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MATH ENG

MATH TERM 2

Mr. Ahmed Ata
The Featured Program
2024-2025

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4 - Applications of Differentiation

1. Linear Approximations and Newton's Method
2. Indeterminate Forms and L'Hôpital's Rule
3. Maximum and Minimum Values
4. Increasing and Decreasing Functins
5. Concavity and the seconed Derivative Test
6. Overview of Curve Sketching
7. Optimization
8. Related Rates
9. Rates of Change in Economic and the Science

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LESSON 4-3

Maximum and Minimum Values

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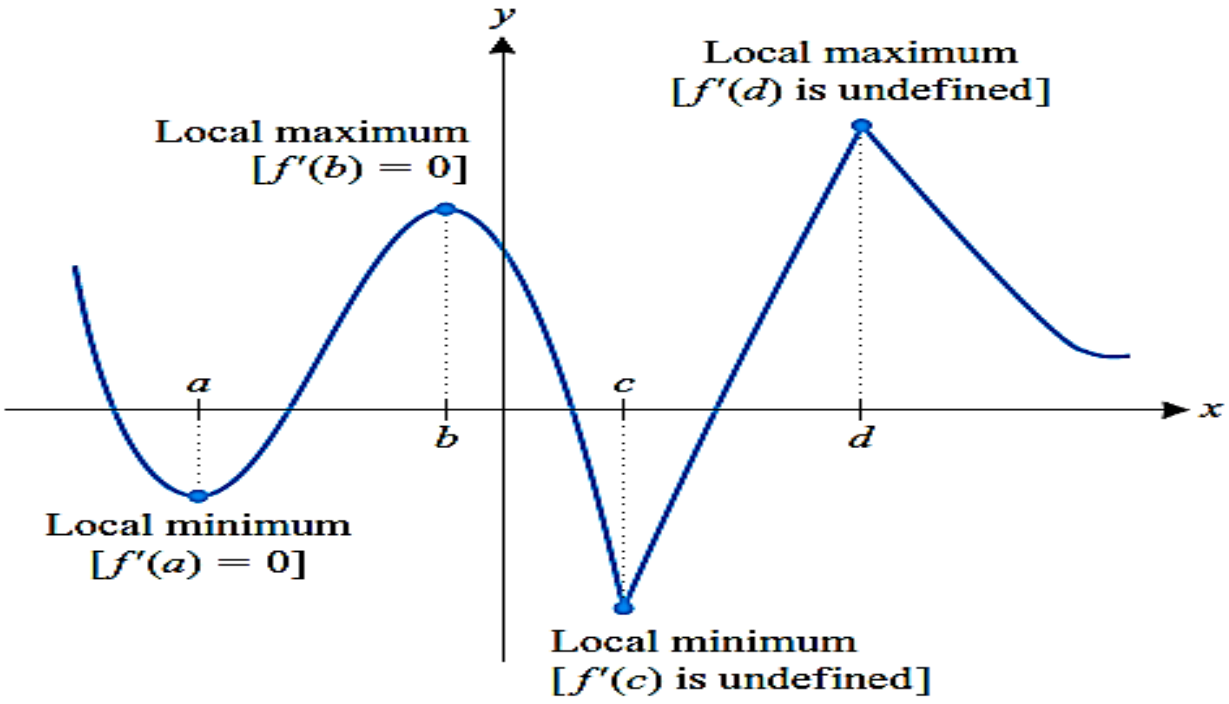
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Lesson (4-3)

Maximum and Minimum Values

DEFINITION

A number c in the domain of a function f is called a critical number of f if $f'(c) = 0$ or $f'(c)$ is undefined

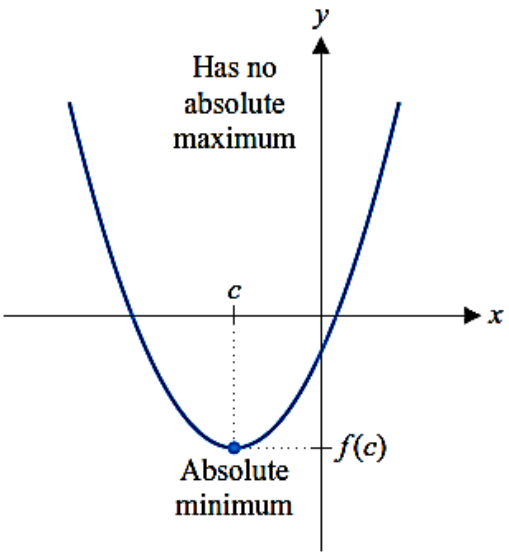


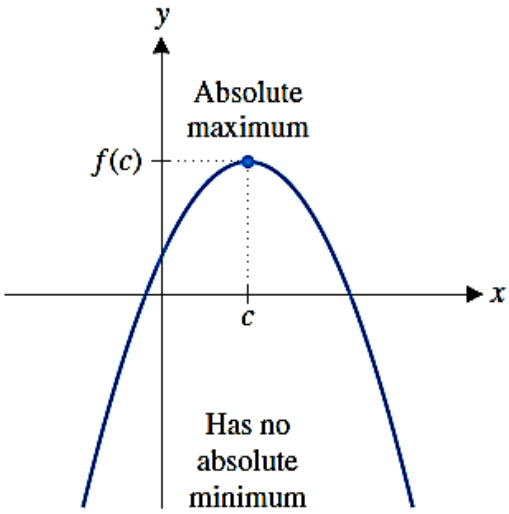
DEFINITION 3.1

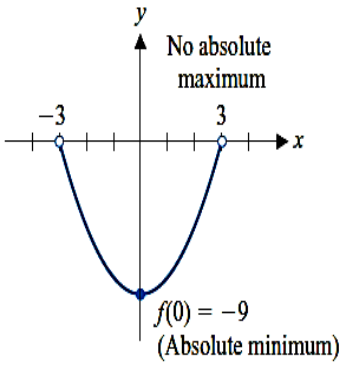
For a function f defined on a set S of real numbers and a number $c \in S$,

(i) $f(c)$ is the **absolute maximum** of f on S if $f(c) \geq f(x)$ for all $x \in S$ and
(ii) $f(c)$ is the **absolute minimum** of f on S if $f(c) \leq f(x)$ for all $x \in S$.

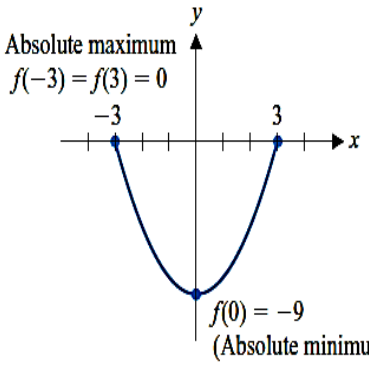
An absolute maximum or an absolute minimum is referred to as an **absolute extremum.** (The plural form of extremum is **extrema.**)



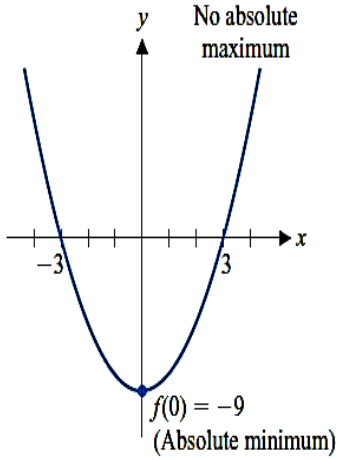




$y = x^2 - 9$ on $(-3, 3)$



$y = x^2 - 9$ on $[-3, 3]$



$f(x) = x^2 - 9$ on the interval $(-\infty, \infty)$

THEOREM 3.1 (Extreme Value Theorem)

A continuous function f defined on a *closed, bounded* interval $[a, b]$ attains both an absolute maximum and an absolute minimum on that interval.

DEFINITION 3.2

(i) $f(c)$ is a **local maximum** of f if $f(c) \geq f(x)$ for all x in some *open* interval containing c.

(ii) $f(c)$ is a **local minimum** of f if $f(c) \leq f(x)$ for all x in some *open* interval containing c.

In either case, we call $f(c)$ a **local extremum** of f.

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Find all critical numbers

1. $f(x) = x^2 + 5x - 1$

2. $f(x) = x^3 - 3x + 1$

3. $f(x) = x^4 - 2x^2 + 1$

4. $f(x) = x^{3/4} - 4x^{1/4}$

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5 $f(x) = \sin x \cos x, [0, 2\pi]$

6 $f(x) = \dfrac{x^2 - 2}{x + 2}$ (Recall that find the domain)

7 $f(x) = \frac{1}{2}(e^x + e^{-x})$

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8 $f(x) = 2x\sqrt{x+1}$

9 $f(x) = |x^2 - 1|$

10 $f(x) = (3x+1)^{2/3}$

11 $f(x) = \dfrac{2x^2}{x+2}$.

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12 $f(x) = \begin{cases} x^2 + 2x - 1 & \text{if } x < 0 \\ x^2 - 4x + 3 & \text{if } x \geq 0 \end{cases}$

13 $f(x) = \begin{cases} 7 - 2x^2, & x \leq 1 \\ x^2 - 4x, & x > 1 \end{cases}$

Find the absolute extrema of the given function on each indicated interval

14 $f(x) = x^3 - 3x + 1 \quad \text{on } [0, 2]$

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15 $f(x) = x^{2/3}$ on $[-4, -2]$

16 $f(x) = \sin x + \cos x$ on $[0, 2\pi]$

17 $f(x) = e^{-x^2}$ on $[0, 2]$

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18 $f(x) = \dfrac{3x^2}{x-3}$ on $[-2, 2]$

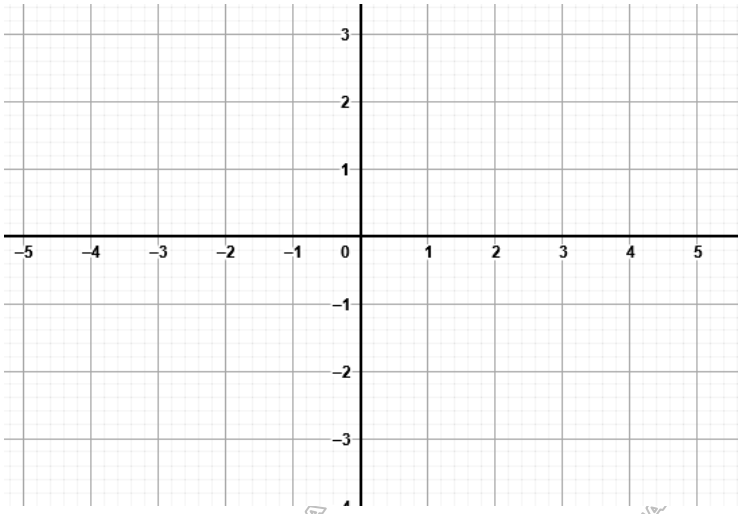
19 $f(x) = \tan^{-1}(x^2)$ on $[0, 1]$

20 $f(x) = \dfrac{x}{x^2+1}$ on $[0, 2]$

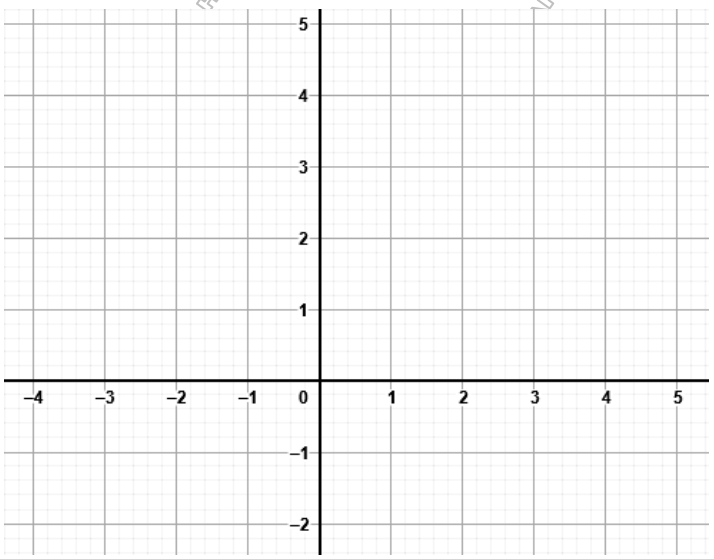
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21 $f(x) = 4x^{5/4} - 8x^{1/4}$ on $[0, 4]$

22 **Sketch a graph of a function f such that the absolute maximum of $f(x)$ on the interval $[-2, 2]$ equals 3 and the absolute minimum does not exist**

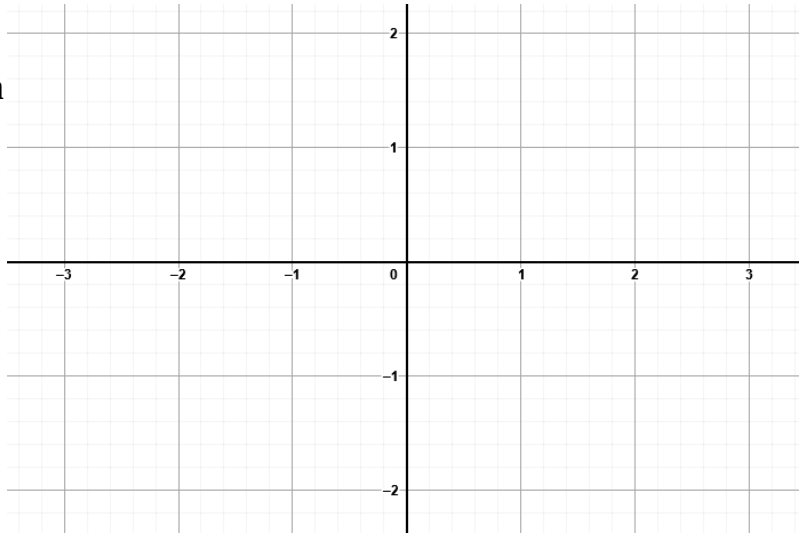


23 **Sketch a graph of a continuous function f such that the absolute maximum of $f(x)$ on the interval $(-2, 2)$ does not exist and the absolute minimum equals 2.**

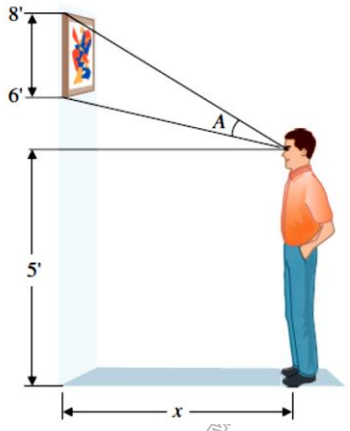


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24 **Sketch a graph of a function f such that the absolute maximum of $f(x)$ on the interval $[-2, 2]$ does not exist and the absolute minimum does not exist**



25 **Suppose a painting hangs on a wall as in the figure. The frame extends from 6 feet to 8 feet above the floor. A person whose eyes are 5 feet above the ground stands x feet from the wall and views the painting, with a viewing angle A formed by the ray from the person's eye to the top of the frame and the ray from the person's eye to the bottom of the bottom of the frame. Find the value of x that maximizes the viewing angle A.**



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LESSON 4-4

Increasing and Decreasing Functions

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Lesson (4-4)

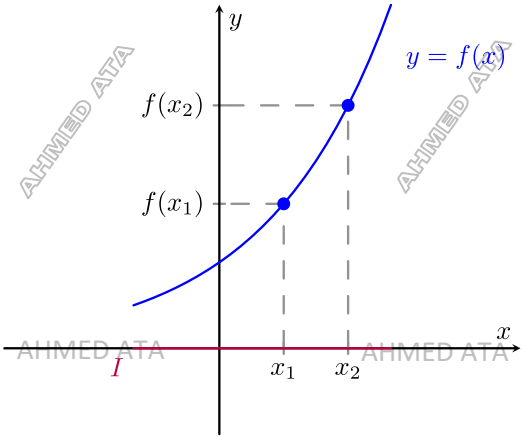
Increasing and Decreasing Functions

DEFINITION 4.1

A function f is **increasing** on an interval I if for every $x_1, x_2 \in I$ with $x_1 < x_2$, $f(x_1) < f(x_2)$ [i.e., $f(x)$ gets larger as x gets larger].

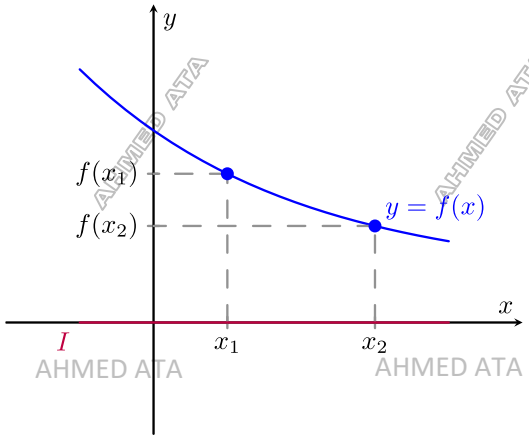
A function f is **decreasing** on the interval I if for every $x_1, x_2 \in I$ with $x_1 < x_2$, $f(x_1) > f(x_2)$ [i.e., $f(x)$ gets smaller as x gets larger].

$f(x)$ is increasing on I



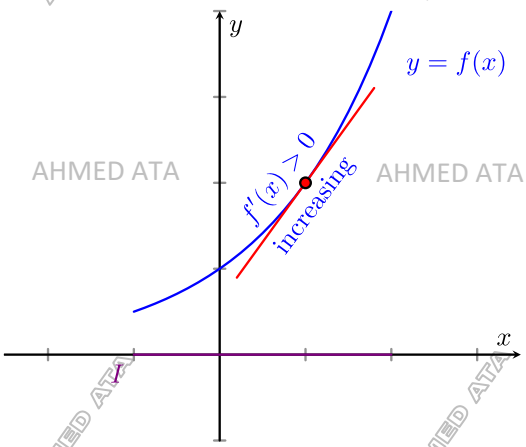
$x_1 < x_2 \implies f(x_1) < f(x_2)$

$f(x)$ is decreasing on I



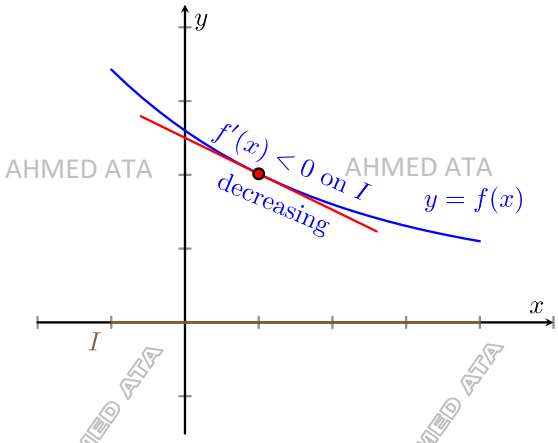
$x_1 < x_2 \implies f(x_1) > f(x_2)$

$f'(x) > 0$ on an interval I



$f'(x) > 0 \implies f(x)$ is increasing

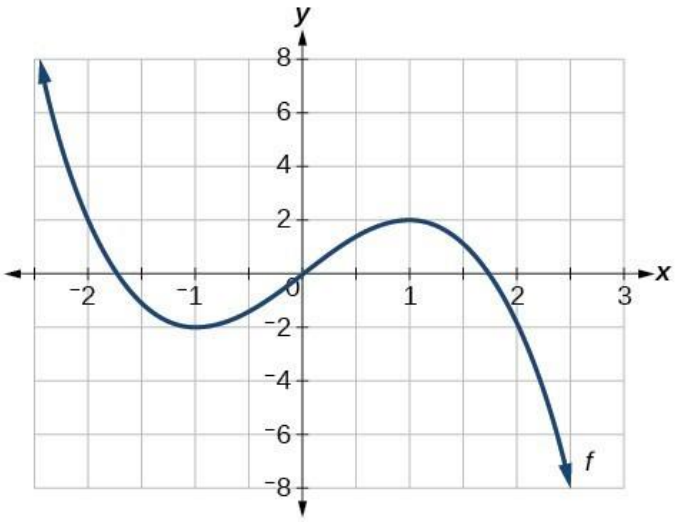
$f'(x) < 0$ on an interval I



$f'(x) < 0 \implies f(x)$ is decreasing

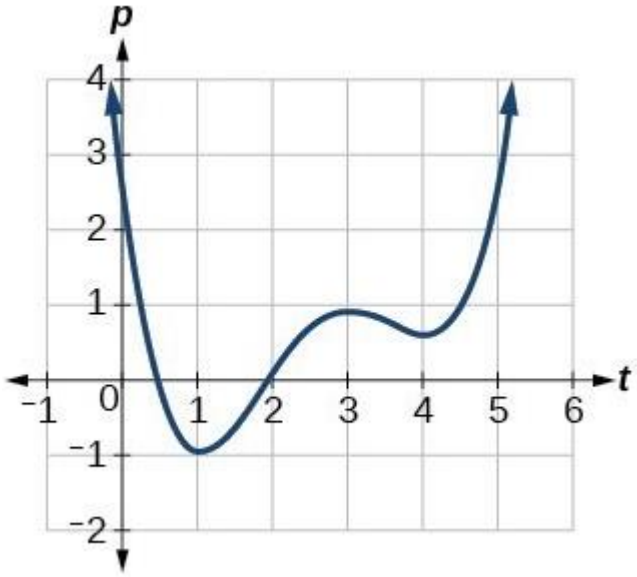
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1 in the graph Shown, identify



- Critical numbers
- Intervals increasing
- Intervals decreasing
- Absolute maximum
- Absolute minimum
- Relative (local) maximum
- Relative (local) minimum

2 in the graph Shown, identify



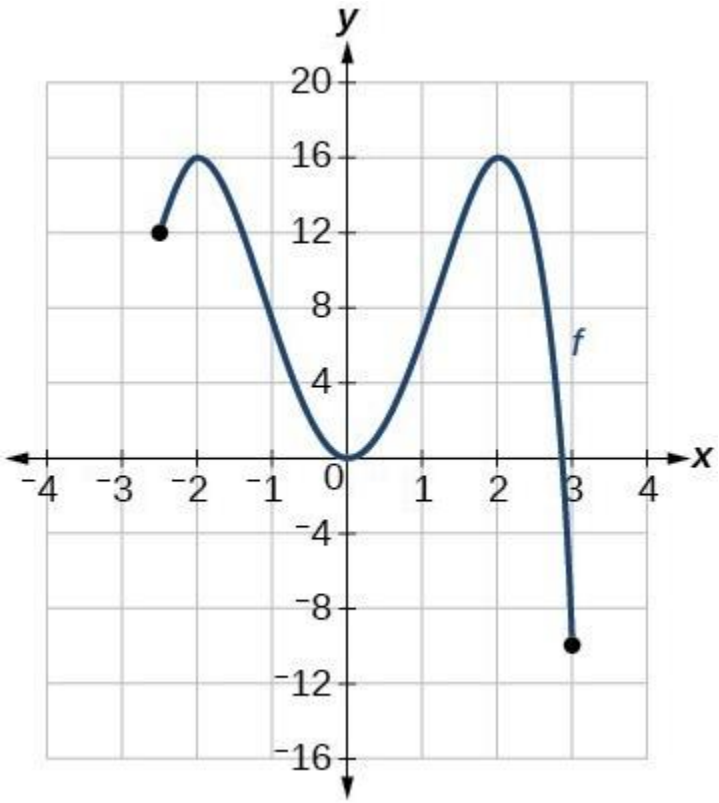
- Critical numbers
- Intervals increasing
- Intervals decreasing
- Absolute maximum
- Absolute minimum
- Relative (local) maximum
- Relative (local) minimum

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3 in the graph Shown, identify

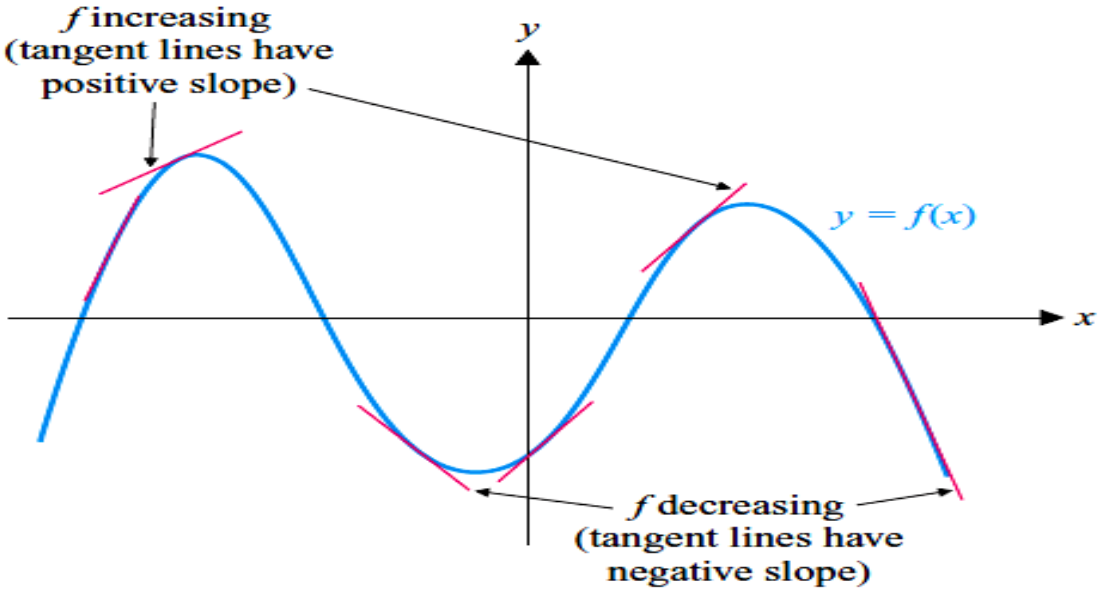
- Critical numbers
- Intervals increasing
- Intervals decreasing
- Absolute maximum
- Absolute minimum
- Relative (local) maximum
- Relative (local) minimum



THEOREM 4.1

Suppose that f is differentiable on an interval I.

(i) If $f'(x) > 0$ for all $x \in I$, then f is increasing on I.
(ii) If $f'(x) < 0$ for all $x \in I$, then f is decreasing on I.



Increasing and decreasing

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THEOREM 4.2 (First Derivative Test)

Suppose that f is continuous on the interval $[a, b]$ and $c \in (a, b)$ is a critical number.

(i) If $f'(x) > 0$ for all $x \in (a, c)$ and $f'(x) < 0$ for all $x \in (c, b)$ (i.e., f changes from increasing to decreasing at c), then $f(c)$ is a local maximum.
(ii) If $f'(x) < 0$ for all $x \in (a, c)$ and $f'(x) > 0$ for all $x \in (c, b)$ (i.e., f changes from decreasing to increasing at c), then $f(c)$ is a local minimum.
(iii) If $f'(x)$ has the *same* sign on (a, c) and (c, b), then $f(c)$ is *not* a local extremum.

Find (by hand)

a) All critical numbers
b) The intervals where the function is increasing and decreasing
c) Use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

4 $y = x^3 - 3x + 2$

5 $y = x^4 - 8x^2 + 1$

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6 $y = (x+1)^{2/3}$

7 $y = \sin x + \cos x$

8 $y = e^{x^2-1}$

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9 $y = \ln(x^2 - 1)$

Find (by hand) a) All critical numbers

b) Use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

10 $y = \tan^{-1}(x^2)$

11 $f(x) = x^{5/3} - 3x^{2/3}$.

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12 $y = \sqrt{x^3 + 3x^2}$

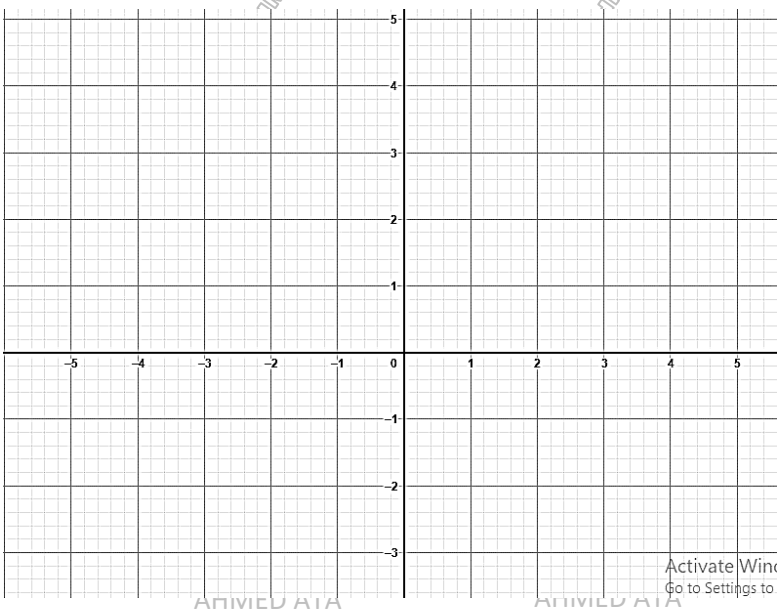
13 $y = \dfrac{x}{1 + x^3}$

14 **Sketch a graph of a function with the given properties.**

$f(0) = 1, \quad f(2) = 5,$

$f'(x) < 0 \quad \text{for} \quad x < 0 \quad \text{and} \quad x > 2,$

$f'(x) > 0 \text{ for } 0 < x < 2.$



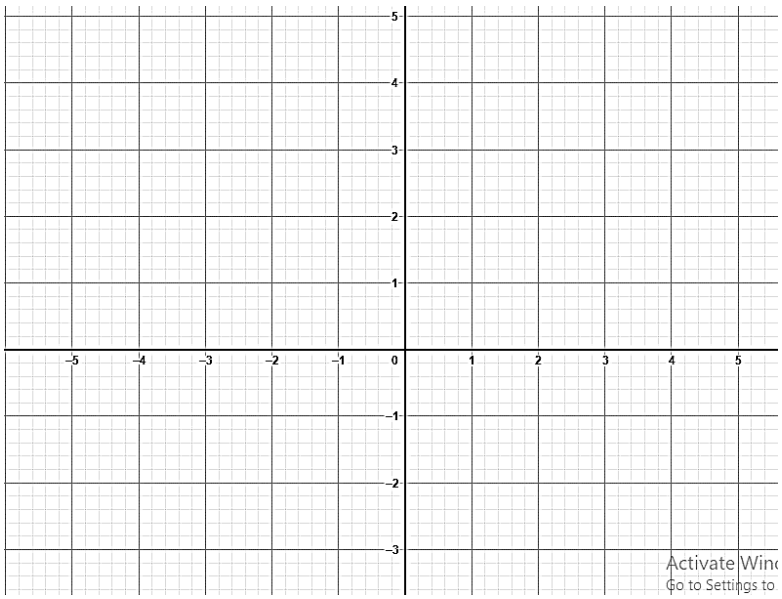
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15 $f(-1)=1,\ f(2)=5,$

$f'(x)<0$ for $x<-1$ and $x>2,$

$f'(x)>0$ for $-1<x<2,$

$f'(-1)=0,\ f'(2)$ does not exist.

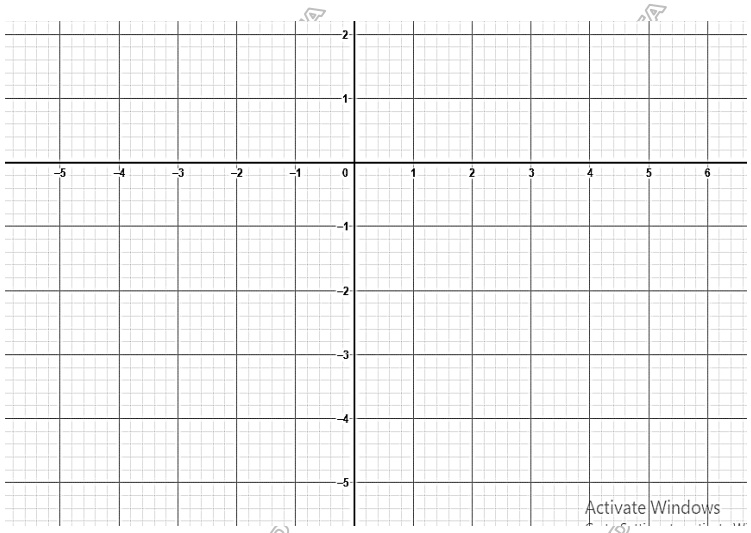


16 $f(3)=0,\ f'(x)<0$ for $x<0$ and $x>3,$

$f'(x)>0$ for $0<x<3,$

$f'(3)=0,$

$f(0)$ and $f'(0)$ do not exist.



17 $f(1)=0,\ \lim_{x\to\infty} f(x)=2,$

$f'(x)<0$ for $x<1,$

$f'(x)>0$ for $x>1,$

$f'(1)=0.$



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18 **Find (by hand) all asymptotes and extrema**

$$y = \frac{x}{x^2 - 1}$$

19

$$y = \frac{x^2}{x^2 - 1}$$

20 State the domain for $\sin^{-1} x$ and determine where it is increasing and decreasing.

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LESSON 4-5

Concavity and the Second Derivative Test

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Lesson (4-5)

Concavity and the Second Derivative Test

DEFINITION 5.1

For a function f that is differentiable on an interval I, the graph of f is

(i) **concave up** on I if f' is increasing on I or
(ii) **concave down** on I if f' is decreasing on I.

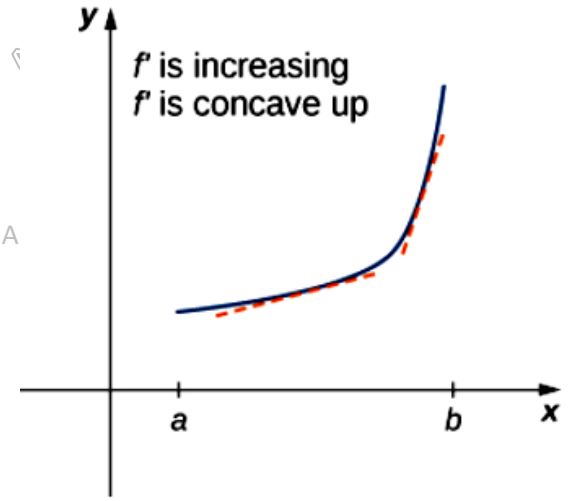
THEOREM 5.1

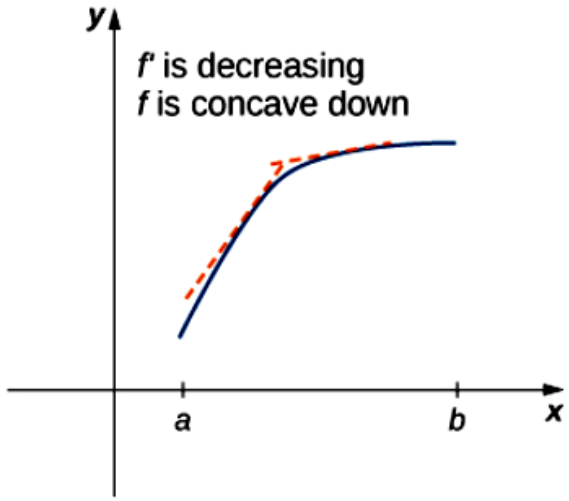
Suppose that f'' exists on an interval I.

(i) If $f''(x) > 0$ on I, then the graph of f is concave up on I.
(ii) If $f''(x) < 0$ on I, then the graph of f is concave down on I.

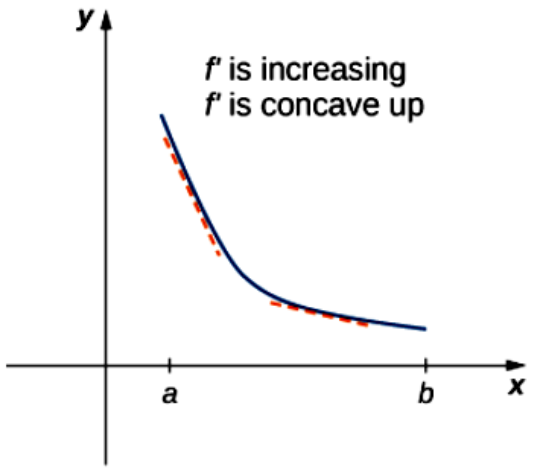
DEFINITION 5.2

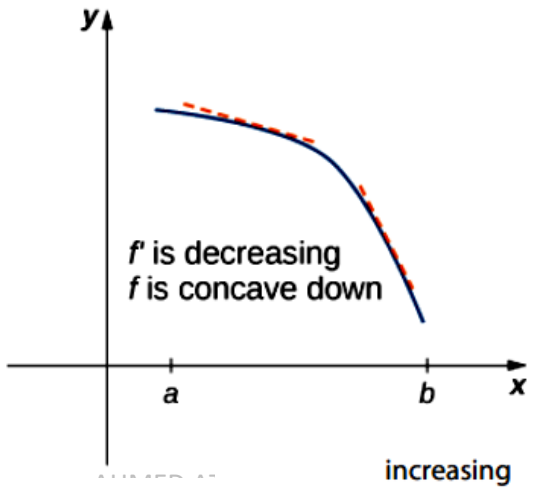
Suppose that f is continuous on the interval (a, b) and that the graph changes concavity at a point $c \in (a, b)$ (i.e., the graph is concave down on one side of c and concave up on the other). Then, the point $(c, f(c))$ is called an **inflection point** of f.

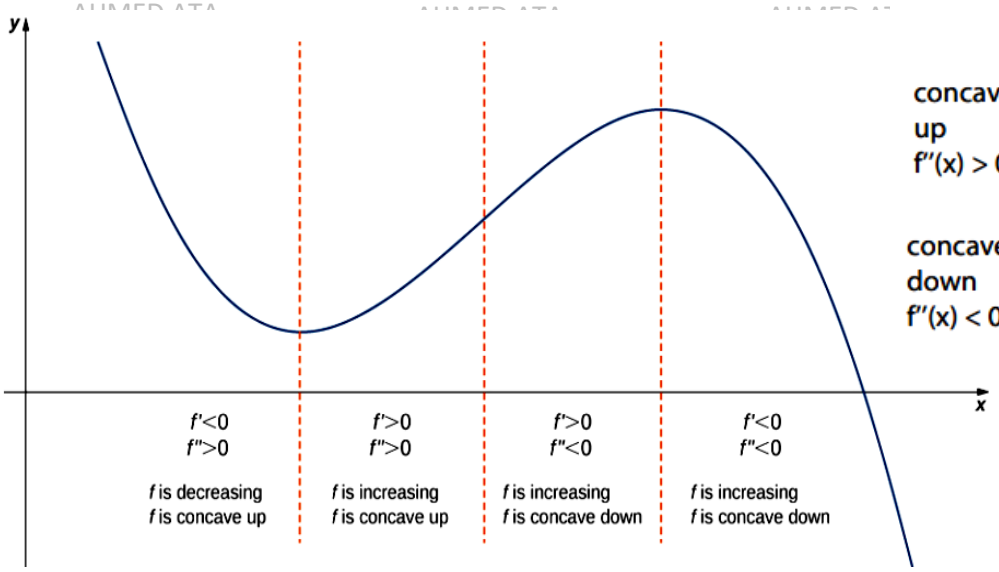


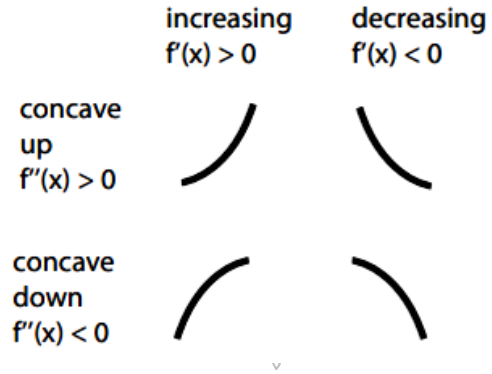


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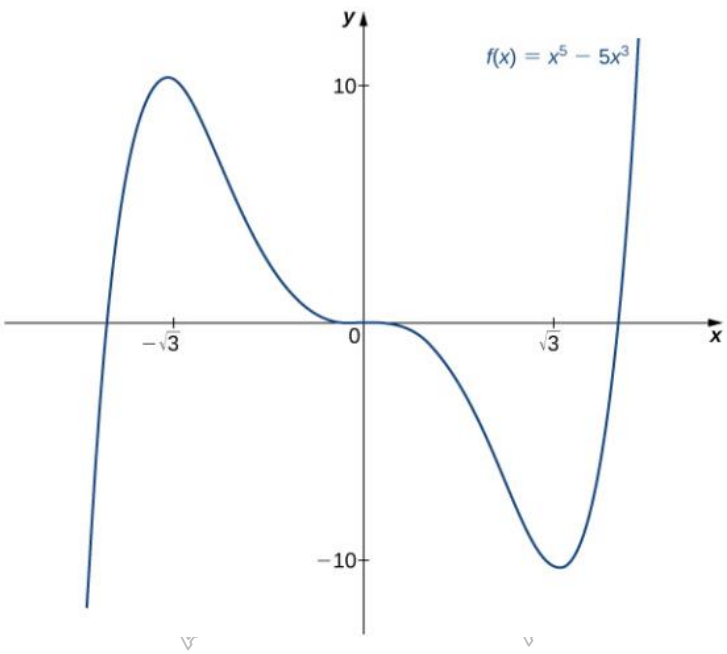






1 **in the graph Shown, identify**

- **Critical numbers**
- **Intervals increasing**
- **Intervals decreasing**
- **Intervals concave up**
- **Intervals concave down**
- **inflection point**



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2 **Function is concave up between points (choose one or more)**

(i) A to C

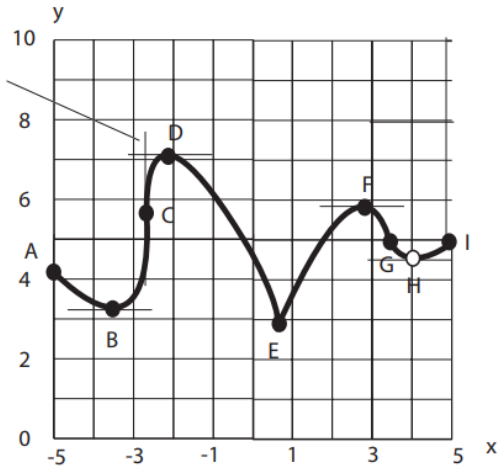
(ii) C to E

(iii) E to F

(iv) F to G

(v) G to I

(vi) none



3 **Function is concave down between points (choose one or more)**

(i) A to C

(ii) C to E

(iii) E to F

(iv) F to G

(v) G to I

(vi) none

4 **Use the numbered points on the graph of $f(x)$ at the right to answer parts (a) through (g).**

(a) At which points is the function increasing?

(b) At which points is there a relative maximum?

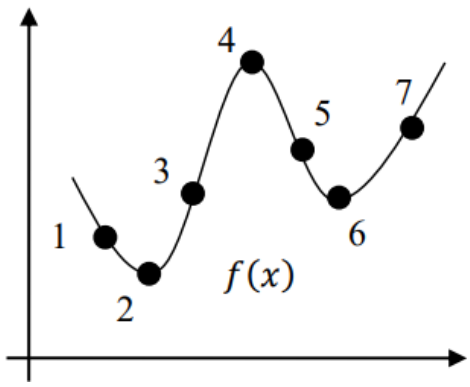
(c) At which points is the slope negative?

(d) At which points is the slope zero?

(e) At which points is there a relative minimum?

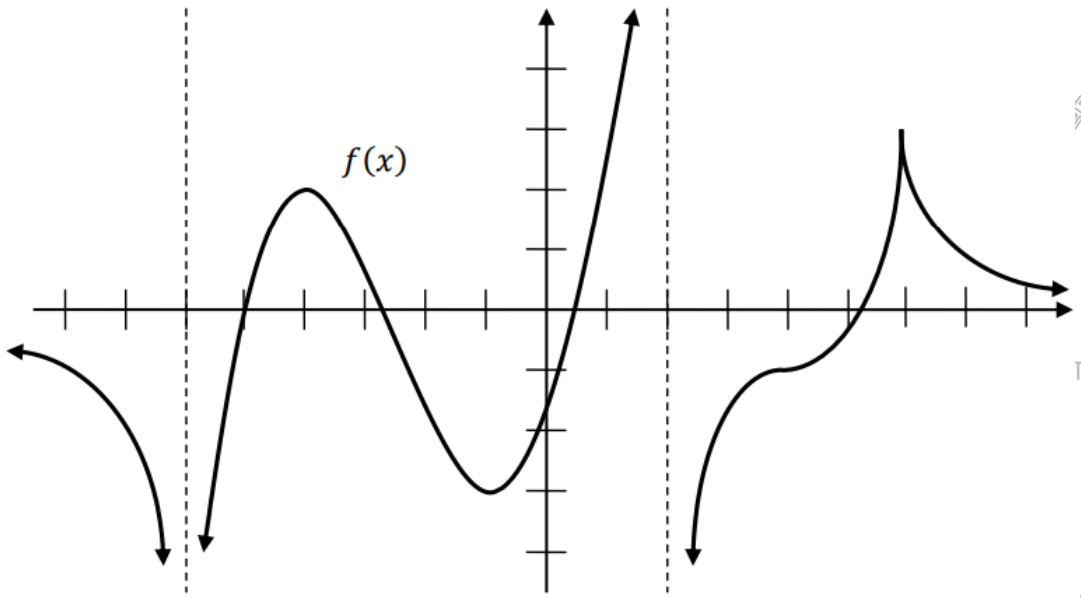
(f) At which points is the function decreasing?

(g) At which points is the slope positive?



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5 **Use the graph of $f(x)$ below to answer parts (a) through (h).**



(a) Find the intervals where $f(x)$ is increasing.

(b) Find the intervals where $f(x)$ is decreasing.

(c) Find all the x-values where the slope of $f(x)$ is zero.

(d) Find all the x-values where the derivative of $f(x)$ does not exist.

(e) Find all the critical points of $f(x)$.

(f) Find the coordinates of all the relative maxima of $f(x)$.

(g) Find the coordinates of all the relative minima of $f(x)$.

(h) Find all the x-values where $f(x)$ changes from increasing to decreasing or decreasing to increasing.

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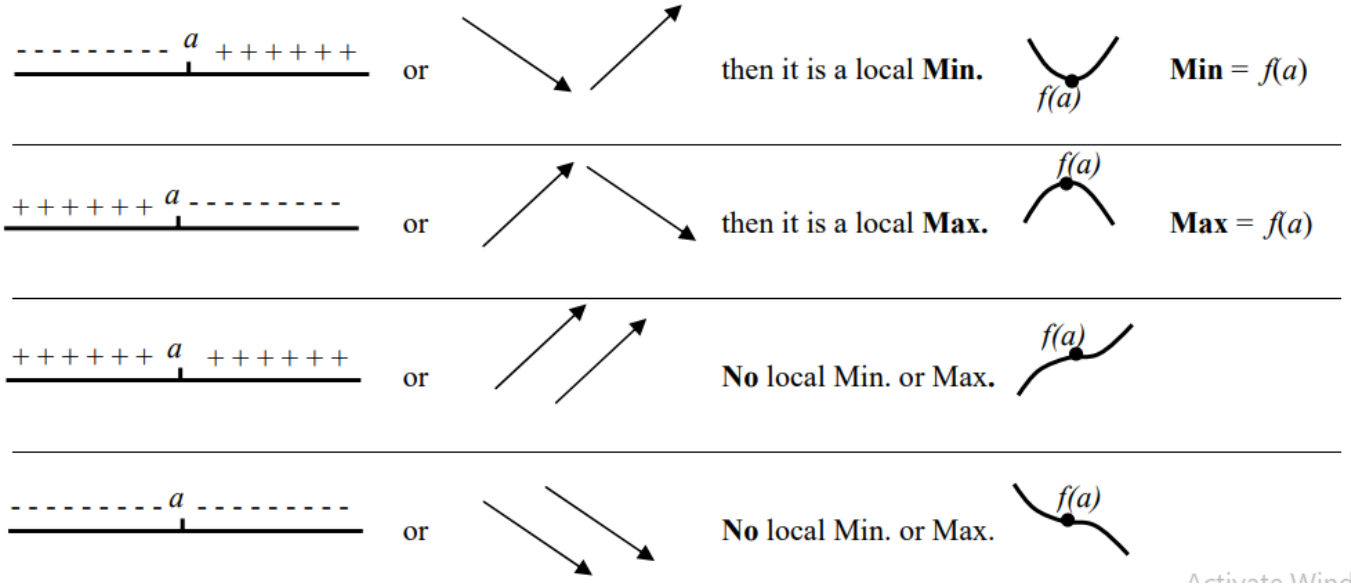
Applications of The First Derivative:

Step 1: Finding the critical points

Step 2: Determining the intervals where the function is increasing or decreasing

Step 3: Finding the local maxima and local minima

Step 4: Look at both sides of each critical point, take point for example:



Applications of The Second Derivative

Step 5: Locate the inflection points where the second derivative is = 0

Step 6: Divide f '' (x) into intervals using the inflection points found in the previous step

Step 7: Find the second derivative for function in each test point

Step 8: Summarize all results in the following table:

Increasing in the intervals:	
Decreasing in the intervals:	
Local Max. points and Max values:	
Local Min. points and Min values:	
Inflection points at:	
Concave Up in the intervals:	
Concave Down in the intervals:	

Step 9: Sketch the graph using the information from steps 3,4 and 7 showing the critical points, inflection points, intervals of increasing or decreasing, local maxima and minima and the intervals of concave up or down

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6 **determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points**

$$f(x) = x^3 - 3x^2 + 4x - 1$$

7 $f(x) = x^4 - 6x^2 + 2x + 3$

8 $f(x) = x + 3(1 - x)^{1/3}$

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9 $f(x) = \sin x - \cos x$

10 $f(x) = x + 1/x$

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Find all critical numbers and use the Second Derivative Test to determine all local extrema.

11 $f(x) = x^4 + 4x^3 - 1$

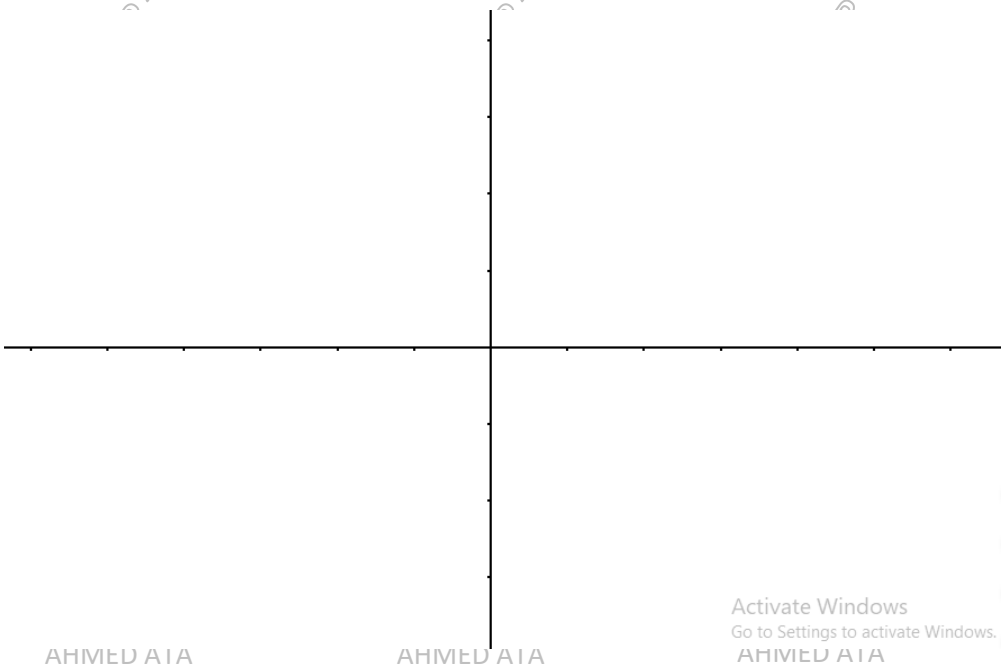
12 $f(x) = xe^{-x}$

13 $f(x) = \dfrac{x^2 - 5x + 4}{x}$

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Determine all significant features (approximately if necessary) and sketch a graph

14 $f(x) = x^4 - 26x^3 + x$



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Activate Windows
Go to Settings to activate Windows.

15 $if\ f(x) = ax^3 - 6x^2\ has\ inflection\ point\ at\ x = 2\ find\ value\ a$

16 $if\ f(x) = x^3 + ax^2 + bx\ \ has\ local\ minimum\ at\ x = 4$
$and\ inflection\ point\ at\ x = 1\ find\ value\ a\ and\ b$

17 $if\ f(x) = ax^2 + bx + c\ \ pass\ through\ point\ (1, 2)\ and\ has$
$local\ maximum\ at\ point\ (0, 3) find\ \ values\ a, b\ and\ c$

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18 $if\ f(x) = x^3 + ax^2 + bx$ $has\ maximun\ value\ at\ x = -1$ $and\ minimum\ value\ at\ x = 3\ find$ $values\ a\ and\ b$

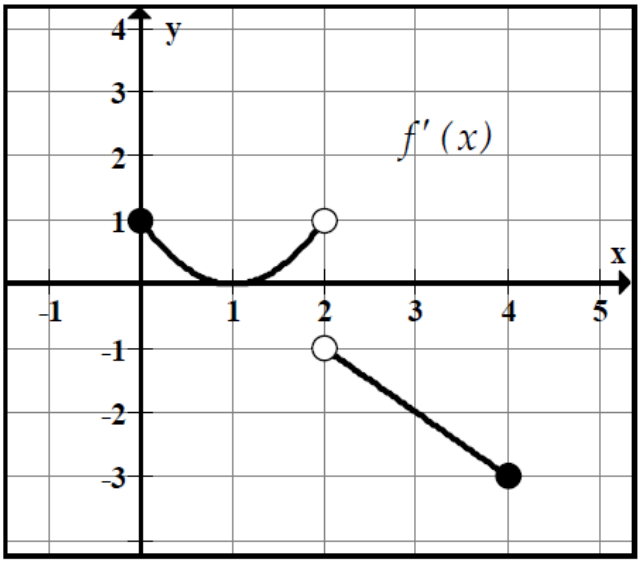
19 $if\ f(x)\ has\ minimum\ value\ at\ x = 1\ such\ that$

$$f'(x) = \frac{(x^2+1)(x-a)}{x^4+1}\ find\ value\ a$$

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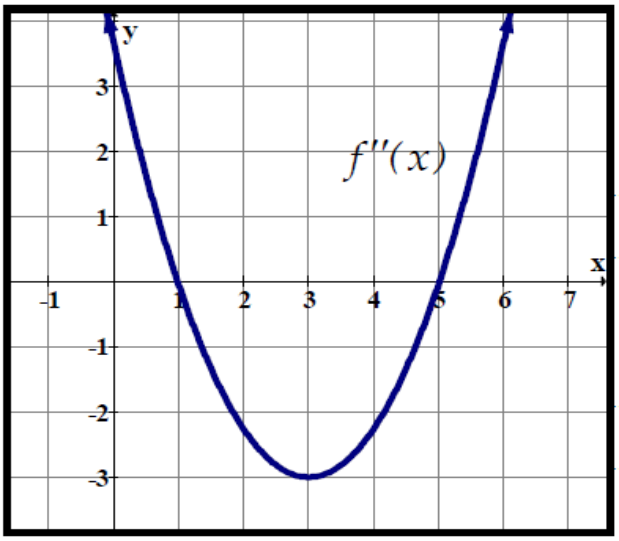
20 in the graph Shown, such that f(x) continuous on $[0,4]$ identify

- Critical numbers
- Intervals increasing
- Intervals decreasing
- Intervals concave up
- Intervals concave down
- inflection point



21 in the graph Shown, $f''(x)$ *defined for all real number*, identify

- Intervals concave up
- Intervals concave down
- inflection point



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determine the intervals where the graph of the given function is concave up and concave down, and identify inflection points

22 $f(x) = x + 3(1 - x)^{1/3}$

Find all critical numbers and use the Second Derivative Test to determine all local extrema.

23 $f(x) = 6x^2 + 5x - 11$

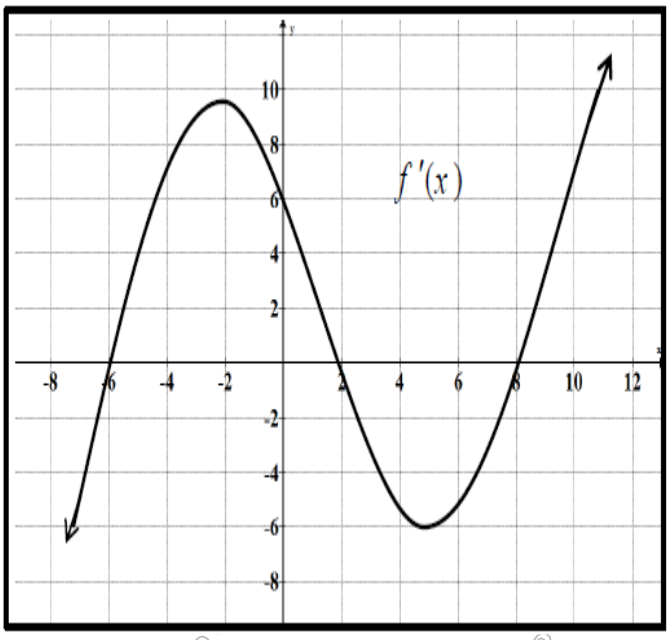
24 $f(x) = -3x^2 - 7x - 1$

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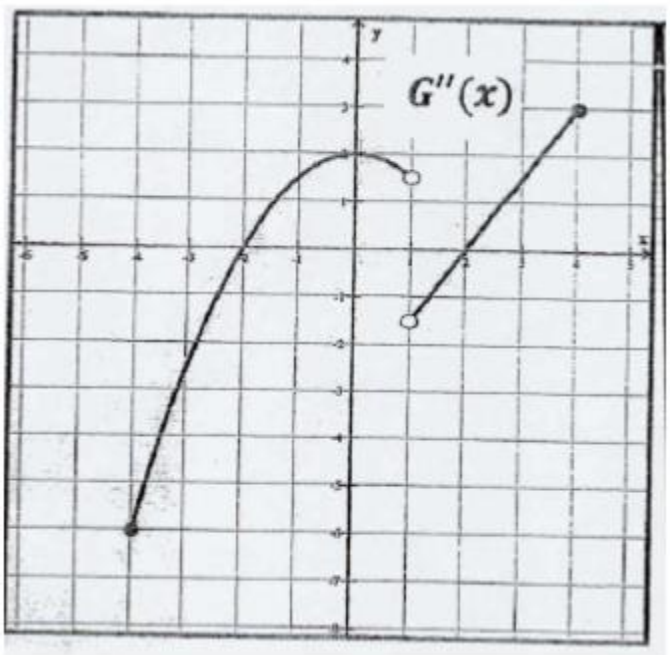
25 in the graph Shown, such that f(x) continuous identify

- Critical numbers
- Intervals increasing
- Intervals decreasing
- Intervals concave up
- Intervals concave down
- inflection point



26 in the graph Shown $G''(x)$, such that G(x) continuous on $[-4, 4]$ identify

- Intervals concave up
- Intervals concave down
- inflection point



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MATH ENG

LESSON 4-6

Overview of Curve Sketching

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

Prepared by :

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Lesson (4-6)

Overview of Curve Sketching

We begin by summarizing the steps that you should take when trying to draw a graph of y = f (x).

- **Domain:**

  Always determine the domain of *f first*.

- **Vertical Asymptotes:**

  For any isolated point not in the domain of *f*(x), check the limit of *f* (*x*) as *x* approaches that point, to see if there is a vertical asymptote or a jump or removable discontinuity at that point.

- **First Derivative Information:**

  Determine where *f* is increasing and decreasing, and find any local extrema.

- **Vertical Tangent Lines:**

  At any isolated point not in the domain of $f'(x)$, *but in the domain of f(x),* check the limit of $f'(x)$, to determine whether there is a vertical tangent line at that point.

- **Second Derivative Information:**

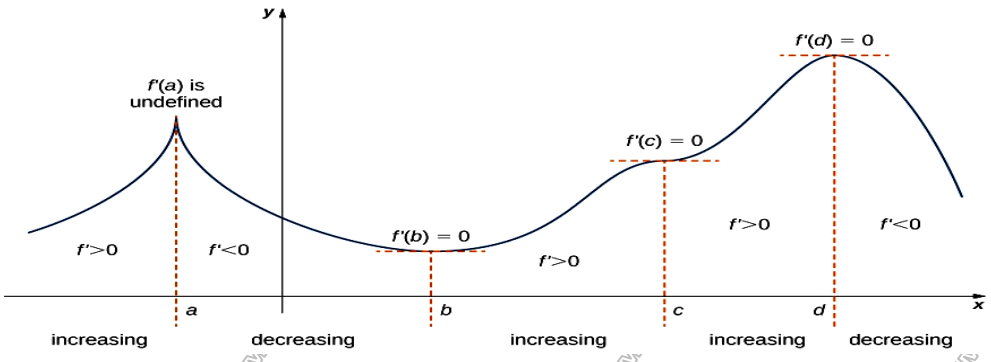
  Determine where the graph is concave up and concave down and locate any inflection points.

- **Horizontal Asymptotes:**

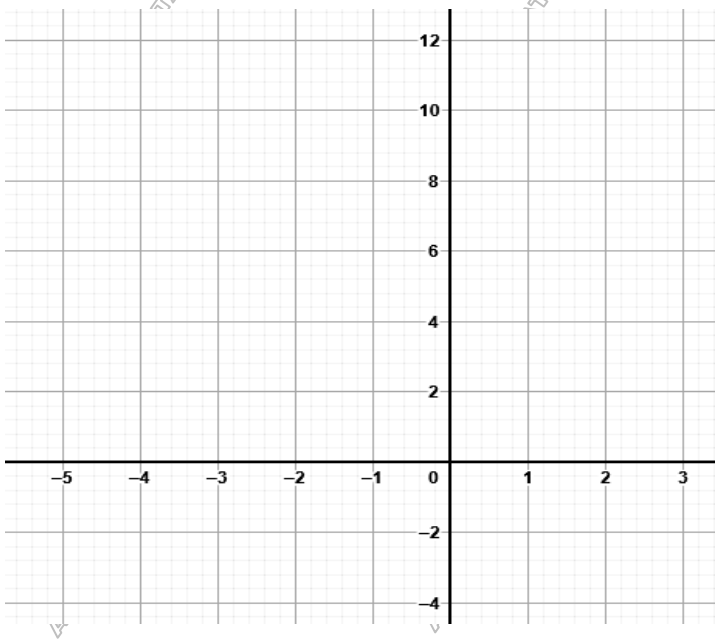
  Check the limit of *f* (*x*) as $x \to \infty$ and as $x \to -\infty$.

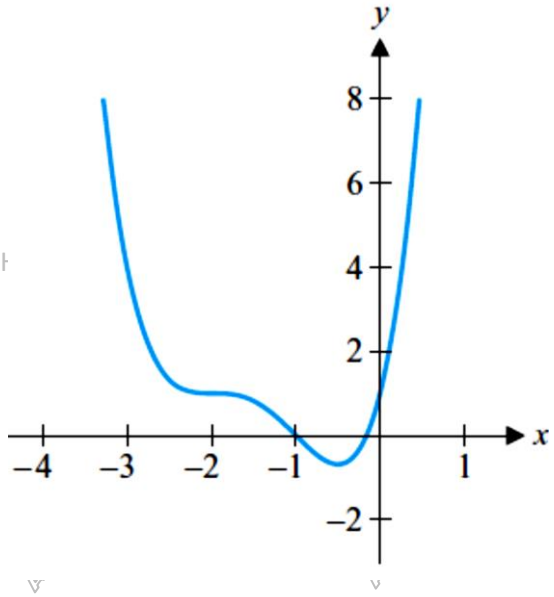
- **Intercepts:**

  Locate *x*- and *y*-intercepts, if any. If this can't be done exactly, then do so approximately (e.g., using Newton's method).



1 Draw a graph of $f(x) = x^4 + 6x^3 + 12x^2 + 8x + 1$, showing all significant features



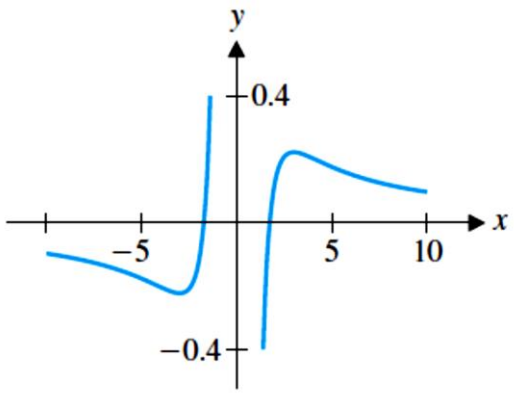


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2 Draw a graph of $f(x) = \dfrac{x^2 - 3}{x^3}$, showing all significant features.

$$f'(x) = \frac{(3-x)(3+x)}{x^4}.$$

$$f''(x) = \frac{2(x-\sqrt{18})(x+\sqrt{18})}{x^5}.$$

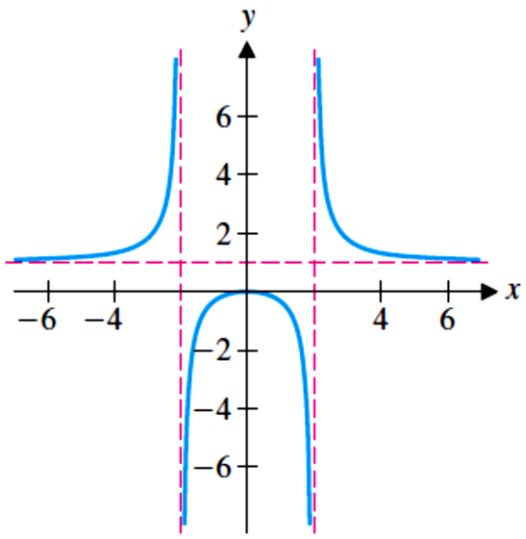


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3 Draw a graph of $f(x) = \dfrac{x^2}{x^2 - 4}$ showing all significant features.

$$f'(x) = \frac{-8x}{(x^2 - 4)^2}.$$

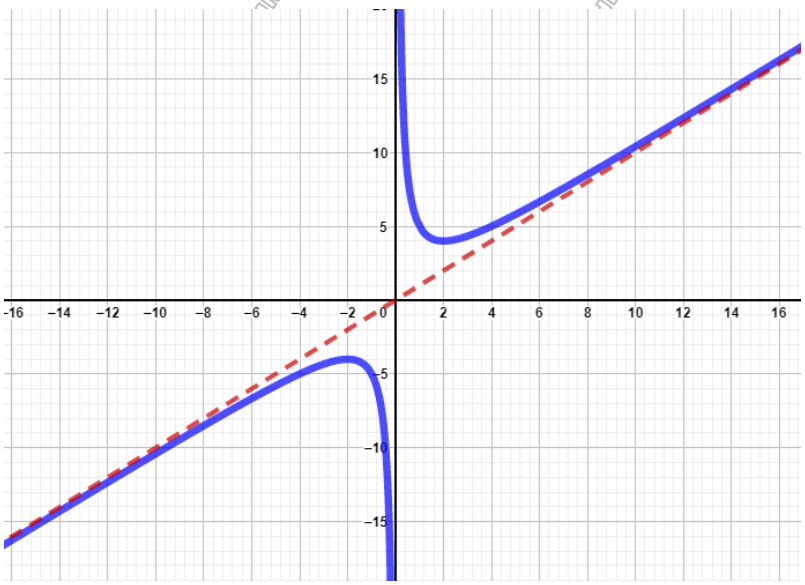
$$f''(x) = \frac{8(3x^2 + 4)}{(x - 2)^3(x + 2)^3}.$$



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determine all significant features (approximately if necessary) and sketch a graph

4 $f(x) = x + \frac{4}{x}$



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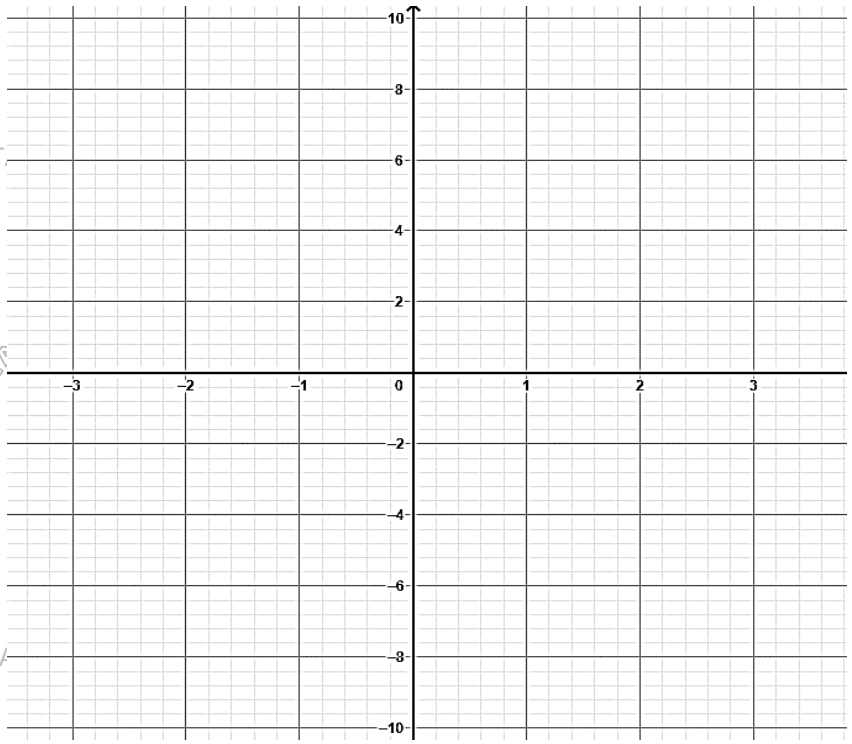
5 **Sketch a graph with the given properties**

$f(0)=0,\ f'(x)>0$ for $x<-1$ and $-1<x<1$,

$f'(x)<0$ for $x>1$

$f''(x)>0$ for $x<-1,\ 0<x<1$ and $x>1$,

$f''(x)<0$ for $-1<x<0$



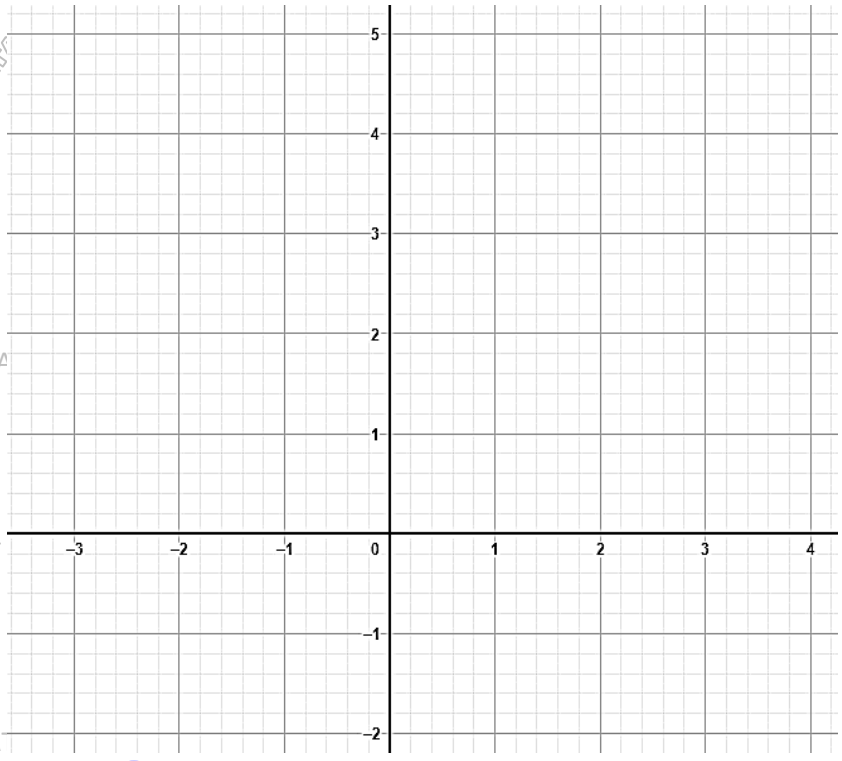
6

$f(0)=2,\ f'(x)>0$ for all x,

$f'(0)=1$,

$f''(x)>0$ for $x<0$,

$f''(x)<0$ for $x>0$



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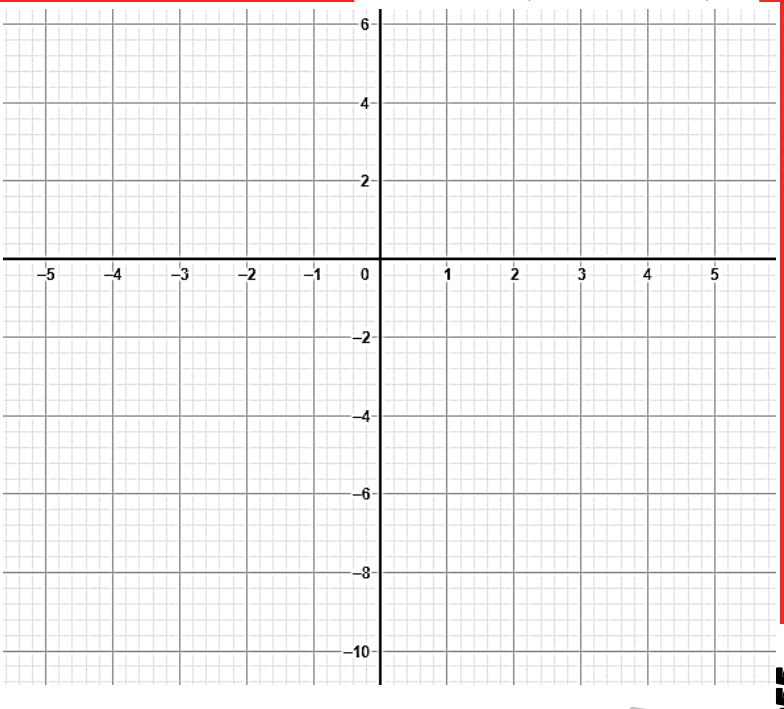
7

$f(0) = 0,\ f(-1) = -1,\ f(1) = 1,$

$f'(x) > 0$ for $x < -1$ and $0 < x < 1,$

$f'(x) < 0$ for $-1 < x < 0$ and $x > 1,$

$f''(x) < 0$ for $x < 0$ and $x > 0$



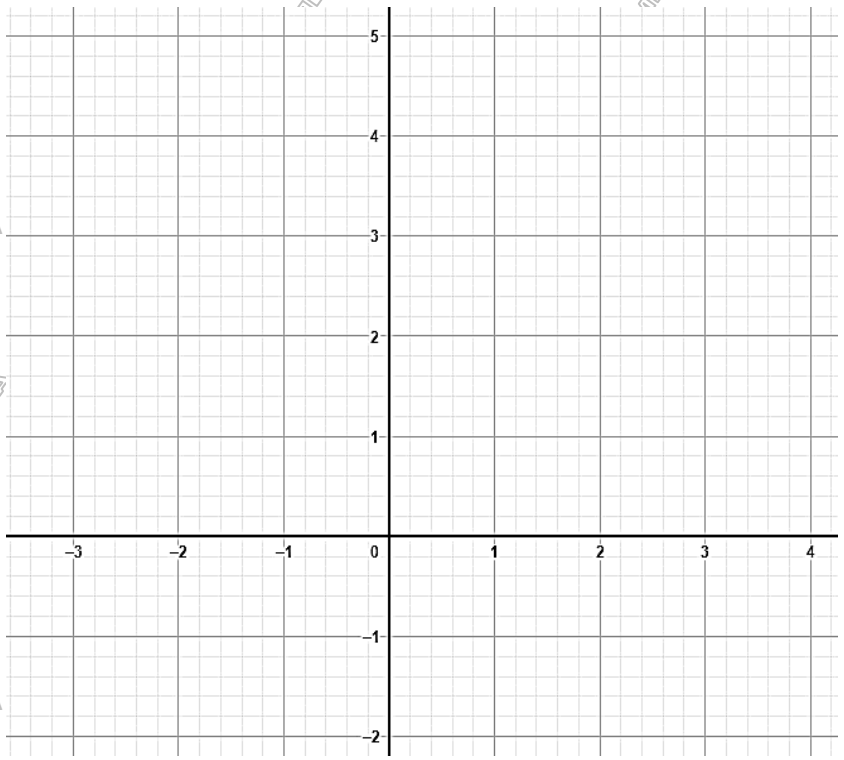
8

$f(1) = 0,$

$f'(x) < 0$ for $x < 1,$

$f'(x) > 0$ for $x > 1,$

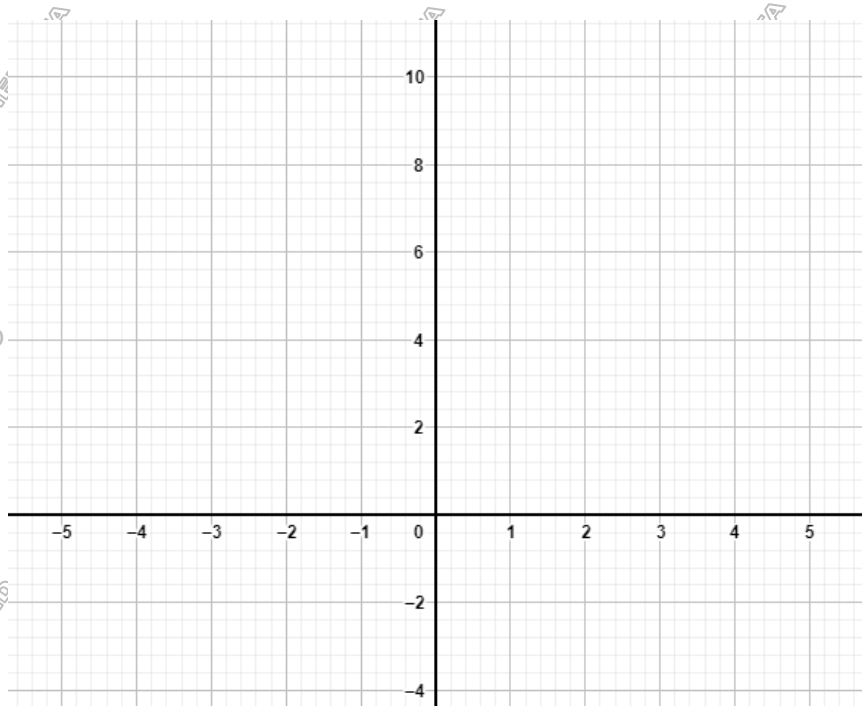
$f''(x) < 0$ for $x < 1$ and $x > 1$



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9 $f(2) = 2, f'(x) < 0$ *and* $f''(x) > 0$ *when* $x < 2$

$f'(x) > 0$ *and* $f''(x) < 0$ *when* $x > 2$

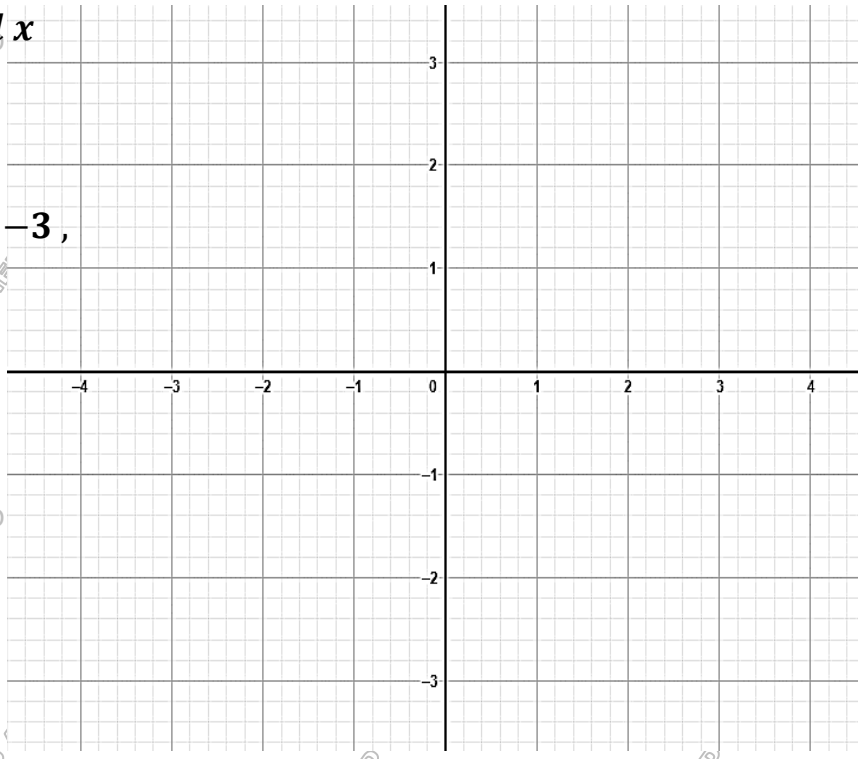


10 $f(0) = 0$ $f'(x) > 0$ *for all* x

$f''(x) > 0$ *where* $x < 0$

$f''(x) < 0$ *where* $x > 0$

$\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to-\infty} f(x) = -3$,



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11 Find a function whose graph has the given asymptotes.

$$x = 1, x = 2 \text{ and } y = 3$$

$a)\ f(x) = \dfrac{2x^2}{(x-1)(x-3)}$ $b)\ f(x) = \dfrac{2x}{\sqrt{(x-1)(x+1)}}$

$c)\ f(x) = \dfrac{x}{x^2-1}$ $d)\ f(x) = \dfrac{3x^2}{(x-1)(x-2)}$

12 Find a function whose graph has the given asymptotes.

$$x = -1, x = 1 \text{ and } y = 0$$

$a)\ f(x) = \dfrac{2x^2}{(x-1)(x+1)}$ $b)\ f(x) = \dfrac{2x}{\sqrt{(x-1)(x+1)}}$

$c)\ f(x) = \dfrac{x}{x^2-1}$ $d)\ f(x) = \dfrac{3x^2}{(x-1)(x+1)}$

13 Find a function whose graph has the given asymptotes.

$$x = -1, x = 1 \text{ and } y = -2, y = 2$$

$a)\ f(x) = \dfrac{2x^2}{(x-1)(x+1)}$ $b)\ f(x) = \dfrac{2x}{\sqrt{(x-1)(x+1)}}$

$c)\ f(x) = \dfrac{x}{x^2-1}$ $d)\ f(x) = \dfrac{3x^2}{(x-1)(x+1)}$

14 Find a function whose graph has the given asymptotes.

$$x = 1, x = 3 \text{ and } y = 2$$

$a)\ f(x) = \dfrac{2x^2}{(x-1)(x-3)}$ $b)\ f(x) = \dfrac{2x}{\sqrt{(x-1)(x-3)}}$

$c)\ f(x) = \dfrac{x}{x^2-1}$ $d)\ f(x) = \dfrac{3x^2}{(x-1)(x-3)}$

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MATH ENG

LESSON 4-7

Optimization

Mr. Ahmed Ata
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2024-2025

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Lesson (4-7)

Optimization

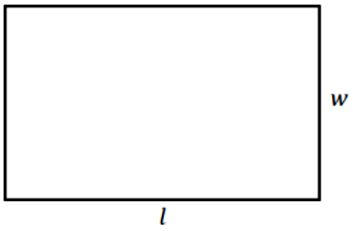
Perimeter and Area

Square



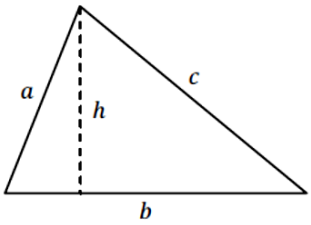
Perimeter: $P = 4s$
Area: $A = s^2$

Rectangle



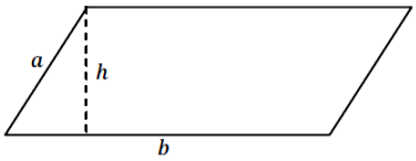
Perimeter: $P = 2l + 2w$
Area: $A = lw$

Triangle



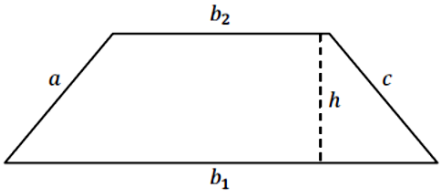
Perimeter: $P = a + b + c$
Area: $A = \frac{bh}{2}$

Parallelogram



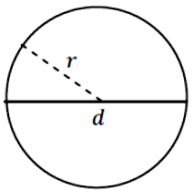
Perimeter: $P = 2a + 2b$
Area: $A = bh$

Trapezoid



Perimeter: $P = a + b_1 + c + b_2$
Area: $\frac{1}{2}(b_1 + b_2) \cdot h$

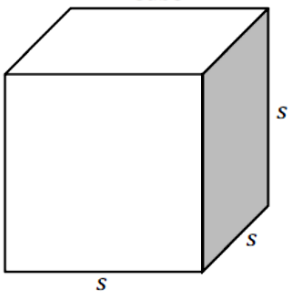
Circle



Circumference: $C = 2\pi r$ or $C = \pi d$
Area: $A = \pi r^2$

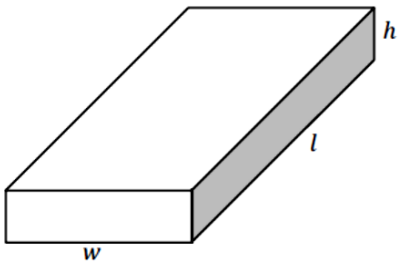
Volume & Surface Area

Cube



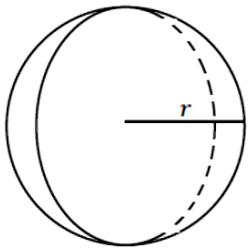
Volume: $V = s^3$
Surface Area: $S = 6s^2$

Rectangular Prism (Box)



Volume: $V = lwh$
Surface Area: $S = 2lh + 2wh + 2wl$

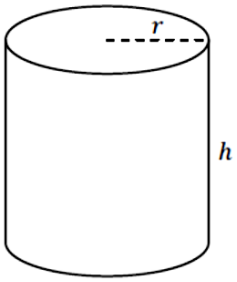
Sphere



Volume: $V = \frac{4}{3}\pi r^3$
Surface Area: $S = 4\pi r^2$

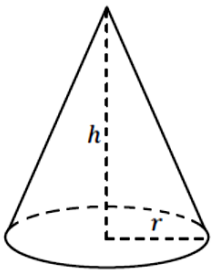
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Cylinder



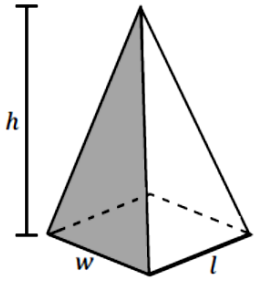
Volume: $V = \pi r^2 h$
Surface Area: $S = 2\pi rh + 2\pi r^2$

Cone

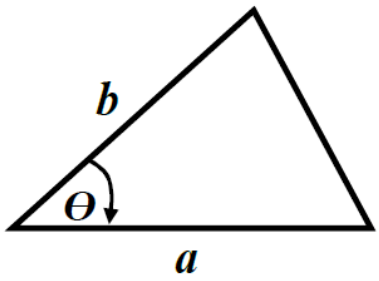


Volume: $V = \frac{1}{3}\pi r^2 h$
Surface Area: $S = \pi r\sqrt{r^2 + h^2}$

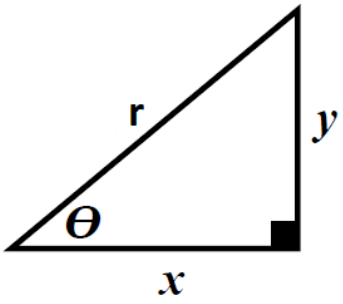
Square or Rectangle Pyramid



Volume: $V = \frac{1}{3}lwh$

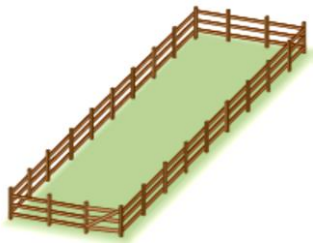


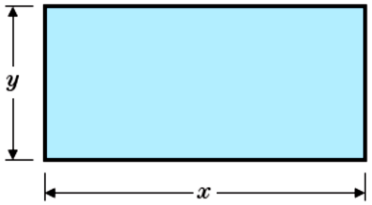
$A = \frac{1}{2} a\, b \sin\theta$



$r^2 = x^2 + y^2$

1 **You have 40 (linear) feet of fencing with which to enclose a rectangular space for a garden. Find the *largest* area that can be enclosed with this much fencing and the dimensions of the corresponding garden.**





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2 **A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 ft^2. Find the minimum perimeter and the dimensions of the corresponding enclosure.**

3 **A three-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. There is 96 feet of fencing available. Find the maximum enclosed area and the dimensions of the corresponding enclosure.**

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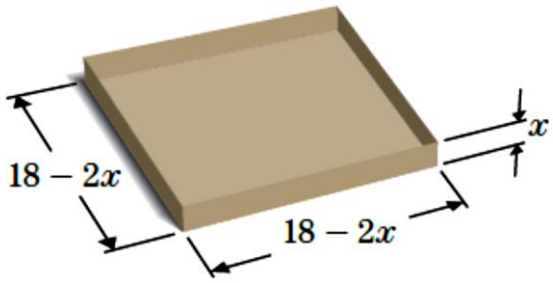
4 **A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 ft of fencing available, what dimensions of the corral will maximize the enclosed area?**

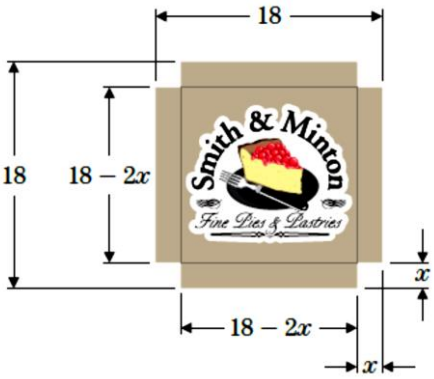
5 **A showroom for a department store is to be rectangular with walls on three sides, 6-ft door openings on the two facing sides and a 10-ft door opening on the remaining wall. The showroom is to have 800 ft^2 of floor space. What dimensions will minimize the length of wall used?**

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6 **Show that the rectangle of maximum area for a given perimeter P is always a square**

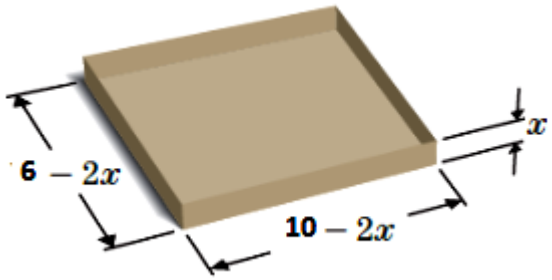
7 **A square sheet of cardboard 18 in on a side is made into an open box (there's no top), by cutting squares of equal size out of each corner (see Figure) and folding up the sides along the dotted lines. (See Figure) Find the dimensions of the box with the maximum volume.**





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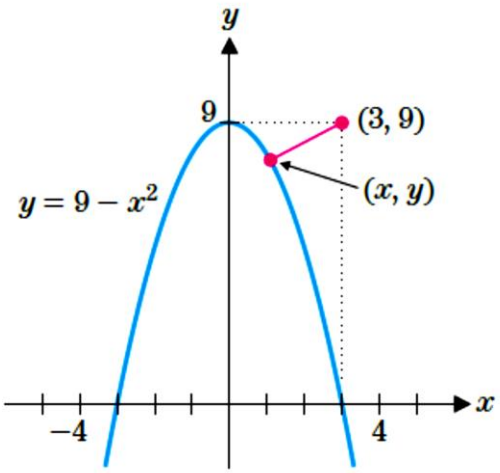
8 **A box with no top is to be built by taking a 6 in by 10 in sheet of cardboard, cutting x-in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.**



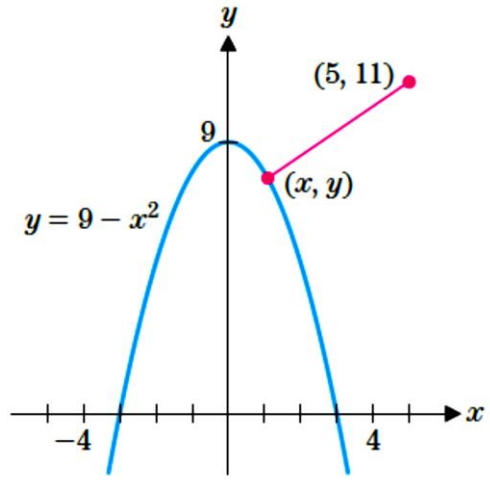
9 **A box with no top is built by taking a 6 in-by-6 in piece of cardboard, cutting x-in. squares out of each corner, and folding up the sides. The four x-in. squares are then taped together to form a second box (with no top or bottom). Find the value of x that maximizes the sum of the volumes of the boxes.**

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10 **Find the point on the parabola $y = 9 - x^2$ closest to the point (3, 9). (See Figure)**

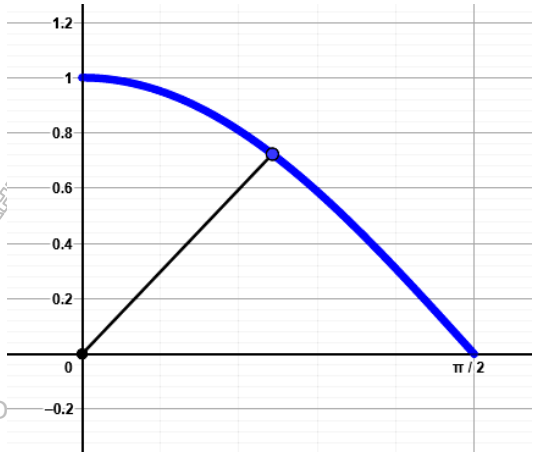


11 **Find the point on the parabola $y = 9 - x^2$ closest to the point (5, 11). (See Figure)**

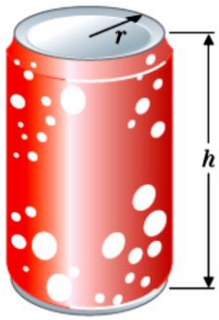


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12 **Find the point on the curve $y = \cos x$ closest to the point (0, 0).**



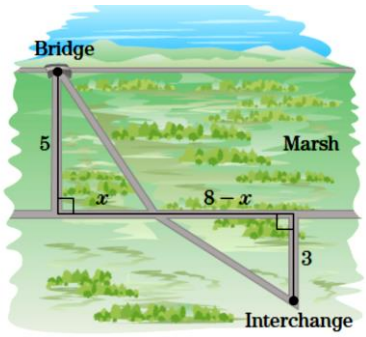
13 **A soda can is to hold 12 fluid ounces. Find the dimensions that will minimize the amount of material used in its construction, assuming that the thickness of the material is uniform (i.e., the thickness of the aluminum is the same everywhere in the can).**



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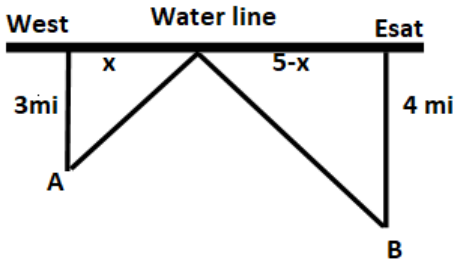
14 **A soda can is to hold 12 fluid ounces. Suppose that the bottom and top are twice as thick as the sides. Find the dimensions of the can that minimize the amount of material used. (Hint: Instead of minimizing surface area, minimize the cost, which is proportional to the product of the thickness and the area.)**

15 **The state wants to build a new stretch of highway to link an existing bridge with a turnpike interchange, located 8 miles to the east and 8 miles to the south of the bridge. There is a 5-mile-wide stretch of marshland adjacent to the bridge that must be crossed. (See Figure) Given that the highway costs $10 million per mile to build over the marsh and only $7 million per mile to build over dry land, how far to the east of the bridge should the highway be when it crosses out of the marsh?**

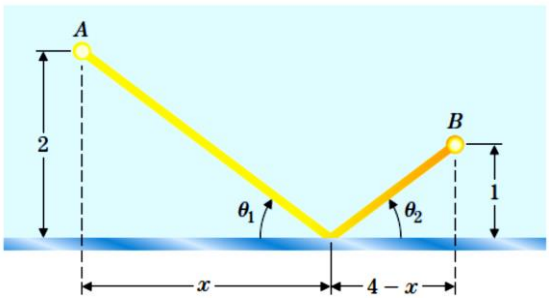


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16 **A water line runs east-west. A town wants to connect two new housing developments to the line by running lines from a single point on the existing line to the two developments. One development is 3 miles south of the existing line; the other development is 4 miles south of the existing line and 5 miles east of the first development. Find the place on the existing line to make the connection to minimize the total length of new line.**

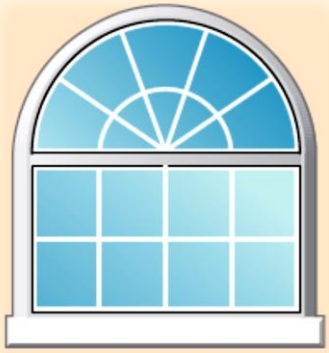


17 **Suppose that light reflects off a mirror to get from point A to point B as indicated in the figure. Assuming a constant velocity of light, we can minimize time by minimizing the distance traveled. Find the point on the mirror that minimizes the distance traveled. Show that the angles in the figure are equal (the angle of incidence equals the angle of reflection).**

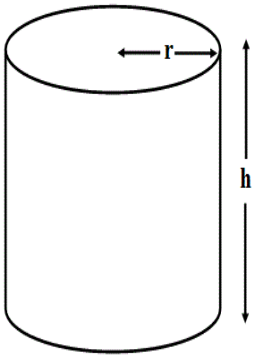


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18 **A Norman window has the outline of a semicircle on top of a rectangle. Suppose there is $8 + \pi$ feet of wood trim available. Discuss why a window designer might want to maximize the area of the window. Find the dimensions of the rectangle (and, hence, the semicircle) that will maximize the area of the window**

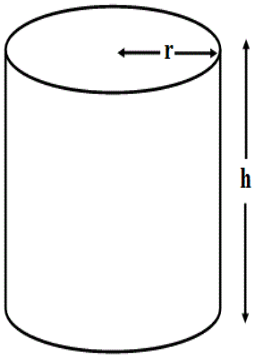


19 **A cylinder with an open top capacity $125\pi cm^3$. Find its dimensions when the area of the metal is the user made it as minimum as possible**



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20 A cylinder with an open top capacity $1000cm^3$. Find its dimensions when the area of the metal is the user made it as minimum as possible



21 Suppose a wire 2 ft long is to be cut into two pieces, each of which will be formed into a square. Find the size of each piece to minimize the total area of the two squares.

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22 **An advertisement consists of a rectangular printed region plus 1-in margins on the sides and 2-in margins at top and bottom. If the area of the printed region is to be 92 in^2, find the dimensions of the printed region and overall advertisement that minimize the total area.**

23 **An advertisement consists of a rectangular printed region plus 1-in margins on the sides and 1.5-in margins at top and bottom. If the total area of the advertisement is to be 120 in^2, what dimensions should the advertisement be to maximize the area of the printed region?**

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البرنامج المميز

12 ADVANCED

MATH ENG

LESSON 4-8

RELATED RATES

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

Prepared by :

MR- AHMED ATA

@AHMEDATACHAT

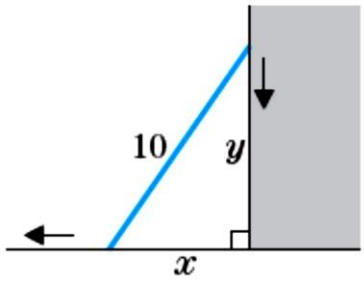
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UAE - ABU DHABI

Lesson (4-8)

RELATED RATES

1. Make a simple sketch, if appropriate.
2. Set up an equation relating all of the relevant quantities.
3. Differentiate (implicitly) both sides of the equation with respect to time (t).
4. Substitute in values for all known quantities and derivatives.
5. Solve for the remaining rate.

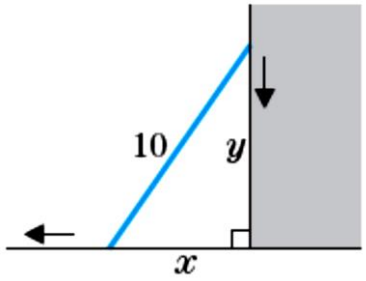
1 **A 10-feet ladder leans against the side of a building. If the top of the ladder begins to slide down the wall at the rate of 2 ft/sec, how fast is the bottom of the ladder sliding away from the wall when the top of the ladder is 8 feet off the ground?**



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2 **A 10-feet ladder leans against the side of a building If the bottom of the ladder is pulled away from the wall at the rate of 3 ft/s and the ladder remains in contact with the wall,**

(a) find the rate at which the top of the ladder is dropping when the bottom is 6 feet from the wall.



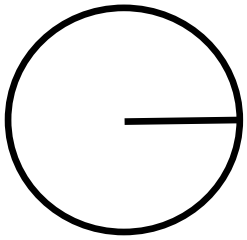
(b) Find the rate at which the angle between the ladder and the horizontal is changing when the bottom of the ladder is 6 feet from the wall.

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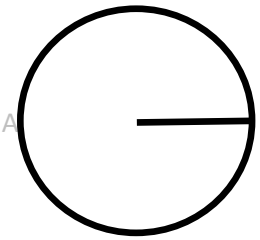
3 **Assume that the infected area of an injury is circular.**

(a) If the radius of the infected area is 3 mm and growing at a rate of 1 mm/hr, at what rate is the infected area increasing?



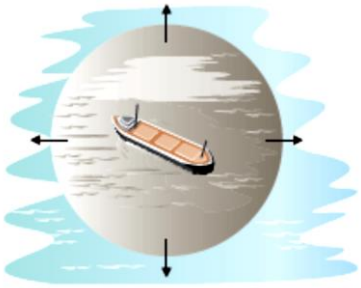
(b) Find the rate of increase of the infected area when the radius reaches 6 mm.

4 **Suppose a forest fire spreads in a circle with radius changing at a rate of 5 feet per minute. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?**



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5 **An oil tanker has an accident and oil pours out at the rate of 150 gallons per minute. Suppose that the oil spreads onto the water in a circle at a thickness of $\frac{1}{10}in$ (See Figure) Given that 1 ft^3 equals 7.5 gallons, determine the rate at which the radius of the spill is increasing when the radius reaches 500 feet.**



6 **Oil spills out of a tanker at the rate of 120 gallons per minute. The oil spreads in a circle with a thickness of $\frac{1}{4}in$ Given that 1 ft^3 equals 7.5 gallons, determine the rate at which the radius of the spill is increasing when the radius reaches 200 ft**

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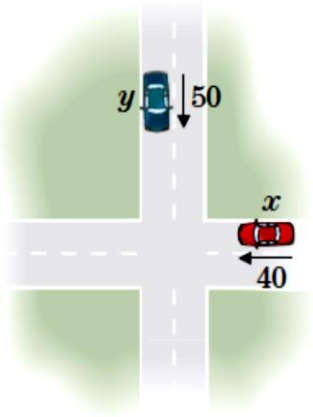
7 **Oil spills out of a tanker at the rate of 90 gallons per minute. The oil spreads in a circle with a thickness of $\frac{1}{8}$ in Determine the rate at which the radius of the spill is increasing when the radius reaches 100 feet.**

8 **Oil spills out of a tanker at the rate of g gallons per minute. The oil spreads in a circle with a thickness of $\frac{1}{4}$ in Given that the radius of the spill is increasing at a rate of 0.6 ft/min when the radius equals 100 feet, determine the value of g.**

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9 A car is traveling at 50 mph due south at a point $\frac{1}{2}$ mile north of an intersection. A police car is traveling at 40 mph due west at a point $\frac{1}{4}$ mile east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing.

a What does the radar gun register?

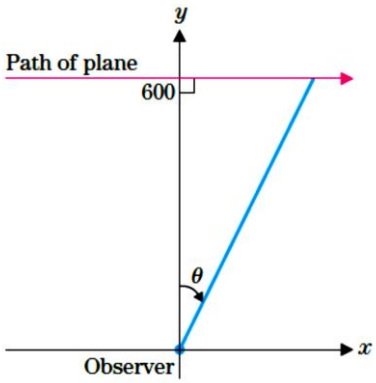


b (if the police car is not moving)

Does this make the radar gun's measurement more accurate?

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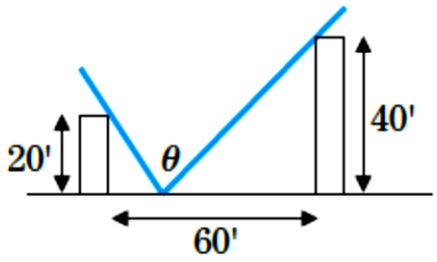
10 **A spectator at an air show is trying to follow the flight of a jet. The jet follows a straight path in front of the observer at 792 ft/sec. At its closest approach, the jet passes 600 feet in front of the person. Find the maximum rate of change of the angle between the spectator's line of sight and a line perpendicular to the flight path, as the jet flies by.** **(hint , $-1 \leq cos\theta \leq 1$)**



11 **A plane is located x = 40 miles (horizontally) away from an airport at an altitude of h miles. Radar at the airport detects that the distance s(t) between the plane and airport is changing at the rate of $s'(t) = -240$ mph.**
If the plane flies toward the airport at the constant altitude h = 4, what is the speed $|x'(t)|$ of the airplane?

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12 **Two buildings of height 20 feet and 40 feet, respectively, are 60 feet apart. Suppose that the intensity of light at a point between the buildings is proportional to the angle θ in the figure. If a person is moving from left to right at 4 ft/s, at what rate is θ changing when the person is exactly halfway between the two buildings?**



13 A small company estimates that when it spends x thousand dollars for advertising in a year, its annual sales will be described by $s = 60 - 40e^{-0.05x}$ thousand dollars. The four most recent annual advertising totals are given in the following table.

Year	1	2	3	4
Advertising Dollars	14,500	16,000	18,000	20,000

Estimate the current (year 4) value of $x'(t)$ and the current rate of change of sales.

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14 Suppose that the average yearly cost per item for producing x items of a business product is $\bar{C}(x) = 12 + \frac{94}{x}$. The three most recent yearly production figures are given in the table.

Year	0	1	2
Prod. (x)	8.2	8.8	9.4

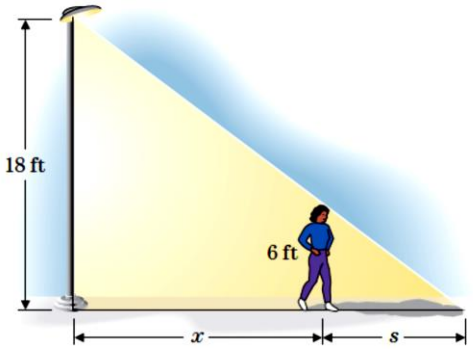
Estimate the value of $x'(2)$ and the current (year 2) rate of change of the average cost.

15 Suppose that the average yearly cost per item for producing x items of a business product is $\bar{C}(x) = 10 + \frac{100}{x}$. If the current production is $x = 10$ and production is increasing at a rate of 2 items per year, find the rate of change of the average cost

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16 **Suppose a 6-ft-tall person is 12 ft away from an 18-ft-tall lamppost (See the figure). If the person is moving away from the lamppost at a rate of 2 ft/s, at what rate is the length of the shadow changing?**

(Hint: Show that $\frac{18}{6} = \frac{x+s}{s}$)



17 **A dock is 6 feet above water. Suppose you stand on the edge of the dock and pull a rope attached to a boat at the constant rate of 2 ft/s. Assume that the boat remains at water level. At what speed is the boat approaching the dock when it is 20 feet from the dock**

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18 Sand is poured into a conical pile with the height of the pile equalling the diameter of the pile. If the sand is poured at a constant rate of 5 m^3/s, at what rate is the height of the pile increasing when the height is 2 meters?

19 Suppose that you are blowing up a balloon by adding air at the rate of 1 ft^3/s. If the balloon maintains a spherical shape, the volume and radius are related by $V = \frac{4}{3}\pi r^3$. what is the rate when radius to be r = 0.01 ft

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20 Sand is dumped such that the shape of the sandpile remains a cone with height equal to twice the radius.
If the sand is dumped at the constant rate of 20 ft^3/s, find the rate at which the radius is increasing when the height reaches 6 feet.

21 Suppose that a raindrop evaporates in such a way that it maintains a spherical shape. Given that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$ and its surface area is $A = 4\pi r^2$ if the radius changes in time, show that $V' = Ar'$ If the rate of evaporation (V') is proportional to the surface area, show that the radius changes at a constant rate.

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البرنامج المميز

MATH ENG

12 ADVANCED

LESSON 4-9

Rates of Change in Economics and the Sciences

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

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Lesson (4-9)

Rates of Change in Economics and the Sciences

In economics, the term ***marginal*** is used to indicate a rate. Thus, ***marginal cost*** is the derivative of the cost function, ***marginal profit*** is the derivative of the profit function and so on.

1 Suppose that $C(x) = 0.02x^2 + 2x + 4000$ is the total cost (in AED) for a company to produce x units of a certain product. **Compute** the marginal cost at $x = 100$ and compare this to the actual cost of producing the 100th unit

2 If the cost of manufacturing x items is $C(x) = x^3 + 20x^2 + 90x + 15$, find the marginal cost function and compare the marginal cost at $x = 50$ with the actual cost of manufacturing the $50th$ item.

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3 If the cost of manufacturing x items is $C(x) = x^4 + 14x^2 + 60x + 35$, find the marginal cost function and compare the marginal cost at $x = 50$ with the actual cost of manufacturing the $50th$ item

4 If the cost of manufacturing x items is $C(x) = x^3 + 21x^2 + 110x + 20$, find the marginal cost function and compare the marginal cost at $x = 100$ with the actual cost of manufacturing the $100th$ item

5 If the cost of manufacturing x items is $C(x) = x^3 + 11x^2 + 40x + 10$, find the marginal cost function and compare the marginal cost at $x = 100$ with the actual cost of manufacturing the $100th$ item

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6 Suppose that $C(x) = 0.02x^2 + 2x + 4000$ is the total cost (in dollars) for a company to produce x units of a certain product. Find the production level x that minimizes the average cost.

7 Suppose the cost of manufacturing x items is $C(x) = x^3 - 30x^2 + 300x + 100$ dollars. Find the inflection point and

8 find the production level that minimizes the average cost.

a $C(x) = 0.1x^2 + 3x + 2000$

b $C(x) = 10e^{0.02x}$

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9 **(a)** Let $C(x)$ be the cost function and $\overline{C}(x)$ be the average cost function. Suppose that $C(x) = 0.01x^2 + 40x + 3600$. Show that $C'(100) < \overline{C}(100)$ and show that increasing the production (x) by 1 will decrease the average cost.

(b) Show that $C'(1000) > \overline{C}(1000)$ and show that increasing the production (x) by 1 will increase the average cost.

(c) Prove that average cost is minimized at the x-value where $C'(x) = \overline{C}(x)$.

10 Let $R(x)$ be the revenue and $C(x)$ be the cost from manufacturing x items. Profit is defined as $P(x) = R(x) - C(x)$.

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(a) Show that at the value of x that maximizes profit, marginal revenue equals marginal cost.

(b) Find the maximum profit if $R(x) = 10x - 0.001x^2$ dollars and $C(x) = 2x + 5000$ dollars.

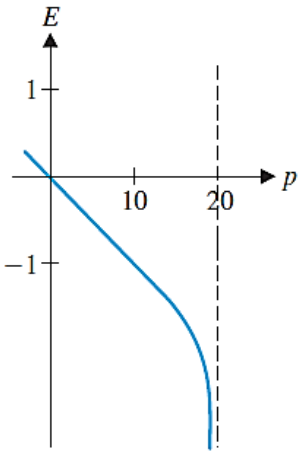
11 Suppose that $f(p) = 400(20 - p)$ is the demand for an item at price p (in dirhams) with p < 20.

(a) Find the elasticity of demand.

$$\boxed{E = \frac{p}{f(p)} f'(p)}$$

(b) Find the range of prices for which E < −1.

$$0 < p < 20$$



$$E = \frac{p}{p - 20}$$

Elastic Increase in price will decrease demand and decrease the revenue
Inelastic Increase in price will not affect demand, it will increase revenue

Find (a) the elasticity of demand and
(b) the range of prices for which the demand is elastic (E < −1).

a $f(p) = 200(30 - p)$ $0 < p < 30$

b $f(p) = 100p(20 - p)$ $0 < p < 20$

13 If the concentration of a chemical changes according to the equation

$$x'(t) = 2x(t)[4 - x(t)]$$

(a) find the concentration $x(t)$ for which the reaction rate is a maximum,
(b) find the limiting concentration

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14 If the concentration of a chemical changes according to the equation

$$x'(t) = 0.5x(t)[5 - x(t)]$$

(a) find the concentration $x(t)$ for which the reaction rate is a maximum,
(b) find the limiting concentration

linear density at $x = x_1$ is defined as $\rho(x_1) = \lim_{x \to x_1} \frac{f(x) - f(x_1)}{x - x_1} = f'(x_1)$,

15 Suppose that the mass of the first x meters of a thin rod is given by $f(x) = \sqrt{2x}$ Compute the linear density at x = 2 and at x = 8 and compare the densities at the two points.

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16 the mass of the first x meters of a thin rod is given by the function m(x) on the indicated interval. Find the linear mass density function for the rod

a $m(x) = 4x - \sin x \quad grams \quad for\ 0 \le x \le 6$

b $m(x) = (x-1)^3 + 6x \quad grams \quad for\ 0 \le x \le 2$

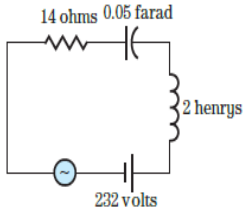
The **instantaneous current** $I(t)$ at any time t_1 can then be found by computing the limit

$$I(t_1) = \lim_{t \to t_1} \frac{Q(t) - Q(t_1)}{t - t_1} = Q'(t_1),$$

electrical charge Q(t)

17 The electrical circuit shown in Figure includes a 14-ohm resistor, a 2-henry inductor, a 0.05-farad capacitor, and a battery supplying 232 volts of AC current modeled by the oscillating function 232 sin 2t, where t is measured in seconds. Find the current in the circuit at any time t.

$Q(t) = 10e^{-5t} + 2te^{-2t} + 3\sin 2t - 7\cos 2t \quad coulombs$



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18 Suppose that the charge in an electrical circuit is $Q(t) = e^{-2t}(cos\ 3t - 2\ sin\ 3t)$ coulombs. Find the current.

19 Suppose that the charge in an electrical circuit is $Q(t) = e^{t}(3\ cos\ 2t + sin\ 2t)$ coulombs. Find the current

20 Suppose that a population grows according to the equation

$$p'(t) = 2p(t)[1 - p(t)]$$

(The logistic equation with r =2). Find the population for which the growth rate is a maximum.

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21 Suppose that a population grows according to the logistic equation

$$p'(t) = 4p(t)[5 - p(t)].$$

Find the population at which the population growth rate is a maximum

22 Suppose that a population grows according to the logistic equation

$$p'(t) = 2p(t)[7 - 2p(t)].$$

Find the population at which the population growth rate is a maximum.

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5 - Integration

1. Antiderivatives
2. Sums and Sigma Notation
3. Area
4. The Definite Integral
5. The Fundamental Theorem of Calculus
6. Integration by Substitution
7. Numerical Integration
8. The Natural Logarithm as an Integral

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MATH ENG

LESSON 5-1

Antiderivatives

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2024-2025

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Lesson (5-1)

Antiderivatives

THEOREM 1.1

Suppose that F and G are both antiderivatives of f on an interval I. Then,

$$G(x) = F(x) + c,$$

for some constant c.

DEFINITION 1.1

Let F be any antiderivative of f on an interval I. The **indefinite integral** of $f(x)$ (with respect to x) on I, is defined by

$$\int f(x)\,dx = F(x) + c,$$

where c is an arbitrary constant (the **constant of integration**).

THEOREM 1.2 (Power Rule)

For any rational power $r \neq -1$,

$$\int x^r dx = \frac{x^{r+1}}{r+1} + c.$$

Here, if $r < -1$, the interval I on which this is defined can be any interval that does not include $x = 0$.

THEOREM 1.3

Suppose that $f(x)$ and $g(x)$ have antiderivatives. Then, for any constants, a and b,

$$\int [af(x) + bg(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx.$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c, \text{ for } r \neq -1 \text{ (power rule)} \qquad \int \sec x \tan x\,dx = \sec x + c$$

$$\int \sin x\,dx = -\cos x + c \qquad \int \csc x \cot x\,dx = -\csc x + c$$

$$\int \cos x\,dx = \sin x + c \qquad \int e^x\,dx = e^x + c$$

$$\int \sec^2 x\,dx = \tan x + c \qquad \int e^{-x}\,dx = -e^{-x} + c$$

$$\int \csc^2 x\,dx = -\cot x + c \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c \qquad \int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$$

THEOREM 1.4

For $x \neq 0$, $\dfrac{d}{dx} \ln|x| = \dfrac{1}{x}$.

COROLLARY 1.1

In any interval not containing 0,

$$\int \frac{1}{x}\,dx = \ln|x| + c.$$

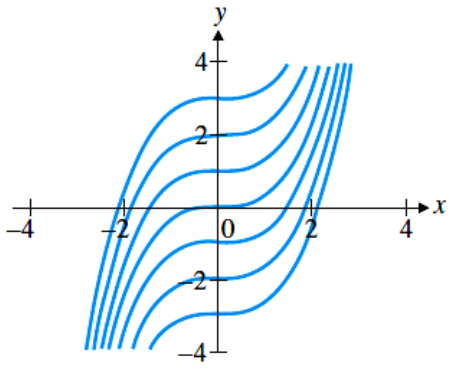
COROLLARY 1.2

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c,$$

in any interval in which $f(x) \neq 0$.

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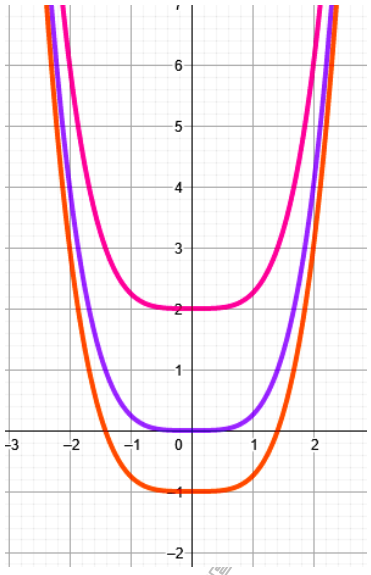
1 **Find an antiderivative of $f(x) = x^2$**



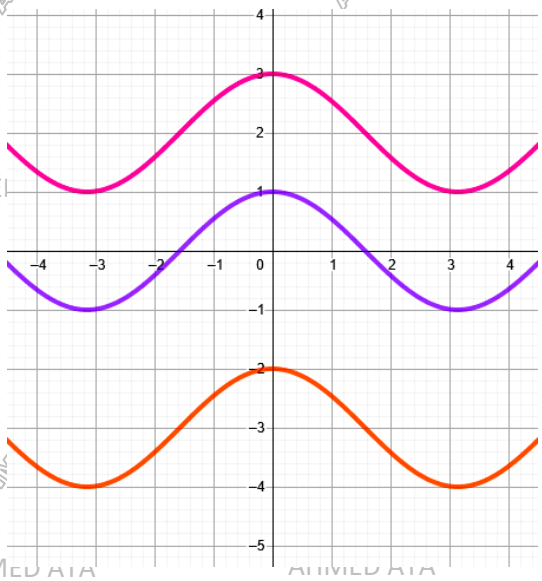
A family of antiderivative curves

Sketch several members of the family of functions defined by the antiderivative.

2 $\int x^3\,dx$



3 $\int -\sin x\,dx$



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find the general antiderivative.

4 $\int (3x^4 - 3x)\,dx$

5 $\int \left(3\sqrt{x} - \frac{1}{x^4}\right)dx$

6 $\int \frac{x^{1/3} - 3}{x^{2/3}}\,dx$

7 $\int (2\sin x + \cos x)\,dx$

8 $\int 2\sec x \tan x\,dx$

9 $\int \frac{4}{\sqrt{1 - x^2}}\,dx$

10 $\int 4\frac{\cos x}{\sin^2 x}\,dx$

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11 $\int 5\sec^2 x\,dx$

12 $\int (3e^x - 2)\,dx$

13 $\int (3\cos x - 1/x)\,dx$

14 $\int (2x^{-1} + \sin x)\,dx$

15 $\int \frac{4x}{x^2+4}\,dx$

16 $\int \frac{3}{4x^2+4}\,dx$

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17 $\int \frac{\cos x}{\sin x}\, dx$

18 $\int \left(2\cos\ x - \sqrt{e^{2x}}\right) dx$

19 $\int \frac{e^x}{e^x + 3}\, dx$

20 $\int \frac{e^x + 3}{e^x}\, dx$

21 $\int x^{1/4}(x^{5/4} - 4)\, dx$

22 $\int x^{2/3}(x^{-4/3} - 3)\, dx$

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23 $\int \left(\sqrt{x^3}+4\right) dx$

24 $\int \frac{3x^2-4}{x^2}\, dx$

25 $\int \sec^2 x\, dx$

26 $\int \left(\frac{1}{x^2}-1\right) dx$

27 **Find the derivative.**

a $\frac{d}{dx} \ln|\sec x + \tan x|$

b $\frac{d}{dx} \ln|\sin x \cdot 2|$

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Find the function $f(x)$ satisfying the given conditions.

28 $f'(x) = 3e^x + x, \quad f(0) = 4$

29 $f'(x) = 4\cos x, \quad f(0) = 3$

30 $f''(x) = 12x^2 + 2e^x, \quad f'(0) = 2, \quad f(0) = 3$

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31 $f''(x) = 2 + 2t, \quad f(0) = 2, \quad f(3) = 2$

32 $f''(x) = 4 + 6t, \quad f(1) = 3, \quad f(-1) = -2$

33 $f''(x) = 3\sin x + 4x^2$

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34 Determine the position function if the velocity function is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$.

35 Determine the position function if the velocity function is $v(t) = 3e^{-t} - 2$ and the initial position is $s(0) = 0$.

36 Determine the position function if the acceleration function is $a(t) = 3\sin t + 1$, the initial velocity is $v(0) = 0$ and the initial position is $s(0) = 4$.

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37 Determine the position function if the acceleration function is $a(t) = t^2 + 1$, the initial velocity is $v(0) = 4$ and the initial position is $s(0) = 0$.

38 Find an antiderivative by reversing the chain rule, product rule or quotient rule

a $\int xsin2x + x^2cos2x\,dx$

b $\int \frac{2xe^{3x} - 3x^2e^{3x}}{e^{6x}}\,dx$

c $\int \frac{xcosx^2}{sinx^2}\,dx$

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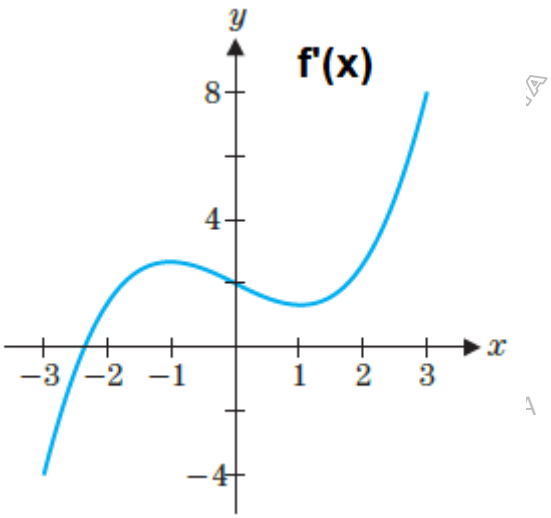
39 Find a function $f(x)$ such that the point $(1, 2)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(1, 2)$ is 3 and $f''(x) = x - 1$

40 Find a function $f(x)$ such that the point $(-1, 1)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(-1, 1)$ is 2 and $f''(x) = 6x + 4$.

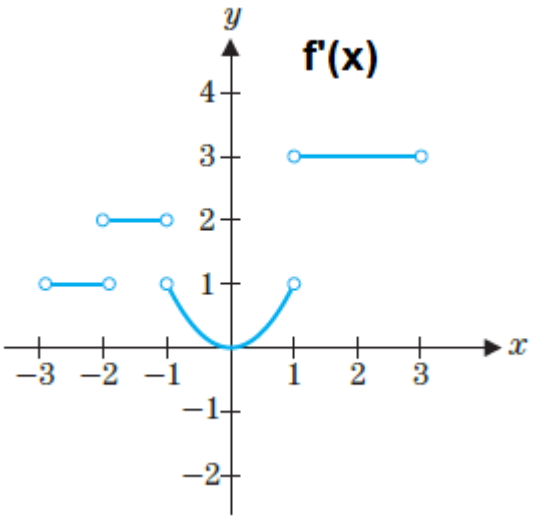
41 If an object's downward acceleration is given by $y''(t) = -9.8\ m/s^2$, find the position function $y(t)$. Assume that the initial velocity is $y'(0) = -30\ m/s$ and the initial position is $y(0) = 30,000\ m$.

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42 Sketch the graph of functions f (x) corresponding to the given graph of y = f ′(x).



43 Sketch the graph of function f (x) corresponding to the given graph of y = f ′(x).



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Lesson (5-1) part 2

Find the general antiderivative.

1. $\int \frac{x-4}{\sqrt{x}-2}\,dx$

2. $\int secx(tanx - secx)dx$

3. $\int \frac{sin^2x + cos^2x}{cosx\ cotx}\,dx$

4. $\int \frac{2sinx}{cos^2x} + \frac{cosx}{sin^2x}\,dx$

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5 $\int \frac{sec^2 x}{tanx} dx$

6 $\int \frac{x^2 + 2x}{x^3 + 3x^2 - 5} dx$

7 $\int \frac{2x + 5}{2x(x + 5)} dx$

8 $\int \frac{2e^x - e^{-x}}{4e^x + 2e^{-x}} dx$

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9 $\int (tanx - cotx)dx$

10 $\int 5e^{7x-1}\,dx$

11 $\int 3e^{x}\,(e^{x} - 5)dx$

12 $\int \frac{1}{5 + e^{-x}}\,dx$

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13 $\int \frac{1}{1+e^x}\,dx$

14 $\int \frac{5x}{2x^3+2x}\,dx$

15 $\int \frac{3}{\sqrt{9-9x^2}}\,dx$

16 $\int \frac{1}{\sqrt{x^4-x^2}}\,dx$

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17 $\int cos5x\,dx$

18 $\int \frac{sin^2x}{1-cosx}\,dx$

19 $\int \frac{x+1}{x^2+1}\,dx$

20 $\int secx dx$

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MATH ENG

LESSON 5-2

Sums and Sigma Notation

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2024-2025

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Lesson (5-2)

Sums and Sigma Notation

$$\sum_{i=1}^{20} i^2 = 1^2 + 2^2 + 3^2 + \cdots + 20^2$$

to indicate that we add together terms of the form i^2, starting with $i = 1$ and ending with $i = 20$. The variable i is called the index of summation

In general, for any real numbers $a_1, a_2, \ldots\ldots, a_n$, we have

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

Write in summation notation

1. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{10}$

2. $3^3 + 4^3 + 5^3 + \cdots + 45^3.$

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3 $2(1)^2 + 2(2)^2 + 2(3)^2 + \cdots + 2(14)^2$

4 $\sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \cdots + \sqrt{15-1}$

5 **Write in summation notation: the sum of the first 200 odd positive integers**

6 **The sum of the squares of the first 50 positive integers**

7 **The square of the sum of the first 50 positive integers**

8 **The sum of the square roots of the first 10 positive integers.**

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9 The square root of the sum of the first 10 positive integers

write out all terms and compute the sums

10 $\sum_{i=1}^{8}(2i+1)$

11 $\sum_{i=2}^{6}\sin(2\pi i)$

12 $\sum_{i=4}^{10}5$

13 $\sum_{i=1}^{6}3i^2$

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14 $\sum_{i=3}^{7}(i^2+i)$

15 $\sum_{i=6}^{10}(4i+2)$

16 $\sum_{i=6}^{8}(i^2+2)$

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THEOREM 2.1

If n is any positive integer and c is any constant, then

(i) $\sum_{i=1}^{n} c = cn$ **(sum of constants),**

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THEOREM 2.2

For any constants c and d,

$$\sum_{i=1}^{n}(ca_i + db_i) = c\sum_{i=1}^{n} a_i + d\sum_{i=1}^{n} b_i.$$

Computing Sums Using Theorems 2.1 and 2.2

17 $\sum_{i=1}^{8}(2i+1)$

18 $\sum_{i=1}^{800}(2i+1)$

19 $\sum_{i=1}^{20} i^2$

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20 $\sum_{i=1}^{20} \left(\frac{i}{20}\right)^2$

21 $\sum_{i=1}^{70} (3i - 1)$

22 $\sum_{i=1}^{40} (4 - i^2)$

23 $\sum_{n=1}^{100} (n^2 - 3n + 2)$

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24 $\sum_{i=4}^{20}(i-3)(i+3)$

25 $\sum_{k=3}^{n}(k^2-3)$

26 $\sum_{i=3}^{30}[(i-3)^2+i-3]$

compute sums of the form $\sum_{i=1}^{n} f(x_i)\Delta x$ for the given values of x_i.

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27 $f(x) = x^2 + 4x;\quad x = 0.2, 0.4, 0.6, 0.8, 1.0;\quad \Delta x = 0.2;\quad n = 5$

28 $f(x) = 3x + 5;\quad x = 0.4, 0.8, 1.2, 1.6, 2.0;\quad \Delta x = 0.4;\quad n = 5$

29 $f(x) = x^3 + 4;\quad x = 2.05, 2.15, 2.25, 2.35, \ldots, 2.95;\quad \Delta x = 0.1;\quad n = 10$

30 *Sum the values* of $f(x) = x^2 + 3$

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Evaluated at $x = 0.1, x = 0.2, \ldots, x = 1.0$.

31 *Sum the values of* $f(x) = 3x^2 - 4x + 2$ *evaluated at* $x = 1.05, x = 1.15, x = 1.25, \ldots, x = 2.95$.

compute the sum and the limit of the sum as $n \to \infty$

32 $$\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$$

$$\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 - 5\left(\frac{i}{n}\right)\right]$$

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33

34 $$\sum_{i=1}^{n} \frac{1}{n}\left[4\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right)\right]$$

35 $$\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{2i}{n}\right)^2 + 4\left(\frac{i}{n}\right)\right]$$

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36 **Suppose that a car has velocity 50 mph for 2 hours, velocity 60 mph for 1 hour, velocity 70 mph for 30 minutes and velocity 60 mph for 3 hours. Find the distance traveled**

37 **Suppose that a car has velocity 50 mph for 1 hour, velocity 40 mph for 1 hour, velocity 60 mph for 30 minutes and velocity 55 mph for 3 hours. Find the distance traveled**

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MATH ENG

LESSON 5-3

Area

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Lesson (5-3)

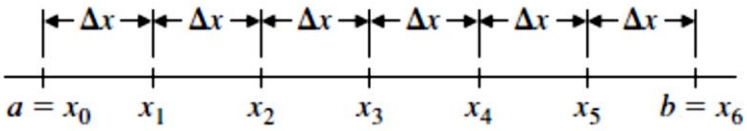
Area

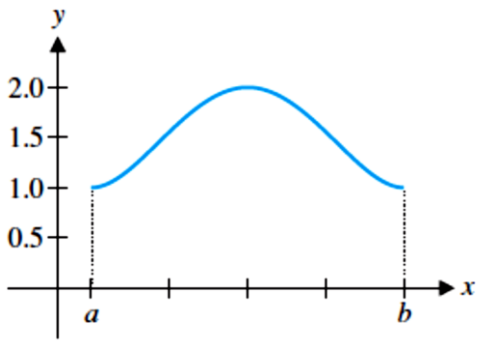
Assume that $f(x) \geq 0$ and f is continuous on the interval $[a, b]$, as in Figure

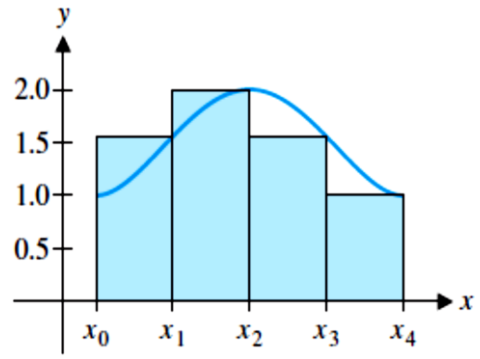
We start by dividing the interval $[a, b]$ into n equal pieces. This is called a regular partition of $[a, b]$. The width of each subinterval in the partition is then $\frac{b-a}{n}$ which we denote by Δx (Meaning a small change in x). The points in the partition are denoted by

$x_0 = a, \quad x_1 = x_0 + \Delta x, \quad x_2 = x_1 + \Delta x$ **and so on. In general,**

$$x_i = x_0 + i\Delta x, \qquad for\ i = 1, 2, \ldots, n$$







$$A \approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x + f(x_4)\,\Delta x = A_4.$$

$$A \approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x$$

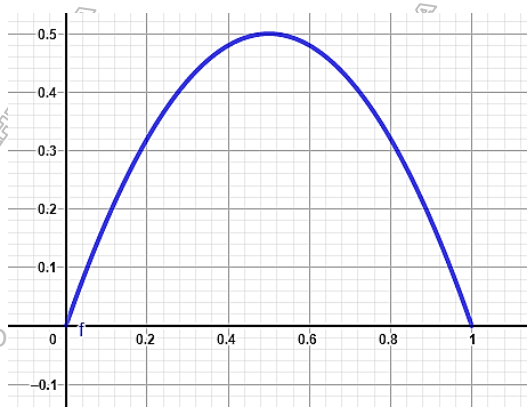
$$= \sum_{i=1}^{n} f(x_i)\,\Delta x = A_n.$$

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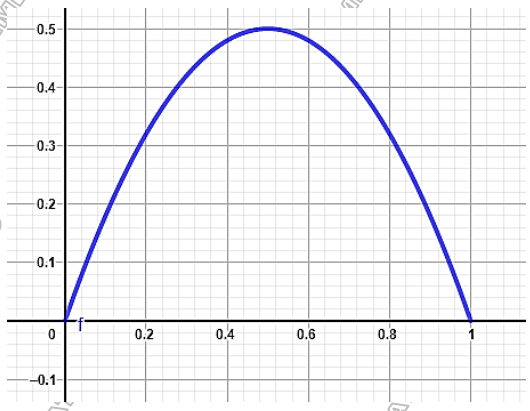
1 **Approximate the area under the curve $f(x) = 2x - 2x^2$ on the interval [0, 1], using 5 rectangles and the evaluation rules**

(a) right endpoint

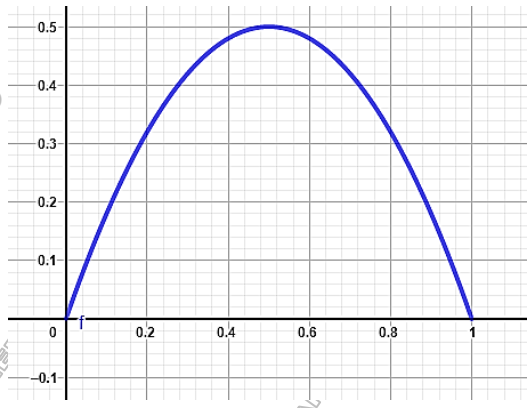
$$A \approx A_5 = \sum_{i=1}^{5} f(x_i)\Delta x =$$



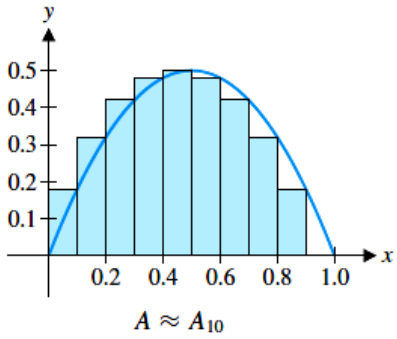
(b) left endpoint



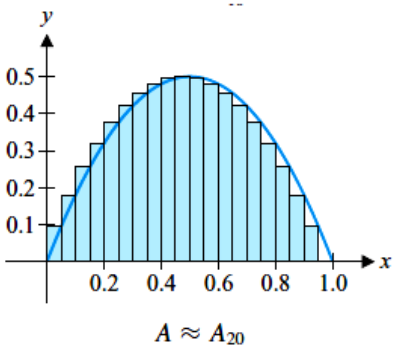
(c) midpoint



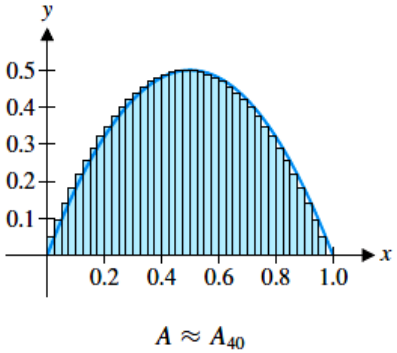
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$A \approx A_{10}$



$A \approx A_{20}$

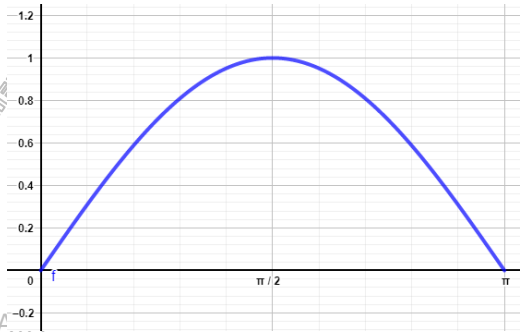


$A \approx A_{40}$

Notice that as *n* gets larger and larger, A_n seems to be approaching $\frac{1}{3}$

2 **Approximate the area under the curve $f(x) = \sin x$ on the interval $[0, \pi], n = 4$**

a **right endpoint**



b **left endpoint**

c **midpoint**

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use the given function values to estimate the area under the curve using

3

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	2.0	2.4	2.6	2.7	2.6	2.4	2.0	1.4	0.6

a **left endpoint**

b **right-endpoint**

4

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$f(x)$	2.0	2.2	1.6	1.4	1.6	2.0	2.2	2.4	2.0

a **left endpoint**

b **right-endpoint**

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5 **Approximate the area under the curve on the given interval using n rectangles and the evaluation rules**

$$A \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

$R;\ x_i = a + \Delta x i \qquad L;\ x_i = a + \Delta x(i-1) \qquad M;\ x_i = a + \Delta x\left(i - \frac{1}{2}\right)$

$y = x^2 + 1 \qquad on\ [0,1], \qquad n = 16$

(a) Right endpoint

(b) Left endpoint

(c) Midpoint

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6 $y = \sqrt{x + 2}\ on\ [1,4], n = 16$

(a) Right endpoint

(b) Left endpoint

7 $y = cos\ x \quad on\ [0, \frac{\pi}{2}], \quad n = 50$

(a) Right endpoint

(b) Midpoint

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DEFINITION 3.1

For a function f defined on the interval $[a, b]$, if f is continuous on $[a, b]$ and $f(x) \geq 0$ on $[a, b]$, the **area A under the curve** $y = f(x)$ on $[a, b]$ is given by

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x.$$

DEFINITION 3.2

Let $\{x_0, x_1, \ldots, x_n\}$ be a regular partition of the interval $[a, b]$, with $x_i - x_{i-1} = \Delta x = \dfrac{b-a}{n}$, for all i. Pick points $c_1, c_2, \ldots, c_n$, where c_i is any point in the subinterval $[x_{i-1}, x_i]$, for $i = 1, 2, \ldots, n$. (These are called **evaluation points.**) The **Riemann sum** for this partition and set of evaluation points is

$$\sum_{i=1}^{n} f(c_i)\,\Delta x.$$

Use Riemann sums and a limit to compute the exact area under the curve

8 $y = 3x$ *on* $[0, 3]$

9 $y = x^2 + 1$ *on* $[0, 1]$

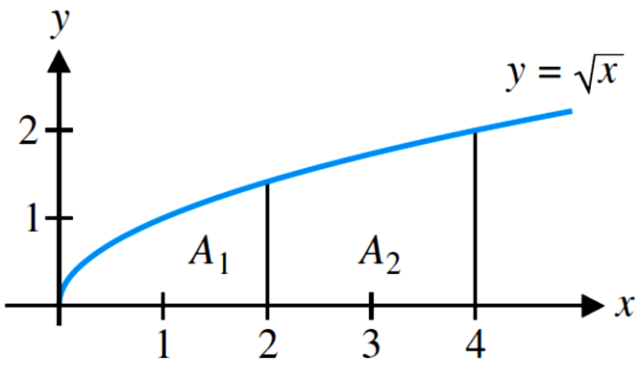
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10 $y = x^2 + 3x$ $on\ [0, 2]$

11 $y = 3x^2$ $on\ [1, 3]$

12 **In the figure**

which area equals $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2}\sqrt{1 + i/n}\,\frac{2}{n}$?



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13 Show that the area under $y = ax^2$ for $0 \leq x \leq b$ is $\frac{1}{3}$ of the base times the height $(\frac{1}{3}b \cdot ab^2)$

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MATH ENG

LESSON 5-4

The Definite Integral

Mr. Ahmed Ata
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2024-2025

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Lesson (5-4)

The Definite Integral

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x,$$

DEFINITION 4.1

For any function f defined on $[a, b]$, the **definite integral** of f from a to b is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x,$$

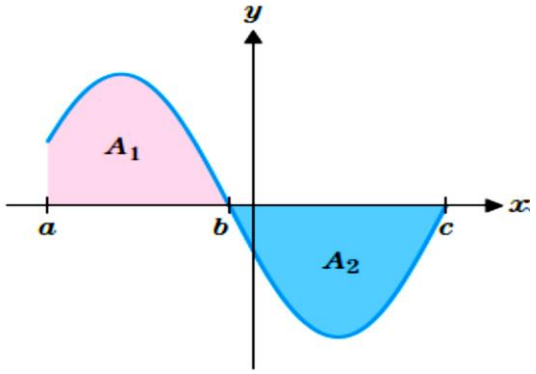
whenever the limit exists and is the same for every choice of evaluation points, $c_1, c_2, \ldots, c_n$. When the limit exists, we say that f is **integrable** on $[a, b]$.

THEOREM 4.1

If f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

DEFINITION 4.2

Suppose that $f(x) \geq 0$ on the interval $[a, b]$ and A_1 is the area bounded between the curve $y = f(x)$ and the x-axis for $a \leq x \leq b$. Further, suppose that $f(x) \leq 0$ on the interval $[b, c]$ and A_2 is the area bounded between the curve $y = f(x)$ and the x-axis for $b \leq x \leq c$. The **signed area** between $y = f(x)$ and the x-axis for $a \leq x \leq c$ is $A_1 - A_2$, and the **total area** between $y = f(x)$ and the x-axis for $a \leq x \leq c$ is $A_1 + A_2$. (See Figure 5.16.)



Use the Midpoint Rule with n =6 to estimate the value of the integral

1 $\int_0^3 (x^3 + x)\,dx$

Use the Right endpoint Rule with n =6 to estimate the value of the integral

2 $\int_0^{\pi} \sin x^2\,dx$

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Evaluate the integral by computing the limit of Riemann sums.

3 $\int_0^1 2x\,dx$

4 $\int_1^2 2x\,dx$

5 $\int_0^2 x^2\,dx$

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Properties of Definite Integrals

Linearity of Definite Integrals

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b (f(x)-g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Additive Interval Property of Definite Integrals

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Reversed Interval Property of Definite Integrals

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Simple Maximum and Minimum Values for Definite Integrals

If a function $f(x)$ is continuous and bounded between $y = m$ and $y = M$ on the interval $[a, b]$, i.e. $m \le f(x) \le M$ on the interval, then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Comparison of Definite Integrals

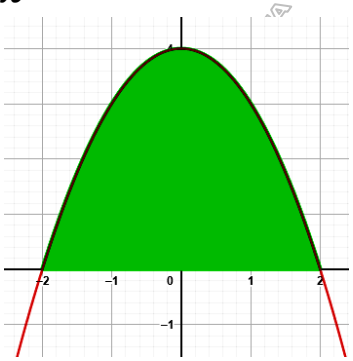
If $f(x) \le g(x)$ on an interval $[a, b]$, then

$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$$

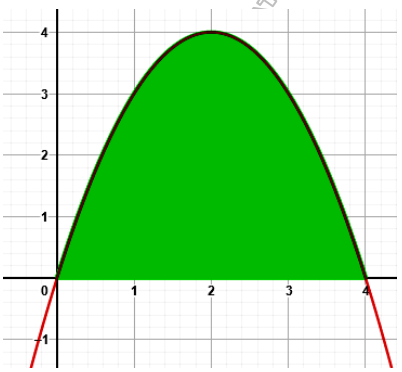
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write the given (total) area as an integral or sum of integrals.

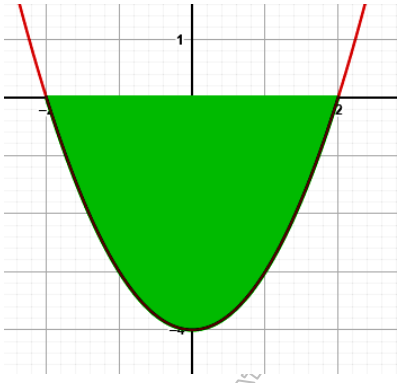
6 **The area above the x-axis and below** $y = 4 - x^2$



7 **The area above the x-axis and below** $y = 4x - x^2$

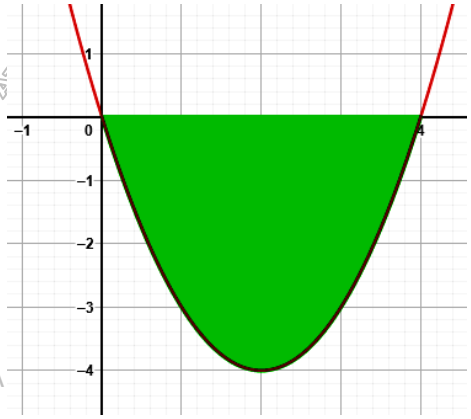


8 **The area below the x-axis and above** $y = x^2 - 4$

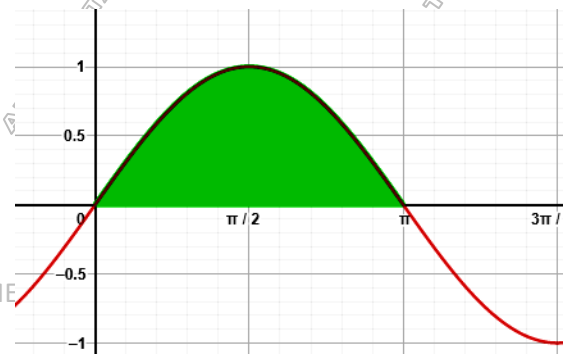


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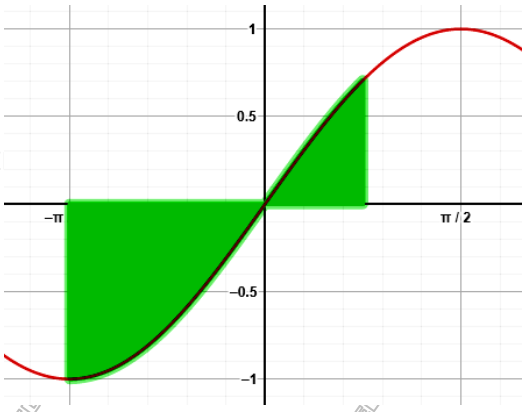
9 The area below the x-axis and above $y = x^2 - 4x$



10 The area between $y = \sin x$ and the $x - axis$ for $0 \leq x \leq \pi$



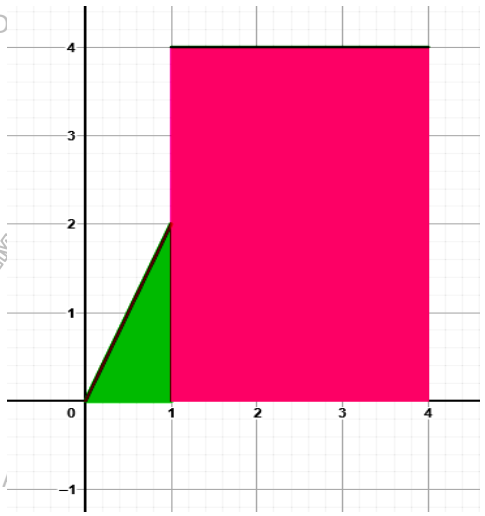
11 The area between $y = \sin x$ and the $x - axis$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{4}$



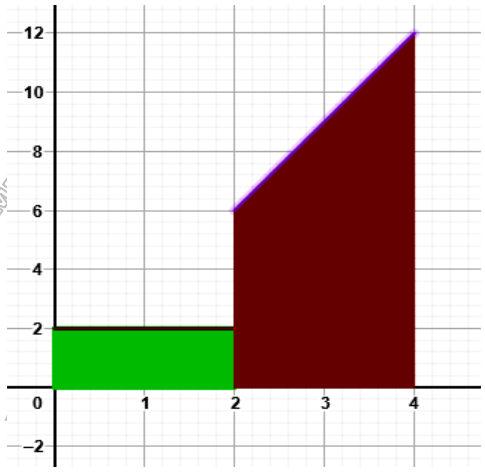
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compute $\int_0^4 f(x)\,dx$

12 $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 & \text{if } x \geq 1 \end{cases}$

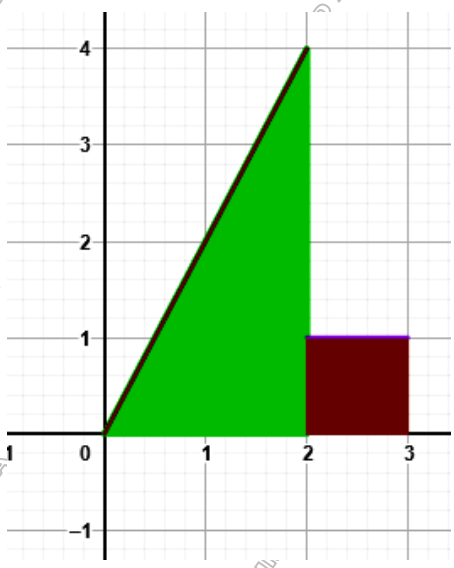


13 $f(x) = \begin{cases} 2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$



Evaluate $\int_0^3 f(x)\,dx$, where $f(x)$ is defined by

14 $f(x) = \begin{cases} 2x, & \text{if } x \leq 2 \\ 1, & \text{if } x > 2 \end{cases}$.



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An integral representing average value:

$$f_{\text{ave}} = \lim_{n\to\infty}\left[\frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\,\Delta x\right] = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

THEOREM 4.4 (Integral Mean Value Theorem)

If f is continuous on $[a, b]$, then there is a number $c \in (a, b)$ for which

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Compute the average value of the function on the given interval.

15 $f(x) = 2x + 1, [0, 4]$

16 $f(x) = x^2 + 2x, [0, 1]$

17 $f(x) = x^2 - 1, [1, 3]$

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Simple Maximum and Minimum Values for Definite Integrals

If a function $f(x)$ is continuous and bounded between $y = m$ and $y = M$ on the interval $[a, b]$, i.e. $m \leq f(x) \leq M$ on the interval, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

Use the Integral Mean Value Theorem to estimate the value of the integral

18 A $\int_0^1 \sqrt{x^2+1}\,dx$

19 $\int_{\pi/3}^{\pi/2} 3\cos x^2\,dx$

20 A $\int_0^{1/2} e^{-x^2}\,dx$

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Find a value of c that satisfies the conclusion of the Integral Mean Value Theorem

21 $\int_0^2 3x^2\,dx\ (=8)$

22 $\int_{-1}^1 (x^2-2x)\,dx\ (=\frac{2}{3})$

Use properties of definite integrals to write the expression as a single integral

23 $\int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$

24 $\int_0^3 f(x)\,dx - \int_2^3 f(x)\,dx$

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25 $\int_0^2 f(x)\,dx + \int_2^1 f(x)\,dx$

26 $\int_{-1}^2 f(x)\,dx + \int_2^3 f(x)\,dx$

assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ and find

27 $\int_1^3 [f(x) + g(x)]\,dx$

28 $\int_1^3 [2f(x) - g(x)]\,dx$

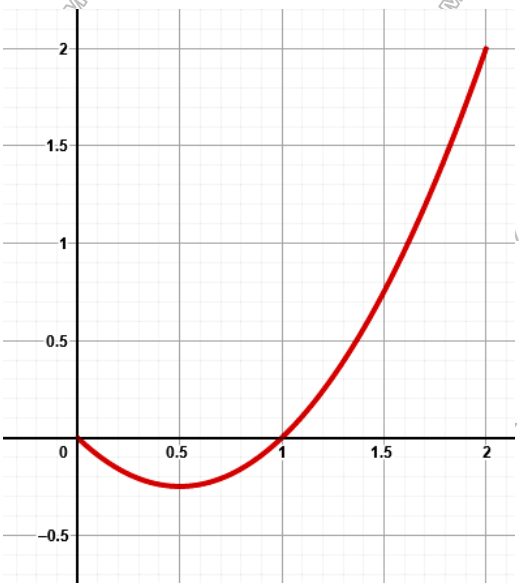
29 $\int_1^3 [f(x) - g(x)]\,dx$

30 $\int_1^3 [4g(x) - 3f(x)]\,dx$

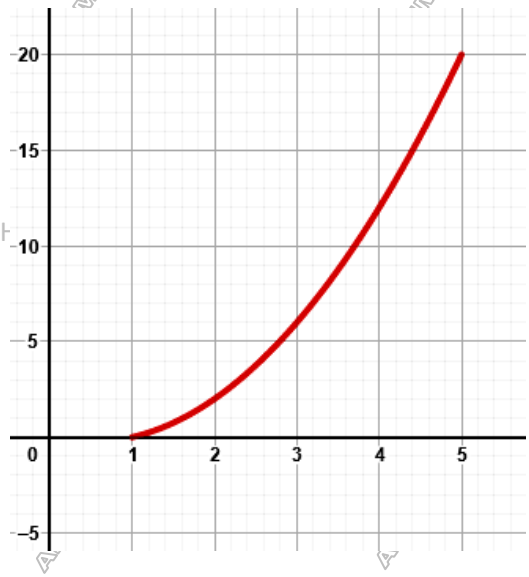
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Sketch the area corresponding to the integral.

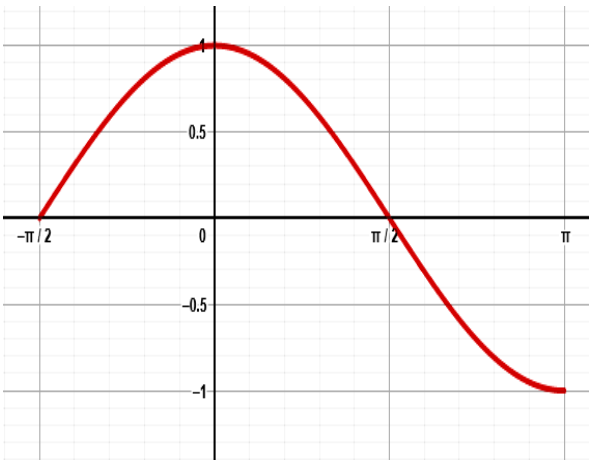
31 $\int_1^2 (x^2 - x)\,dx$



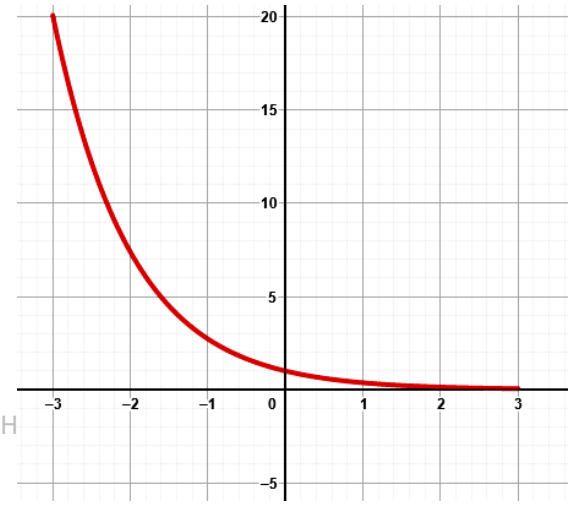
32 $\int_2^4 (x^2 - x)\,dx$



33 $\int_0^{\pi/2} \cos x\,dx$



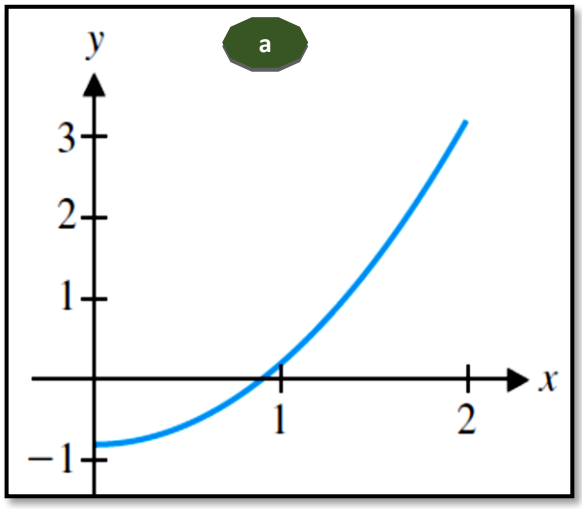
34 $\int_{-2}^{2} e^{-x}\,dx$



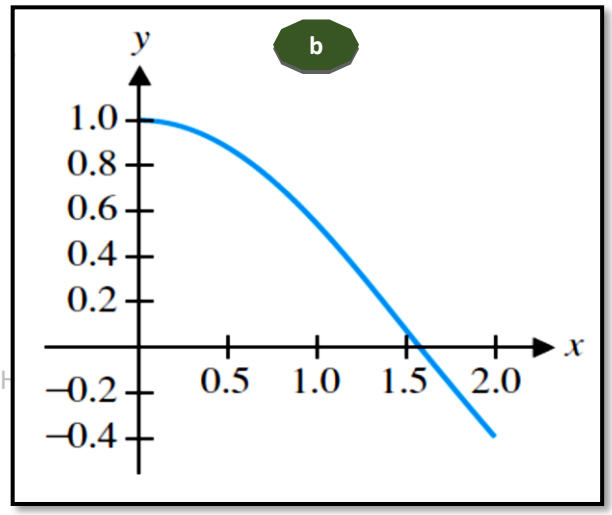
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35 Use the graph to determine whether $\int_0^2 f(x)dx$ is positive or negative

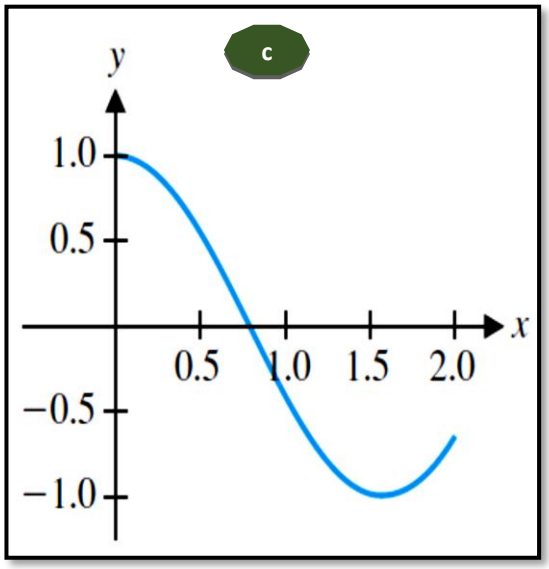
a



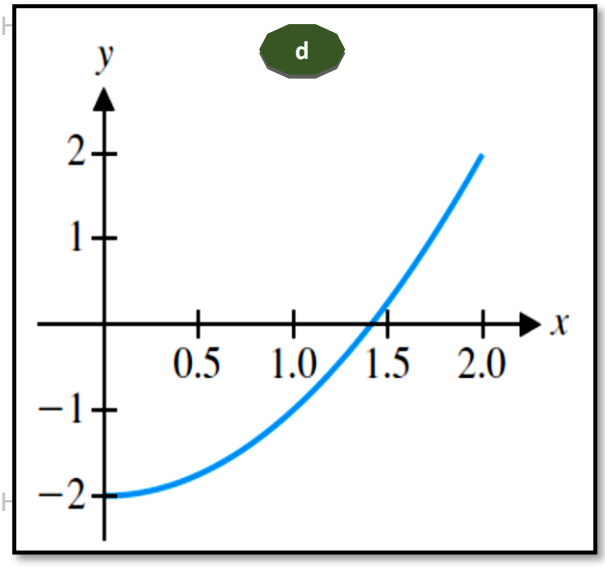
b



c



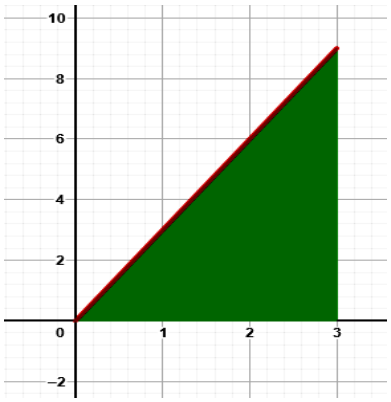
d



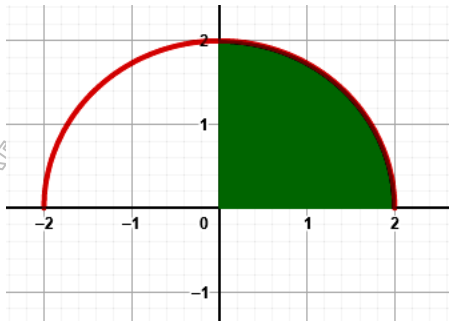
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36 **Use a geometric formula to compute the integral.**

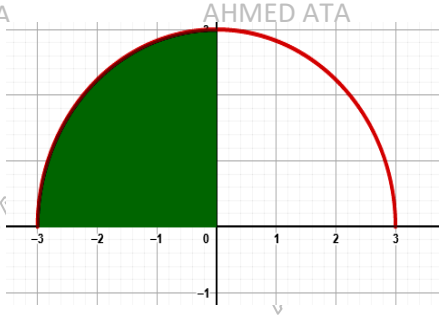
a $\int_0^2 3x dx$



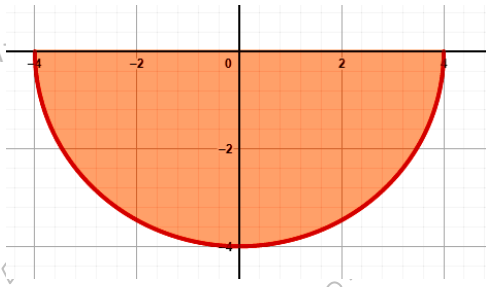
b $\int_0^2 \sqrt{4-x^2}\,dx$



c $\int_{-3}^0 \sqrt{9-x^2}\,dx$



d $-\int_{-4}^4 \sqrt{16-x^2}\,dx$



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MATH ENG

LESSON 5-4

The Fundamental Theorem of Calculus

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Lesson (5-5)

The Fundamental Theorem of Calculus

THEOREM 5.1 (The Fundamental Theorem of Calculus, Part I)

If f is continuous on $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a). \tag{5.1}$$

Using the Fundamental Theorem to Compute each integral exactly

1. $\int_0^2 (2x - 3)\,dx$

2. $\int_0^3 (x^2 - 2)\,dx$

3. $\int_{-1}^1 (x^3 + 2x)\,dx$

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4 $\int_1^4 \left(\sqrt{x} - \frac{1}{x^2}\right) dx.$

5 $\int_1^4 \left(x\sqrt{x} + \frac{3}{x}\right) dx$

6 $\int_0^1 (6e^{-3x} + 4)\, dx$

7 $\int_{\pi/2}^{\pi} (2\sin x - \cos x)\, dx$

8 $\int_0^{\pi/4} \sec t \tan t\, dt$

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9 $\int_0^{\pi/4} \sec^2 t \, dt$

10 $\int_0^{1/2} \frac{3}{\sqrt{1-x^2}} \, dx$

11 $\int_{-1}^{1} \frac{4}{1+x^2} \, dx$

12 $\int_1^4 \frac{t-3}{t} \, dt$

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13 $\int_0^4 t(t-2)\,dt$

14 $\int_0^4 e^{-2x}\,dx$

15 $\int_1^x 12t^5\,dt.$

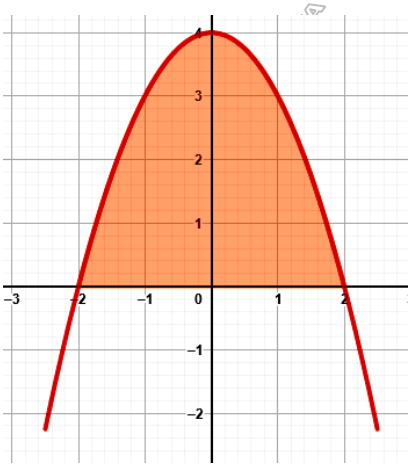
16 $\int_0^t (e^{x/2})^2\,dx$

17 $\int_0^t (\sin^2 x + \cos^2 x)\,dx$

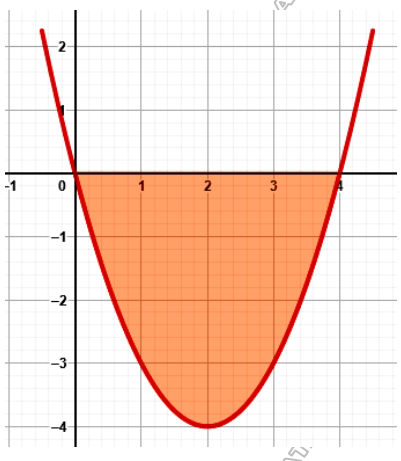
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Find the given area

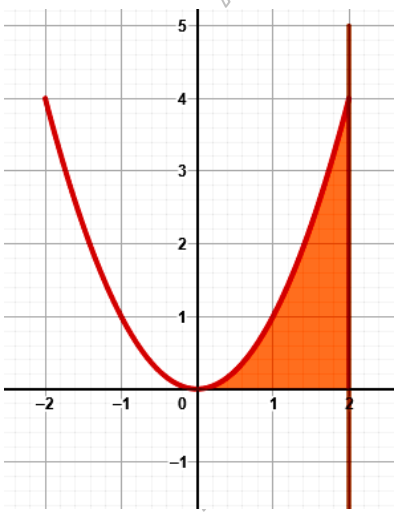
18 The area above the x-axis and below $y = 4 - x^2$



19 The area below the x-axis and above $y = x^2 - 4x$

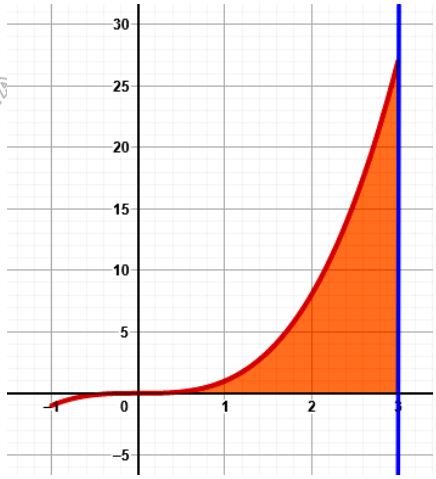


20 The area of the region bounded by $y = x^2, x = 2$ and the $x - axis$

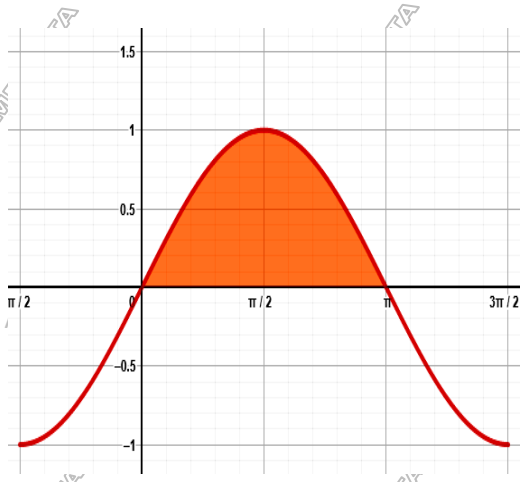


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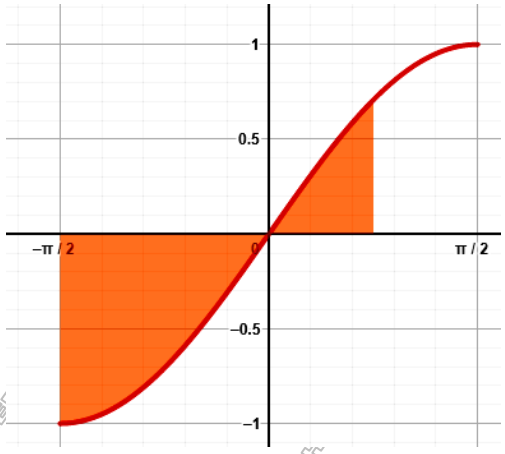
21 The area of the region bounded by $y = x^3, x = 3$ *and the* $x-$*axis*



22 The area between $y = \sin x$ *and the* $x-$*axis for* $0 \leq x \leq \pi$



23 The area between $y = \sin x$ *and the* $x-$*axis for* $-\frac{\pi}{2} \leq x \leq \frac{\pi}{4}$



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THEOREM 5.2 (The Fundamental Theorem of Calculus, Part II)

If f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$, on $[a, b]$.

Find the derivative $f'(x)$.

24 $f(x) = \int_0^x (t^2 - 3t + 2)\,dt$

25 $f(x) = \int_2^x (t^2 - 3t - 4)\,dt$

26 $f(x) = \int_0^{x^2} (e^{-t^2} + 1)\,dt$

27 $f(x) = \int_x^2 \sec t\,dt$

28 $f(x) = \int_{e^x}^{2-x} \sin t^2 dt$

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29 $f(x) = \int_{2-x}^{xe^x} e^{2t}\,dt$

30 $f(x) = \int_{x^2}^{x^3} \sin(3t)\,dt$

31 $f(x) = \int_{3x}^{\sin x} (t^2 + 4)\,dt$

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Find the position function $s(t)$ from the given velocity or acceleration function and initial value(s). Assume that units are feet and seconds

32 $v(t) = 40 - \sin t, s(0) = 2$

33 $v(t) = 10e^{-t}, s(0) = 2$

34 $a(t) = 4 - t, v(0) = 8, s(0) = 0$

35 $a(t) = 16 - t^2, v(0) = 0, s(0) = 30$

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36 **Suppose that the rate of change of water in a storage tank is $f(t) = 10 \sin t$ gallons per minute.**

(a) For $0 \leq t \leq 2\pi$, determine when the water level is increasing and when the water level is decreasing.

(b) If the tank has 100 gallons of water at time $t = 0$, determine how many gallons are in the tank at $t = \pi$.

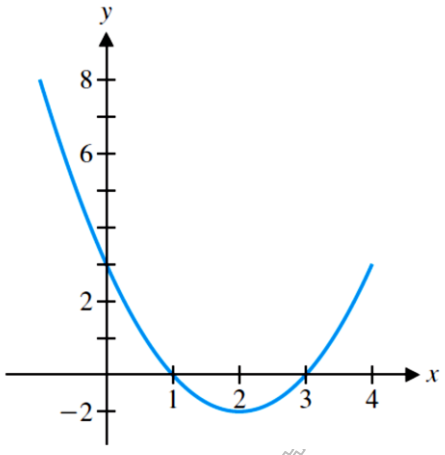
Find an equation of the tangent line at the given value of x.

37 $F(x) = \int_4^{x^2} \ln(t^3 + 4)\,dt$ $\quad$ $at\ x = 2$

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38 $F(x) = \int_0^x \sin\sqrt{t^2 + \pi^2}\,dt$ $\quad at\ x = 0$

39 **Use the graph to list $\int_0^1 f(x)dx$; $\int_0^2 f(x)dx$ and $\int_0^3 f(x)dx$ in order, from smallest to largest. For $g(x) = \int_0^x f(t)\,dt$ determine intervals where g is increasing and identify critical points for g.**



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MATH ENG

LESSON 5-6

Integration by Substitution

Mr. Ahmed Ata
The Featured Program
2024-2025

البرنامج المميز طريقك للتميز

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Lesson (5-6)

Integration by Substitution

Key Point

To evaluate

$$\int f(g(x))g'(x)\mathrm{d}x$$

substitute $u = g(x)$, and $\mathrm{d}u = g'(x)\mathrm{d}x$ to give

$$\int f(u)\,\mathrm{d}u$$

Integration is then carried out with respect to u, before reverting to the original variable x.

Use the given substitution to evaluate the indicated integral

1. $\int x^2\sqrt{x^3+2}\,dx,\ u = x^3+2$

2. $\int x^3(x^4+1)^{-2/3}dx,\ u = x^4+1$

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3 $\int \frac{(\sqrt{x}+2)^3}{\sqrt{x}}\,dx,\ u=\sqrt{x}+2$

4 $\int \sin x \cos x\,dx,\ u=\sin x$

evaluate the indicated integral.

5 $\int x^3\sqrt{x^4+3}\,dx$

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6 $\int \frac{\sin x}{\sqrt{\cos x}}\, dx$

7 $\int \sin^3 x \cos x\, dx$

8 $\int t^2 \cos t^3\, dt$

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9 $\int xe^{x^2+1}\,dx$

10 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

11 $\int \frac{\cos(1/x)}{x^2}\,dx$

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12 $\int \sec^2 x \sqrt{\tan x}\, dx$

13 $\int \frac{v}{v^2+4}\, dv$

14 $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}}\, dx$

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15 $\int x^2 \sec^2 x^3 \, dx$

16 $\int \frac{x}{\sqrt{1-x^4}} \, dx$

17 $\int \frac{x^3}{\sqrt{1-x^4}} \, dx$

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18 $\int \frac{x^2}{1+x^6}\,dx$

19 $\int \frac{x^5}{1+x^6}\,dx$

20 $\int \frac{1+x}{1+x^2}\,dx$

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21 $\int \frac{1+x}{1-x^2}\,dx$

22 $\int \frac{3\sqrt{x}}{1+x^3}\,dx$

23 $\int \frac{2t+3}{t+7}\,dt$

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24 $\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$

25 $\int \frac{t^2}{\sqrt[3]{t+3}}\,dt$

26 $\int_0^2 x\sqrt{x^2+1}\,dx$

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27 $\int_1^3 x \sin(\pi x^2)\, dx$

28 $\int_{-1}^1 \frac{t}{(t^2+1)^2}\, dt$

29 $\int_0^2 \frac{e^x}{1+e^{2x}}\, dx$

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30 $\int_{\pi/4}^{\pi/2} \cot x \, dx$

31 $\int_{1}^{e} \frac{\ln x}{x} \, dx$

32 $\int_{1}^{4} \frac{x-1}{\sqrt{x}} \, dx$

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33 $\int_0^1 \frac{x}{\sqrt{x^2+1}}\,dx$

Make the indicated substitution for an unspecified function $f(x)$.

34 $u = x^2$ for $\int_0^2 xf(x^2)\,dx$

35 $u = x^3$ for $\int_1^2 x^2 f(x^3)\,dx$

36 $u = \sin x$ for $\int_0^{\pi/2} (\cos x) f(\sin x)\,dx$

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MATH ENG

LESSON 5-7

Numerical Integration

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2024-2025

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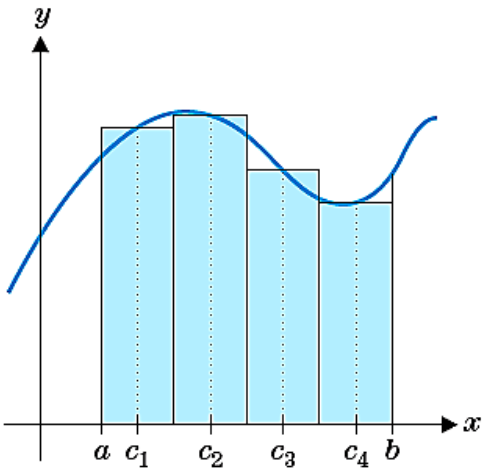
Lesson (5-7)

Numerical Integration

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(c_i)\,\Delta x,$$

where c_i is the midpoint of the subinterval $[x_{i-1}, x_i]$,

$$c_i = \frac{1}{2}(x_{i-1} + x_i), \quad \text{for } i = 1, 2, \ldots, n.$$



Midpoint Rule

1 Write out the Midpoint Rule $M_n(f)$ approximation of $\int_0^1 x^2\ dx$ with $n = 4$.

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2 Write out the Midpoint Rule $M_n(f)$ approximation of $\int_0^2 x^2 + 1 \, dx$ with $n = 4$.

3 Use the Midpoint Rule $M_n(f)$ to approximate $\int_0^2 \sqrt{x^2+1} \, dx$ accurate to three decimal places.

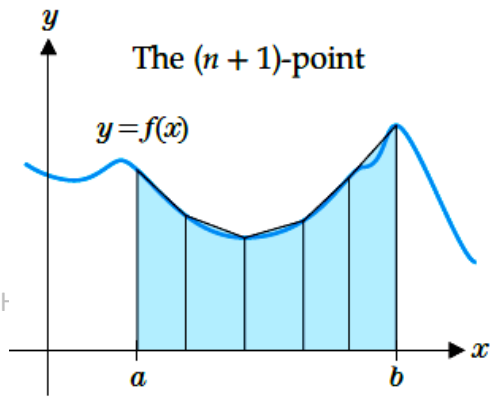
$let\ n = (10, 20, 30)$

$Recall\ that\ x_i = \Delta x(i - \frac{1}{2})$

Trapezoidal Rule, $T_n(f)$

$$\int_a^b f(x)\,dx \approx T_n(f) = \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

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Trapezoidal Rule

4 Compute the Trapezoidal Rule $T_n(f)$ approximations with $n = 4$ for $\int_0^1 3x^2\ dx$

5 Compute the Trapezoidal Rule $T_n(f)$ approximations with $n = 4$ for $\int_0^3 \frac{1}{x}\ dx$

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Simpson's Rule $S_n(f)$

SIMPSON'S RULE

$$\int_a^b f(x)\,dx \approx S_n(f) = \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3)$$

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6 Approximate the value of $\int_0^1 3x^2\ dx$ using Simpson's Rule $S_n(f)$ with $n = 4$.

7 Compute Simpson's Rule approximations $S_n(f)$ with $n = 4$ for $\int_0^2 \sqrt{x^2+1}\ dx$

8 Use (a) Trapezoidal Rule and (b) Simpson's Rule to estimate $\int_0^2 f(x)\ dx$ from the given data.

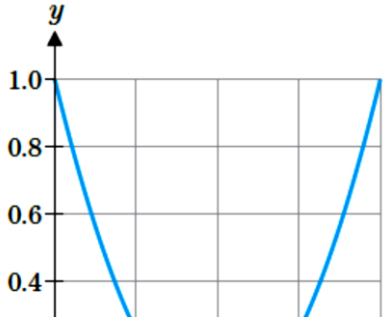
x	0.0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
$f(x)$	4.0	4.6	5.2	4.8	5.0	4.6	4.4	3.8	4.0

9 Use (a) Trapezoidal Rule and (b) Simpson's Rule to estimate $\int_0^2 f(x)\ dx$ from the given data.

x	0.0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
$f(x)$	1.0	0.6	0.2	-0.2	-0.4	0.4	0.8	1.2	2.0

10 Use the graph to estimate with n = 4 of $\int_0^2 f(x)\ dx$

(a) Riemann sum with left-endpoint evaluation



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(b) Midpoint Rule

(c) Trapezoidal Rule

(d) Simpson's Rule approximations

Error Bounds for Numerical Integration

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$$ET_n = \text{exact} - \text{approximate} = \int_a^b f(x)\,dx - T_n(f).$$

THEOREM 7.1

Suppose that f'' is continuous on $[a, b]$ and that $|f''(x)| \le K$, for all x in $[a, b]$. Then,

$$|ET_n| \le K\frac{(b-a)^3}{12n^2}$$

and

$$|EM_n| \le K\frac{(b-a)^3}{24n^2}.$$

THEOREM 7.2

Suppose that $f^{(4)}$ is continuous on $[a, b]$ and that $|f^{(4)}(x)| \le L$, for all x in $[a, b]$. Then,

$$|ES_n| \le L\frac{(b-a)^5}{180n^4}.$$

11 **Find bounds on the error to approximate the value of the integral $\int_0^3 \frac{1}{x}\,dx$ using $n = 10$ in using each of the**

a) Midpoint Rule

b) Trapezoidal Rule

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c) Simpson's Rule

12 Find bounds on the error to approximate the value of the integral $\int_0^{\frac{\pi}{4}} cosx \ dx$ using $n = 10$ in using each of the

A) Midpoint Rule

B) Trapezoidal Rule

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C) Simpson's Rule

13 Find bounds on the error to approximate the value of the integral $\int_0^1 5x^4\,dx$ using $n = 10$ in using each of the

A) Midpoint Rule

B) Trapezoidal Rule

C) Simpson's Rule

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14 Determine the number of steps that will guarantee an accuracy of at least 10^{-7} for to approximate $\int_1^3 \frac{1}{x}\,dx$ using each of

a) Trapezoidal Rule

b) Simpson's Rule

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15 Determine the number of steps that will guarantee an accuracy of at least 10^{-7} for to approximate $\int_0^1 cosx\ dx$ using each of

a) Trapezoidal Rule

b) Simpson's Rule

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16 Determine the number of steps to guarantee an accuracy of 10^{-6} to approximate $\int_1^4 xlnx\,dx$ using

(a) the Trapezoidal Rule

(b) Midpoint Rule

(c) Simpson's Rule

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MATH ENG

LESSON 5-8

The Natural Logarithm as an Integral

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The Featured Program
2024-2025

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Lesson (5-8)

The Natural Logarithm as an Integral

DEFINITION 8.1

For $x > 0$, we define the **natural logarithm** function, written $\ln x$, by

$$\ln x = \int_1^x \frac{1}{t}\, dt.$$

$$\int \frac{1}{x}\, dx = \ln|x| + c.$$

THEOREM 8.1

For any real numbers $a, b > 0$ and any rational number r,

(i) $\ln 1 = 0$

(ii) $\ln(ab) = \ln a + \ln b$

(iii) $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ and

(iv) $\ln(a^r) = r \ln a$.

DEFINITION 8.2

We define e to be that number for which

$$\ln e = 1.$$

DEFINITION 8.3

For x irrational, we define $y = e^x$ to be that number for which

$$\ln y = \ln(e^x) = x.$$

$$e^{\ln x} = x, \quad \text{for } x > 0.$$

$$\ln(e^x) = x, \quad \text{for all } x \in (-\infty, \infty).$$

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The Natural Logarithm as an Integral

THEOREM 8.2

For r, s any real numbers and t any rational number,

(i) $e^r e^s = e^{r+s}$

(ii) $\frac{e^r}{e^s} = e^{r-s}$ and

(iii) $(e^r)^t = e^{rt}$.

$$b^x = e^{\ln(b^x)} = e^{x \ln b}.$$

THEOREM 8.3

For any base $a > 0$ $(a \neq 1)$ and any $x > 0$, $\log_a x = \frac{\ln x}{\ln a}$.

THEOREM

For any constant $a > 0$, $\frac{d}{dx} a^x = a^x \ln a$.

THEOREM

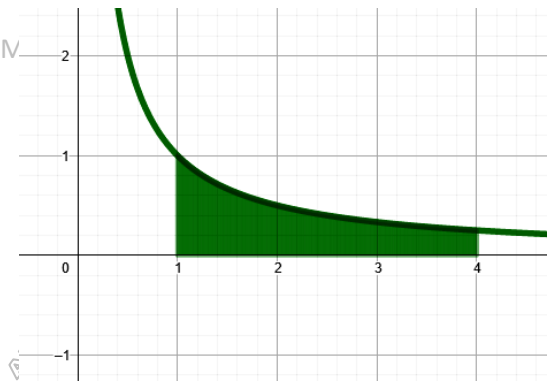
$$\frac{d}{dx} e^x = e^x.$$

THEOREM

For $x > 0$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

Express the number as an integral and sketch the corresponding area.

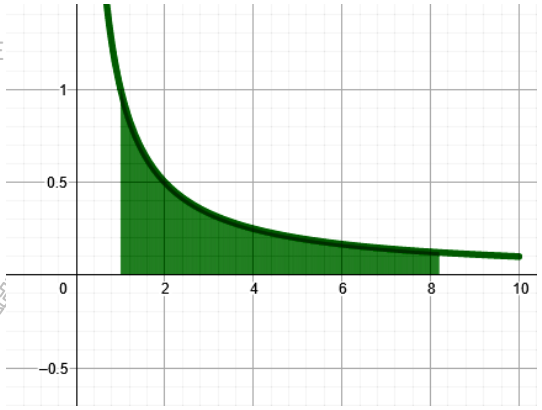
1 $\ln 4$



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The Natural Logarithm as an Integral

2 $ln\,8.2$



3 Use Simpson's Rule with $n = 4$ to estimate ln 4.

4 Use Simpson's Rule with n = 4 to estimate ln 5.

Use the properties of logarithms to rewrite the expression as a single term.

5 $\ln\sqrt{2} + 3\ln 2$

6 $\ln 8 - 2\ln 2$

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The Natural Logarithm as an Integral

7 $2\ln 3 - \ln 9 + \ln\sqrt{3}$

8 $2\ln\left(\frac{1}{3}\right) - \ln 3 + \ln\left(\frac{1}{9}\right)$

Evaluate the derivative using properties of logarithms where needed.

9 $\frac{d}{dx}(\ln\sqrt{x^2+1})$

10 $\frac{d}{dx}[\ln(x^5 \sin x \cos x)]$

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The Natur... al

11 $\frac{d}{dx}\left(\ln\frac{x^4}{x^5+1}\right)$

12 $\frac{d}{dx}\left(\ln\sqrt{\frac{x^3}{x^5+1}}\right)$

13 $\frac{d}{dx}\log_7\sqrt{x^2+1}$

14 $\frac{d}{dx}\log_{10}(2^x)$

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15 $\frac{d}{dx}\left(3^{\sin x}\right)$

16 $\frac{d}{dx}\left(4^{\sqrt{x}}\right)$

Evaluate the integral

17 $\int \frac{1}{x \ln x} dx$

18 $\int \frac{1}{\sqrt{1-x^2}\sin^{-1} x} dx$

19 $\int x3^{x^2} dx$

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The Natural Logarithm as an Integral

20 $\int 2^x \sin(2^x)\, dx$

21 $\int \frac{e^{2/x}}{x^2} dx$

22 $\int \frac{\sin(\ln x^3)}{x} dx$

23 $\int_0^1 \frac{x^2}{x^3 - 4} dx$

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24 $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

25 $\int_0^1 \tan x \, dx$

26 $\int_1^2 \frac{\ln x}{x} dx$

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