6 Direct Current Circuits

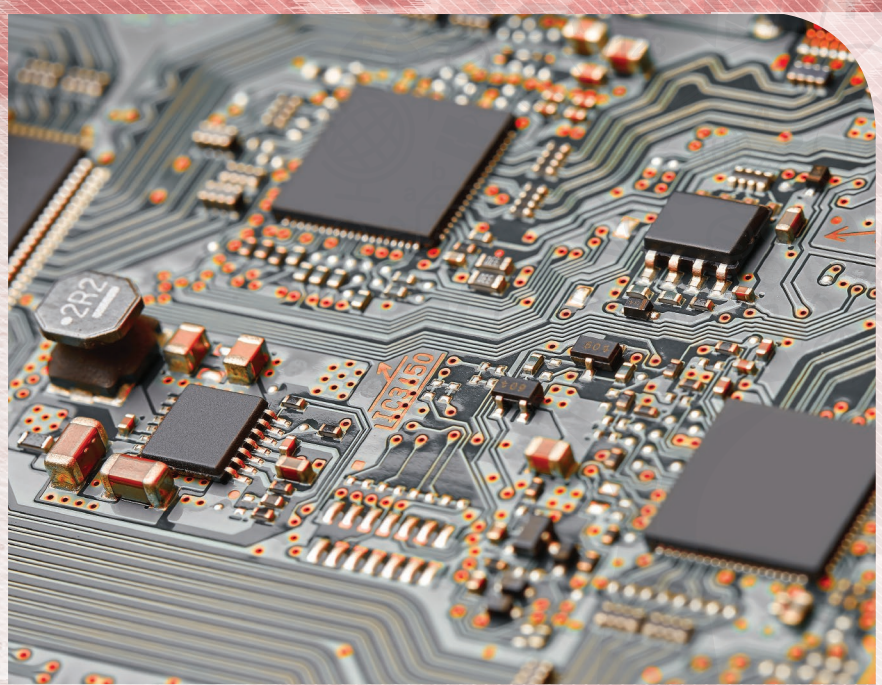


FIGURE 6.1 A circuit board can have hundreds of circuit components connected by metallic conducting paths.

The electric circuit, such as the one shown in Figure 6.1, undoubtedly changed the world. Modern electronics continues to change human society, at a faster and faster pace. It took 38 years for radio to reach 50 million users in the United States. However, it took only 13 years for television to reach that number of users, 10 years for cable TV, 5 years for the Internet, and 3 years for cell phones.

This chapter examines the techniques used to analyze circuits that cannot be broken down into simple series and parallel connections. Modern electronics design depends on millions of different circuits, each with its own purpose and configuration. However, regardless of how complicated a circuit becomes, the basic rules for analyzing it are the ones presented in this chapter.

Some of the circuits analyzed in this chapter contain not only resistors and emf devices but also capacitors. In these circuits, the current is not steady, but changes with time. Time-varying currents will be covered more thoroughly in later chapters, which introduce additional circuit components.

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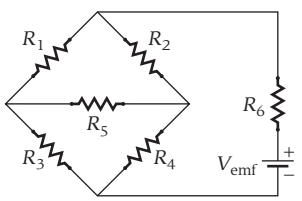
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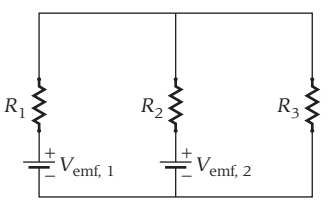
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WHAT WE WILL LEARN

- Some circuits cannot be reduced to a single loop; complex circuits can be analyzed using Kirchhoff's rules.
- Kirchhoff's Junction Rule states that the algebraic sum of the currents at any junction in a circuit must be zero.
- Kirchhoff's Loop Rule states that the algebraic sum of the potential changes around any closed loop in a circuit must be zero.
- Single-loop circuits can be analyzed using Kirchhoff's Loop Rule.
- Multiloop circuits must be analyzed using both Kirchhoff's Junction Rule and Kirchhoff's Loop Rule.
- The current in a circuit that contains a resistor and a capacitor varies exponentially with time, with a characteristic time constant given by the product of the resistance and the capacitance.



(a)



(b)

FIGURE 6.2 Two examples of circuits that cannot be reduced to simple combinations of parallel and series resistors.

6.1 Kirchhoff's Rules

In Chapter 5, we considered several kinds of direct current (DC) circuits, each containing one emf device along with resistors connected in series or in parallel. Some seemingly complicated circuits contain multiple resistors in series or in parallel that can be replaced with an equivalent resistance. However, we did not consider circuits containing multiple sources of emf. In addition, there are single-loop and multiloop circuits with emf devices and resistors that cannot be reduced to simple circuits containing parallel or series connections. Figure 6.2 shows two examples of such circuits. This chapter explains how to analyze these kinds of circuits using **Kirchhoff's rules.**

Kirchhoff's Junction Rule

A **junction** is a place in a circuit where three or more wires are connected to each other. Each connection between two junctions in a circuit is called a **branch.** A branch can contain any number of different circuit elements and the wires between them. Each branch can have a current flowing, and this current is the same everywhere in the branch. This fact leads to **Kirchhoff's Junction Rule:**

> The sum of the currents entering a junction must equal the sum of the currents leaving the junction.

With a positive sign assigned (arbitrarily) to currents entering the junction and a negative sign to those exiting the junction, Kirchhoff's Junction Rule is expressed mathematically as

$$\text{Junction:} \quad \sum_{k=1}^{n} i_k = 0. \tag{6.1}$$

How do you know which currents enter a junction and which exit the junction when you make a drawing like the one shown in Figure 6.3? You don't; you simply assign a direction for each current along a given wire. If an assigned direction turns out to be wrong, you will obtain a negative number for that particular current in your final solution.

Kirchhoff's Junction Rule is a direct consequence of the conservation of electric charge. Junctions do not have the capability of storing charge. Thus, charge conservation requires that all charges streaming into a junction also leave the junction, which is exactly what Kirchhoff's Junction Rule states.

According to Kirchhoff's Junction Rule, at each junction in a multiloop circuit, the current flowing into the junction must equal the current flowing out of the junction. For example, Figure 6.3 shows a single junction, *a*, with a current, i_1, entering the junction and two currents, i_2 and i_3, leaving the junction. According to Kirchhoff's Junction Rule, in this case,

$$\sum_{k=1}^{3} i_k = i_1 - i_2 - i_3 = 0 \Rightarrow i_1 = i_2 + i_3.$$

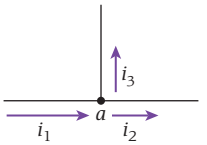


FIGURE 6.3 A single junction from a multiloop circuit.

Kirchhoff's Loop Rule

A **loop** in a circuit is any set of connected wires and circuit elements forming a closed path. If you follow a loop, eventually you will get to the same point from which you started. For example, in the circuit diagram shown in Figure 6.2b, three possible loops can be identified. These three loops are shown in different colors (red, green, and blue) in Figure 6.4. The blue loop includes resistors 1 and 2, emf sources 1 and 2, and their connecting wires. The red loop includes resistors 2 and 3, emf source 2, and their connecting wires. Finally, the green loop includes resistors 1 and 3, emf source 1, and their connecting wires. Note that any given wire or circuit element can be and usually is part of more than one loop.

You can move through any loop in a circuit in either a clockwise or a counterclockwise direction. Figure 6.4 shows a clockwise path through each of the loops, as indicated by the arrows. But the direction of the path taken around the loop is irrelevant as long as your choice is followed consistently all the way around the loop.

Summing the potential differences from all circuit elements encountered along any given loop yields the total potential difference of the complete path along the loop. **Kirchhoff's Loop Rule** then states:

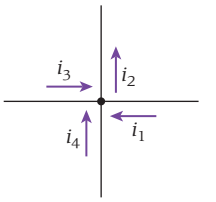
> The potential difference around a complete circuit loop must sum to zero.

Kirchhoff's Loop Rule is a direct consequence of the fact that electric potential is single-valued. This means that the electric potential energy of a conduction electron at a point in the circuit has one specific value. Suppose this rule were not valid. Then we could analyze the potential changes of a conduction electron in going around a loop and find that the electron had a different potential energy when it returned to its starting point. The potential energy of this electron would change at a point in the circuit, in obvious contradiction of energy conservation. In other words, Kirchhoff's Loop Rule is simply a consequence of the law of conservation of energy.

Application of Kirchhoff's Loop Rule requires conventions for determining the potential drop across each element of the circuit. This depends on the assumed direction of the current and the direction of the analysis. For emf sources, the rules are straightforward, since minus and plus signs (as well as short and long lines) indicate which side of the emf source is at the higher potential. The potential drop for an emf source is in the direction from minus to plus or from short line to long line. As noted earlier, the assignment of the current directions and the choice of a clockwise or counterclockwise path around a loop are arbitrary. Any direction will give the same information, as long as it is applied consistently around a loop. The conventions used to analyze circuit elements in a loop are summarized in Table 6.1 and Figure 6.5, where the magnitude of the current through the circuit element is *i*. (The labels in the rightmost column of Table 6.1 correspond to the parts of Figure 6.5.)

Concept Check 6.1

For the junction shown in the figure, which equation correctly expresses the sum of the currents?



a) $i_1 + i_2 + i_3 + i_4 = 0$

b) $i_1 - i_2 + i_3 + i_4 = 0$

c) $-i_1 + i_2 + i_3 - i_4 = 0$

d) $i_1 - i_2 - i_3 - i_4 = 0$

e) $i_1 + i_2 - i_3 - i_4 = 0$

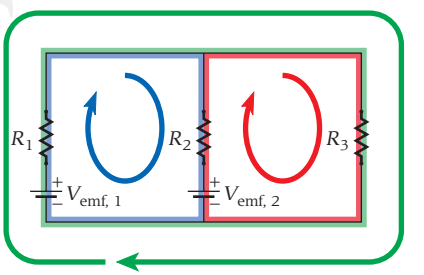


FIGURE 6.4 The three possible loops (indicated in red, green, and blue) for the circuit diagram shown in Figure 6.2b.

Table 6.1 Conventions Used to Determine the Sign of Potential Changes Around a Single-Loop Circuit Containing Several Resistors and Sources of emf

Element	Direction of Analysis	Potential Change	
R	Same as current	$-iR$	(a)
R	Opposite to current	$+iR$	(b)
V_{emf}	Same as emf	$+V_{emf}$	(c)
V_{emf}	Opposite to emf	$-V_{emf}$	(d)

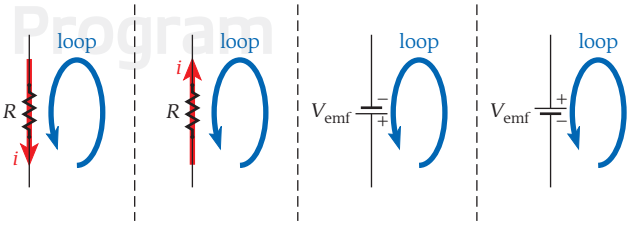


FIGURE 6.5 Sign convention for potential changes in analyzing loops.

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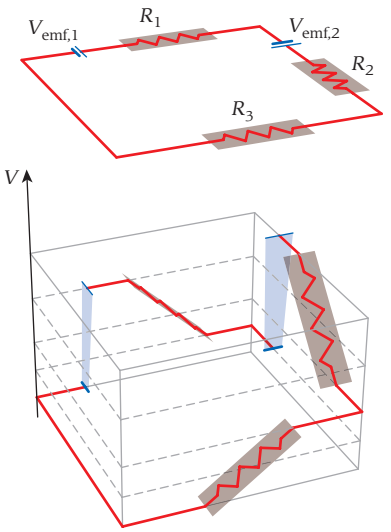


FIGURE 6.6 Loop with multiple sources of emf and multiple resistors.

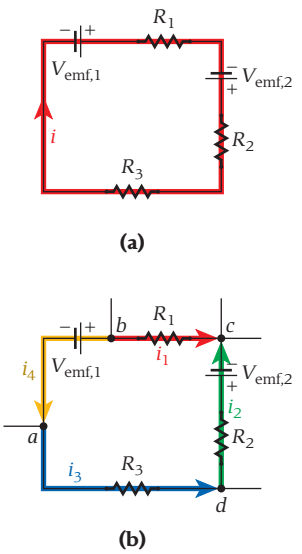


FIGURE 6.7 The same circuit loop as in Figure 6.6: (a) as an isolated single loop; (b) as a loop connected to other circuit branches.

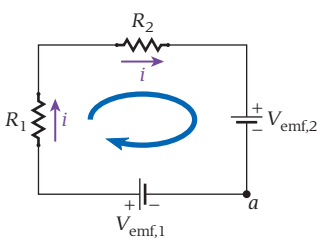


FIGURE 6.8 A single-loop circuit containing two resistors and two sources of emf in series.

If we move around a loop in a circuit in the same direction as the current, the potential changes across resistors will be negative. If we move around the loop in the opposite direction from the current, the potential changes across the resistors will be positive. If we move around a loop in such a way that we pass through an emf source from the negative to the positive terminal, this component contributes a positive potential difference. If we pass through an emf source from the positive to the negative terminal, that component contributes a negative potential difference.

With these conventions, Kirchhoff's Loop Rule is written in mathematical form as

$$\text{Closed loop:} \quad \sum_{j=1}^{m} V_{\text{emf},j} - \sum_{k=1}^{n} i_k R_k = 0. \tag{6.2}$$

The most important point visualized by Figure 6.6 is that one complete turn around the loop always ends up at the same value of the potential as at the starting point. This is exactly what Kirchhoff's Loop Rule (equation 6.2) claims.

A final point about loops is illustrated by Figure 6.7. Figure 6.7a reproduces the circuit shown in Figure 6.6 as a single isolated loop. Since this loop does not have junctions and thus has only one branch (which is the entire loop), the same current i flows everywhere in the loop. In Figure 6.7b, this loop is connected at four junctions (labeled *a*, *b*, *c*, and *d*) to other parts of a more extended circuit. Now this loop has four branches, each of which can have a different current flowing through it, as illustrated by the different-colored arrows in the figure. The point is this: In both parts of the figure, Kirchhoff's Loop Rule holds for the loop shown. The relative values of the electric potential between any two circuit elements in Figure 6.7b are the same as those shown in Figure 6.6, independent of the currents that are forced through the different branches of the loop by the rest of the circuit.

6.2 Single-Loop Circuits

Let's begin analyzing general circuits by considering a circuit containing two sources of emf, $V_{emf,1}$ and $V_{emf,2}$, and two resistors, R_1 and R_2, connected in series in a single loop, as shown in Figure 6.8. Note that $V_{emf,1}$ and $V_{emf,2}$ have opposite polarity. In this single-loop circuit, there are no junctions, and so the entire circuit consists of a single branch. The current is the same everywhere in the loop. To illustrate the potential changes across the components of this circuit, Figure 6.9 shows a three-dimensional view.

Although we could arbitrarily pick any point in the circuit of Figure 6.8 and assign it the value 0 V (or any other value of the potential, because we can always add a global additive constant to all potential values without changing the physical outcome), we start at point *a* with $V = 0$ V and proceed around the circuit in a clockwise direction (indicated by blue elliptical arrow in the figure). Because the components of the circuit are in series, the current, i, is the same in each component, and we assume that the current is flowing in the clockwise direction (purple arrows in the figure). The first circuit component along the clockwise path from point *a* is the source of emf, $V_{emf,1}$, which produces a positive potential

gain of $V_{emf,1}$. Next is resistor R_1, which produces a potential drop given by $\Delta V_1 = iR_1$. Continuing around the loop, the next component is resistor R_2, which produces a potential drop given by $\Delta V_2 = iR_2$. Next, we encounter a second source of emf, $V_{emf,2}$. This source of emf is wired into the circuit with its polarity opposite that of $V_{emf,1}$. Thus, this component produces a potential *drop* with magnitude $V_{emf,2}$, rather than a potential *gain*. We have now completed the loop and are back at $V = 0$ V. Using equation 6.2, we sum the potential changes of this loop as follows:

$$V_{emf,1} - \Delta V_1 - \Delta V_2 - V_{emf,2} = V_{emf,1} - iR_1 - iR_2 - V_{emf,2} = 0.$$

To show that the direction in which we move through a loop, clockwise or counterclockwise, is arbitrary, let's analyze the same circuit in the counterclockwise direction, starting at point *a* (see Figure 6.10). The first circuit element is $V_{emf,2}$, which produces a positive potential gain. The next element is R_2. Because we have assumed that the current is in the clockwise direction and we are analyzing the loop in the counterclockwise direction, the potential change for R_2 is $+iR_2$, according to the conventions listed in Table 6.1. Proceeding to the next element in the loop, R_1, we use a similar argument to designate the potential change for this resistor as $+iR_1$. The final element in the circuit is $V_{emf,1}$, which is aligned in a direction opposite to that of our analysis, so the potential change across this element is $-V_{emf,1}$. Kirchhoff's Loop Rule then gives us

$$+V_{emf,2} + iR_2 + iR_1 - V_{emf,1} = 0.$$

You can see that the clockwise and counterclockwise loop directions give the same information, which means that the direction in which we choose to analyze the circuit does not matter.

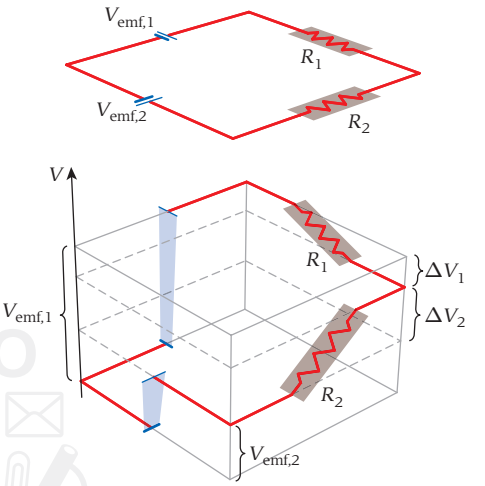


FIGURE 6.9 Three-dimensional representation of the single-loop circuit in Figure 6.8, containing two resistors and two sources of emf in series.

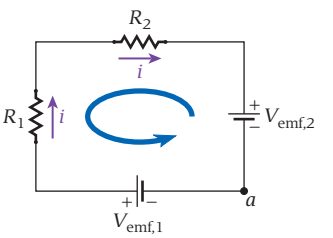


FIGURE 6.10 The same loop as in Figure 6.8, but analyzed in the counterclockwise direction.

SOLVED PROBLEM 6.1 Charging a Battery

A 12.0 V battery with internal resistance $R_i = 0.200\ \Omega$ is being charged by a battery charger that is capable of delivering a current of magnitude $i = 6.00$ A.

PROBLEM

What is the minimum emf the battery charger must supply to be able to charge the battery?

SOLUTION

THINK The battery charger, which is an external source of emf, must have enough potential difference to overcome the potential difference of the battery and the potential drop across the battery's internal resistance. The battery charger must be hooked up so that its positive terminal is connected to the positive terminal of the battery to be charged. We can think of the battery's internal resistance as a resistor in a single-loop circuit that also contains two sources of emf with opposite polarities.

SKETCH Figure 6.11 shows a diagram of the circuit, consisting of a battery with potential difference V_t and internal resistance R_i connected to an external source of emf, V_e. The yellow shaded area represents the battery's physical dimensions. Note that the positive terminal of the battery charger is connected to the positive terminal of the battery.

RESEARCH We can apply Kirchhoff's Loop Rule to this circuit. We assume a current flowing counterclockwise around the circuit, as shown in Figure 6.11. The potential changes around the circuit must sum to zero. We sum the potential changes starting at point *b* and moving in a counterclockwise direction:

$$-iR_i - V_t + V_e = 0.$$

– Continued

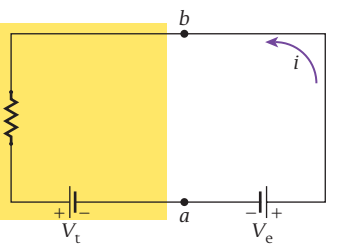


FIGURE 6.11 Circuit consisting of a battery with internal resistance connected to an external source of emf.



SIMPLIFY We can solve this equation for the required potential difference of the charger:

$$V_e = iR_i + V_t,$$

where i is the current that the charger supplies.

CALCULATE Putting in the numerical values gives us

$$V_e = iR_i + V_t = (6.00\ \text{A})(0.200\ \Omega) + 12.0\ \text{V} = 13.20\ \text{V}.$$

ROUND We report our result to three significant figures:

$$V_e = 13.2\ \text{V}.$$

DOUBLE-CHECK Our result indicates that the battery charger has to have a higher potential difference than the specified potential difference of the battery, which is reasonable. A typical charger for a 12 V battery has a potential difference of around 14 V.



6.3 Multiloop Circuits

Analyzing multiloop circuits requires both Kirchhoff's Loop Rule and Kirchhoff's Junction Rule. The procedure for analyzing a multiloop circuit consists of identifying complete loops and junction points in the circuit and applying Kirchhoff's rules to these parts of the circuit separately. Analyzing the single loops in a multiloop circuit with Kirchhoff's Loop Rule and the junctions with Kirchhoff's Junction Rule results in a system of coupled equations in several unknown variables. These equations can be solved for the quantities of interest using various techniques, including direct substitution. Example 6.1 illustrates the analysis of a multiloop circuit.

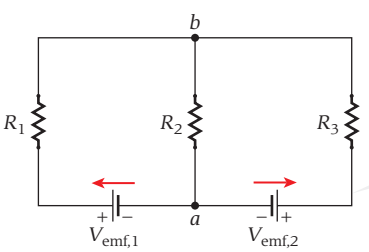


FIGURE 6.12 Multiloop circuit with three resistors and two sources of emf.

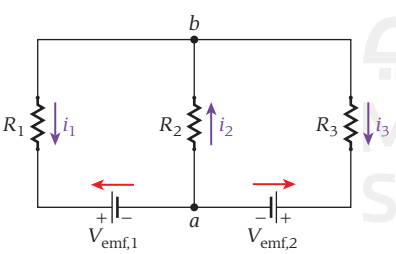


FIGURE 6.13 Multiloop circuit with the assumed direction of the current through the resistors indicated.

EXAMPLE 6.1 Multiloop Circuit

Consider the circuit shown in Figure 6.12. This circuit has three resistors, R_1, R_2, and R_3, and two sources of emf, $V_{emf,1}$ and $V_{emf,2}$. The red arrows show the direction of potential drop across the emf sources. This circuit cannot be resolved into simple series or parallel connections. To analyze this circuit, we need to assign directions to the currents flowing through the resistors. We can choose these directions arbitrarily (knowing that if we choose the wrong direction, the resulting current value will be negative). Figure 6.13 shows the circuit with the assigned directions for the currents shown by the purple arrows.

Let's consider junction b first. The current entering the junction must equal the current leaving it, so we can write

$$i_2 = i_1 + i_3. \qquad \text{(i)}$$

Looking at junction a, we again equate the incoming current and the outgoing current to get

$$i_1 + i_3 = i_2,$$

which provides the same information obtained for junction b. Note that this is a typical result: If a circuit has n junctions, it is possible to obtain at most $n - 1$ independent equations from application of Kirchhoff's Junction Rule. (In this case, $n = 2$, so we can get only one independent equation.)

At this point we cannot determine the currents in the circuit because we have three unknown values and only one equation. Therefore, we need two more independent equations. To get these equations, we apply Kirchhoff's Loop Rule. We can identify three loops in the circuit shown in Figure 6.13:

1. the left half of the circuit, including the elements R_1, R_2, and $V_{emf,1}$;
2. the right half of the circuit, including the elements R_2, R_3, and $V_{emf,2}$; and
3. the outer loop, including the elements R_1, R_3, $V_{emf,1}$, and $V_{emf,2}$.

Applying Kirchhoff's Loop Rule to the left half of the circuit, using the assumed directions for the currents and analyzing the loop in a counterclockwise direction starting at junction b, we obtain

$$-i_1R_1 - V_{emf,1} - i_2R_2 = 0,$$

or

$$i_1R_1 + V_{\text{emf},1} + i_2R_2 = 0. \quad \text{(ii)}$$

Applying the Loop Rule to the right half of the circuit, again starting at junction b and analyzing the loop in a clockwise direction, we get

$$-i_3R_3 - V_{\text{emf},2} - i_2R_2 = 0,$$

or

$$i_3R_3 + V_{\text{emf},2} + i_2R_2 = 0. \quad \text{(iii)}$$

Applying the Loop Rule to the outer loop, starting at junction b and working in a clockwise direction, gives us

$$-i_3R_3 - V_{\text{emf},2} + V_{\text{emf},1} + i_1R_1 = 0.$$

This equation provides no new information because we can also obtain it by subtracting equation (iii) from equation (ii). For all three loops, we obtain equivalent information if we analyze them in either a counterclockwise direction or a clockwise direction or if we start at any other point and move around the loops from there.

With three equations, (i), (ii), and (iii), and three unknowns, i_1, i_2, and i_3, we can solve for the unknown currents in several ways. For example, we could put the three equations in matrix format and then solve them using Kramer's method on a calculator. This is a recommended method for complicated circuits with many equations and many unknowns. However, for this example, we can proceed by substituting from equation (i) into the other two, thus eliminating i_2. We then solve one of the two resulting equations for i_1 and substitute from that into the other to obtain an expression for i_3. Substituting back then gives solutions for i_2 and i_1:

$$i_1 = -\frac{(R_2 + R_3)V_{\text{emf},1} - R_2V_{\text{emf},2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

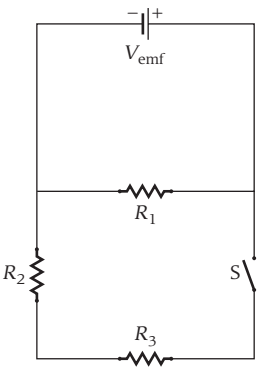
$$i_2 = -\frac{R_3V_{\text{emf},1} + R_1V_{\text{emf},2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$i_3 = -\frac{-R_2V_{\text{emf},1} + (R_1 + R_2)V_{\text{emf},2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

Note: You do not need to remember this particular solution or the linear algebra used to get there. However, the general method of applying Kirchhoff's rules for loops and junctions, and assigning currents in arbitrary directions is the central idea of circuit analysis.

Concept Check 6.2

In the circuit in the figure, there are three identical resistors. The switch, S, is initially open. When the switch is closed, what happens to the current flowing in R_1?



a) The current in R_1 decreases.

b) The current in R_1 increases.

c) The current in R_1 stays the same.

SOLVED PROBLEM 6.2 The Wheatstone Bridge

The Wheatstone bridge is a particular circuit used to measure unknown resistances. The circuit diagram of a Wheatstone bridge is shown in Figure 6.14. This circuit consists of three known resistances, R_1, R_3, and a variable resistor, R_v, as well as an unknown resistance, R_u. A source of emf, V, is connected across junctions a and c. A sensitive ammeter (a device used to measure current, discussed in Section 6.4) is connected between junctions b and d. The Wheatstone bridge is used to determine R_u by varying R_v until the ammeter between b and d shows no current flowing. When the ammeter reads zero, the bridge is said to be balanced.

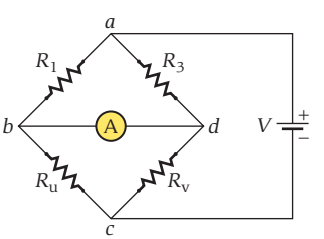


FIGURE 6.14 Circuit diagram of a Wheatstone bridge.

PROBLEM

Determine the unknown resistance, R_u, in the Wheatstone bridge shown in Figure 6.14. The known resistances are $R_1 = 100.0\ \Omega$ and $R_3 = 110.0\ \Omega$, and $R_v = 15.63\ \Omega$ when the current through the ammeter is zero, indicating that the bridge is balanced.

SOLUTION

THINK The circuit has four resistors and an ammeter, and each component can have a current flowing through it. However, in this case, with $R_v = 15.63\ \Omega$, there is no current flowing through the ammeter. Setting this current to zero leaves four unknown currents through the four resistors, and we therefore need four equations. We can use Kirchhoff's rules to analyze two loops, *adb* and *cbd*, and two junctions, *b* and *d*.

SKETCH Figure 6.15 shows the Wheatstone bridge with the assumed directions for currents i_1, i_3, i_u, i_v, and i_A.

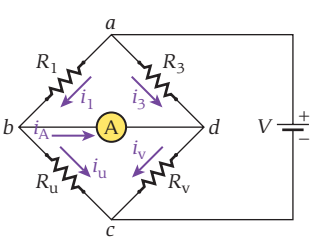


FIGURE 6.15 The Wheatstone bridge with the assumed current directions indicated.

– *Continued*

RESEARCH We first apply Kirchhoff's Loop Rule to loop *adb*, starting at *a* and going clockwise, to obtain

$$-i_3R_3 + i_AR_A + i_1R_1 = 0, \tag{i}$$

where R_A is the resistance of the ammeter. We apply Kirchhoff's Loop Rule again to loop *cbd*, starting at *c* and going clockwise, to get

$$+i_uR_u - i_AR_A - i_vR_v = 0. \tag{ii}$$

Now we can use Kirchhoff's Junction Rule at junction *b* to obtain

$$i_1 = i_A + i_u. \tag{iii}$$

Another application of Kirchhoff's Junction Rule, at junction *d*, gives

$$i_3 + i_A = i_v. \tag{iv}$$

SIMPLIFY When the current through the ammeter is zero ($i_A = 0$), we can rewrite equations (i) through (iv) as follows:

$$i_1R_1 = i_3R_3 \tag{v}$$

$$i_uR_u = i_vR_v \tag{vi}$$

$$i_1 = i_u \tag{vii}$$

and

$$i_3 = i_v. \tag{viii}$$

Dividing equation (vi) by equation (v) gives us

$$\frac{i_uR_u}{i_1R_1} = \frac{i_vR_v}{i_3R_3},$$

which we can rewrite using equations (vii) and (viii):

$$R_u = \frac{R_1}{R_3}R_v.$$

CALCULATE Putting in the numerical values, we get

$$R_u = \frac{R_1}{R_3}R_v = \frac{100.0\ \Omega}{110.0\ \Omega}15.63\ \Omega = 14.20901\ \Omega.$$

ROUND We report our result to four significant figures:

$$R_u = 14.21\ \Omega.$$

DOUBLE-CHECK Our result for the resistance of the unknown resistor is similar to the value for the variable resistor. Thus, our answer seems reasonable, because the other two resistors in the circuit also have resistances that are approximately equal.

Self-Test Opportunity 6.1

Show that when the current through the ammeter is zero, the equivalent resistance for the resistors in the Wheatstone bridge in Figure 6.14 is given by

$$R_{eq} = \frac{(R_1 + R_u)(R_3 + R_v)}{R_1 + R_u + R_3 + R_v}.$$

General Observations on Circuit Networks

An important observation about solving circuit problems is that, in general, the complete analysis of a circuit requires knowing the current flowing in each branch of the circuit. We use Kirchhoff's Junction Rule and Loop Rule to establish equations relating the currents, and we need as many linearly independent equations as there are branches to guarantee that we can obtain a solution to the system.

Let's consider the abstract example shown in Figure 6.16, where all circuit components except the wires have been omitted. This circuit has four junctions, shown in blue in Figure 6.16a. Six branches connect these junctions, shown in Figure 6.16b. We therefore need six linearly independent equations relating the currents in these branches. It was noted earlier that not all equations obtained by applying Kirchhoff's rules to a circuit are linearly independent. This fact is worth repeating: If a circuit has *n* junctions, it is possible to obtain at most $n - 1$ independent equations from application of the Junction Rule. (For the circuit in Figure 6.16, $n = 4$, so we can get only three independent equations.)

The Junction Rule alone is not enough for the complete analysis of any circuit. It is generally best to write as many equations as possible for junctions and then augment them with equations obtained from loops. Figure 6.16c shows that there are six possible loops in

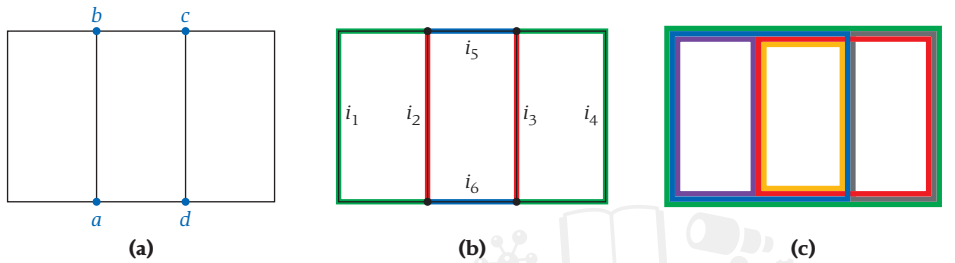
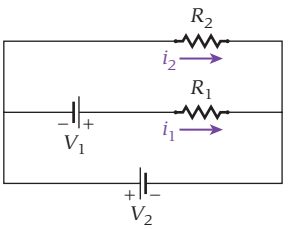


FIGURE 6.16 Circuit network consisting of (a) four junctions, (b) six branches, (c) six possible loops.

this network, which are marked in different colors. Clearly, there are more loops than we need to analyze to obtain three equations. This is again a general observation: The system of equations that can be set up by considering all possible loops is overdetermined. Thus, you'll always have the freedom to select particular loops to augment the equations obtained from analyzing the junctions. As a general rule of thumb, it is best to choose loops with fewer circuit elements, which often makes the subsequent linear algebra considerably simpler. In particular, if you are asked to find the current in a particular branch of a network, choosing the appropriate loop may allow you to avoid setting up a lengthy set of equations and let you solve the problem with just one equation. So it pays to devote some attention to the selection of loops! Do not write down more equations than you need to solve for the unknowns in any particular problem. This will only complicate the algebra. However, once you have the solution, you can use one or more of the unused loops to check your values.

Concept Check 6.3

In the multiloop circuit shown in the figure, $V_1 = 6.00$ V, $V_2 = 12.0$ V, $R_1 = 10.0\ \Omega$, and $R_2 = 12.0$ V. What is the magnitude of current i_2?



a) 0.500 A
b) 0.750 A
c) 1.00 A
d) 1.25 A
e) 1.50 A

6.4 Ammeters and Voltmeters

A device used to measure current is called an **ammeter.** A device used to measure potential difference is called a **voltmeter.** To measure the current, an ammeter must be wired in a circuit in *series.* Figure 6.17 shows an ammeter connected in a circuit in a way that allows it to measure the current *i.* To measure the potential difference, a voltmeter must be wired in *parallel* with the component across which the potential difference is to be measured. Figure 6.17 shows a voltmeter placed in the circuit to measure the potential drop across resistor R_1.

It is important to realize that these instruments must be able to make measurements while disturbing the circuit as little as possible. Thus, ammeters are designed to have as low a resistance as possible, usually on the order of 1 Ω, so they do not have an appreciable effect on the currents they measure. Voltmeters are designed to have as high a resistance as possible, usually on the order of 10 MΩ (10^7 Ω), so they have a negligible effect on the potential differences they are measuring.

In practice, measurements of current and potential difference are made with a digital multimeter that can switch between functioning as an ammeter and functioning as a voltmeter. It displays the results with a numerical digital display, which includes the sign of the potential difference or current. Most digital multimeters can also measure the resistance of a circuit component; that is, they can function as an **ohmmeter.** The digital multimeter performs this task by applying a known potential difference and measuring the resulting current. This test is useful for determining circuit continuity and the status of fuses, as well as measuring the resistance of resistors.

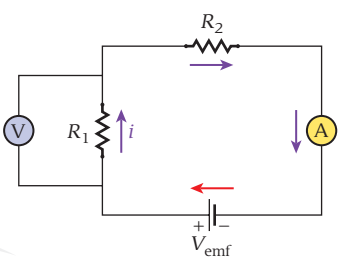


FIGURE 6.17 Placement of an ammeter and a voltmeter in a simple circuit.



EXAMPLE 6.2 Voltmeter in a Simple Circuit

Consider a simple circuit consisting of a source of emf with voltage $V_{emf} = 150.$ V and a resistor with resistance $R = 100.\ \text{k}\Omega$ (Figure 6.18). A voltmeter with resistance $R_V = 10.0$ MΩ is connected across the resistor.

PROBLEM 1

What is the current in the circuit before the voltmeter is connected? – *Continued*

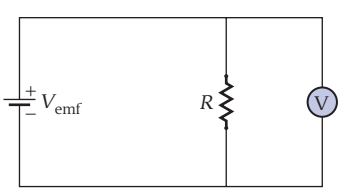


FIGURE 6.18 A simple circuit with a voltmeter connected in parallel across a resistor.

SOLUTION 1

Ohm's Law says that $V = iR$, so we can find the current in the circuit:

$$i = \frac{V_{emf}}{R} = \frac{150.\ \text{V}}{100.\times10^{3}\,\Omega} = 1.50\times10^{-3}\ \text{A} = 1.50\ \text{mA}.$$

PROBLEM 2

What is the current in the circuit when the voltmeter is connected across the resistor?

SOLUTION 2

The equivalent resistance of the resistor and the voltmeter connected in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R_V}.$$

Solving for the equivalent resistance and putting in the numerical values, we get

$$R_{eq} = \frac{RR_V}{R + R_V} = \frac{\left(100.\times10^{3}\,\Omega\right)\left(10.0\times10^{6}\,\Omega\right)}{100.\times10^{3}\,\Omega + 10.0\times10^{6}\,\Omega} = 9.90\times10^{4}\,\Omega = 99.0\ \text{k}\Omega.$$

The current is then

$$i = \frac{V_{emf}}{R_{eq}} = \frac{150.\ \text{V}}{9.90\times10^{4}\,\Omega} = 1.52\times10^{-3}\ \text{A} = 1.52\ \text{mA}.$$

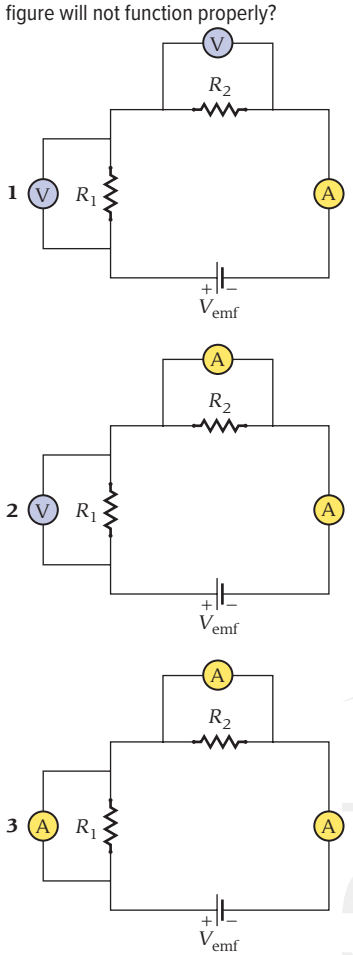
The current in the circuit increases by 0.02 mA when the voltmeter is connected because the parallel combination of the resistor and the voltmeter has a lower resistance than that of the resistor alone. However, the effect is small, even with this relatively large resistance ($R = 100.\ \text{k}\Omega$).

Self-Test Opportunity 6.2

When the starter of a car is engaged while the headlights are on, the headlights dim. Explain.

Concept Check 6.4

Which of the circuits shown in the figure will not function properly?



a) 1
b) 2
c) 3
d) 1 and 2
e) 2 and 3

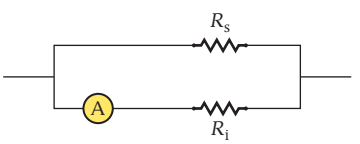


FIGURE 6.19 An ammeter with a shunt resistor connected across it in parallel.

SOLVED PROBLEM 6.3 Increasing the Range of an Ammeter

PROBLEM

An ammeter can be used to measure different ranges of current by adding a current divider in the form of a shunt resistor connected in parallel with the ammeter. A *shunt resistor* is simply a resistor with a very small resistance. Its name arises from the fact that when it is connected in parallel with the ammeter, whose resistance is larger, most of the current is shunted through it, bypassing the meter. The sensitivity of the ammeter is therefore decreased allowing it to measure larger currents. Suppose an ammeter produces a full-scale reading when a current of $i_{int} = 5.10$ mA passes through it. The ammeter has an internal resistance of $R_i = 16.8\ \Omega$. To use this ammeter to measure a maximum current of $i_{max} = 20.2$ A, what should be the resistance of the shunt resistor, R_s, connected in parallel with the ammeter?

SOLUTION

THINK The shunt resistor connected in parallel with the ammeter needs to have a substantially lower resistance than the internal resistance of the ammeter. Most of the current will then flow through the shunt resistor rather than the ammeter.

SKETCH Figure 6.19 shows a shunt resistor, R_s, connected in parallel with an ammeter.

RESEARCH The two resistors are connected in parallel, so the potential difference across each resistor is the same. The potential difference that gives a full-scale reading on the ammeter is

$$\Delta V_{fs} = i_{int}R_i. \qquad \text{(i)}$$

From Chapter 5, we know that the equivalent resistance of the two resistors in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_i} + \frac{1}{R_s}. \qquad \text{(ii)}$$

The voltage drop across the equivalent resistance must equal the voltage drop across the ammeter that gives a full-scale reading when current i_{max} is flowing through the circuit. Therefore, we can write

$$\Delta V_{fs} = i_{max}R_{eq}. \qquad \text{(iii)}$$

SIMPLIFY Combining equations (i) and (iii) for the potential difference gives

$$\Delta V_{fs} = i_{int}R_i = i_{max}R_{eq}. \qquad \text{(iv)}$$

We can rearrange equation (iv) and substitute for R_{eq} in equation (ii):

$$\frac{i_{max}}{i_{int}R_i} = \frac{1}{R_{eq}} = \frac{1}{R_i} + \frac{1}{R_s}. \quad \text{(v)}$$

Solving equation (v) for the shunt resistance gives us

$$\frac{1}{R_s} = \frac{i_{max}}{i_{int}R_i} - \frac{1}{R_i} = \frac{1}{R_i}\left(\frac{i_{max}}{i_{int}} - 1\right) = \frac{1}{R_i}\left(\frac{i_{max} - i_{int}}{i_{int}}\right),$$

or

$$R_s = R_i \frac{i_{int}}{i_{max} - i_{int}}.$$

CALCULATE Putting in the numerical values, we get

$$R_s = R_i \frac{i_{int}}{i_{max} - i_{int}} = (16.8\ \Omega)\frac{5.10\times10^{-3}\ \text{A}}{20.2\ \text{A} - 5.10\times10^{-3}\ \text{A}}$$
$$= 0.00424266\ \Omega.$$

ROUND We report our result to three significant figures:

$$R_s = 0.00424\ \Omega.$$

DOUBLE-CHECK The equivalent resistance of the ammeter and the shunt resistor connected in parallel is given by equation (ii). Solving that equation for the equivalent resistance and putting in the numbers gives

$$R_{eq} = \frac{R_i R_s}{R_i + R_s} = \frac{(16.8\ \Omega)(0.00424\ \Omega)}{16.8\ \Omega + 0.00424\ \Omega} = 0.00424\ \Omega.$$

Thus, the equivalent resistance of the ammeter and the shunt resistor connected in parallel is approximately equal to the resistance of the shunt resistor. This low equivalent resistance is necessary for a current-measuring instrument, which must be placed in series in a circuit. If the current-measuring device has a high resistance, its presence will disturb the measurement of the current.

6.5 RC Circuits

So far in this chapter, we have dealt with circuits containing sources of emf and resistors. The currents in these circuits do not vary in time. Now we consider circuits that contain capacitors, as well as sources of emf and resistors. Called **RC circuits,** these circuits have currents that *do* vary with time. The simplest circuit operations that involve time-dependent currents are the charging and discharging of a capacitor. Understanding these time-dependent processes involves the solution of some simple differential equations.

Charging a Capacitor

Consider a circuit with a source of emf, V_{emf}, a resistor, R, and a capacitor, C (Figure 6.20). Initially, the switch is open and the capacitor is uncharged, as shown in Figure 6.20a. When the switch is closed (Figure 6.20b), current begins to flow in the circuit, building up opposite charges on the plates of the capacitor and thus creating a potential difference, ΔV, across the capacitor. Current flows because of the source of emf, which maintains a constant voltage. When the capacitor is fully charged (Figure 6.20c), no more current flows in the circuit. The potential difference across the plates is then equal to the voltage provided by the source of emf, and the magnitude of the total charge, q_{tot}, on each plate of the capacitor is $q_{tot} = CV_{emf}$.

While the capacitor is charging, we can analyze the current, i, flowing in the circuit (assumed to flow from the negative to the positive terminal inside the voltage source) by applying Kirchhoff's Loop Rule to the loop in Figure 6.20b in the counterclockwise direction:

$$V_{emf} - V_R - V_C = V_{emf} - i(t)R - q(t)/C = 0,$$

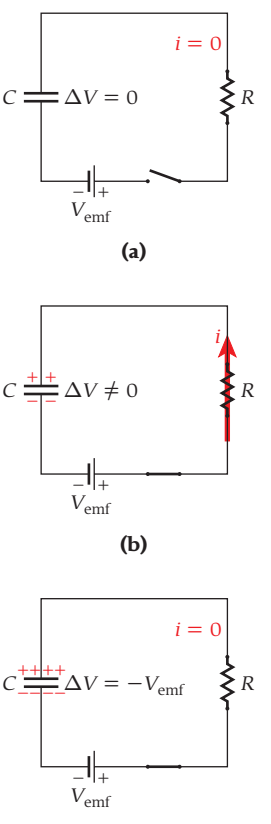


FIGURE 6.20 A basic RC circuit, containing a source of emf, a resistor, and a capacitor: (a) with the switch open; (b) a short time after the switch is closed; (c) a long time after the switch is closed.

where V_C is the potential drop across the capacitor and $q(t)$ is the charge on the capacitor at a given time t. The change of the charge on the capacitor plates due to the current is $i(t) = dq(t)/dt$, and we can rewrite the preceding equation as

$$R\frac{dq(t)}{dt} + \frac{q(t)}{C} = V_{\text{emf}},$$

or

$$\frac{dq(t)}{dt} + \frac{q(t)}{RC} = \frac{V_{\text{emf}}}{R}. \tag{6.3}$$

This differential equation relates the charge to its time derivative. It seems appropriate to try an exponential form for the solution of equation 6.3 because an exponential function is the only type that has the property of having a derivative that is identical to itself. Because equation 6.3 also has a constant term, the trial solution needs to have a constant term. We therefore try a solution with a constant and an exponential and for which $q(0) = 0$:

$$q(t) = q_{\max}\left(1 - e^{-t/\tau}\right),$$

where the constants $q_{\max}$ and τ are to be determined. Substituting this trial solution back into equation 6.3, we obtain

$$q_{\max}\frac{1}{\tau}e^{-t/\tau} + \frac{1}{RC}q_{\max}\left(1 - e^{-t/\tau}\right) = \frac{V_{\text{emf}}}{R}.$$

Now we collect the time-dependent terms on the left-hand side and the time-independent terms on the right-hand side:

$$q_{\max}e^{-t/\tau}\left(\frac{1}{\tau} - \frac{1}{RC}\right) = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}q_{\max}.$$

This equation can only be true for all times if both sides are equal to zero. From the left-hand side, we then find

$$\tau = RC. \tag{6.4}$$

Thus, the constant τ (called the **time constant**) is simply the product of the capacitance and the resistance. From the right-hand side, we find an expression for the constant $q_{\max}$:

$$q_{\max} = CV_{\text{emf}}.$$

Thus, the differential equation for charging the capacitor (equation 6.3) has the solution

$$q(t) = CV_{\text{emf}}\left(1 - e^{-t/RC}\right). \tag{6.5}$$

Note that at $t = 0$, $q = 0$, which is the initial condition before the circuit components were connected. At $t = \infty$, $q = q_{\max} = CV_{\text{emf}}$, which is the steady-state condition in which the capacitor is fully charged. The time dependence of the charge on the capacitor is shown in Figure 6.21a for three different values of the time constant τ.

The current flowing in the circuit is obtained by differentiating equation 6.5 with respect to time:

$$i = \frac{dq}{dt} = \left(\frac{V_{\text{emf}}}{R}\right)e^{-t/RC}. \tag{6.6}$$

From equation 6.6, at $t = 0$, the current in the circuit is V_{emf}/R, and at $t = \infty$, the current is zero, as shown in Figure 6.21b.

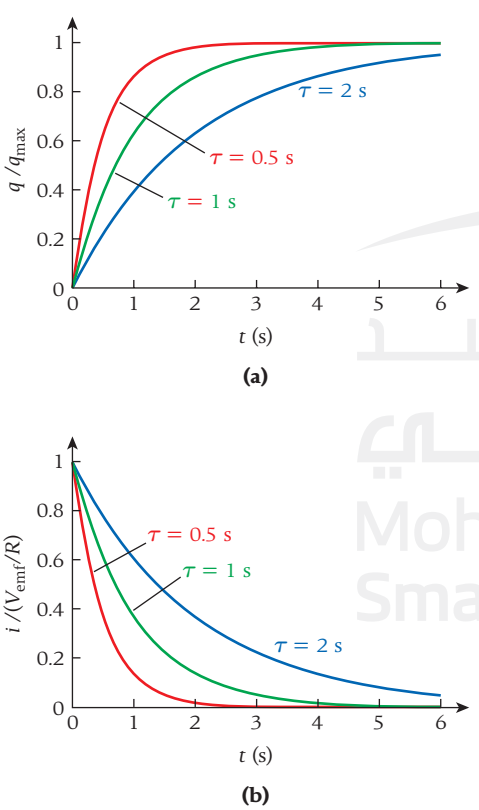


FIGURE 6.21 Charging a capacitor: (a) charge on the capacitor as a function of time; (b) current flowing through the resistor as a function of time.

Discharging a Capacitor

Now let's consider a circuit containing only a resistor, R, and a fully charged capacitor, C, obtained by moving the switch in Figure 6.22 from position 1 to position 2. The charge on the capacitor before the switch is moved is q_{max}. In this case, current will flow in the circuit until the capacitor is completely discharged. While the capacitor is discharging, we can apply Kirchhoff's Loop Rule around the loop in the clockwise direction and obtain

$$-i(t)R - V_C = -i(t)R - \frac{q(t)}{C} = 0.$$

We can rewrite this equation using the definition of current:

$$\frac{R\,dq(t)}{dt} + \frac{q(t)}{C} = 0. \tag{6.7}$$

The solution to equation 6.7 is obtained using the same method as for equation 6.3, except that equation 6.7 has no constant term and $q(0) > 0$. Thus, we try a solution of the form $q(t) = q_{max}e^{-t/\tau}$, which leads to

$$q(t) = q_{max}e^{-t/RC}. \tag{6.8}$$

At $t = 0$, the charge on the capacitor is q_{max}. At $t = \infty$, the charge on the capacitor is zero.

We can obtain the current by differentiating equation 6.8 as a function of time:

$$i(t) = \frac{dq}{dt} = -\left(\frac{q_{max}}{RC}\right)e^{-t/RC}. \tag{6.9}$$

At $t = 0$, the current in the circuit is $-q_{max}/RC$. At $t = \infty$, the current in the circuit is zero. Plotting the time dependence of the charge on the capacitor and the current flowing through the resistor for the discharging process would result in exponentially decreasing curves like those in Figure 6.21b.

The equations describing the time dependence of the charging and the discharging of a capacitor all involve the exponential factor $e^{-t/RC}$. Again, the product of the resistance and the capacitance is defined as the time constant of an RC circuit: $\tau = RC$. According to equation 6.5, after an amount of time equal to the time constant, the capacitor will have been charged to 63% of its maximum value. Thus, an RC circuit can be characterized by specifying the time constant. A large time constant means that it takes a long time to charge the capacitor; a small time constant means that it takes a short time to charge the capacitor.

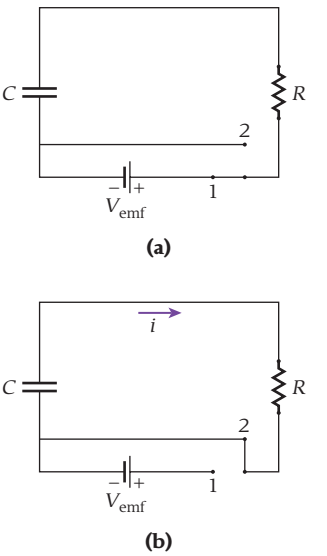


FIGURE 6.22 RC circuit containing an emf source, a resistor, a capacitor, and a switch. The capacitor is (a) charged with the switch in position 1 and (b) discharged with the switch in position 2.

Concept Check 6.5

To discharge a capacitor in an RC circuit very quickly, what should the values of the resistance and the capacitance be?

a) Both should be as large as possible.

b) Resistance should be as large as possible, and capacitance as small as possible.

c) Resistance should be as small as possible, and capacitance as large as possible.

d) Both should be as small as possible.

EXAMPLE 6.3 Time Required to Charge a Capacitor

Consider a circuit consisting of a 12.0 V battery, a 50.0 Ω resistor, and a 100.0 μF capacitor wired in series. The capacitor is initially completely discharged.

PROBLEM

How long after the circuit is closed will it take to charge the capacitor to 90% of its maximum charge?

SOLUTION

The charge on the capacitor as a function of time is given by

$$q(t) = q_{max}\left(1 - e^{-t/RC}\right),$$

where q_{max} is the maximum charge on the capacitor. We want to know the time until $q(t)/q_{max} = 0.90$, which can be obtained from

$$\left(1 - e^{-t/RC}\right) = \frac{q(t)}{q_{max}} = 0.90,$$

or

$$0.10 = e^{-t/RC}. \tag{i}$$

– Continued

Self-Test Opportunity 6.3

A 1.00-mF capacitor is fully charged, and a 100.0-Ω resistor is connected across the capacitor. How long will it take to remove 99.0% of the charge stored in the capacitor?

Taking the natural log of both sides of equation (i), we get

$$\ln 0.10 = -\frac{t}{RC},$$

or

$$t = -RC\ln 0.10 = -(50.0\ \Omega)\left(100\times10^{-6}\ \text{F}\right)(-2.30) = 0.0115\ \text{s} = 11.5\ \text{ms}.$$

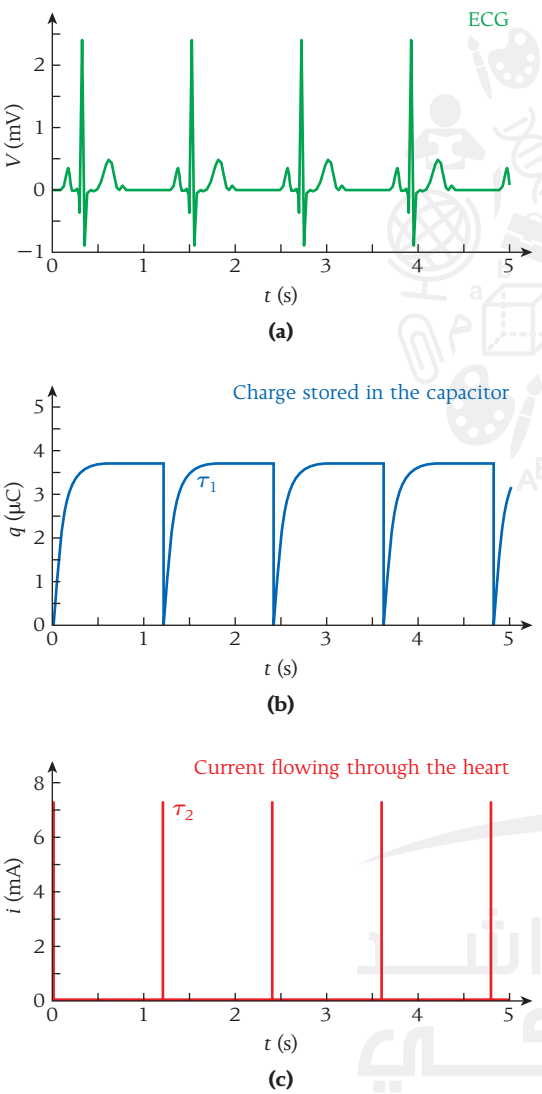


FIGURE 6.23 (a) An electrocardiogram (ECG) showing four regular heartbeats. (b) The charge stored in the pacemaker's capacitor as a function of time. (c) The current flowing through the heart due to the discharging of the pacemaker's capacitor.

Pacemaker

A normal human heart beats at regular intervals, sending blood through the body. The heart's own electrical signals regulate its beating. These electrical signals can be measured through the skin using an electrocardiograph. This device produces a graph of potential difference versus time, which is called an *electrocardiogram*, or *ECG* (sometimes *EKG*, from the German word *Electrokardiograph*). Figure 6.23a shows how an ECG would represent four regular heartbeats occurring at a rate of 72 beats per minute. Doctors and medical personnel can use an ECG to diagnose the health of the heart.

Sometimes the heart does not beat regularly and needs help to maintain its proper rhythm. This help can be provided by a pacemaker, an electrical circuit that sends electrical pulses to the heart at regular intervals, replacing the heart's usual electrical signals and stimulating the heart to beat at prescribed intervals. A pacemaker is implanted in the patient and connected directly to the heart, as illustrated in Figure 6.24.

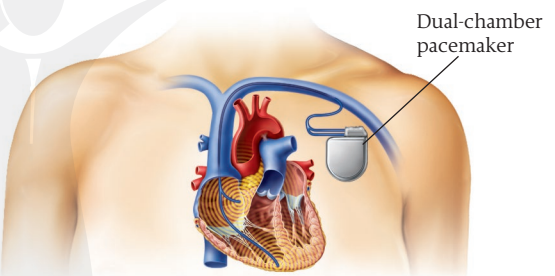


FIGURE 6.24 A modern pacemaker implanted in a patient. The pacemaker sends electrical pulses to two chambers of the heart to help keep it beating regularly.

Concept Check 6.6

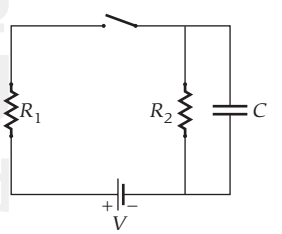
In the circuit shown in the figure, the capacitor, C, is initially uncharged. Immediately after the switch is closed,

a) the current flowing through R_1 is zero.

b) the current flowing through R_1 is larger than that through R_2.

c) the current flowing through R_2 is larger than that through R_1.

d) the current flowing through R_1 is the same as that through R_2.



Concept Check 6.7

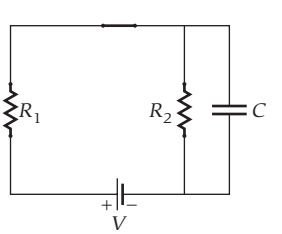
In the circuit shown in the figure, the switch is closed. After a long time,

a) the current through R_1 is zero.

b) the current flowing through R_1 is larger than that through R_2.

c) the current flowing through R_2 is larger than that through R_1.

d) the current flowing through R_1 is the same as that through R_2.



EXAMPLE 6.4 Circuit Elements of a Pacemaker

Let's analyze the circuit shown in Figure 6.25, which simulates the function of a pacemaker. This pacemaker circuit operates by charging a capacitor, C, for some time using a battery with voltage V_{emf} and a resistor R_1, as illustrated in Figure 6.25a, where the switch is open. Closing the switch, as in Figure 6.25b, shorts the capacitor across the heart, and the capacitor discharges through the heart in a short time to stimulate the heart to beat. Thus, this circuit operates as a pacemaker by keeping the switch open for the time between heartbeats, closing the switch for a short time to stimulate a heartbeat, and then opening the switch again.

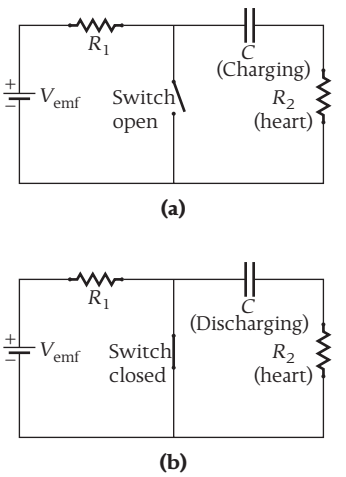


FIGURE 6.25 (a) A simplified pacemaker circuit in charging mode. (b) The pacemaker circuit in discharging mode.

PROBLEM

What values of the capacitance, C, and the resistance, R_1, should be used in a pacemaker?

SOLUTION

We assume that the heart acts as a resistor with a value $R_2 = 500\ \Omega$ and that the source of emf is a lithium ion battery. Such a battery has a very high energy density and a voltage of 3.7 V. A normal heart rate is in the range from 60 to 100 beats per minute. However, the pacemaker might need to stimulate the heart to beat faster, so it should be capable of running at 180 beats per minute, which means that the capacitor may be charged up to 180 times a minute. Thus, the minimum time between discharges, t_{min}, is

$$t_{min} = \frac{1}{180\ \text{beats/min}} = \left(\frac{1\ \text{min}}{180}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 0.333\ \text{s}.$$

Equation 6.5 gives the charge, q, as a function of time, t, for the maximum charge, q_{max}, for a given time constant, $\tau_1 = R_1C$:

$$q = q_{max}\left(1 - e^{-t/\tau_1}\right).$$

Rearranging this equation gives us

$$\frac{q}{q_{max}} = f = 1 - e^{-t/\tau_1},$$

where f is the fraction of the charge capacity of the capacitor. Let's assume the capacitor must be charged to 95% of its maximum charge in time t_{min}. Solving for the time constant gives us

$$\tau_1 = R_1C = -\frac{t_{min}}{\ln(1-f)} = -\frac{0.333\ \text{s}}{\ln(1-0.95)} = 0.111\ \text{s}.$$

Thus, the time constant for the charging should be $\tau_1 = R_1C = 100$ ms. The time constant for discharging, τ_2, needs to be small to produce short pulses of a high current to stimulate the heart. Letting $\tau_2 = 0.500$ ms, which is on the order of the narrow electrical pulse in the ECG, we have

$$\tau_2 = R_2C = 0.500\ \text{ms}.$$

We can solve for the required capacitance and substitute the value 500 Ω for R_2:

$$C = \frac{0.000500\ \text{s}}{500\ \Omega} = 1.00\ \mu\text{F}.$$

The resistance required for the charging circuit can now be related to the time constant for the charging and the value of the capacitance we just calculated:

$$\tau_1 = R_1C = 0.100\ \text{s} = R_1(1.00\ \mu\text{F}),$$

which gives us

$$R_1 = \frac{0.100\ \text{s}}{1.00\times10^{-6}\ \text{F}} = 100\ \text{k}\Omega.$$

Figure 6.23b shows the charge in the capacitor of the simulated pacemaker as a function of time for a heart rate of 72 heartbeats per minute. You can see that the capacitor charges to nearly full capacity before it is discharged. Figure 6.23c shows the current that flows through the heart when the capacitor discharges. The current pulse is narrow, lasting less than a millisecond. This pulse stimulates the heart to beat, as illustrated by the ECG in Figure 6.23a. The rate at which the heart beats is controlled by the rate at which the pacemaker's switch is closed and opened, which is controlled by a microprocessor.

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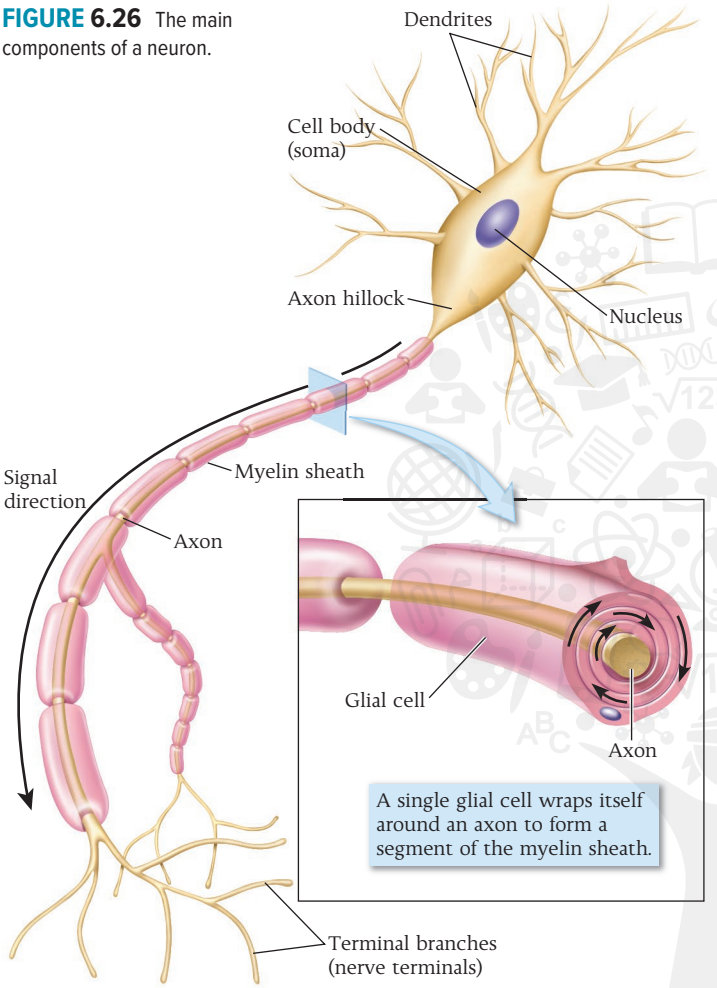


FIGURE 6.26 The main components of a neuron.

Neuron

The type of cell responsible for transmitting and processing signals in the nervous systems and brains of humans and other animals is a neuron (Figure 6.26). The neuron conducts the necessary currents by electrochemical means, via the movement of ions (mainly Na^+, K^+, and Cl^-). Neurons receive signals from other neurons through dendrites and send signals to other neurons through an axon. The axon can be quite long (for example, in the spinal cord) and is covered with an insulating myelin sheath. The signals are received from and sent to other neurons or cells in the sense organs and other tissues. All of this, and much more, can be learned in an introductory biology or physiology course. Here we'll take a look at a neuron as a basic circuit that processes signals.

An input signal has to be strong enough to get a neuron to fire, that is, to send an output signal down the axon. It is a crude but reasonable approximation to represent the main cell body of a neuron, the soma, as a basic RC circuit that processes these signals. A diagram of this RC circuit is shown in Figure 6.27. A capacitor and a resistor are connected in parallel to an input and an output potential. The typical potential values for neurons are on the order of ±50 mV relative to the background of the surrounding tissue. If $\Delta V = V_{in} - V_{out} \neq 0$, a current flows through the circuit. Part of this current flows through the resistor, but part of the current simultaneously charges the capacitor until it reaches the potential difference between input and output potentials. The potential difference between the capacitor plates rises exponentially, according to $V_C(t) = (V_{in} - V_{out})(1 - e^{-t/RC})$, just as for the process of charging a capacitor in an RC circuit. (The time constant, $\tau = RC$, is on the order of 10 ms, assuming a capacitance of 1 nF and a resistance of 10 MΩ.) If the external source of potential difference is then removed from the circuit, the capacitor discharges with the same time constant, and the potential across the capacitor decays exponentially, according to $V_C(t) = V_0 e^{-t/RC}$. This simple time dependence captures the basic response of a neuron. Figure 6.28 shows the potential difference across the capacitor of this model neuron, while it is charged for 30 ms and then discharged.

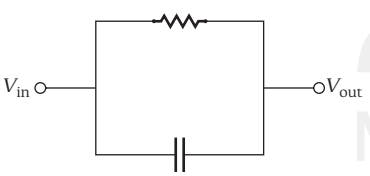


FIGURE 6.27 Simplified model of a neuron as an RC circuit.

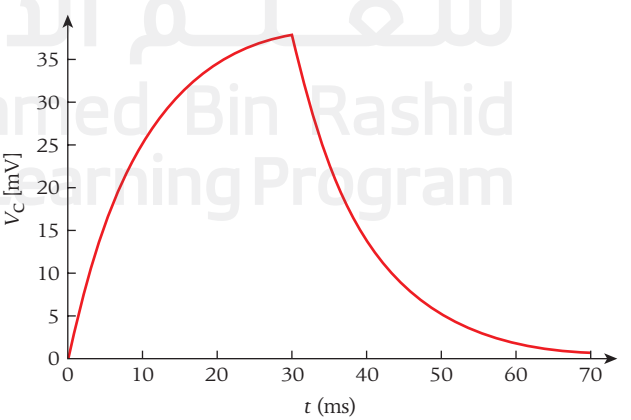


FIGURE 6.28 Potential difference on the capacitor in a model neuron.

SOLVED PROBLEM 6.4 Rate of Energy Storage in a Capacitor

A resistor with $R = 2.50\ \text{M}\Omega$ and a capacitor with $C = 1.25\ \mu\text{F}$ are connected in series with a battery for which $V_{\text{emf}} = 12.0$ V. At $t = 2.50$ s after the circuit is closed, what is the rate at which energy is being stored in the capacitor?

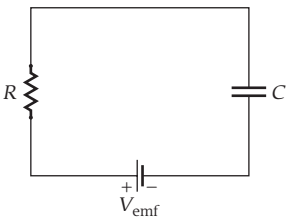


FIGURE 6.29 Series circuit containing a battery, a resistor, and a capacitor.

THINK When the circuit is closed, the capacitor begins to charge. The rate at which energy is stored in the capacitor is given by the time derivative of the amount of energy stored in the capacitor, which is a function of the charge on the capacitor.

SKETCH Figure 6.29 shows a diagram of the series circuit containing a battery, a resistor, and a capacitor.

RESEARCH The charge on the capacitor as a function of time is given by equation 6.5:

$$q(t) = CV_{\text{emf}}\left(1 - e^{-t/RC}\right).$$

The energy stored in a capacitor that has charge q is given by

$$U = \tfrac{1}{2}\frac{q^2}{C}. \tag{i}$$

The time derivative of the energy stored in the capacitor is then

$$\frac{dU}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}\frac{q^2(t)}{C}\right) = \frac{q(t)}{C}\frac{dq(t)}{dt}. \tag{ii}$$

The time derivative of the charge is the current, i. Thus, we can replace dq/dt with the expression given by equation 6.6:

$$i(t) = \frac{dq(t)}{dt} = \left(\frac{V_{\text{emf}}}{R}\right)e^{-t/RC}. \tag{iii}$$

SIMPLIFY We can express the rate of change of the energy stored in the capacitor by combining equations (i) through (iii):

$$\frac{dU}{dt} = \frac{q(t)}{C}i(t) = \frac{CV_{\text{emf}}\left(1 - e^{-t/RC}\right)}{C}\left(\frac{V_{\text{emf}}}{R}\right)e^{-t/RC} = \frac{V_{\text{emf}}^2}{R}e^{-t/RC}\left(1 - e^{-t/RC}\right).$$

CALCULATE We first calculate the value of the time constant, $\tau = RC$:

$$RC = \left(2.50\times10^{6}\ \Omega\right)\left(1.25\times10^{-6}\ \text{F}\right) = 3.125\ \text{s}.$$

We can then calculate the rate of change of the energy stored in the capacitor:

$$\frac{dU}{dt} = \frac{(12.0\ \text{V})^2}{2.50\times10^{6}\Omega}e^{-(2.50\ \text{s})/(3.125\ \text{s})}\left(1 - e^{-(2.50\ \text{s})/(3.125\ \text{s})}\right) = 1.42521\times10^{-5}\ \text{W}.$$

ROUND We report our result to three significant figures:

$$\frac{dU}{dt} = 1.43\times10^{-5}\ \text{W}.$$

DOUBLE-CHECK The current at $t = 2.50$ s is

$$i(2.50\ \text{s}) = \left(\frac{12.0\ \text{V}}{2.50\ \text{M}\Omega}\right)e^{-(2.50\ \text{s})/(3.125\ \text{s})} = 2.16\times10^{-6}\ \text{A}.$$

The rate of energy dissipation in the resistor at this time is

$$P = \frac{dU}{dt} = i^2R = \left(2.16\times10^{-6}\ \text{A}\right)^2\left(2.50\times10^{6}\ \Omega\right) = 1.16\times10^{-5}\ \text{W}.$$

The rate at which the battery delivers energy to the circuit at this time is given by

$$P = \frac{dU}{dt} = iV_{\text{emf}} = \left(2.16\times10^{-6}\ \text{A}\right)(12.0\ \text{V}) = 2.59\times10^{-5}\ \text{W}.$$

Energy conservation dictates that at any time the energy supplied by the battery is either dissipated as heat in the resistor or stored in the capacitor. In this case, the power supplied by the battery, 2.59×10^{-5} W, is equal to the power dissipated as heat in the resistor, 1.16×10^{-5} W, plus the rate at which energy is stored in the capacitor, 1.43×10^{-5} W. Thus, our answer is consistent.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- Kirchhoff's rules for analyzing circuits are as follows:
  - Kirchhoff's Junction Rule: The sum of the currents entering a junction must equal the sum of the currents leaving a junction.
  - Kirchhoff's Loop Rule: The potential difference around a complete loop must sum to zero.
- In applying Kirchhoff's Loop Rule, the sign of the potential change for each circuit element is determined by the direction of the current and the direction of analysis. The conventions are
  - Sources of emf in the same direction as the direction of analysis provide potential gains, while sources opposite to the analysis direction give potential drops.
  - For resistors, the magnitude of the potential change is $|iR|$, where i is the assumed current and R is the resistance. The sign of the potential change depends on the (known or assumed) direction of the current as well as the direction of analysis. If these directions are the same, the resistor produces a potential drop. If the directions are opposite, the resistor produces a potential gain.
- An RC circuit contains a resistor of resistance R and a capacitor of capacitance C. The time constant, τ, is given by $\tau = RC$.
- In an RC circuit, the charge, q, as a function of time for a charging capacitor with capacitance C is given by $q(t) = CV_{\text{emf}}(1 - e^{-t/RC})$, where V_{emf} is the voltage supplied by the source of emf and R is the resistance of the resistor.
- In an RC circuit, the charge, q, as a function of time for a discharging capacitor with capacitance C is given by $q(t) = q_{\max}e^{-t/RC}$, where $q_{\max}$ is the size of the charge on the capacitor plates at $t = 0$ and R is the resistance of the resistor.

ANSWERS TO SELF-TEST OPPORTUNITIES

6.1 The resistors R_1 and R_u are in series and have an equivalent resistance of $R_{1u} = R_1 + R_u$. The resistors R_3 and R_v are in series and have an equivalent resistance of $R_{3v} = R_3 + R_v$. The equivalent resistances R_{1u} and R_{3v} are in parallel. Thus, we can write

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_{1u}} + \frac{1}{R_{3v}},$$

or

$$R_{\text{eq}} = \frac{R_{1u}R_{3v}}{R_{1u} + R_{3v}} = \frac{(R_1 + R_u)(R_3 + R_v)}{R_1 + R_u + R_3 + R_v}.$$

6.2 When the headlights are on, the battery is supplying a modest amount of current to the lights, and the potential drop across the internal resistance of the battery is small. The starter motor is wired in parallel with the lights. When the starter motor engages, it draws a large current, producing a noticeable drop in potential across the internal resistance of the battery and causing less current to flow to the headlights.

6.3

$$q = q_{\max}e^{-t/RC}$$

$$\frac{q}{q_{\max}} = 0.01 = e^{-t/RC} \Rightarrow \ln 0.01 = -\frac{t}{RC}$$

$$t = -RC \ln 0.01 = -(100\ \Omega)\left(1.00 \times 10^{-3}\ \text{F}\right)(\ln 0.01) = 0.461\ \text{s}.$$

PROBLEM-SOLVING GUIDELINES

1. It is always helpful to label everything in a circuit diagram, including all the given information and all the unknowns, as well as pertinent currents, branches, and junctions. Redraw the diagram at a larger scale if you need more space for clarity.

2. Remember that the directions you choose for currents and for the path around a circuit loop are arbitrary. If your choice turns out to be incorrect, a negative value for the current will result.

3. Review the signs for potential changes given in Table 6.1. Moving through a circuit loop in the same direction as the assumed current means that an emf device produces a positive potential change in the direction from negative to positive within the device and that the potential change across a resistor is negative. Sign errors are common, and it pays to stick to the conventions to avoid such errors.

4. Sources of emf or resistors may be parts of two separate loops. Count each circuit component as a part of each loop it is in, according to the sign conventions you've adopted for that loop. A resistor may yield a potential drop in one loop and a potential gain in the other loop.

5. It is always possible to use Kirchhoff's rules to write more equations than you need to solve for unknown currents in a circuit's branches. Write as many equations as possible for junctions, and then augment them with equations representing loops. But not all loops are created equal; you need to select them carefully. As a rule of thumb, choose loops with fewer circuit elements.

MULTIPLE-CHOICE QUESTIONS

6.1 A resistor and a capacitor are connected in series. If a second identical capacitor is connected in series in the same circuit, the time constant for the circuit will

a) decrease. b) increase. c) stay the same.

6.2 A resistor and a capacitor are connected in series. If a second identical resistor is connected in series in the same circuit, the time constant for the circuit will

a) decrease. b) increase. c) stay the same.

6.3 A circuit consists of a source of emf, a resistor, and a capacitor, all connected in series. The capacitor is fully charged. How much current is flowing through it?

a) $i = V/R$ b) zero c) neither (a) nor (b)

6.4 Which of the following will reduce the time constant in an RC circuit?

a) increasing the dielectric constant of the capacitor

b) adding an additional 20 m of wire between the capacitor and the resistor

c) increasing the voltage of the battery

d) adding an additional resistor in parallel with the first resistor

e) none of the above

6.5 Kirchhoff's Junction Rule states that

a) the algebraic sum of the currents at any junction in a circuit must be zero.

b) the algebraic sum of the potential changes around any closed loop in a circuit must be zero.

c) the current in a circuit with a resistor and a capacitor varies exponentially with time.

d) the current at a junction is given by the product of the resistance and the capacitance.

e) the time for the current development at a junction is given by the product of the resistance and the capacitance.

6.6 How long will it take, as a multiple of the time constant, τ, for the capacitor in an RC circuit to be 98% charged?

a) 9τ c) 90τ e) 0.98τ

b) 0.9τ d) 4τ

6.7 A capacitor C is initially uncharged. At time $t = 0$, the capacitor is attached through a resistor R to a battery. The energy stored in the capacitor increases, eventually reaching a value U as $t \to \infty$. After a time equal to the time constant $\tau = RC$, the energy stored in the capacitor is given by

a) U/e. c) $U(1 - 1/e)^2$.

b) U/e^2. d) $U(1 - 1/e)$.

6.8 Which of the following has the same unit as the electromotive force (emf)?

a) current

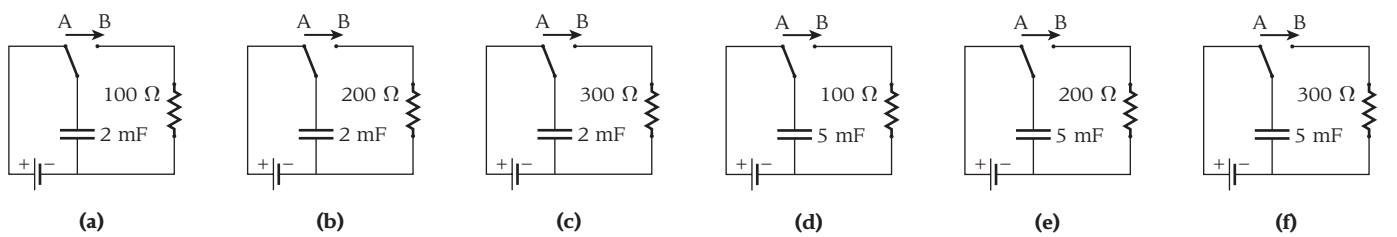
b) electric potential

c) electric field

d) electric power

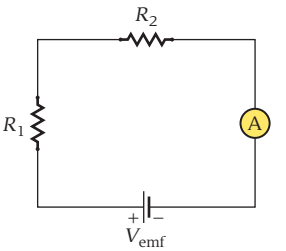
e) none of the above

6.9 The capacitor in each circuit in the figure is first charged by a 10-V battery with no internal resistance. Then, the switch is flipped from position A to position B, and the capacitor is discharged through various resistors. For which circuit is the total energy dissipated by the resistor the largest?



(a) (b) (c) (d) (e) (f)

6.10 Two resistors, $R_1 = 3.00\ \Omega$ and $R_2 = 5.00\ \Omega$, are connected in series with a battery and an ammeter, as shown in the figure. The battery supplies $V_{emf} = 8.00$ V, and the ammeter has the resistance $R_A = 1.00\ \Omega$. What is the current measured by the ammeter?



a) 0.500 A

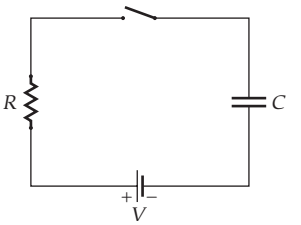
b) 0.750 A

c) 0.889 A

d) 1.00 A

e) 1.50 A

6.11 An uncharged capacitor ($C = 14.9\ \mu$F), a resistor ($R = 24.3\ \text{k}\Omega$), and a battery ($V = 25.7$ V) are connected in series, as shown in the figure. What is the charge on the capacitor at $t = 0.3621$ s after the switch is closed?



a) 5.48×10^{-5} C

b) 7.94×10^{-5} C

c) 1.15×10^{-5} C

d) 1.66×10^{-4} C

e) 2.42×10^{-4} C

6.12 Kirchhoff's Loop Rule states that

a) the algebraic sum of the currents around a complete circuit loop must be zero.

b) the resistances around a complete circuit loop must sum to zero.

c) the sources of emf around a complete circuit loop must sum to zero.

d) the sum of the potential differences around a complete circuit loop must be greater than zero.

e) the potential differences around a complete circuit loop must sum to zero.

6.13 Which of the following statements are true?

1. An ideal ammeter should have infinite resistance.
2. An ideal ammeter should have zero resistance.
3. An ideal voltmeter should have infinite resistance.
4. An ideal voltmeter should have zero resistance.

a) 1 and 3

b) 2 and 4

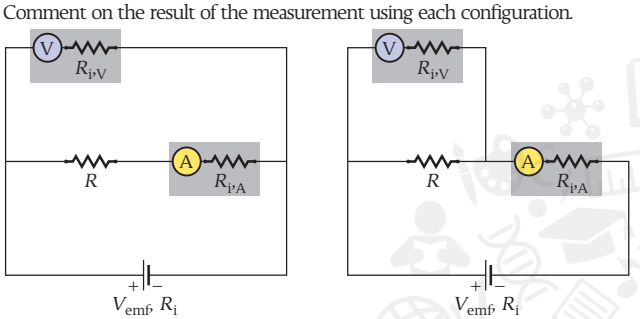
c) 2 and 3

d) 1 and 4

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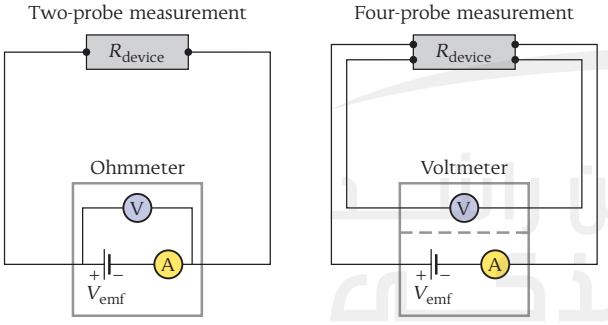
CONCEPTUAL QUESTIONS

6.14 You want to measure simultaneously the potential difference across and the current through a resistor, R. As the circuit diagrams show, there are two ways to connect the two instruments–ammeter and voltmeter–in the circuit. Comment on the result of the measurement using each configuration.



6.15 If the capacitor in an RC circuit is replaced with two identical capacitors connected in series, what happens to the time constant for the circuit?

6.16 You want to accurately measure the resistance, R_{device}, of a new device. The figure shows two ways to accomplish this task. On the left, an ohmmeter produces a current through the device and measures that current, i, and the potential difference, ΔV, across the device. This potential difference includes the potential drops across the wires leading to and from the device and across the contacts that connect the wires to the device. These extra resistances cannot always be neglected, especially if the device has low resistance. This technique is called *two-probe measurement* since two probe wires are connected to the device. The resulting current, i, is measured with an ammeter. The total resistance is then determined by dividing ΔV by i. For this configuration, what is the resistance that the ohmmeter measures? In the alternative configuration, shown on the right, a similar current source is used to produce and measure the current through the device, but the potential difference, ΔV, is measured directly across the device with a nearly ideal voltmeter with extremely large internal resistance. This technique is called a *four-probe measurement* since four probe wires are connected to the device. What resistance is being measured in this four-probe configuration? Is it different from that being assessed by the two-probe measurement? Why or why not? (*Hint:* Four-probe measurements are used extensively by scientists and engineers and are especially useful for accurate measurements of the resistance of materials or devices with low resistance.)



6.17 Explain why the time constant for an RC circuit increases with R and with C. (The answer "That's what the formula says" is not sufficient.)

6.18 A battery, a resistor, and a capacitor are connected in series in an RC circuit. What happens to the current through a resistor after a long time? Explain using Kirchhoff's rules.

6.19 How can you light a 1.0 W, 1.5 V bulb with your 12.0 V car battery?

6.20 A multiloop circuit contains a number of resistors and batteries. If the emf values of all the batteries are doubled, what happens to the currents in all the components of the circuit?

6.21 A multiloop circuit of resistors, capacitors, and batteries is switched on at $t = 0$, at which time all the capacitors are uncharged. The initial distribution of currents and potential differences in the circuit can be analyzed by treating the capacitors as if they were connecting wires or closed switches. The final distribution of currents and potential differences, which occurs after a long time has passed, can be analyzed by treating the capacitors as open segments or open switches. Explain why these tricks work.

6.22 Voltmeters are always connected in parallel with a circuit component, and ammeters are always connected in series. Explain why.

6.23 You wish to measure both the current through and the potential difference across some component of a circuit. It is not possible to do this simultaneously and accurately with ordinary voltmeters and ammeters. Explain why not.

6.24 Two light bulbs for use at 110 V are rated at 60 W and 100 W, respectively. Which has the filament with lower resistance?

6.25 Two capacitors in series are charged through a resistor. Identical capacitors are instead connected in parallel and charged through the same resistor. How do the times required to fully charge the two sets of capacitors compare?

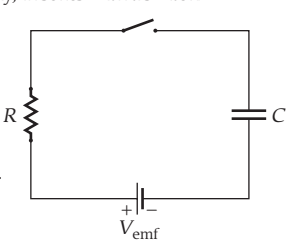
6.26 The figure shows a circuit consisting of a battery connected to a resistor and a capacitor, which is fully discharged initially, in series with a switch.

a) What is the current in the circuit at any time t?

b) Calculate the total energy provided by the battery from $t = 0$ to $t = \infty$.

c) Calculate the total energy dissipated from the resistor over the same time period.

d) Is energy conserved in this circuit?



EXERCISES

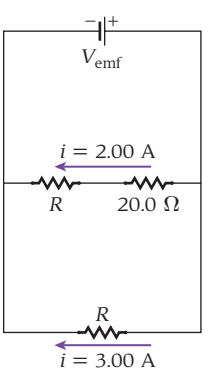
A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Sections 6.1 through 6.3

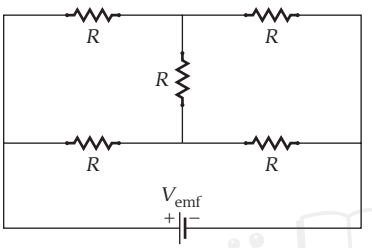
6.27 Two resistors, R_1 and R_2, are connected in series across a potential difference, ΔV_0. Express the potential drop across each resistor individually, in terms of these quantities. What is the significance of this arrangement?

6.28 A battery has $V_{emf} = 12.0$ V and internal resistance $r = 1.00\ \Omega$. What resistance, R, can be put across the battery to extract 10.0 W of power from it?

6.29 Three resistors are connected across a battery as shown in the figure. What values of R and V_{emf} will produce the indicated currents?



•6.30 Find the equivalent resistance for the circuit in the figure.

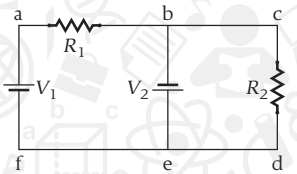


•6.31 The dead battery of your car provides a potential difference of 9.950 V and has an internal resistance of 1.100 Ω. You charge it by connecting it with jumper cables to the live battery of another car. The live battery provides a potential difference of 12.00 V and has an internal resistance of 0.01000 Ω, and the starter resistance is 0.07000 Ω.

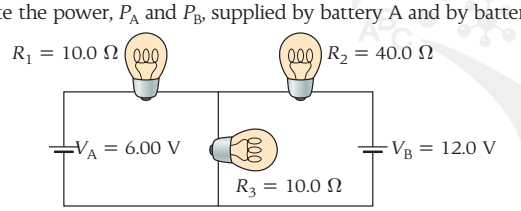
a) Draw the circuit diagram for the connected batteries.

b) Determine the current in the live battery, in the dead battery, and in the starter immediately after you closed the circuit.

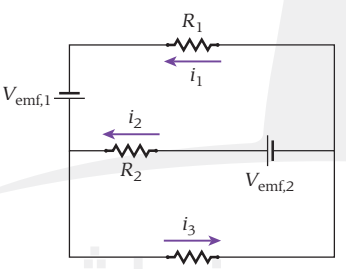
•6.32 In the circuit shown in the figure, $V_1 = 1.50$ V, $V_2 = 2.50$ V, $R_1 = 4.00$ Ω, and $R_2 = 5.00$ Ω. What is the magnitude of the current, i_1, flowing through resistor R_1?

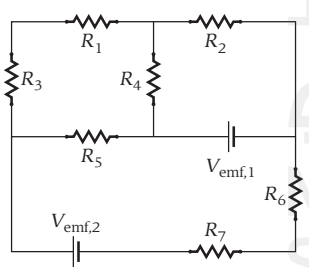


•6.33 The circuit shown in the figure consists of two batteries supplying voltages V_A and V_B and three light bulbs with resistances R_1, R_2, and R_3. Calculate the magnitudes of the currents i_1, i_2, and i_3 flowing through the bulbs. Indicate the correct directions of current flow on the diagram. Calculate the power, P_A and P_B, supplied by battery A and by battery B.



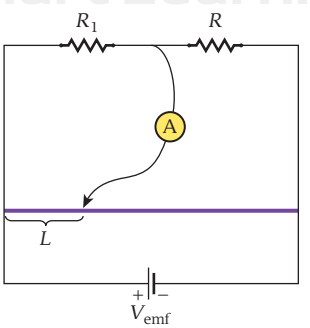
•6.34 In the circuit shown in the figure, $R_1 = 5.00\ \Omega$, $R_2 = 10.0\ \Omega$, $R_3 = 15.0\ \Omega$, $V_{emf,1} = 10.0$ V, and $V_{emf,2} = 15.0$ V. Using Kirchhoff's Loop and Junction Rules, determine the currents i_1, i_2, and i_3 flowing through R_1, R_2, and R_3, respectively, in the direction indicated in the figure.



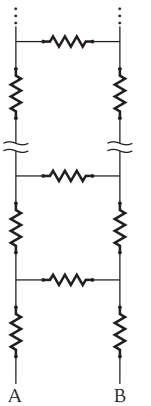


•6.35 For the circuit shown in the figure, find the magnitude and the direction of the current through each resistor and the power supplied by each battery, using the following values: $R_1 = 4.00\ \Omega$, $R_2 = 6.00\ \Omega$, $R_3 = 8.00\ \Omega$, $R_4 = 6.00\ \Omega$, $R_5 = 5.00\ \Omega$, $R_6 = 10.0\ \Omega$, $R_7 = 3.00\ \Omega$, $V_{emf,1} = 6.00$ V, and $V_{emf,2} = 12.0$ V.

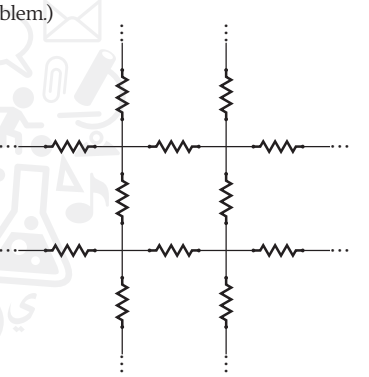
•6.36 A Wheatstone bridge is constructed using a 1.00 m long Nichrome wire (the purple line in the figure) with a conducting contact that can slide along the wire. A resistor, $R_1 = 100.\ \Omega$, is placed on one side of the bridge, and another resistor, R, of unknown resistance, is placed on the other side. The contact is moved along the Nichrome wire, and it is found that the ammeter reading is zero for $L = 25.0$ cm. Knowing that the wire has a uniform cross section throughout its length, determine the unknown resistance.



••6.37 A "resistive ladder" is constructed with identical resistors, R, making up its legs and rungs, as shown in the figure. The ladder has "infinite" height; that is, it extends very far in one direction. Find the equivalent resistance of the ladder, measured between its "feet" (points A and B), in terms of R.

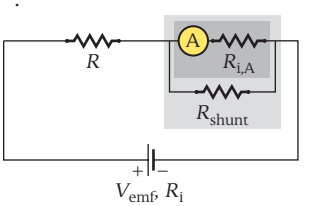


••6.38 Consider an "infinite," that is, very large, two-dimensional square grid of identical resistors, R, as shown in the figure. Find the equivalent resistance of the grid, as measured across any individual resistor. (*Hint:* Symmetry and superposition are very helpful in solving this problem.)

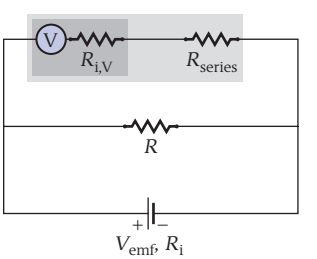


Section 6.4

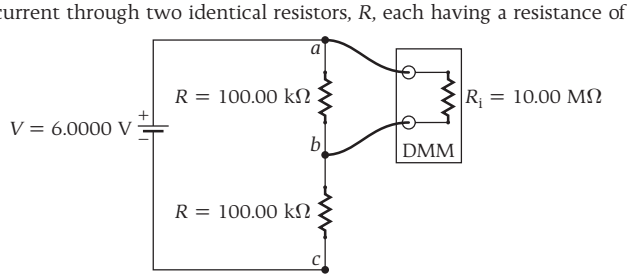
6.39 To extend the useful range of an ammeter, a shunt resistor, R_{shunt}, is placed in parallel with the ammeter as shown in the figure. If the internal resistance of the ammeter is $R_{i,A}$, determine the resistance that the shunt resistor has to have to extend the useful range of the ammeter by a factor N. Then, calculate the resistance the shunt resistor has to have to allow an ammeter with an internal resistance of 1.00 Ω and a maximum range of 1.00 A to measure currents up to 100. A. What fraction of the total 100.-A current flows through the ammeter, and what fraction flows through the shunt resistor?



6.40 To extend the useful range of a voltmeter, an additional resistor, R_{series}, is placed in series with the voltmeter as shown in the figure. If the internal resistance of the voltmeter is $R_{i,V}$, determine the resistance that the added series resistor has to have to extend the useful range of the voltmeter by a factor N. Then, calculate the resistance the series resistor has to have to allow a voltmeter with an internal resistance of 1.00 MΩ (10^6 Ω) and a maximum range of 1.00 V to measure potential differences up to 100. V. What fraction of the total 100.-V potential drop occurs across the voltmeter, and what fraction of that drop occurs across the added series resistor?



6.41 As shown in the figure, a 6.0000-V battery is used to produce a current through two identical resistors, R, each having a resistance of



100.00 kΩ. A digital multimeter (DMM) is used to measure the potential difference across the first resistor. DMMs typically have an internal resistance of 10.00 MΩ. Determine the potential differences V_{ab} (the potential difference between points *a* and *b*, which is the difference the DMM measures) and V_{bc} (the potential difference between points *b* and *c*, which is the difference across the second resistor). Nominally, $V_{ab} = V_{bc}$, but this may not be the case here. How can this measurement error be reduced?

6.42 You want to make an ohmmeter to measure the resistance of unknown resistors. You have a battery with voltage $V_{emf} = 9.00$ V, a variable resistor, *R*, and an ammeter that measures current on a linear scale from 0 to 10.0 mA.

a) What resistance should the variable resistor have so that the ammeter gives its full-scale (maximum) reading when the ohmmeter is shorted?

b) Using the resistance from part (a), what is the unknown resistance if the ammeter reads $\frac{1}{4}$ of its full scale?

•6.43 A circuit consists of two 1.00 kΩ resistors in series with an ideal 12.0 V battery.

a) Calculate the current flowing through each resistor.

b) A student trying to measure the current flowing through one of the resistors inadvertently connects an ammeter in parallel with that resistor rather than in series with it. How much current will flow through the ammeter, assuming that it has an internal resistance of 1.00 Ω?

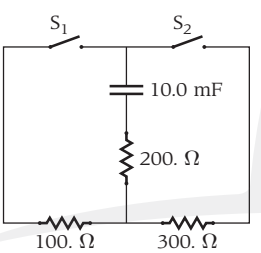
•6.44 A circuit consists of two 100. kΩ resistors in series with an ideal 12.0-V battery.

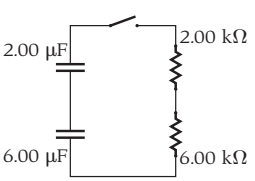
a) Calculate the potential drop across one of the resistors.

b) A voltmeter with internal resistance 10.0 MΩ is connected in parallel with one of the two resistors in order to measure the potential drop across the resistor. By what percentage will the voltmeter reading deviate from the value you determined in part (a)? (*Hint:* The difference is rather small so it is helpful to solve algebraically first to avoid a rounding error.)

Section 6.5

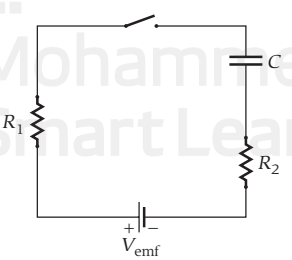
6.45 Initially, switches S_1 and S_2 in the circuit shown in the figure are open and the capacitor has a charge of 100. mC. About how long will it take after switch S_1 is closed for the charge on the capacitor to drop to 5.00 mC?





6.46 What is the time constant for the discharging of the capacitors in the circuit shown in the figure? If the 2.00 μF capacitor initially has a potential difference of 10.0 V across its plates, how much charge is left on it after the switch has been closed for a time equal to half of the time constant?

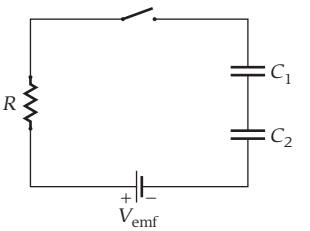
6.47 The circuit shown in the figure has a switch, S, two resistors, $R_1 = 1.00\ \Omega$ and $R_2 = 2.00\ \Omega$, a 12.0-V battery, and a capacitor with $C = 20.0\ \mu$F. After the switch is closed, what will the maximum charge on the capacitor be? How long after the switch has been closed will the capacitor have 50.0% of this maximum charge?



6.48 In the movie *Back to the Future*, time travel is made possible by a flux capacitor, which generates 1.21 GW of power. Assuming that a 1.00 F capacitor is charged to its maximum capacity with a 12.0 V car battery and is discharged through a resistor, what resistance is necessary to produce a peak power output of 1.21 GW in the resistor? How long would it take for a 12.0 V car battery to charge the capacitor to 90.0% of its maximum capacity through this resistor?

6.49 During a physics demonstration, a fully charged 90.0-μF capacitor is discharged through a 60.0 Ω resistor. How long will it take for the capacitor to lose 80.0% of its initial energy?

•6.50 Two parallel plate capacitors, C_1 and C_2, are connected in series with a 60.0-V battery and a 300. kΩ resistor, as shown in the figure. Both capacitors have plates with an area of 2.00 cm^2 and a separation of 0.100 mm. Capacitor C_1 has air between its plates, and capacitor C_2 has the gap filled with a certain porcelain (dielectric constant of 7.00 and dielectric strength of 5.70 kV/mm). The switch is closed, and a long time passes.



a) What is the charge on capacitor C_1?

b) What is the charge on capacitor C_2?

c) What is the total energy stored in the two capacitors?

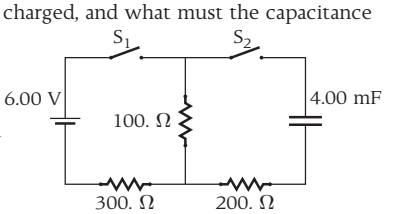
d) What is the electric field inside capacitor C_2?

•6.51 A parallel plate capacitor with $C = 0.0500\ \mu$F has a separation between its plates of $d = 50.0\ \mu$m. The dielectric that fills the space between the plates has dielectric constant $\kappa = 2.50$ and resistivity $r = 4.00 \times 10^{12}\ \Omega$ m. What is the time constant for this capacitor? (*Hint:* First calculate the area of the plates for the given *C* and κ, and then determine the resistance of the dielectric between the plates.)

•6.52 A 12.0-V battery is attached to a 2.00 mF capacitor and a 100. Ω resistor. Once the capacitor is fully charged, what is the energy stored in it? What is the energy dissipated as heat by the resistor as the capacitor is charging?

•6.53 A capacitor bank is designed to discharge 5.00 J of energy through a 10.0 kΩ resistor array in under 2.00 ms. To what potential difference must the bank be charged, and what must the capacitance of the bank be?

•6.54 The circuit in the figure has a capacitor connected to a battery, two switches, and three resistors. Initially, the capacitor is uncharged and both of the switches are open.



a) Switch S_1 is closed. What is the current flowing out of the battery immediately after switch S_1 is closed?

b) After about 10.0 min, switch S_2 is closed. What is the current flowing out of the battery immediately after switch S_2 is closed?

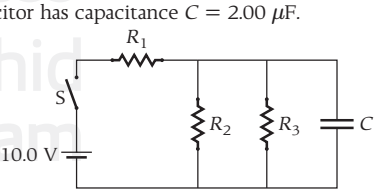
c) What is the current flowing out of the battery about 10.0 min after switch S_2 has been closed?

d) After another 10.0 min, switch S_1 is opened. How long will it take until the current in the 200.-Ω resistor is below 1.00 mA?

•6.55 In the circuit shown in the figure, $R_1 = 10.0\ \Omega$, $R_2 = 4.00\ \Omega$, and $R_3 = 10.0\ \Omega$, and the capacitor has capacitance $C = 2.00\ \mu$F.

a) Determine the potential difference, ΔV_C, across the capacitor after switch S has been closed for a long time.

b) Determine the energy stored in the capacitor when switch S has been closed for a long time.

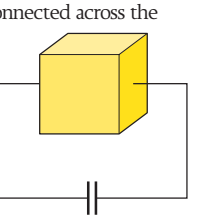


c) After switch S is opened, how much energy is dissipated through R_3?

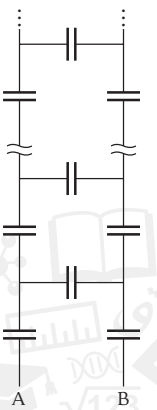
••6.56 A cube of gold that is 2.50 mm on a side is connected across the terminals of a 15.0 μF capacitor that initially has a potential difference of 100.0 V between its plates.

a) What time is required to fully discharge the capacitor?

b) When the capacitor is fully discharged, what is the temperature of the gold cube?



••**6.57** A "capacitive ladder" is constructed with identical capacitors, C, making up its legs and rungs, as shown in the figure. The ladder has "infinite" height; that is, it extends very far in one direction. Calculate the equivalent capacitance of the ladder, measured between its "feet" (points A and B), in terms of C.



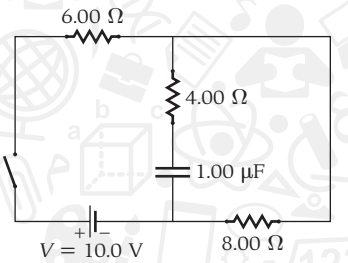
Additional Exercises

6.58 In the circuit in the figure, the capacitors are completely uncharged. The switch is then closed for a long time.

a) Calculate the current through the 4.00 Ω resistor.

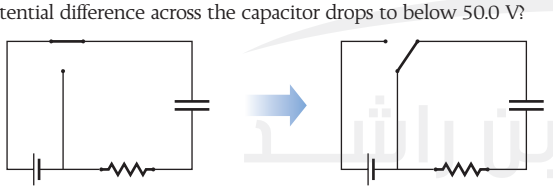
b) Find the potential difference across the 4.00 Ω, 6.00 Ω, and 8.00 Ω resistors.

c) Find the potential difference across the 1.00 μF capacitor.

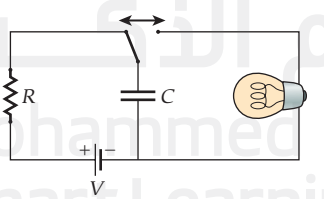


6.59 The ammeter your physics instructor uses for in-class demonstrations has internal resistance $R_i = 75.0\ \Omega$ and measures a maximum current of 1.50 mA. The same ammeter can be used to measure currents of much greater magnitudes by wiring a shunt resistor of relatively small resistance, R_{shunt}, in parallel with the ammeter. (a) Sketch the circuit diagram, and explain why the shunt resistor connected in parallel with the ammeter allows it to measure larger currents. (b) Calculate the resistance the shunt resistor has to have to allow the ammeter to measure a maximum current of 15.0 A.

6.60 Many electronics devices can be dangerous even after they are shut off. Consider an RC circuit with a 150. μF capacitor and a 1.00 MΩ resistor connected to a 200. V power source for a long time and then disconnected and shorted, as shown in the figure. How long will it be until the potential difference across the capacitor drops to below 50.0 V?

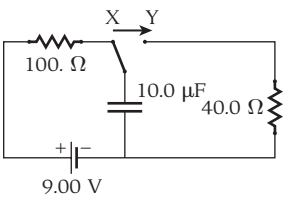


6.61 Design a circuit like that shown in the figure to operate a strobe light. The capacitor discharges power through the light bulb filament (resistance of 2.50 kΩ) in 0.200 ms and charges through a resistor R, with a repeat cycle of 1.00 kHz. What capacitor and resistor should be used?



6.62 An ammeter with an internal resistance of 53.0 Ω measures a current of 5.25 mA in a circuit containing a battery and a total resistance of 1130 Ω. The insertion of the ammeter alters the resistance of the circuit, and thus the measurement does not give the actual value of the current in the circuit without the ammeter. Determine the actual value of the current.

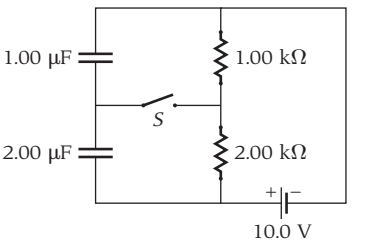
•**6.63** In the circuit shown in the figure, a 10.0 μF capacitor is charged by a 9.00 V battery with the two-way switch kept in position X for a long time. Then the switch is suddenly flicked to position Y. What current flows through the 40.0 Ω resistor a) immediately after the switch moves to position Y? b) 1.00 ms after the switch moves to position Y?

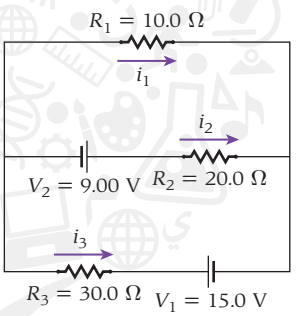


•**6.64** How long will it take for the current in a circuit to drop from its initial value to 1.50 mA if the circuit contains two 3.80 μF capacitors that are initially uncharged, two 2.20 kΩ resistors, and a 12.0-V battery all connected in series?

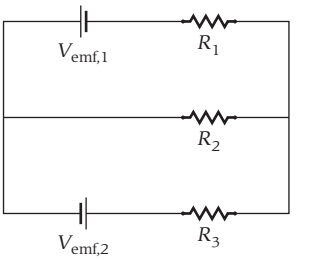
6.65 An RC circuit has a time constant of 3.10 s. At $t = 0$, the process of charging the capacitor begins. At what time will the energy stored in the capacitor reach half of its maximum value?

•**6.66** For the circuit shown in the figure, determine the charge on each capacitor when (a) switch S has been closed for a long time and (b) switch S has been open for a long time.

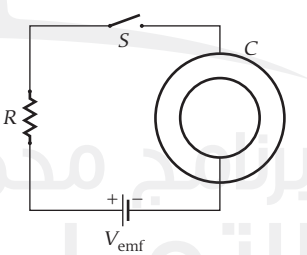




•**6.67** Three resistors, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, and $R_3 = 30.0\ \Omega$, are connected in a multiloop circuit, as shown in the figure. Determine the amount of power dissipated in the three resistors.



•**6.68** The figure shows a circuit containing two batteries and three resistors. The batteries provide $V_{emf,1} = 12.0$ V and $V_{emf,2} = 16.0$ V and have no internal resistance. The resistors have resistances of $R_1 = 30.0\ \Omega$, $R_2 = 40.0\ \Omega$, and $R_3 = 20.0\ \Omega$. Find the magnitude of the potential drop across R_2.

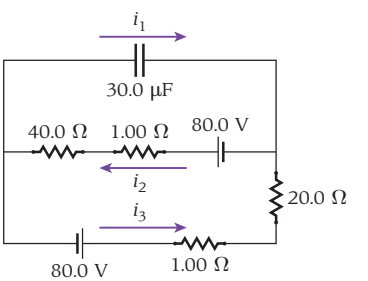


6.69 The figure shows a spherical capacitor. The inner sphere has radius $a = 1.00$ cm, and the outer sphere has radius $b = 1.10$ cm. The battery supplies $V_{emf} = 10.0$ V, and the resistor has a value of $R = 10.0$ MΩ.

a) Determine the time constant of the RC circuit.

b) Determine how much charge has accumulated on the capacitor after switch S has been closed for 0.1 ms.

•**6.70** Write the set of equations that determines the three currents in the circuit shown in the figure. (Assume that the capacitor is initially uncharged.)



•**6.71** Consider a series RC circuit with $R = 10.0\ \Omega$, $C = 10.0\ \mu$F and $V = 10.0$ V.

a) How much time, expressed as a multiple of the time constant, does it take for the capacitor to be charged to half of its maximum value?

b) At this instant, what is the ratio of the energy stored in the capacitor to its maximum possible value?

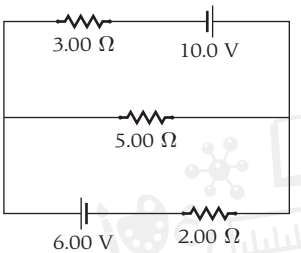
c) Now suppose the capacitor is fully charged. At time $t = 0$, the original circuit is opened and the capacitor is allowed to discharge across another

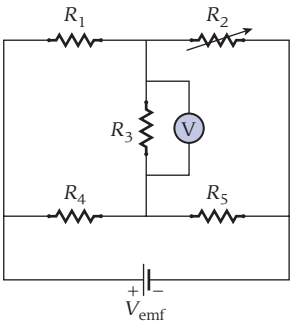
resistor, $R' = 1.00\ \Omega$, that is connected across the capacitor. What is the time constant for the discharging of the capacitor?

d) How many seconds does it take for the capacitor to discharge half of its maximum stored charge, Q?

•6.72 a) What is the current in the 5.00 Ω resistor in the circuit shown in the figure?

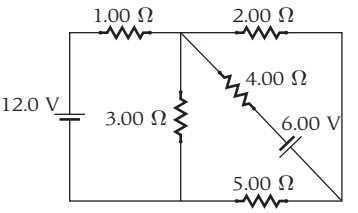
b) What is the power dissipated in the 5.00 Ω resistor?





•6.73 In the Wheatstone bridge shown in the figure, the known resistances are $R_1 = 8.00\ \Omega$, $R_4 = 2.00\ \Omega$, and $R_5 = 6.00\ \Omega$, and the battery supplies $V_{emf} = 15.0$ V. The variable resistance R_2 is adjusted until the potential difference across R_3 is zero ($V = 0$). Find i_2 (the current through resistor R_2) at this time.

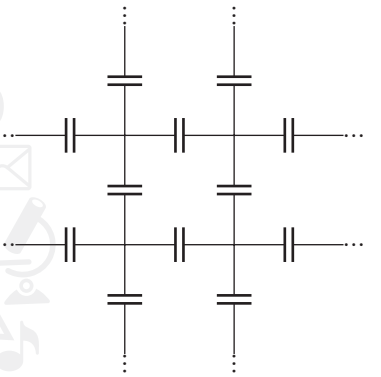
••6.74 Consider the circuit with five resistors and two batteries (with no internal resistance) shown in the figure.



a) Write a set of equations that will allow you to solve for the current in each of the resistors.

b) Solve the equations from part (a) for the current in the 4.00-Ω resistor.

••6.75 Consider an "infinite," that is, very large, two-dimensional square grid of identical capacitors, C, shown in the figure. Find the effective capacitance of the grid, as measured across any individual capacitor.



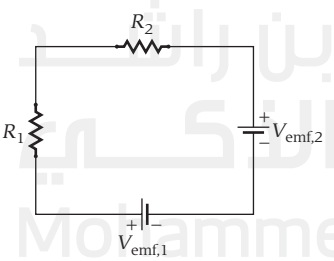
MULTI-VERSION EXERCISES

6.76 A 11.45 V battery with internal resistance $R_i = 0.1373\ \Omega$ is to be charged by a battery charger that is capable of delivering a current $i = 9.759$ A. What is the minimum emf the battery charger must be able to supply in order to charge the battery?

6.77 A battery with internal resistance $R_i = 0.1415\ \Omega$ is being charged by a battery charger that delivers a current $i = 5.399$ A. The battery charger supplies an emf of 14.51 V. What is the potential difference across the terminals of the battery?

6.78 A 16.05 V battery with internal resistance R_i is being charged by a battery charger that is capable of delivering a current $i = 6.041$ A. The battery charger supplies an emf of 16.93 V. What is the internal resistance, R_i, of the battery?

6.79 The single-loop circuit shown in the figure has $V_{emf,1} = 21.01$ V, $V_{emf,2} = 10.75$ V, $R_1 = 23.37\ \Omega$, and $R_2 = 11.61\ \Omega$. What is the current flowing in the circuit?



6.80 The single-loop circuit shown in the figure has $V_{emf,1} = 16.37$ V, $V_{emf,2} = 10.81$ V, and $R_1 = 24.65\ \Omega$. The current flowing in the circuit is 0.1600 A. What is the resistance R_2?

6.81 The single-loop circuit shown in the figure has $V_{emf,1} = 17.75$ V, $R_1 = 25.95\ \Omega$, and $R_2 = 13.59\ \Omega$. The current flowing in the circuit is 0.1740 A. What is $V_{emf,2}$?

6.82 A 15.19 mF capacitor is fully charged using a battery that supplies $V_{emf} = 131.1$ V. The battery is disconnected, and a 616.5 Ω resistor is connected across the capacitor. What current will be flowing through the resistor after 3.871 s?

6.83 A capacitor is fully charged using a battery that supplies $V_{emf} = 133.1$ V. The battery is disconnected, and a 655.1 Ω resistor is connected across the capacitor. The current flowing through the resistor after 1.743 s is 0.1745 A. What is the capacitance of the capacitor?

6.84 A 19.79 mF capacitor is fully charged using a battery. The battery is disconnected, and a 693.5 Ω resistor is connected across the capacitor. The current flowing through the resistor after 6.615 s is 0.1203 A. What is the emf supplied by the battery?

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