

Grade 12 Advanced

EoT3 – 2023/2024



الدرس 6-1 | المساحة المحصورة بين منحنين

Lesson 6-1 | Area Between Curves

MCG ← FRQ



1 Find the area between two curves using definite integrations.

إيجاد مساحة المنطقة المحصورة بين منحنيين باستخدام التكامل المحدود

Exercises (1-18)

P414

جد المساحة المحصورة بين المنحنيين على الفترة المعطاة

find the area between the curves on the given interval.

1. $y = x^3, y = x^2 - 1, 1 \leq x \leq 3$

- A) $A = \frac{10}{3}$
 B) $A = \frac{40}{3}$
 C) $A = \frac{4}{3}$
 D) $A = \frac{40}{31}$

$$\begin{aligned}
 A &= \int_1^3 (\text{upper} - \text{lower}) dx \\
 &= \int_1^3 x^3 - (x^2 - 1) dx \\
 &= \int_1^3 x^3 - x^2 + 1 dx \\
 &= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 + x \right]_1^3 \\
 &= \frac{40}{3}
 \end{aligned}$$



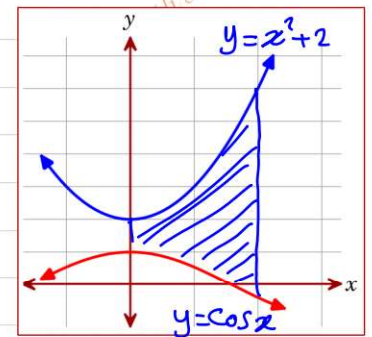
جد المساحة المحصورة بين المنحنيين على الفترة المعطاة

find the area between the curves on the given interval.

2. $y = \cos x, y = x^2 + 2, 0 \leq x \leq 2$

- A) $A = \frac{20}{3} - \sin 2$
 B) $A = \frac{20}{3} - \cos 2$
 C) $A = \frac{20}{3} + \sin 2$
 D) $A = \frac{20}{3} - \cos 2$

$$\begin{aligned}
 A &= \int_0^2 x^2 + 2 - \cos x dx \\
 &= \left[\frac{1}{3}x^3 + 2x - \sin x \right]_0^2 \\
 &= \left(\frac{8}{3} + 4 - \sin 2 \right) - (0 + 0 + 0) \\
 &= \frac{20}{3} - \sin 2
 \end{aligned}$$

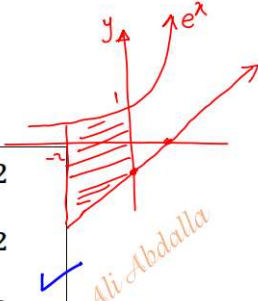


جد المساحة المحصورة بين المنحنيين على الفترة المعطاة

find the area between the curves on the given interval.

3. $y = e^x, y = x - 1, -2 \leq x \leq 0$

- A) $A = 4 + e^{-2}$
 B) $A = 5 - e^{-2}$
 C) $A = 5 + e^{-2}$
 D) $A = 4 - e^{-2}$



$$\begin{aligned} A &= \int_{-2}^0 e^x - (x - 1) dx \\ &= \int_{-2}^0 e^x - x + 1 dx \\ &= \left[e^x - \frac{1}{2}x^2 + x \right]_{-2}^0 \\ &= e^0 - \left(\frac{1}{2}(-2)^2 - 2 - 2 \right) \\ &= 5 - e^{-2} \end{aligned}$$

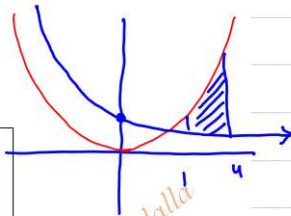


جد المساحة المحصورة بين المنحنيين على الفترة المعطاة

find the area between the curves on the given interval.

4. $y = e^{-x}, y = x^2, 1 \leq x \leq 4$

- A) $A = 21.54$
 B) $A = 14.72$
 C) $A = 65.20$
 D) $A = 20.65$



$$\begin{aligned} A &= \int_1^4 x^2 - e^{-x} dx \\ &= 20.65 \end{aligned}$$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

5. $y = x^2 - 1, y = 7 - x^2$

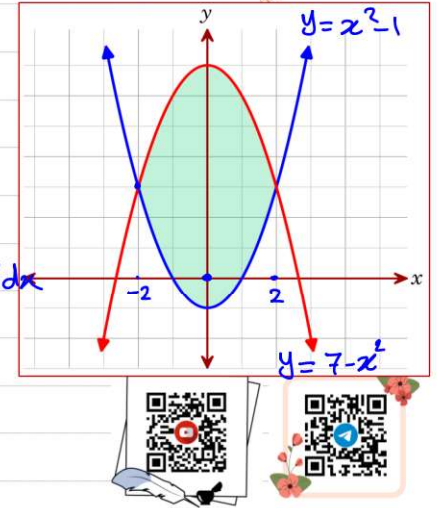
A) $A = \int_{-2}^2 (7 - x^2) - (x^2 - 1) dx$

B) $A = \int_{-2}^2 7 - 2x^2 dx$

C) $A = \int_{-2}^2 (x^2 - 1) - (7 - x^2) dx$

D) $A = \pi \int_{-2}^2 (x^2 - 1) - (7 - x^2) dx$

• $A = \int_{-2}^2 7 - x^2 - (x^2 - 1) dx$
 $= \int_{-2}^2 7 - x^2 - x^2 + 1 dx$
 $= \int_{-2}^2 8 - 2x^2 dx = 2 \int_0^2 8 - 2x^2 dx$
 $x^2 - 1 = 7 - x^2$
 $2x^2 = 8 \Rightarrow x^2 = 4$
 $\Rightarrow x = \pm 2$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

6. $y = x^2 - 1, y = \frac{1}{2}x^2$

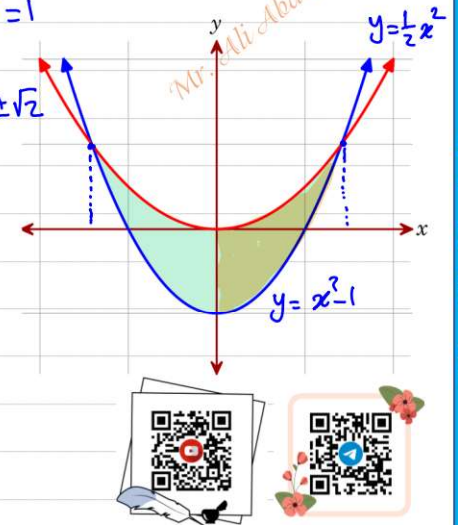
A) $A = \int_{-\sqrt{2}}^{\sqrt{2}} (x^2 - 1) - \frac{1}{2}x^2 dx$ X

B) $A = \int_0^{\sqrt{2}} \frac{1}{2}x^2 - (x^2 - 1) dx$ X

C) $A = 2 \int_0^{\sqrt{2}} \frac{1}{2}x^2 - (x^2 - 1) dx$

D) $A = \pi \int_{-\sqrt{2}}^{\sqrt{2}} \frac{1}{2}x^2 - (x^2 - 1) dx$

$x^2 - 1 = \frac{1}{2}x^2$ $x^2 - \frac{1}{2}x^2 = 1$
 $\frac{1}{2}x^2 = 1 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$
 $A = \int_{-\sqrt{2}}^{\sqrt{2}} \frac{1}{2}x^2 - (x^2 - 1) dx$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

7. $y = x^3, y = 3x + 2$

$$x^3 = 3x + 2$$

$$x^3 - 3x - 2 = 0$$

A) $A = \frac{1}{3}$

B) $A = \pi - 1$

C) $A = 2\pi - 1$

D) $A = \frac{27}{4}$

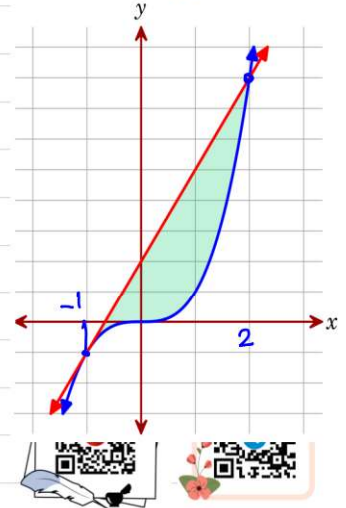
$$\Rightarrow A = \int_{-1}^2 (3x + 2 - x^3) dx$$

$$= \left[\frac{3}{2}x^2 + 2x - \frac{1}{4}x^4 \right]_{-1}^2$$

$$= (6 + 4 - 4) - \left(\frac{3}{2} - 2 - \frac{1}{4} \right)$$

$$= 6 + \frac{3}{4}$$

$$= \frac{27}{4}$$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

8. $y = \sqrt{x}, y = x^2$

A) $A = \frac{1}{3}$

B) $A = \pi - 1$

C) $A = 2\pi - 1$

D) $A = \frac{27}{4}$

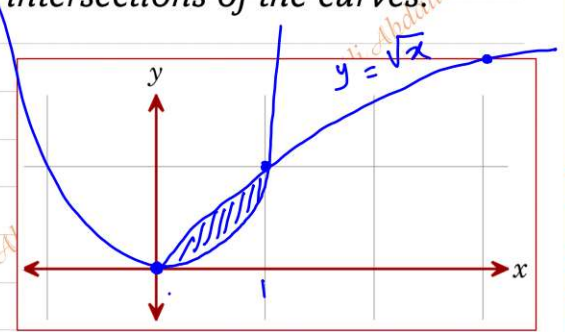
$$A = \int_0^1 (\sqrt{x} - x^2) dx$$

$$= \int_0^1 (x^{\frac{1}{2}} - x^2) dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3 \right]_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3} \right) - (0)$$

$$= \left(\frac{1}{3} \right)$$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

9. $y = 4xe^{-x^2}$, $y = |x|$

A) $A = \int_0^{\ln 4} (4xe^{-x^2} - x) dx$

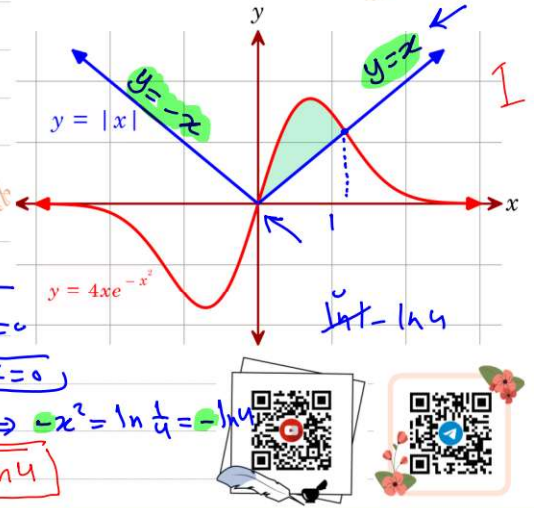
B) $A = \int_0^{\sqrt{\ln 4}} (4xe^{-x^2} - x) dx$

C) $A = \int_0^{\frac{1}{2}} (4xe^{-x^2} - x) dx$

D) $A = \pi \int_0^{\frac{3}{2}} (4xe^{-x^2} - x) dx$

$A = \int_0^{\sqrt{\ln 4}} 4xe^{-x^2} - x dx$

$4xe^{-x^2} = x \Rightarrow 4xe^{-x^2} - x = 0$
 $\Rightarrow x(4e^{-x^2} - 1) = 0 \Rightarrow x = 0$
 $4e^{-x^2} = 1 \Rightarrow e^{-x^2} = \frac{1}{4} \Rightarrow -x^2 = \ln \frac{1}{4} = -\ln 4$
 $x^2 = \ln 4 \Rightarrow x = \sqrt{\ln 4}$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

10. $y = \frac{2}{x^2 + 1}$, $y = |x|$

A) $A = \frac{1}{3}$

B) $A = \pi - 1$

C) $A = 2\pi - 1$

D) $A = \frac{27}{4}$

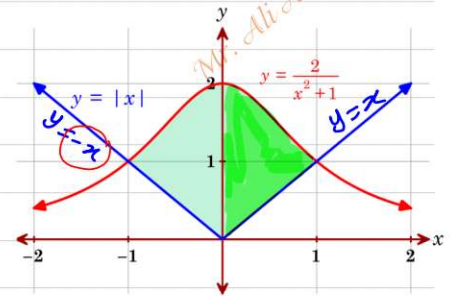
$A = \int_{-1}^0 \frac{2}{x^2+1} - (-x) dx$

$+ \int_0^1 \frac{2}{x^2+1} - x dx$

$A = 2 \left[\int_0^1 \frac{2}{x^2+1} - x dx \right]$

$= 2 \left[2 \tan^{-1} x - \frac{1}{2} x^2 \right]_0^1$

$= 2 \left[\left(\frac{\pi}{2} - \frac{1}{2} \right) - 0 \right] = \pi - 1$



ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

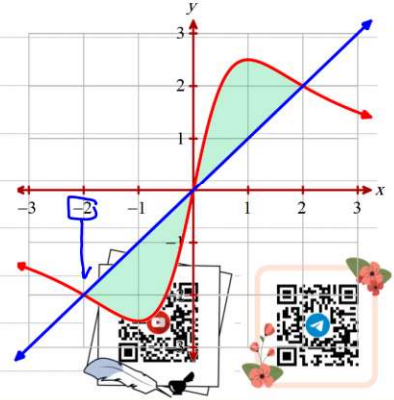
11. $y = \frac{5x}{x^2 + 1}, y = x$

$$A = \int_{-2}^0 x - \frac{5x}{x^2 + 1} dx + \int_0^2 \frac{5x}{x^2 + 1} - x dx$$

$$= 2.02359 + 2.02359$$

$$A = 2 \left[\int_0^2 \frac{5x}{x^2 + 1} - x dx \right]$$

$$= 2 \left[\frac{5}{2} \ln|x^2 + 1| - \frac{1}{2} x^2 \right]_0^2$$



- ☒ A) $A = 5 \ln(5) - 4$
☐ B) $A = 5 \ln(5) + 4$
☐ C) $A = 4 \ln(5) - 5$
☐ D) $A = 5 \ln(5)$

ارسم وجد مساحة المنطقة التي تحددها تقاطعات المنحنيات

Sketch and find the area of the region determined by the intersections of the curves.

12. $y = \sin x (0 \leq x \leq 2\pi), y = \cos x$

$$\sin x = \cos x \div \cos x$$

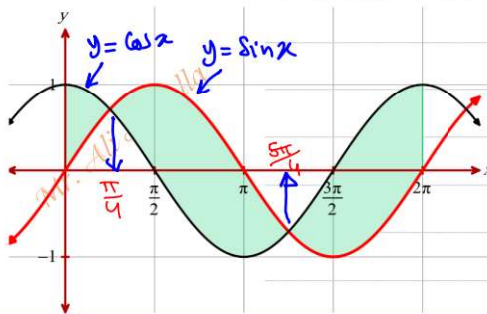
$$\tan x = 1 \Rightarrow I, III$$

$$x = \frac{\pi}{4}$$

$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$A = \int_0^{\pi/4} \cos x - \sin x dx + \int_{\pi/4}^{5\pi/4} \sin x - \cos x dx + \int_{5\pi/4}^{2\pi} \cos x - \sin x dx$$

=



- ☐ A) $A = 4\sqrt{3}$
☐ B) $A = 4\sqrt{5}$
☒ C) $A = 4\sqrt{2}$
☐ D) $A = 5\sqrt{7}$

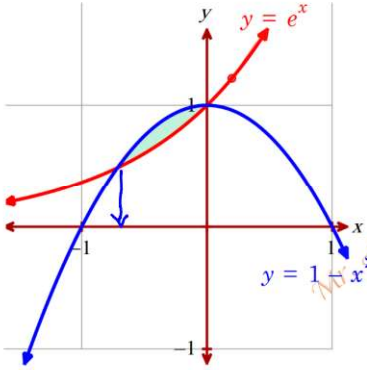
Radian



ارسم وقد ر المساحة التي تحددها تقاطعات المنحنيات

sketch and estimate the area determined by the intersections of the curves.

13. $y = e^x, y = 1 - x^2$



$$A = \int_{-0.7146}^0 (1 - x^2 - e^x) dx$$

$$= 0.08235$$

☒ A) $A = 0.08235$

☐ B) $A = 0.8235$

☐ C) $A = 8.235$

☐ D) $A = 1.8235$

$$e^x = 1 - x^2 \quad \downarrow$$

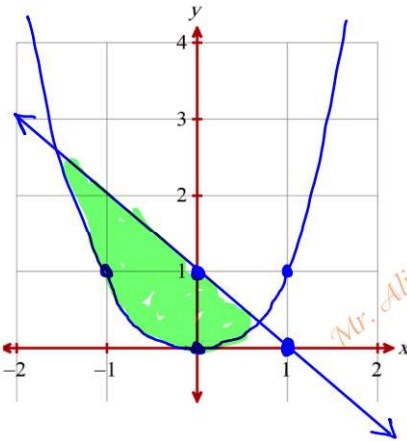
$$x = -0.7146 \quad x = 0$$



ارسم وقد ر المساحة التي تحددها تقاطعات المنحنيات

sketch and estimate the area determined by the intersections of the curves.

14. $y = x^4, y = 1 - x$



$$A = \int_{-1.2207}^{0.72449} (1 - x - x^4) dx$$

$$= 1.846$$

☒ A) $A = 1.846$

☐ B) $A = 1.821$

☐ C) $A = 1.486$

☐ D) $A = 1.832$

$$x^4 = 1 - x$$

$$x^4 + x - 1 = 0$$

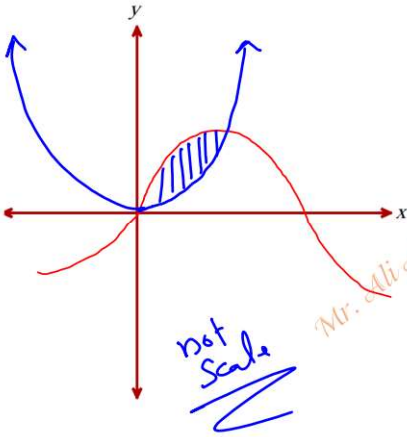
$$x = 0.72449 \quad x = -1.2207$$



ارسم وقدّر المساحة التي تحددها تقاطعات المنحنيات

sketch and estimate the area determined by the intersections of the curves.

15. $y = \sin x, y = x^2$



$$A = \int_0^{0.8767} \sin x - x^2 dx$$

$$= 0.135697$$

$$\approx 0.136$$

- ☒ A) $A = 0.135697$
☐ B) $A = 1.35697$
☐ C) $A = 2.35697$
☐ D) $A = 0.315697$

$$\sin x = x^2$$

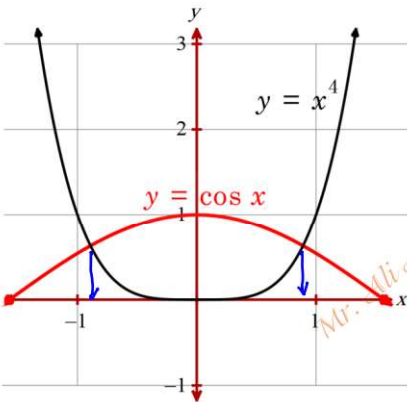
$$x = 0.8767, x = 0$$



ارسم وقدّر المساحة التي تحددها تقاطعات المنحنيات

sketch and estimate the area determined by the intersections of the curves.

16. $y = \cos x, y = x^4$



$$A = \int_{-0.89055}^{0.89055} \cos x - x^4 dx$$

$$= 1.331$$

- ☒ A) $A = 1.331$
☐ B) $A = 1.131$
☐ C) $A = 1.341$
☐ D) $A = 1.133$

$$\cos x = x^4$$

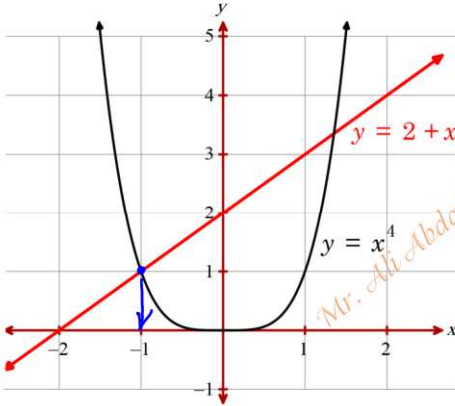
$$x = \pm 0.89055$$



ارسم وقد ر المساحة التي تحددها تقاطعات المنحنيات

sketch and estimate the area determined by the intersections of the curves.

17. $y = x^4$, $y = 2 + x$



$$A = \int_{-1}^{1.3532} (2 + x - x^4) dx$$

$$= 4.014$$

- ☒ A) $A = 4.014$
☐ B) $A = 4.104$
☐ C) $A = 3.014$
☐ D) $A = 3.104$

$$x^4 = 2 + x$$

$$x^4 - x - 2 = 0$$

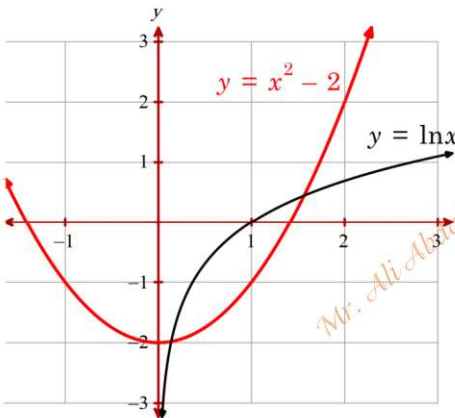
$$x = 1.3532, x = -1$$



ارسم وقد ر المساحة التي تحددها تقاطعات المنحنيات

sketch and estimate the area determined by the intersections of the curves.

18. $y = \ln x$, $y = x^2 - 2$



$$A = \int_{0.13793}^{1.56446} (\ln x - (x^2 - 2)) dx$$

integration by part

- ☒ A) $A = 1.214$
☐ B) $A = 1.124$
☐ C) $A = 1.012$
☐ D) $A = 2.124$

$$\ln x = x^2 - 2$$

$$x = 1.56446, x = 0.13793$$



16

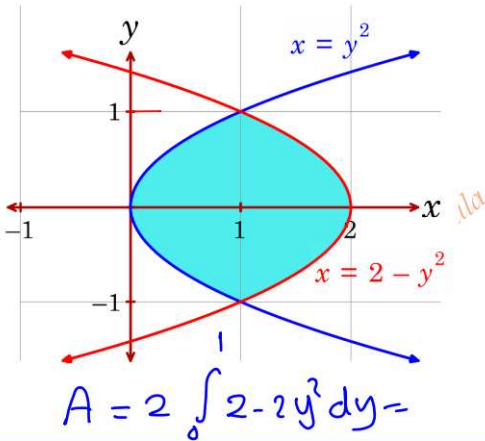
Compute the area of a region using definite integration with y as a variable x إيجاد مساحة منطقة متكامل محدود بمعلومية y عوضاً عن x

Example 1.6

Ex (19,20,22,24)

P413

P414

جد مساحة المنطقة المحصورة بين التمثيلين البيانيين $x = y^2$ و $x = 2 - y^2$.Find the area bounded by the graphs of $x = y^2$ and $x = 2 - y^2$.

$$A = \int (\text{Right} - \text{left}) dy$$

$$A = \int_{-1}^1 (2 - y^2) - (y^2) dy$$

$$\rightarrow \int_{-1}^1 (2 - 2y^2) dy = 2y - \frac{2}{3}y^3 \Big|_{-1}^1$$

$$= (2 - \frac{2}{3}) - (-2 + \frac{2}{3}) \leftarrow$$

$$= \frac{8}{3}$$

مقالتي
FRQ

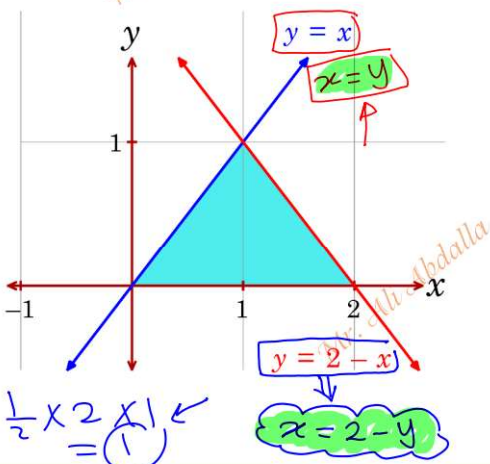
$$\begin{aligned} 2 - y^2 &= y^2 \\ 2 &= 2y^2 \\ 1 &= y^2 \\ y &= \pm 1 \end{aligned}$$



sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

ارسم وجهد مساحة المنطقة المحصورة بين المنحنيات العطاة. اختر متغير التكامل بحيث تتم كتابة المساحة كتكامل واحد

$$19. y = x, y = 2 - x, y = 0$$



$$A = \int_0^1 (2 - y) - y dy$$

$$= \int_0^1 (2 - 2y) dy$$

$$= 2y - y^2 \Big|_0^1$$

$$= (2 - 1) - (0 - 0)$$

$$= 1$$

مقالتي
FRQ

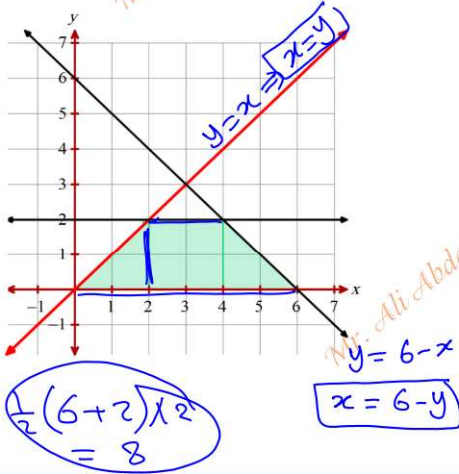
$$\begin{aligned} y &= 2 - y \\ 2y &= 2 \\ y &= 1 \end{aligned}$$



sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

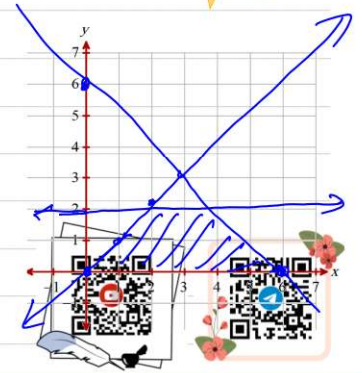
ارسم وجه مساحة المنطقة المحصورة بين المنحنيات العطاء. اختر متغير التكامل بحيث تتم كتابة المساحة كتكامل واحد

20. $y = x$, $y = 2$, $y = 6 - x$, $y = 0$



$$\begin{aligned}
 A &= \int_0^2 (6-y-y) dy \\
 &= \int_0^2 (6-2y) dy \\
 &= 6y - y^2 \Big|_0^2 \\
 &= (12-4) - (0) \\
 &= 8
 \end{aligned}$$

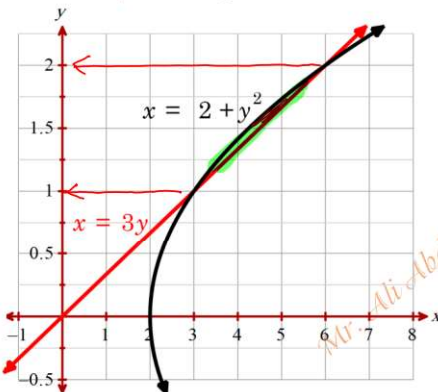
مقالتي
FRQ



sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

ارسم وجه مساحة المنطقة المحصورة بين المنحنيات العطاء. اختر متغير التكامل بحيث تتم كتابة المساحة كتكامل واحد

22. $x = 3y$, $x = 2 + y^2$



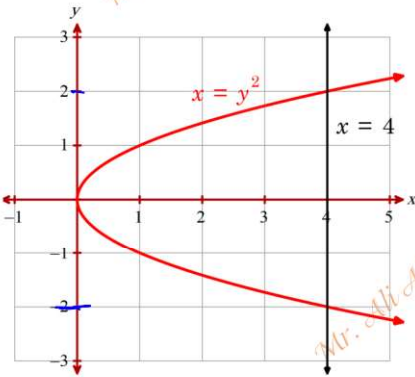
$$\begin{aligned}
 A &= \int_1^2 (3y - (2 + y^2)) dy \\
 &= \int_1^2 (3y - 2 - y^2) dy \\
 &= \left[\frac{3}{2}y^2 - 2y - \frac{1}{3}y^3 \right]_1^2 \\
 &= \left(6 - 4 - \frac{8}{3} \right) - \left(\frac{3}{2} - 2 - \frac{1}{3} \right) \\
 &= \left[\frac{1}{6} \right]
 \end{aligned}$$

مقالتي
FRQ



sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

24. $x = y^2$, $x = 4$



ارسم وجه مساحة المنطقة المحصورة بين المنحنيات المعطاة. اختر متغير التكامل بحيث تتم كتابة المساحة كتكامل واحد

$$A = \int_{-2}^2 (4 - y^2) dy = 2 \int_0^2 (4 - y^2) dy$$

مقالتي
FRQ

$$= \left(4y - \frac{1}{3}y^3 \right) \Big|_{-2}^2$$

$$= \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right)$$

$$= 16 - \frac{16}{3}$$

$$= \boxed{\frac{32}{3}}$$

$$\begin{aligned} y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

