

Definition

A **solid revolution** is formed by revolving a plane region about a line. The line is called the axis of revolution.

The Disk Method طريقة القرص

The volume of the solid formed by revolving the region bounded by the graph of f and the x - axis $a \leq x \leq b$ about the axis is

$$\text{Volume} = \pi \int_a^b (f(x))^2 dx$$

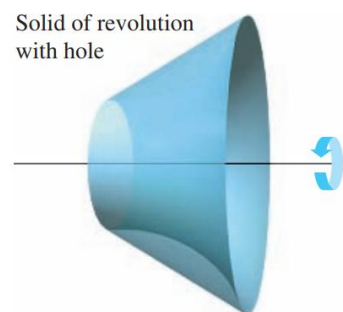
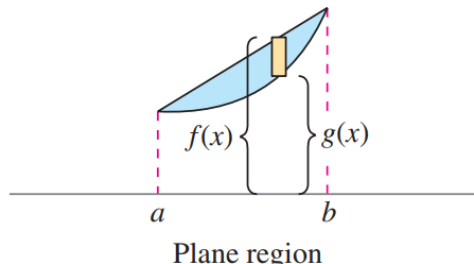
The Washer Method طريقة الحلقة (أو طريقة القرص الحلقى)

Let f and g be continuous and nonnegative on the closed interval as shown.

If $g(x) \leq f(x)$ for all x in the interval, then the volume of the solid formed by revolving the region bounded by the graphs of f and g ($a \leq x \leq b$) about the axis is

$$\text{Volume} = \pi \int_a^b ([f(x)]^2 - [g(x)]^2) dx$$

$f(x)$ is the outer radius and $g(x)$ is the inner radius.



Volumes of revolution around the x-axis

- Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = -x^2 + x$ and the x -axis about the x -axis.

$$\text{Volume} = \pi \int_0^1 [f(x)]^2 dx$$

Disk Method

$$= \pi \int_0^1 (-x^2 + x)^2 dx$$

Substitute for $f(x)$.

$$= \pi \int_0^1 (x^4 - 2x^3 + x^2) dx$$

Expand integrand.

$$= \pi \left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} \right]_0^1$$

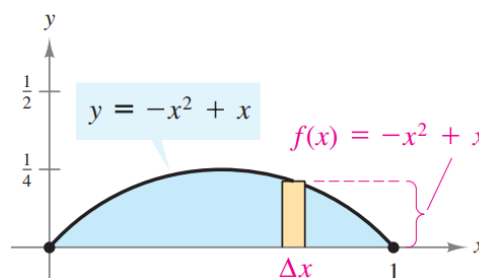
Find antiderivative.

$$= \frac{\pi}{30}$$

Apply Fundamental Theorem.

$$\approx 0.105$$

Round to three decimal places.



Radius: $f(x) = -x^2 + x$

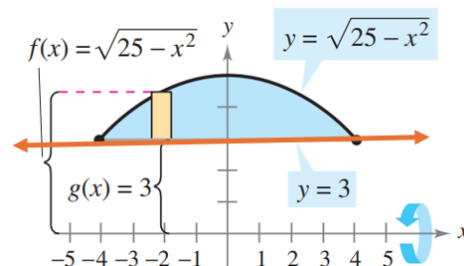
- 2) Find the volume of the solid formed by revolving the region bounded by the graphs of $f(x) = \sqrt{25 - x^2}$ and $g(x) = 3$ about the x-axis

Finding the points of intersection of f and g

$$f(x) = g(x)$$

$$\sqrt{25 - x^2} = 3$$

$$x = \pm 4$$



$$\text{Volume} = \pi \int_{-4}^4 \{[f(x)]^2 - [g(x)]^2\} dx$$

Washer Method

$$= \pi \int_{-4}^4 [(\sqrt{25 - x^2})^2 - (3)^2] dx$$

Substitute for $f(x)$ and $g(x)$.

$$= \pi \int_{-4}^4 (16 - x^2) dx$$

Simplify.

$$= \pi \left[16x - \frac{x^3}{3} \right]_{-4}^4$$

Find antiderivative.

$$= \frac{256\pi}{3}$$

Apply Fundamental Theorem.

$$\approx 268.08$$

Round to two decimal places.

- 3) Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = -x^2 + 4$ about the x-axis.

Volumes of revolution around the y-axis

Let R be the region bounded by $y = 4 - x^2$ and $y = 0$. Find the volume of the solids obtained by revolving R about the following:

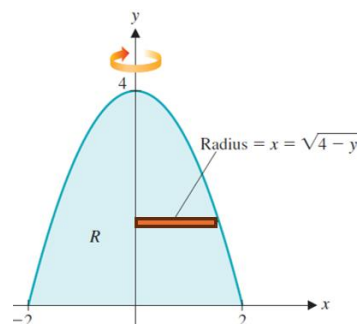
a) the line the y-axis

$$y = 4 - x^2$$

$$x = \sqrt{4 - y} \Rightarrow f(y) = \sqrt{4 - y}$$

$$V = \pi \int_a^b ([f(y)]^2) dy$$

$$= \pi \int_0^4 ([\sqrt{4 - y}]^2) dy = \pi \int_0^4 (4 - y) dy = 8\pi$$



b) line $x = 3$

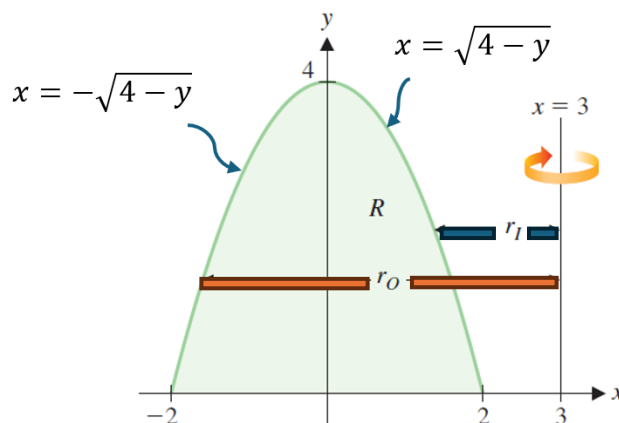
$$y = 4 - x^2$$

$$x = \pm\sqrt{4 - y}$$

$$r_0 = 3 - (-\sqrt{4 - y}) = 3 + \sqrt{4 - y}$$

$$r_1 = 3 - \sqrt{4 - y}$$

$$\text{Volume} = \pi \int_a^b ([r_1]^2 - [r_0]^2) dy$$



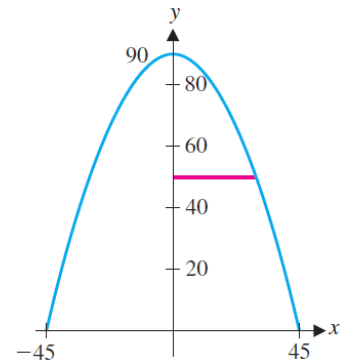
$$= \pi \int_0^4 ([3 + \sqrt{4 - y}]^2 - [3 - \sqrt{4 - y}]^2) dy$$

$$= \int_0^4 \pi (3 + \sqrt{4 - y})^2 dy - \int_0^4 \pi (3 - \sqrt{4 - y})^2 dy$$

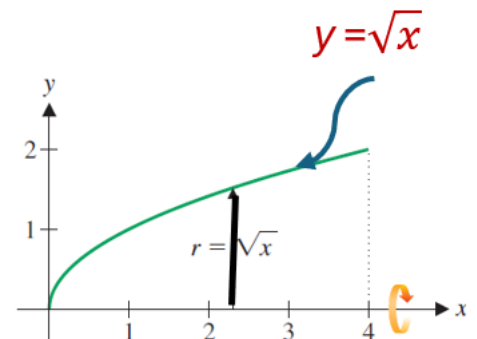
$$= 64\pi$$

4) Find the volume of the solid formed by revolving the region bounded by the graph(s) of the equation(s) about the y-axis. $y = x^2$, $y = 4$, $0 \leq x \leq 2$

5) Suppose that a dome has circular cross sections, with outline $y = -\frac{2}{45}x^2 + 90$ for $-45 \leq x \leq 45$. Find the volume of the dome



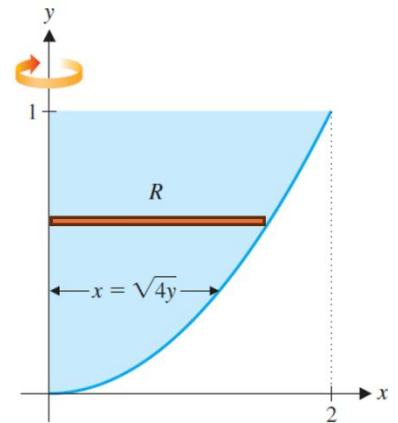
6) Revolve the region under the curve $y = \sqrt{x}$ on the interval $[0, 4]$ about the x-axis and find the volume of the resulting solid of revolution.



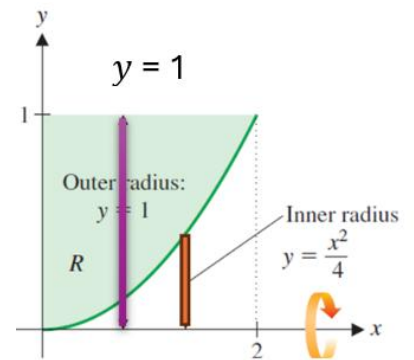
7) Let R be the region bounded by the graphs of $y = \frac{1}{4}x^2$, $x = 0$ and $y = 1$.

Compute the volume of the solid formed by revolving R about

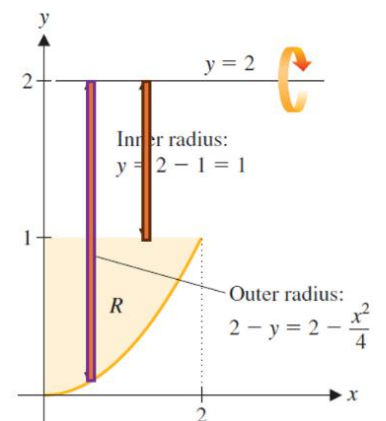
(a) the y -axis.



(b) the x -axis.

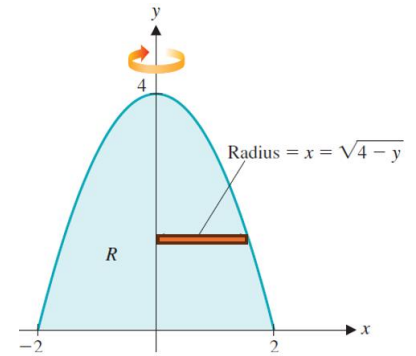


(c) the line $y = 2$.

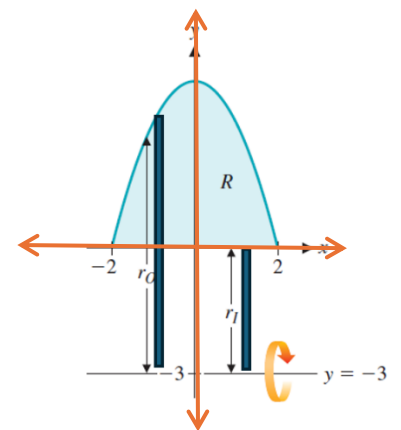


8) Let R be the region bounded by $y = 4 - x^2$ and $y = 0$. Find the volume of the solids obtained by revolving R about each of the following:

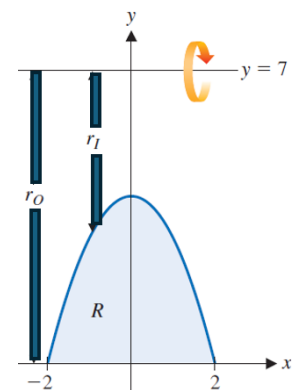
(a) The y -axis.



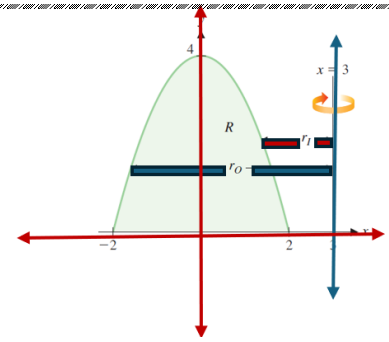
(b) The line $y = -3$.



(c) The line $y = 7$



(d) The line $x = 3$.



9) Estimate the volume from the cross-sectional areas.

x (ft)	0.0	0.5	1.0	1.5	2.0
$A(x)$ (ft ²)	1.0	1.2	1.4	1.3	1.2

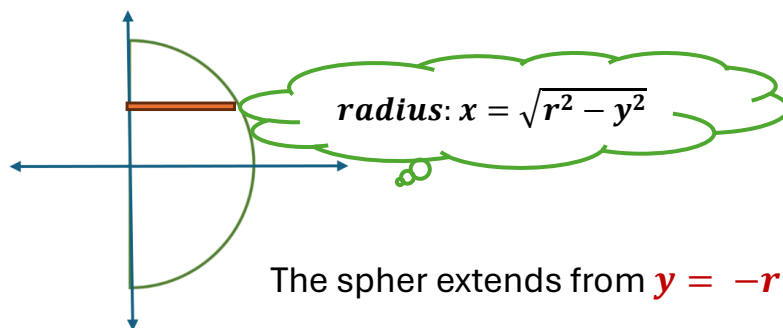
Answer: **Simpson's rule**

$$\Delta x = \frac{b-a}{n} = 0.5$$

$$V \approx \frac{\Delta x}{3} (A_1 + 4A_2 + 2A_3 + 4A_4 + \dots + A_n)$$

$$= \frac{0.5}{3} (1 + 4(1.2) + 2(1.4) + 4(1.3) + 1.2) = 2.5 \text{ ft}^3$$

10) Verify the formula for the volume of a sphere by revolving the circle $x^2 + y^2 = r^2$ about the y -axis.



$$x^2 + y^2 = r^2$$

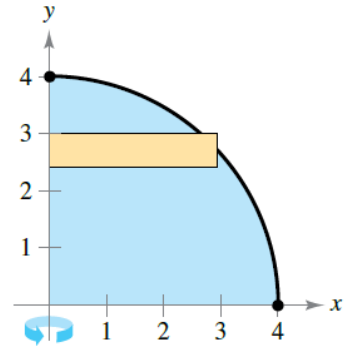
The sphere extends from $y = -r$ to $y = r$

$$V = \int_{-r}^r A(y) dy = \int_{-r}^r \left(\sqrt{r^2 - y^2} \right)^2 dy = \frac{4}{3} \pi r^3$$

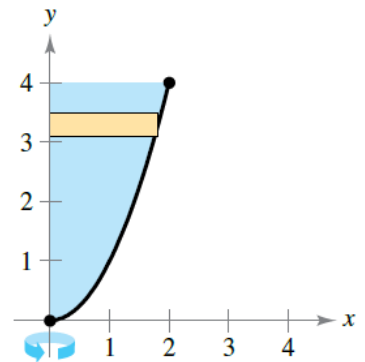
Homework

Write and evaluate the definite integral that represents the volume of the solid formed by revolving the region about the y -axis.

1) $y = \sqrt{16 - x^2}$

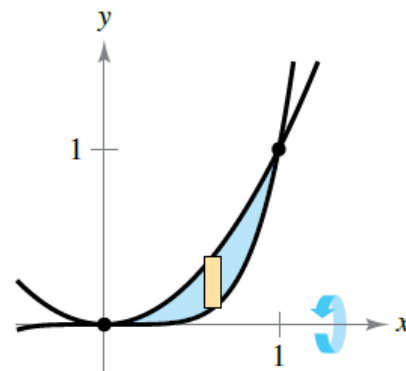


2) $y = x^2$

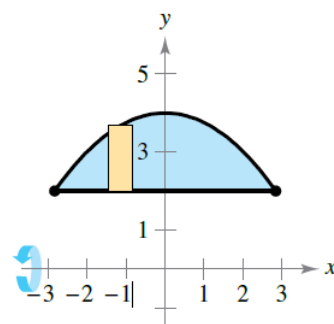


Write and evaluate the definite integral that represents the volume of the solid formed by revolving the region about the x-axis

1) $y = x^2$, $y = x^5$



2) $y = 2$, $y = 4 - \frac{x^2}{4}$



اللهم اغفر لي، وارحمني، واهدني، واجبرني، وعافني، وارزقني. (كان النبي ﷺ يدعو به بين السجدين)