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MATH ENG

MATH TERM 2

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The Featured Program

2024-2025

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LESSON 6-1

Multivariable Linear Systems and Row Operations

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Multivariable Linear Systems and Row Operations

1 Gaussian Elimination A **multivariable linear system**, or *multivariate* linear system, is a system of linear equations in two or more variables. In previous courses, you may have used the *elimination method* to solve such systems. One elimination method begins by rewriting a system using an inverted triangular shape in which the leading coefficients are 1.

The substitution and elimination methods you have previously learned can be used to convert a multivariable linear system into an equivalent system in *triangular* or **row-echelon form**.

System in Row-Echelon Form

$$\begin{aligned} x - y - 2z &= 5 \\ y + 4z &= -5 \\ z &= -2 \end{aligned}$$

Notice that the left side of the system forms a triangle in which the leading coefficients are 1. The last equation contains only one variable, and each equation above it contains the variables from the equation immediately below it.

KeyConcept Operations that Produce Equivalent Systems

Each of the following operations produces an equivalent system of linear equations.

- Interchange any two equations.
- Multiply one of the equations by a nonzero real number.
- Add a multiple of one equation to another equation.

1 **Write each system of equations in triangular form using Gaussian elimination. Then solve the system.**

$$\begin{aligned} 5x &= -3y - 31 \\ 2y &= -4x - 22 \end{aligned}$$

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2 **Write each system of equations in triangular form using Gaussian elimination. Then solve the system.**

$$4y + 17 = -7x$$
$$8x + 5y = -19$$

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$$12x = 21 - 3y$$
$$2y = 6x + 7$$

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4 **Write each system of equations in triangular form using Gaussian elimination. Then solve the system.**

$$5x - 5y - 5z = 35$$
$$-x + 2y - 3z = -12$$
$$3x - 2y + 7z = 30$$

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The **augmented matrix** of a system is derived from the coefficients and constant terms of the linear equations, each written in standard form with the constant terms to the right of the equals sign. If the column of constant terms is not included, the matrix reduces to that of the **coefficient matrix** of the system. You will use this type of matrix in Lesson 6-3.

System of Equations	Augmented Matrix	Coefficient Matrix
$5x - 5y - 5z = 35$ $-x + 2y - 3z = -12$ $3x - 2y + 7z = 30$	$\left[\begin{array}{rrr\|r} 5 & -5 & -5 & 35 \\ -1 & 2 & -3 & -12 \\ 3 & -2 & 7 & 30 \end{array}\right]$	$\begin{bmatrix} 5 & -5 & -5 \\ -1 & 2 & -3 \\ 3 & -2 & 7 \end{bmatrix}$

5 **Write the augmented matrix for the system of linear equations.**

A

$12x - 5y = -9$
$-3x + 8y = 10$

$3x - 5y + 7z = 9$
$-10x + y + 8z = 6$
$4x - 15z = -8$

B

$w + 4x + z = 2$
$x + 2y - 3z = 0$
$w - 3y - 8z = -1$
$3w + 2x + 3y = 9$

C

$4w - 5x + 7z = -11$
$-w + 8x + 3y = 6$
$15x - 2y + 10z = 9$

D

$-3w + 7x + y = 21$
$4w - 12y + 8z = 5$
$16w - 14y + z = -2$
$w + x + 2y = 7$

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KeyConcept Elementary Row Operations

Each of the following row operations produces an equivalent augmented matrix.

- Interchange any two rows.
- Multiply one row by a nonzero real number.
- Add a multiple of one row to another row.

KeyConcept Row-Echelon Form

A matrix is in row-echelon form if the following conditions are met.

- Rows consisting entirely of zeros (if any) appear at the bottom of the matrix.
- The first nonzero entry in a row is 1, called a *leading 1*.
- For two successive rows with nonzero entries, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row.

$$\left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 0 \end{array}\right]$$

6 Determine whether each matrix is in row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 4 & 2 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 6 & 2 & -11 & 10 \\ 0 & 1 & -5 & 8 & -7 \\ 0 & 0 & 0 & 1 & 14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 5 & -6 & 10 \\ 0 & 1 & 9 & -3 \\ 0 & 1 & 0 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -6 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 9 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 19 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -20 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 0 & 1 & 0 & 4 & 10 \\ 1 & 0 & -3 & 10 & -7 \\ 0 & 1 & 6 & 0 & 8 \\ 0 & 0 & 1 & -2 & -4 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 8 & 7 \\ 0 & 1 & 3 \end{array}\right] \qquad \left[\begin{array}{cc|c} 1 & -8 & 12 \\ 1 & 3 & -7 \\ 0 & 1 & 4 \end{array}\right] \qquad \left[\begin{array}{cc|c} 1 & -4 & 10 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{array}\right]$$

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Use Gauss-Jordan Elimination

7 Solve the system of equations.

$$x - y + z = 0$$
$$-x + 2y - 3z = -5$$
$$2x - 3y + 5z = 8$$

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Solve the system of equations.

$$-5x - 2y + z = 2$$
$$4x - y - 6z = 2$$
$$-3x - y + z = 1$$

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9 **Write the augmented matrix for each system of linear equations**

$$12x - 5y = -9 \qquad -3x + 8y = 10$$

a) $\left[\begin{array}{cc:c} 12 & -5 & -9 \\ -3 & 8 & 10 \end{array}\right]$

b) $\left[\begin{array}{cc:c} 12 & -5 & -9 \\ 8 & -3 & 10 \end{array}\right]$

c) $\left[\begin{array}{cc:c} 12 & -5 & 10 \\ -3 & 8 & 9 \end{array}\right]$

d) $\left[\begin{array}{cc:c} -5 & 12 & -9 \\ -3 & 8 & 10 \end{array}\right]$

10 **Write the augmented matrix for each system of linear equations**

$$3x - 5y + 7z = 9$$
$$-10x + y + 8z = 6$$
$$4x - 15z = -8$$

a) $\left[\begin{array}{ccc|c} 3 & 5 & 7 & 9 \\ -10 & 1 & 8 & 6 \\ 4 & 15 & 0 & -8 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 3 & 5 & 7 & 9 \\ 10 & 1 & 8 & 6 \\ 4 & 15 & 0 & -8 \end{array}\right]$

c) $\left[\begin{array}{ccc|c} 3 & -5 & 7 & 9 \\ -10 & 1 & 8 & 6 \\ 4 & 0 & -15 & -8 \end{array}\right]$

d) $\left[\begin{array}{ccc|c} 3 & -5 & 7 & 9 \\ -10 & 1 & 8 & 6 \\ 4 & 0 & 15 & -8 \end{array}\right]$

11 **Write the augmented matrix for each system of linear equations**

$$4x - z = 27$$
$$-8x + 7y - 6z = -35$$
$$12x - 3y + 5z = 20$$

a) $\left[\begin{array}{ccc|c} 4 & 5 & 0 & 27 \\ -8 & 1 & -6 & -35 \\ 12 & 15 & 0 & 20 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 4 & 0 & -1 & 27 \\ -8 & 7 & -6 & -35 \\ 12 & -3 & 5 & 20 \end{array}\right]$

c) $\left[\begin{array}{ccc|c} 3 & -5 & 7 & 9 \\ -10 & 1 & 8 & 6 \\ 4 & 0 & -15 & -8 \end{array}\right]$

d) $\left[\begin{array}{ccc|c} 3 & -5 & 7 & 9 \\ -10 & 1 & 8 & 6 \\ 4 & 0 & 15 & -8 \end{array}\right]$

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12 Solve each system of equations using Gaussian or Gauss-Jordan elimination.

$$2x = -10y + 11$$
$$-8y = -9x + 23$$

$a)\ x = 3\ , y = 2$

$b)\ x = -1\ , y = 2$

$c)\ x = \frac{1}{2}\ , y = 3$

$d)\ x = 3\ , y = \frac{1}{2}$

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13 Solve each system of equations using Gaussian or Gauss-Jordan elimination.

$$4y + 17 = -7x$$
$$8x + 5y = -19$$

$a)\ x = -2\ , y = -7$

$b)\ x = -1\ , y = 2$

$c)\ x = -11\ , y = 3$

$d)\ x = -3\ , y = 1$

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14 Solve each system of equations using Gaussian or Gauss-Jordan elimination.

$$x + 7y = 10$$
$$3x + 9y = -6$$

$a)\ x = -2\ , y = -7$

$b)\ x = -1\ , y = 2$

$c)\ x = -11\ , y = 3$

$d)\ x = -3\ , y = 1$

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15 Solve each system of equations using Gaussian or Gauss-Jordan elimination.

$$7y = 9 - 5x$$
$$8x = 2 - 5y$$

a) $x = -2\,, y = -7$

b) $x = -1\,, y = 2$

c) $x = -11\,, y = 3$

d) $x = -3\,, y = 1$

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16 Solve each system of equations using Gaussian or Gauss-Jordan elimination.

$$3x - 4y + 8z = 27$$
$$9x - y - z = 3$$
$$x + 8y - 2z = 9$$

a) $x = -1\,, y = -2\,, z = 4$

b) $x = -1\,, y = 3\,, z = 4$

c) $x = 1\,, y = 2\,, z = 4$

d) $x = 10\,, y = -2\,, z = 1$

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17 Solve each system of equations using Gaussian or Gauss-Jordan elimination.

$$x + 9y + 8z = 0$$
$$5x + 8y + z = 35$$
$$x - 4y - z = 17$$

a) $x = -1\,, y = -2\,, z = 4$

b) $x = -1\,, y = 3\,, z = 4$

c) $x = 1\,, y = 2\,, z = 4$

d) $x = 10\,, y = -2\,, z = 1$

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LESSON 6-2

Matrix Multiplication, Inverses, and Determinants

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6-2

Matrix Multiplication, Inverses, and Determinants

LESSON 2

Outcomes

1. Multiply matrices.
2. Find determinants and inverses of 2×2 and 3×3 matrices.

---

matrix A $\cdot$ matrix B $=$ AB

$3 \times \mathbf{2}$ $\times$ $\mathbf{2}$ 4 $=$ 3×4

Equal

Dimensions of AB

1 Use matrices $A = \begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 6 \\ 3 & 5 & 1 \end{bmatrix}$ to find each product, if possible.

a. AB

b. BA

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2 **Find AB and BA, if possible.**

$$A = \begin{bmatrix} 3 & 1 & -5 \\ -2 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -6 & 1 & 7 \\ 4 & -5 & 3 \end{bmatrix}$$

3 $$A = \begin{bmatrix} -2 & 0 & 3 \\ 5 & -7 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ -1 & 0 \\ 9 & 3 \end{bmatrix}$$

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4 The percent of voter of different ages who were registered as members of one of the three political parties, Democratic, Republican, or Independent in a recent election are shown. Use this information to determine whether there were more male voters registered as Democrats than there were female voters registered as Republicans.

Distribution by Political Party and Age (%)

Party	18–25	26–40	41–50	50+
Democrat	0.55	0.50	0.35	0.40
Republican	0.30	0.40	0.45	0.55
Independent	0.15	0.10	0.20	0.05

Distribution by Age and Gender

Age	Female	Male
18–25	18,500	16,000
26–40	20,000	24,000
41–50	24,500	22,500
50+	16,500	14,000

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2 Inverses and Determinants

You know that if a is a nonzero real number, then $\frac{1}{a}$ or a^{-1} is the multiplicative inverse of a because $a\left(\frac{1}{a}\right) = a \cdot a^{-1} = 1$. The multiplicative inverse of a square matrix is called its **inverse matrix**.

KeyConcept Inverse of a Square Matrix

Let A be an $n \times n$ matrix. If there exists a matrix B such that $AB = BA = I_n$, then B is called the **inverse** of A and is written as A^{-1}. So, $AA^{-1} = A^{-1}A = I_n$.

5 Determine whether $A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$ are inverse matrices.

6 $A = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

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KeyConcept Inverse and Determinant of a 2 × 2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. A is invertible if and only if $ad - cb \neq 0$.

If A is invertible, then $A^{-1} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

The number $ad - cb$ is called the **determinant** of the 2 × 2 matrix and is denoted

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$$

Find A^{-1}, if it exists. If A^{-1} does not exist, write *singular*.

7 $A = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

8 $A = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$

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Find the determinant of each matrix. Then find the inverse of the matrix, if it exists.

9 $A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

10 $B = \begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$

11 $\begin{bmatrix} -1 & -2 & -2 \\ 3 & 7 & 9 \\ 1 & 4 & 7 \end{bmatrix}$

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Find the determinant of each matrix. Then find the inverse of the matrix, if it exists.

12 $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

13 $\begin{bmatrix} -1 & -2 & 1 \\ 4 & 0 & 3 \\ -3 & 1 & -2 \end{bmatrix}$

Given A and AB, find B.

14 $A = \begin{bmatrix} 8 & -4 \\ 3 & 6 \end{bmatrix}, AB = \begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix}$

15 **Find x and y.** $A = \begin{bmatrix} 2x & -y \\ -3y & 5x \end{bmatrix}, B = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, and $AB = \begin{bmatrix} -2 \\ 31 \end{bmatrix}$

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16 Evaluate each expression.

$$A = \begin{bmatrix} 4 & 1 & -3 \\ 0 & 2 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & 9 & -6 \\ 7 & 5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & -2 \end{bmatrix} \qquad D = \begin{bmatrix} 7 & 2 \\ -4 & -1 \end{bmatrix}$$

$BD + B$ $DC - A$

.....

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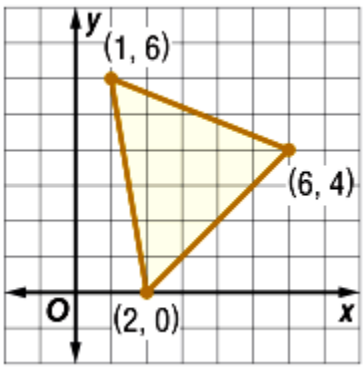
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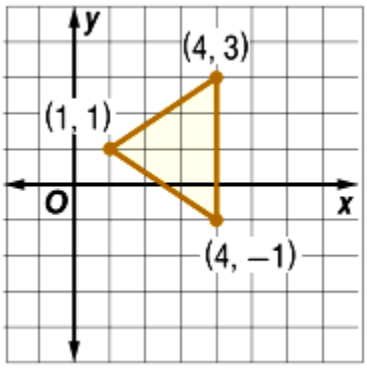
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17 Find the area A of each triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3), by using $A = \frac{1}{2}|\det(X)|$, where X is $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$.





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18 Find AB if possible.

$$A=\begin{bmatrix} 2 & 9 \\ -7 & 3 \end{bmatrix} \qquad B=\begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$$

a) $\begin{bmatrix} \mathbf{12} & \mathbf{19} \\ \mathbf{42} & \mathbf{37} \end{bmatrix}$ ***b***) $\begin{bmatrix} \mathbf{12} & \mathbf{19} \\ \mathbf{42} & \mathbf{-37} \end{bmatrix}$

c) $\begin{bmatrix} \mathbf{12} & \mathbf{19} \\ \mathbf{-42} & \mathbf{37} \end{bmatrix}$ ***d***) $\begin{bmatrix} \mathbf{12} & \mathbf{-19} \\ \mathbf{42} & \mathbf{37} \end{bmatrix}$

19 Find AB if possible.

$$A=\begin{bmatrix} 2 & 0 \\ -4 & -3 \\ 1 & -2 \end{bmatrix} \qquad B=\begin{bmatrix} 0 & 6 & -5 \\ 2 & -7 & 1 \end{bmatrix}$$

a) $\begin{bmatrix} \mathbf{0} & \mathbf{12} & \mathbf{-10} \\ \mathbf{-6} & \mathbf{-3} & \mathbf{17} \\ \mathbf{-4} & \mathbf{20} & \mathbf{-7} \end{bmatrix}$ ***b***) $\begin{bmatrix} \mathbf{0} & \mathbf{12} & \mathbf{-10} \\ \mathbf{-6} & \mathbf{3} & \mathbf{17} \\ \mathbf{4} & \mathbf{20} & \mathbf{7} \end{bmatrix}$

c) $\begin{bmatrix} \mathbf{0} & \mathbf{12} & \mathbf{10} \\ \mathbf{-6} & \mathbf{-3} & \mathbf{17} \\ \mathbf{-4} & \mathbf{20} & \mathbf{7} \end{bmatrix}$ ***d***) $\begin{bmatrix} \mathbf{0} & \mathbf{12} & \mathbf{-10} \\ \mathbf{6} & \mathbf{-3} & \mathbf{17} \\ \mathbf{4} & \mathbf{20} & \mathbf{-7} \end{bmatrix}$

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20 Find AB if possible.

$$A=\begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix} \qquad B=\begin{bmatrix} 5 & 2 & -8 \\ -6 & 0 & 9 \end{bmatrix}$$

a) $\begin{bmatrix} \mathbf{-9} & \mathbf{6} & \mathbf{-12} \\ \mathbf{-41} & \mathbf{-14} & \mathbf{-65} \end{bmatrix}$ ***b***) $\begin{bmatrix} \mathbf{-9} & \mathbf{6} & \mathbf{12} \\ \mathbf{-41} & \mathbf{-14} & \mathbf{65} \end{bmatrix}$

c) $\begin{bmatrix} \mathbf{-9} & \mathbf{6} & \mathbf{12} \\ \mathbf{41} & \mathbf{14} & \mathbf{65} \end{bmatrix}$ ***d***) $\begin{bmatrix} \mathbf{-9} & \mathbf{0} & \mathbf{-2} \\ \mathbf{-41} & \mathbf{-14} & \mathbf{65} \end{bmatrix}$

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21 Find BA, if possible.

$$A = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$$

a) $\begin{bmatrix} 0 & 12 & -10 \\ -6 & -3 & 17 \\ -4 & 20 & -7 \end{bmatrix}$ ***b)*** $\begin{bmatrix} 18 \\ -11 \end{bmatrix}$

c) $\begin{bmatrix} -9 & 6 & 12 \\ 41 & 14 & 65 \end{bmatrix}$ ***d) Impossible***

22 Find BA, if possible.

$$A = \begin{bmatrix} 6 & -9 & 10 \\ 4 & 3 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & -8 \\ 3 & -9 \\ -2 & 5 \\ 4 & 1 \end{bmatrix}$$

a) $\begin{bmatrix} 4 & 78 & 4 \\ 18 & 54 & 42 \\ 8 & 33 & 20 \\ 28 & 33 & 48 \end{bmatrix}$ ***b)*** $\begin{bmatrix} 4 & -78 & -4 \\ -18 & -54 & -42 \\ 8 & 33 & 20 \\ 28 & -33 & 48 \end{bmatrix}$

c) $\begin{bmatrix} -9 & 6 & 12 \\ 41 & 14 & 65 \end{bmatrix}$ ***d) Impossible***

23 Find A^{-1}, if it exists. $A = \begin{bmatrix} -4 & 2 \\ -6 & 3 \end{bmatrix}$

a) $\begin{bmatrix} -4 & 2 \\ -6 & -3 \end{bmatrix}$ ***b)*** $\begin{bmatrix} -12 & 8 \\ 6 & -4 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 2 \\ -6 & 3 \end{bmatrix}$ ***d) singular***

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24 Find A^{-1}, if it exists. $A = \begin{bmatrix} -4 & 8 \\ 1 & -2 \end{bmatrix}$

$a) \begin{bmatrix} 2 & 8 \\ 1 & 4 \end{bmatrix}$ $b) \begin{bmatrix} 2 & -8 \\ -1 & 4 \end{bmatrix}$

$c) \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}$ $d)$ *singular*

25 Find A^{-1}, if it exists. $A = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

$a) \begin{bmatrix} -3 & -5 \\ 2 & 3 \end{bmatrix}$ $b) \begin{bmatrix} 2 & -\frac{5}{2} \\ -3 & 4 \end{bmatrix}$

$c) \begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix}$ $d)$ *singular*

26 Find A^{-1}, if it exists. $A = \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}$

$a) \begin{bmatrix} -4 & 5 \\ 6 & 8 \end{bmatrix}$ $b) \begin{bmatrix} 2 & -\frac{5}{2} \\ -3 & 4 \end{bmatrix}$

$c) \begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix}$ $d)$ *singular*

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27 Find A^{-1}, if it exists $A = \begin{bmatrix} -1 & -1 & -3 \\ 3 & 6 & 4 \\ 2 & 1 & 8 \end{bmatrix}$

$a)$ $\begin{bmatrix} -44 & 12 & -10 \\ -6 & -3 & 17 \\ -4 & 20 & -7 \end{bmatrix}$

$b)$ $\begin{bmatrix} -44 & -5 & -14 \\ 16 & 2 & 2 \\ 9 & 1 & 3 \end{bmatrix}$

$c)$ $\begin{bmatrix} -44 & 5 & 14 \\ 6 & 2 & 2 \\ 9 & 1 & 3 \end{bmatrix}$

$d)$ $singular$

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28 Find A^{-1}, if it exists. $A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & 3 & 5 \\ 6 & -1 & -4 \end{bmatrix}$

$a)$ $\begin{bmatrix} -44 & 12 & -10 \\ -6 & -3 & 17 \\ -4 & 20 & -7 \end{bmatrix}$

$b)$ $\begin{bmatrix} -44 & -5 & -14 \\ 16 & 2 & 2 \\ 9 & 1 & 3 \end{bmatrix}$

$c)$ $\begin{bmatrix} -44 & 5 & 14 \\ 6 & 2 & 2 \\ 9 & 1 & 3 \end{bmatrix}$

$d)$ $singular$

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LESSON 6-3

Solving Linear Systems using Inverse and Cramer's Rule

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6-3

Solving Linear Systems using Inverses and Cramer's Rule

LESSON 3

Outcomes

1 Solve systems of linear equations using inverse matrices.

2 Solve systems of linear equations using Cramer's Rule.

KeyConcept Invertible Square Linear Systems

Let A be the coefficient matrix of a system of n linear equations in n variables given by $AX = B$, where X is the matrix of variables and B is the matrix of constants. If A is invertible, then the system of equations has a unique solution given by $X = A^{-1}B$.

Use an inverse matrix to solve the system of equations, if possible.

1 $2x - 3y = -1$
$-3x + 5y = 3$

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2 $6x + y = -8$
$-4x - 5y = -12$

3 $-3x + 9y = 36$
$7x - 8y = -19$

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KeyConcept Cramer's Rule

Let A be the coefficient matrix of a system of n linear equations in n variables given by $AX = B$. If $\det(A) \neq 0$, then the unique solution of the system is given by

$$x_1 = \frac{|A_1|}{|A|}, x_2 = \frac{|A_2|}{|A|}, x_3 = \frac{|A_3|}{|A|}, \ldots, x_n = \frac{|A_n|}{|A|},$$

where A_i is obtained by replacing the ith column of A with the column of constant terms B. If $\det(A) = 0$, then $AX = B$ has either no solution or infinitely many solutions.

Use Cramer's Rule to find the solution of the system of linear equations, if a unique solution exists.

4 $3x_1 + 2x_2 = 6$
$-4x_1 - x_2 = -13$

5 $2x - y = 4$
$5x - 3y = -6$

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6

$$-9x + 3y = 8$$
$$2x - y = -3$$

7

$$-x - 2y = -4z + 12$$
$$3x - 6y + z = 15$$
$$2x + 5y + 1 = 0$$

8

$$8x + 12y - 24z = -40$$
$$3x - 8y + 12z = 23$$
$$2x + 3y - 6z = -10$$

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MATH ENG

LESSON 6-4

Modeling Motion with Matrices

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2024-2025

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Modeling Motion with Matrices

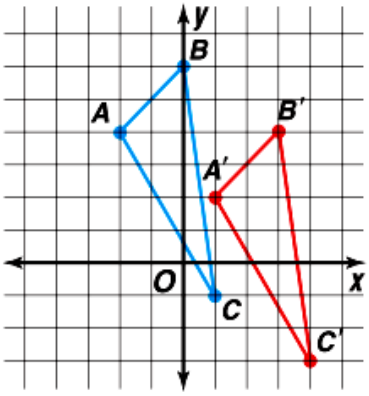
LESSON 4

Outcomes

1 Use matrices to determine the coordinates of polygons under a given transformation.

1 Even though large matrices are used for computer animation, you can use a simple matrix to describe many of the motions called **transformations** that you learned about in geometry. Some of the transformations we will examine in this lesson are **translations** (slides), **reflections** (flips), **rotations** (turns), and **dilations** (enlargements or reductions).

A $2 \times n$ matrix can be used to express the vertices of an n-gon with the first row of elements representing the x-coordinates and the second row the y-coordinates of the vertices.



Triangle ABC can be represented by the following **vertex matrix**.

$$\begin{array}{l} \\ x\text{-coordinate} \\ y\text{-coordinate} \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} -2 & 0 & 1 \\ 4 & 6 & -1 \end{bmatrix} \end{array}$$

Triangle $A'B'C'$ is congruent to and has the same orientation as ΔABC, but is moved 3 units right and 2 units down from ΔABC's location. The coordinates of $\Delta A'B'C'$ can be expressed as the following vertex matrix.

$$\begin{array}{l} \\ x\text{-coordinate} \\ y\text{-coordinate} \end{array} \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & -3 \end{bmatrix} \end{array}$$

Compare the two matrices. If you add $\begin{bmatrix} 3 & 3 & 3 \\ -2 & -2 & -2 \end{bmatrix}$ to the first matrix, you get the second matrix. Each 3 represents moving 3 units right for each x-coordinate. Likewise, each -2 represents moving 2 units down for each y-coordinate. This type of matrix is called a **translation matrix**. In this transformation, ΔABC is the **pre-image**, and $\Delta A'B'C$, is the **image** after the translation.

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1 *Suppose quadrilateral ABCD with vertices A(-1, 1), B(4, 0), C(4, -5), and D(1, -3) is translated 2 units left and 4 units up.*

a. Represent the vertices of the quadrilateral as a matrix.

b. Write the translation matrix.

c. Use the translation matrix to find the vertices of A'B'C'D', the translated image of the quadrilateral.

d. Graph quadrilateral ABCD and its image

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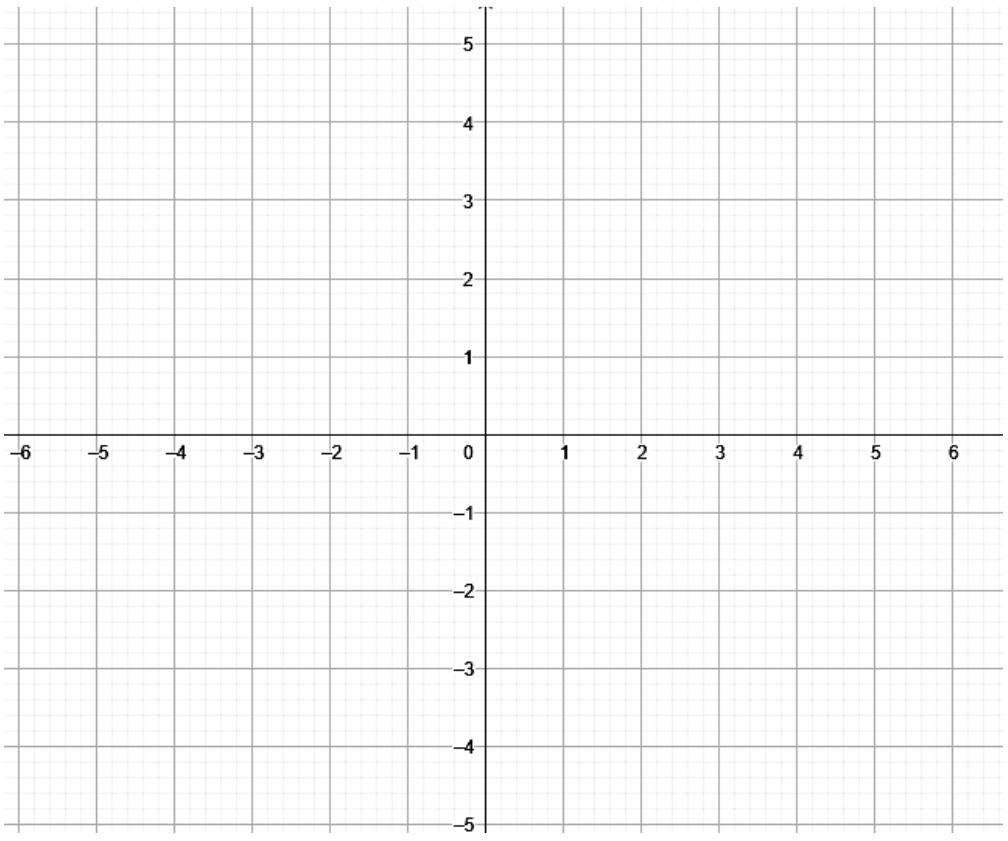
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Reflection Matrices		
For a reflection over the:	Symbolized by:	Multiply the vertex matrix by:
x-axis	$R_{x\text{-axis}}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
y-axis	$R_{y\text{-axis}}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
line $y = x$	$R_{y=x}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Reflection Matrices		
For a counterclockwise rotation about the origin of:	Symbolized by:	Multiply the vertex matrix by:
90°	Rot_{90}	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
180°	Rot_{180}	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
270°	Rot_{270}	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

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2 *To create an image that appears to be reflected in a mirror, an animator will use a matrix to reflect an image over the y-axis. Use a reflection matrix to find the coordinates of the vertices of a star reflected in a mirror (the y-axis) if the coordinates of the points connected to create the star are (-2, 4), (-3.5, 4), (4, 5), (-4.5, 4), (-6, 4), (-5, 3), (-5, 1), (-4, 2), (-3, 1), and (-3, 3)*

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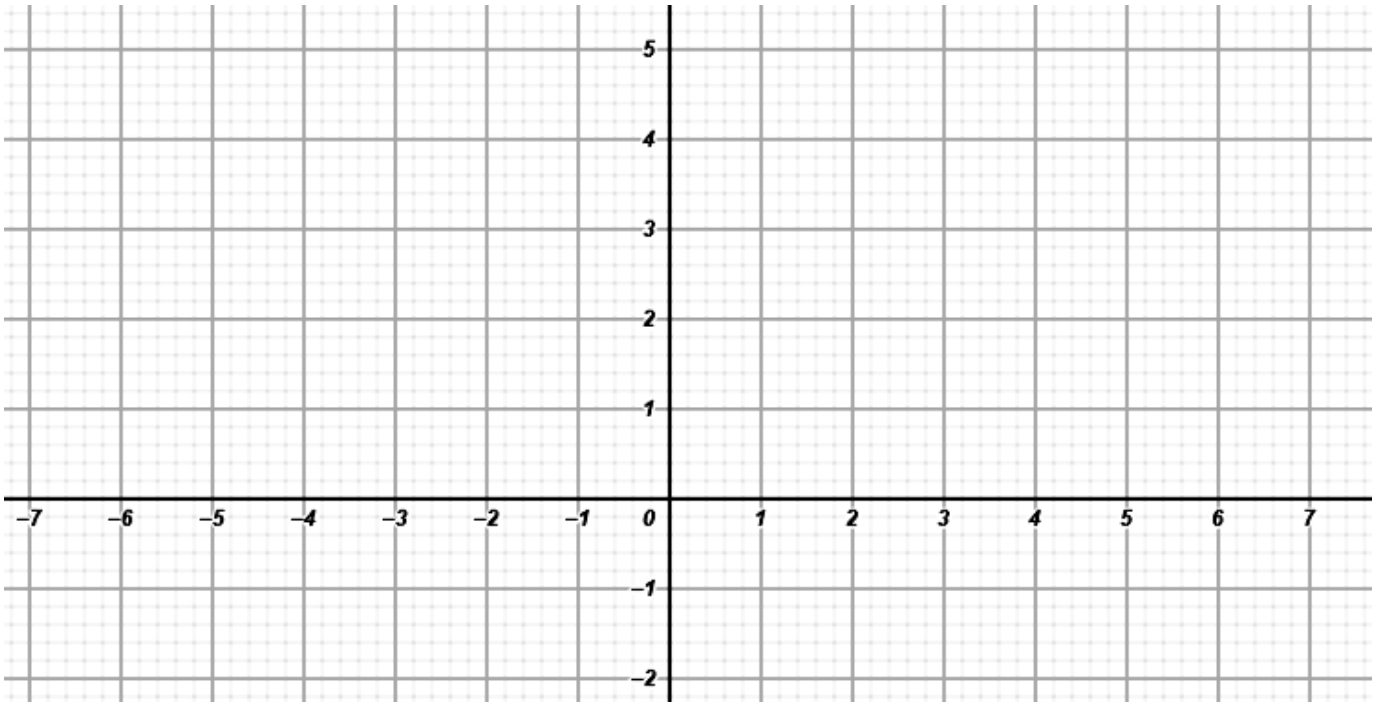
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3 *Suppose a figure is animated to spin around a certain point. Numerous rotation images would be necessary to make a smooth movement image. If the image has key points at (1, 1), (-1, 4), (-2, 4), and, (-2, 3) and the rotation is about the origin, find the location of these points at the 90°, 180°, and 270° counterclockwise rotations.*

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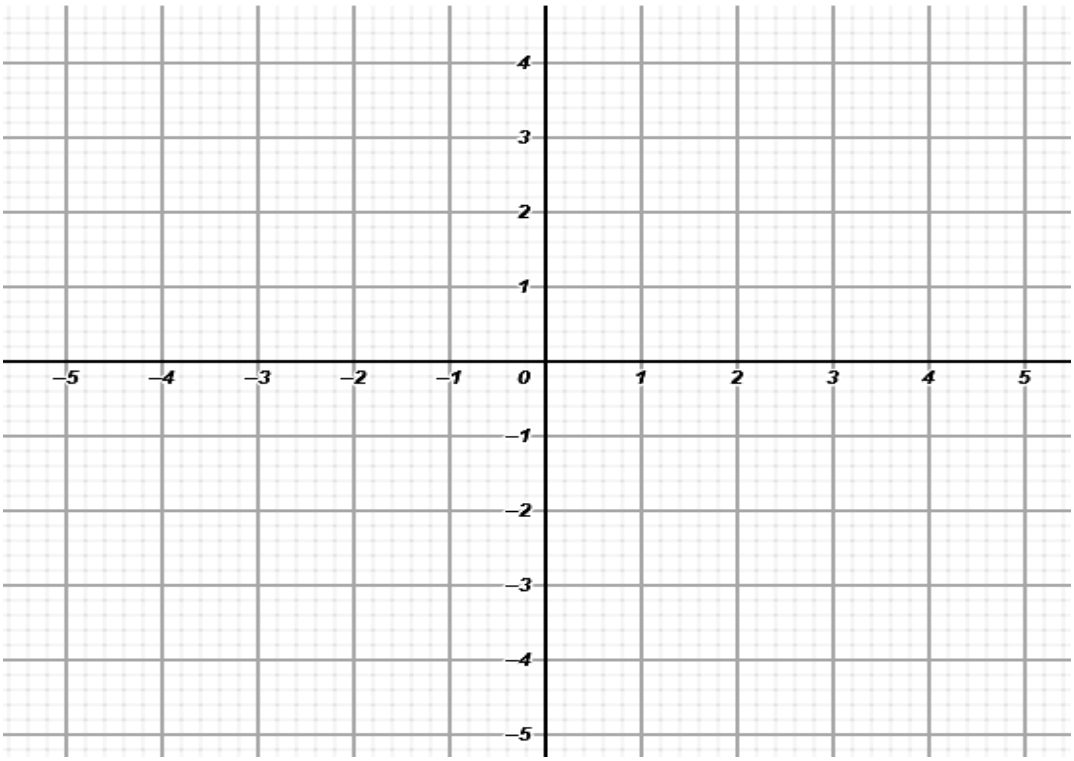
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4 *A trapezoid has vertices at L(-4, 1), M(1, 4), N(7, 0), and P(-3, -6). Find the coordinates of the dilated trapezoid L'M'N'P' for a scale factor of 0.5. Describe the dilation.*

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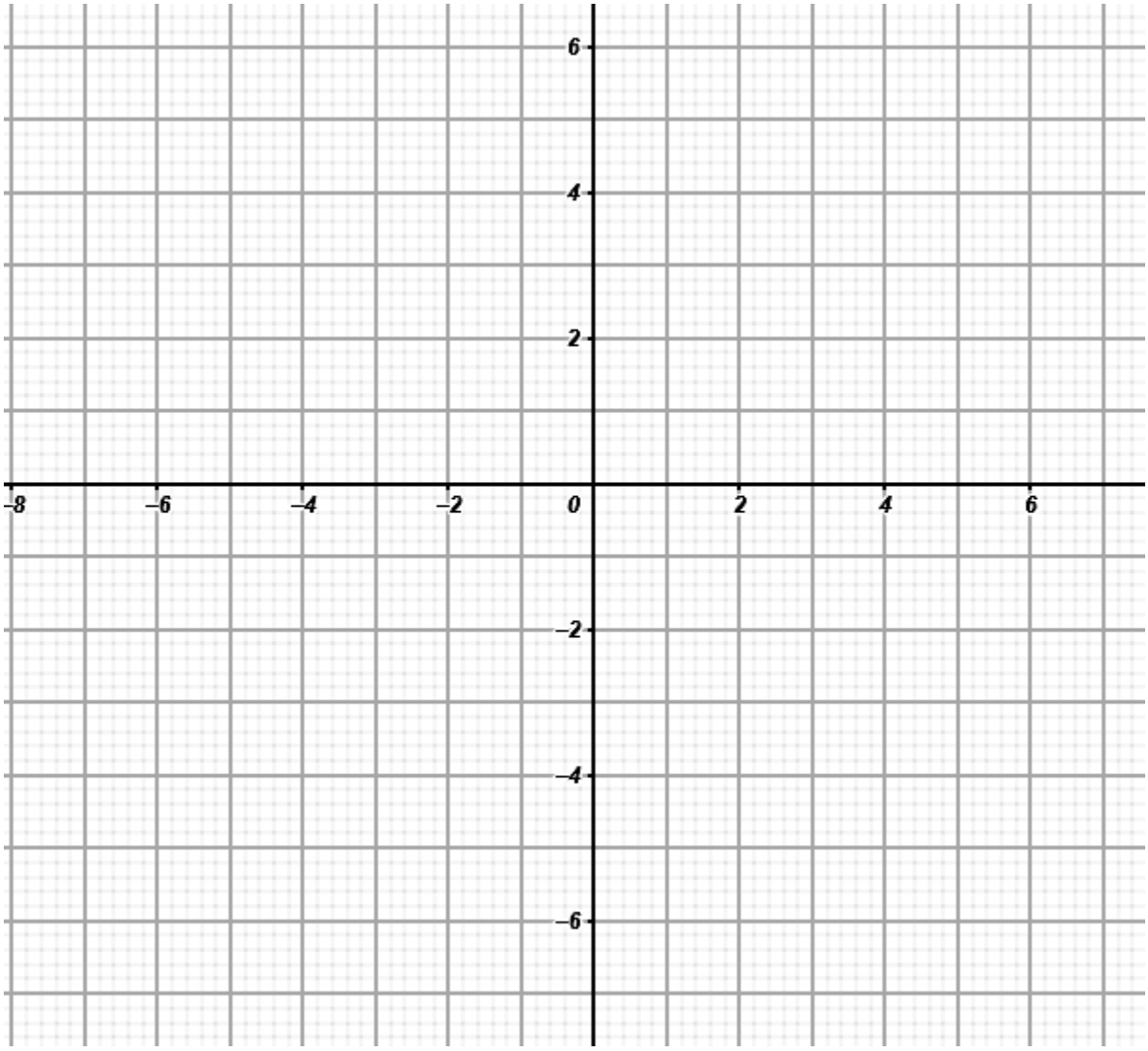
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5 **Triangle JKL has vertices J(-2, 5), K(1, 3), and L(0, -2). Use scalar multiplication to find the coordinates of the triangle after a dilation of scale factor 1.5.**

$a)$ $(-3, 7.5), (1.5, 4.5), (0, -3)$

$b)$ $(3, 7.5), (1.5, 4.5), (0, 3)$

$c)$ $(3, 7.5), (1.5, 4.5), (0, -3)$

$d)$ $(-3, 7.5), (-1.5, 4.5), (0, 3)$

6 **Square ABCD has vertices A(-1, 3), B(3, 3), C(3, -1), and D(-1, -1). Find the coordinates of the square after a translation of 1 unit left and 2 units down**

$a)$ $(2, -1), (2, 1), (2, -3), (-2, -3)$

$b)$ $(-2, 1), (-2, -1), (2, -3), (-2, -3)$

$c)$ $(-2, 1), (2, 1), (2, -3), (2, 3)$

$d)$ $(-2, 1), (2, 1), (2, -3), (-2, -3)$

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7 **Square ABCD has vertices at (-1, 2), (-4, 1), (-3, -2), and (0, -1). Find the image of the square after a reflection over the y-axis.**

$a)$ $(1, -2), (4, -1), (3, 2), (0, 1)$

$b)$ $(1, 2), (4, 1), (3, -2), (0, -1)$

$c)$ $(-1, -2), (-4, -1), (-3, 2), (0, -1)$

$d)$ $(2, 1), (1, 4), (-2, 3), (-1, 0)$

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8 Triangle PQR is represented by the matrix $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix}$ Find the image of the triangle after a rotation of 270° counterclockwise about the origin

$a)$ $\begin{bmatrix} -2 & -4 & 2 \\ -3 & 1 & -1 \end{bmatrix}$

$b)$ $\begin{bmatrix} 3 & -1 & 1 \\ -2 & -4 & 2 \end{bmatrix}$

$c)$ $\begin{bmatrix} 2 & 4 & -2 \\ -3 & 1 & -1 \end{bmatrix}$

$d)$ $\begin{bmatrix} 2 & 4 & -2 \\ 3 & -1 & 1 \end{bmatrix}$

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9 Find the image of ΔLMN after Rot $_{180}$ · R $_{y\text{-axis}}$ if the vertices are L(-6, 4), M(-3, 2), and N(-1, -2).

$a)$ $(-6,-4),(-3,-2),(-1,-2)$

$b)$ $(6,-4),(3,-2),(1,-2)$

$c)$ $(4,-6),(2,-3),(-2,-1)$

$d)$ $(4,6),(2,3),(-2,1)$

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10 Triangle with vertices A(1, 1), B(1, 4), and C(5, 1); scale factor 3

$a)$ $(-3,-3),(3,12),(15,3)$

$b)$ $(3,3),(3,12),(15,3)$

$c)$ $(3,3),(-3,-12),(15,3)$

$d)$ $(3,3),(3,12),(-15,-3)$

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11 Triangle with vertices $X(0,8), Y(-5,9), and\ Z(-3,2)$; scale factor $\frac{3}{4}$

$a)$ $(0,-6),\left(-\frac{15}{4},\frac{27}{4}\right),(-\frac{9}{4},\frac{3}{2})$

$b)$ $(0,6),(-15,27),(-9,6)$

$c)$ $(0,6),\left(\frac{15}{4},\frac{27}{4}\right),(\frac{9}{4},\frac{3}{2})$

$d)$ $(0,6),\left(-\frac{15}{4},\frac{27}{4}\right),(-\frac{9}{4},\frac{3}{2})$

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12 quadrilateral PQRS with vertex matrix $\begin{bmatrix}-3 & -2 & 1 & 4\\ 0 & 2 & 3 & 2\end{bmatrix}$ scale factor 2

$a)$ $\begin{bmatrix}-2 & -4 & 2 & 8\\ 0 & -4 & -6 & -4\end{bmatrix}$

$b)$ $\begin{bmatrix}-6 & -4 & 2 & 8\\ 0 & 4 & 6 & 4\end{bmatrix}$

$c)$ $\begin{bmatrix}-6 & -4 & 2 & 8\\ 0 & 2 & 3 & 2\end{bmatrix}$

$d)$ $\begin{bmatrix}6 & 4 & 2 & 8\\ 0 & 4 & 6 & 4\end{bmatrix}$

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LESSON 7-1

Midpoint and Distance Formulas

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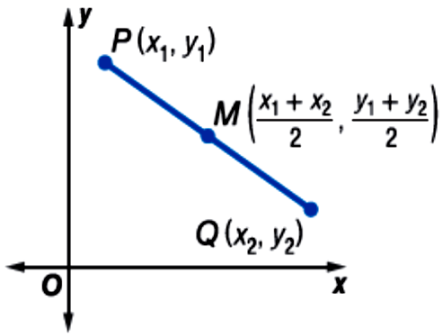
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Midpoint and Distance Formulas

KeyConcept Midpoint Formula

Words If a line segment has endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the midpoint of the segment has coordinates $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Model



1 Find the coordinates of M, the midpoint of $\overline{JK}$, for $J(-1, 2)$ and $K(6, 1)$.

2 Find the coordinates of the midpoint of $\overline{AB}$ for $A(5, 12)$ and $B(-4, 8)$.

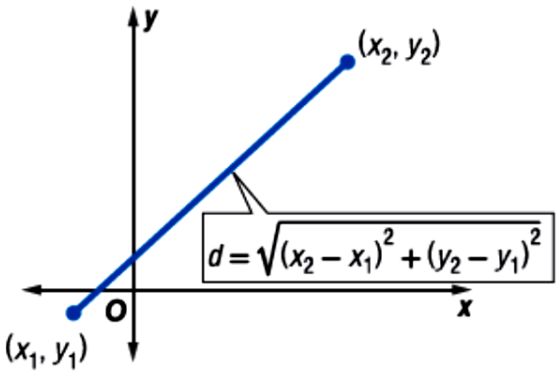
3 Find the coordinates of the midpoint of $\overline{CD}$ for $C(4, 5)$ and $D(14, 13)$.

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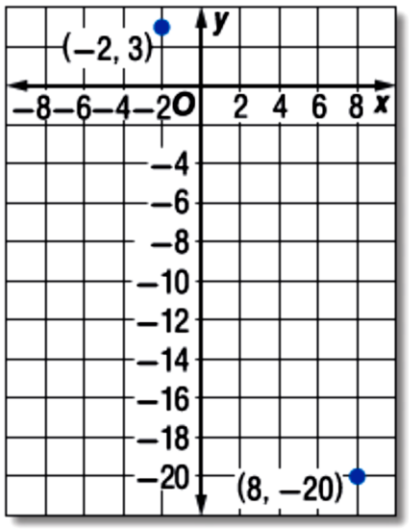
KeyConcept Distance Formula

Words The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Model



4 **DISC GOLF** Saleh's disc is 20 feet short and 8 feet to the right of the basket. On his first putt, the disc lands 2 feet to the left and 3 feet beyond the basket. If the disc went in a straight line, how far did it go?



5 Find the distance between each pair of points with the given coordinates.

(1, 2), (6, 3)

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6 A coordinate grid is placed over a map. City A is located at (3, 13), and City B is located at (8, −1). If City C is halfway between City A and City B, which is closest to the distance in coordinate units from City A to City C?

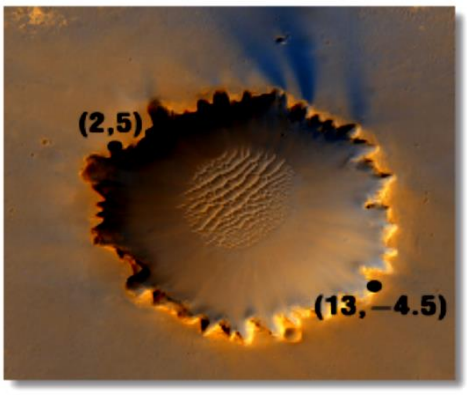
A 4.75 B 7.43 C 14.9 D 19

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7 The coordinates for points A and B are $(-4, -5)$ and $(10, -7)$, respectively. Find the distance between the midpoint of A and B and point B.

F $\sqrt{10}$ units G $5\sqrt{10}$ units H $\sqrt{50}$ units J $10\sqrt{5}$ units

8 **SPACE** Use the labeled points on the outline of the circular crater on Mars to estimate its diameter in kilometers. Assume each unit on the coordinate system is 1 kilometer.



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9 Find the midpoint of the line segment with endpoints at the given coordinates.

$(-4, 7), (3, 9)$

a) (-0.5 , 8)

b) (3.5 , -1.5)

c) (14.5 , 9.75)

d) (-11.25 , -4)

10 Find the midpoint of the line segment with endpoints at the given coordinates.

$(11, 6), (18, 13.5)$

a) (-0.5 , 8)

b) (3.5 , -1.5)

c) (14.5 , 9.75)

d) (-11.25 , -4)

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11 Find the midpoint of the line segment with endpoints at the given coordinates.

$(-12, -2), (-10.5, -6)$

a) (-0.5 , 8)

b) (3.5 , -1.5)

c) (14.5 , 9.75)

d) (-11.25 , -4)

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12 Find the midpoint of the line segment with endpoints at the given coordinates.

$$(8, 2), (-1, -5)$$

a) (-0.5 , 8)

b) (3.5 , -1.5)

c) (14.5 , 9.75)

d) (-11.25 , -4)

13 **Find the distance between each pair of points with the given coordinates.**

$$(1, 2), (6, 3)$$

$a)\ 5.099$

$b)\ 17.720$

$c)\ 6.083$

$d)\ 29.069$

14 **Find the distance between each pair of points with the given coordinates.**

$$(-6, -7), (11, -12)$$

$a)\ 5.099$

$b)\ 17.720$

$c)\ 6.083$

$d)\ 29.069$

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15 **Find the distance between each pair of points with the given coordinates.**

$$(4, 0), (5, -6)$$

$a)$ 5.099

$b)$ 17.720

$c)$ 6.083

$d)$ 29.069

16 **Find the distance between each pair of points with the given coordinates.**

$$(-4, -5), (15, 17)$$

$a)$ 5.099

$b)$ 17.720

$c)$ 6.083

$d)$ 29.069

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LESSON 7-2

Parabolas

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Parabolas

LESSON 2

Outcomes

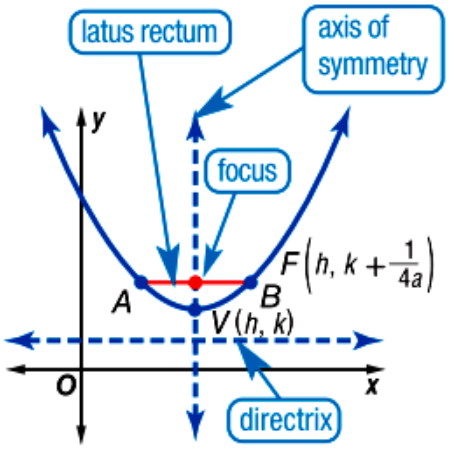
1 Write equations of parabolas in standard form.

2 Graph parabolas.

1 Equations of Parabolas

A **parabola** can be defined as the set of all points in a plane that are the same distance from a given point called the **focus** and a given line called the **directrix**.

The line segment through the focus of a parabola and perpendicular to the axis of symmetry is called the **latus rectum**. The endpoints of the latus rectum lie on the parabola.



KeyConcept Equations of Parabolas

Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Vertex	(h, k)	(h, k)
Axis of Symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Length of Latus Rectum	$\left\|\frac{1}{a}\right\|$ units	$\left\|\frac{1}{a}\right\|$ units

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1 Write $y = 2x^2 - 12x + 6$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

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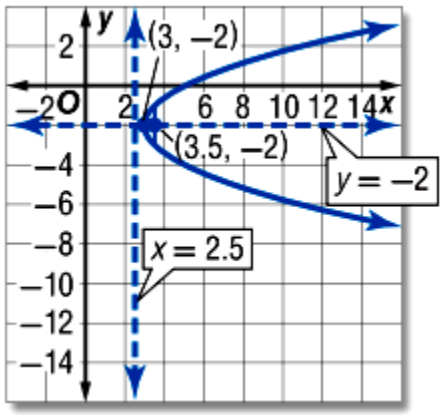
2 Write $y = 4x^2 + 16x + 34$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

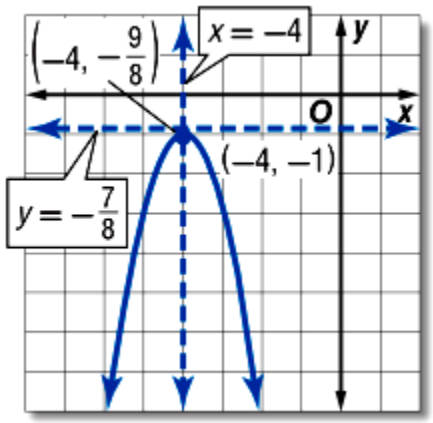
3 $x + 3y^2 + 12y = 18$

Graph each equation.

4 $2x - y^2 = 4y + 10$



5 $y + 2x^2 + 32 = -16x - 1$



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6 $3x - y^2 = 4x + 25$

7 $y = x^2 + 6x - 4$

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8 Write an equation for a parabola with vertex at (-2, -4) and directrix y = 1. Then graph the equation.

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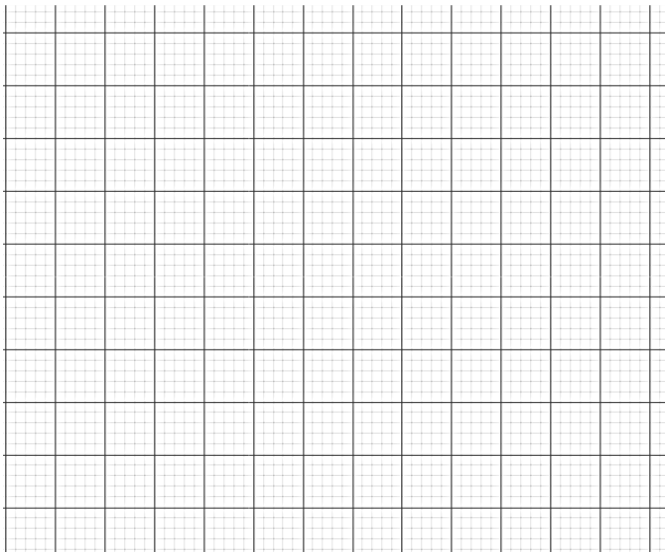
Write an equation for each parabola described below.

9 vertex (1, 3), focus (1, 5)

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10 focus (5, 6), directrix $x = -2$

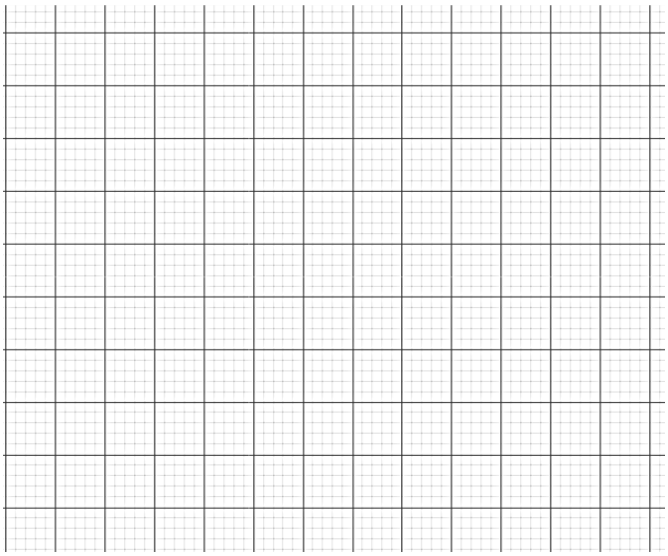
11 Solar energy may be harnessed by using parabolic mirrors. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of each parabolic mirror at the facility described at the left is 1.9 meters above the vertex. The latus rectum is 7.6 meters long.



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12 Write and graph an equation for a parabolic mirror that has a focus 1.4 meters above the vertex and a latus rectum that is 5.5 meters long, when the focus is at the origin.



13 A beach umbrella has an arch in the shape of a parabola that opens downward. The umbrella spans 1.8 meters across and is 0.45 meters high. Write an equation of a parabola to model the arch, assuming that the origin is at the point where the pole and umbrella meet at the vertex of the arch.

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14 **Consider a parabolic mercury mirror like the one described at the beginning of the lesson. The focus is 1.8 meters above the vertex and the latus rectum is 7.3 meters long. Assume that the focus is at the origin. Write an equation for the parabola formed by the parabolic microphone.**

EOT هيكل الاختبارات السابقة

15 Write each equation in standard form.

$$y = 2x^2 - 24x + 40$$

a) $y = 2(x-6)^2 - 32$ ***b)*** $y = 3(x-1)^2 - 7$

c) $x = (y-4)^2 - 27$ ***d)*** $x = -3(y+2)^2 + 30$

16 Write each equation in standard form.

$$x + 3y^2 + 12y = 18$$

a) $y = 2(x-6)^2 - 32$ ***b)*** $y = 3(x-1)^2 - 7$

c) $x = (y-4)^2 - 27$ ***d)*** $x = -3(y+2)^2 + 30$

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17 Write each equation in standard form.

$$x = y^2 - 8y - 11$$

a) $y = 2(x-6)^2 - 32$ ***b)*** $y = 3(x-1)^2 - 7$

c) $x = (y-4)^2 - 27$ ***d)*** $x = -3(y+2)^2 + 30$

18 Write each equation in standard form.

$$y = 3x^2 - 6x - 4$$

a) $y = 2(x-6)^2 - 32$ ***b)*** $y = 3(x-1)^2 - 7$

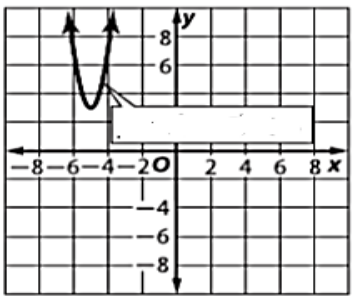
c) $x = (y-4)^2 - 27$ ***d)*** $x = -3(y+2)^2 + 30$

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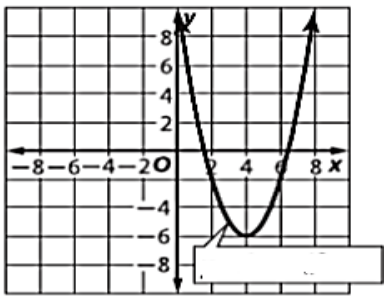
19 Graph each equation.

$y = (x-4)^2 - 6$

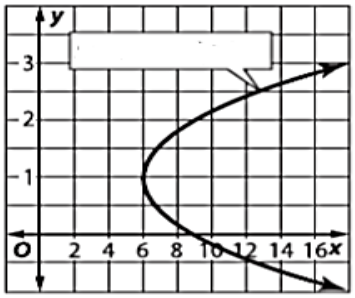
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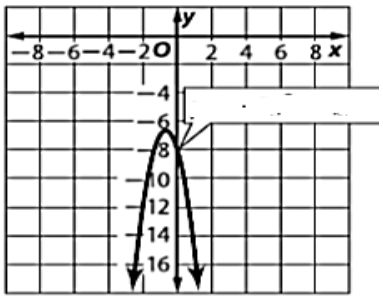
b)



c)



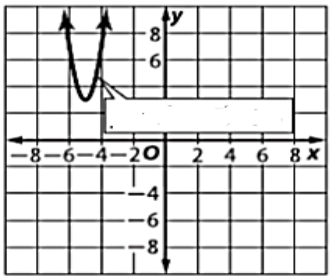
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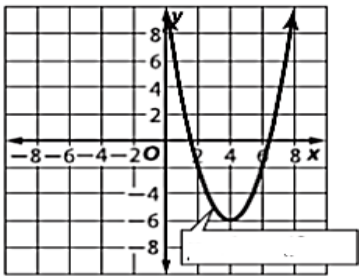
20 **Graph each equation.**

$y = 4(x + 5)^2 + 3$

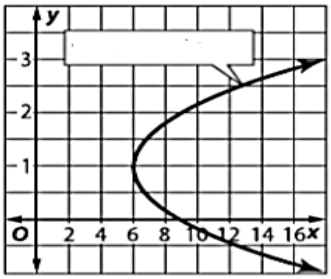
a)



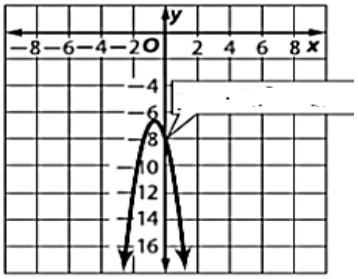
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c)



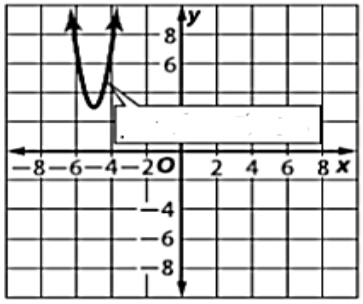
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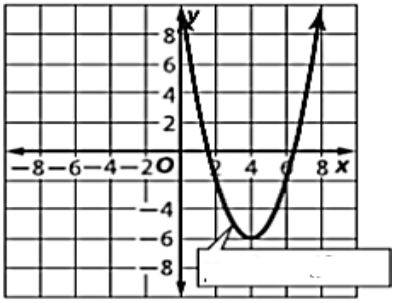
21 **Graph each equation.**

$y = -3x^2 - 4x - 8$

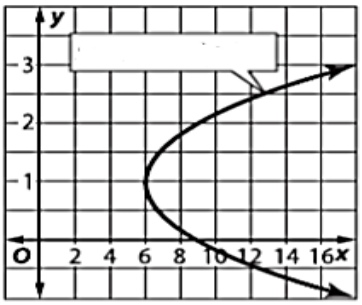
a)



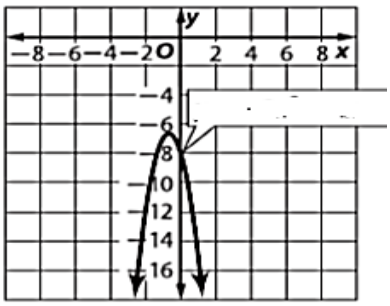
b)



c)



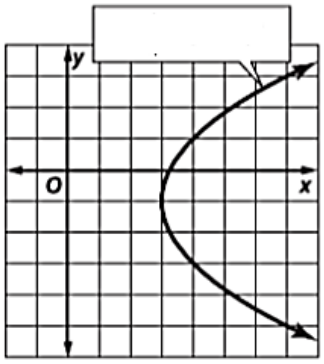
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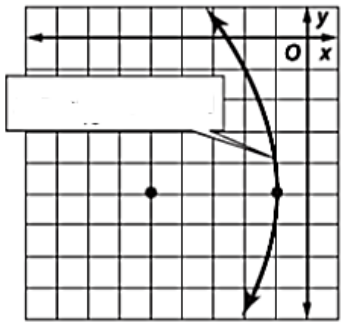
22 **Graph each equation.**

$x = 3y^2 - 6y + 9$

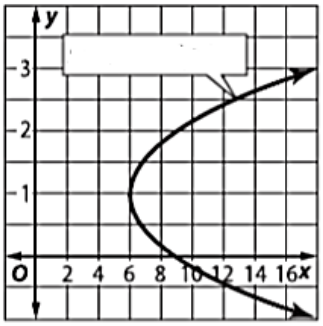
a)



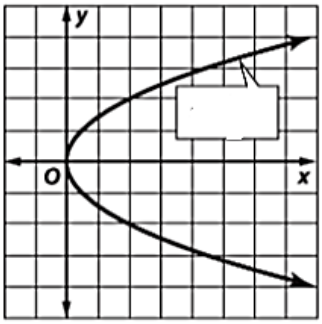
b)



c)



d)



23 **Write an equation for each parabola described below. Then graph the equation.**

vertex (0, 1), focus (0, 4)

$a)\ y = \frac{-1}{12}x^2 + 1$

$b)\ y = \frac{1}{12}x^2 + 1$

$c)\ x = \frac{-1}{12}y^2 + 1$

$d)\ x = \frac{1}{12}y^2 + 1$

24 **Write an equation for each parabola described below. Then graph the equation.**

vertex (1, 8), directrix y = 3

$a)\ y = \frac{1}{20}(x-1)^2 + 8$

$b)\ y = \frac{1}{20}(x+1)^2 + 8$

$c)\ x = \frac{1}{20}(y+8)^2 - 1$

$d)\ x = \frac{1}{20}(y+8)^2 + 1$

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25 Write an equation for each parabola described below. Then graph the equation.

focus (-2, -4), directrix x = -6

$a)\ y = \frac{1}{8}(x-1)^2 + 4$ $b)\ y = \frac{1}{8}(x+1)^2 + 4$

$c)\ x = \frac{1}{8}(y+4)^2 - 4$ $d)\ x = \frac{1}{8}(y+4)^2 + 4$

26 Write an equation for each parabola described below. Then graph the equation.

focus (2, 4), directrix x = 10

$a)\ y = \frac{1}{8}(x-1)^2 + 4$ $b)\ x = -\frac{1}{16}(y+4)^2 + 6$

$c)\ x = \frac{1}{16}(y-4)^2 - 6$ $d)\ x = -\frac{1}{16}(y-4)^2 + 6$

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27 Write an equation for each parabola described below. Then graph the equation.

vertex (-6, 0), directrix x = 2

$a)\ x = \frac{-1}{32}y^2 - 6$ $b)\ x = \frac{-1}{32}y^2 + 6$

$c)\ y = \frac{-1}{32}x^2 + 6$ $d)\ y = \frac{-1}{32}x^2 - 6$

28 **Write an equation for each parabola described below. Then graph the equation.**

vertex (9, 6), focus (9, 5)

$a)\ x = \frac{-1}{4}(y-9)^2 + 6$

$b)\ y = \frac{-1}{4}(x-9)^2 + 6$

$c)\ y = \frac{1}{4}(x-9)^2 + 6$

$d)\ y = \frac{-1}{4}(x+9)^2 + 6$

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12 GENERAL

MATH ENG

LESSON 7-3

Circles

Mr. Ahmed Ata
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2024-2025

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7-3

Circles

LESSON 3

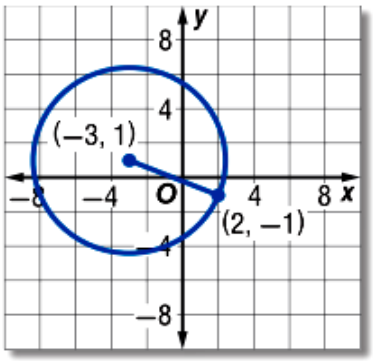
Outcomes

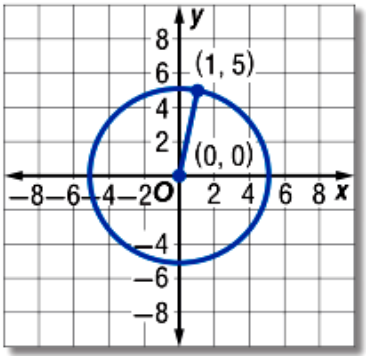
1 Write equations of circles.

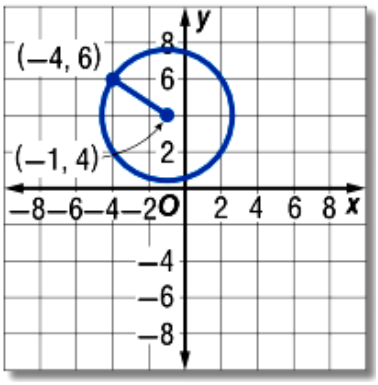
2 Graph circles.

KeyConcept Equations of Circles		
Standard Form of Equation	$x^2 + y^2 = r^2$	$(x - h)^2 + (y - k)^2 = r^2$
Center	(0, 0)	(h, k)
Radius	r	r

1 Write an equation for the graph





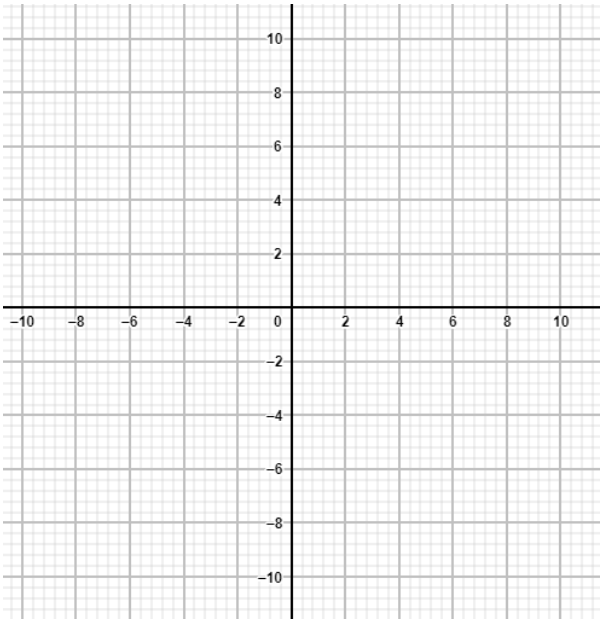


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2 Write an equation for a circle if the endpoints of a diameter are at (7, 6) and (-1, -8).

3 Write an equation for a circle if the endpoints of a diameter are at (3, -3) and (1, 5)

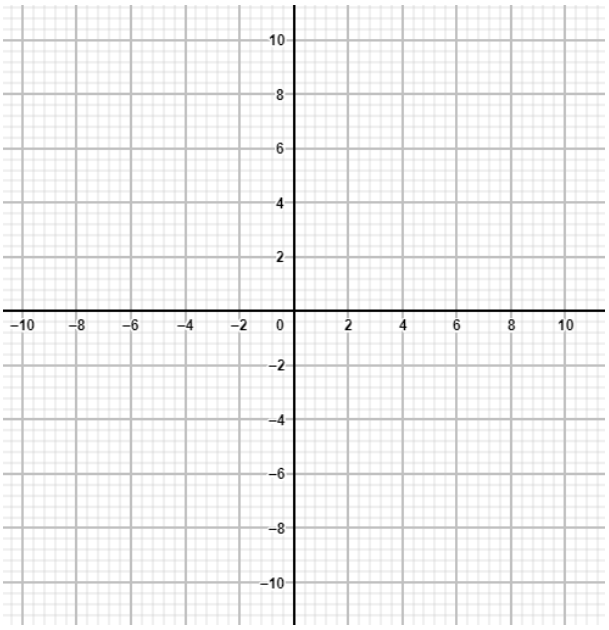
4 Find the center and radius of the circle with equation $x^2 + y^2 = 100$. Then graph the circle.



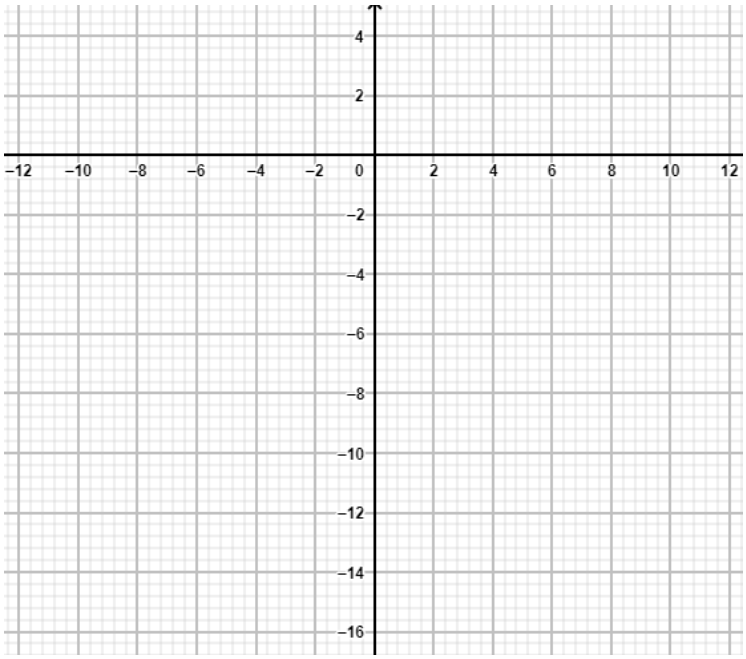
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5 Find the center and radius of the circle with equation $x^2 + y^2 = 81$. Then graph the circle.

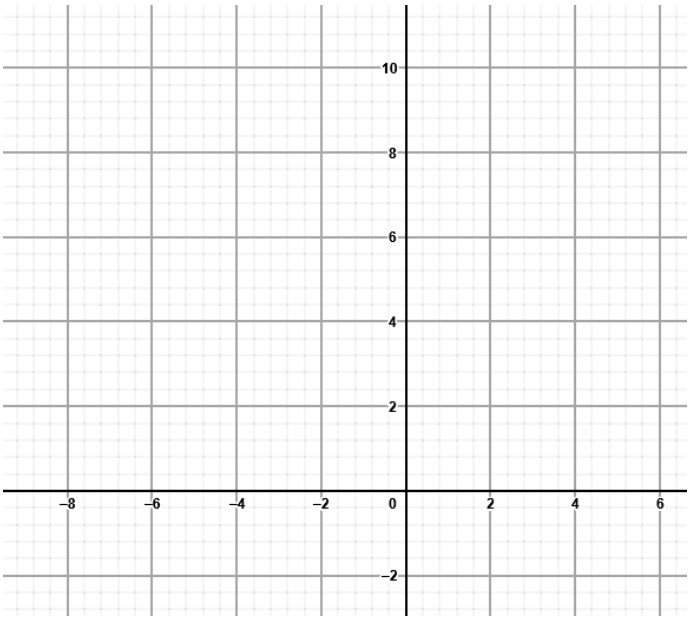


6 **Find the center and radius of the circle with equation $x^2 + y^2 - 8x + 12y - 12 = 0$. Then graph the circle.**



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7 Find the center and radius of the circle with equation $x^2 + y^2 + 4x - 10y - 7 = 0$. Then graph the circle.



8 **A sprinkler waters a circular section of lawn.**

a. Write an equation to represent the boundary of the sprinkler area if the endpoints of a diameter are at (-12, 16) and (12, -16). $x^2 + y^2 = 400$

b. What is the area of the lawn that the sprinkler waters?

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9 **Write an equation for each circle given the center and radius.**

$$center: (4, 9), r = 6$$

a) $(x-4)^2+(y-9)^2=6$

b) $(x-4)^2+(y-9)^2=36$

c) $(x+4)^2+(y+9)^2=36$

d) $(x-4)^2+(y+9)^2=36$

10 **Write an equation for each circle given the center and radius.**

$$center: (-3, 1), r = 4$$

a) $(x+3)^2+(y-1)^2=4$

b) $(x-3)^2+(y-1)^2=16$

c) $(x+3)^2+(y-1)^2=16$

d) $(x-3)^2+(y+1)^2=16$

11 **Write an equation for each circle given the center and radius.**

$$center: (-2, -1), r = 9$$

a) $(x+2)^2+(y+1)^2=18$

b) $(x+2)^2+(y+1)^2=9$

c) $(x-2)^2+(y-1)^2=81$

d) $(x+2)^2+(y+1)^2=81$

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12 **Write an equation for each circle given the center and radius.**

$$center: (1, 0), r = \sqrt{15}$$

$a)$ $(x+1)^2 + y^2 = 225$

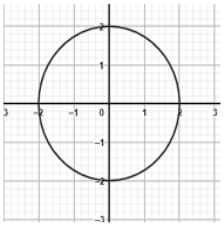
$b)$ $(x+1)^2 + y^2 = 15$

$c)$ $(x+1)^2 + y^2 = 30$

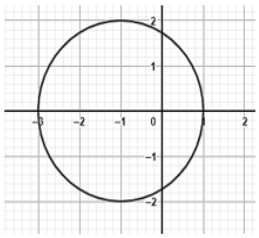
$d)$ $(x-1)^2 + y^2 = 15$

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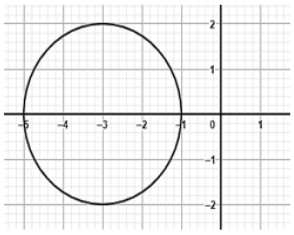
13 **Graph the circle.** $(x-3)^2 + y^2 = 4$



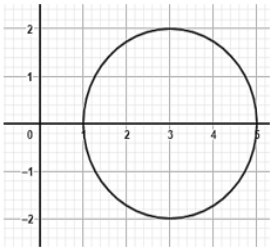
a)



b)



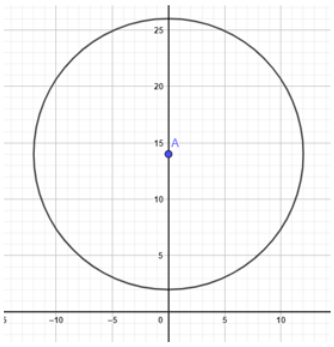
c)



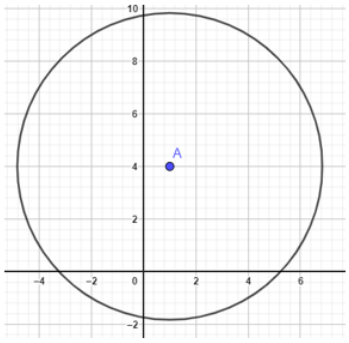
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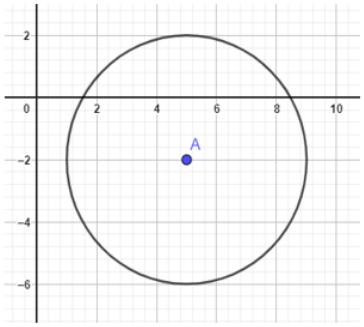
14 **Graph the circle.** $(x-1)^2 + (y-4)^2 = 34$



a)



b)

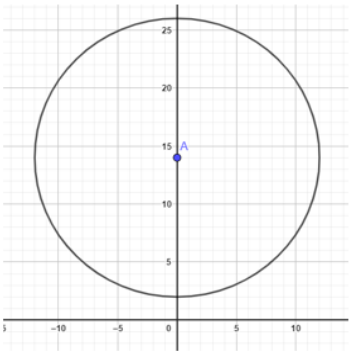


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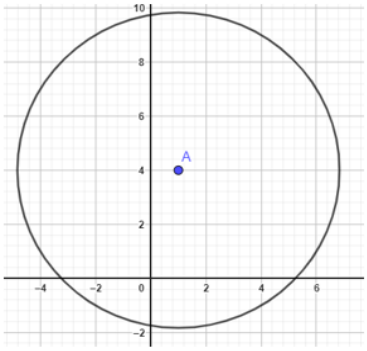
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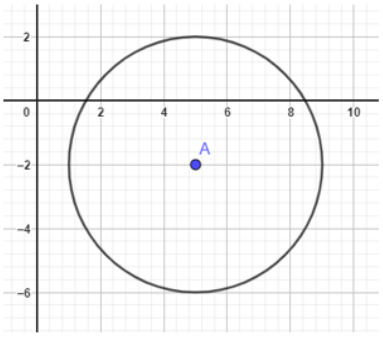
15 Graph the circle. $x^2 + (y - 14)^2 = 144$



a)

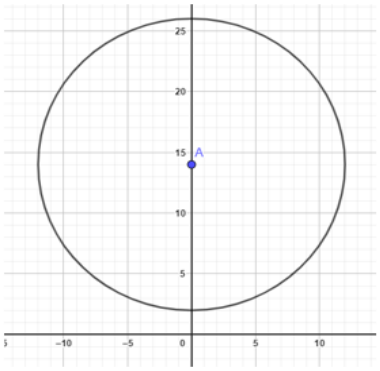


b)

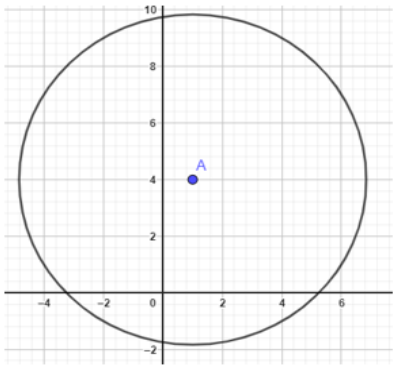


c)

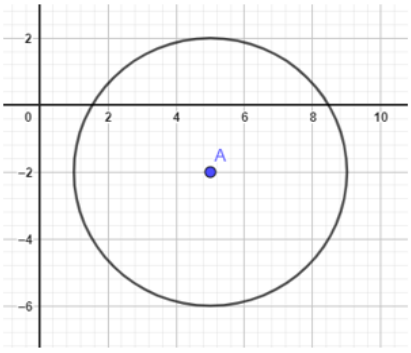
16 Graph the circle. $(x - 5)^2 + (y + 2)^2 = 16$



a)



b)



c)

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17 Write an equation for the circle that satisfies each set of conditions.

$$center\ (9, -8), passes\ through\ (19, 22)$$

$a)\ (x - 9)^2 + (y + 8)^2 = 100$

$b)\ (x - 9)^2 + (y + 8)^2 = 1000$

$c)\ (x + 9)^2 + (y + 8)^2 = 1000$

$d)\ (x - 9)^2 + (y - 8)^2 = 1000$

18 Write an equation for the circle that satisfies each set of conditions.

$$center\ (-\sqrt{15}, 30), passes\ through\ the\ origin$$

$a)\ (x+\sqrt{15})^2+(y+30)^2=915$

$b)\ (x+\sqrt{15})^2+(y-30)^2=\sqrt{915}$

$c)\ (x+\sqrt{15})^2+(y-30)^2=915^2$

$d)\ (x+\sqrt{15})^2+(y-30)^2=915$

19 Write an equation for the circle that satisfies each set of conditions.

$$center\ at\ (8,-9), tangent\ to\ y-axis$$

$a)\ (x-8)^2-(y+9)^2=64$

$b)\ (x-8)^2+(y-9)^2=64$

$c)\ (x-8)^2+(y+9)^2=64$

$d)\ (x-8)^2+(y+9)^2=8$

20 Write an equation for the circle that satisfies each set of conditions.

$$center\ at\ (2,4), tangent\ to\ x-axis$$

$a)\ (x-2)^2+(y+4)^2=16$

$b)\ (x-2)^2+(y-4)^2=16$

$c)\ (x+2)^2+(y-4)^2=16$

$d)\ (x-2)^2+(y-4)^2=4$

21 Write an equation for the circle that satisfies each set of conditions.

$center\ in\ the\ first\ quadrant;\ tangent\ to\ x = 5, the\ x-axis, and\ the\ y-axis$

$a)\ (x-2.5)^2+(y-2.5)^2=6.25$

$b)\ (x-2.5)^2+(y-2.5)^2=2.5$

$c)\ (x+2.5)^2+(y+2.5)^2=6.25$

$d)\ (x-5)^2+(y-5)^2=6.25$

22 Write an equation for the circle that satisfies each set of conditions.

$center\ in\ the\ second\ quadrant;\ tangent\ to\ y = 1, y = 5, and\ the\ y-axis$

$a)\ (x-2)^2+(y+3)^2=4$

$b)\ (x+2)^2+(y-3)^2=2$

$c)\ (x+2)^2+(y-3)^2=16$

$d)\ (x+2)^2+(y-3)^2=4$

23 A satellite is in a circular orbit 25,000 miles above Earth. Write an equation for the orbit of this satellite if the origin is at the center of Earth. Use 8000 miles as the diameter of Earth.

$a)\ x^2-y^2=(29000)^2$

$b)\ x^2+y^2=8410000$

$c)\ x^2+y^2=29000$

$d)\ x^2+y^2=(29000)^2$

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24 Suppose an unobstructed radio station broadcast could travel 120 kilometers. Assume the station is centered at the origin Write an equation to represent the boundary of the broadcast area with the origin as the center.

$a)\ x^2 + y^2 = 120$ $b)\ x^2 + y^2 = 1440$

$c)\ x^2 + y^2 = 14000$ $d)\ x^2 + y^2 = 14400$

25 Suppose an unobstructed radio station broadcast could travel 120 kilometers. Assume the station is centered at the origin.

If the transmission tower is relocated 40 kilometers east and 10 kilometers south of the current location, and an increased signal will transmit signals an additional 80 kilometers, what is an equation to represent the new broadcast area?

$a)\ (x-40)^2 + (y+10)^2 = 40000$ $b)\ (x-40)^2 + (y+10)^2 = 40000$

$c)\ (x-40)^2 + (y+10)^2 = 14400$ $d)\ (x-40)^2 + (y+10)^2 = 200$

25 Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.

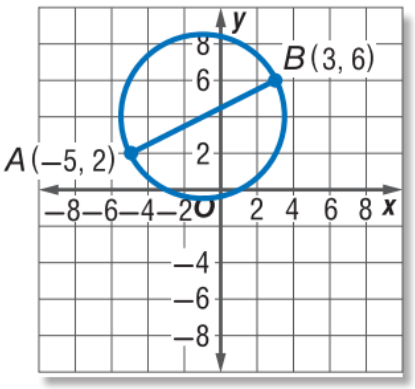
Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.

$a)\ (x-1)^2 + (y-4)^2 = 36 + 16\sqrt{5}$

$b)\ (x+1)^2 + (y-4)^2 = 36 + 16\sqrt{5}$

$c)\ (x-1)^2 + (y-4)^2 = 36 - 16\sqrt{5}$

$d)\ (x-1)^2 + (y-4)^2 = 36$



26 **Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.**

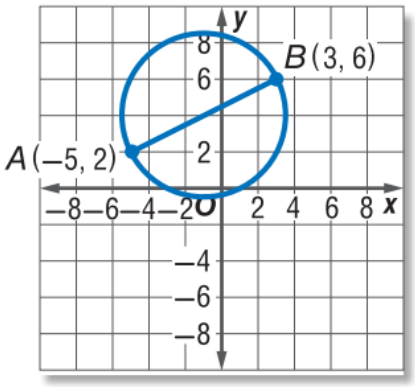
Write an equation of the circle concentric with the circle at the right, with radius 2 units less.

$a)\ (x-1)^2+(y-4)^2=24+16\sqrt{5}$

$b)\ (x+1)^2+(y-4)^2=24+8\sqrt{5}$

$c)\ (x+1)^2+(y-4)^2=24-8\sqrt{5}$

$d)\ (x-1)^2+(y-4)^2=24$



27 **A stadium is located about 35 kilometers west and 40 kilometers north of a city Suppose an earthquake occurs with its epicenter about 55 kilometers from the stadium. Assume that the origin of a coordinate plane is located at the center of the city. Write an equation for the set of points that could be the epicenter of the earthquake.**

$a)\ (x-35)^2+(y-40)^2=3025$

$b)\ (x-35)^2+(y+40)^2=3025$

$c)\ (x+35)^2+(y-40)^2=55$

$d)\ (x+35)^2+(y-40)^2=3025$

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البرنامج المميز

12 GENERAL

MATH ENG

LESSON 7-4

Ellipses

Mr. Ahmed Ata
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2024-2025

البرنامج المميز طريقك للتميز

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UAE - ABU DHABI

7-4

Ellipses

LESSON 4

Outcomes

1 Write equations of ellipses.

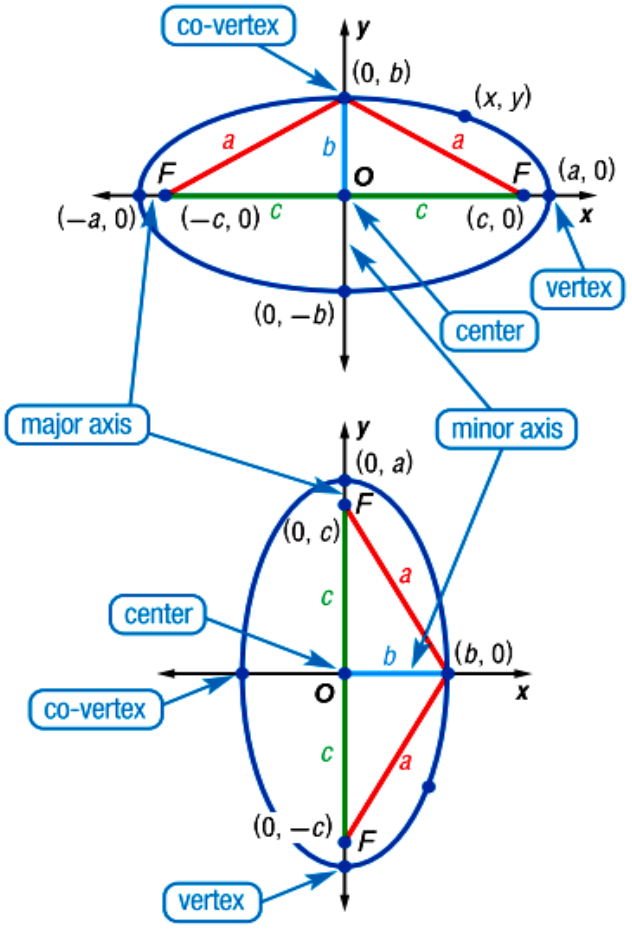
2 Graph ellipses.

1 Equations of Ellipses

An **ellipse** is the set of all points in a plane such that the sum of the distances from two fixed points is constant. These two points are called the **foci** of the ellipse.

Every ellipse has two axes of symmetry, the **major axis** and the **minor axis**. The axes are perpendicular at the **center** of the ellipse.

The foci of an ellipse always lie on the major axis. The endpoints of the major axis are the **vertices** of the ellipse and the endpoints of the minor axis are the **co-vertices** of the ellipse.

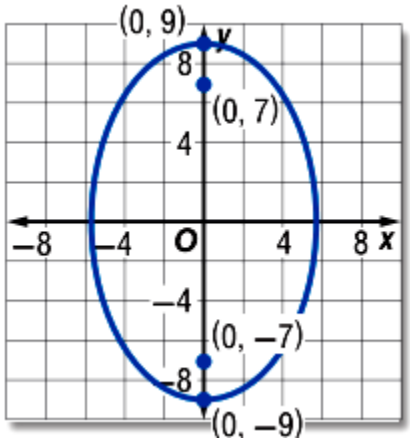


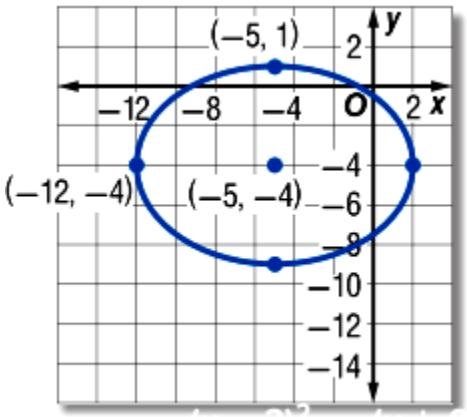
KeyConcept Equations of Ellipses Centered at (h, k)

Standard Form	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Orientation	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-vertices	$(h, k \pm b)$	$(h \pm b, k)$

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1 Write an equation for the ellipse.





2 Write an equation for an ellipse with vertices at (-4, 0) and (4, 0) and foci at (2, 0) and (-2, 0).

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3 Write an equation for the ellipse with vertices at (6, -8) and (6, 4) and co-vertices at (3, -2) and (9, -2).

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4 Write an equation for the ellipse with vertices at (-3, 8) and (9, 8) and co-vertices at (3, 12) and (3, 4).

Write an equation for an ellipse that satisfies each set of conditions.

5 center at (-2, 6), vertex at (-2, 16), co-vertex at (1, 6)

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6 The opening of a tunnel in the mountains can be modeled by semiellipses, or halves of ellipses. If the opening is 14.6 meters wide and 8.6 meters high, determine an equation to represent the opening with the center at the origin.

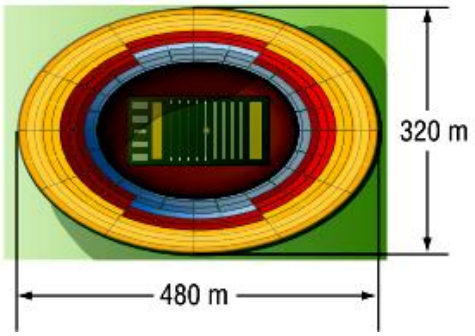


7 An architectural firm sent a proposal to a city for building a coliseum, shown at the right.

a. Determine the values of a and b. $a = 240$, $b = 160$

b. Assuming that the center is at the origin, write an equation to represent the ellipse.

c. Determine the coordinates of the foci.

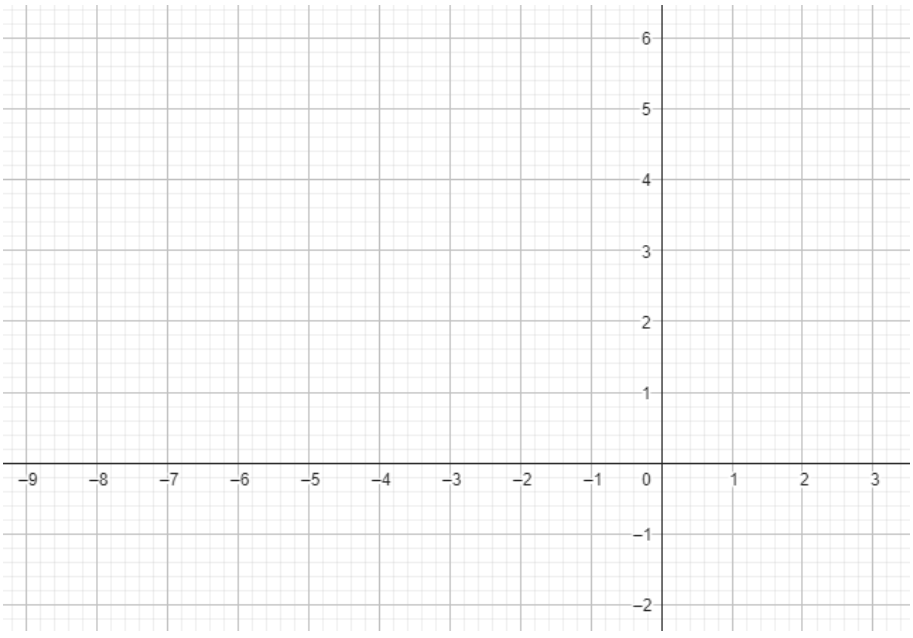


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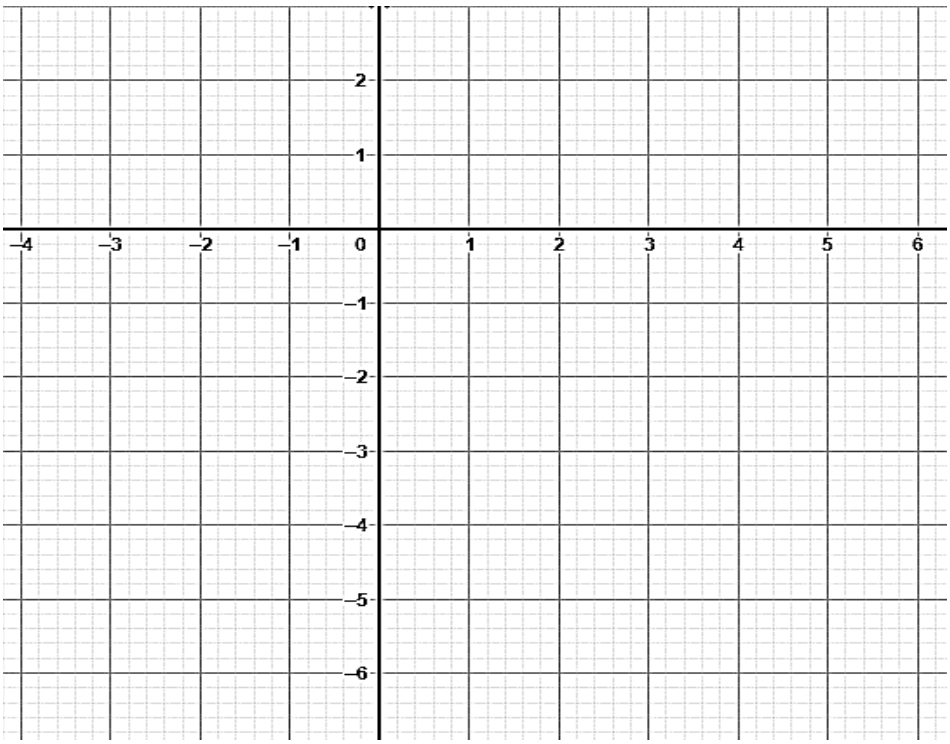
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The Featured program 0566010255
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8 Find the coordinates of the center and foci, and the lengths of the major and minor axes of an ellipse with equation $9x^2 + 25y^2 + 72x - 150y + 144 = 0$. Then graph the ellipse.



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9 Find the coordinates of the center and foci, and the lengths of the major and minor axes of an ellipse with equation $x^2 + 4y^2 - 2x - 24y + 21 = 0$. Then graph the ellipse.



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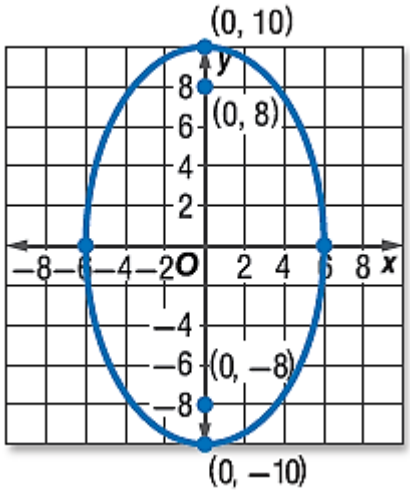
10 Write an equation for each ellipse.

$a)\ \frac{y^2}{36}+\frac{x^2}{100}=1$

$b)\ \frac{y^2}{100}+\frac{x^2}{36}=1$

$c)\ \frac{y^2}{36}+\frac{x^2}{81}=1$

$d)\ \frac{y^2}{81}+\frac{x^2}{100}=1$



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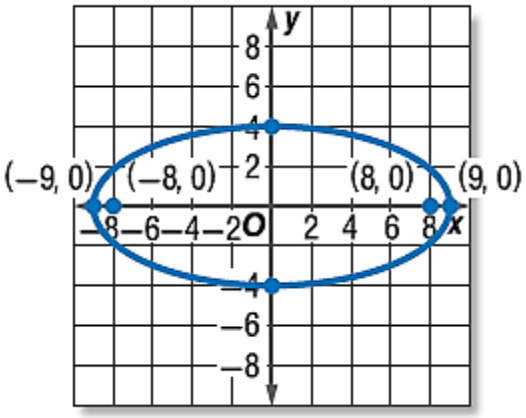
11 Write an equation for each ellipse.

$a)\ \frac{y^2}{36}+\frac{x^2}{17}=1$

$b)\ \frac{y^2}{17}+\frac{x^2}{36}=1$

$c)\ \frac{x^2}{16}+\frac{y^2}{81}=1$

$d)\ \frac{x^2}{81}+\frac{y^2}{17}=1$



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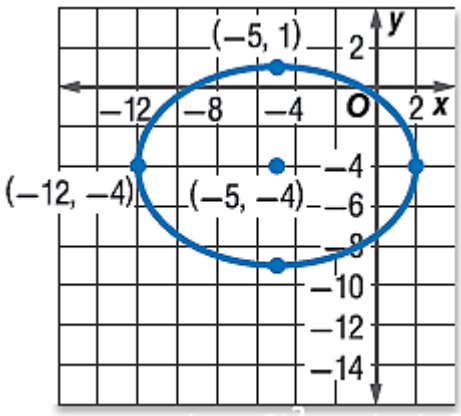
12 Write an equation for each ellipse.

$a)\ \frac{(x+5)^2}{49}+\frac{(y+4)^2}{25}=1$

$b)\ \frac{(x-5)^2}{49}+\frac{(y-4)^2}{25}=1$

$c)\ \frac{(x+5)^2}{25}+\frac{(y+4)^2}{49}=1$

$d)\ \frac{(x-5)^2}{25}+\frac{(y-4)^2}{49}=1$



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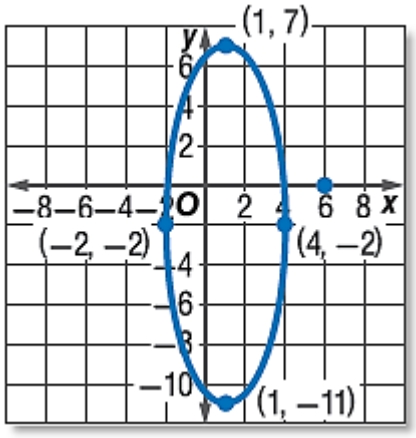
13 Write an equation for each ellipse.

$a)\ \dfrac{(y+2)^2}{36}+\dfrac{(x-1)^2}{9}=1$

$b)\ \dfrac{(y-2)^2}{81}+\dfrac{(x-1)^2}{9}=1$

$c)\ \dfrac{(y+2)^2}{81}+\dfrac{(x-1)^2}{9}=1$

$d)\ \dfrac{(y+2)^2}{9}+\dfrac{(x-1)^2}{81}=1$



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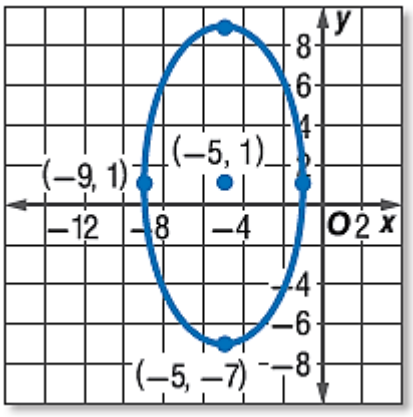
14 Write an equation for each ellipse.

$a)\ \dfrac{(y+1)^2}{64}+\dfrac{(x-5)^2}{16}=1$

$b)\ \dfrac{(y-1)^2}{16}+\dfrac{(x+5)^2}{64}=1$

$c)\ \dfrac{(y-1)^2}{64}+\dfrac{(x+5)^2}{16}=1$

$d)\ \dfrac{(y+1)^2}{64}+\dfrac{(x+5)^2}{16}=1$



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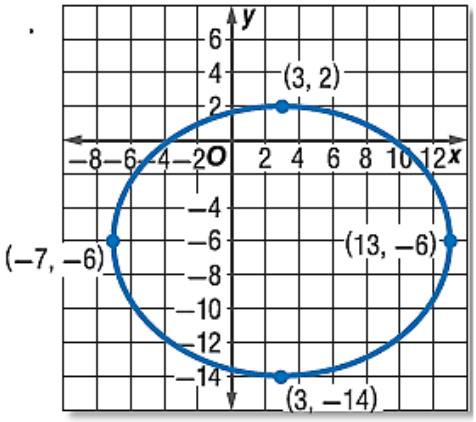
15 Write an equation for each ellipse.

$a)\ \dfrac{(x-3)^2}{100}+\dfrac{(y+6)^2}{64}=1$

$b)\ \dfrac{(x+3)^2}{100}+\dfrac{(y+6)^2}{64}=1$

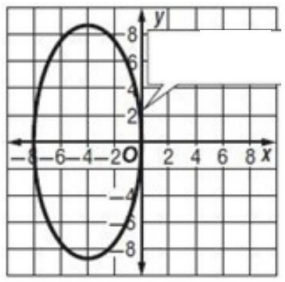
$c)\ \dfrac{(x-3)^2}{100}+\dfrac{(y-6)^2}{64}=1$

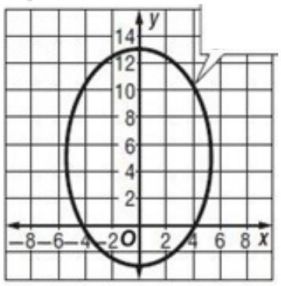
$d)\ \dfrac{(x-3)^2}{64}+\dfrac{(y+6)^2}{100}=1$

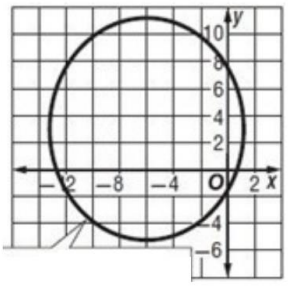


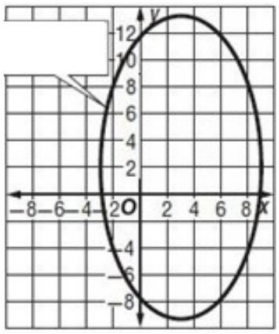
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16 Graph the ellipse. $\frac{(x-3)^2}{36}+\frac{(y-2)^2}{128}=1$

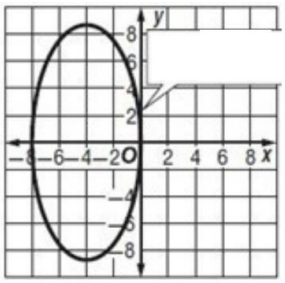

a)

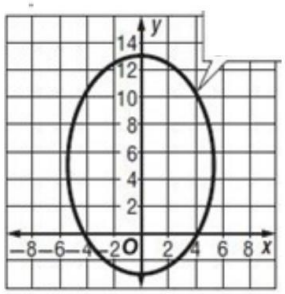

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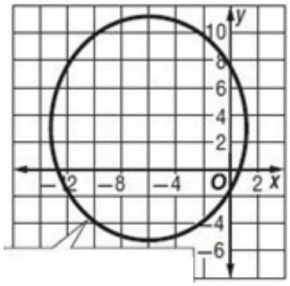

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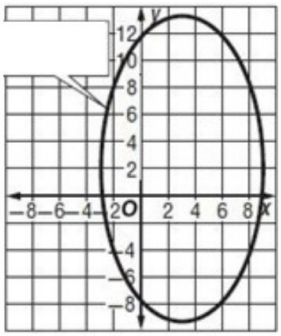

d)

17 Graph the ellipse. $\frac{(x+6)^2}{50}+\frac{(y-3)^2}{72}=1$

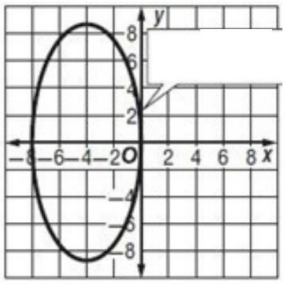

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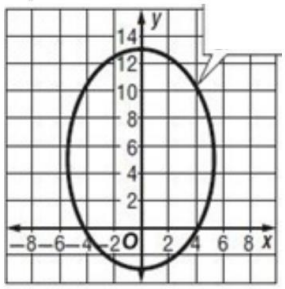

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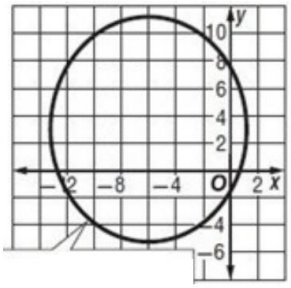

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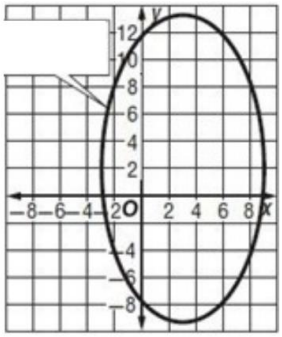

d)

18 Graph the ellipse. $\frac{x^2}{27}+\frac{(y-5)^2}{64}=1$


a)


b)

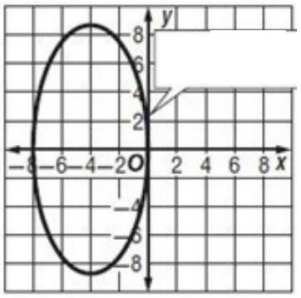

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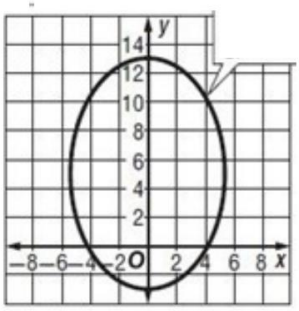

d)

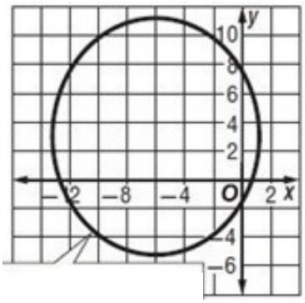
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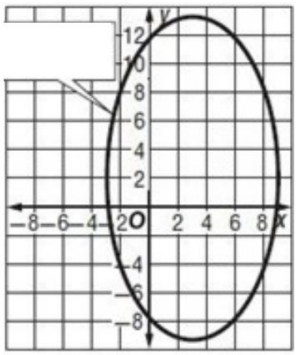
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19 Graph the ellipse. $\frac{(x+4)^2}{36}+\frac{y^2}{75}=1$

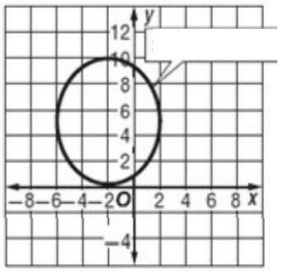

a)

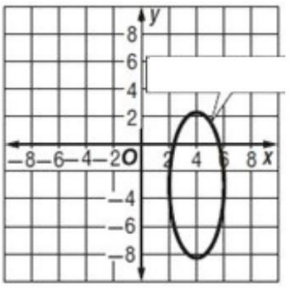

b)

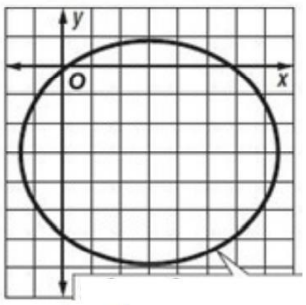

c)

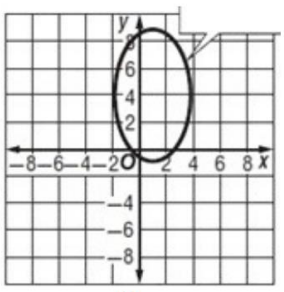

d)

20 Graph the ellipse. $3x^2+y^2-6x-8y-5=0$

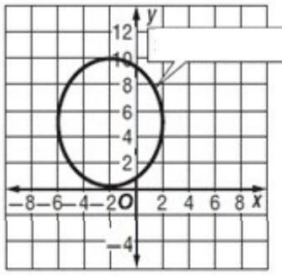

a)

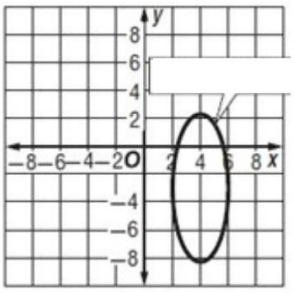

b)

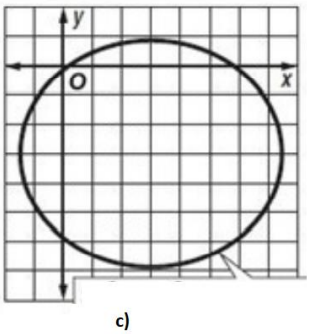

c)

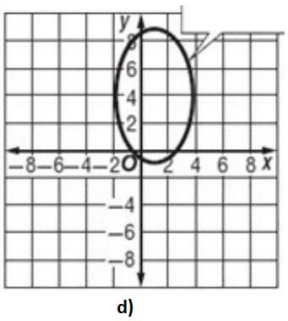

d)

21 Graph the ellipse. $3x^2+4y^2-18x+24y+3=0$


a)


b)


c)


d)

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Write an equation for an ellipse that satisfies each set of conditions.

vertices at (-6, 4) and (12, 4), co-vertices at (3, 12) and (3, -4)

$a)\ \frac{(x+3)^2}{81}+\frac{(y+4)^2}{64}=1$

$b)\ \frac{(x-3)^2}{81}+\frac{(y-4)^2}{25}=1$

$c)\ \frac{(x-3)^2}{81}+\frac{(y-4)^2}{64}=1$

$d)\ \frac{(y-6)^2}{100}+\frac{(x+2)^2}{9}=1$

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Write an equation for an ellipse that satisfies each set of conditions.

vertices at (-1, 11) and (-1, 1), co-vertices at (-4, 6) and (2, 6)

$a)\ \frac{(y-6)^2}{9}+\frac{(x+1)^2}{25}=1$

$b)\ \frac{(y-6)^2}{25}+\frac{(x-1)^2}{9}=1$

$c)\ \frac{(y+6)^2}{25}+\frac{(x+1)^2}{9}=1$

$d)\ \frac{(y-6)^2}{25}+\frac{(x+1)^2}{9}=1$

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Write an equation for an ellipse that satisfies each set of conditions.

center at (-2, 6), vertex at (-2, 16), co-vertex at (1, 6)

$a)\ \frac{(x-3)^2}{100}+\frac{(y+6)^2}{9}=1$

$b)\ \frac{(x+3)^2}{100}+\frac{(y+6)^2}{9}=1$

$c)\ \frac{(x+2)^2}{100}+\frac{(y-6)^2}{9}=1$

$d)\ \frac{(y-6)^2}{100}+\frac{(x+2)^2}{9}=1$

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Write an equation for an ellipse that satisfies each set of conditions.

center at (3, -4), vertex at (8, -4), co-vertex at (3, -2)

$a)\ \frac{(x-3)^2}{25}+\frac{(y+4)^2}{4}=1$

$b)\ \frac{(x+3)^2}{25}+\frac{(y+6)^2}{4}=1$

$c)\ \frac{(x-3)^2}{25}+\frac{(y+4)^2}{16}=1$

$d)\ \frac{(y-6)^2}{100}+\frac{(x+2)^2}{9}=1$

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Write an equation for an ellipse that satisfies each set of conditions.

vertices at (4, 12) and (4, -4), co-vertices at (1, 4) and (7, 4)

$a)\ \frac{(y+4)^2}{64}+\frac{(x+4)^2}{9}=1$

$b)\ \frac{(y-4)^2}{64}+\frac{(x-4)^2}{9}=1$

$c)\ \frac{(y-4)^2}{9}+\frac{(x+4)^2}{64}=1$

$d)\ \frac{(y-4)^2}{64}+\frac{(x+4)^2}{9}=1$

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Write an equation for an ellipse that satisfies each set of conditions.

vertices at (-11, 2) and (-1, 2), co-vertices at (-6, 0) and (-6, 4)

$a)\ \frac{(x+6)^2}{4}+\frac{(y-2)^2}{25}=1$

$b)\ \frac{(x+3)^2}{100}+\frac{(y+6)^2}{9}=1$

$c)\ \frac{(x+6)^2}{25}+\frac{(y-2)^2}{4}=1$

$d)\ \frac{(y-6)^2}{100}+\frac{(x+2)^2}{9}$

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12 GENERAL

MATH ENG

LESSON 7-5

Hyperbolas

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2024-2025

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7-5

Hyperbolas

LESSON 5

Outcomes

1 Write equations of hyperbolas.

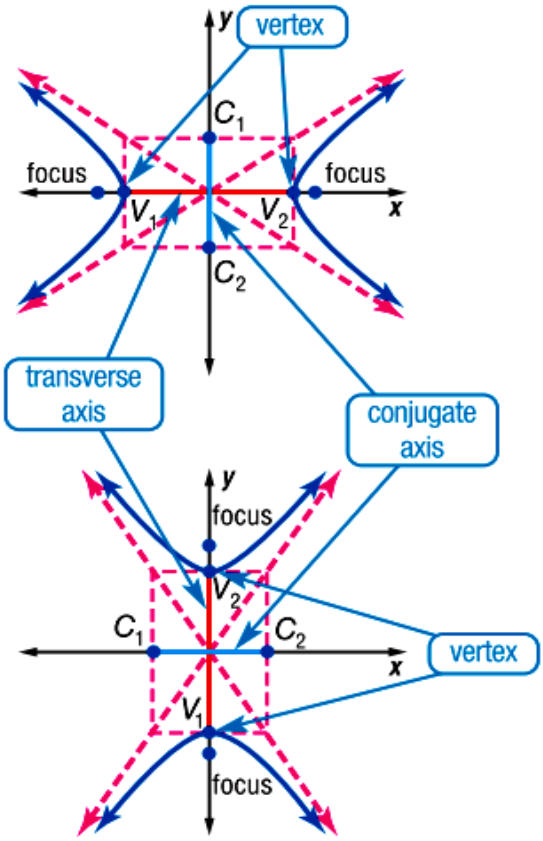
2 Graph hyperbolas.

1 Equations of Hyperbolas

Similar to an ellipse, a **hyperbola** is the set of all points in a plane such that the absolute value of the differences of the distances from the foci is constant.

Every hyperbola has two axes of symmetry, the **transverse axis** and the **conjugate axis**. The axes are perpendicular at the center of the hyperbola.

The **foci** of a hyperbola always lie on the transverse axis. The **vertices** are the endpoints of the transverse axis. The **co-vertices** are the endpoints of the conjugate axis.



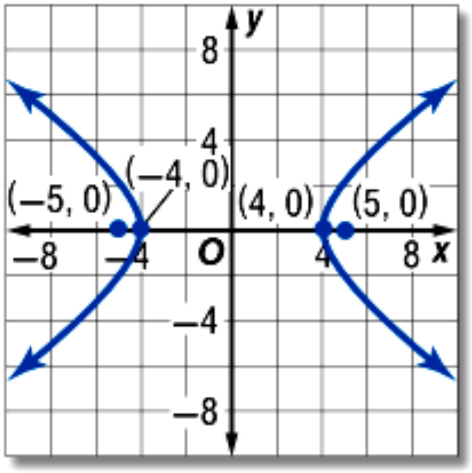
KeyConcept Equations of Hyperbolas Centered at (h, k)

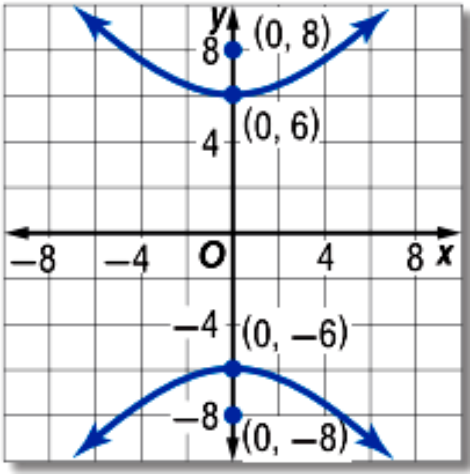
Standard Form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-vertices	$(h, k \pm b)$	$(h \pm b, k)$
Equations of Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$

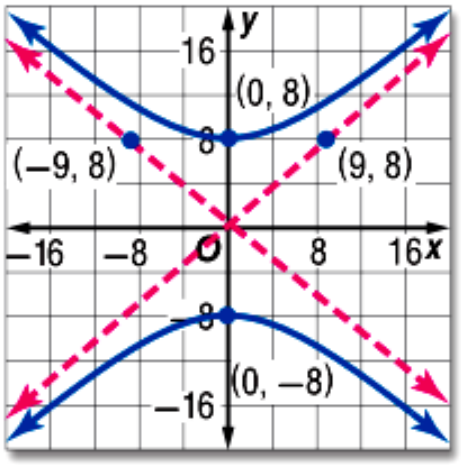
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1 Write an equation for the hyperbola shown in the graph.







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2 Write an equation for a hyperbola with vertices at (6, o) and (-6, o) and foci at (8, o) and (-8, o).

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3 The asymptotes for a vertical hyperbola are $y = \frac{5}{3}x$ and $y = -\frac{5}{3}x$ and the vertices are at (0, 5) and (0, −5). Write the equation for the hyperbola.

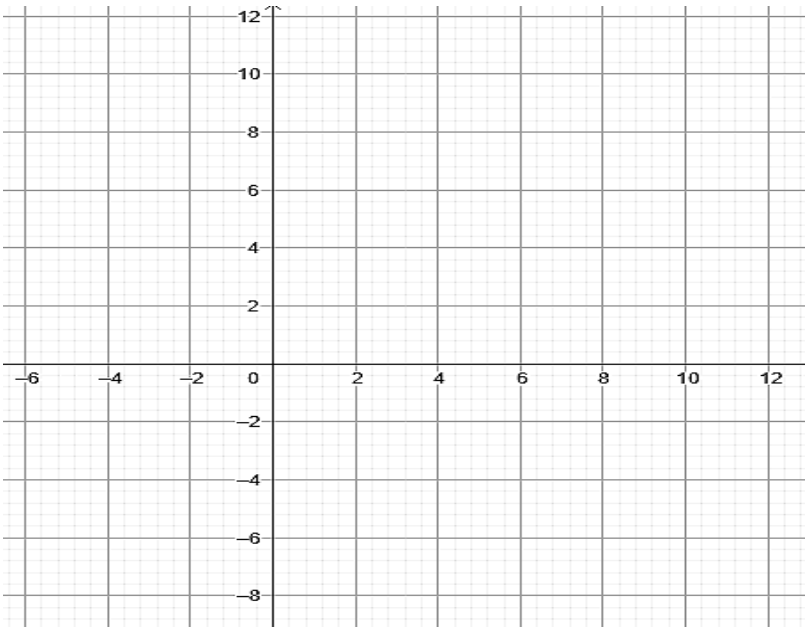
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4 The asymptotes for a horizontal hyperbola are $y = \frac{7}{9}x$ and $y = -\frac{7}{9}x$. The vertices are (9, 0) and (−9, 0). Write an equation for the hyperbola.

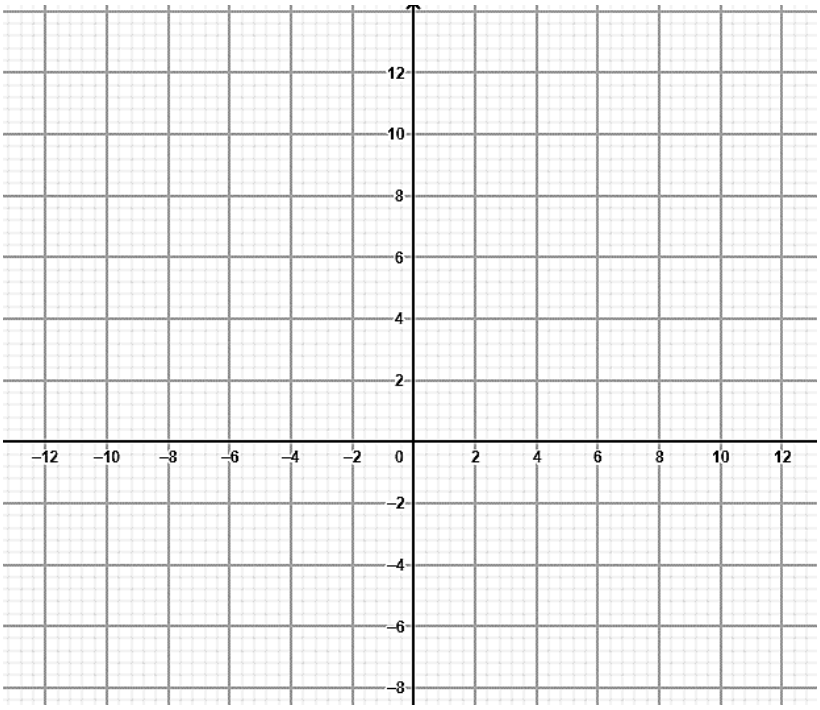
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5 Graph $\frac{(x-3)^2}{4} - \frac{(y+2)^2}{16} = 1$. Identify the vertices, foci, and asymptotes.



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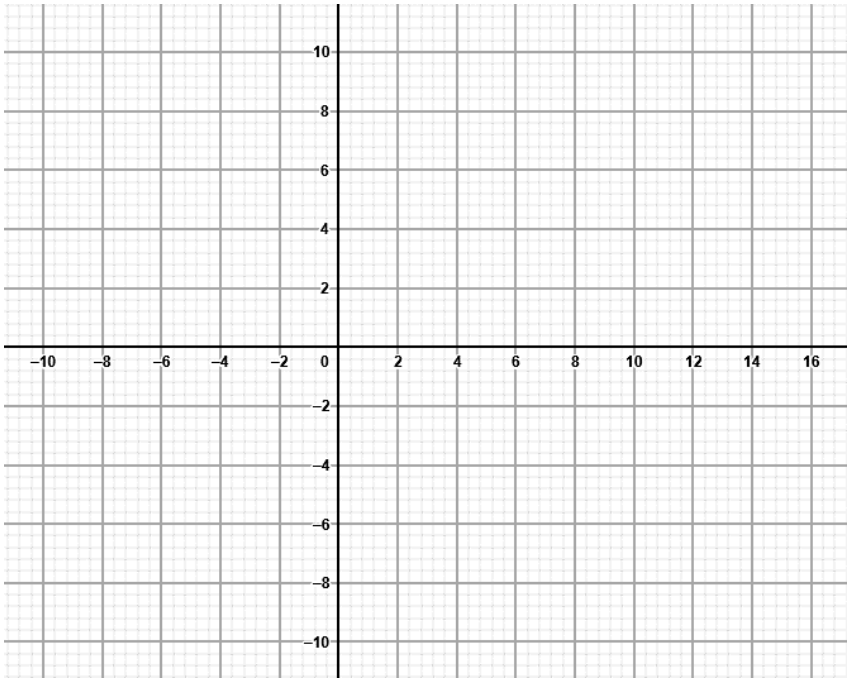
5 Graph $\frac{(y-4)^2}{9} - \frac{(x+3)^2}{25} = 1$. Identify the vertices, foci, and asymptotes.



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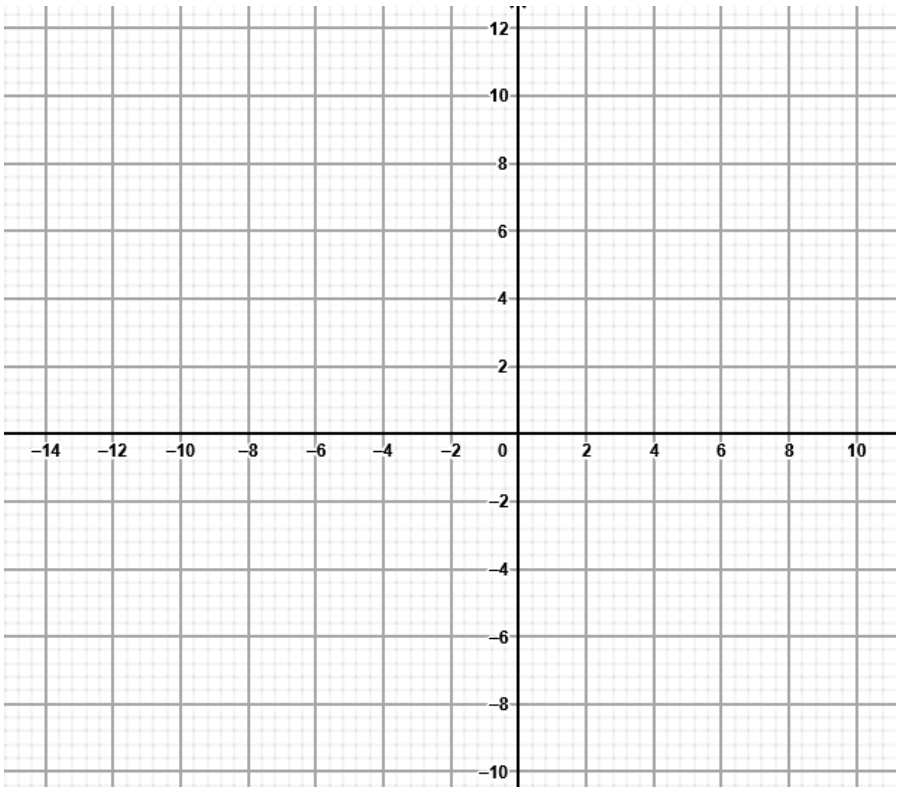
Graph each hyperbola. Identify the vertices, foci, and asymptotes.

6 $9y^2 - 4x^2 - 54y + 32x - 19 = 0$



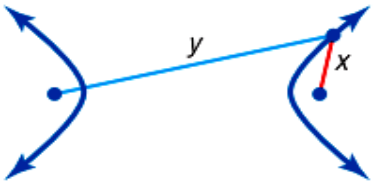
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7 $16x^2 - 9y^2 + 128x + 36y + 76 = 0$

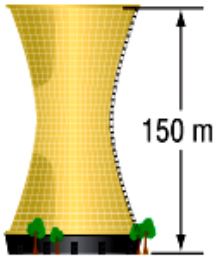


In the equation for any hyperbola, the value of $2a$ represents the **constant difference**. This is the absolute value of the difference between the distances from any point on the hyperbola to the foci of the hyperbola.

Any point on the hyperbola at the right will have the same constant difference, $|y - x|$ or $2a$.



8 **COOLING** Natural draft cooling towers are shaped like hyperbolas for more efficient cooling of power plants. The hyperbola in the tower at the right can be modeled by $\frac{x^2}{16} - \frac{y^2}{225} = 1$, where the units are in meters. Find the width of the tower at the top and at its narrowest point in the middle.



9 **Write an equation for the hyperbola that satisfies each set of conditions.**

vertices (-8, 0) and (8, 0), conjugate axis of length 20 units

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10 centered at the origin with a vertical transverse axis of length 16 units and a conjugate axis of length 12 units

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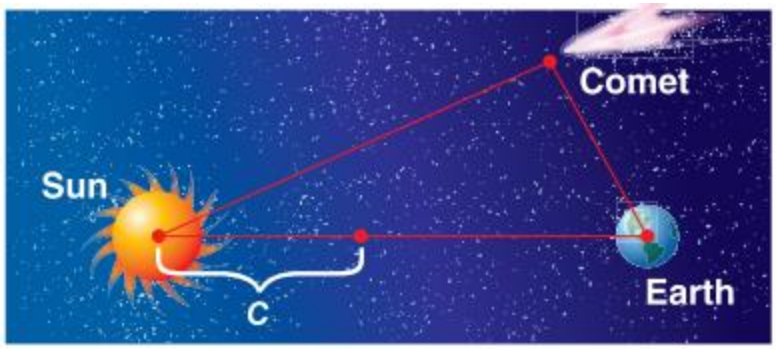
11 vertices (6, -2) and (-2, -2), foci (10, -2) and (-6, -2)

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12 *centered at the origin with a horizontal transverse axis of length 10 units and a conjugate axis of length 4 units*

13 **Earth and the Sun are 146 million kilometers apart. A comet follows a path that is one branch of a hyperbola. Suppose the comet is 30 million kilometers farther from the Sun than from Earth. Determine the equation of the hyperbola centered at the origin for the path of the comet.**



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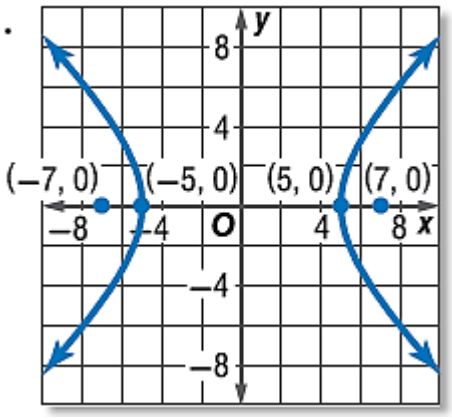
14 Write an equation for the hyperbola shown in the graph.

$a)\ \frac{y^2}{24} - \frac{x^2}{25} = 1$

$b)\ \frac{x^2}{25} - \frac{y^2}{24} = 1$

$c)\ \frac{x^2}{25} + \frac{y^2}{24} = 1$

$d)\ \frac{x^2}{24} + \frac{y^2}{25} = 1$



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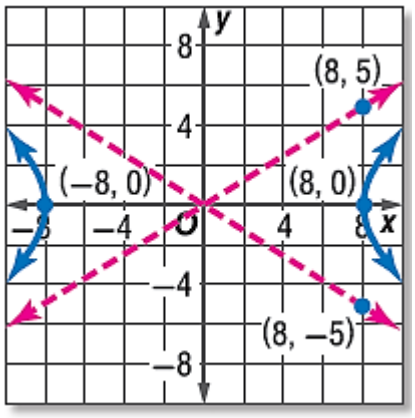
15 Write an equation for the hyperbola shown in the graph.

$a)\ \frac{y^2}{25} - \frac{x^2}{64} = 1$

$b)\ \frac{y^2}{64} - \frac{x^2}{25} = 1$

$c)\ \frac{x^2}{64} - \frac{y^2}{25} = 1$

$d)\ \frac{x^2}{64} + \frac{y^2}{25} = 1$



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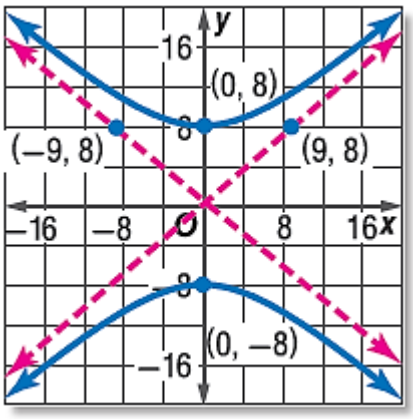
16 Write an equation for the hyperbola shown in the graph.

$a)\ \frac{y^2}{81} - \frac{x^2}{64} = 1$

$b)\ \frac{y^2}{64} - \frac{x^2}{81} = 1$

$c)\ \frac{x^2}{81} - \frac{y^2}{64} = 1$

$d)\ \frac{x^2}{64} - \frac{y^2}{81} = 1$



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12 GENERAL

MATH ENG

LESSON 7-6

Identifying Conic Sections

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2024-2025

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7-6

Identifying Conic Sections

LESSON 6

Outcomes

1 Write equations of conic sections in standard form.

2 Identify conic sections from their equations.

ConceptSummary Standard Forms of Conic Sections

Conic Section	Standard Form of Equation	
Circle	$(x-h)^2+(y-k)=r^2$	
	Horizontal Axis	Vertical Axis
Parabola	$y=a(x-h)^2+k$	$x=a(y-k)^2+h$
Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Hyperbola	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

1 Write $16x^2-25y^2-128x-144=0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

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2 Identify Conic Sections You can determine the type of conic without having to write $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ in standard form. When there is an xy-term ($B \neq 0$), you can use the discriminant to identify the conic. $B^2 - 4AC$ is the discriminant of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

ConceptSummary Classify Conics with the Discriminant

Discriminant	Conic Section
$B^2 - 4AC < 0$; $B = 0$ and $A = C$	circle
$B^2 - 4AC < 0$; either $B \neq 0$ or $A \neq C$	ellipse
$B^2 - 4AC = 0$	parabola
$B^2 - 4AC > 0$	hyperbola

Without writing in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

2 $y^2 + 4x^2 - 3xy + 4x - 5y - 8 = 0$

3 $3x^2 - 6x + 4y - 5y^2 + 2xy - 4 = 0$

4 $4y^2 - 8x + 6y - 14 = 0$

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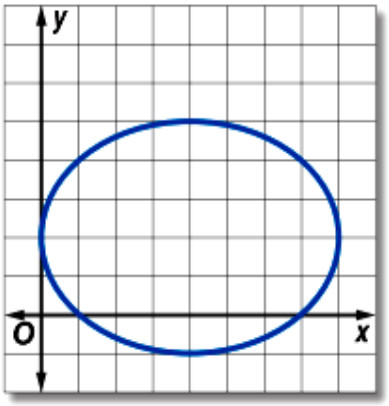
5 $8y^2 - 6x^2 + 4xy - 6x + 2y - 4 = 0$

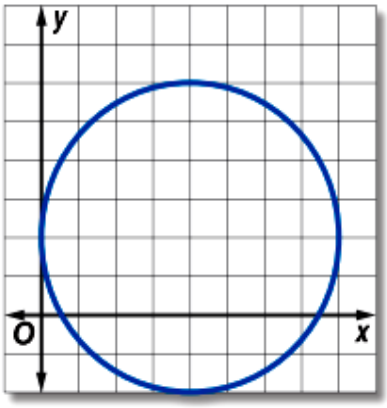
6 $3xy + 4x^2 - 2y + 9x - 3 = 0$

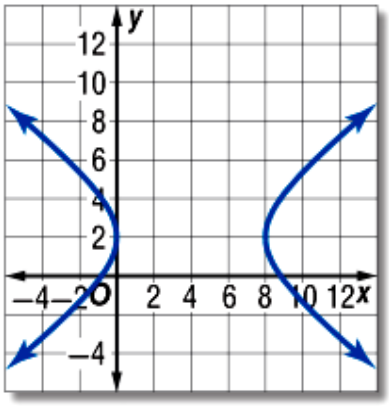
7 $3x^2 + 16x - 12y + 2y^2 - 6 = 0$

8 Match each graph with its corresponding equation.

a. $x^2 + y^2 - 8x - 4y = -4$ b. $9x^2 - 16y^2 - 72x + 64y = 64$ c. $9x^2 + 16y^2 = 72x + 64y - 64$







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9 The graph of equation represents.

$$4x^2 + 6y^2 - 3x - 2y = 12$$

$a)$ *parabola*

$b)$ *circle*

$c)$ *ellipse*

$d)$ *hyperbola*

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10 The graph of equation represents.

$$5y^2 = 2x + 6y - 8 + 3x^2$$

$a)$ *parabola*

$b)$ *circle*

$c)$ *ellipse*

$d)$ *hyperbola*

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11 The graph of equation represents.

$$8x^2 + 8y^2 + 16x + 24 = 0$$

$a)$ *parabola*

$b)$ *circle*

$c)$ *ellipse*

$d)$ *hyperbola*

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12 **The graph of equation represents.**

$$4x^2 - 3y^2 + 8xy - 12 = 2x + 4y$$

$a)$ $parabola$

$b)$ $circle$

$c)$ $ellipse$

$d)$ $hyperbola$

13 **The graph of equation represents.**

$$8x^2 + 12xy + 16y^2 + 4y - 3x = 12$$

$a)$ $parabola$

$b)$ $circle$

$c)$ $ellipse$

$d)$ $hyperbola$

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12 GENERAL

MATH ENG

LESSON 7-7

Solving Linear-Nonlinear Systems

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2024-2025

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Solving Linear-Nonlinear Systems

LESSON 7

Outcomes

1 Solve systems of linear and nonlinear equations algebraically and graphically.

2 Solve systems of linear and nonlinear inequalities graphically.

1 Systems of Equations When a system of equations consists of a linear and a nonlinear equation, the system may have zero, one, or two solutions. Some of the possible solutions are shown below.



You can solve linear-quadratic systems by using graphical or algebraic methods.

Solve the system of equations.

1 $x^2 + 3x = -4y - 2$
$y = -2x + 1$

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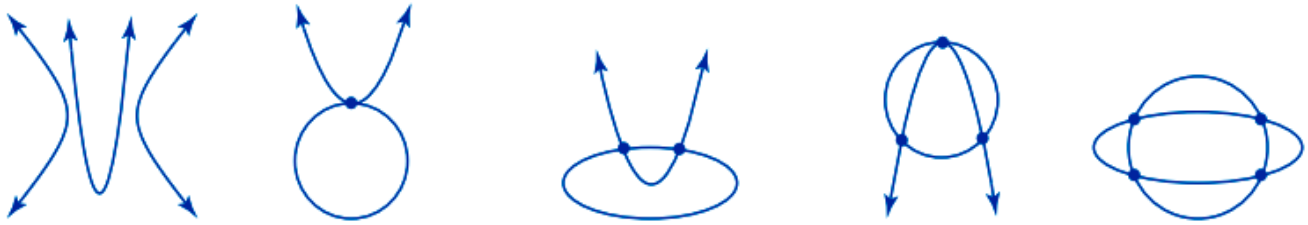
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2 $y = 2x$
$4x^2 - 2y^2 = -36$

3 $y = 12x - 30$
$4x^2 - 3y = 18$

4 $6y^2 - 27 = 3x$
$6y - x = 13$

If a quadratic system contains two conic sections, the system may have anywhere from zero to four solutions. Some graphical representations are shown below.



You can use elimination to solve quadratic-quadratic systems.

Solve the system of equations.

5 $x^2 + y^2 = 45$ (1)
$y^2 - x^2 = 27$ (2)

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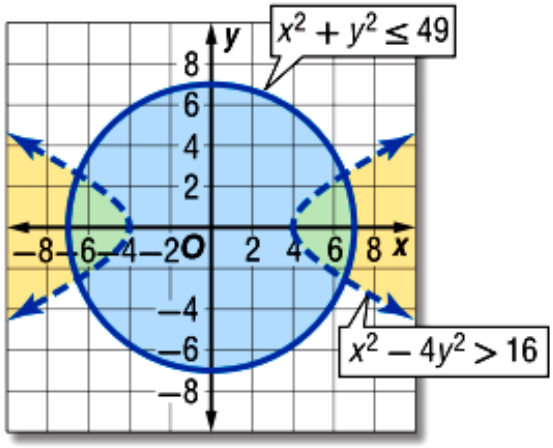
6 $x^2 + y^2 = 16$
$x^2 - y^2 = 20$

7 $2y^2 + 5x^2 = 26$
$2x^2 - y^2 = 5$

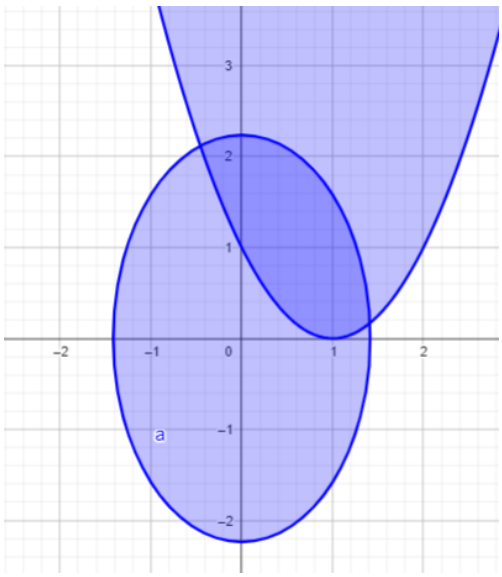
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Solve the system of inequalities by graphing.

8 $x^2 + y^2 \leq 49$
$x^2 - 4y^2 > 16$



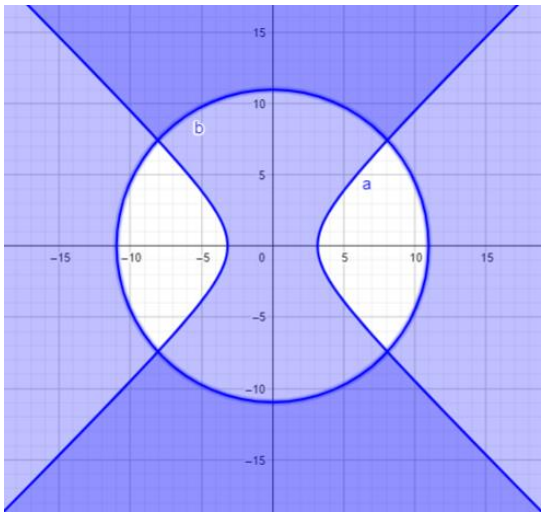
9 $5x^2 + 2y^2 \leq 10$
$y \geq x^2 - 2x + 1$



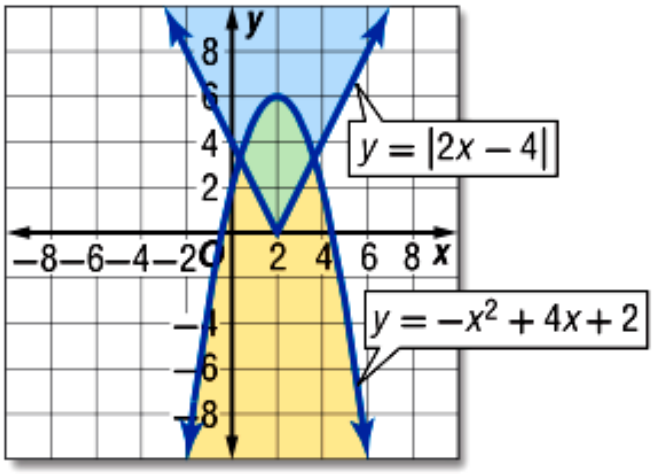
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Solve the system of inequalities by graphing.

10 $x^2 - y^2 \leq 8$
$x^2 + y^2 \geq 120$



11 $y \geq |2x - 4|$
$y \leq -x^2 + 4x + 2$



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12 Solve each system of equations

$$8y = -10x$$
$$y^2 = 2x^2 - 7$$

$a)$ $(4, -5)$; $(-4, 5)$

$b)$ $(-4, -5)$; $(-4, 5)$

$c)$ $(2, -5)$; $(-2, 5)$

$d)$ $(4, -6)$; $(-4, 6)$

13 Solve each system of equations

$$x^2 + y^2 = 68$$
$$5y = -3x + 34$$

$a)$ $(-2, -5)$; $(8, 5)$

$b)$ $(8, -2)$; $(2, 5)$

$c)$ $(2, -8)$; $(-2, 8)$

$d)$ $(-2, 8)$; $(8, 2)$

14 Solve each system of equations

$$4x^2 - 3y = 18$$
$$y = 12x - 30$$

$a)$ $(3, 6)$; $(6, 42)$

$b)$ $(3, 6)$; $(-6, 42)$

$c)$ $(3, -6)$; $(6, 42)$

$d)$ $(3, 8)$; $(6, 2)$

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15 Solve each system of equations

$$6y^2 - 27 = 3x$$

$$6y - x = 13$$

$a)$ $(-7, 1)$; $(-1, -2)$

$b)$ $(7, 1)$; $(1, 2))$

$c)$ $(-7, 1)$; $(-1, 2)$

$d)$ $(-7, -1)$; $(-1, -2)$

16 Solve each system of equations

$$x^2 + y^2 = 16$$

$$x^2 - y^2 = 20$$

$a)$ $(4, 1)$; $(-4, -2)$

$b)$ $(4, 1)$; $(4, 2))$

$c)$ $(-4, 1)$; $(7, 2)$

$d)$ $no\ solution$

17 Solve each system of equations

$$y^2 - 2x^2 = 8$$

$$3y^2 + x^2 = 52$$

$a)$ $(2, 4)$; $(2, -4)$; $(-2, 4)$; $(-2, -4)$;

$b)$ $(2, 4)$; $(-2, 4)$

$c)$ $(-2, -4)$; $(2, -4)$

$d)$ $(2, 4)$; $(-2, 4)$; $(-2, -4)$

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12 GENERAL

MATH ENG

LESSON 7-8

Parametric Equations



البرنامج المميز طريقك للتميز

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7-8

Parametric Equations

LESSON 8

Outcomes

1. Graph parametric equations.
2. Solve problems related to the motion of projectiles.

KeyConcept Parametric Equations

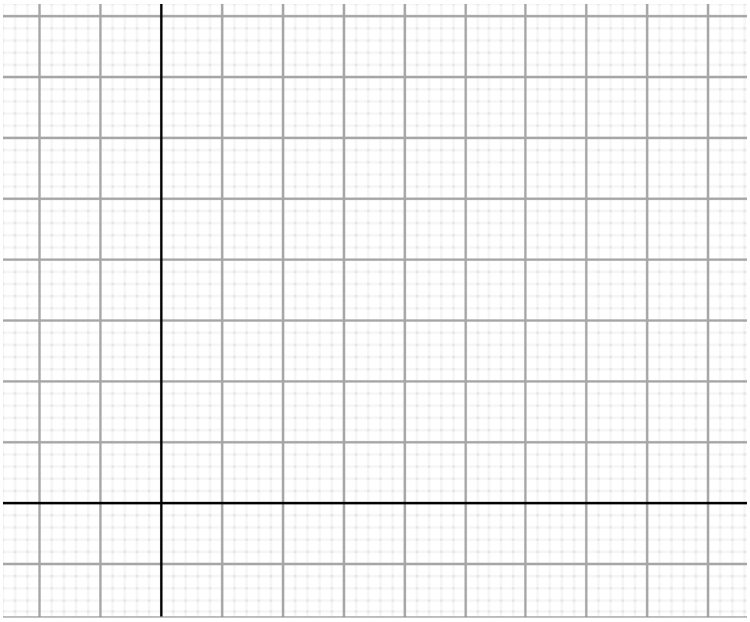
If f and g are continuous functions of t on the interval I, then the set of ordered pairs $(f(t), g(t))$ represent a **parametric curve**. The equations

$$x = f(t) \text{ and } y = g(t)$$

are parametric equations for this curve, t is the parameter, and I is the parameter interval.

Sketch the curve given by each pair of parametric equations over the given interval.

1 $x = t^2 + 5$ and $y = \frac{t}{2} + 4; -4 \le t \le 4$

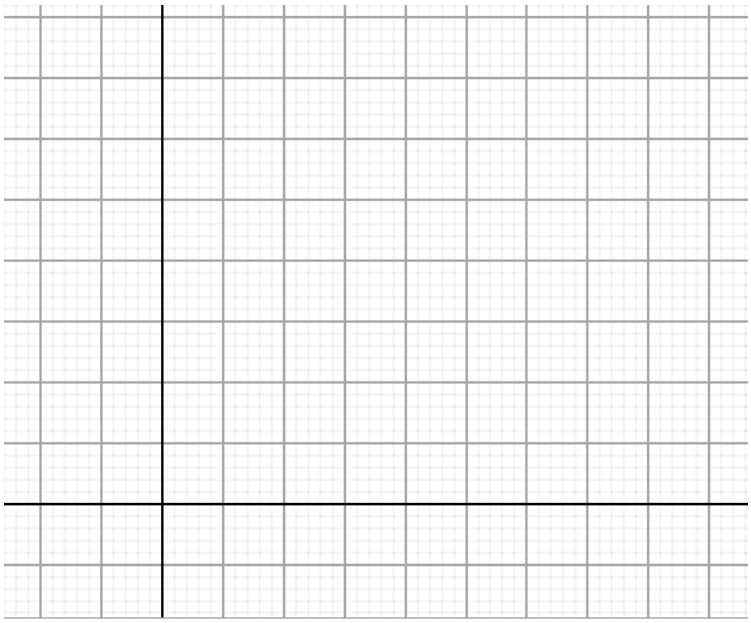


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2 $x = \frac{t^2}{4} + 5$ and $y = \frac{t}{4} + 4; -8 \leq t \leq 8$



3 Write $x = -3t$ and $y = t^2 + 2$ in rectangular form.

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4 Write $x = t^2 - 5$ and $y = 4t$ in rectangular form.

Write each pair of parametric equations in rectangular form. Then graph the equation and state any restrictions on the domain.

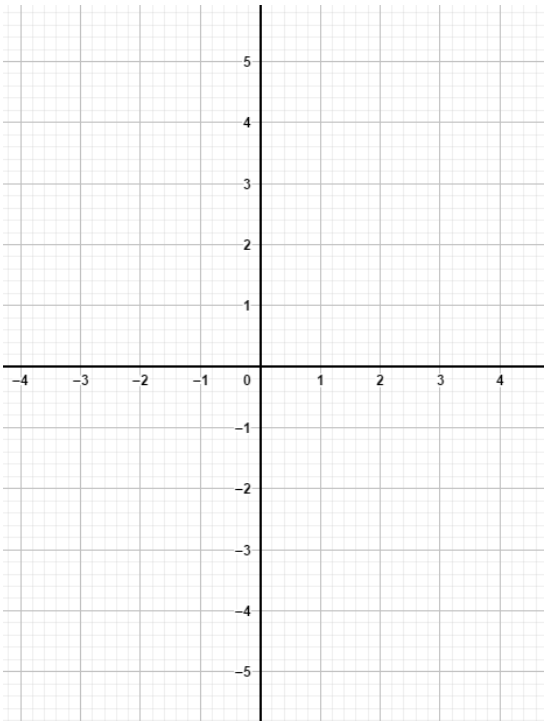
5 $x = 2t - 5, y = t^2 + 4$

6 $x = \sqrt{t} + 4$ $y = 4t + 3$

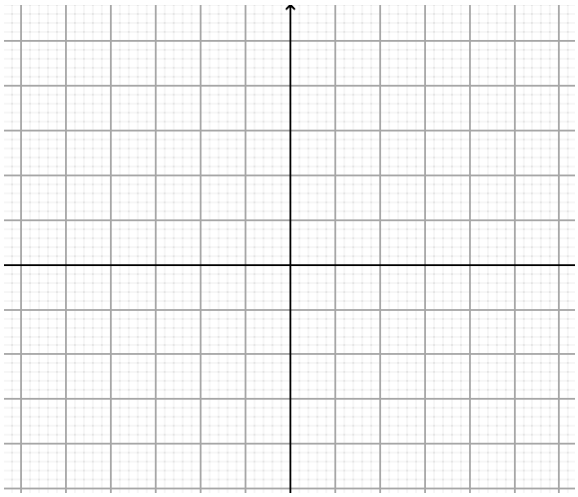
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7 Write $x = 2\cos\theta$ and $y = 4\sin\theta$ in rectangular form. Then graph the equation.



8 Write $x = 3\sin\theta$ and $y = 8\cos\theta$ in rectangular form. Then graph the equation



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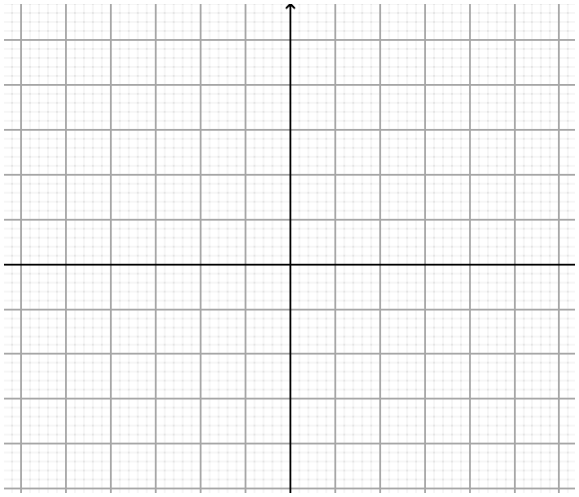
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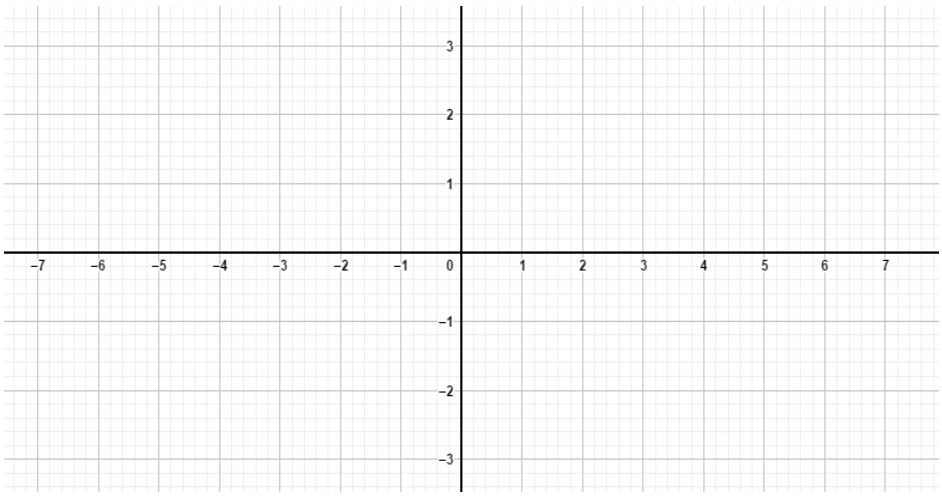
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9 Write each pair of parametric equations in rectangular form. Then graph the equation.

$x = 3\cos\theta \text{ and } y = 5\sin\theta$



10 $x = 7\sin\theta \text{ and } y = 2\cos\theta$

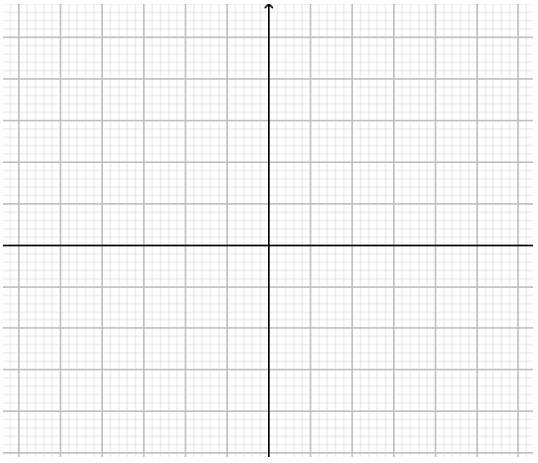


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Use each parameter to write the parametric equations that can represent $y = x^2 - 4$. Then graph the equation, indicating the speed and orientation.

11 $t = x$



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2 Projectile Motion

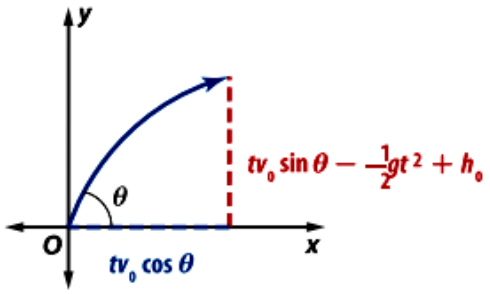
Parametric equations are often used to simulate projectile motion. The path of a projectile launched at an angle other than 90° with the horizontal can be modeled by the following parametric equations.

KeyConcept Projectile Motion

For an object launched at an angle θ with the horizontal at an initial velocity v_0, where g is the gravitational constant, t is time, and h_0 is the initial height:

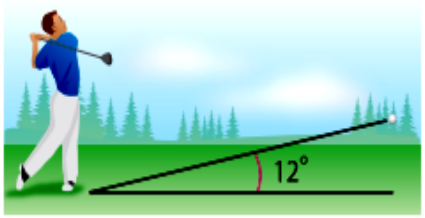
Horizontal Distance $x = tv_0 \cos \theta$

Vertical Position $y = tv_0 \sin \theta - \frac{1}{2}gt^2 + h_0$



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12 Saeed drives a golf ball with an initial velocity of 56 meters per second at an angle of 12° down a flat driving range. How far away will the golf ball land?

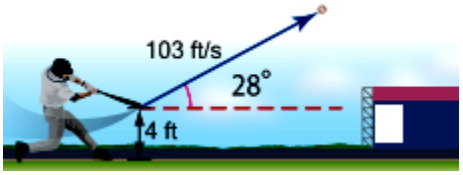


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13 Obaid attempts a 39.3-meter goal. He kicks the ball at a 41° angle with an initial speed of 21.3 meters per second. The goal is 4.6 meters high. Is the kick long enough to make the goal?

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14 A baseball player hits the ball at a 28° angle with an initial speed of 103 feet per second. The bat is 4 feet from the ground at the time of impact. Assuming that the ball is not caught, determine the distance traveled by the ball.



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15 Write a set of parametric equations for the line or line segment with the given characteristics.
line with a slope of 3 that passes through (4, 7)

$a)\ x = t + 4 \ \ and\ y = 3t + 7$

$b)\ x = t + 3 \ \ and\ y = 0.5t - 2$

$c)\ x = t - 2 \ \ and\ y = 4t - 6$

$d)\ x = t + 7 \ \ and\ y = -\frac{1}{3}t + 13$

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16 **Write a set of parametric equations for the line or line segment with the given characteristics.**
line with a slope of -0.5 that passes through (3, -2)

$a)\ x = t + 4 \ \ and\ y = 3t + 7$

$b)\ x = t + 3 \ \ and\ y = -0.5t - 2$

$c)\ x = t - 2 \ \ and\ y = 4t - 6$

$d)\ x = t + 7 \ \ and\ y = -\frac{1}{3}t + 13$

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17 **Write a set of parametric equations for the line or line segment with the given characteristics.**
line segment with endpoints (-2, -6) and (2, 10)

$a)\ x = t + 4 \ \ and\ y = 3t + 7$

$b)\ x = t + 3 \ \ and\ y = 0.5t - 2$

$c)\ x = t - 2 \ \ and\ y = 4t - 6$

$d)\ x = t + 7 \ \ and\ y = -\frac{1}{3}t + 13$

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18 **Write each pair of parametric equations in rectangular form**

$$x = -3t \ \text{ and } y = t^2 + 2$$

$a)\ y = -\frac{1}{9}x^2 + 2$

$b)\ y = \frac{1}{3}x^2 + 2$

$c)\ y = \frac{1}{9}x^2 + 2$

$d)\ y = \frac{1}{9}x^2 - 2$

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19 **Write each pair of parametric equations in rectangular form.**

$$x = 2t - 5 \text{ and } y = t^2 + 4$$

$a)\ y = \left(\frac{x+5}{2}\right)^2 + 4$

$b)\ y = \left(\frac{x-9}{3}\right)^2 - 7$

$c)\ x = \left(\frac{y}{2}\right)^2 - 2$

$d)\ x = \left(\frac{3-y}{4}\right)^2 + 1$

20 **Write each pair of parametric equations in rectangular form.**

$$x = 3t + 9 \text{ and } y = t^2 - 7$$

$a)\ y = \left(\frac{x+5}{2}\right)^2 + 4$

$b)\ y = \left(\frac{x-9}{3}\right)^2 - 7$

$c)\ x = \left(\frac{y}{2}\right)^2 - 2$

$d)\ x = \left(\frac{3-y}{4}\right)^2 + 1$

21 **Write each pair of parametric equations in rectangular form.**

$$x = t^2 - 2 \text{ and } y = 5t$$

$a)\ y = \left(\frac{x+5}{5}\right)^2 + 2$

$b)\ y = \left(\frac{x-9}{5}\right)^2 - 7$

$c)\ x = \left(\frac{y}{5}\right)^2 - 2$

$d)\ x = \left(\frac{3-y}{5}\right)^2 + 1$

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22 **Write each pair of parametric equations in rectangular form.**

$$x = t^2 + 1 \text{ and } y = -4t + 3$$

a) $y = \left(\frac{x+5}{2}\right)^2 + 4$

b) $y = \left(\frac{x-9}{3}\right)^2 - 7$

c) $x = \left(\frac{y}{2}\right)^2 - 2$

d) $x = \left(\frac{3-y}{4}\right)^2 + 1$

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23 **Write each pair of parametric equations in rectangular form.**

$$x = -t - 4 \text{ and } y = 3t^2$$

a) $y = 2\left(\frac{x+1}{5}\right)^2 + 8$

b) $y = 3x^2 + 24x$

c) $y = 3x^2 + 24x + 48$

d) $y = 2\left(\frac{x+1}{8}\right)^2 + 5$

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24 **Write each pair of parametric equations in rectangular form.**

$$x = 5t - 1 \text{ and } y = 2t^2 + 8$$

a) $y = 2\left(\frac{x+1}{5}\right)^2 + 8$

b) $y = 3x^2 + 24x$

c) $y = 3x^2 + 24x + 48$

d) $y = 2\left(\frac{x+1}{8}\right)^2 + 5$

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12 GENERAL

MATH ENG

LESSON 8-1

Introduction to Vectors

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

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Introduction to Vectors

Vectors single real number called a Many physical quantities, such as speed, can be completely described by a *scalar*. This number indicates the *magnitude* or *size* of the quantity. **A vector** is a quantity that has both magnitude and *direction*. The velocity of a football is a vector that describes both the speed and direction of the ball.

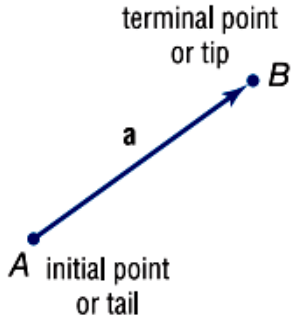
1 State whether each quantity described is a *vector* quantity or a *scalar* quantity.

- a boat traveling at 15 kilometers per hour
- a hiker walking 25 paces due west
- a person's weight on a bathroom scale
- a car traveling 60 kilometers per hour 15° east of south
- a parachutist falling straight down at 12.5 miles per hour
- a child pulling a sled with a force of 40 newtons
- a box being pushed with a force of 125 newtons
- wind blowing at 20 km/h
- a deer running 15 meters per second due west
- a baseball thrown with a speed of 136 km/h
- a 15-Newton tire hanging from a rope
- a rock thrown straight up at a velocity of 15 meters per second

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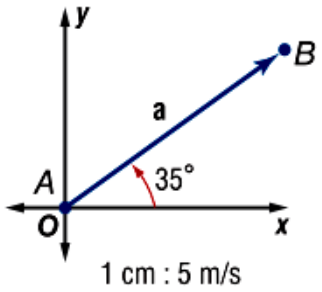
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A vector can be represented geometrically by a directed line segment, or arrow diagram, that shows both magnitude and direction. Consider the directed line segment with an **initial point** A (also known as the *tail*) and **terminal point** B (also known as the *head* or *tip*) shown. This vector is denoted by $\overrightarrow{AB}$, $\vec{a}$, or **a**.

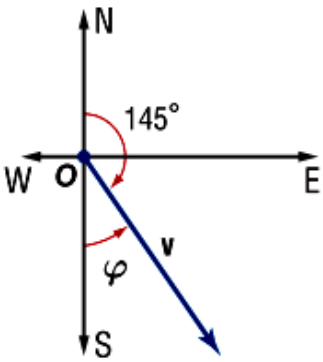


If a vector has its initial point at the origin, it is in **standard position**. The **direction** of a vector is the directed angle between the vector and the horizontal line that could be used to represent the positive x-axis. The direction of **a** is 35°.

The length of the line segment represents, and is proportional to, the **magnitude** of the vector. If the scale of the arrow diagram for **a** is 1 cm = 5 m/s, then the magnitude of **a**, denoted $|\mathbf{a}|$, is 2.6×5 or 13 meters per second.



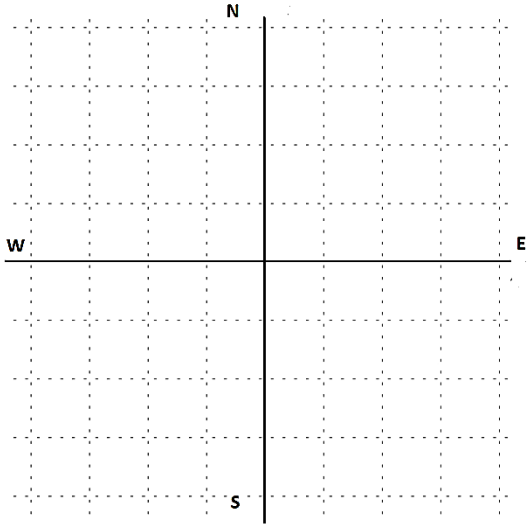
The direction of a vector can also be given as a bearing. A **quadrant bearing** φ, or *phi*, is a directional measurement between 0° and 90° east or west of the north-south line. The quadrant bearing of vector **v** shown is 35° east of south or southeast, written S35°E.



A **true bearing** is a directional measurement where the angle is measured clockwise from north. True bearings are always given using three digits. So, a direction that measures 25° clockwise from north would be written as a true bearing of 025°.

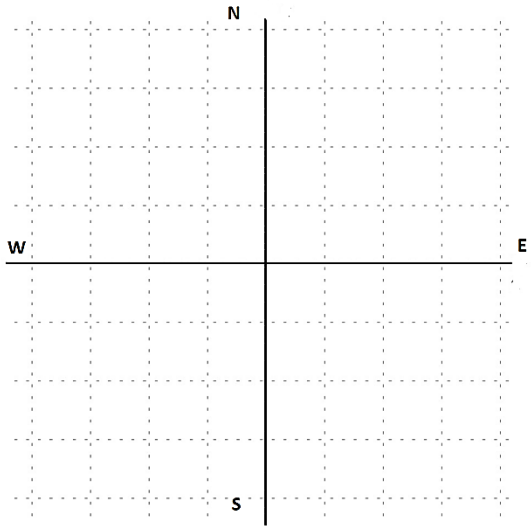
Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

4 **a = 20 feet per second at a bearing of 030°**



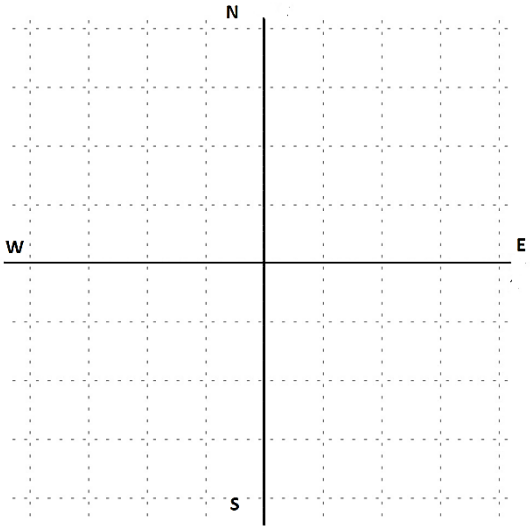
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5 v = 75 Newtons of force at 140° to the horizontal



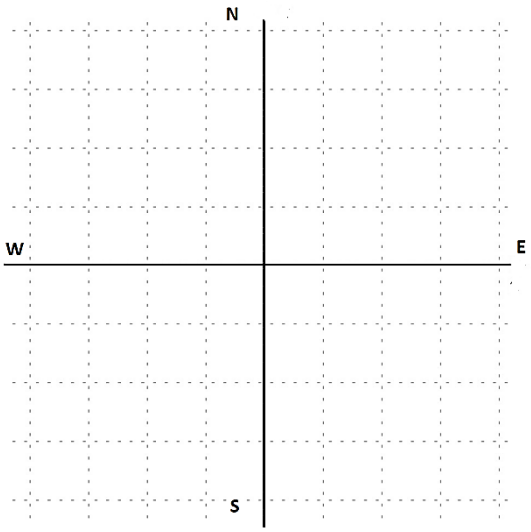
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6 z = 30 miles per hour at a bearing of S60°W



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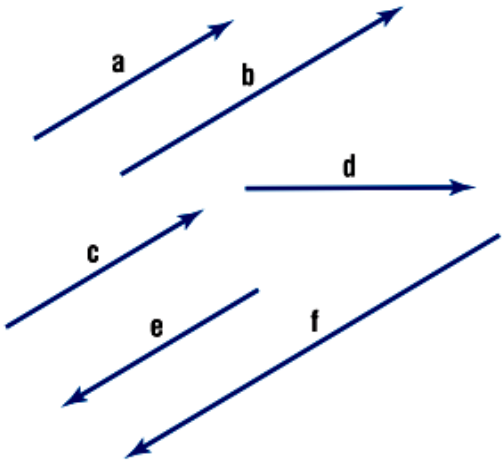
7 u = 15 kilometers per hour at a bearing of S25°E



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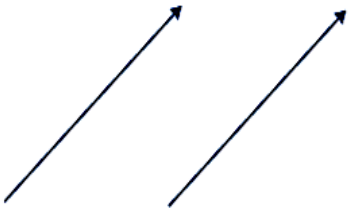
In your operations with vectors, you will need to be familiar with the following vector types.

- **Parallel vectors** have the same or opposite direction but not necessarily the same magnitude. In the figure, $\mathbf{a} \parallel \mathbf{b} \parallel \mathbf{c} \parallel \mathbf{e} \parallel \mathbf{f}$.
- **Equivalent vectors** have the same magnitude and direction. In the figure, $\mathbf{a} = \mathbf{c}$ because they have the same magnitude and direction. Notice that $\mathbf{a} \neq \mathbf{b}$, since $|\mathbf{a}| \neq |\mathbf{b}|$, and $\mathbf{a} \neq \mathbf{d}$, since $\mathbf{a}$ and $\mathbf{d}$ do not have the same direction.
- **Opposite vectors** have the same magnitude but opposite direction. The vector opposite $\mathbf{a}$ is written $-\mathbf{a}$. In the figure, $\mathbf{e} = -\mathbf{a}$.



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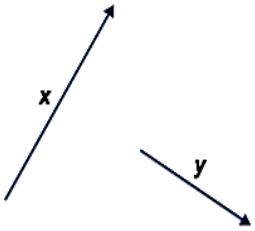
8 **Which statement best describes the pair of vectors?**



a) equivalent
b) parallel
c) opposite
d) neither opposite, parallel, nor equivalent

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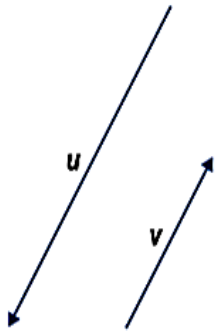
9 **Which statement best describes the pair of vectors?**



a) equivalent
b) parallel
c) opposite
d) neither opposite, parallel, nor equivalent

10 **Choose the statement that best describes the pair of vectors.**

a) equivalent
b) parallel
c) opposite
d) neither opposite, parallel, nor equivalent



11 **Choose the statement that best describes the pair of vectors.**

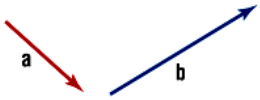
e) equivalent
f) parallel
g) opposite
h) neither opposite, parallel, nor equivalent



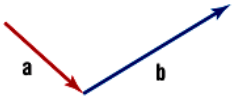
KeyConcept Finding Resultants

Triangle Method (Tip-to-Tail)

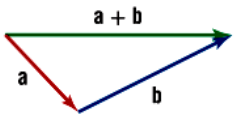
To find the resultant of **a** and **b**, follow these steps.



Step 1 Translate **b** so that the tail of **b** touches the tip of **a**.



Step 2 The resultant is the vector from the tail of **a** to the tip of **b**.

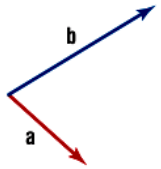


Parallelogram Method (Tail-to-Tail)

To find the resultant of **a** and **b**, follow these steps.

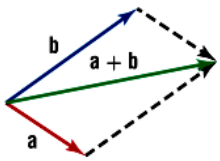


Step 1 Translate **b** so that the tail of **b** touches the tail of **a**.



Step 2 Complete the parallelogram that has **a** and **b** as two of its sides.

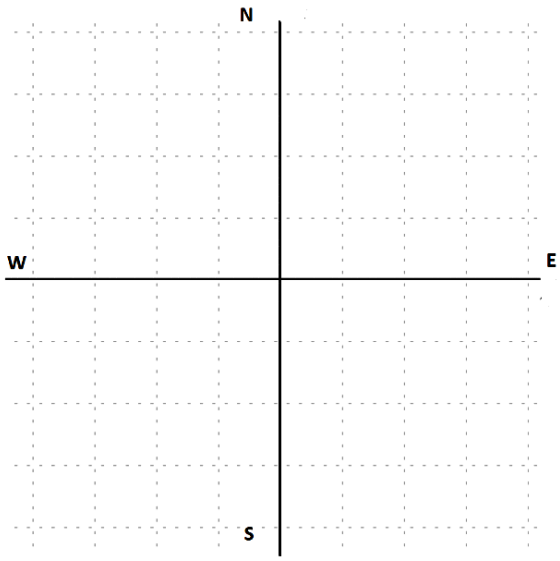
Step 3 The resultant is the vector that forms the indicated diagonal of the parallelogram.



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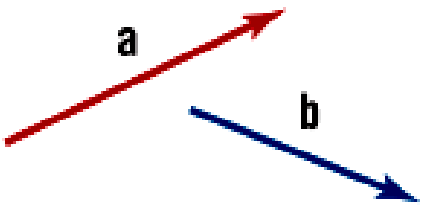
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12 In an orienteering competition, Noura walks N50°E for 120 meters and then walks 80 meters due east. How far and at what quadrant bearing is Noura from her starting position?



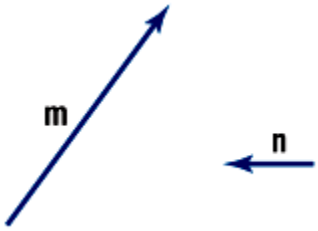
13 Find the resultant of each pair of vectors using either the triangle or parallelogram method. State the magnitude of the resultant to the nearest centimeter and its direction relative to the horizontal.



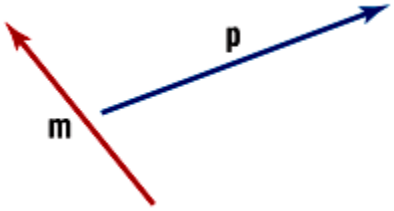


14 **Use the set of vectors to draw a vector diagram of each expression.**

$\mathbf{m} - 2\mathbf{n}$



$\mathbf{m} - \frac{1}{4}\mathbf{p}$



15 **AVIATION An airplane is flying with an airspeed of 310 km/h on a heading of 050°. If a 78-km/h wind is blowing from a true heading of 125°, determine the speed and direction of the plane relative to the ground.**

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16 **Ibrahim swims due east at a speed of 3.5 feet per second across a river directly toward the opposite bank. At the same time, the current of the river is carrying him due south at a rate of 2 feet per second. Find Ibrahim's speed and direction relative to the shore.**

Two or more vectors with a sum that is a vector **r** are called **components** of **r**. While components can have any direction, it often useful to express or *resolve* a vector into two perpendicular components. The **rectangular components** of a vector are horizontal and vertical.

In the diagram, the force **r** exerted to pull the wagon can be thought of as the sum of a horizontal component force **x** that moves the wagon forward and a vertical component force **y** that pulls the wagon upward.



17 **Eiman is pushing the handle of a lawn mower with a force of 450 newtons at an angle of 56° with the ground.**



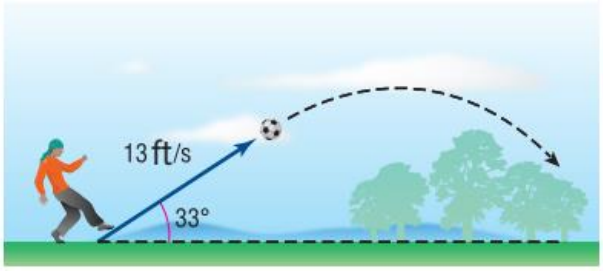
a. Draw a diagram that shows the resolution of the force that Eiman exerts into its rectangular components.

b. Find the magnitudes of the horizontal and vertical components of the force.

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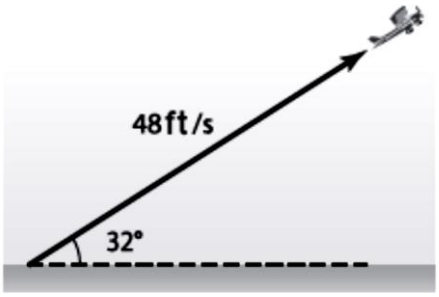
18 **A player kicks a football so that it leaves the ground with a velocity of 44 feet per second at an angle of 33° with the ground.**

- **Draw a diagram that shows the resolution of this force into its rectangular components.**
- **Find the magnitude of the horizontal and vertical components of the velocity.**



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19 A remote control airplane flew along an initial path of 32° to the horizontal at a velocity of 48 feet per second as shown. Which of the following represent the magnitudes of the horizontal and vertical components of the velocity?



A) 25.4 ft/s, 40.7 ft/s
B) 56.6 ft/s, 90.6 ft/s
C) 40.7 ft/s, 25.4 ft/s
D) 90.6 ft/s, 56.6 ft/s

20 **Buthaina is pushing the handle of a push broom with a [illegible] at an angle of 33° with the ground.**

a) Find the magnitudes of the horizontal and vertical components.



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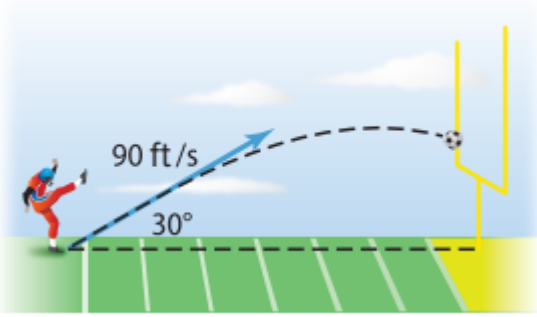
21 **For a field goal attempt, a ball is kicked with the velocity shown in the diagram below. Find the magnitudes of the horizontal and vertical components of the force.**

$a)\ x = 45\sqrt{3}\ ft/s \ \ and\ y = 45\ ft/s$

$b)\ x = 45ft/s \ \ and\ y = 45\sqrt{3}ft/s$

$c)\ x = 45\sqrt{3}\ ft/s \ \ and\ y = 45\sqrt{3}\ ft/s$

$d)\ x = 45\sqrt{2}ft/s \ \ and\ y = 45ft/s$



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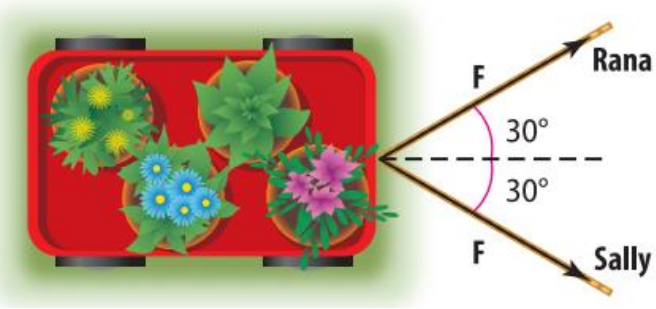
22 **Rana and Sally are pulling a wagon full of plants. Each person pulls on the wagon with equal force at an angle of 30° with the axis of the wagon. The resultant force is 120 newtons.**
How much force is each person exerting?

$a)$ *about* $96\ N$

$b)$ *about* $138\ N$

$c)$ *about* $34.5\ N$

$d)$ *about* $69\ N$



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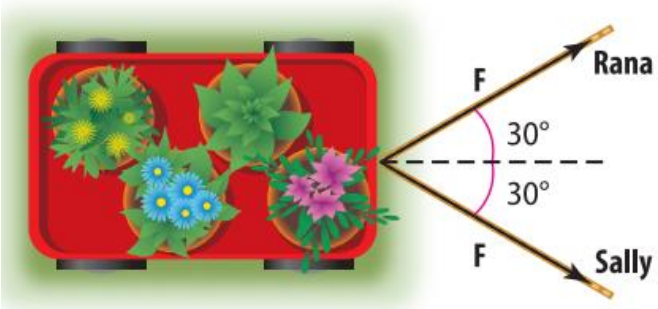
23 **Rana and Sally are pulling a wagon full of plants. Each person pulls on the wagon with equal force at an angle of 30° with the axis of the wagon.**
If each person exerts a force of 75 newtons, what is the resultant force?

$a)$ *about* $96\ N$

$b)$ *about* $130\ N$

$c)$ *about* $34.5\ N$

$d)$ *about* $69\ N$



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12 GENERAL

MATH ENG

LESSON 8-2

Vectors in the Coordinate Plane

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

Prepared by :

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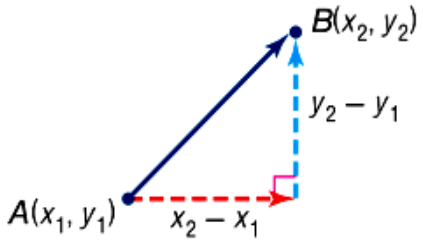
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Vectors in the Coordinate Plane

KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\langle x_2 - x_1, y_2 - y_1 \rangle.$$

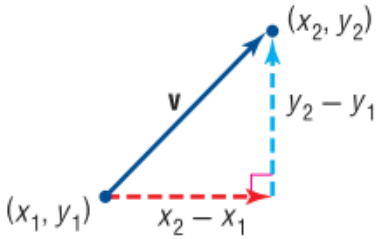


KeyConcept Magnitude of a Vector in the Coordinate Plane

If **v** is a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), then the magnitude of **v** is given by

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

If **v** has a component form of $\langle a, b \rangle$, then $|\mathbf{v}| = \sqrt{a^2 + b^2}$.



1 **Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. And find the magnitude $\overrightarrow{AB}$**

A(-3, 1), B(4, 5)

***A*(2, -7), *B*(-6, 9)**

A(-2, -7), B(6, 1)

A(0, 8), B(-9, -3)

A(-4, 2), B(3, -5)

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2 Find each of the following for $\mathbf{w} = \langle -4, 1 \rangle$, $\mathbf{y} = \langle 2, 5 \rangle$, and $\mathbf{z} = \langle -3, 0 \rangle$.

$\mathbf{w} + \mathbf{y}$

$\mathbf{z} - 2\mathbf{y}$

$4\mathbf{w} + \mathbf{z}$

$-3\mathbf{w}$

$2\mathbf{w} + 4\mathbf{y} - \mathbf{z}$

2 Unit Vectors

A vector that has a magnitude of 1 unit is called a **unit vector**. It is sometimes useful to describe a nonzero vector $\mathbf{v}$ as a scalar multiple of a unit vector $\mathbf{u}$ with the same direction as $\mathbf{v}$. To find $\mathbf{u}$, divide $\mathbf{v}$ by its magnitude $|\mathbf{v}|$.

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad \text{or} \quad \frac{1}{|\mathbf{v}|}\mathbf{v}$$

3 Find a unit vector with the same direction as the given vector.

$\mathbf{w} = \langle 6, -2 \rangle$

$\mathbf{x} = \langle -4, -8 \rangle$

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4 **Find a unit vector with the same direction as the given vector.**

$\mathbf{v} = \langle -8, -5 \rangle$ $\mathbf{v} = \langle 9, -3 \rangle$

The vector sum $a\mathbf{i} + b\mathbf{j}$ is called a **linear combination** of the vectors **i** and **j**.



Figure 8.2.3

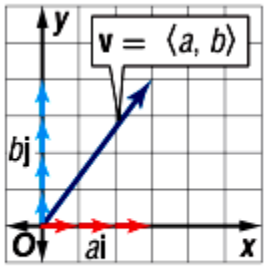


Figure 8.2.4

5 **Let $\overrightarrow{DE}$ be the vector with initial point *D*(-2, 3) and terminal point *E*(4, 5). Write $\overrightarrow{DE}$ as a linear combination of the vectors i and j.**

6 **Let $\overrightarrow{DE}$ be the vector with the given initial and terminal points. Write $\overrightarrow{DE}$ as a linear combination of the vectors i and j.**

$D(-6, 0), E(2, 5)$ $D(-3, -8), E(-7, 1)$

A way to specify the direction of a vector $\mathbf{v} = \langle a, b \rangle$ is to state the direction angle θ that $\mathbf{v}$ makes with the positive x-axis. From Figure 8.2.5, it follows that $\mathbf{v}$ can be written in component form or as a linear combination of $\mathbf{i}$ and $\mathbf{j}$ using the magnitude and direction angle of the vector.

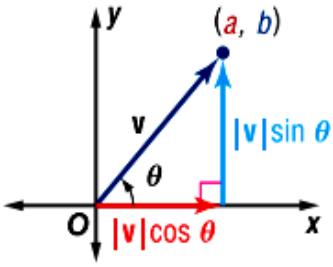


Figure 8.2.5

$\mathbf{v} = \langle a, b \rangle$ Component form

$= \langle |\mathbf{v}| \cos \theta, |\mathbf{v}| \sin \theta \rangle$ Substitution

$= |\mathbf{v}| (\cos \theta)\mathbf{i} + |\mathbf{v}| (\sin \theta)\mathbf{j}$ Linear combination of i and j

7 **Find the component form of v with the given magnitude and direction angle.**

$|\mathbf{v}| = 8, \theta = 45^\circ$

$|\mathbf{v}| = 24, \theta = 210^\circ$

$$\tan \theta = \frac{|\mathbf{v}| \sin \theta}{|\mathbf{v}| \cos \theta} \text{ or } \tan \theta = \frac{b}{a}.$$

8 **Find the direction angle of each vector to the nearest tenth of a degree.**

$8\mathbf{i} - 2\mathbf{j}$

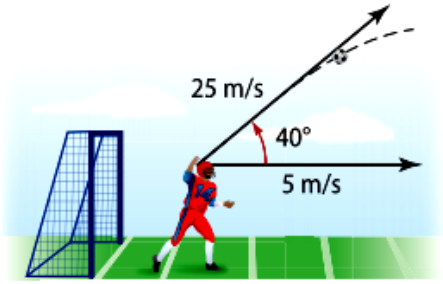
$-2\mathbf{i} + 5\mathbf{j}$

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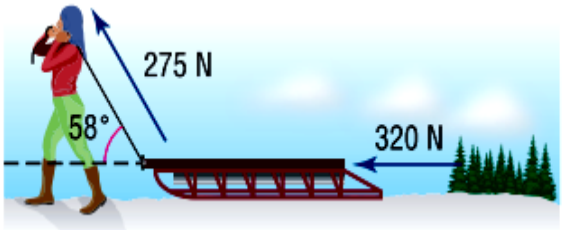
9 Find the direction angle of each vector to the nearest tenth of a degree.

$\langle 7, 7 \rangle$ $\langle -6, -4 \rangle$

10 A goalkeeper running forward at 5 meters per second throws a ball with a velocity of 25 meters per second at an angle of 40° with the horizontal. What is the resultant speed and direction of the pass?



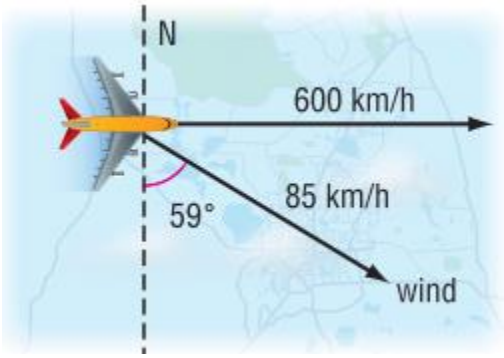
11 Houriyya is pulling a sled with a force of 275 newtons by holding its rope at a 58° angle. Her brother is pushing the sled with a force of 320 newtons. Determine the magnitude and direction of the resultant force on the sled.



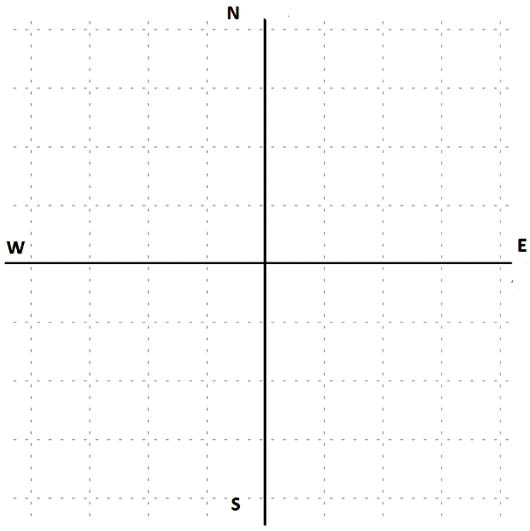
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12 **An airplane is traveling due east with a speed of 600 miles per hour. The wind blows at 85 miles per hour at an angle of S59°E**

a. Determine the speed of the airplane's flight.
b. Determine the angle of the airplane's flight.



13 **In an orienteering competition, Noura walks N50°E for 120 meters and then walks 80 meters due east. How far and at what quadrant bearing is Noura from her starting position?**



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14 **AVIATION** An airplane is flying with an airspeed of 310 km/h on a heading of 050°. If a 78-km/h wind is blowing from a true heading of 125°, determine the speed and direction of the plane relative to the ground.

15 Find each of the following for
$\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$.

$$4h - g$$

$a)$ $\langle -4, 4 \rangle$

$b)$ $\langle -55, -13 \rangle$

$c)$ $\langle 31, -11 \rangle$

$d)$ $\langle -21, 13 \rangle$

16 Find each of the following for
$\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$.

$$f + 2h$$

$a)$ $\langle -4, 4 \rangle$

$b)$ $\langle -55, -13 \rangle$

$c)$ $\langle 31, -11 \rangle$

$d)$ $\langle -21, 13 \rangle$

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17 Find each of the following for
$\mathbf{f} = \langle 8,0 \rangle$, $\mathbf{g} = \langle -3,-5 \rangle$, and $\mathbf{h} = \langle -6,2 \rangle$.

$$3g - 5f + h$$

a) $\langle -4,4 \rangle$

b) $\langle -55,-13 \rangle$

c) $\langle 31,-11 \rangle$

d) $\langle -21,13 \rangle$

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18 Find each of the following for
$\mathbf{f} = \langle 8,0 \rangle$, $\mathbf{g} = \langle -3,-5 \rangle$, and $\mathbf{h} = \langle -6,2 \rangle$.

$$2f + g - 3h$$

a) $\langle -4,4 \rangle$

b) $\langle -55,-13 \rangle$

c) $\langle 31,-11 \rangle$

d) $\langle -21,13 \rangle$

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19 Find each of the following for
$\mathbf{w} = \langle 8,0 \rangle$, $\mathbf{y} = \langle -3,-5 \rangle$, and $\mathbf{z} = \langle -6,2 \rangle$.

$$w + y$$

a) $\langle -19,4 \rangle$

b) $\langle 3,22 \rangle$

c) $\langle 5,-5 \rangle$

d) $\langle -5,5 \rangle$

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20 Find each of the following for
$\mathbf{w} = \langle 8,0 \rangle$, $\mathbf{y} = \langle -3,-5 \rangle$, and $\mathbf{z} = \langle -6,2 \rangle$.

$$z - 2y$$

a) $\langle -19,4 \rangle$

b) $\langle 0,12 \rangle$

c) $\langle -2,6 \rangle$

d) $\langle -7,-10 \rangle$

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21 Find each of the following for
$\mathbf{w} = \langle 8,0 \rangle$, $\mathbf{y} = \langle -3,-5 \rangle$, and $\mathbf{z} = \langle -6,2 \rangle$.

$$4w + z$$

a) $\langle -19,4 \rangle$

b) $\langle 3,22 \rangle$

c) $\langle -2,2\,6 \rangle$

d) $\langle 26,2 \rangle$

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22 Find each of the following for
$\mathbf{w} = \langle 8,0 \rangle$, $\mathbf{y} = \langle -3,-5 \rangle$, and $\mathbf{z} = \langle -6,2 \rangle$.

$$2w + 4y - z$$

a) $\langle -9,4 \rangle$

b) $\langle 10,-22 \rangle$

c) $\langle 5,6 \rangle$

d) $\langle 22,-10 \rangle$

23 Find the component form of v with the given magnitude and direction angle.

$$|v| = 16, \theta = 330°$$

$a)$ $\langle 6, 6\sqrt{3} \rangle$

$b)$ $\langle -2\sqrt{2}, -2\sqrt{2} \rangle$

$c)$ $\langle -3, -3\sqrt{3} \rangle$

$d)$ $\langle 8\sqrt{3}, -8 \rangle$

24 Find the component form of v with the given magnitude and direction angle.

$$|v| = 28, \theta = 273°$$

$a)$ $\langle -6, 2\sqrt{3} \rangle$

$b)$ $\langle 1.47, -27.96 \rangle$

$c)$ $\langle 5, -3\sqrt{3} \rangle$

$d)$ $\langle -8.6, 12.29 \rangle$

25 Find the component form of v with the given magnitude and direction angle.

$$|v| = 15, \theta = 125°$$

$a)$ $\langle -6, 2\sqrt{3} \rangle$

$b)$ $\langle 1.47, -27.96 \rangle$

$c)$ $\langle 5, -3\sqrt{3} \rangle$

$d)$ $\langle -8.6, 12.29 \rangle$

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12 GENERAL

MATH ENG

LESSON 8-3

Dot Products and Vector Projections

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2024-2025

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Dot Products and Vector Projections

KeyConcept Dot Product of Vectors in a Plane

The dot product of $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$ is defined as $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2$.

KeyConcept Orthogonal Vectors

The vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Find the dot product of u and v. Then determine if u and v are orthogonal.

1 $\mathbf{u} = \langle 3, -5 \rangle, \mathbf{v} = \langle 6, 2 \rangle$

$\mathbf{u} = \langle 9, -3 \rangle, \mathbf{v} = \langle 1, 3 \rangle$

2 $\mathbf{u} = 8\mathbf{i} + 6\mathbf{j}; \mathbf{v} = -\mathbf{i} + 2\mathbf{j}$

$\mathbf{u} = 11\mathbf{i} + 7\mathbf{j}; \mathbf{v} = -7\mathbf{i} + 11\mathbf{j}$

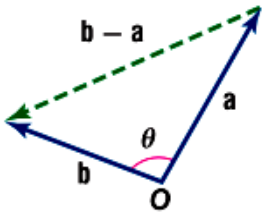
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KeyConcept Angle Between Two Vectors

If θ is the angle between nonzero vectors **a** and **b**, then

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}.$$



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Find the angle θ between vectors u and v to the nearest tenth of a degree.

3 $\mathbf{u} = \langle 7, 10 \rangle, \mathbf{v} = \langle 4, -4 \rangle$

4 $\mathrm{u} = \langle 6, 2 \rangle$ and $\mathrm{v} = \langle -4, 3 \rangle$

5 $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

6 $\mathbf{u} = -10\mathbf{i} + \mathbf{j}, \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

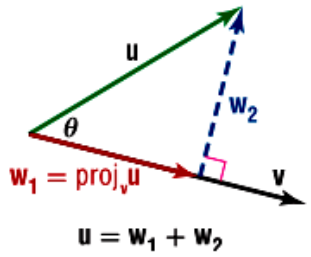
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KeyConcept Projection of u onto v

Let **u** and **v** be nonzero vectors, and let $\mathbf{w}_1$ and $\mathbf{w}_2$ be vector components of **u** such that $\mathbf{w}_1$ is parallel to **v** as shown. Then vector $\mathbf{w}_1$ is called the **vector projection** of **u** onto **v**, denoted $\text{proj}_{\mathbf{v}}\mathbf{u}$, and

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}.$$



7 **Find the projection of u = ⟨3, 2⟩ onto v = ⟨5, -5⟩. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.**

8 **Find the projection of u = ⟨1, 2⟩ onto v = ⟨8, 5⟩. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.**

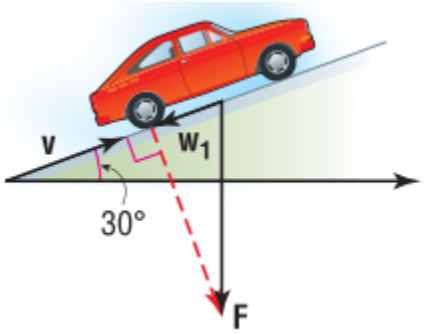
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9 Find the projection of u = ⟨4, -3⟩ onto v = ⟨2, 6⟩. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.

10 Find the projection of u = ⟨-3, 4⟩ onto v = ⟨6, 1⟩. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.

11 A 13,000-newton car sits on a hill inclined at 30° as shown. Ignoring the force of friction, what force is required to keep the car from rolling down the hill?



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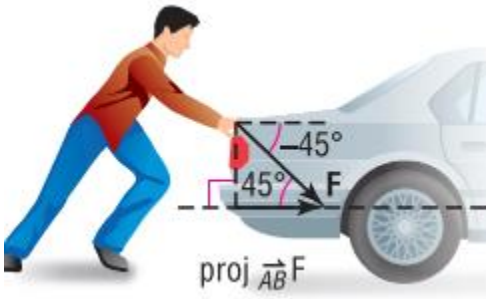
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12 **Khadija sits on a sled on the side of a hill inclined at 60°. What force is required to keep the sled from sliding down the hill if the weight of Khadija and the sled is 550 newtons?**

$$W = |\text{proj}_{\overrightarrow{AB}}\ \mathbf{F}|\ |\overrightarrow{AB}| \qquad W = |\mathbf{F}|\ (\cos\theta)\ |\overrightarrow{AB}|$$

$$W = \mathbf{F} \cdot \overrightarrow{AB}$$

13 **A person pushes a car with a constant force of 120 newtons at a constant angle of 45° as shown. Find the work done in joules moving the car 10 meters.**

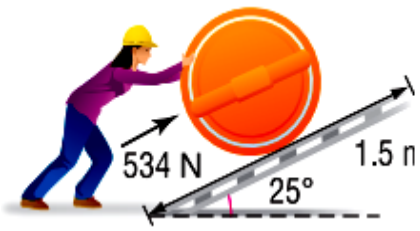


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14 **Khalid is pushing a vacuum cleaner with a force of 375 newtons. The handle of the vacuum cleaner makes a 60° angle with the floor. How much work in newton-meters does he do if he pushes the vacuum cleaner 2 meters?**



15 **Rana is pushing a construction barrel up a ramp 1.5 meters long into the back of a truck. She is using a force of 534 newtons and the ramp is 25° from the horizontal. How much work in joules is Rana doing?**



16 A student rolls her backpack from her Chemistry classroom to her English classroom using a force of 175 newtons.

a. If she exerts 3060 joules to pull her backpack 31 meters, what is the angle of her force?



b. If she exerts 1315 joules at an angle of 60°, how far did she pull her backpack?

17 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = \langle 0, -5 \rangle \ , \mathbf{v} = \langle 1, -4 \rangle$$

$a)\ 14.0^\circ$

$b)\ 14.2^\circ$

$c)\ 14.8^\circ$

$d)\ 14.7^\circ$

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18 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = \langle 7, 10 \rangle \ , \mathbf{v} = \langle 4, -4 \rangle$$

$a)\ 100.2^\circ$

$b)\ 100.0^\circ$

$c)\ 100.1^\circ$

$d)\ 100.7^\circ$

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19 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = \langle -2, 4 \rangle \ , \mathbf{v} = \langle 2, -10 \rangle$$

$a)\ 164.2^\circ$

$b)\ 146.7^\circ$

$c)\ 164.1^\circ$

$d)\ 164.7^\circ$

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20 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}, \mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$$

$a)\ 82.2^\circ$

$b)\ 82.7^\circ$

$c)\ 82.1^\circ$

$d)\ 82.9^\circ$

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21 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = \langle -9, 0 \rangle \ , \mathbf{v} = \langle -1, -1 \rangle$$

$a)\ 45.2^\circ$

$b)\ 45.7^\circ$

$c)\ 45.0^\circ$

$d)\ 45.9^\circ$

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22 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = -\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$$

$a)\ 48.2^\circ$

$b)\ 48.4^\circ$

$c)\ 84.4^\circ$

$d)\ 48.9^\circ$

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23 Find the angle θ between u and v to the nearest tenth of a degree

$$\mathbf{u} = \langle 6, 0 \rangle \ , \mathbf{v} = \langle -10, 8 \rangle$$

$a)\ 141.1^\circ$

$b)\ 141.2^\circ$

$c)\ 141.3^\circ$

$d)\ 114.3^\circ$

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24 Determine whether each pair of vectors are *parallel*, *perpendicular*, or *neither*

$$\mathbf{u} = \left\langle -\frac{2}{3}, \frac{3}{4} \right\rangle, \mathbf{v} = \langle 9, 8 \rangle$$

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25 Determine whether each pair of vectors are *parallel*, *perpendicular*, or *neither*

$$\mathbf{u} = \langle -1, -4 \rangle, \mathbf{v} = \langle 3, 6 \rangle$$

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26 Determine whether each pair of vectors are *parallel*, *perpendicular*, or *neither*

$$\mathbf{u} = \langle 5, 7 \rangle, \mathbf{v} = \langle -15, -21 \rangle$$

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12 GENERAL

MATH ENG

LESSON 8-4

Vectors in Three-Dimensional Space

Mr. Ahmed Ata
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2024-2025

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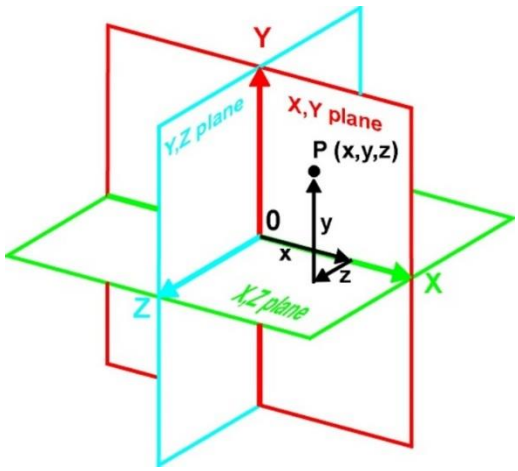
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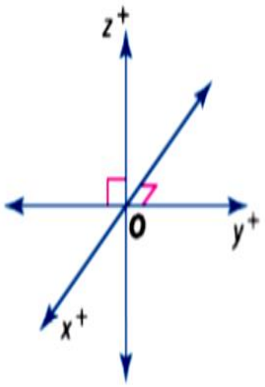
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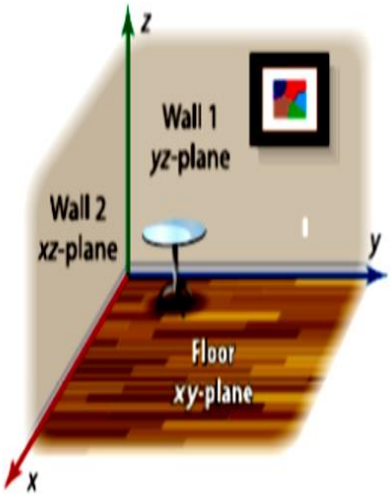
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Vectors in Three-Dimensional Space



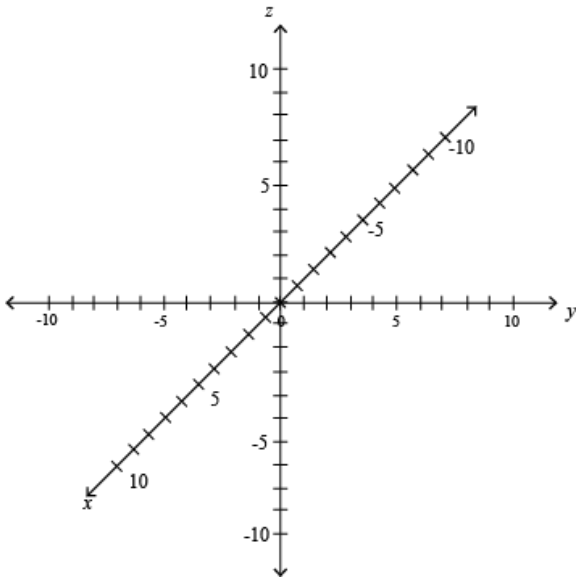




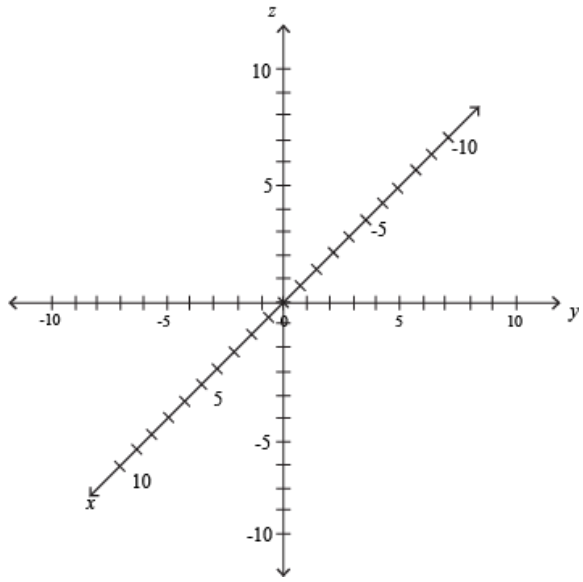
Plot each point in a three-dimensional coordinate system.

1

(4, 6, 2)



(−2, 4, −5)



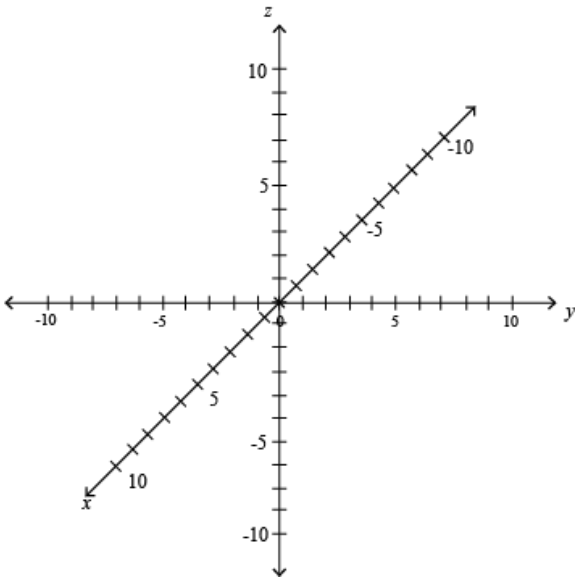
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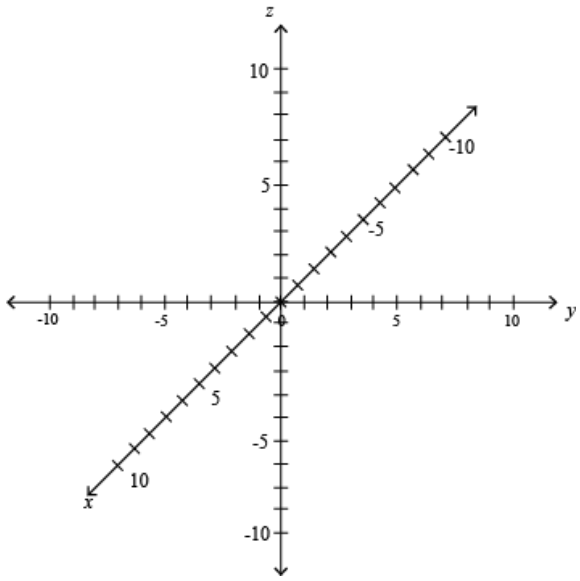
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2

$(-3, -4, 2)$



$(5, -4, -1)$



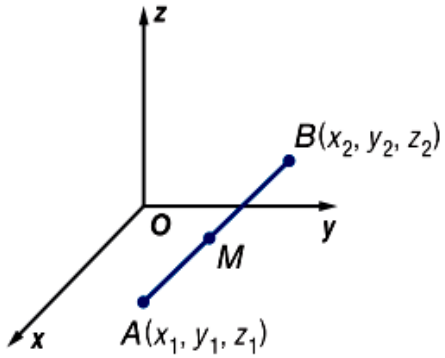
KeyConcept Distance and Midpoint Formulas in Space

The distance between points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

The midpoint M of AB is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$



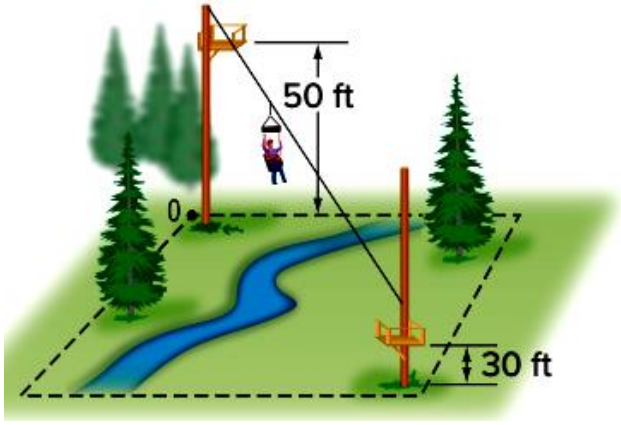
3 **Find the length and midpoint of the segment with the given endpoints.**

$(-4, 10, 4), (1, 0, 9)$

$(8, 3, 4), (-4, -7, 5)$

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4 A tour of the Sierra Madre Mountains lets guests experience nature by zip-lining from one platform to another over the scenic surroundings. Two platforms that are connected by a zip-line are represented by the coordinates (10, 12, 50) and (70, 92, 30), where the coordinates are given in feet.



a. Find the length of the zip-line needed to connect the two platforms.

b. An additional platform is to be built halfway between the existing platforms. Find the coordinates of the new platform.

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5 Safety regulations require airplanes to be at least 0.8 kilometer apart when in the sky. Two planes are flying above Dubai with the coordinates (90, 45, 9000) and (135, -75, 8400), where the coordinates are given in meters.

A. Are the two planes in violation of the safety regulations? Explain.

B. If a firework was launched and exploded directly in between the two planes, what are the coordinates of the firework explosion?

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6 During a training session, the location of two F-18 fighter jets are represented by the coordinates (203, -36, 1990) and (-87, 215, 4,830), where the coordinates are given in meters.

a. Determine the distance between the two jets.

b. To what location would one of the fighter pilots have to fly the F-18 in order to reduce the distance between the two jets by half?

7 Find each of the following for y = ⟨3, -6, 2⟩ , w = ⟨-1, 4, -4⟩ , and z = ⟨-2, 0, 5⟩

$4y + 2z$

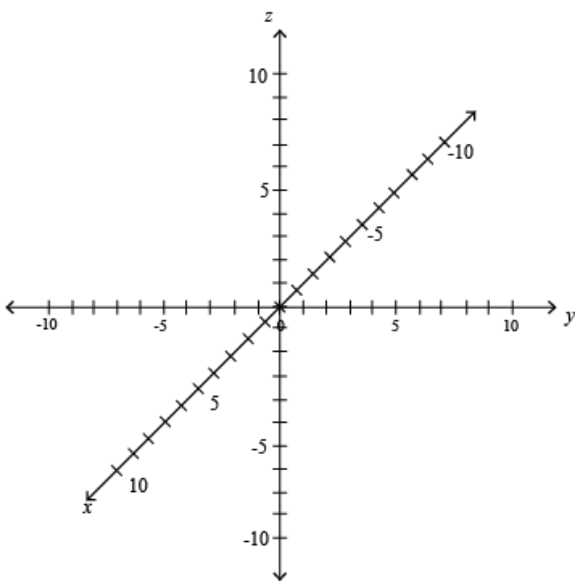
$4w - 8z$

$2w - z + 3y$

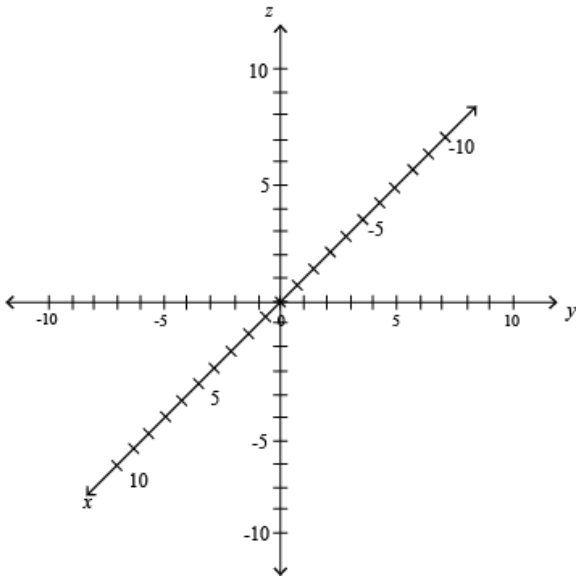
$3y + 3z - 6w$

8 Locate and graph each vector in space.

$\mathbf{d} = \langle 4, -2, -3 \rangle$



$\mathbf{m} = 7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$



9 Find the component form and magnitude of $\overrightarrow{AB}$ with initial point $A(-4, -2, 1)$ and terminal point $B(3, 6, -6)$. Then find a unit vector in the direction of $\overrightarrow{AB}$.

10 Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points. Then find a unit vector in the direction of $\overrightarrow{AB}$.

$A(-2, -5, -5)$, $B(-1, 4, -2)$

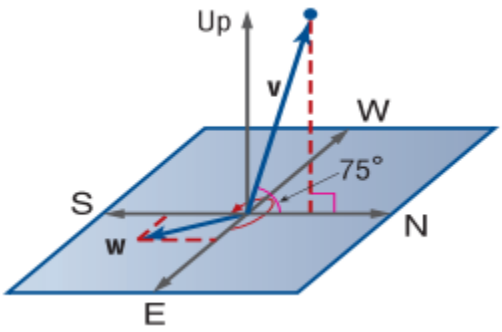
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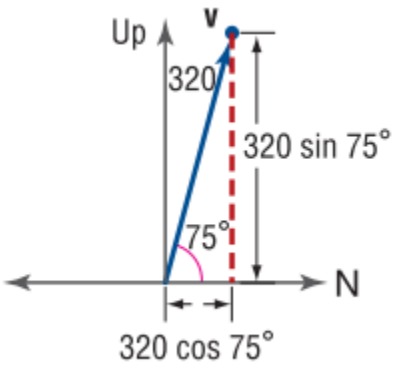
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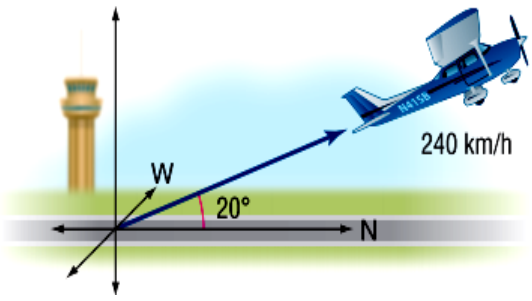
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11 **After liftoff, a model rocket is headed due north and climbing at an angle of 75° relative to the horizontal at 320 kilometers per hour. If the wind blows from the northwest at 8 kilometers per hour, find a vector for the resultant velocity of the rocket relative to the point of liftoff.**





12 **An airplane is taking off headed due north with an air speed of 240 kilometers per hour at an angle of 20° relative to the horizontal. The wind is blowing with a velocity of 13 kilometers per hour from the southwest.**
Find a vector that represents the resultant velocity of the plane relative to the point of takeoff. Let i point east, j points north, and k poin



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13 **After takeoff, an airplane is headed east and is climbing at an angle of 18° relative to the horizontal. Its air speed is 400 kilometers per hour. If the wind blows from the northeast at 16 kilometers per hour, find a vector that represents the resultant velocity of the plane relative to the point of takeoff. Let i point east, j points north, and k point up.**

Determine whether the triangle with the given vertices is *isosceles* or *scalene*.

14 $A(3, 1, 2)$, $B(5, -1, 1)$, $C(1, 3, 1)$

15 $A(4, 3, 4)$, $B(4, 6, 4)$, $C(4, 3, 6)$

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If N is the midpoint of $\overline{MP}$, find P.

16 $M(-1, -4, -9); N(-2, 1, -5)$

17 $M(3, 4, 5); N\left(\frac{7}{2}, 1, 2\right)$

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18 **Find each of the following for**

$\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$.

$$6\boldsymbol{a} - 7\boldsymbol{b} + 8\boldsymbol{c}$$

a) $\langle -65, -18, 56 \rangle$

b) $\langle 38, -36, -65 \rangle$

c) $\langle -88, 6, 99 \rangle$

d) $\langle 48, 12, -38 \rangle$

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19 Find each of the following for
$\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$.

$$7a - 5b$$

$a)$ $\langle -65, -18, 56 \rangle$

$b)$ $\langle 38, -36, -65 \rangle$

$c)$ $\langle -88, 6, 99 \rangle$

$d)$ $\langle 48, 12, -38 \rangle$

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20 Find each of the following for
$\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$.

$$2a + 5b - 9c$$

$a)$ $\langle -65, -18, 56 \rangle$

$b)$ $\langle 38, -36, -65 \rangle$

$c)$ $\langle -88, 6, 99 \rangle$

$d)$ $\langle 48, 12, -38 \rangle$

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21 Find each of the following for
$\mathbf{a} = \langle -5, -4, 3 \rangle$, $\mathbf{b} = \langle 6, -2, -7 \rangle$, and $\mathbf{c} = \langle -2, 2, 4 \rangle$.

$$6b + 4c - 4a$$

$a)$ $\langle -65, -18, 56 \rangle$

$b)$ $\langle 38, -36, -65 \rangle$

$c)$ $\langle -88, 6, 99 \rangle$

$d)$ $\langle 48, 12, -38 \rangle$

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22 Find each of the following for

$$x = -9i + 4j + 3k, y = 6i - 2j - 7k, and\ z = -2i + 2j + 4k.$$

$$4x + 3y + 2z$$

$a)\ -22i + 14j + -k$

$b)\ 50i - 18j + 10k$

$c)\ -18i - 6j + 6k$

$d)\ -13i + 2j + 21k$

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23 Find each of the following for

$$x = -9i + 4j + 3k, y = 6i - 2j - 7k, and\ z = -2i + 2j + 4k.$$

$$-8x - 2y + 5z$$

$a)\ -22i + 14j + -k$

$b)\ 50i - 18j + 10k$

$c)\ -18i - 6j + 6k$

$d)\ -13i + 2j + 21k$

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24 Find each of the following for

$$x = -9i + 4j + 3k, y = 6i - 2j - 7k, and\ z = -2i + 2j + 4k.$$

$$-6y - 9z$$

$a)\ -22i + 14j + -k$

$b)\ 50i - 18j + 10k$

$c)\ -18i - 6j + 6k$

$d)\ -13i + 2j + 21k$

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12 GENERAL

MATH ENG

LESSON 8-5

Dot and Cross Products of Vectors in Space

Mr. Ahmed Ata
The Featured Program
2024-2025

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Dot and Cross Products of Vectors in Space

KeyConcept Dot Product and Orthogonal Vectors in Space

The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is defined as $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$. The vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Find the dot product of u and v. Then determine if u and v are orthogonal.

1 $\mathbf{u} = \langle 3, -9, 6 \rangle, \mathbf{v} = \langle -8, 2, 7 \rangle$

$\mathbf{u} = \langle 3, -5, 4 \rangle, \mathbf{v} = \langle 5, 7, 5 \rangle$

2 $\mathbf{u} = \langle 4, -2, -3 \rangle, \mathbf{v} = \langle 1, 3, -2 \rangle$

$\mathbf{u} = 3\mathbf{i} - 10\mathbf{j} + \mathbf{k}, \mathbf{v} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

As with vectors in a plane, if θ is the angle between nonzero vectors **a** and **b**, then $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$.

Find the angle θ between vectors u and v to the nearest tenth of a degree.

3 $\mathbf{u} = \langle 3, -2, 2 \rangle, \mathbf{v} = \langle 1, 4, -7 \rangle$

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4 Find the angle θ between vectors u and v to the nearest tenth of a degree.

$\mathbf{u} = -6\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}, \mathbf{v} = -4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

5 Find the cross product of u and v. Then show that u × v is orthogonal to both u and v.

$\mathbf{u} = \langle -1, 3, 5 \rangle, \mathbf{v} = \langle 2, -6, -3 \rangle$

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6 Find the cross product of u and v. Then show that u × v is orthogonal to both u and v.

$\mathbf{u} = -4\mathbf{i} + \mathbf{j} + 8\mathbf{k}, \mathbf{v} = 3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$

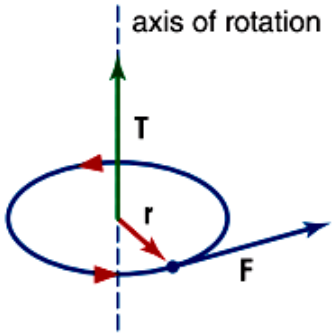
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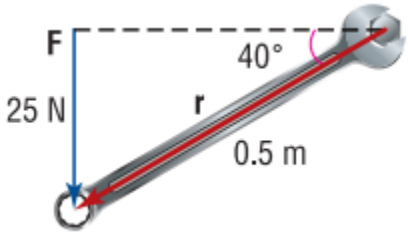
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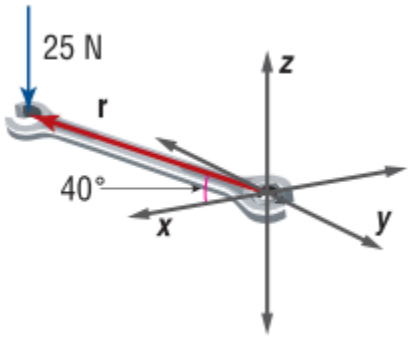
You can use the cross product to find a vector quantity called **torque**. Torque measures how effectively a force applied to a lever causes rotation along the axis of rotation. The torque vector **T** is perpendicular to the plane containing the directed distance **r** from the axis of rotation to the point of the applied force and the applied force **F** as shown. Therefore, the torque vector is $\mathbf{T} = \mathbf{r} \times \mathbf{F}$ and is measured in newton-meters (N • m).



7 **Tarek uses a lug wrench to tighten a lug nut. The wrench he uses is 50 centimeters or 0.5 meter long. Find the magnitude and direction of the torque about the lug nut if he applies a force of 25 newtons straight down to the end of the handle when it is 40° below the positive *x*-axis as shown.**



8 **Find the magnitude of the torque if Tarek applied the same amount of force to the end of the handle straight down when the handle makes a 40° angle above the positive *x*-axis as shown in Figure**



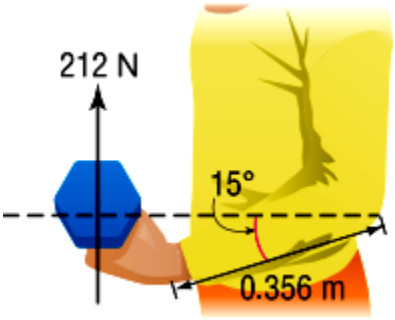
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9 **A weightlifter doing bicep curls applies 212 newtons of force to lift the dumbbell. The weightlifter's forearm is 0.356 meters long and he begins the bicep curl with his elbow bent at a 15° angle below the horizontal in the direction of the positive x-axis.**

a. Find the vector representing the torque about the weightlifter's elbow in component form.



b. Find the magnitude and direction of the torque.

Find the area of the parallelogram with adjacent sides u and v.

10 $\mathbf{u} = \langle 2, -5, 3 \rangle$, $\mathbf{v} = \langle 4, 6, -1 \rangle$

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11 $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$.

12 $\mathbf{u} = -6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

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13 Find the volume of the parallelepiped with adjacent edges $\mathbf{t} = 4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, and $\mathbf{v} = \mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$.

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14 Find the volume of the parallelepiped with adjacent edges $\mathbf{t} = 2\mathbf{j} - 5\mathbf{k}$, $\mathbf{u} = -6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

15 **Find a vector that is orthogonal to each vector.** $\langle 3, -8, 4 \rangle$

16 **Given v and u • v, find u.** $\mathbf{v} = \langle 2, -4, -6 \rangle$, $\mathbf{u} \cdot \mathbf{v} = -22$

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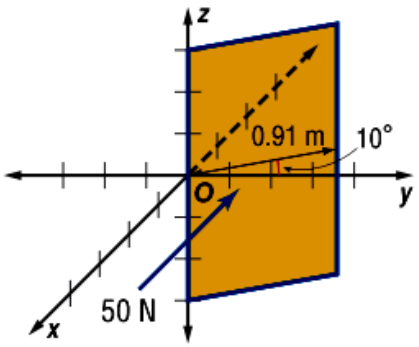
17 Determine whether each pair of vectors are parallel.

$\mathbf{m} = \langle 2, -10, 6 \rangle$, $\mathbf{n} = \langle 3, -15, 9 \rangle$

18 Determine whether the points are collinear.

$(-1, 7, 7)$, $(-3, 9, 11)$, $(-5, 11, 13)$

19 A restaurant server applies 50 newtons of force to open a door. Find the magnitude and direction of the torque about the door's hinge.



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12 GENERAL

MATH ENG

LESSON 8-6

Transformation Matrices in Three-Dimensional Space

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

Prepared by :

MR- AHMED ATA

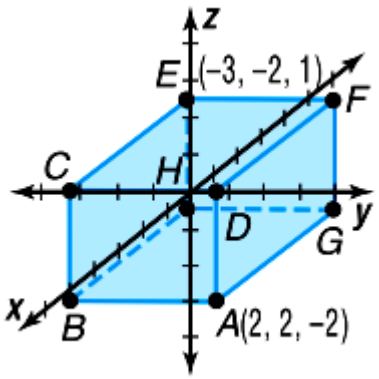
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Transformation Matrices in Three-Dimensional Space

1 **Find the coordinates of the vertices of the rectangular prism and represent them as a vertex matrix.**

$\overrightarrow{AE} = \langle -3\ -2, -2\ -2, 1, -(-2) \rangle$ or $\langle -5\ -4, 3 \rangle$

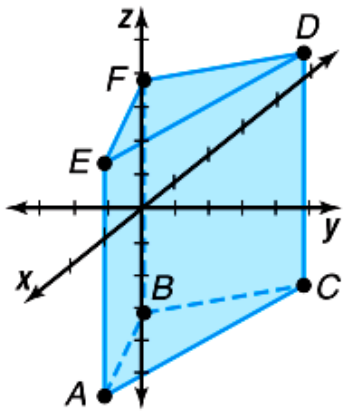


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2 She needs to translate a prism using the vector $\vec{a} = \langle 3, 3, 0 \rangle$. The vertices of the prism have the following coordinates.

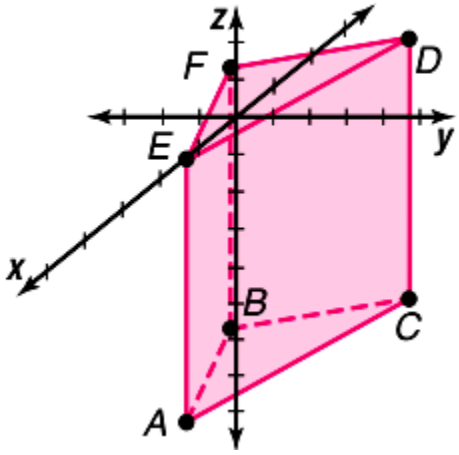
$A(2, 1, -4)$ $B(-1, -1, -4)$ $C(-2, 3, -4)$
$D(-2, 3, 3)$ $E(2, 1, 3)$ $F(-1, -1, 3)$



a. Write a matrix that will have such an effect on the figure.

b. Find the coordinates of the vertices of the translated image.

c. Graph the translated image.



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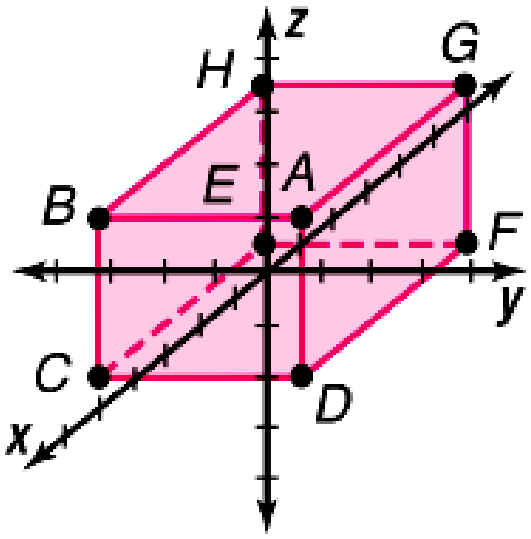
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3 Let M represent the vertex matrix of the rectangular prism in Example 1.

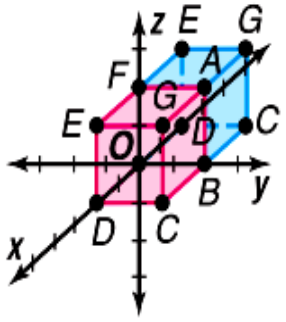
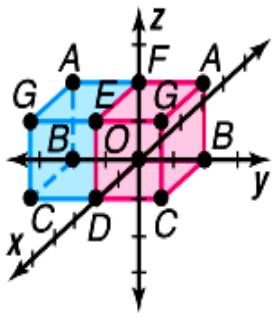
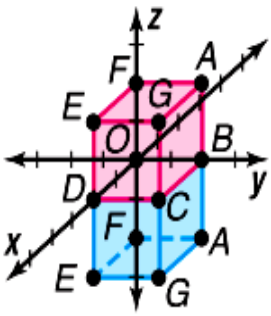
a. Find TM if $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

b. Graph the resulting image.

c. Describe the transformation represented by matrix T.



Reflection Matrices

Reflection Matrices		
For a reflection over the:	Multiply the vertex matrix by:	Resulting image
yz-plane	$R_{yz\text{-plane}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
xz-plane	$R_{xz\text{-plane}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
xy-plane	$R_{xy\text{-plane}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	

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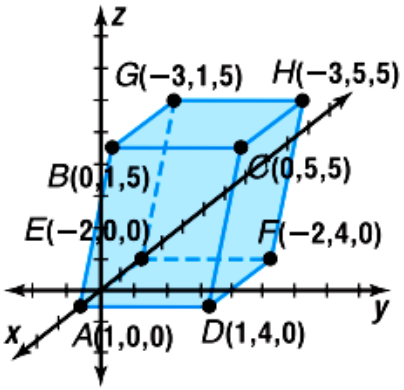
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4 A parallelepiped is a prism whose faces are all parallelograms as shown in the graph.

a. Find the vertex matrix for the transformation D where $k = 2$.

b. Draw a graph of the resulting figure.

c. What effect does transformation D have on the original figure?



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