

Full Revision

Transformations on Coordinate Plane

Summary of transformations Rules

1. Translation: $(x, y) \rightarrow (x \mp k, y \mp k)$, where k is the number of units the figure moved (*Translated*)

A) If a **horizontal translation** is applied, then the x coordinate will change. (*The figure goes right or left*)

B) If a **vertical translation** is applied, then the y coordinate will change. (*The figure goes up or down*)

2. Dilation: $(x, y) \rightarrow (kx, ky)$, where k is the scale factor applied for the dilation.

A) If an **Enlargement** is applied, then $k > 1$

B) If a **Reduction** is applied, then $0 < k < 1$

3. Reflection:

A) $(x, y) \rightarrow (-x, y)$: **Reflection across y axis**

B) $(x, y) \rightarrow (x, -y)$: **Reflection across x axis**

C) $(x, y) \rightarrow (2k - x, y)$: **Reflection across $x = k$**

D) $(x, y) \rightarrow (x, 2k - y)$: **Reflection across $y = k$**

E) $(x, y) \rightarrow (y, x)$: **Reflection across $y = x$**

4. Rotation:

A) $(x, y) \rightarrow (-y, x)$: **Rotation 90° counter clockwise or equivalently 270° clockwise**

B) $(x, y) \rightarrow (y, -x)$: **Rotation 270° counter clockwise or equivalently 90° clockwise**

C) $(x, y) \rightarrow (-x, -y)$: **Rotation 180° counter clockwise or equivalently 180° clockwise**

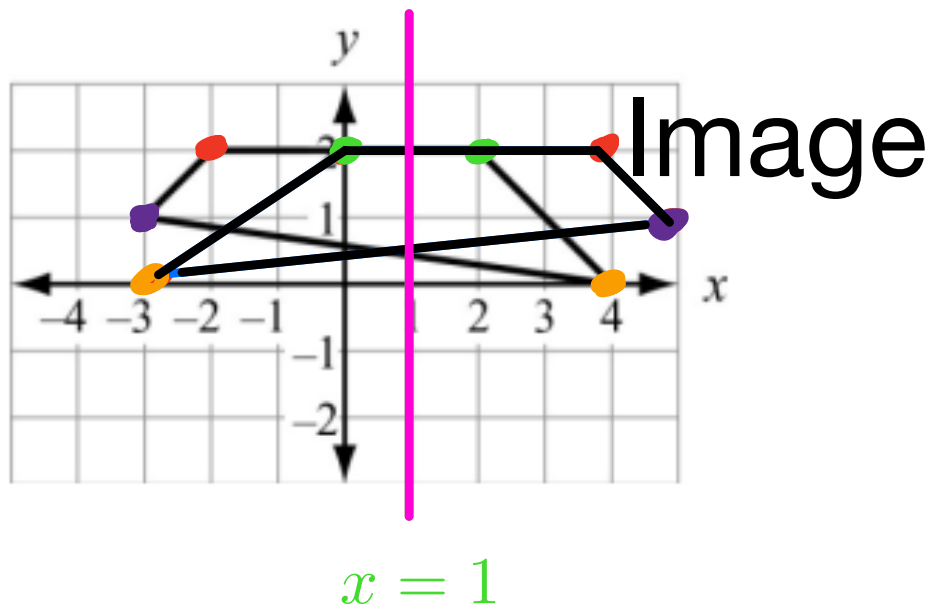
Ideas:

1. Given the Preimage (original figure), Graph the image on the coordinate plane.
2. Given the Preimage and the image on coordinate plane, describe the transformation.
3. Given the Preimage and the image on coordinate plane, write the rule of the transformation.

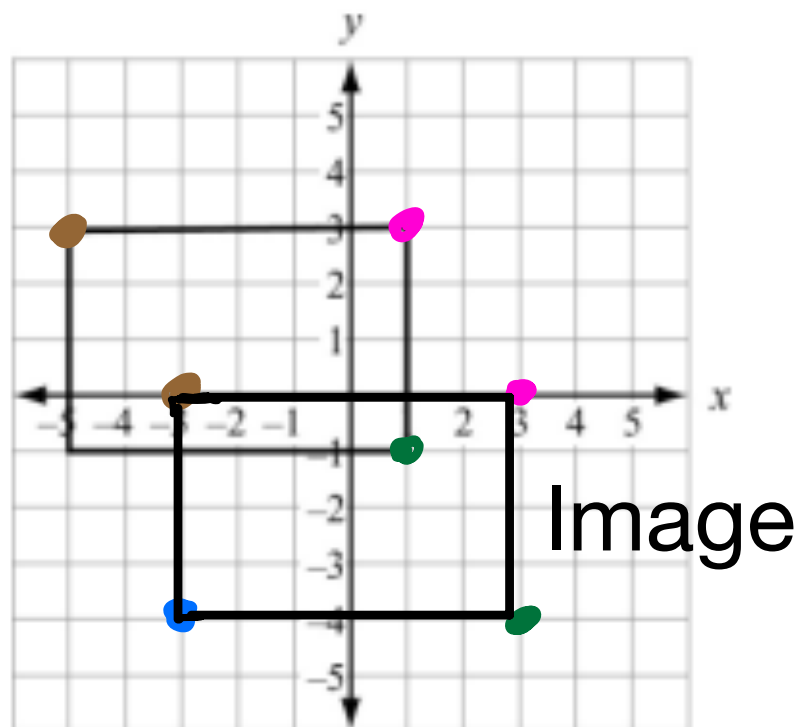
Exercises

Idea 1 Given the Preimage (original figure), Graph the image on the coordinate plane.

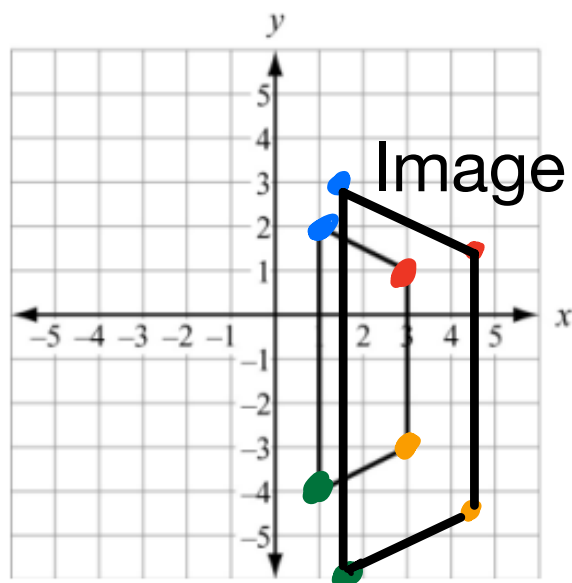
Apply Reflection across $x = 1$



Apply Translation 2 units right then 3 units down



Apply Dilation with scale factor 1.5



$$3 \times 1.5 = 4.5$$

$$1 \times 1.5 = 1.5$$

$$1 \times 1.5 = 1.5$$

$$2 \times 1.5 = 3$$

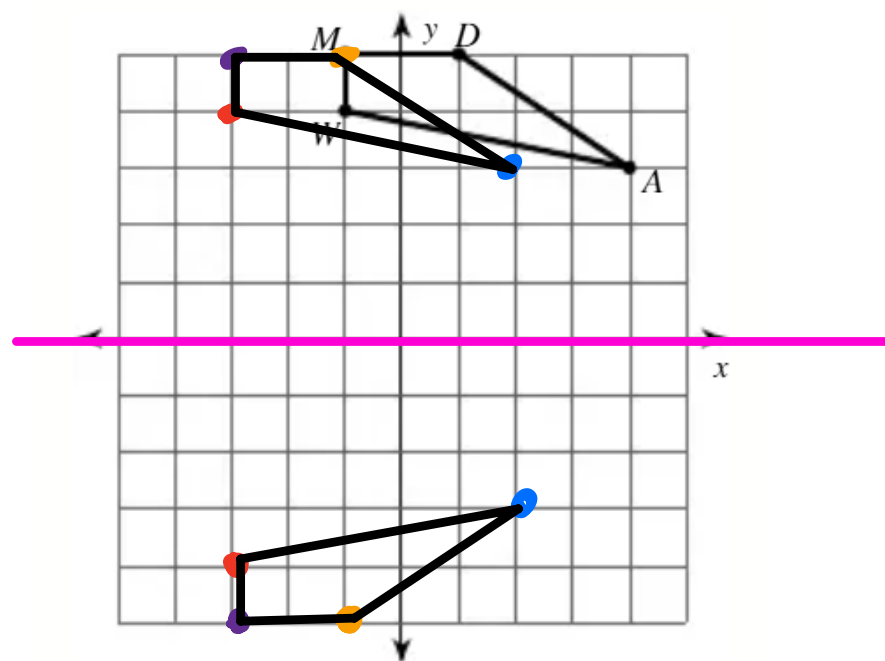
$$3 \times 1.5 = 4.5$$

$$-3 \times 1.5 = -4.5$$

$$1 \times 1.5 = 1.5$$

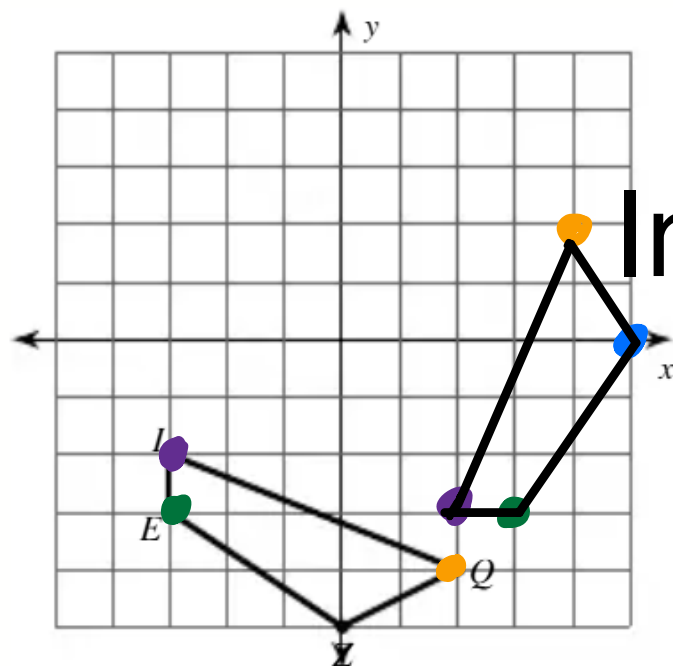
$$-4 \times 1.5 = -6$$

Translate the figure 2 units to the left , then apply reflection across x - axis



Image

Rotate the figure about the origin 90° clockwise.



Image

$$(x, y) \rightarrow (-y, x)$$

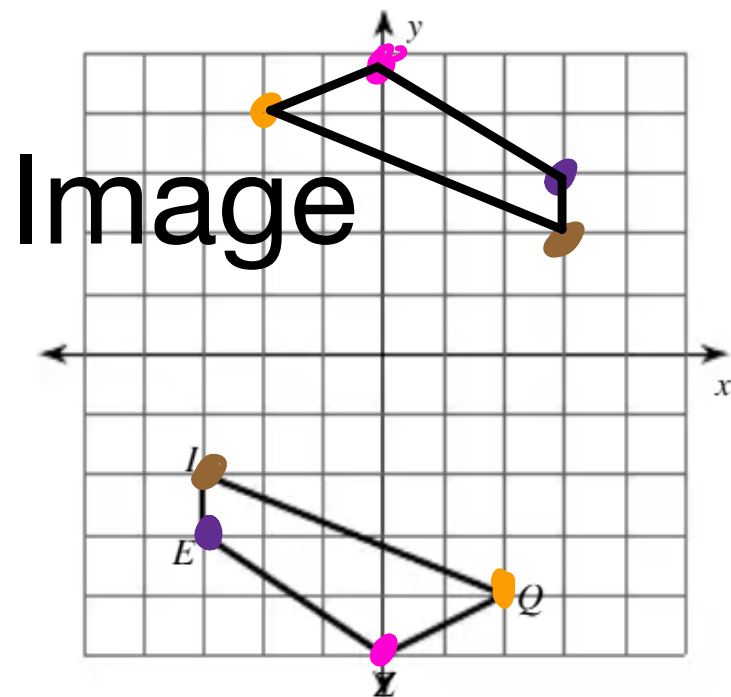
$$Q' (4, 2)$$

$$Z' (5, 0)$$

$$E' (3, -3)$$

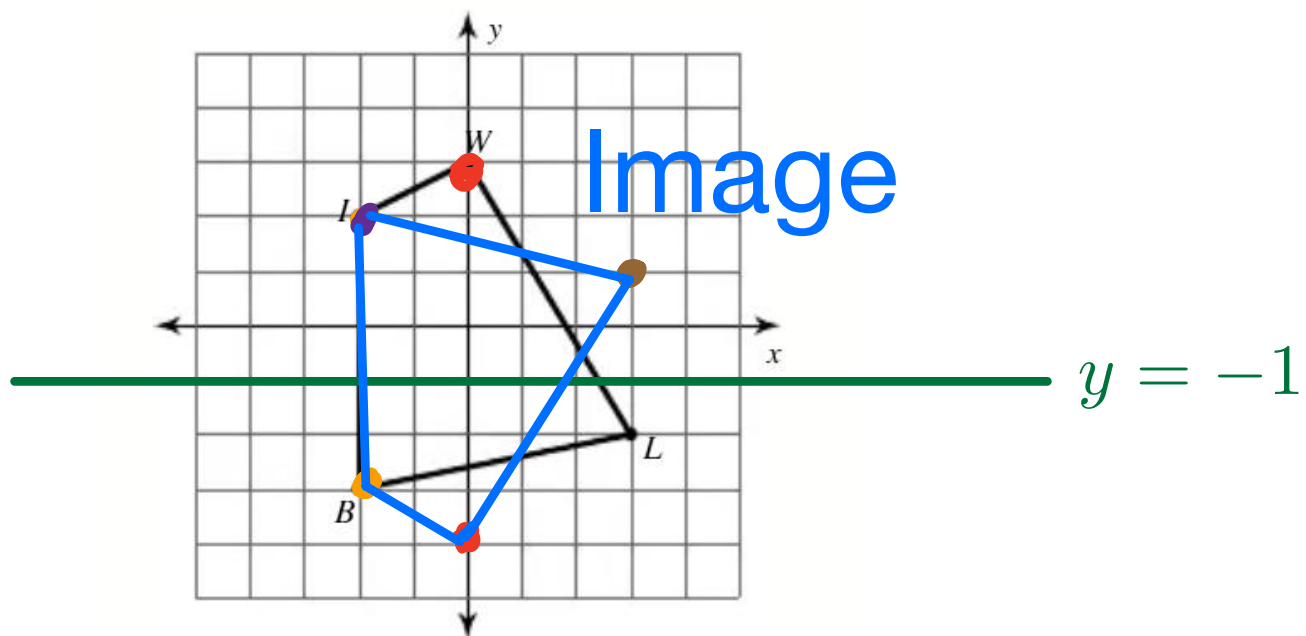
$$I' (2, -3)$$

Rotate the figure about the origin 180° counterclockwise

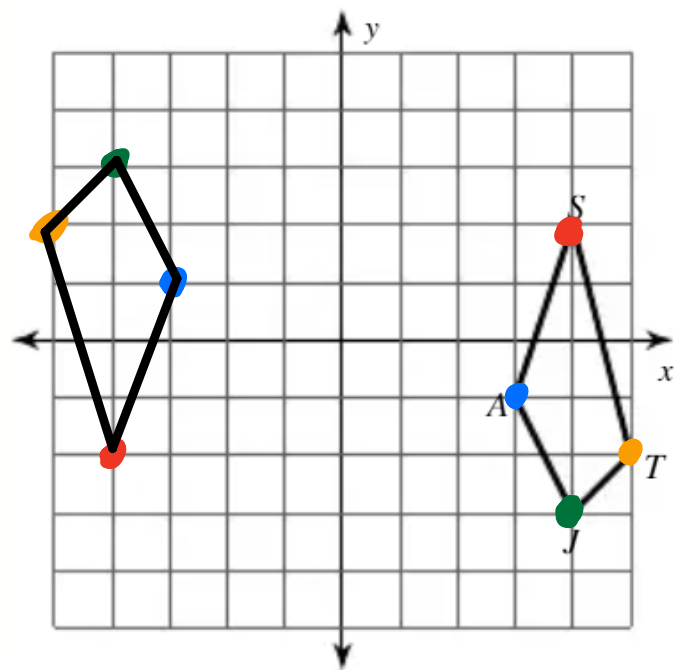


$$(x, y) \rightarrow (-x, -y)$$

Reflect the figure across $y = -1$, then translate 1 units up.

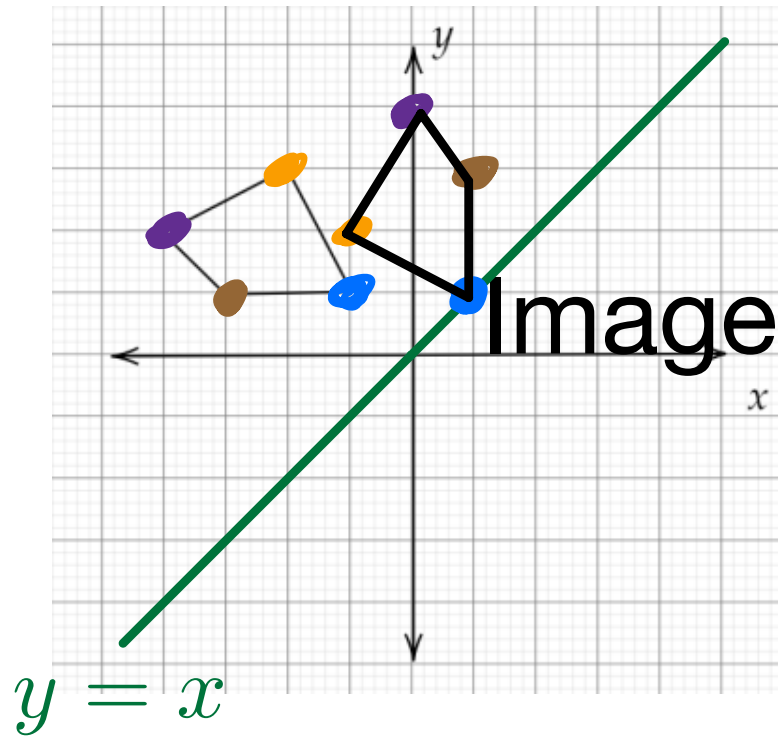


Reflect the figure about the origin 180° .



$$(x, y) \rightarrow (-x, -y)$$

Reflect the quadrilateral over the line $y = x$, then apply translation 2 units left, then rotate 180°

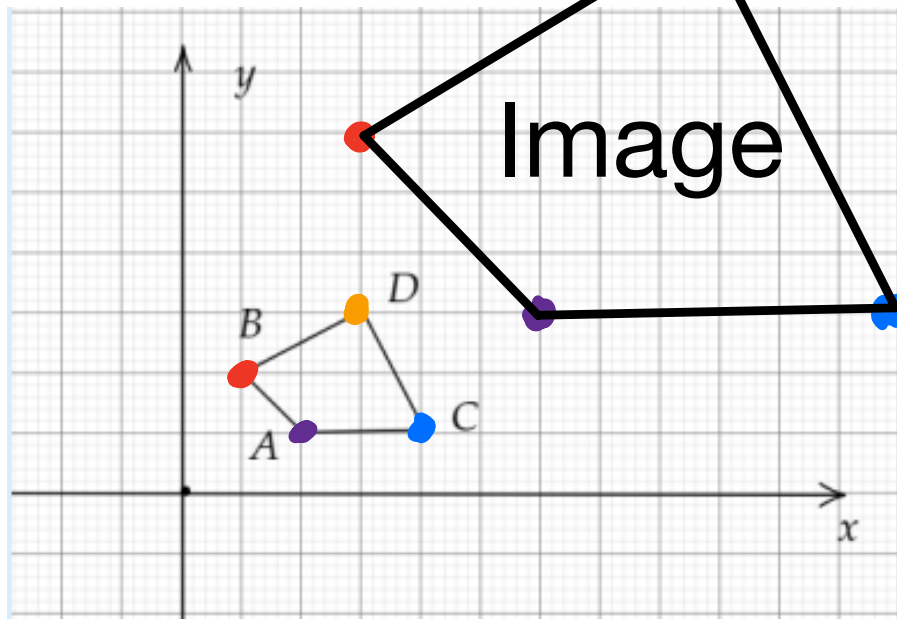


Reflection $(x, y) \rightarrow (y, x)$

Translation $(x, y) \rightarrow (x - 2, y)$

Rotation $180^\circ \rightarrow (-x, -y)$

Enlarge the shape by scale factor of 3 about the origin.



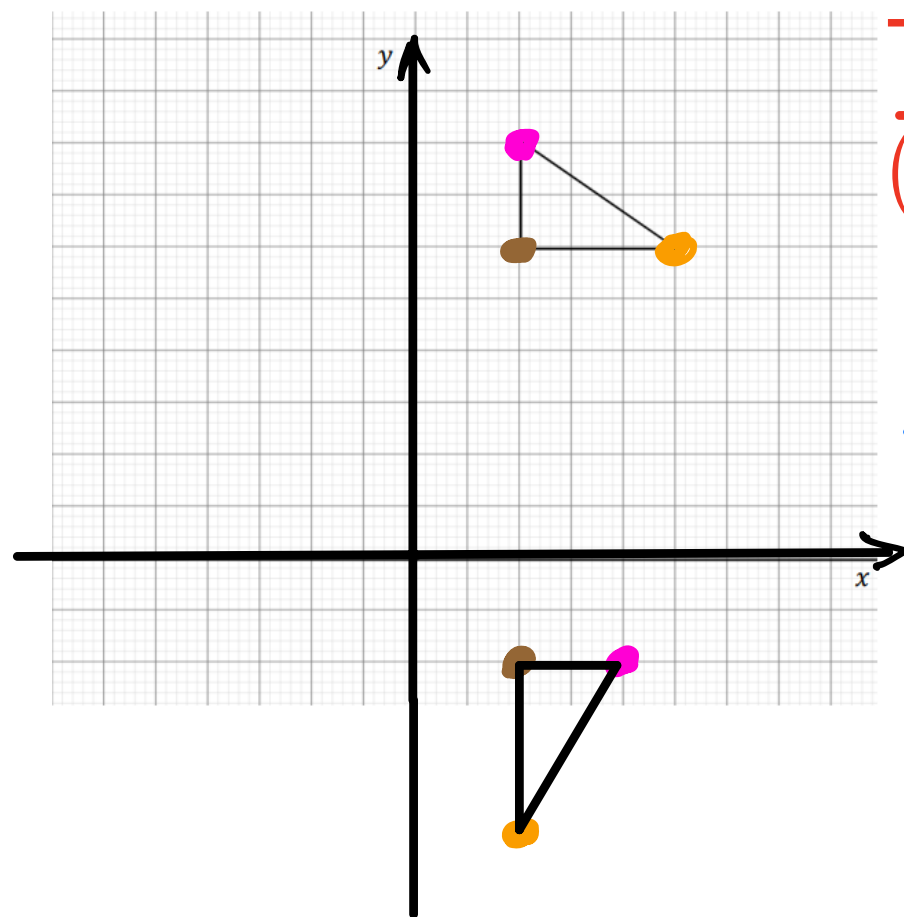
$$(1, 2) \rightarrow (3, 6)$$

$$(2, 1) \rightarrow (6, 3)$$

$$(4, 1) \rightarrow (12, 3)$$

$$(3, 3) \rightarrow (9, 9)$$

Translate the figure 4 units down, then rotate 270° counter clockwise about the origin.



Translation

$$(x, y) \rightarrow (x, y - 4)$$

Rotation

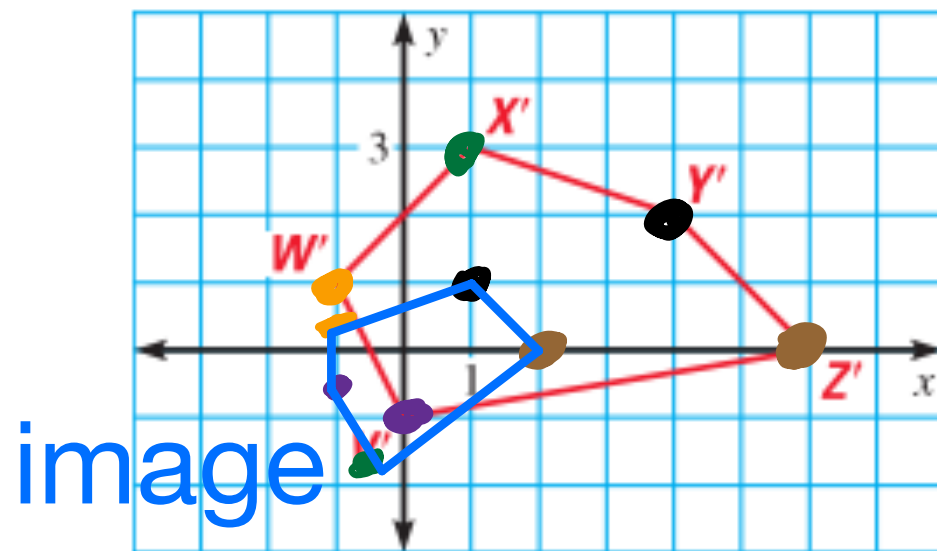
$$(x, y) \rightarrow (y, -x)$$

$$(2, 6) \rightarrow (2, 2) \rightarrow (2, -2)$$

$$(2, 8) \rightarrow (2, 4) \rightarrow (4, -2)$$

$$(5, 6) \rightarrow (5, \mathbf{2}) \rightarrow (2, -5)$$

Translate the figure 2 units left, then apply dilation with scale factor of $\frac{1}{2}$



Translation

$$(x, y) \rightarrow (x - 2, y)$$

Dilation

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y \right)$$

$$(0, -1) \rightarrow (-2, -1) \rightarrow (-1, -0.5)$$

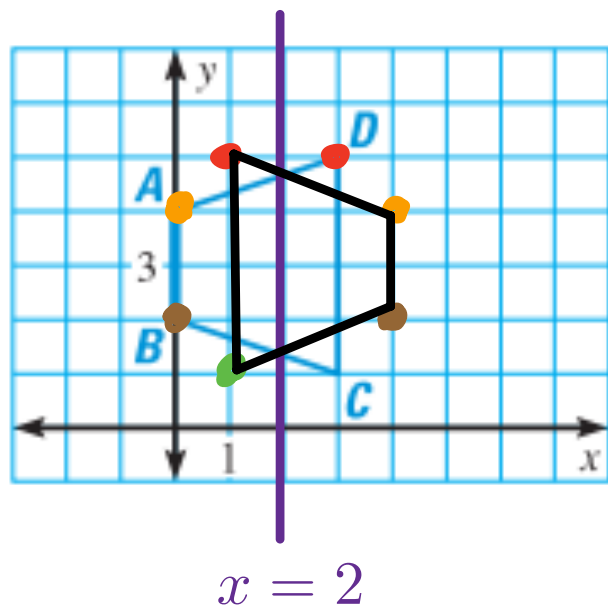
$$(-1, 1) \rightarrow (-3, 1) \rightarrow (-1.5, 0.5)$$

$$(1, 3) \rightarrow (-1, 3) \rightarrow (-0.5, 1.5)$$

$$(4, 2) \rightarrow (2, 2) \rightarrow (1, 1)$$

$$(6, 0) \rightarrow (4, 0) \rightarrow (2, 0)$$

Reflect the figure across the vertical line $x = 2$



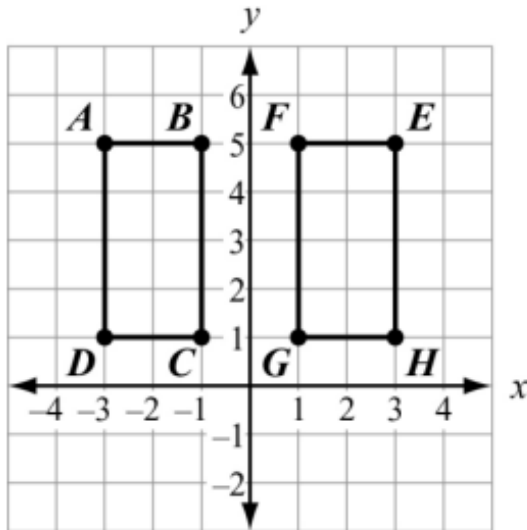
image

Idea 2 Given the Preimage and the image on coordinate plane, **describe** the transformation.

Idea 23 Given the Preimage and the image on coordinate plane, write the rule of the transformation.

* In each of the following figures, **describe** the transformation, then write the rule of the transformation.

Preimage is $ABCD$



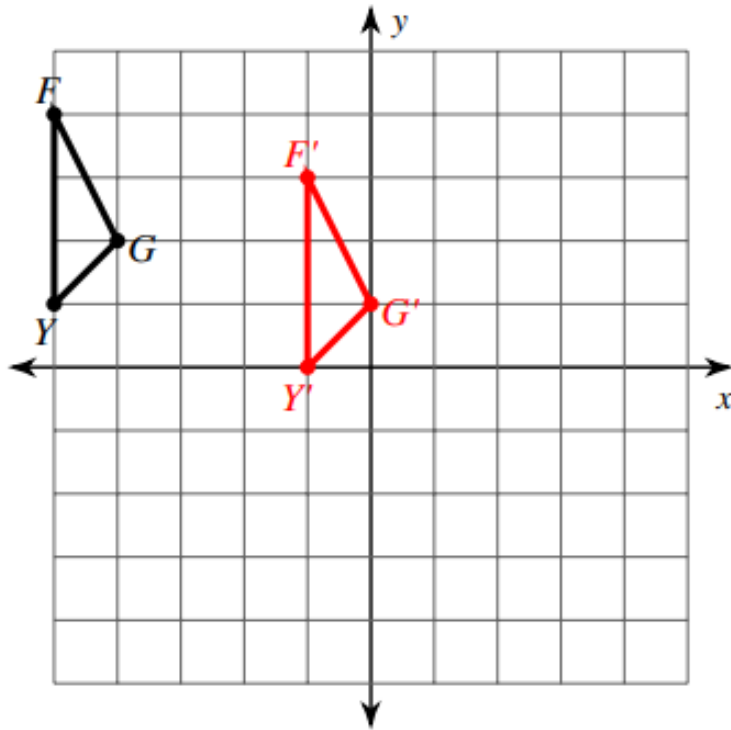
Reflection across y -axis
Rule: $(x, y) \rightarrow (-x, y)$

OR

Translation (4) units right

Rule: $(x, y) \rightarrow (x + 4, y)$

Preimage is FGY

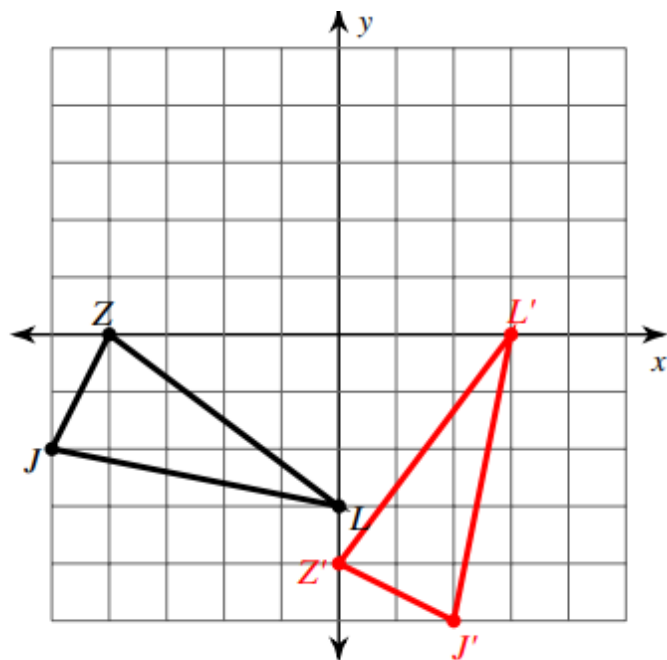


Translation
(4) units right
(1) unit down

Rule

$$(x, y) \rightarrow (x + 4, y - 1)$$

Preimage is ZLJ



Rotation 270° clockwise

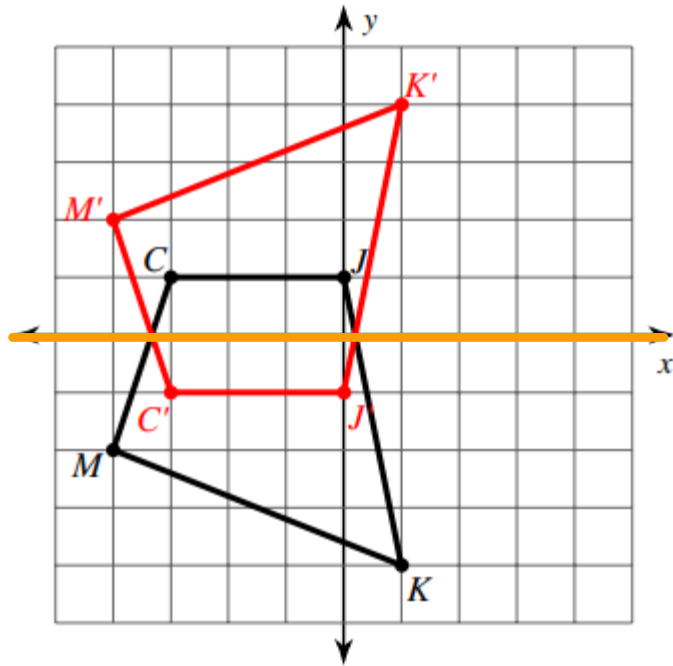
Rule

$$(x, y) \rightarrow (-y, x)$$

OR

90° counter clockwise

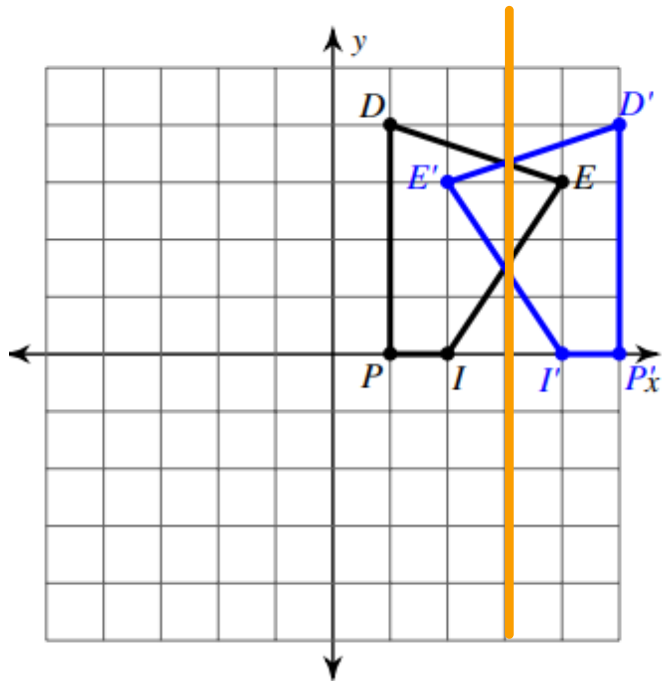
Preimage is $KJCM$



Reflection across x-axis

$$(x, y) \rightarrow (x, -y)$$

Preimage is $DEIP$

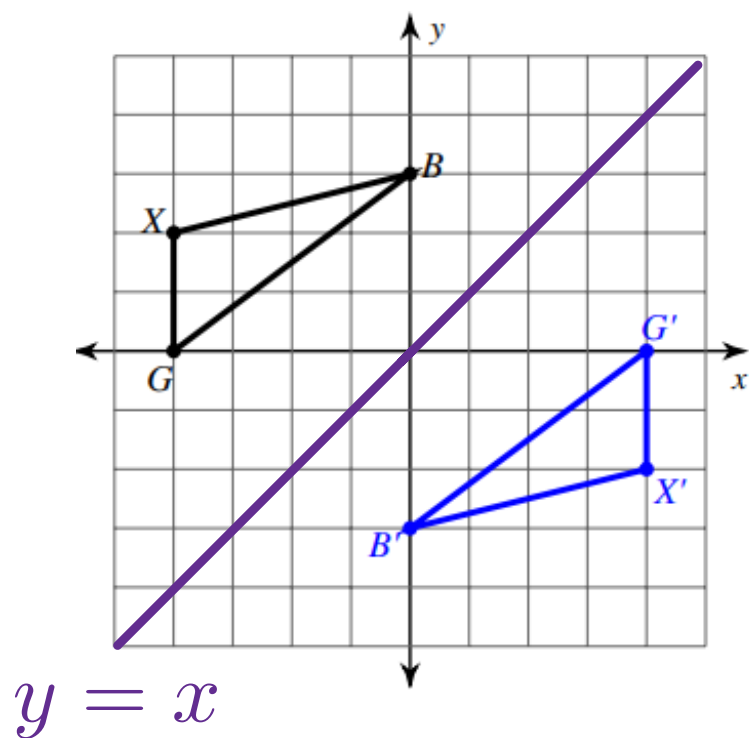


Reflection across $x = 3$

$$(x, y) \rightarrow (2k - x, y)$$

$$(x, y) \rightarrow (6 - x, y)$$

Preimage is XBG

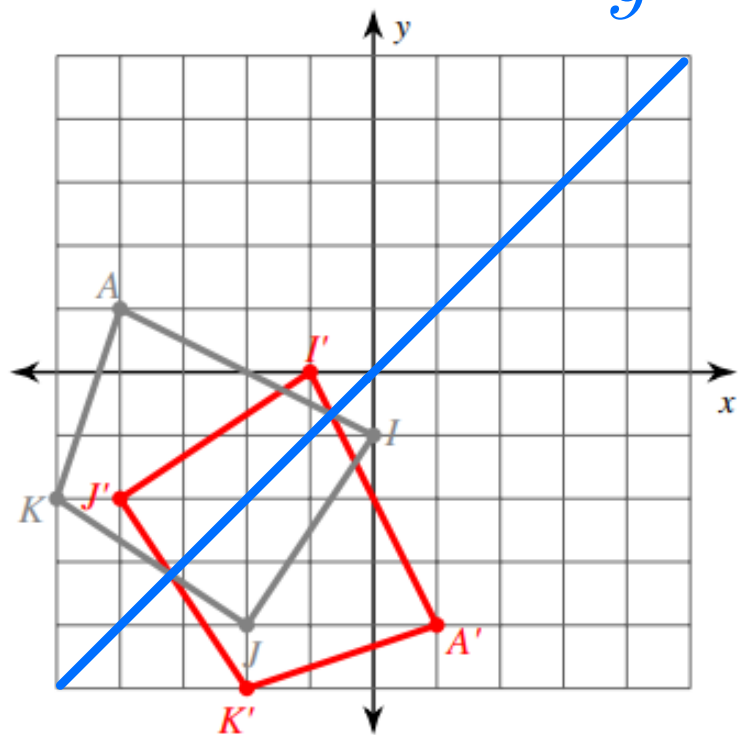


Reflection across $y=x$

$$(x, y) \rightarrow (y, x)$$

Preimage is $A'IJK$

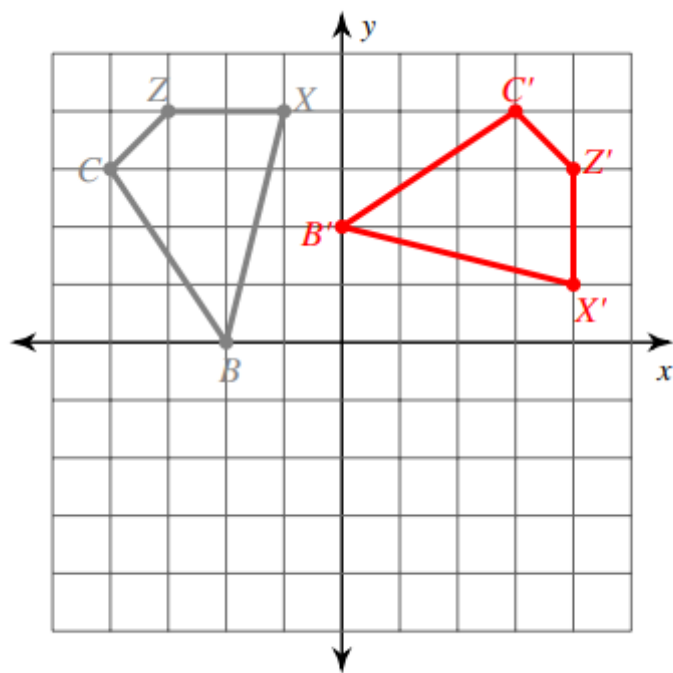
$$y = x$$



Reflection across $y=x$

$$(x, y) \rightarrow (y, x)$$

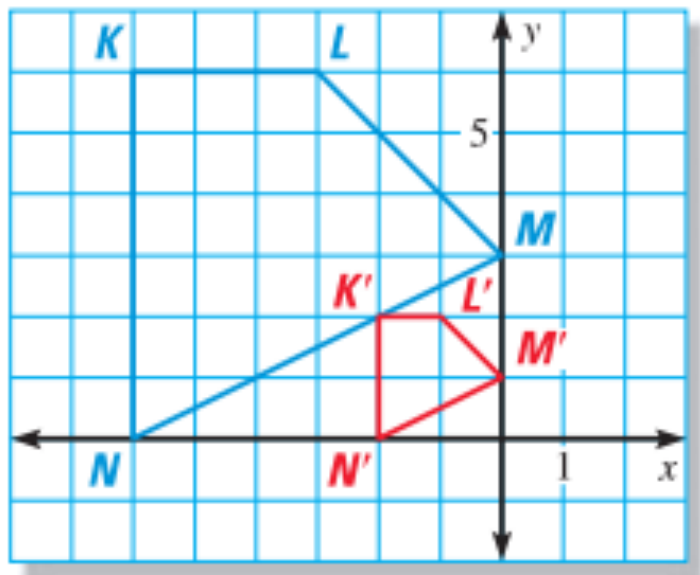
Preimage is $ZXBC$



Rotation 90° clockwise

$$(x, y) \rightarrow (y, -x)$$

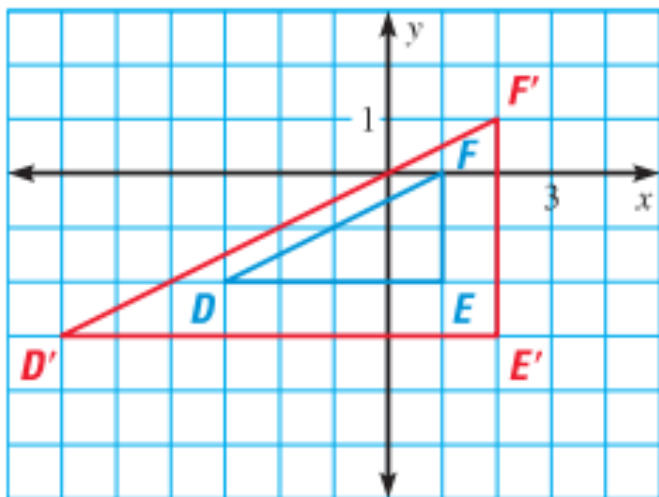
Preimage is the blue figure.



Dilation (scale factor $\frac{1}{3}$)

$$(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$$

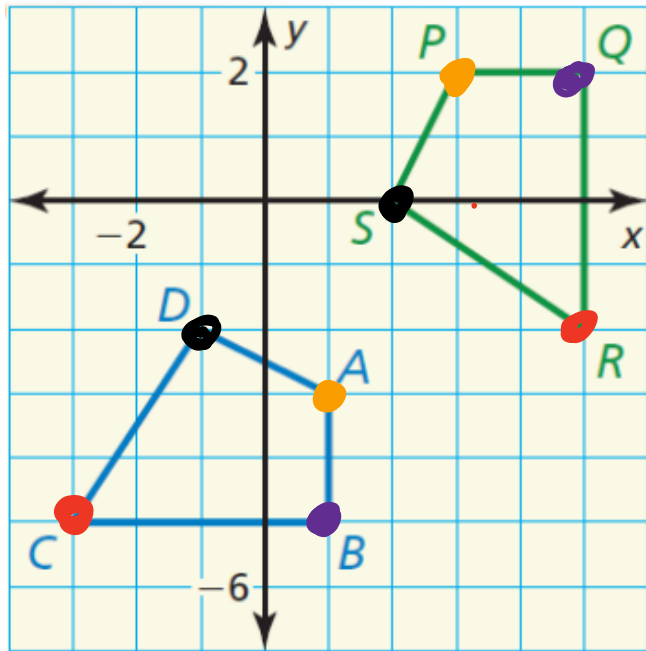
Preimage is the blue figure.



Dilation with scale factor 2

$$(x, y) \rightarrow (2x, 2y)$$

Preimage is $ABCD$



$$(-3, -5) \rightarrow (5, -2)$$

$$(1, -5) \rightarrow (5, 2)$$

$$(1, -3) \rightarrow (3, 2)$$

$$(-1, -2) \rightarrow (2, 0)$$

Rule

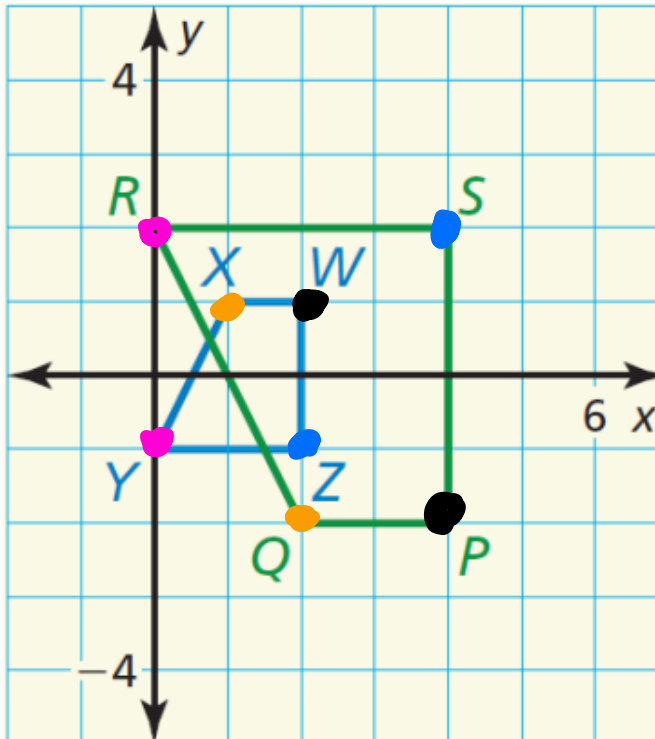
$$(x, y) \rightarrow (-y, x + 1)$$

Rotation 270° clockwise

OR

Rotation 90° counterclockwise

Preimage is $XWZY$



$$(2, 1) \rightarrow (4, -2)$$

$$(1, 1) \rightarrow (2, -2)$$

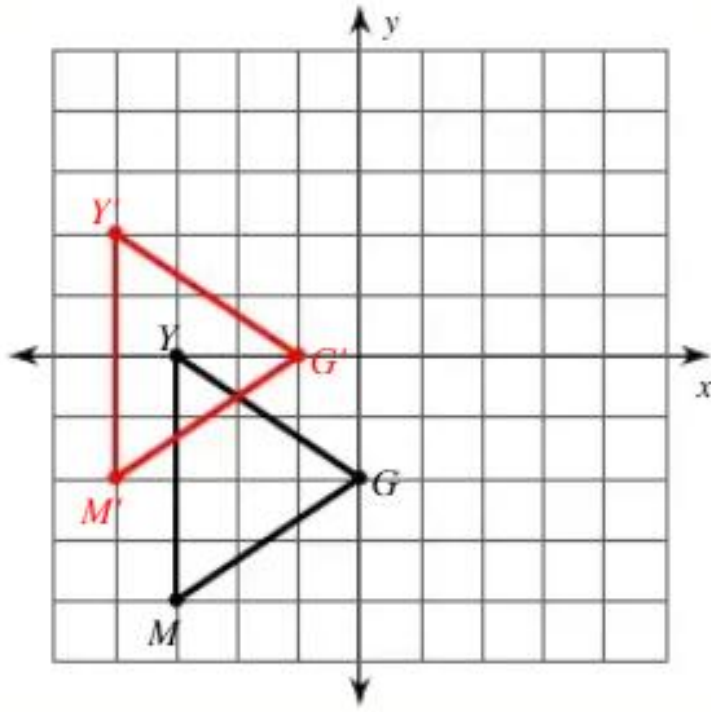
$$(0, -1) \rightarrow (0, 2)$$

$$(2, -1) \rightarrow (4, 2)$$

Rule $\boxed{(x, y) \rightarrow (2x, -2y)}$

Reflection across x-axis followed by
Dilation with scale factor 2

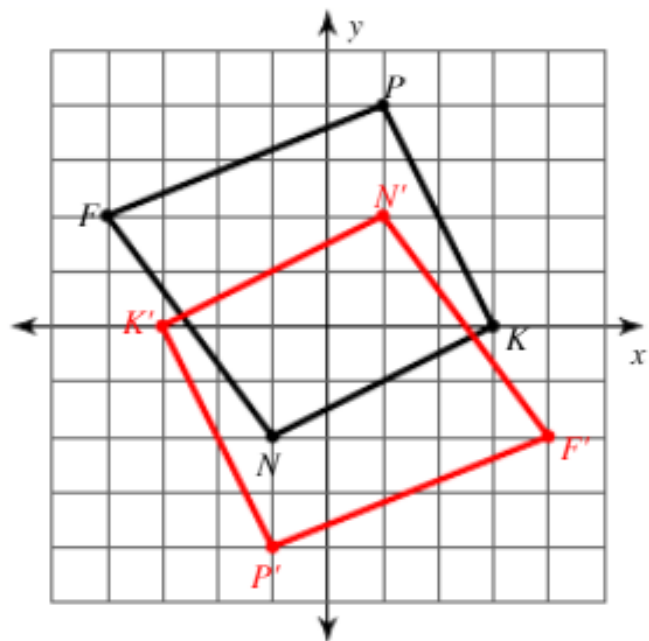
Preimage is YGM



Translation (1) unit left
then (2) units up

$$(x, y) \rightarrow (x - 1, y + 2)$$

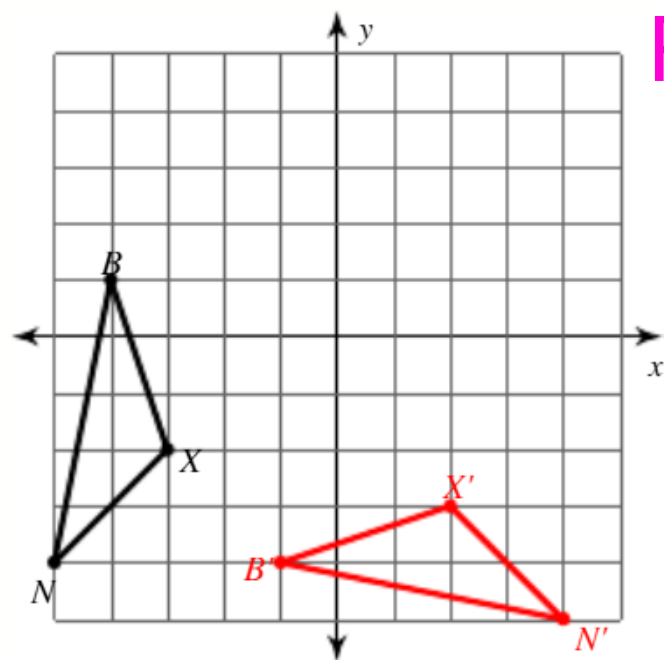
Preimage is the black figure.



Rotation 180° about the origin

$$(x, y) \rightarrow (-x, -y)$$

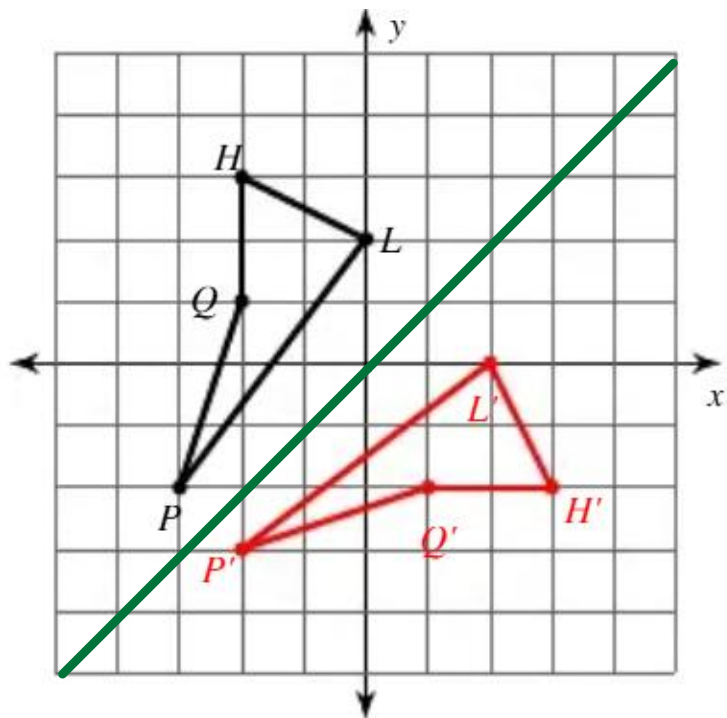
Preimage is the black figure.



Rotation 90° counterclockwise

$$(x, y) \rightarrow (-y, x)$$

Preimage is the black figure.

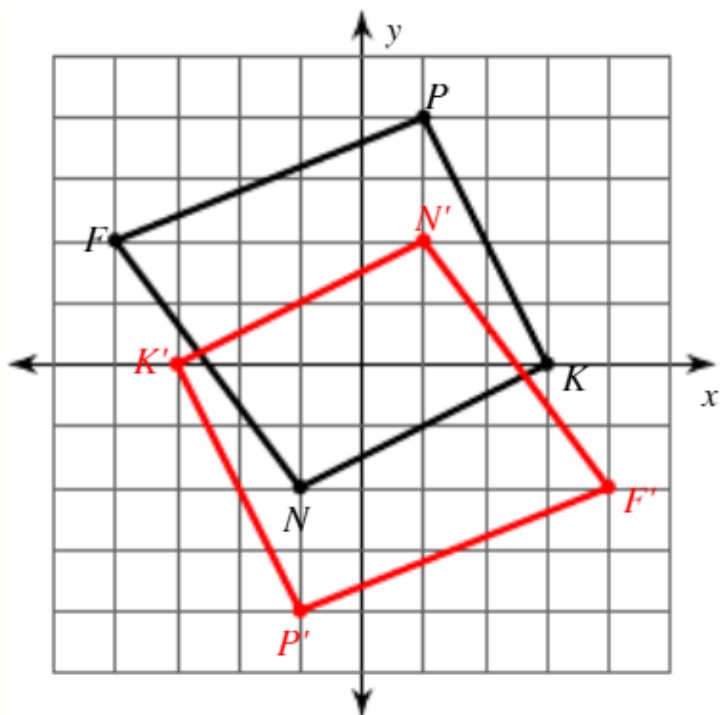


Reflection across $y = x$

$$(x, y) \rightarrow (y, x)$$

$$y = x$$

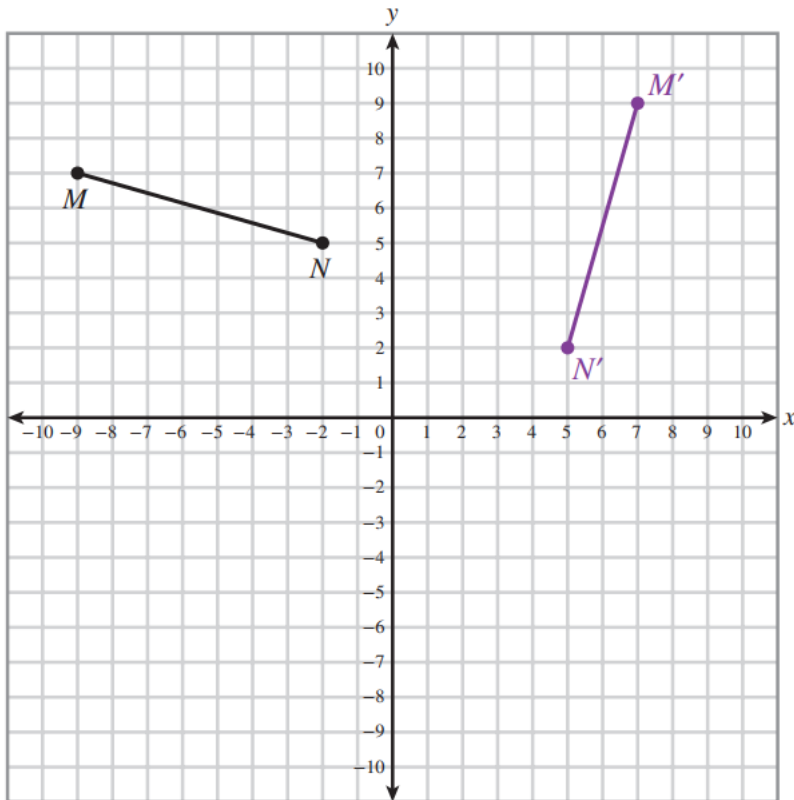
Preimage is the black figure.



Rotation 180° about the origin

$$(x, y) \rightarrow (-x, -y)$$

Preimage is the MN



Rotation 90° clockwise

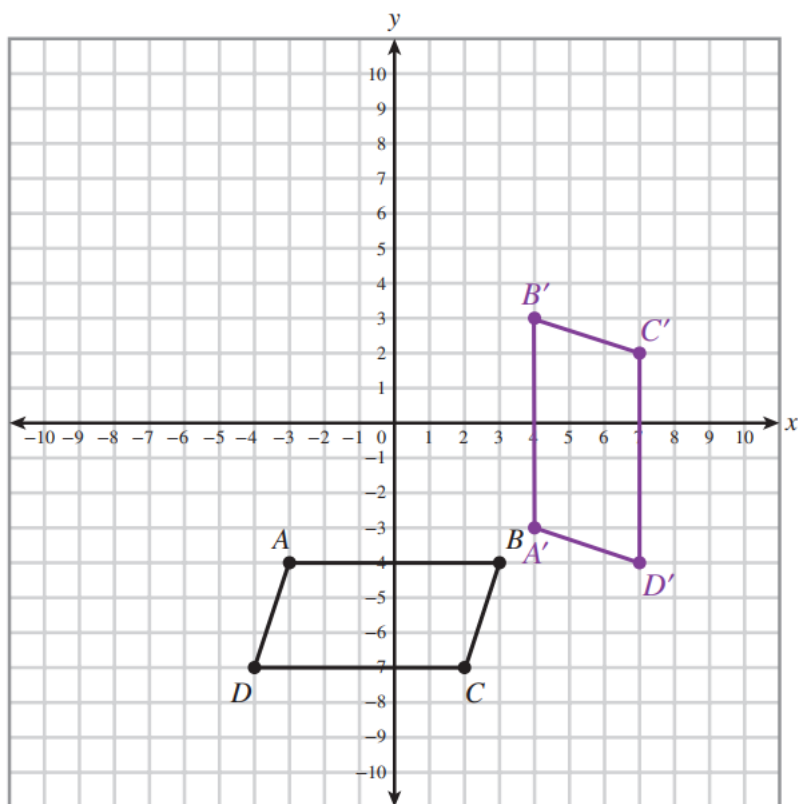
$$(-2, 5) \rightarrow (5, 2)$$

$$(-9, 7) \rightarrow (7, 9)$$

Rule

$$(x, y) \rightarrow (y, -x)$$

Preimage is the black figure.



Rotation 270° Clockwise

$$(x, y) \rightarrow (-y, x)$$