

مدرسة زايد الخير / حلقة ثالثة

قسم الرياضيات

Grade 12 General

الوحدة الرابعة

Trigonometric

Functions

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تنويه: هذه الأوراق لا تغنيك عن الكتاب المدرسي وهي فقط لتخفيف أعباء نقل السؤال
عن السبورة

Student Name:..... Section:.....

4-1

Right Triangle Trigonometry

- OUTCOMES:**
- 1 Find values of trigonometric functions for acute angles of right triangles.
 - 2 Solve right triangles.

1 Values of Trigonometric Ratios

Key Concept Trigonometric Functions

Let θ be an acute angle in a right triangle and the abbreviations opp, adj, and hyp refer to the length of the side opposite θ , the length of the side adjacent to θ , and the length of the hypotenuse, respectively.



Then the six trigonometric functions of θ are defined as follows.

$$\text{sine } (\theta) = \sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosecant } (\theta) = \csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\text{cosine } (\theta) = \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{secant } (\theta) = \sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\text{tangent } (\theta) = \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\text{cotangent } (\theta) = \cot \theta = \frac{\text{adj}}{\text{opp}}$$

reciprocal functions

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

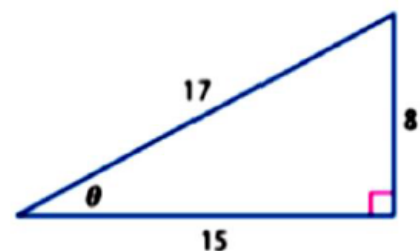
Example

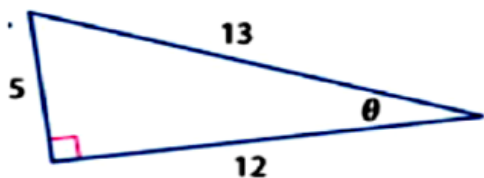
Find the exact values of the six trigonometric functions of θ .

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Example

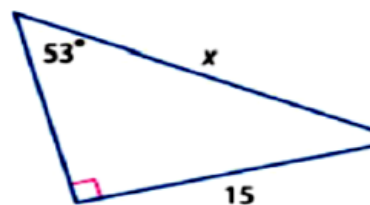
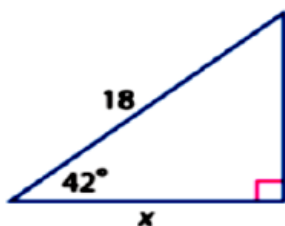
If $\cos \theta = \frac{2}{5}$, find the exact values of the five remaining trigonometric functions for the acute angle θ .

Example

If $\tan \theta = \frac{1}{2}$, find the exact values of the five remaining trigonometric functions for the acute angle θ .

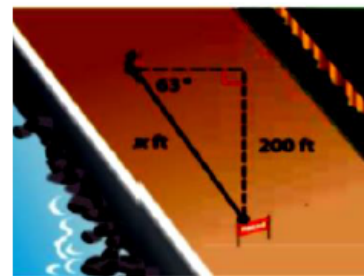
Example

Find the value of x . Round to the nearest tenth, if necessary.



Example

TRIATHLONS A competitor in a triathlon is running along the course shown. Determine the length in feet that the runner must cover to reach the finish line.



Key Concept Inverse Trigonometric Functions

Inverse Sine

If θ is an acute angle and the sine of θ is x , then the **inverse sine** of x is the measure of angle θ . That is, if $\sin \theta = x$, then $\sin^{-1} x = \theta$.

Inverse Cosine

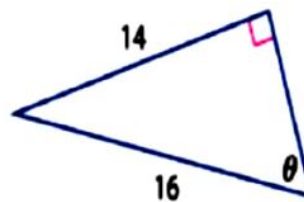
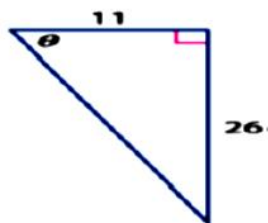
If θ is an acute angle and the cosine of θ is x , then the **inverse cosine** of x is the measure of angle θ . That is, if $\cos \theta = x$, then $\cos^{-1} x = \theta$.

Inverse Tangent

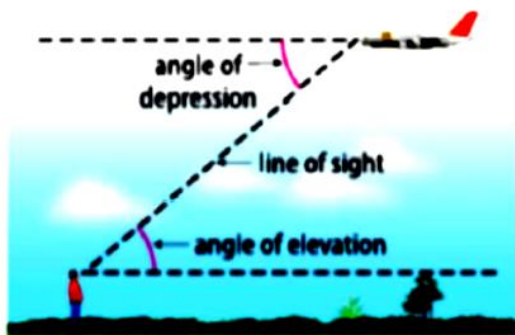
If θ is an acute angle and the tangent of θ is x , then the **inverse tangent** of x is the measure of angle θ . That is, if $\tan \theta = x$, then $\tan^{-1} x = \theta$.

Example

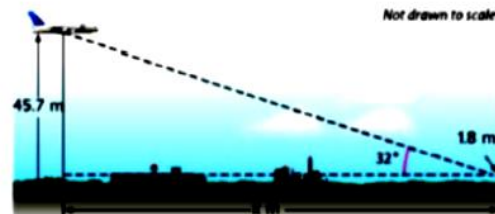
Use a trigonometric function to find the measure of θ . Round to the nearest degree, if necessary.



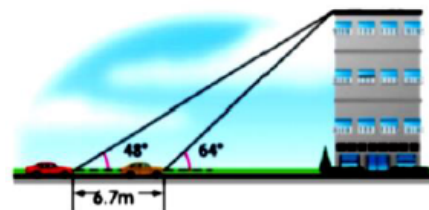
Some applications of trigonometry use an angle of elevation or depression. An **angle of elevation** is the angle formed by a horizontal line and an observer's line of sight to an object above. An **angle of depression** is the angle formed by a horizontal line and an observer's line of sight to an object below.

**Example**

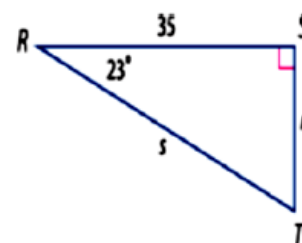
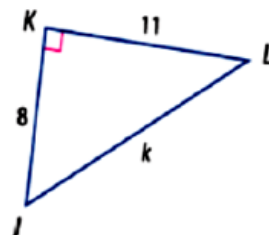
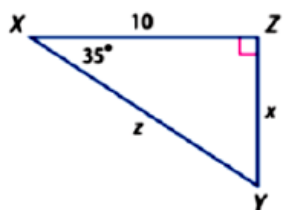
AIRPLANES A ground crew worker who is 1.8 meters tall is directing a plane on a runway. If the worker sights the plane at an angle of elevation of 32° , what is the horizontal distance from the worker to the plane?



Example **BUILDINGS** The angle of elevation from a car to the top of an apartment building is 48° . If the angle of elevation from another car that is 6.7 m directly in front of the first car is 64° , how tall is the building?



Example Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

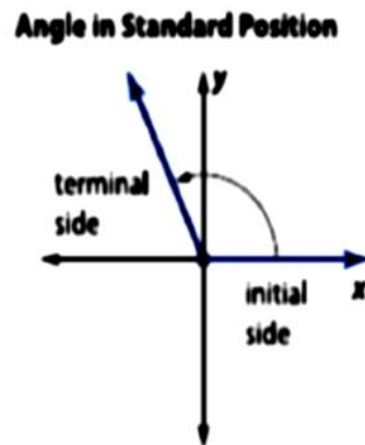
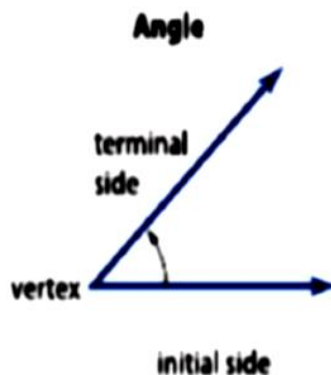


OUTCOMES:

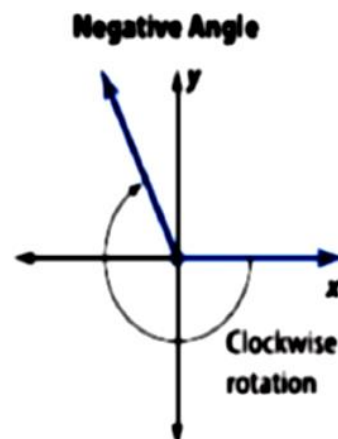
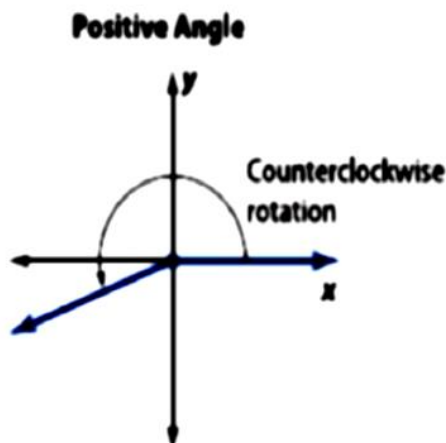
1 Convert degree measures of angles to radian measures, and vice versa.

2 Use angle measures to solve real-world problems.

1 Angles and Their Measures From geometry, you may recall an angle being defined as two noncollinear rays that share a common endpoint known as a **vertex**. An angle can also be thought of as being formed by the action of rotating a ray about its endpoint. From this dynamic perspective, the starting position of the ray forms the **initial side** of the angle, while the ray's position after rotation forms the angle's **terminal side**. In the coordinate plane, an angle with its vertex at the origin and its initial side along the positive x-axis is said to be in **standard position**.



The measure of an angle describes the amount and direction of rotation necessary to move from the initial side to the terminal side of the angle. A *positive angle* is generated by a counterclockwise rotation and a *negative angle* by a clockwise rotation.



Example Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

$$56.735^\circ$$

$$32^\circ 5' 28''$$

$$213.875^\circ$$

KeyConcept Radian Measure

Words

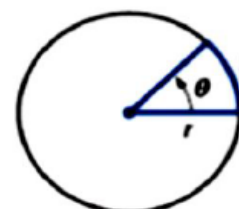
The measure θ in radians of a central angle of a circle is equal to the ratio of the length of the intercepted arc s to the radius r of the circle.

Symbols

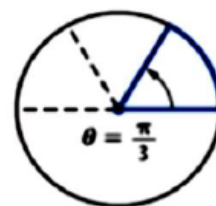
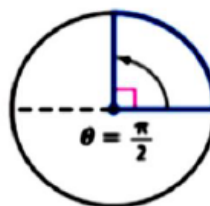
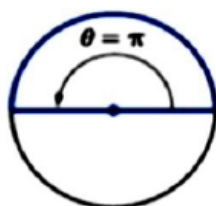
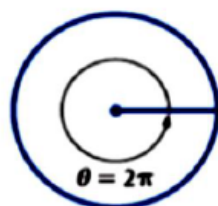
$\theta = \frac{s}{r}$, where θ is measured in radians (rad)

Example

A central angle has a measure of 1 radian if it intercepts an arc with the same length as the radius of the circle.



$\theta = 1$ radian when $s = r$.



KeyConcept Degree/Radian Conversion Rules

1. To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.
2. To convert a radian measure to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

Example Write each degree measure in radians as a multiple of π and each radian measure in degrees

$$120^\circ$$

$$-45^\circ$$

$$\frac{5\pi}{6}$$

$$-\frac{3\pi}{2}$$

$$210^\circ$$

$$-60^\circ$$

$$\frac{4\pi}{3}$$

$$-\frac{\pi}{6}$$

KeyConcept Coterminal Angles

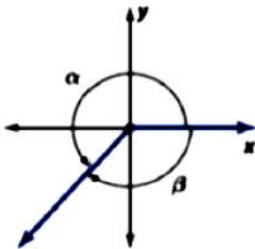
Degrees

If α is the degree measure of an angle, then all angles measuring $\alpha + 360n^\circ$, where n is an integer, are coterminal with α .

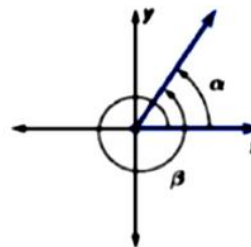
Radians

If α is the radian measure of an angle, then all angles measuring $\alpha + 2n\pi$, where n is an integer, are coterminal with α .

Positive and Negative Coterminal Angles



Positive Coterminal Angles



Example

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

$$45^\circ$$

$$-\frac{\pi}{3}$$

$$-30^\circ$$

$$\frac{3\pi}{4}$$

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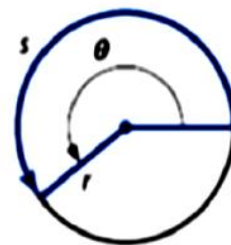
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KeyConcept Arc Length

If θ is a central angle in a circle of radius r , then the length of the intercepted arc s is given by

$$s = r\theta,$$

where θ is measured in radians.



Example

Find the length of the intercepted arc in each circle with the given central angle measure and radius. Round to the nearest tenth.

$$\frac{\pi}{4}, r = 5 \text{ cm}$$

$$60^\circ, r = 2 \text{ in}$$

$$\frac{2\pi}{3}, r = 2 \text{ m}$$

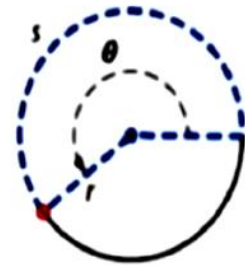
$$135^\circ, r = 0.5 \text{ ft}$$

KeyConcept Linear and Angular Speed

Suppose an object moves at a constant speed along a circular path of radius r .

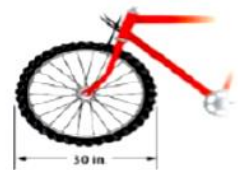
If s is the arc length traveled by the object during time t , then the object's *linear speed* v is given by $v = \frac{s}{t}$.

If θ is the angle of rotation (in radians) through which the object moves during time t , then the *angular speed* ω of the object is given by $\omega = \frac{\theta}{t}$.



Example **BICYCLING** A bicycle messenger rides the bicycle shown.

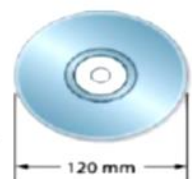
- a. During one delivery, the tires rotate at a rate of 140 revolutions per minute. Find the angular speed of the tire in radians per minute.



- b. On part of the trip to the next delivery, the tire turns at a constant rate of 2.5 revolutions per second. Find the linear speed of the tire in kilometers per hour.

Example **MEDIA** Consider the DVD shown.

- 5A. Find the angular speed of the DVD in radians per second if the disc rotates at a rate of 3.5 revolutions per second.
- 5B. If the DVD player overheats and the disc begins to rotate at a slower rate of 3 revolutions per second, find the disc's linear speed in meters per minute.



KeyConcept Area of a Sector

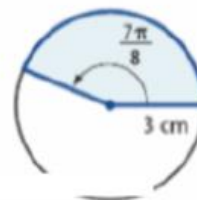
The area A of a sector of a circle with radius r and central angle θ is

$$A = \frac{1}{2}r^2\theta.$$

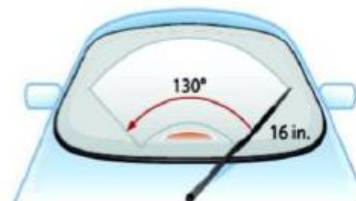
where θ is measured in radians.



Example a. Find the area of the sector of the circle.



b. WIPERS Find the approximate area swept by the wiper blade shown, if the total length of the windshield wiper mechanism is 26 inches.

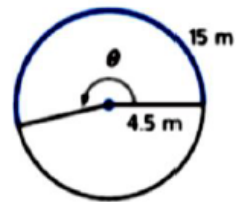
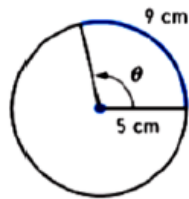


Example Find the area of the sector of a circle with the given central angle θ and radius r .

$$\theta = \frac{3\pi}{4}, r = 1.5 \text{ ft}$$

$$\theta = 50^\circ, r = 6 \text{ m}$$

Example Find the measure of angle θ in radians and degrees.



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4-3

Trigonometric Functions on the Unit Circle

OUTCOMES

1 Find values of trigonometric functions for any angle.

2 Find values of trigonometric functions using the unit circle.

Key Concept Trigonometric Functions of Any Angle

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ . Let r represent the nonzero distance from P to the origin.

That is, let $r = \sqrt{x^2 + y^2} \neq 0$. Then the trigonometric functions of θ are as follows.

$$\sin \theta = \frac{y}{r}$$

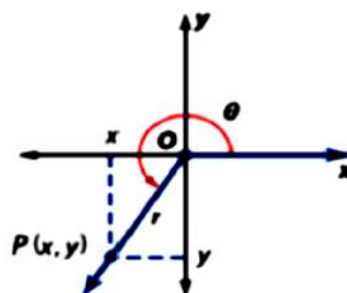
$$\csc \theta = \frac{r}{y}, y \neq 0$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}, x \neq 0$$

$$\tan \theta = \frac{y}{x}, x \neq 0$$

$$\cot \theta = \frac{x}{y}, y \neq 0$$



Example

Let $(8, -6)$ be a point on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ .

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The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ .

$(4, 3)$

$(-2, -1)$

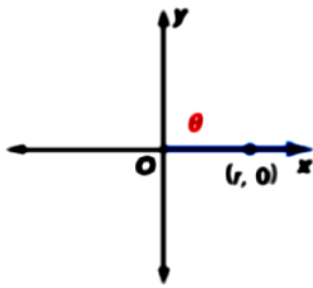
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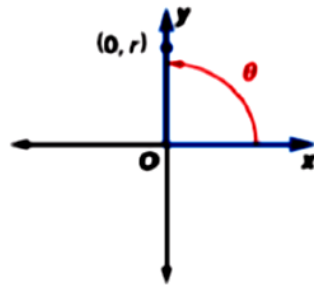
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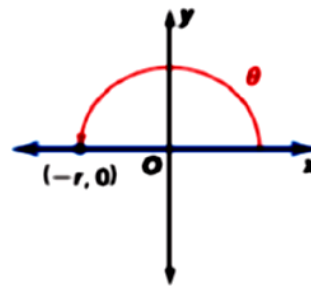
KeyConcept Common Quadrantal Angles



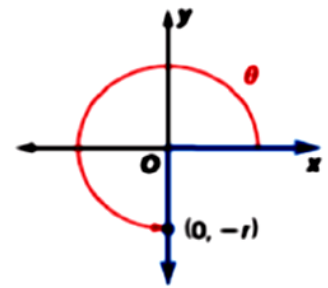
$$\theta = 0^\circ \text{ or } 0 \text{ radians}$$



$$\theta = 90^\circ \text{ or } \frac{\pi}{2} \text{ radians}$$



$$\theta = 180^\circ \text{ or } \pi \text{ radians}$$



$$\theta = 270^\circ \text{ or } \frac{3\pi}{2} \text{ radians}$$

Example

Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*.

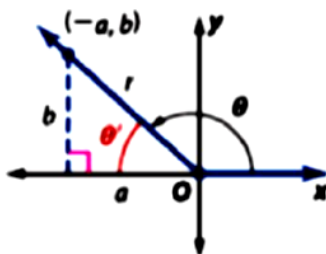
a. $\sin(-180^\circ)$

b. $\tan \frac{3\pi}{2}$

c. $\sec 4\pi$

$\cos 270^\circ$

Quadrant II



$$\sin \theta = \frac{b}{r}$$

$$\sin \theta' = \frac{b}{r}$$

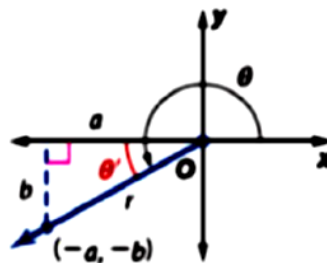
$$\cos \theta = -\frac{a}{r}$$

$$\cos \theta' = \frac{a}{r}$$

$$\tan \theta = -\frac{b}{a}$$

$$\tan \theta' = \frac{b}{a}$$

Quadrant III



$$\sin \theta = -\frac{b}{r}$$

$$\sin \theta' = \frac{b}{r}$$

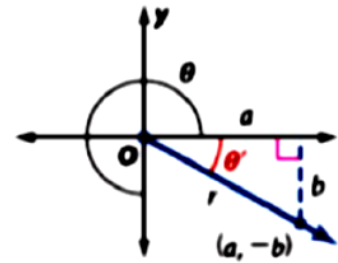
$$\cos \theta = -\frac{a}{r}$$

$$\cos \theta' = \frac{a}{r}$$

$$\tan \theta = \frac{b}{a}$$

$$\tan \theta' = \frac{b}{a}$$

Quadrant IV



$$\sin \theta = -\frac{b}{r}$$

$$\sin \theta' = \frac{b}{r}$$

$$\cos \theta = \frac{a}{r}$$

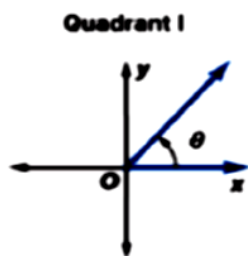
$$\cos \theta' = \frac{a}{r}$$

$$\tan \theta = -\frac{b}{a}$$

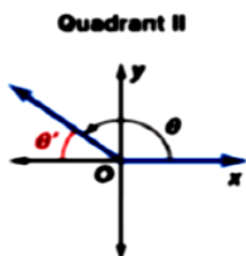
$$\tan \theta' = \frac{b}{a}$$

KeyConcept Reference Angle Rules

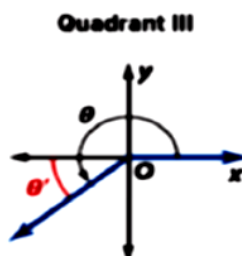
If θ is an angle in standard position, its reference angle θ' is the acute angle formed by the terminal side of θ and the x -axis. The reference angle θ' for any angle θ , $0^\circ < \theta < 360^\circ$ or $0 < \theta < 2\pi$, is defined as follows.



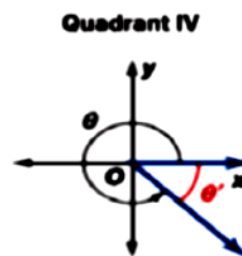
$$\theta' = \theta$$



$$\begin{aligned}\theta' &= 180^\circ - \theta \\ \theta' &= \pi - \theta\end{aligned}$$



$$\begin{aligned}\theta' &= \theta - 180^\circ \\ \theta' &= \theta - \pi\end{aligned}$$



$$\begin{aligned}\theta' &= 360^\circ - \theta \\ \theta' &= 2\pi - \theta\end{aligned}$$

Example Sketch each angle. Then find its reference angle.

A. $\frac{5\pi}{4}$

B. -240°

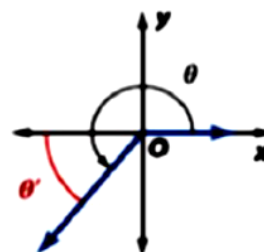
C. 390°

KeyConcept Evaluating Trigonometric Functions of Any Angle

Step 1 Find the reference angle θ' .

Step 2 Find the value of the trigonometric function for θ' .

Step 3 Using the quadrant in which the terminal side of θ lies, determine the sign of the trigonometric function value of θ .



Example Find the exact value of each expression.

a. $\cos 120^\circ$

b. $\tan \frac{7\pi}{6}$

c. $\csc \frac{15\pi}{4}$

A. $\tan \frac{5\pi}{3}$

B. $\sin \frac{5\pi}{6}$

C. $\sec(-135^\circ)$

Example Find the exact values of the five remaining trigonometric functions of θ .

Let $\tan \theta = \frac{5}{12}$, where $\sin \theta < 0$.

$\sec \theta = \sqrt{3}$, $\tan \theta < 0$

KeyConcept Trigonometric Functions on the Unit Circle

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$$\sin t = y$$

$$\cos t = x$$

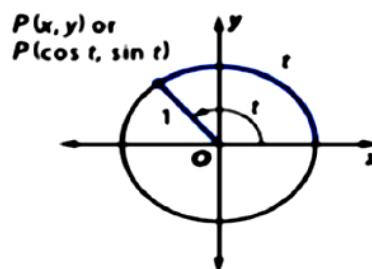
$$\tan t = \frac{y}{x}, x \neq 0$$

$$\csc t = \frac{1}{y}, y \neq 0$$

$$\sec t = \frac{1}{x}, x \neq 0$$

$$\cot t = \frac{x}{y}, y \neq 0$$

Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.



Example Find the exact value of each expression. If undefined, write *undefined*.

a. $\sin \frac{\pi}{3}$

b. $\cos 135^\circ$

c. $\tan 270^\circ$

A. $\cos \frac{\pi}{4}$

B. $\sin 120^\circ$

C. $\cot 210^\circ$

Key Concept Periodic Functions

A function $y = f(t)$ is periodic if there exists a positive real number c such that $f(t + c) = f(t)$ for all values of t in the domain of f .

The smallest number c for which f is periodic is called the **period** of f .

Example Find the exact value of each expression.

a. $\cos \frac{11\pi}{4}$

b. $\sin \left(-\frac{2\pi}{3} \right)$

c. $\tan \frac{19\pi}{6}$

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OUTCOMES

1 Graph transformations of the sine and cosine functions.

2 Use sinusoidal functions to solve problems.

Key Concept Properties of the Sine and Cosine Functions

Sine Function

Domain: $(-\infty, \infty)$ **Range:** $[-1, 1]$

y-intercept: 0

x-intercepts: $n\pi, n \in \mathbb{Z}$

Continuity: continuous on $(-\infty, \infty)$

Symmetry: origin (odd function)

Extrema: maximum of 1 at

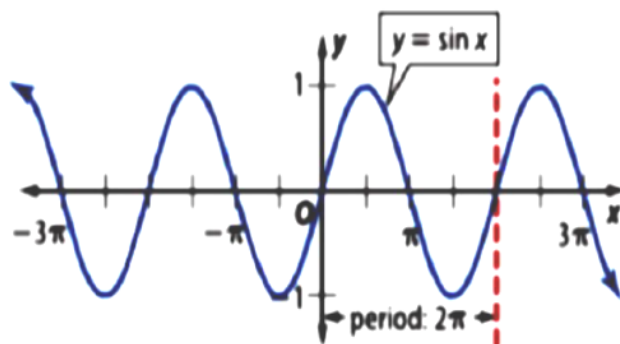
$$x = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$$

minimum of -1 at

$$x = \frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z}$$

End Behavior: $\lim_{x \rightarrow -\infty} \sin x$ and $\lim_{x \rightarrow \infty} \sin x$ do not exist.

Oscillation: between -1 and 1



Cosine Function

Domain: $(-\infty, \infty)$ **Range:** $[-1, 1]$

y-intercept: 1

x-intercepts: $\frac{\pi}{2}n, n \in \mathbb{Z}$

Continuity: continuous on $(-\infty, \infty)$

Symmetry: y-axis (even function)

Extrema: maximum of 1 at $x = 2n\pi,$

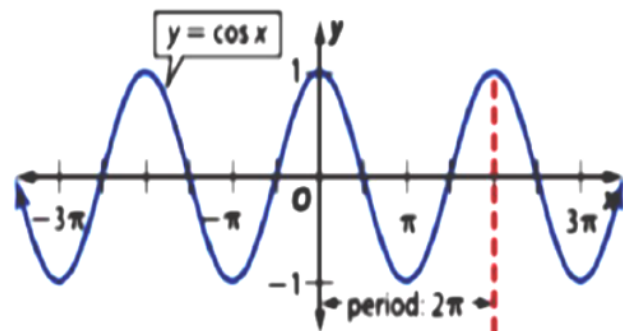
$$n \in \mathbb{Z}$$

minimum of -1 at $x = \pi + 2n\pi,$

$$n \in \mathbb{Z}$$

End Behavior: $\lim_{x \rightarrow -\infty} \cos x$ and $\lim_{x \rightarrow \infty} \cos x$ do not exist.

Oscillation: between -1 and 1



Example

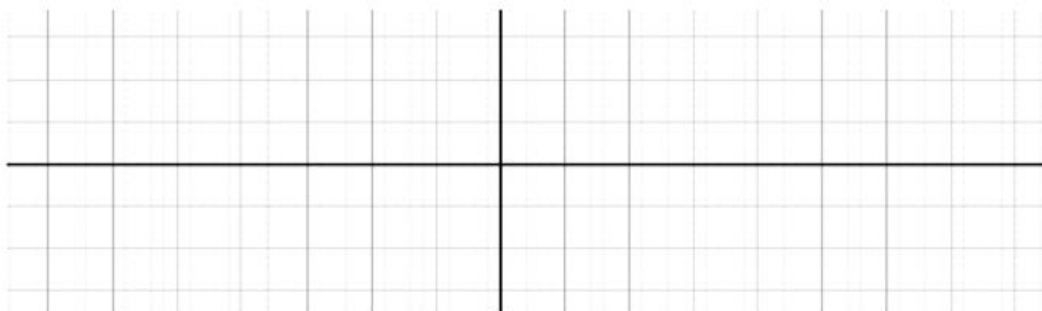
Describe how the graphs of $f(x)$ and $g(x)$ are related. Then find the amplitude of $g(x)$, and sketch two periods of both functions on the same coordinate axes.

$$f(x) = \sin x, g(x) = \frac{1}{4} \sin x$$



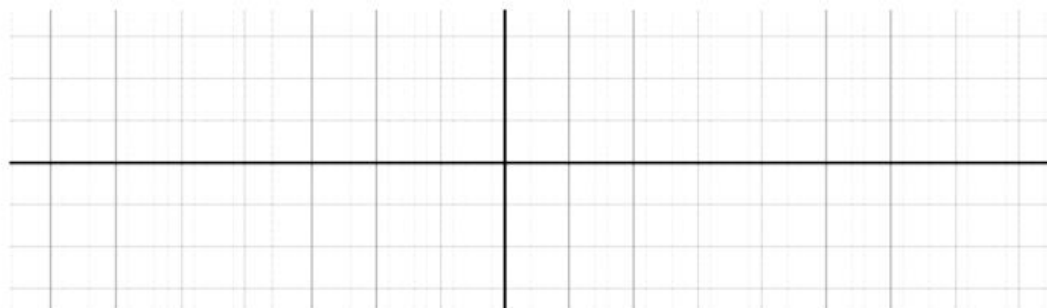
Example

$$f(x) = \cos x$$
$$g(x) = \frac{1}{3} \cos x$$



Example

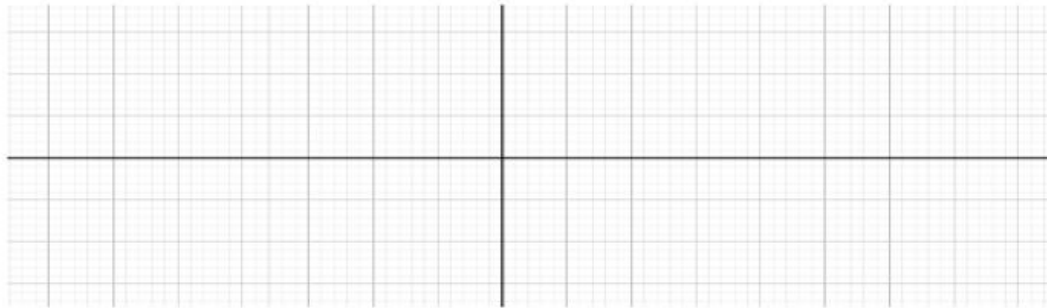
$$f(x) = \cos x, g(x) = -3 \cos x$$



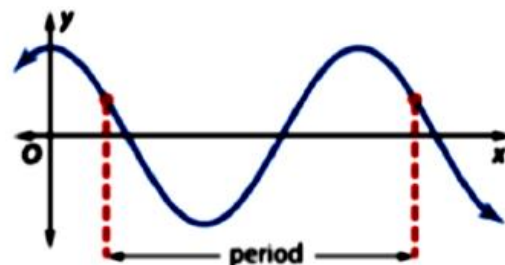
Example

$$f(x) = \sin x$$

$$g(x) = \sin 3x$$

**KeyConcept Periods of Sine and Cosine Functions****Words**

The period of a sinusoidal function is the distance between any two sets of repeating points on the graph of the function.

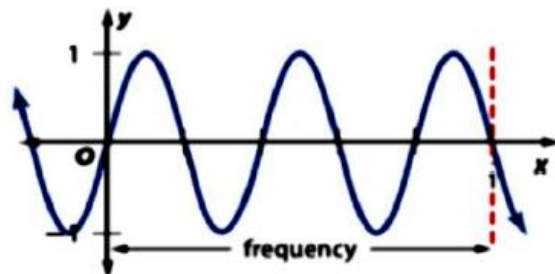
Model**Symbols**

For $y = a \sin (bx + c) + d$ and $y = a \cos (bx + c) + d$, where $b \neq 0$,

$$\text{period} = \frac{2\pi}{|b|}$$

KeyConcept Frequency of Sine and Cosine Functions**Words**

The **frequency** of a sinusoidal function is the number of cycles the function completes in a one unit interval. The frequency is the reciprocal of the period.

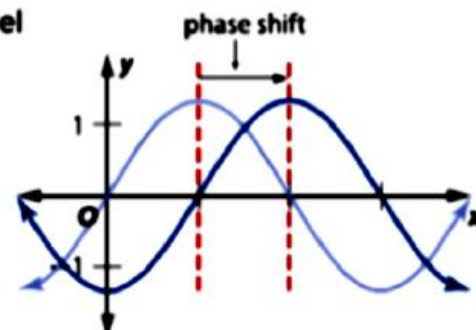
Model**Symbols**

For $y = a \sin (bx + c) + d$ and $y = a \cos (bx + c) + d$,

$$\text{frequency} = \frac{1}{\text{period}} \text{ or } \frac{|b|}{2\pi}$$

KeyConcept Phase Shift of Sine and Cosine Functions**Words**

The **phase shift** of a sinusoidal function is the difference between the horizontal position of the function and that of an otherwise similar sinusoidal function.

Model**Symbols**

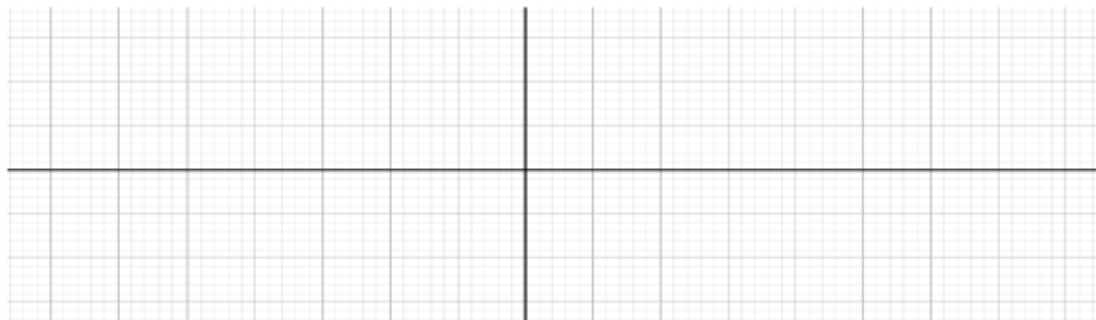
For $y = a \sin (bx + c) + d$ and $y = a \cos (bx + c) + d$, where $b \neq 0$,

$$\text{phase shift} = -\frac{c}{|b|}$$

Example

State the amplitude, period, frequency, and phase shift of each function. Then graph two periods of the function.

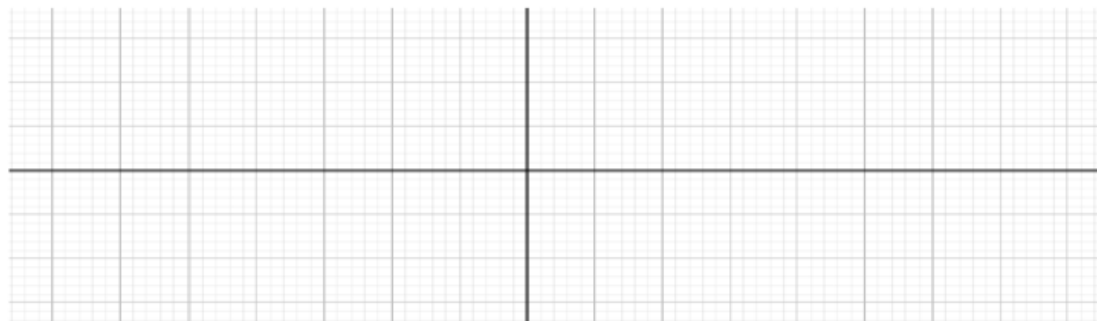
$$y = \sin \left(3x - \frac{\pi}{2} \right)$$



Example

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

$$y = \sin(x + 2\pi) - 1$$



ConceptSummary Graphs of Sinusoidal Functions

The graphs of $y = a \sin(bx + c) + d$ and $y = a \cos(bx + c) + d$, where $a \neq 0$ and $b \neq 0$, have the following characteristics.

Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Frequency: $\frac{|b|}{2\pi}$ or $\frac{1}{\text{Period}}$

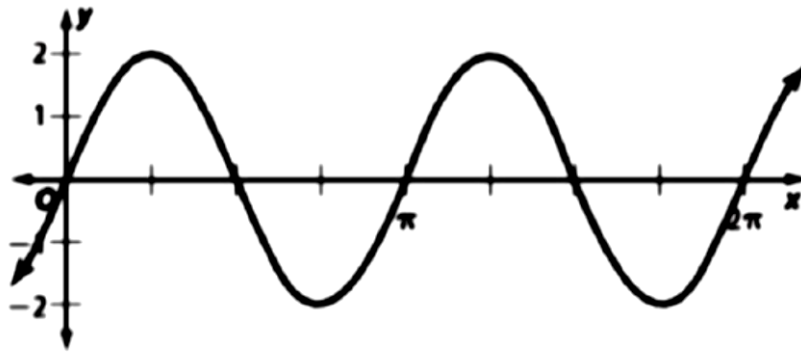
Phase shift: $-\frac{c}{|b|}$

Vertical shift: d

Midline: $y = d$

Example

Identify the equation represented by the graph.



A $y = \frac{1}{2} \sin 4x$

B $y = \frac{1}{4} \sin 2x$

C $y = 2 \sin 2x$

D $y = 4 \sin \frac{1}{2} x$

4-6

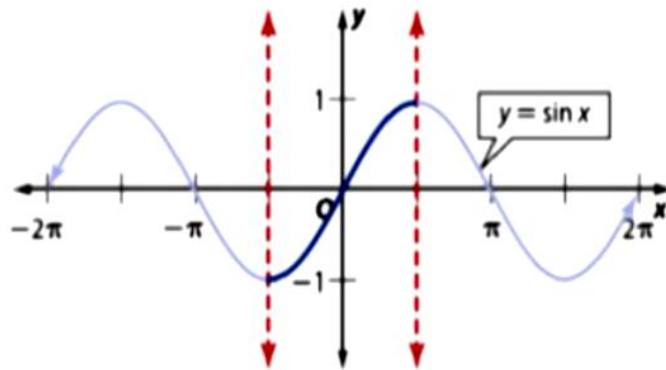
Inverse Trigonometric Functions

OUTCOMES:

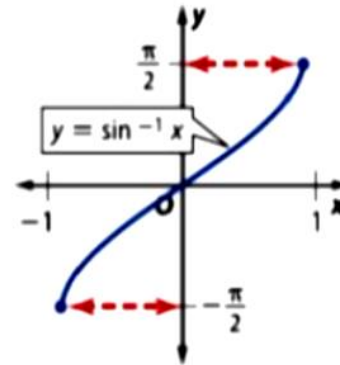
1 Evaluate and graph inverse trigonometric functions.

2 Find compositions of trigonometric functions.

Restricted Sine Function

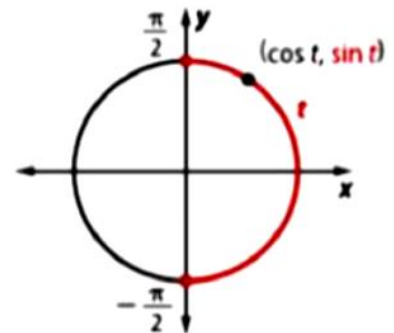


Inverse Sine Function



Recall that $\sin t$ is the y -coordinate of the point on the unit circle corresponding to the angle or arc length t . Because the range of the inverse sine function is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the possible angle measures of the inverse sine function are located on the right half of the unit circle, as shown.

Inverse Sine Values



Example Find the exact value of each expression, if it exists.

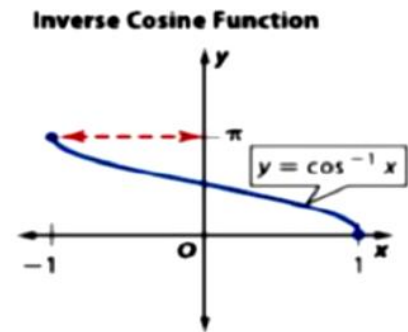
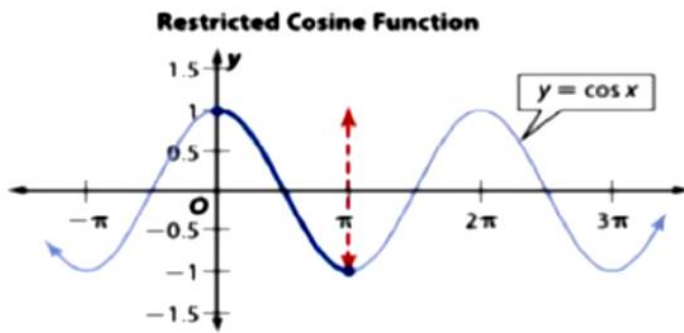
$$\sin^{-1} \frac{1}{2}$$

$$\arcsin \left(-\frac{\sqrt{2}}{2} \right)$$

$$\arcsin \left(\frac{\sqrt{3}}{2} \right)$$

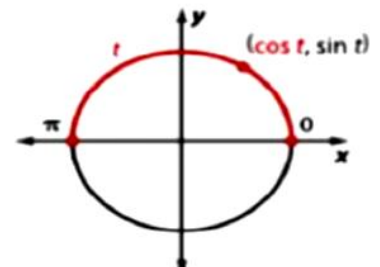
$$\sin^{-1} (-2\pi)$$

$$\arcsin (-1)$$



Recall that $\cos t$ is the x -coordinate of the point on the unit circle corresponding to the angle or arc length t . Because the range of $y = \cos^{-1} x$ is restricted to $[0, \pi]$, the values of an inverse cosine function are located on the upper half of the unit circle.

Inverse Cosine Values



Example Find the exact value of each expression, if it exists.

$$\cos^{-1} \left(-\frac{\sqrt{2}}{2} \right)$$

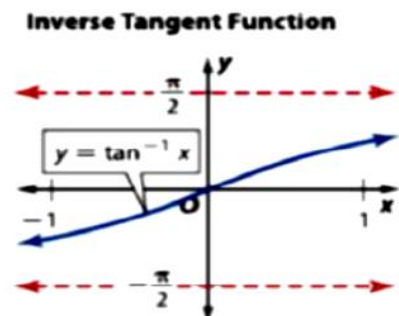
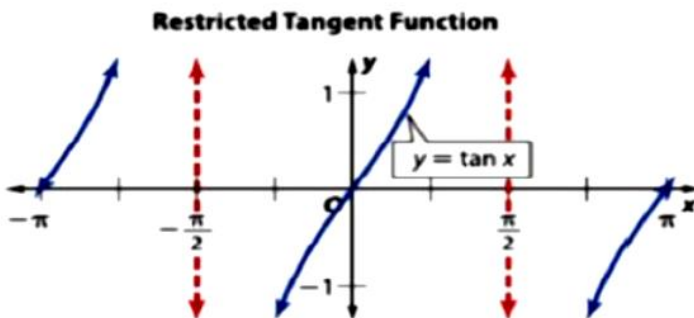
$$\cos^{-1} 0$$

$$\arccos(-2)$$

$$\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right)$$

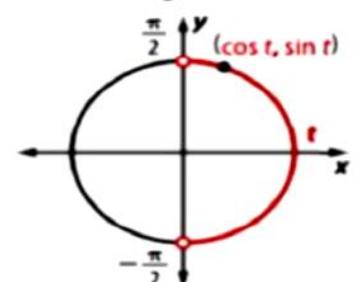
$$\arccos 2.5$$

$$\cos^{-1} \left(-\frac{1}{2} \right)$$



You can also use the unit circle to find the value of an inverse tangent expression. On the unit circle, $\tan t = \frac{\sin t}{\cos t}$ or $\frac{y}{x}$. The values of $y = \tan^{-1} x$ will be located on the right half of the unit circle, not including $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, because the tangent function is undefined at those points.

Inverse Tangent Values



Example Find the exact value of each expression, if it exists.

$$\tan^{-1} \sqrt{3}$$

$$\arctan 0$$

$$\arctan \left(-\frac{\sqrt{3}}{3} \right)$$

$$\tan^{-1} (-1)$$

KeyConcept Inverse Trigonometric Functions

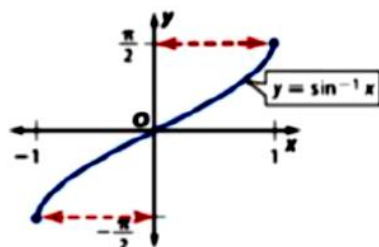
Inverse Sine of x

Words The angle (or arc) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ with a sine of x .

Symbols $y = \sin^{-1} x$ if and only if $\sin y = x$, for $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



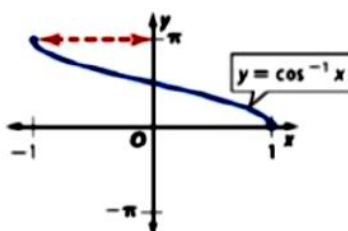
Inverse Cosine of x

Words The angle (or arc) between 0 and π with a cosine of x .

Symbols $y = \cos^{-1} x$ if and only if $\cos y = x$, for $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$.

Domain: $[-1, 1]$

Range: $[0, \pi]$



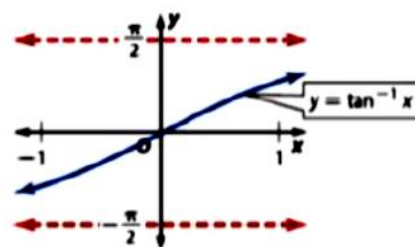
Inverse Tangent of x

Words The angle (or arc) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ with a tangent of x .

Symbols $y = \tan^{-1} x$ if and only if $\tan y = x$, for $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



KeyConcept Domain of Compositions of Trigonometric Functions

$$f[f^{-1}(x)] = x$$

$$f^{-1}[f(x)] = x$$

If $-1 \leq x \leq 1$, then $\sin(\sin^{-1} x) = x$.

If $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, then $\sin^{-1}(\sin x) = x$.

If $-1 \leq x \leq 1$, then $\cos(\cos^{-1} x) = x$.

If $0 \leq x \leq \pi$, then $\cos^{-1}(\cos x) = x$.

If $-\infty < x < \infty$, then $\tan(\tan^{-1} x) = x$.

If $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then $\tan^{-1}(\tan x) = x$.

Example Find the exact value of each expression, if it exists.

$$\sin \left[\sin^{-1} \left(-\frac{1}{4} \right) \right]$$

$$\arctan \left(\tan \frac{\pi}{2} \right)$$

$$\arcsin \left(\sin \frac{7\pi}{4} \right)$$

$$\tan \left(\tan^{-1} \frac{\pi}{3} \right)$$

$$\cos^{-1} \left(\cos \frac{3\pi}{4} \right)$$

$$\arcsin \left(\sin \frac{2\pi}{3} \right)$$

Example Find the exact value of $\cos \left[\tan^{-1} \left(-\frac{3}{4} \right) \right]$.

$$\cos^{-1} \left(\sin \frac{\pi}{3} \right)$$

Example Write each expression as an algebraic expression of x that does not involve trigonometric functions.

A. $\sin (\arccos x)$

B. $\cot [\sin^{-1} x]$

END OF UNIT 4

مع تمنياتي لكم بالتوفيق والنجاح
أ/ طارق معاينة