range.

1. $f(x) = 3x^4$

2. $f(x) = -2x^3$

3. $f(x) = -\frac{1}{2}x^5$

4. $f(x) = \frac{3}{4}x^6$

Example 2

5. USE A MODEL The shape of a parabolic reflector inside a flashlight can be modeled by the function $f(x) = \frac{4}{3}x^2$. Graph the function $f(x)$, and state the domain and range.

6. MACHINE EFFICIENCY A company uses the function $f(x) = x^3 + 3x^2 - 18x - 40$ to model the change in efficiency of a machine based on its position x. Graph the function and state the domain and range.

2.1

Polynomial Functions

Example 3

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

7. $n + 8$

8. $(2x - 1)(4x^2 + 3)$

9. $-5x^5 + 3x^3 - 8$

10. $18 - 3y + 5y^2 - y^5 + 7y^6$

11. $u^3 + 4u^2t^2 + t^4$

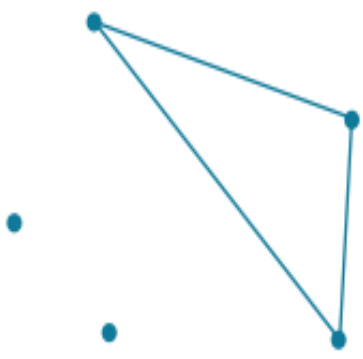
12. $2r - r^2 + \frac{1}{r^2}$

Example 4

13. **TRIANGLES** Dylan drew n dots on a piece of paper making sure that no set of 3 points were collinear. The number of triangles that can be made using the dots as vertices is equal to $f(n) = \frac{1}{6}(n^3 - 3n^2 + 2n)$, when $n \geq 0$.

a. If Dylan drew 15 dots, how many triangles can be made?

b. Sketch a graph of the function.



2.1

Polynomial Functions

14. **DRILLING** The volume of a drill bit can be estimated by the formula for a cone, $V = \frac{1}{3}\pi h r^2$, where h is the height of the bit and r is its radius. Substituting $\frac{\sqrt{3}}{3}r$ for h, the volume of the drill bit can be estimated by $V = \frac{\sqrt{3}}{9}\pi r^3$.

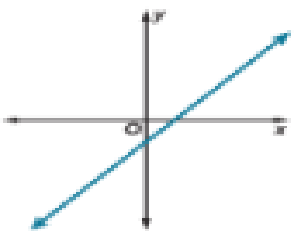
a. What is the volume of a drill bit with a radius of 3 centimeters?

b. Sketch a graph of the function.

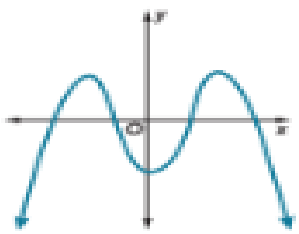
Example 5

Use the graph to state the number of real zeros of the function.

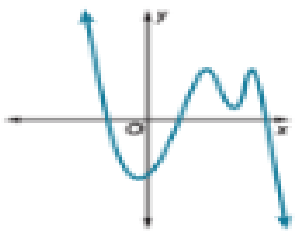
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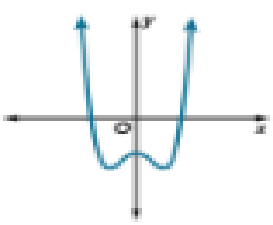
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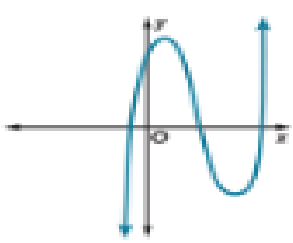
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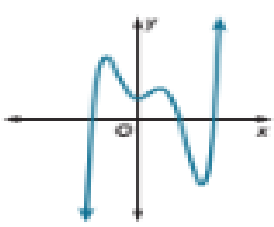
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19.



20.

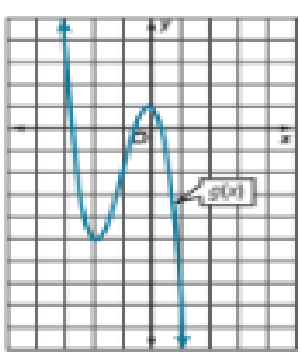


Example 6

21. Examine $f(x) = x^3 - 2x^2 - 4x + 1$ and $g(x)$ shown in the graph.

a. Which function has the greater relative maximum?

b. Compare the zeros, x- and y-intercepts, and end behavior of $f(x)$ and $g(x)$.



2.2

Analyzing Graphs of Polynomial Functions

Example 2

Use a table to graph each function. Then estimate the *x*-coordinates at which relative maxima and relative minima occur.

5. $f(x) = -2x^3 + 12x^2 - 8x$

6. $f(x) = 2x^3 - 4x^2 - 3x + 4$

7. $f(x) = x^4 + 2x - 1$

8. $f(x) = x^4 + 8x^2 - 12$

Example 3

9. BUSINESS A banker models the expected value v of a company in millions of dollars by using the formula $v = n^3 - 3n^2$, where n is the number of years in business. Graph the function and describe its key features over the relevant domain.

2.2

Analyzing Graphs of Polynomial Functions

10. **HEIGHT** A plant's height is modeled by the function $f(x) = 1.5x^3 - 20x^2 + 85x - 84$, where x is the number of weeks since the seed was planted and $f(x)$ is the height of the plant. Graph the function and describe its key features over its relevant domain.

Example 4

11. **USE ESTIMATION** The table shows U.S. car sales in millions of cars. Use a graphing calculator to make a scatter plot and a curve of best fit to show the trend over time. Then use the equation to estimate the car sales in 2017. Let 2008 be represented by year 0. Round the coefficients of the regression equation to the thousandths place.

Year	Cars (millions)	Year	Cars (millions)	Year	Cars (millions)
2008	7.659	2011	6.769	2014	6.089
2009	7.761	2012	5.400	2015	7.243
2010	7.562	2013	5.635	2016	7.780

2.3

Operations with Polynomials

Examples 4, 5 and 6

Multiply.

13. $3p(np - z)$

14. $4x(2x^2 + y)$

15. $-5(2c^2 - d^2)$

16. $x^2(2x + 9)$

17. $(a - 5)^2$

18. $(2x - 3)(3x - 5)$

19. $(x - y)(x^2 + 2xy + y^2)$

20. $(a + b)(a^3 - 3ab - b^2)$

2.3

Operations with Polynomials

21. $(x - y)(x + y)(2x + y)$

22. $(a + b)(2a + 3b)(2x - y)$

23. $(r - 2t)(r + 2t)$

24. $(3y + 4)(2y - 3)$

25. $(x^3 - 3x^2 + 1)(2x^2 - x + 2)$

26. $(4x^5 + x^3 - 7x^2 + 2)(3x - 1)$

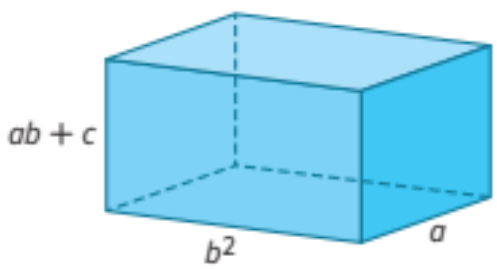
2.3

Operations with Polynomials

Example 7

27. **CONSTRUCTION** A rectangular deck is built around a square pool. The pool has side length s. The length of the deck is 5 units longer than twice the side length of the pool. The width of the deck is 3 units longer than the side length of the pool. What is the area of the deck in terms of s?

28. **VOLUME** The volume of a rectangular prism is given by the product of its length, width, and height. A rectangular prism has a length of b^2 units, a width of a units, and a height of $ab + c$ units. What is the volume of the rectangular prism? Express your answer in simplified form.



2.4

Dividing Polynomials

Examples 4 and 5

Simplify using synthetic division.

11. $(3v^2 - 7v - 10)(v - 4)^{-1}$

12. $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

13. $\frac{y^3 + 6}{y + 2}$

14. $\frac{2x^3 - x^2 - 18x + 32}{2x - 6}$

15. $(4p^3 - p^2 + 2p) \div (3p - 1)$

16. $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

3.1

Solving Polynomial Equations by Graphing

Example 1

Use a graphing calculator to solve each equation by graphing. If necessary, round to the nearest hundredth.

1. $\frac{2}{3}x^3 + x^2 - 5x = -9$

2. $x^3 - 9x^2 + 27x = 20$

3. $x^3 + 1 = 4x^2$

4. $x^6 - 15 = 5x^4 - x^2$

5. $\frac{1}{2}x^5 = \frac{1}{5}x^2 - 2$

6. $x^8 = -x^7 + 3$

Example 2

7. SHIPPING A shipping company will ship a package for \$7.50 when the volume is no more than 15,000 cm^3. Grace needs to ship a package that is $3x - 5$ cm long, $2x$ cm wide, and $x + 20$ cm tall.

a. Write a polynomial equation to represent the situation.

b. Write and solve a system of equations.

c. What should the dimensions of the package be to have the maximum volume?

3.1

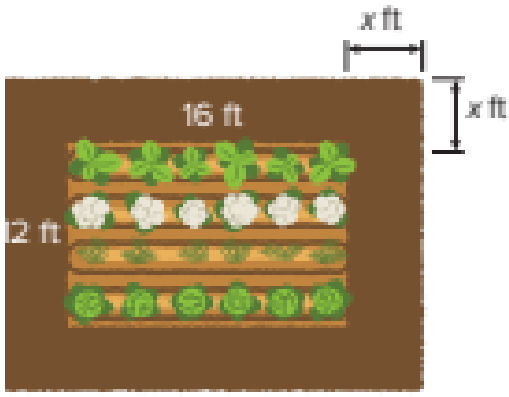
Solving Polynomial Equations by Graphing

8. **GARDEN** A rectangular garden is 12 feet across and 16 feet long. It is surrounded by a border of mulch that is a uniform width, x. The maximum area for the garden, plus border, is 285 ft^2.

a. Write a polynomial equation to represent the situation.

b. Write and solve a system of equations.

c. What are the dimensions of the garden plus border?



9. **PACKAGING** A juice manufacturer is creating new cylindrical packaging. The height of the cylinder is to be 3 inches longer than the radius of the can. The cylinder is to have a volume of 628 cubic inches. Use 3.14 for π.

a. Write a polynomial equation to support the model.

b. Write and solve a system of equations.

c. What are the radius and height of the new packaging?

3.2

Solving Polynomial Equations Algebraically

Example 6

Write each expression in quadratic form, if possible.

16. $x^4 + 12x^2 - 8$

17. $-15x^4 + 18x^2 - 4$

18. $8x^6 + 6x^3 + 7$

19. $5x^6 - 2x^2 + 8$

20. $9x^8 - 21x^4 + 12$

21. $16x^{10} + 2x^5 + 6$

Example 7

Solve each equation.

22. $x^4 + 6x^2 + 5 = 0$

23. $x^4 - 3x^2 - 10 = 0$

3.2

Solving Polynomial Equations Algebraically

24. $4x^4 - 14x^2 + 12 = 0$

25. $9x^4 - 27x^2 + 20 = 0$

26. $4x^4 - 5x^2 - 6 = 0$

27. $24x^4 + 14x^2 - 3 = 0$

3.4

The Remainder and Factor Theorems

Mixed Exercises

45. **REASONING** Jessica evaluates the polynomial $p(x) = x^3 - 5x^2 + 3x + 5$ for a factor using synthetic substitution. Some of her work is shown below. Find the values of a and b.

a	1	−5	3	5
		11	66	759
	1	6	69	b

46. **STATE YOUR ASSUMPTION** The revenue from streaming music services in the United States from 2005 to 2016 can be modeled by $y = 0.26x^5 - 7.48x^4 + 79.20x^3 - 333.33x^2 + 481.68x + 99.13$, where x is the number of years since 2005 and y is the revenue in millions of U.S. dollars.

a. Estimate the revenue from streaming music services in 2010.

b. What might the revenue from streaming music services be in 2020? What assumption did you make to make your prediction?

47. **NATURAL EXPONENTIAL FUNCTION** The natural exponential function $y = e^x$ is a special function that is applied in many fields such as physics, biology, and economics. It is not a polynomial function, however for small values of x, the value of e^x is very closely approximated by the polynomial function $f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$. Use synthetic substitution to determine $f(0.1)$.

3.4

The Remainder and Factor Theorems

Find values of *k* so that each remainder is 3.

48. $(x^2 - x + k) \div (x - 1)$

49. $(x^2 + kx - 17) \div (x - 2)$

50. $(x^2 + 5x + 7) \div (x + k)$

51. $(x^3 + 4x^2 + x + k) \div (x + 2)$

52. If $f(-8) = 0$ and $f(x) = x^3 - x^2 - 58x + 112$, find all the zeros of $f(x)$ and use them to graph the function.

53. **REASONING** If $P(1) = 0$ and $P(x) = 10x^3 + kx^2 - 16x + 3$, find all the factors of $P(x)$ and use them to graph the function. Explain your reasoning.

3.4

The Remainder and Factor Theorems

54. **GEOMETRY** The volume of a box with a square base is $V(x) = 2x^3 + 15x^2 + 36x + 27$. If the height of the box is $(2x + 3)$ units, what are the measures of the sides of the base in terms of x?

55. **SPORTS** The average value of a franchise in the National Football League from 2000 to 2018 can be modeled by $y = -0.037x^5 + 1.658x^4 - 24.804x^3 + 145.100x^2 - 207.594x + 482.008$, where x is the number of years since 2000 and y is the value in millions of U.S. dollars.

a. Copy and complete the table of estimated values. Round to the nearest million.

Year	2003	2012	2021	2025
Estimated Average Franchise Value (millions $)				

b. What assumption did you make to make your predictions? Do you think the assumption is valid? Explain.

3.4

The Remainder and Factor Theorems

56. **CONSTRUCT ARGUMENTS** Divide the polynomial function $f(x) = 4x^3 - 10x + 8$ by the factor $(x + 5)$. Then state and confirm the Remainder Theorem for this particular polynomial function and factor.

57. **REGULARITY** The polynomial function $P(x)$ is symmetric in the y-axis and contains the point $(2, -5)$. What is the remainder when $P(x)$ is divided by $(x + 2)$? Explain your reasoning.

58. **STRUCTURE** Verify the Remainder Theorem for the polynomial $x^2 + 3x + 5$ and the factor $(x - \sqrt{3})$ by first using synthetic division and then evaluating for $x = \sqrt{3}$.

3.4

The Remainder and Factor Theorems

Example 3

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

23. $x^3 - 3x + 2; x + 2$

24. $x^4 + 2x^3 - 8x - 16; x + 2$

25. $x^3 - x^2 - 10x - 8; x + 2$

26. $x^3 - x^2 - 5x - 3; x - 3$

27. $2x^3 + 17x^2 + 23x - 42; x - 1$

28. $2x^3 + 7x^2 - 53x - 28; x - 4$

29. $x^4 + 2x^3 + 2x^2 - 2x - 3; x - 1$

30. $x^3 + 2x^2 - x - 2; x + 2$

3.5

roots and zeros

Example 1

Solve each equation. State the number and type of roots.

1. $5x + 12 = 0$

2. $x^2 - 4x + 40 = 0$

3. $x^5 + 4x^3 = 0$

4. $x^4 - 625 = 0$

5. $4x^2 - 4x - 1 = 0$

6. $x^5 - 81x = 0$

7. $2x^2 + x - 6 = 0$

8. $4x^2 + 1 = 0$

3.5

roots and zeros

9. $x^3 + 1 = 0$

10. $2x^2 - 5x + 14 = 0$

11. $-3x^2 - 5x + 8 = 0$

12. $8x^3 - 27 = 0$

13. $16x^4 - 625 = 0$

14. $x^3 - 6x^2 + 7x = 0$

15. $x^5 - 8x^3 + 16x = 0$

16. $x^5 + 2x^3 + x = 0$

3.5

roots and zeros

Example 2

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

17. $g(x) = 3x^3 - 4x^2 - 17x + 6$

18. $h(x) = 4x^3 - 12x^2 - x + 3$

19. $f(x) = x^3 - 8x^2 + 2x - 4$

20. $p(x) = x^3 - x^2 + 4x - 6$

21. $q(x) = x^4 + 7x^2 + 3x - 9$

22. $f(x) = x^4 - x^3 - 5x^2 + 6x + 1$

3.5

roots and zeros

23. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$

24. $f(x) = 2x^3 - 7x^2 - 2x + 12$

25. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$

26. $f(x) = x^4 - 2x^2 - 5x + 19$

27. $f(x) = 4x^6 - 5x^4 - x^2 + 24$

28. $f(x) = -x^5 + 14x^3 + 18x - 36$

3.5

roots and zeros

Example 3

Find all of the zeros of each function and use them to sketch a rough graph.

29. $h(x) = x^3 - 5x^2 + 5x + 3$

30. $g(x) = x^3 - 6x^2 + 13x - 10$

31. $h(x) = x^3 + 4x^2 + x - 6$

32. $q(x) = x^3 + 3x^2 - 6x - 8$

33. $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$

34. $f(x) = x^4 - 21x^2 + 80$

3.5

roots and zeros

35. $f(x) = x^3 + 7x^2 + 4x - 12$

36. $f(x) = x^3 + x^2 - 17x + 15$

37. $f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$

38. $f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$

39. $f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$

40. $f(x) = x^5 - 8x^3 - 9x$

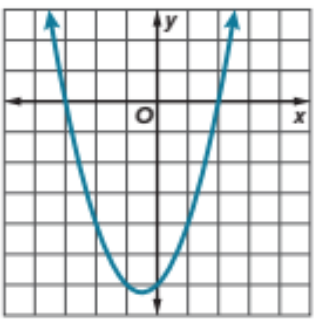
3.5

roots and zeros

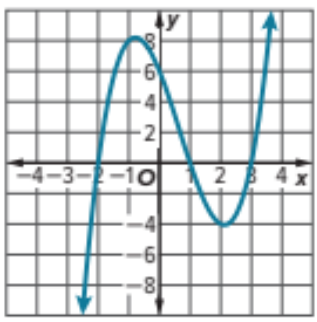
Example 4

Write a polynomial that could be represented by each graph.

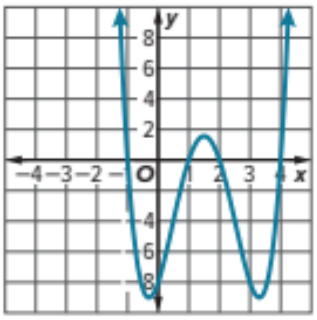
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42.



43.



Example 5

44. **FISH** Some fish jump out of the water. When a fish is out of the water, its location is above sea level. When a fish dives back into the water, its location is below sea level. A biologist can use polynomial functions to model the location of fish compared to sea level. A biologist noticed that a fish is at sea level at −3, −2, −1, 1, 2, and 3 minutes from noon. Graph a polynomial function that could represent the location of the fish compared to sea level y, in centimeters, x seconds from noon.

45. **BUSINESS** After introducing a new product, a company's profit is modeled by a polynomial function. In 2012 and 2017, the company's profit on the product was $0. Graph a polynomial function that could represent the amount of profit $p(x)$, in thousands of dollars, x years since 2010.

4.1

Operations on Functions

Example 4

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each.

9. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
$g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$

10. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
$g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$

11. $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$
$g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$

12. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
$g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$

Example 5

Find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

13. $f(x) = 2x$
$g(x) = x + 5$

14. $f(x) = -3x$
$g(x) = -x + 8$

15. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

16. $f(x) = 2x^2 - x + 1$
$g(x) = 4x + 3$

4.2

Inverse Relations and Functions

Example 5

Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

17. $f(x) = x - 1$

$g(x) = 1 - x$

18. $f(x) = 2x + 3$

$g(x) = \frac{1}{2}(x - 3)$

19. $f(x) = 5x - 5$

$g(x) = \frac{1}{5}x + 1$

20. $f(x) = 2x$

$g(x) = \frac{1}{2}x$

21. $h(x) = 6x - 2$

$g(x) = \frac{1}{6}x + 3$

22. $f(x) = 8x - 10$

$g(x) = \frac{1}{8}x + \frac{5}{4}$

4.2

Inverse Relations and Functions

Example 6

23. **GEOMETRY** The formula for the volume of a right circular cone with a height of 2 feet is $V = \frac{2}{3}\pi r^2$. Determine whether $r = \sqrt{\frac{3V}{2\pi}}$ is the inverse of the original function.

24. **GEOMETRY** The formula for the area of a trapezoid is $A = \frac{h}{2}(a + b)$. Determine whether $h = 2A - (a + b)$ is the inverse of the original function.

Example 1

For each polygon, find the inverse of the relation. Then, graph both the original relation and its inverse.

1. $\triangle MNP$ with vertices at $\{(-8, 6), (6, -2), (4, -6)\}$

4.2

Inverse Relations and Functions

2. $\triangle XYZ$ with vertices at {(7, 7), (4, 9), (3, −7)}

3. trapezoid $QRST$ with vertices at {(8, −1), (−8, −1), (−2, −8), (2, −8)}

4. quadrilateral $FGHJ$ with vertices at {(4, 3), (−4, −4), (−3, −5), (5, 2)}

Examples 2 and 3

Find the inverse of each function. Then graph the function and its inverse. If necessary, restrict the domain of the inverse so that it is a function.

5. $f(x) = x + 2$

6. $g(x) = 5x$

4.2

Inverse Relations and Functions

7. $f(x) = -2x + 1$

8. $h(x) = \frac{x-4}{3}$

9. $f(x) = -\frac{5}{3}x - 8$

10. $g(x) = x + 4$

11. $f(x) = 4x$

12. $f(x) = -8x + 9$

13. $f(x) = 5x^2$

14. $h(x) = x^2 + 4$

4.2

Inverse Relations and Functions

Example 4

15. **WEIGHT** The formula to convert weight in pounds to stones is $p(x) = \frac{x}{14}$, where x is the weight in pounds.

a. Find the inverse of $p(x)$, and describe its meaning.

b. Graph $p(x)$ and $p^{-1}(x)$. Use the graph to find the weight in pounds of a dog that weighs about 2.5 stones.

16. **CRYPTOGRAPHY** DeAndre is designing a code to send secret messages. He assigns each letter of the alphabet to a number, where A = 1, B = 2, C = 3, and so on. Then he uses $c(x) = 4x - 9$ to create the secret code.

a. Find the inverse of $c(x)$, and describe its meaning.

b. Make tables of $c(x)$ and $c^{-1}(x)$. Use the table to decipher the message: 15, 75, 47, 3, 71, 27, 51, 47, 67.

4.3

nth Roots and Rational Exponents

Examples 3

Write each expression in radical form, or write each radical in exponential form.

13. $8^{\frac{1}{5}}$

14. $4^{\frac{2}{7}}$

15. $(x^3)^{\frac{3}{2}}$

16. $\sqrt{17}$

17. $\sqrt[3]{5xy^2}$

18. $\sqrt[4]{625x^2}$

Examples 4

19. ORBITING The distance in millions of miles a planet is from the Sun in terms of t, the number of Earth days it takes for the planet to orbit the Sun, can be modeled by the expression $\sqrt[3]{6t^2}$. Write the expression in exponential form.

4.3

nth Roots and Rational Exponents

20. **DEPRECIATION** The depreciation rate is calculated by the expression $1-\left(\frac{T}{P}\right)^{\frac{1}{n}}$, where n is the age of the item in years, T is the resale price in dollars, and P is the original price in dollars. Write the expression in radical form for an 8 year old car that was originally purchased for $52,425.

Example 5

Evaluate each expression.

21. $27^{\frac{1}{3}}$ **22.** $256^{\frac{1}{4}}$ **23.** $16^{-\frac{3}{2}}$

24. $81^{-\frac{1}{4}}$ **25.** $1024^{\frac{3}{5}}$ **26.** $16^{-\frac{5}{4}}$

4.3

nth Roots and Rational Exponents

Example 6

Simplify each expression.

27. $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$

28. $a^{\frac{4}{9}} \cdot a^{\frac{1}{4}}$

29. $b^{-\frac{3}{4}}$

30. $y^{-\frac{4}{5}}$

WRITING PART

2.4

Dividing Polynomials

Example 1

Simplify each expression.

1. $\frac{15y^3 + 6y^2 + 3y}{3y}$

2. $(4f^5 - 6f^4 + 12f^3 - 8f^2)(4f^2)^{-1}$

3. $(6j^2k - 9jk^2) \div (3jk)$

4. $(4a^2h^2 - 8a^3h + 3a^4) \div (2a^2)$

2.4

Dividing Polynomials

Examples 2 and 3

Simplify by using long division.

5. $(n^2 + 7n + 10) \div (n + 5)$

6. $(d^2 + 4d + 3)(d + 1)^{-1}$

7. $(2t^2 + 13t + 15) \div (t + 6)$

8. $(6y^2 + y - 2)(2y - 1)^{-1}$

9. $(4g^2 - 9) \div (2g + 3)$

10. $(2x^2 - 5x - 4) \div (x - 3)$

2.5

Powers of Binomials

Example 1

Use Pascal's triangle to expand each binomial.

1. $(x - y)^3$

2. $(a + b)^4$

3. $(g - h)^4$

4. $(m + 1)^4$

5. $(y - z)^6$

6. $(d + 2)^8$

2.5

Powers of Binomials

Example 2

7. **BAND** A school band went to 4 competitions during the year and received a superior rating 2 times. If the band is as likely to receive a superior rating as to not receive a superior rating, find the probability of this outcome by expanding $(s + n)^4$. Round to the nearest percent if necessary.

8. **BASKETBALL** Oliver shot 8 free throws at practice, making 6 free throws and missing 2 free throws. If Oliver is equally likely to make a free throw as he is to miss a free throw, find the probability of this outcome by expanding $(m + n)^8$. Round to the nearest percent if necessary.

Example 3

Expand each binomial.

9. $(3x + 4y)^5$

10. $(2c - 2d)^7$

2.5

Powers of Binomials

11. $(8h - 3j)^4$

12. $(4a + 3b)^6$

Mixed Exercises

Expand each binomial.

13. $\left(x + \frac{1}{2}\right)^5$

14. $\left(x - \frac{1}{3}\right)^4$

15. $\left(2b + \frac{1}{4}\right)^5$

16. $\left(3c + \frac{1}{3}d\right)^3$

3.2

Solving Polynomial Equations Algebraically

Examples 1-3

Factor completely. If the polynomial is not factorable, write *prime*.

1. $8c^3 - 27d^3$

2. $64x^4 + xy^3$

3. $a^8 - a^2b^6$

4. $x^6y^3 + y^9$

5. $18x^6 + 5y^6$

6. $w^3 - 2y^3$

3.2

Solving Polynomial Equations Algebraically

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$ **8.** $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

9. $a^3x^2 - 16\,a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

4.1

Operations on Functions

Examples 1 and 2

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 2x$

$g(x) = -4x + 5$

2. $f(x) = x - 1$

$g(x) = 5x - 2$

3. $f(x) = x - 2$

$g(x) = 2x - 7$

4. $f(x) = x^2$

$g(x) = x - 5$

5. $f(x) = -x^2 + 6$

$g(x) = 2x^2 + 3x - 5$

6. $f(x) = 3x^2 - 4$

$g(x) = x^2 - 8x + 4$

4.1

Operations on Functions

Example 3

7. **FINANCE** Trevon opens a checking account that he only uses to pay fixed bills, which are expenses that are the same each month, such as car loans or rent. The checking account has an initial balance of $1750 and Trevon deposits $925 each month. The balance of the account can be modeled by $a(x) = 1750 + 925x$, where x is the number of months since the account was opened. The total of Trevon's fixed bills is modeled by $b(x) = 840x$. Define and graph the function that represents the account balance after he pays his bills.

a. Identify and write a new function to represent the account balance.

b. Graph the combined function.

8. **BASEBALL** A coach is ordering custom practice T-shirts and game jerseys for each of the team members. The coach orders T-shirts from a local shop that charges $7.50 for each, plus a $35 initial printer fee. The cost of the T-shirts is modeled by $t(x) = 7.5x + 35$, where x is the number of team members. He orders jerseys online, which cost $18 each with $20 shipping. The cost of the jerseys is modeled by $j(x) = 18x + 20$. Define and graph the function that represents the total cost of the T-shirts and jerseys.

a. Identify and write a new function to represent total cost.

4.1

Operations on Functions

38. **REASONING** The National Center for Education Statistics reports data showing that since 2006, college enrollment for men in thousands can be modeled by $f(x) = 389x + 7500$, where x represents the number of years since 2006. Similarly, enrollment for women can be modeled by $g(x) = 480x + 10{,}075$. Write a function for $(f + g)(x)$ and interpret what it represents.

39. **STRUCTURE** The table shows various values of functions $f(x)$, $g(x)$, and $h(x)$.

x	−1	0	1	2	3	4
$f(x)$	7	−2	0	2	4	1
$g(x)$	−3	−4	−5	0	1	1
$h(x)$	0	4	1	1	5	5

Use the table to find the following values:

a. $(f + g)(-1)$ **b.** $(h - g)(0)$ **c.** $(f \cdot h)(4)$

d. $\left(\frac{f}{g}\right)(3)$ **e.** $\left(\frac{g}{h}\right)(2)$ **f.** $\left(\frac{g}{f}\right)(1)$

4.1

Operations on Functions

40. **PERSEVERE** If $(f + g)(3) = 5$ and $(f \cdot g)(3) = 6$, find $f(3)$ and $g(3)$. Explain.

41. **CREATE** Write two functions $f(x)$ and $g(x)$ such that $(f \circ g)(4) = 0$.

42. **FIND THE ERROR** Chris and Tobias are finding $(f \circ g)(x)$, where $f(x) = x^2 + 2x - 8$ and $g(x) = x^2 + 8$. Is either of them correct? Explain your reasoning.

Chris

$$\begin{aligned}(f \circ g)(x) &= f[g(x)] \\ &= (x^2 + 8)^2 + 2x - 8 \\ &= x^4 + 16x^2 + 64 + 2x - 8 \\ &= x^4 + 16x^2 + 2x + 58\end{aligned}$$

Tobias

$$\begin{aligned}(f \circ g)(x) &= f[g(x)] \\ &= (x^2 + 8)^2 + 2(x^2 + 8) - 8 \\ &= x^4 + 16x^2 + 64 + 2x^2 + 16 - 8 \\ &= x^4 + 18x^2 + 72\end{aligned}$$

43. **PERSEVERE** Given $f(x) = \sqrt{x^3}$ and $g(x) = \sqrt{x^6}$, determine each domain.

a. $g(x) \cdot g(x)$

b. $f(x) \cdot f(x)$

44. **ANALYZE** State whether the following statement is *sometimes*, *always*, or *never* true. Justify your argument.

The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $g(x)$.

4.3

nth Roots and Rational Exponents

Examples 1 and 2

Simplify.

1. $\pm\sqrt{121x^4y^{16}}$

2. $\pm\sqrt{225a^{16}b^{36}}$

3. $\pm\sqrt{49x^4}$

4. $-\sqrt{16c^4d^2}$

5. $-\sqrt{81a^{16}b^{20}c^{12}}$

6. $-\sqrt{400x^{32}y^{40}}$

7. $\sqrt[4]{16(x-3)^{12}}$

8. $\sqrt[8]{x^{16}y^8}$

9. $\sqrt[4]{81(x-4)^4}$

10. $\sqrt[6]{x^{18}}$

11. $\sqrt[4]{a^{12}}$

12. $\sqrt[3]{a^{12}}$