



وزارة التربية والتعليم
مكتب العين التعليمي - مدرسة البدع للتعليم الأساسي والثانوي
الصف / الثاني عشر المتقدم

EOT (12-Advanced)

هيكل الرياضيات

الصف الثاني عشر المتقدم

الفصل الدراسي الأول

2025 – 2026 م

إعداد الأستاذ /
محمد عبدالحميد الطحاوي

- تم مراعاة الهيكل وترتيب أسئلته
- تم وضع بعض الأسئلة الاسترشادية لكل أسئلة الهيكل
- تم تغيير بعض الأسئلة مع الاحتفاظ بفكرة السؤال لكيلا يحفظ الطالب الأسئلة
- يجب حل الهيكل أولاً ثم بعد ذلك حل الامتحانات التجريبية والتي تعتبر وسيلة سريعة للمراجعة

MCQ - الأسئلة الموضوعية

1) Estimate an arc length of a given function (7-12) page 68

تقدير طول القوس لدالة معينة (7-12) الصفحة 68

In exercises 7–12, estimate the length of the curve $y = f(x)$ on the given interval using (a) $n = 4$ and (b) $n = 8$ line segments. (c) If you can program a calculator or computer, use larger n 's and conjecture the actual length of the curve.

7. $f(x) = \cos x, 0 \leq x \leq \pi/2$

8. $f(x) = \sin x, 0 \leq x \leq \pi/2$

9. $f(x) = \sqrt{x+1}, 0 \leq x \leq 3$

10. $f(x) = 1/x, 1 \leq x \leq 2$

11. $f(x) = x^2 + 1, -2 \leq x \leq 2$

12. $f(x) = x^3 + 2, -1 \leq x \leq 1$

1) Estimate the length of the curve $f(x) = 2 + \sin x$ on $[0, \frac{\pi}{2}]$ by using $n = 4$

1) أوجد بطريقة تقريبية طول المنحنى $f(x) = 2 + \sin x$ على الفترة $[0, \frac{\pi}{2}]$ مستخدماً $n = 4$

- A) 1.9
- B) 3.8
- C) 0.95
- D) 2.2

1) Estimate the length of the curve $y = f(x) = x^2 + 2x$ on $[0, 2]$ by using $n = 4$

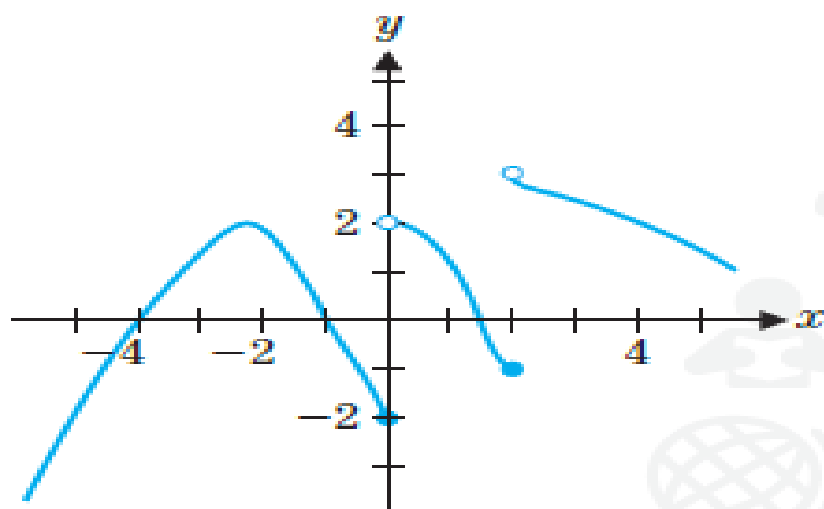
1) أوجد بطريقة تقريبية طول المنحنى $y = f(x) = x^2 + 2x$ على الفترة $[0, 2]$ مستخدماً $n = 4$

- A) 8.3
- B) 4.4
- C) 2.7
- D) 7.2

2) Find a limit algebraically or graphically, if it exists. (7-8) page 75

إيجاد نهاية جبريا أو بيانيا ، إذا كانت موجودة (7-8). صفحة 75

In exercises 7 and 8, identify each limit or state that it does not exist.



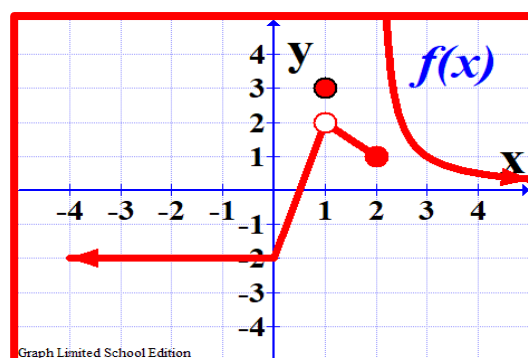
7. (a) $\lim_{x \rightarrow 0^-} f(x)$ (b) $\lim_{x \rightarrow 0^+} f(x)$ (c) $\lim_{x \rightarrow 0} f(x)$
 (d) $\lim_{x \rightarrow -2^-} f(x)$ (e) $\lim_{x \rightarrow -2^+} f(x)$ (f) $\lim_{x \rightarrow -2} f(x)$
 (g) $\lim_{x \rightarrow -1} f(x)$ (h) $\lim_{x \rightarrow 1^-} f(x)$
8. (a) $\lim_{x \rightarrow 1^-} f(x)$ (b) $\lim_{x \rightarrow 1^+} f(x)$ (c) $\lim_{x \rightarrow 1} f(x)$
 (d) $\lim_{x \rightarrow 2^-} f(x)$ (e) $\lim_{x \rightarrow -2^+} f(x)$ (f) $\lim_{x \rightarrow 2} f(x)$
 (g) $\lim_{x \rightarrow 3^-} f(x)$ (h) $\lim_{x \rightarrow -3} f(x)$

2) Use the shown graph represented $f(x)$ to answer the following question

استخدم الرسم المجاور للدالة في الإجابة $f(x)$ عن السؤال الآتي

$$\lim_{x \rightarrow 2^+} f(x)$$

- A) -2
 B) 1
 C) 0
 D) ∞



3) Find limits of polynomial, rational, and trigonometric functions using theorems. (21-28) page 85

إيجاد نهاية الدوال كثيرة الحدود والنسبية والمثلثية باستخدام نظريات (21-28) صفحة 85

21. $\lim_{x \rightarrow 2} f(x)$, where $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$

22. $\lim_{x \rightarrow -1} f(x)$, where $f(x) = \begin{cases} x^2 + 1 & \text{if } x < -1 \\ 3x + 1 & \text{if } x \geq -1 \end{cases}$

23. $\lim_{x \rightarrow -1} f(x)$, where $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

24. $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

25. $\lim_{h \rightarrow 0} \frac{(2 + h)^2 - 4}{h}$

26. $\lim_{h \rightarrow 0} \frac{(1 + h)^3 - 1}{h}$

27. $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x^2 - 4}$

28. $\lim_{x \rightarrow 0} \frac{\tan x}{5x}$

3) $\lim_{x \rightarrow -1} f(x)$, $f(x) = \begin{cases} 3x^2 + 2, & x < -1 \\ 2x - 7, & x \geq -1 \end{cases}$

A) 5

B) -9

C) -1

D) غير موجودة DNE

3) $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$

A) -9

B) 9

C) 6

D) 3

4) Use the squeeze theorem to find limits (29-32) page 85

استخدم نظرية الشطيرة لإيجاد النهايات

29. Use numerical and graphical evidence to conjecture the value of $\lim_{x \rightarrow 0} x^2 \sin(1/x)$. Use the Squeeze Theorem to prove that you are correct: identify the functions f and h , show graphically that $f(x) \leq x^2 \sin(1/x) \leq h(x)$ and justify $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} h(x)$.
30. Why can't you use the Squeeze Theorem as in exercise 29 to prove that $\lim_{x \rightarrow 0} x^2 \sec(1/x) = 0$? Explore this limit graphically.
31. Use the Squeeze Theorem to prove that $\lim_{x \rightarrow 0^+} [\sqrt{x} \cos^2(1/x)] = 0$. Identify the functions f and h , show graphically that $f(x) \leq \sqrt{x} \cos^2(1/x) \leq h(x)$ for all $x > 0$, and justify $\lim_{x \rightarrow 0^+} f(x) = 0$ and $\lim_{x \rightarrow 0^+} h(x) = 0$.
32. Suppose that $f(x)$ is bounded: that is, there exists a constant M such that $|f(x)| \leq M$ for all x . Use the Squeeze Theorem to prove that $\lim_{x \rightarrow 0} x^2 f(x) = 0$.

4) Find $\lim_{x \rightarrow 0} 4 + x^3 \sin\left(\frac{1}{x}\right)$

- A) 4
- B) -4
- C) 0
- D) *DNE* غير موجودة

4) If $|f(x) - 5| \leq x^2$ Find $\lim_{x \rightarrow 0} f(x)$

- A) 5
- B) -5
- C) 0
- D) *DNE* غير موجودة

5) Use the continuity properties to study the continuity of a function or a composition of functions at a given point (21-28) page 95

استخدم خصائص الاتصال لدراسة اتصال دالة أو مجموعة من الدوال عند نقطة معينة (21-28) صفحة 95

In exercises 21–28, determine the intervals on which f is continuous.

21. $f(x) = \sqrt{x+3}$

22. $f(x) = \sqrt{x^2 - 4}$

23. $f(x) = \sqrt[3]{x+2}$

24. $f(x) = (x-1)^{3/2}$

25. $f(x) = \sin^{-1}(x+2)$

26. $f(x) = \ln(\sin x)$

27. $f(x) = \frac{\sqrt{x+1} + e^x}{x^2 - 2}$

28. $f(x) = \frac{\ln(x^2 - 1)}{\sqrt{x^2 - 2x}}$

5) Determine the interval(s) where $f(x)$ is continuous, for

5) حدد الفترات التي تكون عندها الدالة $f(x)$ متصلة

$$f(x) = \frac{\ln(6-2x)}{\sqrt{x^2+2x}}$$

A) $(-\infty, -2] \cup (0, 3)$

B) $(-2, 3)$

C) $(-\infty, -2) \cup (0, 3)$

D) $[2, \infty) \cup (-5, \infty)$

5) Determine the interval(s) where f is continuous, for $f(x) = \sin^{-1}(x+2)$

A) $(-3, -1)$

B) $[-3, -1]$

C) $(-\infty, -3] \cup [-1, \infty)$

D) $(-\infty, -3) \cup (-1, \infty)$

6) Find horizontal, vertical, and slant asymptotes using limits. (23-32) page 106

إيجاد خطوط التقارب الأفقية والرأسية والمائلة باستخدام النهايات (23-32). صفحة 106

In exercises 23–28, determine all horizontal and vertical asymptotes. For each side of each vertical asymptote, determine whether $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$.

23. (a) $f(x) = \frac{x}{4 - x^2}$

(b) $f(x) = \frac{x^2}{4 - x^2}$

24. (a) $f(x) = \frac{x}{\sqrt{4 + x^2}}$

(b) $f(x) = \frac{x}{\sqrt{4 - x^2}}$

25. $f(x) = \frac{3x^2 + 1}{x^2 - 2x - 3}$

26. $f(x) = \frac{1 - x}{x^2 + x - 2}$

27. $f(x) = 4 \tan^{-1} x - 1$

28. $f(x) = \ln(1 - \cos x)$

6) The horizontal asymptote of $f(x) = \frac{3x^2+5}{x-x^2}$ is:

A) $y = 3$

B) $y = -3$

C) $y = 5$

D) $x = -3$

6) Determine all vertical asymptotes

(6) حدد خطوط التقارب الرأسية

$$y = \frac{3x+6}{x^2-4}$$

A) $x = -2$

B) $x = 2$

C) $x = -2, 2$

D) $x = 4, 0$

7) b) Find limits at infinity and limits that are infinite. (9-22) page

إيجاد النهايات التي تؤول إلى اللانهاية والنهايات عند اللانهاية

In exercises 5-22, determine each limit (answer as appropriate, with a number, ∞ , $-\infty$ or does not exist).

9. $\lim_{x \rightarrow \infty} \frac{x^2 + 3x - 2}{3x^2 + 4x - 1}$

10. $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1}$

11. $\lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{4 + x^2}}$

12. $\lim_{x \rightarrow \infty} \frac{2x^2 - 1}{4x^3 - 5x - 1}$

13. $\lim_{x \rightarrow \infty} \ln \left(\frac{x^2 + 1}{x - 3} \right)$

14. $\lim_{x \rightarrow 0^+} \ln(x \sin x)$

15. $\lim_{x \rightarrow 0^+} e^{-2/x^3}$

16. $\lim_{x \rightarrow \infty} e^{-(x+1)/(x^2+2)}$

17. $\lim_{x \rightarrow \infty} \cot^{-1} x$

18. $\lim_{x \rightarrow \infty} \sec^{-1} \frac{x^2 + 1}{x + 1}$

19. $\lim_{x \rightarrow 0} \sin(e^{-1/x^2})$

20. $\lim_{x \rightarrow \infty} \sin(\tan^{-1} x)$

21. $\lim_{x \rightarrow \pi/2} e^{-\tan x}$

22. $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x)$

4) $\lim_{x \rightarrow 0} x \cot x$

A) 0

B) 1

C) -1

D) ∞

7) $\lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{2+x^2}}$

A) 1

B) $-\infty$

C) $-\frac{1}{2}$

D) -1

8) Link and interpret the slope of a secant line and tangent line Example 3 3 page 136

رابط وتفسير ميل الخط القاطع والخط المماس

EXAMPLE 1.3 Graphical and Numerical Approximation of Tangent Lines

Graphically and numerically approximate the slope of the tangent line to $y = \frac{x-1}{x+1}$ at $x = 0$.

Solution A graph of $y = \frac{x-1}{x+1}$ is shown in Figure 3.10a. We sketch the tangent line at the point $(0, -1)$ in Figure 3.10b, where we have zoomed in to provide better detail. To approximate the slope, we estimate the coordinates of one point on the tangent line other than $(0, -1)$. In Figure 3.10b, it appears that the tangent line passes through the point $(1, 1)$. An estimate of the slope is then

$m_{\tan} \approx \frac{1 - (-1)}{1 - 0} = 2$. To approximate the slope numerically, we choose several points near $(0, -1)$ and compute the slopes of the secant lines. For example, rounding the y -values to four decimal places, we have

Second Point	$m_{\sec}$	Second Point	$m_{\sec}$
$(1, 0)$	$\frac{0 - (-1)}{1 - 0} = 1$	$(-0.5, -3)$	$\frac{-3 - (-1)}{-0.5 - 0} = 4.0$
$(0.1, -0.8182)$	$\frac{-0.8182 - (-1)}{0.1 - 0} = 1.818$	$(-0.1, -1.2222)$	$\frac{-1.2222 - (-1)}{-0.1 - 0} = 2.222$
$(0.01, -0.9802)$	$\frac{-0.9802 - (-1)}{0.01 - 0} = 1.98$	$(-0.01, -1.0202)$	$\frac{-1.0202 - (-1)}{-0.01 - 0} = 2.02$

In both columns, as the second point gets closer to $(0, -1)$, the slope of the secant line gets closer to 2. A reasonable estimate of the slope of the tangent line at the point $(0, -1)$ is then 2. ■

8) Estimate the slope of $y = \frac{x}{x-3}$ at $x = 2$

- A) -3
- B) 3
- C) -2
- D) $-\frac{1}{3}$

9) Find the derivative of a function at a given point - Example 3 page 145

إيجاد المشتقة للدالة عند نقطة معينة

EXAMPLE 2.3 Finding the Derivative of a Simple Rational Function

If $f(x) = \frac{1}{x}$ ($x \neq 0$), find $f'(x)$.

Solution We have

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left(\frac{1}{x+h} - \frac{1}{x} \right)}{h} \end{aligned}$$

Since $f(x+h) = \frac{1}{x+h}$.

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\left[\frac{x - (x+h)}{x(x+h)} \right]}{h} && \text{Add fractions and cancel.} \\ &= \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} && \text{Cancel } h\text{'s.} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}, \end{aligned}$$

so that $f'(x) = -x^{-2}$. ■

9) $\lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$

A) $-\frac{2}{x^2}$

B) $-\frac{2}{x}$

C) $-2x$

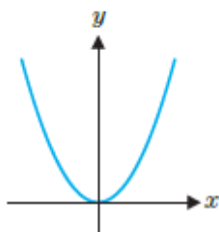
D) $\frac{2}{x}$

10) Understand the relationship between continuity and differentiability (13-18) page 151

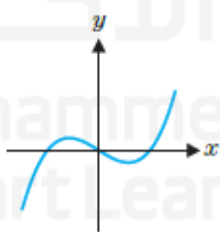
فهم العلاقة بين الاتصال والاشتقاق

In exercises 13–16, use the graph of f to sketch a graph of f' .

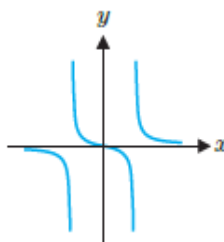
13. (a)



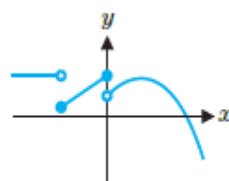
(b)



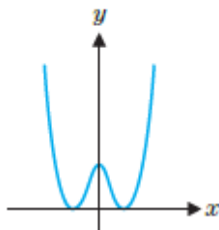
15. (a)



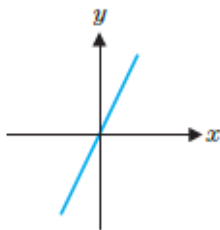
(b)



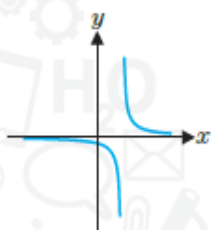
14. (a)



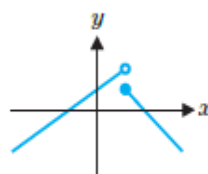
(b)



16. (a)

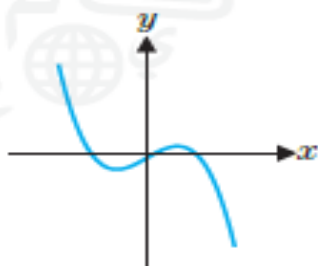


(b)

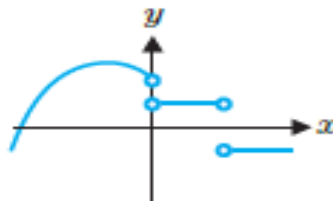


In exercises 17 and 18, use the given graph of f' to sketch a plausible graph of a continuous function f .

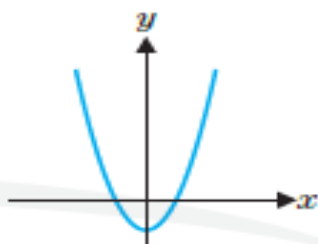
17. (a)



(b)



18. (a)

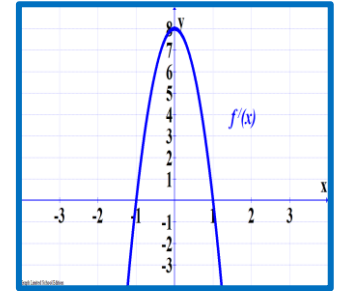
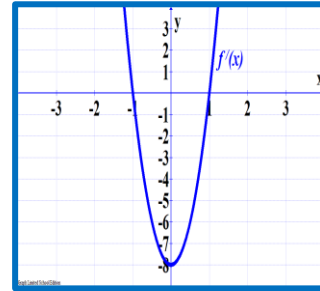
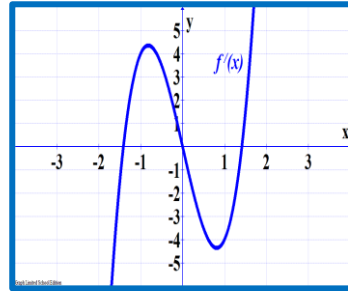
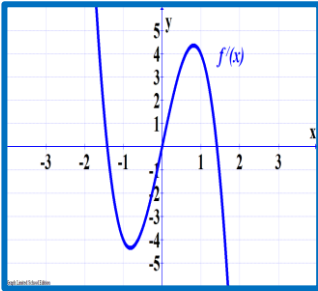
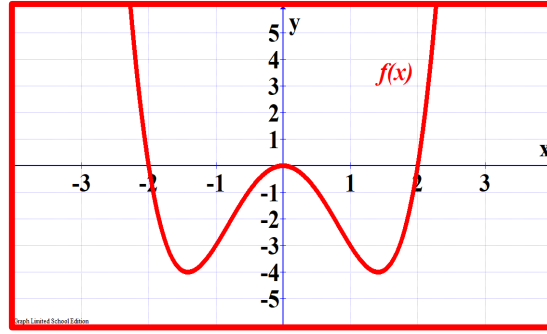


(b)



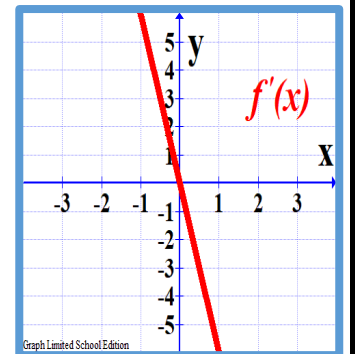
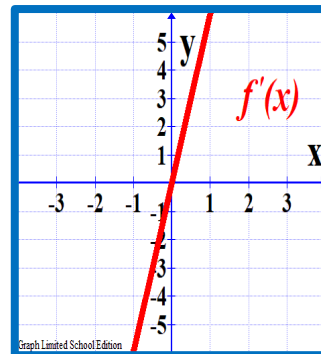
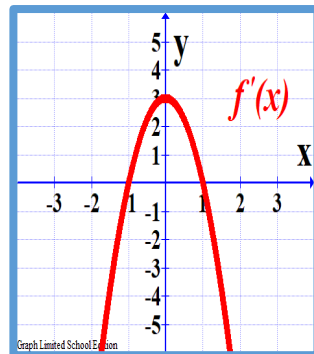
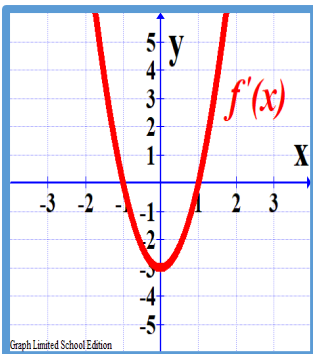
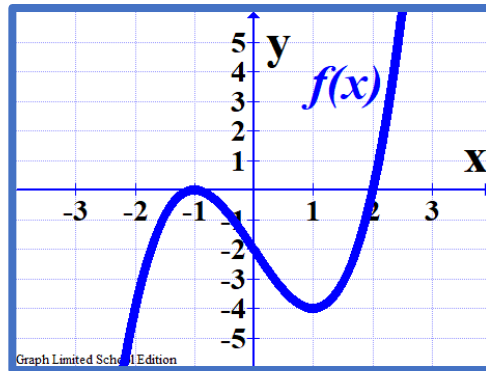
10) Use the shown graph represented $f(x)$ to determine the graph of the function derivative $f'(x)$

10) استخدم الشكل المرسوم الذي يمثل بيان الدالة $f(x)$ لتحديد الرسم المناسب لمشتقة الدالة $f'(x)$



11) Use the shown graph represented $f(x)$ to determine the graph of the function $(f'(x))$

11) استخدم الرسم المجاور للدالة $f(x)$ لإيجاد الرسم المناسب للمشتقة الأولى $(f'(x))$



11) Use differentiation rules and higher derivatives in solving real-life problems (21-24) . Page161

استخدام قواعد الاشتقاق والمشتقات العليا في حل المسائل الحياتية

In exercises 21–24, use the given position function to find the velocity and acceleration functions.

21. $s(t) = -16t^2 + 40t + 10$

22. $s(t) = -4.9t^2 + 12t - 3$

23. $s(t) = \sqrt{t} + 2t^2$

24. $s(t) = 10 - \frac{10}{t}$

11) the function represented the position in feet of an object at time t seconds $s(t) = -16t^2 + 64t + 5$
Find the instantaneous velocity at $t = 1$

11) افترض $s(t)$ تقيس الإزاحة بالقدم لمدة t ثانية

$s(t) = -16t^2 + 64t + 5$
أوجد السرعة اللحظية $t = 1$

- A) instantaneous velocity = 32 ft/sec
السرعة اللحظية
- B) instantaneous velocity = -32 ft/sec
السرعة اللحظية
- C) instantaneous velocity = 53 ft/sec
السرعة اللحظية
- D) instantaneous velocity = 37 ft/sec
السرعة اللحظية

12) Use differentiation rules and higher derivatives in solving real-life problems. (37-42) page 161

استخدام قواعد الاشتقاق والمشتقات العليا في حل المشكلات الحياتية

37. Find all values of x for which the tangent line to $y = x^3 - 3x + 1$ is (a) at an angle of 45° with the x -axis; (b) at an angle of 30° with the x -axis, assuming that the angles are measured counterclockwise.
38. Find all values of x for which the tangent lines to $y = x^3 + 2x + 1$ and $y = x^4 + x^3 + 3$ are (a) parallel; (b) perpendicular.
39. Find a second-degree polynomial (of the form $ax^2 + bx + c$) such that (a) $f(0) = -2, f'(0) = 2$ and $f''(0) = 3$. (b) $f(0) = 0, f'(0) = 5$ and $f''(0) = 1$.
40. Find a general formula for the n th derivative $f^{(n)}(x)$ for
(a) $f(x) = \sqrt{x}$ (b) $f(x) = \frac{2}{x}$
41. Find the area of the triangle bounded by $x = 0, y = 0$ and the tangent line to $y = \frac{1}{x}$ at $x = 1$. Repeat with the triangle bounded by $x = 0, y = 0$ and the tangent line to $y = \frac{1}{x}$ at $x = 2$. Show that you get the same area using the tangent line to $y = \frac{1}{x}$ at any $x = a > 0$.
42. Show that the result of exercise 41 does not hold for $y = \frac{1}{x^2}$. That is, the area of the triangle bounded by $x = 0, y = 0$ and the tangent line to $y = \frac{1}{x^2}$ at $x = a > 0$ does depend on the value of a .

12) Find all values of x for which the tangent line to $y = x^3 - 3x^2 - 8x$ at an angle of 45° with the x -axis

أوجد جميع قيم x إذا كانت معادلة المنحنى $y = x^3 - 3x^2 - 8x$ ، فالمماس عند النقطة التي يكون ميله بزاوية 45° مع محور x

- A) $x = 3$
B) $x = 3, -1$
C) $x = -3, 1$
D) $x = 9, -1$

13) Apply the chain rule for differentiation (31-38) page 177

تطبيق قاعدة السلسلة في الاشتقاق

In exercises 31 and 32, use the relevant information to compute the derivative for $h(x) = f(g(x))$.

31. $h'(1)$, where $f(1) = 3, g(1) = 2, f'(1) = 4, f'(2) = 3, g'(1) = -2$ and $g'(3) = 5$
32. $h'(2)$, where $f(2) = 1, g(2) = 3, f'(2) = -1, f'(3) = -3, g'(1) = 2$ and $g'(2) = 4$
33. A function f is an **even function** if $f(-x) = f(x)$ for all x and is an **odd function** if $f(-x) = -f(x)$ for all x . Prove that the derivative of an even function is odd and the derivative of an odd function is even.
34. If the graph of a differentiable function f is symmetric about the line $x = a$, what can you say about the symmetry of the graph of f' ?

In exercises 35–38, find the derivative where f is an unspecified differentiable function.

35. (a) $f(x^2)$ (b) $[f(x)]^2$ (c) $f(f(x))$
36. (a) $f(\sqrt{x})$ (b) $\sqrt{f(x)}$ (c) $f(xf(x))$
37. (a) $f(1/x)$ (b) $1/f(x)$ (c) $f\left(\frac{x}{f(x)}\right)$
38. (a) $1 + f(x^2)$ (b) $[1 + f(x)]^2$ (c) $f(1 + f(x))$

13) If $f(2) = 2$, $f'(2) = 3$, $f(2) = 2$, $f'(2) = 3$ إذا كانت $h(x) = f(f(x))$ Evaluate $h'(2)$ $h'(2)$ أوجد $h(x) = f(f(x))$ ،

- A) $h'(2) = 3$
- B) $h'(2) = 2$
- C) $h'(2) = 6$
- D) $h'(2) = 9$

14) Find the derivative of an inverse function using the Chain Rule.

(17-22) page 176

إيجاد مشتقة معكوس الدالة باستخدام قاعدة السلسلة

In exercises 17–22, f has an inverse g . Use Theorem 5.2 to find $g'(a)$.

17. $f(x) = x^3 + 4x - 1, a = -1$

18. $f(x) = x^5 + 4x - 2, a = -2$

19. $f(x) = x^5 + 3x^3 + x, a = 5$

20. $f(x) = x^3 + 2x + 1, a = -2$

21. $f(x) = \sqrt{x^3 + 2x + 4}, a = 2$

22. $f(x) = \sqrt{x^5 + 4x^3 + 3x + 1}, a = 3$

14) If $f(x) = \sqrt{x^3 + 2x + 4}$ have an inverse function $g(x)$,

What is the value of $g'(4)$?

14) إذا كانت $f(x) = \sqrt{x^3 + 2x + 4}$ لها دالة عكسية $g(x)$ أوجد $g'(4)$

- A) $g'(2) = \frac{4}{7}$
B) $g'(2) = \frac{7}{4}$
C) $g'(2) = \frac{8}{7}$
D) $g'(2) = \frac{7}{8}$

15) Find the derivatives of trigonometric functions using differentiation rules (9-18) page 184

إيجاد مشتقة الدوال المثلثية باستخدام قواعد التفاضل

9. $f(t) = \sin 3t \sec 3t$

10. $f(t) = \sqrt{\cos 5t \sec 5t}$

11. $f(w) = \frac{1}{\sin 4w}$

12. $f(w) = w^2 \sec^2 3w$

13. $f(x) = 2 \sin 2x \cos 2x$

14. $f(x) = 4 \sin^2 3x + 4 \cos^2 3x$

15. $f(x) = \tan \sqrt{x^2 + 1}$

16. $f(x) = 4x^2 \sin x \sec 3x$

17. $f(x) = \sin^3 (\cos \sqrt{x^3 + 2x^2})$

18. $f(x) = \tan^4(\sin^2(x^3 + 2x))$

15) Find the derivative of $y = 5\sin^2 4x + 5\cos^2 4x$

A) $y' = 10 \sin 4x + 10 \cos 4x$

B) $y' = 40 \sin 4x + 40 \cos 4x$

C) $y' = 5$

D) $y' = 0$

16) Find the derivatives of natural logarithmic functions (39-44) page 194

إيجاد مشتقات الدوال اللوغاريتمية الطبيعية

In exercises 39–44, use logarithmic differentiation to find the derivative.

39. $f(x) = x^{\sin x}$

40. $f(x) = x^{4-x^2}$

41. $f(x) = (\sin x)^x$

42. $f(x) = (x^2)^{4x}$

43. $f(x) = x^{\ln x}$

44. $f(x) = x^{\sqrt{x}}$

16) Use logarithm differentiation to find the derivative of

(16) استخدم تفاضل اللوغاريتم في إيجاد مشتقة الدالة

$$f(x) = x^{\ln x}$$

A) $f'(x) = \frac{(2\ln x)x^{\ln x}}{x}$

B) $f'(x) = \frac{(\ln x)x^{\ln x}}{x}$

C) $f'(x) = \frac{2\ln x}{x}$

D) $f'(x) = (2\ln x)x^{\ln x}$

17) Find the derivatives of exponential functions (1-14) page 193

إيجاد مشتقات الدوال الأسية

In exercises 1-24, differentiate each function.

1. $f(x) = x^3 e^x$

2. $f(x) = e^{2x} \cos 4x$

3. $f(t) = t + 2^t$

4. $f(t) = t4^{3t}$

5. $f(x) = 2e^{4x+1}$

6. $f(x) = (1/e)^x$

7. $h(x) = (1/3)^{x^2}$

8. $h(x) = 4^{-x^2}$

9. $f(u) = e^{u^2+4u}$

10. $f(u) = 3e^{\tan u}$

11. $f(w) = \frac{e^{4w}}{w}$

12. $f(w) = \frac{w}{e^{6w}}$

13. $f(x) = \ln 2x$

14. $f(x) = \ln \sqrt{8x}$

17) Find the derivative

$$f(x) = \frac{e^x}{2^x}$$

(17) أوجد المشتقة

A) $f'(x) = \frac{e^x(1 - \ln 2)}{2^x}$

B) $f'(x) = \frac{e^x(1 + \ln 2)}{2^x}$

C) $f'(x) = \frac{e^x(1 - \ln 2)}{4^x}$

D) $f'(x) = \frac{e^x}{2^x(\ln 2)}$

18) Use implicit differentiation to find derivatives of inverse trigonometric functions (29-32) page 204

استخدام الاشتقاق الضمني لإيجاد مشتقات الدوال المثلثية العكسية

In exercises 29–34, find the derivative of the given function.

29. (a) $f(x) = \sin^{-1}(x^3 + 1)$

(b) $f(x) = \sin^{-1}(\sqrt{x})$

30. (a) $f(x) = \cos^{-1}(x^2 + x)$

(b) $f(x) = \cos^{-1}(2/x)$

31. (a) $f(x) = \tan^{-1}(\sqrt{x})$

(b) $f(x) = \tan^{-1}(1/x)$

32. (a) $f(x) = \sqrt{2 + \tan^{-1} x}$

(b) $f(x) = e^{\tan^{-1} x}$

18) Find the derivative of
 $f(x) = \csc^{-1}(2x)$

(18) أوجد مشتقة الدالة
 $f(x) = \csc^{-1}(2x)$

A) $f'(x) = \frac{-2x}{|x|\sqrt{4x^2-1}}$

B) $f'(x) = \frac{-1}{|x|\sqrt{4x^2-1}}$

C) $f'(x) = \frac{-2}{|x|\sqrt{1-4x^2}}$

D) $f'(x) = \frac{-1}{|x|\sqrt{1-4x^2}}$

19) Learn Rolle's Theorem and use it in applications Example1 page 214
فهم نظرية رول واستخدامها في التطبيقات

EXAMPLE 10.1 An Illustration of Rolle's Theorem

Find a value of c satisfying the conclusion of Rolle's Theorem for

$$f(x) = x^3 - 3x^2 + 2x + 2$$

on the interval $[0, 1]$.

Solution First, we verify that the hypotheses of the theorem are satisfied: f is differentiable and continuous for all x [since $f(x)$ is a polynomial and all polynomials are continuous and differentiable everywhere]. Also, $f(0) = f(1) = 2$. We have

$$f'(x) = 3x^2 - 6x + 2.$$

We now look for values of c such that

$$f'(c) = 3c^2 - 6c + 2 = 0.$$

By the quadratic formula, we get $c = 1 + \frac{1}{3}\sqrt{3} \approx 1.5774$ [not in the interval $(0, 1)$] and $c = 1 - \frac{1}{3}\sqrt{3} \approx 0.42265 \in (0, 1)$. ■

19) Find the value c satisfying the conclusion of Rolle's Theorem
 $f(x) = x^2 + 2$ on $[-1, 1]$

19) أوجد قيمة c التي تحقق شروط نظرية رول
للدالة $f(x) = x^2 + 2$ في الفترة $[-1, 1]$

- A) $c = -1$
- B) $c = 1$
- C) $c = 0$
- D) $c = 2$

20) Learn the Mean Value Theorem and use it in applications -Example5

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تعلم نظرية المتوسطة للدالة واستخدمها في التطبيقات

EXAMPLE 10.5 Proving an Inequality for $\sin x$

Prove that $|\sin a| \leq |a|$ for all a .

Solution First, note that $f(x) = \sin x$ is continuous and differentiable on any interval and that for any a ,

$$|\sin a| = |\sin a - \sin 0|,$$

since $\sin 0 = 0$. From the Mean Value Theorem, we have that (for $a \neq 0$)

$$\frac{\sin a - \sin 0}{a - 0} = f'(c) = \cos c, \quad (10.6)$$

for some number c between a and 0 . Notice that if we multiply both sides of (10.6) by a and take absolute values, we get

$$|\sin a| = |\sin a - \sin 0| = |\cos c| |a - 0| = |\cos c| |a|. \quad (10.7)$$

But, $|\cos c| \leq 1$, for all real numbers c and so, from (10.6), we have

$$|\sin a| = |\cos c| |a| \leq (1) |a| = |a|,$$

as desired. ■

20) Which of the following methods can be used to prove the inequality $|\sin a| \leq |a|$ for all a ?

A) By using the Mean Value Theorem to show that

باستخدام نظرية القيمة المتوسطة لإثبات ذلك

$$|\sin a - \sin 0| \leq |a - 0|.$$

B) By differentiating $\sin a$ twice and comparing with $\cos a$.

بالتفاضل مرتين للدالة $\sin a$ والمقارنة مع $\cos a$

C) By assuming $\sin a = a$ for all

بافتراض $\sin a = a$ لكل a

D) By integrating both sides from 0 to a

بأخذ التكامل للطرفين من 0 إلى a

FRQ - الأسئلة المقالية

21) a) Determine the continuity of a function at a given point. (39-41)

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البحث في اتصال دالة عند نقطة معينة

b) Find limits at infinity and limits that are infinite. (33-37) page 106

إيجاد النهايات التي تؤول إلى اللانهاية والنهايات عند اللانهاية

In exercises 39–41, determine values of a and b that make the given function continuous.

$$39. f(x) = \begin{cases} \frac{2 \sin x}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b \cos x & \text{if } x > 0 \end{cases}$$

$$40. f(x) = \begin{cases} ae^x + 1 & \text{if } x < 0 \\ \sin^{-1} \frac{x}{2} & \text{if } 0 \leq x \leq 2 \\ x^2 - x + b & \text{if } x > 2 \end{cases}$$

$$41. f(x) = \begin{cases} a(\tan^{-1} x + 2) & \text{if } x < 0 \\ 2e^{bx} + 1 & \text{if } 0 \leq x \leq 3 \\ \ln(x - 2) + x^2 & \text{if } x > 3 \end{cases}$$

33. Suppose that the size of the pupil of a certain animal is given by $f(x)$ (mm), where x is the intensity of the light on the pupil. If $f(x) = \frac{80x^{-0.3} + 60}{2x^{-0.3} + 5}$, find the size of the pupil with no light and the size of the pupil with an infinite amount of light.
34. Repeat exercise 33 with $f(x) = \frac{80x^{-0.3} + 60}{8x^{-0.3} + 15}$.
35. Modify the function in exercise 33 to find a function f such that $\lim_{x \rightarrow 0^+} f(x) = 8$ and $\lim_{x \rightarrow \infty} f(x) = 2$.
36. Find a function of the form $f(x) = \frac{20x^{-0.4} + 16}{g(x)}$ such that $\lim_{x \rightarrow 0^+} f(x) = 5$ and $\lim_{x \rightarrow \infty} f(x) = 4$.
37. Suppose that the velocity of a skydiver t seconds after jumping is given by $v(t) = -\sqrt{\frac{32}{k}} \frac{1 - e^{-2t\sqrt{32k}}}{1 + e^{-2t\sqrt{32k}}}$. Find the limiting velocity with $k = 0.00064$ and $k = 0.00128$. By what factor does a skydiver have to change the value of k to cut the limiting velocity in half?

22) Apply the Product Rule on derivatives (25-30) page 169

تطبيقات حياتية على قاعدة الضرب على المشتقات

25. Suppose that for some toy, the quantity sold $Q(t)$ at time t years decreases at a rate of 4%; explain why this translates to $Q'(t) = -0.04Q(t)$. Suppose also that the price increases at a rate of 3%; write out a similar equation for $P'(t)$ in terms of $P(t)$. The revenue for the toy is $R(t) = Q(t)P(t)$. Substituting the expressions for $Q'(t)$ and $P'(t)$ into the product rule $R'(t) = Q'(t)P(t) + Q(t)P'(t)$, show that the revenue decreases at a rate of 1%. Explain why this is "obvious."
26. As in exercise 25, suppose that the quantity sold decreases at a rate of 4%. By what rate must the price be increased to keep the revenue constant?
27. Suppose the price of an object is \$20 and 20,000 units are sold. If the price increases at a rate of \$1.25 per year and the quantity sold increases at a rate of 2000 per year, at what rate will revenue increase?
28. Suppose the price of an object is \$14 and 12,000 units are sold. The company wants to increase the quantity sold by 1200 units per year, while increasing the revenue by \$20,000 per year. At what rate would the price have to be increased to reach these goals?
29. A baseball with mass 0.15 kg and speed 45 m/s is struck by a baseball bat of mass m kg and speed 40 m/s (in the opposite direction of the ball's motion). After the collision, the ball has initial speed $u(m) = \frac{82.5m - 6.75}{m + 0.15}$ m/s. Show that $u'(m) > 0$ and interpret this in baseball terms. Compare $u'(1)$ and $u'(1.2)$.
30. In exercise 29, if the baseball has mass M kg at speed 45 m/s and the bat has mass 1.05 kg and speed 40 m/s, the ball's initial speed is $u(M) = \frac{86.625 - 45M}{M + 1.05}$ m/s. Compute $u'(M)$ and interpret its sign (positive or negative) in baseball terms.

23) Solve real-life problems using derivatives of exponential and logarithmic functions (25-30) page 194

حل المشكلات الحياتية باستخدام مشتقات الدوال الأسية واللوغاريتمية

In exercises 25–28, find an equation of the tangent line to $y = f(x)$ at $x = 1$.

25. $f(x) = 3e^{x^2}$

26. $f(x) = 3^{x^e}$

27. $f(x) = x^2 \ln x$

28. $f(x) = 2 \ln x^3$

In exercises 29 and 30, find all values of x for which the tangent line to $y = f(x)$ is horizontal.

29. (a) $f(x) = xe^{-2x}$

(b) $f(x) = xe^{-3x}$

30. (a) $f(x) = x^2 e^{-2x}$

(b) $f(x) = x^2 e^{-3x}$

EXAMPLE 8.4 Finding a Second Derivative Implicitly

Find $y''(x)$ implicitly for $y^2 + 2e^{-xy} = 6$. Then find the value of y'' at the point $(0, 2)$.

Solution We begin by differentiating both sides of the equation with respect to x . We have

$$\frac{d}{dx}(y^2 + 2e^{-xy}) = \frac{d}{dx}(6).$$

By the chain rule and the product rule, we have

$$2yy'(x) + 2e^{-xy}[-y - xy'(x)] = 0. \quad (8.3)$$

Notice that we don't need to solve this for $y'(x)$. Dividing out the common factor of 2 and differentiating again, we get

$$\begin{aligned} y'(x)y'(x) + yy''(x) - e^{-xy}[-y - xy'(x)][y + xy'(x)] \\ - e^{-xy}[y'(x) + y'(x) + xy''(x)] = 0. \end{aligned}$$

Grouping all the terms involving $y''(x)$ on one side of the equation gives us

$$yy''(x) - xe^{-xy}y''(x) = -[y'(x)]^2 - e^{-xy}[y + xy'(x)]^2 + 2e^{-xy}y'(x).$$

Factoring out the $y''(x)$ on the left, we get

$$(y - xe^{-xy})y''(x) = -[y'(x)]^2 - e^{-xy}[y + xy'(x)]^2 + 2e^{-xy}y'(x),$$

$$\text{so that} \quad y''(x) = \frac{-[y'(x)]^2 - e^{-xy}[y + xy'(x)]^2 + 2e^{-xy}y'(x)}{y - xe^{-xy}}. \quad (8.4)$$

25) Use implicit differentiation to find derivatives of inverse trigonometric functions page (29-32) page 20

إيجاد مشتقات الدوال المثلثية العكسية

In exercises 29–34, find the derivative of the given function.

29. (a) $f(x) = \sin^{-1}(x^3 + 1)$ (b) $f(x) = \sin^{-1}(\sqrt{x})$

30. (a) $f(x) = \cos^{-1}(x^2 + x)$ (b) $f(x) = \cos^{-1}(2/x)$

31. (a) $f(x) = \tan^{-1}(\sqrt{x})$ (b) $f(x) = \tan^{-1}(1/x)$

32. (a) $f(x) = \sqrt{2 + \tan^{-1} x}$ (b) $f(x) = e^{\tan^{-1} x}$

26) Understand the Mean Value Theorem and use it in applications.

(1-6) page 219

التعرف على نظرية القيمة المتوسطة واستخدامها في التطبيقات

In exercises 1–6, check the hypotheses of Rolle's Theorem and the Mean Value Theorem and find a value of c that makes the appropriate conclusion true. Illustrate the conclusion with a graph.

1. $f(x) = x^2 + 1, [-2, 2]$

2. $f(x) = x^2 + 1, [0, 2]$

3. $f(x) = x^3 + x^2, [0, 1]$

4. $f(x) = x^3 + x^2, [-1, 1]$

5. $f(x) = \sin x, [0, \pi/2]$

6. $f(x) = \sin x, [-\pi, 0]$

26) Find a value of c satisfying the conclusion of the mean value theorem for $f(x) = x^3 - 2x^2 + x + 1$ on $[0, 2]$

26) أوجد قيمة c التي تحقق نظرية القيمة المتوسطة للدالة $f(x) = x^3 - 2x^2 + x + 1$ في الفترة $[0, 2]$