

احجز مكانك واستعد للامتحان بثقة كاملة

ملزمة الرياضيات الفصل الثاني

2026 Reveal 8 عام لعام

تنويه:

تم إنشاء هذه الملزمة لمساعدتك، ولكن المرجع الرئيسي هو الكتاب، وسيكون هناك ملف إضافي للأمثلة.

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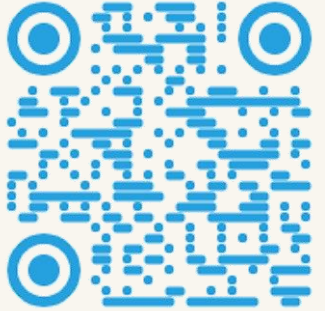




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Module (5): Functions

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Module (5): Functions

01
/
02

First & Second Lessons:
Identify Functions &
Function Tables



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Mastering Functions: The Complete Guide

From the Core Rule to Practical Application

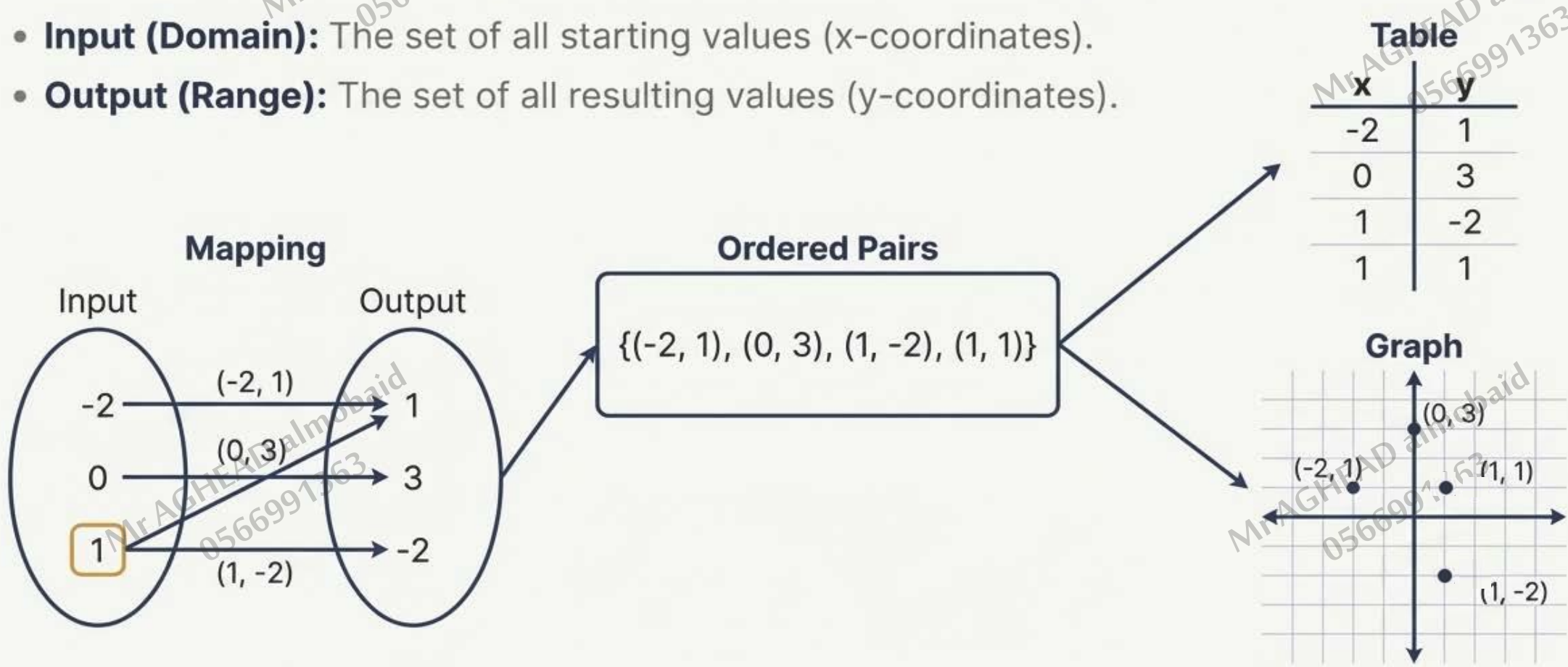


Prepared for you by mr.aghead

All Connections Begin with a Relation

Core Idea: A relation is any rule that connects a set of inputs with a set of outputs.

- **Input (Domain):** The set of all starting values (x-coordinates).
- **Output (Range):** The set of all resulting values (y-coordinates).



This specific example is intentionally **not** a function. Notice the problem? We'll solve it next.

The Golden Rule of Functions: Predictability

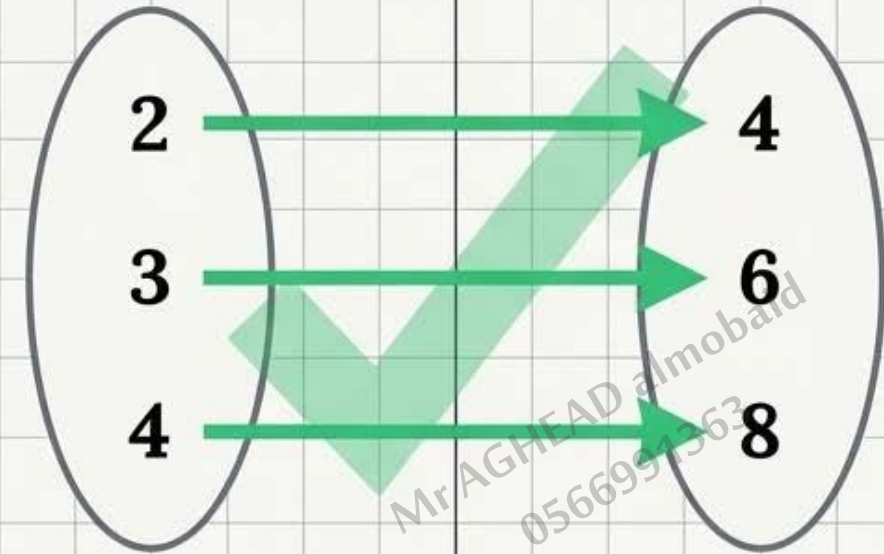
A relation becomes a function when it is predictable. The rule is simple and absolute: **Every single input must have exactly one output.**

Not a Function



AMBIGUOUS. If the input is 1, what's the output?
We can't be certain. This is not a function.

Is a Function



PREDICTABLE. If the input is 2, the output is *always* 4. This is a function.

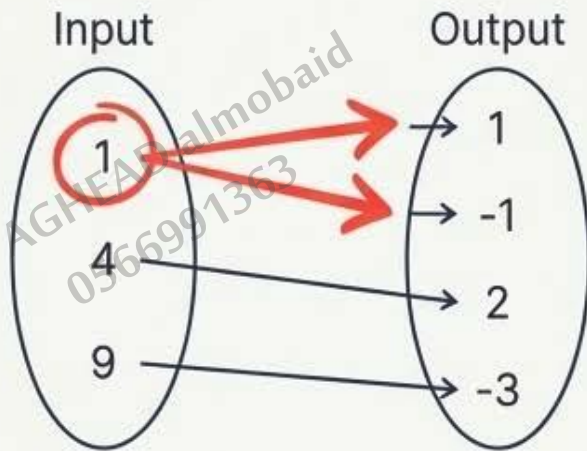
The Investigator's Toolkit: Finding the Evidence

Here are two ways to check for the Golden Rule in diagrams and tables.



Method 1: The Mapping Diagram Test

Clue: Look for any input with more than one arrow pointing away from it.



Input 1 maps to both 1 and -1. Fails the test.



Method 2: The Table Test

Clue: Look for a repeated x-value in the input column that corresponds to different y-values.

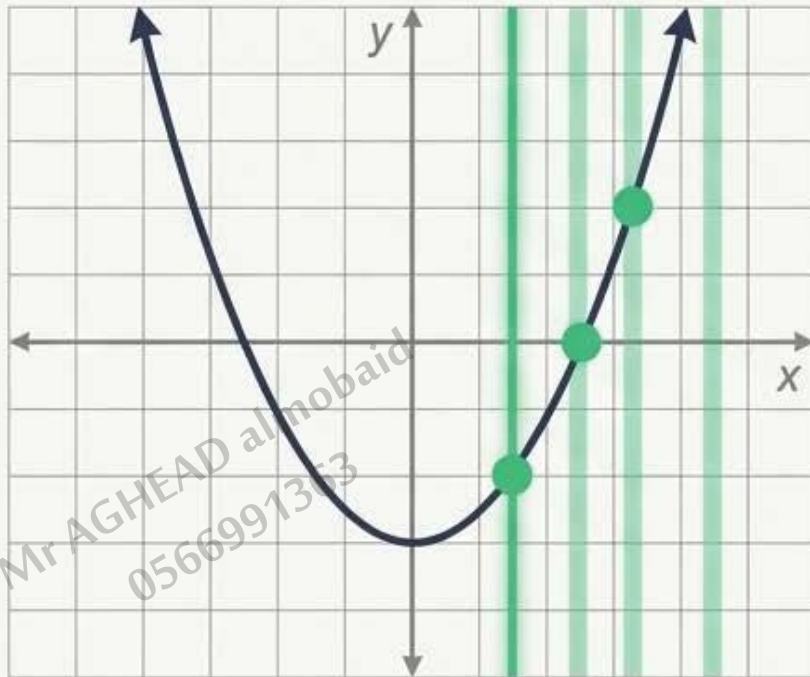
Input, x	Output, y
1	2
1	4
3	6
3	8

Input 1 corresponds to both 2 and 4. Fails the test.

The Visual Toolkit: The Vertical Line Test

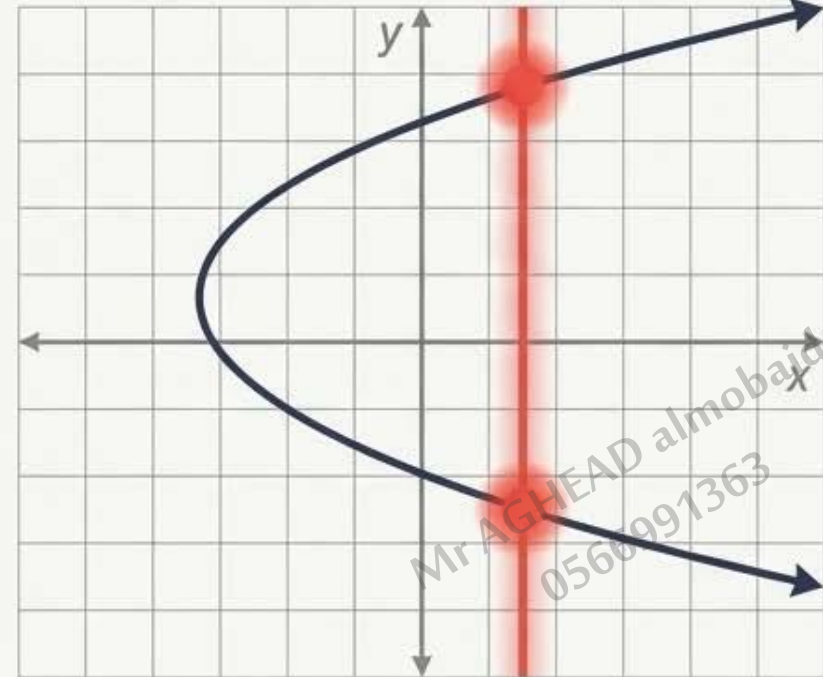
A fast, powerful test for graphs. If a vertical line can touch the graph in more than one place *at the same time*, it is not a function.

Parabola graph



PASSES. Each input (x-value) corresponds to exactly one output.

Sideways graph



FAILS. An input corresponds to more than one output.

Workshop: Are These Functions? You Decide.

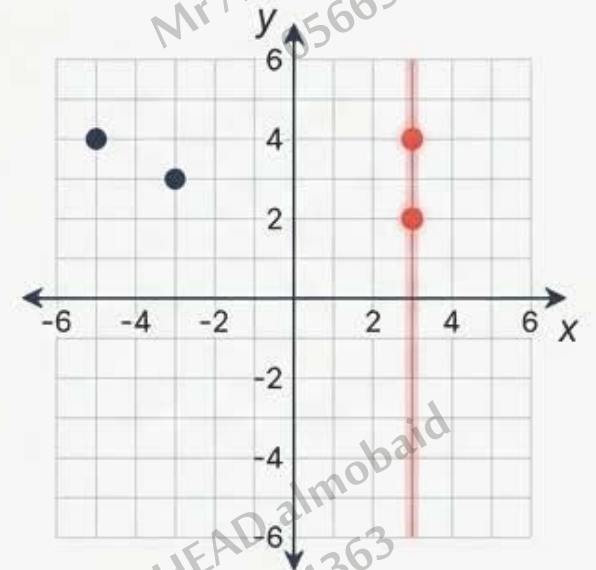
Use your toolkit. For each relation below, determine if it is a function and explain why.

Input	Output
2	1
5	-1
9	5
12	5

✓ **Function.** Each input has exactly one output. It's okay for multiple inputs to point to the same output.

Input, x	Output, y
1	2
1	3
1	4
1	5

✗ **Not a function.** The input '1' is repeated and corresponds to different outputs (3, 4, and 5).



✗ **Not a function.** A vertical line drawn at $x=3$ would pass through two points.

From Recipe to Results: Using a Function Rule

Core Idea: A function rule, like an equation $y = -3x + 4$, is the “recipe” for building a function. We use a function table to organize the inputs and their calculated results.

Function Rule: $y = -3x + 4$

Input (x)	Rule ($-3x + 4$)	Output (y)
-1	$-3(-1) + 4$	7
0	$-3(0) + 4$	4
1	$-3(1) + 4$	1
2	$-3(2) + 4$	-2

Real-World Functions: Inputs Must Make Sense

Core Idea: In practical scenarios, the situation determines which numbers are valid inputs for the domain.

Example: The Peanut Butter Factory

The number of peanuts needed (n) is given by the function $n = 770p$, where p is the number of jars.

What are appropriate input values for p ? Can you make -10 jars? Can you make 2.5 jars?

The Answer: Only positive whole numbers make sense as inputs. You cannot make a negative or a fractional number of jars.

Valid Domain

Input (p)	Output (n)	
1	770	✓
2	1540	✓
3	2310	✓
100	77000	✓

Invalid Domain

~~-10~~
~~2.5~~
<del></del>

Negative jars, half-jars, or an irrational number of jars are not possible in this context.

Bringing Functions to Life: Graphing

Core Idea: Every row in a function table is an ordered pair (x, y) that you can plot as a point. Connecting these points reveals the function's visual shape.

$$y = x + 2$$

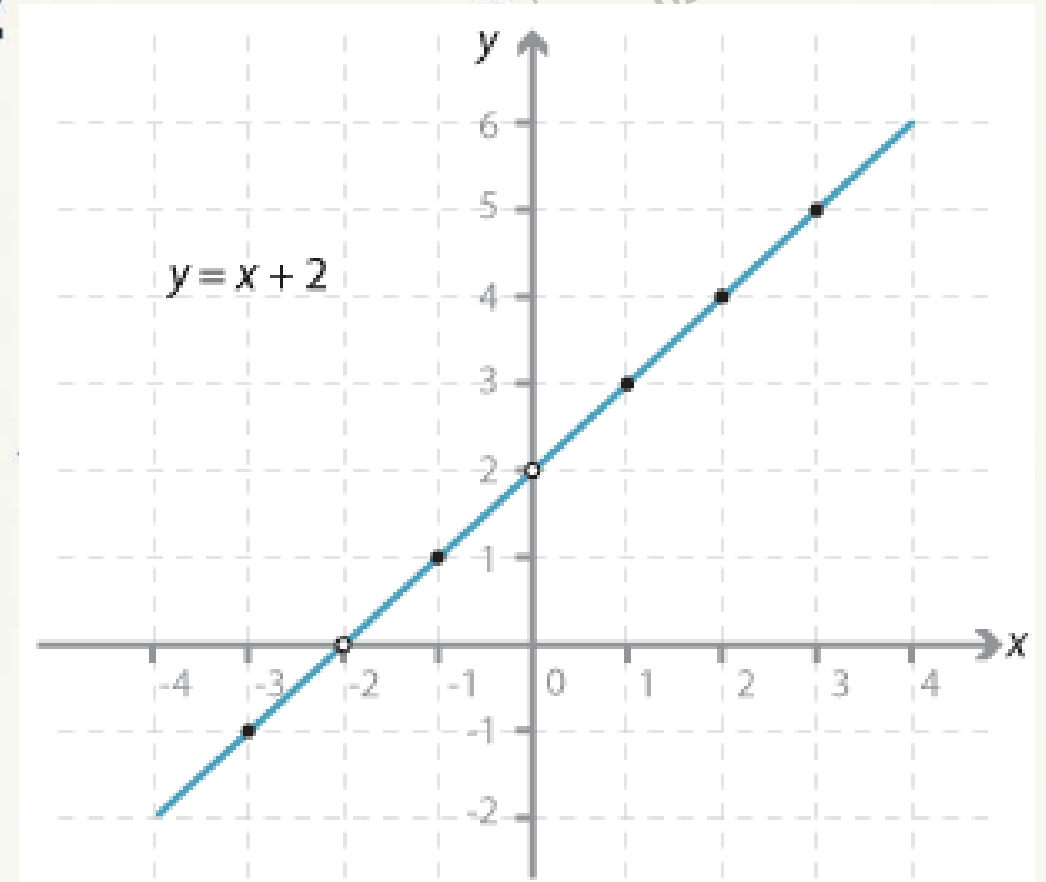
$(0, 2)$

$(1, 3)$

$(2, 4)$

$(3, 5)$

Input (x)	Rule (x + 2)	Output (y)	Ordered Pair (x, y)
0	$0 + 2$	2	$(0, 2)$
1	$1 + 2$	3	$(1, 3)$
2	$2 + 2$	4	$(2, 4)$
3	$3 + 2$	5	$(3, 5)$



Workshop: Let's Build a Graph from Scratch

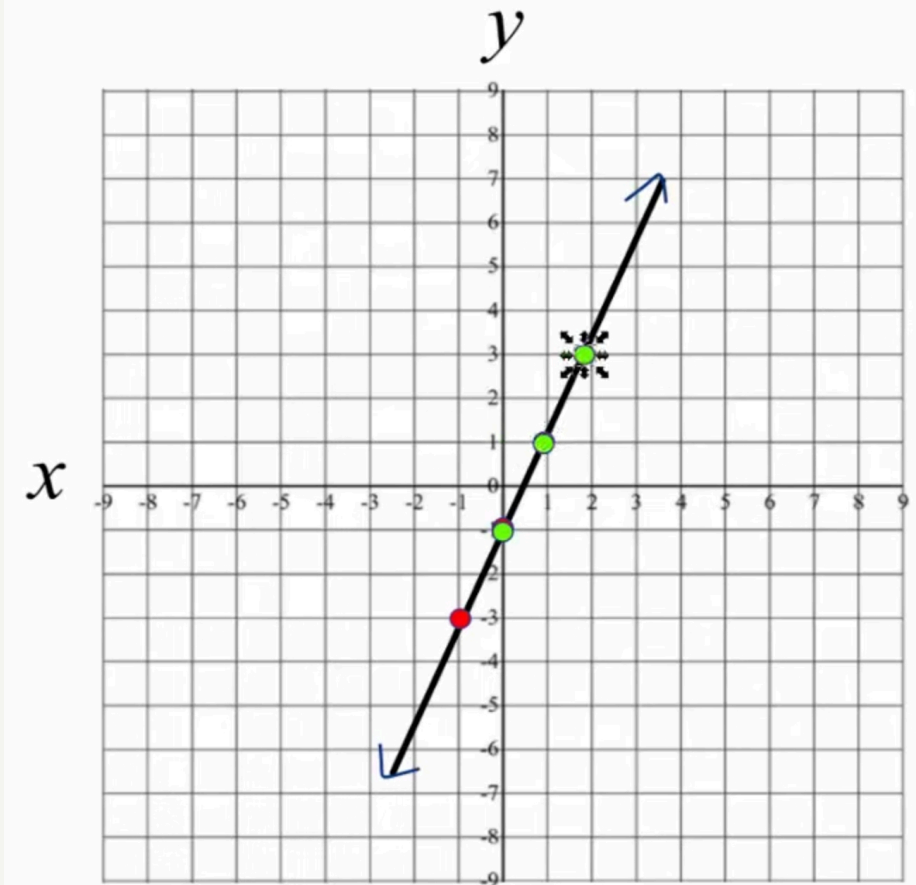
Create a function table and graph for the function $y = 2x - 1$.

Step 1: Build the Table

$$y = 2x - 1$$

Input (x)	Rule ($2x - 1$)	Output (y)	Ordered Pair (x, y)
-1	$2(-1) - 1$	-3	$(-1, -3)$
0	$2(0) - 1$	-1	$(0, -1)$
1	$2(1) - 1$	1	$(1, 1)$
2	$2(2) - 1$	3	$(2, 3)$

Step 2: Plot and Connect



Putting It All Together: The Taxi Fare

Scenario

A taxi service charges a \$2.50 flat fee plus \$0.75 per mile. The function for the total cost (C) is $C = 0.75m + 2.50$, where m is the number of miles.

Task

Your Task:

1. Create a function table showing the cost for rides of 0, 2, 5, and 10 miles.
2. Graph the function using the data from your table.

Function Table

Miles (m)	Cost (C)
0	2.50
2	4.00
5	6.25
10	10.00

Graph

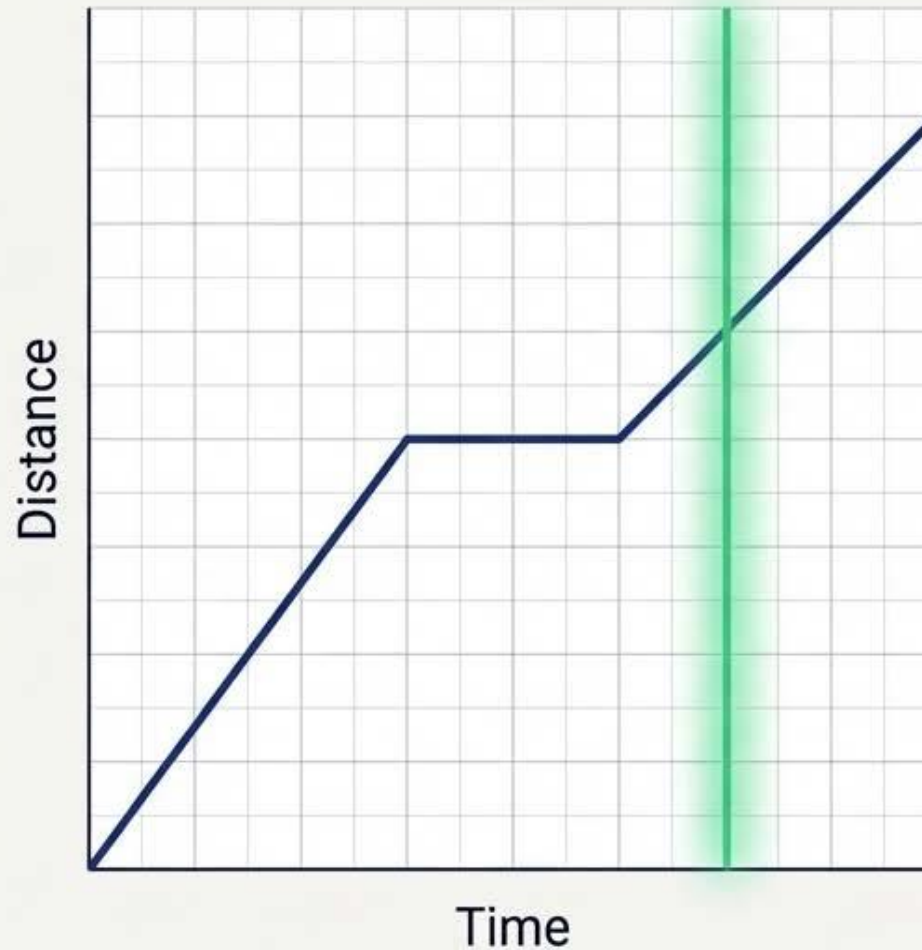


Thinking Like a Mathematician

Scenario

Camilla walks at a steady pace, stops for a few minutes to minutes to rest, then continues at a faster pace. Is the relationship between time spent walking and distance traveled a function?

Graph



Analysis

Question: At any single moment in time (**the input**), can Camilla be in two different locations (**the output**)?

Answer: No. Therefore, it **is a function**. Even when she is stopped, time moves forward, and her distance output simply stays the same for a period of time. It never fails the vertical line test.

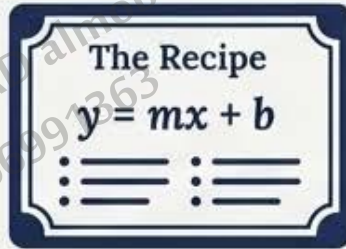
Your Functions Mastery Guide



A function is predictable:
1 Input → Exactly 1 Output.



Check the rule using **Mapping Diagrams, Tables,** and the **Vertical Line Test.**



A function rule (equation) lets you
build a table of values.



A **graph visualizes** a function, turning
table rows into **points** on a **line** or **curve.**

You've Mastered the Fundamentals

The concepts of **relations** and **functions** are the bedrock of **algebra** and beyond. Understanding them is a critical skill.

Continue **practicing** to build speed and **confidence**.

Challenge yourself: Find **functions** in the world around you. Think about your phone bill (minutes used vs. cost), the **speed of a car** (time vs. distance), or a **cooking recipe** (ingredients vs. servings).



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A presentation by **mr.aghead**

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Module (5): Functions

03

04

Third & Fourth Lessons:
Construct Linear Functions
& Compare Functions



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Mastering Linear Functions

A Practical Guide to Building and Comparing Functions from Any Source

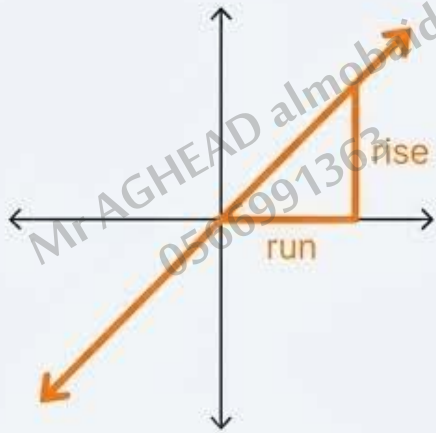
A presentation by mr.aghead

The Code to Every Linear Function: $y = mx + b$

m = Rate of Change (The Slope)

This is the "action" part of the function. It tells you how much y changes for every one unit x changes.

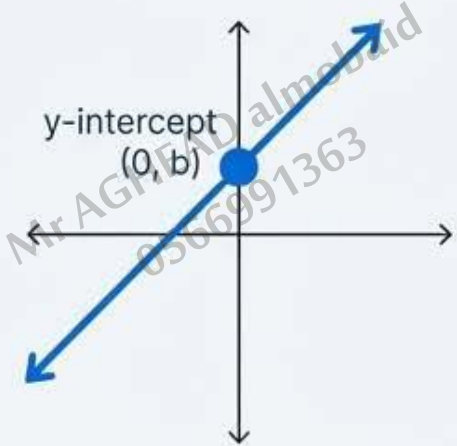
per, each, rate



b = Initial Value (The y-intercept)

This is the "starting point" of the function. It's the value of y when x is zero.

starting fee, initial amount, flat rate



$$y = mx + b$$

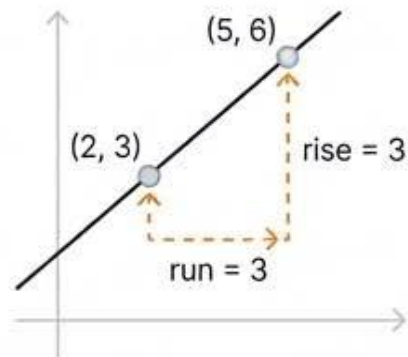
Level 1: Constructing Functions from Graphs

A graph gives you a visual story. Here's how to read it.

1. Find the Rate of Change (m)

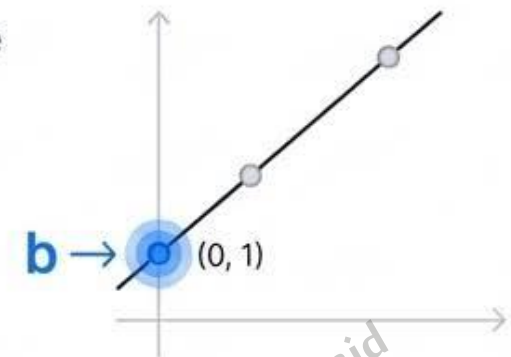
Pick any two clear points on the line. Calculate the "rise" (change in y) and the "run" (change in x).

$$m = \frac{\text{change in } y}{\text{change in } x}$$



2. Find the Initial Value (b)

Locate the point where the line crosses the vertical y -axis. That's your initial value.



3. Write the Equation

Substitute your m and b values into the $y = mx + b$ template.

$$y = 1x + 1 \rightarrow y = 1x + 1$$

Example: The Shoe Store Rewards Program

A shoe store offers free points for signing up for its rewards card. You then earn additional points for each pair of shoes purchased. The graph shows the total points earned.



Step 1: Find m .

$$\frac{(90 - 60) \text{ points}}{(4 - 2) \text{ pairs}} = \frac{30 \text{ points}}{2 \text{ pairs}} = \mathbf{m = 15}$$

Step 2: Find b .

The line crosses the y-axis at 30. $\mathbf{b = 30}$

Step 3: Write the Equation.

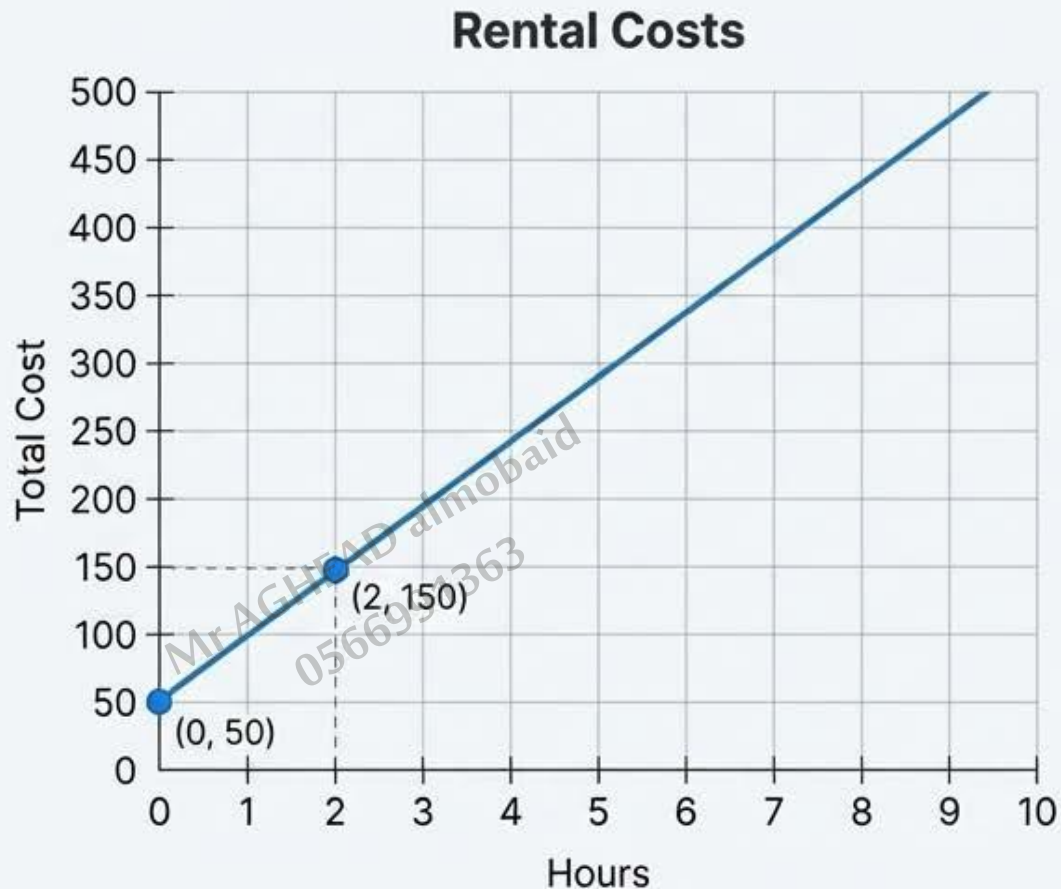
$$\mathbf{y = 15x + 30}$$

Interpretation is Power

- What m means: You earn **15 points** per pair of shoes purchased.
- What b means: You start with **30 free points** just for signing up.

Practice: The Conference Center Rental

A conference center charges an initial deposit plus an hourly rate. The graph shows the total cost for different numbers of hours.



Step 1: Find m .

Use points $(0, 50)$ and $(2, 150)$.

$$(150 - 50) / (2 - 0) = 100 / 2 = 50 \quad \mathbf{m = 50}$$

Step 2: Find b .

The line crosses the y -axis at 50. $\mathbf{b = 50}$

Step 3: Write the Equation.

$$\mathbf{y = 50x + 50}$$

Interpretation is Power

- What m means: The rental cost is **\$50 per hour**
- What b means: There is an initial **\$50 deposit**

Level 2: Constructing Functions from Tables

A table gives you precise data points. Here's how to unlock the function inside.

Step 1: Find the Rate of Change (m)

Pick any two rows (points) from the table. Calculate the change in **y** and divide it by the change in **x**.

x	y
0	50
2	60
2	60
4	90
6	80

Change in **y**:
 $90 - 60 = 30$

Change in **x**:
 $4 - 2 = 2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30}{2} = 15$$

Step 2: Find the Initial Value (b)

Substitute the **m** you just found and any (**x**, **y**) pair from the table into the $y = mx + b$ equation. Then, solve for **b**.

$$(y) = (m)(x) + b$$

$$60 = 15(2) + b$$

$$60 = 30 + b$$

$$b = 60 - 30$$

$$b = 30$$

Step 3: Write the Equation

Combine your **m** and newly found **b** into the final equation.

$$y = mx + b$$
$$y = 15x + 30$$

Example: Ava's Savings

The table shows how much money Ava has saved over several months.

Number of Months, x	Money Saved (\$), y
3	110
4	130
5	150
6	170

Step 1: Find m .

Calculation: $(130 - 110) / (4 - 3)$
 $= 20 / 1 = 20$.

$$m = 20$$

Step 2: Find b .

Substitution: Start with $y = 20x + b$.
Use the point $(3, 110)$.

$$110 = 20(3) + b$$

$$110 = 60 + b$$

$$50 = b$$

$$b = 50$$

Step 3: Write the Equation.

$$y = 20x + 50$$

Interpretation is Power

- What m means: Ava saves **\$20 each month**.
- What b means: Ava initially had **\$50 saved**.

Practice: The Smartphone Data Plan

The table shows the cost of a smartphone plan based on the gigabytes of data used per month.

Number of Gigabytes of Data, x	Cost (\$), y
2 GB	\$50
3 GB	\$65
4 GB	\$80
5 GB	\$95

Step 1: Find m .

Use points (2, 50) and (3, 65). $\frac{65 - 50}{3 - 2} = \frac{15}{1} = 15$

$$m = 15$$

Step 2: Find b .

Start with $y = 15x + b$. Use the point (2, 50).

$$50 = 15(2) + b$$

$$50 = 30 + b$$

$$20 = b$$

$$b = 20$$

Step 3: Write the Equation.

$$y = 15x + 20$$

Interpretation is Power

- What m means: The cost is **\$15 per gigabyte** of data.
- What b means: There is a **\$20 monthly base fee**.

Level 3: Constructing Functions from Words

A word problem is a function in disguise. Here are the clues to look for.

Clue #1: Finding the Rate of Change (m)

Look for phrases that indicate a repeating action or cost.

per (hour, item, month)

each

for every

rate of

Clue #2: Finding the Initial Value (b)

Look for phrases that indicate a one-time fee or a starting amount.

initial fee

starting value

flat rate

deposit

base charge

Pro Tip

If the problem gives you two data points (e.g., 'cost is \$10 for 3 hours and \$15 for 5 hours'), treat them just like two rows in a table to find m first.

Example: Joan's Photo Album

Joan plans to add **12 photos** to her photo album **each week**. After **8 weeks**, there are **120 photos** in the album.

Step 1: Find m.	The phrase "12 photos... each week" directly tells us the rate of change. m = 12
Step 2: Find b.	We know $y = 12x + b$. We're given a point: (x=8 weeks, y=120 photos). $120 = 12(8) + b$ $120 = 96 + b$ $24 = b$ b = 24
Step 3: Write the Equation.	$y = 12x + 24$

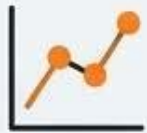
Interpretation is Power

- **What m means:** Joan adds **12 photos per week**.
- **What b means:** Joan started with **24 photos** in the album.

Practice: Tracking Plant Growth

Julie is tracking a plant for a science project. On the **2nd day**, she measured its height at **8 inches**. On the **7th day**, it was **20.5 inches**.

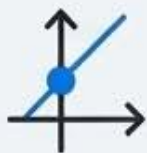
Step 1: Find m.



We have two points: (2, 8) and (7, 20.5).

$$\frac{20.5 - 8}{7 - 2} = \frac{12.5}{5} = \mathbf{2.5} \quad m = \mathbf{2.5}$$

Step 2: Find b.



Use $y = \mathbf{2.5}x + b$ and the point (2, 8).

$$8 = 2.5(2) + b$$

$$8 = 5 + b$$

$$3 = b$$

$$b = \mathbf{3}$$

Step 3: Write the Equation.

$$y = \mathbf{2.5}x + \mathbf{3}$$



💡 Interpretation is Power

- **What m means**
The plant grows **2.5 inches per day**.
- **What b means**
The plant's initial height at day 0 was **3 inches**.



The Boss Level: Comparing Functions to Make Decisions

You will be given multiple options (e.g., cell phone plans, job offers, service companies) presented in different ways: an equation, a table, a graph, or a story.

Your Mission

1. **Standardize:** For EACH option, find its rate of change (***m***) and initial value (***b***) and write its equation in **$y = mx + b$** form.
2. **Analyze:** Compare the '***m***' values (Who has the higher rate?) and the '***b***' values (Who has the higher starting cost?).
3. **Decide:** Use the equations to answer a specific question (e.g., 'Which is cheaper for 10 months?' or 'Who saves more after 20 weeks?').



1	16	15
2	14	30
3	18	35
4	10	35
5	20	40



$$y = mx + b$$

Comparison Scenario: Cell Phone Bills

Inter Regular: Angela's plan costs a **\$49 initial fee** plus **\$0.15 per minute**. Benjamin's plan is shown in the graph. Who has the greater initial cost? Who pays more per minute?

Team Angela (Verbal)

Find b

initial fee of \$49

$$b = 49$$

Find m

\$0.15 per minute

$$m = 0.15$$

Equation

$$y = 0.15x + 49$$

Team Benjamin (Graph)

Find b

Graph starts at $y=60$

$$b = 60$$

Find m

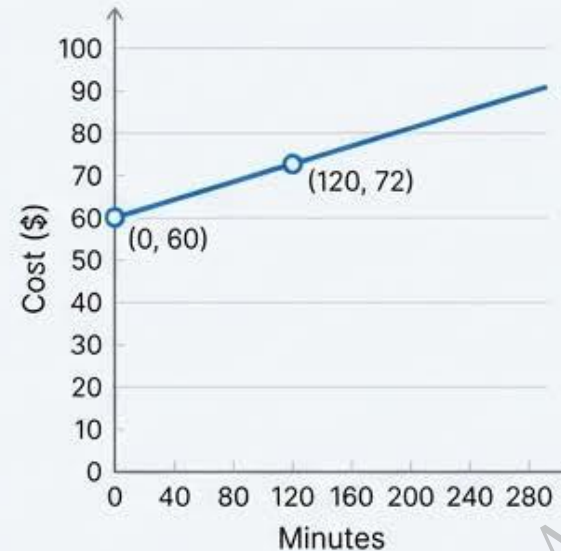
Use points (0, 60) and (120, 72).

$$(72 - 60) / (120 - 0) = 12 / 120 = 0.10$$

$$m = 0.10$$

Equation

$$y = 0.10x + 60$$



Conclusion

Initial Cost: Benjamin's initial cost (**\$60**) is greater than Angela's (**\$49**).

Rate: Angela's rate (**\$0.15/min**) is greater than Benjamin's (**\$0.10/min**).

Final Challenge: Choosing an eBook Subscription

Problem

Aisha is researching three different online book subscription companies. She plans to download 10 books. Which company should she choose to spend the lesser amount?

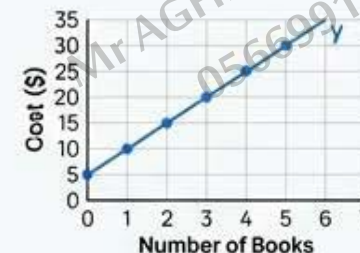
The Bookshelf

$$y = 4.5x + 20$$

We-Reads

Number of Books	Cost (\$)
0	18
1	23.75
2	29.50
3	35.25
4	41

Bookworm



Analysis Table

Company	How to Find m and b	Rate of Change (m)	Initial Value (b)	Equation
The Bookshelf	From equation $y = 4.5x + 20$	4.5	20	$y = 4.5x + 20$
We-Reads	From table: $(23.75 - 18) / (1 - 0)$	5.75	18	$y = 5.75x + 18$
Bookworm	From graph: $(15 - 5) / (2 - 0)$ and y-int	5	5	$y = 5x + 5$

The Decision: Cost for 10 Books ($x=10$)

The Bookshelf: $y = 4.5(10) + 20 = \$65.00$

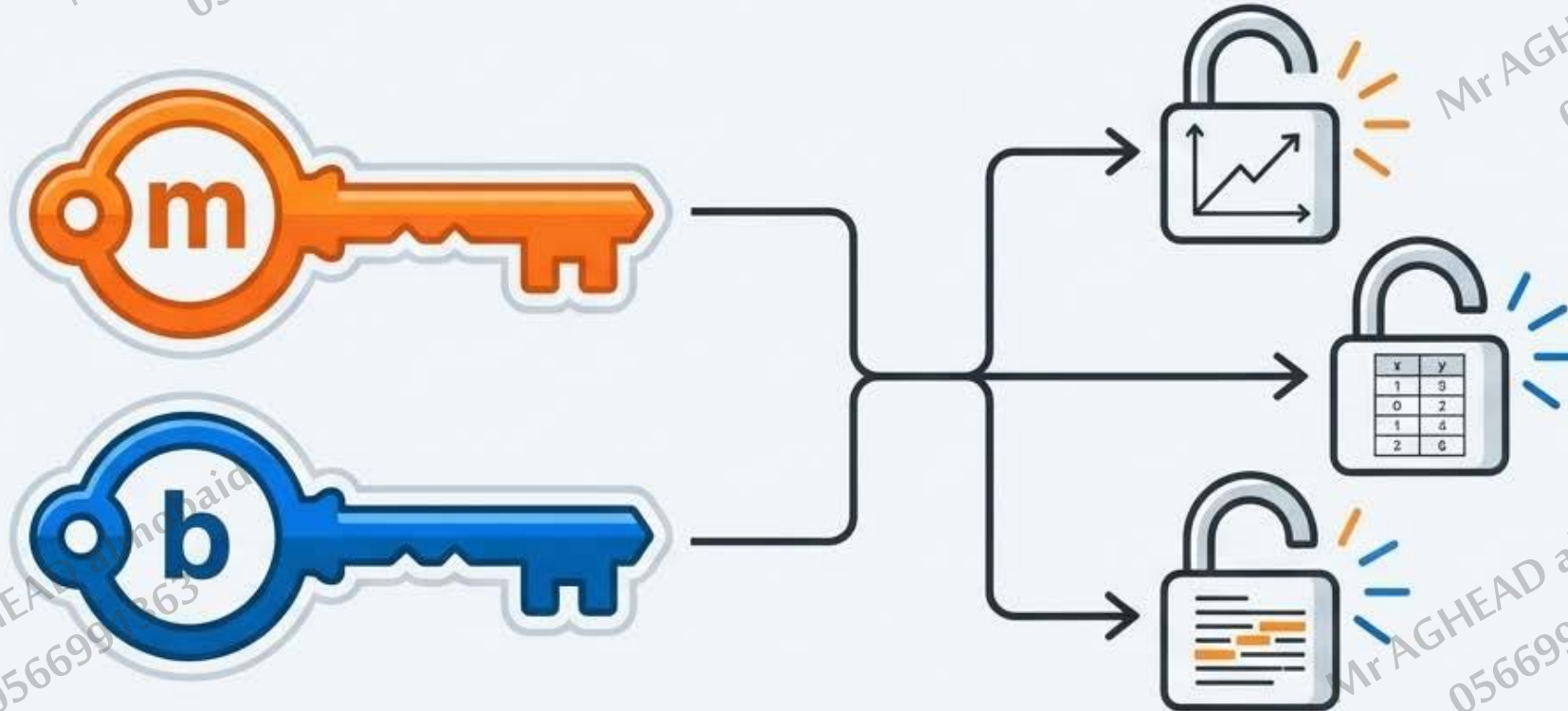
We-Reads: $y = 5.75(10) + 18 = \$75.50$

Bookworm: $y = 5(10) + 5 = \$55.00$

Final Answer: To spend the lesser amount, Aisha should choose **Bookworm**.

You've Mastered the Code

Every linear story, whether shown in a graph, a table, or words, is built from the same two components.



By learning to find the **rate of change** (highlighted in Tangerine Orange) and the **initial value** (highlighted in Ocean Blue), you have gained a powerful analytical tool. You can now confidently compare options, predict outcomes, and make smarter, data-driven decisions in any situation that follows a linear pattern. This isn't just math—it's a practical life skill.



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Module (5): Functions

05
/ 06

Fifth & Sixth Lessons:
Nonlinear Functions &
Qualitative Graphs

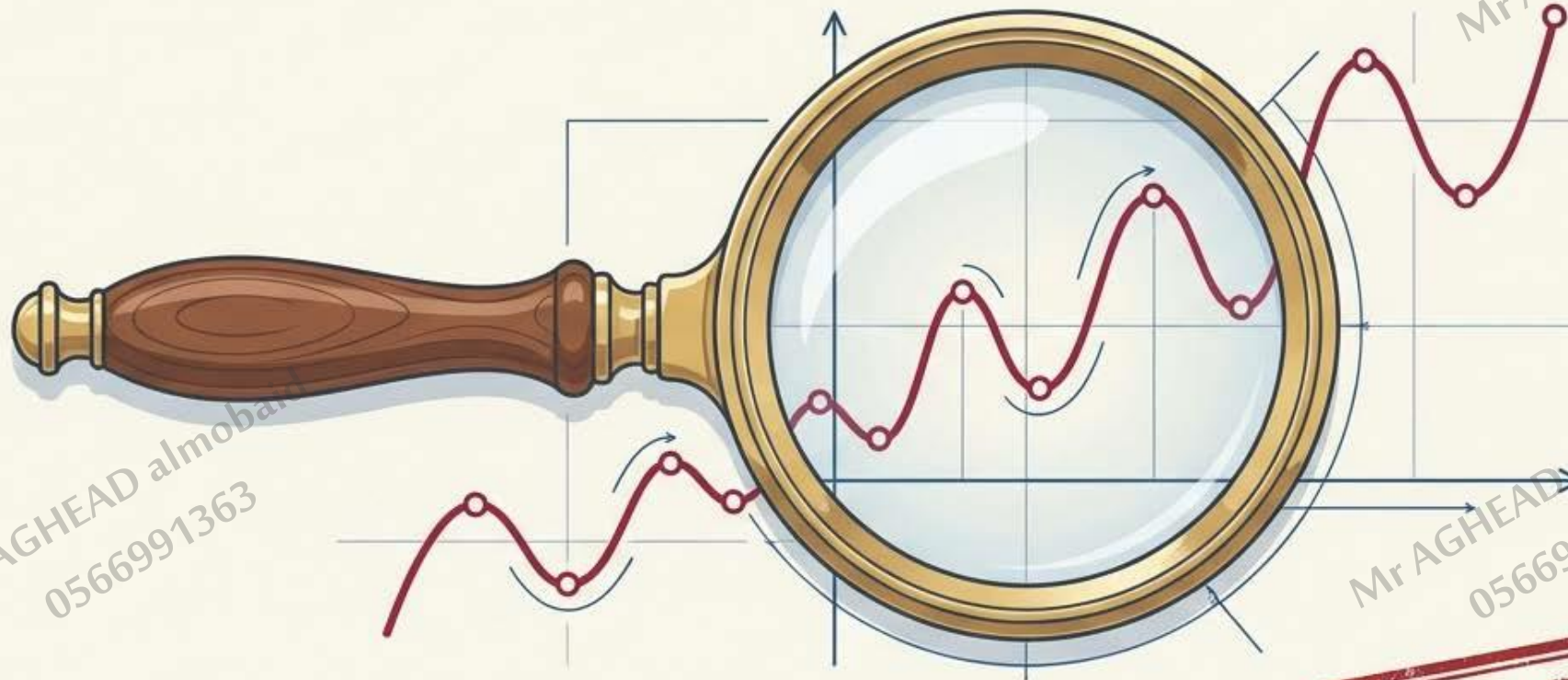


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The Function Detective's Handbook

Uncovering the Stories Behind the Graphs



TOP SECRET

Prepared for you by mr.aghead



Your Mission, Should You Choose to Accept It...

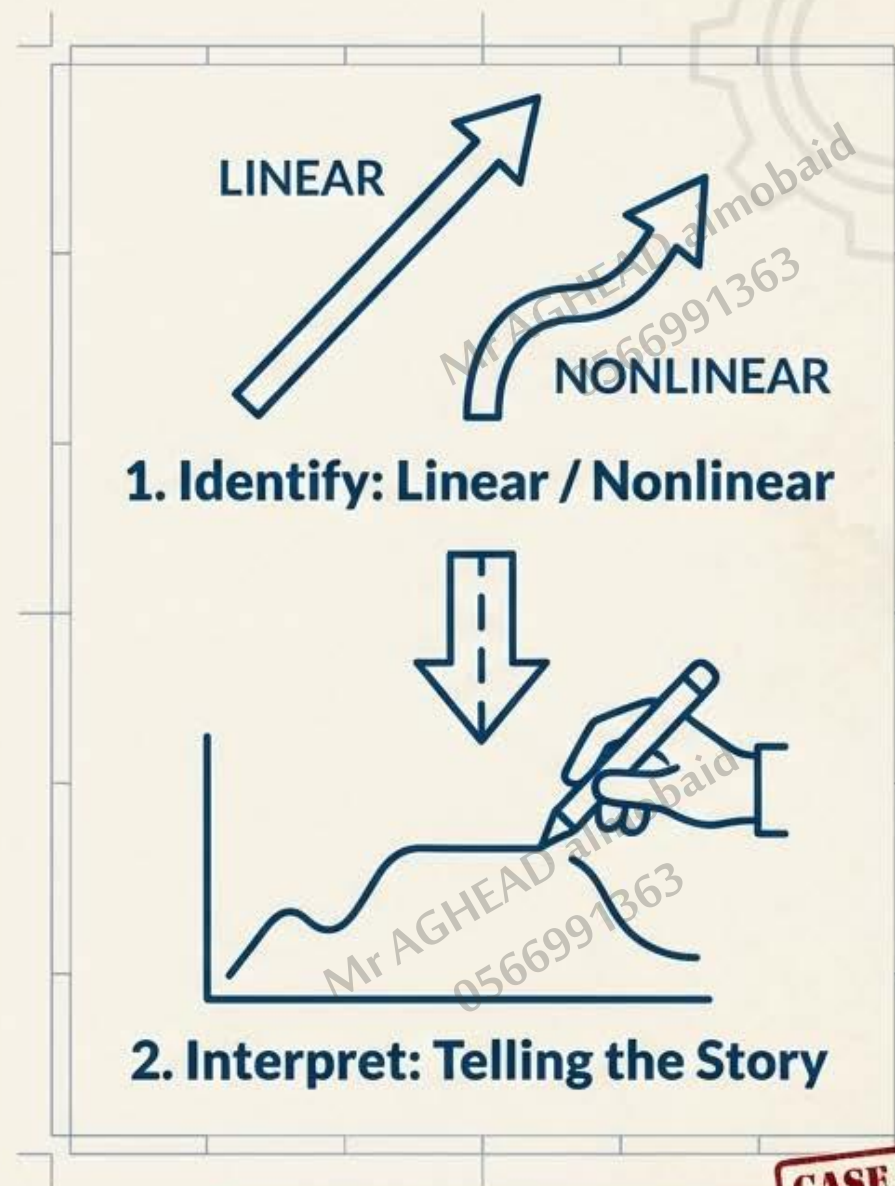
Functions are everywhere, telling stories about the world. Some stories are simple and predictable (a straight line), while others are full of twists and turns (a curve).

Your Mission:

- ✓ To become an expert Function Detective. By the end of this briefing, you will be able to:
- ✓ 1. **Identify** a function as either Linear or Nonlinear using 3 key investigative methods.
- ✓ 2. **Interpret & Sketch** qualitative graphs that tell a story, even without numbers.

Note: A Function Detective always observes the data's pattern before drawing conclusions.

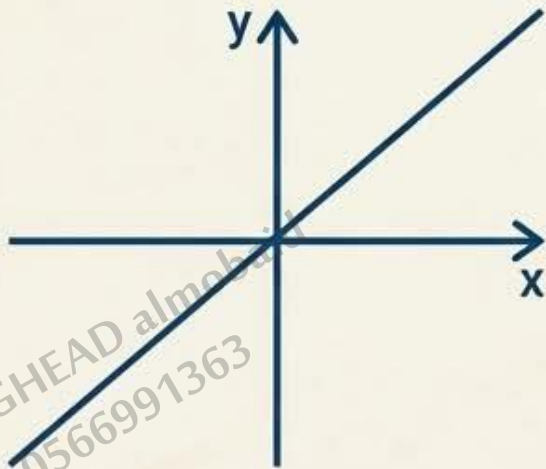
CONFIDENTIAL BRIEFING



Meet the Prime Suspects

The Predictable Professional

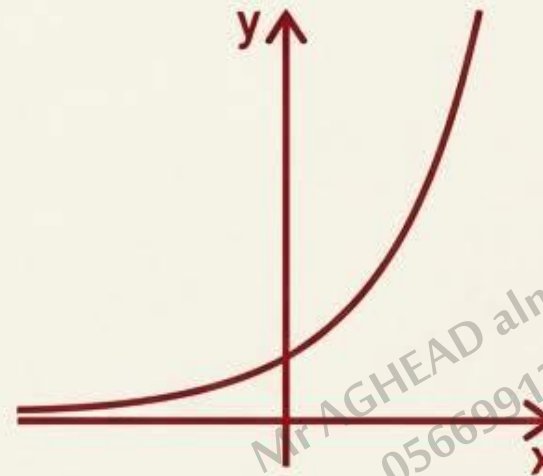
Their rate of change is **always constant**.
They **never deviate**.



Motive: Always fits the clean $y = mx + b$ profile.

The Unpredictable Artist

Their rate of change is **variable**. They are **always** changing their pace.



Motive: Breaks the $y = mx + b$ rule. Look for powers, roots, or variables in the denominator.

CASE FILE

Toolkit Part 1: The Lineup (Visual Analysis)

A function is **Linear** if its graph is a single, unbroken straight line. It is **Nonlinear** if it curves, bends, breaks, or is composed of multiple straight line segments.



Detective's Note: Heads up! A graph made of multiple straight sections, like a 'V' shape, is still **NONLINEAR**. Why? Because the rate of change (the slope) isn't constant across the *entire* graph.

CASE FILE

Toolkit Part 2: Data Interrogation (Tables)

Calculate the rate of change (change in y / change in x) between each set of points. If the rate is **constant**, the function is **Linear**. If the rate **changes**, the function is **Nonlinear**.

Case File #1: Steady Payout

x	y
5	8
10	6
15	4
20	2

Diagram illustrating the rate of change for Case File #1. The x-values increase by +5 between consecutive rows, and the y-values decrease by -2 between consecutive rows.

Rate of change is consistently $-2/5$.

**Verdict:
LINEAR**

Case File #2: Tiger Cub's Diet

Age (weeks)	Min. Calories
1	825
2	1000
3	1185
4	1320

Diagram illustrating the rate of change for Case File #2. The age increases by +1 week between consecutive rows, and the minimum calories increase by +175, +185, and +135 between consecutive rows.

The rate of change is not the same.
It changes from +175 to +185 to +135.

**Verdict:
NONLINEAR**

CASE FILE

Your Turn: Analyze These Case Files

Examine the evidence in each table. Is the rate of change constant or variable? Determine if the function is Linear or Nonlinear and state your reasoning.

Case File #1: Runner's Pace

Time (min)	Distance (mi)
15	2.2
30	4.4
45	6.6

Linear or Nonlinear? Why?

Verdict: LINEAR

Case File #2: Phone Battery

Time (hr)	Battery %
0	100
1	90
2	60
3	25

Linear or Nonlinear? Why?

Verdict: NONLINEAR

Case File #3: Monthly Savings

Month	Account Balance (\$)
1	250
2	325
3	400
4	475

Linear or Nonlinear? Why?

Verdict: LINEAR

Answer Key:

1. **Linear:** Rate is constant. The distance increases by 2.2 miles for every 15 minutes.
2. **Nonlinear:** Rate is not constant. The battery loss is -10% in the first hour, then -30%, then -35%.
3. **Linear:** Rate is constant. The balance increases by exactly \$75 each month.

CASE FILE

Toolkit Part 3: Cracking the Code (Equations)

A function is **Linear** if its equation can be written in the form **$y = mx + b$** , where **m** is the slope and **b** is the y-intercept. Any deviation makes it nonlinear.

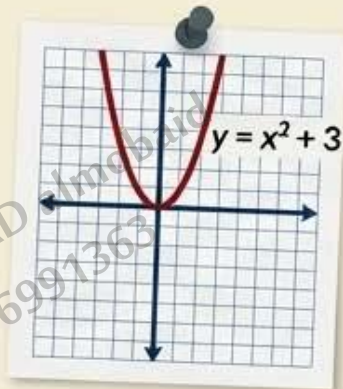
The “Most Wanted” List for Nonlinear Equations

Be on the lookout for these telltale signs:

$$x^n$$

Illicit Powers

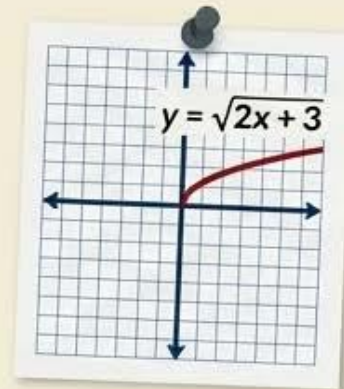
Any variable with an exponent other than 1 (e.g., x^2 , x^3).



$$\sqrt{\quad}$$

Hidden Roots

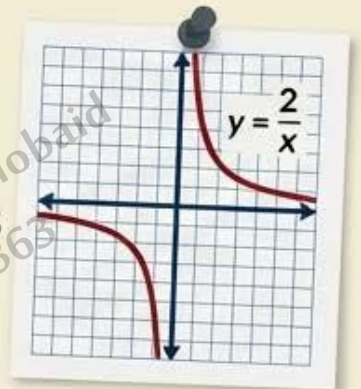
Any variable under a square root or other root symbol (e.g., $\sqrt{x}$).



$$\frac{1}{x}$$

Fugitive Variables

Any variable hiding in the denominator of a fraction (e.g., $\frac{4}{x}$).



CASE FILE

Your Turn: Interrogate These Equations

Examine the structure of each equation. Does it fit the $y = mx + b$ profile, or is it on the “Most Wanted” list? Label each as **LINEAR** or **NONLINEAR**.

Suspects

1. $y = -2x + 5$

2. $y = x^2$

3. $y = \frac{4}{x}$

4. $2.3 = y - 5.8x$

5. $y = 7 + 2x$

6. $y = \sqrt{8x}$

Challenge Case!

Answer Key & Explanations

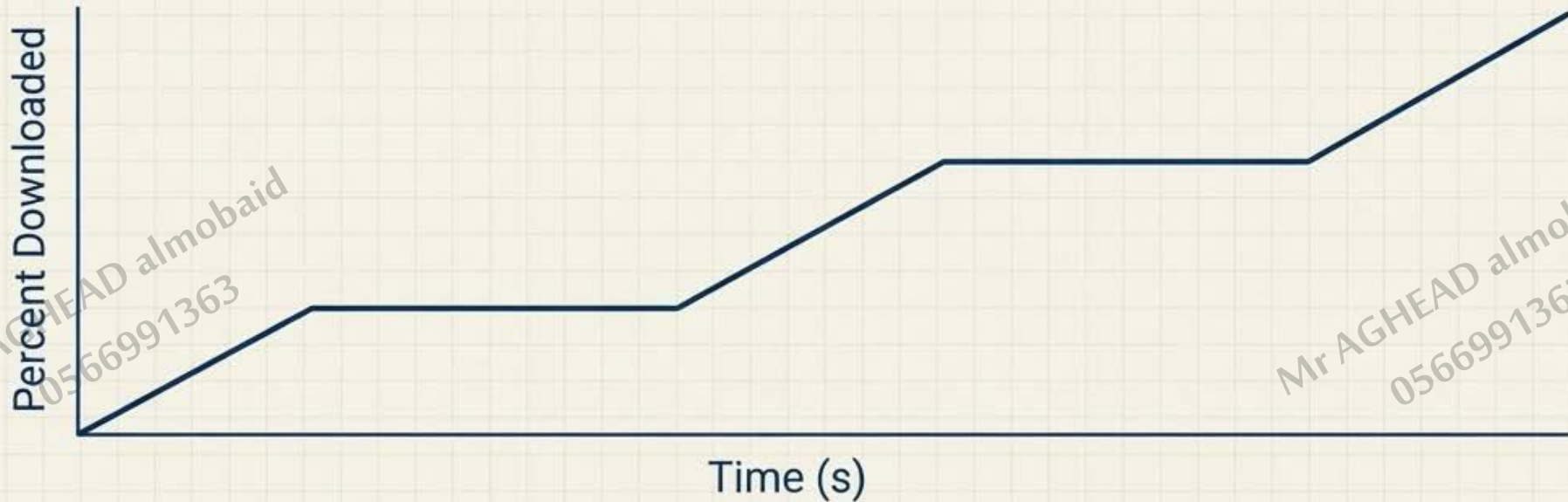
1. **LINEAR**: Perfect $y = mx + b$ form.
2. **NONLINEAR**: Power of 2 on the variable.
3. **NONLINEAR**: Variable in the denominator.
4. **LINEAR**: This is a trick! It can be rearranged to $y = 5.8x + 2.3$, which fits the profile.
5. **LINEAR**: Just a rearranged $y = 2x + 7$, which is in $y = mx + b$ form.
6. **NONLINEAR**: Variable is under a square root.

CASE FILE

Part 2: From Identifying Suspects to Telling Their Stories



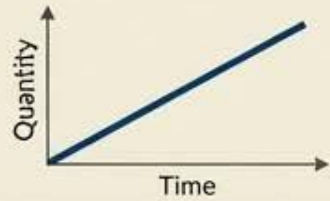
The Big Idea: "You've learned to identify functions. Now, you'll learn to interpret their stories. A **qualitative graph** shows the relationship between two quantities without using exact numbers."



Key Insight: "The **SHAPE** of the line tells the story."



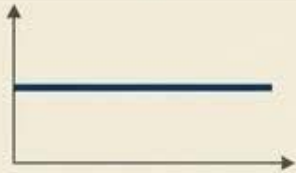
The Language of the Line: Decoding Narrative Shapes



Meaning: Increasing at a constant rate (e.g., driving at a steady speed).



Meaning: Decreasing at a constant rate (e.g., a candle burning steadily).



Meaning: No change over time (e.g., a parked car).



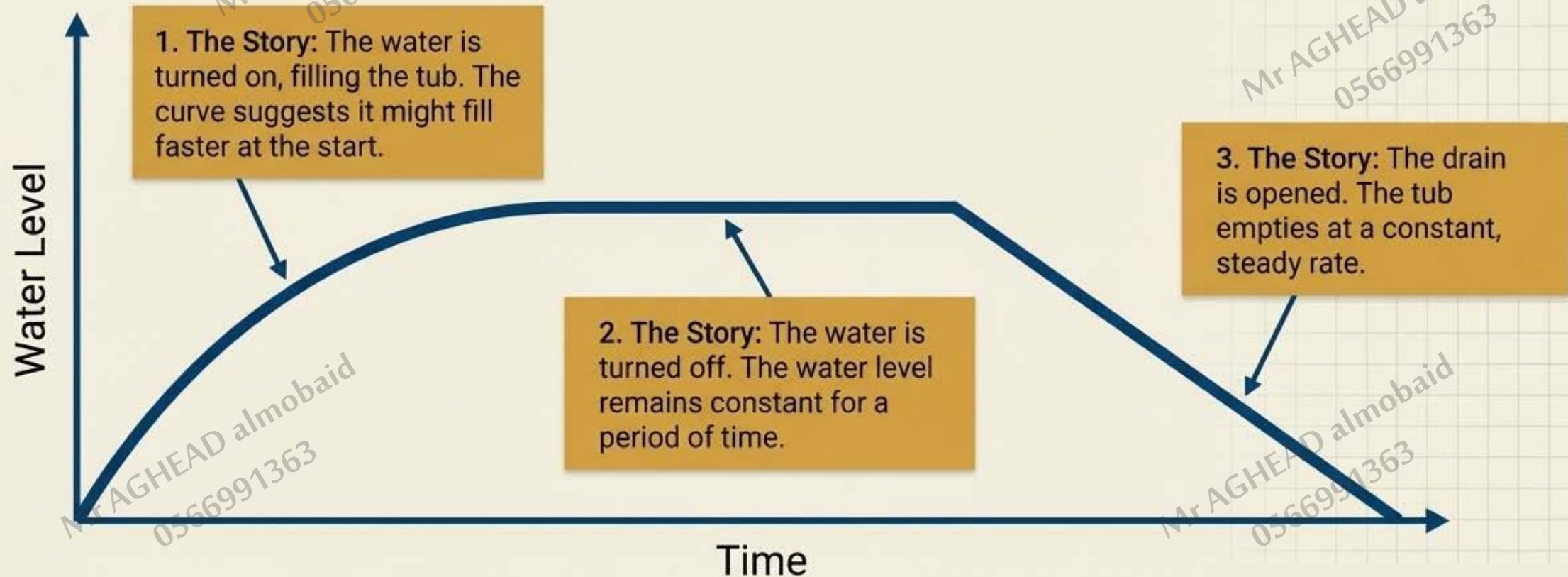
Increasing at a faster rate (e.g., accelerating).



Increasing at a slower rate (e.g., a phone charging, which slows near 100%).



Case Study: The Story of the Bathtub



Final Report: The graph tells the complete story of a bathtub being filled, left to sit, and then drained.

Your Turn: You're the Sketch Artist

Read the following scenarios. On a piece of paper, sketch a qualitative graph that tells the story. Don't forget to label your axes!



Scenario 1: Charlie's Bike Ride

Charlie is riding his bike. He speeds up, rides at a constant rate, then slows down to a stop.



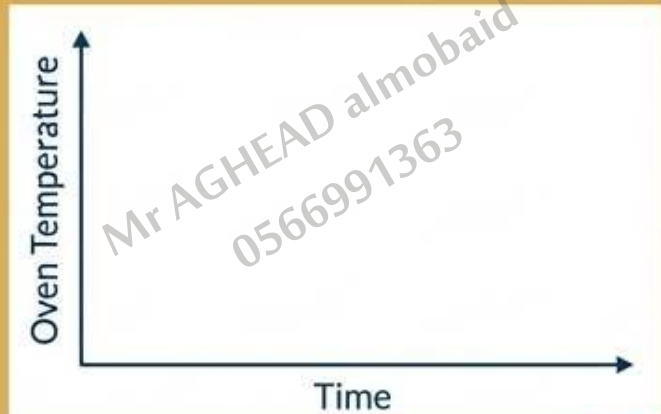
Scenario 2: The Bouncing Ball

A tennis ball is dropped onto the floor. On each successive bounce, it rebounds to a height less than its previous bounce until it comes to rest.

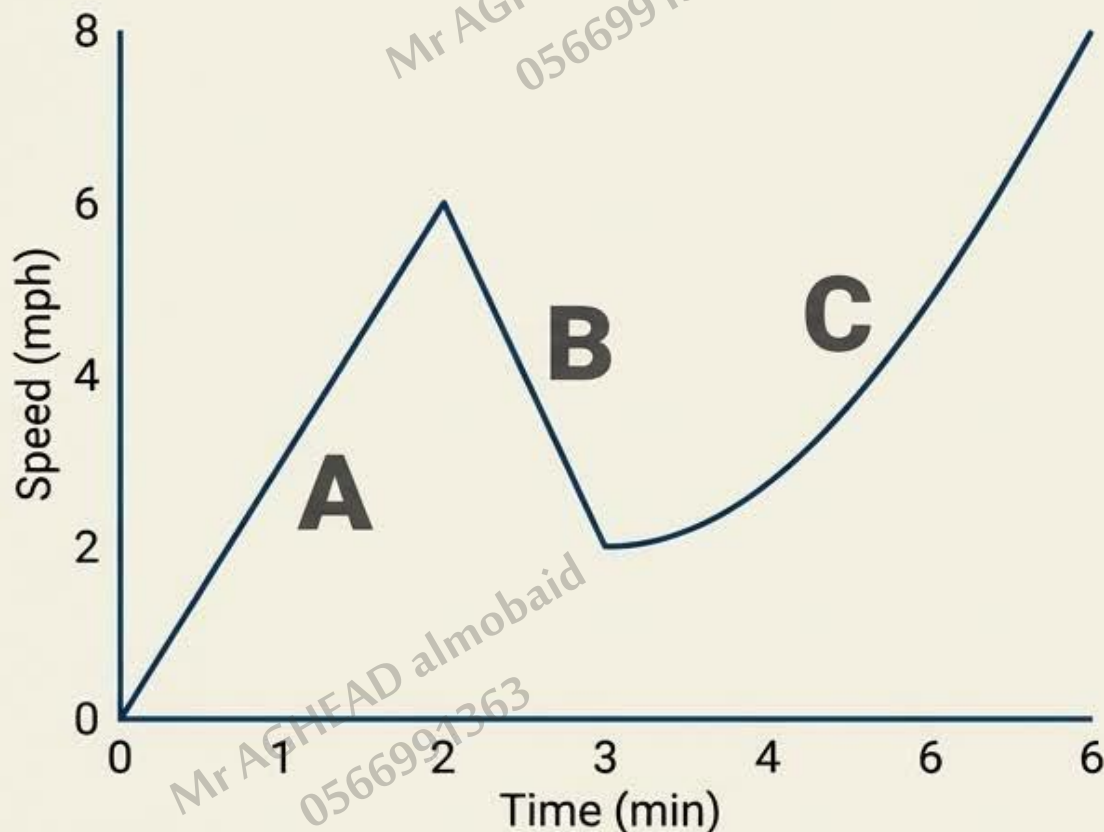


Scenario 3: Baking Cookies

You preheat an oven to a target temperature. When you open the door to put cookies in, the temperature drops slightly before returning to the target temperature, where it stays for the rest of the baking time.



Advanced Analysis: The Case of Luis's Bike Trip



The Official Report

A: He increases his speed at a constant rate for 2 minutes until he reaches 6 mph.

B: He then decreases his speed at a varied rate for 1 minute until he reaches 2 mph.

C: Finally, he increases his speed at a varied rate for 3 minutes until he reaches 8 mph.

Your Task, Detective: Match the story segments (A, B, C) to the corresponding parts of the graph.

Key Question: Looking at the entire journey, is Luis's trip a linear or nonlinear function of time?
(Answer: Nonlinear, because the rate of change is not constant throughout).



The Function Detective's Master Guide

Identity Check: Linear or Nonlinear?

From a GRAPH



Linear

A single, straight line.



Nonlinear

Any curve, bend, break, or multiple segments.

From a TABLE

x	y
1	Δy
2	Δy
3	Δx

Linear

The rate of change ($\Delta y / \Delta x$) is constant.

x	y
1	$-x$
2	Δy
3	Δy

Nonlinear

The rate of change varies between points.

From an EQUATION

$$y = mx + b$$

Linear

Can be written as $y = mx + b$.

$$x^2 \quad \sqrt{x} \quad \div \quad \frac{1}{x}$$

Nonlinear

Contains powers (x^2), roots ($\sqrt{x}$), or variables in the denominator ($1/x$).

The Story of the Shapes (A Visual Glossary)



Constant Increase



No Change



Constant Decrease



Increasing Faster (Accelerating)



Increasing Slower (Decelerating)



Case Closed!



You've mastered the art of function detection. You now have the skills to analyze graphs, tables, and equations to uncover the hidden stories they tell. Keep your eyes sharp and continue practicing your new detective skills!

Well done, Detective. —mr.aghead

Module (6): Systems of Linear Equations

	Lesson Title	Page
6-1+2+3	Solve Systems of Equations by Graphing + Determine Number of Solutions + Solve Systems of Equations by Substitution	53
6-4+5	Solve Systems of Equations by Elimination + Write and Solve Systems of Equations	69

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Module (6): Systems of Linear Equations

01

First, Second & Third Lessons:

02

Solve Systems of Equations by Graphing, Determine Number of Solutions & Solve Systems of

03

Equations by Substitution



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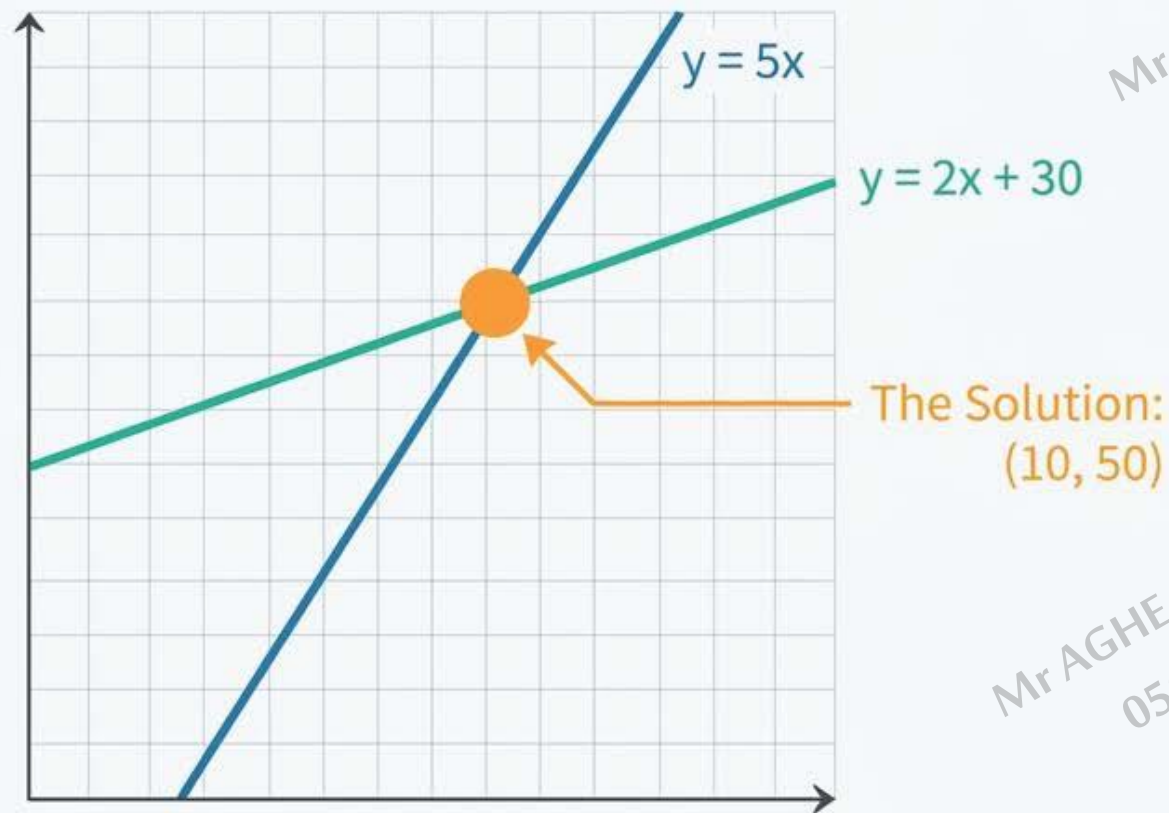
Solving Systems of Equations: The Complete Guide

Master the methods to find where paths cross.

Presented by mr.aghead

Two Equations. One Story.

A system of equations is just two or more equations working together. The “solution” is the single point (x, y) that makes *both* equations true at the same time.



The solution $(10, 50)$ means that when $x = 10$, both expressions equal 50.



Tool #1: The Visual Method (Solving by Graphing)

How it Works

1. Graph the first line on a coordinate plane.
2. Graph the second line on the same plane.
3. The point where they intersect is your solution.

When to Use It

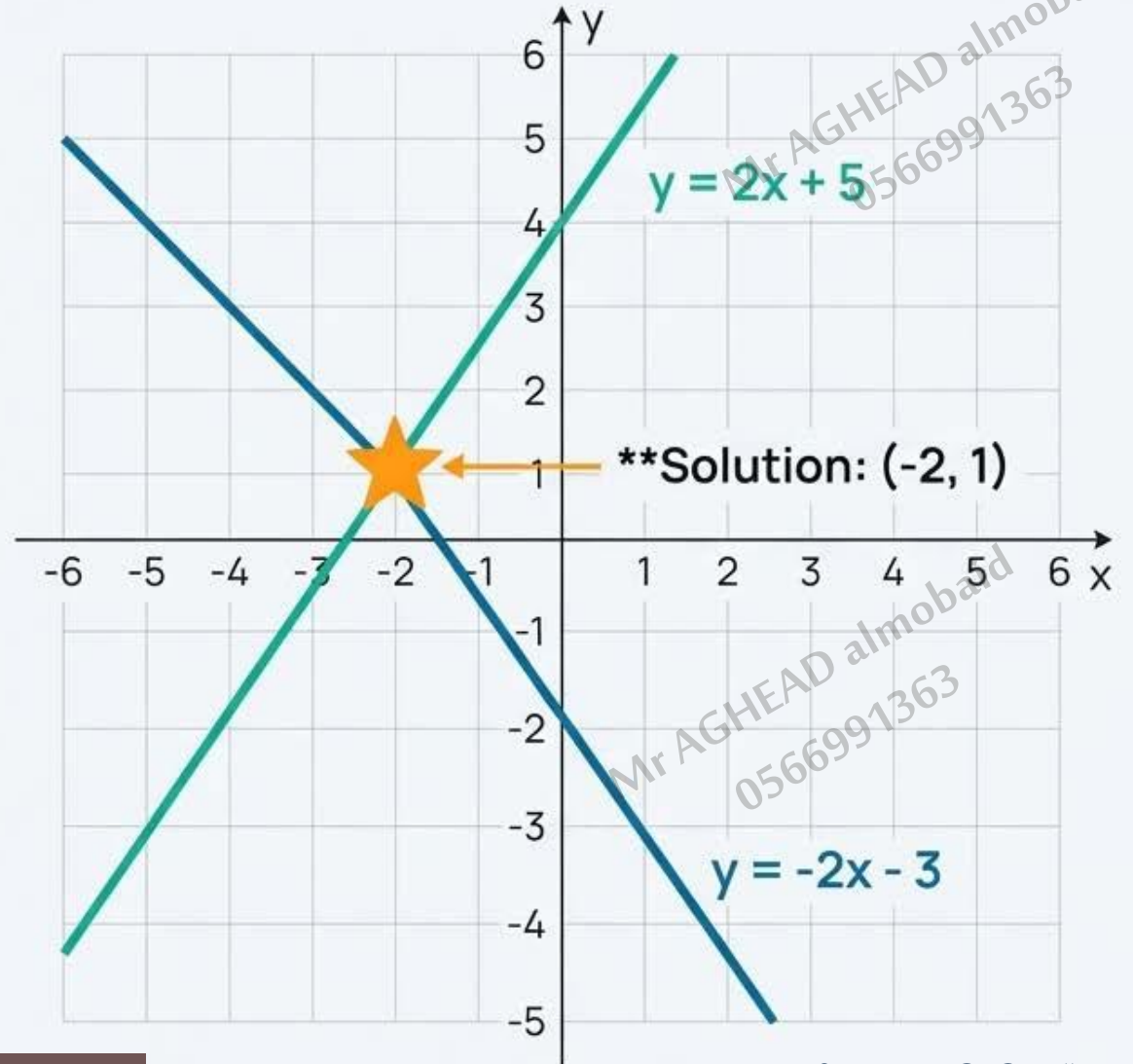
Best for visualizing the problem and finding quick estimates. It can be less precise if the solution involves decimals or fractions.

Graphing in Action: A Single Meeting Point

$$y = -2x - 3$$

$$y = 2x + 5$$

Graph both lines on the same plane to find the solution.

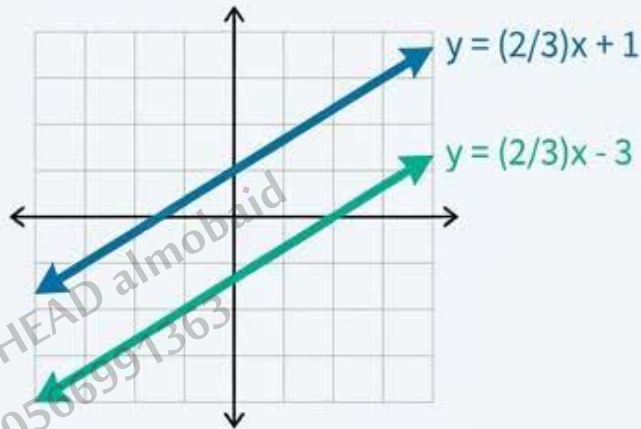


What If Lines Don't Cross? (Or Always Do?)

/// No Solution

$$y = (2/3)x + 1$$

$$y = (2/3)x - 3$$



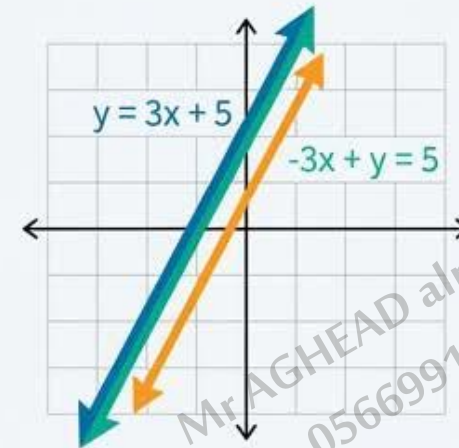
Parallel Lines: They have the same slope but different y-intercepts. They never intersect.

Result: No Solution.

/ Infinite Solutions

$$y = 3x + 5$$

$$-3x + y = 5$$



Same Line: They have the same slope and the same y-intercept. They intersect everywhere.

Result: Infinitely Many Solutions.



Tool #2: The Analytical Method (Predicting the Outcome)

You can determine the number of solutions by comparing the slopes (m) and y-intercepts (b) of your equations when they are in slope-intercept form ($y = mx + b$).

Condition	Result
 Different Slopes	One Solution (Lines will intersect at one point).
 Same Slope, Different y-intercepts	No Solution (Lines are parallel).
 Same Slope, Same y-intercept	Infinite Solutions (It's the exact same line).

Practice: Predict the Number of Solutions

For each system, rewrite the equations in $y = mx + b$ form and compare their slopes and y-intercepts.

$$y = 5x - 1$$

A

$$-5x + y = 10$$

B

Rewrite second as $y = 5x + 10$.

Same slope (5), different y-intercepts (-1 and 10).

No Solution.

$$y = -4x + 9$$

A

$$y = \frac{2}{3}x - 5$$

B

Different slopes (-4 and $\frac{2}{3}$).

One Solution.

$$y + 1 = 3x$$

A

$$2y = 6x - 2$$

B

Rewrite first as $y = 3x - 1$.

Rewrite second as $y = 3x - 1$.

Same slope (3) and same y-intercept (-1).

Infinite Solutions.



Tool #3: The Algebraic Method (Solving by Substitution)

The “3 S’s” - How it Works

1. **Solve:** Solve one of the equations for one variable (e.g., get it into the form $y = \dots$ or $x = \dots$).
2. **Substitute:** Plug this entire expression into the *other* equation in place of the variable.
3. **Solve Again:** Solve this new single-variable equation. Then, use that answer to find the value of the second variable.

When to Use It

The most precise and reliable method. Perfect for when graphing is difficult or when one variable is already isolated.

Substitution in Action: Finding the Precise Point

$$y = 3x + 8 \quad | \quad 8x + 4y = 12$$

Step 1: Isolate.

The first equation is already solved for y.

$$y = 3x + 8$$

Step 2: Substitute.

We replace y in the second equation with $(3x + 8)$.

$$8x + 4(3x + 8) = 12$$

Step 3: Solve for x.

$$8x + 12x + 32 = 12$$

Distribute

$$20x + 32 = 12$$

Combine like terms

$$20x = -20$$

Subtract 32

$$x = -1$$

Divide by 20

Step 4: Solve for y.

Substitute $x = -1$ back into the easy equation.

$$y = 3(-1) + 8$$

$$y = -3 + 8$$

$$y = 5$$

Solution: $(-1, 5)$

Substitution: What Happens in Special Cases?

Sometimes, the variables cancel out completely. The final statement tells you the answer.

No Solution



$$x - y = -4$$

$$x - y = 2/5$$

1. Solve the first for x: $x = y - 4$
2. Substitute into the second: $(y - 4) - y = 2/5$
3. Variables cancel: $-4 = 2/5$

A False Statement. It's impossible.
This means the lines never meet.
Result: No Solution.

Infinite Solutions



$$y = x + 5$$

$$2y - 2x = 10$$

1. Substitute $(x + 5)$ for y: $2(x + 5) - 2x = 10$
2. Distribute: $2x + 10 - 2x = 10$
3. Variables cancel: $10 = 10$

A True Statement. It's always true.
This means the lines are the same.
Result: Infinite Solutions.

Application: Solving the Bake Sale Problem

The Scenario

The Spanish Club is having a bake sale. A bag of trail mix is x dollars and a loaf of pumpkin bread is y dollars.

- **Equation 1:** $x + y = 11$ (One of each costs \$11)
- **Equation 2:** $3x + 2y = 24$ (Three trail mixes and two breads cost \$24)

Choosing the Best Tool

- **Graphing?** Possible, but we'd have to rewrite both equations first.
- **Substitution?** Excellent choice! It's easy to rewrite the first equation as $y = 11 - x$.

The Solution

1. Substitute:

$$3x + 2(11 - x) = 24$$

2. Solve:

$$3x + 22 - 2x = 24$$

$$\rightarrow x + 22 = 24 \rightarrow x = 2$$

$$x = 2$$

3. Back-substitute:

$$y = 11 - 2 \rightarrow y = 9$$

Trail mix costs \$2 per bag, and pumpkin bread costs \$9 per loaf.

Your Turn: Choose Your Method & Solve

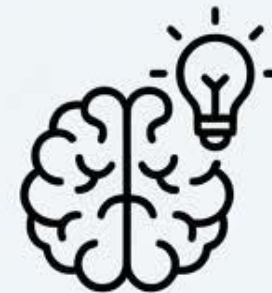
The Scenario

Kaylee and Sophia are riding scooters. Kaylee rides for x hours and Sophia for y hours.

Equation 1: $x + y = 5$ (Their total time is 5 hours)

Equation 2: $15x + 10y = 60$ (Kaylee's speed is 15 mph, Sophia's is 10 mph, total distance is 60 miles)

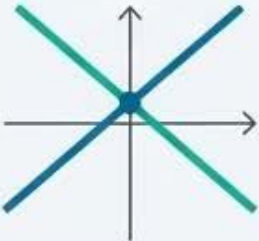
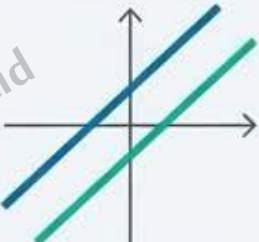
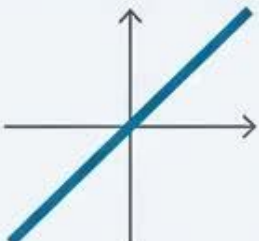
The Challenge



Which method is most efficient here?
Why?

Use your chosen method to find the solution.

The Complete Guide to Solving Systems

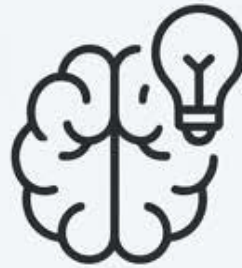
	What it Looks Like (The Graph)	How to Predict It (The Analysis)	What Happens in Algebra (The Substitution Result)
One Solution		Different Slopes	You find a unique value for x and y (e.g., $x = -1$, $y = 5$).
No Solution		Same Slope, Different y-intercepts	Variables disappear, leaving a false statement (e.g., $-4 = 2/5$).
Infinite Solutions		Same Slope, Same y-intercept	Variables disappear, leaving a true statement (e.g., $10 = 10$).

You've Mastered the Toolkit



Visualize with Graphing:

See the big picture and estimate solutions.



Predict with Analysis: Use slopes and y-intercepts as a powerful shortcut.



Calculate with Substitution:

Get precise, exact answers using algebra.

True mastery is choosing the most efficient tool for the job.

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mr.aghead

“Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding.” — William Paul Thurston

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Module (6): Systems of Linear Equations

04
/ 05

Fourth & Fifth Lessons:
Solve Systems of Equations by
Elimination & Write and Solve
Systems of Equations



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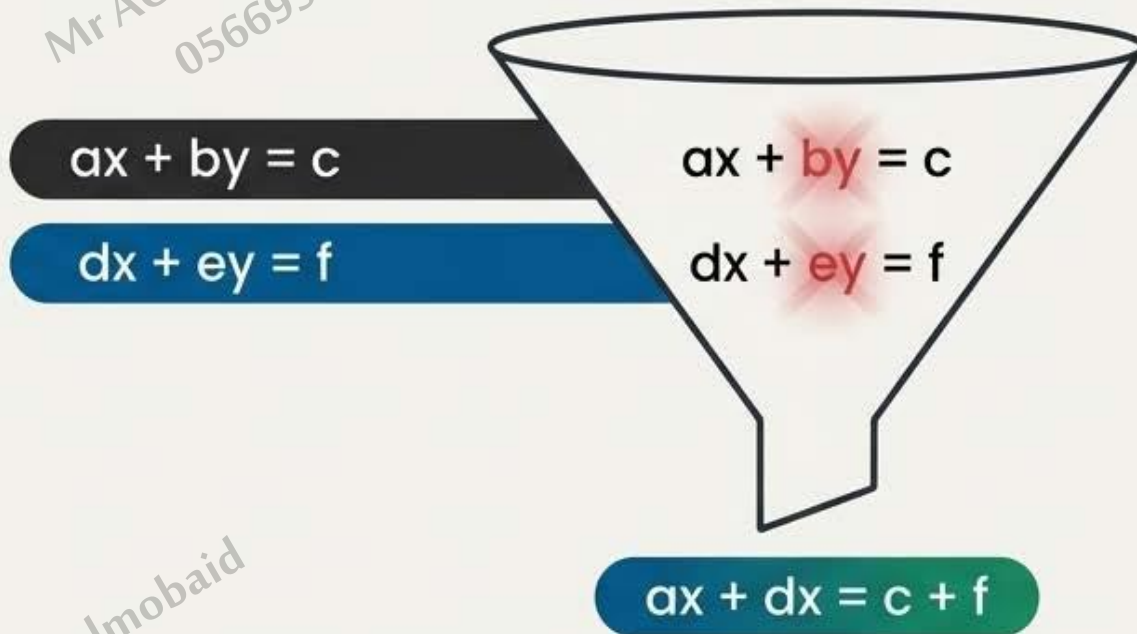
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Mastering Systems of Equations: The Elimination Method

A Comprehensive & Practical Guide

Prepared for you by mr.aghead

The Goal: Make One Variable Disappear.



The use of addition or subtraction to eliminate one variable in a system of equations is called **elimination** (in Inter Medium).

It's an algebraic method you can use to solve a system of equations by combining them in a way that makes one of the variables cancel out.

$$+y + -y$$

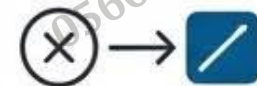
Elimination by Addition

Opposite Coefficients

$$+y - +y$$

Elimination by Subtraction

Identical Coefficients



Elimination by Multiplication

No Matching Coefficients

Level 1: Elimination by Addition

The Easiest Case: When Variables are Perfect Opposites.

Doing

Step 1: Add the equations.

$$\begin{array}{r} 2x + 5y = 3 \\ + 4x - 5y = 21 \\ \hline 6x = 24 \end{array}$$

Notice how $+5y$ and $-5y$ cancel out perfectly!

Step 2: Solve for x .

$$\frac{6x}{6} = \frac{24}{6}$$

$$x = 4$$

Step 3: Substitute and solve for y .

$$\begin{array}{r} 2(4) + 5y = 3 \\ 8 + 5y = 3 \\ 5y = -5 \\ y = -1 \end{array}$$

Step 4: The Solution.

The solution is the ordered pair $(4, -1)$.

Explaining

First, we add the two equations vertically. The terms with ' y ' have opposite coefficients ($+5$ and -5), so they add up to zero and are eliminated. This leaves us with an equation in only one variable, ' x '.

Now, we have a simple one-step equation. We divide both sides by 6 to isolate ' x ' and find its value.

We take the value of x we just found ($x=4$) and substitute it back into either of the original equations. In this case, we used the first equation. This allows us to solve for the remaining variable, ' y '.

The final step is to state the solution as an ordered pair (x, y) . The values we found for x and y make both original equations true.

Learn: Lock in the Skill: Solve by Addition



Your Turn: Solve the system of equations:

$$4x - y = -8$$

$$7x + y = -14$$



Pro Tip: Always align the equations by like terms before you begin. Look for the variable with opposite coefficients—that's your target for **elimination**!



Show Solution

Encourage students to solve before reviewing.

Step 1: Add.

$$(4x - y) + (7x + y) = -8 + (-14)$$

$$11x = -22$$

Step 2: Solve for x.

$$x = -2$$

Step 3: Substitute.

$$4(-2) - y = -8$$

$$-8 - y = -8$$

$$-y = 0$$

$$y = 0$$

Answer:

The solution is $(-2, 0)$.

Level 2: Elimination by Subtraction

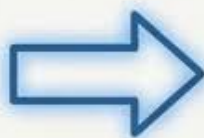
The Next Step: When Coefficients are Identical.

When the **coefficients** of a variable are the same, subtracting the equations will eliminate the variable.
Remember: subtracting an equation is the same as adding its opposite.

Worked Example: Solve $8x + 2y = 12$ and $8x + 4y = 16$

Step 1: Subtract the second equation.

$$\begin{array}{r} 8x + 2y = 12 \\ - (8x + 4y = 16) \end{array}$$



$$\begin{array}{r} 8x + 2y = 12 \\ + -8x - 4y = -16 \end{array}$$

Subtracting the entire equation flips every sign.

Step 2: Add the new opposite equation.

$$\begin{array}{r} \cancel{8x} + 2y = 12 \\ + \cancel{-8x} - 4y = -16 \\ \hline -2y = -4 \end{array}$$

Step 3: Solve for y.

$$\begin{array}{r} -2y = -4 \\ \hline -2 \quad -2 \\ \hline y = 2 \end{array}$$

Step 4: Substitute and solve for x.

$$\begin{array}{r} 8x + 2y = 12 \\ 8x + 2(2) = 12 \\ 8x + 4 = 12 \\ 8x = 8 \\ x = 1 \end{array}$$

✓ The solution is (1, 2). ✓

Test Your Understanding: Solve by Subtraction

Your Turn: Solve the system of equations:

$$-6x + 5y = 26$$

$$-6x + 2y = 14$$



Pro Tip: Be careful with signs! When you subtract the second equation, distribute the negative to *every single term* before combining.

Show Solution

Step 1: Subtract.

$$(-6x + 5y) - (-6x + 2y) = 26 - 14$$

Step 2: Simplify.

$$\cancel{-6x} + 5y + \cancel{6x} - 2y = 12$$

Step 3: Solve for y.

$$y = 4$$

Step 4: Substitute.

$$-6x + 5(4) = 26$$

$$-6x + 20 = 26$$

$$-6x = 6$$

$$x = -1$$

The solution is $(-1, 4)$. 

The Power Move: Elimination by Multiplication (Part 1)

What If Nothing Matches? Multiply ONE Equation.

Sometimes, you need to multiply each term in one equation by a constant to create opposite coefficients. The goal is to make one variable's coefficients match or be opposites.

Worked Example:

$$2x + 5y = -13$$

$$x + 3y = -5$$

Step 1: Multiply.

$$x + 3y = -5 \xrightarrow{-2} -2x - 6y = 10$$

Step 2: Add the new equation to the first.

$$\begin{array}{r} \cancel{2x} + 5y = -13 \\ + \cancel{-2x} - 6y = 10 \\ \hline -1y = -3 \end{array}$$



Strategic Thought Process

The x-coefficients are 2 and 1. We can multiply the second equation by **-2** to make the x-coefficients opposites (2 and -2).

Step 3: Solve for y.

$$\begin{array}{l} -1y = -3 \\ y = 3 \end{array}$$

Step 4: Substitute.

$$\begin{array}{l} x + 3y = -5 \\ x + 3(\mathbf{3}) = -5 \\ x + 9 = -5 \\ x = -14 \end{array}$$



The solution is (-14, 3).

The Ultimate Power Move: Multiply BOTH Equations

When one multiplication isn't enough, transform both.

In some cases, you need to multiply both equations by different constants to create opposite coefficients. The key is to find the Least Common Multiple (LCM) of the coefficients for the variable you want to eliminate.

Worked Example:

$$6x + 2y = 14$$

$$4x + 3y = 1$$



Strategic Thought Process: To eliminate y , the coefficients are 2 and 3. The LCM is 6. We can multiply the first equation by **3** (to get $+6y$) and the second equation by **-2** (to get $-6y$).

Step 1: Multiply both.

$$3(6x + 2y = 14)$$

$$18x + 6y = 42$$

$$3(6x + 2y = 14)$$

$$18x + 6y = 42$$

$$-2(4x + 3y = 1)$$

$$-8x - 6y = -2$$

Step 4: Substitute.

$$6(4) + 2y = 14$$

$$24 + 2y = 14$$

$$2y = -10$$

$$y = -5$$

Step 3: Solve for x .

$$x = 4$$

Step 2: Add the new equations.

$$18x + \cancel{6y} = 42$$

$$+ -8x - \cancel{6y} = -2$$

$$\hline 10x = 40$$



The solution is **(4, -5)**.

Your Turn: Master Multiplication

Solve both systems using elimination by multiplication. For the first, you only need to multiply one equation. For the second, you'll need to multiply both.

Problem 1 (Multiply One)

$$\begin{aligned}x + 2y &= -1 \\ 7x - 3y &= 10\end{aligned}$$

✓ Show Solution

Strategy: Multiply the first equation by -7 to eliminate x.

$$-7(x + 2y = -1) \rightarrow -7x - 14y = 7$$

Add to second eq:

$$\begin{array}{r} -7x - 14y = 7 \\ + \quad 7x - 3y = 10 \\ \hline -17y = 17 \\ y = -1 \end{array}$$

Substitute: $x + 2(-1) = -1 \rightarrow x - 2 = -1 \rightarrow x = 1$.

Answer: ✓ (1, -1).

Problem 2 (Multiply Both)

$$\begin{aligned}-6x - 4y &= 6 \\ 5x + 3y &= -4\end{aligned}$$

✓ Show Solution

Strategy: Multiply first eq by 3 and second by 4 to eliminate y.

$$3(-6x - 4y = 6) \rightarrow -18x - 12y = 18$$

$$4(5x + 3y = -4) \rightarrow 20x + 12y = -16$$

$$\begin{array}{r} \text{Add them:} \quad -18x - 12y = 18 \\ + \quad 20x + 12y = -16 \\ \hline 2x = 2 \\ x = 1 \end{array}$$

Substitute: $5(1) + 3y = -4 \rightarrow 5 + 3y = -4 \rightarrow 3y = -9 \rightarrow y = -3$.

Answer: ✓ (1, -3).

From Words to Equations: A 3-Step Strategy

The true power of algebra is solving real-world problems. The elimination method is a fantastic tool for this. Here's how to translate any word problem into a solvable system.



1. Define Your Variables

Read the problem to identify the two unknown quantities. Assign a variable (like x and y) to each one.

Example: "Let x = the cost of a notebook, and y = the cost of a folder."



2. Write the Equations

Translate the sentences and relationships in the problem into two separate linear equations. Look for two distinct pieces of information (e.g., total cost and total items).



3. Solve and Interpret

Use the best elimination method (addition, subtraction, or multiplication) to solve the system. Crucially, state your final answer in the context of the problem.

*"The solution is (2, 3)" is not enough.
"A notebook costs \$2 and a folder costs \$3" is a complete answer.**

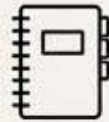
Case Study 1: The Shopping Trip



Problem: Caroline spent \$15.50 on 5 notebooks and 3 folders. Jose bought 4 notebooks and 4 folders and spent \$14. What is the cost of one notebook and one folder?

1. Define Variables

Let x = the cost of one notebook.



Let y = the cost of one folder.



2. Write Equations

Caroline's purchase: $5x + 3y = 15.50$

Jose's purchase: $4x + 4y = 14$

3. Solve and Interpret

Strategy: Multiply both equations. Multiply the top by 4 and the bottom by -5 to eliminate x .

$$\begin{array}{rclcl} 4(5x + 3y = 15.50) & \rightarrow & 20x + 12y = 62 \\ -5(4x + 4y = 14) & \rightarrow & -20x - 20y = -70 \\ \hline & & -8y = -8 & & y = 1 \\ 4x + 4(1) = 14 & \rightarrow & 4x = 10 & \rightarrow & x = 2.5 \end{array}$$



Interpretation: The solution is (2.5, 1). This means a notebook costs **\$2.50** and a folder costs **\$1.00**.

Case Study 2: The Vehicle Puzzle



Problem: A store has a total of 45 motorcycles (two wheels) and ATVs (four wheels) in stock. Together, they have 130 wheels. How many of each vehicle are in the store?

1. Define Variables



Let `x` = the number of motorcycles.



Let `y` = the number of ATVs.

2. Write Equations



Total vehicles: $x + y = 45$



Total wheels: $2x + 4y = 130$

3. Solve and Interpret

Strategy: Multiply the first equation by **-2** to eliminate x.

$$-2(x + y = 45) \longrightarrow -2x - 2y = -90$$

$$\begin{array}{r} (-2x - 2y = -90) \\ + (2x + 4y = 130) \\ \hline \end{array}$$

$$2y = 40$$

$$y = 20$$

$$x + 20 = 45$$

$$x = 25$$



Interpretation: The solution is (25, 20). This means the store has **25 motorcycles** and **20 ATVs**.

Case Study 3: The Number Game

Problem: The sum of two numbers is 39. Their difference is 7. Find the two numbers.

1. Define Variables

- #1 Let 'x' = the first number.
- #2 Let 'y' = the second number.

2. Write Equations

- ⊕ Sum: 'x + y = 39'
- ⊖ Difference: 'x - y = 7'

3. Solve and Interpret

Strategy: This system is perfectly set up for addition.

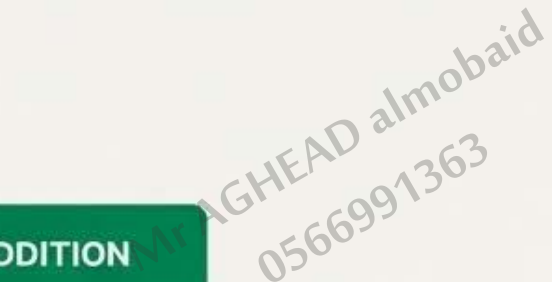
$$\begin{array}{r} x + y = 39 \\ + x - y = 7 \\ \hline 2x = 46 \\ x = 23 \end{array}$$

Substitute x into the first equation: $23 + y = 39$
 $y = 16$



The solution is (23, 16). The two numbers are **23** and **16**.

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IMPLICATION
(question)

You've Mastered the Elimination Method.

- ✓ Solved systems using **Addition** for opposite coefficients.
- ✓ Solved systems using **Subtraction** for identical coefficients.
- ✓ Mastered the 'Power Move' of **Multiplication** for any system.
- ✓ Translated real-world problems into solvable equations.
- ✓ Developed a **strategic toolkit** to choose the best method.

This is a powerful tool in algebra. Keep practicing, and you'll be able to solve any problem that comes your way.

Module (7): Triangles and the Pythagorean Theorem

	Lesson Title	Page
7-1+2+3	Angle Relationships and Parallel Lines + Angle Relationships and Triangles + The Pythagorean Theorem	86
7-4+5	Converse of the Pythagorean Theorem + Distance on the Coordinate Plane	102

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Module (7): Triangles and the Pythagorean Theorem

01

02

03

First, Second & Third Lessons:
Angle Relationships and
Parallel Lines, Angle
Relationships and Triangles &
The Pythagorean Theorem



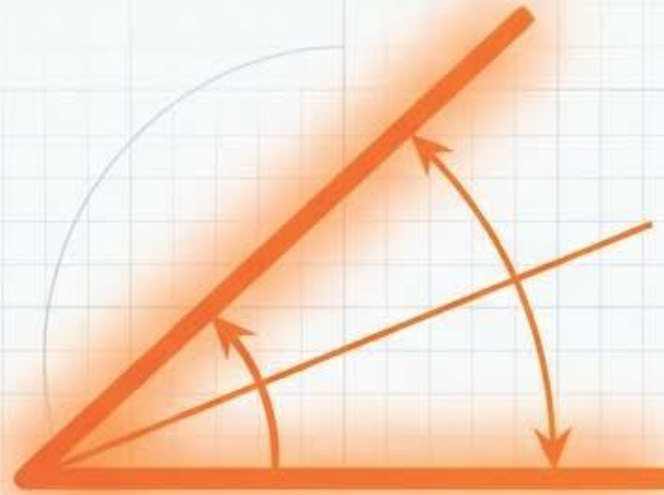
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Mastering the Geometric Code

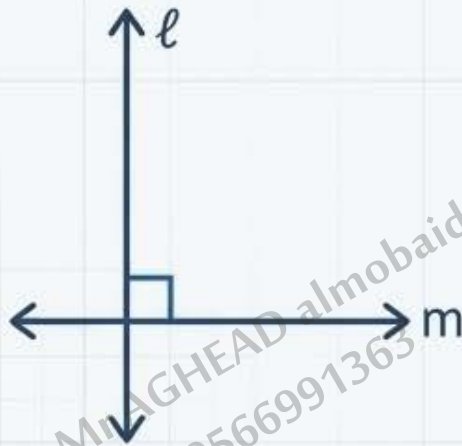
A Practical Guide to Angles, Parallel Lines, and Triangles



Chapter 1: The Language of Lines

Before we can crack the code, we must understand its language. Every geometric problem is built on three key elements: parallel lines, perpendicular lines, and transversals.

Perpendicular Lines



Two lines that intersect to form a right angle (90°).

Symbol: $\ell \perp m$

Parallel Lines



Two lines in a plane that never intersect. They always remain the same distance apart. Symbol: $s \parallel t$

Transversal



A line that intersects two or more other lines in a plane.

The Codebook: Identifying the Angles

Alternate Interior Angles

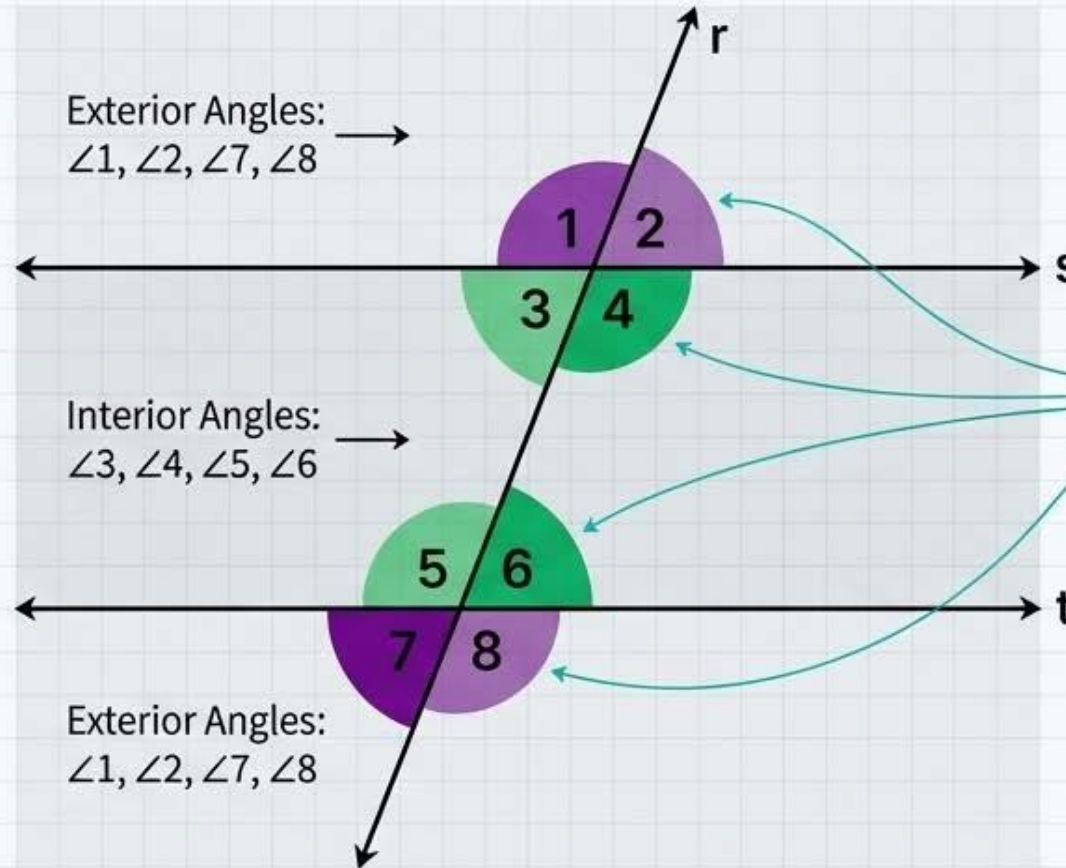
A pair of interior angles on **opposite sides** of the transversal. They are non-adjacent.

Examples:
($\angle 4$ and $\angle 6$), ($\angle 3$ and $\angle 5$)

Alternate Exterior Angles

A pair of exterior angles on **opposite sides** of the transversal. They are non-adjacent.

Examples:
($\angle 1$ and $\angle 7$), ($\angle 2$ and $\angle 8$)



Corresponding Angles

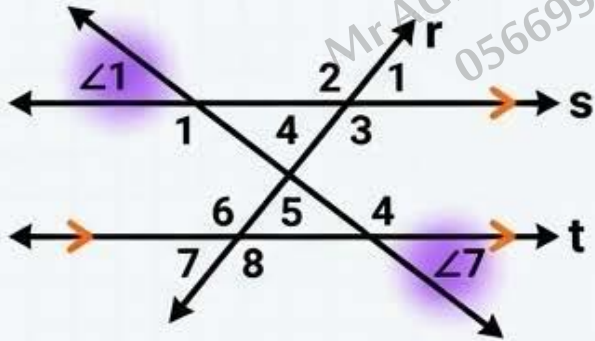
Angles that are in the **same position** on the two lines in relation to the transversal.

Examples:
($\angle 1$ and $\angle 5$), ($\angle 2$ and $\angle 6$),
($\angle 3$ and $\angle 7$), ($\angle 4$ and $\angle 8$)

The Playbook: Practice Classifying Angle Pairs

Use your Codebook knowledge to classify the relationship between the highlighted angle pairs.

1.

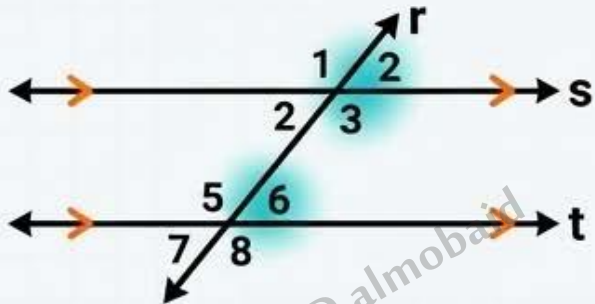


What is the relationship between $\angle 1$ and $\angle 7$?

They are exterior angles on opposite sides of the transversal.

They are **alternate exterior angles**.

2.



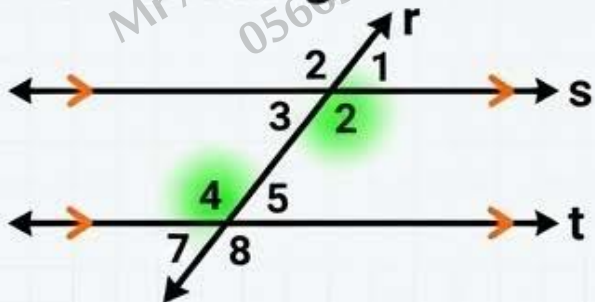
What is the relationship between $\angle 2$ and $\angle 6$?

They are in the same relative position at each intersection.

They are **corresponding angles**.

Your Turn Challenge

3.



Now you try: Classify the relationship between $\angle 2$ and $\angle 4$.

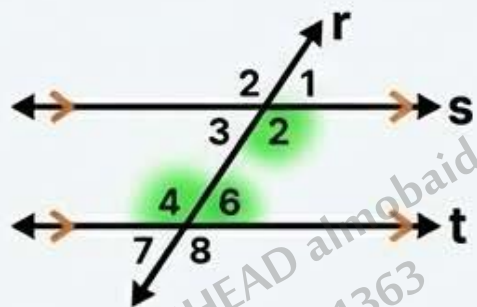
They are interior angles on opposite sides of the transversal.

They are **alternate interior angles**.

The Master Key: Unlocking Angle Measures

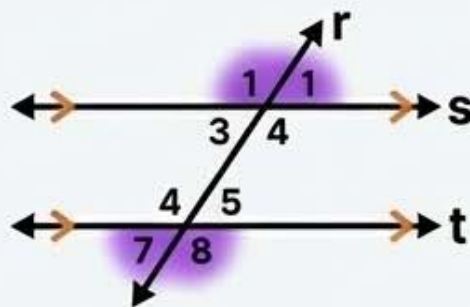
When two **parallel lines** are intersected by a transversal, special relationships emerge. The angles are no longer just related by name, but by measure.

Alternate Interior Angles are Equal.



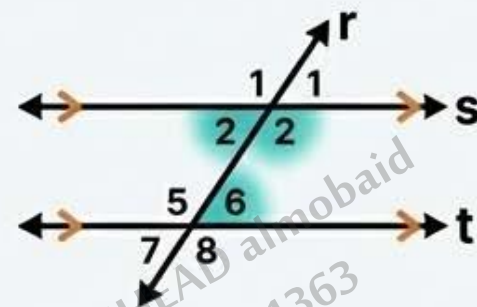
$$m\angle 4 = m\angle 6, m\angle 3 = m\angle 5$$

Alternate Exterior Angles are Equal.



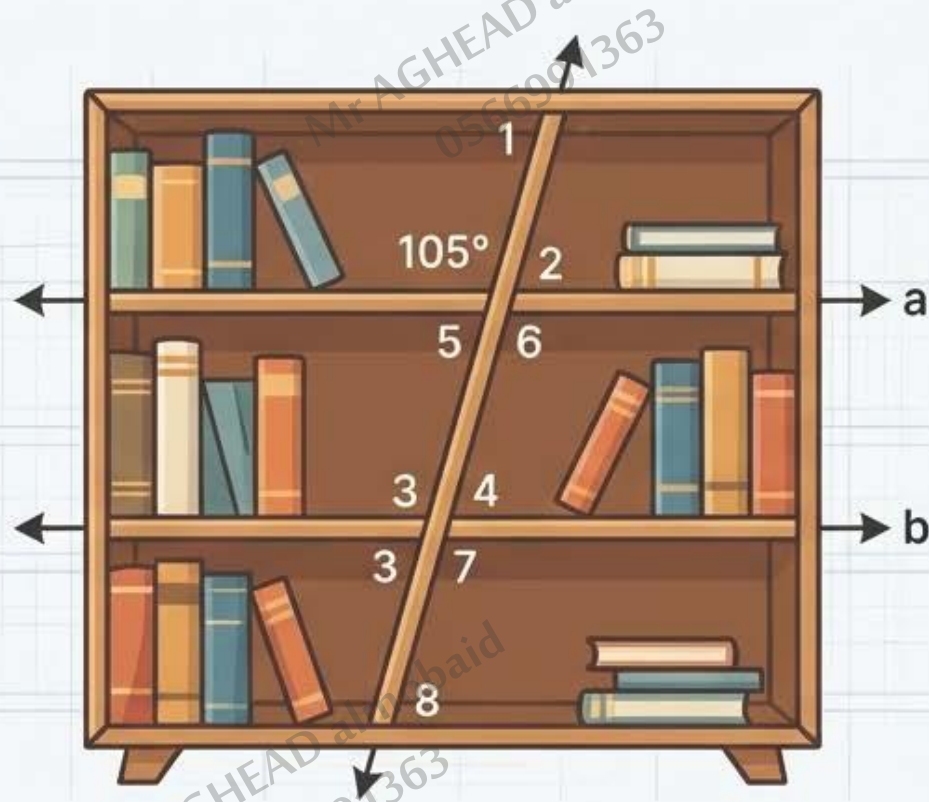
$$m\angle 1 = m\angle 7, m\angle 2 = m\angle 8$$

Corresponding Angles are Equal.



$$m\angle 1 = m\angle 5, m\angle 2 = m\angle 6, \\ m\angle 3 = m\angle 7, m\angle 4 = m\angle 8$$

The Playbook: Finding Missing Angles in a Bookcase



Mrs. Kumar designed the bookcase shown, where shelf line **a** is **parallel** to shelf line **b**. Let's use our rules to find the missing angle measures.

Problem Statement: If $m\angle 2 = 105^\circ$, find $m\angle 6$ and $m\angle 3$. Justify your answer.

Part A: Find $m\angle 6$

Logic: First, we identify the relationship. $\angle 2$ and $\angle 6$ are in the same relative position, making them **Corresponding Angles**.

Rule: The 'Master Key' states that when lines are parallel, corresponding angles are equal.

Calculation: Therefore, $m\angle 6 = m\angle 2 = 105^\circ$.

Part B: Find $m\angle 3$

Logic: Next, we relate $\angle 3$ to an angle we know. $\angle 3$ and $\angle 6$ are interior angles on opposite sides of the transversal, making them **Alternate Interior Angles**.

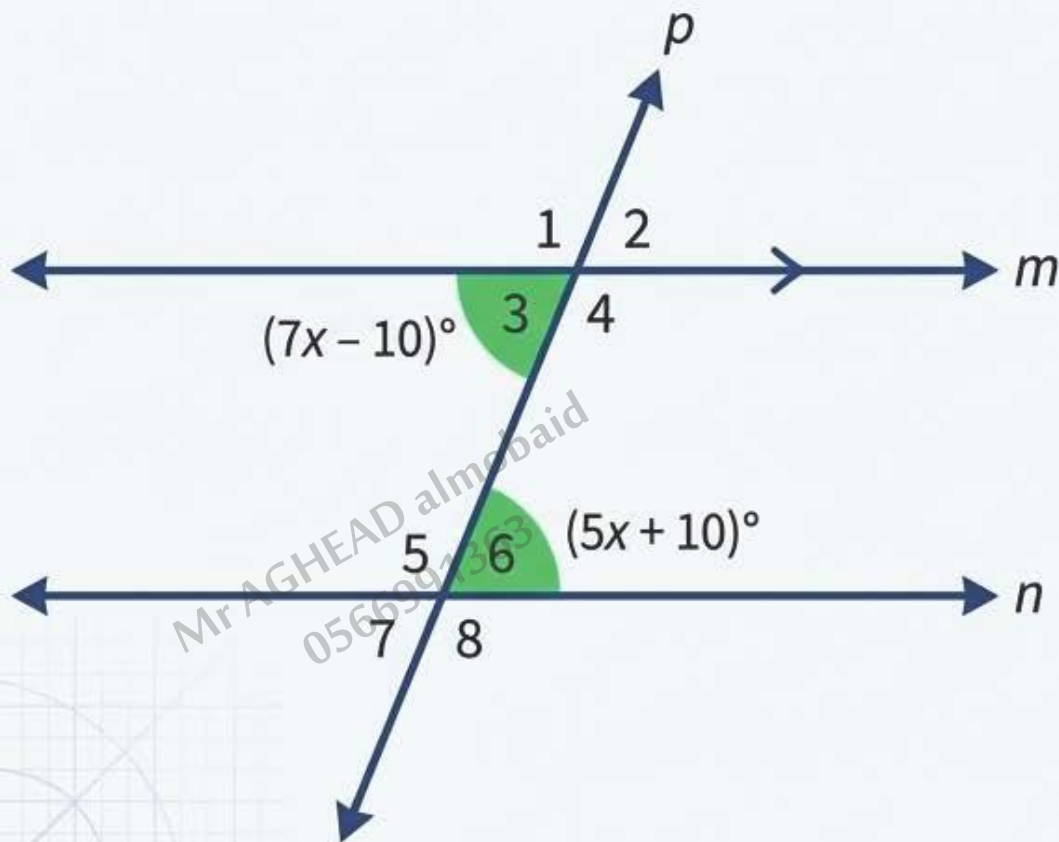
Rule: The 'Master Key' states that alternate interior angles are equal.

Calculation: Therefore, $m\angle 3 = m\angle 6 = 105^\circ$.

Final Answer: $m\angle 6 = 105^\circ$ and $m\angle 3 = 105^\circ$.

The Playbook: Using Algebra to Crack the Code

The rules of geometry are most powerful when combined with algebra. They allow us to solve for unknown variables.



In the figure, line $m \parallel$ line n . If $m\angle 3 = (7x - 10)^\circ$ and $m\angle 6 = (5x + 10)^\circ$, find the measures of $\angle 3$ and $\angle 6$.

1. Identify the Relationship

$\angle 3$ and $\angle 6$ are **alternate interior angles**.

2. Form the Equation

Our “Master Key” rule states that alternate interior angles are equal. We can set their measures equal to each other:
 $7x - 10 = 5x + 10$

3. Solve for x

$$2x - 10 = 10$$

Subtract $5x$ from both sides

$$2x = 20$$

Add 10 to both sides

$$x = 10$$

Divide by 2

4. Find the Angle Measures

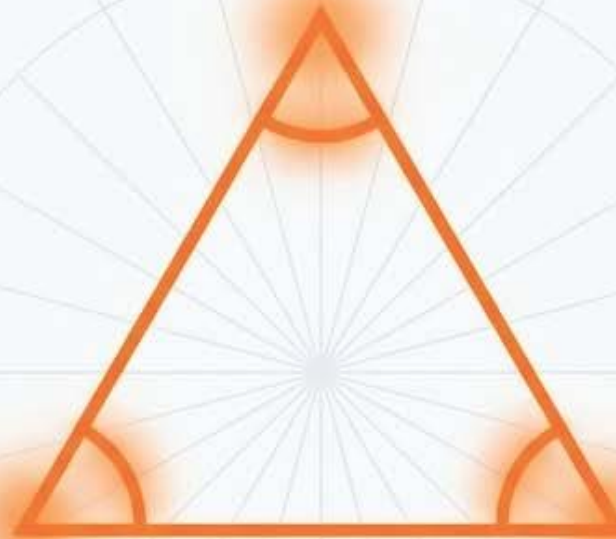
Substitute $x = 10$ back into the original expressions.

$$m\angle 3 = 7(10) - 10 = 70 - 10 = \mathbf{60^\circ}$$

$$m\angle 6 = 5(10) + 10 = 50 + 10 = \mathbf{60^\circ}$$

The measure of both $\angle 3$ and $\angle 6$ is 60° .

Chapter 2: The Secrets of the Triangle



We've mastered the code of lines. Now, we turn to one of the most fundamental shapes in the universe: the triangle. Understanding the relationship between its angles is key to solving a new class of problems.

Vertex:

The point where two sides of a triangle meet.



Interior Angles:

The three angles inside the triangle.



Exterior Angle:

An angle formed by one side of the triangle and the extension of an adjacent side.

The Codebook: The 180° Rule

The Theorem

In Words:

The sum of the measures of the interior angles of a triangle is always 180° .

In Variables:

$$x + y + z = 180$$

Visual Proof



The three angles join together to form a straight line, which is always 180° . This is true for any triangle!

The Playbook: Applying the 180° Rule

Example 1: Simple Missing Angle



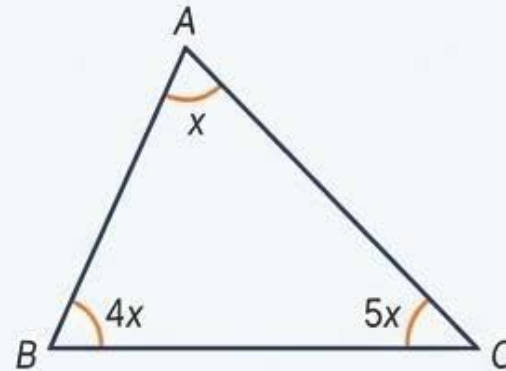
A right angle (90°) and a 56° angle are known. Find the value of x .

$$x + 56 + 90 = 180$$

$$x + 146 = 180$$

$$x = 34^\circ$$

Example 2: Using Ratios



The angles in $\triangle ABC$ have a ratio of 1:4:5. Find the measure of each angle.

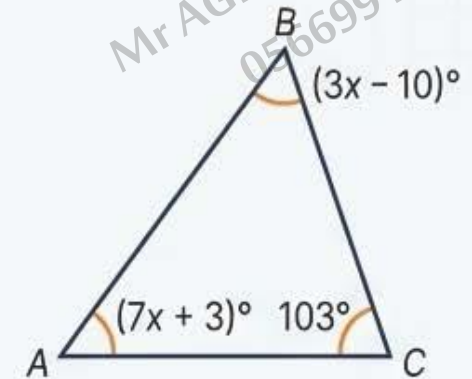
Represent angles as: x , $4x$, and $5x$.

$$\text{Equation: } x + 4x + 5x = 180$$

$$\text{Solve: } 10x = 180 \rightarrow x = 18$$

$$\text{Final Angles: } 18^\circ, 72^\circ, \text{ and } 90^\circ$$

Example 3: Using Algebra



Find the measures of $\angle CAB$ and $\angle ACB$ in the figure.

$$\text{Equation: } (7x + 3) + (3x - 10) + 103 = 180$$

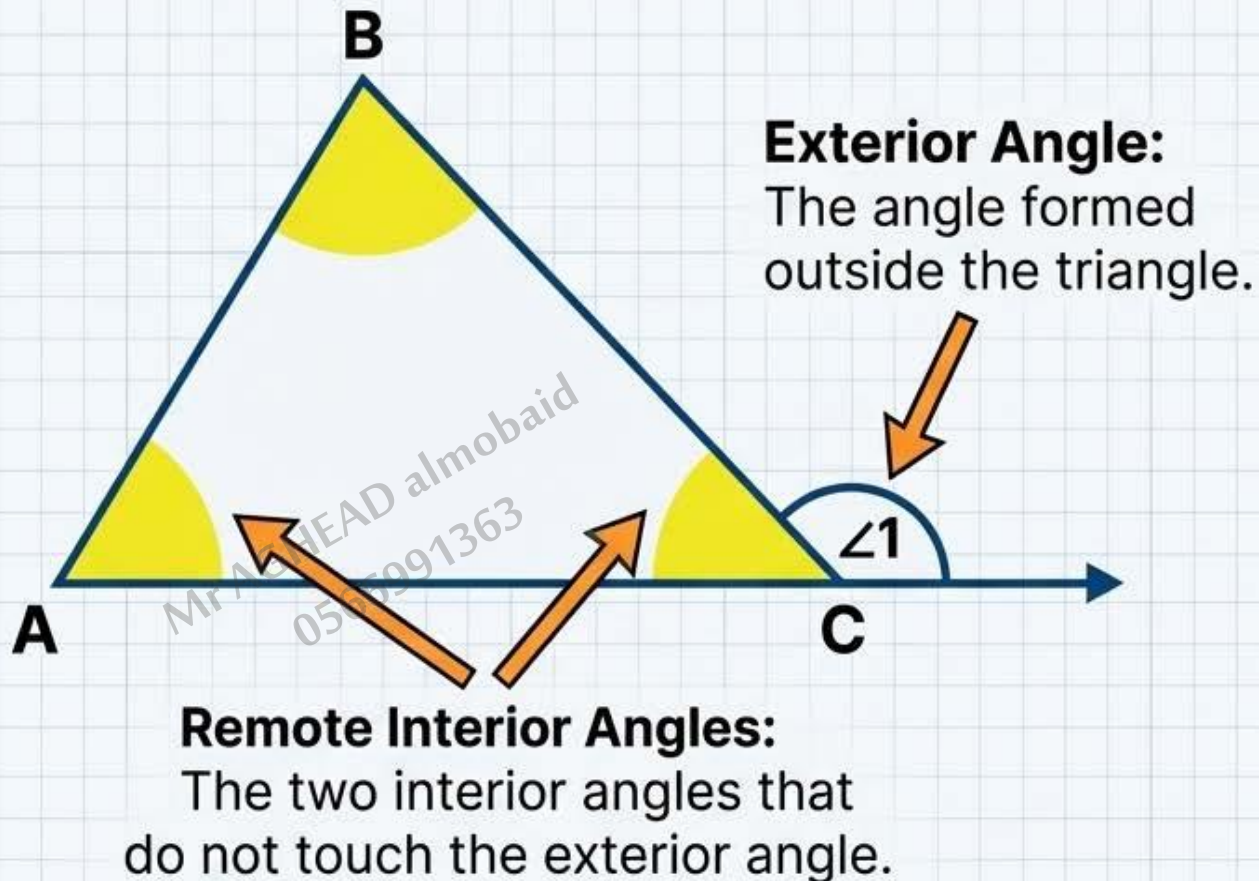
$$\text{Solve: } 10x + 96 = 180 \rightarrow 10x = 84 \rightarrow x = 8.4$$

$$\text{Find Angles: } m\angle CAB = 7(8.4) + 3 = 61.8^\circ$$

$$m\angle ACB = 3(8.4) - 10 = 15.2^\circ$$

The Codebook: The Exterior Angle Theorem

An **exterior angle** of a triangle has a special relationship with the two **interior** angles that are farthest away from it.



The Theorem

In Words:

The measure of an **exterior angle of a triangle** is equal to the **sum** of the measures of its two **remote interior angles**.

In Symbols:

$$m\angle A + m\angle B = m\angle 1$$

The Playbook: Using the Exterior Angle Shortcut

Example 1: Simple Application



In the beach chair shown, $m\angle 2 = 55^\circ$ and $m\angle 3 = 60^\circ$. Find the measure of the exterior angle, $m\angle 1$.

Logic:

$\angle 1$ is an exterior angle, and $\angle 2$ and $\angle 3$ are its remote interior angles.

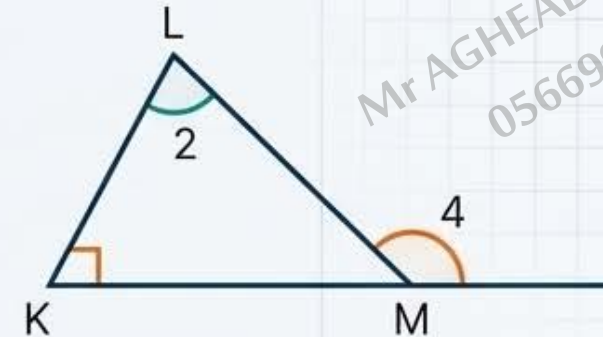
Solution:

$$m\angle 1 = m\angle 2 + m\angle 3$$

$$m\angle 1 = 55 + 60 = 115^\circ$$

The measure of $\angle 1$ is 115° .

Example 2: Finding a Remote Interior Angle



In the figure, the exterior angle $m\angle 4 = 135^\circ$. Find $m\angle 2$, given that $\angle LKM$ is a right angle (90°).

Logic:

The exterior angle ($\angle 4$) is the sum of the two remote interior angles ($\angle 2$ and $\angle LKM$).

Solution:

$$m\angle 2 + m\angle LKM = m\angle 4$$

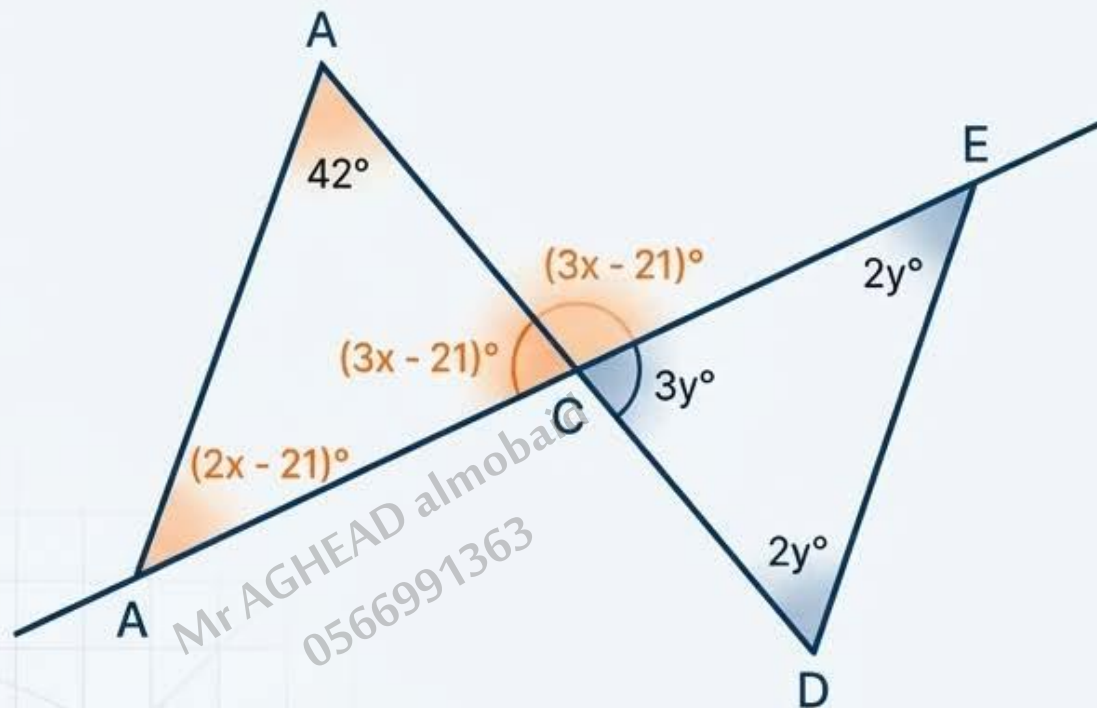
$$m\angle 2 + 90 = 135$$

$$m\angle 2 = 45^\circ$$

The measure of $\angle 2$ is 45° .

The Playbook: Advanced Algebraic Problems" in Inter Bold, Primary Blue (#003366)

True mastery comes from combining geometric rules to solve complex algebraic puzzles.



Find the measures of $\angle CDE$ and $\angle BCE$.

Step 1: Find x using the Exterior Angle Theorem.

The angle $(3x - 21)^\circ$ is an exterior angle to $\triangle ABC$. It equals the sum of the two remote interior angles.

$$(3x - 21) = 42 + (2x - 21)$$

$$\text{Solution: } 3x - 21 = 2x + 21 \rightarrow x = 42$$

Step 2: Find the angles in $\triangle ABC$.

Substitute $x = 42$ to find the angles.

$$m\angle ABC = 2(42) - 21 = 63^\circ$$

$$\text{Find } m\angle BCA \text{ using the } 180^\circ \text{ Rule: } 42 + 63 + m\angle BCA = 180 \rightarrow m\angle BCA = 75^\circ.$$

Step 3: Find $m\angle BCE$.

$\angle BCA$ and $\angle BCE$ are supplementary.

$$\text{Solution: } m\angle BCE = 180 - 75 = 105^\circ$$

Step 4: Solve for y and find $m\angle CDE$.

$\angle DCE$ and $\angle BCA$ are vertical angles, so $m\angle DCE = 75^\circ$.

$$\text{Equation for } \triangle CDE: 3y + 2y + 75 = 180$$

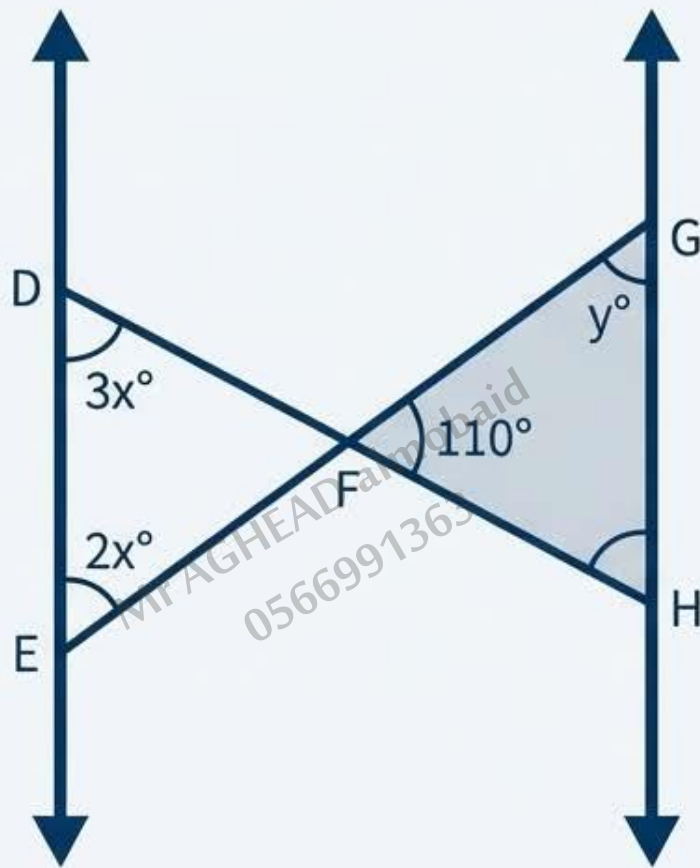
$$\text{Solution: } 5y = 105 \rightarrow y = 21$$

$$m\angle CDE = 3y = 3(21) = 63^\circ$$

Solution: $m\angle CDE = 63^\circ$ and $m\angle BCE = 105^\circ$

Final Challenge: The Codebreaker's Test

You have all the keys. It's time to unlock the final puzzle. In the diagram, the two vertical lines are parallel. Use all your knowledge to find the measures of $\angle DEF$, $\angle FHG$, and $\angle FDE$.



Step 1: Find the angles inside the triangle.

The angle supplementary to 110° is $m\angle EFG$. So, $m\angle EFG = 180 - 110 = 70^\circ$. The angle $m\angle EGF$ is alternate interior to the angle marked $2x$ (at vertex D), so $m\angle EGF = 2x$.

Step 2: Solve for x using the 180° Rule in $\triangle EFG$.

The sum of angles in $\triangle EFG$ is 180° .

$$(3x) + (70) + (2x) = 180$$

$$5x + 70 = 180 \rightarrow 5x = 110 \rightarrow x = 22$$

Step 3: Calculate the required angles.

$m\angle DEF$: This is the angle at vertex E, $3x$. So, $m\angle DEF = 3 \cdot 22 = 66^\circ$.

$m\angle FHG$: This angle is supplementary to $m\angle EGF$ (which is $2x$).

$m\angle EGF = 2 \cdot 22 = 44^\circ$. So, $m\angle FHG = 180 - 44 = 136^\circ$.

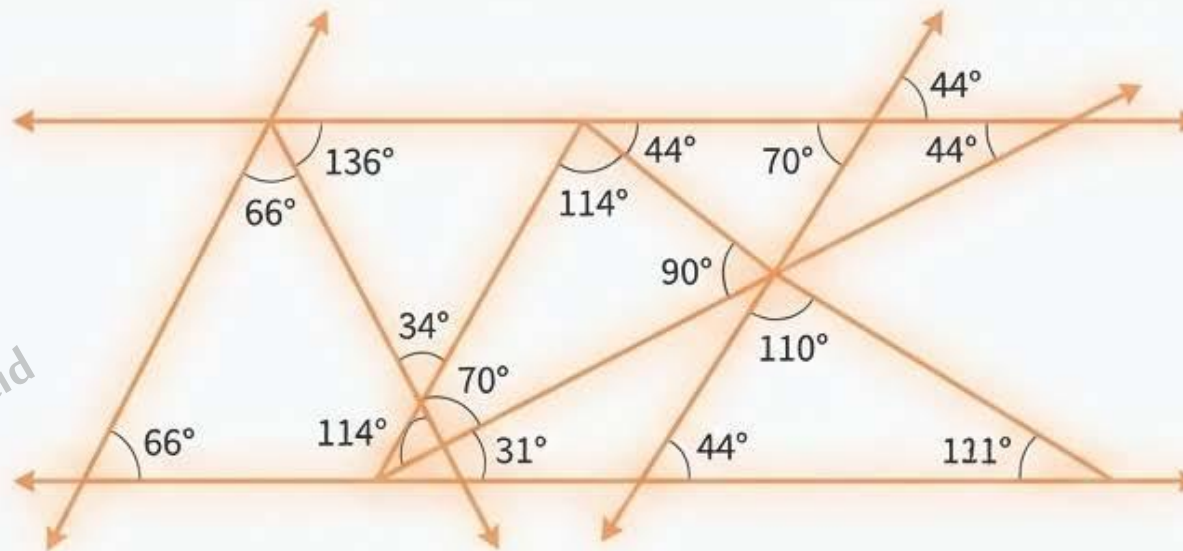
$m\angle FDE$: This angle is supplementary to $m\angle DEF$ on the straight line.

So, $m\angle FDE = 180 - 66 = 114^\circ$.

Final Answer: $m\angle DEF = 66^\circ$ $m\angle FHG = 136^\circ$ $m\angle FDE = 114^\circ$

You've Mastered the Code

Geometry is not a set of random rules, but a logical and powerful system. By identifying relationships and applying the correct theorems, you can solve any puzzle. The code is unlocked.



Identify

Recognize key angle relationships (Alternate Interior, Corresponding, etc.).

Formulate

Translate geometric relationships into algebraic equations.

Solve

Confidently find missing angles and variables in any diagram.



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Module (7): Triangles and the Pythagorean Theorem

04

05

Fourth & Fifth Lessons:
Converse of the Pythagorean
Theorem & Distance on the
Coordinate Plane



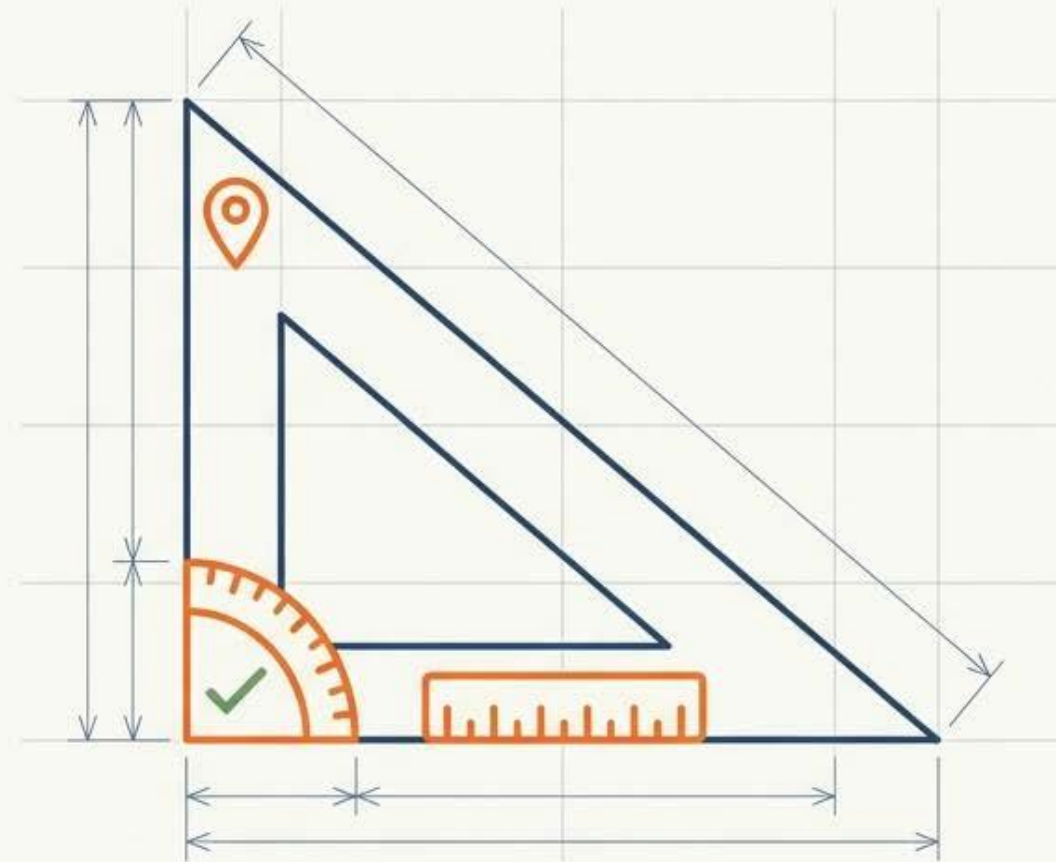
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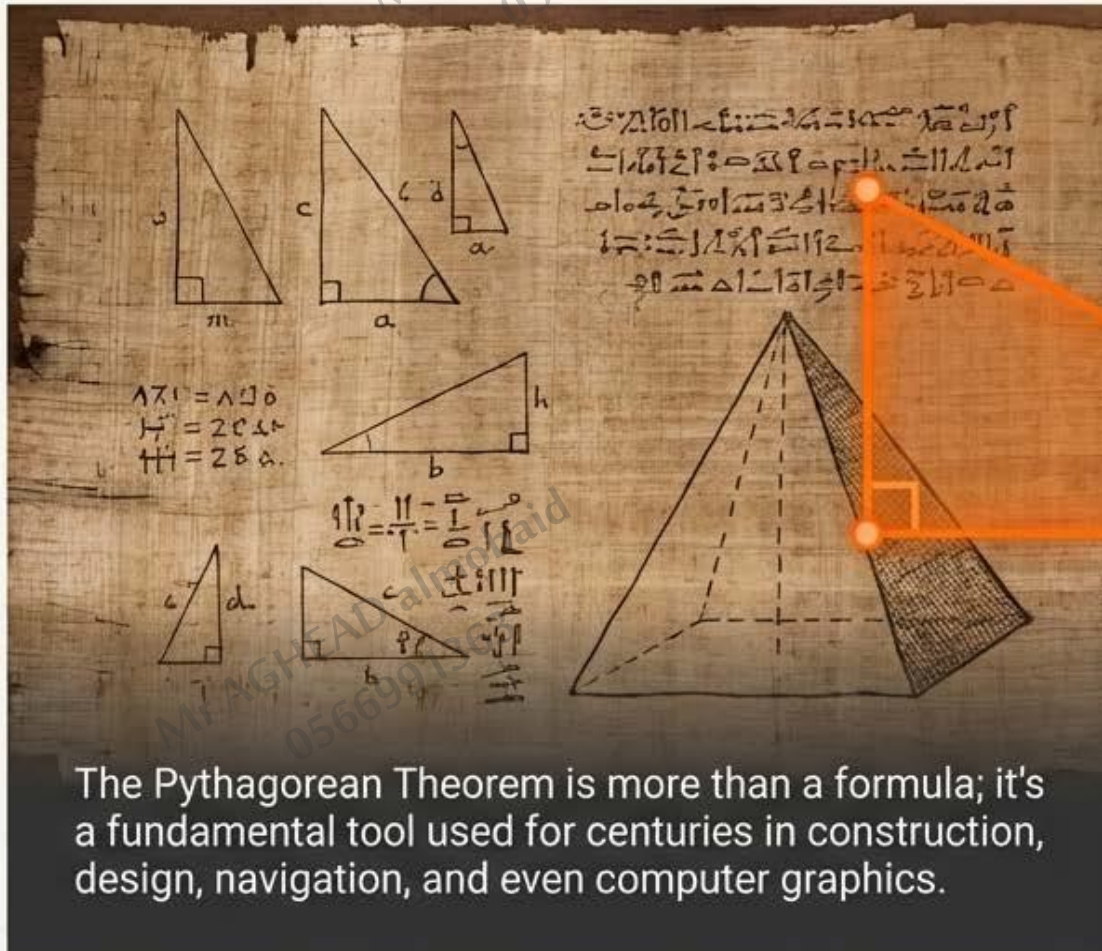
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The Right Triangle Toolkit: A Master Guide

Mastering the Pythagorean Theorem and Its Applications



Why This Matters: From Ancient Secret to Modern Tool



Meet Your Tools: The Three Essentials in Your Kit

Tool #1: The Length Finder



Tool #1: The Length Finder (The Pythagorean Theorem)

Use it to find any missing side of a right triangle.

Tool #2: The Angle Verifier



Tool #2: The Angle Verifier (The Converse of the Theorem)

Use it to check if a triangle has a perfect 90° angle.

Tool #3: The Distance Mapper



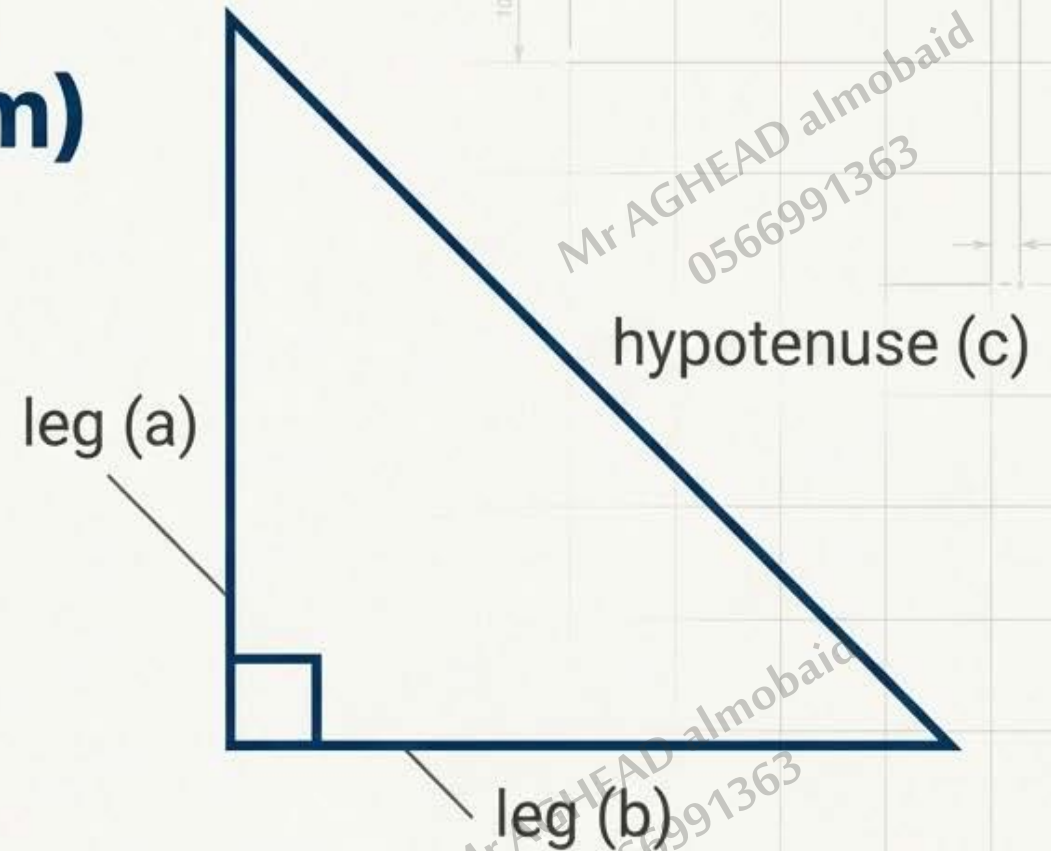
Tool #3: The Distance Mapper (Application on the Coordinate Plane)

Use it to find the direct distance between any two points on a map or grid.

Tool #1: The Length Finder (The Pythagorean Theorem)

The Core Formula: $a^2 + b^2 = c^2$

In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.



Pro-Tip: Always identify the hypotenuse (c) first! It's the longest side and always opposite the right angle. Getting this right is the key to using the formula correctly.

The Length Finder in Action: Finding the Hypotenuse



The Job

A 15-foot tall support mast on a sailboat needs a support rope. The rope attaches to the deck 8 feet away from the base of the mast. How long must the support rope be?

1. **Identify:** Legs (a, b) = 15 ft, 8 ft.
Hypotenuse (c) = x (the unknown).
2. **Formula:** $a^2 + b^2 = c^2$
3. **Substitute:** $15^2 + 8^2 = x^2$
4. **Solve:** $225 + 64 = x^2 \rightarrow 289 = x^2 \rightarrow x = \sqrt{289} = 17$

The support rope must be 17 feet long.

The Length Finder in Action: Finding a Missing Leg

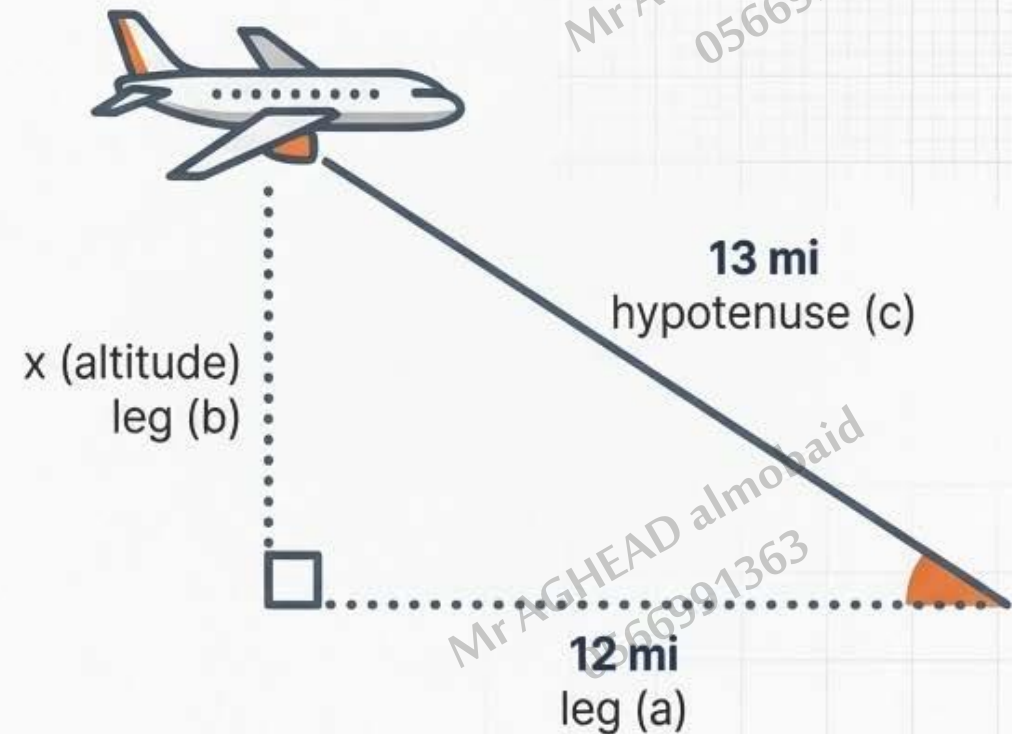
The Job

A plane takes off and travels 13 miles. If it has covered a horizontal distance of 12 miles from its takeoff point, what is its current altitude?

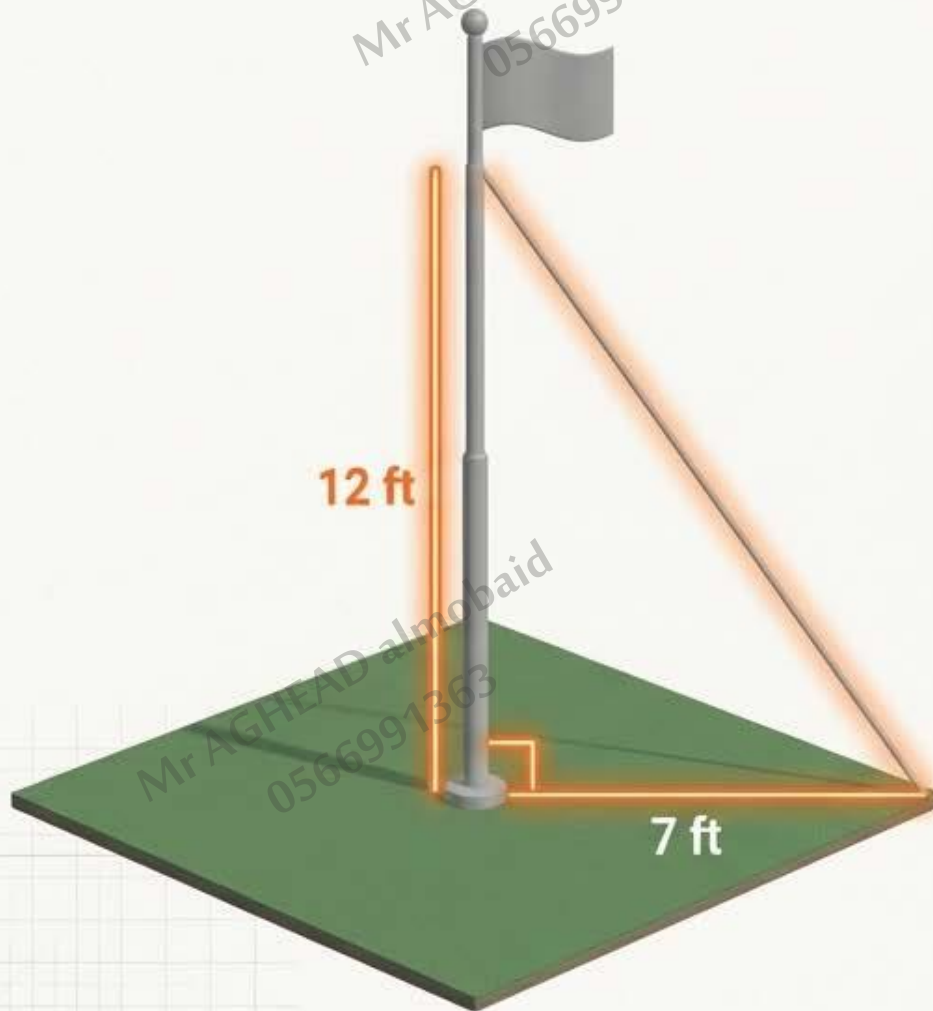
Step-by-Step Solution

1. Identify: Leg (a) = 12 mi.
Hypotenuse (c) = 13 mi.
Leg (b) = x (unknown altitude).
2. Formula: $a^2 + b^2 = c^2$
3. Substitute: $12^2 + x^2 = 13^2$
4. Solve: $144 + x^2 = 169 \rightarrow x^2 = 169 - 144 \rightarrow x^2 = 25 \rightarrow x = 5$.

The plane's current altitude is 5 miles.



Advanced Application: The Length Finder in 3D



The Job

A 12-foot flagpole is placed in the center of a square. To stabilize it, wires will stretch from the top of the pole to each corner of the square. Each corner is 7 feet from the base of the pole. **What is the length of each wire?**

- Notice the hidden right triangle formed by the pole (height), the ground (distance from center), and the wire (hypotenuse).
- **Legs:** 12 ft (pole height), 7 ft (ground distance).
- **Hypotenuse:** The wire (c).
- **Calculation:** $12^2 + 7^2 = c^2 \rightarrow 144 + 49 = c^2 \rightarrow 193 = c^2 \rightarrow c \approx 13.9$ ft.

Each wire needs to be approximately 13.9 feet long.

Tool #2: The Angle Verifier (The Converse)

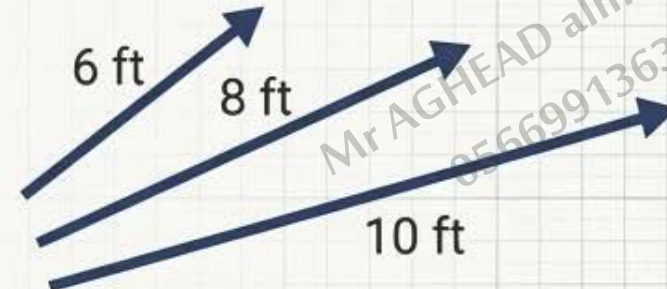
If the sides fit the formula, the angle is right.

This is the "reverse" of the theorem. If a triangle has side lengths a , b , and c , where c is the longest side, and $a^2 + b^2 = c^2$, then you can be 100% certain the triangle is a right triangle.



Pro-Tip: When testing a triangle, always use the longest side as "c" in the formula.

Stage 1



Stage 2



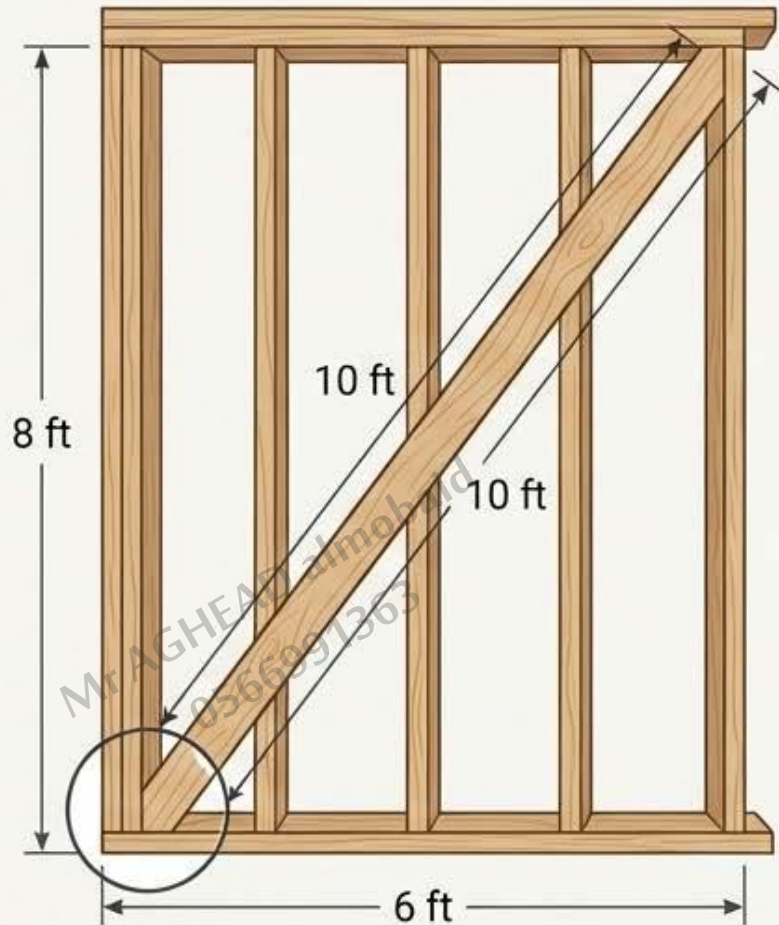
Stage 3



$$6^2 + 8^2 = 10^2$$



The Angle Verifier in Action: Construction & Carpentry



The Job

A carpenter is framing a wall. To ensure the corners are perfectly square (90°), they measure the sides. The sides are 6 ft and 8 ft, and the diagonal is 10 ft. Does the frame form a perfect right angle?

Step-by-Step Verification

1. **Identify:** Potential Legs (a, b) = 6, 8.
Potential Hypotenuse (c) = 10.
2. **Test:** Does $a^2 + b^2 = c^2$?
3. **Substitute:** Does $6^2 + 8^2 = 10^2$?
4. **Check:** $36 + 64 = 100 \rightarrow 100 = 100$.

Yes. Since the sides satisfy the formula, the frame forms a perfect 90° angle.

The Angle Verifier: When It's NOT a Right Angle

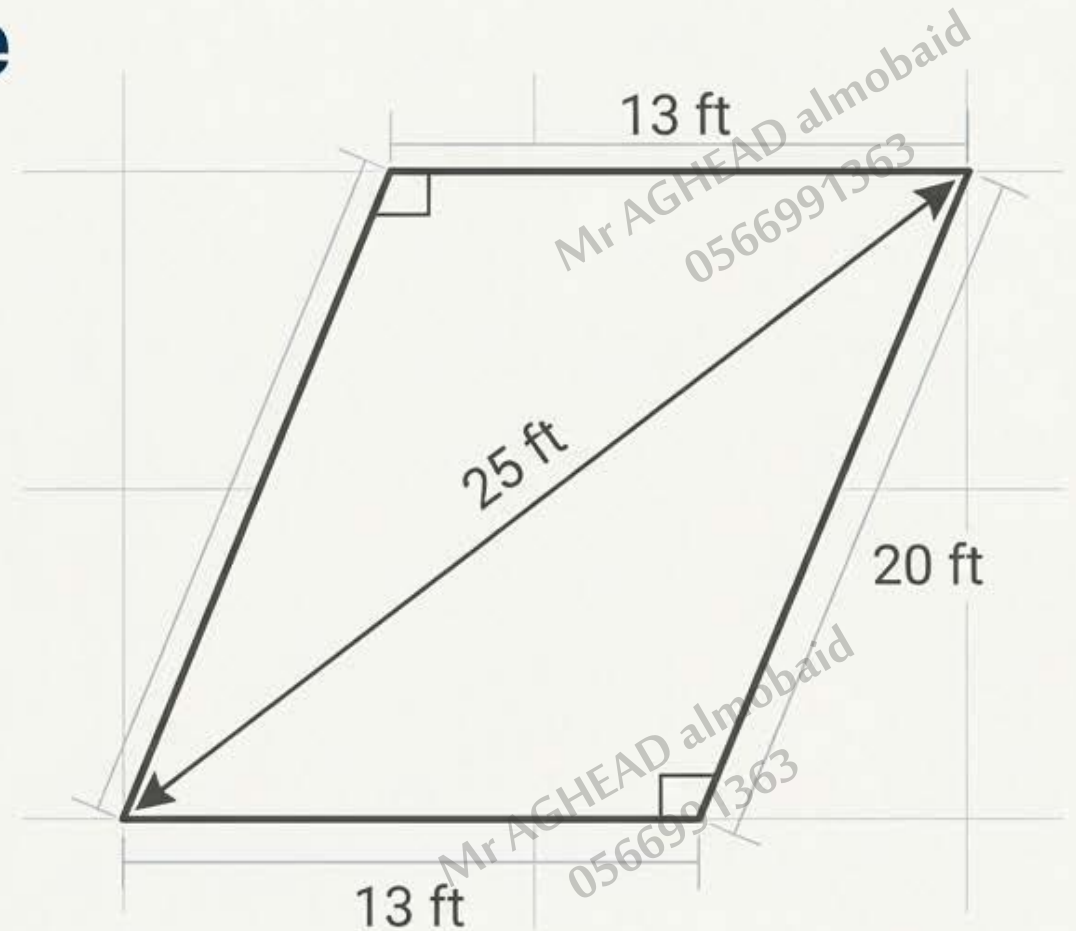
The Job

You are laying out a rectangular patio. You measure two sides as 13 ft and 20 ft, and the diagonal as 25 ft. Is your corner a true 90° angle?

Step-by-Step Verification

1. **Identify:** Potential Legs (a, b) = 13, 20.
Potential Hypotenuse (c) = 25.
2. **Test:** Does $a^2 + b^2 = c^2$?
3. **Substitute:** Does $13^2 + 20^2 = 25^2$?
4. **Check:** $169 + 400 = 625 \rightarrow 569 \neq 625$.

No. The sides do NOT satisfy the formula. The corner is not a 90° angle, and the patio is not a true rectangle.



Tool #3: The Distance Mapper

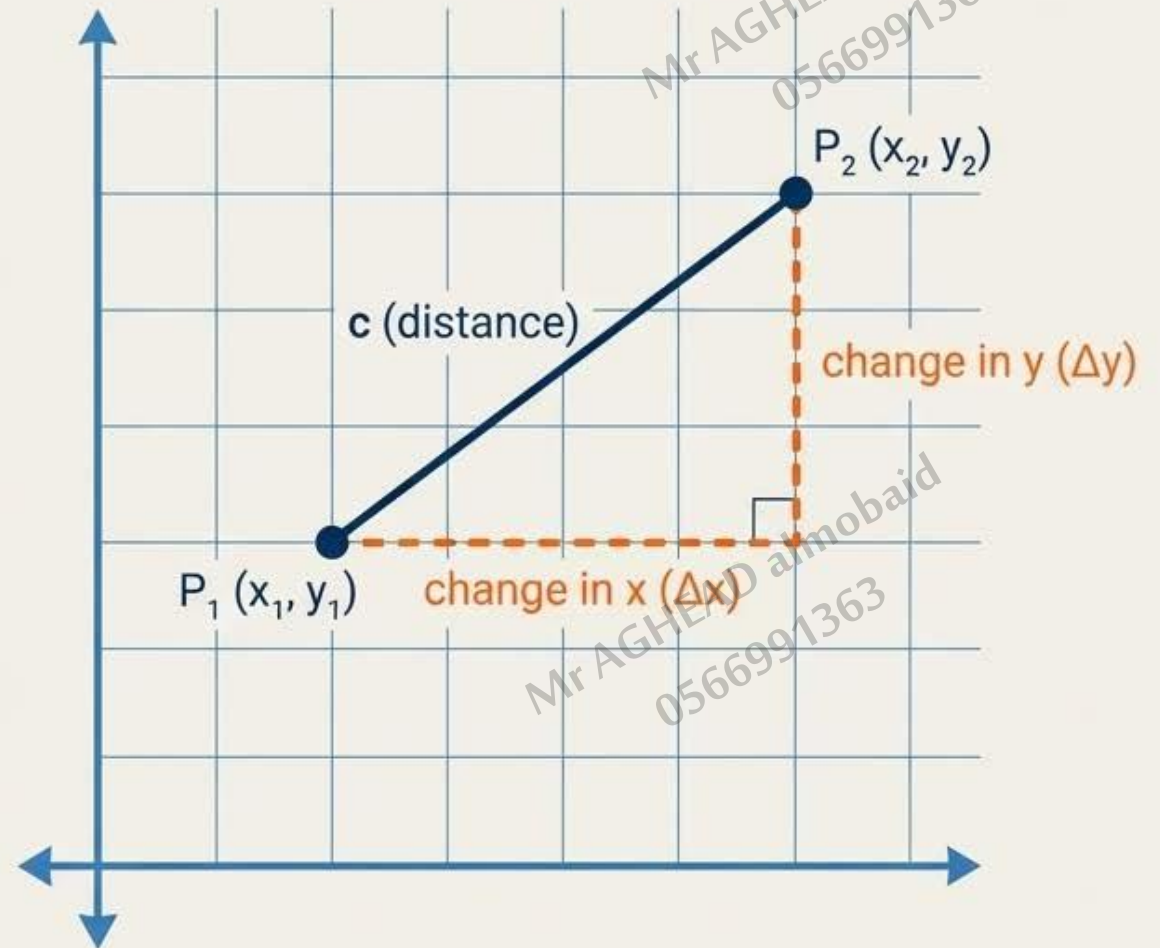
Finding the Shortest Path on Any Grid

Core Concept

The direct distance between any two points on a coordinate plane is simply the hypotenuse of a right triangle that you can draw between them. The lengths of the legs are the horizontal change ("change in x") and the vertical change ("change in y").

Connection formula is Inter Bold:

$$(\text{change in } x)^2 + (\text{change in } y)^2 = (\text{distance})^2$$



The Distance Mapper in Action: From Coordinates to Distance

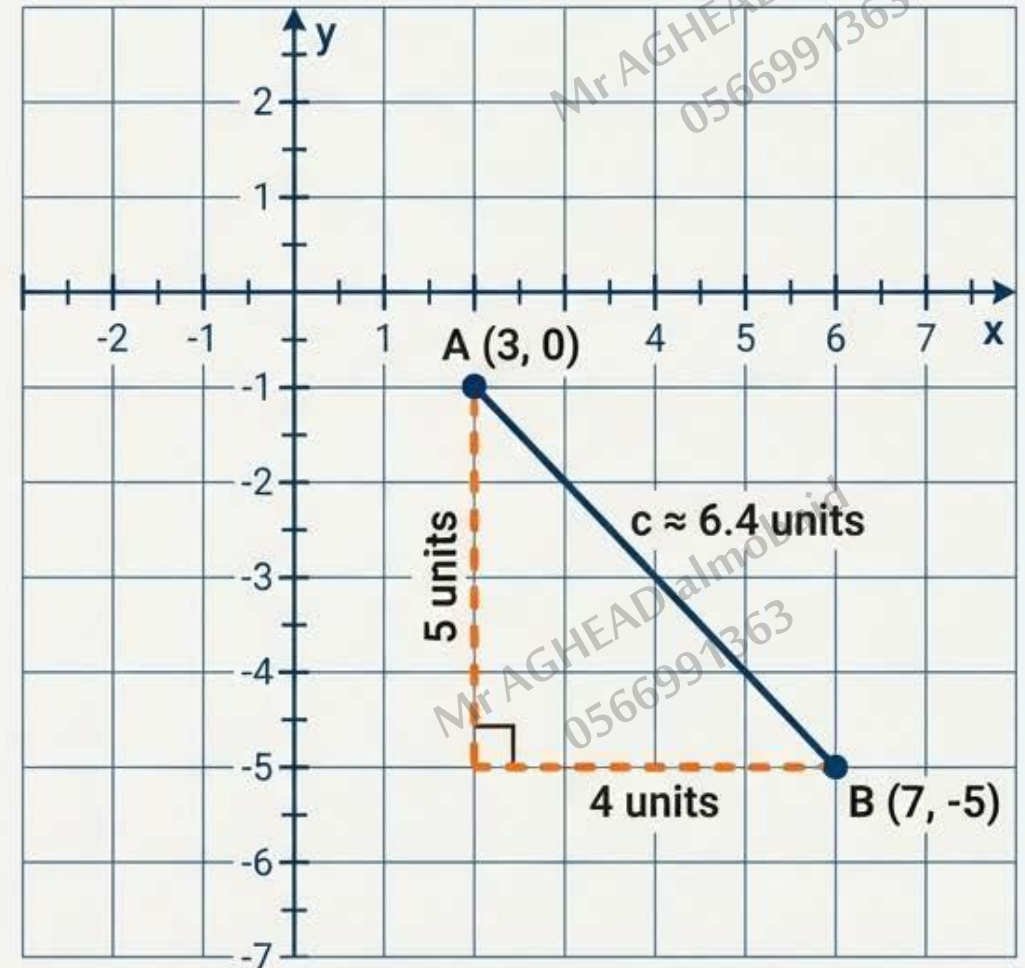
The Job

Find the direct distance between point A (3, 0) and point B (7, -5) on a coordinate plane.

Step-by-Step Solution

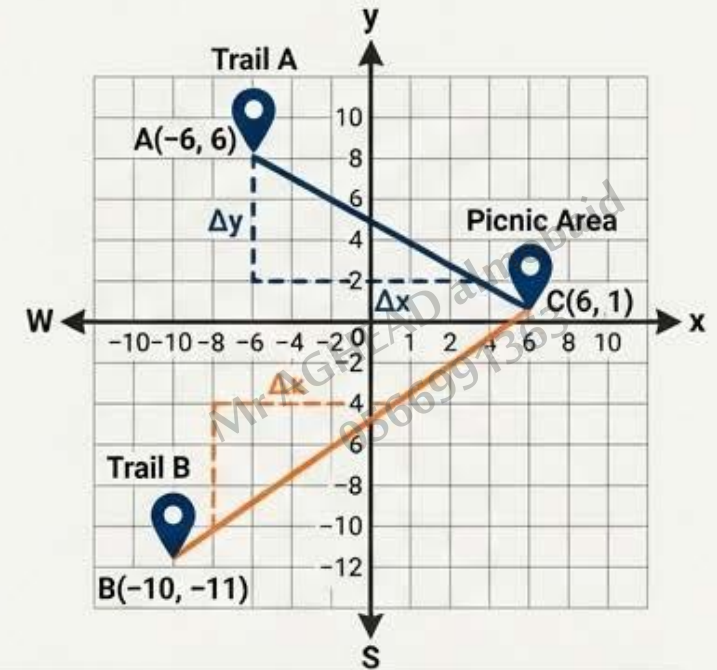
1. **Form Triangle:** Draw a right triangle with the segment AB as the hypotenuse.
2. **Find Leg Lengths:**
 - Horizontal leg (change in x) = $7 - 3 = 4$ units.
 - Vertical leg (change in y) = $0 - (-5) = 5$ units.
3. **Use Tool #1:** $a^2 + b^2 = c^2$
4. **Solve:** $4^2 + 5^2 = c^2 \rightarrow 16 + 25 = c^2 \rightarrow 41 = c^2 \rightarrow c \approx 6.4$ units.

✓ The distance between the points is approximately 6.4 units.



Practical Application: Trail Map Challenge

The Job: A map shows two trails starting from a picnic area at C(6, 1). Trail A ends at A(-6, 6). Trail B ends at B(-10, -11). If each grid unit represents 0.5 miles, how many miles longer is Trail B than Trail A?



Trail A (Distance AC)

Horizontal change (Δx) = $6 - (-6) = 12$

Vertical change (Δy) = $6 - 1 = 5$

Distance² = $12^2 + 5^2 = 144 + 25 = 169 \rightarrow$ Distance = **13 units.**

Trail B (Distance BC)

Horizontal change (Δx) = $6 - (-10) = 16$

Vertical change (Δy) = $1 - (-11) = 12$

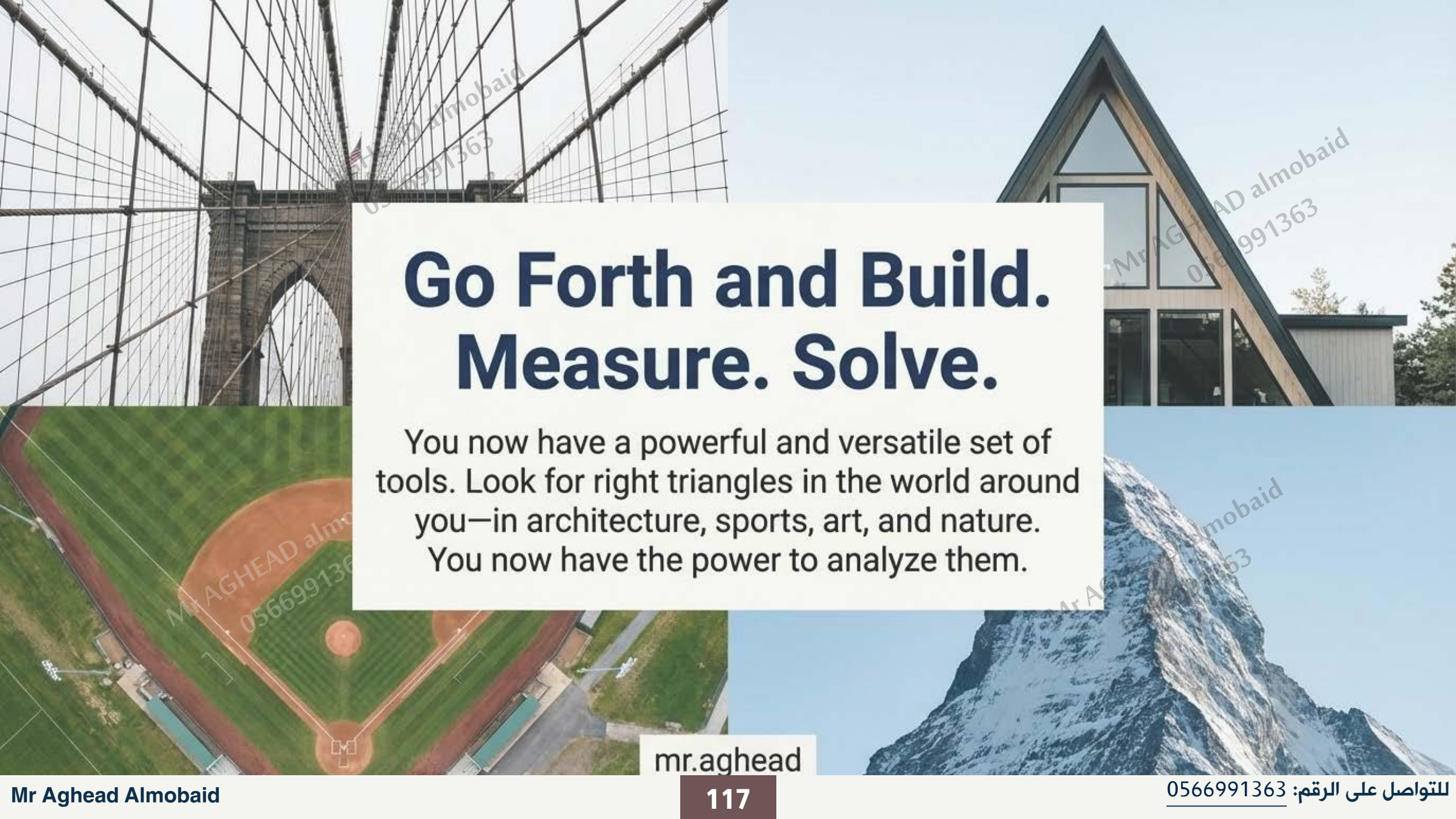
Distance² = $16^2 + 12^2 = 256 + 144 = 400 \rightarrow$ Distance = **20 units.**

Difference in units: $20 \text{ units} - 13 \text{ units} = 7 \text{ units}.$
Difference in miles: $7 \text{ units} \times 0.5 \text{ miles/unit} = 3.5 \text{ miles}.$

Trail B is 3.5 miles longer than Trail A.

Your Complete Right Triangle Toolkit: A Quick Reference Guide

The Tool	The Formula / Method	Primary Job
 The Length Finder	$a^2 + b^2 = c^2$	To find a missing side length in any right triangle.
 The Angle Verifier	Test if $a^2 + b^2 = c^2$ holds true.	To confirm if a corner or angle is exactly 90°
 The Distance Mapper	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ <i>*This is the 'shortcut' version of the theorem for coordinates.</i>	To calculate the direct distance between two points on a grid.



Go Forth and Build. Measure. Solve.

You now have a powerful and versatile set of tools. Look for right triangles in the world around you—in architecture, sports, art, and nature. You now have the power to analyze them.

mr.aghead

Mr Aghead Almobaaid
0566991363

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قم بمسح رمز الـ QR



ختاماً، نسأل الله أن يوفقكم، وأن
تكون هذه الملزمة قد حققت
الفائدة المرجوة ♥



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