

9-2 Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. $y = 2x^2 - 24x + 40$

SOLUTION:

$$\begin{aligned} y &= 2x^2 - 24x + 40 \\ &= 2(x^2 - 12x + 36) + 40 - 72 \\ &= 2(x - 6)^2 - 32 \end{aligned}$$

The vertex of the equation is (6, -32); axis of symmetry: $x = 6$; The graph opens upward.

ANSWER:

$$\begin{aligned} y &= 2(x - 6)^2 - 32; \text{ vertex} \\ &(6, -32); \text{ axis of symmetry:} \\ &x = 6; \text{ opens upward} \end{aligned}$$

2. $y = 3x^2 - 6x - 4$

SOLUTION:

$$\begin{aligned} y &= 3x^2 - 6x - 4 \\ &= 3(x^2 - 2x + 1) - 4 - 3 \\ &= 3(x - 1)^2 - 7 \end{aligned}$$

The vertex of the equation is (1, -7); axis of symmetry: $x = 1$; The graph opens upward.

ANSWER:

$$\begin{aligned} y &= 3(x - 1)^2 - 7; \text{ vertex} \\ &(1, -7); \text{ axis of symmetry:} \\ &x = 1; \text{ opens upward} \end{aligned}$$

3. $x = y^2 - 8y - 11$

SOLUTION:

$$\begin{aligned} x &= y^2 - 8y - 11 \\ &= (y^2 - 8y + 16) - 11 - 16 \\ &= (y - 4)^2 - 27 \end{aligned}$$

The vertex of the equation is (-27, 4); axis of symmetry: $y = 4$; The graph opens right.

ANSWER:

$$\begin{aligned} x &= (y - 4)^2 - 27; \text{ vertex} \\ &(-27, 4); \text{ axis of symmetry:} \\ &y = 4; \text{ opens right} \end{aligned}$$

4. $x + 3y^2 + 12y = 18$

SOLUTION:

$$\begin{aligned} x + 3y^2 + 12y &= 18 \\ x &= -3y^2 - 12y + 18 \\ x &= -3(y^2 + 4y + 4) + 18 + 12 \\ x &= -3(y + 2)^2 + 30 \end{aligned}$$

The vertex of the equation is (30, -2); axis of symmetry: $y = -2$; The graph opens left.

ANSWER:

$$\begin{aligned} x &= -3(y + 2)^2 + 30; \text{ vertex} \\ &(30, -2); \text{ axis of symmetry:} \\ &y = -2; \text{ opens left} \end{aligned}$$

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Graph each equation.

5. $y = (x - 4)^2 - 6$

SOLUTION:

vertex: $(4, -6)$

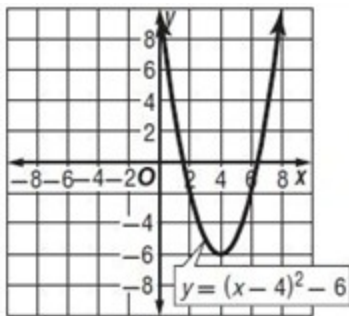
axis of symmetry: $x = 4$

Since $a = 1 (> 0)$, the graph opens up.

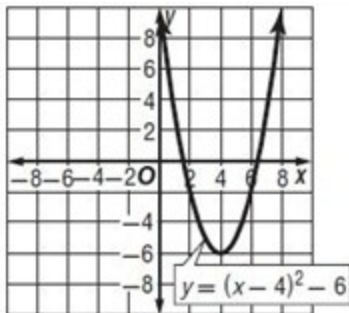
focus: $(4, -5.75)$

directrix: $y = -6.25$

length of the latus rectum: 1



ANSWER:



6. $y = 4(x + 5)^2 + 3$

SOLUTION:

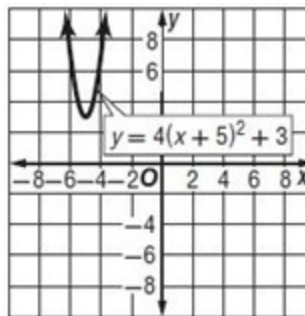
vertex: $(-5, 3)$

axis of symmetry: $x = -5$

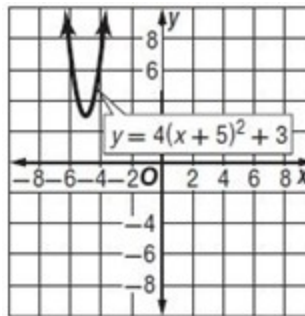
focus: $\left(-5, \frac{49}{16}\right)$

directrix: $y = \frac{47}{16}$

length of the latus rectum: $\frac{1}{4}$



ANSWER:



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7. $y = -3x^2 - 4x - 8$

SOLUTION:

$$\begin{aligned} y &= -3x^2 - 4x - 8 \\ &= -3\left(x^2 + \frac{4}{3}x + \left(\frac{2}{3}\right)^2\right) - 8 + \frac{4}{3} \\ &= -3\left(x + \frac{2}{3}\right)^2 - \frac{20}{3} \end{aligned}$$

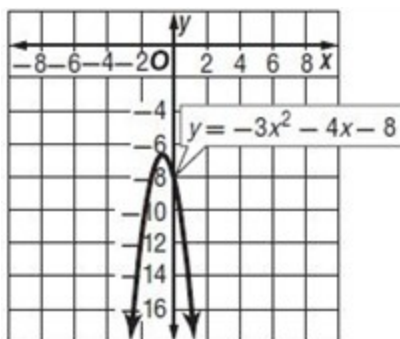
vertex: $\left(-\frac{2}{3}, -\frac{20}{3}\right)$

axis of symmetry: $x = -\frac{2}{3}$

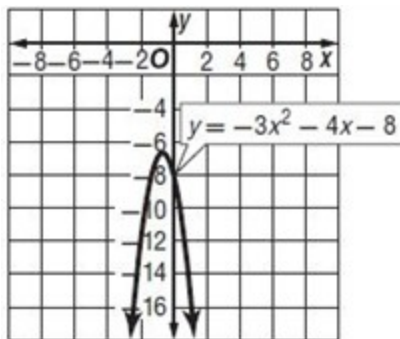
focus: $\left(-\frac{2}{3}, -\frac{27}{4}\right)$

directrix: $y = -\frac{79}{12}$

length of the latus rectum: $\frac{1}{3}$



ANSWER:



8. $x = 3y^2 - 6y + 9$

SOLUTION:

$$\begin{aligned} x &= 3y^2 - 6y + 9 \\ &= 3(y^2 - 2y + 1) + 9 - 3 \\ &= 3(y - 1)^2 + 6 \end{aligned}$$

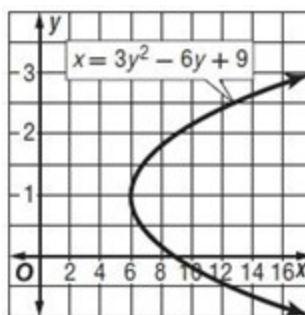
vertex: $(6, 1)$

axis of symmetry: $y = 1$

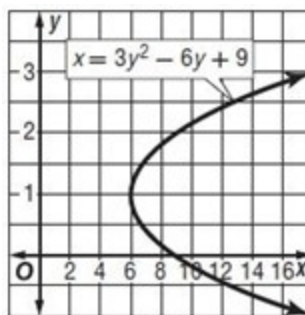
focus: $\left(\frac{73}{12}, 1\right)$

directrix: $x = \frac{71}{12}$

length of the latus rectum: $\frac{1}{3}$



ANSWER:



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Write an equation for each parabola described below. Then graph the equation.

9. vertex (0, 2), focus (0, 4)

SOLUTION:

The vertex is at (0, 2), so $h = 0$ and $k = 2$.

The focus is at (0, 4), so $h = 0$ and $k + \frac{1}{4a} = 4$.

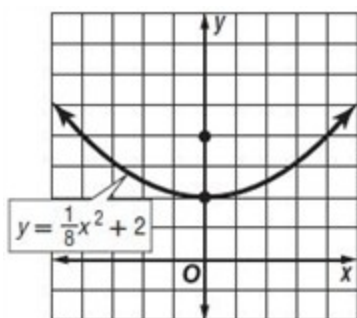
Thus, $a = \frac{1}{8}$.

So, the equation of the parabola is $y = \frac{1}{8}x^2 + 2$.

axis of symmetry: $x = 0$

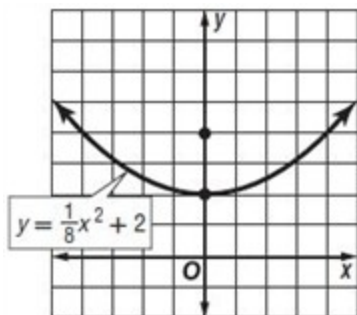
directrix: $y = 0$

length of the latus rectum: 8



ANSWER:

$$y = \frac{1}{8}x^2 + 2$$



10. vertex (-2, 4), directrix $x = -1$

SOLUTION:

The directrix is a vertical line, so the equation of the parabola is of the form $x = a(y - k)^2 + h$.

The vertex is at (-2, 4), so $h = -2$ and $k = 4$.

Use the equation of the directrix to find a .

$$x = h - \frac{1}{4a}$$

$$-1 = -2 - \frac{1}{4a}$$

$$4a = -1$$

$$a = -\frac{1}{4}$$

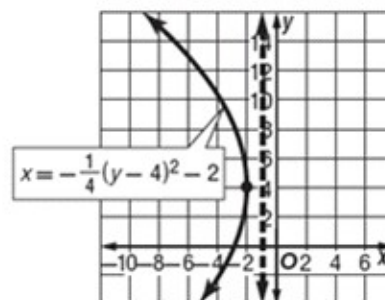
So, the equation of the parabola is

$$x = -\frac{1}{4}(y - 4)^2 - 2.$$

axis of symmetry: $y = 4$

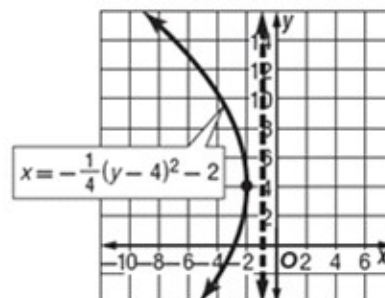
focus: (-3, 4)

length of latus rectum is 4.



ANSWER:

$$x = -\frac{1}{4}(y - 4)^2 - 2$$



11. focus (3, 2), directrix $y = 8$

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SOLUTION:

The directrix is a horizontal line, so the equation of the parabola is of the form $y = a(x - h)^2 + k$.

The focus is at $(3, 2)$, so $h = 3$ and $k + \frac{1}{4a} = 2$.

Thus, $k = 2 - \frac{1}{4a}$.

Use the equation of the directrix to find a .

$$y = k - \frac{1}{4a}$$

$$8 = 2 - \frac{1}{4a} - \frac{1}{4a}$$

$$6 = -\frac{1}{2a}$$

$$a = -\frac{1}{12}$$

Find k by substituting the value of a in $k = 2 - \frac{1}{4a}$.

$$\begin{aligned} k &= 2 - \frac{1}{4\left(-\frac{1}{12}\right)} \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

So, the equation of the parabola is

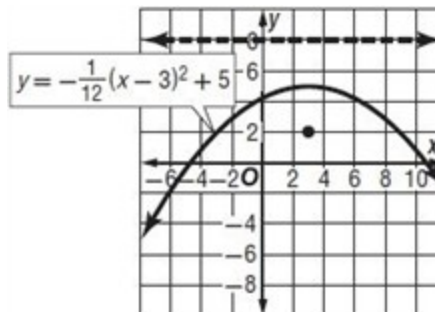
$$y = -\frac{1}{12}(x - 3)^2 + 5.$$

vertex: $(3, 5)$

axis of symmetry: $x = 3$

length of the latus rectum: 12

$$y = -\frac{1}{12}(x - 3)^2 + 5$$



ANSWER:

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12. vertex $(-1, -5)$, focus $(-5, -5)$

SOLUTION:

The vertex is at $(-1, -5)$, so $h = -1$ and $k = -5$.

The focus is at $(-5, -5)$, so $h + \frac{1}{4a} = -5$ and $k = -5$.

Thus, $a = -\frac{1}{16}$.

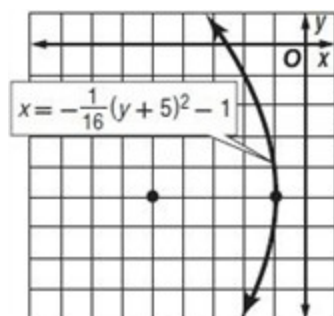
So, the equation of the parabola is

$$x = -\frac{1}{16}(y + 5)^2 - 1.$$

axis of symmetry: $y = -5$

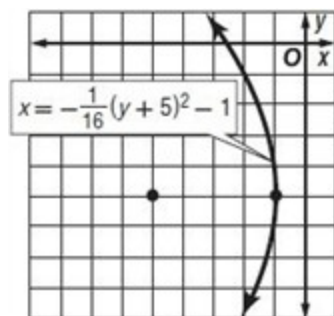
directrix: $x = 3$

length of latus rectum: 16



ANSWER:

$$x = -\frac{1}{16}(y + 5)^2 - 1$$



13. **ASTRONOMY** Consider a parabolic mercury mirror like the one described at the beginning of the lesson. The focus is 6 feet above the vertex and the latus rectum is 24 feet long.

a. Assume that the focus is at the origin. Write an equation for the parabola formed by the parabolic microphone.

b. Graph the equation

SOLUTION:

a.

The vertex is at $(0, -6)$, so $h = 0$ and $k = -6$.

The latus rectum is 24 feet long, so $a = \frac{1}{24}$.

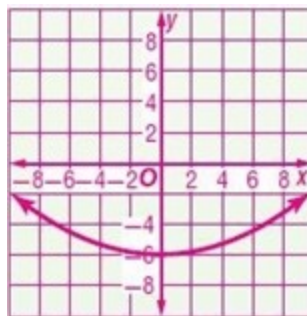
So the equation of the parabola is $y = \frac{1}{24}x^2 - 6$.

b.

axis of symmetry: $x = 0$

focus: $(0, 0)$

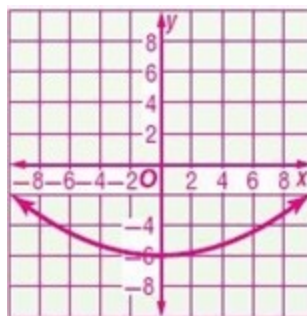
directrix: $y = -12$



ANSWER:

a. $y = \frac{1}{24}x^2 - 6$

b.



9-2 Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

14. $y = x^2 - 8x + 13$

SOLUTION:

Equation in standard form:

$$\begin{aligned}y &= x^2 - 8x + 13 \\&= (x^2 - 8x + 16) + 13 - 16 \\&= (x - 4)^2 - 3\end{aligned}$$

The vertex of the equation is (4, -3); axis of symmetry: $x = 4$; The graph opens upward.

ANSWER:

$y = (x - 4)^2 - 3$; vertex = (4, -3); axis of symmetry: $x = 4$; opens upward

15. $y = 3x^2 + 42x + 149$

SOLUTION:

Equation in standard form:

$$\begin{aligned}y &= 3x^2 + 42x + 149 \\&= 3(x^2 + 14x + 49) + 149 - 147 \\&= 3(x + 7)^2 + 2\end{aligned}$$

The vertex of the equation is (-7, 2); axis of symmetry: $x = -7$; The graph opens upward.

ANSWER:

$y = 3(x + 7)^2 + 2$; vertex = (-7, 2); axis of symmetry: $x = -7$; opens upward

16. $y = -6x^2 - 36x - 8$

SOLUTION:

Equation in standard form:

$$\begin{aligned}y &= -6x^2 - 36x - 8 \\&= -6(x^2 + 6x + 9) - 8 + 54 \\&= -6(x + 3)^2 + 46\end{aligned}$$

The vertex of the equation is (-3, 46); axis of symmetry: $x = -3$; The graph opens downward.

ANSWER:

$y = -6(x + 3)^2 + 46$; vertex = (-3, 46); axis of symmetry: $x = -3$; opens downward

17. $y = -3x^2 - 9x - 6$

SOLUTION:

Equation in standard form:

$$\begin{aligned}y &= -3x^2 - 9x - 6 \\&= -3\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) - 6 + \frac{27}{4} \\&= -3\left(x + \frac{3}{2}\right)^2 + \frac{3}{4}\end{aligned}$$

The vertex of the equation is $\left(-\frac{3}{2}, \frac{3}{4}\right)$; axis of symmetry: $x = -\frac{3}{2}$; The graph opens downward.

ANSWER:

$y = -3\left(x + \frac{3}{2}\right)^2 + \frac{3}{4}$; vertex = $\left(-\frac{3}{2}, \frac{3}{4}\right)$; axis of symmetry: $x = -\frac{3}{2}$; opens downward

9-2 Parabolas

18. $x = \frac{1}{3}y^2 - 3y + 4$

SOLUTION:

Equation in standard form:

$$\begin{aligned}x &= \frac{1}{3}y^2 - 3y + 4 \\&= \frac{1}{3}(y^2 - 9y + (4.5)^2) + 4 - 6.75 \\&= \frac{1}{3}(y - 4.5)^2 - 2.75\end{aligned}$$

The vertex of the equation is $(-2.75, 4.5)$; axis of symmetry: $y = 4.5$; The graph opens right.

ANSWER:

$x = \frac{1}{3}(y - 4.5)^2 - 2.75$; vertex = $(-2.75, 4.5)$; axis of symmetry: $y = 4.5$; opens right

19. $x = \frac{2}{3}y^2 - 4y + 12$

SOLUTION:

Equation in standard form:

$$\begin{aligned}x &= \frac{2}{3}y^2 - 4y + 12 \\&= \frac{2}{3}(y^2 - 6y + 9) + 12 - 6 \\&= \frac{2}{3}(y - 3)^2 + 6\end{aligned}$$

The vertex of the equation is $(6, 3)$; axis of symmetry: $y = 3$; The graph opens right.

ANSWER:

$x = \frac{2}{3}(y - 3)^2 + 6$; vertex = $(6, 3)$; axis of symmetry: $y = 3$; opens right

Graph each equation.

20. $y = \frac{1}{3}x^2$

SOLUTION:

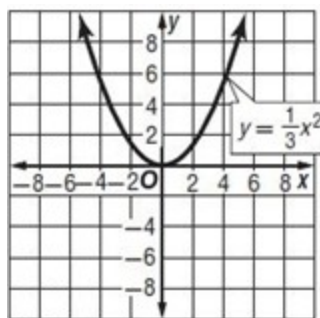
vertex: $(0, 0)$

axis of symmetry: $x = 0$

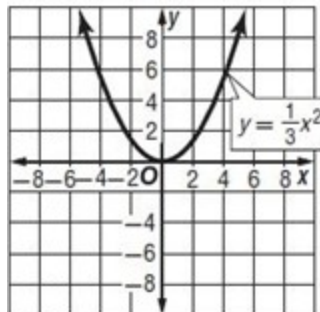
focus: $\left(0, \frac{3}{4}\right)$

directrix: $y = -\frac{3}{4}$

length of the latus rectum: 3



ANSWER:



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21. $y = -2x^2$

SOLUTION:

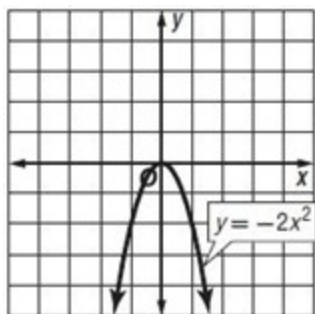
vertex: $(0, 0)$

axis of symmetry: $x = 0$

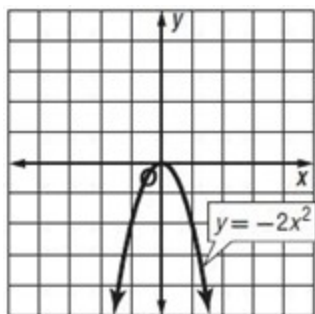
focus: $\left(0, -\frac{1}{8}\right)$

directrix: $y = \frac{1}{8}$

length of the latus rectum: $\frac{1}{2}$



ANSWER:



22. $y = -2(x - 2)^2 + 3$

SOLUTION:

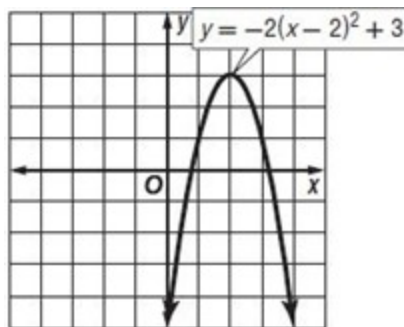
vertex: $(2, 3)$

axis of symmetry: $x = 2$

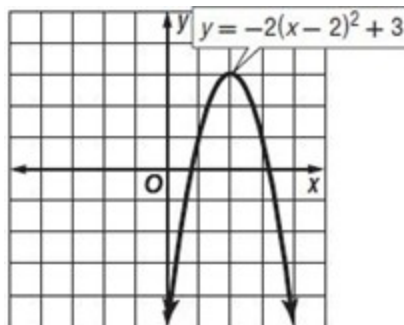
focus: $\left(2, \frac{23}{8}\right)$

directrix: $y = \frac{25}{8}$

length of the latus rectum: $\frac{1}{2}$



ANSWER:



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23. $y = 3(x - 3)^2 - 5$

SOLUTION:

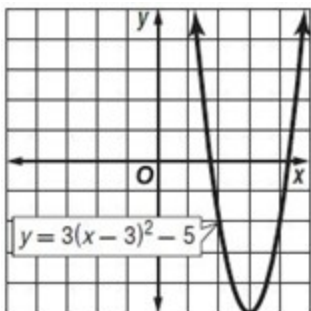
vertex: $(3, -5)$

axis of symmetry: $x = 3$

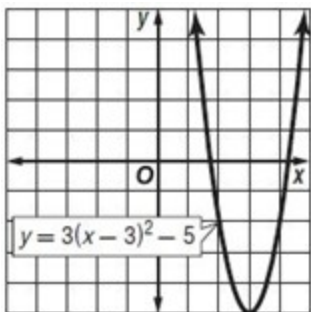
focus: $\left(3, -\frac{59}{12}\right)$

directrix: $y = -\frac{61}{12}$

length of the latus rectum: $\frac{1}{3}$



ANSWER:



24. $x = \frac{1}{2}y^2$

SOLUTION:

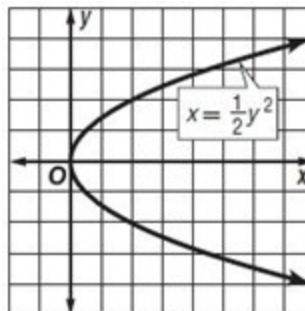
vertex: $(0, 0)$

axis of symmetry: $y = 0$

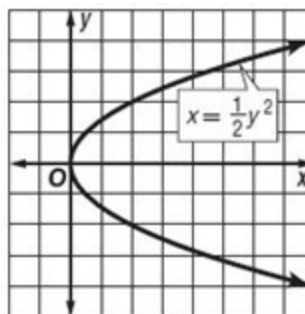
focus: $(0.5, 0)$

directrix: $x = -0.5$

length of the latus rectum: 2



ANSWER:



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25. $4x - y^2 = 2y + 13$

SOLUTION:

$$4x - y^2 = 2y + 13$$

$$4x = (y^2 + 2y + 1) + 13 - 1$$

$$4x = (y + 1)^2 + 12$$

$$x = \frac{1}{4}(y + 1)^2 + 3$$

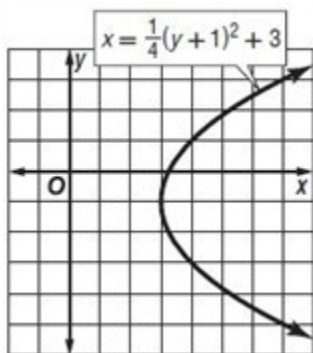
vertex: $(3, -1)$

axis of symmetry: $y = -1$

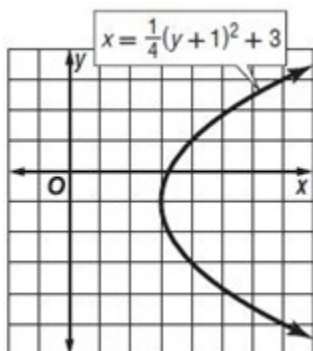
focus: $(4, -1)$

directrix: $x = 2$

length of the latus rectum: 4



ANSWER:



Write an equation for each parabola described below. Then graph the equation.

26. vertex $(0, 1)$, focus $(0, 4)$

SOLUTION:

The vertex is at $(0, 1)$, so $h = 0$ and $k = 1$.

The focus is at $(0, 4)$, so $h = 0$ and $k + \frac{1}{4a} = 4$.

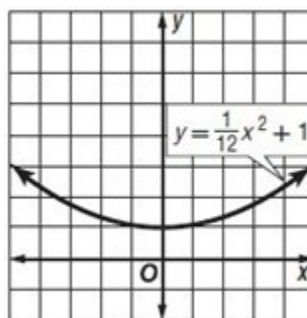
Thus, $a = \frac{1}{12}$.

So, the equation of the parabola is $y = \frac{1}{12}x^2 + 1$.

axis of symmetry: $x = 0$

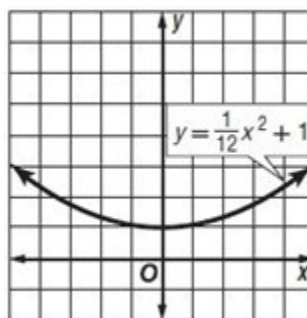
directrix: $y = -2$

length of the latus rectum: 12



ANSWER:

$$y = \frac{1}{12}x^2 + 1$$



27. vertex $(1, 8)$, directrix $y = 3$

SOLUTION:

The directrix is a horizontal line, so the equation of the parabola is of the form $y = a(x - h)^2 + k$.

The vertex is at $(1, 8)$, so $h = 1$ and $k = 8$.

Use the equation of the directrix to find a .

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$$y = k - \frac{1}{4a}$$

$$3 = 8 - \frac{1}{4a}$$

$$-20a = -1$$

$$a = \frac{1}{20}$$

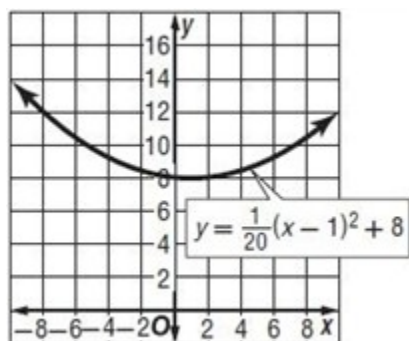
So, the equation of the parabola is

$$y = \frac{1}{20}(x-1)^2 + 8.$$

axis of symmetry: $x = 1$

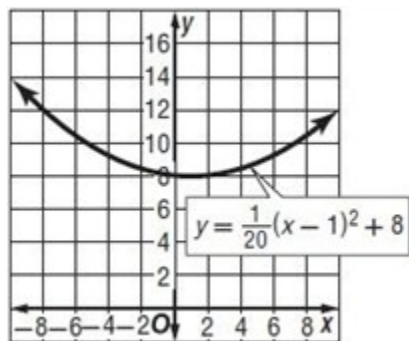
focus: $(1, 13)$

length of the latus rectum: 20



ANSWER:

$$y = \frac{1}{20}(x-1)^2 + 8$$



28. focus $(-2, -4)$, directrix $x = -6$

SOLUTION:

The directrix is a vertical line, so the equation of the parabola is of the form $x = a(y-k)^2 + h$.

The focus is at $(-2, -4)$, so $h + \frac{1}{4a} = -2$ and $k = -4$.

Thus, $h = -2 - \frac{1}{4a}$.

Use the equation of the directrix to find a .

$$x = h - \frac{1}{4a}$$

$$-6 = \left(-2 - \frac{1}{4a}\right) - \frac{1}{4a}$$

$$-4 = -\frac{1}{2a}$$

$$a = \frac{1}{8}$$

Find h by substituting the value of a in $h = -2 - \frac{1}{4a}$.

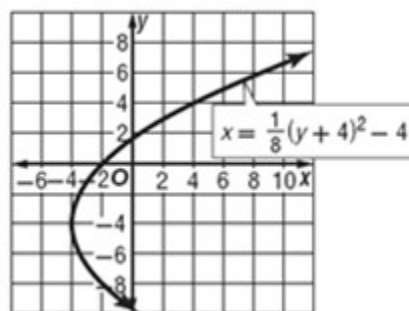
$$\begin{aligned} h &= -2 - \frac{1}{4\left(\frac{1}{8}\right)} \\ &= -2 - 2 \\ &= -4 \end{aligned}$$

So, the equation of the parabola is $x = \frac{1}{8}(y+4)^2 - 4$.

vertex: $(-4, -4)$

axis of symmetry: $y = -4$

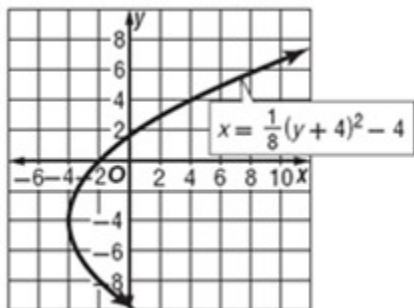
length of the latus rectum: 8



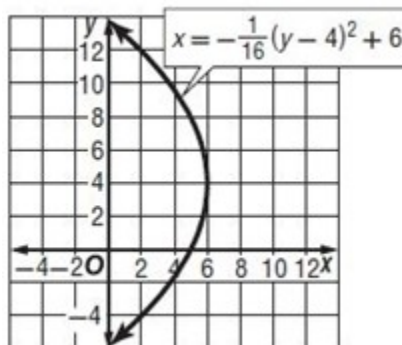
ANSWER:

$$x = \frac{1}{8}(y+4)^2 - 4$$

9-2 Parabolas



vertex: (6, 4)
axis of symmetry: $y = 4$
length of the latus rectum: 16



29. focus (2, 4), directrix $x = 10$

SOLUTION:

The directrix is a vertical line, so the equation of the parabola is of the form $x = a(y - k)^2 + h$.

The focus is at (2, 4), so $h + \frac{1}{4a} = 2$ and $k = 4$.

Thus, $h = 2 - \frac{1}{4a}$.

Use the equation of the directrix to find a .

$$x = h - \frac{1}{4a}$$

$$10 = 2 - \frac{1}{4a} - \frac{1}{4a}$$

$$8 = -\frac{1}{2a}$$

$$a = -\frac{1}{16}$$

Find h by substituting the value of a in $h = 2 - \frac{1}{4a}$.

$$h = 2 - \frac{1}{4\left(-\frac{1}{16}\right)}$$

$$= 2 + 4$$

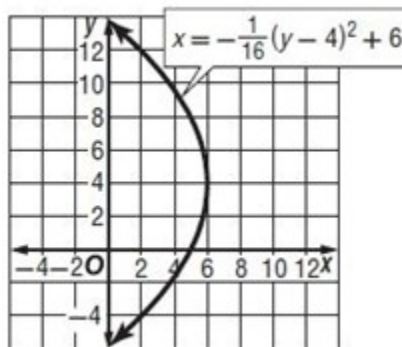
$$= 6$$

So, the equation of the parabola is

$$x = -\frac{1}{16}(y - 4)^2 + 6.$$

ANSWER:

$$x = -\frac{1}{16}(y - 4)^2 + 6$$



9-2 Parabolas

30. vertex $(-6, 0)$, directrix $x = 2$

SOLUTION:

The directrix is a vertical line, so the equation of the parabola is of the form $x = a(y - k)^2 + h$.

The vertex is at $(-6, 0)$, so $h = -6$ and $k = 0$.

Use the equation of the directrix to find a .

$$x = h - \frac{1}{4a}$$

$$2 = -6 - \frac{1}{4a}$$

$$8 = -\frac{1}{4a}$$

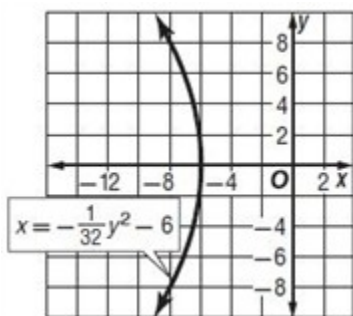
$$a = -\frac{1}{32}$$

So, the equation of the parabola is $x = -\frac{1}{32}y^2 - 6$.

axis of symmetry: $y = 0$

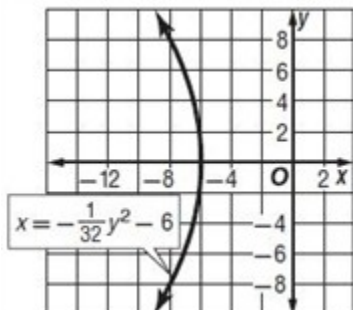
focus: $(-14, 0)$

length of the latus rectum: 32



ANSWER:

$$x = -\frac{1}{32}y^2 - 6$$



31. vertex $(9, 6)$, focus $(9, 5)$

SOLUTION:

The vertex is at $(9, 6)$, so $h = 9$ and $k = 6$.

The focus is at $(9, 5)$, so $h = 9$ and $k + \frac{1}{4a} = 5$

Thus, $a = -\frac{1}{4}$.

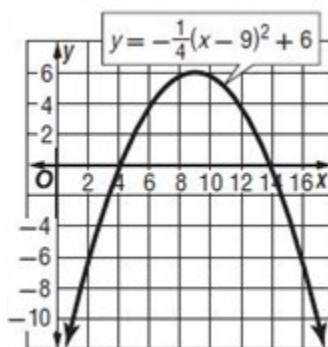
So, the equation of the parabola is

$$y = -\frac{1}{4}(x - 9)^2 + 6$$

axis of symmetry: $x = 9$

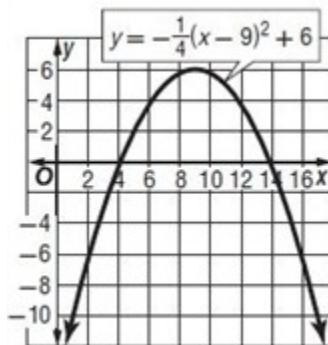
directrix: $y = 7$

length of the latus rectum: 4



ANSWER:

$$y = -\frac{1}{4}(x - 9)^2 + 6$$



9-2 Parabolas

32. **BASEBALL** When a ball is thrown, the path it travels is a parabola. Suppose a baseball is thrown from ground level, reaches a maximum height of 50 feet, and hits the ground 200 feet from where it was thrown. Assuming this situation could be modeled on a coordinate plane with the focus of the parabola at the origin, find the equation of the parabolic path of the ball. Assume the focus is on ground level.

SOLUTION:

The parabola must open downward.

Therefore, the vertex must be above the focus.

The vertex is $(0, 50)$, so $h = 0$ and $k = 50$.

The focus is at $(0, 0)$.

The measure of the latus rectum is 200, so $a =$

$$-\frac{1}{200}.$$

The equation of the parabola is $y = -\frac{1}{200}x^2 + 50$.

ANSWER:

$$y = -\frac{1}{200}x^2 + 50$$

33. **CCSS PERSEVERANCE** Ground antennas and satellites are used to relay signals between the NASA Mission Operations Center and the spacecraft it controls. One such parabolic dish is 146 feet in diameter. Its focus is 48 feet from the vertex.

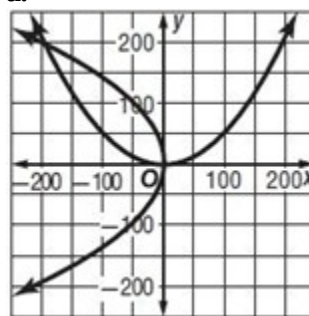
a. Sketch two options for the dish, one that opens up and one that opens left.

b. Write two equations that model the sketches in part a.

c. If you wanted to find the depth of the dish, does it matter which equation you use? Why or why not?

SOLUTION:

a.



b.

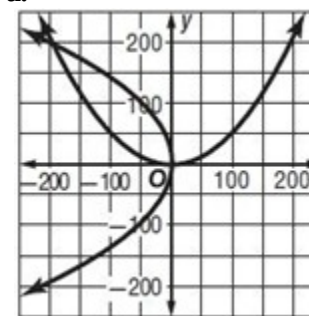
$$y = \frac{x^2}{192} \text{ and } x = \frac{y^2}{-192}$$

c.

Sample answer: No; except for the direction in which they open, the graphs are identical.

ANSWER:

a.



$$b. y = \frac{x^2}{192} \text{ and } x = \frac{y^2}{-192}$$

c. Sample answer: No; except for the direction in which they open, the graphs are identical.

9-2 Parabolas

34. **UMBRELLAS** A beach umbrella has an arch in the shape of a parabola that opens downward. The umbrella spans 6 feet across and is $1\frac{1}{2}$ feet high.

Write an equation of a parabola to model the arch, assuming that the origin is at the point where the pole and umbrella meet, beneath the vertex of the arch.

SOLUTION:

The vertex is at $\left(0, \frac{3}{2}\right)$, so $h = 0$ and $k = \frac{3}{2}$.

The focus is at $(0, 0)$, so $h = 0$ and $k + \frac{1}{4a} = 0$

Thus, $a = -\frac{1}{6}$.

The equation of the parabola is $y = -\frac{1}{6}x^2 + \frac{3}{2}$.

ANSWER:

$$y = -\frac{1}{6}x^2 + \frac{3}{2}$$

35. **AUTOMOBILES** An automobile headlight contains a parabolic reflector. The light coming from the source bounces off the parabolic reflector and shines out the front of the headlight. The equation of the cross section of the reflector is $y = \frac{1}{12}x^2$. How far from the vertex should the filament for the high beams be placed?

SOLUTION:

From the equation $y = \frac{1}{12}x^2$, $h = 0$, $k = 0$ and $a = \frac{1}{12}$.

Substitute these values in the equation of the directrix and simplify.

$$\begin{aligned}\left(h, k + \frac{1}{4a}\right) &= \left(0, 0 + \frac{1}{4\left(\frac{1}{12}\right)}\right) \\ &= (0, 3)\end{aligned}$$

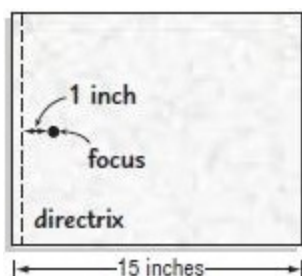
Thus, the filament should be placed 3 units from the vertex.

ANSWER:

3 units

9-2 Parabolas

36. **MULTIPLE REPRESENTATIONS** Start with a sheet of wax paper that is about 15 inches long and 12 inches wide.
- a. CONCRETE** Make a line that is perpendicular to the sides of the sheet by folding the sheet near one end. Open up the paper again. This line is the directrix. Mark a point about midway between the sides of the sheet so that the distance from the directrix is about 1 inch. This is the focus.
- b. CONCRETE** Start with a new sheet of wax paper. Form another outline of a parabola with a focus that is about 3 inches from the directrix.
- c. CONCRETE** On a new sheet of a wax paper, form a third outline of a parabola with a focus that is about 5 inches from the directrix.
- d. VERBAL** Compare the shapes of the three parabolas. How does the distance between the focus and the directrix affect the shape of a parabola?



SOLUTION:

a–c. See students' work.

d. As the distance between the directrix and the focus increases, the parabola becomes wider.

ANSWER:

a–c. See students' work.

d. As the distance between the directrix and the focus increases, the parabola becomes wider.

37. **REASONING** How do you change the equation of the parent function $y = x^2$ to shift the graph to the right

SOLUTION:

Rewrite it as $y = (x - h)^2$, where $h > 0$.

ANSWER:

Rewrite it as $y = (x - h)^2$, where $h > 0$.

38. **OPEN ENDED** Two different parabolas have their vertex at $(-3, 1)$ and contain the point with coordinates $(-1, 0)$. Write two possible equations for these parabolas.

SOLUTION:

Sample answers:

$$y = -\frac{1}{4}(x + 3)^2 + 1 \text{ and } x = 2(y - 1)^2 - 3$$

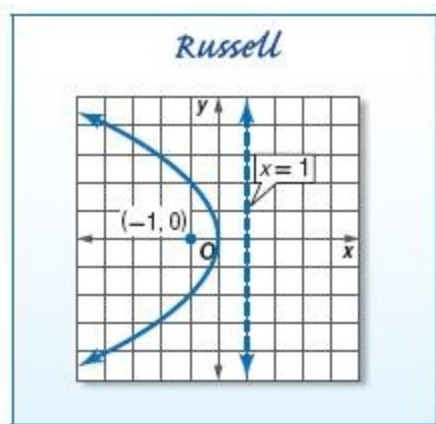
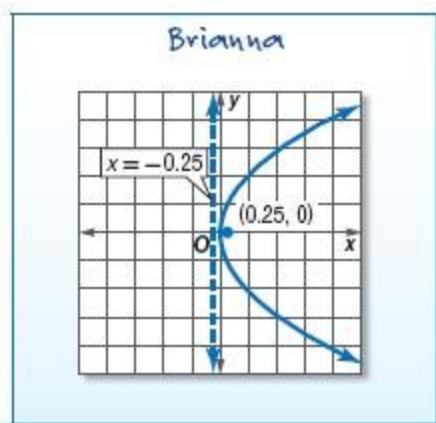
ANSWER:

Sample answers:

$$y = -\frac{1}{4}(x + 3)^2 + 1 \text{ and } x = 2(y - 1)^2 - 3$$

9-2 Parabolas

39. **CCSS CRITIQUE** Brianna and Russell are graphing $\frac{1}{4}y^2 + x = 0$. Is either of them correct? Explain your reasoning.



SOLUTION:

Russell; the parabola should open to the left rather than to the right.

ANSWER:

Russell; the parabola should open to the left rather than to the right.

40. **WRITING IN MATH** Why are parabolic shapes used in the real world?

SOLUTION:

Sample answer: When rays which are parallel to the axis of symmetry of a parabolic mirror are reflected off the mirror, they are directed to the focus. This focuses all of the rays at one specific point. Also, a parabolic microphone can be used to make capturing sound more effective, because reflected sound waves are focused at the particular point where the microphone is located.

ANSWER:

Sample answer: When rays which are parallel to the axis of symmetry of a parabolic mirror are reflected off the mirror, they are directed to the focus. This focuses all of the rays at one specific point. Also, a parabolic microphone can be used to make capturing sound more effective, because reflected sound waves are focused at the particular point where the microphone is located.

41. A gardener is placing a fence around a 1320-square-foot rectangular garden. He ordered 148 feet of fencing. If he uses all the fencing, what is the length of the longer side of the garden?
- A** 30 ft
B 34 ft
C 44 ft
D 46 ft

SOLUTION:

Let l and w be the length and width of the garden respectively.

The equations that represent the situation are

$$w = \frac{1320}{l};$$

$$2l + 2w = 148$$

Thus, the length of the garden is 44 ft.
C is the correct option.

ANSWER:

C

9-2 Parabolas

42. **SAT/ACT** When a number is divided by 5, the result is 7 more than the number. Find the number.

F $-\frac{35}{4}$

G $-\frac{35}{6}$

H $\frac{35}{6}$

J $\frac{28}{4}$

K $\frac{35}{4}$

SOLUTION:

Let the unknown number be x .

The equation that represents the situation is

$$\frac{x}{5} = x + 7.$$

The value of x is $-\frac{35}{4}$.

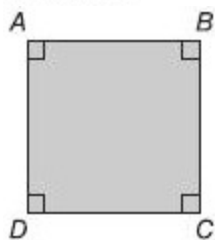
F is the correct option.

ANSWER:

F

43. **GEOMETRY** What is the area of the following square, if the length of

$\overline{BD}$ is $2\sqrt{2}$?



A 1

B 2

C 3

D 4

SOLUTION:

The length of the sides of the square is 2 units.

So, the area of the square is 4 square units.

D is the correct option.

ANSWER:

D

44. **SHORT RESPONSE** The measure of the smallest angle of a triangle is two thirds the measure of the middle angle. The measure of the middle angle is three sevenths of the measure of the largest angle. Find the largest angle's measure.

SOLUTION:

Let the largest angle be x .

Writing all the angles in terms of largest angle, we

get the equation $x + \frac{3}{7}x + \frac{2}{7}x = 180$.

Thus the angle of the largest measure is 105° .

ANSWER:

105°

45. **GEOMETRY** Find the perimeter of a triangle with vertices at (2, 4), (−1, 3), and (1, −3).

SOLUTION:

The distance between (2, 4) and (−1, 3) is $\sqrt{10}$.

The distance between (−1, 3) and (1, −3) is $2\sqrt{10}$.

The distance between (2, 4) and (1, −3) is $5\sqrt{2}$.

Perimeter of the triangle:

$$\begin{aligned} &\sqrt{10} + 2\sqrt{10} + 5\sqrt{2} \\ &= 3\sqrt{10} + 5\sqrt{2} \text{ units} \end{aligned}$$

ANSWER:

$$5\sqrt{2} + 3\sqrt{10} \text{ units}$$

9-2 Parabolas

46. **WORK** A worker can powerwash a wall of a certain size in 5 hours. Another worker can do the same job in 4 hours. If the workers work together, how long would it take to do the job? Determine whether your answer is reasonable.

SOLUTION:

$$\begin{aligned}\frac{x}{5} + \frac{x}{4} &= 1 \\ \frac{4x + 5x}{20} &= 1 \\ \frac{9x}{20} &= 1 \\ x &= \frac{20}{9} \\ x &= 2\frac{2}{9}\end{aligned}$$

So, the workers will take $2\frac{2}{9}$ hours to complete the job.

The answer is reasonable. The time to complete the job when working together must be less than the time it would take either person working alone.

ANSWER:

$2\frac{2}{9}$ h ; The answer is reasonable. The time to complete the job when working together must be less than the time it would take either person working alone.

Solve each equation or inequality. Round to the nearest ten-thousandth.

47. $\ln(x + 1) = 1$

SOLUTION:

$$\begin{aligned}\ln(x + 1) &= 1 \\ \log_e(x + 1) &= 1 \\ x + 1 &= e^1 \\ x &\approx 1.7183\end{aligned}$$

ANSWER:

1.7183

48. $\ln(x - 7) = 2$

SOLUTION:

$$\begin{aligned}\ln(x - 7) &= 2 \\ \log_e(x - 7) &= 2 \\ x - 7 &= e^2 \\ x &\approx 14.3891\end{aligned}$$

ANSWER:

14.3891

49. $e^x > 1.6$

SOLUTION:

$$\begin{aligned}e^x &> 1.6 \\ \ln e^x &> \ln 1.6 \\ x &> \ln 1.6 \\ x &> 0.4700\end{aligned}$$

ANSWER:

$$x > 0.4700$$

50. $e^{5x} \geq 25$

SOLUTION:

$$\begin{aligned}e^{5x} &\geq 25 \\ \ln e^{5x} &\geq \ln 25 \\ 5x &\geq \ln 25 \\ x &\geq 0.6438\end{aligned}$$

ANSWER:

$$x \geq 0.6438$$

Simplify.

51. $\sqrt{0.25}$

SOLUTION:

$$\begin{aligned}\sqrt{0.25} &= \sqrt{0.5 \times 0.5} \\ &= 0.5\end{aligned}$$

ANSWER:

0.5

9-2 Parabolas

52. $\sqrt[3]{-0.064}$

SOLUTION:

$$\begin{aligned}\sqrt[3]{-0.064} &= \sqrt[3]{-0.4 \times -0.4 \times -0.4} \\ &= -0.4\end{aligned}$$

ANSWER:

$$-0.4$$

53. $\sqrt[4]{z^8}$

SOLUTION:

$$\begin{aligned}\sqrt[4]{z^8} &= \sqrt[4]{(z^2)^4} \\ &= z^2\end{aligned}$$

ANSWER:

$$z^2$$

54. $-\sqrt[6]{x^6}$

SOLUTION:

$$-\sqrt[6]{x^6} = -|x|$$

ANSWER:

$$-|x|$$

List all of the possible rational zeros of each function.

55. $h(x) = x^3 + 8x + 6$

SOLUTION:

$$\pm 1, \pm 2, \pm 3, \pm 6$$

ANSWER:

$$\pm 1, \pm 2, \pm 3, \pm 6$$

56. $p(x) = 3x^3 - 5x^2 - 11x + 3$

SOLUTION:

$$\pm 1, \pm \frac{1}{3}, \pm 3$$

ANSWER:

$$\pm 1, \pm \frac{1}{3}, \pm 3$$

57. $h(x) = 9x^6 - 5x^3 + 27$

SOLUTION:

$$\pm 1, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm 3, \pm 9, \pm 27$$

ANSWER:

$$\pm 1, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm 3, \pm 9, \pm 27$$

Simplify each expression.

58. $\sqrt{24}$

SOLUTION:

$$\begin{aligned}\sqrt{24} &= \sqrt{2 \times 2 \times 2 \times 3} \\ &= 2\sqrt{6}\end{aligned}$$

ANSWER:

$$2\sqrt{6}$$

59. $\sqrt{45}$

SOLUTION:

$$\begin{aligned}\sqrt{45} &= \sqrt{3 \times 3 \times 5} \\ &= 3\sqrt{5}\end{aligned}$$

ANSWER:

$$3\sqrt{5}$$

60. $\sqrt{252}$

SOLUTION:

$$\begin{aligned}\sqrt{252} &= \sqrt{4 \times 9 \times 7} \\ &= 6\sqrt{7}\end{aligned}$$

ANSWER:

$$6\sqrt{7}$$

61. $\sqrt{512}$

SOLUTION:

$$\begin{aligned}\sqrt{512} &= \sqrt{256 \times 2} \\ &= 16\sqrt{2}\end{aligned}$$

ANSWER:

$$16\sqrt{2}$$

9-3 Circles

1. **WEATHER** On average, the eye of a tornado is about 200 feet across. Suppose the center of the eye is at the point $(72, 39)$. Write an equation to represent the boundary of the eye.

SOLUTION:

The center of the circle is at $(72, 39)$, so $h = 72$ and $k = 39$.

Radius of the circle is 100.

So, the equation of the circle is

$$(x - 72)^2 + (y - 39)^2 = 10,000$$

ANSWER:

$$(x - 72)^2 + (y - 39)^2 = 10,000$$

Write an equation for each circle given the center and radius.

2. center: $(-2, -6)$, $r = 4$ units

SOLUTION:

$h = -2$ and $k = -6$

The equation of the circle is $(x + 2)^2 + (y + 6)^2 = 16$.

ANSWER:

$$(x + 2)^2 + (y + 6)^2 = 16.$$

3. center: $(1, -5)$, $r = 3$ units

SOLUTION:

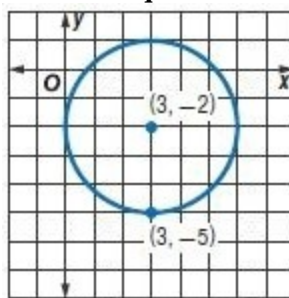
$h = 1$ and $k = -5$

The equation of the circle is $(x - 1)^2 + (y + 5)^2 = 9$.

ANSWER:

$$(x - 1)^2 + (y + 5)^2 = 9.$$

Write an equation for each graph.



4.

SOLUTION:

The center of the circle is at $(3, -2)$, so $h = 3$ and $k = -2$.

Radius of the circle is 3.

So, the equation of the circle is

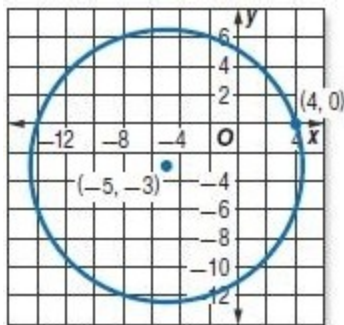
$$(x - 3)^2 + (y + 2)^2 = 9$$

ANSWER:

$$(x - 3)^2 + (y + 2)^2 = 9$$

9-3 Circles

5.



SOLUTION:

The center of the circle is at $(-5, -3)$, so $h = -5$ and $k = -3$.

Find the radius.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 + 5)^2 + (0 + 3)^2} \\ &= \sqrt{81 + 9} \\ &= \sqrt{90} \end{aligned}$$

Thus, the radius of the circle is $\sqrt{90}$.

So, the equation of the circle is

$$(x + 5)^2 + (y + 3)^2 = 90$$

ANSWER:

$$(x + 5)^2 + (y + 3)^2 = 90$$

Write an equation for each circle given the endpoints of a diameter.

6. $(-1, -7)$ and $(0, 0)$

SOLUTION:

Find the center.

$$(x_1, y_1) = (-1, -7), (x_2, y_2) = (0, 0)$$

$$\begin{aligned} (h, k) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-1 + 0}{2}, \frac{-7 + 0}{2} \right) \\ &= \left(-\frac{1}{2}, -\frac{7}{2} \right) \end{aligned}$$

Find the radius.

$$(x_1, y_1) = \left(-\frac{1}{2}, -\frac{7}{2} \right), (x_2, y_2) = (0, 0)$$

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(0 + \frac{1}{2} \right)^2 + \left(0 + \frac{7}{2} \right)^2} \\ &= \sqrt{\frac{50}{4}} \\ &= \frac{5\sqrt{2}}{2} \end{aligned}$$

The equation of the circle is

$$\left(x + \frac{1}{2} \right)^2 + \left(y + \frac{7}{2} \right)^2 = \frac{25}{2}$$

ANSWER:

$$\left(x + \frac{1}{2} \right)^2 + \left(y + \frac{7}{2} \right)^2 = \frac{25}{2}$$

9-3 Circles

7. $(4, -2)$ and $(-4, -6)$

SOLUTION:

Find the center.

$$(x_1, y_1) = (4, -2), (x_2, y_2) = (-4, -6)$$

$$(h, k) = \left(\frac{4 - 4}{2}, \frac{-2 - 6}{2} \right) \\ = (0, -4)$$

Find the radius.

$$(x_1, y_1) = (0, -4), (x_2, y_2) = (-4, -6)$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ = \sqrt{(-4 - 0)^2 + (-6 + 4)^2} \\ = \sqrt{20}$$

The equation of the circle is $x^2 + (y + 4)^2 = 20$.

ANSWER:

$$x^2 + (y + 4)^2 = 20$$

Find the center and radius of each circle. Then graph the circle.

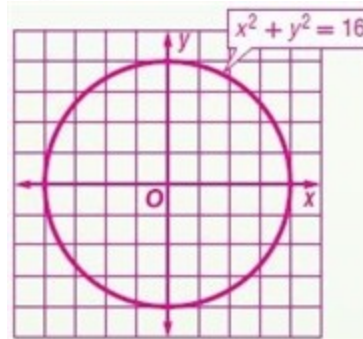
8. $x^2 + y^2 = 16$

SOLUTION:

$h = 0$, and $k = 0$, so center of the circle is $(0, 0)$.

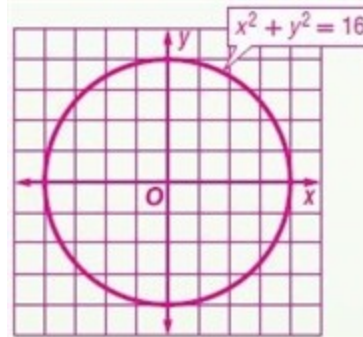
$$r^2 = 16$$

So, $r = 4$.



ANSWER:

center: $(0, 0)$; radius: 4



9-3 Circles

9. $x^2 + (y - 7)^2 = 9$

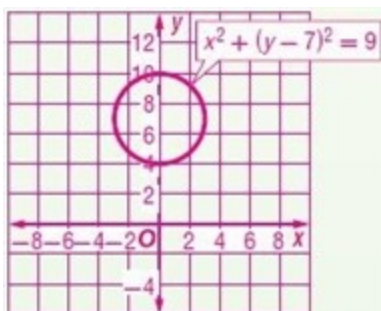
SOLUTION:

$h = 0$, and $k = 7$, so center of the circle is $(0, 7)$.

Find the radius.

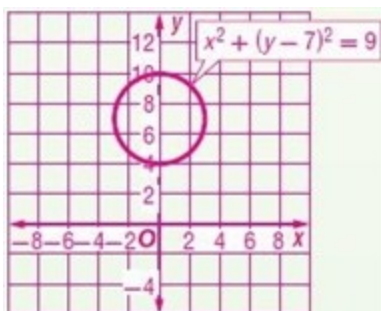
$$r^2 = 9$$

$$r = 3$$



ANSWER:

center: $(0, 7)$; radius: 3



10. $(x - 4)^2 + (y - 4)^2 = 25$

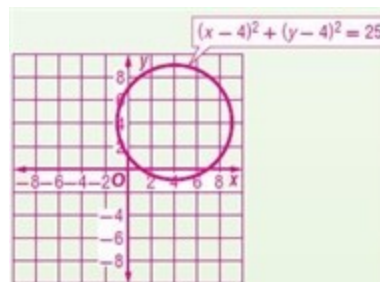
SOLUTION:

$h = 4$, and $k = 4$, so center of the circle is $(4, 4)$.

Find the radius.

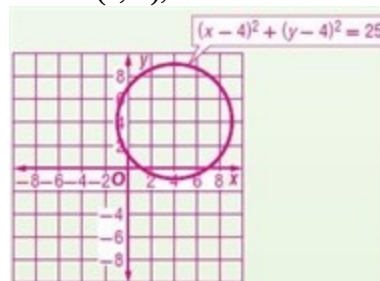
$$r^2 = 25$$

$$r = 5$$



ANSWER:

center: $(4, 4)$; radius: 5



9-3 Circles

11. $x^2 + y^2 - 4x + 8y - 5 = 0$

SOLUTION:

Complete the squares:

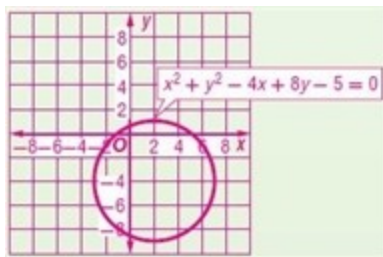
$$\begin{aligned}x^2 + y^2 - 4x + 8y - 5 &= 0 \\(x^2 - 4x + 4) + (y^2 + 8y + 16) - 5 - 4 - 16 &= 0 \\(x - 2)^2 + (y + 4)^2 &= 25\end{aligned}$$

$h = 2$, and $k = -4$, so center of the circle is $(2, -4)$.

Find the radius.

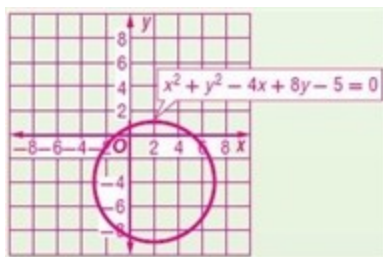
$$r^2 = 25$$

$$r = 5$$



ANSWER:

center: $(2, -4)$; radius: 5



Write an equation for each circle given the center and radius.

12. center: $(4, 9)$, $r = 6$

SOLUTION:

$$h = 4 \text{ and } k = 9$$

The equation of the circle is $(x - 4)^2 + (y - 9)^2 = 36$.

ANSWER:

$$(x - 4)^2 + (y - 9)^2 = 36.$$

13. center: $(-3, 1)$, $r = 4$

SOLUTION:

$$h = -3 \text{ and } k = 1$$

The equation of the circle is $(x + 3)^2 + (y - 1)^2 = 16$.

ANSWER:

$$(x + 3)^2 + (y - 1)^2 = 16$$

14. center: $(-7, -3)$, $r = 13$

SOLUTION:

$$h = -7 \text{ and } k = -3$$

The equation of the circle is

$$(x + 7)^2 + (y + 3)^2 = 169.$$

ANSWER:

$$(x + 7)^2 + (y + 3)^2 = 169$$

15. center: $(-2, -1)$, $r = 9$

SOLUTION:

$$h = -2 \text{ and } k = -1$$

The equation of the circle is $(x + 2)^2 + (y + 1)^2 = 81$.

ANSWER:

$$(x + 2)^2 + (y + 1)^2 = 81$$

16. center: $(1, 0)$, $r = \sqrt{15}$

SOLUTION:

$$h = 1 \text{ and } k = 0$$

The equation of the circle is $(x - 1)^2 + y^2 = 15$.

ANSWER:

$$(x - 1)^2 + y^2 = 15$$

17. center: $(0, -6)$, $r = \sqrt{35}$

SOLUTION:

$$h = 0 \text{ and } k = -6$$

The equation of the circle is $x^2 + (y + 6)^2 = 35$.

ANSWER:

$$x^2 + (y + 6)^2 = 35$$

9-3 Circles

18. **CCSS MODELING** The radar for a county airport control tower is located at (5, 10) on a map. It can detect a plane up to 20 miles away. Write an equation for the outer limits of the detection area.

SOLUTION:

The center of the circle is at (5, 10), so $h = 5$ and $k = 10$.

Radius of the circle is 20.

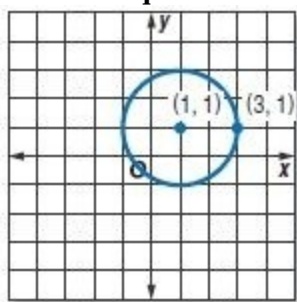
So, the equation of the circle is

$$(x - 5)^2 + (y - 10)^2 = 400.$$

ANSWER:

$$(x - 5)^2 + (y - 10)^2 = 400$$

Write an equation for each graph.



19.

SOLUTION:

The center of the circle is at (1, 1), so $h = 1$ and $k = 1$.

Find the radius.

$$(x_1, y_1) = (1, 1), (x_2, y_2) = (3, 1)$$

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 1)^2 + (1 - 1)^2} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

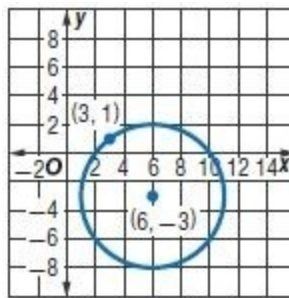
Thus, the radius of the circle is 2.

So, the equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = 4.$$

ANSWER:

$$(x - 1)^2 + (y - 1)^2 = 4$$



20.

SOLUTION:

The center of the circle is at (6, -3), so $h = 6$ and $k = -3$.

Find the radius.

$$(x_1, y_1) = (6, -3), (x_2, y_2) = (3, 1)$$

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 6)^2 + (1 + 3)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Thus, the radius of the circle is 5.

So, the equation of the circle is

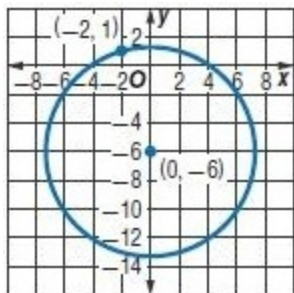
$$(x - 6)^2 + (y + 3)^2 = 25.$$

ANSWER:

$$(x - 6)^2 + (y + 3)^2 = 25$$

9-3 Circles

21.



SOLUTION:

The center of the circle is at $(0, -6)$, so $h = 0$ and $k = -6$.

Find the radius.

$$(x_1, y_1) = (0, -6), (x_2, y_2) = (-2, 1)$$

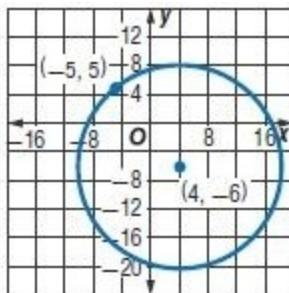
$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 0)^2 + (1 + 6)^2} \\ &= \sqrt{53} \end{aligned}$$

So, the equation of the circle is $x^2 + (y + 6)^2 = 53$.

ANSWER:

$$x^2 + (y + 6)^2 = 53$$

22.



SOLUTION:

The center of the circle is at $(4, -6)$, so $h = 4$ and $k = -6$.

Find the radius.

$$(x_1, y_1) = (4, -6), (x_2, y_2) = (-5, 5)$$

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 4)^2 + (5 + 6)^2} \\ &= \sqrt{202} \end{aligned}$$

So, the equation of the circle is

$$(x - 4)^2 + (y + 6)^2 = 202$$

ANSWER:

$$(x - 4)^2 + (y + 6)^2 = 202$$

9-3 Circles

Write an equation for each circle given the endpoints of a diameter.

23. (2, 1) and (2, -4)

SOLUTION:

Find the center.

$$(x_1, y_1) = (2, 1), (x_2, y_2) = (2, -4)$$

$$\begin{aligned}(h, k) &= \left(\frac{2+2}{2}, \frac{1-4}{2} \right) \\ &= \left(2, -\frac{3}{2} \right)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = \left(2, -\frac{3}{2} \right), (x_2, y_2) = (2, -4)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-2)^2 + \left(-4 + \frac{3}{2}\right)^2} \\ &= \frac{5}{2}\end{aligned}$$

The equation of the circle is

$$(x-2)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}.$$

ANSWER:

$$(x-2)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}$$

24. (-4, -10) and (4, -10)

SOLUTION:

Find the center.

$$(x_1, y_1) = (-4, -10), (x_2, y_2) = (4, -10)$$

$$\begin{aligned}(h, k) &= \left(\frac{-4+4}{2}, \frac{-10-10}{2} \right) \\ &= (0, -10)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = (0, -10), (x_2, y_2) = (4, -10)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-0)^2 + (-10+10)^2} \\ &= 4\end{aligned}$$

The equation of the circle is $x^2 + (y+10)^2 = 16$.

ANSWER:

$$x^2 + (y+10)^2 = 16$$

9-3 Circles

25. $(5, -7)$ and $(-2, -9)$

SOLUTION:

Find the center.

$$(x_1, y_1) = (5, -7), (x_2, y_2) = (-2, -9)$$

$$\begin{aligned}(h, k) &= \left(\frac{5-2}{2}, \frac{-7-9}{2} \right) \\ &= \left(\frac{3}{2}, -8 \right)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = \left(\frac{3}{2}, -8 \right), (x_2, y_2) = (-2, -9)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(-2 - \frac{3}{2}\right)^2 + (-9 + 8)^2} \\ &= \sqrt{\frac{53}{4}} \\ &= \frac{\sqrt{53}}{2}\end{aligned}$$

The equation of the circle is

$$\left(x - \frac{3}{2}\right)^2 + (y + 8)^2 = \frac{53}{4}.$$

ANSWER:

$$\left(x - \frac{3}{2}\right)^2 + (y + 8)^2 = \frac{53}{4}$$

26. $(-6, 4)$ and $(4, 8)$

SOLUTION:

Find the center.

$$(x_1, y_1) = (-6, 4), (x_2, y_2) = (4, 8)$$

$$\begin{aligned}(h, k) &= \left(\frac{-6+4}{2}, \frac{4+8}{2} \right) \\ &= (-1, 6)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = (-1, 6), (x_2, y_2) = (4, 8)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4+1)^2 + (8-6)^2} \\ &= \sqrt{29}\end{aligned}$$

The equation of the circle is $(x+1)^2 + (y-6)^2 = 29$.

ANSWER:

$$(x+1)^2 + (y-6)^2 = 29$$

9-3 Circles

27. $(2, -5)$ and $(6, 3)$

SOLUTION:

Find the center.

$$(x_1, y_1) = (2, -5), (x_2, y_2) = (6, 3)$$

$$\begin{aligned}(h, k) &= \left(\frac{2+6}{2}, \frac{-5+3}{2} \right) \\ &= (4, -1)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = (4, -1), (x_2, y_2) = (6, 3)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6-4)^2 + (3+1)^2} \\ &= \sqrt{20} \\ &= 2\sqrt{5}\end{aligned}$$

The equation of the circle is $(x-4)^2 + (y+1)^2 = 20$.

ANSWER:

$$(x-4)^2 + (y+1)^2 = 20$$

28. $(18, 11)$ and $(-19, -13)$

SOLUTION:

Find the center.

$$(x_1, y_1) = (18, 11), (x_2, y_2) = (-19, -13)$$

$$\begin{aligned}(h, k) &= \left(\frac{18-19}{2}, \frac{11-13}{2} \right) \\ &= \left(-\frac{1}{2}, -1 \right)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = \left(-\frac{1}{2}, -1 \right), (x_2, y_2) = (-19, -13)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(-19 + \frac{1}{2} \right)^2 + (-13+1)^2} \\ &= \frac{\sqrt{1945}}{2}\end{aligned}$$

The equation of the circle is

$$\left(x + \frac{1}{2} \right)^2 + (y+1)^2 = 486\frac{1}{4}.$$

ANSWER:

$$\left(x + \frac{1}{2} \right)^2 + (y+1)^2 = 486\frac{1}{4}$$

9-3 Circles

29. **LAWN CARE** A sprinkler waters a circular section of lawn.

a. Write an equation to represent the boundary of the sprinkler area if the endpoints of a diameter are at $(-12, 16)$ and $(12, -16)$.

b. What is the area of the lawn that the sprinkler waters?

SOLUTION:

a.

Find the center.

$$(x_1, y_1) = (-12, 16), (x_2, y_2) = (12, -16)$$

$$\begin{aligned} (h, k) &= \left(\frac{-12 + 12}{2}, \frac{16 - 16}{2} \right) \\ &= (0, 0) \end{aligned}$$

Find the radius.

$$(x_1, y_1) = (0, 0), (x_2, y_2) = (12, -16)$$

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(12 - 0)^2 + (-16 - 0)^2} \\ &= \sqrt{400} \\ &= 20 \end{aligned}$$

The equation of the circle is $x^2 + y^2 = 400$.

b.

$$\text{Area of the circle} = \pi(20)^2$$

$$\approx 1256.64 \text{ units}^2.$$

ANSWER:

a. $x^2 + y^2 = 400$

b. approximately 1256.64 units^2 .

30. **SPACE** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon's surface. Write an equation to model a single circular orbit of the command module if the endpoints of a diameter are at $(1740, 0)$ of the Moon and $(-1740, 0)$. Let the center of the Moon be at the origin of the coordinate plane measured in kilometers.

SOLUTION:

Find the radius.

$$(x_1, y_1) = (0, 0), (x_2, y_2) = (-1925, 0)$$

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1925 - 0)^2 + (0 - 0)^2} \\ &= 1925 \end{aligned}$$

The equation of the circle is $x^2 + y^2 = 3,705,625$.

ANSWER:

$$x^2 + y^2 = 3,705,625$$

9-3 Circles

Find the center and radius of each circle. Then graph the circle.

31. $x^2 + y^2 = 75$

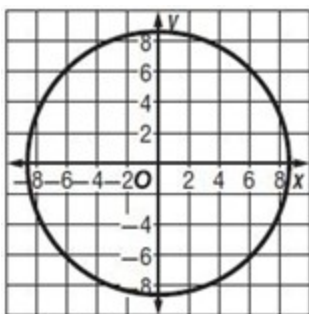
SOLUTION:

$h = 0$ and $k = 0$, so center of the circle is $(0, 0)$.

Find the radius.

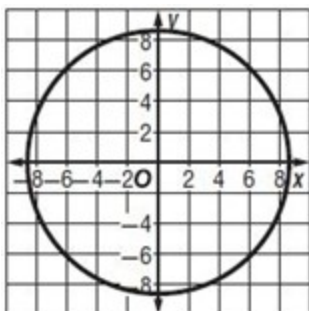
$$r^2 = 75$$

$$r = 5\sqrt{3}$$



ANSWER:

center: $(0, 0)$; radius: $5\sqrt{3}$



32. $(x - 3)^2 + y^2 = 4$

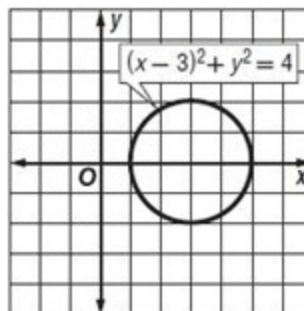
SOLUTION:

$h = 3$ and $k = 0$, so center of the circle is $(3, 0)$.

Find the radius.

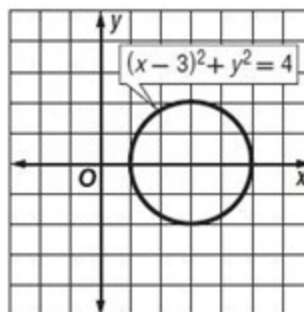
$$r^2 = 4$$

$$r = 2$$



ANSWER:

center: $(3, 0)$; radius: 2



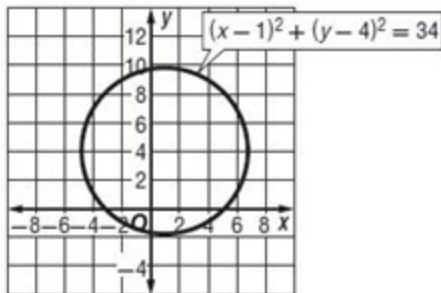
9-3 Circles

33. $(x-1)^2 + (y-4)^2 = 34$

SOLUTION:

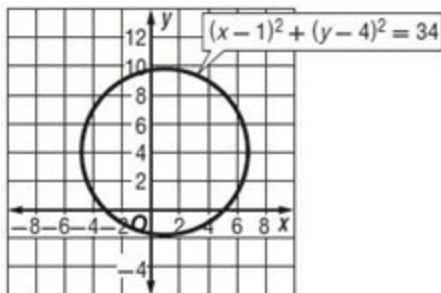
$h = 1$ and $k = 4$, so center of the circle is $(1, 4)$.
Find the radius.

$$r^2 = 34$$
$$r = \sqrt{34}$$



ANSWER:

center: $(1, 4)$; radius: $\sqrt{34}$

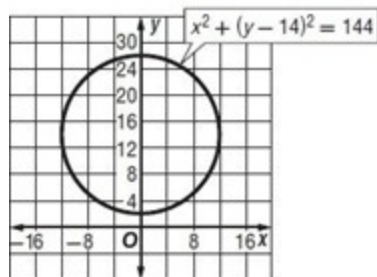


34. $x^2 + (y-14)^2 = 144$

SOLUTION:

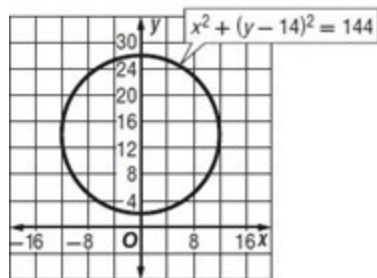
$h = 0$ and $k = 14$, so center of the circle is $(0, 14)$.
Find the radius.

$$r^2 = 144$$
$$r = 12$$



ANSWER:

center: $(0, 14)$; radius: 12



9-3 Circles

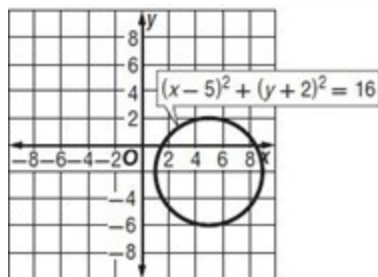
35. $(x - 5)^2 + (y + 2)^2 = 16$

SOLUTION:

$h = 5$ and $k = -2$, so center of the circle is $(5, -2)$.
Find the radius.

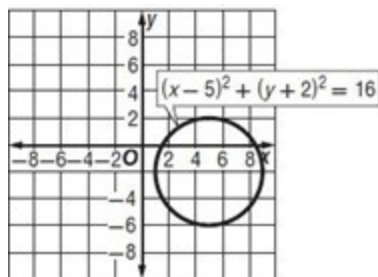
$$r^2 = 16$$

$$r = 4$$



ANSWER:

center: $(5, -2)$; radius: 4



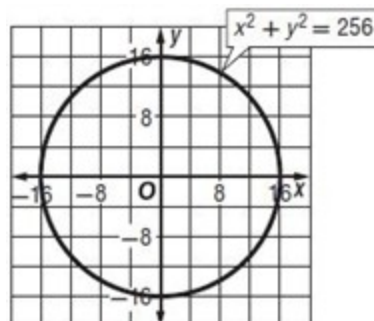
36. $x^2 + y^2 = 256$

SOLUTION:

$h = 0$ and $k = 0$, so center of the circle is $(0, 0)$.
Find the radius.

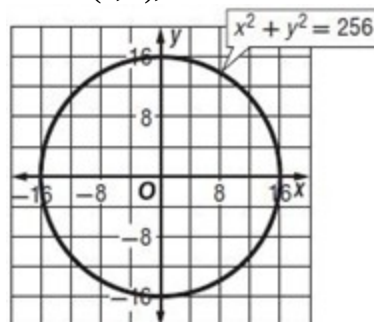
$$r^2 = 256$$

$$r = 16$$



ANSWER:

center: $(0, 0)$; radius: 16



9-3 Circles

37. $(x-4)^2 + y^2 = \frac{8}{9}$

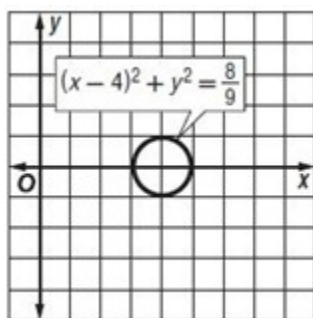
SOLUTION:

$h = 4$ and $k = 0$, so center of the circle is $(4, 0)$.

Find the radius.

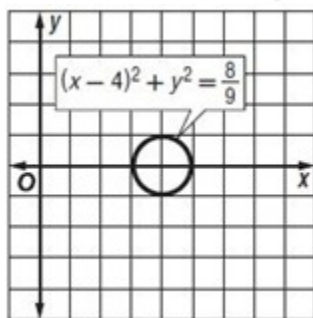
$$r^2 = \frac{8}{9}$$

$$r = \frac{\sqrt{8}}{3}$$



ANSWER:

center: $(4, 0)$; radius: $\frac{\sqrt{8}}{3}$



38. $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{16}{25}$

SOLUTION:

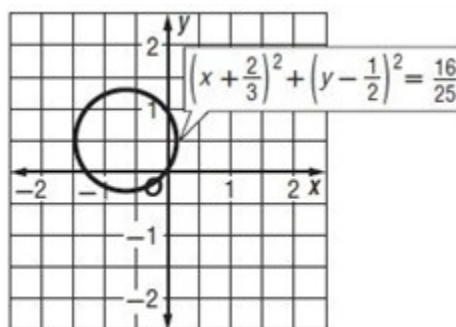
$h = -\frac{2}{3}$, and $k = \frac{1}{2}$, so center of the circle is

$$\left(-\frac{2}{3}, \frac{1}{2}\right).$$

Find the radius.

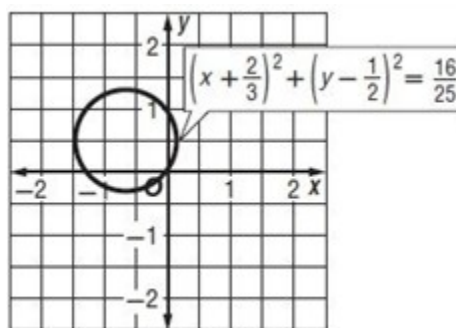
$$r^2 = \frac{16}{25}$$

$$r = \frac{4}{5}$$



ANSWER:

center: $\left(-\frac{2}{3}, \frac{1}{2}\right)$; radius: $\frac{4}{5}$



9-3 Circles

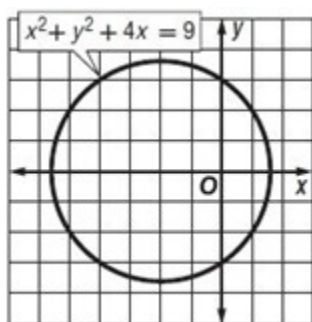
39. $x^2 + y^2 + 4x = 9$

SOLUTION:

$$\begin{aligned}(x^2 + 4x + 4) + y^2 - 4 &= 9 \\ (x + 2)^2 + y^2 &= 13\end{aligned}$$

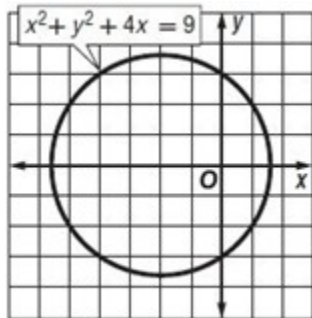
$h = -2$, and $k = 0$, so center of the circle is $(-2, 0)$.
Find the radius.

$$\begin{aligned}r^2 &= 13 \\ r &= \sqrt{13}\end{aligned}$$



ANSWER:

center: $(-2, 0)$; radius: $\sqrt{13}$



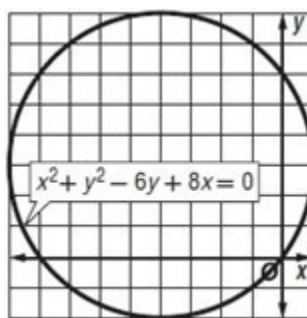
40. $x^2 + y^2 - 6y + 8x = 0$

SOLUTION:

$$\begin{aligned}(x^2 + 8x + 16) + (y^2 - 6y + 9) - 16 - 9 &= 0 \\ (x + 4)^2 + (y - 3)^2 &= 25\end{aligned}$$

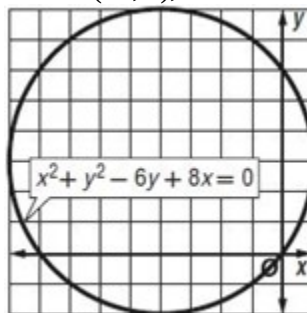
$h = -4$, and $k = 3$, so center of the circle is $(-4, 3)$.
Find the radius.

$$\begin{aligned}r^2 &= 25 \\ r &= 5\end{aligned}$$



ANSWER:

center: $(-4, 3)$; radius: 5



9-3 Circles

41. $x^2 + y^2 + 2x + 4y = 9$

SOLUTION:

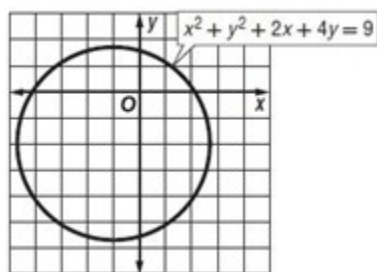
$$(x^2 + 2x + 1) + (y^2 + 4y + 4) - 1 - 4 = 9$$

$$(x+1)^2 + (y+2)^2 = 14$$

$h = -1$ and $k = -2$, so center of the circle is $(-1, -2)$.
Find the radius.

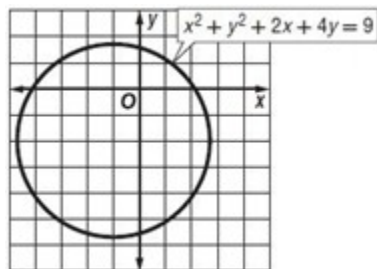
$$r^2 = 14$$

$$r = \sqrt{14}$$



ANSWER:

center: $(-1, -2)$; radius: $\sqrt{14}$



42. $x^2 + y^2 - 3x + 8y = 20$

SOLUTION:

$$\left(x^2 - 3x + \left(\frac{3}{2}\right)^2\right) + (y^2 + 8y + 16) - 16 - \left(\frac{3}{2}\right)^2 = 20$$

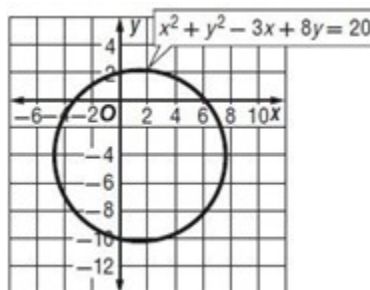
$$\left(x - \frac{3}{2}\right)^2 + (y + 4)^2 = \frac{153}{4}$$

$h = \frac{3}{2}$ and $k = -4$, so center of the circle is $\left(\frac{3}{2}, -4\right)$.

Find the radius.

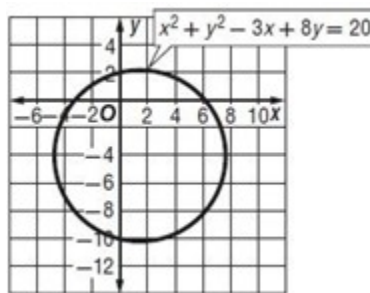
$$r^2 = \frac{153}{4}$$

$$r = \frac{3\sqrt{17}}{2}$$



ANSWER:

center: $\left(\frac{3}{2}, -4\right)$; radius: $\frac{3\sqrt{17}}{2}$



9-3 Circles

43. $x^2 + y^2 + 6y = -50 - 14x$

SOLUTION:

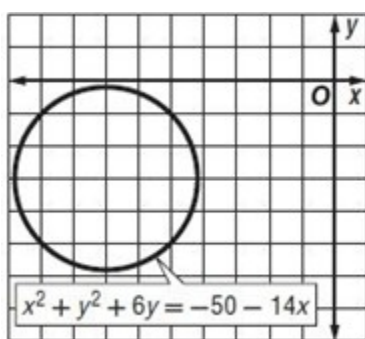
$$(x^2 + 14x + 49) + (y^2 + 6y + 9) - 49 - 9 = -50$$

$$(x + 7)^2 + (y + 3)^2 = 8$$

$h = -7$ and $k = -3$, so center of the circle is $(-7, -3)$.
Find the radius.

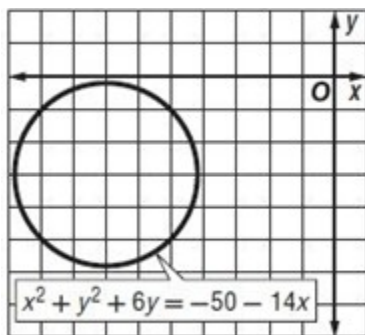
$$r^2 = 8$$

$$r = 2\sqrt{2}$$



ANSWER:

center: $(-7, -3)$; radius: $2\sqrt{2}$ units



44. $x^2 - 18x + 53 = 18y - y^2$

SOLUTION:

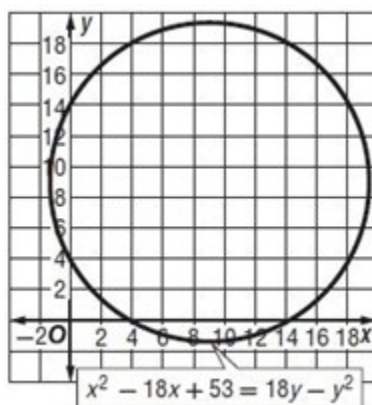
$$(x^2 - 18x + 81) + (y^2 - 18y + 81) + 53 - 81 - 81 = 0$$

$$(x - 9)^2 + (y - 9)^2 = 109$$

$h = 9$ and $k = 9$, so center of the circle is $(9, 9)$.
Find the radius.

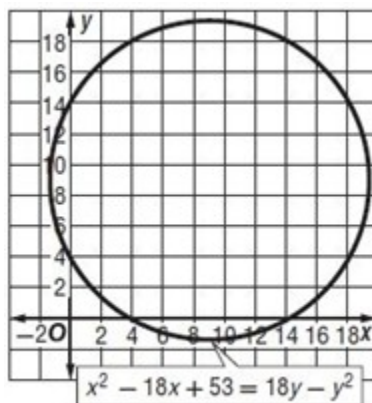
$$r^2 = 109$$

$$r = \sqrt{109}$$



ANSWER:

center: $(9, 9)$; radius: $\sqrt{109}$ units



9-3 Circles

45. $2x^2 + 2y^2 - 4x + 8y = 32$

SOLUTION:

$$(2x^2 - 4x) + (2y^2 + 8y) = 32$$

$$2(x^2 - 2x + 1) + 2(y^2 + 4y + 4) = 32 + 2 + 8$$

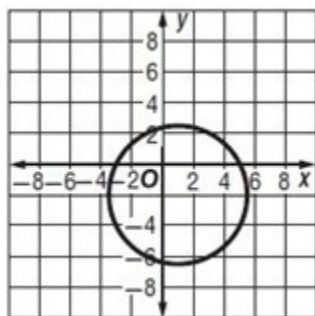
$$2(x-1)^2 + 2(y+2)^2 = 42$$

$$(x-1)^2 + (y+2)^2 = 21$$

$h = 1$ and $k = -2$, so center of the circle is $(1, -2)$.
Find the radius.

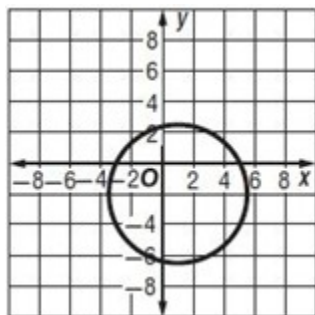
$$r^2 = 21$$

$$r = \sqrt{21}$$



ANSWER:

center: $(1, -2)$; radius: $\sqrt{21}$



46. $3x^2 + 3y^2 - 6y + 12x = 24$

SOLUTION:

$$(3x^2 + 12x) + (3y^2 - 6y) = 24$$

$$3(x^2 + 4x + 4) + 3(y^2 - 2y + 1) = 24 + 12 + 3$$

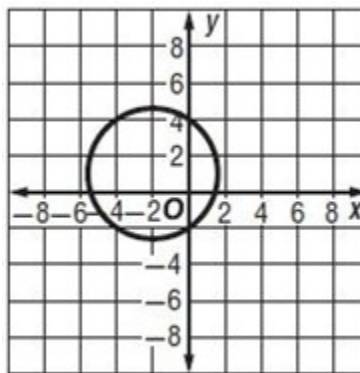
$$3(x+2)^2 + 3(y-1)^2 = 39$$

$$(x+2)^2 + (y-1)^2 = 13$$

$h = -2$ and $k = 1$, so center of the circle is $(-2, 1)$.
Find the radius.

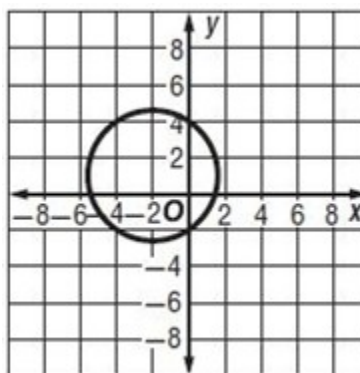
$$r^2 = 13$$

$$r = \sqrt{13}$$



ANSWER:

center: $(-2, 1)$; radius: $\sqrt{13}$



9-3 Circles

47. **SPACE** A satellite is in a circular orbit 25,000 miles above Earth.

a. Write an equation for the orbit of this satellite if the origin is at the center of Earth.

Use 8000 miles as the diameter of Earth.

b. Draw a sketch of Earth and the orbit to scale. Label your sketch.

SOLUTION:

a.

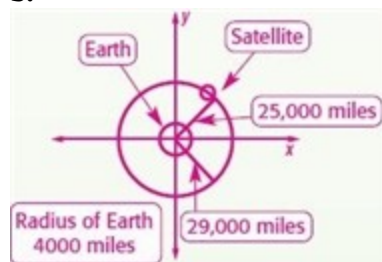
vertex: (0, 0)

radius: 29,000

The equation for the orbit of the satellite is

$$x^2 + y^2 = 841,000,000.$$

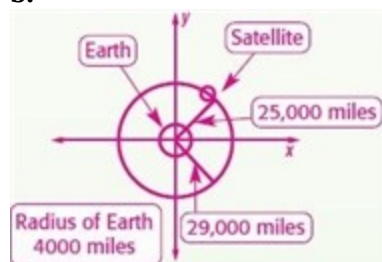
b.



ANSWER:

a. $x^2 + y^2 = 841,000,000$

b.



48. **CCSS SENSE-MAKING** Suppose an unobstructed radio station broadcast could travel 120 miles.

a. Write an equation to represent the boundary of the broadcast area.

b. If the transmission tower is relocated 40 miles east and 10 miles south of the current location, and an increased signal will transmit signals an additional 80 miles, what is an equation to represent the new broadcast area?

SOLUTION:

a.

Given $r = 120$.

The equation that represents the boundary of the broadcast area is $x^2 + y^2 = 14,400$.

b.

$a = 40$, $b = -10$ and $r = 200$ ($120 + 80$).

The equation that represents the new broadcast area is

$$(x - 40)^2 + (y + 10)^2 = 40,000.$$

ANSWER:

a. $x^2 + y^2 = 14,400$

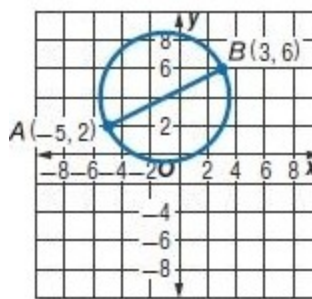
b. $(x - 40)^2 + (y + 10)^2 = 40,000$

49. **GEOMETRY** Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.

a. Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.

b. Write an equation of the circle concentric with the circle at the right, with radius 2 units less.

c. Graph the circles from parts a and b on the same coordinate plane.



SOLUTION:

a.

9-3 Circles

Find the midpoint of $(-5, 2)$ and $(3, 6)$.

$$\begin{aligned} & \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-5 + 3}{2}, \frac{2 + 6}{2} \right) \\ &= (-1, 4) \end{aligned}$$

The center of the circle is $(-1, 4)$.

Find the distance between the coordinates $(-1, 4)$ and $(3, 6)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance formula} \\ d &= \sqrt{(3 + 1)^2 + (6 - 4)^2} \\ &= \sqrt{20} \end{aligned}$$

So the radius of the circle is $\sqrt{20}$.

The equation of the circle larger circle is

$$(x + 1)^2 + (y - 4)^2 = 36 + 16\sqrt{5}.$$

b.

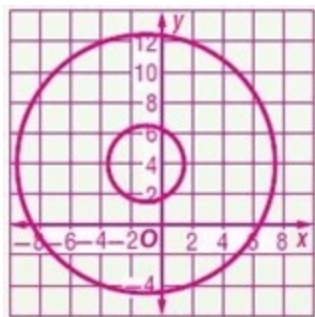
The equation of the smaller circle is

$$\begin{aligned} (x + 1)^2 + (y - 4)^2 &= (\sqrt{20} - 2)^2 \\ (x + 1)^2 + (y - 4)^2 &= 24 - 8\sqrt{5} \end{aligned}$$

c.

Both the circles has the center $(-1, 4)$.

The radius of one circle is about 8.472 and another circle is about 2.472.

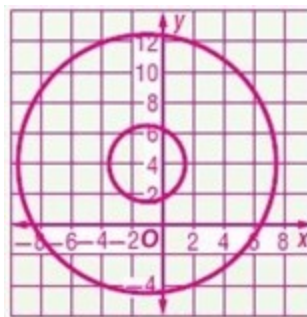


ANSWER:

a. $(x + 1)^2 + (y - 4)^2 = 36 + 16\sqrt{5}$

b. $(x + 1)^2 + (y - 4)^2 = 24 - 8\sqrt{5}$

c.



50. **EARTHQUAKES** The Rose Bowl is located about 35 miles west and about 40 miles north of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 55 miles from the stadium. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.

SOLUTION:

The center of the circle is at $(-35, 40)$, so $h = -35$ and $k = 40$.

The radius is $r = 55$.

Thus, the equation for the set of points that could be the epicenter of the earthquake is

$$(x + 35)^2 + (y - 40)^2 = 3025.$$

ANSWER:

$$(x + 35)^2 + (y - 40)^2 = 3025$$

9-3 Circles

CCSS PRECISION Write an equation for the circle that satisfies each set of conditions.

51. center $(9, -8)$, passes through $(19, 22)$

SOLUTION:

The center of the circle is at $(9, -8)$, so $h = 9$ and $k = -8$.

Find the distance between the coordinates $(9, -8)$ and $(19, 22)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$\begin{aligned} d &= \sqrt{(19 - 9)^2 + (22 + 8)^2} \\ &= 10\sqrt{10} \end{aligned}$$

So the radius of the circle is $10\sqrt{10}$.

The equation of the circle is

$$(x - 9)^2 + (y + 8)^2 = 1000$$

ANSWER:

$$(x - 9)^2 + (y + 8)^2 = 1000$$

52. center $(-\sqrt{15}, 30)$, passes through the origin

SOLUTION:

The center of the circle is at $(-\sqrt{15}, 30)$, so

$h = -\sqrt{15}$ and $k = 30$.

Find the distance between the coordinates

$(-\sqrt{15}, 30)$ and $(0, 0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$\begin{aligned} d &= \sqrt{(0 + \sqrt{15})^2 + (0 - 30)^2} \\ &= \sqrt{915} \end{aligned}$$

So the radius of the circle is $\sqrt{915}$.

The equation of the circle is

$$(x + \sqrt{15})^2 + (y - 30)^2 = 915$$

ANSWER:

$$(x + \sqrt{15})^2 + (y - 30)^2 = 915$$

53. center at $(8, -9)$, tangent to y -axis

SOLUTION:

The center of the circle is at $(8, -9)$, so $h = 8$ and $k = -9$.

Find the distance between the coordinates $(8, -9)$ and $(0, -9)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$\begin{aligned} d &= \sqrt{(0 - 8)^2 + (-9 + 9)^2} \\ &= 8 \end{aligned}$$

So the radius of the circle is 8.

The equation of the circle is $(x - 8)^2 + (y + 9)^2 = 64$

ANSWER:

$$(x - 8)^2 + (y + 9)^2 = 64$$

54. center at $(2, 4)$, tangent to x -axis

SOLUTION:

The center of the circle is at $(2, 4)$, so $h = 2$ and $k = 4$.

Find the distance between the coordinates $(2, 4)$ and $(2, 0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$\begin{aligned} d &= \sqrt{(2 - 2)^2 + (0 - 4)^2} \\ &= 4 \end{aligned}$$

So the radius of the circle is 4.

The equation of the circle is $(x - 2)^2 + (y - 4)^2 = 16$

ANSWER:

$$(x - 2)^2 + (y - 4)^2 = 16$$

9-3 Circles

55. center in the first quadrant; tangent to $x = 5$, the x -axis, and the y -axis

SOLUTION:

The center of the circle is at $(2.5, 2.5)$, so $h = 2.5$ and $k = 2.5$.

The radius is $r = 2.5$.

Thus, the equation of the circle is

$$(x - 2.5)^2 + (y - 2.5)^2 = 6.25.$$

ANSWER:

$$(x - 2.5)^2 + (y - 2.5)^2 = 6.25$$

56. center in the second quadrant; tangent to $y = 1$, $y = 5$, and the y -axis

SOLUTION:

The center of the circle is at $(-2, 3)$, so $h = -2$ and $k = 3$.

The radius is $r = 2$.

Thus, the equation of the circle is

$$(x + 2)^2 + (y - 3)^2 = 4.$$

ANSWER:

$$(x + 2)^2 + (y - 3)^2 = 4$$

57. **MULTIPLE REPRESENTATIONS** Graph

$y = \sqrt{9 - x^2}$ and $y = -\sqrt{9 - x^2}$ on the same graphing calculator screen.

a. VERBAL Describe the graph formed by the union of these two graphs.

b. ALGEBRAIC Write an equation for the union of the two graphs.

c. VERBAL Most graphing calculators cannot graph the equation $x^2 + y^2 = 49$ directly. Describe a way to use a graphing calculator to graph the equation. Then graph the equation.

d. ANALYTICAL Solve $(x - 2)^2 + (y + 1)^2 = 4$ for y . Why do you need two equations to graph a circle on a graphing calculator?

e. VERBAL Do you think that it is easier to graph the equation in part d using graph paper and a pencil or using a graphing calculator? Explain.

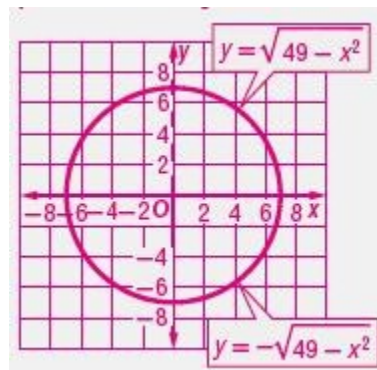
SOLUTION:

a. circle

b. $x^2 + y^2 = 9$

c. Solve the equation for y : $y = \pm\sqrt{49 - x^2}$. Then

graph the positive and negative answers.



d. $y = \pm\sqrt{4 - (x - 2)^2} - 1$; When you solve for y you must take the square root resulting in both a positive and negative answer, so you have to enter the positive equation as Y1 and the negative equation as Y2.

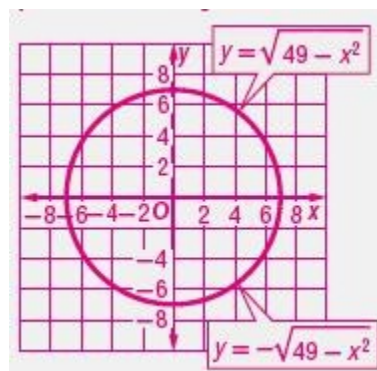
e. See students' work.

ANSWER:

a. circle

b. $x^2 + y^2 = 9$

c. Solve the equation for y : $y = \pm\sqrt{49 - x^2}$. Then graph the positive and negative answers.



d. $y = \pm\sqrt{4 - (x - 2)^2} - 1$; When you solve for y you must take the square root resulting in both a positive and negative answer, so you have to enter the positive equation as Y1 and the negative equation as Y2.

e. See students' work.

9-3 Circles

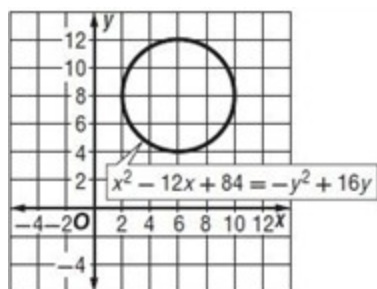
Find the center and radius of each circle. Then graph the circle.

58. $x^2 - 12x + 84 = -y^2 + 16y$

SOLUTION:

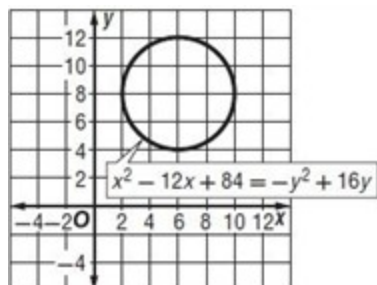
$$\begin{aligned} x^2 - 12x + 84 &= -y^2 + 16y \\ x^2 - 12x + y^2 - 16y &= -84 \\ (x^2 - 12x + 36) + (y^2 - 16y + 64) &= -84 + 36 + 64 \\ (x - 6)^2 + (y - 8)^2 &= 16 \end{aligned}$$

Thus, the center is at (6, 8) and the radius is 4.



ANSWER:

center: (6, 8), radius: 4

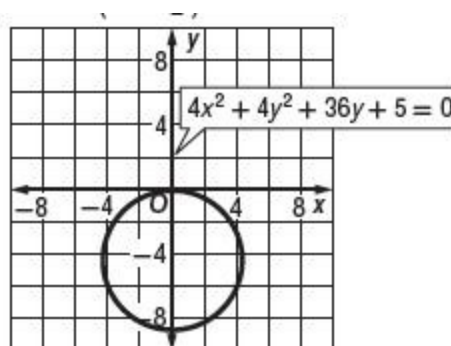


59. $4x^2 + 4y^2 + 36y + 5 = 0$

SOLUTION:

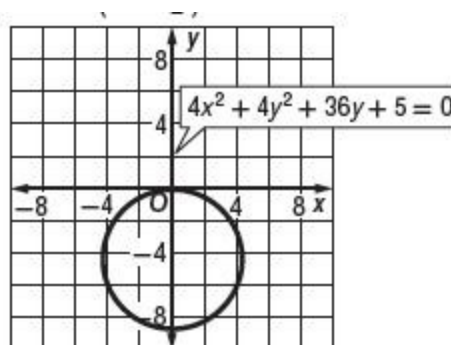
$$\begin{aligned} 4x^2 + 4y^2 + 36y + 5 &= 0 \\ 4x^2 + 4(y^2 + 9y) &= -5 \\ x^2 + \left(y^2 + 9y + \left(\frac{9}{2}\right)^2\right) &= -\frac{5}{4} + \frac{81}{4} \\ (x + 0)^2 + \left(y + \frac{9}{2}\right)^2 &= 19 \end{aligned}$$

Thus, the center is at $\left(0, -\frac{9}{2}\right)$ and the radius is $\sqrt{19}$.



ANSWER:

center: $\left(0, -\frac{9}{2}\right)$ radius: $\sqrt{19}$



9-3 Circles

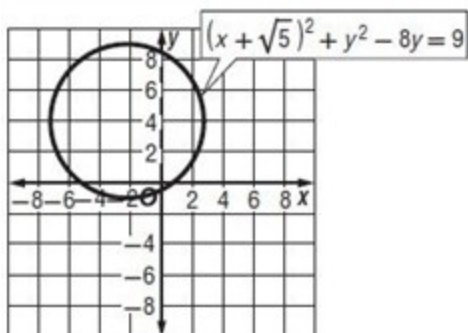
60. $(x + \sqrt{5})^2 + y^2 - 8y = 9$

SOLUTION:

$$(x + \sqrt{5})^2 + y^2 - 8y = 9$$

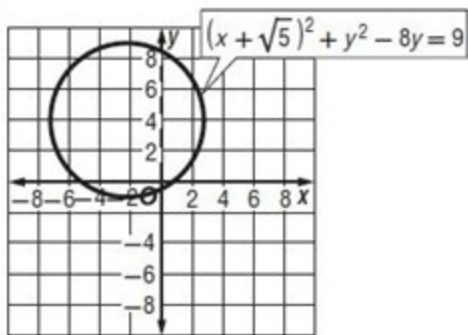
$$(x + \sqrt{5})^2 + (y - 4)^2 = 25$$

Thus, the center is at $(-\sqrt{5}, 4)$ and the radius is 5.



ANSWER:

center: $(-\sqrt{5}, 4)$; radius: 5



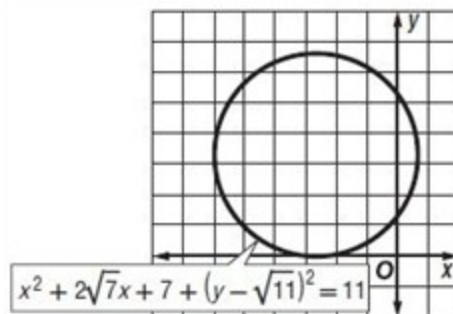
61. $x^2 + 2\sqrt{7}x + 7 + (y - \sqrt{11})^2 = 11$

SOLUTION:

$$x^2 + 2\sqrt{7}x + 7 + (y - \sqrt{11})^2 = 11$$

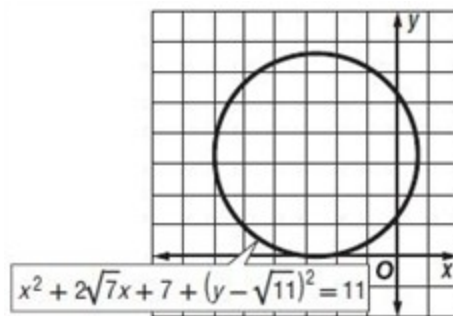
$$(x + \sqrt{7})^2 + (y - \sqrt{11})^2 = 11$$

Thus, the center is at $(-\sqrt{7}, \sqrt{11})$ and the radius is $\sqrt{11}$.



ANSWER:

center: $(-\sqrt{7}, \sqrt{11})$; radius: $\sqrt{11}$



9-3 Circles

62. **ERROR ANALYSIS** Heather says that $(x-2)^2 + (y+3)^2 = 36$ and $(x-2) + (y+3) = 6$ are equivalent equations. Carlota says that the equations are not equivalent. Is either of them correct? Explain your reasoning.

SOLUTION:

Carlota is correct. The square root of

$(x-2)^2 + (y+3)^2$ is not $(x-2) + (y+3)$.

ANSWER:

Carlota; the square root of $(x-2)^2 + (y+3)^2$ is not $(x-2) + (y+3)$.

63. **OPEN ENDED** Consider graphs with equations of the form $(x-3)^2 + (y-a)^2 = 64$. Assign three different values for a , and graph each equation. Describe all graphs with equations of this form.

SOLUTION:

See students' work; circles with a radius of 8 and centers on the graph of $x = 3$.

ANSWER:

See students' work; circles with a radius of 8 and centers on the graph of $x = 3$.

64. **REASONING** Explain why the phrase "in a plane" is included in the definition of a circle. What would be defined if the phrase were *not* included?

SOLUTION:

If the phrase is not included, the figure would be a sphere.

ANSWER:

If the phrase is not included, the figure would be a sphere.

65. **OPEN ENDED** Concentric circles have the same center, but most often, not the same radius. Write equations of two concentric circles. Then graph the circles.

SOLUTION:

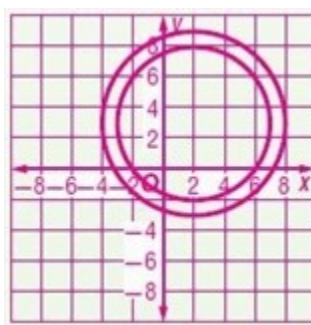
Sample answer:

$$(x-2)^2 + (y-3)^2 = 25 \text{ and}$$

$$(x-2)^2 + (y-3)^2 = 36$$

The center of both the circles is $(2, 3)$.

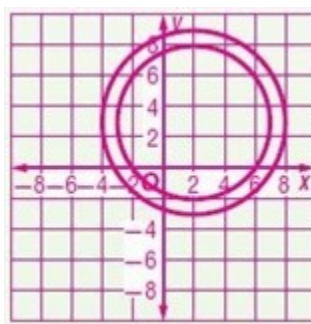
The radius of one circle is 5 and another circle is 6.



ANSWER:

Sample answer:

$$(x-2)^2 + (y-3)^2 = 25 \text{ and}$$
$$(x-2)^2 + (y-3)^2 = 36$$



9-3 Circles

66. **REASONING** Assume that (x, y) are the coordinates of a point on a circle. The center is at (h, k) , and the radius is r . Find an equation of the circle by using the Distance Formula.

SOLUTION:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d \quad \text{Distance formula}$$

$$(x_1, y_1) = (h, k), (x_2, y_2) = (x, y), d = r$$

$$\sqrt{(x - h)^2 + (y - k)^2} = r \quad \text{square each side}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

This is the standard form of the equation of a circle

ANSWER:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

$$(x - h)^2 + (y - k)^2 = r^2$$

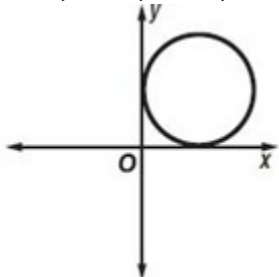
This is the standard form of the equation of a circle.

67. **WRITING IN MATH** The circle with equation $(x - a)^2 + (y - b)^2 = r^2$ lies in the first quadrant and is tangent to both the x -axis and the y -axis. Sketch the circle. Describe the possible values of a , b , and r . Do the same for a circle in Quadrants II, III, and IV. Discuss the similarities among the circles.

SOLUTION:

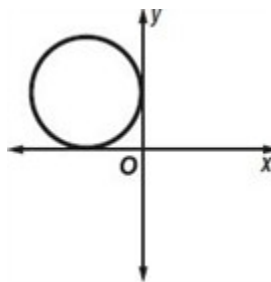
Quadrant I

$$a > 0, b > 0, a = b, r > 0$$



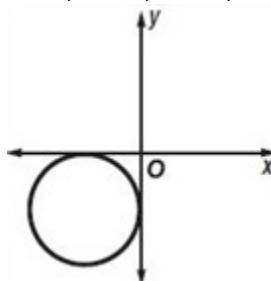
Quadrant II

$$a < 0, b > 0, a = -b, r > 0$$



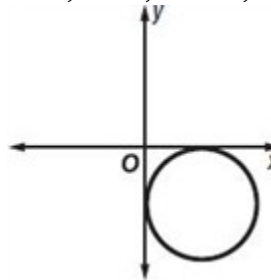
Quadrant III

$$a < 0, b < 0, a = b, r > 0$$



Quadrant IV

$$a > 0, b < 0, a = -b, r > 0$$

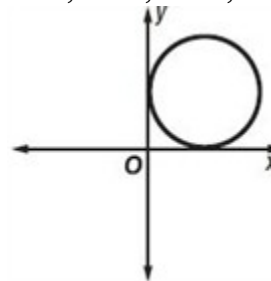


Sample answer: The circle is rotated 90° about the origin from one quadrant to the next.

ANSWER:

Quadrant I

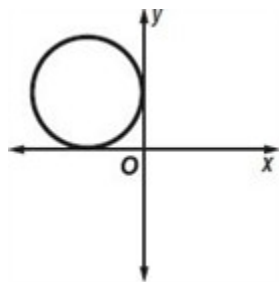
$$a > 0, b > 0, a = b, r > 0$$



Quadrant II

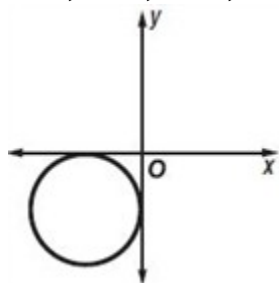
$$a < 0, b > 0, a = -b, r > 0$$

9-3 Circles



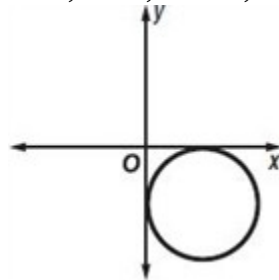
Quadrant III

$$a < 0, b < 0, a = b, r > 0$$



Quadrant IV

$$a > 0, b < 0, a = -b, r > 0$$



Sample answer: The circle is rotated 90° about the origin from one quadrant to the next.

68. **GRIDDED RESPONSE** Two circles, both with radii 6, have exactly one point in common. If A is a point on one circle and B is a point on the other circle, what is the maximum possible length for the line segment $\overline{AB}$?

SOLUTION:

The maximum possible length is the sum of the diameter of both the circles.

Thus, the maximum possible length for the line segment $\overline{AB}$ is 24.

ANSWER:

24

69. In the senior class, there are 20% more girls than boys. If there are 180 girls, how many more girls than boys are there among the seniors?

- A** 30
B 36
C 90
D 144

SOLUTION:

Let the number of boys be x .

The equation that represents the situation is $x + 0.20x = 180$.

The number of boys in the class is 150.

Thus, there are 30 girls more than the boys.

A is the correct option.

ANSWER:

A

70. A \$1000 deposit is made at a bank that pays 2% compounded weekly. How much will you have in your account at the end of 10 years?

- F** \$1200.00
H \$1221.36
G \$1218.99
J \$1224.54

SOLUTION:

Use the compound interest formula.

Substitute \$1000 for P , 0.02 for r , 52 for n and 10 for t and simplify.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = 1000 \left(1 + \frac{0.02}{52} \right)^{52(10)}$$

$$\approx \$1221.36$$

H is the correct option.

ANSWER:

H

9-3 Circles

71. The mean of six numbers is 20. If one of the numbers is removed, the average of the remaining numbers is 15. What is the number that was removed?

A 42
B 43
C 45
D 48

SOLUTION:

Since the mean of six numbers is 20, their sum is 120.

Since the mean of five numbers is 15, their sum is 75.

The removed number can be found by finding the difference between their sums.

Thus, 45 is the number that was removed.

C is the correct option.

ANSWER:

C

Graph each equation.

72. $y = -\frac{1}{2}(x-1)^2 + 4$

SOLUTION:

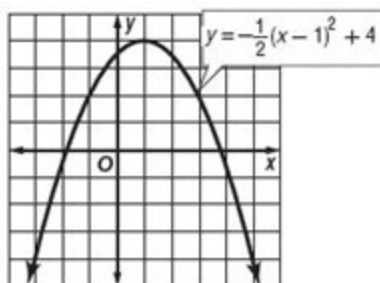
vertex: (1, 4)

axis of symmetry: $x = 1$

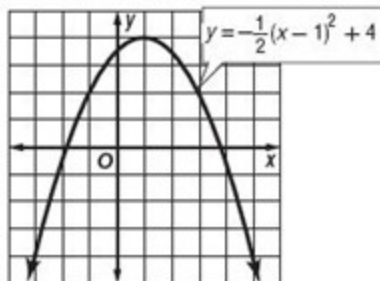
focus: (1, 3.5)

directrix: $y = 4.5$

length of the latus rectum: 2



ANSWER:



73. $4(x-2) = (y+3)^2$

SOLUTION:

$$4(x-2) = (y+3)^2$$

$$x = \frac{1}{4}(y+3)^2 + 2$$

vertex: $(-2, -3)$

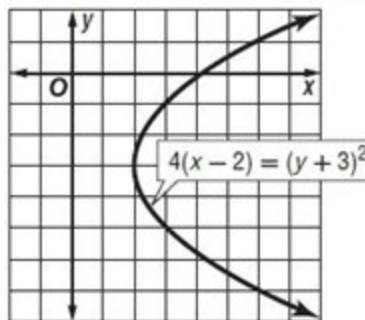
axis of symmetry: $y = -3$

focus: $(-1, -3)$

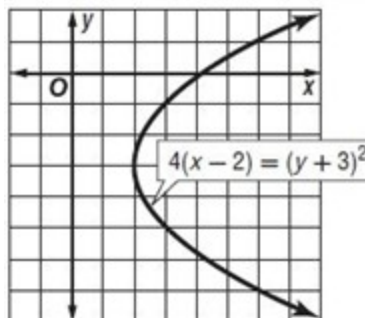
directrix: $x = -3$

length of the latus rectum: 4

The graph opens right.



ANSWER:



9-3 Circles

74. $(y-8)^2 = -4(x-4)$

SOLUTION:

$$(y-8)^2 = -4(x-4)$$

$$4x = -(y-8)^2 + 16$$

$$x = -\frac{1}{4}(y-8)^2 + 4$$

vertex: (4, 8)

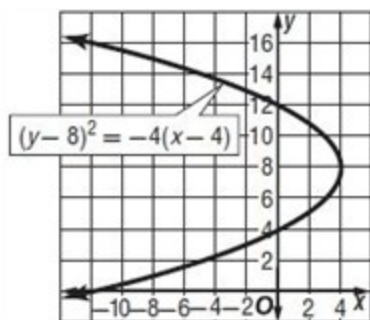
axis of symmetry: $y = 8$

focus: (3, 8)

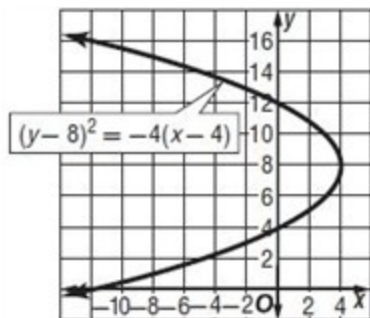
directrix: $x = 5$

length of the latus rectum: 4

The graph opens left.



ANSWER:



Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.

75. $\left(-3, -\frac{2}{11}\right), \left(5, \frac{9}{11}\right)$

SOLUTION:

Find the midpoint of $\left(-3, -\frac{2}{11}\right)$ and $\left(5, \frac{9}{11}\right)$.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \text{ Midpoint formula}$$

$$= \left(\frac{-3+5}{2}, \frac{-\frac{2}{11}+\frac{9}{11}}{2}\right)$$

$$= \left(1, \frac{7}{22}\right)$$

Find the distance between the coordinates

$$\left(-3, -\frac{2}{11}\right) \text{ and } \left(5, \frac{9}{11}\right).$$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \text{ Distance formula}$$

$$d = \sqrt{(5+3)^2 + \left(\frac{9}{11} + \frac{2}{11}\right)^2}$$

$$= \sqrt{65} \text{ units}$$

ANSWER:

$$\left(1, \frac{7}{22}\right); \sqrt{65} \text{ units}$$

9-3 Circles

76. $(2\sqrt{3}, -5), (-3\sqrt{3}, 9)$

SOLUTION:

Find the midpoint of $(2\sqrt{3}, -5)$ and $(-3\sqrt{3}, 9)$.

$$\begin{aligned} & \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \text{ Midpoint formula} \\ & = \left(\frac{2\sqrt{3} - 3\sqrt{3}}{2}, \frac{-5 + 9}{2} \right) \\ & = \left(-\frac{\sqrt{3}}{2}, 2 \right) \end{aligned}$$

Find the distance between the coordinates $(2\sqrt{3}, -5)$ and $(-3\sqrt{3}, 9)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula} \\ d &= \sqrt{(-3\sqrt{3} - 2\sqrt{3})^2 + (9 + 5)^2} \\ &= \sqrt{271} \text{ units} \end{aligned}$$

ANSWER:

$$\left(-\frac{\sqrt{3}}{2}, 2 \right); \sqrt{271} \text{ units}$$

77. $(2.5, 4), (-2.5, 2)$

SOLUTION:

Find the midpoint of $(2.5, 4)$ and $(-2.5, 2)$.

$$\begin{aligned} & \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \text{ Midpoint formula} \\ & = \left(\frac{2.5 - 2.5}{2}, \frac{4 + 2}{2} \right) \\ & = (0, 3) \end{aligned}$$

Find the distance between the coordinates $(2.5, 4)$ and $(-2.5, 2)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula} \\ d &= \sqrt{(-2.5 - 2.5)^2 + (2 - 4)^2} \\ &= \sqrt{29} \text{ units} \end{aligned}$$

ANSWER:

$$(0, 3); \sqrt{29} \text{ units}$$

78. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$.

SOLUTION:

Use a proportion that relates the values.

$$\begin{aligned} \frac{y_1}{x_1} &= \frac{y_2}{x_2} \text{ Direct variation} \\ \frac{8}{6} &= \frac{y_2}{15} \quad x_1 = 6, y_1 = 8 \text{ and } x_2 = 15 \\ y_2 &= 20 \end{aligned}$$

ANSWER:

$$20$$

79. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$.

SOLUTION:

Use a proportion that relates the values.

$$\begin{aligned} \frac{y_1}{x_1 z_1} &= \frac{y_2}{x_2 z_2} \text{ Joint variation} \\ \frac{80}{5(8)} &= \frac{y_2}{16(2)} \quad x_1 = 5, y_1 = 80, z_1 = 8, x_2 = 16 \text{ and } z_2 = 2 \\ y_2 &= 64 \end{aligned}$$

ANSWER:

$$64$$

9-3 Circles

80. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$.

SOLUTION:

Use a proportion that relates the values.

$$\frac{y_1}{x_2} = \frac{y_2}{x_1} \quad \text{Indirect variation}$$

$$\frac{16}{20} = \frac{y_2}{5} \quad x_1=5, y_1=16 \text{ and } x_2=20$$

$$y_2 = 4$$

ANSWER:

4

Evaluate each expression.

81. $\log_9 243$

SOLUTION:

Let the logarithm equal y .

$$\log_9 243 = y$$

$$9^y = 243$$

$$3^{2y} = 3^5$$

$$2y = 5$$

$$y = \frac{5}{2}$$

ANSWER:

$$\frac{5}{2}$$

82. $\log_2 \frac{1}{32}$

SOLUTION:

$$\log_2 \frac{1}{32} = \log_2 2^{-5}$$

$$= -5 \log_2 2$$

$$= -5$$

ANSWER:

-5

83. $\log_3 \frac{1}{81}$

SOLUTION:

$$\log_3 \frac{1}{81} = \log_3 3^{-4}$$

$$= -4 \log_3 3$$

$$= -4$$

ANSWER:

-4

84. $\log_{10} 0.001$

SOLUTION:

Let the logarithm equal y .

$$\log_{10} 0.001 = y$$

$$10^y = 0.001$$

$$10^y = 10^{-3}$$

$$y = -3$$

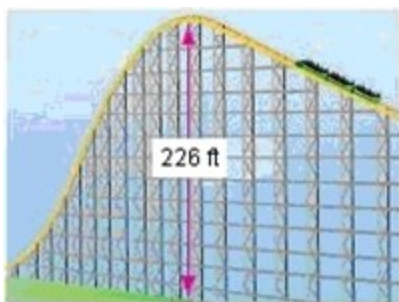
ANSWER:

-3

9-3 Circles

85. **AMUSEMENT PARKS** The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$.

- Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the given formula.
- What velocity must the coaster have at the top of the hill to achieve a velocity of 125 feet per second at the bottom?



SOLUTION:

- The square root of a difference is not the difference of the square roots.
- Substitute $h = 226$ and $v = 125$ in the formula

$$v_0 = \sqrt{v^2 - 64h}$$

$$\begin{aligned} v_0 &= \sqrt{125^2 - 64(226)} \\ &= \sqrt{1161} \\ &\approx 34.1 \end{aligned}$$

The coaster must have a velocity of about 34.1 ft/s at the top of the hill.

ANSWER:

- The square root of a difference is not the difference of the square roots.
- 34.1 ft/s

Solve each equation by completing the square.

86. $x^2 + 3x - 18 = 0$

SOLUTION:

$$x^2 + 3x - 18 = 0$$

$$x^2 + 3x = 18$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 18 + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{81}{4}$$

Use the Square Root Property.

$$x + \frac{3}{2} = \pm \frac{9}{2}$$

$$x = -\frac{3}{2} + \frac{9}{2} \quad \text{or} \quad x = -\frac{3}{2} - \frac{9}{2}$$

$$x = 3 \quad \text{or} \quad x = -6$$

The solution set is $\{-6, 3\}$.

ANSWER:

$$\{-6, 3\}$$

9-3 Circles

87. $2x^2 - 3x - 3 = 0$

SOLUTION:

$$2x^2 - 3x - 3 = 0$$

$$x^2 - \frac{3}{2}x = \frac{3}{2}$$

$$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{3}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{33}{16}$$

Use the Square Root Property.

$$x - \frac{3}{4} = \pm \frac{\sqrt{33}}{4}$$

$$x = \frac{3}{4} + \frac{\sqrt{33}}{4} \quad \text{or} \quad x = \frac{3}{4} - \frac{\sqrt{33}}{4}$$

$$x = \frac{3 + \sqrt{33}}{4} \quad \text{or} \quad x = \frac{3 - \sqrt{33}}{4}$$

The solution set is $\left\{\frac{3 \pm \sqrt{33}}{4}\right\}$.

ANSWER:

$$\left\{\frac{3 \pm \sqrt{33}}{4}\right\}$$

88. $x^2 + 2x + 6 = 0$

SOLUTION:

$$x^2 + 2x + 6 = 0$$

$$x^2 + 2x = -6$$

$$x^2 + 2x + 1 = -5$$

$$(x + 1)^2 = -5$$

Use the Square Root Property.

$$x + 1 = \pm i\sqrt{5}$$

$$x = -1 \pm i\sqrt{5}$$

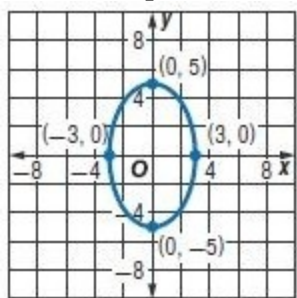
The solution set is $\{-1 \pm i\sqrt{5}\}$.

ANSWER:

$$\{-1 \pm i\sqrt{5}\}$$

9-4 Ellipses

Write an equation of each ellipse.



1.

SOLUTION:

The orientation is vertical. The center of the ellipse is at (0, 0).

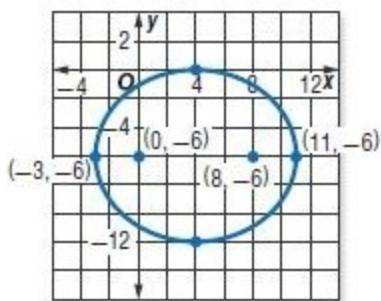
The length of the major axis is 10 units, so $a = 5$.

The length of the minor axis is 6 units, so $b = 3$.

The equation of the ellipse is $\frac{y^2}{25} + \frac{x^2}{9} = 1$.

ANSWER:

$$\frac{y^2}{25} + \frac{x^2}{9} = 1$$



2.

SOLUTION:

The orientation is horizontal. The center of the ellipse is at (4, -6). The length of the major axis is 14 units, so $a = 7$. The length of the minor axis is 12 units, so $b = 6$. The equation of the ellipse is

$$\frac{(x-4)^2}{49} + \frac{(y+6)^2}{36} = 1$$

ANSWER:

$$\frac{(x-4)^2}{49} + \frac{(y+6)^2}{36} = 1$$

Write an equation of an ellipse that satisfies each set of conditions.

3. vertices at $(-2, -6)$ and $(-2, 4)$, co-vertices at $(-5, -1)$ and $(1, -1)$

SOLUTION:

The orientation is vertical. The center of the ellipse is at $(-2, -1)$. The length of the major axis is 10 units, so $a = 5$. The length of the minor axis is 6 units, so $b = 3$. The equation of the ellipse is

$$\frac{(y+1)^2}{25} + \frac{(x+2)^2}{9} = 1$$

ANSWER:

$$\frac{(y+1)^2}{25} + \frac{(x+2)^2}{9} = 1$$

4. vertices at $(-2, 5)$ and $(14, 5)$, co-vertices at $(6, 1)$ and $(6, 9)$

SOLUTION:

The orientation is horizontal. The center of the ellipse is at $(6, 5)$. The length of the major axis is 16 units, so $a = 8$. The length of the minor axis is 8 units, so $b = 4$. The equation of the ellipse is

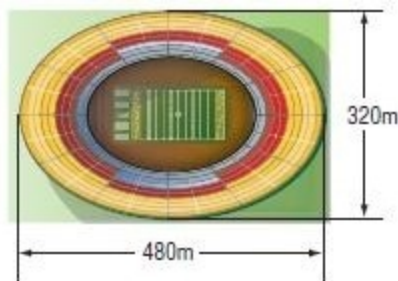
$$\frac{(x-6)^2}{64} + \frac{(y-5)^2}{16} = 1$$

ANSWER:

$$\frac{(x-6)^2}{64} + \frac{(y-5)^2}{16} = 1$$

9-4 Ellipses

5. **CCSS SENSE-MAKING** An architectural firm sent a proposal to a city for building a coliseum, shown at the right.



- Determine the values of a and b .
- Assuming that the center is at the origin, write an equation to represent the ellipse.
- Determine the coordinates of the foci.

SOLUTION:

a.

The center of the ellipse is at $(0, 0)$.

Since the vertices of the ellipse are $(240, 0)$ and $(-240, 0)$, the value of $a = 240$.

Since the co-vertices of the ellipse are $(0, 160)$ and $(0, -160)$, the value of $b = 160$.

b.

Because the major axis is horizontal, a^2 goes with x and b^2 goes with y . The equation of the ellipse is

$$\frac{x^2}{57,600} + \frac{y^2}{25,600} = 1.$$

c.

$$c^2 = a^2 - b^2$$

$$c^2 = 57600 - 25600$$

$$c^2 = 32000$$

$$c \approx 179$$

The coordinates of the foci are about $(179, 0)$ and $(-179, 0)$.

ANSWER:

a. $a = 240, b = 160$

b. $\frac{x^2}{57,600} + \frac{y^2}{25,600} = 1.$

c. about $(179, 0)$ and $(-179, 0)$

6. **SPACE** Earth's orbit is about 91.4 million miles at perihelion and about 94.5 million miles at aphelion. Determine an equation relating Earth's orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

SOLUTION:

The value of a is one half the length of the major axis.

$$\begin{aligned} a &= 0.5(91.4 + 0.87 + 94.5) \\ &= 93.385 \end{aligned}$$

$$\text{So, } a^2 = 8720.8.$$

The value of c is the distance from the center of the ellipse to the focus. This distance is equal to a minus the perihelion and the radius of the sun.

$$c = 93.385 - 91.4 - 0.435$$

$$c = 1.55$$

$$c^2 = a^2 - b^2$$

$$1.55^2 = 93.385^2 - b^2$$

$$b^2 = 93.385^2 - 1.55^2$$

$$b^2 = 8718.356$$

Therefore, the equation of the ellipse is

$$\frac{x^2}{8720.8} + \frac{y^2}{8718.4} = 1.$$

ANSWER:

$$\frac{x^2}{8720.8} + \frac{y^2}{8718.4} = 1$$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

9-4 Ellipses

$$7. \frac{(y+1)^2}{64} + \frac{(x-5)^2}{28} = 1$$

SOLUTION:

The center of the ellipse is $(5, -1)$, so $h = 5$ and $k = -1$.

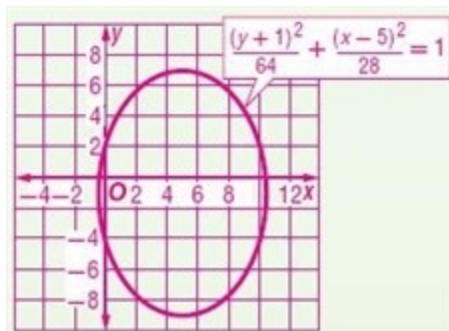
$$a^2 = 64 \text{ and } b^2 = 28$$

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{64 - 28} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

Thus, the coordinates of the foci are $(5, 5)$ and $(5, -7)$.

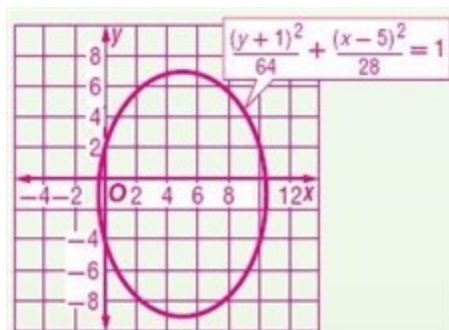
The length of the major axis is $2a$ which is 16 units.

The length of the minor axis is $2b$ which is about 10.58 units.



ANSWER:

center $(5, -1)$; foci $(5, 5)$ and $(5, -7)$; major axis: 16;
minor axis: ≈ 10.58



$$8. \frac{(x+2)^2}{48} + \frac{(y-1)^2}{20} = 1$$

SOLUTION:

The center of the ellipse is $(-2, 1)$, so $h = -2$ and $k = 1$.

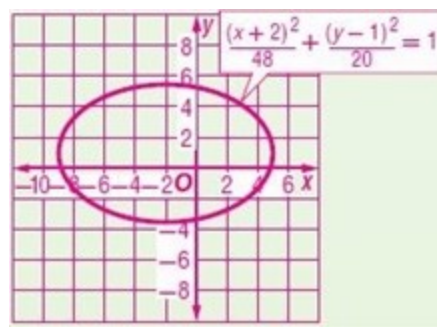
$$a^2 = 48 \text{ and } b^2 = 20$$

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{48 - 20} \\ &\approx 5.29 \end{aligned}$$

Thus, the coordinates of the foci are $(3.29, 1)$ and $(-7.29, 1)$.

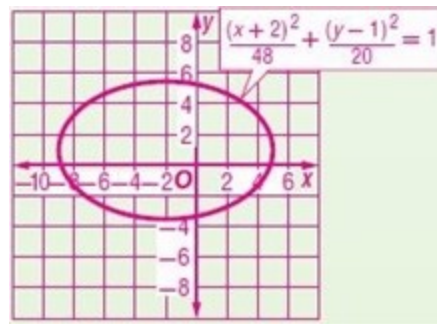
The length of the major axis is $2a$ which is about 13.86 units.

The length of the minor axis is $2b$ which is about 8.94 units.



ANSWER:

center $(-2, 1)$; foci $(3.29, 1)$ and $(-7.29, 1)$; major axis: ≈ 13.86 ; minor axis: ≈ 8.94



9-4 Ellipses

9. $4x^2 + y^2 - 32x - 4y + 52 = 0$

SOLUTION:

$$\begin{aligned} 4x^2 + y^2 - 32x - 4y + 52 &= 0 \\ 4(x^2 - 8x + 16) + (y^2 - 4y + 4) &= -52 + 64 + 4 \\ 4(x - 4)^2 + (y - 2)^2 &= 16 \\ \frac{(x - 4)^2}{4} + \frac{(y - 2)^2}{16} &= 1 \end{aligned}$$

The center of the ellipse is $(4, 2)$ so $h = 4$ and $k = 2$.

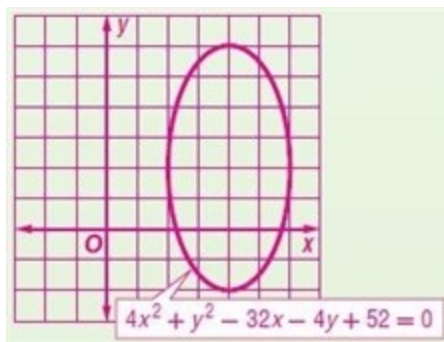
The orientation is vertical, so $a^2 = 16$ and $b^2 = 4$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{16 - 4} \\ &\approx 3.46 \end{aligned}$$

Thus, the coordinates of the foci are $(4, 5.46)$ and $(4, -1.46)$.

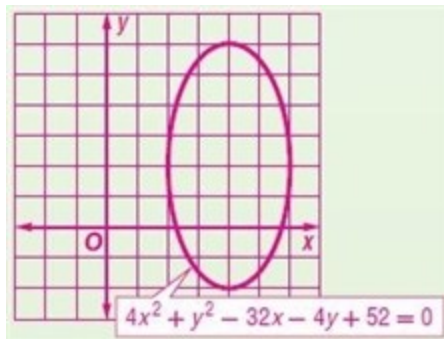
The length of the major axis is $2a$ which is about 8 units.

The length of the minor axis is $2b$ which is about 4 units.



ANSWER:

center $(4, 2)$; foci $(4, 5.46)$ and $(4, -1.46)$; major axis: 8; minor axis: 4



10. $9x^2 + 25y^2 + 72x - 150y + 144 = 0$

SOLUTION:

$$\begin{aligned} 9x^2 + 25y^2 + 72x - 150y + 144 &= 0 \\ 9(x^2 + 8x + 16) + 25(y^2 - 6y + 9) &= -144 + 144 + 225 \\ 9(x + 4)^2 + 25(y - 3)^2 &= 225 \\ \frac{(x + 4)^2}{25} + \frac{(y - 3)^2}{9} &= 1 \end{aligned}$$

The center of the ellipse is $(-4, 3)$, so $h = -4$ and $k = 3$.

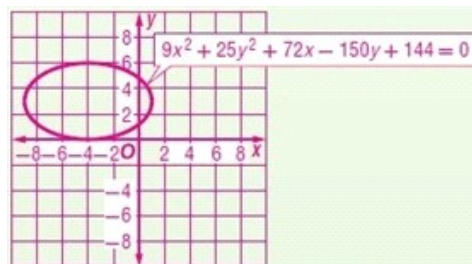
The orientation is horizontal, so $a^2 = 25$ and $b^2 = 9$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{25 - 9} \\ &= 4 \end{aligned}$$

Thus, the coordinates of the foci are $(0, 3)$ and $(-8, 3)$.

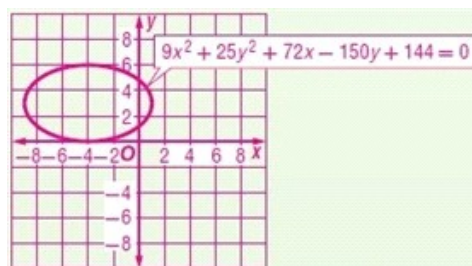
The length of the major axis is $2a$ which is about 10 units.

The length of the minor axis is $2b$ which is about 6 units.



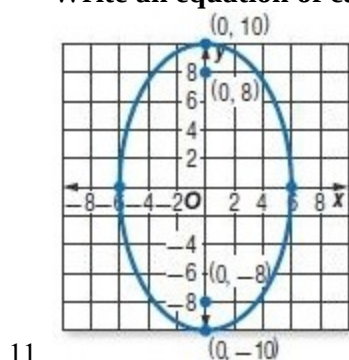
ANSWER:

center $(-4, 3)$; foci $(0, 3)$ and $(-8, 3)$; major axis: 10; minor axis: 6



9-4 Ellipses

Write an equation of each ellipse.



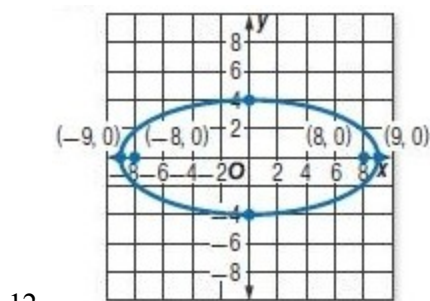
SOLUTION:

The orientation is vertical. The center of the ellipse is at (0, 0). The length of the major axis is 20 units, so $a = 10$. The length of the minor axis is 12 units, so $b =$

6. The equation of the ellipse is $\frac{y^2}{100} + \frac{x^2}{36} = 1$.

ANSWER:

$$\frac{y^2}{100} + \frac{x^2}{36} = 1$$



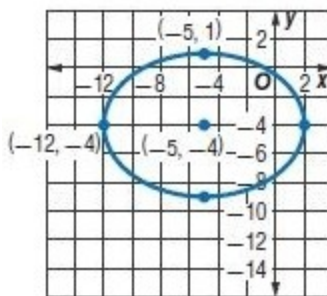
SOLUTION:

The orientation is horizontal. The center of the ellipse is at (0, 0). The length of the major axis is 18 units, so $a = 9$. The length of the minor axis is 8 units, so $b =$

4. The equation of the ellipse is $\frac{x^2}{81} + \frac{y^2}{16} = 1$.

ANSWER:

$$\frac{x^2}{81} + \frac{y^2}{16} = 1$$



13.

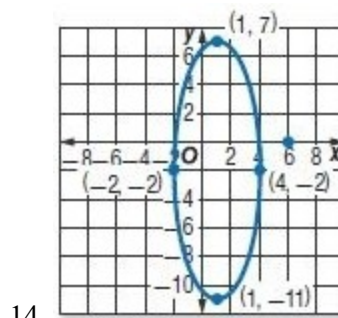
SOLUTION:

The orientation is horizontal. The center of the ellipse is at (-5, -4). The length of the major axis is 14 units, so $a = 7$. The length of the minor axis is 10 units, so $b = 5$. The equation of the ellipse is

$$\frac{(x+5)^2}{49} + \frac{(y+4)^2}{25} = 1.$$

ANSWER:

$$\frac{(x+5)^2}{49} + \frac{(y+4)^2}{25} = 1$$



14.

SOLUTION:

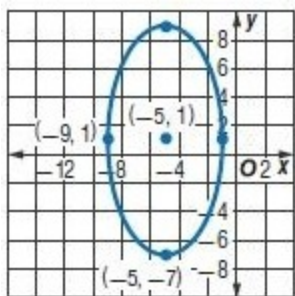
The orientation is vertical. The center of the ellipse is at (1, -2). The length of the major axis is 18 units, so $a = 9$. The length of the minor axis is 6 units, so $b = 3$. The equation of the ellipse is

$$\frac{(y+2)^2}{81} + \frac{(x-1)^2}{9} = 1.$$

ANSWER:

$$\frac{(y+2)^2}{81} + \frac{(x-1)^2}{9} = 1$$

9-4 Ellipses



15.

SOLUTION:

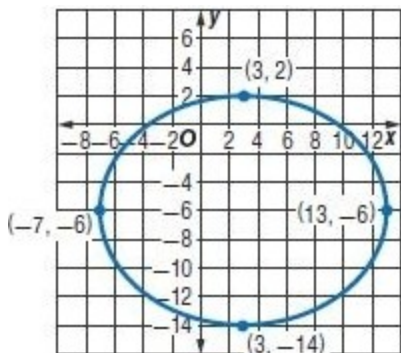
The orientation is vertical. The center of the ellipse is at $(-5, 1)$.

The length of the major axis is 16 units, so $a = 8$. The length of the minor axis is 8 units, so $b = 4$. The

equation of the ellipse is $\frac{(y-1)^2}{64} + \frac{(x+5)^2}{16} = 1$.

ANSWER:

$$\frac{(y-1)^2}{64} + \frac{(x+5)^2}{16} = 1$$



16.

SOLUTION:

The orientation is horizontal. The center of the ellipse is at $(3, -6)$. The length of the major axis is 20 units, so $a = 10$. The length of the minor axis is 16 units, so $b = 8$. The equation of the ellipse is

$$\frac{(x-3)^2}{100} + \frac{(y+6)^2}{64} = 1$$

ANSWER:

$$\frac{(x-3)^2}{100} + \frac{(y+6)^2}{64} = 1$$

Write an equation of an ellipse that satisfies each set of conditions.

17. vertices at $(-6, 4)$ and $(12, 4)$, co-vertices at $(3, 12)$ and $(3, -4)$

SOLUTION:

The orientation is horizontal. The center of the ellipse is at $(3, 4)$. The length of the major axis is 18 units, so $a = 9$. The length of the minor axis is 16 units, so $b = 8$. The equation of the ellipse is

$$\frac{(x-3)^2}{81} + \frac{(y-4)^2}{64} = 1$$

ANSWER:

$$\frac{(x-3)^2}{81} + \frac{(y-4)^2}{64} = 1$$

18. vertices at $(-1, 11)$ and $(-1, 1)$, co-vertices at $(-4, 6)$ and $(2, 6)$

SOLUTION:

The orientation is vertical. The center of the ellipse is at $(-1, 6)$. The length of the major axis is 10 units, so $a = 5$. The length of the minor axis is 6 units, so $b = 3$. The equation of the ellipse is

$$\frac{(y-6)^2}{25} + \frac{(x+1)^2}{9} = 1$$

ANSWER:

$$\frac{(y-6)^2}{25} + \frac{(x+1)^2}{9} = 1$$

19. center at $(-2, 6)$, vertex at $(-2, 16)$, co-vertex at $(1, 6)$

SOLUTION:

The orientation is vertical.

The value of a is 10 and b is 3.

The equation of the ellipse is $\frac{(y-6)^2}{100} + \frac{(x+2)^2}{9} = 1$.

ANSWER:

$$\frac{(y-6)^2}{100} + \frac{(x+2)^2}{9} = 1$$

9-4 Ellipses

20. center at (3, -4), vertex at (8, -4), co-vertex at (3, -2)

SOLUTION:

The orientation is horizontal.

The value of a is 5 and b is 2.

The equation of the ellipse is $\frac{(x-3)^2}{25} + \frac{(y+4)^2}{4} = 1$.

ANSWER:

$$\frac{(x-3)^2}{25} + \frac{(y+4)^2}{4} = 1$$

21. vertices at (4, 12) and (4, -4), co-vertices at (1, 4) and (7, 4)

SOLUTION:

The orientation is vertical. The center of the ellipse is at (4, 4). The length of the major axis is 16 units, so $a = 8$. The length of the minor axis is 6 units, so $b = 3$.

The equation of the ellipse is

$$\frac{(y-4)^2}{64} + \frac{(x-4)^2}{9} = 1.$$

ANSWER:

$$\frac{(y-4)^2}{64} + \frac{(x-4)^2}{9} = 1$$

22. vertices at (-11, 2) and (-1, 2), co-vertices at (-6, 0) and (-6, 4)

SOLUTION:

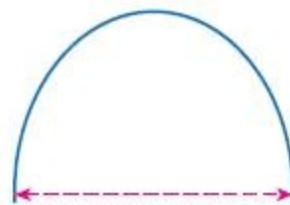
The orientation is horizontal. The center of the ellipse is at (-6, 2). The length of the major axis is 10 units, so $a = 5$. The length of the minor axis is 4 units, so $b = 2$. The equation of the ellipse is

$$\frac{(x+6)^2}{25} + \frac{(y-2)^2}{4} = 1.$$

ANSWER:

$$\frac{(x+6)^2}{25} + \frac{(y-2)^2}{4} = 1$$

23. **CCSS MODELING** The opening of a tunnel in the mountains can be modeled by semiellipses, or halves of ellipses. If the opening is 14.6 meters wide and 8.6 meters high, determine an equation to represent the opening with the center at the origin. Refer to the photo on Page 620.



SOLUTION:

The orientation is vertical. The vertices of the ellipse are (0, 8.6) and (0, -8.6). The center of the ellipse is at (0, 0).

The length of the major axis is 17.2 units, so $a = 8.6$.

The length of the minor axis is 14.6 units, so $b = 7.3$.

The equation of the ellipse is $\frac{y^2}{73.96} + \frac{x^2}{53.29} = 1$.

ANSWER:

$$\frac{y^2}{73.96} + \frac{x^2}{53.29} = 1$$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

9-4 Ellipses

$$24. \frac{(x-3)^2}{36} + \frac{(y-2)^2}{128} = 1$$

SOLUTION:

The center of the ellipse is (3, 2), so $h = 3$ and $k = 2$.

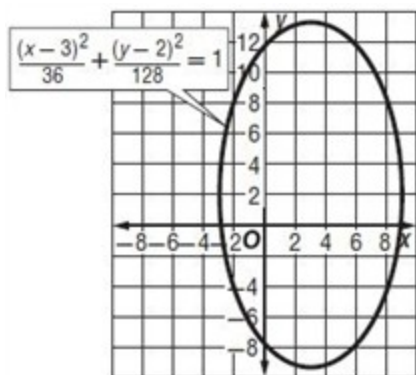
The orientation is vertical, so $a^2 = 128$ and $b^2 = 36$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{128 - 36} \\ &= \sqrt{92} \\ &\approx 9.59 \end{aligned}$$

Thus, the coordinates of the foci are (3, 11.59) and (3, -7.59).

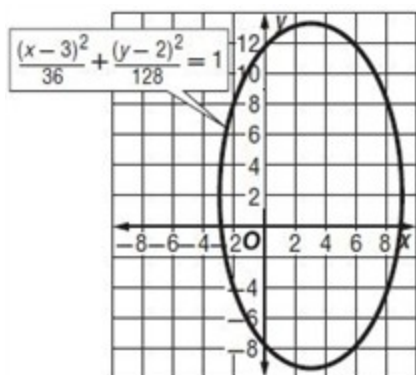
The length of the major axis is $2a$ which is about 22.63 units.

The length of the minor axis is $2b$ which is 12 units.



ANSWER:

center (3, 2); foci (3, 11.59) and (3, -7.59); major axis: ≈ 22.63 ; minor axis: 12



$$25. \frac{(x+6)^2}{50} + \frac{(y-3)^2}{72} = 1$$

SOLUTION:

The center of the ellipse is (-6, 3), so $h = -6$ and $k = 3$.

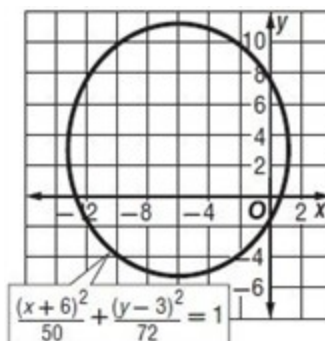
The orientation is vertical, so $a^2 = 72$ and $b^2 = 50$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{72 - 50} \\ &= \sqrt{22} \\ &\approx 4.69 \end{aligned}$$

Thus, the coordinates of the foci are (-6, 7.69) and (-6, -1.69).

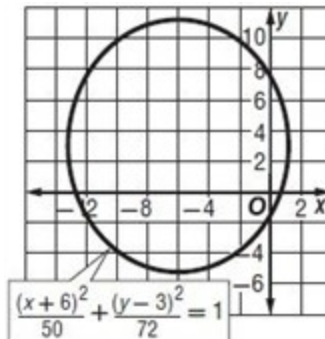
The length of the major axis is $2a$ which is about 16.97 units.

The length of the minor axis is $2b$ which is about 14.14 units.



ANSWER:

center (-6, 3); foci (-6, 7.69) and (-6, -1.69); major axis: ≈ 16.97 ; minor axis: ≈ 14.14



9-4 Ellipses

26. $\frac{x^2}{27} + \frac{(y-5)^2}{64} = 1$

SOLUTION:

The center of the ellipse is (0, 5), so $h = 0$ and $k = 5$.

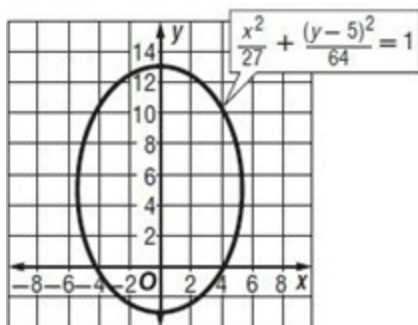
The orientation is vertical, so $a^2 = 64$ and $b^2 = 27$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{64 - 27} \\ &= \sqrt{37} \\ &\approx 6.08 \end{aligned}$$

Thus, the coordinates of the foci are (0, 11.08) and (0, -1.08).

The length of the major axis is $2a$ which is 16 units.

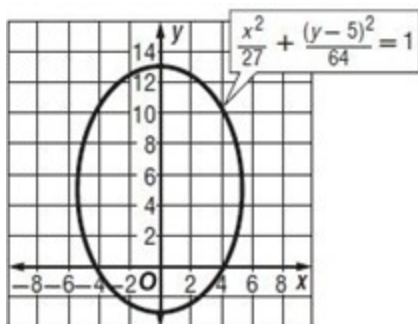
The length of the minor axis is $2b$ which is about 10.39 units.



ANSWER:

center (0, 5); foci (0, 11.08) and

(0, -1.08); major axis: 16; minor axis: ≈ 10.39



27. $\frac{(x+4)^2}{16} + \frac{y^2}{75} = 1$

SOLUTION:

The center of the ellipse is (-4, 0), so $h = -4$ and $k = 0$.

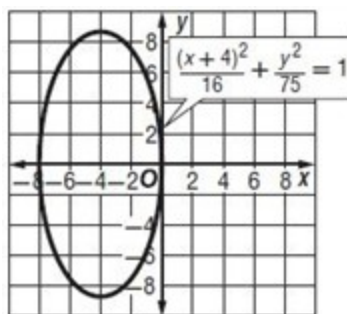
The orientation is vertical, so $a^2 = 75$ and $b^2 = 16$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{75 - 16} \\ &= \sqrt{59} \\ &\approx 7.68 \end{aligned}$$

Thus, the coordinates of the foci are (-4, 7.68) and (-4, -7.68).

The length of the major axis is $2a$ which is about 17.32 units.

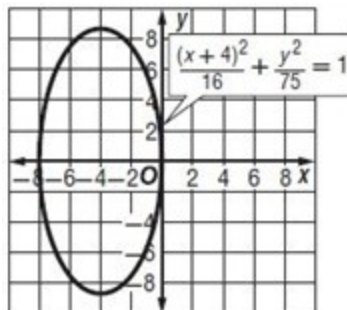
The length of the minor axis is $2b$ which is 8 units.



ANSWER:

center (-4, 0); foci (-4, 7.68) and

(-4, -7.68); major axis: ≈ 17.32 ; minor axis: 8



9-4 Ellipses

28. $3x^2 + y^2 - 6x - 8y - 5 = 0$

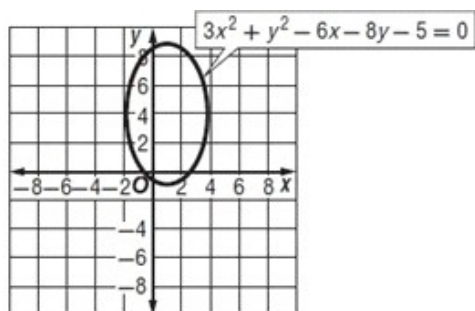
SOLUTION:

$$\begin{aligned} 3x^2 + y^2 - 6x - 8y - 5 &= 0 \\ 3(x^2 - 2x) + (y^2 - 8y) &= 5 \\ 3(x^2 - 2x + 1) + (y^2 - 8y + 16) &= 24 \\ \frac{(x-1)^2}{8} + \frac{(y-4)^2}{24} &= 1 \end{aligned}$$

The center of the ellipse is $(1, 4)$, so $h = 1$ and $k = 4$.
The orientation is vertical, so $a^2 = 24$ and $b^2 = 8$.

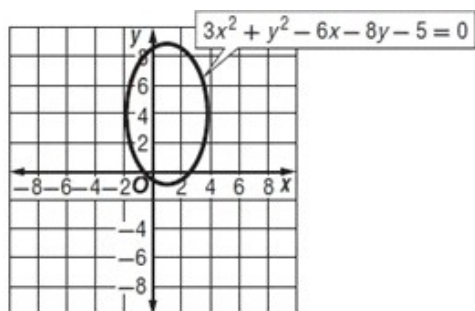
$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{24 - 8} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

Thus, the coordinates of the foci are $(1, 8)$ and $(1, 0)$.
The length of the major axis is $2a$ which is about 9.80 units.
The length of the minor axis is $2b$ which is about 5.66 units.



ANSWER:

center $(1, 4)$; foci $(1, 8)$ and $(1, 0)$; major axis:
 ≈ 9.80 ; minor axis: ≈ 5.66



29. $3x^2 + 4y^2 - 18x + 24y + 3 = 0$

SOLUTION:

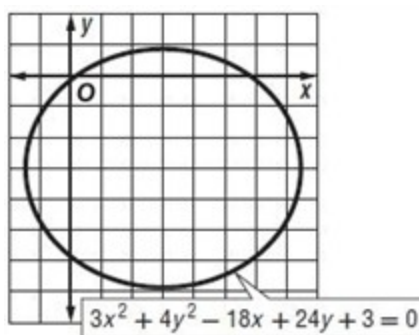
$$\begin{aligned} 3x^2 + 4y^2 - 18x + 24y + 3 &= 0 \\ 3(x^2 - 6x) + 4(y^2 + 6y) &= -3 \\ 3(x^2 - 6x + 9) + 4(y^2 + 6y + 9) &= 60 \\ \frac{(x-3)^2}{20} + \frac{(y+3)^2}{15} &= 1 \end{aligned}$$

The center of the ellipse is $(3, -3)$, so $h = 3$ and $k = -3$.

The orientation is horizontal, so $a^2 = 20$ and $b^2 = 15$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{20 - 15} \\ &= \sqrt{5} \\ &\approx 2.24 \end{aligned}$$

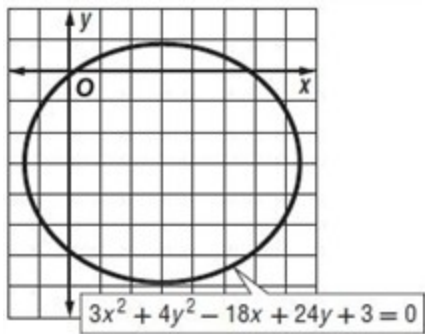
Thus, the coordinates of the foci are $(5.24, -3)$ and $(0.76, -3)$.
The length of the major axis is $2a$ which is about 8.94 units.
The length of the minor axis is $2b$ which is about 7.75 units.



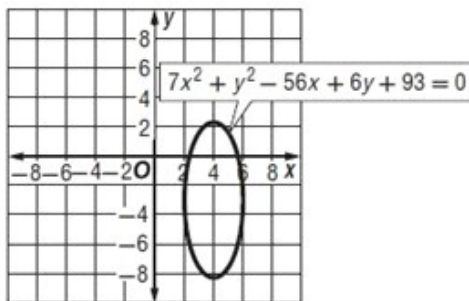
ANSWER:

center $(3, -3)$; foci $(5.24, -3)$ and $(0.76, -3)$; major axis: ≈ 8.94 ;
minor axis: ≈ 7.75

9-4 Ellipses



$(4, -7.90)$; major axis: ≈ 10.58 ; minor axis: 4



30. $7x^2 + y^2 - 56x + 6y + 93 = 0$

SOLUTION:

$$7x^2 + y^2 - 56x + 6y + 93 = 0$$

$$7(x^2 - 8x) + (y^2 + 6y) = -93$$

$$7(x^2 - 8x + 16) + (y^2 + 6y + 9) = 28$$

$$\frac{(x-4)^2}{4} + \frac{(y+3)^2}{28} = 1$$

The center of the ellipse is $(4, -3)$, so $h = 4$ and $k = -3$.

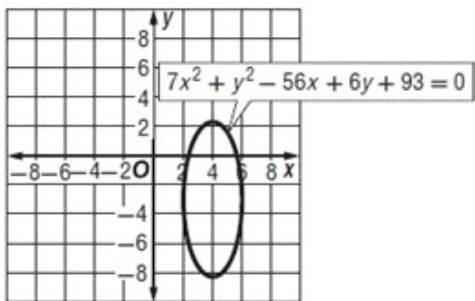
The orientation is vertical, so $a^2 = 28$ and $b^2 = 4$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{28 - 4} \\ &= \sqrt{24} \\ &\approx 4.90 \end{aligned}$$

Thus, the coordinates of the foci are $(4, 1.90)$ and $(4, -7.90)$.

The length of the major axis is $2a$ which is about 10.58 units.

The length of the minor axis is $2b$ which is 4 units.



ANSWER:

center $(4, -3)$; foci $(4, 1.90)$ and

9-4 Ellipses

31. $3x^2 + 2y^2 + 12x - 20y + 14 = 0$

SOLUTION:

$$3x^2 + 2y^2 + 12x - 20y + 14 = 0$$

$$3(x^2 + 4x) + 2(y^2 - 10y) = -14$$

$$3(x^2 + 4x + 4) + 2(y^2 - 10y + 25) = 48$$

$$\frac{(x+2)^2}{16} + \frac{(y-5)^2}{24} = 1$$

The center of the ellipse is $(-2, 5)$, so $h = -2$ and $k = 5$.

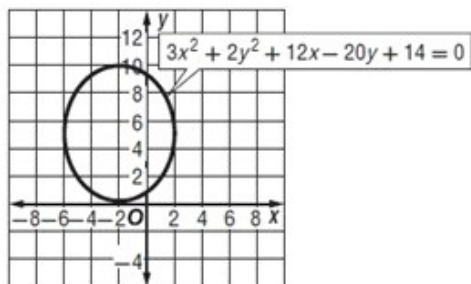
The orientation is vertical, so $a^2 = 24$ and $b^2 = 16$.

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{24 - 16} \\ &= \sqrt{8} \\ &\approx 2.83 \end{aligned}$$

Thus, the coordinates of the foci are $(-2, 7.83)$ and $(-2, 2.17)$.

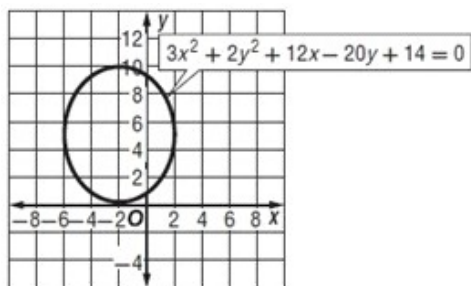
The length of the major axis is $2a$ which is about 9.80 units.

The length of the minor axis is $2b$ which is 8 units.



ANSWER:

center $(-2, 5)$; foci $(-2, 7.83)$ and $(-2, 2.17)$; major axis: ≈ 9.80 ; minor axis: 8



32. **SPACE** Like the planets, Halley's Comet travels around the Sun in an elliptical orbit. The aphelion is 3282.9 million miles and the perihelion is 54.87 million miles. Determine an equation relating the comet's orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

SOLUTION:

The value of a is one half the length of the major axis.

$$\begin{aligned} a &= 0.5(54.87 + 0.87 + 3282.9) \\ &= 1669.32 \end{aligned}$$

$$\text{So, } a^2 = 2786629.262.$$

The value of c is the distance from the center of the ellipse to the focus. This distance is equal to a minus the perihelion and the radius of the sun.

$$\begin{aligned} c &= 1669.32 - 54.87 - 0.435 \\ c &= 1614.015 \end{aligned}$$

$$\begin{aligned} c^2 &= a^2 - b^2 \\ 1614.015^2 &= 1669.32^2 - b^2 \\ b^2 &= 1669.32^2 - 1614.015^2 \\ b^2 &= 181584.8422 \end{aligned}$$

The equation of the ellipse is

$$\frac{x^2}{2,786,629.3} + \frac{y^2}{181,584.8} = 1$$

ANSWER:

$$\frac{x^2}{2,786,629.3} + \frac{y^2}{181,584.8} = 1$$

9-4 Ellipses

Write an equation of an ellipse that satisfies each set of conditions.

33. center at $(-5, -2)$, focus at $(-5, 2)$, co-vertex at $(-8, -2)$

SOLUTION:

The orientation is vertical.

The value of c is 4 and b is 3.

$$c^2 = a^2 - b^2$$

$$16 = a^2 - 9$$

$$a^2 = 25$$

$$a = 5$$

The equation of the ellipse is $\frac{(y+2)^2}{25} + \frac{(x+5)^2}{9} = 1$.

ANSWER:

$$\frac{(y+2)^2}{25} + \frac{(x+5)^2}{9} = 1$$

34. center at $(4, -3)$, focus at $(9, -3)$, co-vertex at $(4, -5)$

SOLUTION:

The orientation is horizontal. The value of c is 5 and b is 2.

$$c^2 = a^2 - b^2$$

$$25 = a^2 - 4$$

$$a = \sqrt{29}$$

The equation of the ellipse is $\frac{(x-4)^2}{29} + \frac{(y+3)^2}{4} = 1$.

ANSWER:

$$\frac{(x-4)^2}{29} + \frac{(y+3)^2}{4} = 1$$

35. foci at $(-2, 8)$ and $(6, 8)$, co-vertex at $(2, 10)$

SOLUTION:

The foci are equidistance from the center. So, the center is at $(2, 8)$.

The orientation is horizontal.

$$b = 2 \text{ and } c = 4$$

$$c^2 = a^2 - b^2$$

$$16 = a^2 - 4$$

$$a^2 = 20$$

$$a = \sqrt{20}$$

The equation of the ellipse is $\frac{(x-2)^2}{20} + \frac{(y-8)^2}{4} = 1$.

ANSWER:

$$\frac{(x-2)^2}{20} + \frac{(y-8)^2}{4} = 1$$

36. foci at $(4, 4)$ and $(4, 14)$, co-vertex at $(0, 9)$

SOLUTION:

The foci are equidistance from the center. So, the center is at $(4, 9)$.

The orientation is vertical.

$$b = 4 \text{ and } c = 5$$

$$c^2 = a^2 - b^2$$

$$25 = a^2 - 16$$

$$a^2 = 41$$

$$a = \sqrt{41}$$

The equation of the ellipse is $\frac{(y-9)^2}{41} + \frac{(x-4)^2}{16} = 1$.

ANSWER:

$$\frac{(y-9)^2}{41} + \frac{(x-4)^2}{16} = 1$$

9-4 Ellipses

37. **GOVERNMENT** The Oval Office is located in the West Wing of the White House. It is an elliptical shaped room used as the main office by the President of the United States. The long axis is 10.9 meters long and the short axis is 8.8 meters long. Write an equation to represent the outer walls of the Oval Office. Assume that the center of the room is at the origin.

SOLUTION:

The center is at (0, 0). the major axis is 10.9 meters and the minor axis is 8.8 meters.

$$2a = 10.9$$

$$a = 5.45$$

$$2b = 8.8$$

$$b = 4.4$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{5.45^2} + \frac{y^2}{4.4^2} = 1$$

$$\frac{x^2}{29.7025} + \frac{y^2}{19.36} = 1$$

The equation of the ellipse is $\frac{x^2}{29.7025} + \frac{y^2}{19.36} = 1$

ANSWER:

$$\frac{x^2}{29.7025} + \frac{y^2}{19.36} = 1$$

38. **SOUND** A whispering gallery is an elliptical room in which a faint whisper at one focus that cannot be heard by other people in the room, can easily be heard by someone at the other focus. Suppose an ellipse is 400 feet long and 120 feet wide. What is the distance between the foci?

SOLUTION:

The center is at (0, 0).

$$2a = 400$$

$$a = 200$$

$$2b = 120$$

$$b = 60$$

$$c^2 = a^2 - b^2$$

$$c^2 = 200^2 - 60^2$$

$$c^2 = 36400$$

$$c \approx 190.79$$

So, the distance between the foci is 190.79×2 which is about 381.58 ft.

ANSWER:

about 381.58 ft

39. **MULTIPLE REPRESENTATIONS** The *eccentricity* of an ellipse measures how circular the ellipse is.

a. GRAPHICAL Graph

$$\frac{x^2}{81} + \frac{y^2}{36} = 1 \text{ and } \frac{x^2}{81} + \frac{y^2}{9} = 1 \text{ on the same graph.}$$

b. VERBAL Describe the difference between the two graphs.

c. ALGEBRAIC The eccentricity of an ellipse is

$\frac{c}{a}$. Find the eccentricity for each.

d. ANALYTICAL Make a conjecture about the relationship between the value of an ellipse's eccentricity and the shape of the ellipse as compared to a circle.

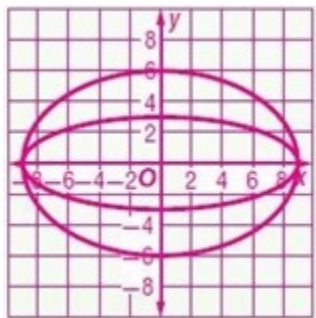
SOLUTION:

a.

The center of the ellipse $\frac{x^2}{81} + \frac{y^2}{36} = 1$ is (0,0) and $a = 9$ and $b = 6$.

9-4 Ellipses

The center of the ellipse $\frac{x^2}{81} + \frac{y^2}{9} = 1$ is $(0,0)$ and $a = 9$ and $b = 3$.



b.

Sample answer: The first graph is more circular than the second graph.

c.

For the equation of ellipse $\frac{x^2}{81} + \frac{y^2}{36} = 1$,

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{81 - 36} \\ &= \sqrt{45} \end{aligned}$$

So, the eccentricity is $\frac{\sqrt{45}}{9} \approx 0.745$.

For the equation of ellipse $\frac{x^2}{81} + \frac{y^2}{9} = 1$,

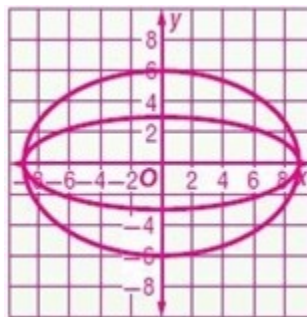
$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{81 - 9} \\ &= \sqrt{72} \end{aligned}$$

So, the eccentricity is $\frac{\sqrt{72}}{9} \approx 0.943$.

d. Sample answer: The closer the eccentricity is to 0, the more circular the ellipse.

ANSWER:

a.



b. Sample answer: The first graph is more circular than the second graph.

c. first graph: 0.745; second graph: 0.943

d. Sample answer: The closer the eccentricity is to 0, the more circular the ellipse.

40. **ERROR ANALYSIS** Serena and Karissa are determining the equation of an ellipse with foci at $(-4, -11)$ and $(-4, 5)$ and co-vertices at $(2, -3)$ and $(-10, -3)$. Is either of them correct? Explain your reasoning.

Serena

$$\frac{(x-4)^2}{64} + \frac{(y+3)^2}{36} = 1$$

Karissa

$$\frac{(x+4)^2}{100} + \frac{(y+3)^2}{36} = 1$$

SOLUTION:

Sample answer: Neither; both are showing horizontal ellipses and answer is vertical.

ANSWER:

Sample answer: Neither; both are showing horizontal ellipses and answer is vertical.

9-4 Ellipses

41. **OPEN ENDED** Write an equation of an ellipse with a focus at the origin.

SOLUTION:

Sample answer: $\frac{(x+4)^2}{40} + \frac{y^2}{24} = 1$

ANSWER:

Sample answer: $\frac{(x+4)^2}{40} + \frac{y^2}{24} = 1$

42. **CHALLENGE** When the values of a and b are equal, an ellipse is a circle. Use this information and your knowledge of ellipses to determine the formula for the area of an ellipse in terms of a and b .

SOLUTION:

Formula to find the area of a circle is πr^2 .

So, the formula to find the area of ellipse is $A = \pi ab$.

ANSWER:

$$A = \pi ab$$

43. **CHALLENGE** Determine an equation of an ellipse with foci at $(2, \sqrt{6})$ and $(2, -\sqrt{6})$ that passes through $(3, \sqrt{6})$.

SOLUTION:

The center of the ellipse is $(2, 0)$, so $h = 2$ and $k = 0$. The ellipse has vertical orientation. Foci is $(h, k \pm c)$, so $c = \sqrt{6}$.

The distance between $(2, \sqrt{6})$ and $(3, \sqrt{6})$ is

$$\sqrt{(3-2)^2 + (\sqrt{6}-\sqrt{6})^2} \text{ or } 1.$$

The distance between $(2, -\sqrt{6})$ and $(3, \sqrt{6})$ is

$$\sqrt{(3-2)^2 + (\sqrt{6}+\sqrt{6})^2} \text{ or } 5.$$

The sum of the distance from the foci to any point on the ellipse is $2a$.

$$\text{So, } 2a = 1 + 5.$$

$$a = 3.$$

Find b .

$$c^2 = a^2 - b^2$$

$$6 = 9 - b^2$$

$$b^2 = 3$$

$$b = \sqrt{3}$$

Thus, the equation of the ellipse is $\frac{y^2}{9} + \frac{(x-2)^2}{3} = 1$.

ANSWER:

$$\frac{y^2}{9} + \frac{(x-2)^2}{3} = 1$$

9-4 Ellipses

44. **CCSS ARGUMENTS** What happens to the location of the foci as an ellipse becomes more circular? Explain your reasoning.

SOLUTION:

Sample answer: As an ellipse becomes more circular, the difference between a and b becomes smaller. This causes the

value of c to become smaller since $c^2 = a^2 - b^2$. The value of $2c$ is the distance between the foci, so the foci get closer together.

ANSWER:

Sample answer: As an ellipse becomes more circular, the difference between a and b becomes smaller. This causes the

value of c to become smaller since $c^2 = a^2 - b^2$. The value of $2c$ is the distance between the foci, so the foci get closer together.

45. **REASONING** An ellipse has foci at $(-7, 2)$ and $(18, 2)$. If $(2, 14)$ is a point on the ellipse, show that $(2, -10)$ is also a point on the ellipse.

SOLUTION:

For any point on an ellipse, the sum of the distances from that point to the foci is constant by the definition of an ellipse. So, if $(2, 14)$ is on the ellipse, then the sum of the distances from it to the foci will be a certain value consistent with every other point on the ellipse.

The distance between $(-7, 2)$ and $(2, 14)$ is $\sqrt{(-7-2)^2 + (2-14)^2}$ or 15.

The distance between $(18, 2)$ and $(2, 14)$ is $\sqrt{(18-2)^2 + (2-14)^2}$ or 20.

The sum of these two distances is 35.

The distance between $(-7, 2)$ and $(2, -10)$ is $\sqrt{(-7-2)^2 + [2-(-10)]^2}$ or 15.

The distance between $(18, 2)$ and $(2, -10)$ is $\sqrt{(18-2)^2 + [2-(-10)]^2}$ or 20.

The sum of these distances is also 35. Thus, $(2, -10)$ also lies on the ellipse.

ANSWER:

For any point on an ellipse, the sum of the distances from that point to the foci is constant by the definition

of an ellipse. So, if $(2, 14)$ is on the ellipse, then the sum of the distances from it to the foci will be a certain value consistent with every other point on the ellipse.

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The sum of these two distances is 35.

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The distance between $(18, 2)$ and $(2, -10)$ is $\sqrt{(18-2)^2 + [2-(-10)]^2}$ or 20.

The sum of these distances is also 35. Thus, $(2, -10)$ also lies on the ellipse.

46. **WRITING IN MATH** Explain why the domain is $\{x | -a \leq x \leq a\}$ and the range is $\{y | -b \leq y \leq b\}$ for an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

SOLUTION:

Sample answer:

The domain is $\{x | -a \leq x \leq a\}$ because, if $|x| > a$, then $\frac{x^2}{a^2}$ would be greater than 1.

This will force $\frac{y^2}{b^2}$ to be negative since

$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ must equal 1.

In order for $\frac{y^2}{b^2}$ to be negative, either y^2 or b^2 must be negative, which cannot happen with real numbers.

For any values of $\{x | -a \leq x \leq a\}$, $\frac{x^2}{a^2}$ will be between 0 and 1.

9-4 Ellipses

The value of $\frac{y^2}{b^2}$ will also be between 0 and 1 for $(y|-b \leq y \leq b)$ and there are infinite combinations of $\frac{x^2}{a^2}$ and $\frac{y^2}{b^2}$ for which $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Thus, the domain must be $(x|-a \leq x \leq a)$. The same method proves that the range must be $(y|-b \leq y \leq b)$.

ANSWER:

Sample answer:

The domain is $(x|-a \leq x \leq a)$ because, if $|x| > a$,

then $\frac{x^2}{a^2}$ would be greater than 1.

This will force $\frac{y^2}{b^2}$ to be negative since

$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ must equal 1.

In order for $\frac{y^2}{b^2}$ to be negative, either y^2 or b^2 must be negative, which cannot happen with real numbers.

For any values of $\{x|-a \leq x \leq a\}$, $\frac{x^2}{a^2}$ will be between 0 and 1.

The value of $\frac{y^2}{b^2}$ will also be between 0 and 1 for $(y|-b \leq y \leq b)$ and there are infinite combinations of $\frac{x^2}{a^2}$ and $\frac{y^2}{b^2}$ for which $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Thus, the domain must be $(x|-a \leq x \leq a)$. The same method proves that the range must be $(y|-b \leq y \leq b)$.

47. Multiply.

$$(2 + 3i)(4 + 7i)$$

A $8 + 21i$

B $-13 + 26i$

$12i$

C $-6 + 10i$

D $13 +$

SOLUTION:

$$\begin{aligned}(2 + 3i)(4 + 7i) &= 8 + 14i + 12i - 21 \\ &= -13 + 26i\end{aligned}$$

B is the correct option.

ANSWER:

B

48. The average lifespan of American women has been tracked, and the model for the data is $y = 0.2t + 73$, where $t = 0$ corresponds to 1960. What is the meaning of the y-intercept?

F In 2007, the average lifespan was 60.

G In 1960, the average lifespan was 58.

H In 1960, the average lifespan was 73.

J The lifespan is increasing 0.2 years every year

SOLUTION:

The y-intercept is 73, which means that in 1960, the average lifespan was 73.

H is the correct option.

ANSWER:

H

49. **GRIDDED RESPONSE** If we decrease a number by 6 and then double the result, we get 5 less than the number. What is the number?

SOLUTION:

Let the unknown number be x .

The equation that represent the situation is

$$2(x - 6) = x - 5.$$

Thus, the value of x is 7.

ANSWER:

7

9-4 Ellipses

50. **SAT/ACT** The length of a rectangular prism is one inch greater than its width. The height is three times the length. Find the volume of the prism.

- A $3x^3 + x^2 + 3x$
 B $x^3 + x^2 + x$
 C $3x^3 + 6x^2 + 3x$
 D $3x^3 + 3x^2 + 3x$
 E $3x^3 + 3x^2$

SOLUTION:

Let the width of the rectangular prism is x .

So, length is $x + 1$ and height is $3x + 3$.

Volume of the rectangular prism = length $\times$ width $\times$ height

$$= x(x+1)(3x+3)$$

$$= 3x^3 + 6x^2 + 3x$$

C is the correct option.

ANSWER:

C

Write an equation of the circle that satisfies each set of conditions.

51. center (8, -9), passes through (21, 22)

SOLUTION:

The center of the circle is at (8, -9), so $h = 8$ and $k = -9$.

Find the distance between the coordinates (8, -9) and (21, 22).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$d = \sqrt{(21 - 8)^2 + (22 + 9)^2}$$

$$= \sqrt{1130}$$

Thus the radius of the circle is $\sqrt{1130}$.

The equation of the circle is

$$(x - 8)^2 + (y + 9)^2 = 1130.$$

ANSWER:

$$(x - 8)^2 + (y + 9)^2 = 1130$$

52. center at (4, 2), tangent to x-axis

SOLUTION:

The center of the circle is at (4, 2), so $h = 4$ and $k = 2$.

Find the distance between the coordinates (4, 2) and (4, 0).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$d = \sqrt{(4 - 4)^2 + (0 - 2)^2}$$

$$= 2$$

Thus the radius of the circle is 2.

The equation of the circle is $(x - 4)^2 + (y - 2)^2 = 4$.

ANSWER:

$$(x - 4)^2 + (y - 2)^2 = 4$$

53. center in the second quadrant; tangent to $y = -1$, $y = 9$, and the y-axis

SOLUTION:

The center of the circle is at (-5, 4), so $h = -5$ and $k = 4$.

Find the distance between the coordinates (-5, 4) and (0, 4).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ Distance formula}$$

$$d = \sqrt{(0 + 5)^2 + (4 - 4)^2}$$

$$= 5$$

Thus the radius of the circle is 5.

The equation of the circle is $(x + 5)^2 + (y - 4)^2 = 25$.

ANSWER:

$$(x + 5)^2 + (y - 4)^2 = 25$$

54. **ENERGY** A parabolic mirror is used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of a particular mirror is 9.75 feet above the vertex, and the latus rectum is 39 feet long.

a. Assume that the focus is at the origin. Write an equation of the parabola formed by the mirror.

b. One foot is exactly 0.3048 meter. Rewrite the equation of the mirror in meters.

c. Graph one of the equations for the mirror.

d. Which equation did you choose to graph? Explain

9-4 Ellipses

why.

SOLUTION:

a.

Focus is at the origin, so the vertex is at $(0, -9.75)$.

Length of the latus rectum is 40, so $a = \frac{1}{39}$.

Thus, the equation of the parabola is

$$y = \frac{1}{39}x^2 - 9.75.$$

b.

The equation of the mirror in meters is

$$y = \frac{1}{11.8872}x^2 - 2.9718.$$

c.

Graph of the equation $y = \frac{1}{39}x^2 - 9.75$:

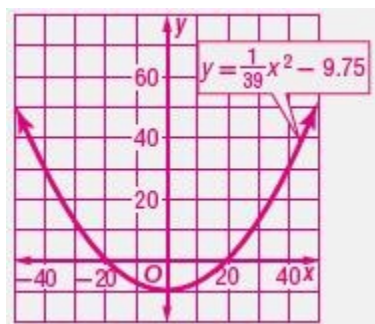
vertex: $(0, -9.75)$

axis of symmetry: $x = 0$

focus: $(0, 0)$

directrix: $y = 0.25$

length of the latus rectum: 40 units opens up



d.

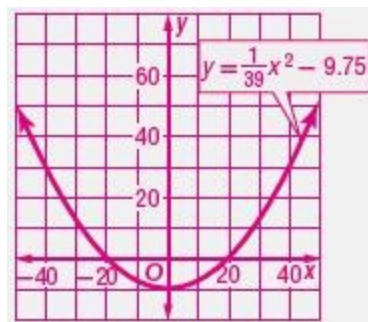
Sample answer: The equation in feet is easier to graph because the numbers have fewer decimal places.

ANSWER:

a. $y = \frac{1}{39}x^2 - 9.75$

b. $y = \frac{1}{11.8872}x^2 - 2.9718$

c.



d. Sample answer: The equation in feet is easier to graph because the numbers have fewer decimal places.

Simplify each expression.

55. $\frac{6}{d^2 + 4d + 4} + \frac{5}{d + 2}$

SOLUTION:

$$\begin{aligned} \frac{6}{d^2 + 4d + 4} + \frac{5}{d + 2} &= \frac{6}{(d + 2)^2} + \frac{5}{d + 2} \\ &= \frac{6 + 5(d + 2)}{(d + 2)^2} \\ &= \frac{5d + 16}{(d + 2)^2} \end{aligned}$$

ANSWER:

$$\frac{5d + 16}{(d + 2)^2}$$

56. $\frac{a}{a^2 - a - 20} + \frac{2}{a + 4}$

SOLUTION:

$$\begin{aligned} \frac{a}{a^2 - a - 20} + \frac{2}{a + 4} &= \frac{a}{(a - 5)(a + 4)} + \frac{2}{a + 4} \\ &= \frac{a + 2(a - 5)}{(a - 5)(a + 4)} \\ &= \frac{3a - 10}{(a - 5)(a + 4)} \end{aligned}$$

ANSWER:

$$\frac{3a - 10}{(a - 5)(a + 4)}$$

9-4 Ellipses

$$57. \frac{x}{x+1} + \frac{3}{x^2 - 4x - 5}$$

SOLUTION:

$$\begin{aligned} \frac{x}{x+1} + \frac{3}{x^2 - 4x - 5} &= \frac{x}{x+1} + \frac{3}{(x-5)(x+1)} \\ &= \frac{x(x-5) + 3}{(x-5)(x+1)} \\ &= \frac{x^2 - 5x + 3}{(x-5)(x+1)} \end{aligned}$$

ANSWER:

$$\frac{x^2 - 5x + 3}{(x-5)(x+1)}$$

Solve each equation.

$$58. \log_{10}(x^2 + 1) = 1$$

SOLUTION:

$$\begin{aligned} \log_{10}(x^2 + 1) &= 1 \\ 10^1 &= x^2 + 1 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

ANSWER:

$$\pm 3$$

$$59. \log_b 64 = 3$$

SOLUTION:

$$\begin{aligned} \log_b 64 &= 3 \\ b^3 &= 64 \\ b^3 &= 4^3 \\ b &= 4 \end{aligned}$$

ANSWER:

$$4$$

$$60. \log_b 121 = 2$$

SOLUTION:

$$\begin{aligned} \log_b 121 &= 2 \\ b^2 &= 121 \\ b &= 11 \end{aligned}$$

ANSWER:

$$11$$

Simplify.

$$61. -5ab^2(-3a^2b + 6a^3b - 3a^4b^4)$$

SOLUTION:

$$\begin{aligned} -5ab^2(-3a^2b + 6a^3b - 3a^4b^4) \\ = 15a^3b^3 - 30a^4b^3 + 15a^5b^6 \end{aligned}$$

ANSWER:

$$15a^3b^3 - 30a^4b^3 + 15a^5b^6$$

$$62. 2xy(3xy^3 - 4xy + 2y^4)$$

SOLUTION:

$$\begin{aligned} 2xy(3xy^3 - 4xy + 2y^4) \\ = 6x^2y^4 - 8x^2y^2 + 4xy^5 \end{aligned}$$

ANSWER:

$$6x^2y^4 - 8x^2y^2 + 4xy^5$$

$$63. (4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$$

SOLUTION:

$$\begin{aligned} (4x^2 - 3y^2 + 5xy) - (8xy + 3y^2) \\ = 4x^2 - 3y^2 + 5xy - 8xy - 3y^2 \\ = 4x^2 - 3xy - 6y^2 \end{aligned}$$

ANSWER:

$$4x^2 - 3xy - 6y^2$$

9-4 Ellipses

64. $(10x^2 - 3xy + 4y^2) - (3x^2 + 5xy)$

SOLUTION:

$$\begin{aligned} & (10x^2 - 3xy + 4y^2) - (3x^2 + 5xy) \\ &= 10x^2 - 3xy + 4y^2 - 3x^2 - 5xy \\ &= 7x^2 - 8xy + 4y^2 \end{aligned}$$

ANSWER:

$$7x^2 - 8xy + 4y^2$$

Write an equation of the line passing through each pair of points.

65. $(-2, 5)$ and $(3, 1)$

SOLUTION:

Substitute $(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (3, 1)$ in the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\begin{aligned} m &= \frac{1 - 5}{3 - (-2)} \\ &= -\frac{4}{5} \end{aligned}$$

Substitute $m = -\frac{4}{5}$ and $(x_1, y_1) = (-2, 5)$ in the point-slope form $y - y_1 = m(x - x_1)$.

$$\begin{aligned} y - 5 &= -\frac{4}{5}(x + 2) \\ y &= -\frac{4}{5}x - \frac{8}{5} + 5 \\ y &= -\frac{4}{5}x + \frac{17}{5} \end{aligned}$$

ANSWER:

$$y = -\frac{4}{5}x + \frac{17}{5}$$

66. $(7, 1)$ and $(7, 8)$

SOLUTION:

The slope of the points $(7, 1)$ and $(7, 8)$ is undefined. So, $x = 7$.

ANSWER:

$$x = 7$$

67. $(-3, 5)$ and $(2, 2)$

SOLUTION:

Substitute $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (2, 2)$ in the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\begin{aligned} m &= \frac{2 - 5}{2 - (-3)} \\ &= -\frac{3}{5} \end{aligned}$$

Substitute $m = -\frac{3}{5}$ and $(x_1, y_1) = (-3, 5)$ in the point-slope form $y - y_1 = m(x - x_1)$.

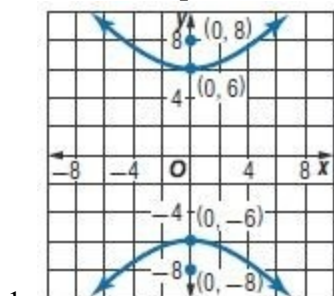
$$\begin{aligned} y - 5 &= -\frac{3}{5}(x + 3) \\ y &= -\frac{3}{5}x - \frac{9}{5} + 5 \\ y &= -\frac{3}{5}x + \frac{16}{5} \end{aligned}$$

ANSWER:

$$y = -\frac{3}{5}x + \frac{16}{5}$$

9-5 Hyperbolas

Write an equation for each hyperbola.



SOLUTION:

The vertices are equidistant from the center. So, the center is at (0, 0).

The value of a is the distance between a vertex and the center, or 6 units.

The value of c is the distance between a focus and the center, or 8 units.

$$c^2 = a^2 + b^2$$

$$64 = 36 + b^2$$

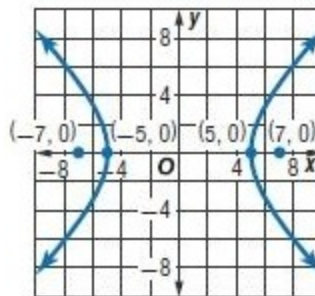
$$b^2 = 28$$

The transversal axis is vertical, so the equation is

$$\frac{y^2}{36} - \frac{x^2}{28} = 1.$$

ANSWER:

$$\frac{y^2}{36} - \frac{x^2}{28} = 1$$



2.

SOLUTION:

The vertices are equidistant from the center. So, the center is at (0, 0).

The value of a is the distance between a vertex and the center, or 5 units.

The value of c is the distance between a focus and the center, or 7 units.

$$c^2 = a^2 + b^2$$

$$49 = 25 + b^2$$

$$b^2 = 24$$

The transversal axis is horizontal, so the equation is

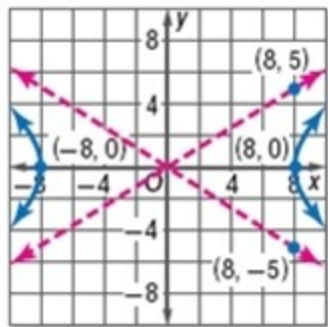
$$\frac{x^2}{25} - \frac{y^2}{24} = 1.$$

ANSWER:

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

9-5 Hyperbolas

3.



SOLUTION:

The vertices are equidistant from the center. So, the center is at $(0, 0)$.

The value of a is the distance between a vertex and the center, or 8 units.

Equation of asymptotes are $y = \pm \frac{5}{8}x$.

From the asymptotes, $b = 5$.

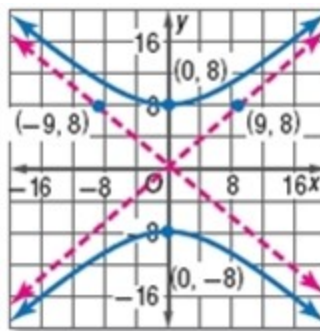
The transversal axis is horizontal, so the equation is

$$\frac{x^2}{64} - \frac{y^2}{25} = 1.$$

ANSWER:

$$\frac{x^2}{64} - \frac{y^2}{25} = 1$$

4.



SOLUTION:

The vertices are equidistant from the center. So, the center is at $(0, 0)$.

The value of a is the distance between a vertex and the center, or 8 units.

Equation of asymptotes are $y = \pm \frac{8}{9}x$.

From the asymptotes, $b = 9$.

The transversal axis is vertical, so the equation is

$$\frac{y^2}{64} - \frac{x^2}{81} = 1.$$

ANSWER:

$$\frac{y^2}{64} - \frac{x^2}{81} = 1$$

CCSS STRUCTURE Graph each hyperbola. Identify the vertices, foci, and asymptotes.

9-5 Hyperbolas

5. $\frac{x^2}{64} - \frac{y^2}{49} = 1$

SOLUTION:

The center of the hyperbola is (0, 0).

$$a^2 = 64 \text{ so } a = 8.$$

$$b^2 = 49 \text{ so } b = 7.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{64 + 49} \\ &= \sqrt{113} \end{aligned}$$

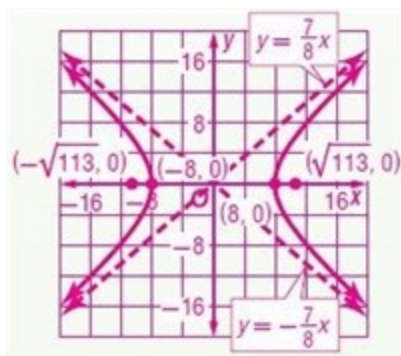
The orientation is horizontal.

vertices: (8, 0) and (-8, 0)

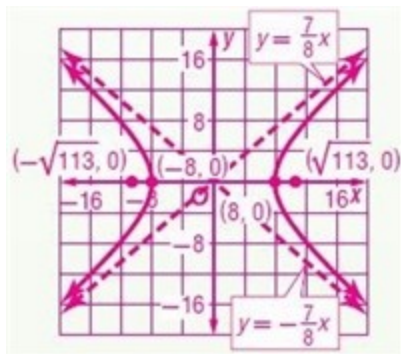
foci: $(\sqrt{113}, 0)$ and $(-\sqrt{113}, 0)$

The equations for the asymptotes are

$$y = \frac{7}{8}x \text{ and } y = -\frac{7}{8}x.$$



ANSWER:



6. $\frac{y^2}{36} - \frac{x^2}{60} = 1$

SOLUTION:

The center of the hyperbola is (0, 0).

$$a^2 = 36 \text{ so } a = 6.$$

$$b^2 = 60 \text{ so } b = 2\sqrt{15}.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{36 + 60} \\ &= \sqrt{96} \\ &\approx 9.8 \end{aligned}$$

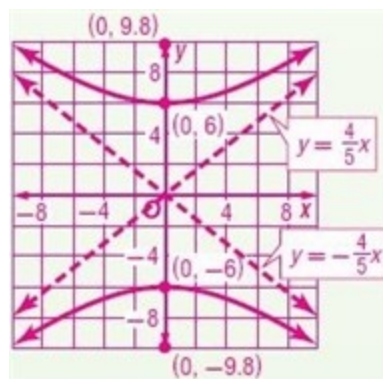
The orientation is vertical.

vertices: (0, 6) and (0, -6)

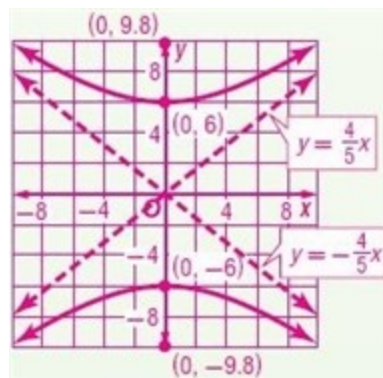
foci: (0, 9.8) and (0, -9.8)

Equations for the asymptotes:

$$\begin{aligned} y &= \frac{\sqrt{15}}{5}x \text{ and } y = -\frac{\sqrt{15}}{5}x \\ y &\approx \frac{4}{5}x \text{ and } y \approx -\frac{4}{5}x \end{aligned}$$



ANSWER:



9-5 Hyperbolas

7. $9y^2 + 18y - 16x^2 + 64x - 199 = 0$

SOLUTION:

$$9y^2 + 18y - 16x^2 + 64x - 199 = 0$$

$$9(y^2 + 2y) - 16(x^2 - 4x) = 199$$

$$9(y^2 + 2y + 1) - 16(x^2 - 4x + 4) = 144$$

$$\frac{(y+1)^2}{16} - \frac{(x-2)^2}{9} = 1$$

The center of the hyperbola is $(2, -1)$.

$$a^2 = 16 \text{ so } a = 4.$$

$$b^2 = 9 \text{ so } b = 3.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 9} \\ &= 5 \end{aligned}$$

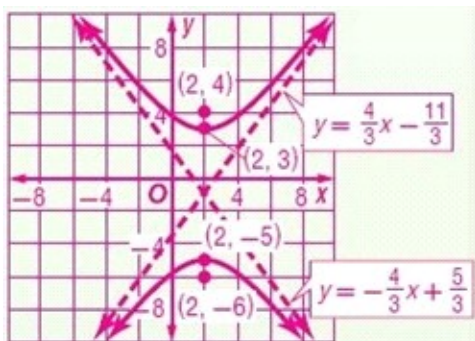
The orientation is vertical.

vertices: $(2, 3)$ and $(2, -5)$

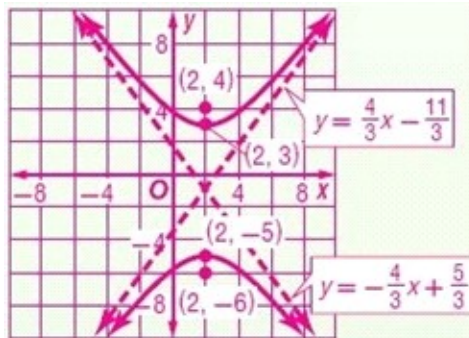
foci: $(2, 4)$ and $(2, -6)$

The equations for the asymptotes are

$$y = \frac{4}{3}x - \frac{11}{3} \text{ and } y = -\frac{4}{3}x + \frac{11}{3}.$$



ANSWER:



8. $4x^2 + 24x - y^2 + 4y - 4 = 0$

SOLUTION:

$$4x^2 + 24x - y^2 + 4y - 4 = 0$$

$$4(x^2 + 6x) - (y^2 - 4y) = 4$$

$$4(x^2 + 6x + 9) - (y^2 - 4y + 4) = 36$$

$$\frac{(x+3)^2}{9} - \frac{(y-2)^2}{36} = 1$$

The center of the hyperbola is $(-3, 2)$.

$$a^2 = 9 \text{ so } a = 3.$$

$$b^2 = 36 \text{ so } b = 6.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \\ &\approx 6.7 \end{aligned}$$

The orientation is horizontal.

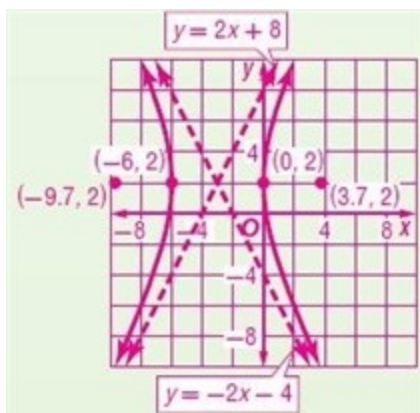
vertices: $(0, 2)$ and $(-6, 2)$

foci: $(3.7, 2)$ and $(-9.7, 2)$

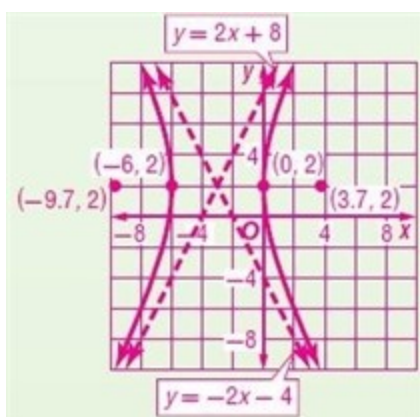
The equations for the asymptotes are

$$y = 2x + 8 \text{ and } y = -2x - 4.$$

9-5 Hyperbolas



ANSWER:



9. **NAVIGATION** A ship determines that the difference of its distances from two stations is 60 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at $(-80, 0)$ and $(80, 0)$.

SOLUTION:

The center of the hyperbola is $(0, 0)$.

The distance between the two stations is 160. The value of c is $160 \div 2$ or 80.

The difference of the distances from the two stations is 60. Therefore, a is $60 \div 2$ or 30 nautical miles.

$$c^2 = a^2 + b^2$$

$$80^2 = 30^2 + b^2$$

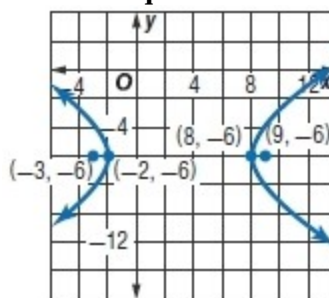
$$b^2 = 5500$$

The equation of the hyperbola is $\frac{x^2}{900} - \frac{y^2}{5500} = 1$.

ANSWER:

$$\frac{x^2}{900} - \frac{y^2}{5500} = 1$$

Write an equation for each hyperbola.



10.

SOLUTION:

The vertices are equidistant from the center. So, the center is at $(3, -6)$.

The value of a is the distance between a vertex and the center, or 5 units.

The value of c is the distance between a focus and the center, or 6 units.

$$c^2 = a^2 + b^2$$

$$36 = 25 + b^2$$

$$b^2 = 11$$

The transversal axis is horizontal, so the equation is

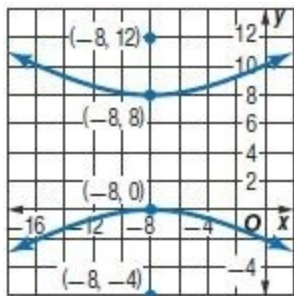
$$\frac{(x-3)^2}{25} - \frac{(y+6)^2}{11} = 1.$$

ANSWER:

$$\frac{(x-3)^2}{25} - \frac{(y+6)^2}{11} = 1$$

9-5 Hyperbolas

11.



SOLUTION:

The vertices are equidistant from the center. So, the center is at $(-8, 4)$.

The value of a is the distance between a vertex and the center, or 4 units.

The value of c is the distance between a focus and the center, or 8 units.

$$c^2 = a^2 + b^2$$

$$64 = 16 + b^2$$

$$b^2 = 48$$

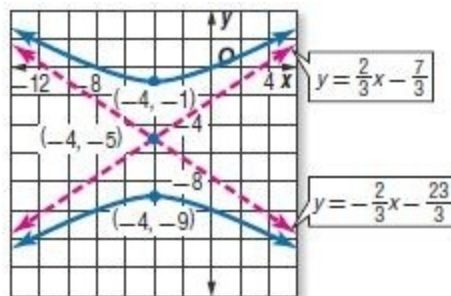
The transversal axis is vertical, so the equation is

$$\frac{(y-4)^2}{16} - \frac{(x+8)^2}{48} = 1.$$

ANSWER:

$$\frac{(y-4)^2}{16} - \frac{(x+8)^2}{48} = 1$$

12.



SOLUTION:

The vertices are equidistant from the center. So, the center is at $(-4, -5)$.

The value of a is the distance between a vertex and the center, or 4 units.

Substitute these values in the general form of the asymptote.

$$y = \pm \frac{a}{b}(x-h) + k$$

$$y = \pm \frac{4}{b}(x+4) - 5$$

$$= \pm \frac{4}{b}x + \frac{16}{b} - 5$$

$$= \pm \frac{4}{b}x + \frac{16-5b}{b}$$

Compare with the equations of the asymptotes.

$$\text{So, } \frac{4}{b} = \frac{2}{3}$$

$$12 = 2b$$

$$b = 6$$

The equation of the hyperbola is

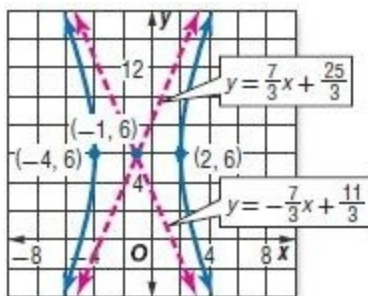
$$\frac{(y+5)^2}{16} - \frac{(x+4)^2}{36} = 1.$$

ANSWER:

$$\frac{(y+5)^2}{16} - \frac{(x+4)^2}{36} = 1$$

9-5 Hyperbolas

13.



SOLUTION:

The vertices are equidistant from the center. So, the center is at $(-1, 6)$.

The value of a is the distance between a vertex and the center, or 3 units.

Substitute these values in the general form of the asymptote.

$$y = \pm \frac{b}{a}(x - h) + k$$

$$y = \pm \frac{b}{3}(x + 1) + 6$$

$$= \pm \frac{b}{3}x + \frac{b}{3} + 6$$

$$= \pm \frac{b}{3}x + \frac{b + 18}{3}$$

Compare with the equations of the asymptotes.

$$\frac{b}{3} = \frac{7}{3}$$

So, $b = 7$.

The equation of the hyperbola is

$$\frac{(x + 1)^2}{9} - \frac{(y - 6)^2}{49} = 1.$$

ANSWER:

$$\frac{(x + 1)^2}{9} - \frac{(y - 6)^2}{49} = 1$$

Graph each hyperbola. Identify the vertices, foci, and asymptotes.

14. $\frac{x^2}{36} - \frac{y^2}{4} = 1$

SOLUTION:

The center of the hyperbola is $(0, 0)$.

$$a^2 = 36 \text{ so } a = 6.$$

$$b^2 = 4 \text{ so } b = 2.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{36 + 4} \\ &= \sqrt{40} \\ &\approx 6.3 \end{aligned}$$

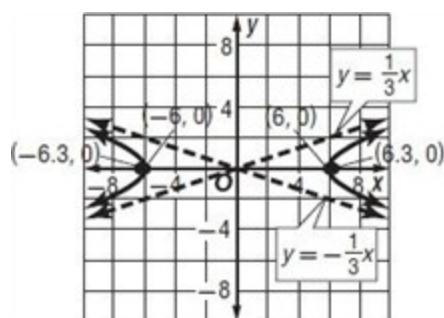
The orientation is horizontal.

vertices: $(6, 0)$ and $(-6, 0)$

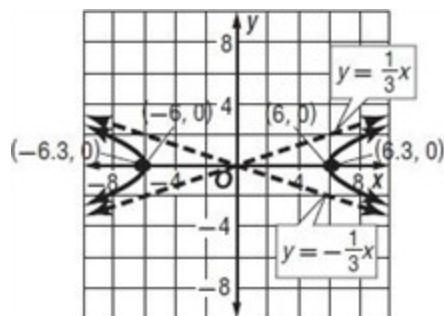
foci: $(6.3, 0)$ and $(-6.3, 0)$

The equations for the asymptotes are

$$y = \frac{1}{3}x \text{ and } y = -\frac{1}{3}x.$$



ANSWER:



9-5 Hyperbolas

15. $\frac{y^2}{9} - \frac{x^2}{49} = 1$

SOLUTION:

The center of the hyperbola is (0, 0).

$$a^2 = 9 \text{ so } a = 3.$$

$$b^2 = 49 \text{ so } b = 7.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{9 + 49} \\ &= \sqrt{58} \\ &\approx 7.6 \end{aligned}$$

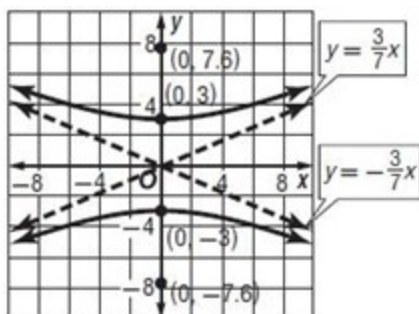
The orientation is vertical.

vertices: (0, 3) and (0, -3)

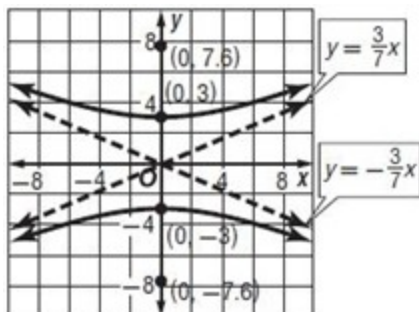
foci: (0, 7.6) and (0, -7.6)

The equations for the asymptotes

are $y = \frac{3}{7}x$ and $y = -\frac{3}{7}x$.



ANSWER:



16. $\frac{y^2}{36} - \frac{x^2}{25} = 1$

SOLUTION:

The center of the hyperbola is (0, 0).

$$a^2 = 36 \text{ so } a = 6.$$

$$b^2 = 25 \text{ so } b = 5.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{36 + 25} \\ &= \sqrt{61} \\ &\approx 7.8 \end{aligned}$$

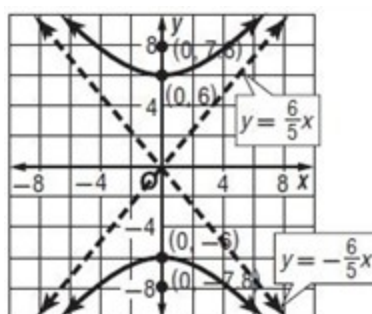
The orientation is vertical.

vertices: (0, 6) and (0, -6)

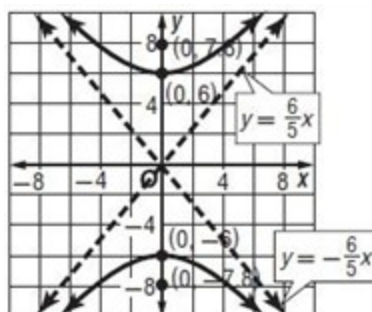
foci: (0, 7.8) and (0, -7.8)

The equations for the asymptotes

are $y = \frac{6}{5}x$ and $y = -\frac{6}{5}x$.



ANSWER:



9-5 Hyperbolas

17. $\frac{x^2}{16} - \frac{y^2}{16} = 1$

SOLUTION:

The center of the hyperbola is (0, 0).

$$a^2 = 16 \text{ so } a = 4.$$

$$b^2 = 16 \text{ so } b = 4.$$

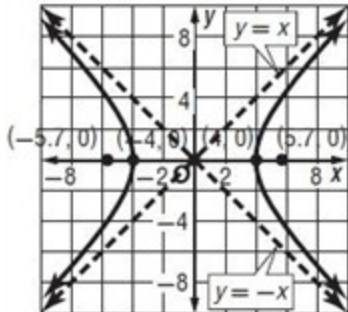
$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \\ &\approx 5.7 \end{aligned}$$

The orientation is horizontal.

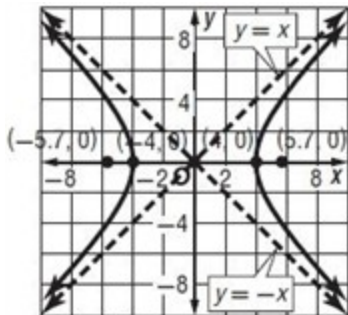
vertices: (4, 0) and (-4, 0)

foci: (5.7, 0) and (-5.7, 0)

The equations for the asymptotes are $y = x$ and $y = -x$.



ANSWER:



18. $\frac{(x-3)^2}{16} - \frac{(y+1)^2}{4} = 1$

SOLUTION:

The center of the hyperbola is (3, -1).

$$a^2 = 16 \text{ so } a = 4.$$

$$b^2 = 4 \text{ so } b = 2.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} \\ &\approx 4.5 \end{aligned}$$

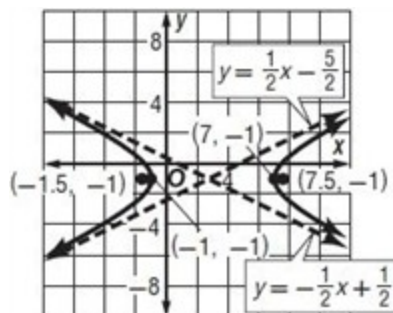
The orientation is horizontal.

vertices: (7, -1) and (-1, -1)

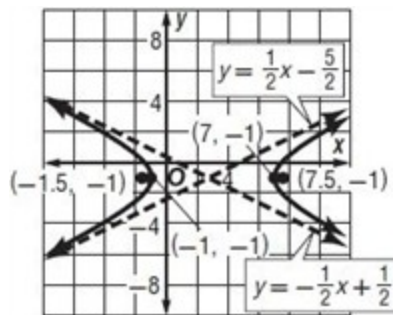
foci: (7.5, -1) and (-1.5, -1)

The equations for the asymptotes

are $y = \frac{1}{2}x - \frac{5}{2}$ and $y = -\frac{1}{2}x + \frac{1}{2}$.



ANSWER:



9-5 Hyperbolas

19. $\frac{(y+5)^2}{16} - \frac{(x+2)^2}{36} = 1$

SOLUTION:

The center of the hyperbola is $(-2, -5)$.

$$a^2 = 16 \text{ so } a = 4.$$

$$b^2 = 36 \text{ so } b = 6.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 36} \\ &= \sqrt{52} \\ &\approx 7.2 \end{aligned}$$

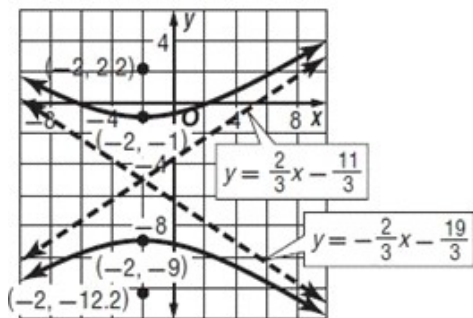
The orientation is vertical.

vertices: $(-2, -1)$ and $(-2, -9)$

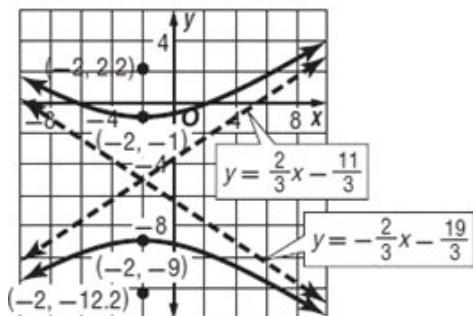
foci: $(-2, 2.2)$ and $(-2, -12.2)$

The equations for the asymptotes

are $y = \frac{2}{3}x - \frac{11}{3}$ and $y = -\frac{2}{3}x - \frac{19}{3}$.



ANSWER:



20. $9y^2 - 4x^2 - 54y + 32x - 19 = 0$

SOLUTION:

$$9y^2 - 4x^2 - 54y + 32x - 19 = 0$$

$$9(y^2 - 6y) - 4(x^2 - 8x) = 19$$

$$9(y^2 - 6y + 9) - 4(x^2 - 8x + 16) = 36$$

$$\frac{(y-3)^2}{4} - \frac{(x-4)^2}{9} = 1$$

The center of the hyperbola is $(4, 3)$.

$$a^2 = 4 \text{ so } a = 2.$$

$$b^2 = 9 \text{ so } b = 3.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{4 + 9} \\ &\approx 3.6 \end{aligned}$$

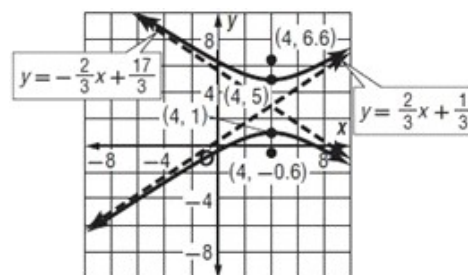
The orientation is vertical.

vertices: $(4, 5)$ and $(4, 1)$

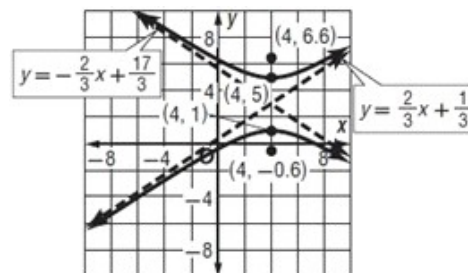
foci: $(4, 6.6)$ and $(4, -0.6)$

The equations for the asymptotes are

$y = \frac{2}{3}x + \frac{1}{3}$ and $y = -\frac{2}{3}x + \frac{17}{3}$.



ANSWER:



9-5 Hyperbolas

21. $16x^2 - 9y^2 + 128x + 36y + 76 = 0$

SOLUTION:

$$16x^2 - 9y^2 + 128x + 36y + 76 = 0$$

$$16(x^2 + 8x) - 9(y^2 - 4y) = -76$$

$$16(x^2 + 8x + 16) - 9(y^2 - 4y + 4) = 144$$

$$\frac{(x+4)^2}{9} - \frac{(y-2)^2}{16} = 1$$

The center of the hyperbola is $(-4, 2)$.

$$a^2 = 9 \text{ so } a = 3.$$

$$b^2 = 16 \text{ so } b = 4.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{9 + 16} \\ &= 5 \end{aligned}$$

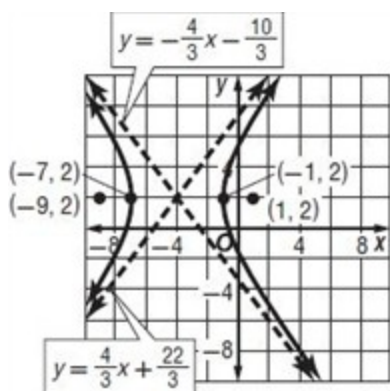
The orientation is horizontal.

vertices: $(-1, 2)$ and $(-7, 2)$

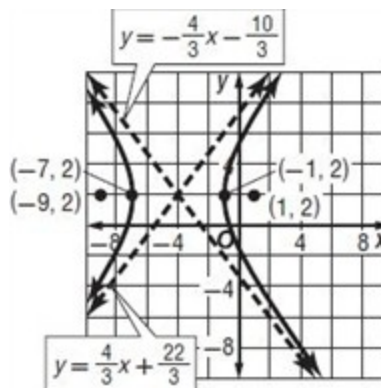
foci: $(1, 2)$ and $(-9, 2)$

The equations for the asymptotes are

$$y = \frac{4}{3}x + \frac{22}{3} \text{ and } y = -\frac{4}{3}x - \frac{10}{3}.$$



ANSWER:



22. $25x^2 - 4y^2 - 100x + 48y - 144 = 0$

SOLUTION:

$$25x^2 - 4y^2 - 100x + 48y - 144 = 0$$

$$25(x^2 - 4x) - 4(y^2 - 12y) = 144$$

$$25(x^2 - 4x + 4) - 4(y^2 - 12y + 36) = 100$$

$$\frac{(x-2)^2}{4} - \frac{(y-6)^2}{25} = 1$$

The center of the hyperbola is $(2, 6)$.

$$a^2 = 4 \text{ so } a = 2.$$

$$b^2 = 25 \text{ so } b = 5.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{4 + 25} \\ &\approx 5.4 \end{aligned}$$

The orientation is horizontal.

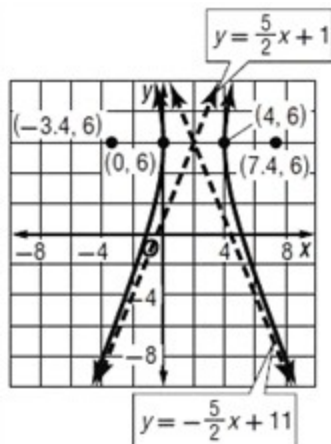
vertices: $(4, 6)$ and $(0, 6)$

foci: $(7.4, 6)$ and $(-3.4, 6)$

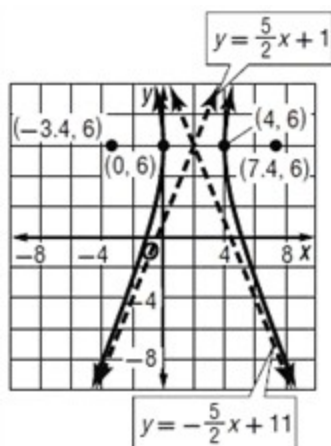
The equations for the asymptotes

$$\text{are } y = \frac{5}{2}x + 1 \text{ and } y = -\frac{5}{2}x + 11.$$

9-5 Hyperbolas



ANSWER:



$$23. 81y^2 - 16x^2 - 810y + 96x + 585 = 0$$

SOLUTION:

$$81y^2 - 16x^2 - 810y + 96x + 585 = 0$$

$$81(y^2 - 10y) - 16(x^2 - 6x) = -585$$

$$81(y^2 - 10y + 25) - 16(x^2 - 6x + 9) = 1296$$

$$\frac{(y-5)^2}{16} - \frac{(x-3)^2}{81} = 1$$

The center of the hyperbola is (3, 5).

$$a^2 = 16 \text{ so } a = 4.$$

$$b^2 = 81 \text{ so } b = 9.$$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 81} \\ &\approx 9.8 \end{aligned}$$

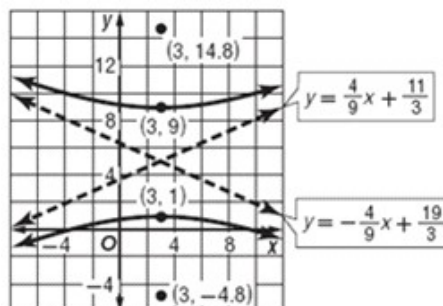
The orientation is vertical.

vertices: (3, 9) and (3, 1)

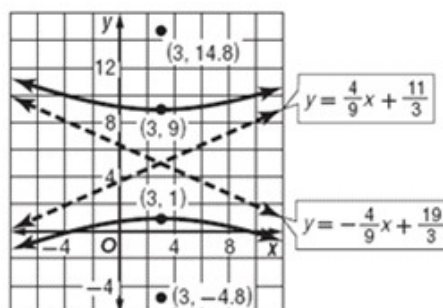
foci: (3, 14.8) and (3, -4.8)

The equations for the asymptotes are

$$y = \frac{4}{9}x + \frac{11}{3} \text{ and } y = -\frac{4}{9}x + \frac{19}{3}.$$



ANSWER:



9-5 Hyperbolas

24. **NAVIGATION** A ship determines that the difference of its distances from two stations is 80 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at $(-100, 0)$ and $(100, 0)$.

SOLUTION:

Find the center and the values of a and b . The center of the hyperbola is $(0, 0)$.

The distance between the two stations is 200. The value of c is $200 \div 2$ or 100.

The difference of the distances from the two stations is 80. Therefore, a is $80 \div 2$ or 40 nautical miles.

$$c^2 = a^2 + b^2$$

$$100^2 = 40^2 + b^2$$

$$b^2 = 8400$$

The equation of the hyperbola is $\frac{x^2}{1600} - \frac{y^2}{8400} = 1$.

ANSWER:

$$\frac{x^2}{1600} - \frac{y^2}{8400} = 1$$

Determine whether the following equations represent ellipses or hyperbolas.

25. $4x^2 = 5y^2 + 6$

SOLUTION:

$$4x^2 - 5y^2 = 6$$

$$\frac{4x^2}{6} - \frac{5y^2}{6} = 1$$

$$\frac{x^2}{\left(\frac{6}{4}\right)} - \frac{y^2}{\left(\frac{6}{5}\right)} = 1$$

This is in the standard form for a hyperbola. Thus, the equation represents a hyperbola.

ANSWER:

hyperbola

26. $8x^2 - 2x = 8y - 3y^2$

SOLUTION:

$$\begin{aligned} 8x^2 - 2x &= 8y - 3y^2 \\ (8x^2 - 2x) + (3y^2 - 8y) &= 0 \\ 8\left(x^2 - \frac{1}{4}x + \left(\frac{1}{8}\right)^2\right) + 3\left(y^2 - \frac{8}{3}y + \left(\frac{2}{3}\right)^2\right) &= \frac{35}{24} \\ \frac{\left(x - \frac{1}{8}\right)^2}{\left(\frac{35}{192}\right)} + \frac{\left(y - \frac{2}{3}\right)^2}{\left(\frac{35}{72}\right)} &= 1 \end{aligned}$$

This is in the standard form for an ellipse. Thus, the equation represents an ellipse.

ANSWER:

ellipse

27. $-5x^2 + 4x = 6y + 3y^2$

SOLUTION:

$$\begin{aligned} -5x^2 + 4x &= 6y + 3y^2 \\ (3y^2 + 6y) + (5x^2 - 4x) &= 0 \\ 3\left(y^2 + 2y + 1\right) + 5\left(x^2 - \frac{4}{5}x + \left(\frac{2}{5}\right)^2\right) &= \frac{19}{5} \\ \frac{(y+1)^2}{\left(\frac{19}{15}\right)} + \frac{\left(x - \frac{2}{5}\right)^2}{\left(\frac{19}{25}\right)} &= 1 \end{aligned}$$

This is in the standard form for an ellipse. Thus, the equation represents an ellipse.

ANSWER:

ellipse

9-5 Hyperbolas

28. $7y - 2x^2 = 6x - 2y^2$

SOLUTION:

$$\begin{aligned}7y - 2x^2 &= 6x - 2y^2 \\2\left(y^2 + \frac{7}{2}y\right) - 2\left(x^2 + 3x\right) &= 0 \\2\left(y^2 + \frac{7}{2}y + \left(\frac{7}{4}\right)^2\right) - 2\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) &= \frac{13}{8} \\ \frac{\left(y + \frac{7}{4}\right)^2}{\left(\frac{13}{16}\right)} - \frac{\left(x + \frac{3}{2}\right)^2}{\left(\frac{13}{16}\right)} &= 1\end{aligned}$$

This is in the standard form for a hyperbola.
Thus, the equation represents a hyperbola.

ANSWER:

hyperbola

29. $6x - 7x^2 - 5y^2 = 2y$

SOLUTION:

$$\begin{aligned}6x - 7x^2 - 5y^2 &= 2y \\(5y^2 + 2y) + (7x^2 - 6x) &= 0 \\5\left(y^2 + \frac{2}{5}y + \left(\frac{1}{5}\right)^2\right) + 7\left(x^2 - \frac{6}{7}x + \left(\frac{3}{7}\right)^2\right) &= \frac{52}{35} \\ \frac{\left(y + \frac{1}{5}\right)^2}{\left(\frac{52}{175}\right)} + \frac{\left(x - \frac{3}{7}\right)^2}{\left(\frac{52}{245}\right)} &= 1\end{aligned}$$

It is in the standard form of the ellipse.
Thus, the equation represents an ellipse.

ANSWER:

ellipse

30. $4x + 6y + 2x^2 = -3y^2$

SOLUTION:

$$\begin{aligned}4x + 6y + 2x^2 &= -3y^2 \\(2x^2 + 4x) + (3y^2 + 6y) &= 0 \\2(x^2 + 2x + 1) + 3(y^2 + 2y + 1) &= 5 \\ \frac{(x+1)^2}{10} + \frac{(y+1)^2}{15} &= 1\end{aligned}$$

This is in the standard form for an ellipse.
Thus, the equation represents an ellipse.

ANSWER:

ellipse

9-5 Hyperbolas

31. **SPACE** Refer to the application at the beginning of the lesson. With the Sun as a focus and the center at the origin, a certain comet's path follows a branch of a hyperbola. If two of the coordinates of the path are (10, 0) and (30, 100) where the units are in millions of miles, determine the equation of the path.

SOLUTION:

The center is at the origin, so $h = 0$ and $k = 0$.

The coordinate (10, 0) is the vertex, so $a = 10$.

The standard form of the equation of a hyperbola

with center at the origin is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Substitute $a = 10$.

$$\frac{x^2}{100} - \frac{y^2}{b^2} = 1$$

Substitute $(x, y) = (30, 100)$ in the equation

$$\frac{x^2}{100} - \frac{y^2}{b^2} = 1.$$

$$\frac{900}{100} - \frac{10000}{b^2} = 1$$

$$9 - \frac{10000}{b^2} = 1$$

$$-\frac{10000}{b^2} = -8$$

$$b^2 = \frac{10000}{8}$$

$$b^2 = 1250$$

Therefore, the equation of the path is

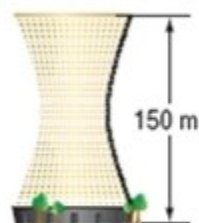
$$\frac{x^2}{100} - \frac{y^2}{1250} = 1.$$

ANSWER:

$$\frac{x^2}{100} - \frac{y^2}{1250} = 1$$

32. **COOLING** Natural draft cooling towers are shaped like hyperbolas for more efficient cooling of power plants. The hyperbola in the tower can be modeled

by $\frac{x^2}{16} - \frac{y^2}{225} = 1$, where the units are in meters. Find the width of the tower at the top and at its narrowest point in the middle.



SOLUTION:

$$\frac{x^2}{16} - \frac{y^2}{225} = 1$$

Here $a^2 = 16$. Therefore, $a = 4$. So, the width of the tower at the narrowest point in the middle is $2(4)$ or 8 meters.

Substitute $y = 75$ in the equation $\frac{x^2}{16} - \frac{y^2}{225} = 1$.

$$\frac{x^2}{16} - \frac{y^2}{225} = 1$$

$$\frac{x^2}{16} - \frac{75^2}{225} = 1$$

$$\frac{x^2}{16} = 1 + \frac{5625}{225}$$

$$\frac{x^2}{16} = 1 + 25$$

$$\frac{x^2}{16} = 26$$

$$x^2 = 26(16)$$

$$x \approx 20.4$$

Therefore, the width of the tower at the top is about $2(20.4)$ or 40.8 meters.

ANSWER:

8 m in the middle and 40.8 m at the top

33. **MULTIPLE REPRESENTATIONS** Consider $xy = 16$.

9-5 Hyperbolas

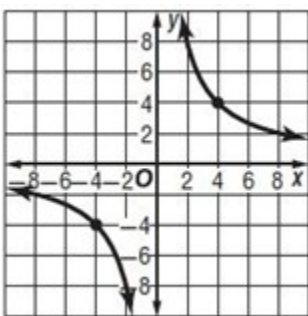
- a. TABULAR** Make a table of values for the equation for $-12 \leq x \leq 12$.
- b. GRAPHICAL** Graph the hyperbola represented by the equation.
- c. LOGICAL** Determine and graph the asymptotes for the hyperbola.
- d. ANALYTICAL** What special property do you notice about the asymptotes? Hyperbolas that represent this property are called *rectangular hyperbolas*.
- e. ANALYTICAL** Without any calculations, what do you think will be the coordinates of the vertices for $xy = 25$? for $xy = 36$?

SOLUTION:

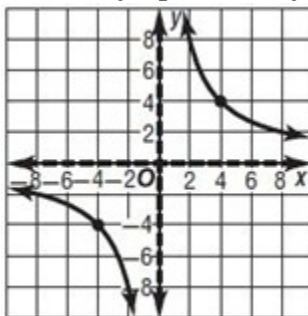
a.

x	y
-12	-1.33
-10	-1.6
-8	-2
-6	-2.67
-4	-4
-2	-8
0	undef
2	8
4	4
6	2.67
8	2
10	1.6
12	1.33

b.



c. The asymptotes are $y = 0$ and $x = 0$.



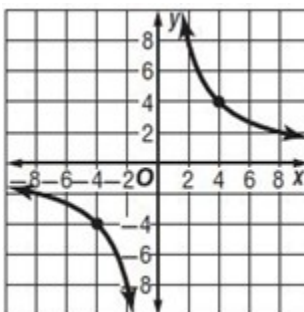
- d.** They are perpendicular.
- e.** For $xy = 25$, the vertices will be at (5, 5) and (-5, -5), and for $xy = 36$, they will be at (-6, -6) and (6, 6).

ANSWER:

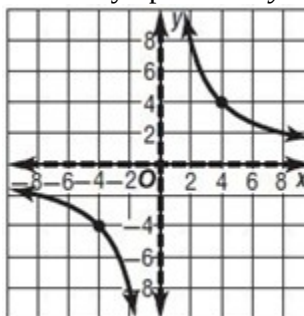
a.

x	y
-12	-1.33
-10	-1.6
-8	-2
-6	-2.67
-4	-4
-2	-8
0	undef
2	8
4	4
6	2.67
8	2
10	1.6
12	1.33

b.



c. The asymptotes are $y = 0$ and $x = 0$.



- d.** They are perpendicular.
- e.** For $xy = 25$, the vertices will be at (5, 5) and (-5, -5), and for $xy = 36$, they will be at (-6, -6) and (6, 6).

9-5 Hyperbolas

34. **CCSS MODELING** Two receiving stations that are 250 miles apart receive a signal from a downed airplane. They determine that the airplane is 70 miles farther from station *B* than from station *A*. Determine the equation of the horizontal hyperbola centered at the origin on which the plane is located.

SOLUTION:

The center of the hyperbola is (0, 0).

The distance between the two stations is 250. So, the value of *c* is $250 \div 2$ or 125.

The difference of the distances from the two stations is 70. Therefore, *a* is $70 \div 2$ or 35 miles.

$$c^2 = a^2 + b^2$$

$$125^2 = 35^2 + b^2$$

$$b^2 = 14400$$

The equation of the hyperbola is $\frac{x^2}{1225} - \frac{y^2}{14400} = 1$.

ANSWER:

$$\frac{x^2}{1225} - \frac{y^2}{14400} = 1$$

35. **WEATHER** Luisa and Karl live exactly 4000 feet apart. While on the phone at their homes, Luisa hears thunder out of her window and Karl hears it 3 seconds later out of his. If sound travels 1100 feet per second, determine the equation for the horizontal hyperbola on which the lightning is located.

SOLUTION:

Since Karl hears the thunder 3 seconds later and the sound travels at 1100 feet per second, the thunder took place $1100(3) = 3300$ feet farther from Karl than from Luisa.

Therefore, the locus of all points that are 3300 feet closer to Luisa than to Karl is one branch of the hyperbola. That is, the position of two friends becomes the foci of the hyperbola.

Therefore, the length of the line joining the two vertices of the hyperbola becomes 3300 feet.

$$2a = 3300$$

$$a = 1650$$

Therefore, $a^2 = 2,722,500$.

The length between the two friends is 4000 feet.

$$2c = 4000$$

$$c = 2000$$

Use the relation $b^2 = c^2 - a^2$ to find the value of b^2 .

$$b^2 = 2000^2 - 1650^2$$

$$= 1,277,500$$

The equation for the horizontal hyperbola where the

lightning is located is $\frac{x^2}{2,722,500} - \frac{y^2}{1,277,500} = 1$.

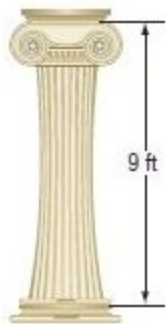
ANSWER:

$$\frac{x^2}{2,722,500} - \frac{y^2}{1,277,500} = 1$$

9-5 Hyperbolas

36. **ARCHITECTURE** Large pillars with cross sections in the shape of hyperbolas were popular in ancient Greece. The curves can be modeled by the equation $\frac{x^2}{0.16} - \frac{y^2}{4} = 1$, where the units are in feet.

If the pillars are 9 feet tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to the nearest hundredth of a foot.



SOLUTION:

$$\frac{x^2}{0.16} - \frac{y^2}{4} = 1$$

Here $a^2 = 0.16$. Therefore, $a = 0.4$. So, the width of pillar at the narrowest point in the middle is $2(0.4)$ or 0.8 feet.

Substitute $y = 4.5$ in the equation $\frac{x^2}{0.16} - \frac{y^2}{4} = 1$.

$$\begin{aligned} \frac{x^2}{0.16} - \frac{4.5^2}{4} &= 1 \\ \frac{x^2}{0.16} &= 1 + \frac{4.5^2}{4} \\ x^2 &= 0.16 \left(\frac{4 + 20.25}{4} \right) \\ x^2 &= 0.04(24.25) \\ x^2 &= 0.97 \\ x &\approx 0.98 \end{aligned}$$

Therefore, the width of the top of each pillar is about $2(0.98)$ or 1.96 feet.

ANSWER:

≈ 1.96 ft, 0.8 ft

Write an equation for the hyperbola that satisfies each set of conditions.

37. vertices $(-8, 0)$ and $(8, 0)$, conjugate axis of length 20 units

SOLUTION:

From the vertices, we know that the orientation is horizontal and $a = 8$.

Conjugate axis, $2b$ is 20 units.

So, $b = 10$.

The equation of the hyperbola is $\frac{x^2}{64} - \frac{y^2}{100} = 1$.

ANSWER:

$$\frac{x^2}{64} - \frac{y^2}{100} = 1$$

38. vertices $(0, -6)$ and $(0, 6)$, conjugate axis of length 24 units

SOLUTION:

From the vertices, we know that the orientation is vertical and $a = 6$.

Conjugate axis, $2b$ is 24 units.

So, $b = 12$.

The equation of the hyperbola is $\frac{y^2}{36} - \frac{x^2}{144} = 1$.

ANSWER:

$$\frac{y^2}{36} - \frac{x^2}{144} = 1$$

9-5 Hyperbolas

39. vertices $(6, -2)$ and $(-2, -2)$, foci $(10, -2)$ and $(-6, -2)$

SOLUTION:

The vertices are equidistant from the center. So, the center is at $(2, -2)$.

The value of a is the distance between a vertex and the center, or 4 units.

The value of c is the distance between a focus and the center, or 8 units.

$$c^2 = a^2 + b^2$$

$$64 = 16 + b^2$$

$$b^2 = 48$$

The transversal axis is horizontal, so the equation is

$$\frac{(x-2)^2}{16} - \frac{(y+2)^2}{48} = 1.$$

ANSWER:

$$\frac{(x-2)^2}{16} - \frac{(y+2)^2}{48} = 1$$

40. vertices $(-3, 4)$ and $(-3, -8)$, foci $(-3, 9)$ and $(-3, -13)$

SOLUTION:

The vertices are equidistant from the center. So, the center is at $(-3, -2)$.

The value of a is the distance between a vertex and the center, or 6 units.

The value of c is the distance between a focus and the center, or 11 units.

$$c^2 = a^2 + b^2$$

$$121 = 36 + b^2$$

$$b^2 = 85$$

The transversal axis is vertical, so the equation is

$$\frac{(y+2)^2}{36} - \frac{(x+3)^2}{85} = 1.$$

ANSWER:

$$\frac{(y+2)^2}{36} - \frac{(x+3)^2}{85} = 1$$

41. centered at the origin with a horizontal transverse axis of length 10 units and a conjugate axis of length 4 units

SOLUTION:

The transverse axis, $2a$ is 10 units, so $a = 5$.

The conjugate axis, $2b$ is 4 units, so $b = 2$.

The equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{4} = 1$.

ANSWER:

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

42. centered at the origin with a vertical transverse axis of length 16 units and a conjugate axis of length 12 units

SOLUTION:

The transverse axis, $2a$ is 16 units, so $a = 8$.

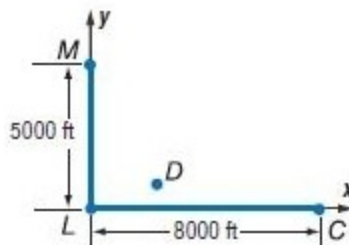
The conjugate axis, $2b$ is 12 units, so $b = 6$.

The equation of the hyperbola is $\frac{y^2}{64} - \frac{x^2}{36} = 1$.

ANSWER:

$$\frac{y^2}{64} - \frac{x^2}{36} = 1$$

43. **TRIANGULATION** While looking for their lost dog in the woods, Lae, Meg, and Cesar hear a bark. Meg hears it 2 seconds before Lae and Cesar hears it 3 seconds before Lae. With Lae at the origin, determine the exact location of their dog if sound travels 1100 feet per second.



SOLUTION:

This is a hyperbola with horizontal orientation.

$$2c = 8000$$

$$c = 4000$$

The positions of Lae and Cesar become the foci of the hyperbola. So, the center of the hyperbola is

9-5 Hyperbolas

(4000, 0).

Since Cesar hears a bark 3 seconds before Lae, $2a$ is $3(1100)$ or 3300.

$$2a = 3300$$

$$a = 1650$$

$$b^2 = c^2 - a^2$$

$$b^2 = 4000^2 - 1650^2$$

$$b^2 = 13277500$$

The equation of the hyperbola is

$$\frac{(x - 4000)^2}{1650^2} - \frac{y^2}{13277500} = 1.$$

Hyperbola with vertical orientation:

$$2c = 5000$$

$$c = 2500$$

The position of Lae and Meg becomes the foci of the hyperbola with vertical orientation. So the center of the hyperbola is (0, 2500).

Since Meg bears a bark 2 seconds before Lae, $2a$ is $2(1100)$ or 2200.

$$2a = 2200$$

$$a = 1100$$

$$b^2 = c^2 - a^2$$

$$= 2500^2 - 1100^2$$

$$= 5040000$$

The equation of the hyperbola is

$$\frac{(y - 2500)^2}{1100^2} - \frac{x^2}{5040000} = 1.$$

The position of the dog can be determined by solving the equation of hyperbolas.

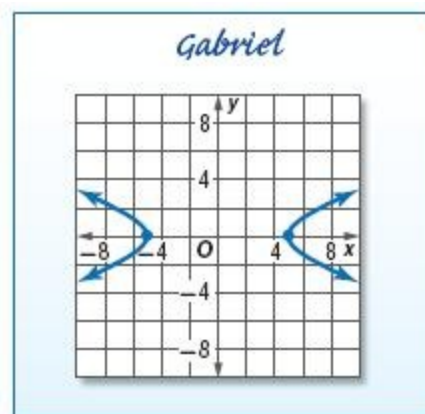
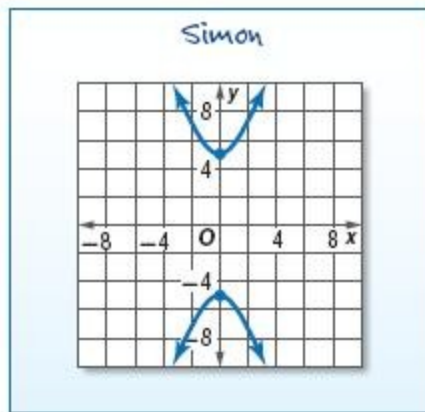
The solution of the system of equations of hyperbolas is (2308, 826). The exact location of dog is (2308, 826).

ANSWER:

(2308, 826)

44. **CCSS CRITIQUE** Simon and Gabriel are graphing

$\frac{y^2}{25} - \frac{x^2}{4} = 1$. Is either of them correct? Explain your reasoning.



SOLUTION:

Simon is correct. The equation indicates a hyperbola that opens up and down. Gabriel drew a horizontal hyperbola.

ANSWER:

Sample answer: Simon; The equation indicates a hyperbola that opens up and down. Gabriel drew a horizontal hyperbola.

45. **CHALLENGE** The origin lies on a horizontal hyperbola. The asymptotes for the hyperbola are $y = -x + 1$ and $y = x - 5$. Find the equation for the hyperbola.

SOLUTION:

Solve the system of equations of the asymptotes.

9-5 Hyperbolas

$$\begin{aligned}-x + 1 &= x - 5 \\ -2x &= -6 \\ x &= 3\end{aligned}$$

Substitute $x = 3$ in anyone of the equation and find y .

$$\begin{aligned}y &= -3 + 1 \\ &= -2\end{aligned}$$

So, the center of the hyperbola is at $(3, -2)$.

Substitute the values of (h, k) in the general form of asymptotes.

$$\begin{aligned}y - k &= \pm \frac{b}{a}(x - h) \\ y + 2 &= \pm \frac{b}{a}(x - 3) \\ y &= \pm \frac{b}{a}x - \frac{3b}{a} - 2\end{aligned}$$

Compare with the equations of the asymptotes.

$$\begin{aligned}\frac{b}{a} &= -1 \\ -\frac{b}{a} &= 1 \\ b &= -a\end{aligned}$$

Substitute the value of (h, k) in the equation of the hyperbola.

$$\begin{aligned}\frac{(x-3)^2}{a^2} - \frac{(y+2)^2}{b^2} &= 1 \\ \frac{(x-3)^2}{a^2} - \frac{(y+2)^2}{a^2} &= 1\end{aligned}$$

The origin lies on the hyperbola. So substitute $x = 0$ and $y = 0$ in the equation.

$$\begin{aligned}\frac{(0-3)^2}{a^2} - \frac{(0+2)^2}{a^2} &= 1 \\ \frac{9}{a^2} - \frac{4}{a^2} &= 1 \\ a^2 &= 5\end{aligned}$$

So, the equation of the hyperbola is

$$\frac{(x-3)^2}{5} - \frac{(y+2)^2}{5} = 1.$$

ANSWER:

$$\frac{(x-3)^2}{5} - \frac{(y+2)^2}{5} = 1$$

46. **REASONING** What happens to the location of the foci of a hyperbola as the value of a becomes increasingly smaller than the value of b ? Explain your reasoning.

SOLUTION:

The foci move farther away from the vertices. When a is only slightly smaller than b , the value of c is only fairly larger than a . However, as b becomes much larger than a , by way of $c^2 = a^2 + b^2$, c becomes much larger than a . When this happens, the distance between the foci c becomes much greater than the distance between the vertices a . Therefore, the foci are much farther away from the vertices.

ANSWER:

Sample answer: The foci move farther away from the vertices. When a is only slightly smaller than b , the value of c is only fairly larger than a . However, as b becomes much larger than a , by way of $c^2 = a^2 + b^2$, c becomes much larger than a . When this happens, the distance between the foci c becomes much greater than the distance between the vertices a . Therefore, the foci are much farther away from the vertices.

9-5 Hyperbolas

47. **REASONING** Consider $\frac{y^2}{36} - \frac{x^2}{16} = 1$. Describe the change in the shape of the hyperbola and the locations of the vertices and foci if 36 is changed to 9. Explain why this happens.

SOLUTION:

When 36 changes to 9, the vertical hyperbola widens (splits out from the y-axis faster). This is due to a smaller value of y being needed to produce the same value of x . The vertices are moved closer together due to the value of a decreasing from 6 to 3. The foci move farther from the vertices because the difference between c and a increased.

ANSWER:

Sample answer: When 36 changes to 9, the vertical hyperbola widens (splits out from the y-axis faster). This is due to a smaller value of y being needed to produce the same value of x . The vertices are moved closer together due to the value of a decreasing from 6 to 3. The foci move farther from the vertices because the difference between c and a increased.

48. **OPEN ENDED** Write an equation for a hyperbola with a focus at the origin.

SOLUTION:

Sample answer: $\frac{(x-5)^2}{9} - \frac{y^2}{16} = 1$

ANSWER:

Sample answer: $\frac{(x-5)^2}{9} - \frac{y^2}{16} = 1$

49. **WRITING IN MATH** Why would you choose a conic section to model a set of data instead of a polynomial function?

SOLUTION:

Sample answer: Conic sections can be used to model phenomena that can't be modeled using functions. For example, parabolas can be used to model paths of comets and ellipses can be used to model planetary orbits.

ANSWER:

Sample answer: Conic sections can be used to model phenomena that can't be modeled using functions. For example, parabolas can be used to model paths of comets and ellipses can be used to model planetary orbits.

9-5 Hyperbolas

50. You have 6 more dimes than quarters. You have a total of \$5.15. How many dimes do you have?

A 13
B 16
C 19
D 25

SOLUTION:

Let the number of dimes be x and the number of quarters be y .

The equations that represents the situation are

$$x = y + 6 \text{ and } 10x + 25y = 515.$$

Substitute $x = y + 6$ in the second equation and solve for y .

$$\begin{aligned} 10x + 25y &= 515 \\ 10(y + 6) + 25y &= 515 \\ 10y + 60 + 25y &= 515 \\ 35y &= 455 \\ y &= 13 \end{aligned}$$

Substitute $y = 13$ in the equation $x = y + 6$ and find x .

$$\begin{aligned} x &= 13 + 6 \\ &= 19 \end{aligned}$$

So, there are 19 dimes.

C is the correct option.

ANSWER:

C

51. How tall is a tree that is 15 feet shorter than a pole three times as tall as the tree?

F 24.5 ft
G 22.5 ft
H 21.5 ft
J 7.5 ft

SOLUTION:

Let the height of the tree be y and the height of the pole be x .

The equations that represents the situation is

$$\begin{aligned} y &= x - 15; \\ x &= 3y. \end{aligned}$$

Substitute $x = 3y$ in the equation $y = x - 15$ and solve for y .

$$\begin{aligned} y &= 3y - 15 \\ -2y &= -15 \\ y &= 7.5 \end{aligned}$$

J is the correct option.

ANSWER:

J

9-5 Hyperbolas

52. **SHORT RESPONSE** A rectangle is 8 feet long and 6 feet wide. If each dimension is increased by the same number of feet, the area of the new rectangle formed is 32 square feet more than the area of the original rectangle. By how many feet was each dimension increased?

SOLUTION:

Let the increased number of feet in each dimension be x .

The equation that represent the given situation is

$$(8 + x)(6 + x) = 32 + 48.$$

Solve the equation for x .

$$(8 + x)(6 + x) = 32 + 48$$

$$48 + 8x + 6x + x^2 = 80$$

$$x^2 + 14x - 32 = 0$$

$$x = 2 \text{ or } -6$$

As the length cannot have a negative value, $x = 2$.
So, each dimension was increased by 2 feet.

ANSWER:

2

53. **SAT/ACT** When the equation $y = 4x^2 - 5$ is graphed in the coordinate plane, the graph is which of the following?

A line

B circle

C ellipse

D hyperbola

E parabola

SOLUTION:

The equation $y = 4x^2 - 5$ is in the standard form of the parabola.

Thus, the equation represents a parabola. Choice E is correct.

ANSWER:

E

Write an equation for an ellipse that satisfies each set of conditions.

54. endpoints of major axis at (2, 2) and (2, -10), endpoints of minor axis at (0, -4) and (4, -4)

SOLUTION:

The orientation is vertical. The length of the major axis is 12 units, so $a = 6$.

The length of the minor axis is 4 units, so $b = 2$.

The center of the ellipse is at (2, -4).

The equation of the ellipse is $\frac{(y + 4)^2}{36} + \frac{(x - 2)^2}{4} = 1$.

ANSWER:

$$\frac{(y + 4)^2}{36} + \frac{(x - 2)^2}{4} = 1$$

55. endpoints of major axis at (0, 10) and (0, -10), foci at (0, 8) and (0, -8)

SOLUTION:

The orientation is vertical.

The center of the ellipse is at (0, 0).

The length of the major axis is 20 units, so $a = 10$.

The length of the foci is 16 units, so $c = 8$.

$$c^2 = a^2 - b^2$$

$$64 = 100 - b^2$$

$$b^2 = 36$$

The equation of the ellipse is $\frac{y^2}{100} + \frac{x^2}{36} = 1$.

ANSWER:

$$\frac{y^2}{100} + \frac{x^2}{36} = 1$$

9-5 Hyperbolas

Find the center and radius of the circle with the given equation. Then graph the circle.

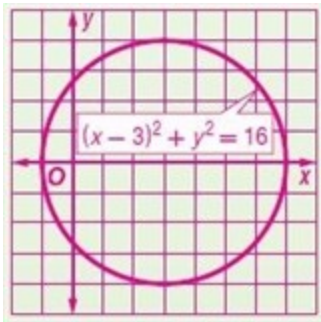
56. $(x - 3)^2 + y^2 = 16$

SOLUTION:

$h = 3$, and $k = 0$, so the center of the circle is $(3, 0)$.
Find the radius.

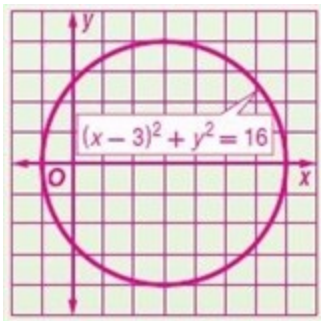
$$r^2 = 16$$

$$r = 4 \text{ units}$$



ANSWER:

$(3, 0)$, 4 units



57. $x^2 + y^2 - 6y - 16 = 0$

SOLUTION:

$$x^2 + y^2 - 6y - 16 = 0$$

$$x^2 + (y^2 - 6y) = 16$$

$$x^2 + (y^2 - 6y + 9) = 25$$

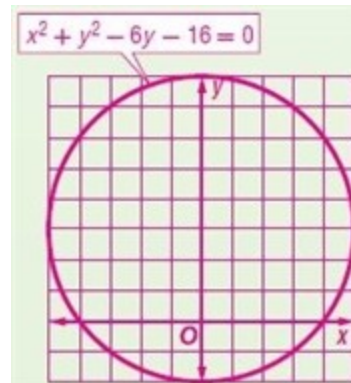
$$x^2 + (y - 3)^2 = 25$$

$h = 0$, and $k = 3$, so the center of the circle is $(0, 3)$.

Find the radius.

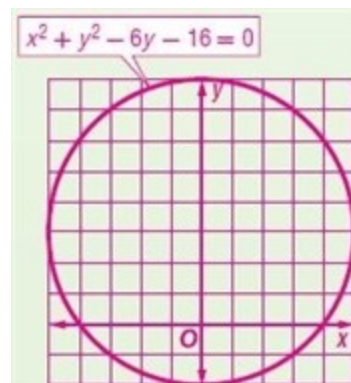
$$r^2 = 25$$

$$r = 5 \text{ units}$$



ANSWER:

$(0, 3)$, 5 units



9-5 Hyperbolas

58. $x^2 + y^2 + 9x - 8y + 4 = 0$

SOLUTION:

$$x^2 + y^2 + 9x - 8y + 4 = 0$$

$$\left(x + \frac{9}{2}\right)^2 + (y - 4)^2 = \frac{129}{4}$$

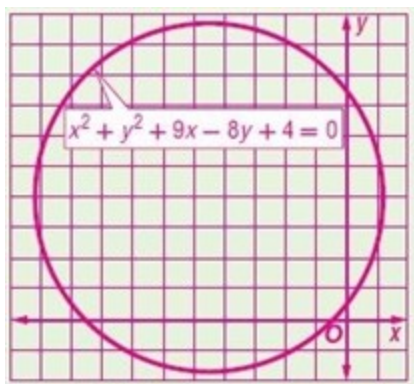
$h = -\frac{9}{2}$, and $k = 4$, so the center of the circle is

$$\left(-\frac{9}{2}, 4\right).$$

Find the radius.

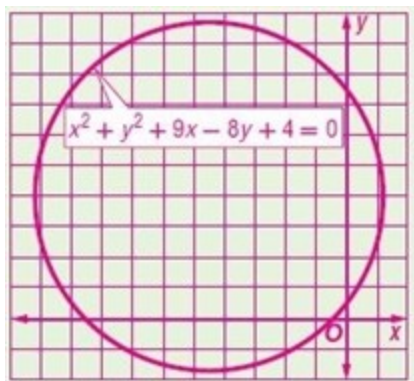
$$r^2 = \frac{129}{4}$$

$$r = \frac{\sqrt{129}}{2} \text{ units}$$



ANSWER:

$$\left(-\frac{9}{2}, 4\right), \frac{\sqrt{129}}{2} \text{ units}$$



Centerville High School. So far this season, she has made 6 out of 10 free-throws. She is determined to improve her free-throw percentage. If she can make x consecutive free throws, her free-throw

percentage can be determined using $P(x) = \frac{6+x}{10+x}$.

a. Graph the function.

b. What part of the graph is meaningful in the context of the problem?

c. Describe the meaning of the y-intercept.

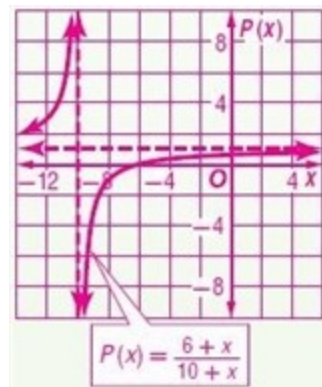
d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Zonta's shooting percentage.

SOLUTION:

a.

The vertical asymptote is $x = -10$.

As the numerator and the denominator has the same degree, the horizontal asymptote is $y = 1$.



b. the part in the first quadrant

c. It represents her original free-throw percentage of 60%.

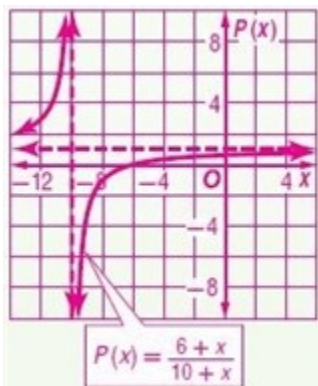
d. $P(x) = 1$; this represents 100%, which she cannot achieve because she has already missed 4 free throws.

ANSWER:

a.

59. **BASKETBALL** Zonta plays basketball for

9-5 Hyperbolas



b. the part in the first quadrant

c. It represents her original free-throw percentage of 60%.

d. $P(x) = 1$; this represents 100%, which she cannot achieve because she has already missed 4 free throws.

Solve each equation.

60. $\left(\frac{1}{7}\right)^{y-3} = 343$

SOLUTION:

$$\left(\frac{1}{7}\right)^{y-3} = 343$$

$$7^{-y+3} = 7^3$$

$$-y + 3 = 3$$

$$y = 0$$

ANSWER:

0

61. $10^{x-1} = 100^{2x-3}$

SOLUTION:

$$10^{x-1} = 100^{2x-3}$$

$$10^{x-1} = 10^{4x-6}$$

$$x - 1 = 4x - 6$$

$$-3x = -5$$

$$x = \frac{5}{3}$$

ANSWER:

$\frac{5}{3}$

62. $36^{2p} = 216^{p-1}$

SOLUTION:

$$36^{2p} = 216^{p-1}$$

$$6^{4p} = 6^{3p-3}$$

$$4p = 3p - 3$$

$$p = -3$$

ANSWER:

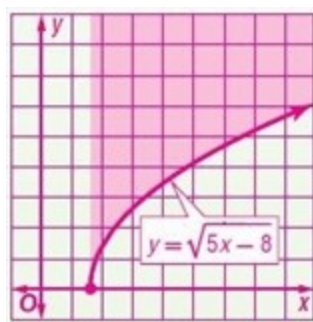
-3

Graph each inequality.

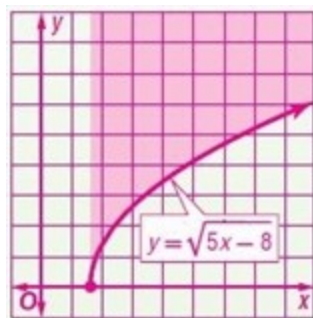
63. $y \geq \sqrt{5x-8}$

SOLUTION:

Graph the boundary $y = \sqrt{5x-8}$. The domain is $\{x \mid x \geq 1.6\}$. Because the inequality is greater than or equal to, the shaded region should be above the boundary and within the domain.



ANSWER:

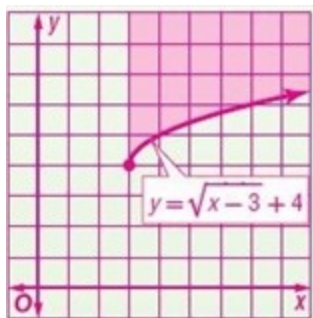


9-5 Hyperbolas

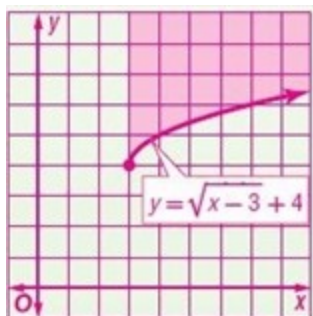
64. $y \geq \sqrt{x-3} + 4$

SOLUTION:

Graph the boundary $y = \sqrt{x-3} + 4$. The domain is $\{x \mid x \geq 3\}$. Because the inequality is greater than or equal to, the shaded region should be above the boundary and within the domain.



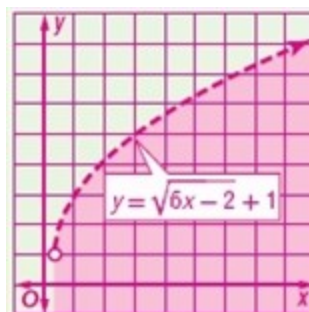
ANSWER:



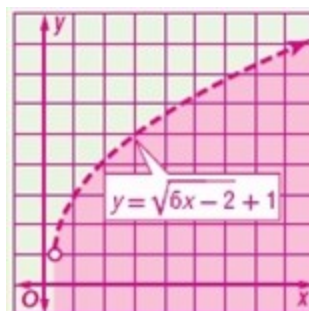
65. $y < \sqrt{6x-2} + 1$

SOLUTION:

Graph the boundary $y = \sqrt{6x-2} + 1$. The domain is $\{x \mid x > 0.4\}$. Because the inequality is less than, the shaded region should be below the boundary and within the domain.



ANSWER:



66. Write an equation for a parabola with vertex at the origin that passes through $(2, -8)$.

SOLUTION:

The vertex is at the origin, so $h = 0$ and $k = 0$.

Let $x = 2$ and $y = -8$.

Substitute these values in the vertex form.

$$y = a(x-h)^2 + k$$

$$-8 = a(2-0)^2 + 0$$

$$-8 = 4a$$

$$a = -2$$

The equation of the parabola is $y = -2x^2$.

ANSWER:

$$y = -2x^2$$

9-5 Hyperbolas

67. Write an equation for a parabola with vertex at $(-3, -4)$ that opens up and has y -intercept 8.

SOLUTION:

The vertex is at $(-3, -4)$, so $h = -3$ and $k = -4$.

Let $x = 0$ and $y = 8$.

Substitute these values in the vertex form.

$$y = a(x - h)^2 + k$$

$$8 = a(0 + 3)^2 - 4$$

$$12 = 9a$$

$$a = \frac{4}{3}$$

The equation of the parabola is $y = \frac{4}{3}(x + 3)^2 - 4$.

ANSWER:

$$y = \frac{4}{3}(x + 3)^2 - 4$$

9-6 Identifying Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

1. $x^2 + 4y^2 - 6x + 16y - 11 = 0$

SOLUTION:

$$x^2 + 4y^2 - 6x + 16y - 11 = 0$$

$$x^2 + 4y^2 - 6x + 16y = 11$$

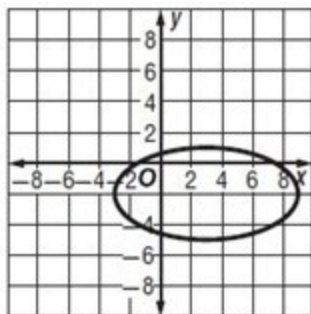
$$(x^2 - 6x) + 4(y^2 + 4y) = 11$$

$$(x^2 - 6x + 9) + 4(y^2 + 4y + 4) = 11 + 16 + 9$$

$$(x - 3)^2 + 4(y + 2)^2 = 36$$

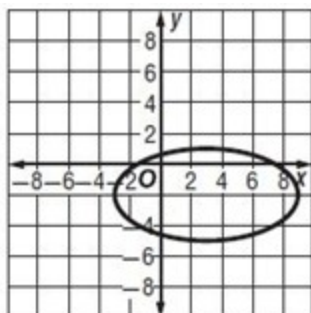
$$\frac{(x - 3)^2}{36} + \frac{(y + 2)^2}{9} = 1$$

The given equation represents an ellipse.
Graph the function.



ANSWER:

$$\frac{(x - 3)^2}{36} + \frac{(y + 2)^2}{9} = 1 ; \text{ ellipse}$$



2. $x^2 + y^2 + 12x - 8y + 36 = 0$

SOLUTION:

$$x^2 + y^2 + 12x - 8y + 36 = 0$$

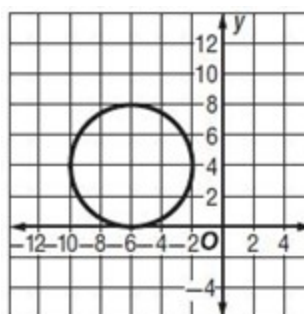
$$x^2 + y^2 + 12x - 8y = -36$$

$$(x^2 + 12x) + (y^2 - 8y) = -36$$

$$(x^2 + 12x + 36) + (y^2 - 8y + 16) = -36 + 36 + 16$$

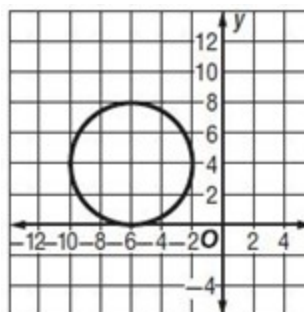
$$(x + 6)^2 + (y - 4)^2 = 16$$

The given equation represents a circle.
Graph the function



ANSWER:

$$(x + 6)^2 + (y - 4)^2 = 16 ; \text{ circle}$$



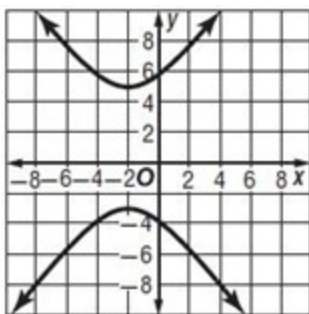
9-6 Identifying Conic Sections

3. $9y^2 - 16x^2 - 18y - 64x - 199 = 0$

SOLUTION:

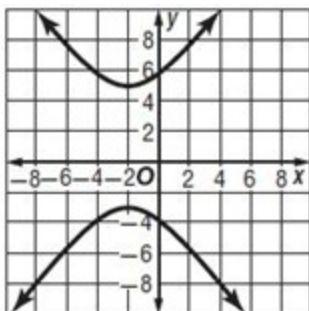
$$\begin{aligned} 9y^2 - 16x^2 - 18y - 64x - 199 &= 0 \\ 9y^2 - 16x^2 - 18y - 64x &= 199 \\ 9(y^2 - 2y) - 16(x^2 + 4x) &= 199 \\ 9(y^2 - 2y + 1) - 16(x^2 + 4x + 4) &= 199 + 9 - 64 \\ 9(y - 1)^2 - 16(x + 2)^2 &= 144 \\ \frac{(y - 1)^2}{16} - \frac{(x + 2)^2}{9} &= 1 \end{aligned}$$

The given equation is a hyperbola.
Graph the function.



ANSWER:

$$\frac{(y - 1)^2}{16} - \frac{(x + 2)^2}{9} ; \text{hyperbola}$$

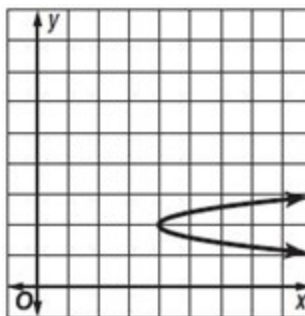


4. $6y^2 - 24y + 28 - x = 0$

SOLUTION:

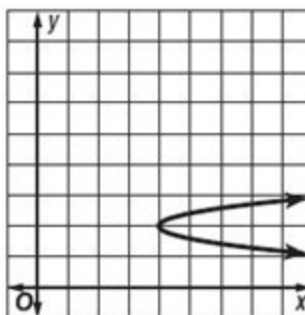
$$\begin{aligned} 6y^2 - 24y + 28 - x &= 0 \\ 6y^2 - 24y - x &= -28 \\ 6(y^2 - 4y) - x &= -28 \\ 6(y^2 - 4y + 4) - x &= -28 + 24 \\ 6(y - 2)^2 - x &= -4 \\ x &= 6(y - 2)^2 + 4 \end{aligned}$$

The given equation is a parabola.
Graph the function.



ANSWER:

$$x = 6(y - 2)^2 + 4 ; \text{parabola}$$



9-6 Identifying Conic Sections

Without writing in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

5. $4x^2 + 6y^2 - 3x - 2y = 12$

SOLUTION:

$$4x^2 + 6y^2 - 3x - 2y = 12$$

$$A = 4, B = 0, \text{ and } C = 6$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(4)(6) = -24.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.

ANSWER:

ellipse

6. $5y^2 = 2x + 6y - 8 + 3x^2$

SOLUTION:

$$5y^2 = 2x + 6y - 8 + 3x^2$$

$$0 = 2x + 6y - 8 + 3x^2 - 5y^2$$

$$A = 3, B = 0, \text{ and } C = -5$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(3)(-5) = 60.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

7. $8x^2 + 8y^2 + 16x + 24 = 0$

SOLUTION:

$$8x^2 + 8y^2 + 16x + 24 = 0$$

$$A = 8, B = 0, \text{ and } C = 8$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(8)(8) = -256.$$

Because the discriminant is less than 0, $B = 0$ and $A = C$, the conic is a circle.

ANSWER:

circle

8. $4x^2 - 6y = 8x + 2$

SOLUTION:

$$4x^2 - 6y = 8x + 2$$

$$4x^2 - 6y - 8x - 2 = 0$$

$$A = 4, B = 0, \text{ and } C = 0$$

$$\text{The discriminant is } B^2 - 4AC = (0^2) - 4(4)(0) = 0.$$

Because the discriminant is equal to 0, the conic is a parabola.

ANSWER:

parabola

9-6 Identifying Conic Sections

9. $4x^2 - 3y^2 + 8xy - 12 = 2x + 4y$

SOLUTION:

$$4x^2 - 3y^2 + 8xy - 12 = 2x + 4y$$

$$4x^2 - 3y^2 + 8xy - 12 - 2x - 4y = 0$$

$$A = 4, B = 8, \text{ and } C = -8$$

The discriminant is

$$B^2 - 4AC = (8^2) - 4(4)(-8) = 192.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

10. $5xy - 3x^2 + 6y^2 + 12y = 18$

SOLUTION:

$$5xy - 3x^2 + 6y^2 + 12y = 18$$

$$5xy - 3x^2 + 6y^2 + 12y - 18 = 0$$

$$A = -3, B = 0, \text{ and } C = 6$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(-3)(6) = 72.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

11. $8x^2 + 12xy + 16y^2 + 4y - 3x = 12$

SOLUTION:

$$8x^2 + 12xy + 16y^2 + 4y - 3x = 12$$

$$8x^2 + 12xy + 16y^2 + 4y - 3x - 12 = 0$$

$$A = 8, B = 12, \text{ and } C = 16$$

The discriminant is

$$B^2 - 4AC = (12^2) - 4(8)(16) = -368.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.

ANSWER:

ellipse

12. $16xy + 8x^2 + 8y^2 - 18x + 8y = 13$

SOLUTION:

$$16xy + 8x^2 + 8y^2 - 18x + 8y = 13$$

$$16xy + 8x^2 + 8y^2 - 18x + 8y - 13 = 0$$

$$A = 8, B = 16, \text{ and } C = 8$$

$$\text{The discriminant is } B^2 - 4AC = (16^2) - 4(8)(8) = 0.$$

Because the discriminant is equal to 0, the conic is a parabola.

ANSWER:

parabola

9-6 Identifying Conic Sections

13. **CCSS MODELING** A military jet performs for an air show. The path of the plane during one trick can be modeled by a conic section with equation $24x^2 + 1000y - 31,680x - 45,600 = 0$, where distances are represented in feet.

- Identify the shape of the curved path of the jet. Write the equation in standard form.
- If the jet begins its path upward or ascent at $x = 0$, what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?
- What is the maximum height of the jet?

SOLUTION:

- Since the equation doesn't have a y^2 -term, it represents a parabola.

$$\begin{aligned} 24x^2 + 1000y - 31680x - 45600 &= 0 \\ 24(x^2 - 1320x) - 45600 &= -1000y \\ 24(x^2 - 1320x + 435600) - 45600 - 10454400 &= -1000y \\ 24(x - 660)^2 - 10500000 &= -1000y \\ -0.024(x - 660)^2 + 10500 &= y \end{aligned}$$

- Substitute 0 for y and solve for x .

$$\begin{aligned} 24x^2 + 1000(0) - 31,680x - 45,600 &= 0 \\ 24x^2 - 31,680x - 45,600 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-(-31680) \pm \sqrt{(-31680)^2 - 4(24)(-45600)}}{2(24)} \\ &= \frac{31680 \pm 31749}{48} \\ &\approx 1321.4 \text{ or } -1.4 \end{aligned}$$

The horizontal distance traveled by the jet is about $1321.4 + (-1.4) = 1320$ ft.

- The vertex of the parabola $y = -0.024(x - 660)^2 + 10,500$ is $(660, 10500)$. Therefore, the maximum height of the jet is 10500.

ANSWER:

- Parabola; $y = -0.024(x - 660)^2 + 10,500$
- about 1320 ft
- 10,500 ft

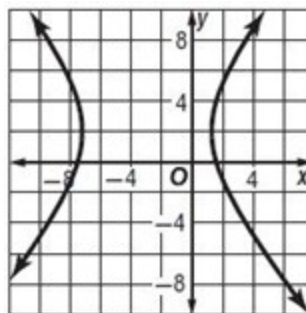
Write each equation in standard form. State whether the graph of the equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.

14. $3x^2 - 2y^2 + 18x + 8y - 35 = 0$

SOLUTION:

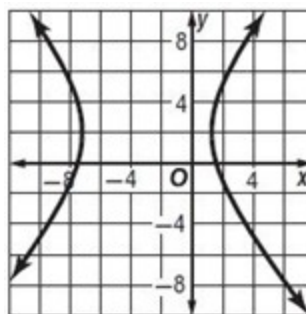
$$\begin{aligned} 3x^2 - 2y^2 + 18x + 8y - 35 &= 0 \\ 3(x^2 + 6x) - 2(y^2 - 4y) &= 35 \\ 3(x^2 + 6x + 9) - 2(y^2 - 4y + 4) &= 35 + 27 - 8 \\ 3(x + 3)^2 - 2(y - 2)^2 &= 54 \\ \frac{(x + 3)^2}{18} - \frac{(y - 2)^2}{27} &= 1 \end{aligned}$$

The given equation is a hyperbola. Graph the function.



ANSWER:

$$\frac{(x + 3)^2}{18} - \frac{(y - 2)^2}{27} = 1 ; \text{ hyperbola}$$



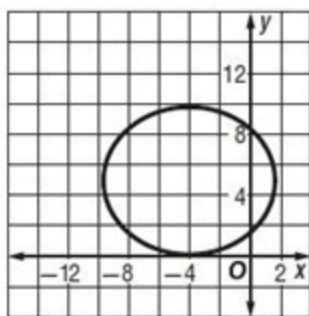
9-6 Identifying Conic Sections

15. $3x^2 + 24x + 4y^2 - 40y + 52 = 0$

SOLUTION:

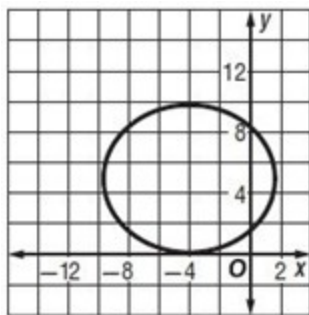
$$\begin{aligned} 3x^2 + 24x + 4y^2 - 40y + 52 &= 0 \\ 3(x^2 + 8x) + 4(y^2 - 10y) &= -52 \\ 3(x^2 + 8x + 16) + 4(y^2 - 10y + 25) &= -52 + 48 + 100 \\ 3(x+4)^2 + 4(y-5)^2 &= 96 \\ \frac{(x+4)^2}{32} + \frac{(y-5)^2}{24} &= 1 \end{aligned}$$

The given equation is an ellipse.
Graph the function.



ANSWER:

$$\frac{(x+4)^2}{32} + \frac{(y-5)^2}{24} = 1 ; \text{ ellipse}$$

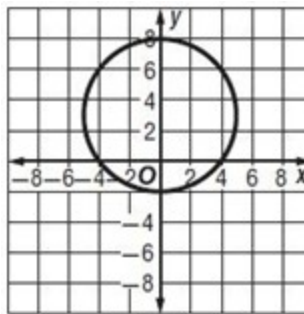


16. $x^2 + y^2 = 16 + 6y$

SOLUTION:

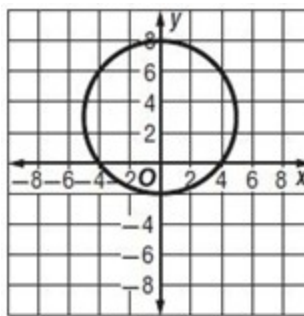
$$\begin{aligned} x^2 + y^2 &= 16 + 6y \\ x^2 + y^2 - 6y &= 16 \\ x^2 + (y^2 - 6y + 9) &= 16 + 9 \\ x^2 + (y-3)^2 &= 25 \\ \frac{x^2}{25} + \frac{(y-3)^2}{25} &= 1 \end{aligned}$$

The given equation is a circle.
Graph the function.



ANSWER:

$$x^2 + (y-3)^2 = 25 ; \text{ circle}$$



9-6 Identifying Conic Sections

17. $32x + 28 = y - 8x^2$

SOLUTION:

$$32x + 28 = y - 8x^2$$

$$8x^2 + 32x - y = -28$$

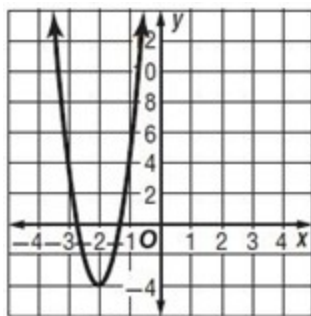
$$8(x^2 + 4x) - y = -28$$

$$8(x^2 + 4x + 4) - y = -28 + 32$$

$$8(x + 2)^2 - y = 4$$

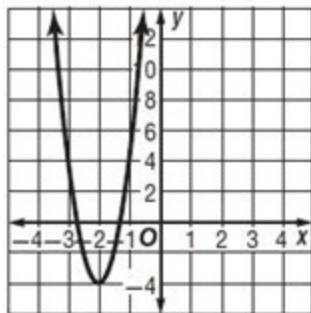
$$8(x + 2)^2 - 4 = y$$

The given equation is a parabola.
Graph the function.



ANSWER:

$$y = 8(x + 2)^2 - 4, \text{ parabola}$$



18. $7x^2 - 8y = 84x - 2y^2 - 176$

SOLUTION:

$$7x^2 - 8y = 84x - 2y^2 - 176$$

$$7x^2 - 84x + 2y^2 - 8y = -176$$

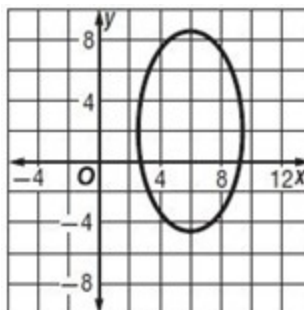
$$7(x^2 - 12x) + 2(y^2 - 4y) = -176$$

$$7(x^2 - 12x + 36) + 2(y^2 - 4y + 4) = -176 + 252 + 8$$

$$7(x - 6)^2 + 2(y - 2)^2 = 84$$

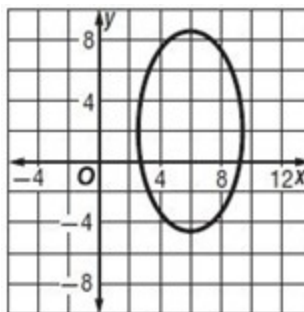
$$\frac{(x - 6)^2}{12} + \frac{(y - 2)^2}{42} = 1$$

The given equation is an ellipse.
Graph the function.



ANSWER:

$$\frac{(y - 2)^2}{42} + \frac{(x - 6)^2}{12} = 1 ; \text{ ellipse}$$



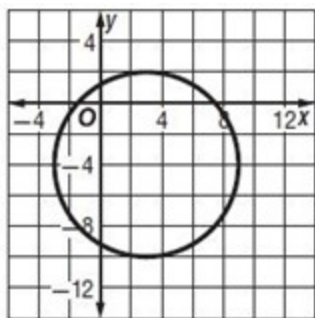
9-6 Identifying Conic Sections

19. $x^2 + 8y = 11 + 6x - y^2$

SOLUTION:

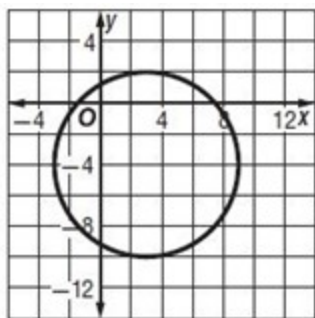
$$\begin{aligned}x^2 + 8y &= 11 + 6x - y^2 \\x^2 - 6x + y^2 + 8y &= 11 \\(x^2 - 6x + 9) + (y^2 + 8y + 16) &= 11 + 9 + 16 \\(x - 3)^2 + (y + 4)^2 &= 36\end{aligned}$$

The given equation is a circle.
Graph the function.



ANSWER:

$(x - 3)^2 + (y + 4)^2 = 36$; circle

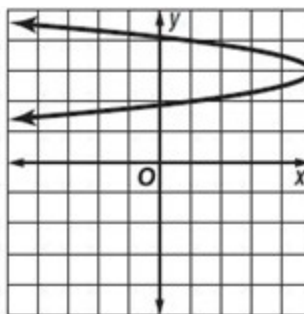


20. $4y^2 = 24y - x - 31$

SOLUTION:

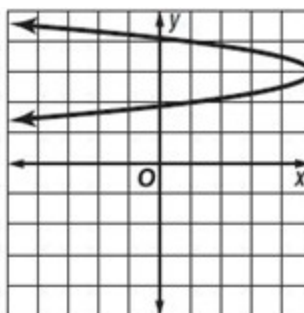
$$\begin{aligned}4y^2 &= 24y - x - 31 \\4y^2 - 24y &= -x - 31 \\4(y^2 - 6y) &= -x - 31 \\4(y^2 - 6y + 9) &= -x - 31 + 36 \\4(y + 3)^2 &= -x + 5 \\x &= -4(y + 3)^2 + 5\end{aligned}$$

The given equation is a parabola.
Graph the function.



ANSWER:

$x = -4(y - 3)^2 + 5$; parabola



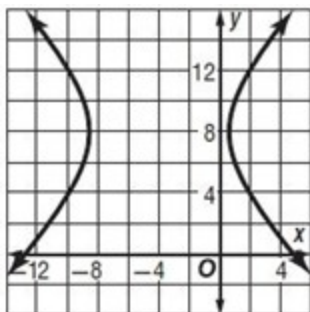
9-6 Identifying Conic Sections

21. $112y + 64x = 488 + 7y^2 - 8x^2$

SOLUTION:

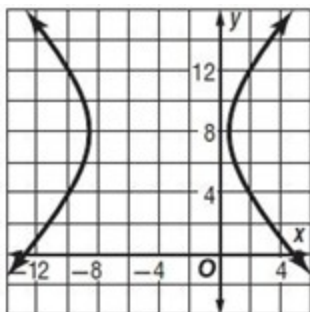
$$\begin{aligned} 112y + 64x &= 488 + 7y^2 - 8x^2 \\ 8x^2 + 64x - 7y^2 + 112y &= 488 \\ 8(x^2 + 8x) - 7(y^2 - 16y) &= 488 \\ 8(x^2 + 8x + 16) - 7(y^2 - 16y + 64) &= 488 + 128 - 448 \\ 8(x+4)^2 - 7(y-8)^2 &= 168 \\ \frac{(x+4)^2}{21} - \frac{(y-8)^2}{24} &= 1 \end{aligned}$$

The given equation is a hyperbola.
Graph the function.



ANSWER:

$$\frac{(x+4)^2}{21} - \frac{(y-8)^2}{24} = 1 ; \text{hyperbola}$$

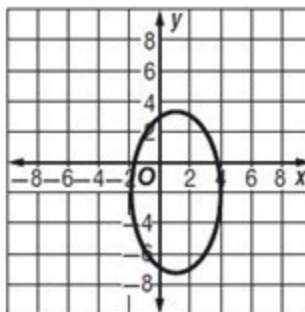


22. $28x^2 + 9y^2 - 188 = 56x - 36y$

SOLUTION:

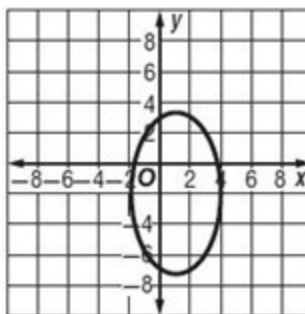
$$\begin{aligned} 28x^2 + 9y^2 - 188 &= 56x - 36y \\ 28x^2 + 9y^2 - 56x + 36y &= 188 \\ 28(x^2 - 2x) + 9(y^2 + 4y) &= 188 \\ 28(x^2 - 2x + 1) + 9(y^2 + 4y + 4) &= 188 + 28 + 36 \\ 28(x^2 - 2x + 1) + 9(y^2 + 4y + 4) &= 252 \\ 28(x-1)^2 + 9(y+2)^2 &= 252 \\ \frac{(x-1)^2}{9} + \frac{(y+2)^2}{28} &= 1 \end{aligned}$$

The given equation is an ellipse.
Graph the function.



ANSWER:

$$\frac{(y+2)^2}{28} + \frac{(x-1)^2}{9} = 1 ; \text{ellipse}$$



9-6 Identifying Conic Sections

23. $25x^2 + 384y - 64y^2 + 200x = 1776$

SOLUTION:

$$25x^2 + 384y - 64y^2 + 200x = 1776$$

$$25x^2 + 200x - 64y^2 + 384y = 1776$$

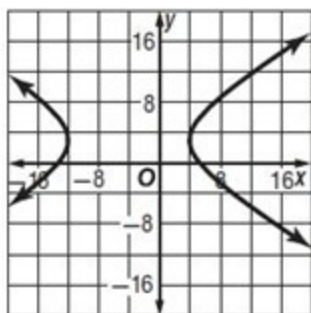
$$25(x^2 + 8x) - 64(y^2 + 6y) = 1776$$

$$25(x^2 + 8x + 16) - 64(y^2 + 6y + 9) = 1776 + 400 - 576$$

$$25(x + 4)^2 - 64(y + 3)^2 = 1600$$

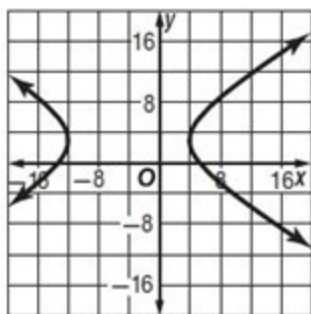
$$\frac{(x + 4)^2}{64} - \frac{(y + 3)^2}{25} = 1$$

The given equation is a hyperbola.
Graph the function.



ANSWER:

$$\frac{(x + 4)^2}{64} - \frac{(y + 3)^2}{25} = 1; \text{ hyperbola}$$



Without writing in standard form, state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

24. $4x^2 - 5y = 9x - 12$

SOLUTION:

$$4x^2 - 5y = 9x - 12$$

$$4x^2 - 5y - 9x + 12 = 0$$

$$A = 4, B = 0, \text{ and } C = 0$$

$$\text{The discriminant is } B^2 - 4AC = (0^2) - 4(4)(0) = 0.$$

Because the discriminant is equal to 0, the conic is a parabola.

ANSWER:

parabola

25. $4x^2 - 12x = 18y - 4y^2$

SOLUTION:

$$4x^2 - 12x = 18y - 4y^2$$

$$4x^2 - 12x + 4y^2 - 18y = 0$$

$$A = 4, B = 0, \text{ and } C = 4$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(4)(4) = -64.$$

Because the discriminant is less than 0, $B = 0$ and $A = C$, the conic is a circle.

ANSWER:

circle

9-6 Identifying Conic Sections

26. $9x^2 + 12y = 9y^2 + 18y - 16$

SOLUTION:

$$\begin{aligned} 9x^2 + 12y &= 9y^2 + 18y - 16 \\ 9x^2 + 12y - 9y^2 - 18y + 16 &= 0 \end{aligned}$$

$A = 9$, $B = 0$, and $C = -9$

The discriminant is

$$B^2 - 4AC = (0)^2 - 4(9)(-9) = 324.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

27. $18x^2 - 16y = 12x - 4y^2 + 19$

SOLUTION:

$$\begin{aligned} 18x^2 - 16y &= 12x - 4y^2 + 19 \\ 18x^2 - 16y - 12x + 4y^2 - 19 &= 0 \end{aligned}$$

$A = 18$, $B = 0$, and $C = 4$

The discriminant is

$$B^2 - 4AC = (0)^2 - 4(18)(4) = -288.$$

Because the discriminant is less than 0 and $B = 0$, the conic is an ellipse.

ANSWER:

ellipse

28. $12y^2 - 4xy + 9x^2 = 18x - 124$

SOLUTION:

$$\begin{aligned} 12y^2 - 4xy + 9x^2 &= 18x - 124 \\ 12y^2 - 4xy + 9x^2 - 18x + 124 &= 0 \end{aligned}$$

$A = 9$, $B = -4$, and $C = 12$

The discriminant is

$$B^2 - 4AC = (-4)^2 - 4(9)(12) = -416.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.

ANSWER:

ellipse

29. $5xy + 12x^2 - 16x = 5y + 3y^2 + 18$

SOLUTION:

$$\begin{aligned} 5xy + 12x^2 - 16x &= 5y + 3y^2 + 18 \\ 5xy + 12x^2 - 16x - 5y - 3y^2 - 18 &= 0 \end{aligned}$$

$A = 12$, $B = 5$, and $C = -3$

The discriminant is

$$B^2 - 4AC = (5)^2 - 4(12)(-3) = 169.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

9-6 Identifying Conic Sections

30. $19x^2 + 14y = 6x - 19y^2 - 88$

SOLUTION:

$$19x^2 + 14y = 6x - 19y^2 - 88$$

$$19x^2 + 14y - 6x + 19y^2 + 88 = 0$$

$$A = 19, B = 0, \text{ and } C = 19$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(19)(19) = -1444.$$

Because the discriminant is less than 0, $B = 0$ and $A = C$, the conic is a circle.

ANSWER:

circle

31. $8x^2 + 20xy + 18 = 4y^2 - 12 + 9x$

SOLUTION:

$$8x^2 + 20xy + 18 = 4y^2 - 12 + 9x$$

$$8x^2 + 20xy + 18 - 4y^2 + 12 - 9x = 0$$

$$A = 8, B = 20, \text{ and } C = -4$$

The discriminant is

$$B^2 - 4AC = (20)^2 - 4(8)(-4) = 528.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

32. $5x - 12xy + 6x^2 = 8y^2 - 24y - 9$

SOLUTION:

$$5x - 12xy + 6x^2 = 8y^2 - 24y - 9$$

$$5x - 12xy + 6x^2 - 8y^2 + 24y + 9 = 0$$

$$A = 6, B = -12, \text{ and } C = -8$$

The discriminant is

$$B^2 - 4AC = (-12)^2 - 4(6)(-8) = 336.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

33. $18x - 24y + 324xy = 27x^2 + 3y^2 - 5$

SOLUTION:

$$18x - 24y + 324xy = 27x^2 + 3y^2 - 5$$

$$18x - 24y + 324xy - 27x^2 - 3y^2 + 5 = 0$$

$$A = -27, B = 324, \text{ and } C = -3$$

The discriminant is

$$B^2 - 4AC = (324)^2 - 4(-27)(-3) = 104652.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

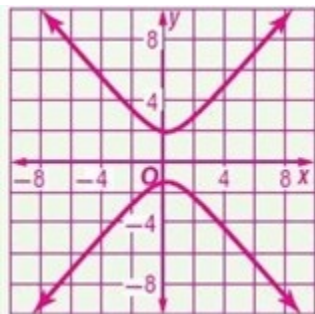
9-6 Identifying Conic Sections

34. **LIGHT** A lamp standing near a wall throws an arc of light in the shape of a conic section. Suppose the edge of the light can be represented by the equation $3y^2 - 2y - 4x^2 + 2x - 8 = 0$. Identify the shape of the edge of the light and graph the equation.

SOLUTION:

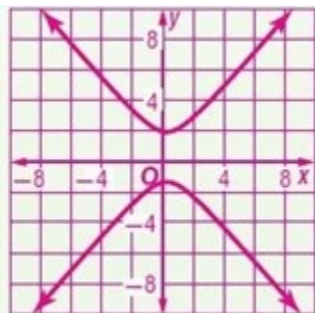
$$\begin{aligned}
 3y^2 - 2y - 4x^2 + 2x - 8 &= 0 \\
 3\left(y^2 - \frac{2}{3}y\right) - 4\left(x^2 - \frac{1}{2}x\right) &= 8 \\
 3\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) - 4\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) &= 8 + \frac{1}{3} - \frac{1}{4} \\
 3\left(y - \frac{1}{3}\right)^2 - 4\left(x - \frac{1}{4}\right)^2 &= \frac{97}{12} \\
 \frac{\left(y - \frac{1}{3}\right)^2}{\frac{97}{36}} - \frac{\left(x - \frac{1}{4}\right)^2}{\frac{97}{48}} &= 1
 \end{aligned}$$

Therefore, the equation represents a hyperbola.
Graph the equation.

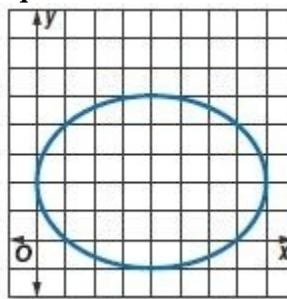


ANSWER:

hyperbola



Match each graph with its corresponding equation.



35.

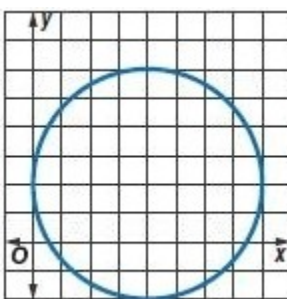
- a. $x^2 + y^2 - 8x - 4y = -4$
- b. $9x^2 - 16y^2 - 72x + 64y = 64$
- c. $9x^2 + 16y^2 = 72x + 64y - 64$

SOLUTION:

Option a, b and c represents a circle, a hyperbola and an ellipse respectively.
Therefore, option c the correct answer.

ANSWER:

c



36.

- a. $x^2 + y^2 - 8x - 4y = -4$
- b. $9x^2 - 16y^2 - 72x + 64y = 64$
- c. $9x^2 + 16y^2 = 72x + 64y - 64$

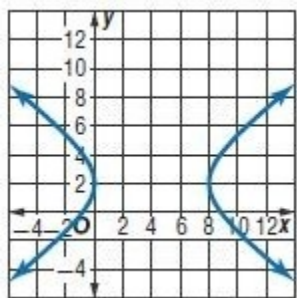
SOLUTION:

Option a, b and c represents a circle, a hyperbola and an ellipse respectively.
Therefore, option a the correct answer.

ANSWER:

a

9-6 Identifying Conic Sections



37.

a. $x^2 + y^2 - 8x - 4y = -4$

b. $9x^2 - 16y^2 - 72x + 64y = 64$

c. $9x^2 + 16y^2 = 72x + 64y - 64$

SOLUTION:

Option a, b and c represents a circle, a hyperbola and an ellipse respectively.

Therefore, option b the correct answer.

ANSWER:

b

For Exercises 38–41, match each situation with an equation that could be used to represent it.

a. $47.25x^2 - 9y^2 + 18y + 33.525 = 0$

b. $25x^2 + 100y^2 - 1900x - 2200y + 45,700 = 0$

c. $16x^2 - 90x + y - 0.25 = 0$

d. $x^2 + y^2 - 18x - 30y - 14,094 = 0$

38. **COMPUTERS** the boundary of a wireless network with a range of 120 feet

SOLUTION:

Option a, b, c and d represents a hyperbola, an ellipse, a parabola and a circle.

Option d is the correct choice.

ANSWER:

d

39. **FITNESS** the oval path of your foot on an exercise machine

SOLUTION:

Option a, b, c and d represents a hyperbola, an ellipse, a parabola and a circle.

Option b is the correct choice.

ANSWER:

b

40. **COMMUNICATIONS** the position of a cell phone between two cell towers

SOLUTION:

Option a, b, c and d represents a hyperbola, an ellipse, a parabola and a circle.

Option a is the correct choice.

ANSWER:

a

41. **SPORTS** the height of a football above the ground after being kicked

SOLUTION:

Option a, b, c and d represents a hyperbola, an ellipse, a parabola and a circle.

Option c is the correct choice.

ANSWER:

c

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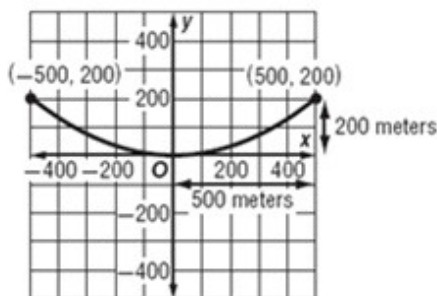
42. **CCSS SENSE-MAKING** The shape of the cables in a suspension bridge is approximately parabolic. If the towers for a planned bridge are 1000 meters apart and the lowest point of the suspension cables is 200 meters below the tops of the towers, write the equation in standard form with the origin at the vertex.

SOLUTION:

The vertex is at the origin, so the center of the cable is at $(0, 0)$. Assuming the towers are equidistant from the center, then they are each 500 meters from the center, at x -coordinates -500 and 500 . The farthest the cables reach down is 200 meters, so the top of the towers are at $(-500, 200)$ and $(500, 200)$.

Since the parabola representing the cable opens up, the equation of the parabola is of the form $y = a(x - h)^2 + k$. The center is at the origin, so $h = k = 0$. Substitute one of the coordinates and solve for a .

$$\begin{aligned} y &= ax^2 \\ 200 &= a(500)^2 \\ 200 &= 250,000a \\ a &= \frac{200}{250000} \text{ or } 0.0008 \end{aligned}$$



The equation for the cable is $y = 0.0008x^2$.

ANSWER:

$$y = 0.0008x^2$$

43. **MULTIPLE REPRESENTATIONS** Consider an ellipse with center $(3, -2)$, vertex $M(-1, -2)$, and co-vertex $N(3, -4)$.

$N(3, -4)$.

a. ANALYTICAL Determine the standard form of the equation of the ellipse.

b. ALGEBRAIC Convert part **a** to $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ form.

c. GRAPHICAL Graph the ellipse.

d. ANALYTICAL If the ellipse is rotated such that M is moved to $(3, -6)$, determine the location of N and the angle of rotation.

SOLUTION:

a. The center (h, k) is $(3, -2)$.

The vertex $(h \pm a, k)$ is $(-1, -2)$.

The co-vertex $(h, k \pm b)$ is $(3, -4)$.

Therefore, $h = 3$, $k = -2$, $a = \pm 4$ and $b = \pm 2$.

The standard form of the ellipse is

$$\frac{(x-3)^2}{16} + \frac{(y+2)^2}{4} = 1.$$

b.

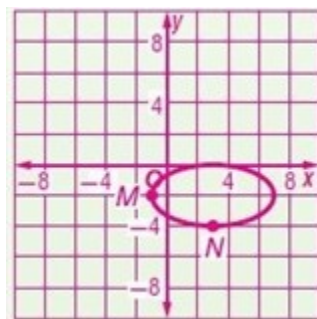
$$4(x^2 - 6x + 9) + 16(y^2 + 4y + 4) = 64$$

$$4x^2 - 24x + 36 + 16y^2 + 64y + 64 = 64$$

$$4x^2 - 24x + 36 + 16y^2 + 64y = 0$$

$$x^2 + 4y^2 - 6x + 16y + 9 = 0$$

c. Graph the ellipse.



d. $N(5, -2)$; 90° counterclockwise

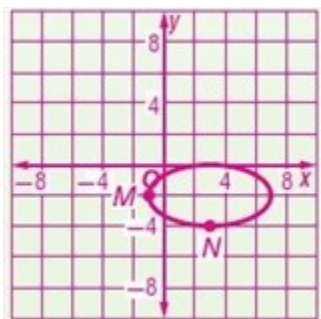
ANSWER:

a.
$$\frac{(x-3)^2}{16} + \frac{(y+2)^2}{4} = 1$$

b.
$$x^2 + 4y^2 - 6x + 16y + 9 = 0$$

c.

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d. $N(5, -2)$; 90° counterclockwise

44. **CHALLENGE** When a plane passes through the vertex of a cone, a degenerate conic is formed.

a. Determine the type of conic represented by

$$4x^2 + 8y^2 = 0.$$

b. Graph the conic.

c. Describe the difference between this degenerate conic and a standard conic.

SOLUTION:

a. $4x^2 + 8y^2 = 0$

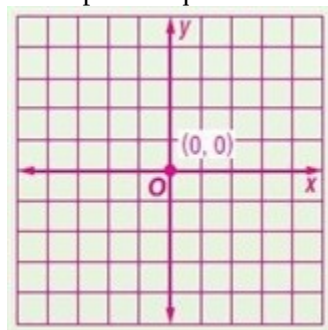
$A = 4$, $B = 0$, and $C = 8$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(4)(8) = -128.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.

b. Graph the equation.

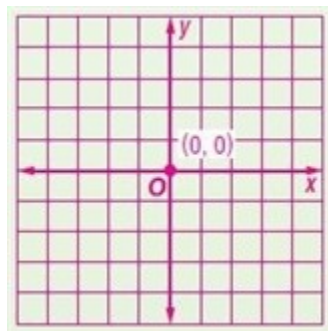


c. The graph of the standard conic is an ellipse, and the graph of the degenerate conic is a single point.

ANSWER:

a. ellipse

b.



c. The graph of the standard conic is an ellipse, and the graph of the degenerate conic is a single point.

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45. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
When a conic is vertical and $A = C$, it is a circle.

SOLUTION:

Sample answer: Always; when a conic is vertical, $B = 0$. When this is true and $A = C$, the conic is a circle.

ANSWER:

Sample answer: Always; when a conic is vertical, $B = 0$. When this is true and $A = C$, the conic is a circle.

46. **OPEN ENDED** Write an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = 9C$, that represents a parabola.

SOLUTION:

Sample answer: $9x^2 + 6xy + y^2 + 2x + 2y + 8 = 0$

ANSWER:

Sample answer: $9x^2 + 6xy + y^2 + 2x + 2y + 8 = 0$

47. **WRITING IN MATH** Compare and contrast the graphs of the four types of conics and their corresponding equations.

SOLUTION:

Sample answer: An ellipse is a flattened circle. Both circles and ellipses are enclosed regions while hyperbolas and parabolas are not. A parabola has one branch, which is a smooth curve that never ends, and a hyperbola has two such branches that are reflections of each other. In standard form and when there is no xy -term: an equation for a parabola consists of only one squared term, an equation for a circle has values for A and C that are equal, an equation for an ellipse has values for A and C that are the same sign but not equal, and an equation for a hyperbola has values of A and C that have opposite signs.

ANSWER:

Sample answer: An ellipse is a flattened circle. Both circles and ellipses are enclosed regions while hyperbolas and parabolas are not. A parabola has one branch, which is a smooth curve that never ends, and a hyperbola has two such branches that are reflections of each other. In standard form and when there is no xy -term: an equation for a parabola consists of only one squared term, an equation for a circle has values for A and C that are equal, an equation for an ellipse has values for A and C that are the same sign but not equal, and an equation for a hyperbola has values of A and C that have opposite signs.

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48. **SAT/ACT** A class of 25 students took a science test. Ten students had a mean score of 80. The other students had an average score of 60. What is the average score of the whole class?

A 66
B 68
C 70
D 72
E 78

SOLUTION:

Total score of the students is

$$10 \times 80 + 15 \times 60 = 1700.$$

Therefore, the average score is $\frac{1700}{25} = 68$.

Option B is the correct answer.

ANSWER:

B

49. Six times a number minus 11 is 43. What is the number?

F 12
G 11
H 10
J 9

SOLUTION:

Six times a number minus 11 means $6x - 11$.

Six times a number minus 11 is 43 means $6x - 11 = 43$.

Solve for x .

$$6x = 54$$

$$x = 9$$

Option J is the correct answer.

ANSWER:

J

50. **EXTENDED RESPONSE** The amount of water remaining in a storage tank as it is drained can be represented by the equation $L = -4t^2 - 10t + 130$, where L represents the number of liters of water remaining and t represents the number of minutes since the drain was opened. How many liters of water were in the tank initially? Determine to the nearest tenth of a minute how long it will take for the tank to drain completely.

SOLUTION:

Substitute 0 for t in the given equation and evaluate.

$$\begin{aligned} L &= -4(0)^2 - 10(0) + 130 \\ &= 130 \end{aligned}$$

Substitute 0 for L and solve for t .

$$0 = -4t^2 - 10t + 130$$

$$\begin{aligned} t &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(-4)(130)}}{2(-4)} \\ &= \frac{10 \pm \sqrt{100 + 2080}}{-8} \\ &\approx -7.1 \text{ or } 4.6 \end{aligned}$$

The value of t should be positive. Therefore, $t = 4.6$ min.

ANSWER:

130 L; 4.6 min

9-6 Identifying Conic Sections

51. Ruben has a square piece of paper with sides 4 inches long. He rolled up the paper to form a cylinder. What is the volume of the cylinder?

- A. $\frac{4}{\pi}$
 B. $\frac{16}{\pi}$
 C. 4π
 D. 16π

SOLUTION:

Since the side of the square is 4 inches, the circumference and the height of the cylinder is 4 inches.

That is $2\pi r = 4$ and $h = 4$.

Therefore, $r = \frac{2}{\pi}$.

The volume of the cylinder is $\pi \left(\frac{2}{\pi} \right) \left(\frac{2}{\pi} \right) 4 = \frac{16}{\pi}$.

Option B is the correct answer.

ANSWER:

B

52. **ASTRONOMY** Suppose a comet's path can be modeled by a branch of the hyperbola with equation $\frac{y^2}{225} - \frac{x^2}{400} = 1$. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola. Then graph the hyperbola.

SOLUTION:

$$\frac{y^2}{225} - \frac{x^2}{400} = 1$$

$$\frac{y^2}{15^2} - \frac{x^2}{20^2} = 1$$

The values of h , k , a and b are 0, 0, 15 and 20.

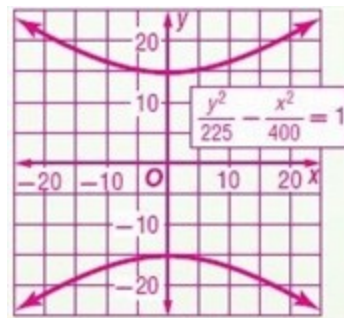
The center of the ellipse is $(0, \pm 15)$.

The foci are $(0, \pm 25)$.

The equations of the asymptotes are

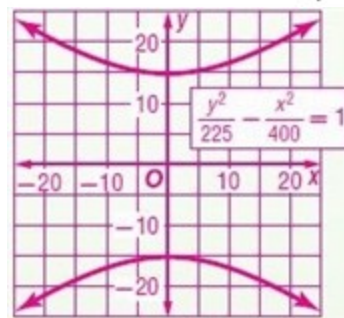
$$y = \pm \frac{a}{b}x = \pm \frac{3}{4}x.$$

Graph the equation.



ANSWER:

$$(0, \pm 15); (0, \pm 25); y = \pm \frac{3}{4}x$$



Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

9-6 Identifying Conic Sections

53. $\frac{y^2}{18} + \frac{x^2}{9} = 1$

SOLUTION:

Rewrite the equation in the standard form.

$$\begin{aligned}\frac{y^2}{18} + \frac{x^2}{9} &= 1 \\ \frac{y^2}{(3\sqrt{2})^2} + \frac{x^2}{3^2} &= 1\end{aligned}$$

The values of h , k , a and b are 0, 0, $3\sqrt{2}$ and 3.

$$\begin{aligned}c &= \sqrt{a^2 - b^2} \\ &= \sqrt{18 - 9} \\ &= 3\end{aligned}$$

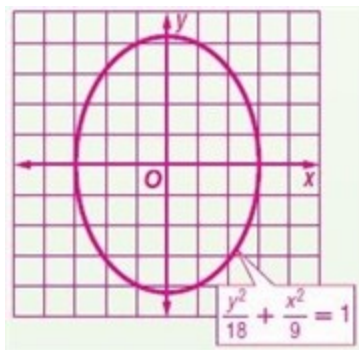
The centre of the ellipse is (0, 0).

The foci are $(0, \pm 3)$.

The length of the major axis is $2a = 2(3\sqrt{2}) = 6\sqrt{2}$.

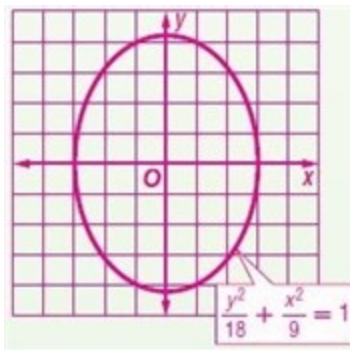
The length of the minor axis is $2b = 2(3) = 6$.

Graph the equation.



ANSWER:

$(0, 0); (0, \pm 3); 6\sqrt{2}; 6$



54. $4x^2 + 8y^2 = 32$

SOLUTION:

Rewrite the equation in the standard form.

$$\begin{aligned}4x^2 + 8y^2 &= 32 \\ \frac{x^2}{8} + \frac{y^2}{4} &= 1 \\ \frac{x^2}{(2\sqrt{2})^2} + \frac{y^2}{2^2} &= 1\end{aligned}$$

The values of h , k , a and b are 0, 0, $2\sqrt{2}$ and 2.

$$\begin{aligned}c &= \sqrt{a^2 - b^2} \\ &= \sqrt{8 - 4} \\ &= 2\end{aligned}$$

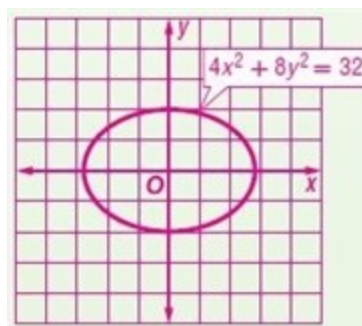
The centre of the ellipse is (0, 0).

The foci are $(\pm 2, 0)$.

The length of the major axis is $2a = 2(2\sqrt{2}) = 4\sqrt{2}$.

The length of the minor axis is $2b = 2(2) = 4$.

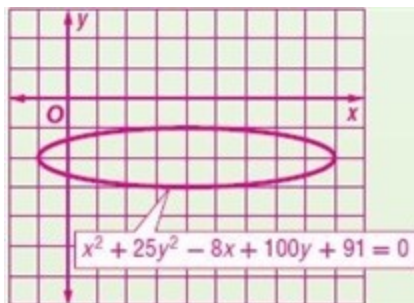
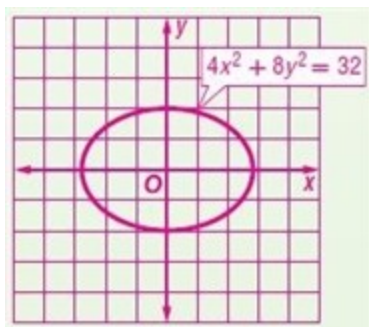
Graph the equation.



ANSWER:

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$$(0, 0); (\pm 2, 0); 4\sqrt{2}; 4$$



ANSWER:

$$(4, -2); (4 \pm 2\sqrt{6}, -2); 10; 2$$

55. $x^2 + 25y^2 - 8x + 100y + 91 = 0$

SOLUTION:

Rewrite the equation in the standard form.

$$\begin{aligned} x^2 + 25y^2 - 8x + 100y + 91 &= 0 \\ (x^2 - 8x) + 25(y^2 + 4y) &= -91 \\ (x^2 - 8x + 16) + 25(y^2 + 4y + 4) &= -91 + 16 + 100 \\ (x - 4)^2 + 25(y + 2)^2 &= 25 \\ \frac{(x - 4)^2}{25} + \frac{(y + 2)^2}{1} &= 1 \\ \frac{(x - 4)^2}{5^2} + \frac{(y + 2)^2}{1^2} &= 1 \end{aligned}$$

The values of h , k , a and b are -4 , -2 , 5 and 1 .

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{25 - 1} \\ &= 2\sqrt{6} \end{aligned}$$

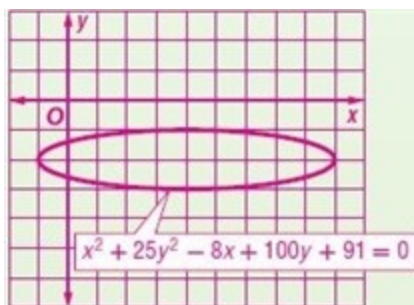
The centre of the ellipse is $(0, 0)$.

The foci are $(4 \pm 2\sqrt{6}, 0)$.

The length of the major axis is $2a = 2(5) = 10$.

The length of the minor axis is $2b = 2(1) = 2$.

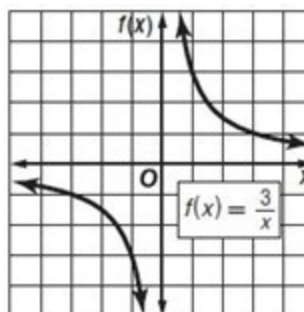
Graph the equation.



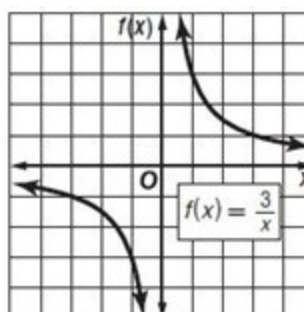
Graph each function.

56. $f(x) = \frac{3}{x}$

SOLUTION:



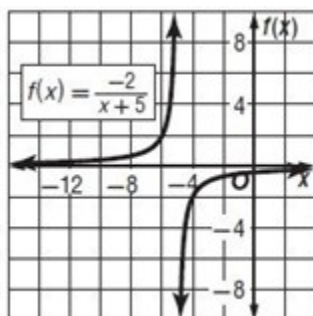
ANSWER:



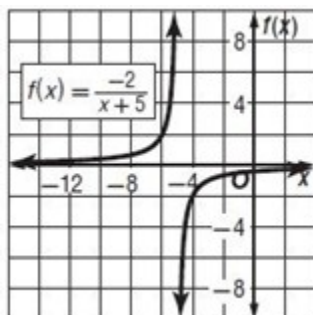
9-6 Identifying Conic Sections

57. $f(x) = \frac{-2}{x+5}$

SOLUTION:

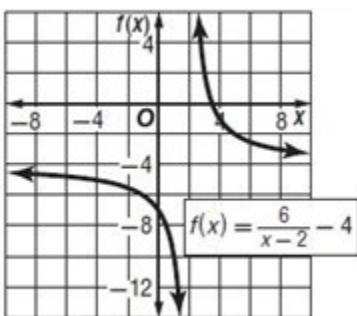


ANSWER:

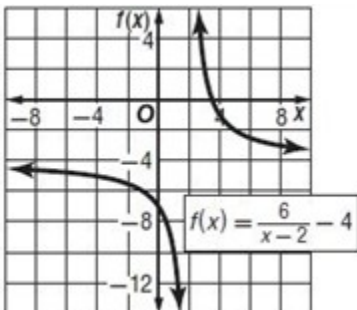


58. $f(x) = \frac{6}{x-2} - 4$

SOLUTION:



ANSWER:



59. **SPACE** A radioisotope is used as a power source for a satellite. The power output P (in watts) is given

by $P = 50e^{-\frac{t}{250}}$, where t is the time in days.

- Is the formula for power output an example of exponential *growth* or *decay*? Explain your reasoning.
- Find the power available after 100 days.
- Ten watts of power are required to operate the equipment in the satellite. How long can the satellite continue to operate?

SOLUTION:

- The formula for power output is an example of exponential decay. Because the exponent is negative.
- Substitute 100 for t and evaluate.

$$\begin{aligned} P &= 50e^{-\frac{100}{250}} \\ &= 50e^{-0.04} \\ &\approx 33.5 \end{aligned}$$

- Substitute 10 for P and solve for t .

$$\begin{aligned} 10 &= 50e^{-\frac{t}{250}} \\ 0.2 &= e^{-\frac{t}{250}} \\ \ln 0.2 &= -\frac{t}{250} \\ t &= -250 \ln 0.2 \\ &\approx 102 \end{aligned}$$

ANSWER:

- Decay; the exponent is negative.
- about 33.5 watts
- about 402 days

9-6 Identifying Conic Sections

Solve each system of equations.

60. $6g - 8h = 50$

$$6h = 22 - 4g$$

SOLUTION:

$$6g - 8h = 50 \rightarrow (1)$$

$$6h = 22 - 4g \rightarrow (2)$$

Multiply the first and the second equation by 2 and 3 respectively then add.

$$(1) \times 2 \quad 12g - 16h = 100 \rightarrow (3)$$

$$(2) \times 3 \quad 18h = 66 - 12g \rightarrow (4)$$

$$(3) + (4) - 34h = 34$$

$$h = -1.$$

Substitute -1 for h and solve for g .

$$6(-1) = 22 - 4g$$

$$4g = 28$$

$$g = 7$$

The solution of the system is $(7, -1)$.

ANSWER:

$$(7, -1)$$

61. $3u + 5v = 6$

$$2u - 4v = -7$$

SOLUTION:

$$3u + 5v = 6 \rightarrow (1)$$

$$2u - 4v = -7 \rightarrow (2)$$

Multiply the first and the second equation by 4 and 5 respectively then add.

$$(1) \times 4 \quad 12u + 20v = 24 \rightarrow (3)$$

$$(2) \times 5 \quad 10u - 20v = -35 \rightarrow (4)$$

$$(3) + (4) \quad 22u = 11$$

$$u = -\frac{1}{2}$$

Substitute $-\frac{1}{2}$ for u and solve for v .

$$3\left(-\frac{1}{2}\right) + 5v = 6$$

$$5v = \frac{15}{2}$$

$$v = \frac{3}{2}$$

The solution of the system is $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

ANSWER:

$$\left(-\frac{1}{2}, \frac{3}{2}\right)$$

9-6 Identifying Conic Sections

$$\begin{aligned} 62. \quad 10m - 9n &= 15 \\ 5m - 4n &= 10 \end{aligned}$$

SOLUTION:

$$10m - 9n = 15 \rightarrow (1)$$

$$5m - 4n = 10 \rightarrow (2)$$

Multiply the first and the second equation by 1 and -2 respectively then add.

$$(1) \times 1 \quad 10m - 9n = 15 \rightarrow (3)$$

$$(2) \times -2 \quad -10m + 8n = -20 \rightarrow (4)$$

$$(3) + (4) \quad -n = -5$$

$$n = 5$$

Substitute 5 for n and solve for m .

$$10m - 9(5) = 15$$

$$10m = 60$$

$$m = 6$$

The solution of the system is $(6, 5)$.

ANSWER:

$$(6, 5)$$

9-7 Solving Linear-Nonlinear Systems

Solve each system of equations.

1.
$$\begin{aligned} 8y &= -10x \\ y^2 &= 2x^2 - 7 \end{aligned}$$

SOLUTION:

Solve the linear equation for y .

$$\begin{aligned} y &= -\frac{10}{8}x \\ &= -\frac{5}{4}x \end{aligned}$$

Substitute $-\frac{5}{4}x$ for y in the quadratic equation and solve for x .

$$\begin{aligned} \left(-\frac{5}{4}x\right)^2 &= 2x^2 - 7 \\ \frac{25}{16}x^2 &= 2x^2 - 7 \\ \frac{7x^2}{16} &= 7 \\ x^2 &= 16 \\ x &= \pm 4 \end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{lll} 8y = -10x & \text{and} & 8y = -10x \\ 8y = -10(4) & \text{and} & 8y = -10(-4) \\ 8y = -40 & \text{and} & 8y = 40 \\ y = -5 & \text{and} & y = 5 \end{array}$$

The solutions are $(4, 5)$ and $(-4, 5)$.

ANSWER:

$(4, -5), (-4, 5)$

2.
$$\begin{aligned} x^2 + y^2 &= 68 \\ 5y &= -3x + 34 \end{aligned}$$

SOLUTION:

Solve the linear equation for y .

$$\begin{aligned} 5y &= -3x + 34 \\ y &= -\frac{3}{5}x + \frac{34}{5} \end{aligned}$$

Substitute $-\frac{3}{5}x + \frac{34}{5}$ for y in the quadratic equation and solve for x .

$$\begin{aligned} x^2 + \left(-\frac{3}{5}x + \frac{34}{5}\right)^2 &= 68 \\ x^2 + \frac{9}{25}x^2 - \frac{204}{25}x + \frac{1156}{25} &= 68 \\ \frac{34}{25}x^2 - \frac{204}{25}x - \frac{544}{25} &= 0 \\ 17x^2 - 102x - 272 &= 0 \\ x &= \frac{-(-102) \pm \sqrt{(-102)^2 - 4(17)(-272)}}{2(17)} \\ &= \frac{102 \pm 170}{34} \\ &= -2 \text{ or } 8 \end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{lll} 5y = -3x + 34 & \text{and} & 5y = -3x + 34 \\ 5y = -3(-2) + 34 & \text{and} & 5y = -3(8) + 34 \\ 5y = 40 & \text{and} & 5y = 10 \\ y = 8 & \text{and} & y = 2 \end{array}$$

The solutions are $(-2, 8)$ and $(8, 2)$.

ANSWER:

$(-2, 8), (8, 2)$

9-7 Solving Linear-Nonlinear Systems

3. $y = 12x - 30$
 $4x^2 - 3y = 18$

SOLUTION:

Substitute $12x - 30$ for y in the quadratic equation and solve for x .

$$4x^2 - 3(12x - 30) = 18$$

$$4x^2 - 36x + 72 = 0$$

$$x^2 - 9x + 18 = 0$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(18)}}{2(1)}$$

$$= \frac{9 \pm 3}{2}$$

$$= 6 \text{ or } 3$$

Substitute the values of x in the linear equation and find the value of y .

$$y = 12x - 30 \quad \text{and} \quad y = 12x - 30$$

$$y = 12(3) - 30 \quad \text{and} \quad y = 12(6) - 30$$

$$y = 6 \quad \text{and} \quad y = 42$$

The solutions are $(3, 6)$ and $(6, 42)$.

ANSWER:

$(3, 6), (6, 42)$

4. $6y^2 - 27 = 3x$
 $6y - x = 13$

SOLUTION:

Solve the linear equation for y .

$$6y - x = 13$$

$$y = \frac{x}{6} + \frac{13}{6}$$

Substitute $\frac{x}{6} + \frac{13}{6}$ for y in the quadratic equation and solve for x .

$$6\left(\frac{x}{6} + \frac{13}{6}\right)^2 - 27 = 3x$$

$$\frac{x^2}{6} + \frac{26}{6}x + \frac{169}{6} - 3x - 27 = 0$$

$$x^2 + 8x + 7 = 0$$

$$(x+1)(x+7) = 0$$

$$x = -1 \text{ or } -7$$

Substitute the values of x in the linear equation and find the value of y .

$$y = \frac{x}{6} + \frac{13}{6}$$

and

$$y = \frac{x}{6} + \frac{13}{6}$$

$$y = \frac{-1}{6} + \frac{13}{6}$$

and

$$y = \frac{-7}{6} + \frac{13}{6}$$

$$y = 2$$

and

$$y = 1$$

The solutions are $(-1, 2)$ and $(-7, 1)$.

ANSWER:

$(-7, 1), (-1, 2)$

9-7 Solving Linear-Nonlinear Systems

$$\begin{aligned} 5. \quad & x^2 + y^2 = 16 \\ & x^2 - y^2 = 20 \end{aligned}$$

SOLUTION:

Subtract the second equation from the first equation.

$$\begin{aligned} (1) - (2) \quad & 2y^2 = -4 \\ & y^2 = -2 \end{aligned}$$

This is impossible.

Therefore, there is no solution.

ANSWER:

no solution

$$\begin{aligned} 6. \quad & y^2 - 2x^2 = 8 \\ & 3y^2 + x^2 = 52 \end{aligned}$$

SOLUTION:

$$\begin{aligned} y^2 - 2x^2 &= 8 \quad \rightarrow (1) \\ 3y^2 + x^2 &= 52 \quad \rightarrow (2) \end{aligned}$$

Multiply the second equation by 2.

$$(2) \times 2 \quad 6y^2 + 2x^2 = 104 \rightarrow (3)$$

$$\begin{aligned} (1) + (3) \quad & 7y^2 = 112 \\ & y^2 = 16 \\ & y = \pm 4 \end{aligned}$$

Substitute the values of y in the first equation and find the values of x .

$$\begin{aligned} y^2 - 2x^2 &= 8 \quad \text{and} \quad y^2 - 2x^2 = 8 \\ (+4)^2 - 2x^2 &= 8 \quad \text{and} \quad (+4)^2 - 2x^2 = 8 \\ 16 - 2x^2 &= 8 \quad \text{and} \quad 16 - 2x^2 = 8 \\ 2x^2 &= 8 \quad \text{and} \quad 2x^2 = 8 \\ x^2 &= 4 \quad \text{and} \quad x^2 = 4 \\ x &= \pm 2 \quad \text{and} \quad x = \pm 2 \end{aligned}$$

The solutions are $(2, 4)$, $(-2, 4)$, $(-2, -4)$ and $(2, -4)$.

ANSWER:

$(2, 4)$, $(-2, 4)$, $(-2, -4)$, $(2, -4)$

$$\begin{aligned} 7. \quad & x^2 + 2y = 7 \\ & y^2 - x^2 = 8 \end{aligned}$$

SOLUTION:

$$\begin{aligned} x^2 + 2y &= 7 \quad \rightarrow (1) \\ y^2 - x^2 &= 8 \quad \rightarrow (2) \end{aligned}$$

$$\begin{aligned} (1) + (2) \quad & y^2 + 2y = 15 \\ & y^2 + 2y - 15 = 0 \\ & (y + 5)(y - 3) = 0 \\ & y = 3 \text{ or } -5 \end{aligned}$$

Substitute the values of y in the first equation and find the values of x .

$$\begin{aligned} x^2 + 2y &= 7 \quad \text{and} \quad x^2 + 2y = 7 \\ x^2 + 2(3) &= 7 \quad \text{and} \quad x^2 + 2(-5) = 7 \\ x^2 &= 1 \quad \text{and} \quad x^2 = 17 \\ x &= \pm 1 \quad \text{and} \quad x = \pm\sqrt{17} \end{aligned}$$

The solutions are

$$(-1, 3), (1, 3), (-\sqrt{17}, -5) \text{ and } (\sqrt{17}, -5).$$

ANSWER:

$$\begin{aligned} & (-1, 3), (1, 3), \\ & (-\sqrt{17}, -5), (\sqrt{17}, -5) \end{aligned}$$

9-7 Solving Linear-Nonlinear Systems

$$8. \quad \begin{aligned} 4y^2 - 3x^2 &= 11 \\ 3y^2 + 2x^2 &= 21 \end{aligned}$$

SOLUTION:

$$4y^2 - 3x^2 = 11 \rightarrow (1)$$

$$3y^2 + 2x^2 = 21 \rightarrow (2)$$

Multiply the first and the second equation by 2 and 3 respectively.

$$(1) \times 2 \quad 8y^2 - 6x^2 = 22 \rightarrow (3)$$

$$(2) \times 3 \quad 9y^2 + 6x^2 = 63 \rightarrow (4)$$

$$(3) + (4) \quad 17y^2 = 85$$

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$

Substitute the values of y in the first equation and find the values of x .

$$\begin{array}{ll} 4y^2 - 3x^2 = 11 & \text{and} \quad 4y^2 - 3x^2 = 11 \\ 4(\sqrt{5})^2 - 3x^2 = 11 & \text{and} \quad 4(-\sqrt{5})^2 - 3x^2 = 11 \\ 3x^2 = 9 & \text{and} \quad 3x^2 = 9 \\ x^2 = 3 & \text{and} \quad x^2 = 3 \\ x = \pm\sqrt{3} & \text{and} \quad x = \pm\sqrt{3} \end{array}$$

The solutions are

$$(-\sqrt{3}, -\sqrt{5}), (-\sqrt{3}, \sqrt{5}), (\sqrt{3}, -\sqrt{5}) \text{ and } (\sqrt{3}, \sqrt{5}).$$

ANSWER:

$$(-\sqrt{3}, -\sqrt{5}), (-\sqrt{3}, \sqrt{5}), (\sqrt{3}, -\sqrt{5}), (\sqrt{3}, \sqrt{5})$$

9. **CCSS PERSEVERANCE** Refer to the beginning of the lesson. A person using a cell phone can be located in respect to three cellular towers. In a coordinate system where one unit represents one mile, the location of the caller is determined to be 50 miles from the tower at the origin. The person is also 40 miles from a tower at (0, 30) and 13 miles from a tower at (35, 18). Where is the caller?

SOLUTION:

The coordinates of the towers are (0, 0), (0, 30) and (35, 18).

Let (x, y) be the coordinates of the caller.

Given:

$$(x-0)^2 + (y-0)^2 = 50^2$$

$$x^2 + y^2 = 2500 \rightarrow (1)$$

$$(x-0)^2 + (y-30)^2 = 40^2$$

$$x^2 + (y-30)^2 = 1600 \rightarrow (2)$$

$$(x-35)^2 + (y-18)^2 = 13^2$$

$$(x-35)^2 + (y-18)^2 = 169 \rightarrow (3)$$

Solve the first two equations.

$$(1) - (2) \quad 60y = 1800$$

$$y = 30$$

Substitute the value of y in the first equation and solve for x .

$$x^2 + 30^2 = 2500$$

$$x^2 = 16$$

$$x = 4$$

The call is at (40, 30).

ANSWER:

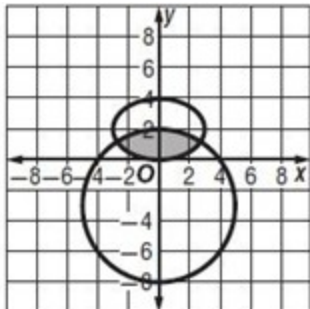
$$(40, 30)$$

9-7 Solving Linear-Nonlinear Systems

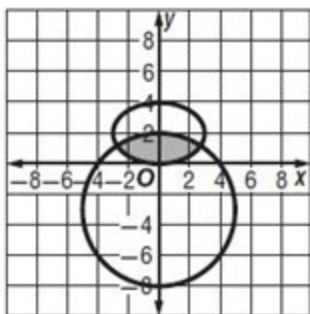
Solve each system of inequalities by graphing.

10. $6x^2 + 9(y-2)^2 \leq 36$
 $x^2 + (y+3)^2 \leq 25$

SOLUTION:

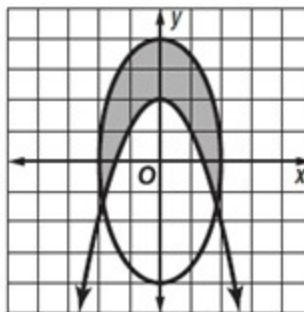


ANSWER:

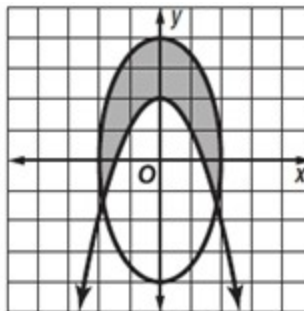


11. $16x^2 + 4y^2 \leq 64$
 $y \geq -x^2 + 2$

SOLUTION:

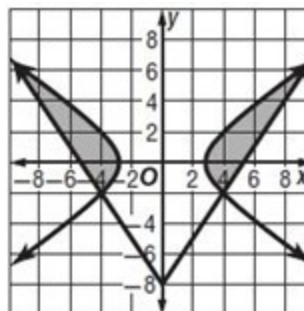


ANSWER:

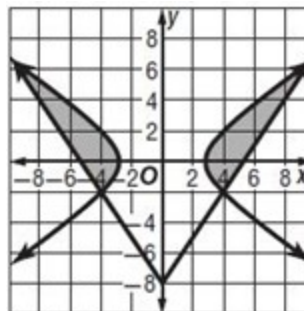


12. $4x^2 - 8y^2 \geq 32$
 $y \geq |1.5x| - 8$

SOLUTION:



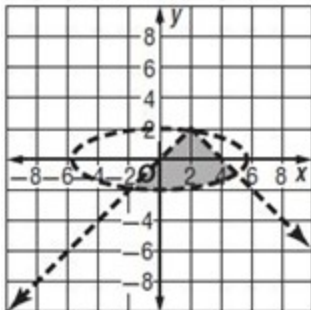
ANSWER:



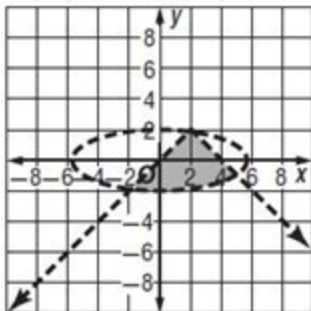
9-7 Solving Linear-Nonlinear Systems

13. $x^2 + 8y^2 < 32$
 $y < -|x - 2| + 2$

SOLUTION:



ANSWER:



Solve each system of equations.

14. $3x^2 - 2y^2 = -24$
 $2y = -3x$

SOLUTION:

Solve the linear equation for y .

$$2y = -3x$$

$$y = -\frac{3}{2}x$$

Substitute $-\frac{3}{2}x$ for y in the quadratic equation and solve for x .

$$3x^2 - 2\left(-\frac{3}{2}x\right)^2 = -24$$

$$3x^2 - \frac{9x^2}{2} = -24$$

$$-\frac{3x^2}{2} = -24$$

$$x^2 = 16$$

$$x = \pm 4$$

Substitute the values of x in the linear equation and find the value of y .

$2y = -3x$	and	$2y = -3x$
$2y = -3(4)$	and	$2y = -3(-4)$
$2y = -12$	and	$2y = 12$
$y = -6$	and	$y = 6$

The solutions are $(4, -6)$ and $(-4, 6)$.

ANSWER:

$(-4, 6), (4, -6)$

9-7 Solving Linear-Nonlinear Systems

15. $5x^2 + 4y^2 = 20$
 $5y = 7x + 35$

SOLUTION:

Solve the linear equation for y .

$$5y = 7x + 35$$
$$y = \frac{7x + 35}{5}$$

Substitute $\frac{7x + 35}{5}$ for y in the quadratic equation and solve for x .

$$5x^2 + 4\left(\frac{7x + 35}{5}\right)^2 = 20$$
$$5x^2 + \frac{196x^2}{25} + \frac{1960x}{25} + \frac{4900}{25} = 20$$
$$321x^2 + 1960x + 4400 = 0$$

$$x = \frac{-1960 \pm \sqrt{1960^2 - 4(4400)(321)}}{2(321)}$$
$$= \frac{-1960 \pm \sqrt{-1808000}}{642}$$

Therefore, there is no solution.

ANSWER:

No solution

16. $x^2 + 3x = -4y - 2$
 $y = -2x + 1$

SOLUTION:

Substitute $-2x + 1$ for y in the quadratic equation and solve for x .

$$x^2 + 3x = -4(-2x + 1) - 2$$
$$x^2 - 5x + 6 = 0$$
$$(x - 3)(x - 2) = 0$$
$$x = 3 \text{ or } 2$$

Substitute the values of x in the linear equation and find the value of y .

$$y = -2x + 1 \quad \text{and} \quad y = -2x + 1$$
$$y = -2(3) + 1 \quad \text{and} \quad y = -2(2) + 1$$
$$y = -5 \quad \text{and} \quad y = -3$$

The solutions are $(3, -5)$ and $(2, -3)$.

ANSWER:

$(3, -5), (2, -3)$

9-7 Solving Linear-Nonlinear Systems

17. $y = 2x$
 $4x^2 - 2y^2 = -36$

SOLUTION:

Substitute $2x$ for y in the quadratic equation and solve for x .

$$\begin{aligned}4x^2 - 2(2x)^2 &= -36 \\4x^2 - 8x^2 &= -36 \\-4x^2 &= -36 \\x &= \pm 3\end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{lll}y = 2x & \text{and} & y = 2x \\y = 2(3) & \text{and} & y = 2(-3) \\y = 6 & \text{and} & y = -6\end{array}$$

The solutions are $(3, 6)$ and $(-3, -6)$.

ANSWER:

$(-3, -6), (3, 6)$

18. $2y = x + 10$
 $y^2 - 4y = 5x + 10$

SOLUTION:

Solve the linear equation for y .

$$\begin{aligned}2y &= x + 10 \\y &= \frac{x + 10}{2}\end{aligned}$$

Substitute $\frac{x + 10}{2}$ for y in the quadratic equation and solve for x .

$$\begin{aligned}\left(\frac{x + 10}{2}\right)^2 - 4\left(\frac{x + 10}{2}\right) &= 5x + 10 \\ \frac{x^2 + 100 + 20x}{4} - 2x - 20 &= 5x + 10 \\ x^2 - 8x - 20 &= 0 \\ (x - 10)(x + 2) &= 0 \\ x &= 10 \text{ or } -2\end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{lll}2y = x + 10 & \text{and} & 2y = x + 10 \\2y = (10) + 10 & \text{and} & 2y = (-2) + 10 \\2y = 20 & \text{and} & 2y = 8 \\y = 10 & \text{and} & y = 4\end{array}$$

The solutions are $(-2, 4)$ and $(10, 10)$.

ANSWER:

$(-2, 4), (10, 10)$

9-7 Solving Linear-Nonlinear Systems

19.
$$\begin{aligned} 9y &= 8x - 19 \\ 8x + 11 &= 2y^2 + 5y \end{aligned}$$

SOLUTION:

Solve the linear equation for y .

$$\begin{aligned} 9y &= 8x - 19 \\ y &= \frac{8x - 19}{9} \end{aligned}$$

Substitute $\frac{8x - 19}{9}$ for y in the quadratic equation and solve for x .

$$\begin{aligned} 8x + 11 &= 2\left(\frac{8x - 19}{9}\right)^2 + 5\left(\frac{8x - 19}{9}\right) \\ 8x + 11 &= \frac{128x^2 - 608x + 722}{81} + \frac{40x - 95}{9} \end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{aligned} 2y &= x + 10 & \text{and} & & 2y &= x + 10 \\ 2y &= (10) + 10 & \text{and} & & 2y &= (-2) + 10 \\ 2y &= 20 & \text{and} & & 2y &= 8 \\ y &= 10 & \text{and} & & y &= 4 \end{aligned}$$

The solutions are $(-2, 4)$ and $(10, 10)$.

ANSWER:

$(-1, -3), (8, 5)$

20.
$$\begin{aligned} 2y^2 + 5x^2 &= 26 \\ 2x^2 - y^2 &= 5 \end{aligned}$$

SOLUTION:

$$\begin{aligned} 2y^2 + 5x^2 &= 26 & \rightarrow (1) \\ 2x^2 - y^2 &= 5 & \rightarrow (2) \end{aligned}$$

Multiply the second equation by 2.

$$(2) \times 2 \quad 4x^2 - 2y^2 = 10 \rightarrow (3)$$

$$\begin{aligned} (3) + (1) \quad 9x^2 &= 36 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Substitute the values of x in the second equation and find the values of y .

$$\begin{aligned} 2x^2 - y^2 &= 5 & \text{and} & & 2x^2 - y^2 &= 5 \\ 2(2)^2 - y^2 &= 5 & \text{and} & & 2(-2)^2 - y^2 &= 5 \\ y^2 &= 3 & \text{and} & & y^2 &= 3 \\ y &= \pm\sqrt{3} & \text{and} & & y &= \pm\sqrt{3} \end{aligned}$$

The solutions are

$(-2, -\sqrt{3}), (-2, \sqrt{3}), (2, -\sqrt{3})$ and $(2, \sqrt{3})$.

ANSWER:

$(-2, -\sqrt{3}), (-2, \sqrt{3}), (2, -\sqrt{3}), (2, \sqrt{3})$

9-7 Solving Linear-Nonlinear Systems

$$\begin{aligned} 21. \quad & x^2 + y^2 = 16 \\ & x^2 - 4x + y^2 = 12 \end{aligned}$$

SOLUTION:

$$x^2 + y^2 = 16 \quad \rightarrow (1)$$

$$x^2 - 4x + y^2 = 12 \quad \rightarrow (2)$$

Substitute 16 for $x^2 + y^2$ in the second equation and solve for x .

$$16 - 4x = 12$$

$$4x = 4$$

$$x = 1$$

Substitute the values of x in the first equation and find the values of y .

$$x^2 + y^2 = 16$$

$$(1)^2 + y^2 = 16$$

$$y^2 = 15$$

$$y = \pm\sqrt{15}$$

The solutions are $(1, -\sqrt{15})$ and $(1, \sqrt{15})$.

ANSWER:

$$(1, -\sqrt{15}), (1, \sqrt{15})$$

$$\begin{aligned} 22. \quad & x^2 + y^2 = 8 \\ & 5y^2 = 3x^2 \end{aligned}$$

SOLUTION:

Solve the second equation for y in terms of x .

$$5y^2 = 3x^2$$

$$y^2 = \frac{3x^2}{5}$$

$$y = \pm \frac{\sqrt{3}}{\sqrt{5}}x$$

Substitute $\pm \frac{\sqrt{3}}{\sqrt{5}}x$ for y in the second equation and solve for x .

$$x^2 + \frac{3}{5}x^2 = 8$$

$$\frac{8x^2}{5} = 8$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

Substitute the values of x in the first equation and find the values of y .

$$x^2 + y^2 = 8 \quad \text{and} \quad x^2 + y^2 = 8$$

$$(\sqrt{5})^2 + y^2 = 8 \quad \text{and} \quad (-\sqrt{5})^2 + y^2 = 8$$

$$y^2 = 3 \quad \text{and} \quad y^2 = 3$$

$$y = \pm\sqrt{3} \quad \text{and} \quad y = \pm\sqrt{3}$$

The solutions are

$$(\sqrt{5}, -\sqrt{3}), (\sqrt{5}, \sqrt{3}), (-\sqrt{5}, -\sqrt{3}) \text{ and } (-\sqrt{5}, \sqrt{3}).$$

ANSWER:

$$(-\sqrt{5}, -\sqrt{3}), (-\sqrt{5}, \sqrt{3}), (\sqrt{5}, -\sqrt{3}), (\sqrt{5}, \sqrt{3})$$

9-7 Solving Linear-Nonlinear Systems

23. $y^2 - x^2 + 3y = 26$
 $x^2 + 2y^2 = 34$

SOLUTION:

$$y^2 - x^2 + 3y = 26 \rightarrow (1)$$

$$x^2 + 2y^2 = 34 \rightarrow (2)$$

Solve the second equation for x^2 in terms of y^2 and solve for y .

$$\begin{aligned}x^2 + 2y^2 &= 34 \\x^2 &= 34 - 2y^2\end{aligned}$$

Substitute $34 - 2y^2$ for x^2 in the second equation and solve for y .

$$\begin{aligned}y^2 - (34 - 2y^2) + 3y &= 26 \\y^2 - 34 + 2y^2 + 3y &= 26 \\3y^2 + 3y - 60 &= 0 \\y^2 + y - 20 &= 0 \\(y + 5)(y - 4) &= 0 \\y &= 4 \text{ or } -5\end{aligned}$$

Substitute the values of y in the second equation and find the values of x .

$$\begin{aligned}x^2 + 2y^2 &= 34 \quad \text{and} \quad x^2 + 2y^2 = 34 \\x^2 + 2(4)^2 &= 34 \quad \text{and} \quad x^2 + 2(-5)^2 = 34 \\x^2 &= 2 \quad \text{and} \quad x^2 = -16 \\x &= \pm\sqrt{2}\end{aligned}$$

The solutions are $(-\sqrt{2}, 4)$ and $(\sqrt{2}, 4)$.

ANSWER:

$$(-\sqrt{2}, 4), (\sqrt{2}, 4)$$

24. $x^2 - y^2 = 25$
 $x^2 + y^2 + 7 = 0$

SOLUTION:

$$x^2 - y^2 = 25 \rightarrow (1)$$

$$x^2 + y^2 + 7 = 0 \rightarrow (2)$$

Add both equations and solve for x .

$$\begin{aligned}(1) + (2) \quad 2x^2 &= 18 \\x^2 &= 9 \\x &= \pm 3\end{aligned}$$

Substitute the values of x in the second equation and find the values of y .

$$\begin{aligned}x^2 + y^2 + 7 &= 0 \quad \text{and} \quad x^2 + y^2 + 7 = 0 \\(3)^2 + y^2 + 7 &= 0 \quad \text{and} \quad (-3)^2 + y^2 + 7 = 0 \\y^2 &= -16 \quad \text{and} \quad y^2 = -16\end{aligned}$$

This is impossible.
Therefore, there is no solution.

ANSWER:

No solution

9-7 Solving Linear-Nonlinear Systems

25.
$$\begin{aligned}x^2 - 10x + 2y^2 &= 47 \\y^2 - 2x^2 &= -14\end{aligned}$$

SOLUTION:

$$x^2 - 10x + 2y^2 = 47 \quad \rightarrow (1)$$

$$y^2 - 2x^2 = -14 \quad \rightarrow (2)$$

Solve the second equation for y^2 in terms of x^2 and solve for x .

$$y^2 = -14 + 2x^2$$

Substitute $-14 + 2x^2$ for y^2 in the second equation and solve for x .

$$x^2 - 10x + 2(-14 + 2x^2) = 47$$

$$x^2 - 10x - 28 + 4x^2 = 47$$

$$5x^2 - 10x - 75 = 0$$

$$x^2 - 2x - 15 = 0$$

$$(x - 5)(x + 3) = 0$$

$$x = 5 \text{ or } -3$$

Substitute the values of x in the second equation and find the values of y .

$$y^2 = -14 + 2x^2 \quad \text{and} \quad y^2 = -14 + 2x^2$$

$$y^2 = -14 + 2(5)^2 \quad \text{and} \quad y^2 = -14 + 2(-3)^2$$

$$y^2 = 36 \quad \text{and} \quad y^2 = 4$$

$$y = \pm 6 \quad \text{and} \quad y = \pm 2$$

The solutions are $(5, -6)$, $(5, 6)$, $(-3, -2)$ and $(-3, 2)$.

ANSWER:

$(5, -6)$, $(5, 6)$, $(-3, -2)$, $(-3, 2)$

26. **FIREWORKS** Two fireworks are set off simultaneously but from different altitudes. The height y in feet of one is represented by $y = -16t^2 + 120t + 10$, where t is the time in seconds. The height of the other is represented by $y = -16t^2 + 60t + 310$.

a. After how many seconds are the fireworks the same height?

b. What is that height?

SOLUTION:

a. Equate the both the equations.

$$-16t^2 + 120t + 10 = -16t^2 + 60t + 310$$

$$60t = 300$$

$$t = 5$$

After 5 seconds, they will be in the same height.

b. Substitute 5 for x in the first equation and evaluate.

$$y = -16(5)^2 + 120(5) + 10$$

$$= -400 + 600 + 10$$

$$= 210$$

The height is 210 ft.

ANSWER:

a. 5 seconds

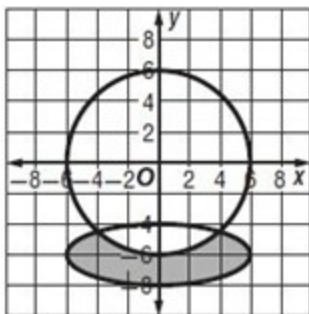
b. 210 ft

9-7 Solving Linear-Nonlinear Systems

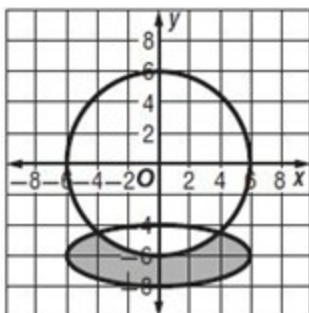
CCSS TOOLS Solve each system of inequalities by graphing.

27. $x^2 + y^2 \geq 36$
 $x^2 + 9(y+6)^2 \leq 36$

SOLUTION:

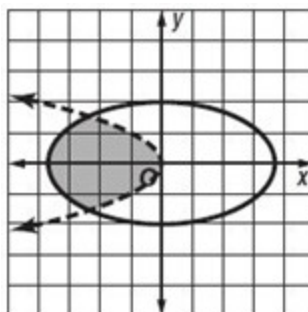


ANSWER:

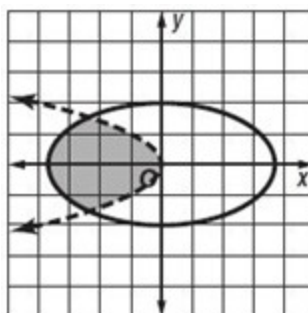


28. $-x > y^2$
 $4x^2 + 14y^2 \leq 56$

SOLUTION:

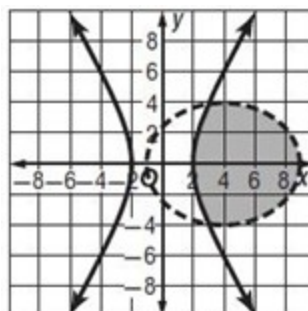


ANSWER:

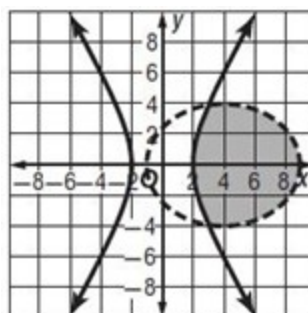


29. $12x^2 - 4y^2 \geq 48$
 $16(x-4)^2 + 25y^2 < 400$

SOLUTION:



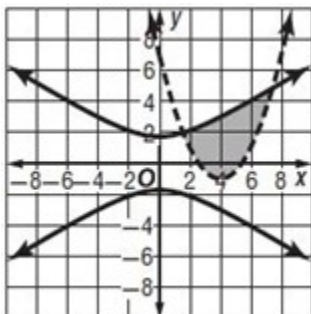
ANSWER:



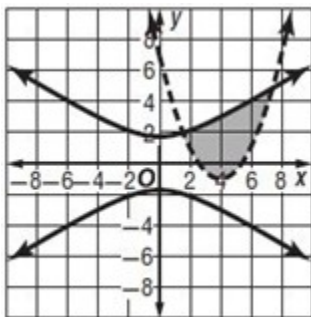
9-7 Solving Linear-Nonlinear Systems

30. $8y^2 - 3x^2 \leq 24$
 $2y > x^2 - 8x + 14$

SOLUTION:

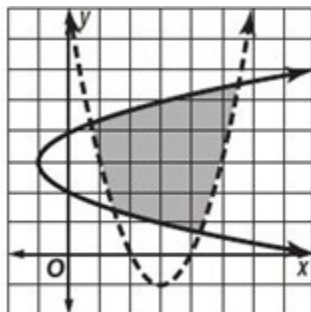


ANSWER:

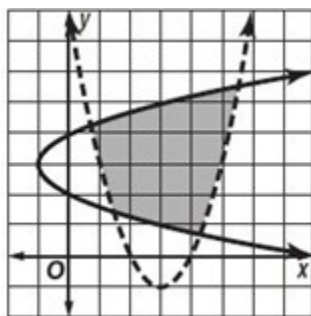


31. $y > x^2 - 6x + 8$
 $x \geq y^2 - 6y + 8$

SOLUTION:

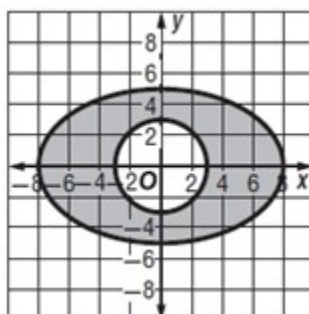


ANSWER:

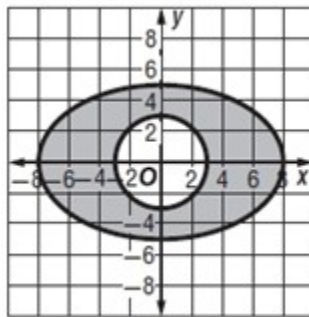


32. $x^2 + y^2 \geq 9$
 $25x^2 + 64y^2 \leq 1600$

SOLUTION:

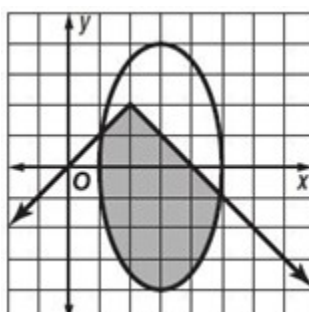


ANSWER:

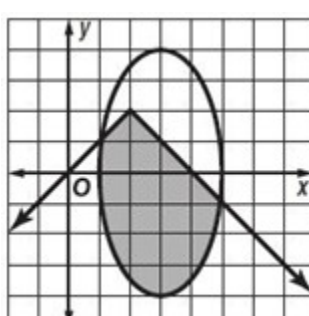


33. $16(x-3)^2 + 4y^2 \leq 64$
 $y \leq -|x-2| + 2$

SOLUTION:



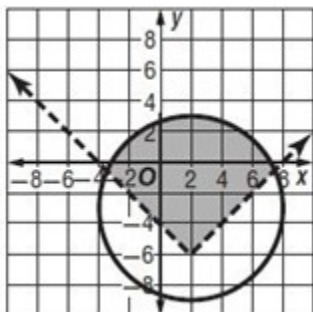
ANSWER:



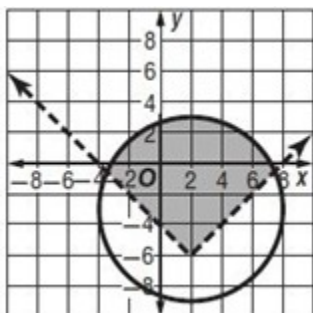
9-7 Solving Linear-Nonlinear Systems

34. $x^2 - 4x + y^2 + 6y \leq 23$
 $y > |x - 2| - 6$

SOLUTION:

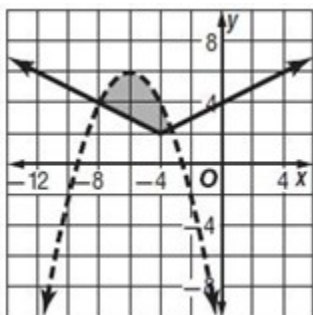


ANSWER:

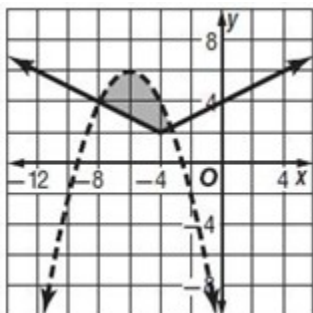


35. $2y - 4 \geq |x + 4|$
 $12 - 2y > x^2 + 12x + 36$

SOLUTION:

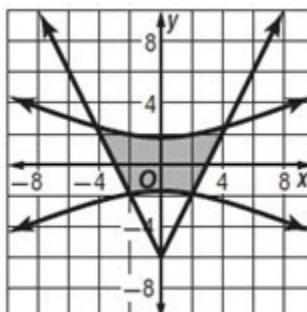


ANSWER:

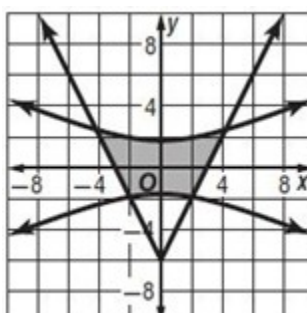


36. $18y^2 - 3x^2 \leq 54$
 $y \geq |2x| - 6$

SOLUTION:



ANSWER:



37. $x^2 + y^2 < 16$
 $y \geq |x - 2| + 6$

SOLUTION:

No solution

ANSWER:

No solution

38. $x^2 < y - 2$
 $y \leq |x + 8| - 7$

SOLUTION:

No solution

ANSWER:

No solution

39. **SPACE** Two satellites are placed in orbit about Earth. The equations of the two orbits are

$$\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1 \text{ and } \frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$$

where distances are in kilometers and Earth is the

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center of each curve.

- Solve each equation for y .
- Use a graphing calculator to estimate the intersection points of the two orbits.
- Compare the orbits of the two satellites.

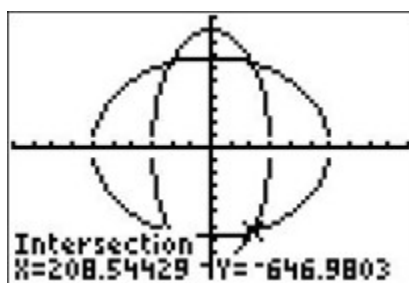
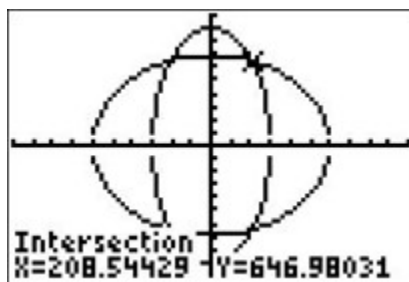
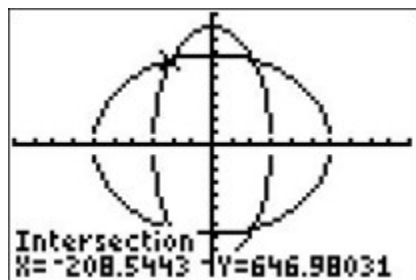
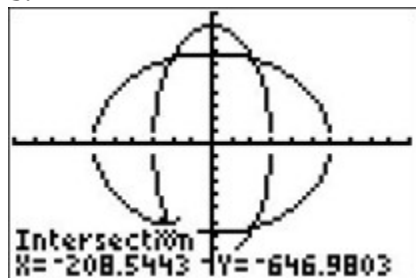
SOLUTION:

a.

$$\begin{aligned}\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} &= 1 \\ \frac{y^2}{(900)^2} &= 1 - \frac{x^2}{(300)^2} \\ y^2 &= 900^2 \left(1 - \frac{x^2}{(300)^2} \right) \\ y &= \pm 900 \sqrt{1 - \frac{x^2}{(300)^2}}\end{aligned}$$

$$\begin{aligned}\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} &= 1 \\ \frac{y^2}{(690)^2} &= 1 - \frac{x^2}{(600)^2} \\ y^2 &= 690^2 \left(1 - \frac{x^2}{(600)^2} \right) \\ y &= \pm 690 \sqrt{1 - \frac{x^2}{(600)^2}}\end{aligned}$$

b.



The intersection points are (209, 647), (−209, 647), (−209, −647), (209, −647).

- Sample answer: The orbit of the satellite modeled by the second equation is closer to a circle than the other orbit. The distance on the x -axis is twice as great for one satellite as the other.

ANSWER:

$$y = \pm 900 \sqrt{1 - \frac{x^2}{(300)^2}};$$

a.

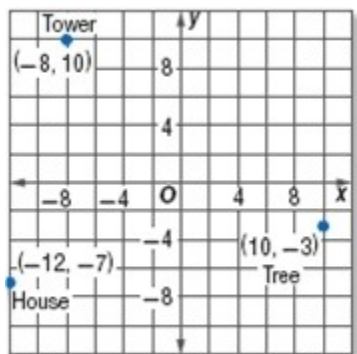
$$y = \pm 690 \sqrt{1 - \frac{x^2}{(600)^2}}$$

- Sample answer: (209, 647), (−209, 647), (−209, −647), (209, −647).

- Sample answer: The orbit of the satellite modeled by the second equation is closer to a circle than the other orbit. The distance on the x -axis is twice as great for one satellite as the other.

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40. **PETS** Taci's dog was missing one day. Fortunately, he was wearing an electronic monitoring device. If the dog is 10 units from the tree, 13 units from the tower, and 20 units from the house, determine the coordinates of his location.



SOLUTION:

Let (x, y) be the coordinates of his location.
Given:

$$(x - 10)^2 + (y - (-3))^2 = 10^2$$

$$(x - 10)^2 + (y + 3)^2 = 100 \quad \rightarrow (1)$$

$$(x - (-12))^2 + (y - (-7))^2 = 20^2$$

$$(x + 12)^2 + (y + 7)^2 = 400 \quad \rightarrow (2)$$

$$(x - (-8))^2 + (y - 10)^2 = 13^2$$

$$(x + 8)^2 + (y - 10)^2 = 169 \quad \rightarrow (3)$$

Simplify.

$$(1) - (2) \quad 11x + 2y = 54 \quad \rightarrow (4)$$

$$(1) - (3) \quad 18x - 13y = 7 \quad \rightarrow (5)$$

The solutions are $(4, 5)$.

Therefore, the coordinates of his location is $(4, 5)$.

ANSWER:

$(4, 5)$

41. **BASEBALL** In 1997, after Mark McGwire hit a home run, the claim was made that the ball would have traveled 538 feet if it had not landed in the stands. The path of the baseball can be modeled by

$$y = -0.0037x^2 + 1.77x - 1.72$$

and the stands can be modeled by

$$y = \frac{3}{7}x - 128.6$$

How far vertically and horizontally from home plate did the ball land in the stands?

SOLUTION:

Substitute $\frac{3}{7}x - 128.6$ for y in the first equation and solve for x .

$$\frac{3}{7}x - 128.6 = -0.0037x^2 + 1.77x - 1.72$$

$$0 = -0.0037x^2 + 1.34x + 126.88$$

$$x = \frac{-1.34 \pm \sqrt{1.34^2 - 4(-0.0037)(126.88)}}{2(-0.0037)}$$

$$\approx 440 \text{ or } -78$$

Since x represents the distance, the value of x is 440 ft.

Substitute 440 for x in the second equation and evaluate.

$$y = \frac{3}{7}(440) - 128.6$$

$$\approx 60$$

Therefore, the ball landed 440 ft from home plate, 60 ft above the playing surface.

ANSWER:

440 ft from home plate, 60 ft above the playing surface

9-7 Solving Linear-Nonlinear Systems

42. **ADVERTISING** The corporate logo for an automobile manufacturer is shown below. Write a system of three equations to model this logo. Refer to the photo on Page 644.

SOLUTION:

Sample answer:

$$\begin{aligned}x^2 + y^2 &= 45, \\ y &= 3|x - 3| - 6, \\ y &= 3|x + 3| - 6\end{aligned}$$

ANSWER:

Sample answer:

$$\begin{aligned}x^2 + y^2 &= 45, \\ y &= 3|x - 3| - 6, \\ y &= 3|x + 3| - 6\end{aligned}$$

Write a system of equations that satisfies each condition.

43. a circle and an ellipse that intersect at one point

SOLUTION:

Sample answer: $\frac{x^2}{16} + \frac{y^2}{36} = 1$ and $(x + 10)^2 + y^2 = 36$

ANSWER:

Sample answer: $\frac{x^2}{16} + \frac{y^2}{36} = 1$ and $(x + 10)^2 + y^2 = 36$

44. a parabola and an ellipse that intersect at two points

SOLUTION:

Sample answer: $y = x^2$ and $\frac{x^2}{16} + \frac{y^2}{36} = 1$

ANSWER:

Sample answer: $y = x^2$ and $\frac{x^2}{16} + \frac{y^2}{36} = 1$

45. a hyperbola and a circle that do not intersect

SOLUTION:

Sample answer: $x^2 + y^2 = 1$ and $\frac{x^2}{16} - \frac{y^2}{36} = 1$

ANSWER:

Sample answer: $x^2 + y^2 = 1$ and $\frac{x^2}{16} - \frac{y^2}{36} = 1$

46. an ellipse and a parabola that intersect at three points

SOLUTION:

Sample answer: $y = x^2$ and $\frac{x^2}{64} + \frac{(y - 4)^2}{16} = 1$

ANSWER:

Sample answer: $y = x^2$ and $\frac{x^2}{64} + \frac{(y - 4)^2}{16} = 1$

47. an ellipse and a hyperbola that intersect at four points

SOLUTION:

Sample answer: $\frac{x^2}{64} + \frac{y^2}{100} = 1$ and $x^2 - y^2 = 1$

ANSWER:

Sample answer: $\frac{x^2}{64} + \frac{y^2}{100} = 1$ and $x^2 - y^2 = 1$

48. **FINANCIAL LITERACY** Prices are often set on an equilibrium curve, where the supply of a certain product equals its corresponding demand by consumers. An economist represents the supply of a product with $y = p^2 + 10p$ and the corresponding demand with $y = -p^2 + 40p$, where p is the price. Determine the equilibrium price.

SOLUTION:

Substitute $-p^2 + 40p$ for y and solve for p .

$$\begin{aligned}-p^2 + 40p &= p^2 + 10p \\ 2p^2 &= 30p \\ p &= 15\end{aligned}$$

The equilibrium price is \$15.

ANSWER:

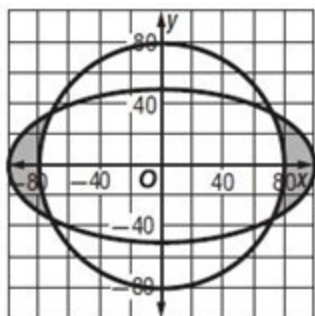
\$15

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49. **PAINTBALL** The shape of a paintball field is modeled by $x^2 + y^2 = 10,000$ in yards where the center is at the origin. The teams are provided with short-range walkie-talkies with a maximum range of 80 yards. Are the teams capable of hearing each other anywhere on the field? Explain your reasoning graphically.

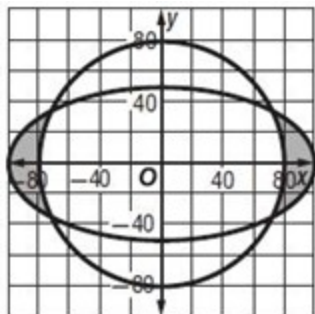
SOLUTION:

Sample answer: No; if one player is in one of the shaded areas and the other player is in the other shaded area, they will not be able to hear each other.



ANSWER:

Sample answer: No; if one player is in one of the shaded areas and the other player is in the other shaded area, they will not be able to hear each other.

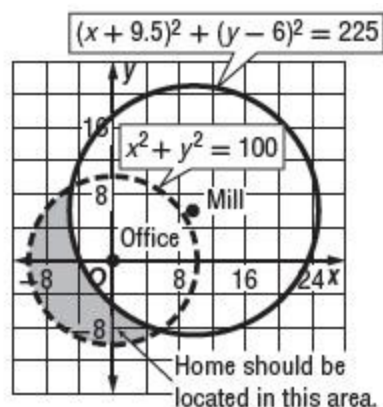


50. **MOVING** Lena is moving to new city and needs for the location of her new home to satisfy the following conditions.

- It must be less than 10 miles from the office where she will work.
 - Because of the terrible smell of the local paper mill, it must be at least 15 miles away from the mill.
- If the paper mill is located 9.5 miles east and 6 miles north of Lena's office, write and graph a system of inequalities to represent the area(s) were she should look for a home.

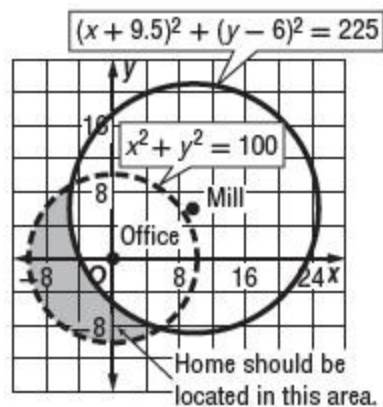
SOLUTION:

If we assume that Lena's office is located at the origin, then a system of equations describing this situation is $x^2 + y^2 < 100$ and $(x + 9.5)^2 + (y - 6)^2 \geq 225$.



ANSWER:

If we assume that Lena's office is located at the origin, then a system of equations describing this situation is $x^2 + y^2 < 100$ and $(x + 9.5)^2 + (y - 6)^2 \geq 225$.



9-7 Solving Linear-Nonlinear Systems

51. **CHALLENGE** Find all values of k for which the following system of equations has two solutions.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad x^2 + y^2 = k^2$$

SOLUTION:

The graph of $x^2 + y^2 = k^2$ intersects the graph of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when either $k = a$ or $k = b$.

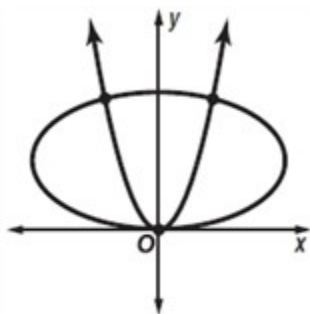
ANSWER:

$$k = a \text{ or } k = b$$

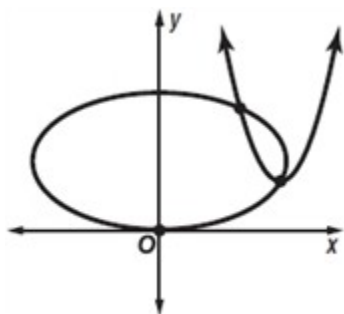
52. **CCSS ARGUMENTS** When the vertex of a parabola lies on an ellipse, how many solutions can the quadratic system represented by the two graphs have? Explain your reasoning using graphs.

SOLUTION:

1, 2, or 3; the parabola could be tangent to the ellipse and have one solution. The parabola could intersect the ellipse at three points like this.

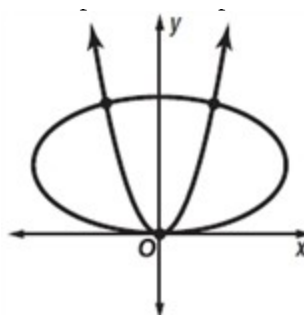


Or it could have two solutions like this.

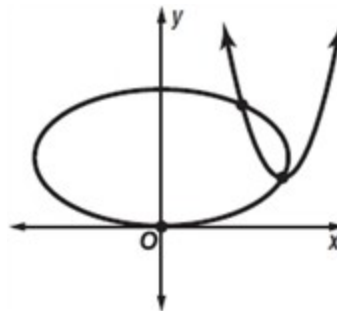


ANSWER:

1, 2, or 3; the parabola could be tangent to the ellipse and have one solution. The parabola could intersect the ellipse at three points like this.



Or it could have two solutions like this.



53. **OPEN ENDED** Write a system of equations, one a hyperbola and the other an ellipse, for which a solution is $(-4, 8)$.

SOLUTION:

Sample answer: $\frac{y^2}{128} + \frac{x^2}{32} = 1$ and $\frac{x^2}{8} - \frac{y^2}{64} = 1$

ANSWER:

Sample answer: $\frac{y^2}{128} + \frac{x^2}{32} = 1$ and $\frac{x^2}{8} - \frac{y^2}{64} = 1$

54. **WRITING IN MATH** Explain how sketching the graph of a quadratic system can help you solve it.

SOLUTION:

Sample answer: By sketching a quick graph of a quadratic system, you can determine how many real solutions there are as well as estimate the values of the solutions. This can help you when confirming the answers that you get algebraically.

ANSWER:

Sample answer: By sketching a quick graph of a quadratic system, you can determine how many real solutions there are as well as estimate the values of the solutions. This can help you when confirming the answers that you get algebraically.

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SHORT RESPONSE

55. Solve.

$$4x - 3y = 0$$

$$x^2 + y^2 = 25$$

SOLUTION:

Solve the linear equation for y .

$$4x - 3y = 0$$

$$y = \frac{4}{3}x$$

Substitute $\frac{4}{3}x$ for y in the quadratic equation and solve for x .

$$x^2 + \left(\frac{4}{3}x\right)^2 = 25$$

$$x^2 + \frac{16x^2}{9} = 25$$

$$\frac{25x^2}{9} = 25$$

$$x^2 = 9$$

$$x = \pm 3$$

Substitute the values of x in the linear equation and find the value of y .

$$y = \frac{4}{3}x$$

and

$$y = \frac{4}{3}x$$

$$y = \frac{4}{3}(3)$$

and

$$y = \frac{4}{3}(-3)$$

$$y = 4$$

and

$$y = -4$$

The solutions are $(3, 4)$ and $(-3, -4)$.

ANSWER:

$(-3, -4), (3, 4)$

56. You have 16 stamps. Some are postcard stamps that cost \$0.23, and the rest cost \$0.41. If you spent a total of \$5.30 on the stamps, how many postcard stamps do you have?

A 7

B 8

C 9

D 10

SOLUTION:

Let x be number of postcard stamps.

$$0.23x + 0.41(16 - x) = 5.30$$

$$0.23x + 6.56 - 0.41x = 5.30$$

$$0.18x = 1.26$$

$$x = 7$$

Option A is the correct answer.

ANSWER:

A

57. Ms. Talbot received a promotion and a 7.2% raise. Her new salary is \$53,600 a year. What was her salary before the raise?

F \$50,000

G \$53,600

H \$55,000

J \$57,500

SOLUTION:

Let x be the salary before the raise.

$$x + 0.072x = 53600$$

$$1.072x = 53600$$

$$x = \frac{53600}{1.072}$$

$$= 50000$$

Option F is the correct answer.

ANSWER:

F

9-7 Solving Linear-Nonlinear Systems

58. **SAT/ACT** When a number is multiplied by $\frac{2}{3}$, the

result is 188. Find the number.

- A 292
B 282
C 272
D 262
E $125\frac{1}{3}$

SOLUTION:

Let x be a number.

$$\begin{aligned}x\left(\frac{2}{3}\right) &= 188 \\x &= 188\left(\frac{3}{2}\right) \\&= 282\end{aligned}$$

Option B is the correct answer.

ANSWER:

B

Match each equation with the situation that it could represent.

- a. $9x^2 + 4y^2 - 36 = 0$
b. $0.004x^2 - x + y - 3 = 0$
c. $x^2 + y^2 - 20x + 30y - 75 = 0$

59. **SPORTS** the flight of a baseball

SOLUTION:

The equations a , b and c are represents an ellipse, a parabola and a circle respectively.

The flight of a baseball forms a parabola.

Therefore, option b is the correct answer.

ANSWER:

b

60. **PHOTOGRAPHY** the oval opening in a picture frame

SOLUTION:

The equations a , b and c are represents an ellipse, a parabola and a circle respectively.

The oval opening in a picture frame represents an ellipse.

Therefore, option a is the correct answer.

ANSWER:

a

61. **GEOGRAPHY** the set of all points 20 miles from a landmark

SOLUTION:

The equations a , b and c are represents an ellipse, a parabola and a circle respectively.

The set of all points 20 miles from a landmark forms a circle.

Therefore, option c is the correct answer.

ANSWER:

c

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

9-7 Solving Linear-Nonlinear Systems

$$62. \frac{y^2}{16} - \frac{x^2}{25} = 1$$

SOLUTION:

$$\frac{y^2}{16} - \frac{x^2}{25} = 1$$

$$\frac{y^2}{4^2} - \frac{x^2}{5^2} = 1$$

From the equation. $a = 4$, $b = 5$, $h = 0$ and $k = 0$.

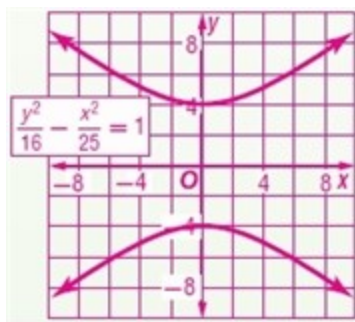
$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

The coordinates of the vertices are $(0, \pm 4)$.

The coordinates of the foci are $(0, \pm \sqrt{41})$.

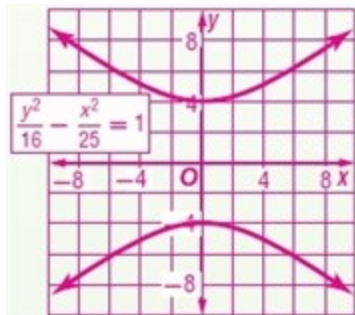
The asymptotes of the hyperbola are $y = \pm \frac{4}{5}x$.

Graph the equation.



ANSWER:

$$(0, \pm 4); (0, \pm \sqrt{41}); y = \pm \frac{4}{5}x$$



$$63. \frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$$

SOLUTION:

$$\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$$

$$\frac{(y-3)^2}{5^2} - \frac{(x-2)^2}{4^2} = 1$$

From the equation. $a = 5$, $b = 4$, $h = 2$ and $k = 3$.

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{25 + 16} \\ &= \sqrt{41} \end{aligned}$$

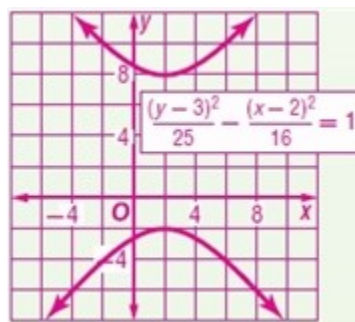
The coordinates of the vertices are $(2, 8)$ and $(2, -2)$.

The coordinates of the foci are $(2, 3 \pm \sqrt{41})$.

The asymptotes of the hyperbola are

$$y - 3 = \pm \frac{5}{4}(x - 2).$$

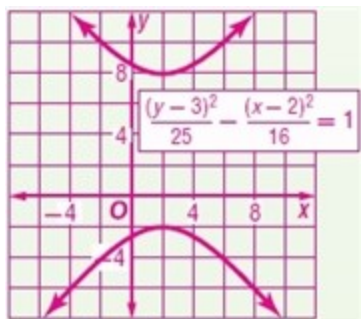
Graph the equation.



ANSWER:

$$(2, -2), (2, 8); (2, 3 \pm \sqrt{41}); y - 3 = \pm \frac{5}{4}(x - 2)$$

9-7 Solving Linear-Nonlinear Systems



64. $6y^2 = 2x^2 + 12$

SOLUTION:

$$6y^2 = 2x^2 + 12$$

$$6y^2 - 2x^2 = 12$$

$$\frac{y^2}{2} - \frac{x^2}{6} = 1$$

From the equation, $a = \sqrt{2}$, $b = \sqrt{6}$, $h = 0$ and $k = 0$.

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{2 + 6} \\ &= 2\sqrt{2} \end{aligned}$$

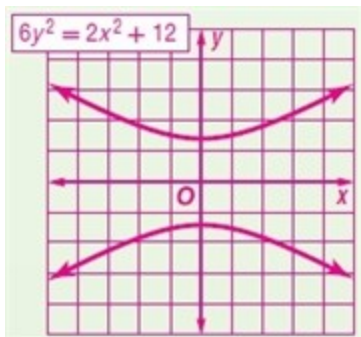
The coordinates of the vertices are $(0, 2\sqrt{2})$.

The coordinates of the foci are $(0, \pm 2\sqrt{2})$.

The asymptotes of the hyperbola are

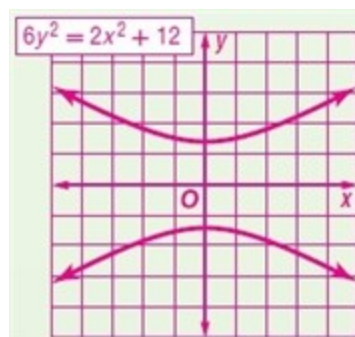
$$y = \pm \frac{\sqrt{2}}{\sqrt{6}}x = \pm \frac{\sqrt{3}}{3}x.$$

Graph the equation.



ANSWER:

$$(0, \pm\sqrt{2}); (0, \pm 2\sqrt{2}); y = \pm \frac{\sqrt{3}}{3}x;$$



Simplify each expression.

65. $\frac{12p^2 + 6p - 6}{4(p+1)^2} \div \frac{6p-3}{2p+10}$

SOLUTION:

$$\begin{aligned} \frac{12p^2 + 6p - 6}{4(p+1)^2} \div \frac{6p-3}{2p+10} &= \frac{12p^2 + 6p - 6}{4(p+1)^2} \times \frac{2p+10}{6p-3} \\ &= \frac{6(p+1)(2p-1)}{4(p+1)^2} \times \frac{2(p+5)}{3(2p-1)} \\ &= \frac{1}{(p+1)} \times \frac{(p+5)}{1} \\ &= \frac{p+5}{p+1} \end{aligned}$$

ANSWER:

$$\frac{p+5}{p+1}$$

66. $\frac{x^2 + 6x + 9}{x^2 + 7x + 6} \div \frac{4x+12}{3x+3}$

SOLUTION:

$$\begin{aligned} \frac{x^2 + 6x + 9}{x^2 + 7x + 6} \div \frac{4x+12}{3x+3} &= \frac{x^2 + 6x + 9}{x^2 + 7x + 6} \times \frac{3x+3}{4x+12} \\ &= \frac{(x+3)(x+3)}{(x+6)(x+1)} \times \frac{3(x+1)}{4(x+3)} \\ &= \frac{3(x+3)}{4(x+6)} \\ &= \frac{3x+9}{4x+24} \end{aligned}$$

ANSWER:

$$\frac{3x+9}{4x+24}$$

9-7 Solving Linear-Nonlinear Systems

67. $\frac{r^2 + 2r - 8}{r^2 + 4r + 3} \div \frac{r - 2}{3r + 3}$

SOLUTION:

$$\begin{aligned} \frac{r^2 + 2r - 8}{r^2 + 4r + 3} \div \frac{r - 2}{3r + 3} &= \frac{r^2 + 2r - 8}{r^2 + 4r + 3} \times \frac{3r + 3}{r - 2} \\ &= \frac{(r + 4)(r - 2)}{(r + 3)(r + 1)} \times \frac{3(r + 1)}{r - 2} \\ &= \frac{3(r + 4)}{r + 3} \end{aligned}$$

ANSWER:

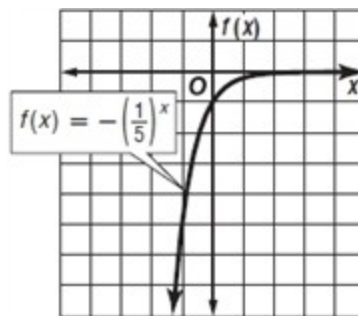
$$\frac{3(r + 4)}{r + 3}$$

Graph each function. State the domain and range.

68. $f(x) = -\left(\frac{1}{5}\right)^x$

SOLUTION:

Graph the function.



The function is defined for all values of x . Therefore, the domain is set of all real numbers.

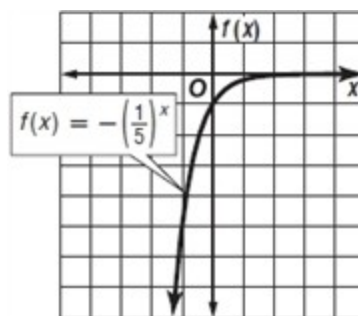
The value of $f(x)$ tends to 0 as x tends to ∞ .

The value of $f(x)$ tends to $-\infty$ as x tends to $-\infty$.

Therefore, the range of the function is $\{f(x) \mid f(x) < 0\}$.

ANSWER:

$$D = \{\text{all real numbers}\}, R = \{f(x) \mid f(x) < 0\}$$

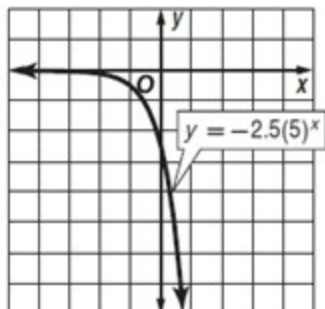


9-7 Solving Linear-Nonlinear Systems

69. $y = -2.5(5)^x$

SOLUTION:

Graph the function.



The function is defined for all values of x . Therefore, the domain is set of all real numbers.

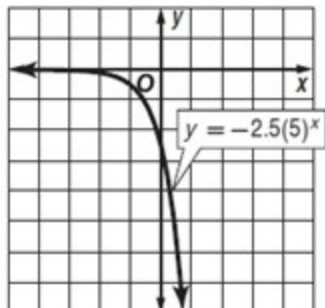
The value of y tends to 0 as x tends to $-\infty$.

The value of y tends to $-\infty$ as x tends to ∞ .

Therefore, the range of the function is $\{y \mid y < 0\}$.

ANSWER:

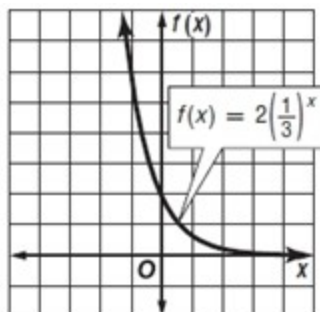
$$D = \{\text{all real numbers}\}, R = \{y \mid y < 0\}$$



70. $f(x) = 2\left(\frac{1}{3}\right)^x$

SOLUTION:

Graph the function.



The function is defined for all values of x . Therefore, the domain is set of all real numbers.

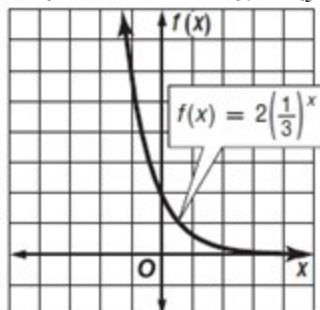
The value of y tends to 0 as x tends to ∞ .

The value of y tends to ∞ as x tends to $-\infty$.

Therefore, the range of the function is $\{f(x) \mid f(x) > 0\}$.

ANSWER:

$$D = \{\text{all real numbers}\}, R = \{f(x) \mid f(x) > 0\}$$



Solve each equation or formula for the specified variable.

71. $d = rt$, for r

SOLUTION:

$$d = rt$$

$$\frac{d}{t} = r$$

ANSWER:

$$\frac{d}{t} = r$$

9-7 Solving Linear-Nonlinear Systems

72. $x = \frac{-b}{2a}$, for a

SOLUTION:

$$x = \frac{-b}{2a}$$

$$a = \frac{-b}{2x}$$

ANSWER:

$$a = \frac{-b}{2x}$$

73. $V = \frac{1}{3}\pi r^2 h$, for h

SOLUTION:

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{3V}{\pi r^2} = h$$

ANSWER:

$$\frac{3V}{\pi r^2} = h$$

74. $A = \frac{1}{2}h(a+b)$, for b

SOLUTION:

$$A = \frac{1}{2}h(a+b)$$

$$\frac{2A}{h} = a+b$$

$$\frac{2A}{h} - a = b$$

ANSWER:

$$\frac{2A}{h} - a = b$$

Practice Test - Chapter 9

Find the midpoint of the line segment with endpoints at the given coordinates.

1. $(8, 3), (-4, 9)$

SOLUTION:

Substitute 8, -4, 3 and 9 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{8 - 4}{2}, \frac{3 + 9}{2} \right) \\ = (2, 6)$$

ANSWER:

$(2, 6)$

2. $\left(\frac{3}{4}, 0 \right), \left(\frac{1}{2}, -1 \right)$

SOLUTION:

Substitute $\frac{3}{4}, \frac{1}{2}, 0$ and -1 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{\frac{3}{4} + \frac{1}{2}}{2}, \frac{0 - 1}{2} \right) \\ = \left(\frac{5}{8}, -\frac{1}{2} \right)$$

ANSWER:

$$\left(\frac{5}{8}, -\frac{1}{2} \right)$$

3. $(-10, 0), (-2, 6)$

SOLUTION:

Substitute -10, -2, 0 and 6 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-10 - 2}{2}, \frac{0 + 6}{2} \right) \\ = (-6, 3)$$

ANSWER:

$(-6, 3)$

Find the distance between each pair of points with the given coordinates.

4. $(-5, 8), (4, 3)$

SOLUTION:

Substitute -5, 4, 8 and 3 for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ = \sqrt{(-5 - 4)^2 + (8 - 3)^2} \\ = \sqrt{81 + 25} \\ = \sqrt{106}$$

ANSWER:

$$\sqrt{106}$$

5. $\left(\frac{1}{3}, \frac{2}{3} \right), \left(-\frac{5}{6}, -\frac{11}{6} \right)$

SOLUTION:

Substitute $\frac{1}{3}, -\frac{5}{6}, \frac{2}{3}$ and $-\frac{11}{6}$ for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ = \sqrt{\left(\frac{1}{3} + \frac{5}{6} \right)^2 + \left(\frac{2}{3} + \frac{11}{6} \right)^2} \\ = \sqrt{\frac{49}{36} + \frac{225}{36}} \\ = \frac{\sqrt{274}}{6}$$

ANSWER:

$$\frac{\sqrt{274}}{6}$$

Practice Test - Chapter 9

6. $(4, -5), (4, 9)$

SOLUTION:

Substitute 4, 4, -5 and 9 for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(4 - 4)^2 + (-5 - 9)^2} \\&= \sqrt{196} \\&= 14\end{aligned}$$

ANSWER:

14

State whether the graph of each equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.

7. $y^2 = 64 - x^2$

SOLUTION:

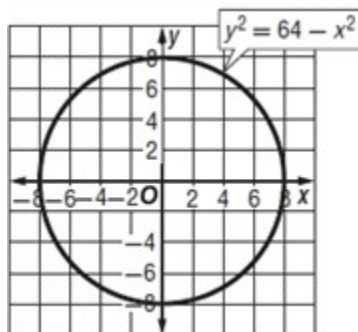
$$y^2 = 64 - x^2$$

$$x^2 + y^2 = 64$$

$$A = 1, B = 0, \text{ and } C = 1$$

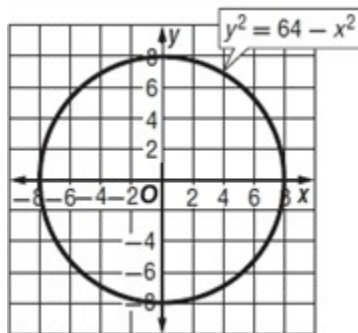
The discriminant is $B^2 - 4AC = (0^2) - 4(1)(1) = -4$.

Because the discriminant is less than 0, $B = 0$ and $A = C$, the conic is a circle.



ANSWER:

circle



Practice Test - Chapter 9

8. $4x^2 + y^2 = 16$

SOLUTION:

$$4x^2 + y^2 = 16$$

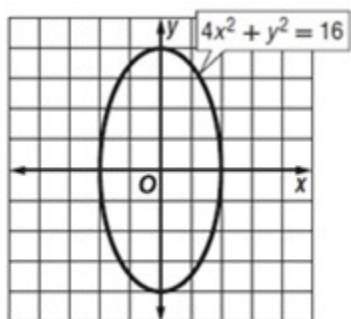
$$4x^2 + y^2 - 16 = 0$$

$$A = 4, B = 0, \text{ and } C = 1$$

The discriminant is

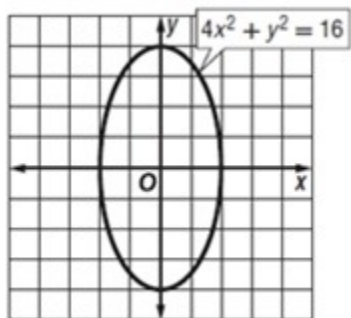
$$B^2 - 4AC = (0^2) - 4(4)(1) = -16.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.



ANSWER:

ellipse



9. $4x^2 - 9y^2 + 8x + 36y = 68$

SOLUTION:

$$4x^2 - 9y^2 + 8x + 36y = 68$$

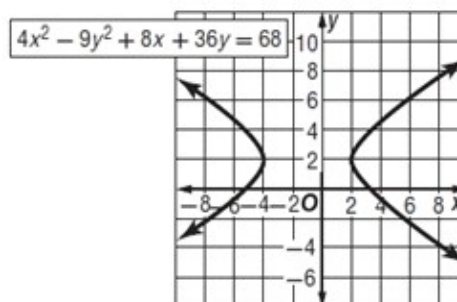
$$4x^2 - 9y^2 + 8x + 36y - 68 = 0$$

$$A = 4, B = 0, \text{ and } C = -9$$

The discriminant is

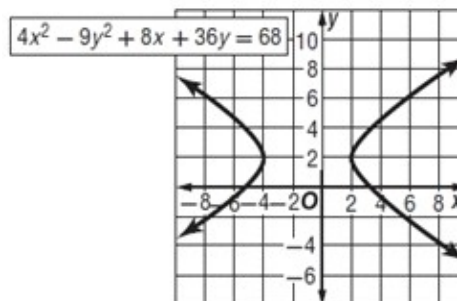
$$B^2 - 4AC = (0^2) - 4(4)(-9) = 144.$$

Because the discriminant is greater than 0, the conic is a hyperbola.



ANSWER:

hyperbola



Practice Test - Chapter 9

10. $\frac{1}{2}x^2 - 3 = y$

SOLUTION:

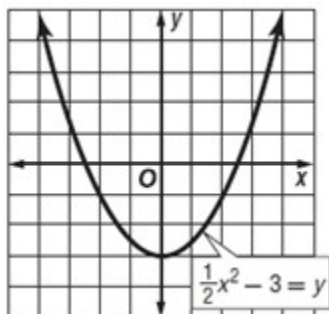
$$\frac{1}{2}x^2 - 3 = y$$

$$\frac{1}{2}x^2 - 3 - y = 0$$

$A = \frac{1}{2}$, $B = 0$, and $C = 0$

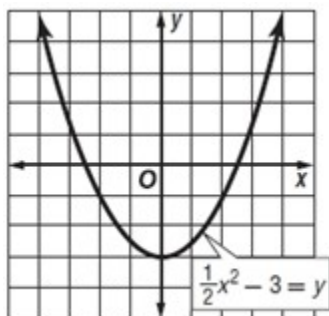
The discriminant is $B^2 - 4AC = (0^2) - 4\left(\frac{1}{2}\right)(0) = 0$.

Because the discriminant is equal to 0, the conic is a parabola.



ANSWER:

parabola



11. $y = -2x^2 - 5$

SOLUTION:

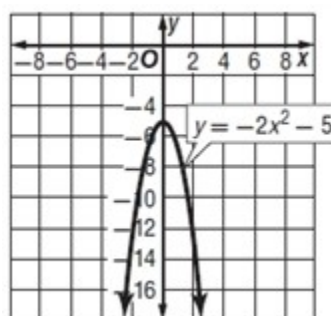
$$y = -2x^2 - 5$$

$$2x^2 + y + 5 = 0$$

$A = 2$, $B = 0$, and $C = 0$

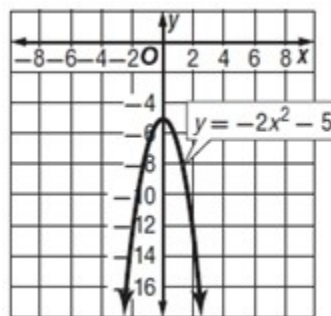
The discriminant is $B^2 - 4AC = (0^2) - 4(2)(0) = 0$.

Because the discriminant is equal to 0, the conic is a parabola.



ANSWER:

parabola



Practice Test - Chapter 9

12. $16x^2 + 25y^2 = 400$

SOLUTION:

$$16x^2 + 25y^2 = 400$$

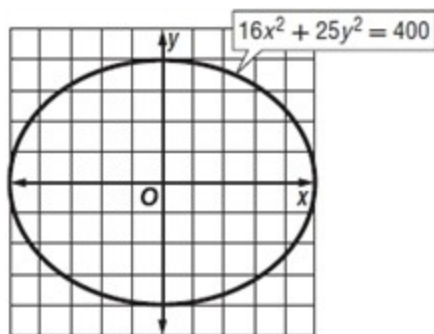
$$16x^2 + 25y^2 - 400 = 0$$

$$A = 16, B = 0, \text{ and } C = 25$$

The discriminant is

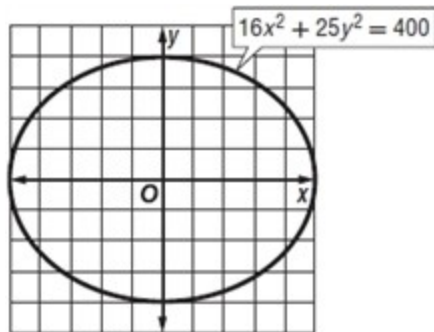
$$B^2 - 4AC = (0^2) - 4(16)(25) = -1600.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.



ANSWER:

ellipse



13. $x^2 + 6x + y^2 = 16$

SOLUTION:

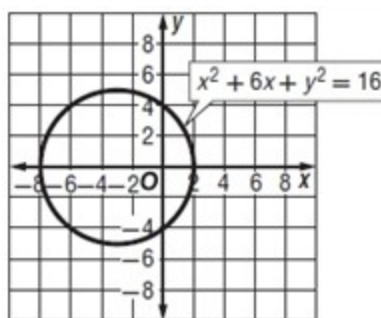
$$x^2 + 6x + y^2 = 16$$

$$x^2 + 6x + y^2 - 16 = 0$$

$$A = 1, B = 0, \text{ and } C = 1$$

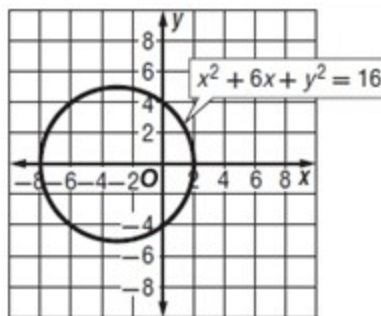
The discriminant is $B^2 - 4AC = (0^2) - 4(1)(1) = -4$.

Because the discriminant is less than 0, $B = 0$ and $A = C$, the conic is a circle.



ANSWER:

circle



Practice Test - Chapter 9

14. $\frac{y^2}{4} - \frac{x^2}{16} = 1$

SOLUTION:

$$\frac{y^2}{4} - \frac{x^2}{16} = 1$$

$$4y^2 - x^2 = 16$$

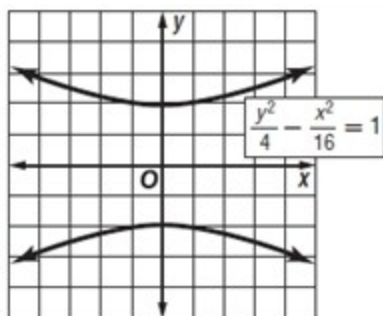
$$4y^2 - x^2 - 16 = 0$$

$A = 1, B = 0,$ and $C = -4$

The discriminant is

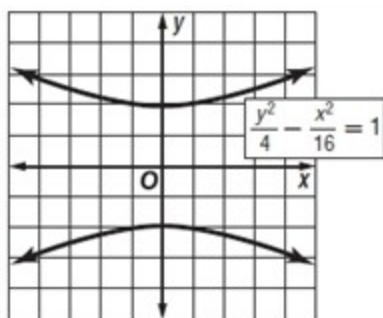
$$B^2 - 4AC = (0^2) - 4(1)(-4) = 16.$$

Because the discriminant is greater than 0, the conic is a hyperbola.



ANSWER:

hyperbola



15. $(x+2)^2 = 3(y-1)$

SOLUTION:

$$(x+2)^2 = 3(y-1)$$

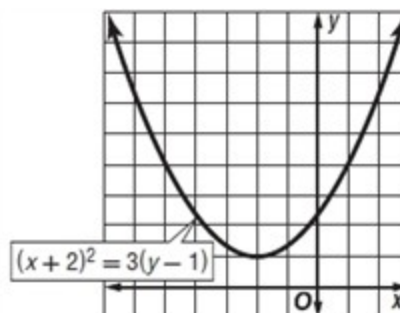
$$x^2 + 4x + 4 = 3y - 3$$

$$x^2 + 4x - 3y + 7 = 0$$

$A = 1, B = 0,$ and $C = 0$

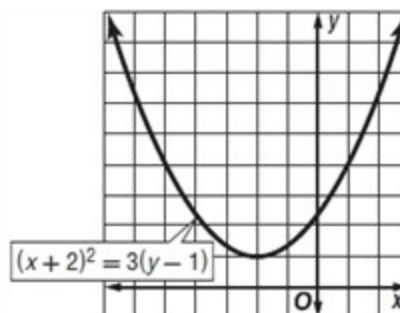
The discriminant is $B^2 - 4AC = (0^2) - 4(1)(0) = 0$.

Because the discriminant is equal to 0, the conic is a parabola.



ANSWER:

parabola



Practice Test - Chapter 9

16. $4x^2 + 16y^2 + 32x + 63 = 0$

SOLUTION:

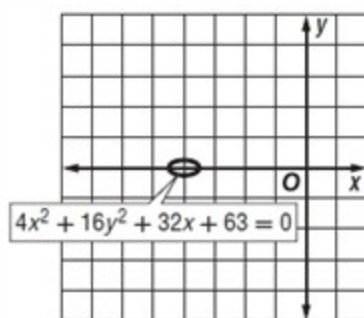
$$4x^2 + 16y^2 + 32x + 63 = 0$$

$A = 4$, $B = 0$, and $C = 16$

The discriminant is

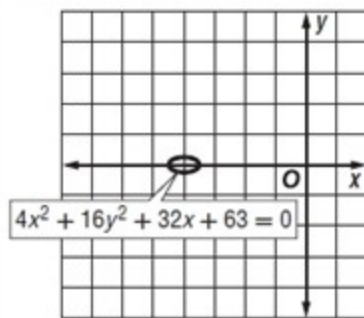
$$B^2 - 4AC = (0^2) - 4(4)(16) = -256.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.



ANSWER:

ellipse



17. **MULTIPLE CHOICE** Which equation represents a hyperbola that has vertices at $(-3, -3)$ and $(5, -3)$ and a conjugate axis of length 6 units?

A $\frac{(y-1)^2}{16} - \frac{(x+3)^2}{9} = 1$

B $\frac{(x-1)^2}{16} - \frac{(y+3)^2}{9} = 1$

C $\frac{(y+1)^2}{16} - \frac{(x-3)^2}{9} = 1$

D $\frac{(x+1)^2}{16} - \frac{(y-3)^2}{9} = 1$

SOLUTION:

Since the y -coordinates of the vertices are same, the orientation should be horizontal. Therefore, option B and D may be the correct choice.

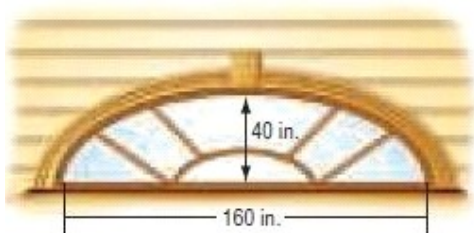
Since the y -coordinate of the vertices is -3 , option B is the correct answer.

ANSWER:

B

Practice Test - Chapter 9

18. **CARPENTRY** Ellis built a window frame shaped like the top half of an ellipse. The window is 40 inches tall at its highest point and 160 inches wide at the bottom. What is the height of the window 20 inches from the center of the base?



SOLUTION:

The length of the major and the minor axis is 160 in. and 80 in.

Therefore, $a = 80$ and $b = 40$.

The equation of the ellipse is $\frac{x^2}{80^2} + \frac{y^2}{40^2} = 1$.

To find the height of the window, substitute 20 for x and solve for y .

$$\begin{aligned}\frac{(20)^2}{80^2} + \frac{y^2}{40^2} &= 1 \\ \frac{1}{4^2} + \frac{y^2}{40^2} &= 1 \\ y^2 &= 40^2 \left(1 - \frac{1}{4^2} \right) \\ y &= 10\sqrt{5} \text{ or } 38.73 \text{ in.}\end{aligned}$$

The height of the window is about 38.73 in.

ANSWER:

$10\sqrt{15}$ or about 38.73 inches

Solve each system of equations.

19.
$$\begin{aligned}x^2 + y^2 &= 100 \\ y &= -x - 2\end{aligned}$$

SOLUTION:

Substitute $-x-2$ for y in the quadratic equation and solve for x .

$$\begin{aligned}x^2 + (-x-2)^2 &= 100 \\ 2x^2 + 4x + 4 &= 100 \\ x^2 + 2x - 48 &= 0 \\ (x-6)(x+8) &= 0\end{aligned}$$

By zero product property:

$$x = -8 \text{ or } 6$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{lll}y = -x - 2 & \text{and} & y = -x - 2 \\ y = -(6) - 2 & \text{and} & y = -(-8) - 2 \\ y = -8 & \text{and} & y = 6\end{array}$$

The solutions are $(6, -8)$ and $(-8, 6)$.

ANSWER:

$(-8, 6), (6, -8)$

Practice Test - Chapter 9

20. $x^2 + 2y^2 = 11$
 $x + y = 2$

SOLUTION:

Solve the linear equation for y .

$$\begin{aligned}x + y &= 2 \\y &= 2 - x\end{aligned}$$

Substitute $2 - x$ for y in the quadratic equation and solve for x .

$$\begin{aligned}x^2 + 2(2 - x)^2 &= 11 \\3x^2 - 8x + 8 &= 11 \\3x^2 - 8x - 3 &= 0 \\(3x + 1)(x - 3) &= 0\end{aligned}$$

By zero product property:

$$x = -\frac{1}{3} \text{ or } 3$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{ll}x + y = 2 & \text{and} \quad x + y = 2 \\3 + y = 2 & \text{and} \quad -\frac{1}{3} + y = 2 \\y = -1 & \text{and} \quad y = \frac{7}{3}\end{array}$$

The solutions are $(3, -1)$ and $\left(-\frac{1}{3}, \frac{7}{3}\right)$.

ANSWER:

$$(3, -1), \left(-\frac{1}{3}, \frac{7}{3}\right)$$

21. $x^2 + y^2 = 34$
 $y^2 - x^2 = 9$

SOLUTION:

$$\begin{aligned}x^2 + y^2 &= 34 & \rightarrow (1) \\y^2 - x^2 &= 9 & \rightarrow (2)\end{aligned}$$

$$\begin{aligned}(1) - (2) \quad 2x^2 &= 25 \\x^2 &= \frac{25}{2} \\x &= \pm \frac{5\sqrt{2}}{2}\end{aligned}$$

Substitute the values of x in either of the equations and find the values of y .

$$\begin{aligned}y^2 - x^2 &= 9 \\y^2 - \left(\frac{5\sqrt{2}}{2}\right)^2 &= 9 \\y^2 - \frac{50}{4} &= 9 \\y^2 &= 9 + \frac{50}{4} \\y^2 &= \frac{86}{4} \\y &= \pm \frac{\sqrt{86}}{2}\end{aligned}$$

The solutions are

$$\left(\frac{5\sqrt{2}}{2}, \frac{\sqrt{86}}{2}\right), \left(\frac{5\sqrt{2}}{2}, -\frac{\sqrt{86}}{2}\right), \left(-\frac{5\sqrt{2}}{2}, \frac{\sqrt{86}}{2}\right), \left(-\frac{5\sqrt{2}}{2}, -\frac{\sqrt{86}}{2}\right)$$

ANSWER:

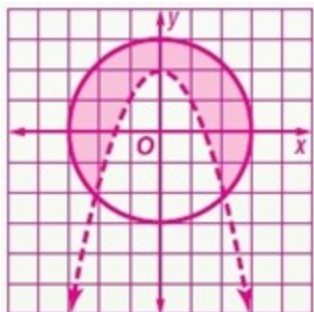
$$\left(\pm \frac{5\sqrt{2}}{2}, \pm \frac{\sqrt{86}}{2}\right)$$

Practice Test - Chapter 9

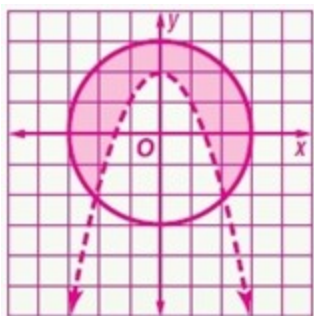
Solve each system of inequalities.

22. $x^2 + y^2 \leq 9$
 $y > -x^2 + 2$

SOLUTION:

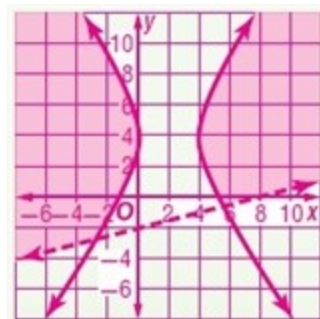


ANSWER:

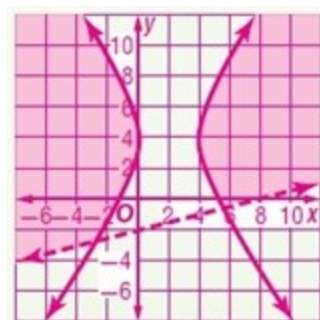


23. $\frac{(x-2)^2}{4} - \frac{(y-4)^2}{9} \geq 1$
 $x - 4y < 8$

SOLUTION:



ANSWER:



24. **MULTIPLE CHOICE** Which is NOT the equation of a parabola?

F $y = 3x^2 + 5x - 3$

G $2y + 3x^2 + x - 9 = 0$

H $x = 3(y+1)^2$

J $x^2 + y^2 + 6x = 10$

SOLUTION:

The equation $x^2 + y^2 + 6x = 10$ has both x^2 and y^2 terms. Therefore, this is not the equation of a parabola.

Option J is the correct answer.

ANSWER:

J

25. **FORESTRY** A forest ranger at an outpost in the Sam Houston National Forest and another ranger at the primary station both heard an explosion. The outpost and the primary station are 6 kilometers apart.

a. If one ranger heard the explosion 6 seconds before

Practice Test - Chapter 9

the other, write an equation that describes all the possible locations of the explosion. Place the two ranger stations on the x -axis with the midpoint between the stations at the origin. The transverse axis is horizontal. (*Hint:* The speed of sound is about 0.35 kilometer per second.)

b. Draw a sketch of the possible locations of the explosion. Include the ranger stations in the drawing.

SOLUTION:

a. Let the ranger in the Sam Houston National Forest hears the explosion first.

The ranger in the primary station hears the sound 6 seconds later than the other.

That is, the explosion place is $6(0.35) = 2.1$ kilometers farther from primary station than from Sam Houston National Forest.

Therefore, the locus of all points that are 2.1 km closer to Sam Houston National Forest than to primary station is one branch of the hyperbola.

$$2a = 2.1$$

$$a = 1.05$$

The distance between the two stations is 6 km.

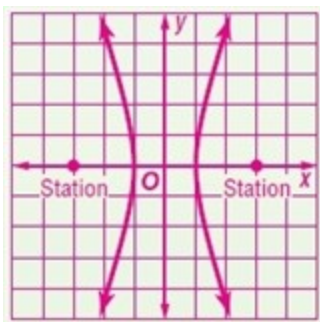
$$2c = 6$$

$$c = 3$$

$$\begin{aligned} b^2 &= c^2 - a^2 \\ &= 9 - 1.1025 \\ &= 7.8975 \end{aligned}$$

The equation is $\frac{x^2}{1.1025} - \frac{y^2}{7.8975} = 1$.

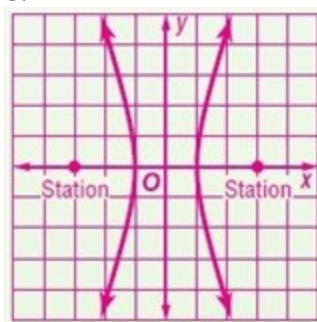
b. Graph the equation.



ANSWER:

a. $\frac{x^2}{1.1025} - \frac{y^2}{7.8975} = 1$

b.



Study Guide and Review - Chapter 9

State whether each sentence is *true* or *false* . If *false*, replace the underlined term to make a true sentence.

1. The set of all points in a plane that are equidistant from a given point in the plane, called the focus, forms a circle.

SOLUTION:

false, center

ANSWER:

false, center

2. A(n) ellipse is the set of all points in a plane such that the sum of the distances from the two fixed points is constant.

SOLUTION:

true

ANSWER:

true

3. The endpoints of the major axis of an ellipse are the foci of the ellipse.

SOLUTION:

false, vertices

ANSWER:

false, vertices

4. The radius is the distance from the center of a circle to any point on the circle.

SOLUTION:

true

ANSWER:

true

5. The line segment with endpoints on a parabola, through the focus of the parabola, and perpendicular to the axis of symmetry is called the latus rectum.

SOLUTION:

true

ANSWER:

true

6. Every hyperbola has two axes of symmetry, the transverse axis and the major axis.

SOLUTION:

false, conjugate axis

ANSWER:

false, conjugate axis

7. A directrix is the set of all points in a plane that are equidistant from a given point in the plane, called the center

SOLUTION:

false, circle

ANSWER:

false, circle

8. A hyperbola is the set of all points in a plane such that the absolute value of the sum of the distances from any point on the hyperbola to two given points is constant.

SOLUTION:

false; difference

ANSWER:

false; difference

9. A parabola can be defined as the set of all points in a plane that are the same distance from the focus and a given line called the directrix.

SOLUTION:

true

ANSWER:

true

10. The major axis is the longer of the two axes of symmetry of an ellipse.

SOLUTION:

true

ANSWER:

true

Study Guide and Review - Chapter 9

Find the midpoint of the line segment with endpoints at the given coordinates.

11. $(-8, 6), (3, 4)$

SOLUTION:

Substitute $-8, 3, 6$ and 4 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-8 + 3}{2}, \frac{6 + 4}{2} \right) \\ = \left(-\frac{5}{2}, 5 \right)$$

ANSWER:

$$\left(-\frac{5}{2}, 5 \right)$$

12. $(-6, 0), (-1, 4)$

SOLUTION:

Substitute $-6, -1, 0$ and 4 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-6 - 1}{2}, \frac{0 + 4}{2} \right) \\ = \left(-\frac{7}{2}, 2 \right)$$

ANSWER:

$$\left(-\frac{7}{2}, 2 \right)$$

13. $\left(\frac{3}{4}, \frac{2}{3} \right), \left(-\frac{1}{3}, \frac{1}{4} \right)$

SOLUTION:

Substitute $\frac{3}{4}, -\frac{1}{3}, \frac{2}{3}$ and $\frac{1}{4}$ for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{\frac{3}{4} - \frac{1}{3}}{2}, \frac{\frac{2}{3} + \frac{1}{4}}{2} \right) \\ = \left(\frac{5}{24}, \frac{11}{24} \right)$$

ANSWER:

$$\left(\frac{5}{24}, \frac{11}{24} \right)$$

14. $(15, 20), (18, 21)$

SOLUTION:

Substitute $15, 18, 20$ and 21 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{15 + 18}{2}, \frac{20 + 21}{2} \right) \\ = \left(\frac{33}{2}, \frac{41}{2} \right)$$

ANSWER:

$$\left(\frac{33}{2}, \frac{41}{2} \right)$$

Study Guide and Review - Chapter 9

Find the distance between each pair of points with the given coordinates.

15. $(10, -3), (1, -5)$

SOLUTION:

Substitute 10, 1, -3 and -5 for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(1 - 10)^2 + (-5 - (-3))^2} \\&= \sqrt{81 + 4} \\&= \sqrt{85}\end{aligned}$$

ANSWER:

$$\sqrt{85}$$

16. $(0, 6), (-9, 7)$

SOLUTION:

Substitute 0, -9, 6 and 7 for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(-9 - 0)^2 + (7 - 6)^2} \\&= \sqrt{81 + 1} \\&= \sqrt{82}\end{aligned}$$

ANSWER:

$$\sqrt{82}$$

17. $\left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{3}{2}, \frac{5}{4}\right)$

SOLUTION:

Substitute $\frac{1}{4}, \frac{2}{3}, \frac{1}{2}$ and $\frac{5}{4}$ for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{\left(\frac{3}{2} - \frac{1}{4}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2} \\&= \sqrt{\frac{25}{16} + \frac{9}{16}} \\&= \frac{\sqrt{34}}{4}\end{aligned}$$

ANSWER:

$$\frac{\sqrt{34}}{4}$$

18. $(5, -3), (7, -1)$

SOLUTION:

Substitute 5, 7, -3 and -1 for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(7 - 5)^2 + (-1 - (-3))^2} \\&= \sqrt{4 + 4} \\&= 2\sqrt{2}\end{aligned}$$

ANSWER:

$$2\sqrt{2}$$

Study Guide and Review - Chapter 9

19. **HIKING** Marc wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

- How far away is the waterfall?
- Marc wants to stop for lunch halfway to the waterfall. Where should he stop?

SOLUTION:

- The coordinates are $(0, -5)$ and $(8, 0)$.
The distance between any two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute 0, 8, -5 and 0 for x_1, x_2, y_1 and y_2 respectively in the distance formula.

$$\begin{aligned} d &= \sqrt{(8 - 0)^2 + (0 - (-5))^2} \\ &= \sqrt{64 + 25} \\ &= \sqrt{89} \end{aligned}$$

- Substitute 0, 8, -5 and 0 for x_1, x_2, y_1 and y_2 respectively in the midpoint formula to find the halfway to the waterfall.

$$\left(\frac{0 + 8}{2}, \frac{-5 + 0}{2} \right) = \left(4, -\frac{5}{2} \right)$$

ANSWER:

- $\sqrt{89} \approx 9.4$ miles
- $\left(4, -\frac{5}{2} \right)$

Graph each equation.

20. $y = 3x^2 + 24x - 10$

SOLUTION:

$$\begin{aligned} y &= 3x^2 + 24x - 10 \\ y + 10 &= 3(x^2 + 8x) \\ y + 10 + 48 &= 3(x^2 + 8x + 16) \\ y + 58 &= 3(x + 4)^2 \end{aligned}$$

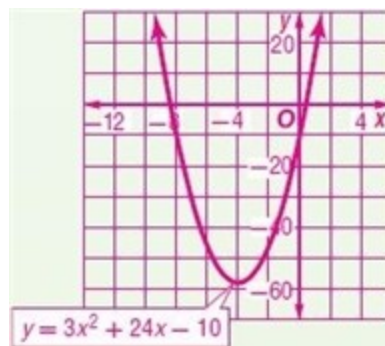
vertex: $(-4, -58)$

axis of symmetry: $x = -4$.

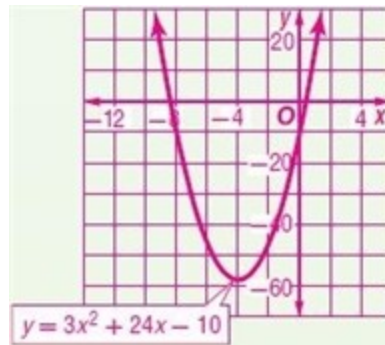
focus: $\left(-4, -57\frac{3}{4} \right)$

directrix: $y = -58\frac{1}{4}$

length of latus rectum: 1 unit
opens up



ANSWER:



Study Guide and Review - Chapter 9

21. $3y - x^2 = 8x - 11$

SOLUTION:

$$3y - x^2 = 8x - 11$$

$$y + 11 = \frac{1}{3}(x^2 + 8x)$$

$$y + 11 + \frac{16}{3} = \frac{1}{3}(x^2 + 8x + 16)$$

$$y + \frac{49}{3} = \frac{1}{3}(x + 4)^2$$

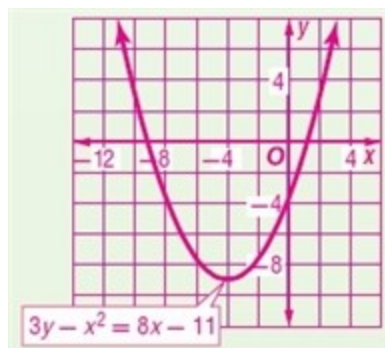
vertex: $\left(-4, -\frac{49}{3}\right)$

axis of symmetry: $x = -4$.

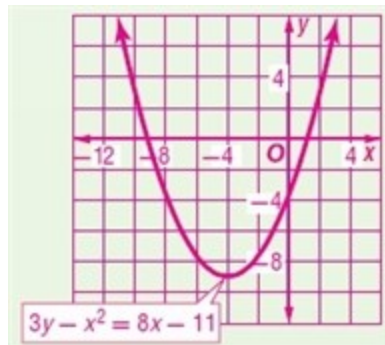
focus: $\left(-4, -15\frac{7}{12}\right)$

directrix: $x = -17\frac{1}{12}$

length of latus rectum: 3 unit
opens up



ANSWER:



22. $x = \frac{1}{2}y^2 - 4y + 3$

SOLUTION:

$$x = \frac{1}{2}y^2 - 4y + 3$$

$$x - 3 = \frac{1}{2}(y^2 - 8y)$$

$$x - 3 + 8 = \frac{1}{2}(y^2 - 8y + 16)$$

$$x + 5 = \frac{1}{2}(y - 4)^2$$

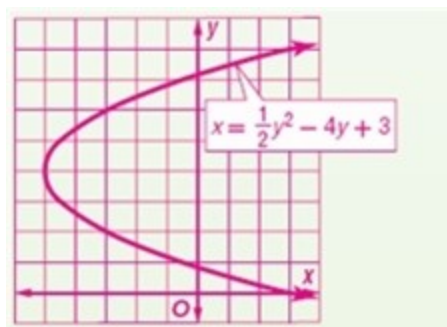
vertex: $(-5, 4)$

axis of symmetry: $y = 4$.

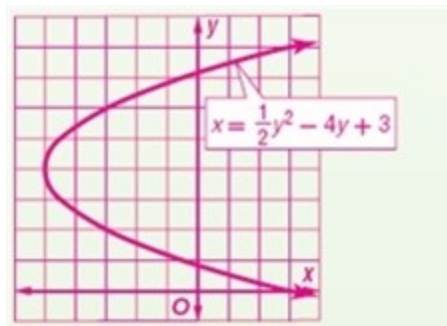
focus: $\left(-4\frac{1}{2}, 4\right)$

directrix: $x = -5\frac{1}{2}$

length of latus rectum: 2 units
opens to the right



ANSWER:



Study Guide and Review - Chapter 9

23. $x = y^2 - 14y + 25$

SOLUTION:

$$x = y^2 - 14y + 25$$

$$x - 25 = y^2 - 14y$$

$$x - 25 + 49 = y^2 - 14y + 49$$

$$x + 24 = (y - 7)^2$$

$$x = (y - 7)^2 - 24$$

vertex: $(-24, 7)$

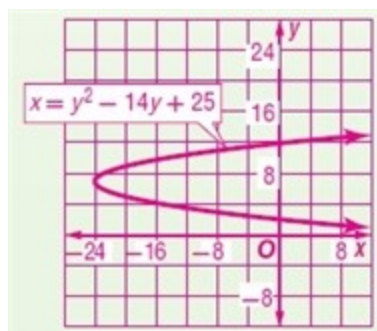
axis of symmetry: $y = 7$.

focus: $(-23.75, 7)$

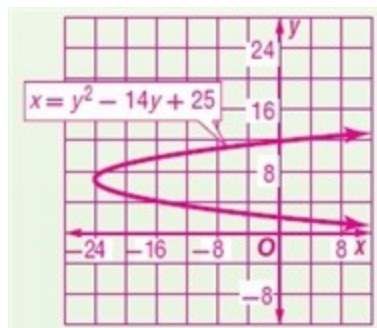
directrix: $x = -24.25$

length of latus rectum: 1 unit

opens to the right



ANSWER:



Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

24. $y = \frac{1}{2}x^2$

SOLUTION:

$$y - 0 = \frac{1}{2}(x - 0)^2$$

vertex: $(0, 0)$

axis of symmetry: $x = 0$

opens up

ANSWER:

$$y = \frac{1}{2}x^2; \text{ vertex: } (0, 0); \text{ axis of symmetry: } x = 0;$$

opens: upward

25. $y = 4x^2 - 16x + 9$

SOLUTION:

$$y = 4x^2 - 16x + 9$$

$$y - 9 = 4(x^2 - 4x)$$

$$y - 9 + 16 = 4(x^2 - 4x + 4)$$

$$y + 7 = 4(x - 2)^2$$

$$y = 4(x - 2)^2 - 7$$

vertex: $(2, -7)$

axis of symmetry: $x = 2$

opens up

ANSWER:

$$y = 4(x - 2)^2 - 7; \text{ vertex: } (2, -7); \text{ axis of symmetry:}$$

$x = 2$; opens: upward

Study Guide and Review - Chapter 9

26. $x - 6y = y^2 + 4$

SOLUTION:

$$x - 6y = y^2 + 4$$

$$x - 4 = y^2 + 6y$$

$$x - 4 + 9 = y^2 + 6y + 9$$

$$x + 5 = (y + 3)^2$$

$$x = (y + 3)^2 - 5$$

vertex: $(-5, -3)$

axis of symmetry: $y = -3$

opens to the right

ANSWER:

$x = (y + 3)^2 - 5$; vertex: $(-5, -3)$; axis of symmetry:

$y = -3$; opens to the right

27. $x = y^2 + 14y + 20$

SOLUTION:

$$x = y^2 + 14y + 20$$

$$x - 20 = y^2 + 14y$$

$$x - 20 + 49 = y^2 + 14y + 49$$

$$x + 29 = (y + 7)^2$$

$$x = (y + 7)^2 - 29$$

vertex: $(-29, -7)$

axis of symmetry: $y = -7$

opens to the right

ANSWER:

$x = (y + 7)^2 - 29$; vertex = $(-29, -7)$; axis of

symmetry: $y = -7$; opens to the right

28. **SPORTS** When a football is kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 50 feet, and lands 200 feet away. Assuming the football was kicked at the origin, write an equation of the parabola that models the flight of the football.

SOLUTION:

The coordinates of the ball when the ball touches the ground are $(0, 0)$ and $(200, 0)$.

The vertex of the path is $(100, 50)$.

The standard form of equation of a parabola is

$$y = a(x - h)^2 + k.$$

Substitute 0, 0, 100 and 50 for y , x , h and k in the standard form then solve for a .

$$0 = a(0 - 100)^2 + 50$$

$$10000a = -50$$

$$a = -\frac{1}{200}$$

The equation of the path is

$$y = -\frac{1}{200}(x - 100)^2 + 50.$$

ANSWER:

$$y = -\frac{1}{200}(x - 100)^2 + 50$$

Write an equation for the circle that satisfies each set of conditions.

29. center $(-1, 6)$, radius 3 units

SOLUTION:

Substitute -1 , 6 and 3 for h , k and d in the standard form of circle equation.

$$(x - (-1))^2 + (y - 6)^2 = (3)^2$$

$$(x + 1)^2 + (y - 6)^2 = 9$$

ANSWER:

$$(x + 1)^2 + (y - 6)^2 = 9$$

Study Guide and Review - Chapter 9

30. endpoints of a diameter (2, 5) and (0, 0)

SOLUTION:

The center is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$$\begin{aligned}(h, k) &= \left(\frac{2-0}{2}, \frac{5-0}{2}\right) \\ &= (1, 2.5)\end{aligned}$$

The radius of the circle is

$$\frac{\sqrt{(2-0)^2 + (5-0)^2}}{2} = \frac{\sqrt{29}}{2}$$

Substitute 1, 2.5 and $\frac{\sqrt{29}}{2}$ for h , k and d in the standard form of circle equation.

$$\begin{aligned}(x-1)^2 + (y-2.5)^2 &= \left(\frac{\sqrt{29}}{2}\right)^2 \\ (x-1)^2 + (y-2.5)^2 &= \frac{29}{4}\end{aligned}$$

ANSWER:

$$(x-1)^2 + (y-2.5)^2 = \frac{29}{4}$$

31. endpoints of a diameter (4, -2) and (-2, -6)

SOLUTION:

$$\begin{aligned}(h, k) &= \left(\frac{4-2}{2}, \frac{-2-6}{2}\right) \\ &= (1, -4)\end{aligned}$$

$$\begin{aligned}d &= \sqrt{(1+2)^2 + (-4+6)^2} \\ &= \sqrt{3^2 + 2^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13}\end{aligned}$$

The radius of the circle is $\sqrt{13}$ units.

Substitute 1, -4 and 5 for h , k and d in the standard form of circle equation.

$$\begin{aligned}(x-1)^2 + (y-(-4))^2 &= (\sqrt{13})^2 \\ (x-1)^2 + (y+4)^2 &= 13\end{aligned}$$

ANSWER:

$$(x-1)^2 + (y+4)^2 = 13$$

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Find the center and radius of each circle. Then graph the circle.

32. $(x+5)^2 + y^2 = 9$

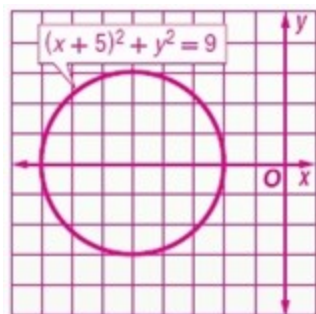
SOLUTION:

$$(x+5)^2 + y^2 = 9$$

$$(x - (-5))^2 + (y - 0)^2 = 3^2$$

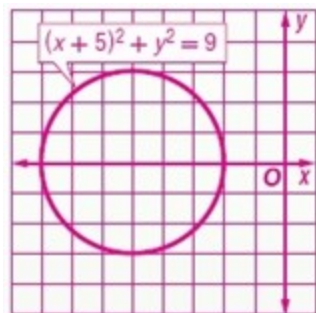
The center of the circle is $(-5, 0)$.

The radius of the circle is 3.



ANSWER:

$(-5, 0); r = 3$



33. $(x-3)^2 + (y+1)^2 = 25$

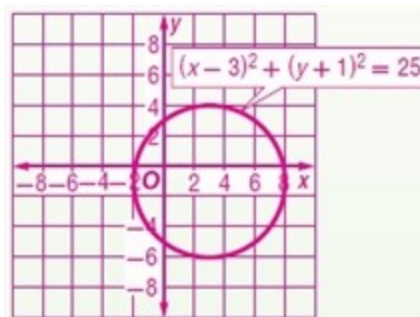
SOLUTION:

$$(x-3)^2 + (y+1)^2 = 25$$

$$(x-3)^2 + (y - (-1))^2 = 5^2$$

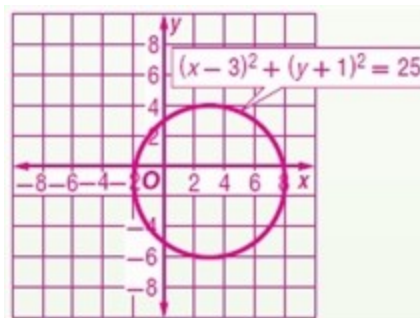
The center of the circle is $(3, -1)$.

The radius of the circle is 5.



ANSWER:

$(3, -1); r = 5$



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34. $(x+2)^2 + (y-8)^2 = 1$

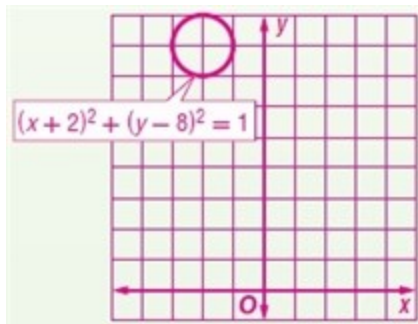
SOLUTION:

$$(x+2)^2 + (y-8)^2 = 1$$

$$(x-(-2))^2 + (y-8)^2 = 1^2$$

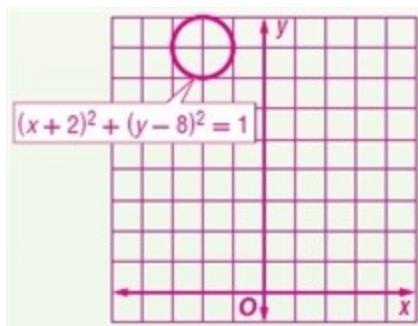
The center of the circle is $(-2, 8)$.

The radius of the circle is 1.



ANSWER:

$(-2, 8); r = 1$



35. $x^2 + 4x + y^2 - 2y - 11 = 0$

SOLUTION:

$$x^2 + 4x + y^2 - 2y - 11 = 0$$

$$x^2 + 4x + y^2 - 2y = 11$$

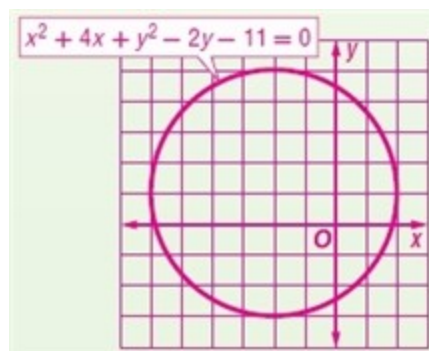
$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 11 + 4 + 1$$

$$(x+2)^2 + (y-1)^2 = 16$$

$$(x-(-2))^2 + (y-1)^2 = 4^2$$

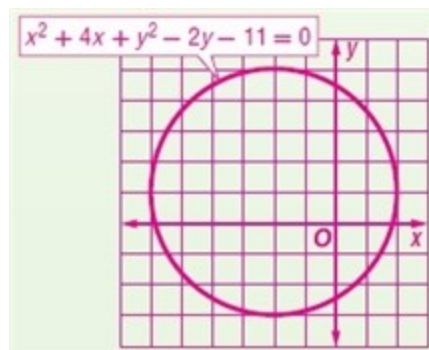
The center of the circle is $(-2, 1)$.

The radius of the circle is 4.



ANSWER:

$(-2, 1); r = 4$



Study Guide and Review - Chapter 9

36. **SOUND** A loudspeaker in a school is located at the point (65, 40). The speaker can be heard in a circle with a radius of 100 feet. Write an equation to represent the possible boundary of the loudspeaker sound.

SOLUTION:

The center of the circle is (65, 40) and the radius is 100.

Substitute 65, 40 and 100 for h , k and r in the standard form of the circle equation.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 65)^2 + (y - 40)^2 = 100^2$$

ANSWER:

$$(x - 65)^2 + (y - 40)^2 = 100^2$$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

37. $\frac{x^2}{9} + \frac{y^2}{36} = 1$

SOLUTION:

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

$$\frac{y^2}{6^2} + \frac{x^2}{3^2} = 1$$

Here, $(h, k) = (0, 0)$, $a = 6$ and $b = 3$.

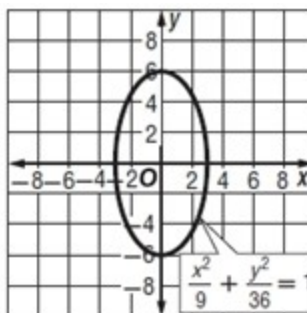
$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{36 - 9} \\ &= 3\sqrt{3} \end{aligned}$$

center: $(0, 0)$

foci: $(0, \pm 3\sqrt{3})$

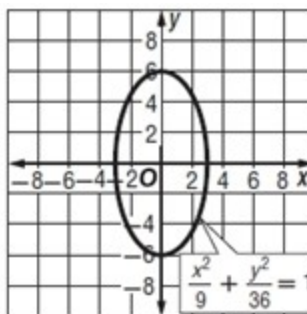
length of the major axis: 12 units

length of the minor axis: 6 units



ANSWER:

$(0, 0); (0, \pm 3\sqrt{3}); 12; 6$



Study Guide and Review - Chapter 9

38. $\frac{y^2}{10} + \frac{x^2}{5} = 1$

SOLUTION:

$$\begin{aligned}\frac{y^2}{10} + \frac{x^2}{5} &= 1 \\ \frac{y^2}{\sqrt{10}^2} + \frac{x^2}{\sqrt{5}^2} &= 1\end{aligned}$$

The values of h , k , a and b are 0, 0, $\sqrt{10}$ and $\sqrt{5}$.

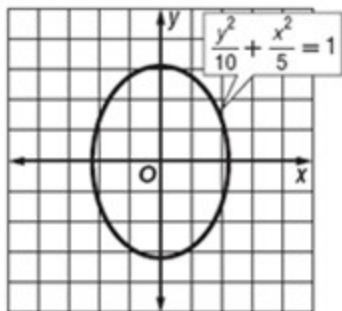
$$\begin{aligned}c &= \sqrt{a^2 - b^2} \\ &= \sqrt{10 - 5} \\ &= \sqrt{5}\end{aligned}$$

center: (0, 0)

foci: $(0, \pm\sqrt{5})$

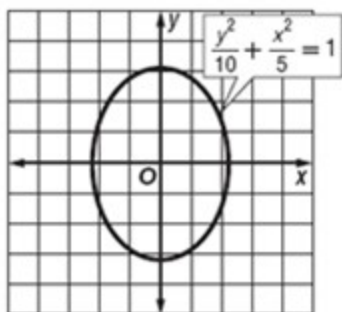
length of the major axis: $2\sqrt{10}$ units

length of the minor axis: $2\sqrt{5}$ units



ANSWER:

$(0,0); (0, \pm\sqrt{5}); 2\sqrt{10}; 2\sqrt{5}$



39. $\frac{x^2}{36} + \frac{(y-4)^2}{4} = 1$

SOLUTION:

$$\begin{aligned}\frac{x^2}{36} + \frac{(y-4)^2}{4} &= 1 \\ \frac{x^2}{6^2} + \frac{(y-4)^2}{2^2} &= 1\end{aligned}$$

Center: $(h, k) = (0, 4)$

Here $a = 6$ and $b = 2$.

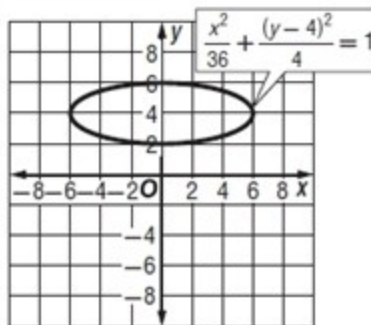
$$\begin{aligned}c &= \sqrt{a^2 - b^2} \\ &= \sqrt{36 - 4} \\ &= 4\sqrt{2}\end{aligned}$$

center: (0, 4)

foci: $(\pm 4\sqrt{2}, 4)$

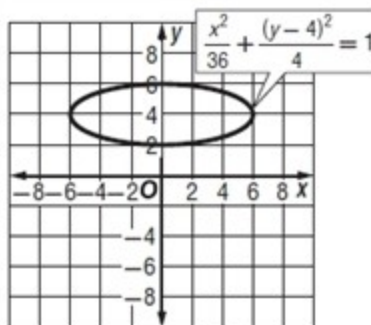
length of the major axis: 12 units

length of the minor axis: 4 units



ANSWER:

$(0,4); (\pm 4\sqrt{2}, 4); 12; 4$



Study Guide and Review - Chapter 9

40. $27x^2 + 9y^2 = 81$

SOLUTION:

$$27x^2 + 9y^2 = 81$$

$$\frac{x^2}{3} + \frac{y^2}{9} = 1$$

$$\frac{y^2}{3^2} + \frac{x^2}{\sqrt{3}^2} = 1$$

Here $a = 3$ and $b = \sqrt{3}$.

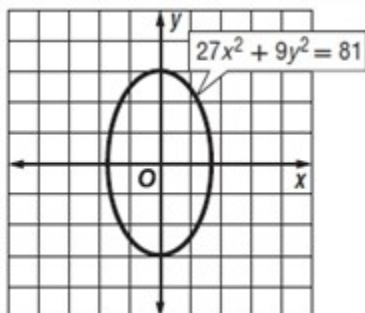
$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{9 - 3} \\ &= \sqrt{6} \end{aligned}$$

center: $(0, 0)$

foci: $(0, \pm \sqrt{6})$

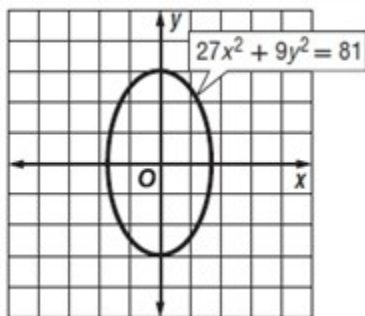
length of the major axis: 6 units

length of the minor axis: $2\sqrt{3}$ units



ANSWER:

$(0, 0); (0, \pm \sqrt{6}); 6; 2\sqrt{3}$



41. $\frac{(x+1)^2}{25} + \frac{(y-2)^2}{16} = 1$

SOLUTION:

$$\frac{(x+1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

$$\frac{(x+1)^2}{5^2} + \frac{(y-2)^2}{4^2} = 1$$

$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{25 - 16} \\ &= 3 \end{aligned}$$

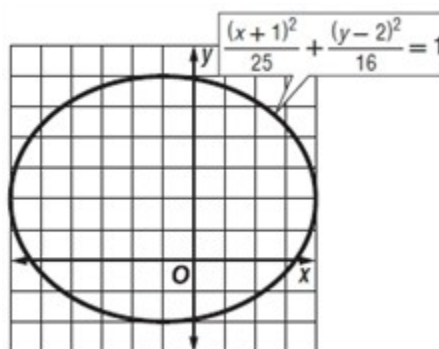
Here $a = 5$ and $b = 4$.

center: $(-1, 2)$

foci: $(-4, 2)$ and $(2, 2)$

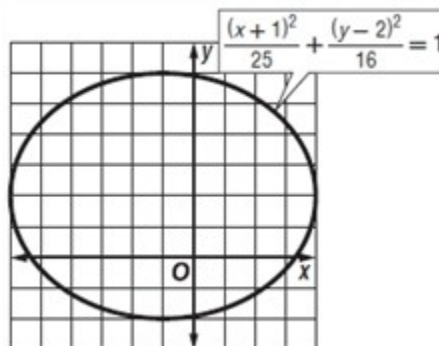
length of the major axis: 10 units

length of the minor axis: 8 units



ANSWER:

$(-1, 2); (-4, 2) \text{ and } (2, 2); 10; 8$



42. $9x^2 + 4y^2 + 54x - 8y + 49 = 0$

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SOLUTION:

$$9x^2 + 4y^2 + 54x - 8y + 49 = 0$$

$$9x^2 + 4y^2 + 54x - 8y = -49$$

$$9(x^2 + 6x) + 4(y^2 - 2y) = -49$$

$$9(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = -49 + 81 + 4$$

$$9(x+3)^2 + 4(y-1)^2 = 36$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{9} = 1$$

$$\frac{(y-1)^2}{3^2} + \frac{(x+3)^2}{2^2} = 1$$

Here $a = 3$ and $b = 2$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{9 - 4}$$

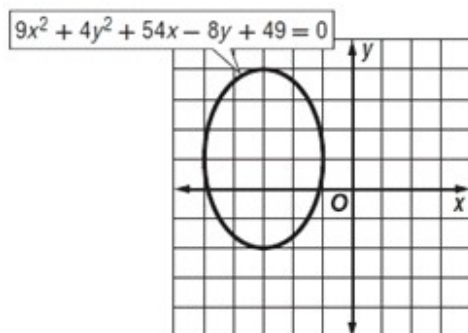
$$= \sqrt{5}$$

center: $(-3, 1)$

foci: $(-3, 1 \pm \sqrt{5})$

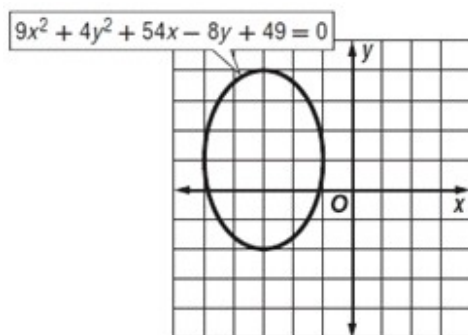
length of the major axis: 6 units

length of the minor axis: 4 units



ANSWER:

$(-3, 1); (-3, 1 \pm \sqrt{5}); 6; 4$



43. $9x^2 + 25y^2 - 18x + 50y - 191 = 0$

SOLUTION:

$$9x^2 + 25y^2 - 18x + 50y - 191 = 0$$

$$9(x^2 - 2x) + 25(y^2 + 2y) = 191$$

$$9(x^2 - 2x + 1) + 25(y^2 + 2y + 1) = 191 + 25 + 9$$

$$9(x-1)^2 + 25(y+1)^2 = 225$$

$$\frac{(x-1)^2}{25} + \frac{(y+1)^2}{9} = 1$$

$$\frac{(x-1)^2}{5^2} + \frac{(y+1)^2}{3^2} = 1$$

Here $a = 5$ and $b = 3$.

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{25 - 9}$$

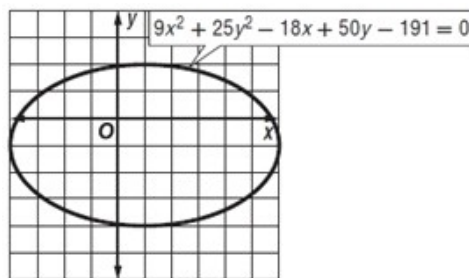
$$= 4$$

center: $(1, -1)$

foci: $(-3, -1)$ and $(5, -1)$

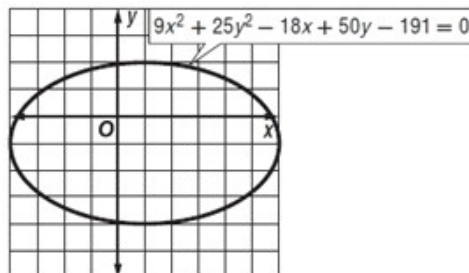
length of the major axis: 10 units

length of the minor axis: 6 units



ANSWER:

$(1, -1); (-3, -1) \text{ and } (5, -1); 10; 6$



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44. $7x^2 + 3y^2 - 28x - 12y = -19$

SOLUTION:

$$7x^2 + 3y^2 - 28x - 12y = -19$$

$$7(x^2 - 4x) + 3(y^2 - 4y) = -19$$

$$7(x^2 - 4x + 4) + 3(y^2 - 4y + 4) = -19 + 28 + 12$$

$$7(x - 2)^2 + 3(y - 2)^2 = 21$$

$$\frac{(x - 2)^2}{3} + \frac{(y - 2)^2}{7} = 1$$

$$\frac{(y - 2)^2}{\sqrt{7}^2} + \frac{(x - 2)^2}{\sqrt{3}^2} = 1$$

The values of h , k , a and b are 2, 2, $\sqrt{7}$ and $\sqrt{3}$.

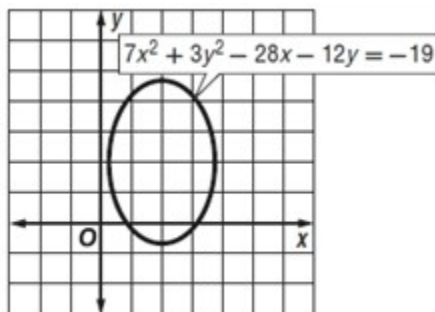
$$\begin{aligned} c &= \sqrt{a^2 - b^2} \\ &= \sqrt{7 - 3} \\ &= 2 \end{aligned}$$

center: $(2, -2)$

foci: $(2, -4)$ and $(2, 0)$

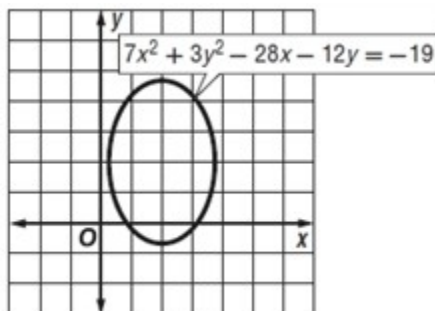
length of the major axis: $2\sqrt{7}$ units

length of the minor axis: $2\sqrt{3}$ units



ANSWER:

$$(2, 2); (2, 4); (2, 0); 2\sqrt{7}; 2\sqrt{3}$$



45. **LANDSCAPING** The Martins have a garden in their front yard that is shaped like an ellipse. The major axis is 16 feet and the minor axis is 10 feet. Write an equation to model the garden. Assume the origin is at the center of the garden.

SOLUTION:

The length of the major axis is 16 ft.

Therefore, $a = 8$.

The length of the minor axis is 10 ft.

Therefore, $b = 5$.

Substitute 8 and 5 for a and b respectively in the standard form of the ellipse equation.

$$\frac{x^2}{8^2} + \frac{y^2}{5^2} = 1$$

$$\frac{x^2}{64} + \frac{y^2}{25} = 1$$

ANSWER:

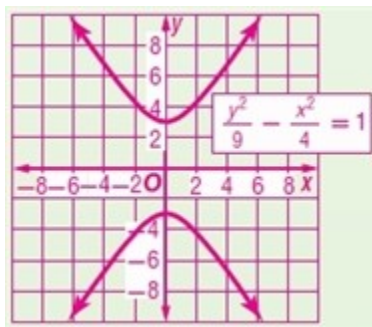
$$\frac{x^2}{64} + \frac{y^2}{25} = 1$$

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Graph each hyperbola. Identify the vertices, foci, and asymptotes.

46. $\frac{y^2}{9} - \frac{x^2}{4} = 1$

SOLUTION:



$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

$$\frac{y^2}{3^2} - \frac{x^2}{2^2} = 1$$

The values of h , k , a and b are 0, 0, 3 and 2.

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{9 + 4}$$

$$= \sqrt{13}$$

The vertices of the hyperbola are $(0, \pm 3)$.

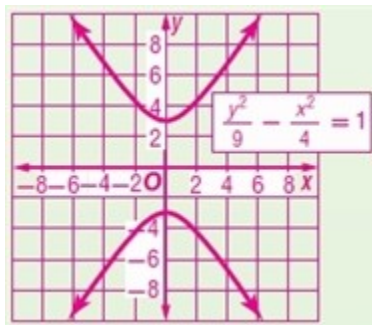
The foci are $(0, \pm \sqrt{13})$.

The equations of the asymptotes are

$$y = \pm \frac{a}{b}x = \pm \frac{3}{2}x.$$

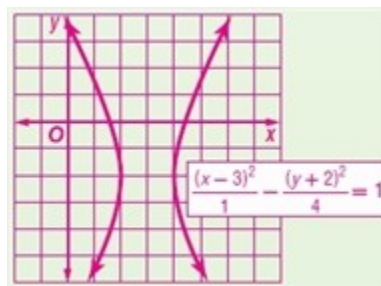
ANSWER:

$$(0, \pm 3); (0, \pm \sqrt{13}); y = \pm \frac{3}{2}x$$



47. $\frac{(x-3)^2}{1} - \frac{(y+2)^2}{4} = 1$

SOLUTION:



$$\frac{(x-3)^2}{1} - \frac{(y+2)^2}{4} = 1$$

$$\frac{(x-3)^2}{1^2} - \frac{(y+2)^2}{2^2} = 1$$

The values of h , k , a and b are 3, -2, 1 and 2.

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{4 + 1}$$

$$= \sqrt{5}$$

The vertices of the hyperbola are $(2, -2)$ and $(4, -2)$.

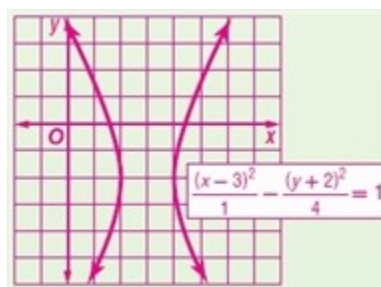
The foci are $(3 \pm \sqrt{5}, -2)$.

The equations of the asymptotes are

$$y + 2 = \pm 2(x - 3).$$

ANSWER:

$$(2, -2); (4, -2); (3 \pm \sqrt{5}, -2); y + 2 = \pm 2(x - 3)$$



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48. $\frac{(y+1)^2}{16} - \frac{(x-4)^2}{9} = 1$

SOLUTION:

$$\begin{aligned}\frac{(y+1)^2}{16} - \frac{(x-4)^2}{9} &= 1 \\ \frac{(y+1)^2}{4^2} - \frac{(x-4)^2}{3^2} &= 1\end{aligned}$$

The values of h , k , a and b are 4, -1 , 4 and 3.

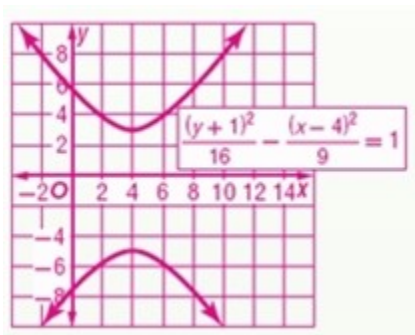
$$\begin{aligned}c &= \sqrt{a^2 + b^2} \\ &= \sqrt{16 + 9} \\ &= 5\end{aligned}$$

The vertices of the hyperbola are (4, -5) and (4, 3).

The foci are (4, -6) and (4, 4).

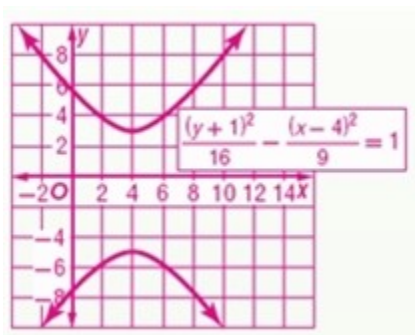
The equations of the asymptotes are

$$y + 1 = \pm \frac{4}{3}(x - 4).$$



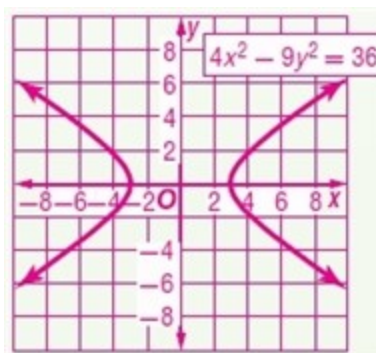
ANSWER:

$$(4, -5); (4, 3); (4, -6); (4, 4); y + 1 = \pm \frac{4}{3}(x - 4)$$



49. $4x^2 - 9y^2 = 36$

SOLUTION:



$$\begin{aligned}4x^2 - 9y^2 &= 36 \\ \frac{x^2}{9} - \frac{y^2}{4} &= 1\end{aligned}$$

The values of h , k , a and b are 0, 0, 3 and 2.

$$\begin{aligned}c &= \sqrt{a^2 + b^2} \\ &= \sqrt{9 + 4} \\ &= \sqrt{13}\end{aligned}$$

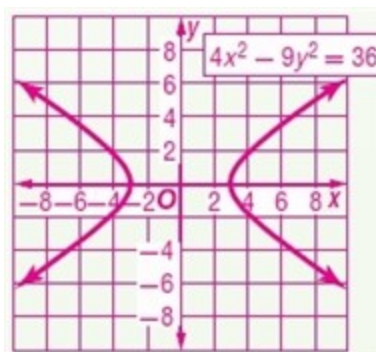
The vertices of the hyperbola are (± 3 , 0).

The foci are ($\pm\sqrt{13}$, 0).

The equations of the asymptotes are $y = \pm \frac{2}{3}x$.

ANSWER:

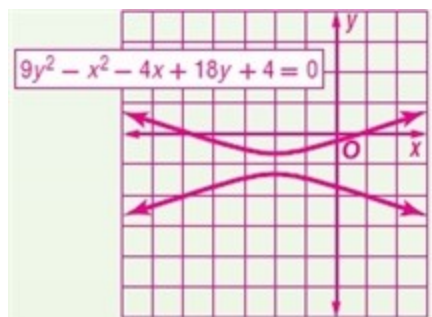
$$(\pm 3, 0); (\pm\sqrt{13}, 0); y = \pm \frac{2}{3}x$$



50. $9y^2 - x^2 - 4x + 18y + 4 = 0$

SOLUTION:

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$$9y^2 - x^2 - 4x + 18y + 4 = 0$$

$$9(y^2 + 2y) - (x^2 + 4x) = -4$$

$$9(y^2 + 2y + 1) - (x^2 + 4x + 4) = -4 + 9 - 4$$

$$9(y+1)^2 - (x+2)^2 = 1$$

$$\frac{(y+1)^2}{\frac{1}{9}} - \frac{(x+2)^2}{1} = 1$$

The values of h , k , a and b are -2 , -1 , $\frac{1}{3}$ and 1 .

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{\frac{1}{9} + 1} \\ &= \frac{\sqrt{10}}{3} \end{aligned}$$

The vertices of the hyperbola are $\left(-2, -1 \pm \frac{1}{3}\right)$.

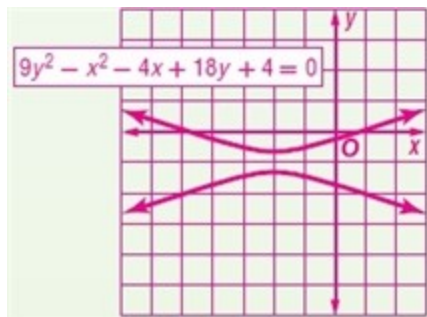
The foci are $\left(-2, -1 \pm \frac{\sqrt{10}}{3}\right)$.

The equations of the asymptotes are

$$y + 1 = \pm \frac{2}{3}(x + 2).$$

ANSWER:

$$\left(-2, -\frac{2}{3}\right)\left(-2, -\frac{4}{3}\right); \left(-2, -1 \pm \frac{\sqrt{10}}{3}\right); y + 1 = \pm \frac{1}{3}(x + 2)$$



51. **MIRRORS** A hyperbolic mirror is shaped like one branch of a hyperbola. It reflects light rays directed at one focus toward the other focus. Suppose a hyperbolic mirror is modeled by the upper branch of the hyperbola $\frac{y^2}{9} - \frac{x^2}{16} = 1$. A light source is located at $(-10, 0)$. Where should the light hit the mirror so that the light will be reflected to $(0, -5)$?

SOLUTION:

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

Foci: $(0, 5)$ and $(0, -5)$

Find the equation of the line joining the points $(0, 5)$ and $(-10, 0)$.

$$\begin{aligned} \frac{y-5}{0-5} &= \frac{x-0}{-10-0} \\ \frac{y-5}{-5} &= \frac{x}{-10} \\ y-5 &= \frac{x}{2} \\ y &= \frac{x}{2} + 5 \\ y &= \frac{x+10}{2} \end{aligned}$$

Find the intersection of the line

$$y = \frac{x+10}{2} \text{ and } \frac{y^2}{9} - \frac{x^2}{16} = 1.$$

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$$\begin{aligned}
 \frac{(x+10)^2}{36} - \frac{x^2}{16} &= 1 \\
 \frac{4(x+10)^2 - 9x^2}{144} &= 1 \\
 4(x+10)^2 - 9x^2 &= 144 \\
 4(x^2 + 100 + 20x) - 9x^2 &= 144 \\
 4x^2 + 400 + 80x - 9x^2 - 144 &= 0 \\
 -5x^2 + 80x + 256 &= 0 \\
 5x^2 - 80x - 256 &= 0 \\
 x &= \frac{80 \pm \sqrt{6400 - 4(5)(-256)}}{10} \\
 &= \frac{80 \pm \sqrt{6400 + 5120}}{10} \\
 &= \frac{80 \pm \sqrt{11520}}{10} \\
 &= \frac{80 \pm \sqrt{2304 \cdot 5}}{10} \\
 &= \frac{80 \pm 48\sqrt{5}}{10} \\
 &= \frac{40 \pm 24\sqrt{5}}{5}
 \end{aligned}$$

Since the mirror is shaped like one branch of hyperbola and the orientation of the hyperbola is

vertical, the value of x is $\frac{40 - 24\sqrt{5}}{5}$.

Substitute $\frac{40 - 24\sqrt{5}}{5}$ for x in the equation

$$y = \frac{x+10}{2}.$$

$$\begin{aligned}
 y &= \frac{\left(\frac{40 - 24\sqrt{5}}{5}\right) + 10}{2} \\
 &= \frac{40 - 24\sqrt{5} + 50}{10} \\
 &= \frac{90 - 24\sqrt{5}}{10} \\
 &= \frac{45 - 12\sqrt{5}}{5}
 \end{aligned}$$

The light should hit the mirror at

$$\left(\frac{40 - 24\sqrt{5}}{5}, \frac{45 - 12\sqrt{5}}{5}\right).$$

ANSWER:

$$\left(\frac{40 - 24\sqrt{5}}{5}, \frac{45 - 12\sqrt{5}}{5}\right)$$

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph.

52. $3x^2 + 12x - y + 8 = 0$

SOLUTION:

$$3x^2 + 12x - y + 8 = 0$$

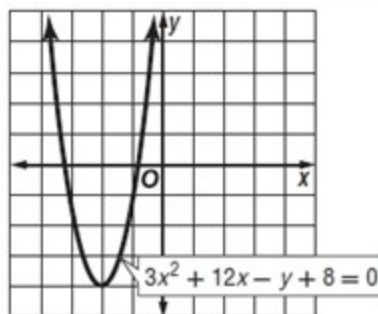
$$3(x^2 + 4x) - y = -8$$

$$3(x^2 + 4x + 4) - y = -8 + 12$$

$$3(x + 2)^2 - y = 4$$

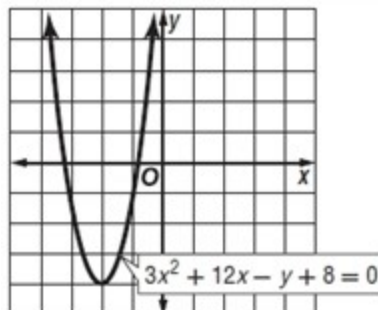
$$3(x + 2)^2 - 4 = y$$

Given equation is a parabola.
Graph the function.



ANSWER:

$$y = 3(x + 2)^2 - 4; \text{ parabola}$$



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53. $9x^2 + 16y^2 = 144$

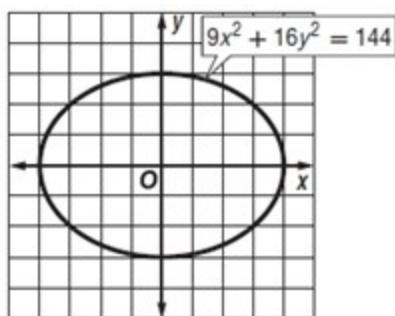
SOLUTION:

$$9x^2 + 16y^2 = 144$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

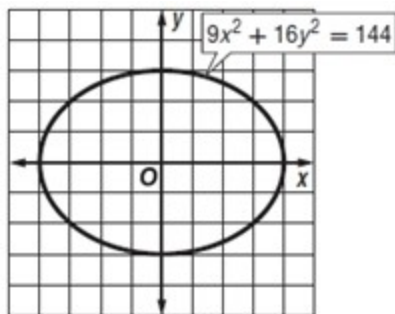
$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$

Given equation is an ellipse.
Graph the function.



ANSWER:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 ; \text{ellipse}$$



54. $x^2 + y^2 - 8x - 2y + 8 = 0$

SOLUTION:

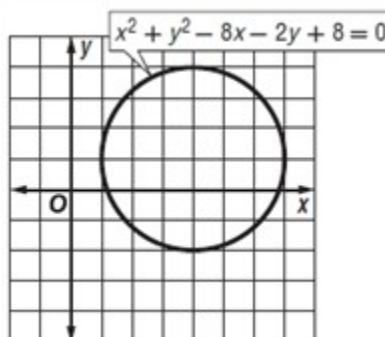
$$x^2 + y^2 - 8x - 2y + 8 = 0$$

$$x^2 - 8x + y^2 - 2y = -8$$

$$x^2 - 8x + 16 + y^2 - 2y + 1 = -8 + 16 + 1$$

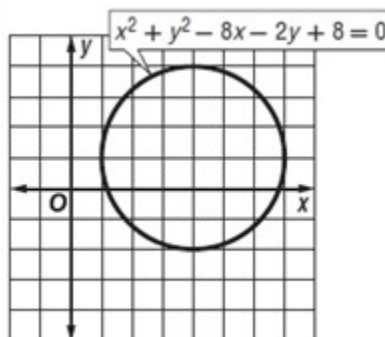
$$(x-4)^2 + (y-1)^2 = 9$$

Given equation is a circle.
Graph the function.



ANSWER:

$$(x-4)^2 + (y-1)^2 = 9 ; \text{circle}$$



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55. $-9x^2 + y^2 + 36x - 45 = 0$

SOLUTION:

$$-9x^2 + y^2 + 36x - 45 = 0$$

$$y^2 - 9x^2 + 36x = 45$$

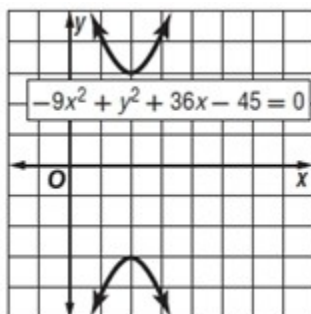
$$y^2 - 9(x^2 - 4x) = 45$$

$$y^2 - 9(x^2 - 4x + 4) = 45 - 36$$

$$y^2 - 9(x - 2)^2 = 9$$

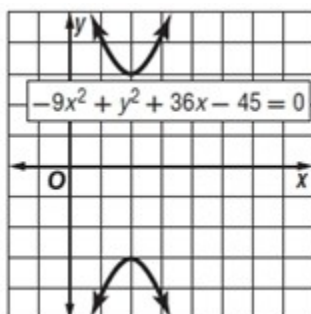
$$\frac{y^2}{9} - \frac{(x - 2)^2}{1} = 1$$

Given equation is an ellipse.
Graph the function.



ANSWER:

$$\frac{y^2}{9} - \frac{(x - 2)^2}{1} = 1 ; \text{hyperbola}$$



Without writing the equation in standard form, state whether the graph of the equation is a parabola, circle, ellipse, or hyperbola.

56. $7x^2 + 9y^2 = 63$

SOLUTION:

$$7x^2 + 9y^2 = 63$$

$$7x^2 + 9y^2 - 63 = 0$$

$$A = 7, B = 0, \text{ and } C = 9$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(7)(9) = -252.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.

ANSWER:

ellipse

57. $5y^2 + 2y + 4x - 13x^2 = 81$

SOLUTION:

$$5y^2 + 2y + 4x - 13x^2 = 81$$

$$5y^2 + 2y + 4x - 13x^2 - 81 = 0$$

$$A = -13, B = 0, \text{ and } C = 5$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(-13)(5) = 260.$$

Because the discriminant is greater than 0, the conic is a hyperbola.

ANSWER:

hyperbola

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58. $x^2 - 8x + 16 = 6y$

SOLUTION:

$$x^2 - 8x + 16 = 6y$$

$$x^2 - 8x + 16 - 6y = 0$$

$$A = 1, B = 0, \text{ and } C = 0$$

The discriminant is $B^2 - 4AC = (0^2) - 4(1)(0) = 0$.

Because the discriminant is equal to 0, the conic is a parabola.

ANSWER:

parabola

59. $x^2 + 4x + y^2 - 285 = 0$

SOLUTION:

$$x^2 + 4x + y^2 - 285 = 0$$

$$A = 1, B = 0, \text{ and } C = 1$$

The discriminant is $B^2 - 4AC = (0^2) - 4(1)(1) = -4$.

Because the discriminant is less than 0, $B = 0$ and $A = C$, the conic is a circle.

ANSWER:

circle

60. **LIGHT** Suppose the edge of a shadow can be represented by the equation

$$16x^2 + 25y^2 - 32x - 100y - 284 = 0.$$

a. What is the shape of the shadow?

b. Graph the equation.

SOLUTION:

a. $16x^2 + 25y^2 - 32x - 100y - 284 = 0$

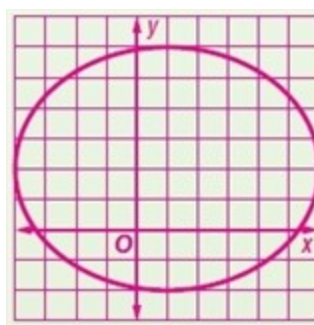
$$A = 16, B = 0, \text{ and } C = 25$$

The discriminant is

$$B^2 - 4AC = (0^2) - 4(16)(25) = -1600.$$

Because the discriminant is less than 0 and $A \neq C$, the conic is an ellipse.

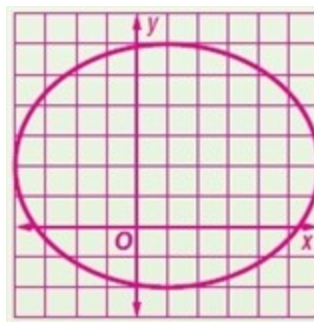
b. Graph the equation.



ANSWER:

a. ellipse

b.



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Solve each system of equations.

61.
$$\begin{aligned}x^2 + y^2 &= 8 \\x + y &= 0\end{aligned}$$

SOLUTION:

Solve the linear equation for y .

$$\begin{aligned}x + y &= 0 \\y &= -x\end{aligned}$$

Substitute $-x$ for y in the quadratic equation and solve for x .

$$\begin{aligned}x^2 + (-x)^2 &= 8 \\2x^2 &= 8 \\x^2 &= 4 \\x &= \pm 2\end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{array}{lll}x + y = 0 & \text{and} & x + y = 0 \\2 + y = 0 & \text{and} & (-2) + y = 0 \\y = -2 & \text{and} & y = 2\end{array}$$

The solutions are $(2, -2)$ and $(-2, 2)$.

ANSWER:

$(2, -2), (-2, 2)$

62.
$$\begin{aligned}x - 2y &= 2 \\y^2 - x^2 &= 2x + 4\end{aligned}$$

SOLUTION:

Solve the linear equation for x .

$$\begin{aligned}x - 2y &= 2 \\x &= 2 + 2y\end{aligned}$$

Substitute $2 + 2y$ for x in the quadratic equation and solve for y .

$$\begin{aligned}y^2 - (2 + 2y)^2 &= 2(2 + 2y) + 4 \\-3y^2 - 4 - 8y &= 4 + 4y + 4 \\y^2 + 4y + 4 &= 0 \\(y + 2)^2 &= 0 \\y &= -2\end{aligned}$$

Substitute the values of y in the linear equation and find the value of x .

$$\begin{aligned}x - 2y &= 2 \\x - 2(-2) &= 2 \\x &= -2\end{aligned}$$

The solution is $(-2, -2)$.

ANSWER:

$(-2, -2)$

Study Guide and Review - Chapter 9

63. $y + x^2 = 4x$
 $y + 4x = 16$

SOLUTION:

Solve the linear equation for y .

$$y = 16 - 4x$$

Substitute $16 - 4x$ for y in the quadratic equation and solve for x .

$$\begin{aligned} 16 - 4x + x^2 &= 4x \\ x^2 - 8x + 16 &= 0 \\ (x - 4)^2 &= 0 \\ x &= 4 \end{aligned}$$

Substitute the values of x in the linear equation and find the value of y .

$$\begin{aligned} y + 4x &= 16 \\ y + 4(4) &= 16 \\ y &= 0 \end{aligned}$$

The solution is $(4, 0)$.

ANSWER:

$(4, 0)$

64. $3x^2 - y^2 = 11$
 $x^2 + 4y^2 = 8$

SOLUTION:

$$\begin{aligned} 3x^2 - y^2 &= 11 & \rightarrow (1) \\ x^2 + 4y^2 &= 8 & \rightarrow (2) \end{aligned}$$

Multiply the first by 4.

$$(1) \times 4 \quad 12x^2 - 4y^2 = 44 \rightarrow (3)$$

$$\begin{aligned} (3) + (2) \quad 13x^2 &= 52 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Substitute the values of x in the first equation and find the values of y .

$$\begin{array}{lll} 3x^2 - y^2 = 11 & \text{and} & 3x^2 - y^2 = 11 \\ 3(2)^2 - y^2 = 11 & \text{and} & 3(-2)^2 - y^2 = 11 \\ y^2 = 1 & \text{and} & y^2 = 1 \\ y = \pm 1 & \text{and} & y = \pm 1 \end{array}$$

The solutions are $(-2, -1)$, $(-2, 1)$, $(2, -1)$ and $(2, 1)$.

ANSWER:

$(-2, -1)$, $(-2, 1)$, $(2, -1)$, $(2, 1)$

Study Guide and Review - Chapter 9

65.
$$\begin{aligned} 5x^2 + y^2 &= 30 \\ 9x^2 - y^2 &= -16 \end{aligned}$$

SOLUTION:

$$5x^2 + y^2 = 30 \rightarrow (1)$$

$$9x^2 - y^2 = -16 \rightarrow (2)$$

$$(1) + (2) \quad 14x^2 = 14$$

$$x^2 = 1$$

$$x = \pm 1$$

Substitute the values of x in the first equation and find the values of y .

$5x^2 + y^2 = 30$	and	$5x^2 + y^2 = 30$
$5(1)^2 + y^2 = 30$	and	$5(-1)^2 + y^2 = 30$
$y^2 = 25$	and	$y^2 = 25$
$y = \pm 5$	and	$y = \pm 5$

The solutions are $(1, \pm 5)$, $(-1, \pm 5)$.

ANSWER:

$$(1, \pm 5), (-1, \pm 5)$$

66.
$$\begin{aligned} \frac{x^2}{30} + \frac{y^2}{6} &= 1 \\ x &= y \end{aligned}$$

SOLUTION:

Substitute x for y in the first equation and solve for y .

$$\begin{aligned} \frac{x^2}{30} + \frac{x^2}{6} &= 1 \\ 6x^2 &= 30 \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \end{aligned}$$

Substitute the values of x in the linear equation and find the values of y .

$$\begin{aligned} x &= y \quad \text{and} \quad x = y \\ \sqrt{5} &= y \quad \text{and} \quad -\sqrt{5} = y \end{aligned}$$

The solutions are $(\sqrt{5}, \sqrt{5})$ and $(-\sqrt{5}, -\sqrt{5})$.

ANSWER:

$$(\sqrt{5}, \sqrt{5}), (-\sqrt{5}, -\sqrt{5})$$

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67. **PHYSICAL SCIENCE** Two balls are launched into the air at the same time. The heights they are launched from are different. The height y in feet of one is represented by $y = -16t^2 + 80t + 25$ where t is the time in seconds. The height of the other ball is represented by $y = -16t^2 + 30t + 100$.
- After how many seconds are the balls at the same height?
 - What is this height?

SOLUTION:

- Equate the both the equations.

$$\begin{aligned} -16t^2 + 80t + 25 &= -16t^2 + 30t + 100 \\ 50t &= 75 \\ t &= 1.5 \end{aligned}$$

- Substitute 1.5 for t in any one of the equation then evaluate.

$$\begin{aligned} y &= -16t^2 + 80t + 25 \\ y &= -16(1.5)^2 + 80(1.5) + 25 \\ &= -36 + 120 + 25 \\ &= 109 \end{aligned}$$

The height is 109 ft.

ANSWER:

- 1.5 seconds
 - 109 ft
68. **ARCHITECTURE** An architect is building the front entrance of a building in the shape of a parabola with the equation $y = -\frac{1}{10}(x - 10)^2 + 20$. While the entrance is being built, the construction team puts in two support beams with equations $y = -x + 10$ and $y = x - 10$. Where do the support beams meet the parabola?

SOLUTION:

Substitute $-x + 10$ for y in the quadratic equation and solve for x .

$$\begin{aligned} -x + 10 &= -\frac{1}{10}(x - 10)^2 + 20 \\ -x - 10 &= -\frac{1}{10}(x - 10)^2 \\ -10x - 100 &= -x^2 + 20x - 100 \\ x^2 - 30x &= 0 \\ x(x - 30) &= 0 \\ x = 0 &\text{ or } x = 30 \end{aligned}$$

Substitute the value of x in the equation $y = -x + 10$ and find the value of y .

$$\begin{array}{ll} y = -x + 10 & \text{or} & y = -x + 10 \\ y = 0 + 10 & \text{or} & y = -30 + 10 \\ y = 10 & \text{or} & y = -20 \end{array}$$

The value of x and y should be positive. Therefore, the intersection point is $(0, 10)$.

Substitute $x - 10$ for y in the quadratic equation and solve for x .

$$\begin{aligned} x - 10 &= -\frac{1}{10}(x - 10)^2 + 20 \\ x - 30 &= -\frac{1}{10}(x - 10)^2 \\ 10x - 300 &= -x^2 + 20x - 100 \\ x^2 - 10x - 200 &= 0 \\ (x + 10)(x - 20) &= 0 \\ x &= -10 \text{ or } 20 \end{aligned}$$

Substitute the value of x in the equation $y = x - 10$ and find the value of y .

$$\begin{array}{ll} y = x - 10 & \text{or} & y = x - 10 \\ y = -10 - 10 & \text{or} & y = 20 - 10 \\ y = -20 & \text{or} & y = 10 \end{array}$$

The value of x and y should be positive. Therefore, the intersection point be $(20, 10)$. Therefore, the support beams meet the parabola at $(0, 10)$ and $(20, 10)$.

ANSWER:

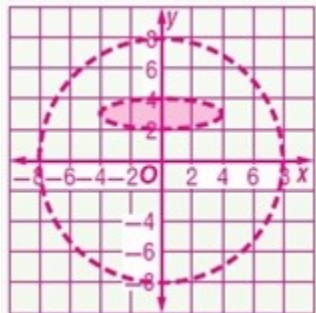
$(0, 10)$ and $(20, 10)$

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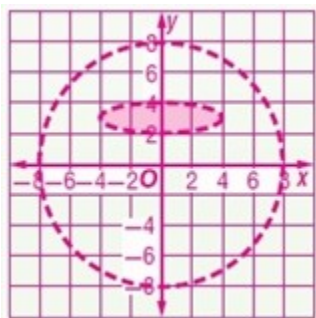
Solve each system of inequalities by graphing.

69. $x^2 + y^2 < 64$
 $x^2 + 16(y - 3)^2 < 16$

SOLUTION:

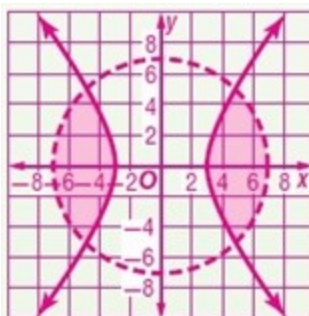


ANSWER:

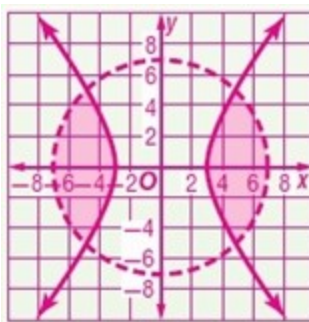


70. $x^2 + y^2 < 49$
 $16x^2 - 9y^2 \geq 144$

SOLUTION:



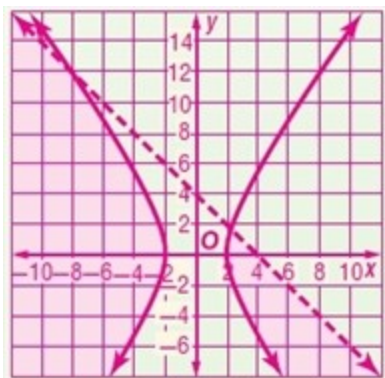
ANSWER:



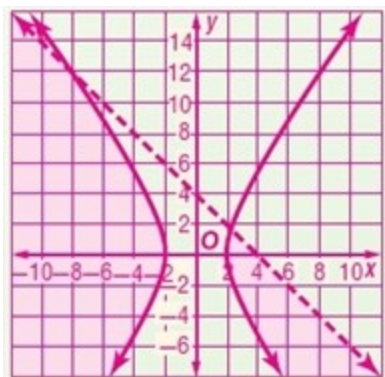
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71. $x + y < 4$
 $9x^2 - 4y^2 \geq 36$

SOLUTION:

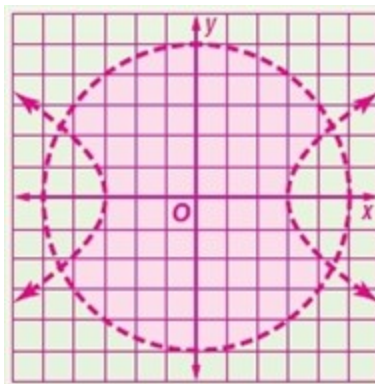


ANSWER:

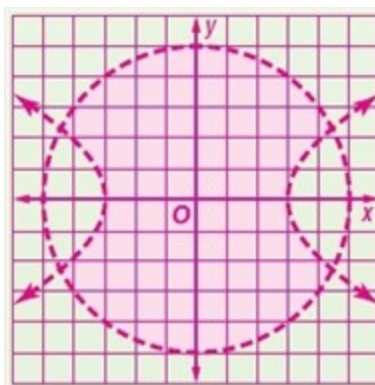


72. $x^2 + y^2 < 25$
 $4x^2 - 9y^2 < 36$

SOLUTION:



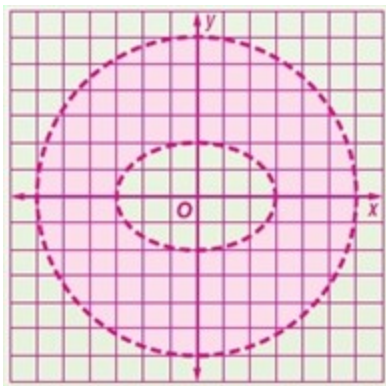
ANSWER:



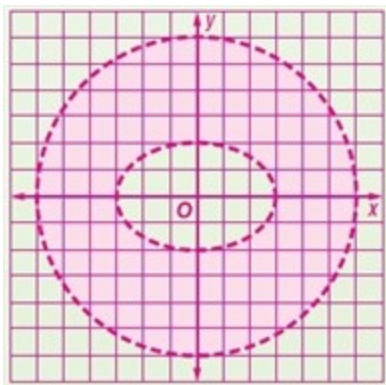
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73. $x^2 + y^2 < 36$
 $4x^2 + 9y^2 > 36$

SOLUTION:

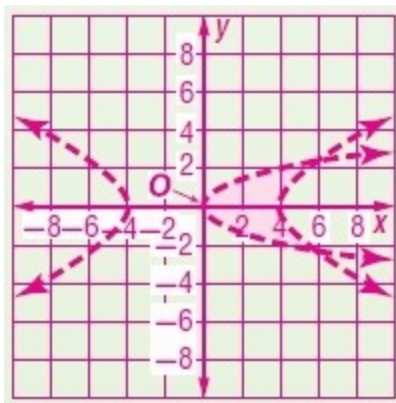


ANSWER:



74. $y^2 < x$
 $x^2 - 4y^2 < 16$

SOLUTION:



ANSWER:

