THE FEATURED
PROGRAM EDUCATION

البرنامج المميز

12 ADVANCED

MATH ENG

CHAPTER 7

Mr. Ahmed Ata
The Featured Program

2025-2026

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7 - Integration Techniques and First-Order Differential Equations

1. Review of Formulas and Techniques
2. Integration by Parts
3. Trigonometric Techniques of Integration
4. Integration of Rational Functions Using Partial Fractions
5. Integration Tables and Computer Algebra Systems
6. Modeling with Differentioal Equations
7. Separable Differential Equations

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LESSON 7-1

Review of Formulas and Techniques

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Lesson (7-1)

Review of Formulas and Techniques

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c, \quad \text{for } r \neq -1 \text{ (power rule)} \qquad \int \frac{1}{x}\,dx = \ln|x| + c, \quad \text{for } x \neq 0$$

$$\int \sin x\,dx = -\cos x + c \qquad \int \cos x\,dx = \sin x + c$$

$$\int \sec^2 x\,dx = \tan x + c \qquad \int \sec x \tan x\,dx = \sec x + c$$

$$\int \csc^2 x\,dx = -\cot x + c \qquad \int \csc x \cot x\,dx = -\csc x + c$$

$$\int e^x\,dx = e^x + c \qquad \int e^{-x}\,dx = -e^{-x} + c$$

$$\int \tan x\,dx = -\ln|\cos x| + c \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c \qquad \int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c \qquad \int sin(ax)dx = -\frac{1}{a}cos(ax) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c \qquad \int \frac{1}{\sqrt{a^2-x^2}} = sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{f'(x)}{f(x)}dx = ln|f(x)| + c \qquad \int \frac{1}{a^2+x^2} = \frac{1}{|a|}tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int a^x dx = \frac{1}{lna}a^x + c \qquad \int \frac{1}{|x|\sqrt{x^2-a^2}} = \frac{1}{|a|}sec^{-1}\left(\frac{x}{a}\right) + c$$

Evaluate the integral

1. $\int e^{ax}\,dx, a \neq 0$

2. $\int \cos(ax)\,dx, a \neq 0$

3. $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx, a > 0$

4. $\int \frac{1}{|x|\sqrt{x^2 - a^2}}\,dx, a > 0$

5. $\int \sin 6t\,dt$

6. $\int \sec 2t \tan 2t\,dt$

7. $\int (x^2 + 4)^2\,dx$

8. $\int x(x^2 + 4)^2\,dx$

9 $\int \frac{3}{16+x^2}\,dx$

10 $\int \frac{2}{4+4x^2}\,dx$

11 $\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$

12 $\int \frac{x+1}{\sqrt{3-2x-x^2}}\,dx$

13 $\int \frac{4}{5+2x+x^2}\,dx$

14 $\int \frac{4x+4}{5+2x+x^2}\,dx$

15 $\int \frac{4t}{5+2t+t^2}\,dt$

16 $\int \frac{t+1}{t^2+2t+4}\,dt$

17 $\int e^{3-2x}\,dx$

18 $\int \frac{3}{e^{6x}}\,dx$

19 $\int \frac{4}{x^{1/3}(1+x^{2/3})}\,dx$

20 $\int \frac{2}{x^{1/4}+x}\,dx$

21 $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

22 $\int \frac{\cos(1/x)}{x^2}\,dx$

23 $\int_0^{\pi} \cos x e^{\sin x}\, dx$

24 $\int_0^{\pi/4} \sec^2 x e^{\tan x}\, dx$

25 $\int_{-\pi/4}^{0} \frac{\sin t}{\cos^2 t}\, dt$

26 $\int \frac{x^2}{1+x^6}\, dx$

27 $\int \frac{x^5}{1+x^6}\,dx$

28 $\int \frac{1}{\sqrt{4-x^2}}\,dx$

29 $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

30 $\int \frac{x}{\sqrt{1-x^4}}\,dx$

31 $\int \frac{2x^3}{\sqrt{1-x^4}}\,dx$

32 $\int \frac{1+x}{1+x^2}\,dx$

33 $\int \frac{\ln x^2}{x}\,dx$

34 $\int_3^4 x\sqrt{x-3}\,dx$

35 $\int_0^1 x(x-3)^2\,dx$

36 $\int_1^4 \frac{x^2+1}{\sqrt{x}}\,dx$

37 $\int_{-2}^{0} xe^{-x^2}\,dx$

38 $\int \frac{5}{3+x^2}\,dx$

39 $\int_0^1 (e^x+4)^2\,dx$

39 $\int \frac{4x^3-1}{x^4-x}\,dx$

41 $\int \frac{e^{\frac{-2}{x^2}}}{x^3} x dx$

42 $\int \frac{3}{\sqrt{-2x - x^2}} dx$

43 $\int a e^{\frac{x}{a}} dx =$

44 Determine m if

$$\int \frac{x^3}{1 + x^m} dx = \frac{1}{4} tan^{-1} x^4 + c \text{ where } m \neq 0$$

45 Determine m if

$$\int \frac{lnx}{mx}\,dx = \frac{1}{8}ln^2x + c \ , m \neq 0$$

46 Determine n if

$$\int 3cos\left(\frac{x}{3n}\right)dx = -45sin\left(\frac{x}{3n}\right) + c \ , n \neq 0$$

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LESSON 7-2

Integration by Parts

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Lesson (7-2)

Integration by Parts

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du. \qquad (2.1)$$

Sign	F(x) Differentiate	F(y) Integration
+ ⇨	F(x)	F(y)
- ⇨	First derivative of F(x)	First Integrate of F(y)
+ ⇨	Second derivative of F(x)	Second Integrate of F(y)
- ⇨	Third derivative of F(x)	Third Integrate of F(y)

Evaluate the integrals

1 $\int x\cos x\,dx$

2 $\int x \sin 4x \, dx$

3 $\int x e^{2x} dx$

4 $\int x^2 \ln x \, dx$

5 $\int x^2 e^{-3x} dx$

6 $\int x^2 e^{x^3}\, dx$

7 $\int e^x \sin 4x\, dx$

8 $\int e^{2x} \cos x\, dx$

9 $\int \cos x \cos 2x \, dx$

10 $\int x \sec^2 x \, dx$

11 $\int x^3 e^{x^2} \, dx$

12 $\int \cos x \ln(\sin x)\, dx$

13 $\int_1^2 x^3 \ln x\, dx.$

14 $\int_0^1 x \sin 2x\, dx$

15 $\int_0^1 x^2 \cos \pi x \, dx$

16 $\int_0^1 x^2 e^{3x} dx$

17 $\int_1^{10} \ln 2x \, dx$

18 $\int_1^2 x \ln x \, dx$

19 $\int e^{ax} x^2 \, dx, a \neq 0$

20 $\int x \sin(ax) \, dx, a \neq 0$

21 $\int x^4 \sin x \, dx$

22 $\int x^4 e^x \, dx$

23 $\int x^5 \cos 2x \, dx$

Evaluate the integral using integration by parts and substitution.

24 $\int \cos^{-1} x \, dx$

25 $\int \tan^{-1} x \, dx$

26 $\int \sin \sqrt{x}\, dx$

27 $\int \sin(\ln x)\, dx$

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LESSON 7-3

Trigonometric Techniques of Integration

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Lesson (7-3)

Trigonometric Techniques of Integration

Case 1: m or n Is an Odd Positive Integer

$$\int sin^m x\, cos^n x\ dx$$

where m and n are positive integers.

Evaluate

1 $\int \cos^4 x \sin x\, dx.$

2 $\int \cos^4 x \sin^3 x\, dx.$

3 $\int \sqrt{\sin x}\cos^5 x\,dx.$

4 $\int \cos x \sin^4 x\,dx$

5 $\int \cos^3 x \sin^4 x\,dx$

6 $\int_0^{\pi/4} \cos 2x \sin^3 2x \, dx$

7 $\int_{\pi/4}^{\pi/3} \cos^3 3x \sin^3 3x \, dx$

8 $\int_0^{\pi/2} \cos^2 x \sin x \, dx$

Case 2: m and n Are Both Even Positive Integers

Evaluate

9 $\int \sin^2 x \, dx.$

10 $\int \cos^2 (x + 1) \, dx$

11 $\int \cos^4 x \, dx.$

12 $\int \sin^4 (x - 3) \, dx$

$\int tan^m x\, sec^n x\, dx$ where m and n are integers.

Case 1: m Is an Odd Positive Integer

Evaluate

13 $\int \tan^3 x \sec^3 x\, dx.$

14 $\int \tan x \sec^3 x\, dx$

15 $\int \cot x \csc^4 x\, dx$

16 $\int \tan(2x+1)\sec^3(2x+1)\,dx$

17 $\int x\tan^3(x^2+1)\sec(x^2+1)\,dx$

Case 2: n Is an Even Positive Integer

18 $\int \tan^2 x \sec^4 x \, dx.$

19 $\int \cot^2 x \csc^4 x \, dx$

20 $\int \cot^2 x \csc^2 x \, dx$

21 $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$

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22 $\int \cos^2 x \sin^2 x \, dx$

23 $\int_{-\pi/3}^{0} \sqrt{\cos x} \sin^3 x \, dx$

24 **Find m if**

أوجد قيمة **m** اذا كان

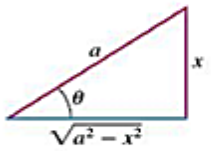
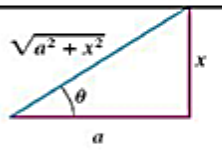
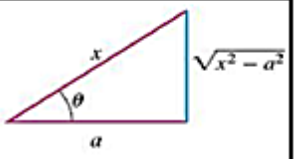
$$\int tanxsec^m x dx = \frac{1}{3} sec^3 x + c \ , m \neq 0$$

$a)$ $\frac{1}{3}$ $b)$ 3

$c)$ $\frac{1}{2}$ $d)$ 2

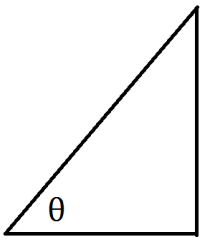
Trigonometric Substitution

TRIGONOMETRIC SUBSTITUTION

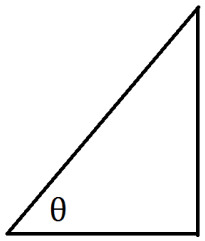
Expression	substitution	Identity	domain	triangle
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$1-\sin^2\theta=\cos^2\theta$	$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$	
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$1+\tan^2\theta=\sec^2\theta$	$-\frac{\pi}{2}<\theta<\frac{\pi}{2}$	
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$\sec^2\theta-1=\tan^2\theta$	$0\le\theta<\frac{\pi}{2}$ or $\pi\le\theta<\frac{3\pi}{2}$	

Evaluate the integral

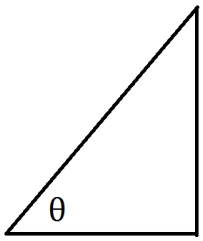
25 $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx.$



26 $\int \frac{1}{\sqrt{9+x^2}}\,dx.$



27 $\int \frac{\sqrt{x^2-25}}{x}\,dx$, for $x \geq 5$.



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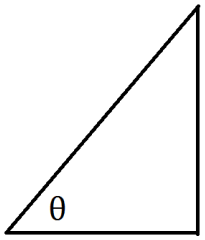
28 $\int \frac{x^2}{\sqrt{16-x^2}}\,dx$



29 $\int \frac{2}{\sqrt{x^2-4}}\,dx$

30 $\int \frac{1}{\sqrt{4+x^2}}\,dx$

31 $\int_0^1 x\sqrt{x^2+8}\,dx$

32 Evaluate the integra أوجد قيمة التكامل

$$\int \frac{1}{x^2\sqrt{9-x^2}}dx$$

$a)\ \frac{\sqrt{9-x^2}}{9x}+c$

$b)\ -\frac{\sqrt{9-x^2}}{9x}+c$

$c)\ \frac{\sqrt{9-x^2}}{x}+c$

$d)\ -\frac{\sqrt{9-x^2}}{x}+c$

33 Evaluate the integra أوجد قيمة التكامل

$$\int \frac{1}{x^2\sqrt{16-x^2}}dx$$

$a)\ \frac{\sqrt{16-x^2}}{16x}+c$

$b)\ -\frac{\sqrt{16-x^2}}{16x}+c$

$c)\ \frac{\sqrt{16-x^2}}{x}+c$

$d)\ -\frac{\sqrt{16-x^2}}{x}+c$

34 Evaluate the integra أوجد قيمة التكامل

$$\int \frac{x^3}{\sqrt{9-x^2}}dx$$

$a)\ 3\sqrt{9-x^2}-\frac{1}{3}\left(\sqrt{9-x^2}\right)^3+c$

$b)\ 9\sqrt{9-x^2}-\frac{1}{9}\left(\sqrt{9-x^2}\right)^3+c$

$c)\ -9\sqrt{9-x^2}+\frac{1}{3}\left(\sqrt{9-x^2}\right)^3+c$

$d)\ 27\sqrt{9-x^2}-\frac{1}{3}\left(\sqrt{9-x^2}\right)^3+c$

35 Evaluate the integra أوجد قيمة التكامل

$$\int_0^2 \sqrt{4-x^2}$$

$a)\ \pi$

$b)\ 2\pi$

$c)\ 4\pi$

$d)\ \frac{\pi}{2}$

36 Evaluate the integra أوجد قيمة التكامل

$$\int_0^1 \frac{x}{\sqrt{4-x^2}}$$

$a)\ 2-\sqrt{2}$

$b)\ 2+\sqrt{3}$

$c)\ 2-\sqrt{5}$

$d)\ 2-\sqrt{3}$

37 Evaluate the integra أوجد قيمة التكامل

$$\int \frac{x}{\sqrt{x^2-4}}dx$$

$a)\ 2\sqrt{x^2-4}+c$

$b)\ 4\sqrt{x^2-4}+c$

$c)\ \sqrt{x^2-4}+c$

$d)\ \frac{1}{2}\sqrt{x^2-4}+c$

38 Evaluate the Evaluate the integra

أوجد قيمة التكامل

$$\int \frac{\sqrt{4x^2-9}}{x}\,dx$$

$a)\ \sqrt{4x^2-9}+3sec^{-1}\left(\frac{2x}{2}\right)+c$

$b)\ \sqrt{4x^2-9}+9cos^{-1}\left(\frac{2x}{3}\right)+c$

$c)\ \sqrt{4x^2-9}-3sec^{-1}\left(\frac{2x}{3}\right)+c$

$d)\sqrt{4x^2-9}-2sec^{-1}\left(\frac{2x}{2}\right)+c$

39 Evaluate the Evaluate the integra

أوجد قيمة التكامل

$$\int sec^3x\,dx$$

40 Determine a if

$$\int_0^a tan^4x\, sec^2x\, dx = \frac{1}{5}$$

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LESSON 7-4

Integration of Rational Functions Using Partial Fractions

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Lesson (7-4)

Integration of Rational Functions Using Partial Fractions

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	• where $x^2 + bx + c$ cannot be factorised further	

Evaluate

1 $\int \frac{1}{x^2+x-2}\,dx.$

Find the partial fractions decomposition and an antiderivative.

2 $\dfrac{x-5}{x^2-1}$

3 $\dfrac{6x}{x^2-x-2}$

4 $\dfrac{-x+5}{x^3-x^2-2x}$

5 $\dfrac{5x-23}{6x^2-11x-7}$

6 $\dfrac{x-1}{x^3+4x^2+4x}$

7 $\dfrac{x+2}{x^3+x}$

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8 $\dfrac{2x+3}{x^2+2x+1}$

9 $\dfrac{2x}{x^2-6x+9}$

10 $\dfrac{x^3+x}{x^2-1}$

Evaluate the integral

11 $\displaystyle\int \frac{x^3+x+2}{x^2+2x-8}\,dx$

12 $\int \frac{x+4}{x^3+3x^2+2x}\,dx$

13 $\int \frac{4x^3-1}{x^4-x}\,dx$

14 $\int \frac{\sin x \cos x}{\sin^2 x - 4}\,dx$

15 $\dfrac{1}{x^3 + 4x}$

16 $\displaystyle\int \frac{2e^x}{e^{3x} + e^x}\, dx$

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LESSON 7-6

Modeling with Differential Equations

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Lesson (7-6)

Modeling with Differential Equations

Growth and Decay Problems

$y'(t) = ky(t)$ $y(t) = A(e)^{kt}$ $A = y(0)$

Find the solution of the given differential equation satisfying the indicated initial condition

1. $y' = 4y, y(0) = 2$

2. $y' = 3y, y(0) = -2$

3. $y' = -3y, y(0) = 5$

4. $y' = -y, y(1) = 2$

5 A freshly inoculated bacterial culture of Streptococcus A (a common group of microorganisms that cause strep throat) contains 100 cells. When the culture is checked 60 minutes later, it is determined that there are 450 cells present. Assuming exponential growth, determine the number of cells present at any time t (measured in minutes) and find the doubling time.

6 Suppose a bacterial culture initially has 400 cells. After 1 hour, the population has increased to 800.

(a) Quickly determine the population after 3 hours.

(b) Find an equation for the population at any time.

(c)What will the population be after 3.5 hours?

7 Suppose a bacterial culture initially has 100 cells. After 2 hours, the population increased to 400.

(a) Quickly find the population after 6 hours.

(b) Find an equation for the population at any time.

(c) What will the population be after 7 hours?

8 Suppose a bacterial culture doubles in population every 4 hours. If the population is initially 100,

(a) quickly determine when the population will reach 400.

(b) Find an equation for the population at any time.

(c) Determine when the population will reach 6000

9 Suppose a bacterial culture triples in population every 5 hours. If the population is initially 200,

(a) quickly determine when the population will reach 5400.

(b) Find an equation for the population at any time.

(c) Determine when the population will reach 20,000.

The doubling time is $\frac{ln\,2}{r}$ or $r = \frac{ln2}{t}$

The half-life is $-\frac{ln\,2}{r}$ or $r = -\frac{ln\,2}{t}$

10 If you have 50 g of ^{14}C today, how much will be left in 100 years? Such that the half-life of carbon-14 (^{14}C) is approximately 5730 years.

11 Strontium-90 is a dangerous radioactive isotope. Because of its similarity to calcium, it is easily absorbed into human bones. The half-life of strontium-90 is 28 years. If a certain amount is absorbed into the bones due to exposure to a nuclear explosion, what percentage will remain after 84 years?

12 The half-life of uranium ^{235}U is approximately 0.7×10^9 years. If 50 grams are buried at a nuclear waste site, how much will remain after 100 years?

13 **The half-life of morphine in the human bloodstream is 3 hours. If initially there is 0.4 mg of morphine in the bloodstream, Find an equation for the amount in the bloodstream at any time. When does the amount drop below 0.1 mg?**

Newton's Law of Cooling

$$y'(t) = k[y(t) - T_a] \qquad y(t) = A(e)^{kt} + T_a \qquad A = y(0) - T_a$$

Find the solution of the given differential equation satisfying the indicated initial condition

14 $y' = y - 50, y(0) = 70$

15 $y' = 0.1y - 10, y(0) = 80$

16 A cup of fast-food coffee is $80^{\circ}C$ when freshly poured. After 2 minutes in a room at $20^{\circ}C$, the coffee has cooled to $75^{\circ}C$. Find the temperature at any time t and find the time at which the coffee has cooled to $50^{\circ}C$

17 A bowl of porridge at $200^{\circ}F$ (too hot) is placed in a $70^{\circ}F$ room. One minute later the porridge has cooled to $180^{\circ}F$. When will the temperature be $120^{\circ}F$ (just right)?

18 A cold drink is poured out at $50^{\circ}F$. After 2 minutes of sitting in a $70^{\circ}F$ room, its temperature has risen to $56^{\circ}F$.

(a) Find the drink's temperature at any time t.

(b) What will the temperature be after 10 minutes?

(c) When will the drink have warmed to $66^{\circ}F$?

19 If you invest AED 7000 at an annual Murabaha rate of 5.75%, compare the value of your investment after 5 years under various forms of compounding.

a) With annual compounding, the value is

b) With monthly compounding, this becomes

c) With daily compounding, these yields

d) With continuous compounding, the value is

20 If you invest AED 1000 at an annual Murabaha rate of 8%, compare the value of the investment after 1 year under the following forms of compounding:

e) With annual compounding, the value is

f) With monthly compounding, this becomes

g) With daily compounding, these yields

h) With continuous compounding, the value is

21 Suppose that the value of a AED 10,000 asset decreases continuously at a constant rate of 24% per year. Find its worth

(a) 10 years

(b) 20 years

22 **Suppose that the value of a AED 40,000 asset decreases at a constant percentage rate of 10%. Find its worth after**

(a) 10 years

(b) 20 years

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PROGRAM EDUCATION

البرنامج المميز

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12 ADVANCED

MATH ENG

LESSON 7-7

Separable Differential Equations

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Lesson (7-7)

Separable Differential Equations

Determine whether the differential equation is separable.

1 $y' = xy^2 - 2xy$

2 $y' = (3x + 1)\cos y$

3 $y' = (3x + y)\cos y$

4 $y' = 2x(\cos y - 1)$

5 $y' = 2x(y - x)$

6 $y' = x^2y + y\cos x$

7 $y' = x^2y - x\cos y$

8 $y' = 2x\cos y - xy^3$

Solve the differential equation

9 $y' = \dfrac{x^2 + 7x + 3}{y^2}$.

10 $y' = (x^2 + 1)y$

11 $y' = 2x^2y^2$

12 $y' = \dfrac{6x^2}{y(1+x^3)}$

13 $y' = \dfrac{2x}{y}e^{y-x}$

14 $y' = \dfrac{\cos x}{\sin y}$

15 $y' = \dfrac{xy}{1+x^2}$

16 $y' = x\cos^2 y$

Find the general solution in an explicit form and sketch several members of the family of solutions

17 $y' = -xy$

18 $y' = \frac{-x}{y}$

19 $y' = \frac{1}{y}$

Solve the IVP (Initial Value Problem) explicitly if possible

20 $y' = 3(x+1)^2 y,\ y(0) = 1$

21 $y' = \frac{x-1}{y^2}, y(0) = 2$

22 $y' = \frac{4x^2}{y}, y(0) = 2$

23 $y' = \frac{x-1}{y}, y(0) = -2$

24 $y' = \dfrac{4y}{x+3}, y(-2) = 1$

25 $y' = \dfrac{3x}{4y+1}, y(1) = 4$

26 $y' = \dfrac{4x}{\cos y}, y(0) = 0$

$$y'(t) = ky(M - y). \quad (7.6)$$

$$y = \frac{AMe^{kMt}}{1 + Ae^{kMt}}.$$

Use equation (7.6) to help solve the IVP.

27 $y' = 3y(2 - y), y(0) = 1$

28 $y' = y(3 - y), y(0) = 2$

29 $y' = 2y(5 - y), y(0) = 4$

30 $y' = y(2 - y), y(0) = 1$

31 **Given a maximum sustainable population of M = 1000 (this could be measured in millions or tons, etc.) and growth rate k = 0.007, find an expression for the population at any time t, given an initial population of y(0) = 350 and assuming logistic growth**

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