

هيكـل الرياضيات للصف الحادي عشر متقدم (ريفيل)

الفصل الدراسي الأول - ٢٠٢٥ - ٢٠٢٦

الوحدة الخامسة

Module 5

Exponential Functions

٢ سؤال الكتروني ١ سؤال كتابي

اولا الاسئلة الالكترونية

Graphing Exponential Functions

Example 1

Graph each function. Find the domain, range, y -intercept, asymptote, and end behavior.

1. $f(x) = 3^x$

2. $f(x) = 5^x$

3. $f(x) = 1.5^x$

4. $f(x) = \left(\frac{5}{2}\right)^x$

Example 2

Graph each function.

5. $f(x) = 2(3)^x$

6. $f(x) = -2(4)^x$

7. $f(x) = 4^{x+1} - 5$

8. $f(x) = 3^{2x} + 1$

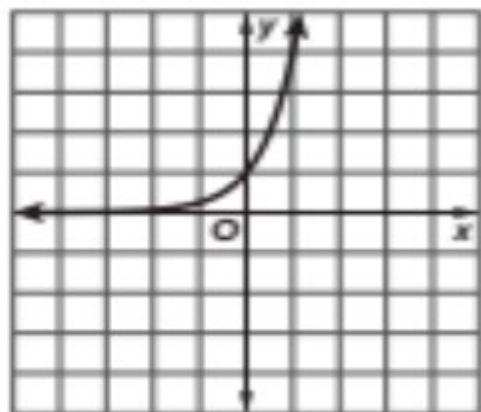
9. $f(x) = -0.4(3)^{x+2} + 4$

10. $f(x) = 1.5(2)^x + 6$

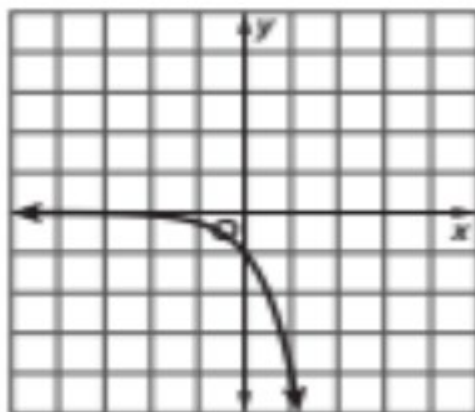
Graphing Exponential Functions

1. **MULTIPLE CHOICE** Select the graph of $f(x) = 4^x$. (Lesson 5-1)

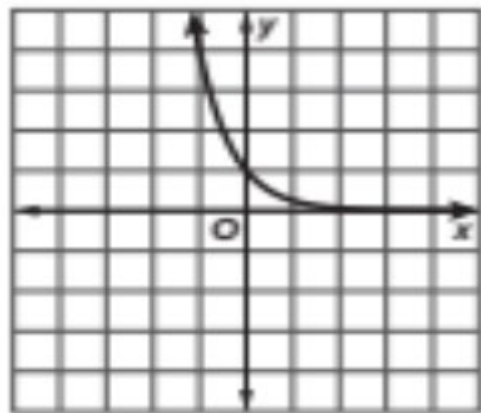
A.



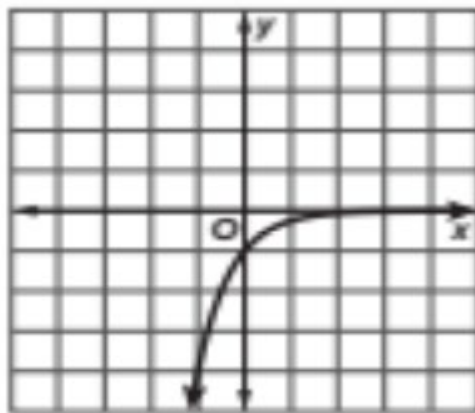
B.



C.



D.



Special Exponential Functions

Example 2 Graph Functions with Base e

Consider $g(x) = -2e^{x-3} + 2$.

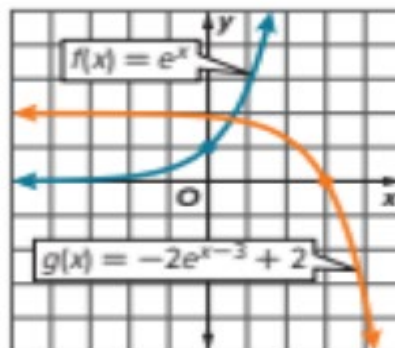
Part A Graph the function.

The function $g(x) = -2e^{x-3} + 2$ represents a transformation of the graph $f(x) = e^x$.

$a = -2$; The graph is reflected in the x -axis and stretched vertically.

$h = 3$; The graph is translated 3 units right.

$k = 2$; The graph is translated 2 units up.

**Part B** Determine the domain and range.

The domain is all real numbers. The range is all real numbers less than 2.

Part C Find the average rate of change.

Determine the average rate of change of $g(x)$ over each interval.

a. $[-4, -1]$

Based on the graph, the graph from -4 to -1 appears approximately horizontal. So, the average rate of change should be close to 0.

$$\frac{g(-1) - g(-4)}{-1 - (-4)} \approx \frac{1.963 - 1.998}{3}$$

Evaluate $g(-1)$ and $g(-4)$.

$$\approx \frac{-0.035}{3} \text{ or } -0.012$$

Simplify.

Special Exponential Functions

b. $[0, 3]$

Based on the graph, the curve appears to pass through approximately $(0, 2)$ and $(3, 0)$. So, the average rate of change for the interval $[0, 3]$ should be about $\frac{0-2}{3-0}$ or $-\frac{2}{3}$.

$$\begin{aligned}\frac{g(3) - g(0)}{3 - 0} &\approx \frac{0 - 1.900}{3} && \text{Evaluate } g(3) \text{ and } g(0). \\ &\approx \frac{-1.900}{3} \text{ or } -0.633 && \text{Simplify.}\end{aligned}$$

c. $[4, 8]$

The graph is very steep for values greater than 4. So, the average rate of change from 4 to 8 should be a negative number with a large absolute value.

$$\begin{aligned}\frac{g(8) - g(4)}{8 - 4} &\approx \frac{-294.826 - (-3.437)}{4} && \text{Evaluate } g(8) \text{ and } g(4). \\ &\approx \frac{-291.389}{4} \text{ or } -72.847 && \text{Simplify.}\end{aligned}$$

اسئلة المقال (الكتابية) عن الوحدة الخامسة

Lesson 5-2

Solving Exponential
Equations and Inequalities

Example 1

Solve each equation.

1. $25^{2x+3} = 25^{5x-9}$

2. $9^{8x-4} = 81^{3x+6}$

3. $4^{x-5} = 16^{2x-31}$

4. $4^{3x-3} = 8^{4x-4}$

5. $9^{-x+5} = 27^{6x-10}$

6. $125^{3x-4} = 25^{4x+2}$

الوحدة السادسة

Module 6

Logarithmic Functions

٤ سؤال الكتروني ١ سؤال كتابي

Logarithms and Logarithmic Functions

Example 1

Write each equation in exponential form.

1. $\log_{15} 225 = 2$

2. $\log_3 \frac{1}{27} = -3$

3. $\log_5 \frac{1}{25} = 2$

4. $\log_3 243 = 5$

5. $\log_4 64 = 3$

6. $\log_4 32 = \frac{5}{2}$

Example 2

Write each equation in logarithmic form.

7. $2^7 = 128$

8. $3^{-4} = \frac{1}{81}$

9. $7^{-2} = \frac{1}{49}$

10. $\left(\frac{1}{7}\right)^3 = \frac{1}{343}$

11. $2^9 = 512$

12. $64^{\frac{2}{3}} = 16$

Example 5 Change of Base Formula

Lesson 6-3

Common Logarithms

Evaluate $\log_2 11$ by writing it in terms of common logarithms. Round to the nearest ten-thousandth.

$$\begin{aligned}\log_2 11 &= \frac{\log_{10} 11}{\log_{10} 2} \\ &\approx 3.4594\end{aligned}$$

Change of Base Formula

Use a calculator.

Example 5

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

21. $\log_4 22$

22. $\log_{12} 200$

23. $\log_2 50$

24. $\log_8 15$

25. $\log_3 2$

26. $\log_5 0.4$

Lesson 6-4

Natural Logarithms

Example 3 Simplify Logarithmic Expressions

Write $\frac{1}{3} \ln 8 - \ln 3 + \ln 9$ as a single logarithm.

$$\frac{1}{3} \ln 8 - \ln 3 + \ln 9$$

Original expression

$$= \ln 8^{\frac{1}{3}} - \ln 3 + \ln 9$$

Power Property of Logarithms

$$= \ln 2 - \ln 3 + \ln 9$$

$$8^{\frac{1}{3}} = \sqrt[3]{8} \text{ or } 2$$

$$= \ln \frac{2}{3} + \ln 9$$

Quotient Property of Logarithms

$$= \ln 6$$

Product Property of Logarithms

Lesson 6-4

Natural Logarithms

Example 3

Write each expression as a single logarithm.

19. $3 \ln 3 - \ln 9$

20. $4 \ln 16 - \ln 256$

21. $2 \ln x + 2 \ln 4$

22. $3 \ln 4 + 3 \ln 3$

23. $\ln 125 - 2 \ln 5$

24. $3 \ln 10 + 2 \ln 100$

25. $4 \ln \frac{1}{3} - 6 \ln \frac{1}{9}$

26. $7 \ln \frac{1}{2} + 5 \ln 2$

27. $8 \ln x - 4 \ln 5$

Using Exponential and Logarithmic Functions

 Example 1 Continuous Exponential Growth

POPULATION In 2016, the population of Florida was 20.61 million people. In 2000, it was 15.98 million.

Part A Write an exponential growth equation.

Because the population is growing as time passes, use the year as the independent variable and the population as the dependent variable. Find the value of k , Florida's relative growth rate. Then write an equation that represents the population of Florida t years after 2000.

Since t is the number of years after 2000, 15.98 represents the initial population, and 20.61 represents the population 16 years after 2000, or at $t = 16$.

(continued on the next page)

Using Exponential and Logarithmic Functions

$$y = ae^{kt}$$

Formula for continuous exponential growth

$$20.61 = 15.98e^{k(16)}$$

 $y = 20.61$, $a = 15.98$, and $t = 2016 - 2000$ or 16

$$\frac{20.61}{15.98} = e^{16k}$$

Divide each side by 15.98.

$$\ln \frac{20.61}{15.98} = \ln e^{16k}$$

Property of Equality for Logarithmic Equations

$$\ln \frac{20.61}{15.98} = 16k$$

 $\ln e^x = x$

$$\frac{\ln \frac{20.61}{15.98}}{16} = k$$

Divide each side by 16.

$$0.0159 \approx k$$

Use a calculator.

Florida's relative rate of growth is about, 0.0159, or about 1.59%.

$$y = ae^{kt}$$

Formula for continuous exponential growth

$$y = 15.98e^{0.0159t}$$

 $a = 15.98$ and $k = 0.0159$

So, the population of Florida t years after 2000 can be modeled by the equation $y = 15.98e^{0.0159t}$.

Part B Predict when the population will reach 25 million people.

Using Exponential and Logarithmic Functions

Use your equation from **Part A** to predict the population.

$$y = 15.98e^{0.0159t} \quad \text{Original equation}$$

$$25 = 15.98e^{0.0159t} \quad y = 25$$

$$1.564 = e^{0.0159t} \quad \text{Divide each side by 15.98.}$$

$$\ln 1.564 = \ln e^{0.0159t} \quad \text{Property of Equality for Logarithmic Equations}$$

$$\ln 1.564 = 0.0159t \quad \ln e^x = x$$

$$\frac{\ln 1.564}{0.0159} = t \quad \text{Divide each side by 0.0159.}$$

$$28.129 \approx t \quad \text{Use a calculator.}$$

Florida's population will reach 25 million people approximately 28.129 years after 2000, or in 2028.

Using Exponential and Logarithmic Functions

Part C Compare the populations of Florida and California.

California's population in 2000 was 33.9 million, and its population growth can be modeled by $y = 33.9e^{0.0092t}$. Determine when Florida's population will surpass California's.

$$15.98e^{0.0159t} > 33.9e^{0.0092t} \quad \text{Formula for exponential growth}$$

$$\ln 15.98e^{0.0159t} > \ln 33.9e^{0.0092t} \quad \text{Prop. of Ineq. for Logarithms}$$

$$\ln 15.98 + \ln e^{0.0159t} > \ln 33.9 + \ln e^{0.0092t} \quad \text{Product Property of Logarithms}$$

$$\ln 15.98 + 0.0159t > \ln 33.9 + 0.0092t \quad \ln e^x = x$$

$$\ln 15.98 + 0.0067t > \ln 33.9 \quad \text{Subtract } 0.0092t \text{ from each side.}$$

$$0.0067t > \ln 33.9 - \ln 15.98 \quad \text{Subtract } \ln 15.98 \text{ from each side.}$$

$$t > \frac{\ln 33.9 - \ln 15.98}{0.0067} \quad \text{Divide each side by } 0.0067.$$

$$t > 112.25 \quad \text{Use a calculator.}$$

Assuming that this trend continues, Florida's population is on track to surpass California's in 2112.

Check

SCIENCE An experiment starts with 20 bacteria A cells. After 45 minutes, there are 710 bacteria A cells.

Part A

Write the equation that models the number of bacteria A cells y after t minutes. Round the value of k to the nearest thousandth.

Part B

After how many minutes will there be 1000 bacteria A cells? Round to the nearest tenth if necessary.

_____ minutes

Part C

Bacteria B grows exponentially according to the model $y = 52e^{0.064t}$. After how many minutes will there be more bacteria A cells than bacteria B cells? Round to the nearest tenth if necessary.

_____ minutes

اسئلة المقال (الكتابية) عن الوحدة السادسة

Properties of Logarithms

Example 2

Solve each equation.

7. $\log_4 (2x^2 - 4) = \log_4 2x$

8. $\log_5 (x^2 - 6) = \log_5 x$

9. $\log_3 (x^2 - 8) = \log_3 2x$

10. $\log_4 (2x^2 - 20) = \log_4 6x$

11. $\log_2 (6x^2 + 1) = \log_2 5x$

12. $\log_6 (6x^2 - 3) = \log_6 7x$

Examples 3 and 4

Use $\log_4 2 = 0.5$, $\log_4 3 \approx 0.7925$, and $\log_4 5 \approx 1.1610$ to approximate the value of each expression.

13. $\log_4 30$

14. $\log_4 20$

15. $\log_4 \frac{2}{3}$

16. $\log_4 \frac{4}{3}$

17. $\log_4 9$

18. $\log_4 8$

Properties of Logarithms

Examples 3 and 4

Use $\log_4 2 = 0.5$, $\log_4 3 \approx 0.7925$, and $\log_4 5 \approx 1.1610$ to approximate the value of each expression.

13. $\log_4 30$

14. $\log_4 20$

15. $\log_4 \frac{2}{3}$

16. $\log_4 \frac{4}{3}$

17. $\log_4 9$

18. $\log_4 8$

22	a) Solve logarithmic equations using properties of equality.	(7-12)	273
	b) Simplify and evaluate expressions by using the properties of logarithms.	(13-24)	

Lesson 6-2

Properties of Logarithms

Example 5

Use $\log_2 3 \approx 1.5850$ and $\log_2 5 \approx 2.3219$ to approximate the value of each expression.

19. $\log_2 25$

20. $\log_2 27$

21. $\log_2 125$

22. $\log_2 625$

23. $\log_2 81$

24. $\log_2 243$

الوحدة السابعة

Module 7

Rational Functions

٥ سؤال الكتروني ١ سؤال كتابي

Example 1

Simplify each expression, and state when the original expression is undefined.

1.
$$\frac{x(x-3)(x+6)}{x^2+x-12}$$

2.
$$\frac{y^2(y^2+3y+2)}{2y(y-4)(y+2)}$$

3.
$$\frac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)}$$

4.
$$\frac{(x^2-16x+64)(x+2)}{(x^2-64)(x^2-6x-16)}$$

5.
$$\frac{x^2(x+2)(x-4)}{6x(x^2+x-20)}$$

6.
$$\frac{3y(y-8)(y^2+2y-24)}{15y^2(y^2-12y+32)}$$

Example 2

Simplify each expression.

7.
$$\frac{x^2-5x-14}{28+3x-x^2}$$

8.
$$\frac{9x^2-x^3}{x^2-3x-54}$$

9.
$$\frac{(x-4)(x^2+2x-48)}{(36-x^2)(x^2+4x-32)}$$

10.
$$\frac{16-c^2}{c^2+c-20}$$

Multiplying and Dividing
Rational Expressions

Adding and Subtracting
Rational Expressions

Examples 1 and 2

Simplify each expression.

1. $\frac{3}{x} + \frac{5}{y}$

2. $\frac{3}{8p^2r} + \frac{5}{4p^2r}$

3. $\frac{2c-7}{3} + 4$

4. $\frac{2}{m^2p} + \frac{5}{p}$

5. $\frac{12}{5y^2} - \frac{2}{5yz}$

6. $\frac{7}{4gh} + \frac{3}{4h^2}$

7. $\frac{3}{w-3} - \frac{2}{w^2-9}$

8. $\frac{3t}{2-x} + \frac{5}{x-2}$

9. $\frac{k}{k-n} - \frac{k}{n-k}$

10. $\frac{4z}{z-4} + \frac{z+4}{z+1}$

11. $\frac{n}{n-3} + \frac{2n+2}{n^2-2n-3}$

12. $\frac{3}{y^2+y-12} - \frac{2}{y^2+6y+8}$

Example 4

Find the zeros and asymptotes of each function. Then graph each function.

11. $f(x) = \frac{(x-4)^2}{x+2}$

12. $f(x) = \frac{(x+3)^2}{x-5}$

13. $f(x) = \frac{6x^2 + 4x + 2}{x+2}$

14. $f(x) = \frac{2x^2 + 7x}{x-2}$

15. $f(x) = \frac{3x^2 + 8}{2x-1}$

16. $f(x) = \frac{2x^2 + 5}{3x+4}$

Example 5

Graph each function. Find the point discontinuity.

17. $f(x) = \frac{x^2 - 2x - 8}{x-4}$

18. $f(x) = \frac{x^2 + 4x - 12}{x-2}$

19. $f(x) = \frac{x^2 - 25}{x+5}$

20. $f(x) = \frac{x^2 - 64}{x-8}$

21. $f(x) = \frac{(x-4)(x^2-4)}{x^2-6x+8}$

22. $f(x) = \frac{(x+5)(x^2+2x-3)}{x^2+8x+15}$

Graphing Rational Functions

Lesson 7-5

Variation

Example 2

If a varies jointly as b and c , find a when $b = 4$ and $c = -3$.

7. $a = -96$ when $b = 3$ and $c = -8$

8. $a = -60$ when $b = -5$ and $c = 4$

9. $a = -108$ when $b = 2$ and $c = 9$

10. $a = 24$ when $b = 8$ and $c = 12$

11. If y varies jointly as x and z , and $y = 18$ when $x = 2$ and $z = 3$, find y when x is 5 and z is 6.

12. If y varies jointly as x and z , and $y = -16$ when $x = 4$ and $z = 2$, find y when x is -1 and z is 7.

Example 1

Solve each equation. Check your solutions.

1. $\frac{2x+3}{x+1} = \frac{3}{2}$

2. $\frac{-12}{y} = y - 7$

3. $\frac{9}{x-7} - \frac{7}{x-6} = \frac{13}{x^2 - 13x + 42}$

4. $\frac{13}{y+3} - \frac{12}{y+4} = \frac{18}{y^2 + 7y + 12}$

5. $\frac{14}{x-2} - \frac{18}{x+1} = \frac{22}{x^2 - x - 2}$

6. $\frac{2}{a+2} + \frac{10}{a+5} = \frac{36}{a^2 + 7a + 10}$

Example 2

7. $\frac{x}{2x-1} + \frac{3}{x+4} = \frac{21}{2x^2 + 7x - 4}$

8. $\frac{2}{y-5} + \frac{y-1}{2y+1} = \frac{2}{2y^2 - 9y - 5}$

9. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

10. $\frac{12p+19}{p^2+7p+12} - \frac{3}{p+3} = \frac{5}{p+4}$

11. $\frac{2f}{f^2-4} + \frac{1}{f-2} = \frac{2}{f+2}$

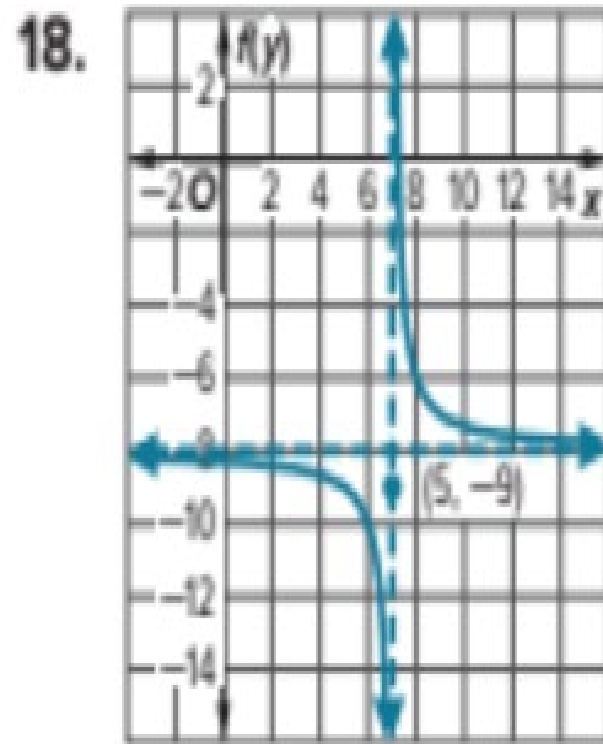
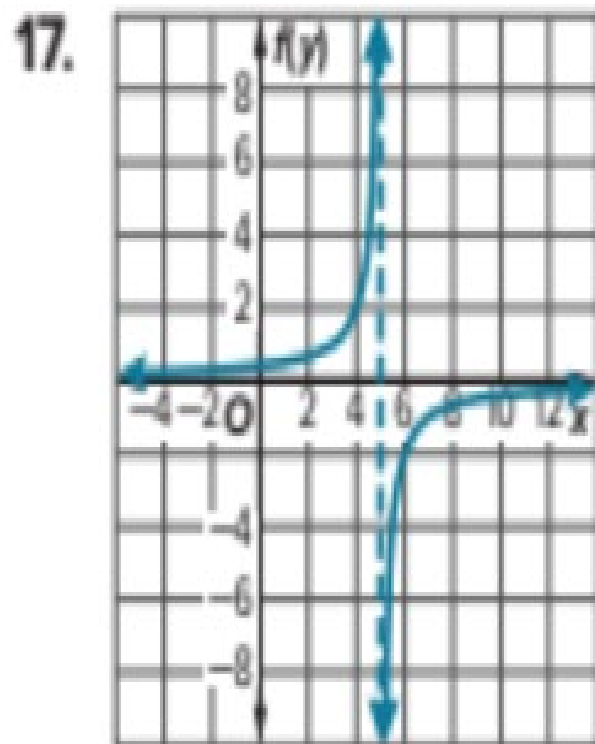
12. $\frac{8}{t^2-9} + \frac{4}{t+3} = 1$

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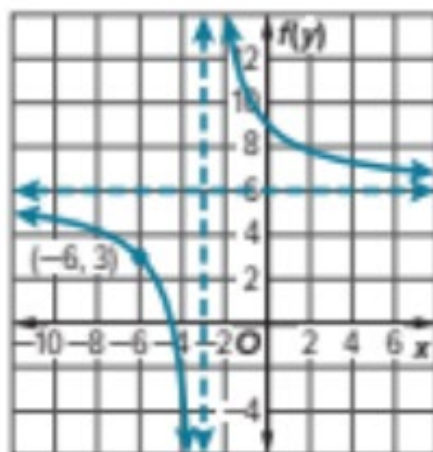
Graphing Reciprocal Functions

Example 5

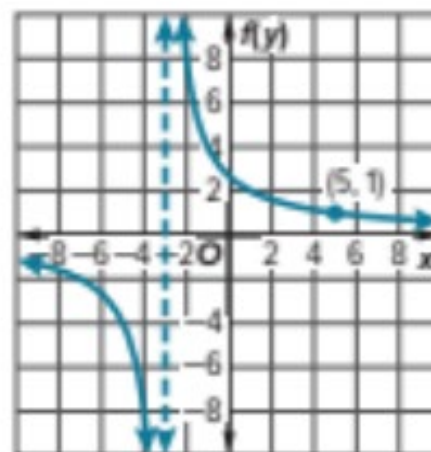
Identify the values of a , h , and k . Then write a function for the graph $g(x) = \frac{a}{x-h} + k$.



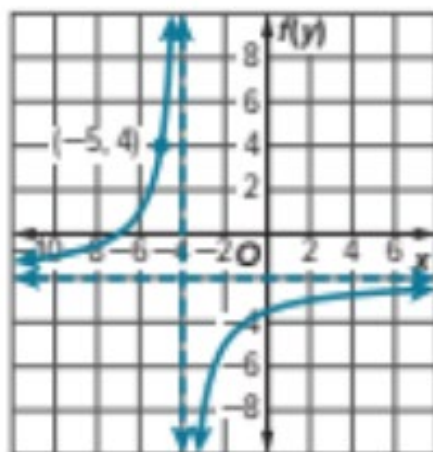
19.



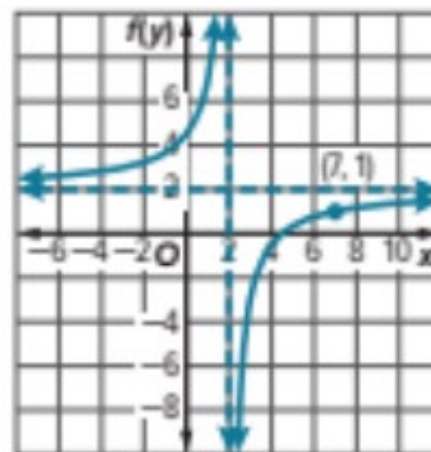
20.



21.



22.



الوحدة الثامنة

Module 8

Inferential Statistics

٣ سؤال الكتروني ١ سؤال كتابي

اولا الاسئلة الالكترونية

Examples 2 and 3

Identify each sample or question as *biased* or *unbiased*. Explain your reasoning.

4. A random sample of eight people is asked to select their favorite food for a survey about Americans' food preferences.
5. Every tenth student at band camp is asked to name his or her favorite band for a survey about the campers.
6. Every fifth person entering a museum is asked to name his or her favorite type of book to read for a survey about reading interests of people in the city.
7. Do you think that the workout facility needs a new treadmill and racquetball court?
8. Which is your favorite type of music, pop, or country?
9. Are you a member of any after-school clubs?
10. Don't you agree that employees should pack their lunch?

Lesson 8-1

Random Sampling

Lesson 8-2

Using Statistical Experiments

Example 1

1. A student spun a spinner with 4 equal sections 100 times and recorded the results.
 - a. Find the theoretical probability of spinning blue.
Write your answer as a percentage rounded to the nearest tenth, if necessary.
 - b. Find the experimental probability of spinning blue.
Write your answer as a percentage rounded to the nearest tenth, if necessary.

Spinner Section	Frequency
Red	35
Blue	38
Green	13
Yellow	14

Lesson 8-2

Using Statistical Experiments

2. A student flipped a coin 125 times and recorded the results.

a. Find the theoretical probability of the coin landing on heads. Write your answer as a percentage rounded to the nearest tenth, if necessary.

b. Find the experimental probability of the coin landing on heads. Write your answer as a percentage rounded to the nearest tenth, if necessary.

Coin Result	Frequency
Heads	73
Tails	52

Lesson 8-2

Using Statistical Experiments

- 3.** A fair 6-sided die is rolled 150 times.
- Find the theoretical probability of rolling a 3. Write your answer as a percentage rounded to the nearest tenth, if necessary.
 - Find the experimental probability of rolling a 3. Write your answer as a percentage rounded to the nearest tenth, if necessary.

Number on Die	Frequency
1	32
2	18
3	27
4	16
5	33
6	24

4. **MULTIPLE CHOICE** Leonard randomly selected a card from a standard deck of playing cards, recorded the suit, and returned the card. He followed this set of steps 120 times. The results are shown.

Suit	Frequency
Heart	28
Diamond	37
Spade	34
Club	21

Which statement about the results is true?

(Lesson 8-2)

- A. The theoretical probability of selecting a heart is less than the experimental probability of selecting a heart.
- B. The theoretical probability of selecting a diamond is less than the experimental probability of selecting a diamond.
- C. The experimental probability of selecting a club is greater than the theoretical probability of selecting a club.
- D. The experimental probability of selecting a heart is equal to the experimental probability of selecting a diamond.

Example 1 Find a Standard Deviation

TRACK A coach recorded the times of each track member for a 400-meter race. Find and interpret the standard deviation of the data.

400m Race Times (seconds)	
57.1	55.9
59.3	54.9
54.6	50.3
55.2	53.5

Step 1 Find the mean, μ .

$$\begin{aligned}\mu &= \frac{57.1 + 59.3 + 54.6 + 55.2 + 55.9 + 54.9 + 50.3 + 53.5}{8} \\ &= 55.1\end{aligned}$$

The mean running time for the team is 55.1 seconds.

Step 2 Find the squares of the differences, $(\mu - x_n)^2$.

$$\begin{array}{ll} (55.1 - 57.1)^2 = 4.00 & (55.1 - 55.9)^2 = 0.64 \\ (55.1 - 59.3)^2 = 17.64 & (55.1 - 54.9)^2 = 0.04 \\ (55.1 - 54.6)^2 = 0.25 & (55.1 - 50.3)^2 = 23.04 \\ (55.1 - 55.2)^2 = 0.01 & (55.1 - 53.5)^2 = 2.56 \end{array}$$

Step 3 Find the sum.

Find the sum of the values from **Step 2**.

$$\begin{aligned}4.00 + 17.64 + 0.25 + 0.01 + 0.64 + 0.04 + 23.04 + 2.56 \\ = 48.18\end{aligned}$$

Analyzing Population Data

Step 4 Divide by the number of values.

Divide the sum from **Step 3** by the number of running times.

$$\frac{48.18}{8} = 6.0225 \quad \text{This is the variance.}$$

Step 5 Take the square root of the variance.

$$\sqrt{6.0225} \approx 2.45 \quad \text{This is the standard deviation.}$$

The standard deviation is about 2.45. This is small compared with the run times, which means that the majority of the team members' times are close to the mean of 55.1 seconds, and almost all of the times will likely fall within 2 standard deviations of the mean, or between 50.2 and 60.0 seconds.

Analyzing Population Data

Example 1

1. **BARBER** A barber wants to purchase new professional shears from a Web site. The prices of all of the shears are shown in the table. Use the standard deviation formula to find and interpret the standard deviation of the data. Round your answers to the nearest cent.

Cost of Shears (\$)			
50	165	55	79
84	68	38	42

2. **READING** Ms. Sanchez keeps track of the total number of books each student in the book club reads during the school year. Use the standard deviation formula to find and interpret the standard deviation of the data. Round your answers to the nearest tenth.

Books Read		
9	6	12
8	9	14
10	13	8

7. MULTI-SELECT Zain is analyzing two sets of data. (Lesson 8-3)

Set A				
35	28	35	33	32
32	31	30	36	49
36	33	29	34	37

Set B				
22	26	26	24	27
38	35	31	22	30
24	33	30	25	38

Select all the true statements about each data set.

- A. Data set A has a higher average value.
- B. Data set B has a higher average value.
- C. Data set A is more varied.
- D. Data set B is more varied.

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Example 6 Use z-Values to Locate Position

Find z if $X = 24$, $\mu = 19$, and $\sigma = 3.8$.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for z-value}$$

$$z = \frac{24 - 19}{3.8} \quad X = 24, \mu = 19, \text{ and } \sigma = 3.8$$

$$z = \frac{5}{3.8} \quad \text{Subtract.}$$

$$z \approx 1.316 \quad \text{Divide.}$$

Check

Find z if $X = 106.3$, $\mu = 88.8$, and $\sigma = 9.6$. Round your answer to the nearest thousandth. $z = \underline{\hspace{1cm}}?$

Example 7 Find Area Under the Standard Normal Curve by Using a Table

Use a Standard Normal Distribution Table to find the area under the normal curve within the interval $z < -1.18$.

Step 1 Identify the probability for the left endpoint, $z = -1.18$.

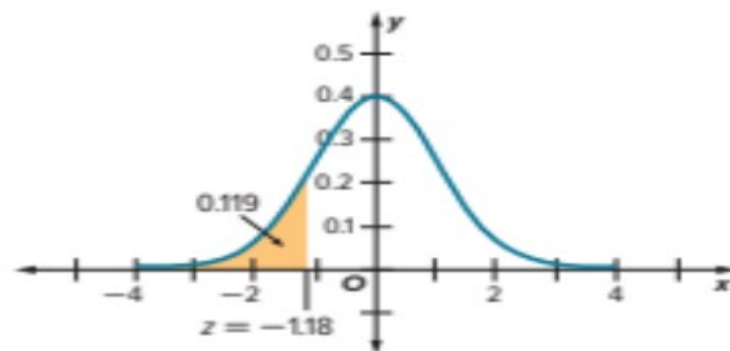
Using a Standard Normal Distribution Table, move down the left column until you reach the correct tenths place, -1.1 . Then move across the row until you reach the hundredths place, 0.08 .

z	0.00	...	0.08	0.09
-3.4	0.0003	...	0.0003	0.0002
$\vdots$	...	...	...	...
-1.1	0.1357	...	0.1190	0.1170

The probability associated with $z = -1.18$ is 0.1190.

Step 2 Identify the area under the curve within the interval.

As shown in the graph, 0.119 represents the area under the curve to the left of $z = -1.18$.



Lesson 8-4

Normal Distributions

Example 8 Find Area Under the Standard Normal Curve by Using a Calculator

INTERNET TRAFFIC The number of daily hits to a local news Web site is normally distributed with $\mu = 98,452$ hits and $\sigma = 10,325$ hits. Find the probability that the Web site will get at least 100,000 hits on a given day, $P(X > 100,000)$.

Visits Summary



Visits Summary - Last 30 Days

Visitor Demographics



Views by Category



Lesson 8-4

Normal Distributions

Step 1 Find the corresponding z-value for $X = 100,000$.

$$z = \frac{X - \mu}{\sigma}$$

Formula for z-value

$$z = \frac{100,000 - 98,452}{10,325}$$

$X = 100,000$, $\mu = 98,452$, and $\sigma = 10,325$

$$z \approx 0.150$$

Simplify.

Step 2 Find the probability.

The area under the curve when $z > 4$ is negligible. So $z = 4$ can be used as an upper bound for finding area. You can use a graphing calculator to find the area between $z = 0.150$ and $z = 4$.

Press **2nd** **[DISTR]** and select **normalcdf**. Enter the interval and press **enter** to display the area.

```
normalcdf(0.150,
4)
.4403506010
```

The area is 0.44. Therefore, the probability of the Web site getting at least 100,000 hits in one day is 44.0%.

Normal Distributions

Example 6

Find the z-value for each standard normal distribution.

8. $\sigma = 9.8$, $X = 55.4$, and $\mu = 68.34$

9. $\sigma = 11.6$, $X = 42.80$, and $\mu = 68.2$

10. $\sigma = 11.9$, $X = 119.2$, and $\mu = 112.4$

Example 7

Use a table to find the area under the normal curve for each interval.

11. $z > 0.58$

12. $z < -1.56$

13. $-2.29 < z < 2.76$

Example 8

14. **TESTING** The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.

a. What percent of the scores are between 70 and 80?

b. What percent of the scores are over 115?

c. If 75 people take the test, how many would you expect to score lower than 75?

الوحدة التاسعة

Module 9

Trigonometric Functions

٦ سؤال الكتروني ١ سؤال كتابي

Lesson 9-1

Angles and Angle Measure

Mixed Exercises

REGULARITY Rewrite each degree measure in radians and each radian measure in degrees.

54. 18°

55. 6°

56. -72°

57. -820°

58. 4π

59. $\frac{5\pi}{2}$

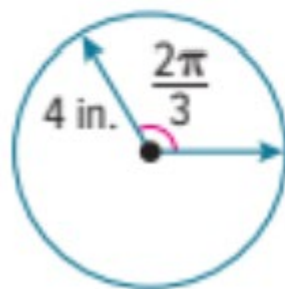
60. $-\frac{9\pi}{2}$

61. $-\frac{7\pi}{12}$

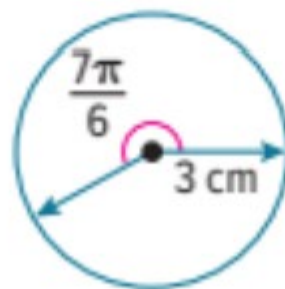
62. -270°

Find the length of each arc. Round to the nearest tenth.

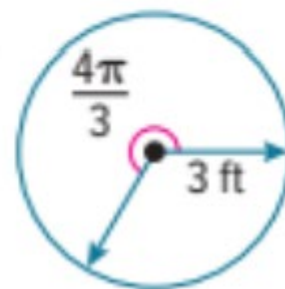
63.



64.



65.



Trigonometric Functions of General Angles

Example 6

PRECISION Find the exact value of each trigonometric function.

40. $\tan 330^\circ$

41. $\cos \left(-\frac{11\pi}{4} \right)$

42. $\cot 30^\circ$

43. $\csc \frac{\pi}{4}$

44. $\sin (-150^\circ)$

45. $\tan \left(-\frac{\pi}{4} \right)$

Lesson 9-3

Circular and Periodic Functions

Example 1

The terminal side of angle θ in standard position intersects the unit circle at each point P . Find $\cos \theta$ and $\sin \theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. $P(0, -1)$

3. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$

5. $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$

6. $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

Example 5 Characteristics of the Sine and Cosine Functions

SPRINGS An object on a spring oscillates according to the function $y = 40 \cos \pi t$, where y is the distance in centimeters above its equilibrium position at time t in seconds.

Part A Find the period and frequency, and describe them in the context of the situation.

The period is $\frac{2\pi}{|b|}$. Since $b = \pi$, the period is $\frac{2\pi}{\pi}$ or 2.

The frequency is $\frac{1}{\text{period}}$ or $\frac{1}{2}$.

Therefore, the object completes $\frac{1}{2}$ of a cycle per second, and it will reach the maximum distance above the equilibrium point every 2 seconds.

Part B Identify the domain and range in the context of the situation.

The domain of $y = 40 \cos \pi t$ is all real numbers. Because time cannot be negative, the relevant domain in the context of the situation is $[0, \infty)$.

The range is $[-40, 40]$. This references the farthest away that the object can get from its point of equilibrium. With the equilibrium at the center, the object can be as much as 40 centimeters from the center in either direction. This is verified by the amplitude.

Graphing Sine and Cosine Functions

Graphing Sine and Cosine Functions

Example 6 Model Periodic Situations

ELECTRICITY The voltage supplied by an electrical outlet can be represented by a periodic function. The voltage oscillates between -120 and 120 volts, with a frequency of 50 cycles per second. Write and graph a function for the voltage v as a function of time t .

Step 1 The voltage oscillates from -120 to 120 .

The maximum is 120 . The minimum is -120 .

The amplitude is 120 . $a = 120$

Step 2 The frequency is 50 cycles per second.

The period is $\frac{1}{\text{frequency}}$ or $\frac{1}{50}$.

$$\text{period} = \frac{2\pi}{b}$$

Period in terms of b

$$\frac{1}{50} = \frac{2\pi}{b}$$

Substitute.

$$b = 100\pi$$

Solve.

Step 3 Use t instead of x to write the function as a function of time.

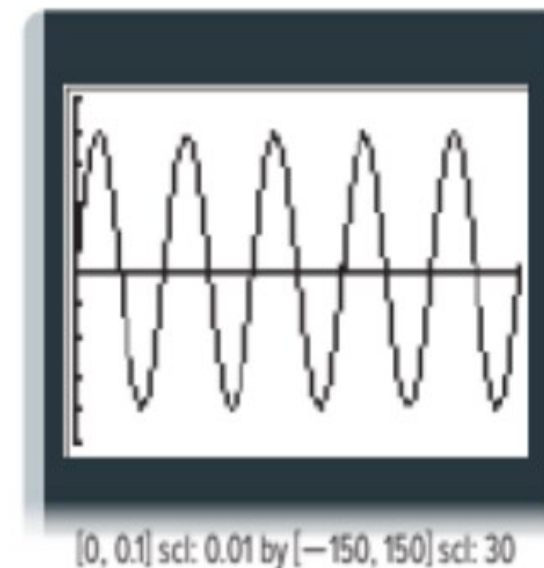
$$y = a \sin bt$$

General equation

$$y = 120 \sin 100\pi t$$

Substitute.

Step 4 Use a graphing calculator to graph the function.



18	Model periodic real-world situations with sine and cosine functions. (No need for graphing calculator)	Example 5 & Example 6 (17-20)	449 & 450 452
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Graphing Sine and Cosine Functions

Example 5

- 17. USE A MODEL** An object on a spring oscillates using the function $y = 25 \cos \pi t$, where y is the distance in inches from its equilibrium position in t seconds.
- Find and describe the period and frequency.
 - Identify the domain and range in the context of the situation.
- 18. SWINGS** Suppose a tire swing is rotated $\frac{\pi}{5}$ radians and released. The function $y = \frac{\pi}{5} \cos 2t$ represents the displacement of the swing at time t for a frequency of radians per second.
- Find the period and frequency, and describe them in the context of the situation.
 - Identify the domain and range in the context of the situation.

Graphing Sine and Cosine Functions

Example 6

19. REASONING A boat that is tied to a dock moves vertically up and down with the waves. Delray watches the boat for 30 seconds and notes that the boat moves up and down a total of 6 times. The difference between the boat's highest point and lowest point is 3 feet. Write and graph a trigonometric function that models the boat's vertical position x seconds after she began watching. Assume that when Delray began watching the boat, it was at its highest point and that its average vertical position was 0 feet.

20. FERRIS WHEELS A Ferris wheel at a state fair has a diameter of 65 feet and makes 4 complete revolutions each minute. Santiago boards a car of the Ferris wheel at the car's lowest point, and he rides for 2 minutes. Write and graph a trigonometric function that models his height above or below the axle of the Ferris wheel θ minutes after the ride starts.



Translations of Trigonometric Graphs

Example 3

State the amplitude, period, phase shift, vertical shift, and midline equation of each function. Then graph the function and state the domain and range.

13. $y = \cos \theta + 3$

14. $y = \tan \theta - 1$

15. $y = \tan \theta + \frac{1}{2}$

16. $y = 2 \cos \theta - 5$

17. $y = 2 \sin \theta - 4$

18. $y = \frac{1}{3} \sin \theta + 7$

Inverse Trigonometric Functions

Example 1 Evaluate Inverse Trigonometric Functions

Find $\tan^{-1} \sqrt{3}$. Write angle measures in degrees and radians.

Find the angle θ for $-90^\circ \leq \theta \leq 90^\circ$ with a tangent value of $\sqrt{3}$.

Because $\tan \theta = \frac{\sin \theta}{\cos \theta}$, use the unit circle to find a point where the ratio of sine to cosine is $\sqrt{3}$ to 1. Every special angle on the unit circle can be written as a value over 2. A point on the unit circle at which

$\sin \theta = \frac{\sqrt{3}}{2}$ and $\cos \theta = \frac{1}{2}$ has a tangent value of $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ or $\sqrt{3}$.

When $\theta = 60^\circ$, $\tan \theta = \sqrt{3}$. So, $\tan^{-1} \sqrt{3} = 60^\circ$ or $\frac{\pi}{3}$.

Inverse Trigonometric Functions

Example 1

Find each value. Write angle measures in degrees and radians.

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

3. $\arccos\left(-\frac{1}{2}\right)$

4. $\arctan \sqrt{3}$

5. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

6. $\tan^{-1}(-1)$

7. $\sin^{-1}\frac{\sqrt{2}}{2}$

8. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

9. $\arcsin 1$

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Graphing Other Trigonometric Functions

Example 1 Graph a Tangent Function with a Dilation

Find the period, asymptotes, x-intercepts, midline, and transformations of $y = \tan 3x$. Then graph the function.

Step 1 Analyze the function.

- For a tangent function in the form $y = a \tan bx$, $\frac{180^\circ}{|b|}$ represents the period. Because $b = 3$, the period is $\frac{180^\circ}{|3|}$ or 60° .
- The asymptotes occur at $x = \frac{(90 + 180n)^\circ}{|3|}$ or $(60n + 30)^\circ$, where n is an odd integer.

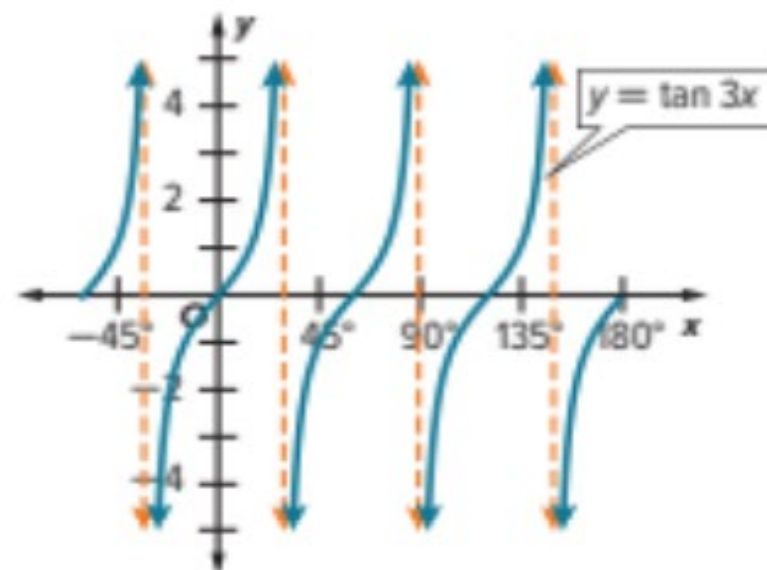
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Graphing Other Trigonometric Functions

- x -intercepts occur at integer multiples of the period. The period of this function is 60° , so the x -intercepts are 0° , 60° , 120° , 180° , ...
- The midline of $y = \tan 3x$ is $y = 0$.
- Because $a = 1$, the function is not vertically dilated in relation to the parent function. Because $b = 3$, the function is compressed horizontally in relation to the parent function.

Step 2 Graph the function.

Use the period, asymptotes, and x -intercepts to graph the function. Notice that $y = \tan 3x$ is compressed horizontally in relation to the parent function.



Example 2 Graph a Tangent Function with a Dilation and a Reflection

Graphing Other Trigonometric Functions

Find the period, asymptotes, x-intercepts, midline, and transformations of $y = -\frac{1}{3} \tan 2x$. Then graph the function.

Step 1 Analyze the function.

- The period is $\frac{\pi}{|2|}$ or $\frac{\pi}{2}$.
- The asymptotes occur at $\frac{\frac{\pi}{2} + \pi n}{|b|}$, where n is an odd integer. The asymptotes are at $x = \frac{\frac{\pi}{2} + \pi n}{|2|}$ or $\frac{\pi}{4} + \frac{\pi n}{2}$, where n is an odd integer.
- x-intercepts occur at integer multiples of the period. The period of this function is $\frac{\pi}{2}$, so the x-intercepts are $-\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots$
- The midline of $y = -\frac{1}{3} \tan 2x$ is $y = 0$.
- Because $a = -\frac{1}{3}$, the function is compressed vertically and reflected in the x-axis in relation to the parent function. Because $b = 2$, the function is compressed horizontally in relation to the parent function.

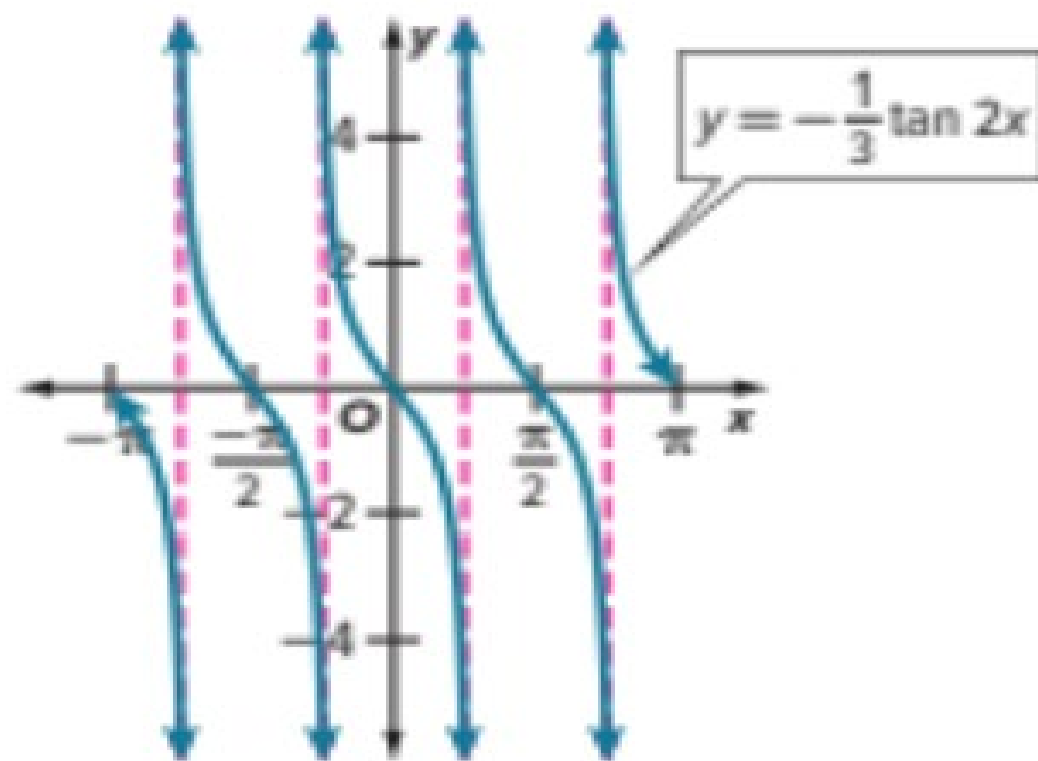
Graphing Other Trigonometric Functions

Step 2 Graph the function.

Use the period, asymptotes, and x -intercepts to graph the function.

Notice that $y = -\frac{1}{3} \tan 2x$ is

compressed vertically, compressed horizontally, and reflected in the x -axis in relation to the parent function.



Graphing Other Trigonometric Functions

Examples 1 and 2

Find the period, asymptotes, x -intercepts, midline, and transformations of each tangent function. Then graph the function.

1. $y = \tan 5x$

2. $y = \tan 4x$

3. $y = \tan 2x$

4. $y = \frac{1}{2} \tan x$

5. $y = 2 \tan \frac{1}{2}x$

6. $y = -\frac{1}{2} \tan 2x$

والحمد لله رب العالمين