



المراجعة النهائية للتاني عشر المتقدم 2020/2019

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المدرسة: _____

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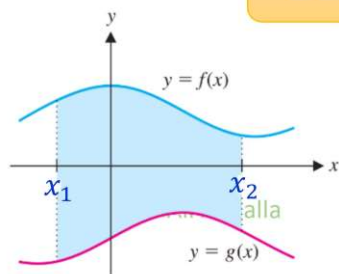
Final Review T3 - 2020

Grade 12 Advanced

وسيلة مساعدة للمراجعة
لا تغني عن الكتاب المدرسي

Unit 6: Application of Integration

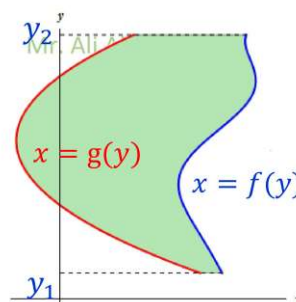
6.1 AREA BETWEEN CURVES



$$\text{Area} = \int_{x_1}^{x_2} (\text{upper function}) - (\text{lower function}) dx$$

$$\text{Area} = \int_{y_1}^{y_2} (\text{right function}) - (\text{left function}) dy$$

$$\int (f(x) - g(x)) dx$$



- 1) Find the area bounded by the graphs of $y = x^2$, $x = y^2$

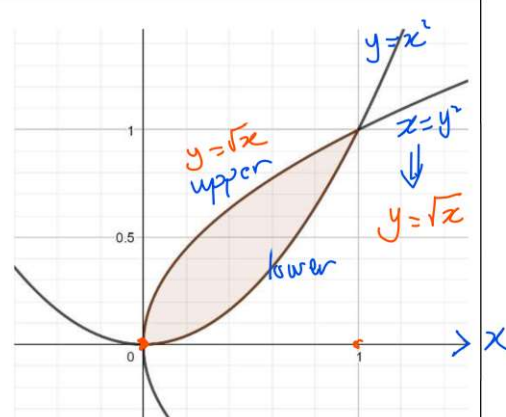
A) $A = \int_0^1 (x - x^2) dx$

B) $A = \int_0^1 (\sqrt{x} - x^2) dx$

C) $A = \int_0^1 (y^2 - y) dx$

D) $A = \pi \int_0^1 (\sqrt{x} - x^2) dx$

$$\int_0^1 (\sqrt{x} - x^2) dx$$



- 2) Find the area bounded by the graphs of $y = 4 - x^2$ and $y = 0$

A) $A = \int_0^2 (4 - x^2) dx$

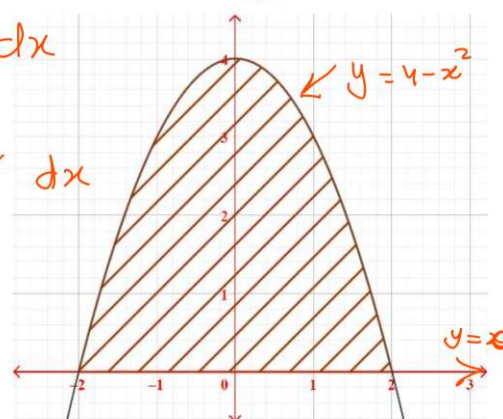
B) $A = \int_0^2 (x^2 - 4) dx$

C) $A = \int_{-2}^2 (4 - x^2) dx$

D) $A = \int_{-2}^2 (\sqrt{4 - y}) dx$

$$\int_{-2}^2 (\text{upper} - \text{lower}) dx$$

$$= \int_{-2}^2 (4 - x^2) dx$$



3) Find the area bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$

Choose two answers

A) $A = \int_0^1 ((2 - y) - \sqrt{y}) dy$

B) $A = \int_0^2 ((2 - y) - \sqrt{y}) dy$

C) $A = \int_0^2 ((2 - x) - x^2) dx$

D) $A = \int_0^1 x^2 dx + \int_1^2 (2 - x) dx$

Handwritten notes: $\int (Right - Left) dy$, $\int_0^1 (2 - y) - \sqrt{y} dy$, $A_1 = \int_0^1 x^2 dx$, $A_2 = \int_1^2 2 - x dx$, $A_1 + A_2$

4) In terms of A_1 , A_2 and A_3 , identify the area given the integral

$\int_0^2 (4 - x^2) dx$

A) A_1

B) $A_1 + A_2$

C) A_3

D) $A_2 + A_3$

Handwritten notes: $A_1 = \int_0^2 (4 - 2x) dx = \int_0^4 \frac{1}{2} y dy$, $A_2 = \int_0^2 (2x - x^2) dx = \int_0^4 \sqrt{y} - \frac{1}{2} y dy$, $A_3 = \int_0^2 x^2 dx = \int_0^4 (2 - \sqrt{y}) dy$

5) In terms of A_1 , A_2 and A_3 , identify the area given the integral

$\int_0^2 (2x - x^2) dx$

A) A_1

B) $A_1 + A_2$

C) A_2

D) A_3

Handwritten notes: $A_1 = \int_0^2 2x dx$, $A_2 = \int_0^2 x^2 dx$, $A_3 = \int_2^4 2x dx$

5)E find

a) $\int_0^4 (2 - \sqrt{y}) dy = A_3$

b) $\int_0^4 (\sqrt{y} - \frac{1}{2} y) dy = A_2$

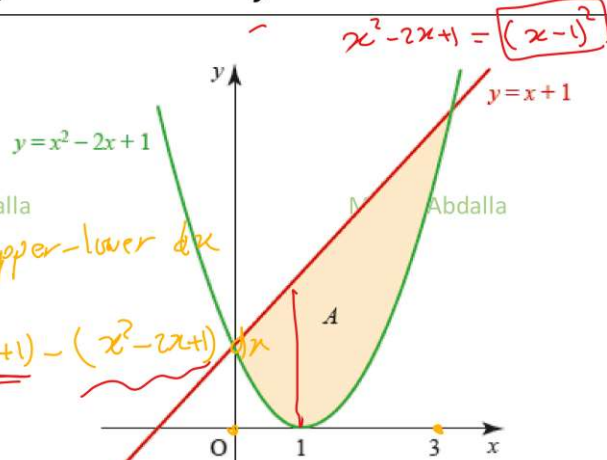
6) Find the area enclosed by the graphs of $y = x + 1$ and $y = x^2 - 2x + 1$

~~A)~~ $A = \int_0^3 ((x+1) - (x-1)^2) dy$

~~B)~~ $A = \int_0^3 ((1+y) - (\sqrt{y}+1)) dy$

~~C)~~ $A = \int_0^3 ((x-1)^2 - (x+1)) dx$

D) $A = \int_1^3 ((x+1) - (x-1)^2) dx$



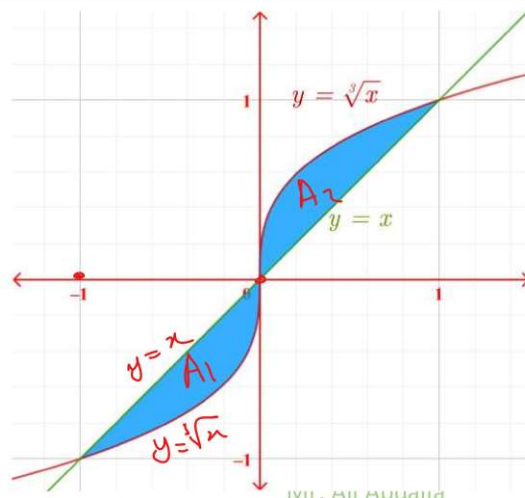
7) Find the area enclosed by the graphs of $g(x) = x$ and $f(x) = \sqrt[3]{x}$

A) $A = \int_{-1}^1 (\sqrt[3]{x} - x) dx$

B) $A = \int_{-1}^0 \sqrt[3]{x} dx - \int_0^1 x dx$

C) $A = \int_{-1}^0 (\sqrt[3]{x} - x) dx - \int_0^1 (x - \sqrt[3]{x}) dx$

~~D)~~ $A = \int_{-1}^0 (x - \sqrt[3]{x}) dx + \int_0^1 (\sqrt[3]{x} - x) dx$



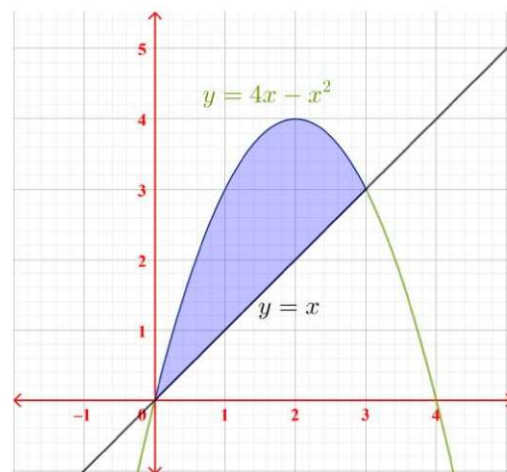
8) Find the area enclosed by the graphs of $y = x$ and $y = 4x - x^2$

A) $A = \int_0^4 (x - (4x - x^2)) dx$

B) $A = \int_0^4 ((4x - x^2) - x) dx$

C) $A = \int_0^4 (y - (\sqrt{4y - y^2})) dy$

~~D)~~ $A = \int_0^3 ((4x - x^2) - x) dx$



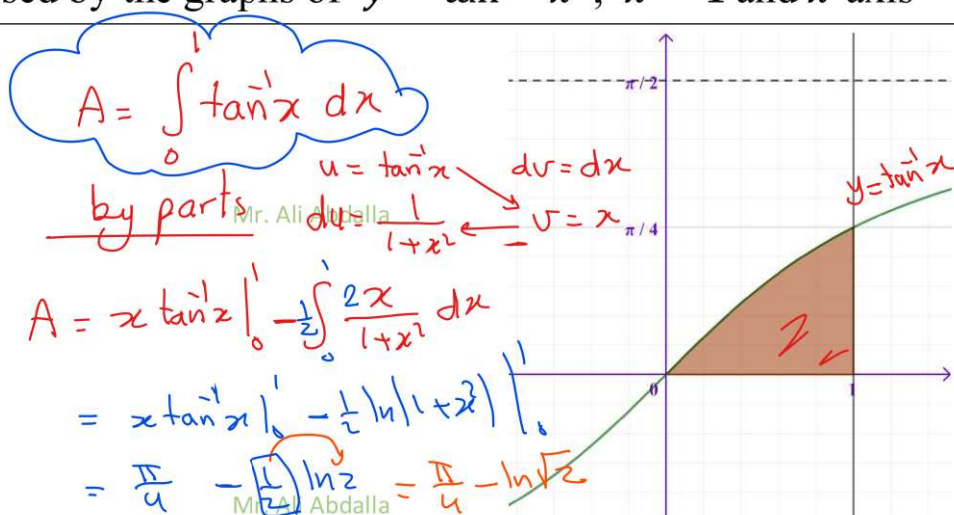
- 9) Find the area enclosed by the graphs of $y = \tan^{-1} x$, $x = 1$ and x -axis

A) $A = \frac{\pi}{4} - \ln \sqrt{2}$

B) $A = \frac{\pi}{4} + \ln \sqrt{2}$

C) $A = \frac{\pi}{2} - \ln \sqrt{2}$

D) $A = \frac{\pi}{4} - \ln 2$



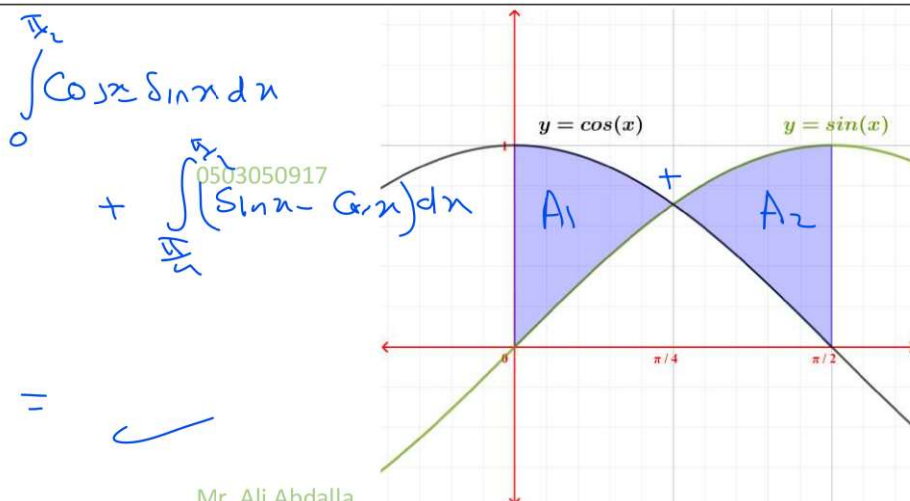
- 10) Find the area enclosed by the graphs of $y = x$ and $y = 4x - x^2$

A) $A = 2\sqrt{2} + 2$

B) $A = 2\sqrt{2} - 2$

C) $A = \sqrt{2} + 2$

D) $A = \sqrt{2} - 2$



- 11) Graph and find the area of the region determined by the intersections of the curves. $y = x$, $y = 6 - x$, $y = 2$, $y = 0$

A) $A = 4$

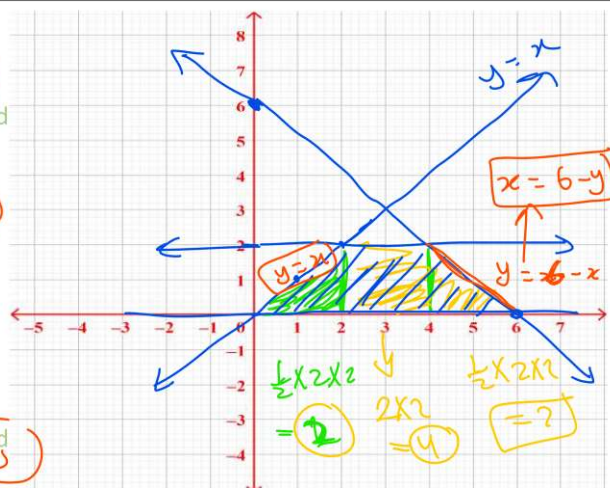
B) $A = 6$

C) $A = 8$

D) $A = 10$

Handwritten solution for Question 11:

$\int_0^2 (\text{Right} - \text{Left}) \, dy$
 $= \int_0^2 (6-y) - y \, dy$
 $= \int_0^2 6-2y \, dy$
 $= 6y - y^2 \Big|_0^2 = 8$



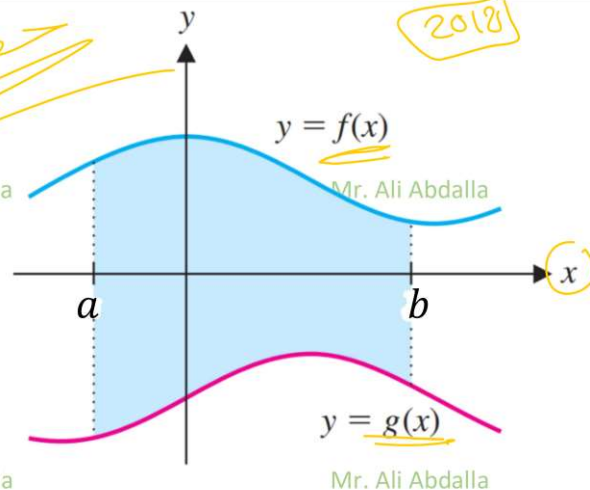
12) Find the area enclosed by two curves in the following graph

☒ A) $A = \int_a^b [f(x) - g(x)] dx$

☐ B) $A = \int_a^b [g(x) - f(x)] dx$

☐ C) $A = \int_a^b f(x) dx$

☐ D) $A = \int_a^b g(x) dx$



13) Find the area enclosed by the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, $1 \leq x \leq 2$

☒ A) $A = \int_1^2 \left[\frac{1}{x^2} - \frac{1}{x} \right] dx$

☒ B) $A = \ln 2 - \frac{1}{2}$

☒ C) $A = \ln 2 + \frac{1}{2}$

☒ D) $A = 2 \ln 2 + 1$

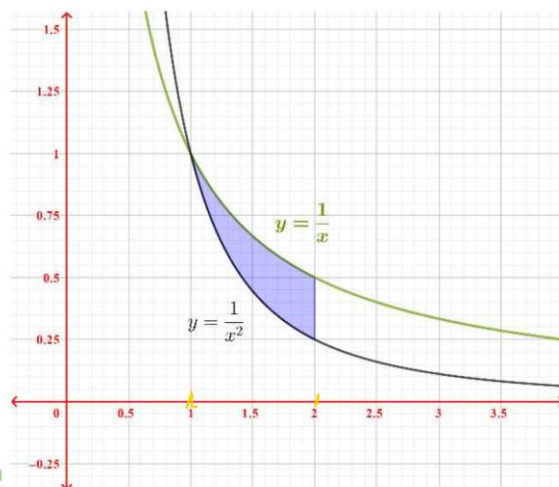
Handwritten work for Question 13:

$$\int_1^2 \left(\frac{1}{x^2} - \frac{1}{x} \right) dx$$

$$= \left(-\frac{1}{x} - \ln x \right) \Big|_1^2$$

$$= \left(-\frac{1}{2} - \ln 2 \right) - \left(-1 - 0 \right)$$

$$= -\frac{1}{2} - \ln 2 + 1 = \frac{1}{2} - \ln 2$$



14) Find the area of the region bounded by $y = \sqrt{x-1}$ and $x - 2y = 1$, $1 \leq x \leq 5$

☒ A) $A = \frac{4}{3}$

☐ B) $A = \frac{3}{4}$

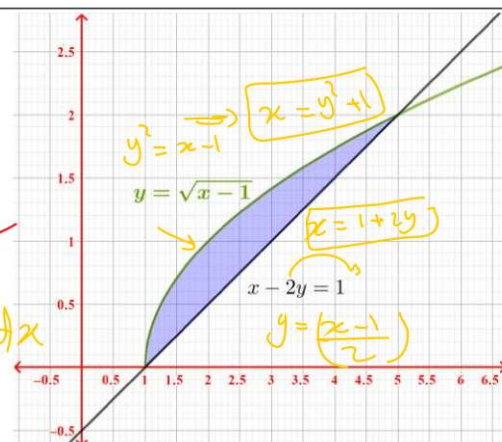
☐ C) $A = \frac{3}{2}$

☐ D) $A = \sqrt{3} - 2$

Handwritten work for Question 14:

$$\int_1^5 (\text{Right} - \text{Left}) dx$$

$$= \int_1^5 \left(\sqrt{x-1} - \left(\frac{x-1}{2} \right) \right) dx$$



Find area of the shaded area in each graph.

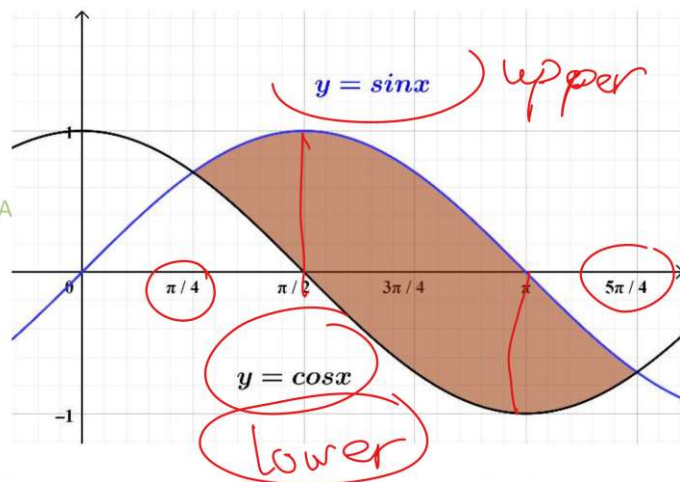
$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx$$

$$= -\cos x - \sin x \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$=$$

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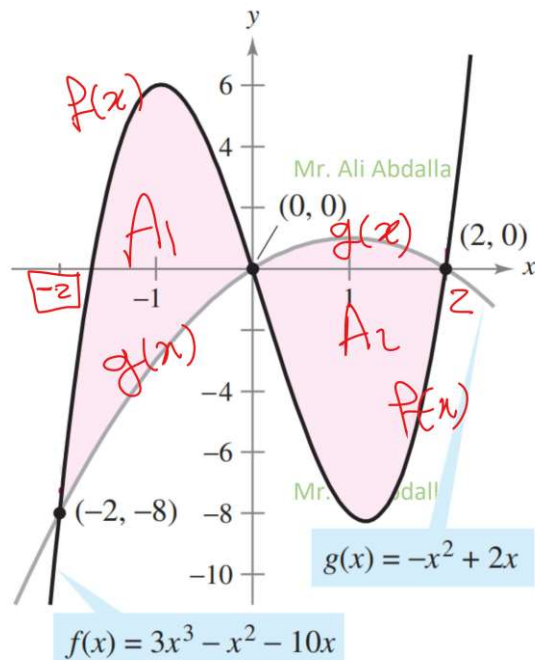
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5.2-3 VOLUME: SLICING, DISKS AND WASHERS

1-Volumes by Slicing

If a solid has a cross section with area $A(x)$ then the volume is:

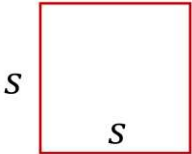
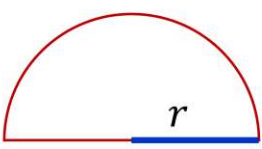
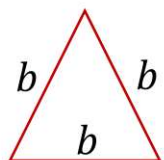
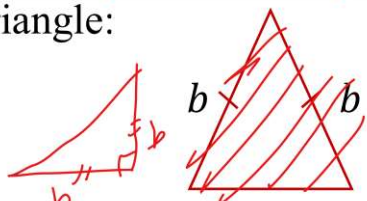
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$$V = \int_a^b A(x) dx$$

cross sectional Area

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Remember: 2D cross sections area:

Square: $A = s^2$ 	Rectangle: $A = \ell w$ 
Semicircle: $A = \frac{1}{2} \pi r^2$ 	Equilateral triangle: $A = \frac{\sqrt{3}}{4} b^2$ 
Right isosceles triangle: $A = \frac{1}{2} b^2$ 	

15) Find the volume of the solid with cross-sectional area $A(x) = \pi(4 - x)^2$, $0 \leq x \leq 2$

~~A) $V = \frac{56}{3} \pi$~~

B) $V = \frac{72}{3} \pi$

C) $V = \frac{56}{3}$

D) $V = 56\pi$

$$\begin{aligned}
 V &= \int_0^2 A(x) dx \\
 &= \int_0^2 \pi(4-x)^2 dx \\
 &= \pi \int_0^2 (4-x)^2 dx
 \end{aligned}$$

16) Find the volume of the solid with cross-sectional area $A(x) = x + 2$, $-1 \leq x \leq 3$

A) $V = \frac{124}{3}$

~~B) $V = 12\pi$~~

C) $V = 12$

D) $V = 10$

$$V = \int_a^b A(x) dx$$

- 17) Find the volume of the solid with cross-sectional area $A(x) = 10e^{0.01x}$, $0 \leq x \leq 10$

A) $V = 1000(e^{0.1} - 1)$

B) $V = 1000(e^{0.1} + 1)$

C) $V = 1000(e^{10} - 1)$

D) $V = 1000(e^1 - 1)$

$$V = \int_0^{10} 10e^{0.01x} dx$$

$$1000 e^{0.01x} \Big|_0^{10} = 10 \frac{e^{0.01x}}{0.01} \Big|_0^{10} = 1000(e^{0.1} - 1)$$

- 18) Use the slicing method to find the volume of the solid whose base is the area between $x = y^2$ and $x = 9$ whose slices perpendicular to the x -axis are semicircles.

A) $V = \frac{81}{4}\pi$

B) $V = \frac{81}{2}\pi$

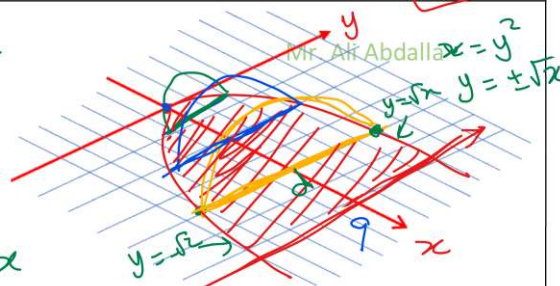
C) $V = 9\pi$

D) $V = \frac{81}{4}$

$$d = \sqrt{x} - (-\sqrt{x}) = 2\sqrt{x}$$

$$r = \frac{d}{2} = \frac{2\sqrt{x}}{2} = \sqrt{x}$$

$$A(x) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi (\sqrt{x})^2 = \frac{1}{2}\pi x$$



- 19) Use the slicing method to find the volume of the solid whose base is the area between $x = y^2$ and $x = 9$ and whose slices perpendicular to the x -axis are an equilateral triangle

A) $V = \frac{81\sqrt{3}}{2}$

B) $V = \frac{81\sqrt{3}}{4}$

C) $V = \frac{27}{2}$

D) $V = \frac{81}{2}$

$$d = \sqrt{x} - (-\sqrt{x}) = 2\sqrt{x}$$

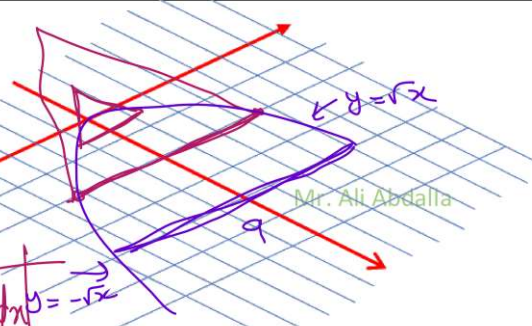
$$A(x) = \frac{\sqrt{3}}{4} b^2$$

$$= \frac{\sqrt{3}}{4} (2\sqrt{x})^2$$

$$= \sqrt{3} x$$

$$A(x) = \sqrt{3} x$$

$$V = \int_0^9 A(x) dx = \int_0^9 \sqrt{3} x dx = \frac{\sqrt{3}}{2} x^2 \Big|_0^9 = \frac{81\sqrt{3}}{2}$$



- 20) Use the given table of values to estimate the volume of the solid formed by revolving $y = f(x)$, $0 \leq x \leq 3$, about the x -axis.

x	0	0.5	1.0	1.5	2.0	2.5	3.0
$f(x)$	2.0	1.2	0.9	0.4	1.0	1.4	1.6

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A) $V = 4.07\pi$

B) $V = 40.7\pi$

C) $V = 4.07$

D) $V = 6.015\pi$

$$[f(x)]^2: 4, 1.44, 0.81, 0.16, 1, 1.96, 2.56$$

Simpson's

$$\frac{0.5}{3} \pi [4 + 4(1.44) + 2(0.81) + 4(0.16) + 2(1) + 4(1.96) + 2.56]$$

21)

Find the volume of a pyramid of height 160 feet that has a square base of side 300 feet.

☒ A) $V = 4,800,000 \text{ ft}^3$

B) $V = 4,000,000 \text{ ft}^3$

C) $V = 480,000 \text{ ft}^3$

D) $V = 24,000 \text{ ft}^3$

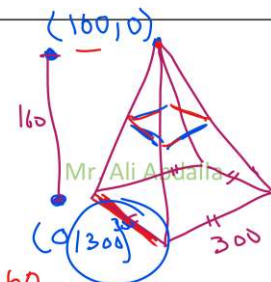
$$V = \frac{1}{3} B h$$

$$= \frac{1}{3} (300)^2 \cdot 160$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 300}{160 - 0} = \left[\frac{-15}{8} \right]$$

$$y = -\frac{15}{8}x + 300$$

$$V = \int_0^{160} \left(-\frac{15}{8}x + 300 \right)^2 dx$$



22)

Suppose that instead of completing a pyramid with a square base of side 750 feet, the builders had stopped at height 250 feet (with a square plateau top of side 375 feet). Compute the volume of this structure.

☒ A) $V = \int_0^{250} \left(-\frac{3}{2}x + 750 \right)^2 dx$

B) $V = \int_0^{250} \left(\frac{3}{2}x + 750 \right)^2 dx$

C) $V = \int_0^{250} \left(-\frac{2}{3}x + 750 \right)^2 dx$

D) $V = \int_0^{250} \left(-\frac{3}{2}x + 375 \right)^2 dx$

$$m = \frac{375 - 750}{250 - 0} = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + 750$$



23)

Use the slicing method to find the volume of the solid whose base is the area between $y = 2x$ and $y = x^2$ and whose slices perpendicular to the x -axis are semicircles.

A) $V = \frac{1}{6} \pi$

☒ B) $V = \frac{4}{15} \pi$

☒ C) $V = \frac{2}{15} \pi$

D) $V = \frac{4}{15}$

$$d = 2x - x^2$$

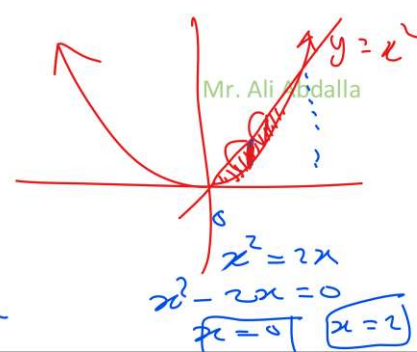
$$r = \frac{d}{2} = \frac{2x - x^2}{2}$$

$$A(x) = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \pi \left(\frac{2x - x^2}{2} \right)^2$$

$$V = \int_0^2 \frac{1}{8} \pi (2x - x^2)^2 dx$$

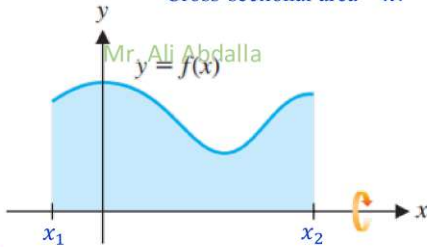
$$= \frac{2}{15} \pi$$



2- The Method of Disks: use one of the following integration:

$$V = \int_{x_1}^{x_2} \pi [f(x)]^2 dx$$

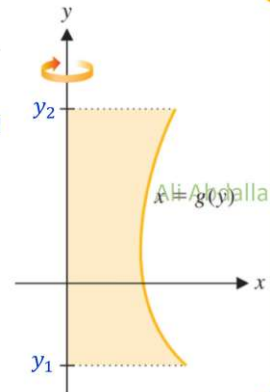
Cross-sectional area = πr^2



or

$$V = \int_{y_1}^{y_2} \pi [g(y)]^2 dy$$

Cross-sectional area = πr^2



3-The Method of Washers

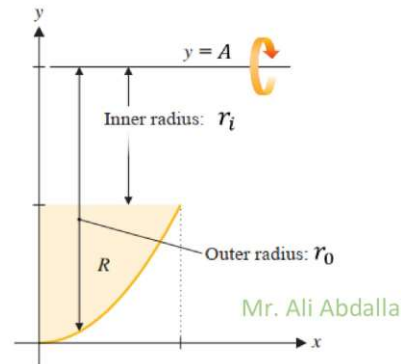
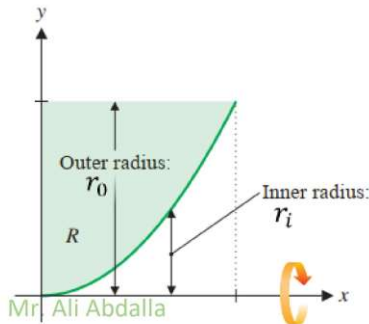
About x -axis ($y = 0$) or $y = A$

The volume of a solid resulting from revolving the region R about the x -axis or $y = A$ given by:

$$V = \int_a^b \pi (r_o^2 - r_i^2) dx$$

where r_o is the **outer** radius of the solid, and r_i is the **inner** radius of the solid

The radius must be as function of x



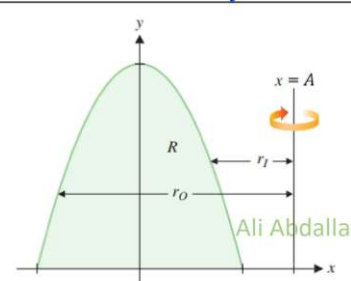
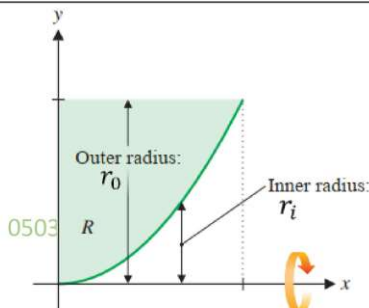
About y -axis ($x = 0$) or $x = A$

The volume of a solid resulting from revolving the region R about the y -axis or $x = A$ given by:

$$V = \int_a^b \pi (r_o^2 - r_i^2) dy$$

Where r_o is the **outer** radius of the solid, and r_i is the **inner** radius of the solid

The radius must be as function of y

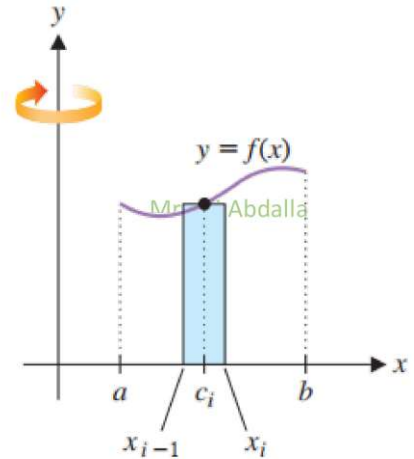


6-3 Volumes by Cylindrical Shells

We find volume using cylindrical shells using the following base:

$$V = \lim_{n \rightarrow \infty} \sum_{i=1}^n 2\pi c_i f(c_i) \Delta x = \int_a^b 2\pi x f(x) dx$$

Where x represent the radius
(distance between the shell and revolving axis)
and $f(x)$ is the height of the shell and **the shell must be parallel to the axis of revolution**



$$V = \int_a^b 2\pi r h dx \quad \text{Or} \quad V = \int_c^d 2\pi r h dy$$

Where r represent the radius (distance between the shell and revolving axis) and h is the height of the shell and **the shell must be parallel to the axis of revolution**

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24)

Let R be the region bounded by $y = \ln x$, $y = 0$, $y = 3$ and $x = 0$. Compute the volume of the solid formed by revolving R about **y-axis**.

A) $V = \pi \int_0^3 [\ln y]^2 dy$

B) $V = \pi \int_0^{e^3} [\ln y]^2 dy$

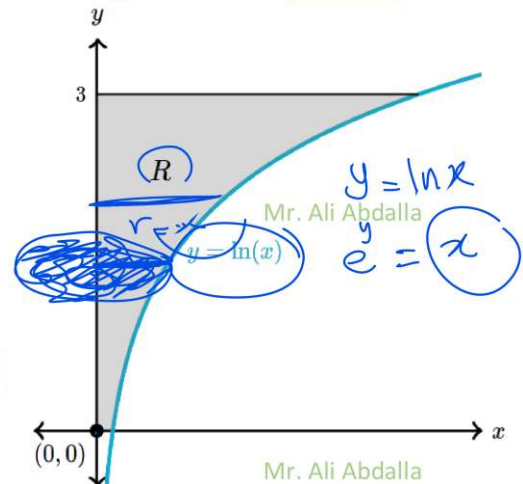
C) $V = \pi \int_0^3 e^{2y} dy$

D) $V = \pi \int_0^{\ln(3)} e^{2y} dy$

$$V = \pi \int_a^b r^2 dy$$

$$= \pi \int_0^3 (e^y)^2 dy$$

$$= \pi \int_0^3 e^{2y} dy$$



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25)

Find the volume of the solid obtained by rotating the region bounded by the curve $y = \sqrt{x+1}$ and the lines $y = 0$ and $x = 4$ about the x -axis.

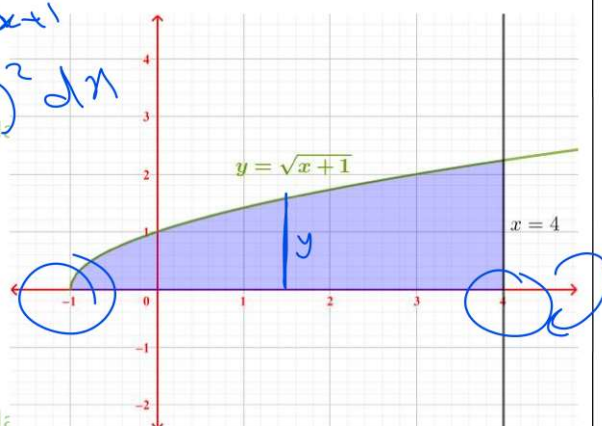
A) $V = \pi \int_{-1}^4 [x+1]^2 dx$

B) $V = \pi \int_{-1}^4 (x+1) dx$

C) $V = \pi \int_0^{\sqrt{5}} (y^2 - 1) dy$

D) $V = \pi \int_{-1}^4 \sqrt{x+1} dx$

Handwritten notes:
 u
 $r = y = \sqrt{x+1}$
 $\pi \int_{-1}^4 (\sqrt{x+1})^2 dx$



26)

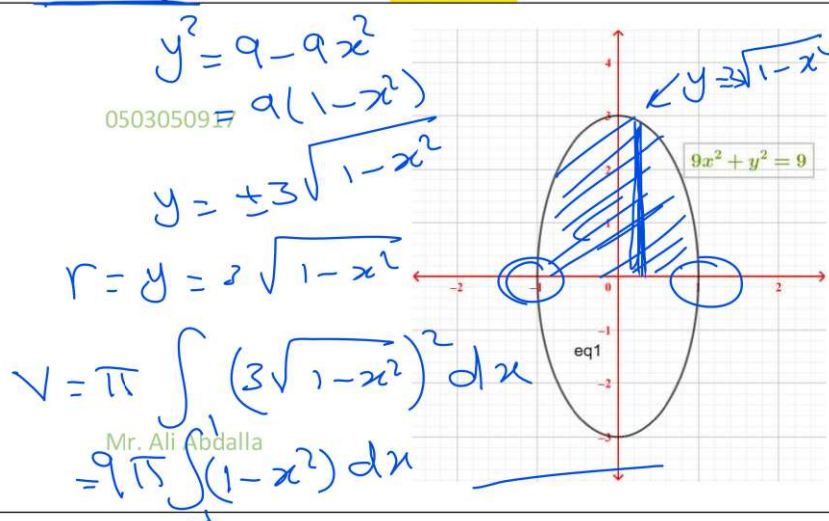
Set up an integral for the volume of the solid obtained by rotating the region bounded by the curve $9x^2 + y^2 = 9$ about x -axis

A) $V = 9\pi \int_{-1}^1 (1 - x^2) dx$

B) $V = 9\pi \int_{-1}^1 \sqrt{1 - x^2} dx$

C) $V = 9 \int_{-1}^1 (1 - x^2) dx$

D) $V = 3 \int_{-1}^1 (1 - x^2) dx$



27)

Let R be the region bounded by $y = x^2$ and $y = 4$. Which of the following gives the volume of the solid formed by revolving R about y -axis.

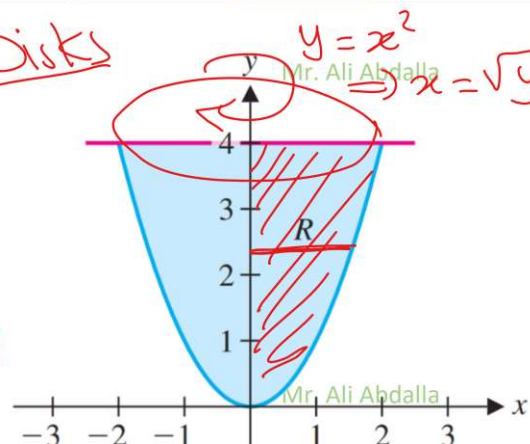
A) $V = \pi \int_{-2}^2 x^4 dx$

B) $V = \pi \int_{-1}^1 \sqrt{x} dy$

C) $V = \pi \int_0^4 y dy$

D) $V = \pi \int_0^2 y dy$

Handwritten notes:
 $r = x = \sqrt{y}$
 $V = \pi \int_0^4 (\sqrt{y})^2 dy$
 $= \pi \int_0^4 y dy$



28)

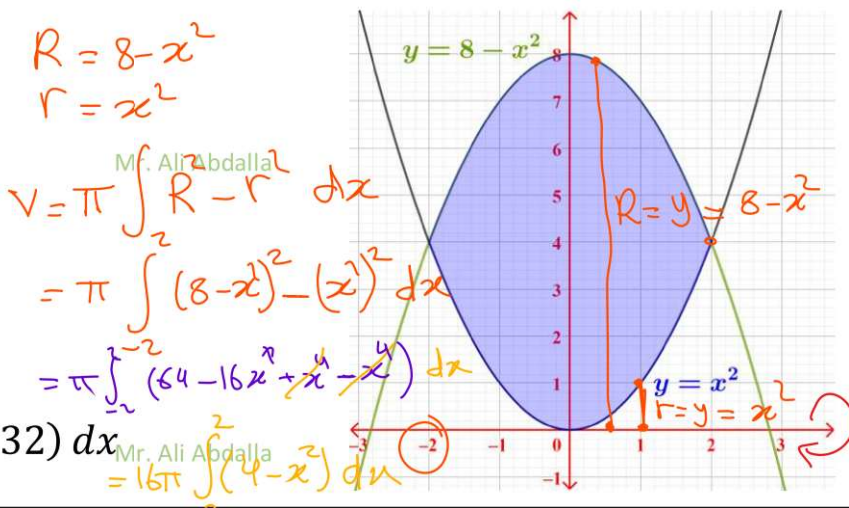
Let R be the region bounded by $y = x^2$ and $y = 8 - x^2$. Which of the following gives the volume of the solid formed by revolving R about x -axis.

A) $V = 16\pi \int_{-2}^2 (2 - x^2) dx$

B) $V = 16\pi \int_{-2}^2 (4 - x^2) dx$

C) $V = 16 \int_{-2}^2 (4 - x^2) dx$

D) $V = 32\pi \int_{-2}^2 (x^4 - 8x^2 + 32) dx$



29)

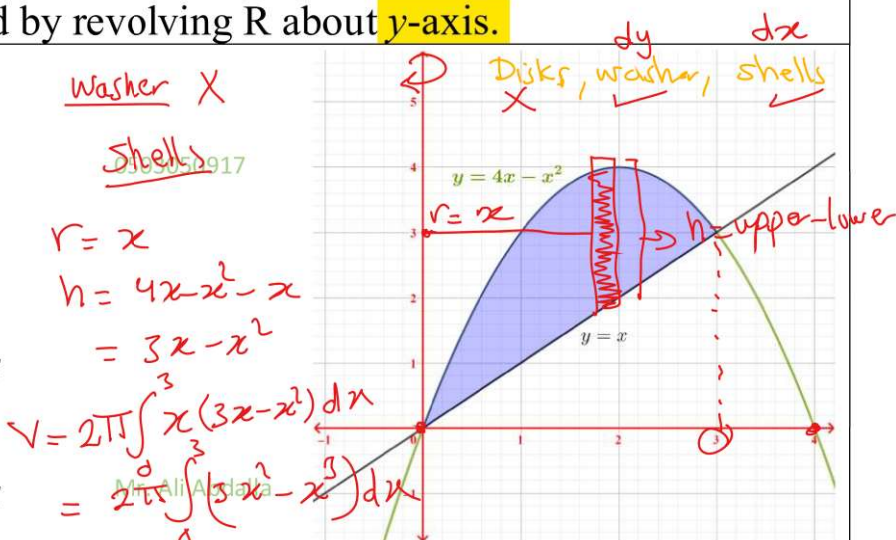
Let R be the region bounded by $y = 4x - x^2$ and $y = x$. Compute the volume of the solid formed by revolving R about y -axis.

A) $V = \pi \int_0^3 (x^3 - 3x^2) dx$

B) $V = \pi \int_0^3 (3x - x^2)^2 dx$

C) $V = 2\pi \int_0^3 (x^3 - 3x^2) dx$

D) $V = 2\pi \int_0^3 (3x^2 - x^3) dx$



30)

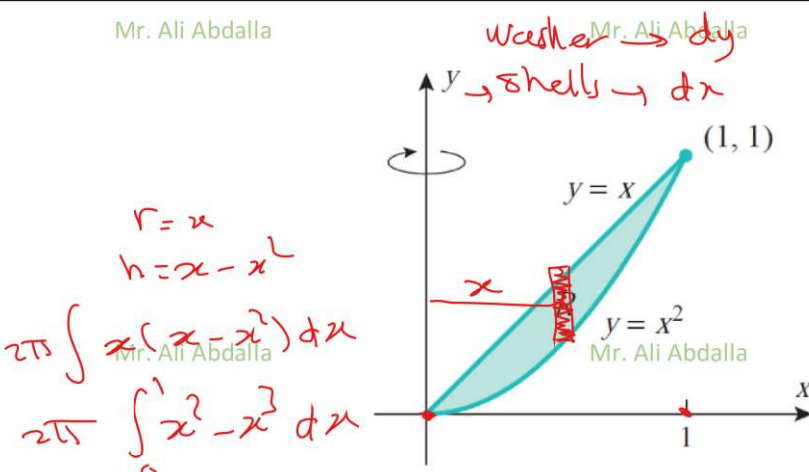
Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y -axis.

A) $V = \pi \int_0^1 (x^2 - x^3) dx$

B) $V = 2\pi \int_0^1 (x^2 - x^3) dx$

C) $V = 2\pi \int_0^1 (x^3 - x^2) dx$

D) $V = 2\pi \int_0^1 (x^3 + x^2) dx$



31)

Find the volume of the solid generated when the region R under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$

$y = A$ $\rightarrow$ z -axis

$\checkmark$ A) $V = 2\pi \int_0^4 (y+1)(2-\sqrt{y}) dy$

B) $V = 2\pi \int_0^4 (y-1)(2-\sqrt{y}) dy$

C) $V = 2\pi \int_0^4 (y+1)(2+\sqrt{y}) dy$

D) $V = 2\pi \int_0^4 (y-1)(2+\sqrt{y}) dy$

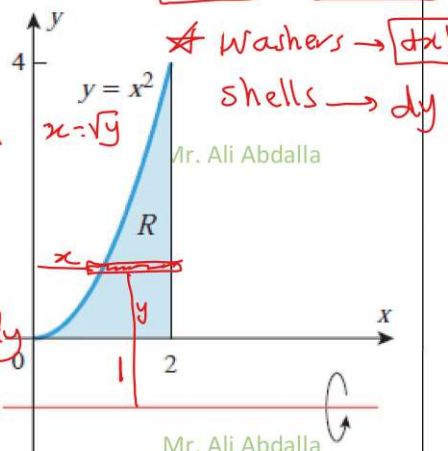
$r = y+1$

$h = \text{Right} - \text{Left}$

$= 2 - x$

$h = 2 - \sqrt{y}$

$2\pi \int_0^4 (y+1)(2-\sqrt{y}) dy$



Washers $\rightarrow dx$
Shells $\rightarrow dy$

32)

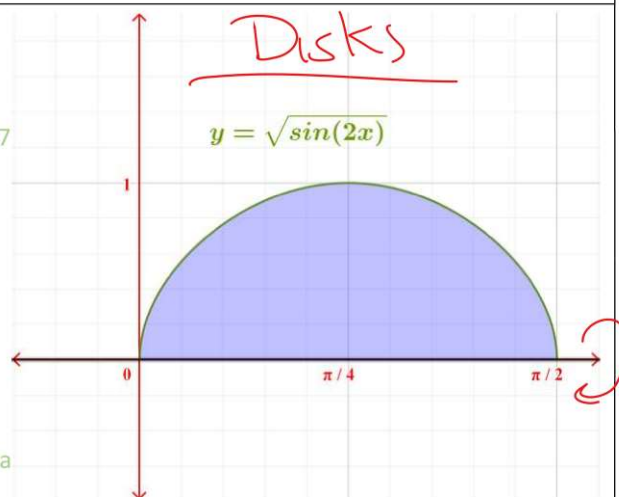
The graph of $y = \sqrt{\sin 2x}$, $0 \leq x \leq \frac{\pi}{2}$, is rotated 360° around x -axis. Find in terms of π , the volume of the solid generated

$\checkmark$ A) $V = \pi \int_0^{\frac{\pi}{2}} \sin 2x dx$ $\pi \int r^2 dx$

B) $V = \pi \int_0^{\frac{\pi}{2}} \sqrt{\sin 2x} dx$

C) $V = \pi \int_0^{\frac{\pi}{2}} (\sin 2x)^2 dx$

D) $V = 2\pi \int_0^{\frac{\pi}{2}} \sin 2x dx$



33)

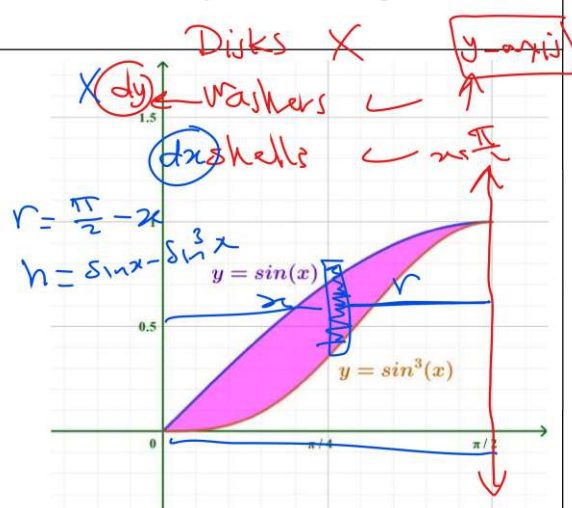
Let R be the region bounded by $y = \sin x$ and $y = \sin^3 x$. Find an expression for the volume of the solid formed by revolving R about the line $x = \pi/2$ and $0 \leq x \leq \pi/2$

A) $V = \pi \int_0^{\pi/2} \left(\frac{\pi^2}{4} - x^2 \right) (\sin^3 x - \sin x) dx$

$\checkmark$ B) $V = 2\pi \int_0^{\pi/2} \left(\frac{\pi}{2} - x \right) (\sin x - \sin^3 x) dx$

C) $V = \pi \int_0^{\pi/2} \left(\frac{\pi}{2} - x \right)^2 (\sin x - \sin^3 x) dx$

D) $V = 4\pi \int_0^{\pi/4} \left(\frac{\pi}{2} - x \right) (\sin^3 x - \sin x) dx$



34)

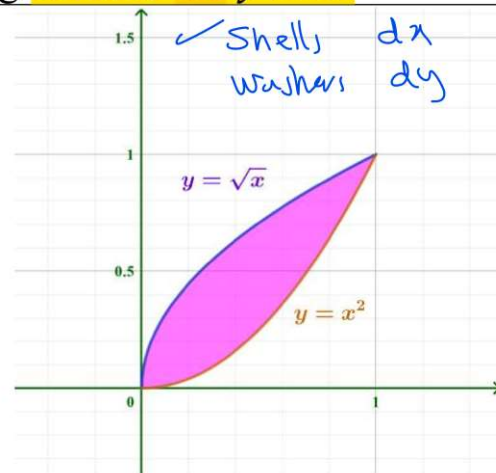
Let R be the region bounded by $y = \sqrt{x}$ and $y = x^2$. Find an expression for the volume of the solid formed by revolving R about the y-axis

A) $V = \pi \int_0^1 x^2 (\sqrt{x} - x^2) dx$

B) $V = \pi \int_0^1 x^2 (x^2 - \sqrt{x}) dx$

~~C) $V = 2\pi \int_0^1 x (\sqrt{x} - x^2) dx$~~

D) $V = 2\pi \int_0^1 x (x^2 - \sqrt{x}) dx$



35)

Find the volume of the solid obtained by rotating the region under the curve $y = 2x^2$ about the x-axis between $x = 1$ and $x = 2$

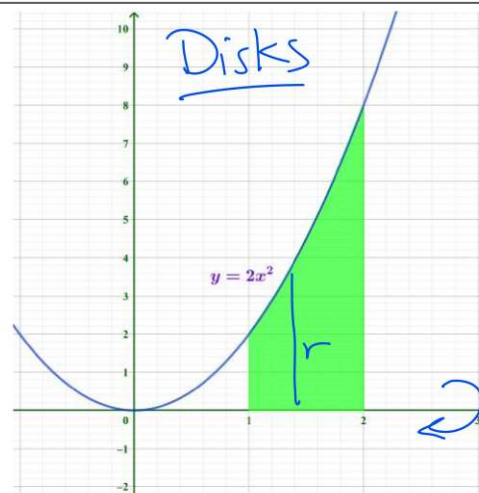
~~A) $V = \frac{124}{5} \pi$~~

B) $V = \frac{208}{5} \pi$

C) $V = \frac{32}{5} \pi$

D) $V = \frac{144}{5} \pi$

Handwritten calculations:
 $r = 2x^2$
 $\pi \int_1^2 (2x^2)^2 dx$
 $\pi \int_1^2 4x^4 dx$
 $= \frac{4\pi}{5} x^5 \Big|_1^2$



36)

Find the volume of the solid obtained by rotating the region under the curve $y = 3x - x^2$ and x-axis about the y-axis

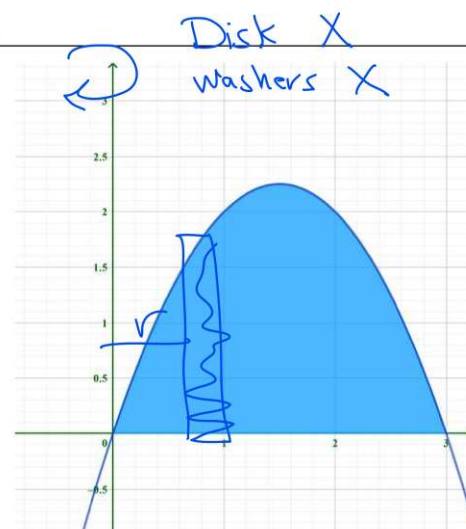
A) $V = 9\pi$

B) $V = \frac{81}{10} \pi$

~~C) $V = \frac{27}{2} \pi$~~

D) $V = \frac{27}{4} \pi$

Handwritten calculations:
Shells
 $r = x$
 $h = 3x - x^2$
 $2\pi \int_0^3 x (3x - x^2) dx$



37)

Triangle ABC , $A(0, h)$, $B(r, h)$, $C(0, 0)$ Find an expression for the volume of the solid resulting by rotating the triangle ABC about the y -axis

A) $V = \frac{\pi r^2}{h^2} \int_0^r x^2 dx$

B) $V = \frac{\pi r^2}{h^2} \int_0^h x^2 dx$

C) $V = \frac{\pi r^2}{h^2} \int_0^r y^2 dy$

D) $V = \frac{\pi r^2}{h^2} \int_0^h y^2 dy$

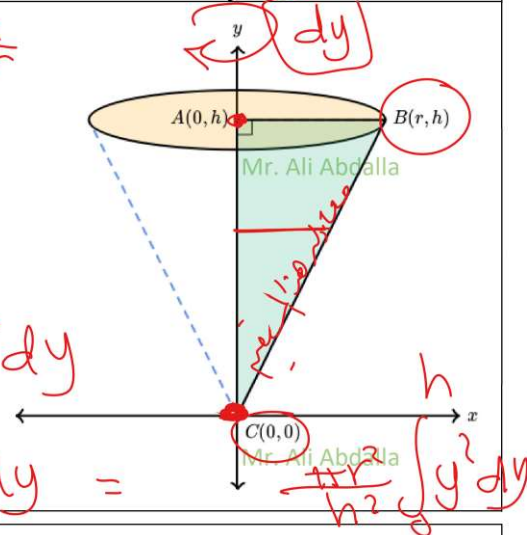
$$m = \frac{h-0}{r-0} = \frac{h}{r}$$

$$y = \frac{h}{r}x$$

$$x = \frac{r}{h}y$$

$$V = \pi \int (g(y))^2 dy$$

$$= \pi \int_0^h \frac{r^2}{h^2} y^2 dy = \frac{\pi r^2}{h^2} \int_0^h y^2 dy$$



38)

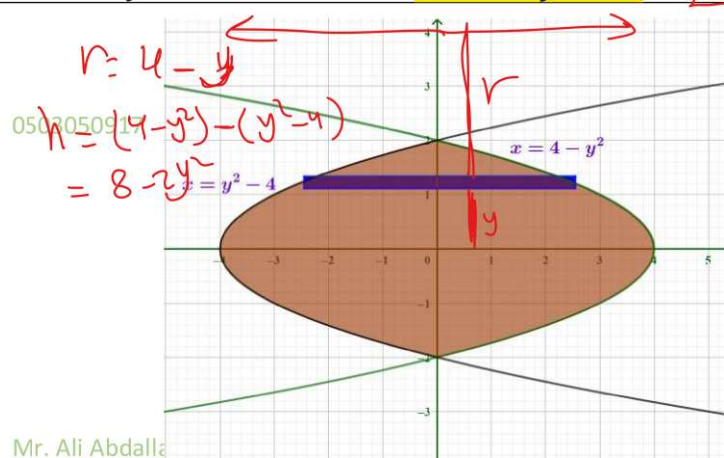
Find an expression for the volume of the indicated solid of revolution. The region bounded by $x = 4 - y^2$ and $x = y^2 - 4$ revolved about $y = 4$

A) $V = 2\pi \int_{-2}^2 (4 - y)(4 - y^2) dy$

B) $V = 4\pi \int_{-2}^2 (4 - y)(4 - y^2) dy$

C) $V = 2\pi \int_{-2}^2 (y - 4)(4 - y^2) dy$

D) $V = 4\pi \int_{-2}^2 (y - 4)(4 - y^2) dy$



39)

Suppose that the triangle with vertices $(-1, -1)$, $(0, 1)$ and $(1, -1)$ is revolved about the y -axis. Find an expression for the volume of the resulting solid

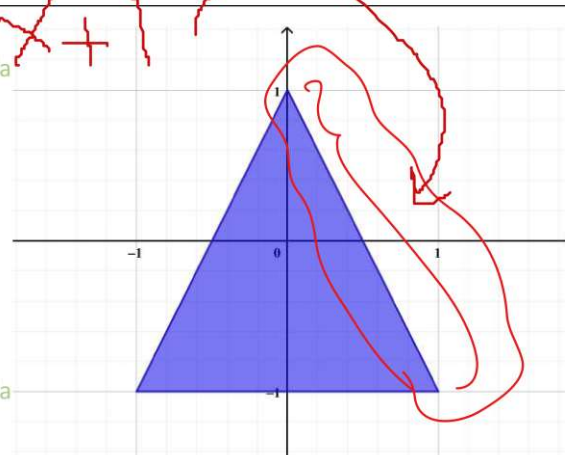
A) $V = \pi \int_{-1}^1 \left(\frac{1-y}{2}\right)^2 dy$

B) $V = 2\pi \int_{-1}^1 \left(\frac{1-y}{2}\right)^2 dy$

C) $V = \pi \int_{-1}^1 \left(\frac{1+y}{2}\right)^2 dy$

D) $V = \frac{\pi}{2} \int_{-1}^1 (1-y)^2 dy$

$$y = -2x + 1$$



$$x^2 + y^2 = 36$$

40)

Find an expression for the volume of a sphere by revolving the circle $x^2 + y^2 = r^2$ about the y -axis

A) $V = \pi \int_{-r}^r \sqrt{r^2 - y^2} dy$

B) $V = \pi \int_{-r}^r (\sqrt{r^2 - y^2})^2 dy$

C) $V = 2\pi \int_{-r}^r (r^2 - y^2) dy$

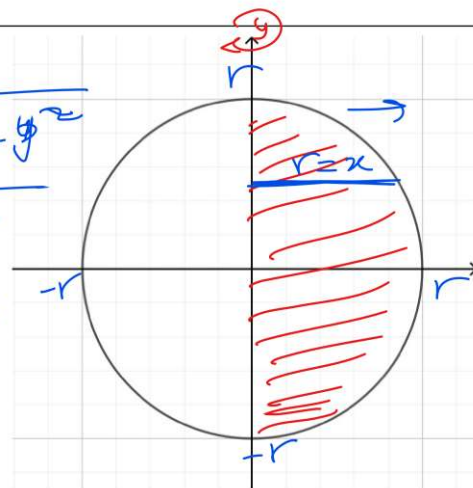
D) $V = \frac{\pi}{2} \int_{-r}^r (r - y)^2 dy$

$$x^2 = r^2 - y^2$$

$$x = \pm \sqrt{r^2 - y^2}$$

$$g(y) = \sqrt{r^2 - y^2}$$

$$\pi \int_{-r}^r (\sqrt{r^2 - y^2})^2 dy$$



41)

Find the volume of the solid resulting from revolving the region bounded by the curves $y = 9 - x^2$ and $y = 2$ from $x = 0$ to $x = \sqrt{7}$ about the y -axis.

A) $V = \pi \int_2^9 (9 - y)^2 dy$

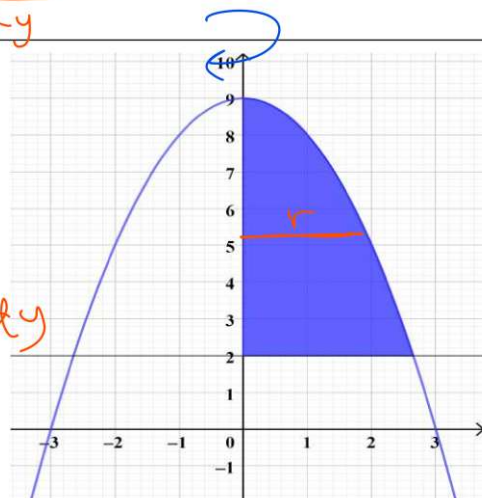
B) $V = \pi \int_2^9 \sqrt{9 - y} dy$

C) $V = \pi \int_2^9 (9 - y) dy$

D) $V = \pi \int_2^9 (9 - y)^2 dy$

$$r = x = \sqrt{9 - y}$$

$$V = \pi \int_2^9 r^2 dy = \pi \int_2^9 (9 - y) dy$$



42)

Find the volume of The region bounded by $y = x$, $y = -x$ and $y = 3$ revolved about $y = 3$

A) $\frac{40}{3}\pi$

A) $V = 4\pi \int_0^2 y(3 - y) dy$

B) $\frac{20}{3}\pi$

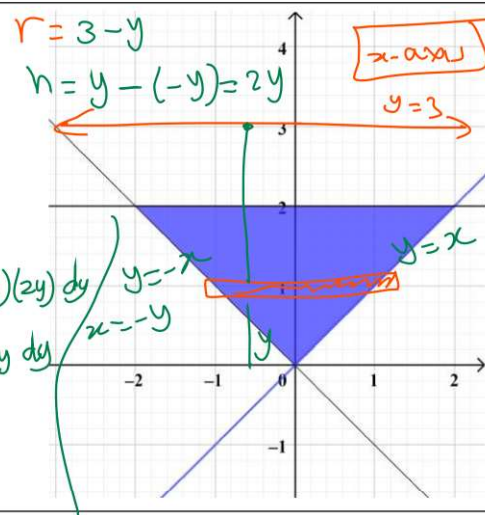
B) $V = 2\pi \int_0^2 y(3 - y) dy$

C) $\frac{10}{3}\pi$

C) $V = \pi \int_0^2 y(3 - y) dy$

D) $\frac{40}{3}$

D) $V = 4 \int_0^2 y(3 - y) dy$



43)

Let R be the region bounded by $y = 4 - 2x$, the x -axis and the y -axis. Compute the volume of the solid formed by revolving R about the given line. (a) the y -axis (b) the x -axis (c) $y = 4$
(d) $y = -4$ (e) $x = 2$ (f) $x = -2$

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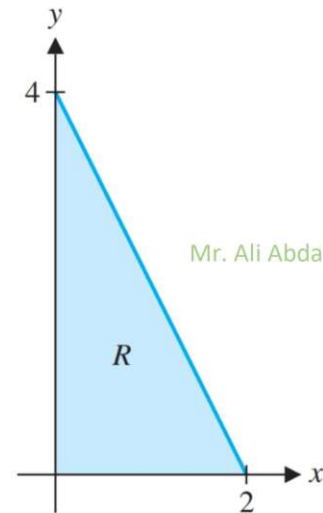
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- 44) Find the volume of the solid resulting from revolving the region bounded by the curves $y = \sec x$ the lines $y = 0$, $x = 0$ on the interval $\left[0, \frac{\pi}{3}\right]$ about the x -axis.

A) $\frac{\pi}{\sqrt{3}}$

B) π

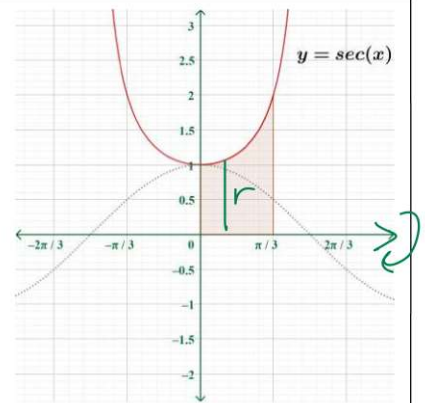
C) $\pi\sqrt{3}$

D) $\frac{8\pi}{3}$

$$r = \sec x$$

$$V = \pi \int_0^{\pi/3} \sec^2 x \, dx$$

$$= \pi \tan x \Big|_0^{\pi/3}$$



- 45) Find the volume of the solid resulting from revolving the region bounded by the curves $y = \sin x$ the lines $y = 0$ on the interval $\left[0, \frac{\pi}{2}\right]$ about the y -axis.

A) 3π

B) 2π

C) π

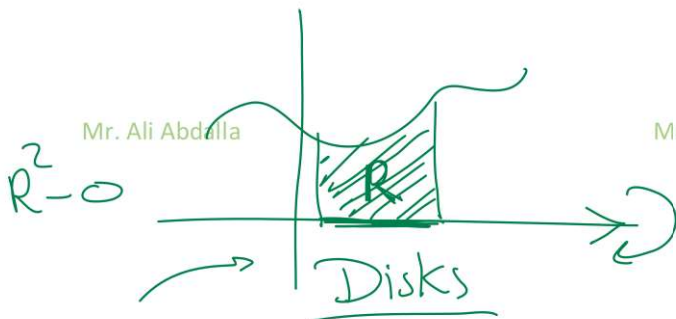
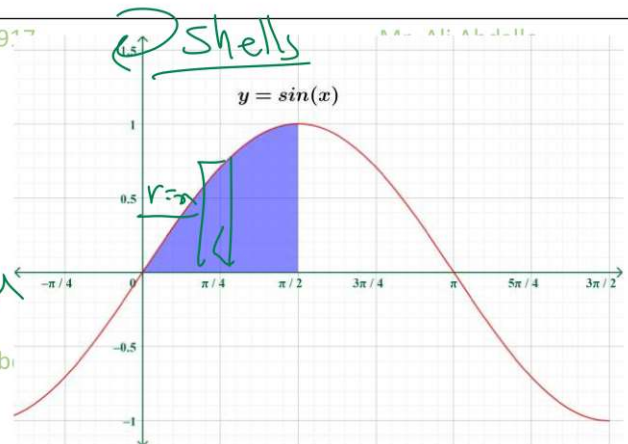
D) $\frac{2}{3}\pi$

$$r = x$$

$$h = \sin x$$

$$2\pi \int_0^{\pi/2} x \sin x \, dx$$

parts



6-4 Arc Length and Surface Area:

Arc length

To find the Arc Length use:

$$\int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Surface Area

To find the Surface Area use:

$$2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} dx$$

46)

Find the arc length of the graph of $y = 4x^{3/2}$ on the interval $[1, 2]$.

A) $s = \pi \int_1^2 \sqrt{1 + 36x} dx$

B) $s = \int_1^2 \sqrt{1 + 36x} dx$

C) $s = 8\pi \int_1^2 x^{2/3} \sqrt{1 + 36x} dx$

D) $s = \int_1^2 \sqrt{1 + 36x^2} dx$

$$s = \int_1^2 \sqrt{1 + [f'(x)]^2} dx$$

$$y = 4x^{3/2} \Rightarrow y' = 4\left(\frac{3}{2}\right)x^{1/2} = 6x^{1/2}$$

$$s = \int_1^2 \sqrt{1 + 36x} dx$$

47)

Find the arc length of the graph of $y = \ln(\cos x)$ on the interval $\left[0, \frac{\pi}{4}\right]$.

A) $s = \int_0^{\pi/4} \sec x dx$

B) $s = \int_0^{\pi/4} \sec^2 x dx$

C) $s = \int_0^{\pi/4} \tan x dx$

D) $s = \int_0^{\pi/4} \sqrt{1 - \tan^2 x} dx$

$$y' = \frac{1}{\cos x} (-\sin x) = \frac{-\sin x}{\cos x}$$

$$y' = -\tan x$$

$$s = \int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx = \int_0^{\pi/4} \sec x dx$$

$$1 + \tan^2 x = \sec^2 x$$

48)

Find the **area of the surface** formed by revolving the graph of $y = x^3$ on the interval $[0, 1]$ about the x -axis.

A) $S = 2\pi \int_0^1 x^3 \sqrt{1 + 3x^4} dx$

B) $S = 6\pi \int_0^1 x^2 \sqrt{1 + 3x^4} dx$

C) $S = 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4} dx$

D) $S = 6\pi \int_0^1 x^2 \sqrt{1 + 9x^4} dx$

$$S = 2\pi \int f(x) \sqrt{1 + [f'(x)]^2} dx$$

$$y = x^3 \Rightarrow (y')^2 = (3x^2)^2 = 9x^4$$

$$S = 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4} dx$$

49)

Find the arc length of the graph of $y = \sin x$ on the interval $[0, \pi]$.

~~A) $S = \int_0^\pi \sqrt{1 + \cos^2 x} dx$~~

B) $S = 2\pi \int_0^\pi \sin x \sqrt{1 + \cos x} dx$

C) $S = \int_0^\pi \sin x \sqrt{1 + \cos^2 x} dx$

D) $S = 2\pi \int_0^\pi \sqrt{1 + \cos^2 x} dx$

50)

Find the area of the surface formed by revolving the graph $y = \cos x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ about x -axis

A) $S = 2\pi \int_{-\pi/2}^{\pi/2} \sqrt{1 + \sin^2 x} dx$

B) $S = 2\pi \int_{-\pi/2}^{\pi/2} \cos x \sqrt{1 + \sin x} dx$

~~C) $S = 2\pi \int_{-\pi/2}^{\pi/2} \cos x \sqrt{1 + \sin^2 x} dx$~~

D) $S = \int_{-\pi/2}^{\pi/2} \sqrt{1 + \sin^2 x} dx$

51)

Find the arc length of the graph of $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[1, 2]$

A) $\frac{17}{12}$

B) $\frac{17}{6}$

C) $\frac{34}{6}$

D) $\frac{17}{24}$

$$S = \int_1^2 \sqrt{1 + [f'(x)]^2} dx$$

$$y = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$$

$$y' = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$$

$$S = \int_1^2 \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2} dx$$

52)

Find the total length of the graph of the asteroid $x^{2/3} + y^{2/3} = 4$

A) 48

B) 12

C) 24

D) 36

Handwritten work for Question 52:

$$x^{2/3} + y^{2/3} = 4$$

$$(y^{2/3})^{3/2} = (4 - x^{2/3})^{3/2}$$

$$y = (4 - x^{2/3})^{3/2}$$

$$y' = \frac{3}{2}(4 - x^{2/3})^{1/2} \cdot \left(-\frac{2}{3}x^{-1/3}\right)$$

$$y'^2 = (4 - x^{2/3})(x^{-2/3})$$

$$= 4x^{-2/3} - 1$$

$$S = 4 \int_0^8 \sqrt{4x^{-2/3} - 1} dx$$

$$= 4 \int_0^8 2x^{-1/3} dx = 8 \cdot \frac{3}{2}x^{2/3} \Big|_0^8 = 24 \cdot 4 = 96$$

Graph of the asteroid $x^{2/3} + y^{2/3} = 4$ is shown. The curve is symmetric about both axes, with vertices at $(\pm 8, 0)$ and $(0, \pm 8)$.

53)

The length of the arc of $f(x) = \sqrt{1 - x^2}$ from $x = 0$ to $x = \frac{1}{2}$ is

A) $s = \frac{\pi}{6}$

B) $s = \frac{\pi}{2}$

C) $s = \frac{\pi}{3}$

D) $s = \frac{\pi}{5}$

$$\int_a^b \sqrt{1 + \frac{1}{4x}} \cos^2(\sqrt{x}) \, dx$$

D) $f(x) = 6 - x^3$

$$\begin{aligned}
 & \downarrow [f'(x)]^2 = 9x^2 \quad \text{off} \quad \downarrow \quad \downarrow \\
 & [f'(x)] = 3x^2 \\
 & f(x) = \int 3x^2 dx \\
 & f(x) = x^3 + C_1 \\
 & 6 = 1 + C_1 \\
 & \boxed{C_1 = 5}
 \end{aligned}
 \quad
 \begin{aligned}
 & \downarrow \quad \downarrow \\
 & f'(x) = -3x^2 \\
 & f(x) = -x^3 + C_2 \\
 & 6 = -1 + C_2 \\
 & C_2 = 7 \\
 & \boxed{f(x) = -x^3 + 7}
 \end{aligned}$$

D) $s = 2 \left(\frac{e+1}{\sqrt{e}} \right)$

$$x' = \frac{1}{2} e^{iy} - \frac{1}{2} e^{-iy} \quad (x')^2 = \left[\frac{1}{2} e^{iy} - \frac{1}{2} e^{-iy} \right]^2$$

57)

Suppose that the circle $x^2 + y^2 = r^2$ is revolved about the y -axis.
Compute the surface area.

A) $S = 4\pi r^2$

B) $S = 4\pi r$

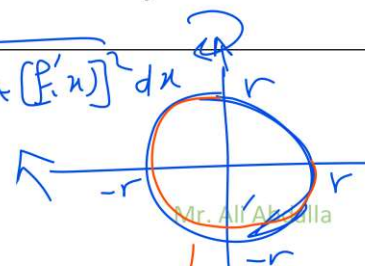
C) $S = 2\pi r^2$

D) $S = \frac{4}{3}\pi r^3$

$$S = 2\pi \int f(x) \sqrt{1 + [f'(x)]^2} dx$$

$$y^2 = r^2 - x^2$$

$$\Rightarrow y = \sqrt{r^2 - x^2}$$



السطح الناتج من دوار
الدائرة

4πr² - دقة

58)

Find the area of the surface formed by revolving the graph $y = \ln x$ on the interval $[1, 2]$ about x -axis

A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}} dx$

B) $\int_1^2 2\pi \sqrt{1 + \frac{1}{x^2}} dx$

C) $\int_1^2 \pi \sqrt{1 + \frac{1}{x^2}} dx$

D) $\int_1^2 2\pi \ln x \sqrt{1 + \frac{1}{x^2}} dx$

$$S = 2\pi \int f(x) \sqrt{1 + [f'(x)]^2} dx$$

$$2\pi \int \ln x \sqrt{1 + \left(\frac{1}{x}\right)^2} dx$$

6-5 Projectile Motion:

$y'(0) = 0 \quad h(0) = 64$

$y'(t_f) = 0 \quad h_f = 0$

59)

A diver drops from a height of 64 feet. Which of the following gives the initial conditions :

A) $h(0) = 64 \text{ ft}, \quad h'(0) = 0$

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B) $h(0) = 64 \text{ ft}, \quad h'(t) = 32 \text{ ft/s}$

C) $h(0) = 64 \text{ ft}, \quad h''(t) = 32 \text{ ft/s}$

D) $h(0) = 0, \quad h''(t) = -32 \text{ ft/s}$

60)

A diver drops from a height of 64 feet. What is the velocity at impact?

A) 32 ft/s

B) 23 ft/s

C) 64 ft/s

D) 46 ft/s

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$h'(0) = 0 \quad h(0) = 64$

$h''(t) = -32$

$h'(t) = -32t + C_1$

$0 = 0 + C_1 \Rightarrow C_1 = 0$

$h'(t) = -32t$

$h(t) = -16t^2 + C_2$

$h(0) = 64$

$64 = 0 + C_2 \Rightarrow C_2 = 64$

$h(t) = -16t^2 + 64$

$(-16) \text{ m/s}^2 \text{ is the acceleration}$

$-16t^2 + 64 = 0 \Rightarrow t = 0 \text{ or } t = 2$

$h'' \rightarrow -32 \text{ feet}$

$\rightarrow -9.8 \text{ m}$

61)

Identify the initial condition $y(0)$ and $y'(0)$ for the vertical motion, if the object is thrown at a velocity of 6 m/s from a height of 30 m. (take the origin to be the ground)

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A) $y(0) = 30, \quad y'(0) = -6$

$y'(t) = -6 \text{ m/s}$

B) $y(0) = 0, \quad y'(0) = -6$

$h(0) = y(0) = 30$

C) $y(0) = 30, \quad y'(0) = 6$

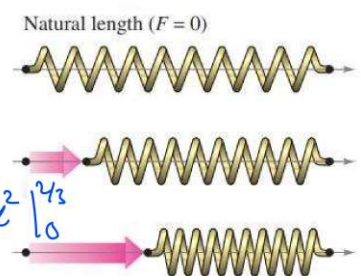
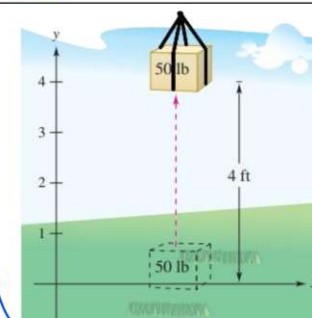
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D) $y(0) = 0, \quad y'(0) = 6$



6-6 Applications of Integration to Physics and Engineering:

62)	A force of 60 pounds stretches a spring 1 foot. Find the work done to stretch the spring 8 inches beyond its natural length.
✓ A) $\frac{40}{3}$ ft-lb B) 40 ft-lb C) 1920 ft-lb D) 192 ft-lb	$F = kx$ $60 = k(1)$ $k = 60$ $F = 60x$ $W = \int_a^b F dx$ $W = \int_0^{\frac{2}{3}} 60x dx$ $= 30x^2 \Big _0^{\frac{2}{3}}$ $= 30 \left(\frac{2}{3} \right)^2 = \frac{40}{3}$ 8 inches = $\frac{8}{12}$ feet = $\frac{2}{3}$ Natural length ($F = 0$) 
63)	A car engine exerts a force of $800 + 2x$ pounds when the car is at position x miles. Find the work done as the car moves from $x = 0$ to $x = 8$.
A) 4646 mils-pound B) 6400 mils-pound ✓ C) 6464 mils-pound D) 7400 mils-pound	$W = \int_0^8 (800 + 2x) dx$ $= 800x + x^2 \Big _0^8$ $= 6400 + 64 = 6464$ mil-pound
64)	If a wall applies a force of $f(t) = 3000t(2 - t)$ pounds to a car for $0 \leq t \leq 2$, find the impulse. If the car (mass $m = 100$ slugs) is motionless after the collision, compute its speed before the collision.
65)	Determine the work done in lifting a 50-pound object 4 feet.
A) 20 foot-pound B) 400 foot-pound ✓ C) 200 foot-pound D) 40 foot-pound	$J = \Delta v \cdot m$ $4000 = 100 \Delta v$ $\Delta v = \frac{4000}{100} = 40$ $J = \int_0^2 f(t) dt$ $= \int_0^2 3000(2t - t^2) dt$ $= 3000 \left[t^2 - \frac{1}{3} t^3 \right]_0^2 = 4000$ F constant $W = F \cdot d$ $= 50 \cdot 4$ $= 200$ ft-pound 

66)

A bucket is lifted a distance of 80 feet at the rate of 4 ft/s. The bucket initially contains 100 pounds of sand but leaks at a rate of 2 lb/s. Compute the work done.

A) 6220 foot-pound

B) 6400 foot-pound

C) 640 foot-pound

D) 4600 foot-pound

$$\frac{dx}{dt} = 4, \quad \frac{dF}{dt} = -2$$

$$\frac{dF}{dx} = \frac{\frac{dF}{dt}}{\frac{dx}{dt}} = \frac{-2}{4} = -\frac{1}{2}$$

$$\int dF = \int -\frac{1}{2} dx$$

$$F = -\frac{1}{2}x + C$$

$$\text{at } x=0, F=100$$

$$C = 100$$

$$F = -\frac{1}{2}x + 100$$

$$W = \int_0^{80} F dx$$



67)

a

A water tower is spherical in shape with radius 20 feet, extending from 200 feet to 240 feet above ground. Compute the work done in filling the tank from the ground

A) 146432000 π foot-pound

B) 146432000 foot-pound

C) 14643200 π foot-poundD) 70720000 π foot-pound

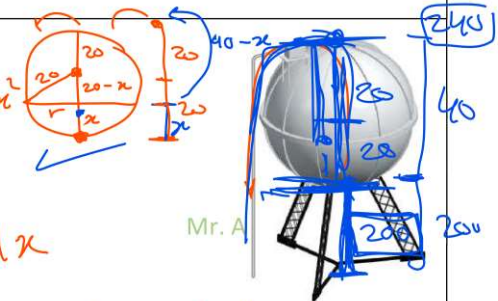
$$r^2 = (20)^2 - (20-x)^2$$

$$= 400 - 400 + 40x - x^2$$

$$r^2 = 40x - x^2$$

$$W = 62.4 \int_0^{40} \pi r^2 h dx$$

$$= 62.4 \int_0^{40} \pi (40x - x^2) (200+x) dx$$



$$W = \int 62.4 \pi r^2 h dx$$

67)b

Compute the work done in filling the tank halfway

view

A) 70720000 π foot-pound

B) 70720000 foot-pound

C) 14643200 π foot-poundD) 14.4×10^6 foot-pound

$$62.4 \int_0^{20} \pi (40x - x^2) (200+x) dx$$



67)c

Compute the work done in pumping all of the water out the top of the tank.

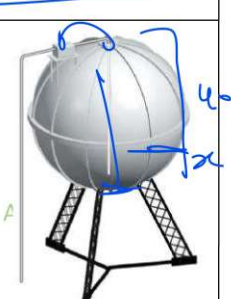
A) 13312000 π foot-pound

B) 9152000 foot-pound

C) 9152000 π foot-pound

D) 13312000 foot-pound

$$62.4 \int_0^{40} \pi (40x - x^2) (40-x) dx$$



- 68) A water tank is in the shape of a right circular cone of altitude 10 feet and base radius 5 feet, with its vertex at the ground. If the tank is full, find the work done in pumping all of the water out the top of the tank.

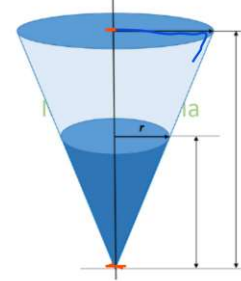
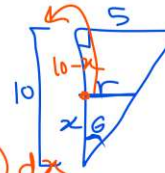
- A) 13000 π foot-pound
 B) 13000 foot-pound
 C) 1300 π foot-pound
 D) 40000 foot-pound

$$\tan \theta = \frac{r}{x} = \frac{5}{10}$$

$$r = \frac{1}{2}x$$

$$W = 62.4 \int_0^{10} \left(\frac{1}{2}x\right)^2 (10-x) dx$$

$$W = \int_a^b (\rho)(\pi r^2)(H-x) dx$$



- 69) Two laborers share the job of digging a rectangular hole 10 feet deep. The dirt from the hole is cleared away by other laborers. Assuming a constant density of dirt, how deep should the first worker dig to do half the work?

- A) $5\sqrt{2}$ feet
 B) 5 feet
 C) $2\sqrt{5}$ feet
 D) 7 feet

$$a^2 = 50$$

$$W = \int_0^{10} kx dx = \frac{1}{2}kx^2 \Big|_0^{10}$$

$$F = kx$$

$$W = 50k$$

$$W = \int F(x) dx$$

$$W_1 = \int_0^a kx dx = \frac{1}{2}kx^2 \Big|_0^a = \frac{1}{2}ka^2 = 25k$$

- 70) A force of 5 pounds stretches a spring 24 inches from its natural length. If the work done in stretching the spring x feet beyond its natural length is 20 ft-lb. find the value of x ?

- A) $x = 4$
 B) $x = 48$
 C) $x = 5$
 D) $x = 15$

$$24 \text{ inches} = \frac{24}{12} = 2 \text{ ft}$$

$$F = kx$$

$$5 = k(2) \Rightarrow k = \frac{5}{2}$$

$$F = \frac{5}{2}x$$

$$W = \int_0^a \frac{5}{2}x dx$$

$$20 = \frac{5}{2} \cdot \frac{1}{2}x^2 \Big|_0^a = \frac{5}{4}a^2$$

$$\frac{5}{4}a^2 = 20 \Rightarrow a^2 = 16$$

$$a = 4 \text{ ft}$$

Natural length ($F = 0$)

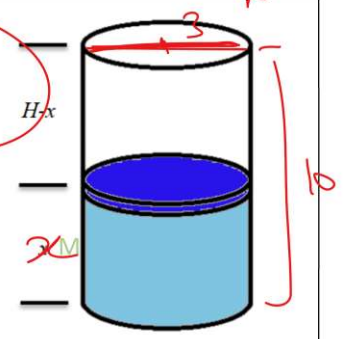


- 71) Find the work done by pumping out water from the top of a cylindrical tank 3 ft in radius and 10 ft tall, if the tank is initially full. (The density of water is 62.4 lb/ft³)

- A) $W = 561.6\pi \int_0^{10} (10-x) dx$
 B) $W = 187.5\pi \int_0^{10} (10-x) dx$
 C) $W = 62.4\pi \int_0^{10} (10-x) dx$
 D) $W = 561.6\pi \int_0^{10} (10-x) dx$

$$W = \int_a^b (\rho)(\pi r^2)(H-x) dx$$

$$62.4 \pi (3)^2 (10-x)$$



- 72) A rocket full of fuel weighs 10,000 pounds at launch. After launch, the rocket gains altitude and loses weight as the fuel burns. Assume that the rocket loses 1 pound of fuel for every 15 feet of altitude gained. Find the work done raising the rocket to an altitude of 30,000 feet.

A) $W = 2.7 \times 10^8$ ft-lb

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B) $W = 2.7 \times 10^7$ ft-lb

C) $W = 2.07 \times 10^7$ ft-lb

D) $W = 27 \times 10^6$ ft-lb

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- 73) A weightlifter lifts 250 pounds a distance of 20 inches. Find the work done (as measured in foot-pounds)

$$20 \text{ inches} = \frac{20}{12} = \frac{5}{3} \text{ ft}$$

A) $W = 1250$ foot-pound

B) $W = \frac{125}{3}$ foot-pound

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C) $W = \frac{1250}{3}$ foot-pound

D) $W = 5000$ foot-pound

$$W = 250 \times \frac{5}{3}$$

$$= \frac{1250}{3}$$

- 74) A thrust-time curve $f(t) = 10te^{-t/3}$ for a model rocket is shown. Compute the impulse ($0 \leq t \leq 6$)

A) $J = 90 - 270e^{-2}$

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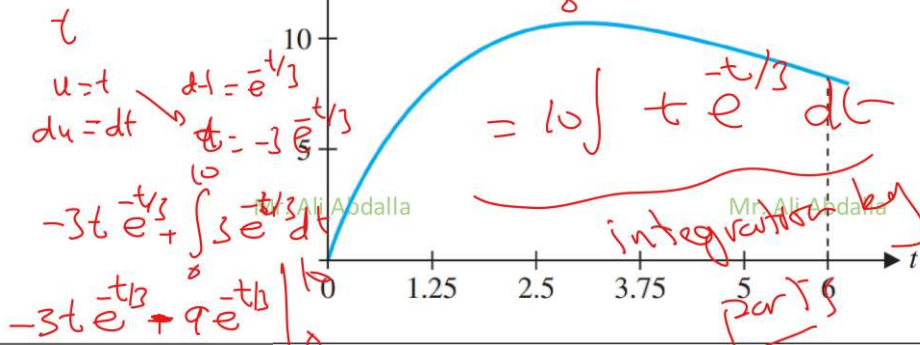
$$J = \int_0^6 f(t) dt$$

B) $J = 90 + 270e^{-2}$

C) $J = 90 - 270e^2$

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D) $J = 90 + 270e^2$



A crash test is performed on a vehicle. The force of the wall on the front bumper is shown in the table. Estimate the impulse and the speed of the vehicle (use $m = 200$ slugs).

t (s)	0	0.1	0.2	0.3	0.4	0.5	0.6
F (lb)	0	8000	16,000	24,000	15,000	9000	0

$$\int P(t) dt = \frac{0.1}{2} [0 + 4(8000) + 2(16000) + 4(24000) + 2(15000) + 4(9000) + 0]$$

6-7 Probability

Suppose that X is a random variable that may assume any value x with $a \leq x \leq b$. A **probability density function (pdf)** for X is a function $f(x)$ satisfying

$$f(x) \geq 0 \quad \text{لكل} \quad a \leq x \leq b$$

$$\int_a^b f(x) dx = 1$$

The probability that the (observed) value of X falls between c and d is given by the area under the graph of the **pdf** on that interval. That is,

$$P(c \leq X \leq d) = \int_c^d f(x) dx$$

The Mean

The mean μ of a random variable with pdf $f(x)$ on the interval $[a, b]$ is given by

$$\mu = \int_a^b x f(x) dx$$

Median (c) – upper (3rd) quartile (Q_3) – lower (1st) quartile (Q_1) given by

$Q \sim$ Median c	$\int_a^c f(x) dx = 0.5$	On the interval $[a, b]$
lower (1 st) quartile Q_1	$\int_a^{Q_1} f(x) dx = 0.25$	
upper (3 rd) quartile Q_3	$\int_a^{Q_3} f(x) dx = 0.75$	

75) Which function is **probability density function** on the interval $[0,1]$

A) $f(x) = 4x^2$

B) $f(x) = \frac{3}{4}x^2$

C) $f(x) = \frac{1}{2}x^2$

D) $f(x) = 4x^3$ ✓

$$\int_0^1 4x^3 dx = \frac{4x^4}{4} \Big|_0^1 = 1$$

76) Find the value of c for which $f(x) = c e^{-4x}$ is a **probability density function** (pdf) on the interval $[0, 1]$

A) $c = \frac{1}{4}(1 - e^{-4})$

B) $c = \frac{4}{1 - e^{-4}}$

C) $c = 1 - e^{-4}$

D) $c = \frac{4}{1 + e^{-4}}$

$$\int_0^1 c e^{-4x} dx = 1$$

$$\frac{c}{-4} e^{-4x} \Big|_0^1 = 1$$

$$c [e^{-4} - 1] = -4 \Rightarrow c = \frac{-4}{e^{-4} - 1} = \frac{4}{1 - e^{-4}}$$

77) Suppose that the lifetime in years of a certain brand of lightbulb is exponentially distributed with pdf $f(x) = 6e^{-6x}$. Find the probability that a given lightbulb lasts less than 3 months.

A) $1 - e^{-3/2}$

B) $1 - e^{-18}$

C) $1 + e^{-3/2}$

D) $1 + e^{-18}$

$$P(a \leq x \leq b) = \int_a^b f(x) dx$$

$$P(0 \leq x \leq \frac{1}{4}) = \int_0^{\frac{1}{4}} 6e^{-6x} dx = ?$$

$$3 \text{ months} = \frac{3}{12} = \frac{1}{4} \text{ year}$$

78) Find the mean of the random variable with the pdf $f(x) = \frac{4/\pi}{1+x^2}$, $[0,1]$

A) $\frac{2 \ln 2}{\pi}$

B) $\frac{\ln 2}{\pi}$

C) $\frac{2 \ln 2}{\pi} - 1$

D) 1

$$\mu = \int_a^b x f(x) dx$$

$$= \frac{4}{\pi} \int_0^1 \frac{x}{1+x^2} dx = \frac{2}{\pi} \ln|1+x^2| \Big|_0^1$$

$$= \frac{2}{\pi} [\ln 2 - 0] = \frac{2 \ln 2}{\pi}$$

79)

Find the value of c for which $f(x) = \frac{c}{1+x^2}$ is a **probability density function** (pdf) on the interval $[0, 1]$

A) $\frac{\pi}{4}$

B) $\frac{4}{\pi}$

C) π

D) 4

$$\int_0^1 \frac{c}{1+x^2} dx = 1$$

$$c \left[\tan^{-1} x \right]_0^1 = 1$$

$$c \left[\frac{\pi}{4} - 0 \right] = 1 \Rightarrow c = \frac{4}{\pi}$$

80)

Find the **Median** of the random variable with the pdf $f(x) = \frac{8}{3}e^{-2x}$ in the interval $[0, \ln 2]$

A) $-2 \ln\left(\frac{5}{8}\right)$

B) $\frac{1}{2} \ln\left(\frac{5}{8}\right)$

C) $2 - \ln\left(\frac{5}{8}\right)$

D) $-\frac{1}{2} \ln\left(\frac{5}{8}\right)$

$$\int_0^c f(x) dx = \frac{1}{2}$$

$$\frac{8}{3} \int_0^c e^{-2x} dx = \frac{1}{2}$$

$$\frac{8}{3} \left(-\frac{1}{2} e^{-2x} \right) \Big|_0^c = \frac{1}{2}$$

$$-e^{-2x} \Big|_0^c = \frac{3}{8}$$

$$-e^{-2c} + 1 = \frac{3}{8} \Rightarrow -e^{-2c} = \frac{3}{8} - 1 = -\frac{5}{8}$$

$$e^{-2c} = \frac{5}{8}$$

$$-2c = \ln\left(\frac{5}{8}\right)$$

81)

Find the **mean** of the random variable with the pdf $f(x) = \frac{1}{2} \sin x$, $[0, \pi]$

A) π

B) $\frac{\pi}{2}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{4}$

$$\mu = \int_0^{\pi} x \cdot \frac{1}{2} \sin x dx$$

$$= \frac{1}{2} \int_0^{\pi} x \sin x dx$$

$$= \frac{1}{2} \left[-x \cos x + \int \cos x dx \right] \Big|_0^{\pi}$$

parts
 $u = x \rightarrow du = dx$
 $dv = \sin x \rightarrow v = -\cos x$

82)

Find the **mean** of the random variable with the pdf $f(x) = x + 2x^3$, $[0, 1]$

A) $\frac{11}{15}$

B) $\frac{3}{8}$

C) $\frac{11}{8}$

D) $\frac{15}{11}$

$$\mu = \int_0^1 x f(x) dx$$

83)

Suppose a basketball player makes 70% of her free throws. If he shoots three free throws and the probability of making each one is 0.7, the probabilities for the total number made are as shown. Assuming that throws follow a **pdf** function find the value of m

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Number made	0	1	2	3
Probability	0.027	m	0.441	0.343

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Then find the probability of each event indicated below.

a) He makes 2 or 3

(b) He makes at least 1

PDF $\rightarrow \int_0^3 f(x) dx = 1$

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$$0.027 + m + 0.441 + 0.343 = 1$$

$$m = 0.189$$

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$$(a) P(X=2) + P(X=3) = 0.441 + 0.343 = 0.784$$

$$(b) P(X=1) + P(X=2) + P(X=3) = 1 - P(X=0)$$

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$$0.973$$

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Find the volume of the indicated solid of revolution.

84)

A) The region bounded by $y = x^2$ and $y = 4$ revolved about

(a) the x -axis (b) the y -axis (c) $x = 2$ (d) $y = -2$

B) The region bounded by $y = x$, $y = 2x$ and $x = 2$ revolved about

(a) the x -axis (b) the y -axis (c) $x = -1$ (d) $y = 4$

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$$A) \frac{256}{5}\pi, 8\pi, \frac{128}{3}\pi, \frac{1408}{15}\pi$$

$$B) 8\pi, \frac{16}{3}\pi, \frac{28}{3}\pi, 8\pi$$

The answers

Q#	Answer		Q#	Answer	Q#	Answer	Q#	Answer
1	B		2	C	3	B, D	4	B
5	C	A_3, A_2	6	A	7	D	8	D
9	A		10	B	11	C	12	A
13	B		14	A	15	A	16	C
17	A		18	A	19	A	20	A
21	A		22	A	23	C	24	C
25	B		26	C	27	C	28	B
29	D		30	B	31	A	32	A
33	B		34	C	35	A	36	C
37	D		38	B	39	A	40	B
41	C		42	A	43	Show your work	44	C
45	B		46	B	47	A	48	C
49	A		50	C	51	A	52	A
53	A		54	D	55	B	56	C
57	A		58	D	59	A	60	C
61	A		62	A	63	C	64	A
65	C		66	B	67a	A	67b	A
67c	A		68	A	69	A	70	A
71	A		72	A	73	C	74	A
75	D		76	B	77	A	78	A
79	B		80	D	81	B	82	A
83	$m = 0.189$		83a	0.784	83b	0.973	84	Show your work