

THE FEATURED
PROGRAM EDUCATION



Mr. Ahmed Ata
The Featured Program

12 ADVANCED

البرنامج المميز

MATH ENG

CHAPTER 6

Mr. Ahmed Ata
The Featured Program

2025-2026

Prepared by : البرنامج المميز طريقك للتميز

MR- AHMED ATA



@AHMEDATACHAT

 <https://t.me/ahmedatachat>

 ahmatta.math@gmail.com

 0566010255 - 0502070147

 UAE - ABU DHABI

6 - Applications of the Definite Integral

1

Area Between Curves

2

Volume: Slicing, Disks and Washers

3

Volumes by Cylindrical Shells

4

Arc Length and Surface Area

5

Projectile Motion

6

Applications of Integration to Physics and Engineering

7

Probability .

THE FEATURED
PROGRAM EDUCATION



البرنامج المميز

MATH ENG

12 ADVANCED

LESSON 6-1

Area Between Curves



البرنامج المميز طريقك للتميز
Prepared by :

MR- AHMED ATA



@AHMEDATACHAT

 <https://t.me/ahmedatachat>
 ahmatta.math@gmail.com

 0566010255 - 0502070147
 UAE - ABU DHABI

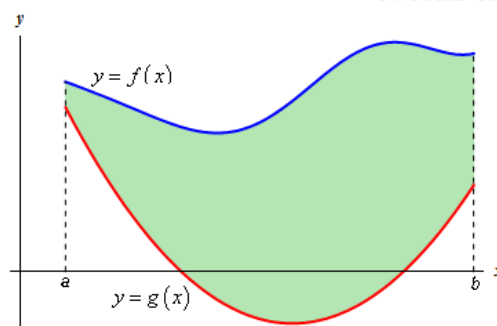
Lesson (6-1)

Area Between Curves

In this section we are going to look at finding the area between two curves. There are actually two cases that we are going to be looking at.

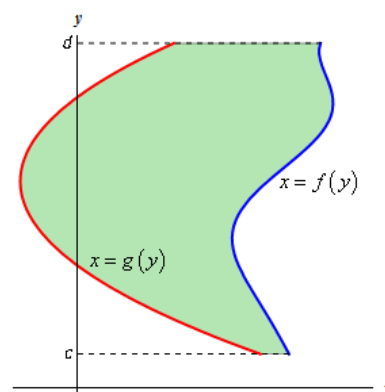
In the first case we want to determine the area between $y = f(x)$ and $y = g(x)$ on the interval $[a, b]$. We are also going to assume that $f(x) \geq g(x)$. Take a look at the following sketch to get an idea of what we're initially going to look at.

$$A = \int_a^b f(x) - g(x) dx$$



The second case is almost identical to the first case. Here we are going to determine the area between $x = f(y)$ and $x = g(y)$ on the interval $[c, d]$ with $f(y) \geq g(y)$.

$$A = \int_c^d f(y) - g(y) dy$$

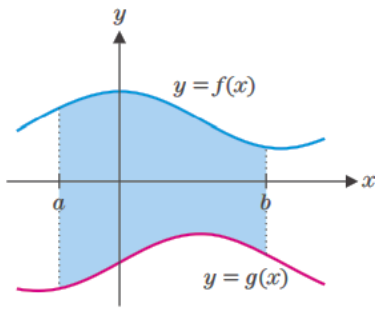


In the first case we will use,

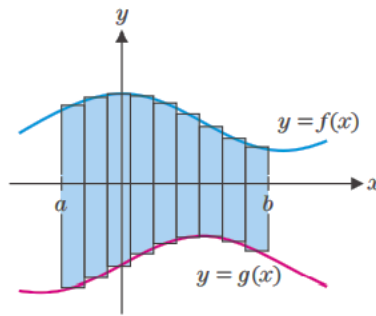
$$A = \int_a^b \left(\begin{array}{c} \text{upper} \\ \text{function} \end{array} \right) - \left(\begin{array}{c} \text{lower} \\ \text{function} \end{array} \right) dx, \quad a \leq x \leq b$$

In the second case we will use,

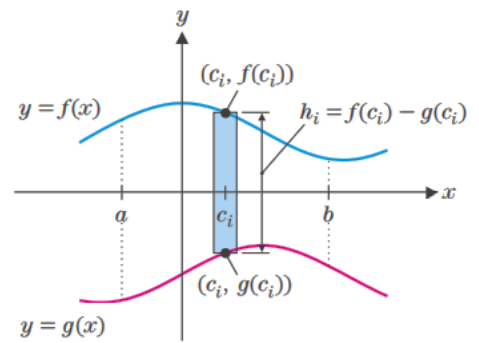
$$A = \int_c^d \left(\begin{array}{c} \text{right} \\ \text{function} \end{array} \right) - \left(\begin{array}{c} \text{left} \\ \text{function} \end{array} \right) dy, \quad c \leq y \leq d$$



Area between two curves



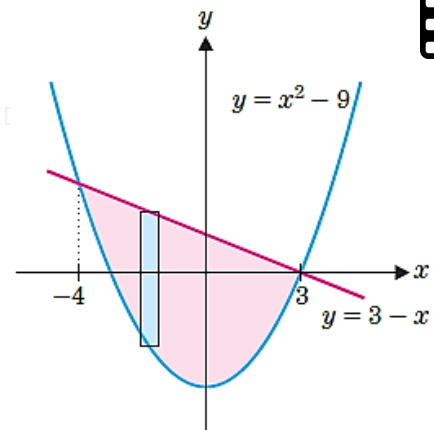
Approximate area

Area of i th rectangle

AREA BETWEEN TWO CURVES

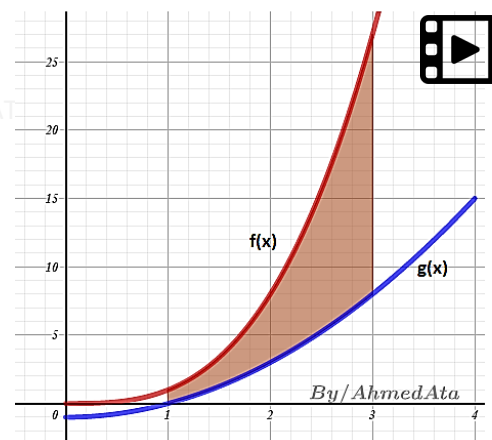
$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n [f(c_i) - g(c_i)] \Delta x = \int_a^b [f(x) - g(x)] dx. \quad (1.1)$$

- 1 Find the area bounded by the graphs of $y = 3 - x$ and $y = x^2 - 9$.



- 2 Find the area between the curves on the given interval.

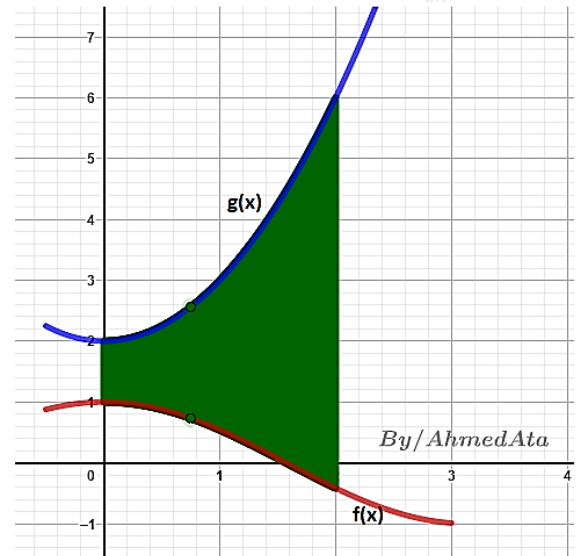
$$f(x) = x^3 \text{ and } g(x) = x^2 - 1, \quad 1 \leq x \leq 3$$



3

Find the area between the curves on the given interval.

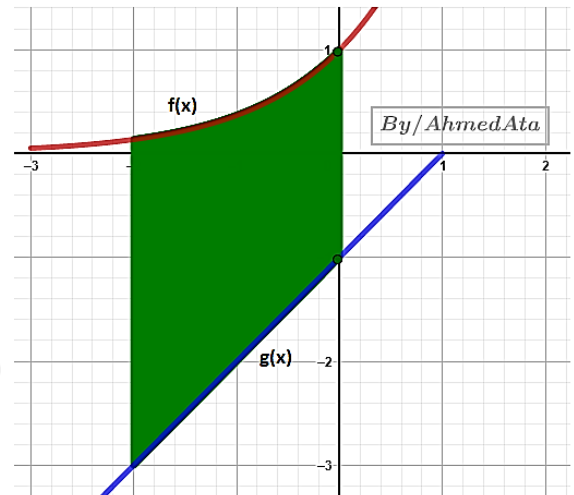
$$f(x) = \cos x \text{ and } g(x) = x^2 + 2, 0 \leq x \leq 2$$



4

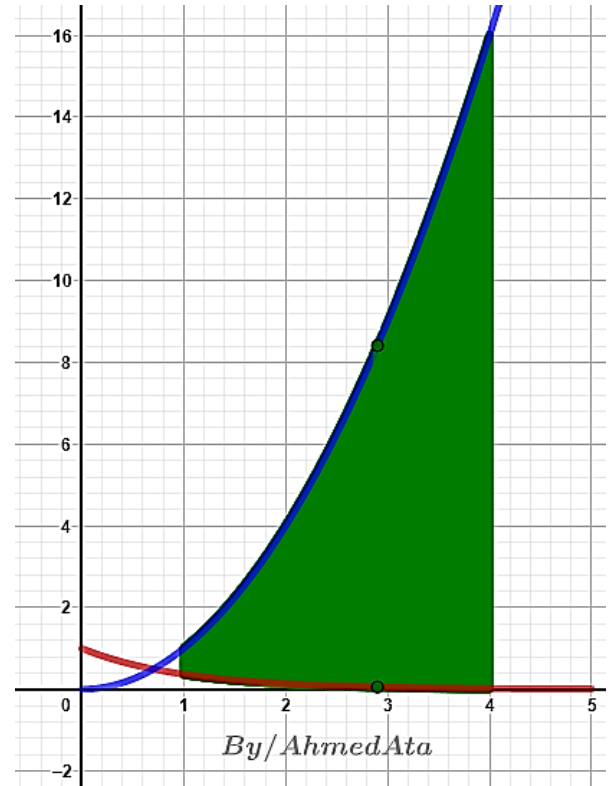
Find the area between the curves on the given interval.

$$f(x) = e^x \text{ and } g(x) = x - 1, -2 \leq x \leq 0$$



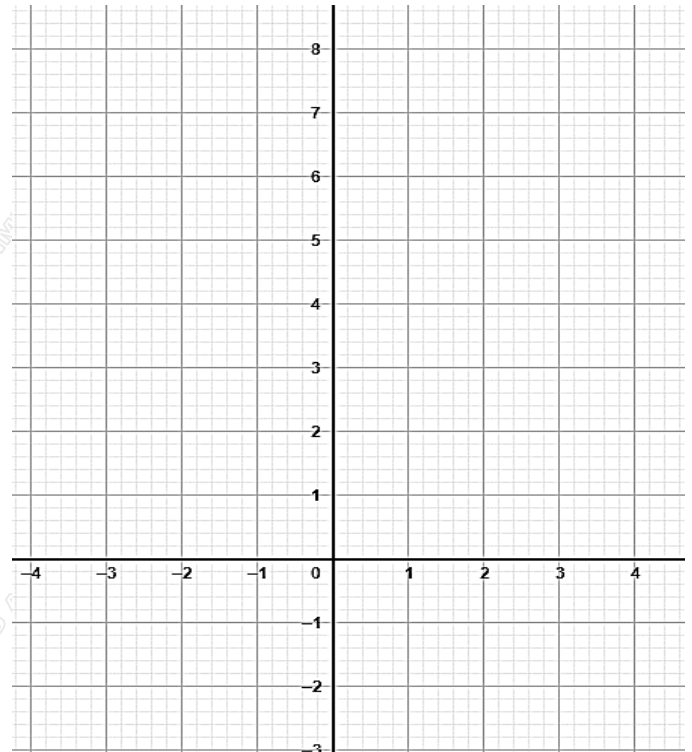
- 5 Find the area between the curves on the given interval.

$$f(x) = e^{-x} \text{ and } g(x) = x^2, 1 \leq x \leq 4$$



- 6 Sketch and find the area of the region determined by the intersections of the curves

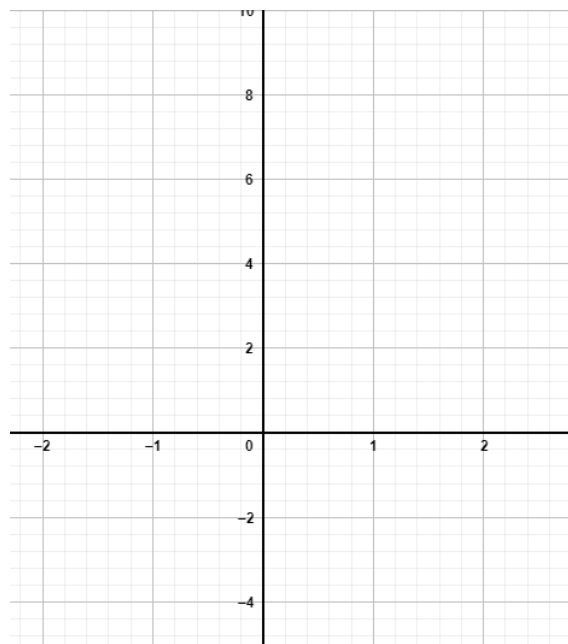
$$y = x^2 - 1, y = 7 - x^2$$



7

$$y = x^3, y = 3x + 2$$

AHMED ATA

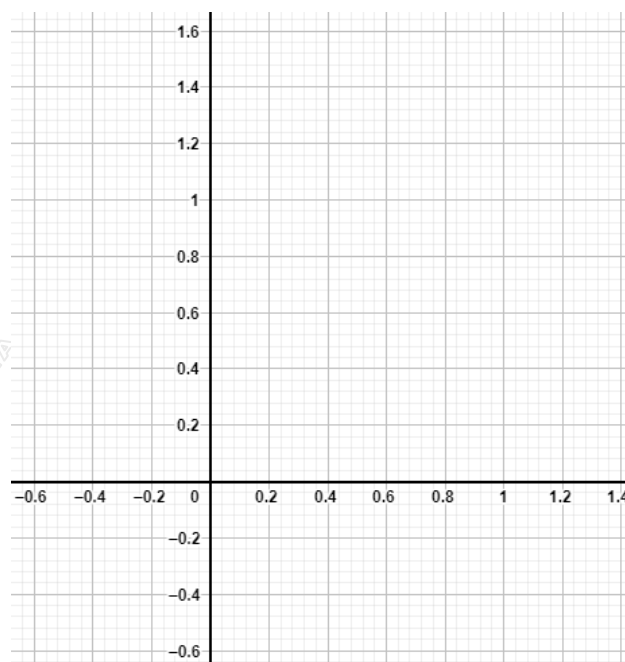


8

$$y = \sqrt{x}, y = x^2$$

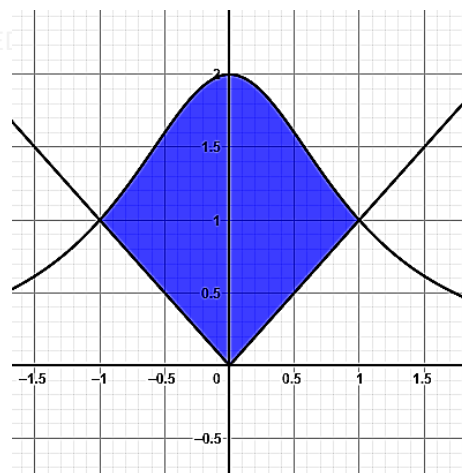
AHMED ATA

AHMED ATA



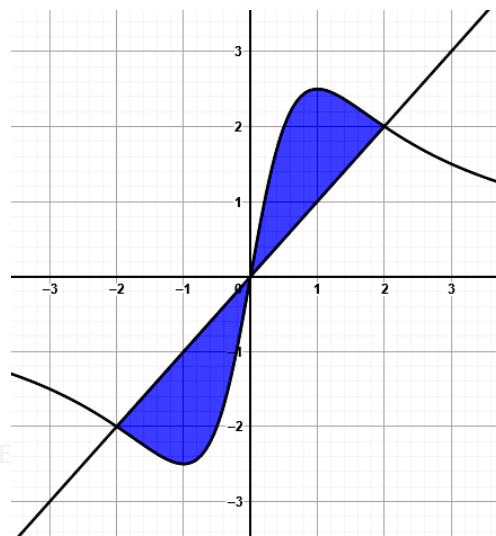
9

$$y = \frac{2}{x^2 + 1}, y = |x|$$



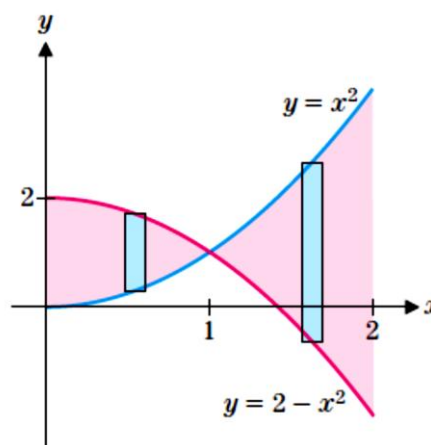
10

$$y = \frac{5x}{x^2 + 1}, y = x$$



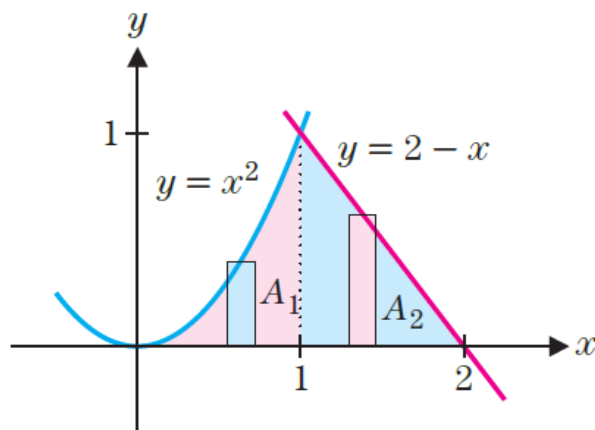
11

Find the area bounded by the graphs of $y = x^2$ and $y = 2 - x^2$ for $0 \leq x \leq 2$.



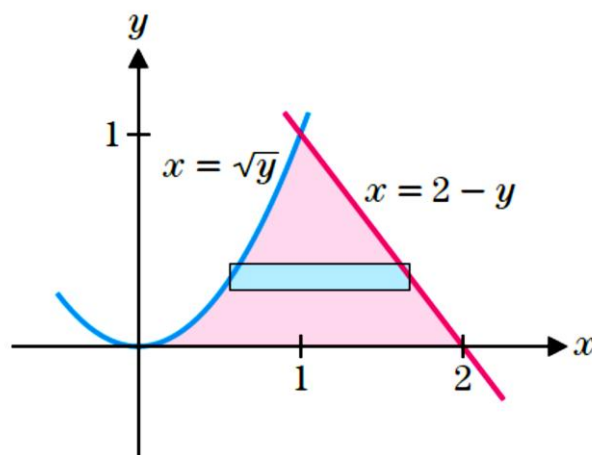
12

Find the area bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$.

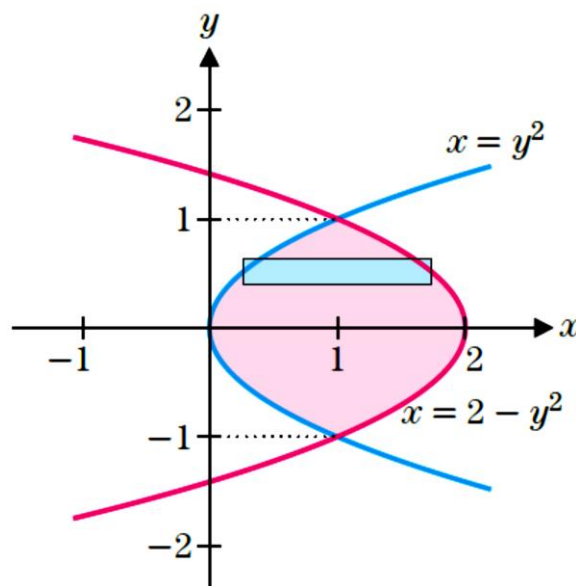


13

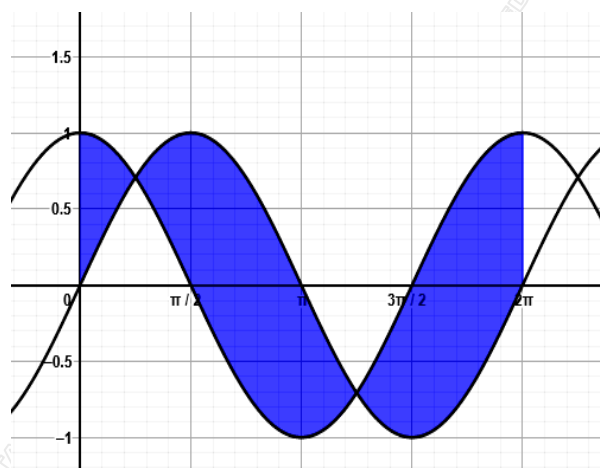
Find the area bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$.



14

Find the area bounded by the graphs of $x = y^2$ and $x = 2 - y^2$. ATA

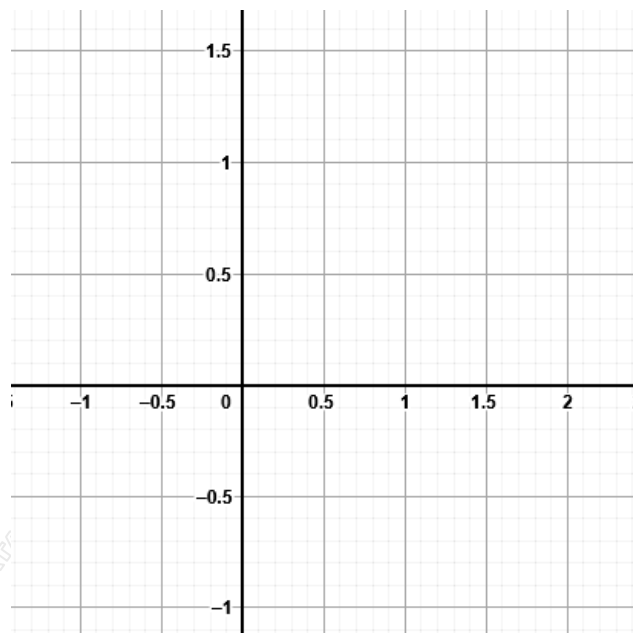
15

 $y = \sin x$ ($0 \leq x \leq 2\pi$), $y = \cos x$ 

Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

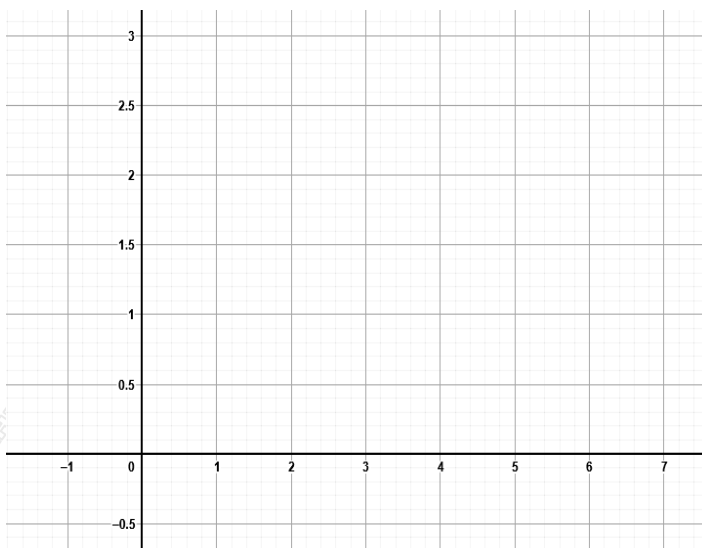
16

$$y = x, y = 2 - x, y = 0$$



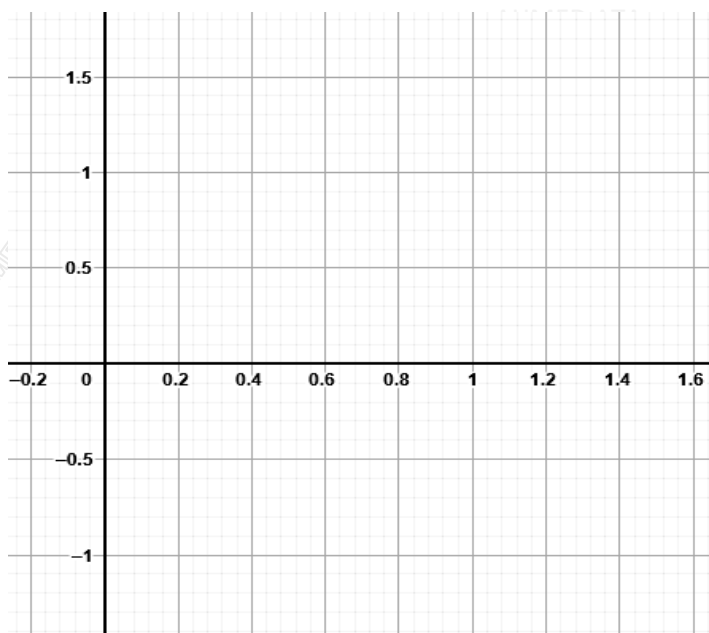
17

$$y = x, y = 2, y = 6 - x, y = 0$$



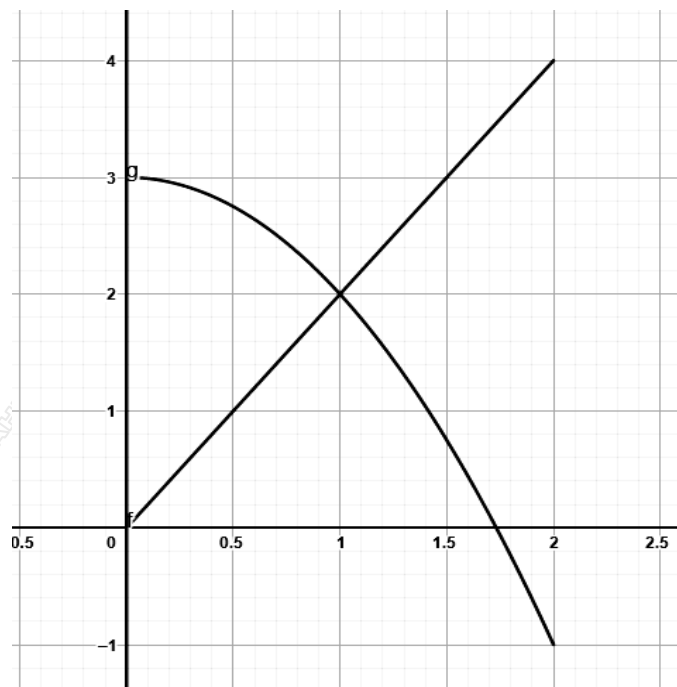
18

$$x = y, x = -y, x = 1$$



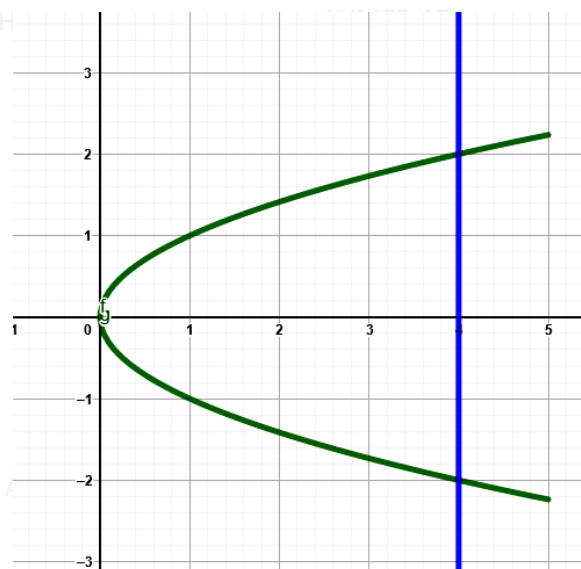
19

$$y = 2x \ (x > 0), y = 3 - x^2, x = 0$$



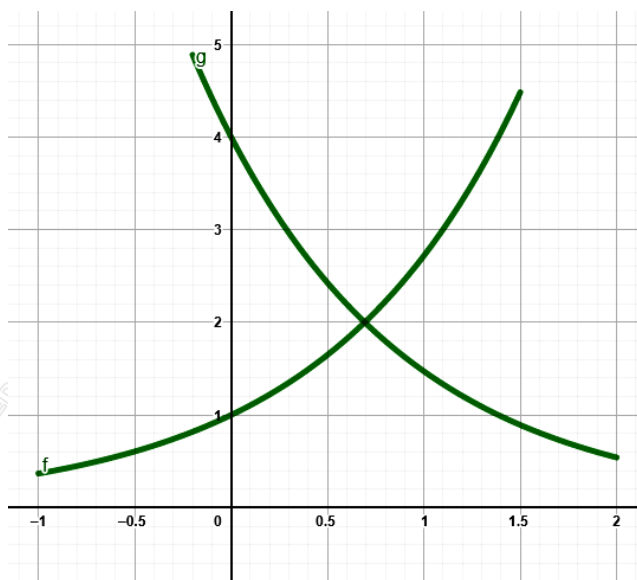
20

$$x = y^2, x = 4$$



21

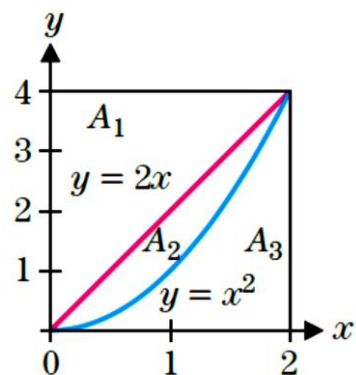
$$y = e^x, y = 4e^{-x}, x = 0$$



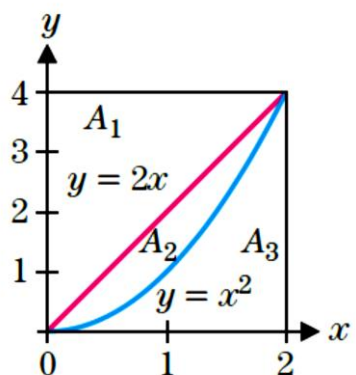
22

In terms of A_1 , A_2 and A_3 , identify the area given by each integral

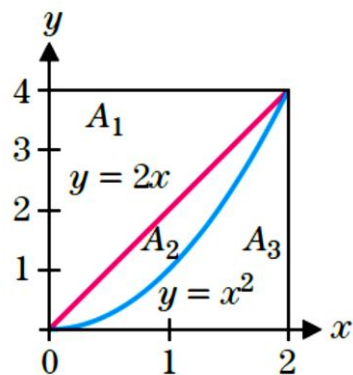
(a) $\int_0^2 (2x - x^2) dx$



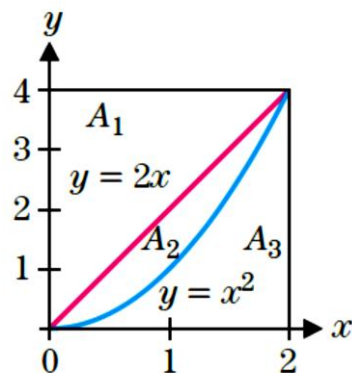
(b) $\int_0^2 (4 - x^2) dx$



(c) $\int_0^4 (2 - \sqrt{y}) dy$



(d) $\int_0^4 (\sqrt{y} - \frac{y}{2}) dy$



THE FEATURED
PROGRAM EDUCATION



البرنامج المميز

MATH ENG

12 ADVANCED

LESSON 6-2

Volume: Slicing, Disks and Washers



البرنامج المميز طريقك للتميز
Prepared by :

MR- AHMED ATA



@AHMEDATACHAT

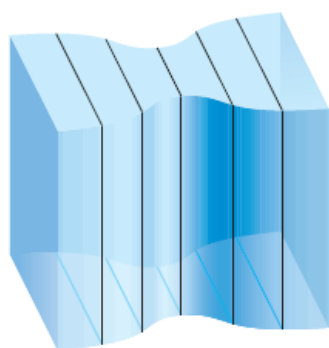
 <https://t.me/ahmedatachat>
 ahmatta.math@gmail.com

 0566010255 -0502070147
 UAE - ABU DHABI

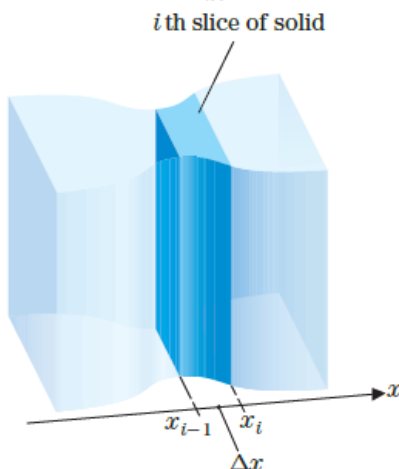
Lesson (6-2)

Volume: Slicing, Disks and Washers

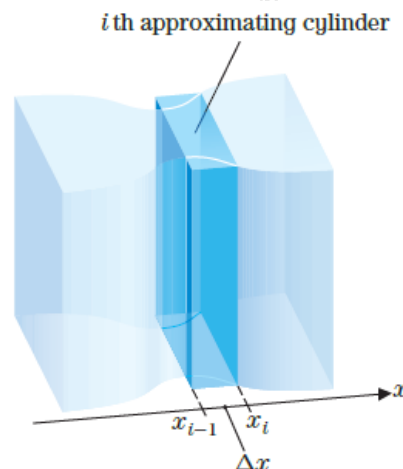
Volumes by Slicing



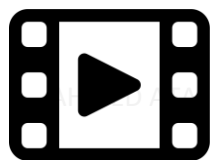
Sliced solid



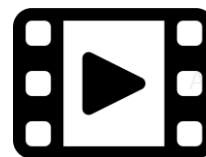
ith slice of solid



ith approximating cylinder

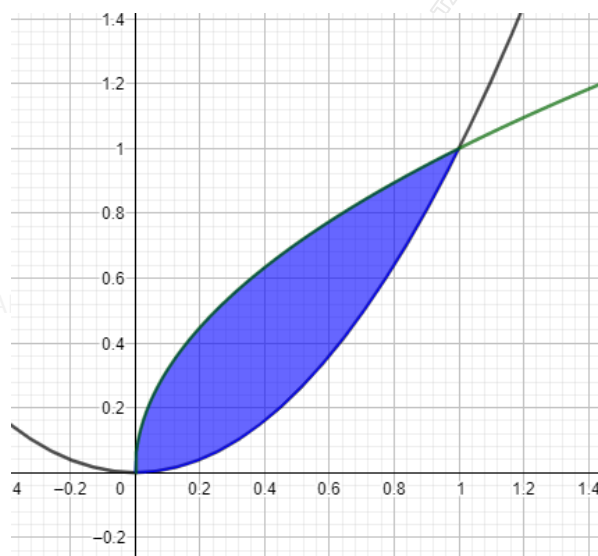


$$V = \int_a^b A(x) dx$$



- 1 Find the Volume of the solid whose base is bounded by $y = x^2$ and $y = \sqrt{x}$ with the indicated cross sections taken perpendicular to the x- axis

a) Square

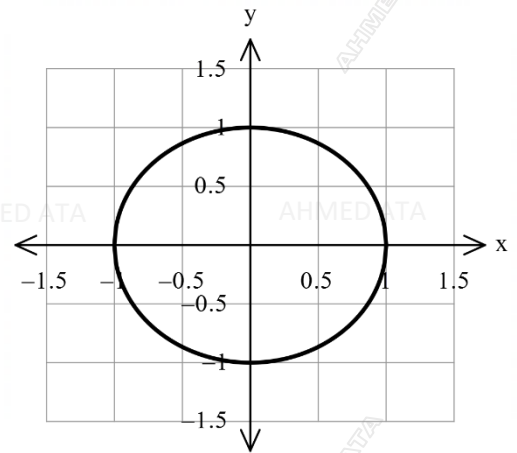


b) Equilateral Triangle

c) Semicircle

2 The base of a solid V is the circle $x^2 + y^2 = 1$. Find the volume if V has cross sections perpendicular to the x-axis.

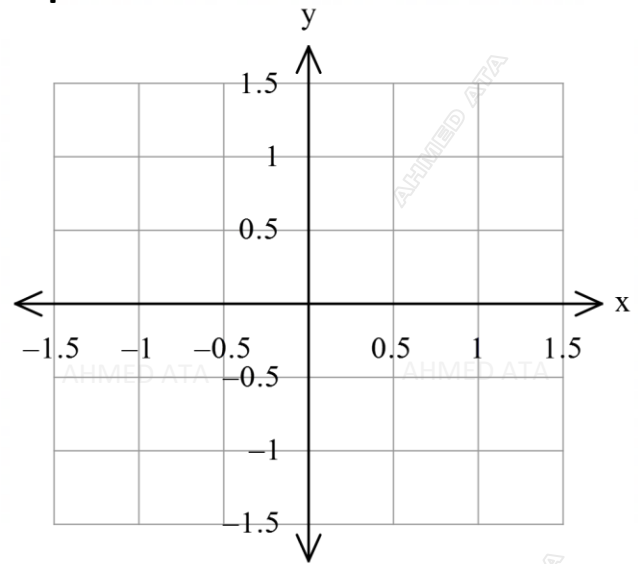
a) square cross sections



b) semicircular cross sections perpendicular to the x-axis

- 3 The base of a solid V is the triangle with vertices $(-1, 0)$, $(0, 1)$ and $(1, 0)$. Find the volume if V has cross sections perpendicular to the x -axis.

a) square cross sections



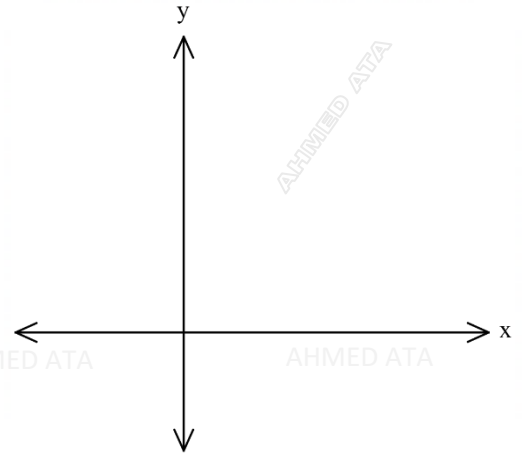
b) semicircular cross sections perpendicular to the x -axis

c) equilateral triangle cross sections perpendicular to the x -axis

4

The base of a solid V is the region bounded by $y = \ln x$, $x = 2$ and $y = 0$. Find the volume if V has cross sections perpendicular to the x -axis.

a) square cross sections

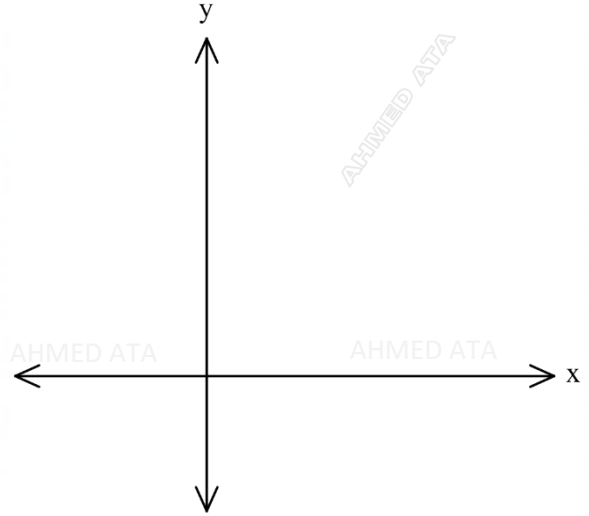


b) semicircular cross sections perpendicular to the x -axis

c) equilateral triangle cross sections perpendicular to the x -axis

5 The base of a solid V is the region bounded by $y = e^{-2x}$, $y = 0$, $x = 0$ and $x = \ln 5$. Find the volume if V has cross sections perpendicular to the x -axis.

a) square cross sections



b) semicircular cross sections perpendicular to the x -axis

Find the volume of the solid with cross sectional area $A(x)$.

AHMED ATA

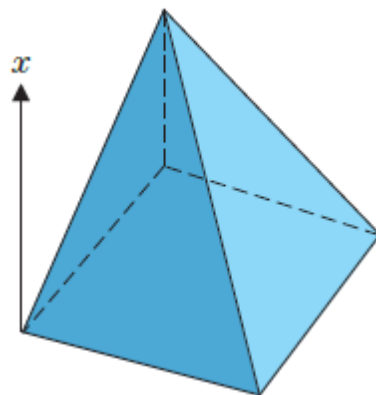
6 $A(x) = x + 2, -1 \leq x \leq 3$

7 $A(x) = 10e^{0.01x}, 0 \leq x \leq 10$

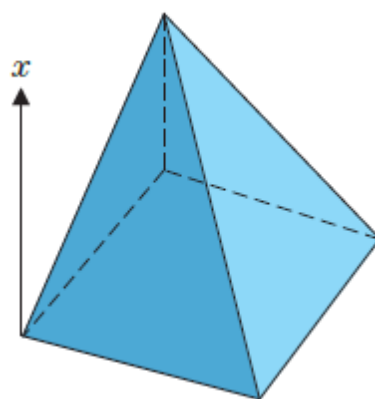
8 $A(x) = \pi(4 - x)^2, 0 \leq x \leq 2$

9 $A(x) = 2(x + 1)^2, 1 \leq x \leq 4$

- 10 The Pyramid Arena in Memphis has a square base of side approximately 180 meters and a height of approximately 100 meters. Find the volume of the pyramid with these measurements.



- 11 The great pyramid at Gizeh is 500 feet high, rising from a square base of side 750 feet. Compute its volume using integration.



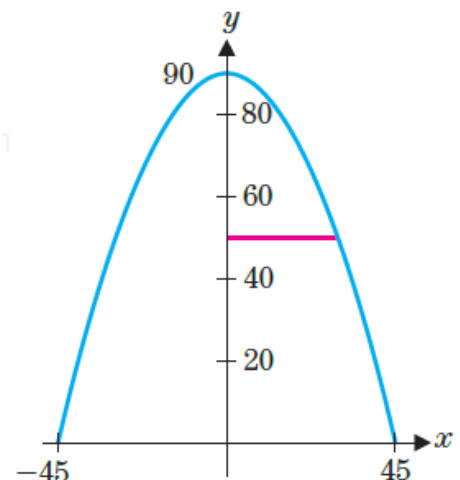
- 12 Suppose that instead of completing a pyramid, the builders at Gizeh had stopped at height 250 feet (with a square plateau top of side 375 feet). Compute the volume of this structure

- 13 Find the volume of a pyramid of height 160 feet that has a square base of side 300 feet.

- 14 Suppose that a dome has circular cross sections, without line

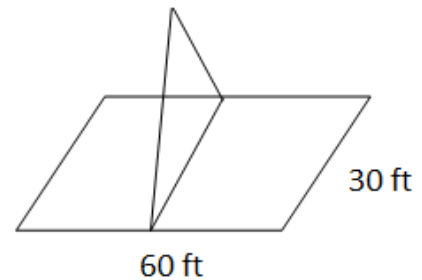
$$y = -\frac{2}{45}x^2 + 90$$

For $-45 \leq x \leq 45$. (In units of centimeters, this gives dimensions of a dome model similar to the Capito I Dome in Figure) Find the volume of the dome.



- 15 A church steeple is 30 feet tall with square cross sections. The square at the base has side 3 feet, the square at the top has side 6 inches and the side varies linearly in between. Compute the volume

- 16 A house attic has rectangular cross sections parallel to the rectangle is 30 feet by 60 feet at the bottom of the attic and the triangles have base 30 feet and height 10 feet. Compute the volume of the attic.



- 17 The outline of a dome is given by $y = 60 - \frac{x^2}{60}$ for $-60 \leq x \leq 60$ (units of feet), with circular cross-sections perpendicular to the y-axis. Find its volume

- 18 A pottery jar has circular cross sections of radius $4 + \sin \frac{x}{2}$ inches for $0 \leq x \leq 2\pi$. compute its volume

- 19 A pottery jar has circular cross sections of radius $4 - \sin \frac{x}{2}$ inches for $0 \leq x \leq 2\pi$. compute its volume.

SIMPSON'S RULE

$$\int_a^b f(x) dx \approx S_n(f) = \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)].$$

- 20 Suppose an MRI scan indicates that cross-sectional areas of adjacent slices of a tumor are as given in the table. Use Simpson's Rule to estimate the volume

x (cm)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$A(x)$ (cm ²)	0.0	0.1	0.2	0.4	0.6	0.4	0.3	0.2	0.2	0.1	0.0

- 21 Suppose an MRI scan indicates that cross-sectional areas of adjacent slices of a tumor are as given in the table. Use Simpson's Rule to estimate the volume

x (cm)	0.0	0.2	0.4	0.6	0.8	1.0	1.2
$A(x)$ (cm ²)	0.0	0.2	0.3	0.2	0.4	0.2	0.0

- 22 Suppose an MRI scan indicates that cross-sectional areas of adjacent slices of a tumor are as given in the table. Use Simpson's Rule to estimate the volume

x (m)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$A(x)$ (m ²)	2.0	1.8	1.7	1.6	1.8	2.0	2.1	2.2	2.4

The Method of Disks

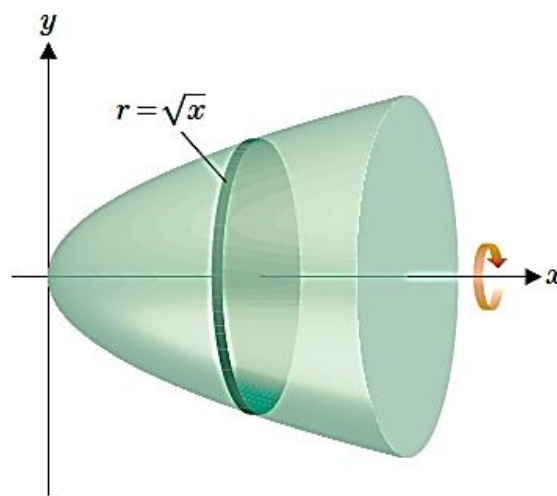
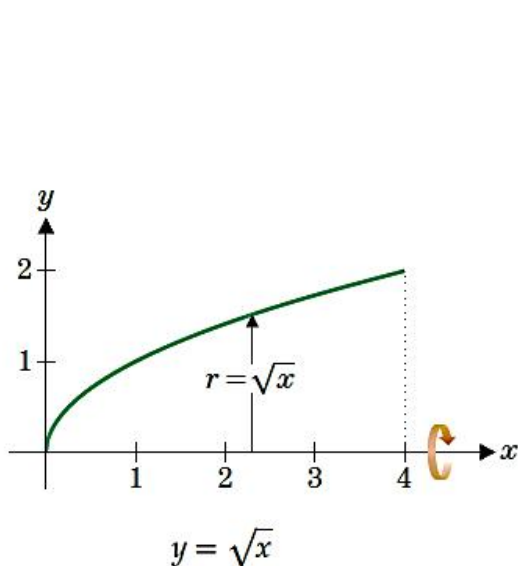


$$V = \int_a^b \underbrace{\pi [f(x)]^2}_{\text{cross-sectional area} = \pi r^2} dx.$$



23

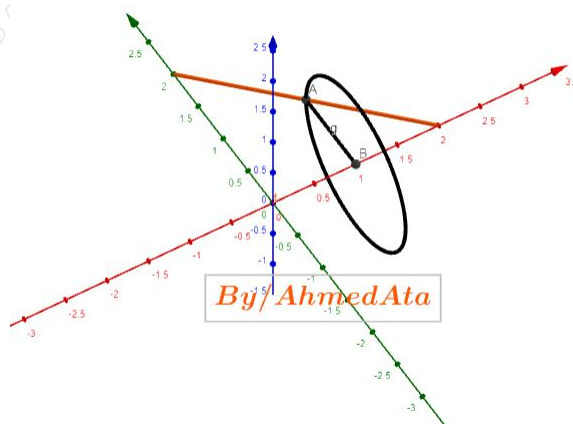
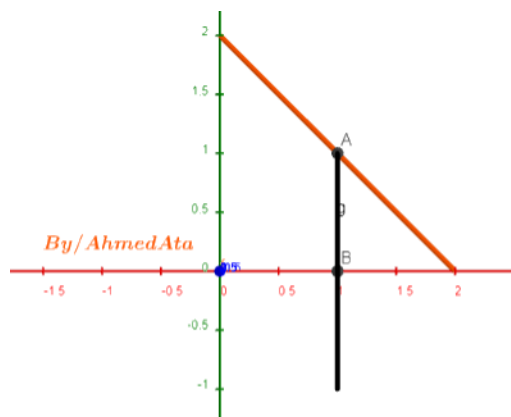
Revolve the region under the curve $y = \sqrt{x}$ on the interval $[0, 4]$ about the x - axis and find the volume of the resulting solid of revolution



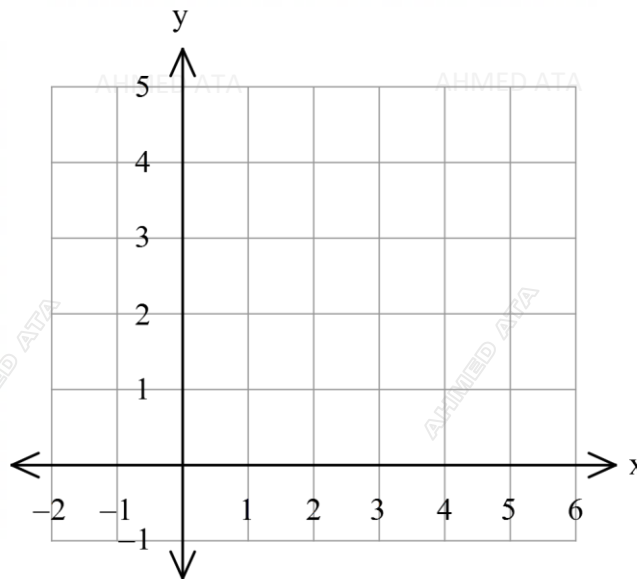
Solid of revolution

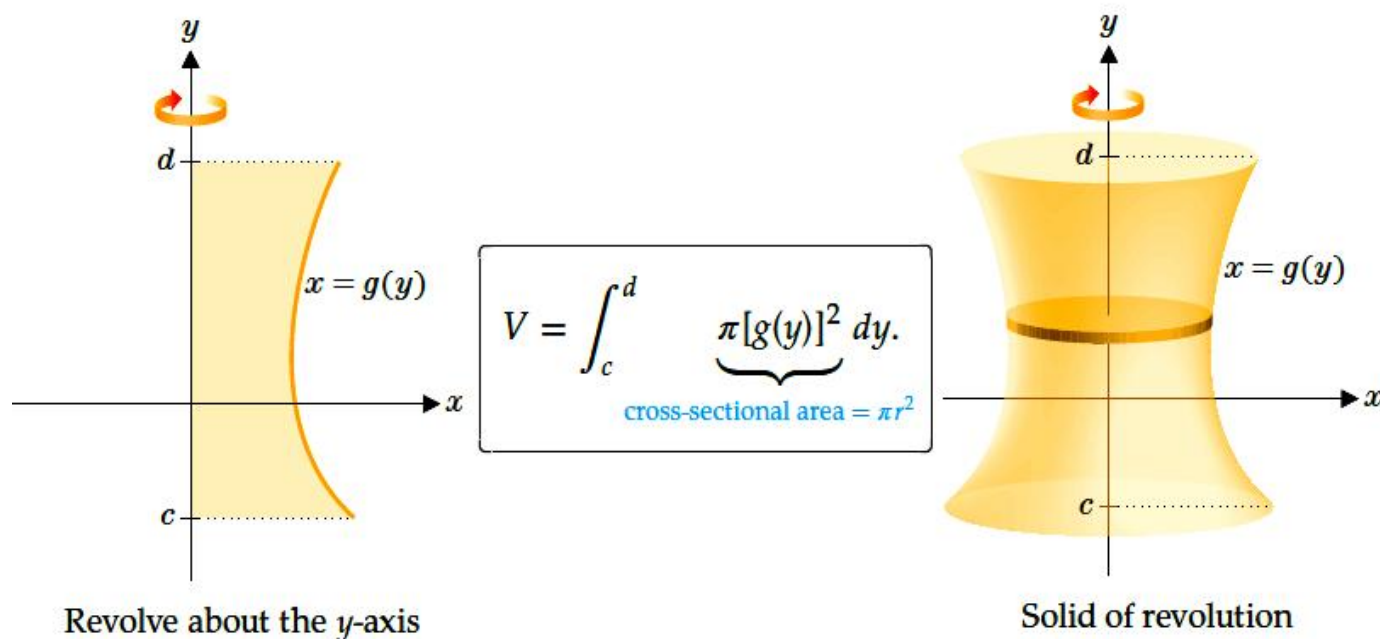
24 Region bounded by $y = 2 - x$, $y = 0$ and $x = 0$ about

(a) the x -axis;



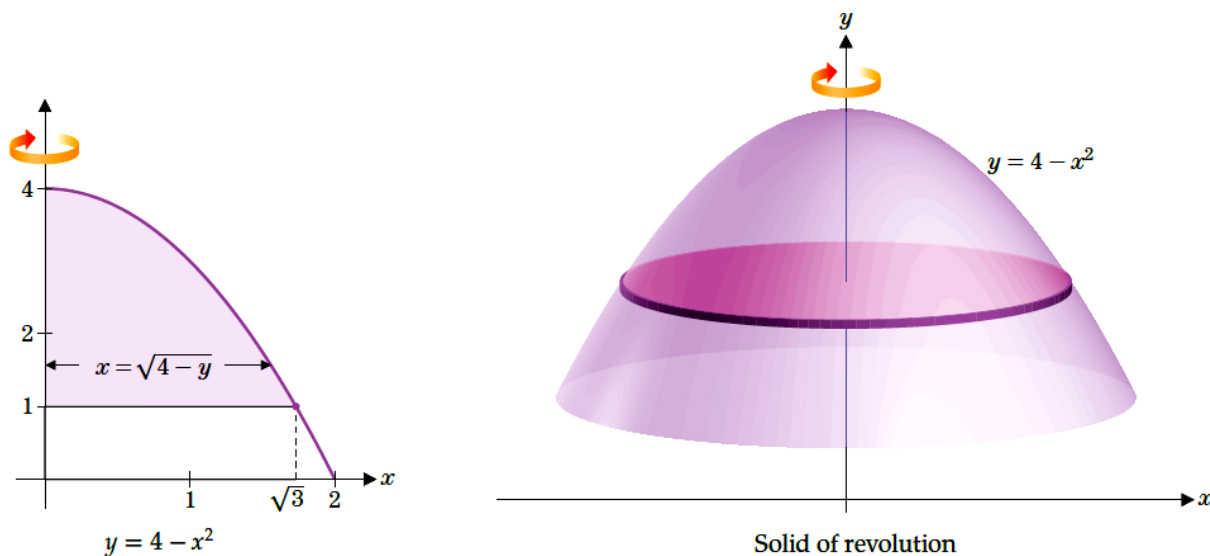
(b) $y = 3$





25

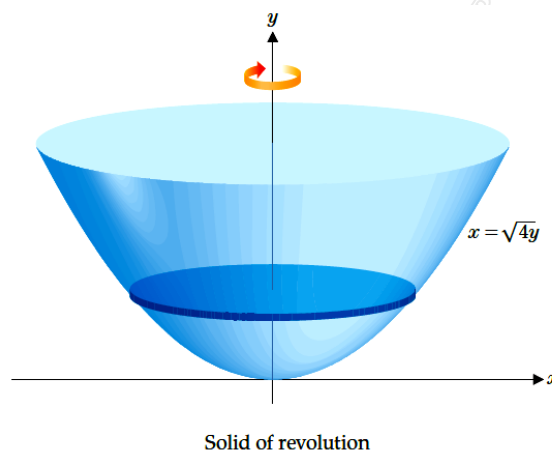
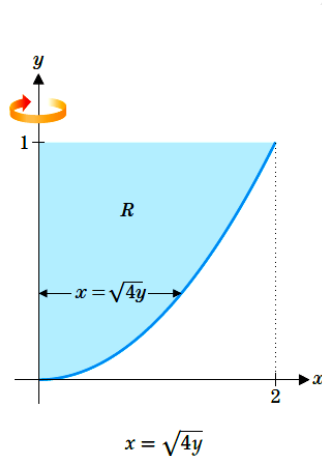
Find the volume of the solid resulting from revolving the region bounded by the curves $y = 4 - x^2$ and $y = 1$ from $x = 0$ to $x = \sqrt{3}$ about the y -axis



The Method of Washers

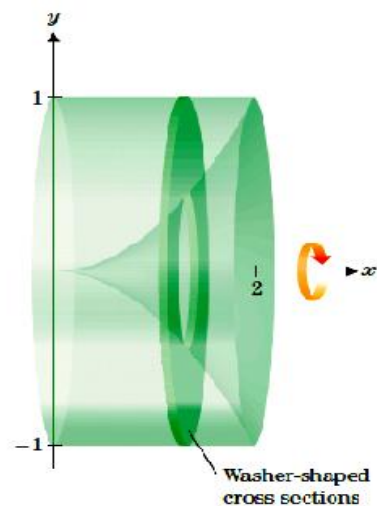
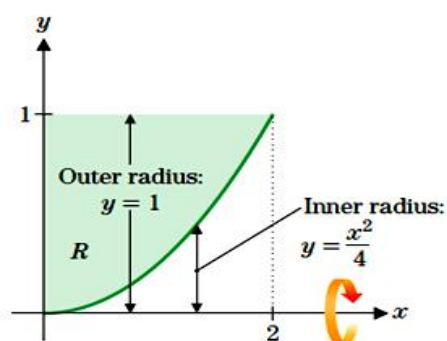
- 26 Let R be the region bounded by the graphs of $y = \frac{1}{4}x^2$, $x = 0$ and $y = 1$.
Compute the volume of the solid formed by revolving R about

(a) the y - axis



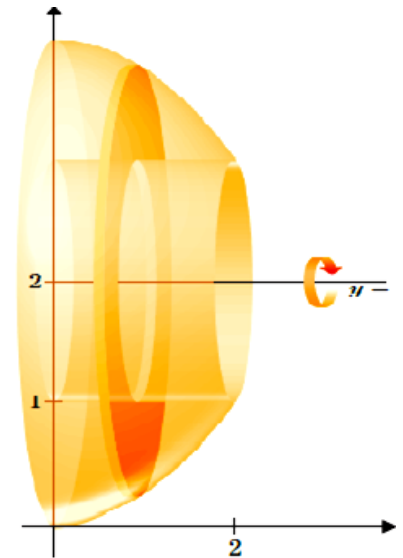
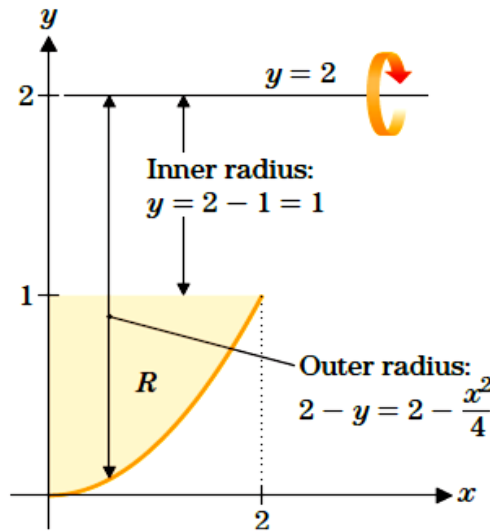
Solid of revolution

(b) the x - axis



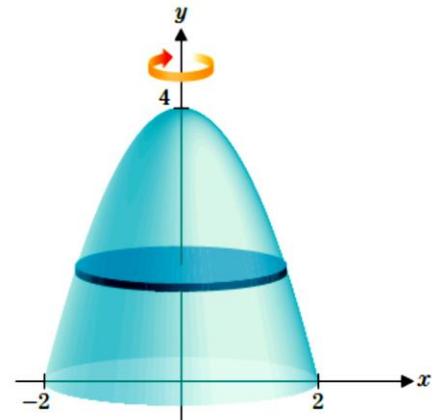
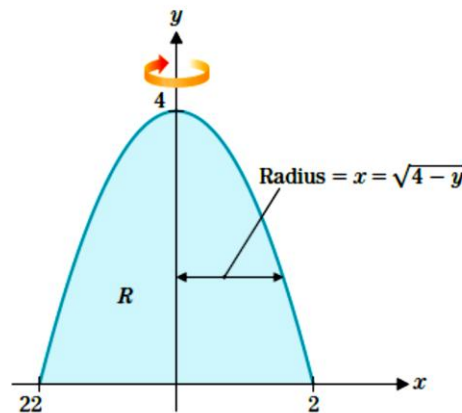
Washer-shaped cross sections

(c) the line $y = 2$.

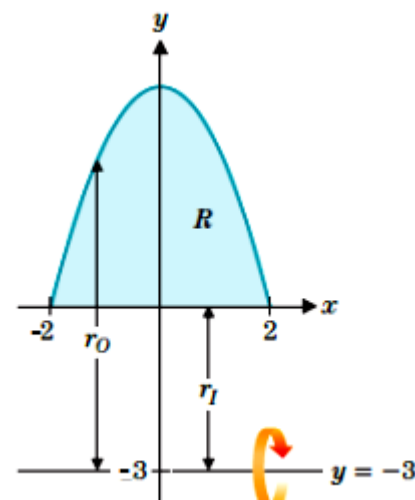


27 Let R be the region bounded by $y = 4 - x^2$ and $y = 0$. Find the volume of the solids obtained by revolving R about each of the following:

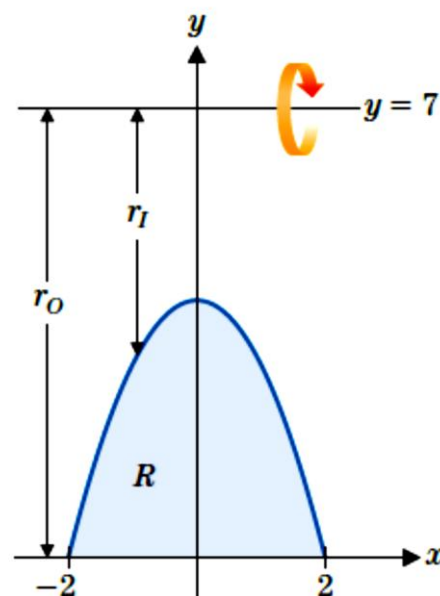
(a) the y -axis



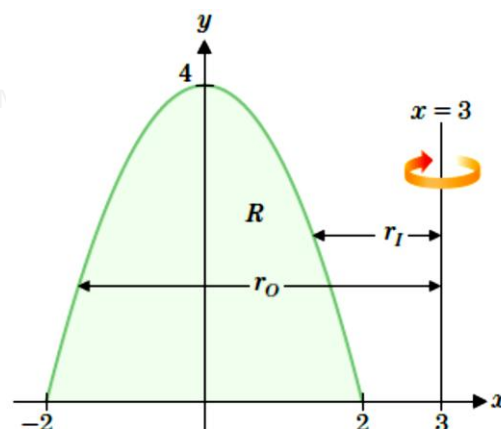
(b) the line $y = -3$



(c) the line $y = 7$



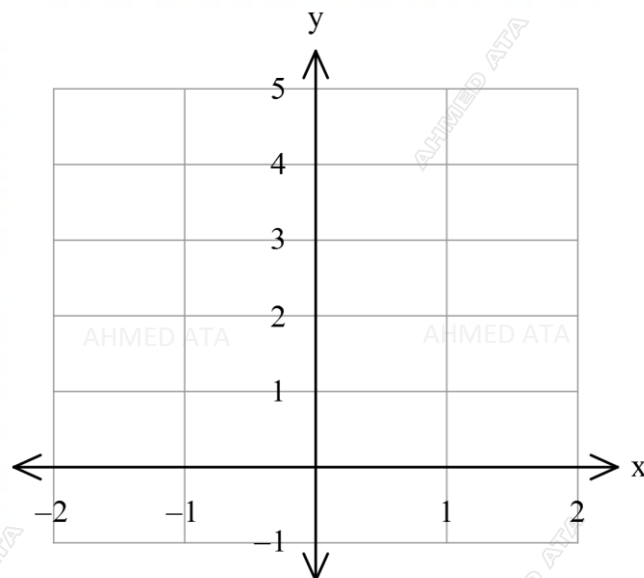
(d) the line $x = 3$



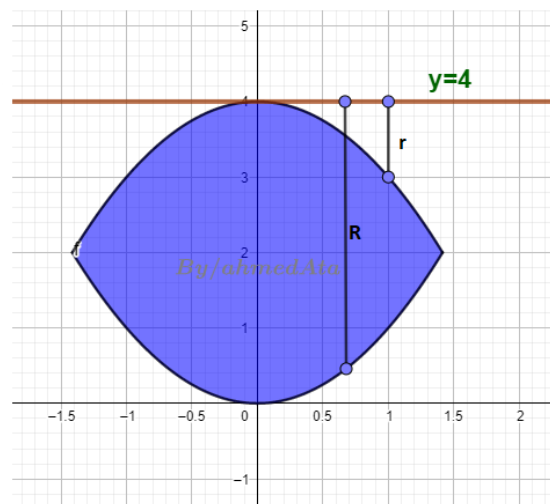
28

compute the volume of the solid formed by revolving the given region about the given line. Region bounded by $y = x^2$, $y = 4 - x^2$ about

(a) the x - axis



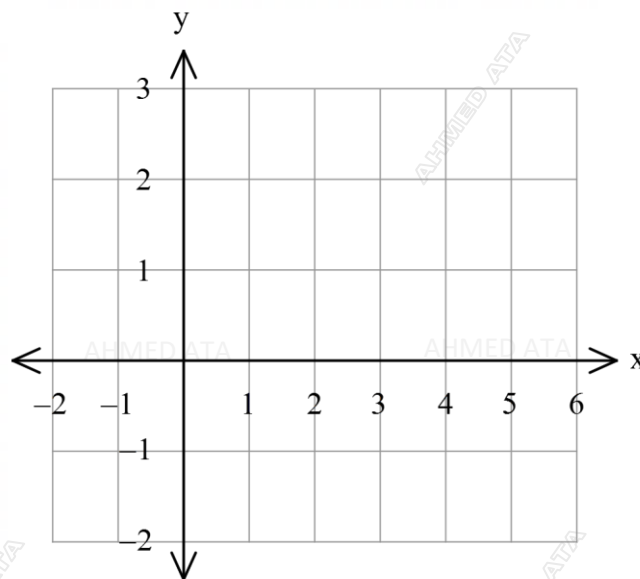
(b) $y = 4$



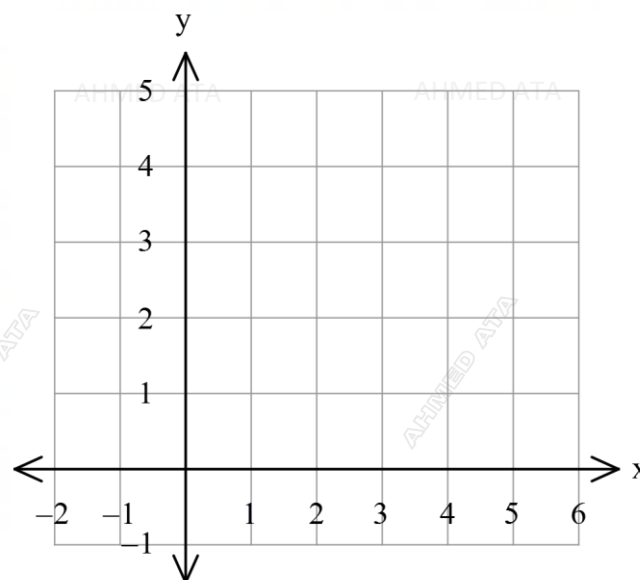
29

compute the volume of the solid formed by revolving the given region about the given line. Region bounded by $y = \sqrt{x}$ and $y = 2$ about

(a) the y -axis



(b) $y = 4$

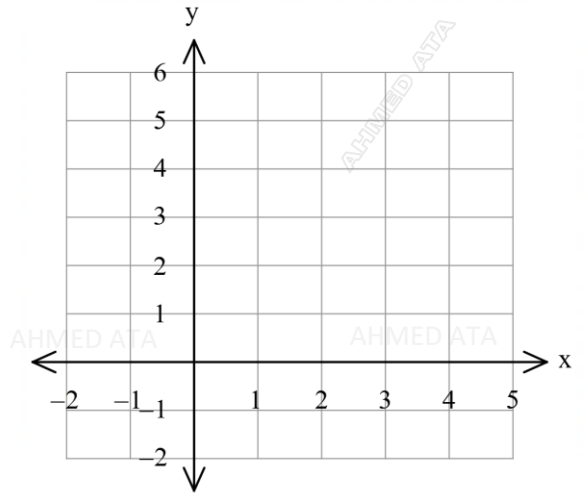


30

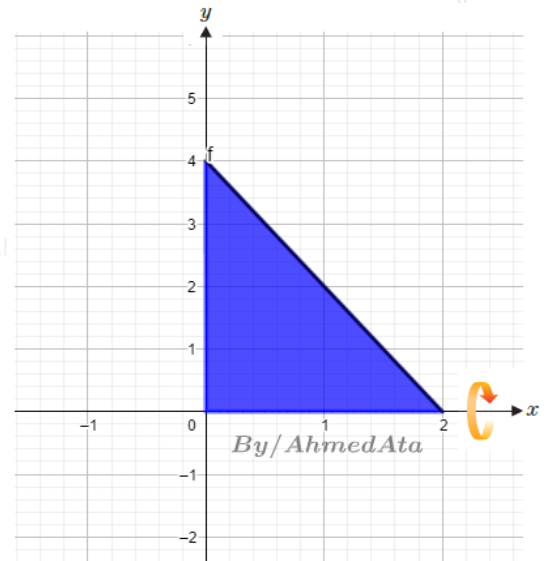
Let R be the region bounded by $y = 4 - 2x$, the x -axis and the y -axis.

Compute the volume of the solid formed by revolving R about the given line

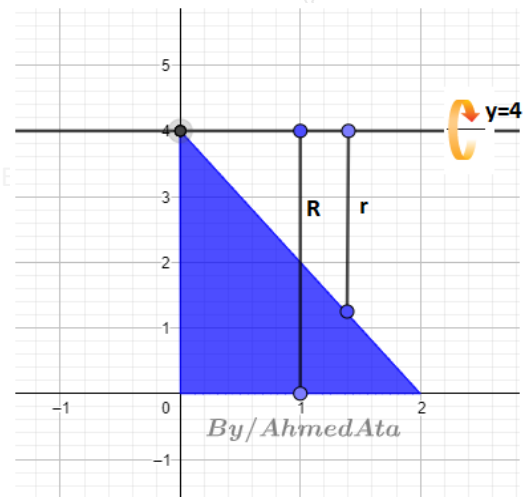
(a) the y -axis



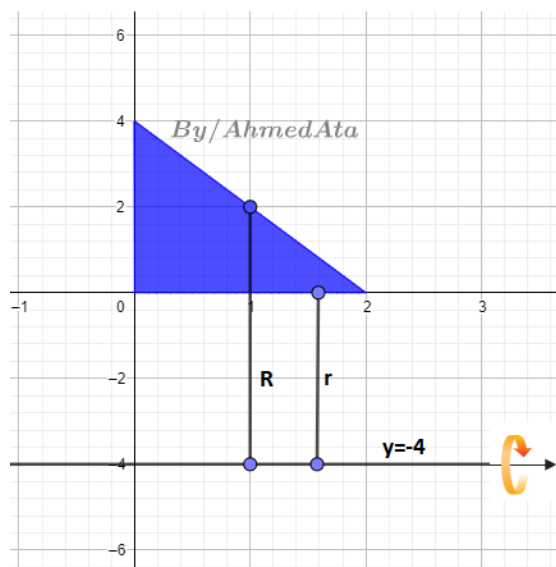
(b) the x -axis



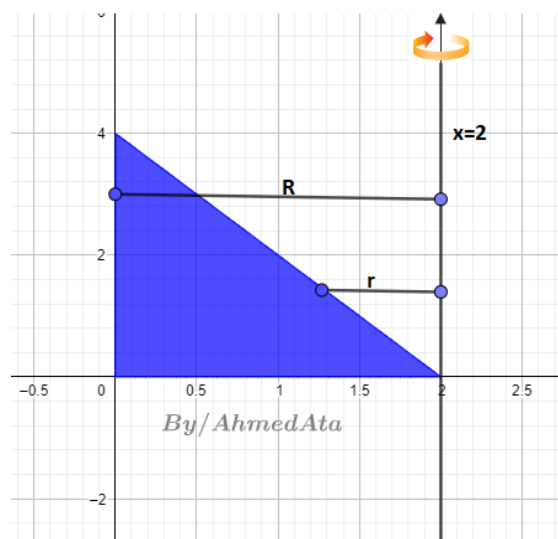
(c) $y = 4$



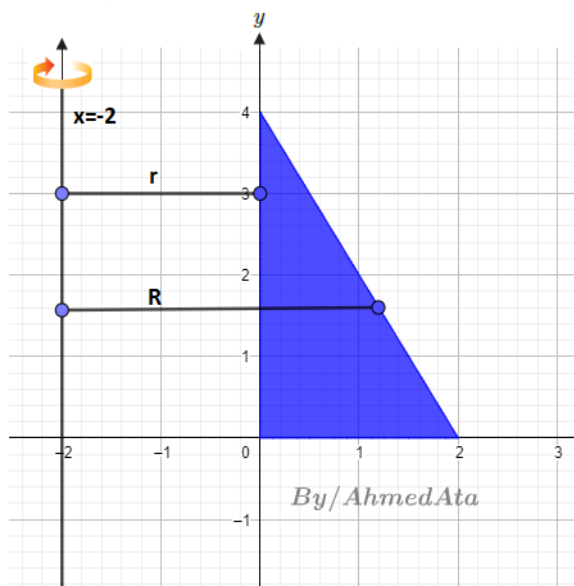
(d) $y = -4$



(e) $x = 2$



(f) $x = -2$

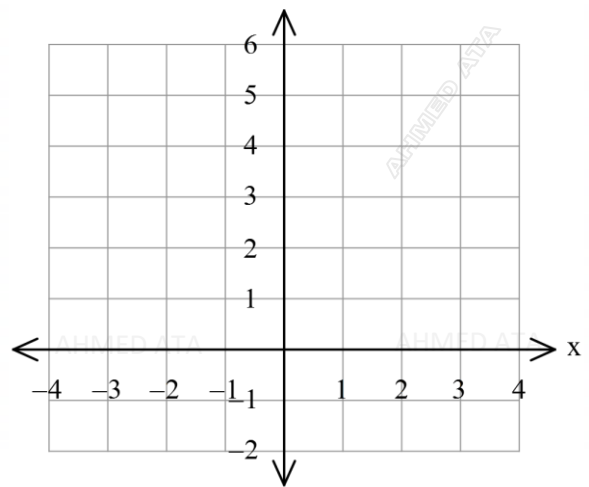


31

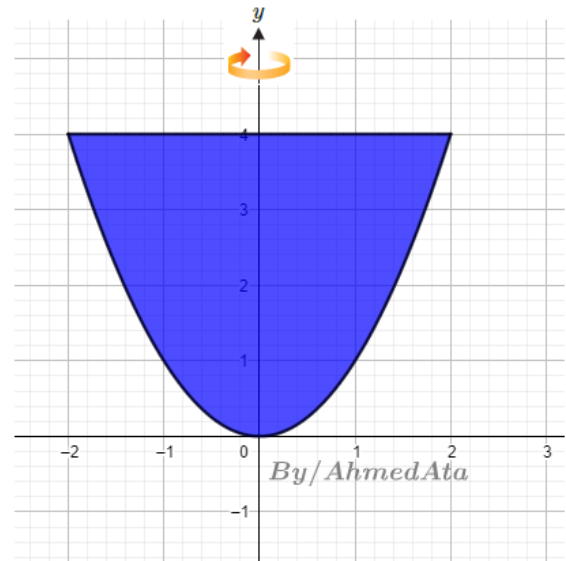
Let R be the region bounded by $y = x^2$, and the $y = 4$.

Compute the volume of the solid formed by revolving R about the given line

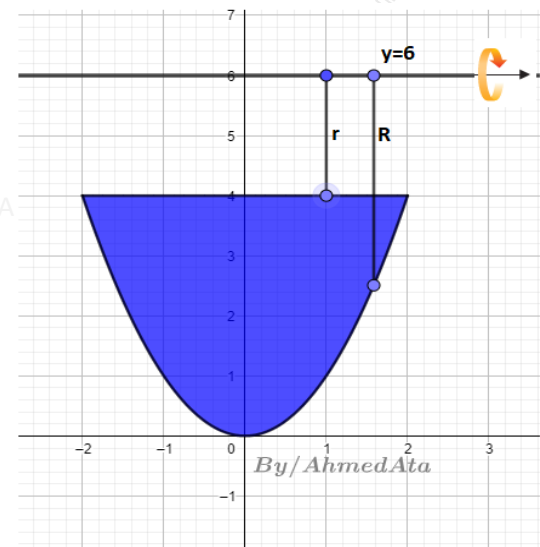
(a) $y = 4$



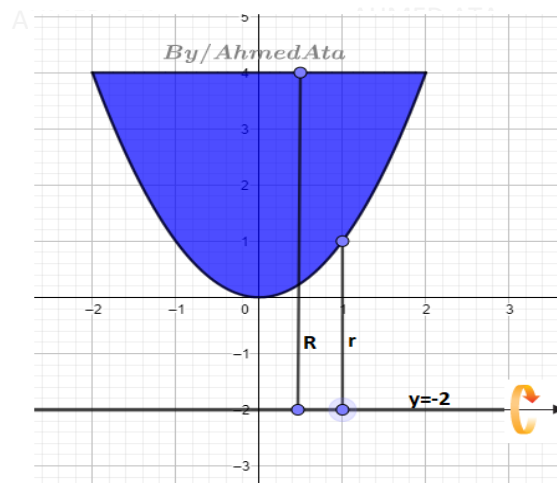
(b) the y - axis



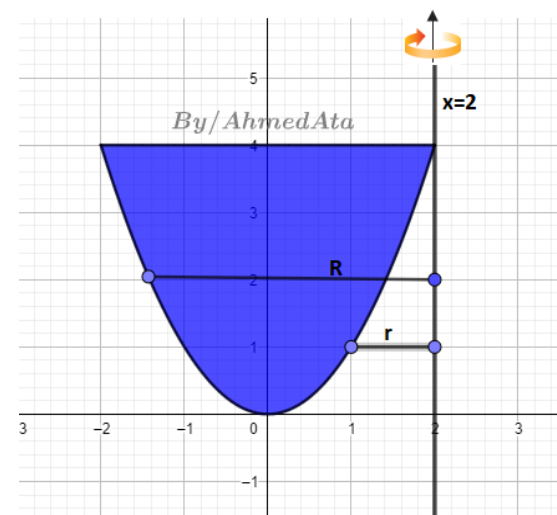
(c) $y = 6$



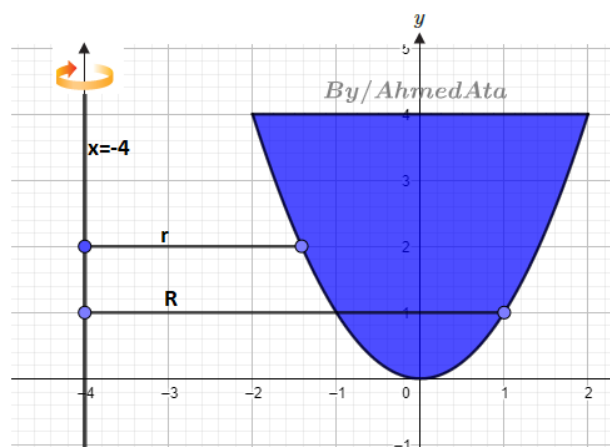
$$(d) y = -2$$



$$(e) x = 2$$



$$(f) x = -4$$



THE FEATURED
PROGRAM EDUCATION



البرنامج المميز

MATH ENG

12 ADVANCED

LESSON 6-4

Arc Length and Surface Area



البرنامج المميز طريقك للتميز
Prepared by :

MR- AHMED ATA



@AHMEDATACHAT

 <https://t.me/ahmedatachat>
 ahmatta.math@gmail.com

 0566010255 - 0502070147
 UAE - ABU DHABI

Lesson (6-4)**Arc Length and Surface Area****Arc Length**

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} dx,$$

1 Find the arc length of the portion of the curve $y = \sin x$ with $0 \leq x \leq \pi$.

2 Find the arc length of the portion of the curve $y = x^2$ with $0 \leq x \leq 1$.

compute the arc length exactly

3 $y = 2x + 1, 0 \leq x \leq 2$

4

$$y = 4x^{3/2} + 1, 1 \leq x \leq 2$$

AHMED ATA

AHMED ATA

5

$$y = \frac{1}{4}(e^{2x} + e^{-2x}), 0 \leq x \leq 1$$

AHMED ATA

AHMED ATA

AHMED ATA

AHMED ATA

6

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, 1 \leq x \leq 2$$

AHMED ATA

AHMED ATA

AHMED ATA

AHMED ATA

7

$$y = \frac{1}{6}x^3 + \frac{1}{2x}, 1 \leq x \leq 3$$

8

$$x = \frac{1}{8}y^4 + \frac{1}{4y^2}, -2 \leq y \leq -1$$

9

$$y = 2 \ln(4 - x^2), 0 \leq x \leq 1$$

10

$$y = x^3, -1 \leq x \leq 1$$

11

$$y = \tan x, 0 \leq x \leq \pi/4$$

12

$$y = \cos x, 0 \leq x \leq \pi$$

13

$$y = \ln x, 1 \leq x \leq 3$$

14

$$y = \int_0^x u \sin u \, du, 0 \leq x \leq \pi$$

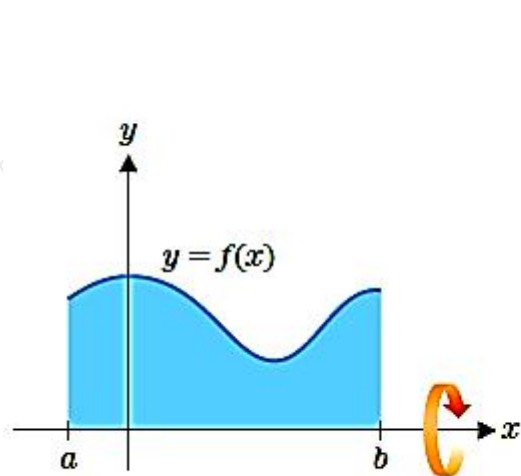
15

$$y = \int_0^x e^{-u} \sin u \, du, 0 \leq x \leq \pi$$

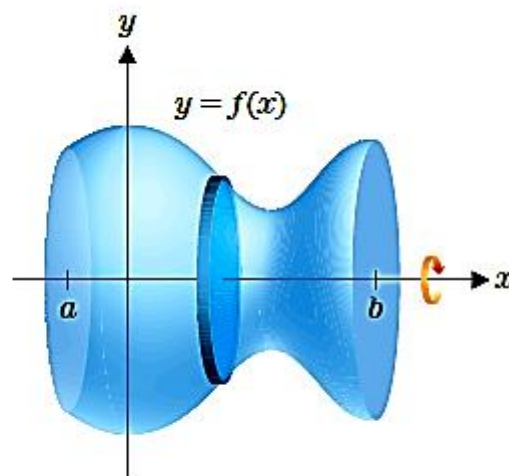
16

A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 10 \left(e^{\frac{x}{20}} + e^{\frac{-x}{20}} \right)$, $-20 \leq x \leq 20$, compute the length of the rope.

- 17 A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 15 \left(e^{\frac{x}{30}} + e^{\frac{-x}{30}} \right)$, $-30 \leq x \leq 30$, compute the length of the rope compute the “sag” in the cable

Surface Area

Revolve about x-axis



Surface of revolution

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx,$$

Find the surface area of the surface generated by revolving

18

$y = x^2, 0 \leq x \leq 1$, revolved about the x-axis

19

$y = \sin x, 0 \leq x \leq \pi$, revolved about the x-axis

20

 $y = 2x - x^2, 0 \leq x \leq 2$, revolved about the x -axis

AHMED ATA

21

 $y = x^3 - 4x, -2 \leq x \leq 0$, revolved about the x -axis

AHMED ATA

22

 $y = e^x, 0 \leq x \leq 1$, revolved about the x -axis

AHMED ATA

23

 $y = \ln x, 1 \leq x \leq 2$, revolved about the x -axis

AHMED ATA

24

 $y = \cos x, 0 \leq x \leq \pi/2$, revolved about the x -axis

AHMED ATA

25

 $y = \sqrt{x}, 1 \leq x \leq 2$, revolved about the x -axis

ATA

AHMED ATA

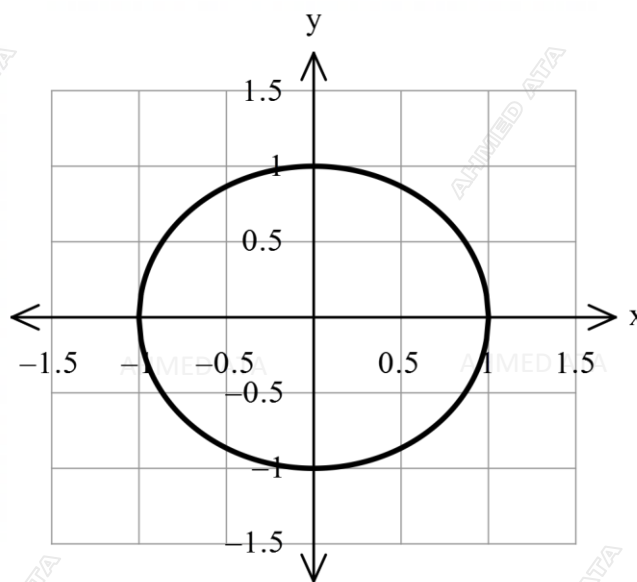
26

compute the arc length $L1$ of the curve and the length $L2$ of the secant line connecting the endpoints of the curve. Compute the ratio $\frac{L2}{L1}$

$$y = \sin x, -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$$

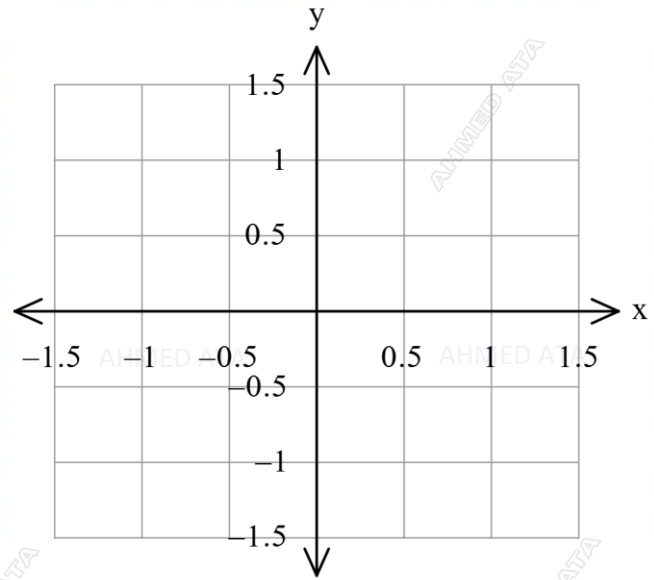
27

Suppose that the circle $x^2 + y^2 = 1$ is revolved about the y-axis. Compute the surface area.



28

Suppose that the triangle with vertices $(-1, -1)$, $(0, 1)$ and $(1, -1)$ is revolved about the y-axis. Compute the surface area.



THE FEATURED
PROGRAM EDUCATION



البرنامج المميز

MATH ENG

12 ADVANCED

LESSON 6-5

Projectile Motion

Mr. Ahmed Ata
The Featured Program
2025-2026

البرنامج المميز طريقك للتميز
Prepared by :

MR- AHMED ATA



@AHMEDATACHAT

 <https://t.me/ahmedatachat>
 ahmatta.math@gmail.com

 0566010255 - 0502070147
 UAE - ABU DHABI

Lesson (6-5)

Projectile Motion

Newton's second law then gives us $h''(t) = -g$

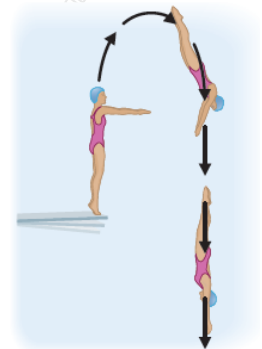
$$g = 9.8 \text{ m/s}^2 \text{ or } g = 32 \text{ ft/s}^2$$

$$\text{hint : } h(t) = -\frac{1}{2}gt^2 + v_0t + h_0 \quad x(t) = v_0t + x_0$$

1 identify the initial conditions $y(0)$ and $y'(0)$

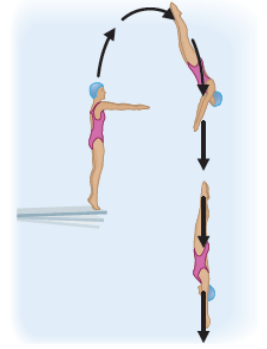
- An object is dropped from a height of 24 meters
- An object is dropped from a height of 30 meters
- An object is released from a height of 18 meters with an upward velocity of 3
- An object is released from a height of 6 meters with a downward velocity of 1.2

2 If a diving board is 4.5 meters above the surface of the water and a diver starts with initial velocity 2.4 m/s (in the upward direction), what is the diver's velocity at impact (assuming no air resistance)?



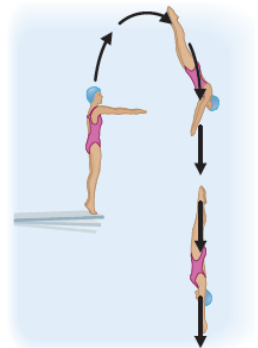
3

A diver drops from 9 meters above the water (about the height of an Olympic platform dive). What is the diver's velocity at impact



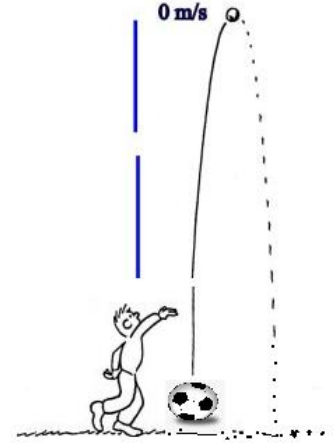
4

A diver drops from 36 meters above the water (about the height of divers at the Acapulco Cliff Diving competition). What is the diver's velocity at impact



5 A ball is propelled straight upward from the ground with initial velocity 19.6 m/s . Ignoring air resistance,

- Find an equation for the height of the ball at any time t .
- Determine the maximum height
- The amount of time the ball spends in the air



6 A certain not-so-wily coyote discovers that he just stepped off the edge of a cliff. Four seconds later, he hits the ground in a puff of dust. How high in meters was the cliff?

7 The coyote's next scheme involves launching himself into the air with an Acme catapult. If the coyote is propelled vertically from the ground with initial velocity 19.6 m/s ,

a) Find an equation for the height of the coyote at any time t .

b) Find his maximum height,

c) The amount of time spent in the air

d) The velocity when he smacks back into the catapult

KeyConcept Projectile Motion

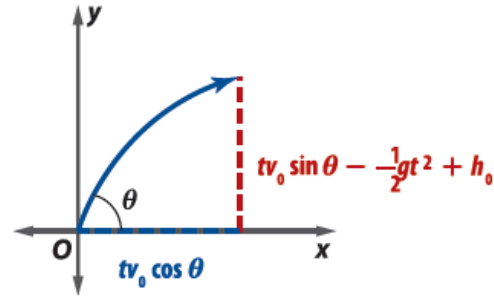
For an object launched at an angle θ with the horizontal at an initial velocity v_0 , where g is the gravitational constant, t is time, and h_0 is the initial height:

Horizontal Distance

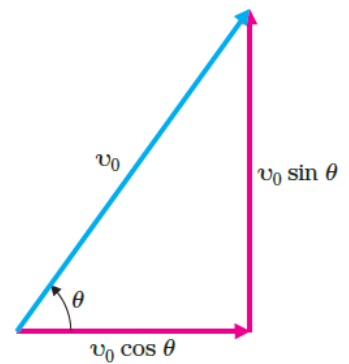
$$x = tv_0 \cos \theta$$

Vertical Position

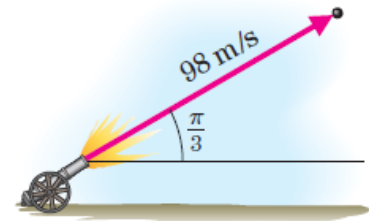
$$y = tv_0 \sin \theta - \frac{1}{2}gt^2 + h_0$$



- 8 An object is launched at angle $\theta = \pi / 6$ from the horizontal with initial speed $v_0 = 98 \text{ m/s}$. Determine the time of flight and the (horizontal) range of the projectile



- 9 An object is launched at angle $\theta = \pi / 3$ from the horizontal with initial speed $v_0 = 98 \text{ m/s}$. Determine the time of flight and the (horizontal) range of the projectile

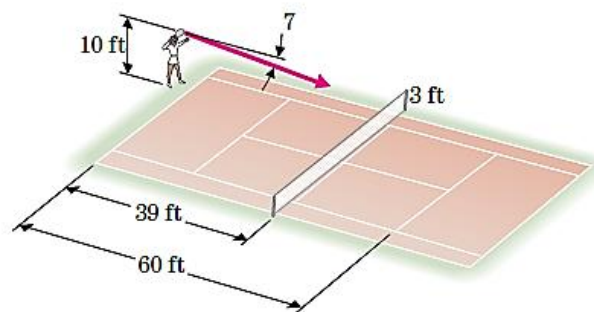


- 10 Find the time of flight and horizontal range of an object launched at angle 30° with initial speed 40 m/s.

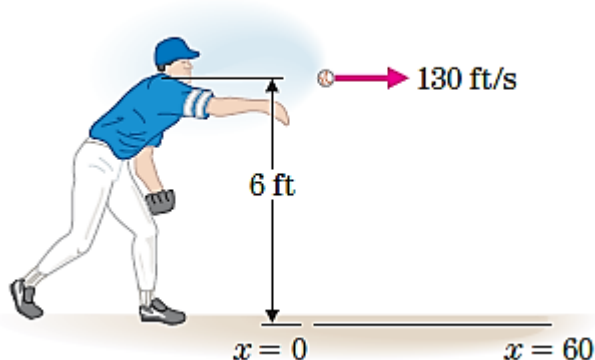
- 11 A daredevil plans to jump over 25 cars. If the cars are all compact cars with a width of 5 feet and the ramp angle is 30° , determine the initial velocity required to complete the jump successfully

12

Venus Williams has one of the fastest serves in women's tennis. Suppose that she hits a serve from a height of 10 feet at an initial speed of 120 mph and at an angle of 7° below the horizontal. The serve is "in" if the ball clears a 3 ft-high net that is 39 ft away and hits the ground in front of the service line 60 ft away. (We illustrate this situation in Figure) Determine whether the serve is in or out.



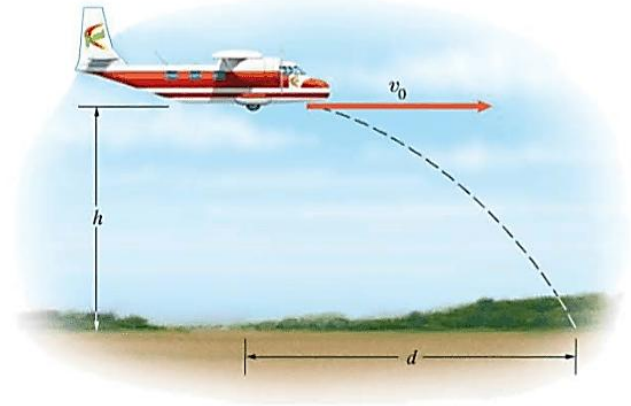
- 13 A baseball pitcher releases the ball horizontally from a height of 6 ft with an initial speed of 130 ft/s. Find the height of the ball when it reaches home plate 60 ft away. (Hint: Determine the time of flight from the x-equation, then use the y-equation to determine the height.)



14

A plane at an altitude of 256 feet wants to drop supplies to a specific location on the ground. If the plane has a horizontal velocity of 100 ft/s, how far away from the target should the plane release the supplies in order to hit the target location?

(Hint: Use the y-equation to determine the time of flight, then use the x-equation to determine how far the supplies will drift)



An Equation for the Motion of a Knuckleball

Newton's second law applied to the lateral motion of the knuckleball gives

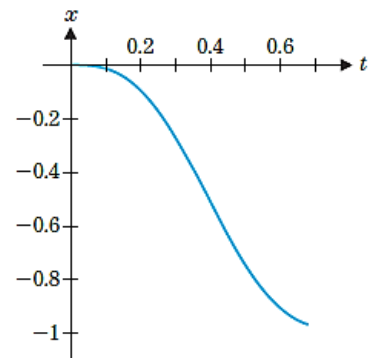
$$mx''(t) = -0.1 \sin(4\theta).$$

The mass of a baseball is about 0.098 kg . We now have

$$x''(t) = -10 \sin(4\omega t + \theta_0)$$

with initial conditions $x'(0) = 0$ and $x(0) = 0$.

- 15 For a spin rate of $\omega = 2$ radians per second and $\theta_0 = 0$, find an equation for the lateral motion of the knuckleball and graph it for $0 \leq t \leq 0.68$.



- 16 Repeat this for $\theta_0 = \frac{\pi}{2}$

