

THE FEATURED
PROGRAM EDUCATION

البرنامج المميز

GRADE 12 GENERAL CHAPTER 1



2024-2025

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zaid Al Khair School

2024-2025

Grade 12 GEN

Lesson 1-1

Chapter 1

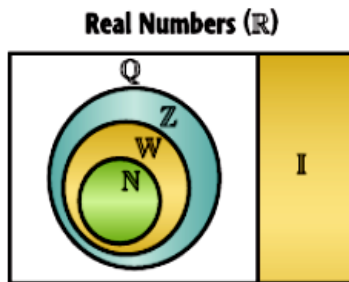


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- 1-Describe subsets of real numbers.
2 -Identify and evaluate functions and state their domains

Key Concept Real Numbers

Letter	Set	Examples
Q	rational numbers	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.666\dots$
I	irrational numbers	$\sqrt{3} = 1.73205\dots$
Z	integers	$-5, 17, -23, 8$
W	wholes	$0, 1, 2, 3\dots$
N	naturals	$1, 2, 3, 4\dots$

$$\{x \mid -3 \leq x \leq 16, x \in \mathbb{Z}\}$$

The set of numbers x such that...

x has the given properties...

and x is an element of the given set of numbers.

Use Set Builder Notation

Describe the set of numbers using set-builder notation.

$\{8, 9, 10, 11, \dots\}$

.....

$\{1, 2, 3, 4, 5, \dots\}$

.....

$x < 7$

.....

$$x \leq -3$$

all multiples of three

all multiples of π

Bounded Intervals		Unbounded Intervals	
Inequality	Interval Notation	Inequality	Interval Notation
$a \leq x \leq b$	$[a, b]$	$x \geq a$	$[a, \infty)$
$a < x < b$	(a, b)	$x \leq a$	$(-\infty, a]$
$a \leq x < b$	$[a, b)$	$x > a$	(a, ∞)
$a < x \leq b$	$(a, b]$	$x < a$	$(-\infty, a)$
		$-\infty < x < \infty$	$(-\infty, \infty)$

1- Write each set of numbers using interval notation.

a. $-8 < x \leq 16$

b. $x < 11$

c. $x \leq -16$ or $x > 5$

Guided Practice

2A. $-4 \leq y < -1$

2B. $a \geq -3$

2C. $x > 9$ or $x < -2$

A **function** is a special type of relation.

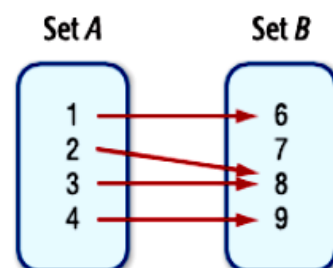
KeyConcept Function

Words A function f from set A to set B is a relation that assigns to each element x in set A *exactly one* element y in set B .

Symbols The relation from set A to set B is a function.

Set A is the domain. $D = \{1, 2, 3, 4\}$

Set B contains the range. $R = \{6, 8, 9\}$

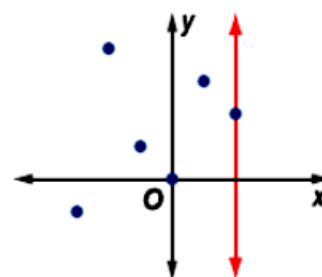


KeyConcept Vertical Line Test

Words

A set of points in the coordinate plane is the graph of a function if each possible vertical line intersects the graph in at most one point.

Model



Determine whether each relation represents y as a function of x .

The input value x is a student's ID number, and the output value y is that student's score on a physics exam.

The input value x is the area code, and the output value y is a phone number in that area code.

x	y
-6	-7
2	3
5	8
5	9
9	22

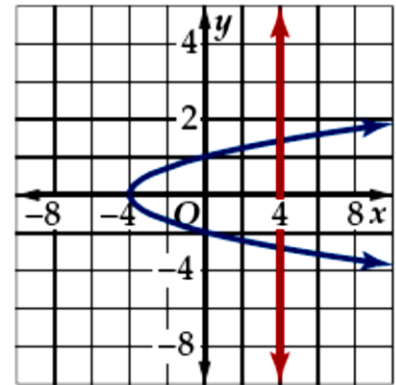
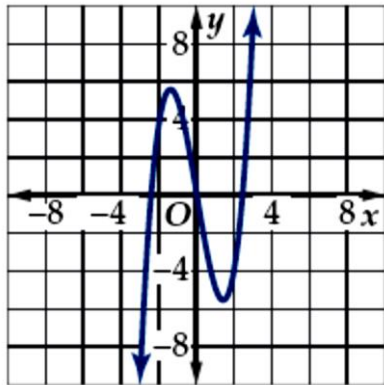
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x	y
-8	-5
-5	-4
0	-3
3	-2
6	-3

.....

$$3y + 6x = 18$$

$$y^2 - 2x = 5$$



If $g(x) = x^2 + 8x - 24$, find each function value.

a. $g(6)$

.....

.....

.....

.....

b. $g(-4x)$

.....

.....

.....

.....

Guided Practice

If $f(x) = \frac{2x + 3}{x^2 - 2x + 1}$, find each function value.

4A. $f(12)$

4B. $f(6x)$

Find Domains Algebraically

State the domain of each function.

$$f(x) = 3x^2 + 5x - 11$$

.....

.....

$$f(x) = \frac{2+x}{x^2-7x}$$

.....

.....

$$f(x) = \frac{5x-2}{x^2+7x+12}$$

.....

.....

$$f(x) = \frac{8x+12}{x^2+5x+4}$$

.....

.....

$$g(x) = \frac{x+1}{x^2-3x-40}$$

.....

.....

$$g(t) = \sqrt{t-5}$$

.....

.....

$$h(a) = \sqrt{a^2-4}$$

.....

.....

Evaluate a Piecewise-Defined Function

HEIGHT The average maximum height of children in inches as a function of their parents' maximum heights in inches can be modeled by the following piecewise function. Find the average maximum heights of children whose parents have the given maximum heights. Use $h(x)$, where x is the independent variable representing the parents' height and $h(x)$ is the dependent variable representing the child's height.

$$h(x) = \begin{cases} 1.6x - 41.6 & \text{if } 63 < x < 66 \\ 3x - 132 & \text{if } 66 \leq x \leq 68 \\ 2x - 66 & \text{if } x > 68 \end{cases}$$

$$h(67)$$

$$h(72)$$

$$h(66)$$

SPEED The speed v of a vehicle in kilometers per hour can be represented by the following piecewise function when t is the time in seconds. Find the speed of the vehicle at each indicated time.

$$v(t) = \begin{cases} 4t & \text{if } 0 \leq t \leq 15 \\ 60 & \text{if } 15 < t < 240 \\ -6t + 1500 & \text{if } 240 \leq t \leq 250 \end{cases}$$

A. $v(5)$

B. $v(15)$

C. $v(245)$

EOT هيكل

if $g(x) = 2x^2 + 18x - 14$ Find $g(9)$

a) 300

b) 310

c) 219

d) 132

if $g(x) = 2x^2 + 18x - 14$ Find $g(3x)$

a) $18x^2 + 54x - 14$

b) $18x^2 - 54x - 14$

c) $18x^2 + 54x + 14$

d) $9x^2 + 54x - 14$

if $h(y) = -3y^3 - 6y + 9$ Find $h(4)$

a) 300

b) 310

c) -207

d) 207

if $h(y) = -3y^3 - 6y + 9$ Find $h(-2y)$

a) $-24y^3 + 12y + 9$

b) $24y^3 + 12y$

c) $12y^3 + 24y + 9$

d) $24y^3 + 12y + 9$

if $f(t) = \frac{4t + 11}{3t^2 + 5t + 1}$ Find $f(-6)$

a) $-\frac{13}{79}$

b) $\frac{13}{79}$

c) $\frac{79}{13}$

d) $-\frac{79}{13}$

if $f(t) = \frac{4t + 11}{3t^2 + 5t + 1}$ Find $f(4t)$

a) $\frac{16t + 11}{48t^2 + 20t + 1}$

b) $\frac{16t - 11}{48t^2 + 20t + 1}$

c) $\frac{t + 11}{48t^2 + 20t + 1}$

d) $\frac{16t + 11}{t^2 + 20t + 1}$

if $g(x) = \frac{3x^3}{x^2 + x - 4}$ Find $g(-2)$

a) 12

b) -0.8

c) 20

d) 3

if $h(x) = 16 - \frac{12}{2x+3}$ Find $h(-3)$

a) 12

b) - 0.8

c) 20

d) 3

if $f(x) = -7 + \frac{6x+1}{x}$ Find $f(5)$

a) 12

b) - 0.8

c) 20

d) 3

if $g(m) = 3 + \sqrt{m^2 - 4}$ Find $g(3m)$

a) $9 + \sqrt{m^2 - 4}$

b) $3 + \sqrt{9m^2 - 4}$

c) $\sqrt{9m^2 + 4}$

d) $3 - \sqrt{9m^2 - 4}$

State the domain of each function.

$$f(x) = \frac{8x + 12}{x^2 + 5x + 4}$$

a) $R/\{-4\}$

b) $R/\{-4, -1\}$

c) $R/\{4, -1\}$

d) $R/\{-1\}$

State the domain of each function.

$$g(x) = \frac{x + 1}{x^2 - 3x - 40}$$

a) $R/\{5, 8\}$

b) $R/\{-5, -8\}$

c) $R/\{-5, 8\}$

d) $R/\{-5, 4\}$

State the domain of each function.

$$g(a) = \sqrt{1 + a^2}$$

$$h(x) = \sqrt{6 - x^2}$$

State the domain of each function.

$$g(x) = \frac{3}{\sqrt{x^2 - 16}}$$

$$f(x) = \frac{2}{x} + \frac{4}{x+1}$$

$$f(a) = \frac{5a}{\sqrt{4a-1}}$$

$$g(x) = \frac{6}{x+3} + \frac{2}{x-4}$$

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Lesson 1-2

Chapter 1



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Outcomes

Analyzing Graphs of Functions and Relations

1-2

- 1-Use graphs of functions to estimate function values and find domains, ranges, y-intercepts, and zeros of functions.
 2 Explore symmetries of graphs and identify even and odd functions.

Estimate Function Values

INTERNET Consider the graph of function R shown.

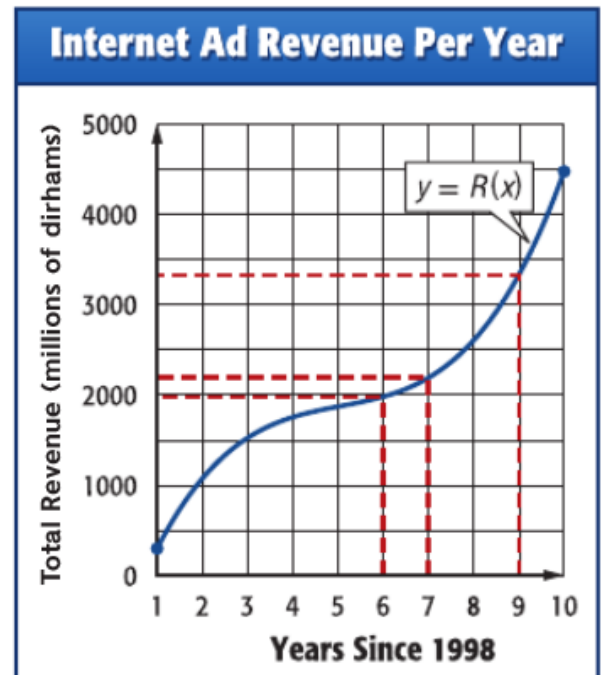
- a. Use the graph to estimate total Internet advertising revenue in 2007. Confirm the estimate algebraically.

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- b. Use the graph to estimate the year in which total Internet advertising revenue reached AED 2 billion. Confirm the estimate algebraically.

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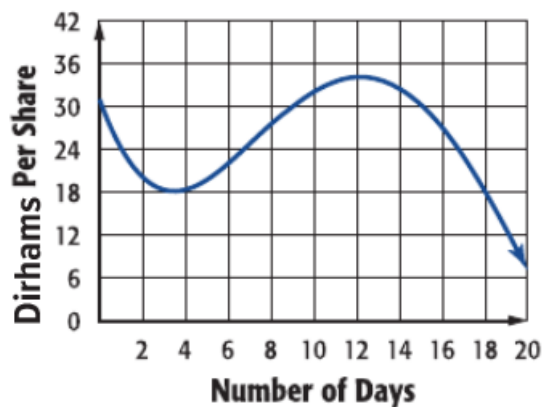
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STOCKS An investor assessed the average daily value of a share of a certain stock over a 20-day period. The value of the stock can be approximated by $v(d) = 0.002d^4 - 0.11d^3 + 1.77d^2 - 8.6d + 31$, $0 \leq d \leq 20$, where d represents the day of the assessment.

- A. Use the graph to estimate the value of the stock on the 10th day. Confirm your estimate algebraically.
- B. Use the graph to estimate the days during which the stock was valued at AED 30 per share. Confirm your estimate algebraically.

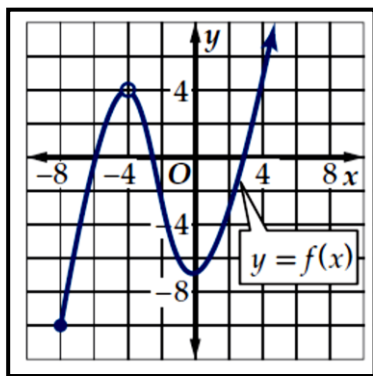
Value of Stock



Find Domain and Range

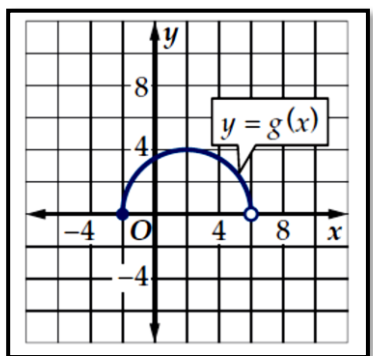
Use the graph of f to find the domain and range of the function.

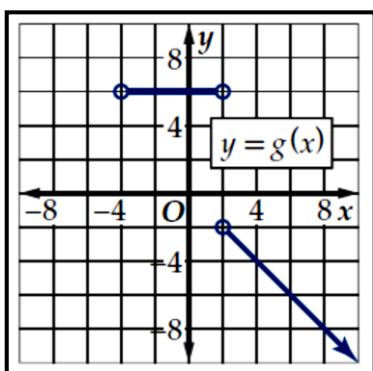
The graph



Domain

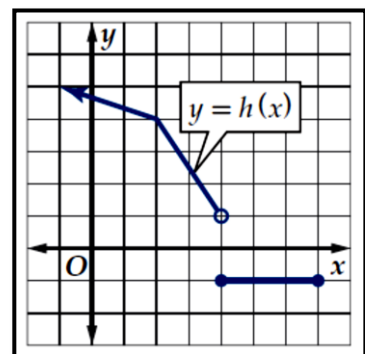
Range





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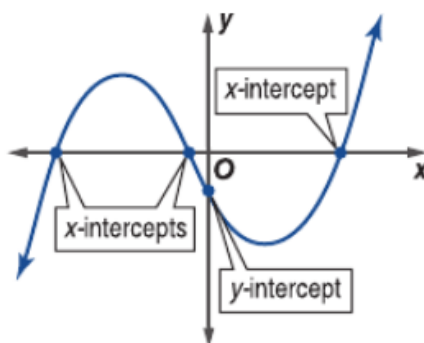
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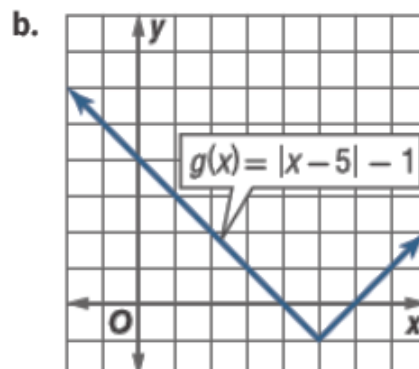
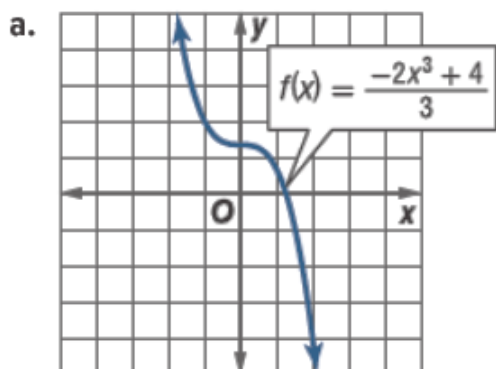
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A point where a graph intersects or meets the x - or y -axis is called an intercept. An x -intercept of a graph occurs where $y = 0$. A y -intercept of a graph occurs where $x = 0$. The graph of a function can have 0, 1, or more x -intercepts, but at most one y -intercept.

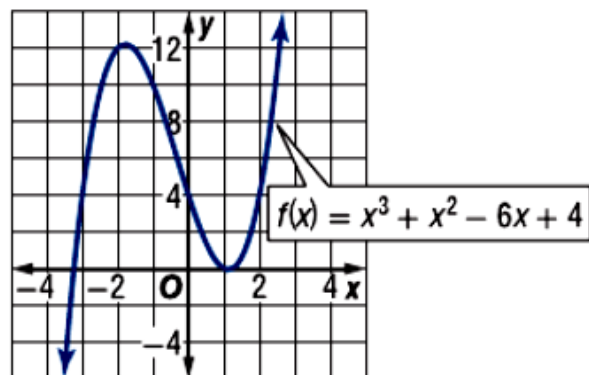


Use the graph of each function to approximate its y -intercept. Then find the y -intercept algebraically.

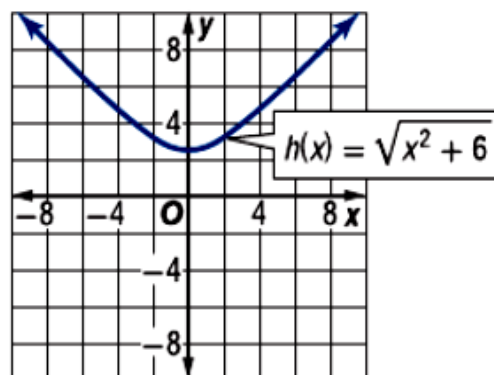


Guided Practice

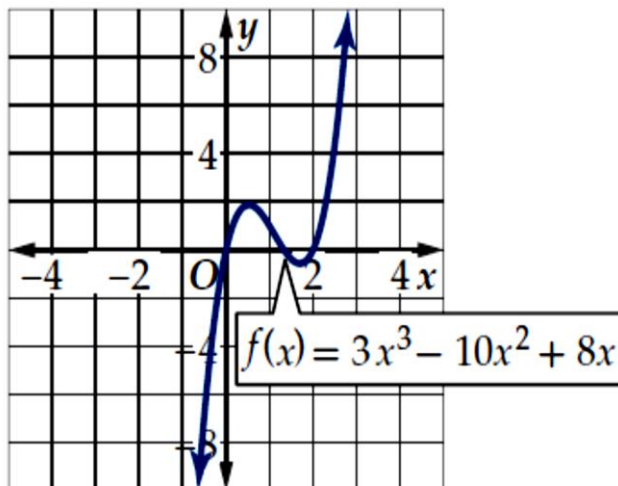
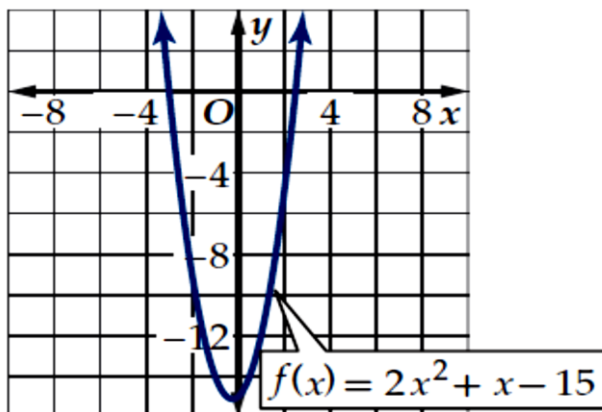
3A.

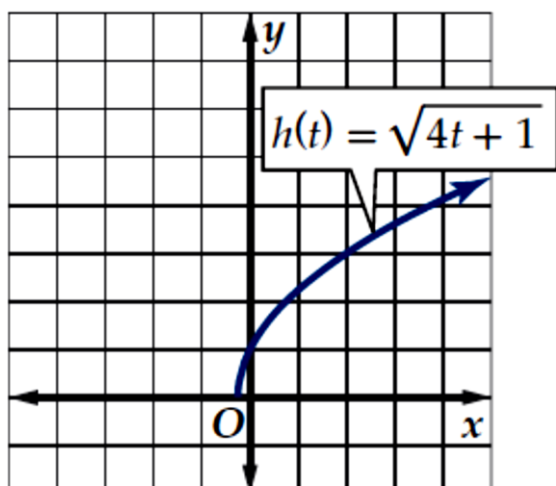


3B.



Use the graph of each function to approximate its zero(s). Then find its zero(s) algebraically.





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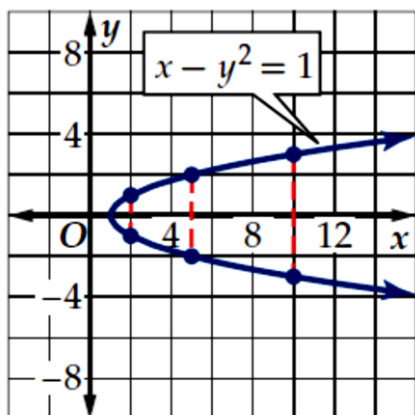
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2 Symmetry of Graphs Graphs of relations can have two different types of symmetry. Graphs with **line symmetry** can be folded along a line so that the two halves match exactly. Graphs that have **point symmetry** can be rotated 180° with respect to a point and appear unchanged. The three most common types of symmetry are shown below.

KeyConcept Tests for Symmetry

Graphical Test	Model	Algebraic Test
The graph of a relation is <i>symmetric with respect to the x-axis</i> if and only if for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.		Replacing y with $-y$ produces an equivalent equation.
The graph of a relation is <i>symmetric with respect to the y-axis</i> if and only if for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.		Replacing x with $-x$ produces an equivalent equation.
The graph of a relation is <i>symmetric with respect to the origin</i> if and only if for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.		Replacing x with $-x$ and y with $-y$ produces an equivalent equation.

Use the graph of each equation to test for symmetry with respect to the x -axis, y -axis, and the origin. Support the answer numerically. Then confirm algebraically.



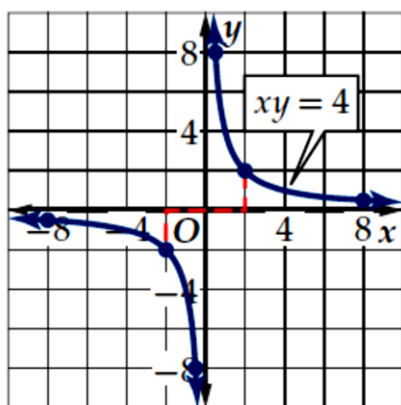
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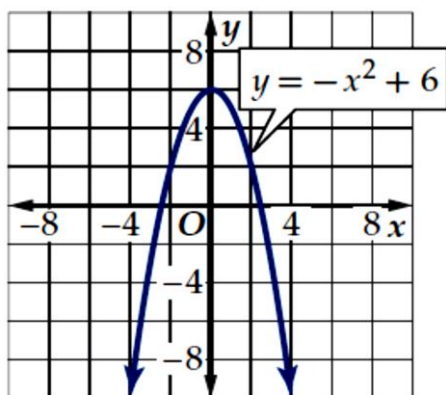
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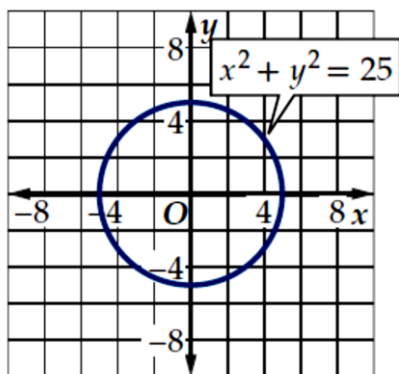
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KeyConcept Even and Odd Functions

Type of Function	Algebraic Test
Functions that are symmetric with respect to the y-axis are called even functions .	For every x in the domain of f , $f(-x) = f(x)$.
Functions that are symmetric with respect to the origin are called odd functions .	For every x in the domain of f , $f(-x) = -f(x)$.

determine whether each function is even, odd, or neither.

$$f(x) = x^3 - 2x$$

.....

.....

$$g(x) = x^4 + 2$$

.....

.....

$$h(x) = x^3 - 0.5x^2 - 3x$$

.....

.....

$$f(x) = x^2 + 6x + 10$$

.....

.....

$$h(x) = x^5 - 2x^3 + x$$

.....

.....

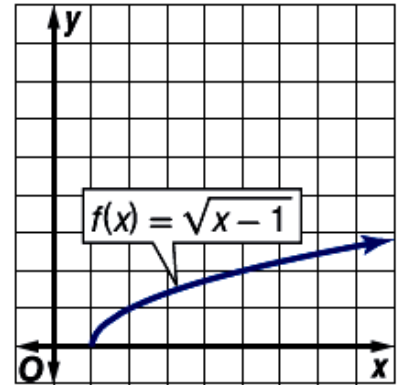
$$g(x) = 4\sqrt{x}$$

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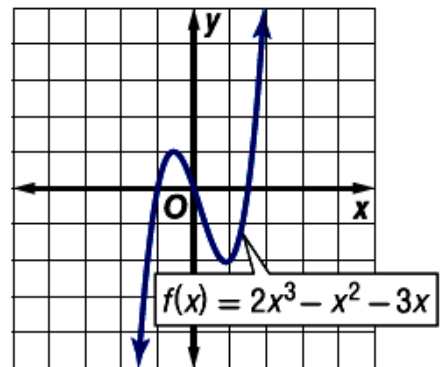
EOT هيكل

- a) y – intercept Non , zeros $x = 1$
 b) y – intercept $y = 0$, zeros $x = 1$
 c) y – intercept $y = 0$, zeros $x = -1, 0, 1.5$
 d) y – intercept $y = 1$, zeros $x = 1.5$



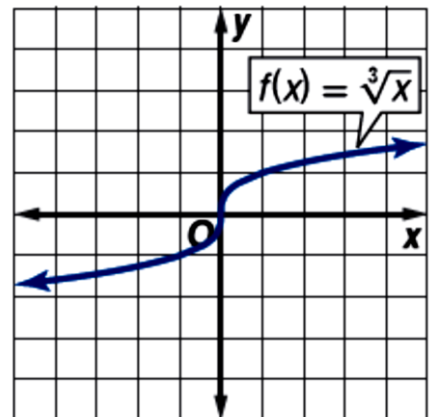
Use the graph of each function to find its y -intercept and zero(s).

- a) y – intercept Non , zeros $x = 1$
 b) y – intercept $y = 0$, zeros $x = 1$
 c) y – intercept $y = 0$, zeros $x = -1, 0, 1.5$
 d) y – intercept $y = 1$, zeros $x = 1.5$



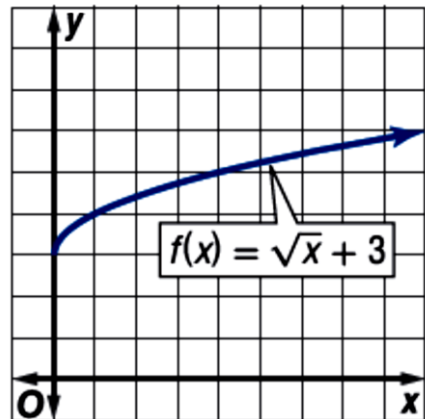
Use the graph of each function to find its y -intercept and zero(s).

- a) y – intercept Non , zeros $x = 1$
 b) y – intercept $y = 0$, zeros $x = 0$
 c) y – intercept $y = 0$, zeros $x = -1, 0$
 d) y – intercept $y = 1$, zeros Non



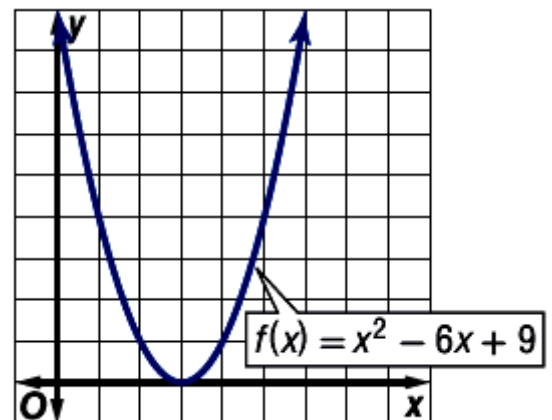
Use the graph of each function to find its y -intercept and zero(s).

- a) y – intercept Non , zeros $x = 3$
- b) y – intercept $y = 0$, zeros $x = 1$
- c) y – intercept $y = 0$, zeros $x = 3$
- d) y – intercept $y = 3$, zeros x Non



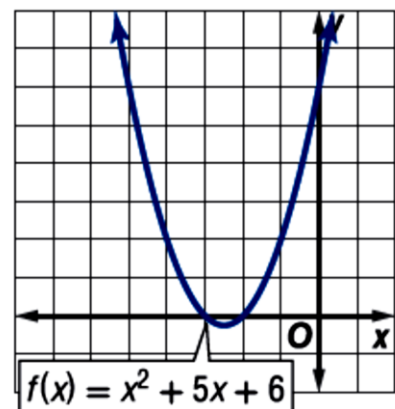
Use the graph of each function to find its y -intercept and zero(s).

- a) y – intercept Non , zeros $x = 3$
- b) y – intercept $y = 0$, zeros $x = 1$
- c) y – intercept $y = 9$, zeros $x = 3$
- d) y – intercept $y = 9$, zeros $x = -3$



Use the graph of each function to find its y -intercept and zero(s).

- a) y – intercept Non , zeros $x = 3$
- b) y – intercept $y = 6$, zeros $x = -3, -2$
- c) y – intercept $y = 6$, zeros $x = -2, -1$
- d) y – intercept $y = 6$, zeros $x = -3, 2$



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Lesson 1-3

Chapter 1



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Outcomes

Continuity, End Behavior, and Limits

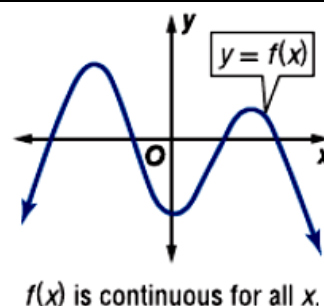
1-3

1-Use limits to determine the continuity of a function and apply the Intermediate Value Theorem to continuous functions.

2- Use limits to describe end behavior of functions

1 Continuity The graph of a **continuous function** has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

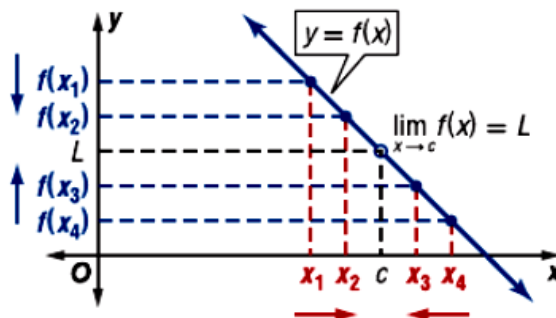
One condition for a function $f(x)$ to be continuous at $x = c$ is that the function must approach a unique function value as x -values approach c from the left and right sides. The concept of approaching a value without necessarily ever reaching it is called a **limit**.



KeyConcept Limits

Words If the value of $f(x)$ approaches a unique value L as x approaches c from each side, then the limit of $f(x)$ as x approaches c is L .

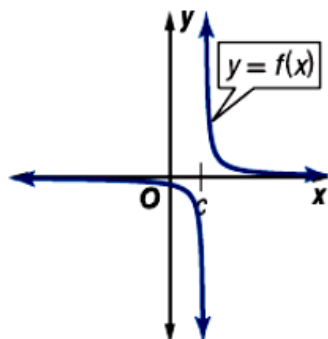
Symbols $\lim_{x \rightarrow c} f(x) = L$, which is read *The limit of $f(x)$ as x approaches c is L .*



KeyConcept Types of Discontinuity

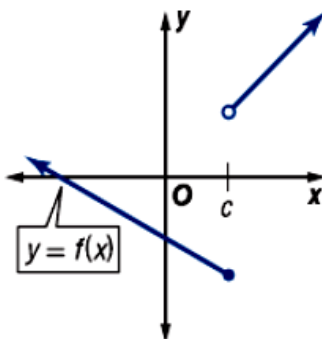
A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example



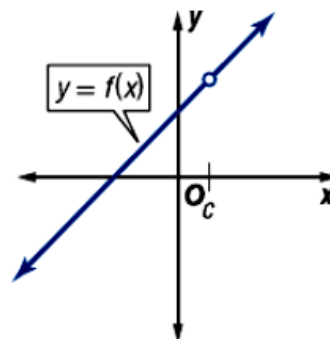
A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example



A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

Example



Concept Summary Continuity Test

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

- $f(x)$ is defined at c . That is, $f(c)$ exists.
- $f(x)$ approaches the same value from either side of c . That is, $\lim_{x \rightarrow c} f(x)$ exists.
- The value that $f(x)$ approaches from each side of c is $f(c)$. That is, $\lim_{x \rightarrow c} f(x) = f(c)$.

Determine whether $f(x) = 2x^2 - 3x - 1$ is continuous at $x = 2$. Justify using the continuity test.

Check the three conditions in the continuity test.

x	1.9	1.99	1.999	2.0	2.001	2.01	2.1
$f(x)$							

.....

.....

.....

.....

.....

.....

Determine whether each function is continuous at $x = 0$. Justify using the continuity test.

x							
$f(x)$							

$$f(x) = \begin{cases} \frac{1}{x} & , \quad x < 0 \\ x & , \quad x \geq 0 \end{cases}$$

.....

.....

.....

.....

Determine whether each function is continuous at the given x -value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

a. $f(x) = \begin{cases} 3x - 2 & \text{if } x > -3 \\ 2 - x & \text{if } x \leq -3 \end{cases}$; at $x = -3$

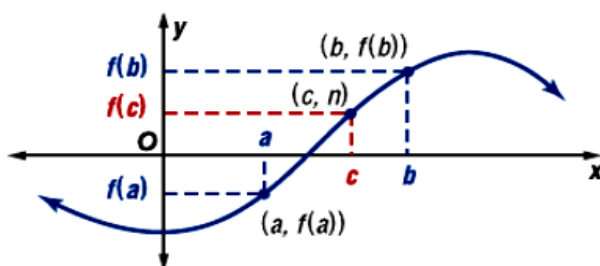
x							
$f(x)$							

$f(x) = \frac{x+3}{x^2-9}$, at $x = 3$

x							
$f(x)$							

KeyConcept Intermediate Value Theorem

If $f(x)$ is a continuous function and $a < b$ and there is a value n such that n is between $f(a)$ and $f(b)$, then there is a number c , such that $a < c < b$ and $f(c) = n$.



Corollary: The Location Principle If $f(x)$ is a continuous function and $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c , such that $a < c < b$ and $f(c) = 0$. That is, there is a zero between a and b .

Determine between which consecutive integers the real zeros of each function are located on the given interval.

a. $f(x) = x^3 - 4x + 2$; $[-4, 4]$

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$									

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2 End Behavior The **end behavior** of a function describes how a function *behaves* at either *end* of the graph. That is, end behavior is what happens to the value of $f(x)$ as x increases or decreases without bound—becoming greater and greater or more and more negative. To describe the end behavior of a graph, you can use the concept of a limit.

Left-End Behavior

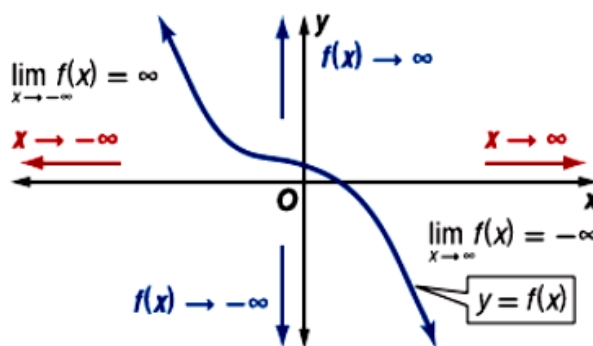
$$\lim_{x \rightarrow -\infty} f(x)$$

One possibility for the end behavior of the graph of a function is for the value of $f(x)$ to increase or decrease without bound.

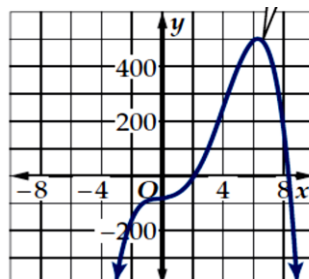
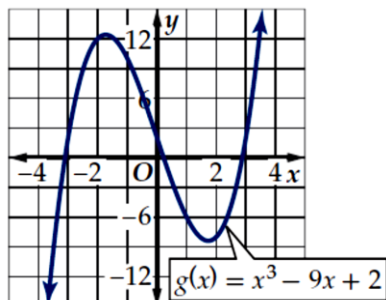
This end behavior is described by saying that $f(x)$ approaches positive or negative infinity.

Right-End Behavior

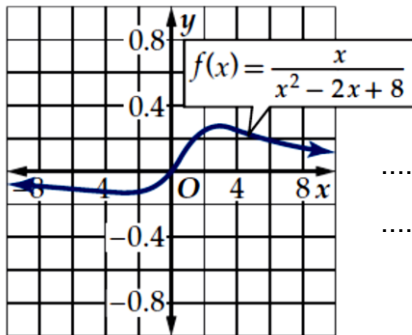
$$\lim_{x \rightarrow \infty} f(x)$$



Use the graph of each function to describe its end behavior. Support the conjecture numerically.



Use the graph of $f(x) = \frac{x}{x^2 - 2x + 8}$ to describe its end behavior. Support the conjecture numerically.



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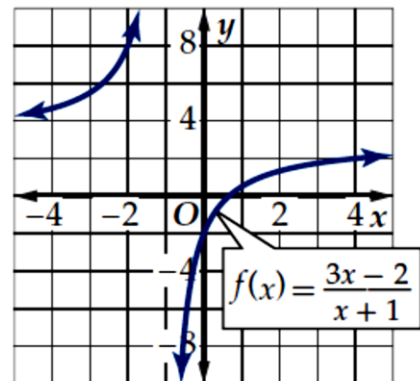
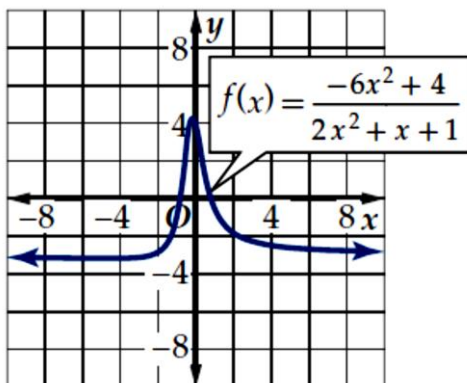
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x	-10000	-1000	-100	0	100	1000	10000
$f(x)$							

.....

.....

Use the graph of each function to describe its end behavior. Support the conjecture numerically.



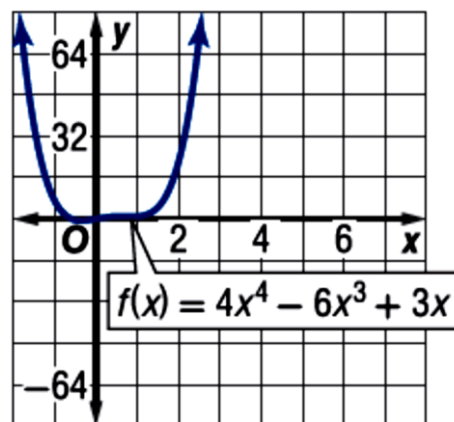
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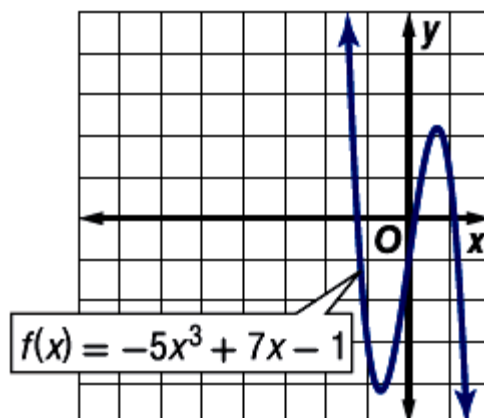
Use the graph of each function to describe its end behavior.

- a) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- d) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$



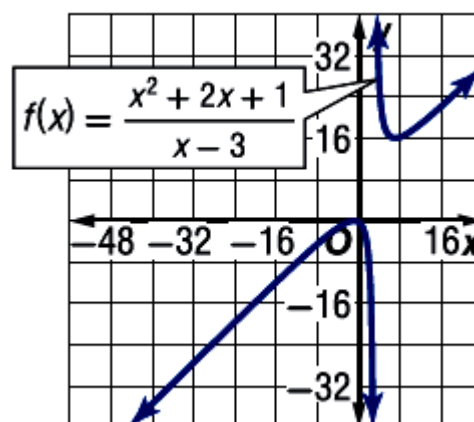
Use the graph of each function to describe its end behavior.

- a) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- d) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$



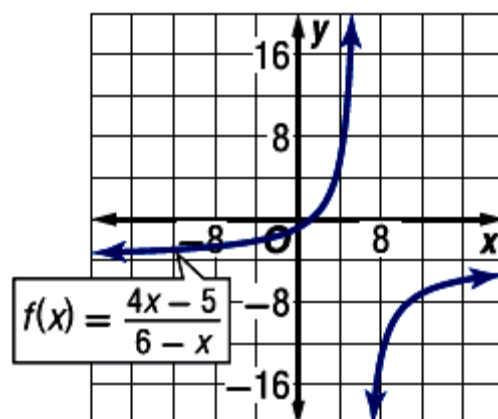
Use the graph of each function to describe its end behavior.

- a) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- d) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$



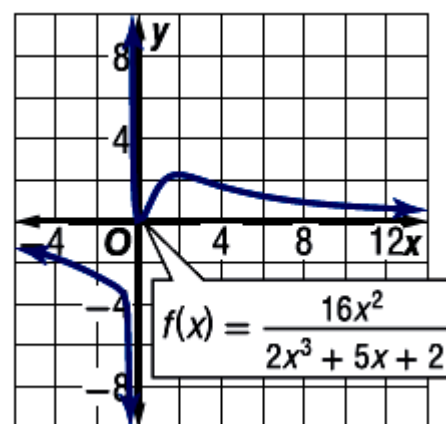
Use the graph of each function to describe its end behavior.

- a) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = \infty$
 b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = 0$
 c) $\lim_{x \rightarrow -\infty} f(x) = -4$, $\lim_{x \rightarrow \infty} f(x) = -4$
 d) $\lim_{x \rightarrow -\infty} f(x) = 3$, $\lim_{x \rightarrow \infty} f(x) = 2$



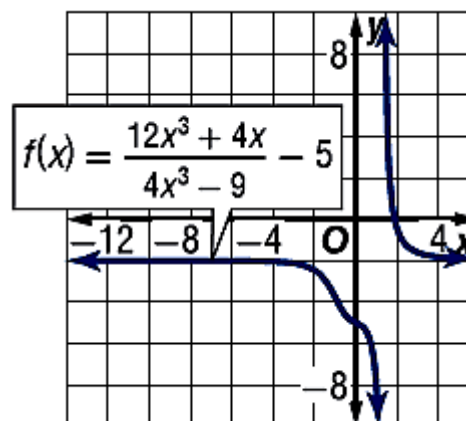
Use the graph of each function to describe its end behavior.

- a) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$
 b) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$
 c) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
 d) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$



Use the graph of each function to describe its end behavior.

- a) $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = \infty$
 b) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$
 c) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 2$
 d) $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = -2$



Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity.

$$q(x) = -\frac{24}{x}$$

- a) $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = \infty$
 b) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$
 c) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 2$
 d) $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = -2$

Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity.

$$p(x) = \frac{x+1}{x-2}$$

- a) $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = \infty$
 b) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$
 c) $\lim_{x \rightarrow -\infty} f(x) = 1$, $\lim_{x \rightarrow \infty} f(x) = 1$
 d) $\lim_{x \rightarrow -\infty} f(x) = -1$, $\lim_{x \rightarrow \infty} f(x) = 1$

Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity.

$$m(x) = \frac{4+x}{2x+6}$$

- a) $\lim_{x \rightarrow -\infty} f(x) = \frac{1}{2}$, $\lim_{x \rightarrow \infty} f(x) = \frac{1}{2}$
 b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
 c) $\lim_{x \rightarrow -\infty} f(x) = 1$, $\lim_{x \rightarrow \infty} f(x) = 1$
 d) $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = 2$

Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity.

$$c(x) = \frac{5x^2}{x^3 + 2x + 1}$$

- a) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = 1$, $\lim_{x \rightarrow \infty} f(x) = 1$
- d) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$

Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity.

$$k(x) = \frac{4x^2 - 3x - 1}{11x}$$

- a) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = 4$, $\lim_{x \rightarrow \infty} f(x) = -4$
- d) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$

Use logical reasoning to determine the end behavior or limit of the function as x approaches infinity.

$$h(x) = 2x^5 + 7x^3 + 5$$

- a) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$
- d) $\lim_{x \rightarrow -\infty} f(x) = -\infty$, $\lim_{x \rightarrow \infty} f(x) = \infty$

$$g(x) = x^4 - 9x^2 + \frac{x}{4}$$

$$g(x) = x^4 - 9x^2 + \frac{x}{4}$$

- $$\begin{aligned} a) \quad & \lim_{x \rightarrow -\infty} f(x) = -\infty, \quad \lim_{x \rightarrow \infty} f(x) = -\infty \\ b) \quad & \lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = -\infty \\ c) \quad & \lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = \infty \\ d) \quad & \lim_{x \rightarrow -\infty} f(x) = -\infty, \quad \lim_{x \rightarrow \infty} f(x) = \infty \end{aligned}$$

Which functions has a removable discontinuity

- $$\begin{array}{ll} \text{a) } f(x) = x^2 - 4 & \text{b) } f(x) = \frac{1}{x-7} \\ \text{c) } f(x) = \frac{x^2 - 25}{x-5} & \text{d) } f(x) = \frac{x-1}{x+3} \end{array}$$

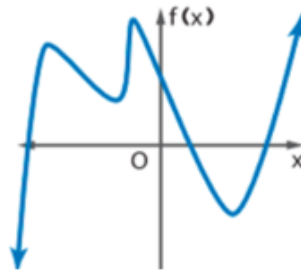
Which of the following functions is discontinuity at $x = -1$

- a. $f(x) = \frac{1}{x+1}$
- b. $f(x) = \sqrt{x^2 + 4}$
- c. $f(x) = \frac{1}{x-1}$
- d. $f(x) = \begin{cases} x-2 & , x \leq -1 \\ 3x & , x > -1 \end{cases}$

Which of the following are the consecutive integers contains the real zeroes of the function $f(x) = x^3 - 2x^2 + x - 1$ on $[-2, 2]$

- a. -2 and -1
- b. 1 and 2
- c. 2 and 3
- d. -1 and 0

Describe the end behavior of $f(x)$.



- a) $\lim_{x \rightarrow -\infty} f(x) = \infty, \lim_{x \rightarrow \infty} f(x) = \infty$
- b) $\lim_{x \rightarrow -\infty} f(x) = \infty, \lim_{x \rightarrow \infty} f(x) = -\infty$
- c) $\lim_{x \rightarrow -\infty} f(x) = -\infty, \lim_{x \rightarrow \infty} f(x) = -\infty$
- d) $\lim_{x \rightarrow -\infty} f(x) = -\infty, \lim_{x \rightarrow \infty} f(x) = \infty$

2024-2025

Grade 12 GEN

Lesson 1-4

Chapter 1



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Outcomes

Extrema and Average Rates of Change

11-4

1-Determine intervals on which functions are increasing, constant, or decreasing, and determine maxima and minima of functions.

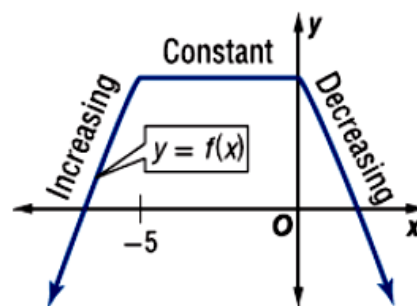
2- Determine the average rate of change of a function

1 Increasing and Decreasing Behavior An analysis of a function can also include a description of the intervals on which the function is increasing, decreasing, or constant.

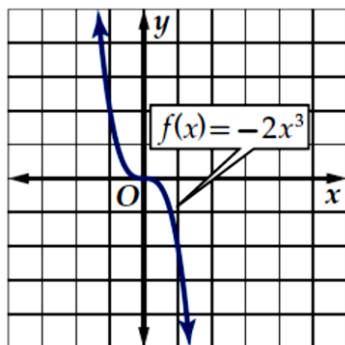
Consider the graph of $f(x)$ shown. As you move from left to right, $f(x)$ is

- increasing or *rising* on $(-\infty, -5)$,
- constant or *flat* on $(-5, 0)$, and
- decreasing or *falling* on $(0, \infty)$.

These graphical interpretations can also be described algebraically.



Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.



a. $f(x) = -2x^3$

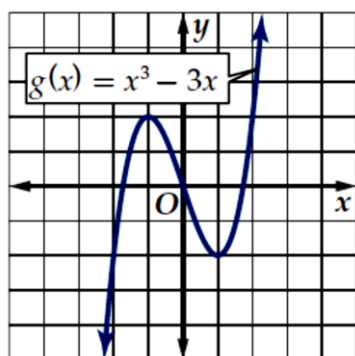
Analyze Graphically

Support Numerically

x	-8	-6	-4	-2	0	2	4	6	8
$f(x)$									

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b. $g(x) = x^3 - 3x$

Analyze Graphically

Support Numerically

x	-11	-9	-7	-5	-3	-1
$f(x)$						

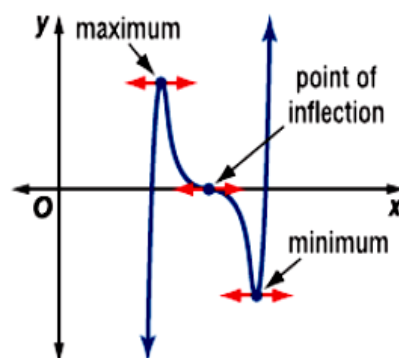
x	-1	-0.5	0	0.5	1
$f(x)$					

x	1	3	5	7	9	11
$f(x)$						

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Critical points of a function are those points at which a line drawn tangent to the curve is horizontal or vertical. **Extrema** are critical points at which a function changes its increasing or decreasing behavior. At these points, the function has a **maximum** or a **minimum** value, either relative or absolute. A **point of inflection** can also be a critical point. At these points, the graph changes its shape, but not its increasing or decreasing behavior. Instead, the curve changes from being bent upward to being bent downward, or vice versa.



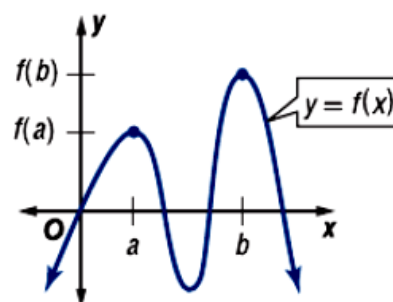
KeyConcept Relative and Absolute Extrema

Words A *relative maximum* of a function f is the greatest value $f(x)$ can attain on some interval of the domain.

Symbols $f(a)$ is a relative maximum of f if there exists an interval (x_1, x_2) containing a such that $f(a) > f(x)$ for every $x \neq a$ in (x_1, x_2) .

Words If a relative maximum is the greatest value a function f can attain over its entire domain, then it is the *absolute maximum*.

Symbols $f(b)$ is the absolute maximum of f if $f(b) > f(x)$ for every $x \neq b$, in the domain of f .

Model

$f(a)$ is a relative maximum of f .

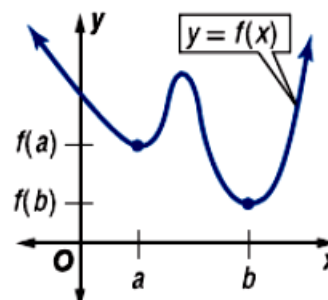
$f(b)$ is the absolute maximum of f .

Words A *relative minimum* of a function f is the least value $f(x)$ can attain on some interval of the domain.

Symbols $f(a)$ is a relative minimum of f if there exists an interval (x_1, x_2) containing a such that $f(a) < f(x)$ for every $x \neq a$ in (x_1, x_2) .

Words If a relative minimum is the least value a function f can attain over its entire domain, then it is the *absolute minimum*.

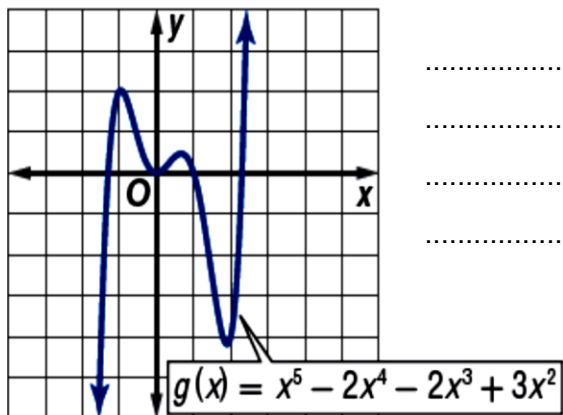
Symbols $f(b)$ is the absolute minimum of f if $f(b) < f(x)$ for every $x \neq b$, in the domain of f .

Model

$f(a)$ is a relative minimum of f .

$f(b)$ is the absolute minimum of f .

Estimate and classify the extrema for the graph of each function.

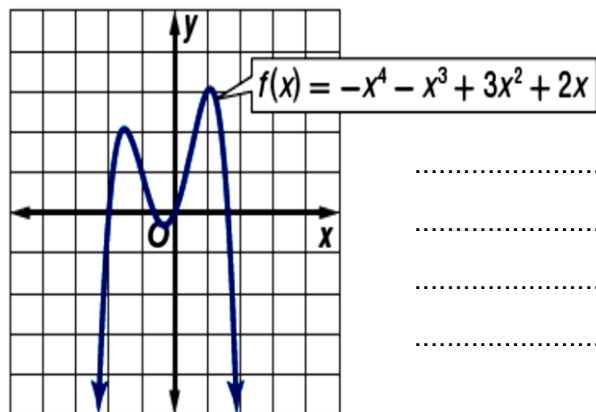


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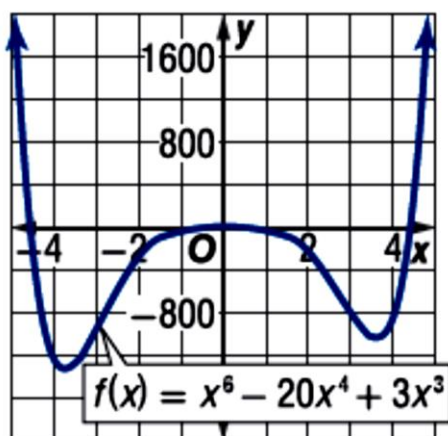


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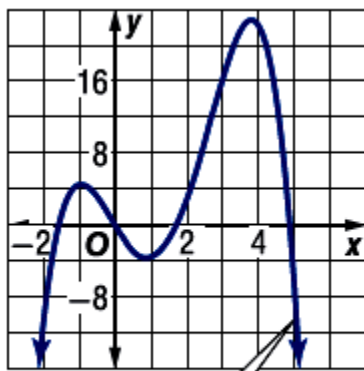


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$$f(x) = -0.5x^4 + 2.5x^3 + x^2 - 6.5x$$

KeyConcept Average Rate of Change

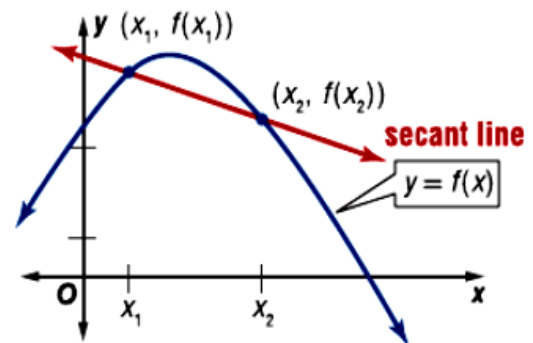
Words The **average rate of change** between any two points on the graph of f is the slope of the line through those points.

Geometry The line through two points on a curve is called a **secant line**. The slope of the secant line is denoted m_{sec} .

Symbols The average rate of change on the interval $[x_1, x_2]$ is

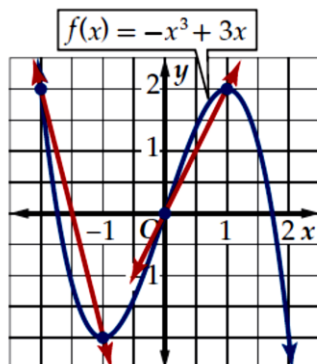
$$m_{\text{sec}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Model



Find the average rate of change of $f(x) = -x^3 + 3x$ on each interval.

a. $[-2, -1]$



b. $[0, 1]$

Find the average rate of change of each function on the given interval.

$$f(x) = x^4 - 6x^2 + 4x, [-5, -3]$$

$$f(x) = x^3 - 2x^2 - 3x + 2, [2, 3]$$

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PHYSICS If wind resistance is ignored, the distance $d(t)$ in feet an object travels when dropped from a high place is given by $d(t) = 16t^2$, where t is the time in seconds after the object is dropped. Find and interpret the average speed of the object from 2 to 4 seconds.

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EOT هيكل

Find the average rate of change of each function on the given interval.

$$g(x) = -4x^2 + 3x - 4; [-1, 3]$$

- a) - 5
- b) 28
- c) 140
- d) - 16

Find the average rate of change of each function on the given interval.

$$g(x) = 3x^2 - 8x + 2; [4, 8]$$

- a) - 5
- b) 28
- c) 140
- d) - 16

Find the average rate of change of each function on the given interval.

$$f(x) = 3x^3 - 2x^2 + 6; [2, 6]$$

- a) - 5
- b) 28
- c) 140
- d) - 16

Find the average rate of change of each function on the given interval.

$$f(x) = -2x^3 - 4x^2 + 2x - 8; [-2, 3]$$

- a) - 5
- b) 28
- c) 140
- d) - 16

Find the average rate of change of each function on the given interval.

$$f(x) = 3x^4 - 2x^2 + 6x - 1; [5, 9]$$

- a) 472
- b) - 2550
- c) - 309
- d) 4430

Find the average rate of change of each function on the given interval.

$$f(x) = \frac{x-3}{x}; [5, 12]$$

- a) 47.2
- b) - 0.45
- c) 0.05
- d) 4.2

Find the average rate of change of each function on the given interval.

$$f(x) = \frac{x+5}{x-4}; [-6, 2]$$

- a) 47.2
- b) - 0.45
- c) 0.05
- d) 4.2

2024-2025

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Lesson 1-5

Chapter 1



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Outcomes

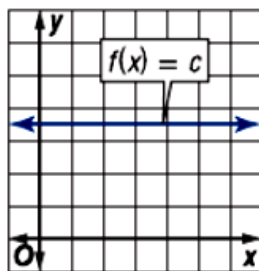
Parent Functions and Transformations

11-5

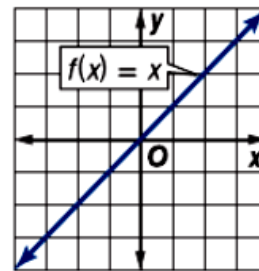
- 1- Identify, graph, and describe parent functions.
- 2- Identify and graph transformations of parent functions

KeyConcept Linear and Polynomial Parent Functions

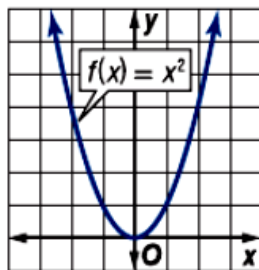
A **constant function** has the form $f(x) = c$, where c is any real number. Its graph is a horizontal line. When $c = 0$, $f(x)$ is the **zero function**.



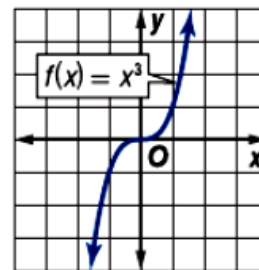
The **identity function** $f(x) = x$ passes through all points with coordinates (a, a) .



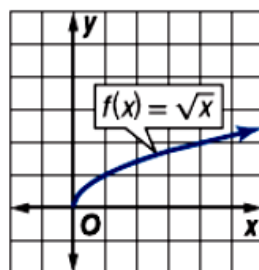
The **quadratic function** $f(x) = x^2$ has a U-shaped graph.



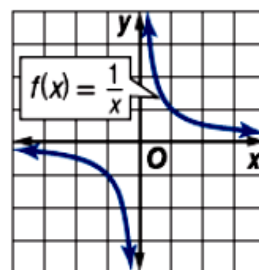
The **cubic function** $f(x) = x^3$ is symmetric about the origin.

**KeyConcept** Square Root and Reciprocal Parent Functions

The **square root function** has the form $f(x) = \sqrt{x}$.



The **reciprocal function** has the form $f(x) = \frac{1}{x}$.



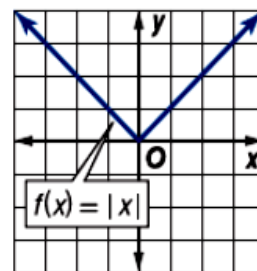
KeyConcept Absolute Value Parent Function

Words The **absolute value function**, denoted $f(x) = |x|$, is a V-shaped function defined as

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Examples $|-5| = 5$, $|0| = 0$, $|4| = 4$

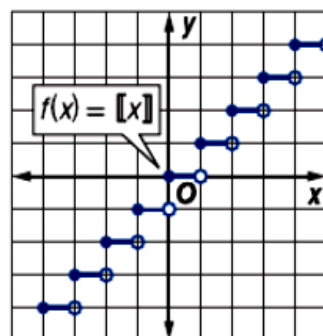
Model

**KeyConcept** Greatest Integer Parent Function

Words The **greatest integer function**, denoted $f(x) = \llbracket x \rrbracket$, is defined as the greatest integer less than or equal to x .

Examples $\llbracket -4 \rrbracket = -4$, $\llbracket -1.5 \rrbracket = -2$, $\llbracket \frac{1}{3} \rrbracket = 0$

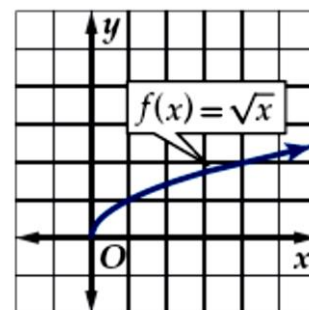
Model



Describe the following characteristics of the graph of the parent function $f(x) = \sqrt{x}$: domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing.

The graph of the square root function (Figure 1.5.1) has the following characteristics.

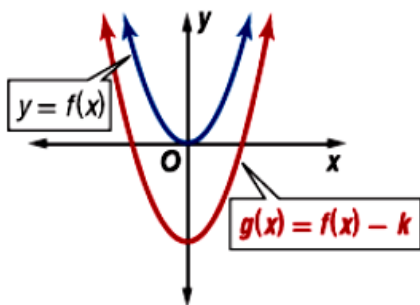
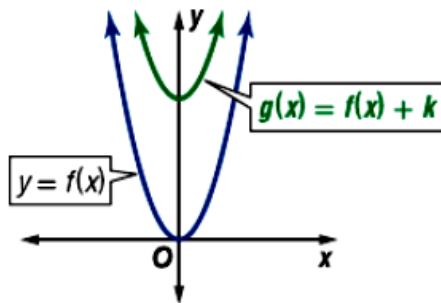
- The domain of the function is $[0, \infty)$, and the range is $[0, \infty)$.
- The graph has one intercept at $(0, 0)$.
- The graph has no symmetry. Therefore, $f(x)$ is neither odd nor even.
- The graph is continuous for all values in its domain.
- The graph begins at $x = 0$ and $\lim_{x \rightarrow \infty} f(x) = \infty$.
- The graph is increasing on the interval $(0, \infty)$.



KeyConcept Vertical and Horizontal Translations**Vertical Translations**

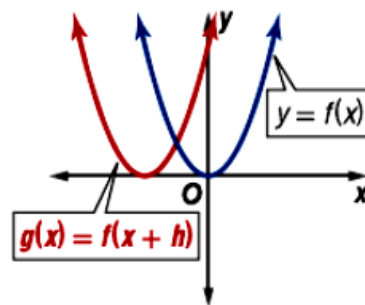
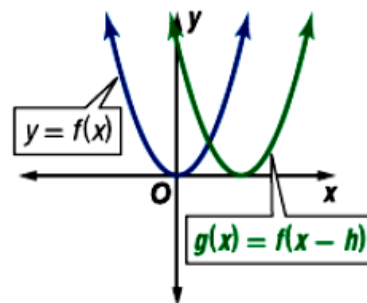
The graph of $g(x) = f(x) + k$ is the graph of $f(x)$ translated

- k units up when $k > 0$, and
- k units down when $k < 0$.

**Horizontal Translations**

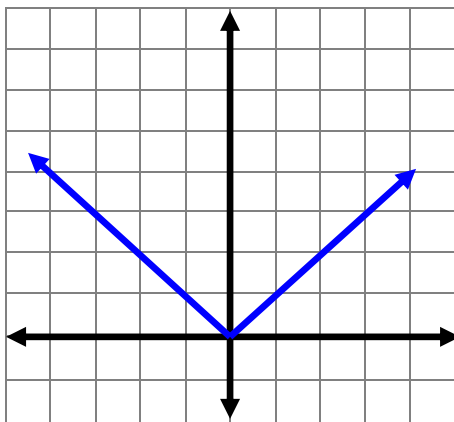
The graph of $g(x) = f(x - h)$ is the graph of $f(x)$ translated

- h units right when $h > 0$, and
- h units left when $h < 0$.

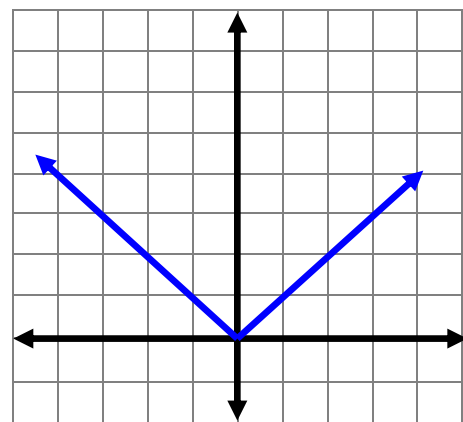


Use the graph of $f(x) = |x|$ to graph each function.

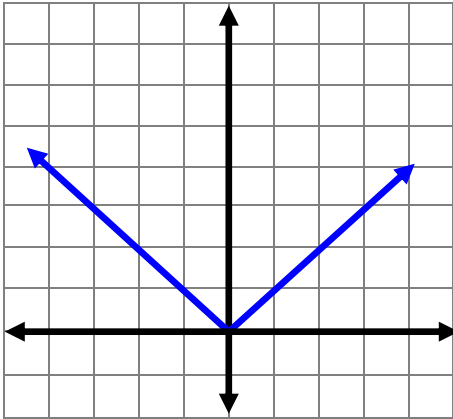
a. $g(x) = |x| + 4$



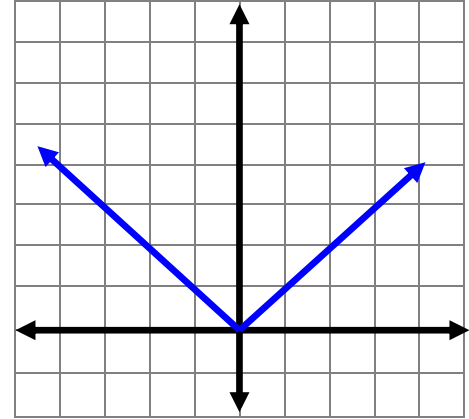
b. $g(x) = |x + 3|$



c. $g(x) = |x - 2| - 1$



$g(x) = |x - 2|$



Guided Practice Use the graph of $f(x) = x^3$ to graph each function.

2A. $h(x) = x^3 - 5$

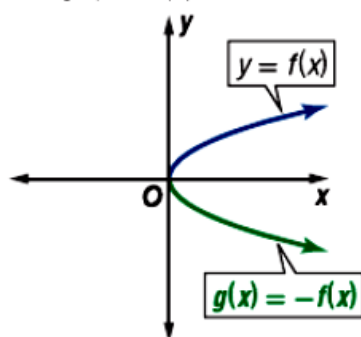
2B. $h(x) = (x - 3)^3$

2C. $h(x) = (x + 2)^3 + 4$

KeyConcept Reflections in the Coordinate Axes

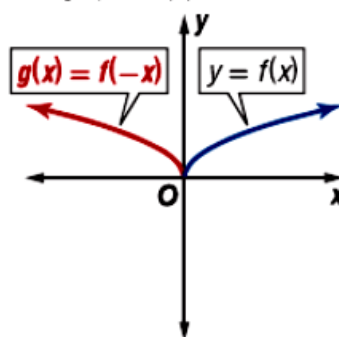
Reflection in x-axis

$g(x) = -f(x)$ is the graph of $f(x)$ **reflected in the x-axis**.

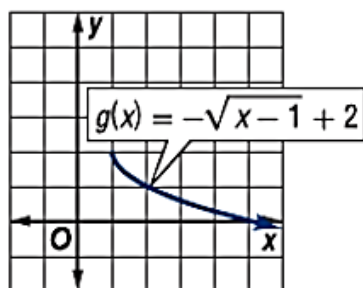


Reflection in y-axis

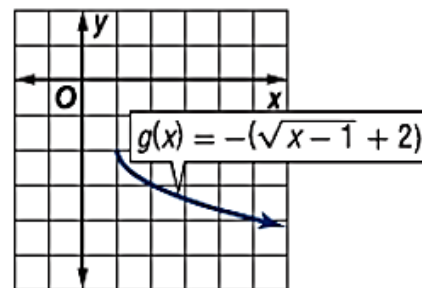
$g(x) = f(-x)$ is the graph of $f(x)$ **reflected in the y-axis**.



When writing an equation for a transformed function, be careful to indicate the transformations correctly. The graph of $g(x) = -\sqrt{x-1} + 2$ is different from the graph of $g(x) = -(\sqrt{x-1} + 2)$.

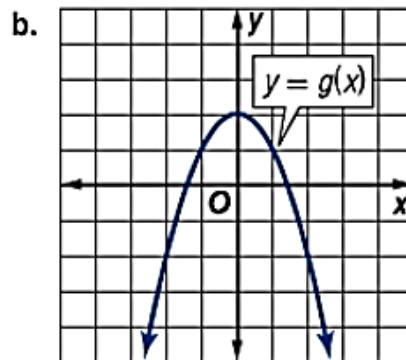
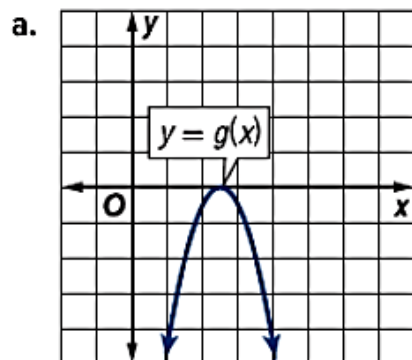


reflection of $f(x) = \sqrt{x}$ in the x-axis, then translated 1 unit to the right and 2 units up



translation of $f(x) = \sqrt{x}$ 1 unit to the right and 2 units up, then reflected in the x-axis

Describe how the graphs of $f(x) = x^2$ and $g(x)$ are related. Then write an equation for $g(x)$.

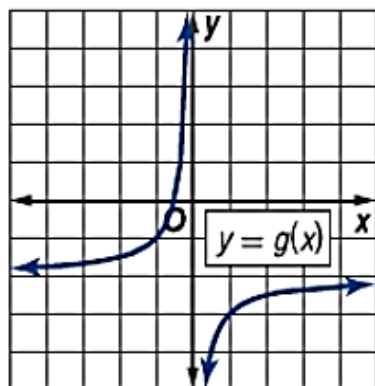


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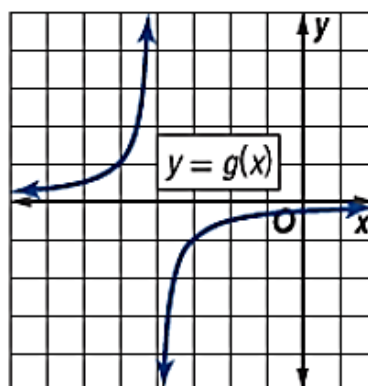
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Describe how the graphs of $f(x) = \frac{1}{x}$ and $g(x)$ are related. Then write an equation for $g(x)$.

3A.



3B.

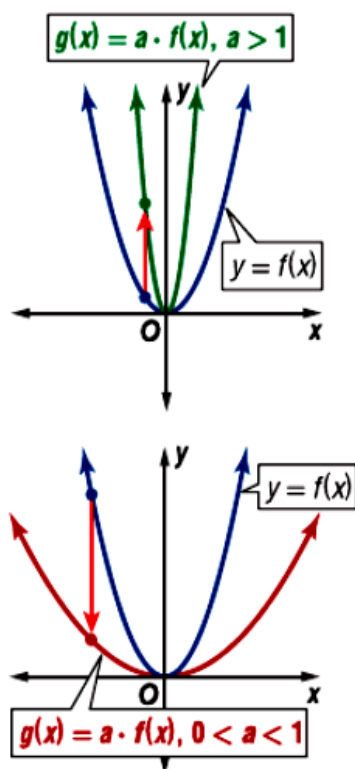


KeyConcept Vertical and Horizontal Translations

Vertical Dilations

If a is a positive real number, then $g(x) = a \cdot f(x)$, is

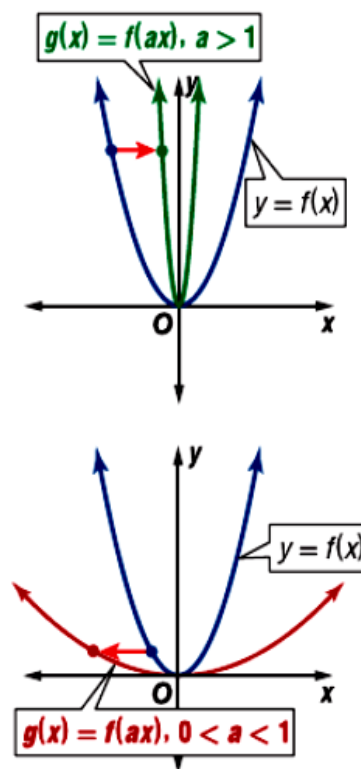
- the graph of $f(x)$ **expanded vertically**, if $a > 1$.
- the graph of $f(x)$ **compressed vertically**, if $0 < a < 1$.



Horizontal Dilations

If a is a positive real number, then $g(x) = f(ax)$, is

- the graph of $f(x)$ **compressed horizontally**, if $a > 1$.
- the graph of $f(x)$ **expanded horizontally**, if $0 < a < 1$.



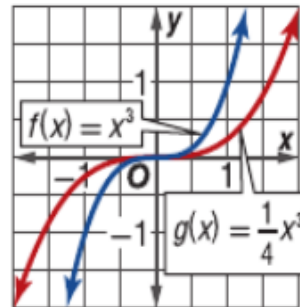
Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

a. $g(x) = \frac{1}{4}x^3$

.....

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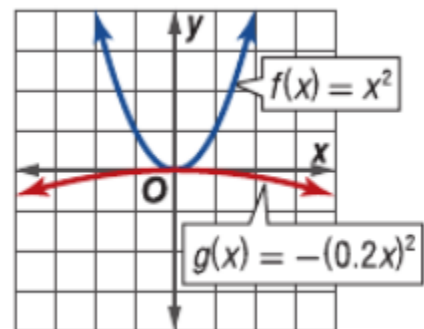


b. $g(x) = -(0.2x)^2$

.....

.....

.....



Guided Practice

4A. $g(x) = \lfloor x \rfloor - 4$

.....

.....

.....

4B. $g(x) = \frac{15}{x} + 3$

.....

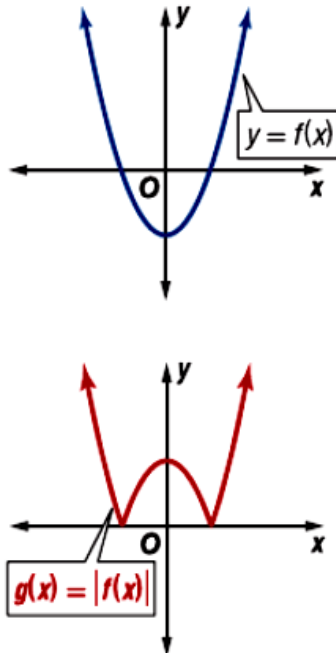
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KeyConcept Transformations with Absolute Value

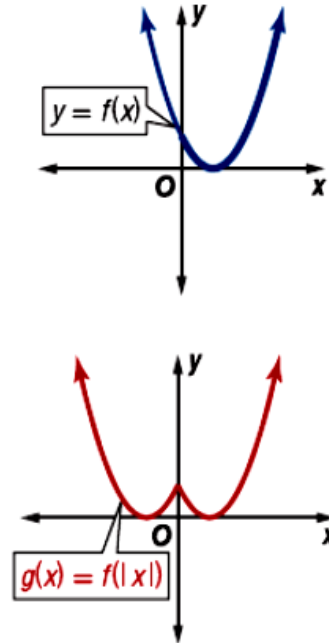
$$g(x) = |f(x)|$$

This transformation reflects any portion of the graph of $f(x)$ that is below the x -axis so that it is above the x -axis.



$$g(x) = f(|x|)$$

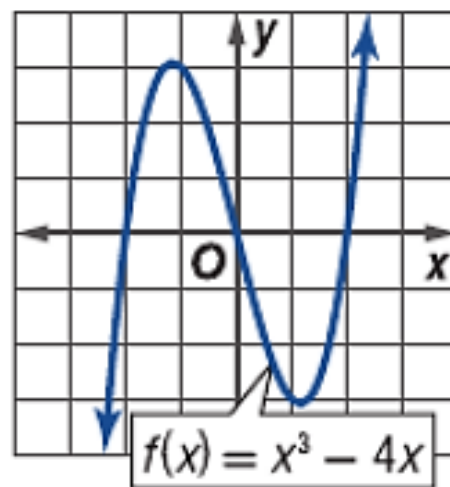
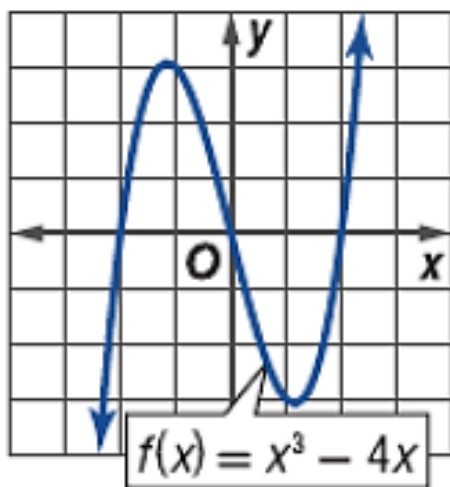
This transformation results in the portion of the graph of $f(x)$ that is to the left of the y -axis being replaced by a reflection of the portion to the right of the y -axis.



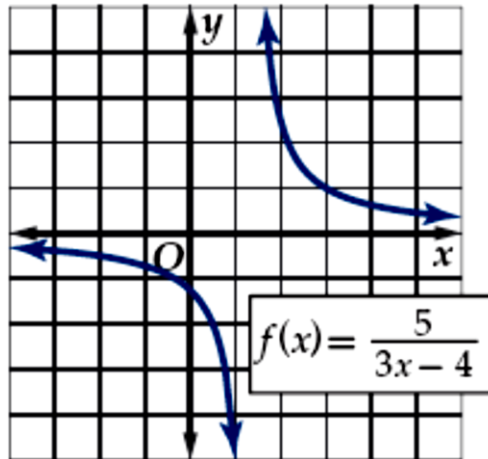
Use the graph of $f(x) = x^3 - 4x$ in Figure 1.5.6 to graph each function.

a. $g(x) = |f(x)|$

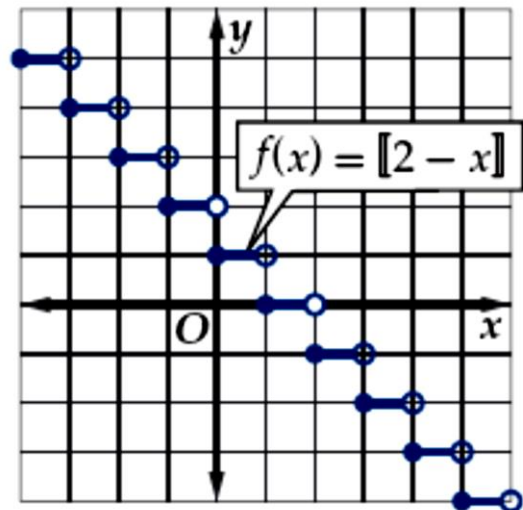
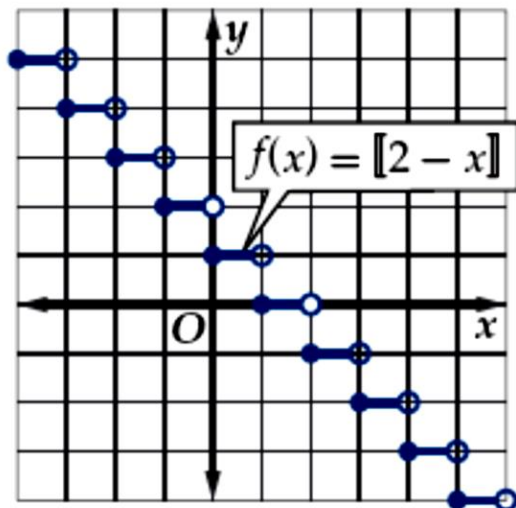
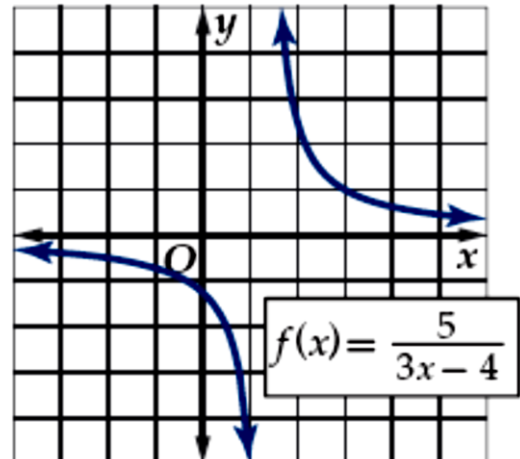
b. $h(x) = f(|x|)$



$$h(x) = f(|x|).$$



$$g(x) = |f(x)|$$



Which function is the parent function for $g(x) = -3|x + 9|$

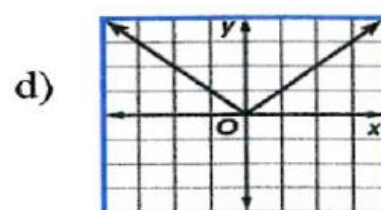
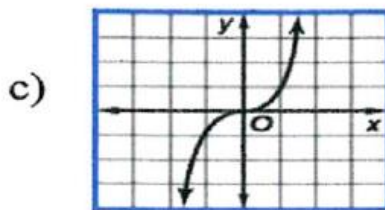
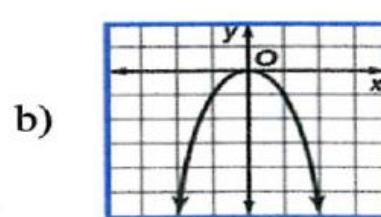
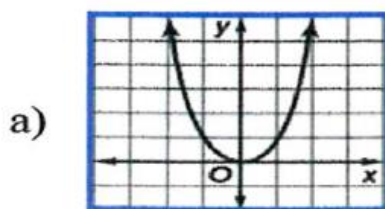
a) $f(x) = 3|x|$

b) $f(x) = |x + 9|$

c) $f(x) = 3|x + 9|$

d) $f(x) = |x|$

Which of the following represents the graph of $f(x) = |x^3|$?



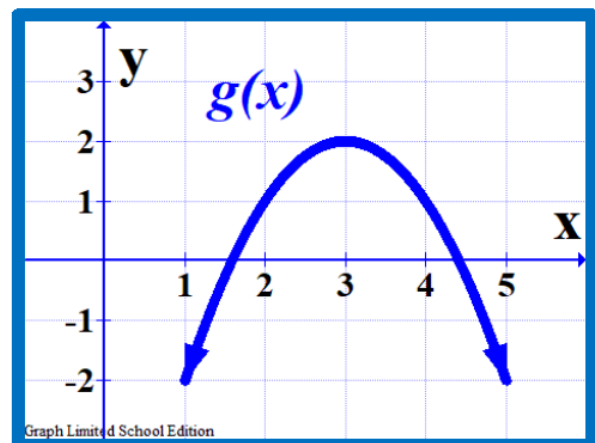
Write the function $g(x)$ it has translation of $f(x) = x^2$

a) $g(x) = (x - 3)^2 + 2$

b) $g(x) = (x - 2)^2 + 3$

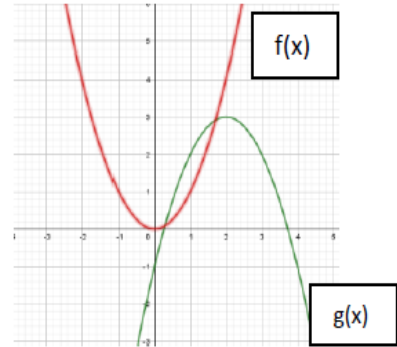
c) $g(x) = -(x - 3)^2 + 2$

d) $g(x) = -(x - 2)^2 + 3$



The equation of the function $g(x)$ in the below graph is:

- a. $g(x) = -(x - 2)^2 + 3$
- b. $g(x) = -(x - 3)^2 + 2$
- c. $g(x) = (x - 2)^2 + 3$
- d. $g(x) = -(x - 2)^2 + 5$



2024-2025

Grade 12 GEN

Lesson 1-6

Chapter 1



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Outcomes

Function Operations and Composition of Functions

11-6

1- Perform operations with functions.

2- Find compositions of functions

Key Concept Operations with Functions

Let f and g be two functions with intersecting domains. Then for all x -values in the intersection, the sum, product, difference, and quotient of f and g are new functions defined as follows.

Sum $(f + g)(x) = f(x) + g(x)$

Product $(f \cdot g)(x) = f(x) \cdot g(x)$

Difference $(f - g)(x) = f(x) - g(x)$

Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Given $f(x) = x^2 + 4x$, $g(x) = \sqrt{x + 2}$, and $h(x) = 3x - 5$, find each function and its domain.

a. $(f + g)(x)$

b. $(f - h)(x)$

.....

.....

c. $(f \cdot h)(x)$

d. $\left(\frac{h}{f}\right)(x)$

.....

.....

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

$f(x) = x - 4$, $g(x) = \sqrt{9 - x^2}$

.....

.....

Given $f(x) = 3x + 4$, $g(x) = x + 8$ and $h(x) = 3x - 5$
find each function and its domain

a. $(f + g)(x)$

b. $(f - h)(x)$

c. $(f \cdot h)(x)$

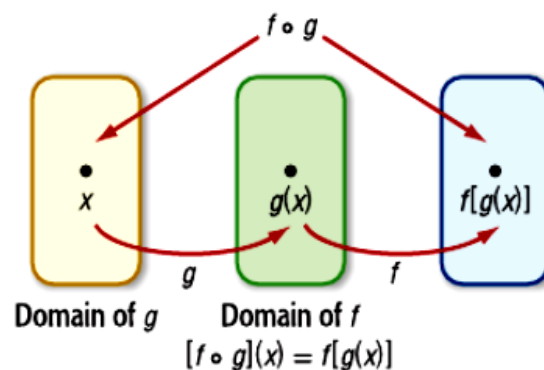
d. $\left(\frac{h}{f}\right)(x)$

KeyConcept Composition of Functions

The **composition** of function f with function g is defined by

$$[f \circ g](x) = f[g(x)].$$

The domain of $f \circ g$ includes all x -values in the domain of g that map to $g(x)$ -values in the domain of f as shown.



Given $f(x) = x^2 + 1$ and $g(x) = x - 4$, find each of the following.

a. $[f \circ g](x)$

.....

.....

.....

b. $[g \circ f](x)$

.....

.....

.....

c. $[f \circ g](2)$

.....

.....

Guided Practice

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](3)$.

$$f(x) = 3x + 1, g(x) = 5 - x^2$$

$$[f \circ g](x)$$

.....

.....

.....

$$[g \circ f](x)$$

.....

.....

.....

$$[f \circ g](3).$$

$$f(x) = 6x^2 - 4, g(x) = x + 2$$

$$[f \circ g](x)$$

$$[g \circ f](x)$$

$$[f \circ g](3).$$

Find a Composite Function with a Restricted Domain

Find $f \circ g$.

a. $f(x) = \frac{1}{x+1}, g(x) = x^2 - 9$

$$f(x) = x^2 - 2, g(x) = \sqrt{x - 3}$$

$$f(x) = \sqrt{x + 1}, g(x) = x^2 - 1$$

$$f(x) = \frac{5}{x}, g(x) = x^2 + x$$

Find two functions f and g such that $h(x) = [f \circ g](x)$. Neither function may be the identity function $f(x) = x$.

a. $h(x) = \sqrt{x^3 - 4}$

.....

.....

.....

b. $h(x) = 2x^2 + 20x + 50$

.....

.....

.....

$h(x) = x^2 - 2x + 1$

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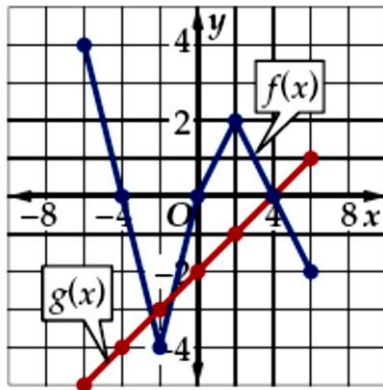
$h(x) = \frac{1}{x+7}$

.....

.....

.....

Use the graphs of $f(x)$ and $g(x)$ to find each function value.



$$(f + g)(2)$$

$$(f - g)(-6)$$

$$(f \cdot g)(4)$$

$$\left(\frac{f}{g}\right)(-2)$$

$$(f \circ g)(-4)$$

$$(g \circ f)(6)$$

Find $f \circ g$. $f(x) = \frac{1}{x+1}$ $g(x) = x^2 - 4$

a) $[f \circ g](x) = \frac{1}{x^2 - 3}$ for $x \neq \pm\sqrt{3}$

b) $[f \circ g](x) = \frac{2}{x^2 + 3}$

c) $[f \circ g](x) = |x|$

d) $[f \circ g](x) = x - 6$ for $x \geq -3$

Find $f \circ g$. $f(x) = \frac{2}{x-3}$ $g(x) = x^2 + 6$

a) $[f \circ g](x) = \frac{1}{x^2 - 3}$ for $x \neq \pm\sqrt{3}$

b) $[f \circ g](x) = \frac{2}{x^2 + 3}$

c) $[f \circ g](x) = |x|$

d) $[f \circ g](x) = x - 6$ for $x \geq -3$

Find $f \circ g$. $f(x) = \sqrt{x+4}$ $g(x) = x^2 - 4$

a) $[f \circ g](x) = \frac{1}{x^2 - 3}$ for $x \neq \pm\sqrt{3}$

b) $[f \circ g](x) = \frac{2}{x^2 + 3}$

c) $[f \circ g](x) = |x|$

d) $[f \circ g](x) = x - 6$ for $x \geq -3$

Find $f \circ g$. $f(x) = x^2 - 9$ $g(x) = \sqrt{x+3}$

a) $[f \circ g](x) = \frac{1}{x^2 - 3}$ for $x \neq \pm\sqrt{3}$

b) $[f \circ g](x) = \frac{2}{x^2 + 3}$

c) $[f \circ g](x) = |x|$

d) $[f \circ g](x) = x - 6$ for $x \geq -3$

Find $f \circ g$. $f(x) = \frac{5}{x}$ $g(x) = \sqrt{6-x}$

a) $[f \circ g](x) = \frac{5\sqrt{6-x}}{6-x}$ for $x < 6$

b) $[f \circ g](x) = \frac{-4\sqrt{x+8}}{x+8}$ for $x > -8$

c) $[f \circ g](x) = |x+2|$

d) $[f \circ g](x) = \sqrt{x^2+6}$

Find $f \circ g$. $f(x) = \sqrt{x-2}$ $g(x) = x^2+8$

a) $[f \circ g](x) = \frac{5\sqrt{6-x}}{6-x}$ for $x < 6$

b) $[f \circ g](x) = \frac{-4\sqrt{x+8}}{x+8}$ for $x > -8$

c) $[f \circ g](x) = |x+2|$

d) $[f \circ g](x) = \sqrt{x^2+6}$

Find $f \circ g$. $f(x) = \sqrt{x+5}$ $g(x) = x^2+4x-1$

a) $[f \circ g](x) = \frac{5\sqrt{6-x}}{6-x}$ for $x < 6$

b) $[f \circ g](x) = \frac{-4\sqrt{x+8}}{x+8}$ for $x > -8$

c) $[f \circ g](x) = |x+2|$

d) $[f \circ g](x) = \sqrt{x^2+6}$

2024-2025

Grade 12 GEN

Lesson 1-7

Chapter 1



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Outcomes

Inverse Relations and Functions

11-7

1-Use the horizontal line test to determine inverse functions.

2- Find inverse functions algebraically and graphically.

Table A

Tickets	1	2	3	4	6
Cost (\$)	2	4	6	8	10

Table B

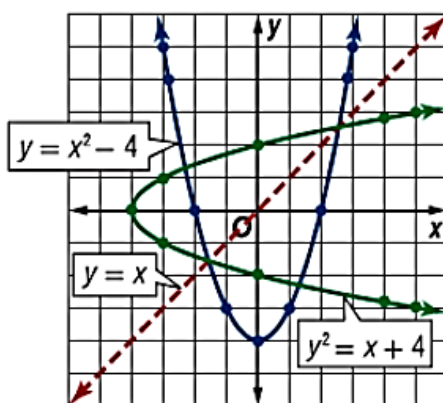
Money Spent (\$)	2	4	6	8	10
Tickets	1	2	3	4	6

1 Inverse Functions The relation shown in Table A is the *inverse relation* of the relation shown in Table B. **Inverse relations** exist if and only if one relation contains (b, a) whenever the other relation contains (a, b) . When a relation is expressed as an equation, its inverse relation can be found by interchanging the independent and dependent variables. Consider the following.

Relation

$$y = x^2 - 4$$

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5



Inverse Relation

$$x = y^2 - 4 \text{ or } y^2 = x + 4$$

x	y
5	-3
0	-2
-3	-1
-4	0
-3	1
0	2
5	3

KeyConcept Horizontal Line Test

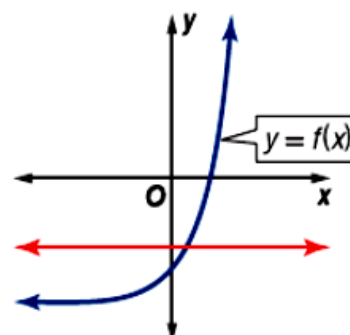
Words

A function f has an inverse function f^{-1} if and only if each horizontal line intersects the graph of the function in at most one point.

Example

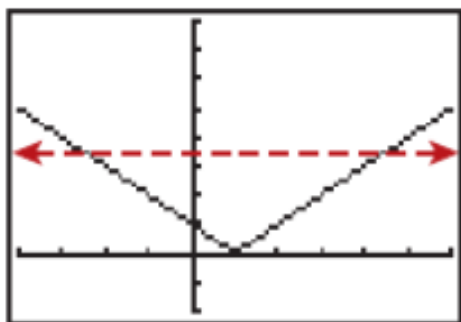
Since no horizontal line intersects the graph of f more than once, the inverse function f^{-1} exists.

Model

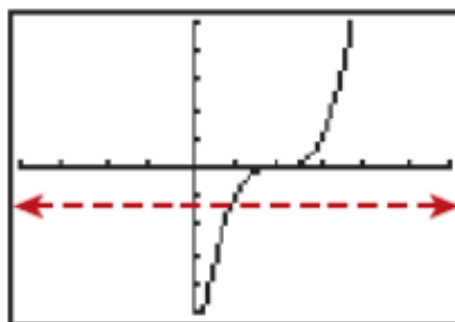


Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

a. $f(x) = |x - 1|$

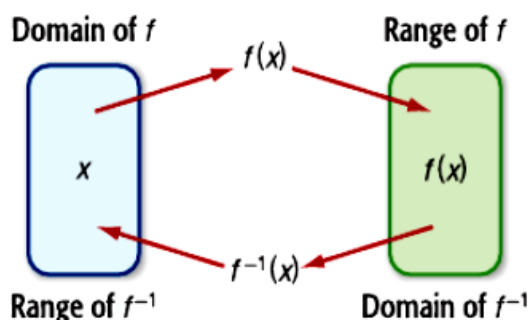


b. $g(x) = x^3 - 6x^2 + 12x - 8$



2 Find Inverse Functions If a function passes the horizontal line test, then it is said to be **one-to-one**, because no x -value is matched with more than one y -value and no y -value is matched with more than one x -value.

If a function f is one-to-one, it has an inverse function f^{-1} such that the domain of f is equal to the range of f^{-1} , and the range of f is equal to the domain of f^{-1} .



KeyConcept Finding an Inverse Function

- Step 1** Determine whether the function has an inverse by checking to see if it is one-to-one using the horizontal line test.
- Step 2** In the equation for $f(x)$, replace $f(x)$ with y and then interchange x and y .
- Step 3** Solve for y and then replace y with $f^{-1}(x)$ in the new equation.
- Step 4** State any restrictions on the domain of f^{-1} . Then show that the domain of f is equal to the range of f^{-1} and the range of f is equal the domain of f^{-1} .

Determine whether f has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

a. $f(x) = \frac{x-1}{x+2}$

$$f(x) = \sqrt{x-4}$$

$$f(x) = -16 + x^3$$

$$f(x) = \frac{x+7}{x}$$

KeyConcept Compositions of Inverse Functions

Two functions, f and g , are inverse functions if and only if

- $f[g(x)] = x$ for every x in the domain of $g(x)$ and
- $g[f(x)] = x$ for every x in the domain of $f(x)$.

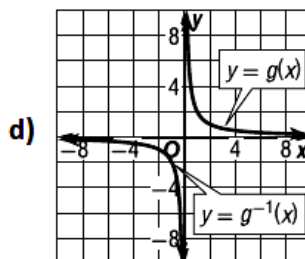
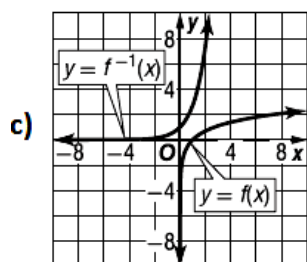
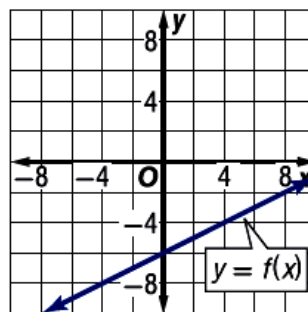
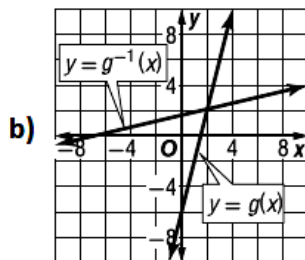
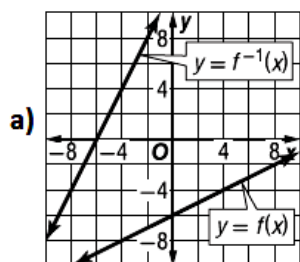
Show that $f(x) = \frac{6}{x-4}$ and $g(x) = \frac{6}{x} + 4$ are inverse functions.

Show that f and g are inverse functions.

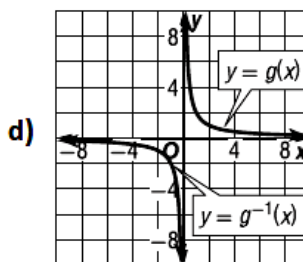
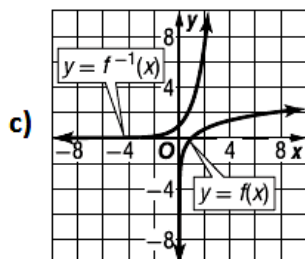
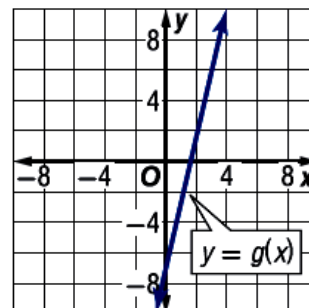
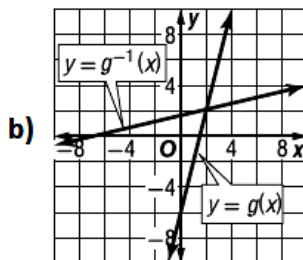
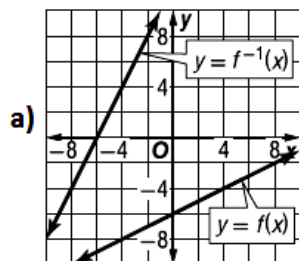
$$f(x) = 18 - 3x, g(x) = 6 - \frac{x}{3}$$

$$f(x) = x^2 + 10, x \geq 0; g(x) = \sqrt{x - 10}$$

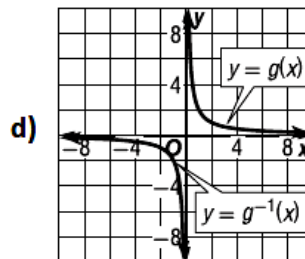
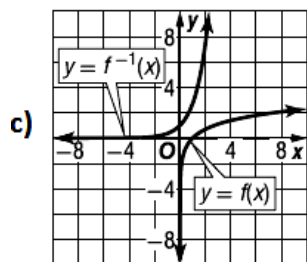
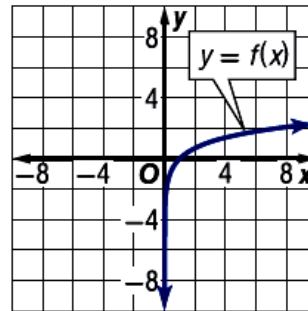
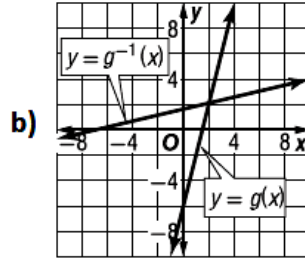
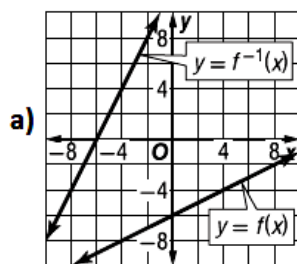
Use the graph of each function to graph its inverse function.



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