

End of Term 1 Revision – Mathematics

Grade 08 / All

Module 1

- 1.1 Powers and Exponents
- 1.2 Multiply and divide monomials
- 1.3 Powers of Monomials
- 1.4 Zero and Negative Exponents
- 1.5 Scientific Notation
- 1.6 Computing with Scientific Notation

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- 2.1 Terminating and Repeating Decimals
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Module 1

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- 1.2 Multiply and divide monomials
- 1.3 Powers of Monomials
- 1.4 Zero and Negative Exponents
- 1.5 Scientific Notation
- 1.6 Computing with Scientific Notation

Hints and Notes

Base and Exponent

a^x : $a = \text{Base}$, $x = \text{Exponent}$

Rules of exponents

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}, \text{ where } a \neq 0$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m \cdot b^m$$

$$a^0 = 1$$

$$a^{-x} = \frac{1}{a^x}$$

Exercises

Power of a Power Rule

1. Simplify: $(x^3)^2$

$$(x^3)^2 = x^{3 \cdot 2} = x^6$$

2. Simplify: $(a^4)^3$

$$(a^4)^3 = a^{4 \cdot 3} = a^{12}$$

3. Simplify: $(m^5)^2$

$$(m^5)^2 = m^{5 \cdot 2} = m^{10}$$

Multiply Powers (same base)

1. Simplify: $6^2 \cdot 6^4$

$$6^2 \cdot 6^4 = 6^{2+4} = 6^6$$

2. Simplify: $b^3 \cdot b^5$

$$b^3 \cdot b^5 = b^{3+5} = b^8$$

3. Simplify: $x^3 \cdot x^2$

$$x^3 \cdot x^2 = x^{3+2} = x^5$$

4. Simplify: $c^4 \cdot c$

$$c^4 \cdot c = c^{4+1} = c^5$$

5. Simplify: $t^6 \cdot t^3$

$$t^6 \cdot t^3 = t^{6+3} = t^9$$

Divide Powers (same base)

1. Simplify: $\frac{p^9}{p^4}$

$$\frac{p^9}{p^4} = p^{9-4} = p^5$$

2. Simplify: $\frac{w^6}{w^5}$

$$\frac{w^6}{w^5} = w^{6-5} = w^1 = w$$

3. Simplify: $\frac{n^5}{n}$

$$\frac{n^5}{n} = n^{5-1} = n^4$$

Power of a Product

1. Simplify: $(2x^2)^2$

$$(2x^2)^2 = 2^2 x^{2 \cdot 2} = 4x^4$$

2. Simplify: $(3y^3)^2$

$$(3y^3)^2 = 3^2 y^{3 \cdot 2} = 9y^6$$

3. Simplify: $(5a)^3$

$$(5a)^3 = 5^3 a^3 = 125a^3$$

Negative Exponents

1. Simplify: $\frac{k^2}{k^5}$

$$\frac{k^2}{k^5} = k^{2-5} = k^{-3} = \frac{1}{k^3}$$

2. Simplify: $\frac{z^3}{z^7}$

$$\frac{z^3}{z^7} = z^{3-7} = z^{-4} = \frac{1}{z^4}$$

3. Simplify: $\frac{u^3}{u^{-7}}$

$$\frac{u^3}{u^{-7}} = u^{3-(-7)} = u^{10}$$

Zero Exponent Rule

1. Simplify: m^0

1

2. Simplify: $(5x)^0$

1

3. Simplify: $1 - (2t)^0$

$$1 - 1 = 0$$

Power of a Negative Exponent

1. Simplify: $(x^{-1})^3$

$$(x^{-1})^3 = x^{-3} = \frac{1}{x^3}$$

2. Simplify: $(m^{-2})^2$

$$(m^{-2})^2 = m^{-4} = \frac{1}{m^4}$$

3. Simplify: $(q^{-3})^2$

$$(q^{-3})^2 = q^{-6} = \frac{1}{q^6}$$

Zero Exponent & Negative Exponent

1. Simplify: $\frac{3a^0}{5b^{-3}}$

$$\frac{3a^0}{5b^{-3}} = \frac{3 \cdot 1}{5 \cdot b^{-3}} = \frac{3}{5} \cdot b^3 = \frac{3b^3}{5}$$

2. Simplify: $5x^{-3} \cdot (4y^0)$

$$5x^{-3} \cdot (4y^0) = 5x^{-3} \cdot 4 \cdot 1 = 20x^{-3} = \frac{20}{x^3}$$

3. Simplify $5y^{-3} \cdot y^3$

$$5y^{-3} \cdot y^3 = 5y^{-3+3} = 5y^0 = 5$$

Expressing as positive powers

1. Express b^{-4} using a positive exponent.

$$b^{-4} = \frac{1}{b^4}$$

2. Express y^{-7} using a positive exponent.

$$y^{-7} = \frac{1}{y^7}$$

3. Express $\frac{1}{m^{-3}}$ using a positive exponent.

$$m^{-3} = \frac{1}{m^3}$$

Expressing as positive powers

1. Write $\frac{1}{d^8}$ using a negative exponent.

$$\frac{1}{d^8} = d^{-8}$$

2. Write $\frac{1}{k^2}$ using a negative exponent.

$$\frac{1}{k^2} = k^{-2}$$

3. Write $\frac{1}{w^6}$ using a negative exponent.

$$\frac{1}{w^6} = w^{-6}$$

Writing a product using exponents

1. Write $(11)(11)(11)(11)(11) \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right)$ using exponents.

$$11^5 \left(-\frac{2}{3}\right)^2$$

2. Write $r \cdot r \cdot r \cdot r \cdot r \cdot r \left(\frac{1}{4}\right) \left(\frac{1}{4}\right) \left(\frac{1}{4}\right)$ using exponents.

$$r^5 \left(\frac{1}{4}\right)^3$$

Evaluating expressions with exponents

1. Evaluate $y^3 + (x^2 - 1)$ if $x = 2, y = 4$

$$4^3 + (2^2 - 1) = 64 + (4 - 1) = 64 + 3 = 67$$

2. Evaluate $a^4 - b^3$ if $a = 2, b = \frac{1}{2}$

$$2^4 - \left(\frac{1}{2}\right)^3 = 16 - \frac{1}{8} = \frac{128}{8} - \frac{1}{8} = \frac{127}{8}$$

3. Evaluate $p^2 + q^3$ if $p = 4, q = \frac{1}{4}$.

$$4^2 + \left(\frac{1}{4}\right)^3 = 16 + \frac{1}{64} = \frac{1025}{64}$$

Write in standard form/ ordinary number

1. Write 3.4×10^5 in ordinary form.

$$3.4 \times 10^5 = 340,000$$

2. Write 7.01×10^3 in ordinary form.

$$7.01 \times 10^3 = 7,010$$

3. Write 9.8×10^6 in ordinary form.

$$9.8 \times 10^6 = 9,800,000$$

Write in scientific notation (Large numbers)

1. Write 6,740,000 in scientific notation.

$$6,740,000 = 6.74 \times 10^6$$

2. Write 21,900,000 in scientific notation.

$$21,900,000 = 2.19 \times 10^7$$

3. Write 902,000,000 in scientific notation.

$$902,000,000 = 9.02 \times 10^8$$

Write in scientific notation (Small numbers)

1. Write 0.000042 in scientific notation.

$$0.000042 = 4.2 \times 10^{-5}$$

2. Write 0.00089 in scientific notation.

$$0.00089 = 8.9 \times 10^{-4}$$

3. Write 0.0000071 in scientific notation.

$$0.0000071 = 7.1 \times 10^{-6}$$

Word Problems

1. The length of Sheikh Zayed Road is about $3^3 \cdot 5^2 \cdot 2$ meters. Yas Island's main route is about $2^4 \cdot 3^3 \cdot 4$ meters. How much longer is Sheikh Zayed Road than the Yas Island route?

Sheikh Zayed Road : $3^3 \cdot 5^2 \cdot 2$

$$3^3 = 27$$

$$5^2 = 25$$

$$\text{So: } 27 \cdot 25 \cdot 2 = 1350 \text{ meters}$$

Yas Island Route $2^4 \cdot 3^3 \cdot 4$

$$2^4 = 16$$

$$3^3 = 27$$

$$\text{So: } 16 \cdot 27 \cdot 4 = 1728 \text{ meters}$$

Now, Find the difference

$$\text{Difference} = 1728 - 1350 = 378 \text{ meters}$$

Sheikh Zayed Road is shorter by 378 meters shorter

2. The Dubai Water Canal walkway is about $4^2 \cdot 6^2 \cdot 3$ meters.

The Sharjah Corniche walkway is $2^5 \cdot 3^2 \cdot 5$ meters.

How much longer is Dubai Water Canal walkway?

3. Dubai's coastline is about 72 miles. Ajman's coastline is about 10 miles.

About how many times longer is Dubai's coastline?

Ratio: $\frac{72}{10} = 7.2$

Dubai's coastline is about 7.2 times longer.

4. Abu Dhabi Corniche is 5 miles long.

Al Khan Beach walkway in Sharjah is 1 mile.

How many times longer is Abu Dhabi Corniche walkway?

Abu Dhabi Corniche vs Al Khan Beach

Ratio: $\frac{5}{1} = 5$

Abu Dhabi Corniche is 5 times longer.

5. The Palm Jumeirah trunk is 2 miles long. Dubai Marina Walk is 7 miles long.

How many times longer is Dubai Marina Walk?

Ratio: $\frac{7}{2} = 3.5$

Dubai Marina Walk is 3.5 times longer.

6. Dubai Frame height is 1.5×10^2 meters.

Burj Al Arab height is 3.21×10^2 meters.

How many times taller is Burj Al Arab?

Dubai Frame: $1.5 \times 10^2 = 150 \text{ m}$

Burj Al Arab: $3.21 \times 10^2 = 321 \text{ m}$

Ratio: $\frac{321}{150} \approx 2.14 \rightarrow$ **Burj Al Arab is about 2.1 times taller.**

7. A mall signboard is 4.5×10^1 meters long.

A shop sign in the mall is 5×10^0 meters.

How many times longer is the mall signboard?

Mall sign: $4.5 \times 10^1 = 45$ m

Shop sign: $5 \times 10^0 = 5$ m

Ratio: $\frac{45}{5} = 9 \rightarrow$ The mall signboard is 9 times longer.

8. A UAE supercomputer runs at 10^{11} operations per second.

A research lab has a computer 10^4 times faster.

A school computer runs at 10^{13} .

Which is fastest?

UAE supercomputer: 10^{11}

Lab computer: 10^4 times faster $\rightarrow 10^{11} \cdot 10^4 = 10^{15}$

School computer: 10^{13}

Fastest: Research lab computer at 10^{15} operations per second.

9. A Dubai tech firm has a computer running at 10^{10} .

Another in Abu Dhabi is 10^3 times faster.

A university computer runs at 10^{14} .

Which is fastest?

Dubai: 10^{10}

Abu Dhabi: 10^3 times faster $\rightarrow 10^{10} \cdot 10^3 = 10^{13}$

University: 10^{14}

Fastest: University computer at 10^{14} ops/sec.

10. A gaming center in Sharjah has a device running at 10^{12} .

A robotics lab uses a machine 10^5 times faster.

Another device runs at 10^{16} .

Which is fastest?

Gaming: 10^{12}

Robotics lab: 10^5 times faster $\rightarrow 10^{12} \cdot 10^5 = 10^{17}$

Another device: 10^{16}

Fastest: Robotics lab machine at 10^{17} .

11. The tiny sensor is 10^{-14} km wide.

The smartwatch screen is 10^{-3} km wide.

How many times larger is the screen?

Sensor: 10^{-14} km

Screen: 10^{-3} km

Ratio: $\frac{10^{-3}}{10^{-14}} = 10^{-3-(-14)} = 10^{11} \rightarrow$ The screen is 10^{11} times larger.

12. A dust particle is 10^{-8} km.

A grain of sand is 10^{-4} km.

How many times larger is the grain of sand?

Dust: 10^{-8} km

Sand: 10^{-4} km

Ratio: $\frac{10^{-4}}{10^{-8}} = 10^4 \rightarrow$ The grain of sand is 10,000 times larger.

13. A micro-LED is 10^{-12} km.

A phone lens opening is 10^{-4} km.

How many times larger is the phone lens?

Micro-LED: 10^{-12} km

Lens: 10^{-4} km

Ratio: $\frac{10^{-4}}{10^{-12}} = 10^8 \rightarrow$ The phone lens opening is 10^8 (100,000,000) times larger.

14. A drone takes 4.2×10^7 seconds to map Dubai.

Is it better to report this in seconds or years? Explain.

Drone: 4.2×10^7 seconds

1 year $\approx 3.15 \times 10^7$ s

$\frac{4.2 \times 10^7}{3.15 \times 10^7} \approx 1.33$ years \rightarrow Better to report in years (it's about 1.3 years).

15. A ship takes 3.5×10^8 seconds to travel from Abu Dhabi to India.

Should you report this in seconds or years? Explain.

Ship: 3.5×10^8 seconds

$\frac{3.5 \times 10^8}{3.15 \times 10^7} \approx 11.1$ years \rightarrow Definitely better in years (about 11 years).

16. A robot in Expo City will run for 6.0×10^6 seconds.

Are seconds or months a better unit? Explain.

Robot in Expo City: 6.0×10^6 seconds

1 month (≈ 30 days) $\approx 2.6 \times 10^6$ s

$\frac{6.0 \times 10^6}{2.6 \times 10^6} \approx 2.3$ months \rightarrow Better to describe in months (about 2-3 months).

17. Mariam said $6.1 \times 10^2 < 5.9 \times 10^1$ because $6.1 > 5.9$

Explain and correct.

She said: $6.1 \times 10^2 < 5.9 \times 10^1$ because $6.1 > 5.9$.

Correct method:

Compare exponents first: 10^2 vs 10^1 .

$10^2 = 100$, $10^1 = 10$, so any number $\times 10^2$ is larger.

$$6.1 \times 10^2 = 610$$

$$5.9 \times 10^1 = 59$$

So $6.1 \times 10^2 > 5.9 \times 10^1$.

Her mistake: she only compared the decimal parts, not the powers of 10.

18. Salem said $3.8 \times 10^4 > 9.1 \times 10^6$ because 3.8 is less than 9.1.

Explain and correct.

He said: $3.8 \times 10^4 > 9.1 \times 10^6$ because 3.8 is less than 9.1.

Compute:

$$3.8 \times 10^4 = 38,000$$

$$9.1 \times 10^6 = 9,100,000$$

So actually $3.8 \times 10^4 < 9.1 \times 10^6$.

His mistake: again, ignoring the exponents.

19. Use the table information to answer the questions.

Object	Diameter (m)
Desert Sand Grain	10^{-4}
Fog Droplet	10^{-6}

- How many times larger is the sand grain than the fog droplet?
- Rewrite both diameters in ordinary form.
- Explain why the sand grain is larger using powers of 10.

Sand grain: 10^{-4} m

Fog droplet: 10^{-6} m

a) Ratio: $\frac{10^{-4}}{10^{-6}} = 10^2 = 100 \rightarrow$ The sand grain is 100 times larger.

b) Ordinary form:

$$10^{-4} = 0.0001 \text{ m}$$

$$10^{-6} = 0.000001 \text{ m}$$

c) Explanation:

The exponent -4 is greater than -6, so 10^{-4} is a larger number than 10^{-6} . That's why the sand grain is bigger by a factor of $10^2 = 100$.

20. The table shows how many steps three students walked inside different UAE malls.

Student	Mall	Steps
Ali	Dubai Mall	3.2×10^4
Sara	Abu Dhabi Mall	1.5×10^4
Khalid	City Centre Sharjah	2.8×10^4

- Who walked the most steps?
- How many more steps did Ali walk than Sara?
- Write all step counts in ordinary numbers.

Ali: 3.2×10^4

Sara: 1.5×10^4

Khalid: 2.8×10^4

a) Largest: Ali (3.2×10^4 steps).

b) How many more than Sara?

$(3.2 - 1.5) \times 10^4 = 1.7 \times 10^4 = 17,000$ steps.

c) Ordinary numbers:

Ali: $3.2 \times 10^4 = 32,000$

Sara: $1.5 \times 10^4 = 15,000$

Khalid: $2.8 \times 10^4 = 28,000$

21. The population of Florida was recently recorded as 20,612,439 people.

Write an estimation in scientific notation for the population.

$20,612,439 \approx 2.06 \times 10^7$ or to one decimal 2.1×10^7 .

A reasonable estimation in scientific notation: 2.1×10^7 people.

22. There are about 3×10^{11} stars in our galaxy and about 100 billion galaxies in the observable universe. Suppose every galaxy has as many stars as ours. How many stars are in the observable universe? Write in scientific notation

Stars per galaxy: 3×10^{11}

Number of galaxies: about 100 billion = 1×10^{11}

Total stars: $(3 \times 10^{11}) \cdot (1 \times 10^{11}) = 3 \times 10^{22} \rightarrow 3 \times 10^{22}$ stars (approx).

Module 2

2.1 Terminating and Repeating Decimals

2.2 Roots

2.3 Real Numbers

2.4 Estimate Irrational Numbers

2.5 Compare and Order Real Numbers

Exercises

Writing fractions and mixed numbers as decimals.

1. Convert $\frac{7}{20}$ to decimal.

$$\frac{7}{20} = \frac{35}{100} = 0.35$$

2. Convert $3\frac{5}{8}$ to decimal.

$$\frac{5}{8} = 0.625 \Rightarrow 3 + 0.625 = 3.625$$

3. Convert $\frac{19}{50}$ to decimal.

$$\frac{19}{50} = \frac{38}{100} = 0.38$$

Write terminating decimals as fractions

1. Write 0.125 as a fraction in simplest form.

$$0.125 = \frac{125}{1000} = \frac{1}{8}$$

2. Write 2.4 as a fraction in simplest form.

$$2.4 = \frac{24}{10} = \frac{12}{5}$$

3. Write 0.0625 as a simplified fraction.

$$0.0625 = \frac{625}{10000}$$

$$625 = 5^4, 10000 = (10)^4 = (2 \cdot 5)^4 = 2^4 \cdot 5^4$$

$$\frac{625}{10000} = \frac{5^4}{2^4 \cdot 5^4} = \frac{1}{2^4} = \frac{1}{16}$$

Writing repeating decimals as fractions

1. Write $0.\overline{8}$ as a fraction in simplest form.

$$\text{Let } x = 0.\overline{8}$$

$$10x = 8.\overline{8}$$

$$10x - x = 8.\overline{8} - 0.\overline{8} = 8$$

$$9x = 8 \Rightarrow x = \frac{8}{9}$$

2. Write $0.7\overline{6}$ as a fraction in simplest form.

$$\text{Let } x = 0.7\overline{6} = 0.767676 \dots$$

$$100x = 76.\overline{76}$$

$$100x - x = 76.\overline{76} - 0.\overline{76} = 76$$

$$99x = 76 \Rightarrow x = \frac{76}{99}$$

3. Write $2.\overline{45}$ as a fraction in simplest form.

$$\text{Let } x = 2.\overline{45} = 2.454545 \dots$$

$$100x = 245.\overline{45}$$

$$100x - x = 245.\overline{45} - 2.\overline{45} = 243$$

$$99x = 243 \Rightarrow x = \frac{243}{99} = \frac{27}{11}$$

4. Write $2.\overline{513}$ as a fraction in simplest form.

$$\text{Let } x = 2.\overline{513} = 2.513513513 \dots$$

$$1000x = 2513.\overline{513}$$

$$1000x - x = 2513.\overline{513} - 2.\overline{513} = 2511$$

$$999x = 2511 \Rightarrow x = \frac{2511}{999} = \frac{93}{37}$$

Finding square roots of perfect squares

1. Find $\sqrt{144}$

12

2. Find $\sqrt{361}$

19

3. Find $\sqrt{0.81}$

$$0.81 = \frac{81}{100}$$

$$\sqrt{0.81} = \sqrt{\frac{81}{100}} = \frac{\sqrt{81}}{\sqrt{100}} = \frac{9}{10} = 0.9$$

4. Find $\sqrt{0.0121}$

$$0.0121 = \frac{121}{10000}$$

$$\sqrt{0.0121} = \sqrt{\frac{121}{10000}} = \frac{11}{100} = 0.11$$

Finding cubic roots of perfect cubes

1. Find $\sqrt[3]{8000}$

$$8000 = 20^3 \Rightarrow \sqrt[3]{8000} = 20$$

2. Find $\sqrt[3]{1331}$

$$11^3 = 1331 \Rightarrow \sqrt[3]{1331} = 11$$

3. Find $\sqrt[3]{0.008}$

$$0.008 =$$

$$\sqrt[3]{0.008} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = \frac{1}{5}$$

4. Find $\sqrt[3]{0.125}$

$$0.125 = \frac{1}{8} = \frac{1}{2^3}$$

$$\sqrt[3]{0.125} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} = 0.5$$

Estimate Square Root to the Nearest Integer

1. Estimate $\sqrt{50}$ to the nearest integer.

$$49 < 50 < 64$$

50 is closer to 49

So, $\sqrt{50}$ is closer to $\sqrt{49}$

$$\sqrt{50} \approx 7$$

2. Estimate $\sqrt{92}$ to the nearest integer.

$$81 < 92 < 100$$

92 is closer to 100

So, $\sqrt{92}$ is closer to $\sqrt{100}$

$$\sqrt{92} \approx 10$$

3. Estimate $\sqrt{130}$ to the nearest integer.

$$121 < 130 < 144$$

130 is closer to 121

So, $\sqrt{130}$ is closer to $\sqrt{121}$

$$\sqrt{130} \approx 11$$

4. Estimate $\sqrt{301}$ to the nearest integer.

$$289 < 301 < 324$$

301 is closer to 289

So, $\sqrt{301}$ is closer to $\sqrt{289}$

$$\sqrt{301} \approx 17$$

Estimate Square Root to the Nearest tenth

1. Estimate $\sqrt{28}$ to the nearest tenth.

5.3

2. Estimate $\sqrt{71}$ to the nearest tenth.

8.4

3. Estimate $\sqrt{119}$ to the nearest tenth.

10.9

4. Estimate $\sqrt{228}$ to the nearest tenth.

15.1

Estimate Cube Root to the Nearest Integer

1. Estimate $\sqrt[3]{11}$ to the nearest integer.

2

2. Estimate $\sqrt[3]{101}$ to the nearest integer.

5

3. Estimate $\sqrt[3]{199}$ to the nearest integer.

6

4. Estimate $\sqrt[3]{1100}$ to the nearest integer.

10

Perfect Square roots word problems

1. A row of **nine equal-sized** square tiles is arranged side by side to form a long rectangular strip on a classroom wall. The total area of the strip is **225** square centimeters. What is the length of the strip?

Let each square tile have side length s cm.

Area of one square = s^2 .

There are 9 tiles, so total area:

$$9s^2 = 225$$

$$s^2 = \frac{225}{9} = 25 \Rightarrow s = 5 \text{ cm}$$

The tiles are in a row, so the length of the strip is: $9 \times 5 = 45$ cm

2. A set of **six equal square** photo frames is placed in a straight line on a hallway display board, forming a rectangle. Their total area is **486** square inches. What is the length of the display?

Let side of each square frame be s inches.

$$6s^2 = 486$$

$$s^2 = \frac{486}{6} = 81 \Rightarrow s = 9 \text{ in}$$

$$\text{Length} = 6 \times 9 = 54 \text{ in}$$

3. A collection of **eight equal-sized** square paintings is hung in a row in an art gallery, forming a rectangular arrangement. The total combined area is **1152** square centimeters. What is the length of the arrangement?

Let side of each painting be s cm.

$$8s^2 = 1152$$

$$s^2 = \frac{1152}{8} = 144 \Rightarrow s = 12 \text{ cm}$$

$$\text{Length of the arrangement} = 8 \times 12 = 96 \text{ cm}$$

Estimating square roots (Word problems)

1. The area of a square playground is **48 m²**.

Estimate the length of one side to the nearest tenth.

$$\text{Side length} = \sqrt{48}$$

$$\text{We know: } 6^2 = 36 \text{ and } 7^2 = 49$$

So $\sqrt{48}$ is between 6 and 7 and very close to 7.

$$\text{Try 6.9: } 6.9^2 = 47.61 \approx 48$$

So, to the nearest tenth:

Answer: **6.9 m (approximately)**

2. The area of a square playground is **115 m²**.

Estimate the length of one side to the nearest tenth.

$$\text{Side length} = \sqrt{115}$$

$$\text{We know: } 10^2 = 100 \text{ and } 11^2 = 121$$

So $\sqrt{115}$ is between 10 and 11.

$$\text{Try 10.7: } 10.7^2 = 114.49 \approx 115$$

$$\text{Try 10.8: } 10.8^2 = 116.64 \text{ (too big)}$$

So, 10.7 is closer.

Answer: **10.7 m (approximately)**

Perfect cubic roots word problems

1. A storage cube has a volume of 1728ft^3 . Find the side length.

$$\text{Storage cube volume} = 1728 \text{ ft}^3$$

$$\text{For a cube: Volume} = s^3$$

$$\text{So: } s^3 = 1728$$

$$\text{We know: } 12^3 = 12 \times 12 \times 12 = 144 \times 12 = 1728$$

$$\text{So: Answer is } \boxed{12 \text{ ft}}$$

2. A toy cube has volume 216 cm^3 . What is the cube's edge length?

$$\text{Toy cube volume} = 216 \text{ cm}^3$$

$$s^3 = 216$$

$$\text{We know: } 6^3 = 6 \times 6 \times 6 = 36 \times 6 = 216$$

$$\text{So: So: Answer is } \boxed{6 \text{ cm}}$$

Identifying Real numbers (Rational, Irrational, Natural, Integers, Whole numbers)

1. Write down all **Natural** number(s): $\sqrt{20}$, 8 , -3 , 0.75 , $\sqrt{25}$

8 only

2. Write down all **Rational** number(s): π , $\frac{3}{7}$, -12 , 4.2

$\frac{3}{7}$, -12 , and 4.2

3. Write down all **Irrational** number(s): $\sqrt{64}$, 0 , -7.5 , $\sqrt{7}$, 3π

$\sqrt{7}$ and 3π

4. Write down all **Integer** number(s): -11 , $\frac{3}{7}$, 12 , 0

All numbers are integers except $\frac{3}{7}$

5. Write down all **Whole** number(s): -11 , $\frac{3}{7}$, 12 , 0

12 and 0

Order a set of real numbers (Rational, Irrational, Natural, Integers, Whole numbers, Percentages)

1. Order from least to greatest: $1.\overline{4}$, $\sqrt{2}$, $\frac{5}{4}$, 1.3

$$\frac{5}{4} = 1.25000 \dots$$

$$1.3 = 1.3000 \dots$$

$$1.\overline{4} = 1.4444 \dots$$

$$\sqrt{2} \approx 1.414$$

$$\text{Ordering: } \frac{5}{4} < 1.3 < \sqrt{2} < 1.\overline{4}$$

$$\text{Answer (least } \rightarrow \text{ greatest): } \frac{5}{4}, 1.3, \sqrt{2}, 1.\overline{4}$$

2. Order from least to greatest: 4.5%, $\sqrt[3]{0.064}$, 0.041, $\frac{1}{2}$

$$4.5\% = 0.045$$

$$\sqrt[3]{0.064} = 0.400$$

$$0.041 = 0.041$$

$$\frac{1}{2} \approx 0.500$$

$$\text{Ordering: } 0.041 < 4.5\% < \sqrt[3]{0.064} < \frac{1}{2}$$

$$\text{Answer (least } \rightarrow \text{ greatest): } 0.041, 4.5\%, \sqrt[3]{0.064}, \frac{1}{2}$$

Real numbers on Number line

1. Answer Parts **a** to **c** below.

a) Write one rational number and one irrational number that lie between 2 and 3

Rational: 2.5

Irrational: $\sqrt{8}$

b) Write an inequality to compare the two numbers obtained above.

$$\sqrt{8} > 2.5$$

c) Graph the numbers obtained above on the number line.



2. Answer Parts **a** to **c** below.

a) Write one rational number and one irrational number that lie between 4 and 5

Rational: 4.1

Irrational: $\sqrt{24}$

b) Write an inequality to compare the two numbers obtained above.

$\sqrt{24} > 4.1$

c) Graph the numbers obtained above on the number line.



Module 3

3.1 Solve Equations with Variables on Each Side

3.2 Write and Solve Equations with Variables on Each Side

3.3 Solve Multi-Step Equations

3.4 write and solve multi-step Equations

3.5 Determine the Number of Solutions

Exercises

Solving Linear equations with variables on both sides (*No brackets to be expanded*)

Coefficients are Integers

1) $14 + 3n = 5n - 6$

Subtract $3n$ from both sides: $14 = 2n - 6$

Add 6 for both sides: $20 = 2n \Rightarrow n = 20 \div 2 = 10$

2) $-2a - 9 = 6a + 15$

Subtract $6a$ from both sides: $-8a - 9 = 15$

Add 9 for both sides: $-8a = 24 \Rightarrow a = 24 \div -8 = -3$

3) $7x - 15 = 15 - 3x$

Add $3x$ for both sides: $10x - 15 = 15$

Add 15 for both sides: $10x = 30 \Rightarrow x = 30 \div 10 = 3$

Coefficients are Fractions

$$1) \frac{1}{2}x - 5 = 10 - \frac{3}{2}x$$

$$\text{Move all } x\text{-terms to one side: } \frac{1}{2}x + \frac{3}{2}x - 5 = 10$$

$$\frac{4}{2}x - 5 = 10 \Rightarrow 2x - 5 = 10$$

$$2x = 15 \Rightarrow x = \frac{15}{2} = 7.5$$

$$2) \frac{2}{3}y + 1 = \frac{1}{6}y + 8$$

$$\text{Bring } x\text{-terms together: } \frac{2}{7}x + \frac{3}{8}x + 2 = 3$$

$$\text{Common denominator 56: } \frac{16}{56}x + \frac{21}{56}x + 2 = 3$$

$$\frac{37}{56}x + 2 = 3$$

$$\frac{37}{56}x = 1 \rightarrow x = \frac{56}{37}$$

$$3) \frac{2}{7}x + 2 = 3 - \frac{3}{8}x$$

Coefficients are Decimals

$$1) 5.4p + 13.1 = -2.6p + 3.5$$

$$5.4p + 2.6p + 13.1 = 3.5$$

$$8.0p + 13.1 = 3.5$$

$$8p = 3.5 - 13.1 = -9.6$$

$$p = \frac{-9.6}{8} = -1.2$$

$$2) 0.15w + 0.35 = 0.22w - 0.14$$

$$0.35 = 0.22w - 0.15w - 0.14$$

$$0.35 = 0.07w - 0.14$$

$$0.35 + 0.14 = 0.07w$$

$$0.49 = 0.07w \Rightarrow w = \frac{0.49}{0.07} = 7$$

$$3) 2.1m + 0.14 = 1.2m + 0.4$$

$$2.1m - 1.2m + 0.14 = 0.4$$

$$0.9m + 0.14 = 0.4$$

$$0.9m = 0.26$$

$$m = \frac{0.26}{0.9} = \frac{26}{90} = \frac{13}{45}$$

Solve Multi-Step Linear Equations (*Brackets to be expanded*)

Coefficients are Integers

$$1) 2(x + 3) = -3(x - 1)$$

$$2(x + 3) = -3(x - 1)$$

Expand:

$$2x + 6 = -3x + 3$$

$$2x + 3x = 3 - 6$$

$$5x = -3 \Rightarrow x = -\frac{3}{5}$$

$$2) 4(x - 2) + 3 = 2(x + 5) - 8$$

$$4(x - 2) + 3 = 2(x + 5) - 8$$

Expand both sides:

$$\text{Left: } 4(x - 2) + 3 = 4x - 8 + 3 = 4x - 5$$

$$\text{Right: } 2(x + 5) - 8 = 2x + 10 - 8 = 2x + 2$$

$$\text{Now solve: } 4x - 5 = 2x + 2$$

$$2x = 7 \Rightarrow x = \frac{7}{2} = 3.5$$

Coefficients are Fractions

$$1) \frac{1}{2}(x + 4) = \frac{1}{3}(3x - 2)$$

$$\text{Expand: } \frac{1}{2}x + 2 = x - \frac{2}{3}$$

$$\text{Bring } x\text{-terms together: } \frac{1}{2}x - x = -\frac{2}{3} - 2$$

$$\frac{1}{2}x = -\frac{8}{3}$$

$$x = -\frac{8}{3} \div \frac{1}{2} = -\frac{8}{3} \times \frac{2}{1} = -\frac{16}{3}$$

$$2) \frac{-4}{5}(x - 1) = \frac{3}{7}(x + 2)$$

$$\text{Expand: } \frac{-4}{5}x + \frac{4}{5} = \frac{3}{7}x + \frac{6}{7}$$

$$\text{Bring } x\text{-terms together: } \frac{-4}{5}x - \frac{3}{7}x = \frac{6}{7} - \frac{4}{5}$$

$$-\frac{43}{35}x = -\frac{2}{35}$$

$$-43x = -2 \rightarrow x = \frac{2}{43}$$

Coefficients are Decimals

$$1) 0.5(x + 6) = 0.2(3x + 1)$$

$$\underline{\text{Left: } 0.5x + 0.5 \cdot 6 = 0.5x + 3}$$

$$\underline{\text{Right: } 0.2 \cdot 3x + 0.2 \cdot 1 = 0.6x + 0.2}$$

$$\text{So, the equation becomes: } 0.5x + 3 = 0.6x + 0.2$$

$$\text{Subtract } 0.5x \text{ from both sides: } 3 = 0.1x + 0.2$$

$$\text{Subtract } 0.2: 2.8 = 0.1x$$

$$\text{Divide by } 0.1: x = 28$$

$$\text{Answer: } \boxed{x = 28}$$

$$2) 0.4(2x - 3) = 0.6(x + 1)$$

$$\underline{\text{Left: } 0.4 \cdot 2x - 0.4 \cdot 3 = 0.8x - 1.2}$$

$$\underline{\text{Right: } 0.6x + 0.6}$$

$$\text{So, } 0.8x - 1.2 = 0.6x + 0.6$$

$$\text{Subtract } 0.6x: 0.2x - 1.2 = 0.6$$

$$\text{Add } 1.2: 0.2x = 1.8$$

$$\text{Divide: } x = \frac{1.8}{0.2} = 9$$

$$\text{Answer: } \boxed{x = 9}$$

$$3) 0.9(7x + 1) - 1.3 = 1.6(3x - 4) + 0.8$$

$$\underline{\text{Left: } 0.9 \cdot 7x + 0.9 \cdot 1 - 1.3 = 6.3x + 0.9 - 1.3 = 6.3x - 0.4}$$

$$\underline{\text{Right: } 1.6 \cdot 3x - 1.6 \cdot 4 + 0.8 = 4.8x - 6.4 + 0.8 = 4.8x - 5.6}$$

$$\text{So, equation becomes: } 6.3x - 0.4 = 4.8x - 5.6$$

$$\text{Subtract } 4.8x: 1.5x - 0.4 = -5.6$$

$$\text{Add } 0.4: 1.5x = -5.2$$

$$\text{Divide: } x = \frac{-5.2}{1.5}$$

$$\text{Simplify: } x = -\frac{52}{15}$$

$$\text{As decimal: } x \approx -3.4667$$

$$\text{Answer: } \boxed{x = -\frac{52}{15} \approx -3.47}$$

Solving and checking the solution of a linear equation

Solve then check your solution.

$$1) 5 - 9x = -15 + 11x$$

$$\text{Add } 9x \text{ to both sides: } 5 = -15 + 20x$$

$$\text{Add } 15 \text{ to both sides: } 20 = 20x$$

$$x = 1$$

Check the solution

Substitute $x = 1$ into the original equation:

$$\text{Left side: } 5 - 9(1) = 5 - 9 = -4$$

$$\text{Right side: } -15 + 11(1) = -15 + 11 = -4$$

→ Both sides are equal.

$$2) 0.3y - 1.4 = 2.7y - 0.2$$

$$\text{Subtract } 0.3y: -1.4 = 2.4y - 0.2$$

$$\text{Add } 0.2: -1.2 = 2.4y$$

$$\text{Solve } y = \frac{-1.2}{2.4} = -0.5$$

Check the solution

Substitute $y = -0.5$:

$$\text{Left side: } 0.3(-0.5) - 1.4 = -0.15 - 1.4 = -1.55$$

$$\text{Right side: } 2.7(-0.5) - 0.2 = -1.35 - 0.2 = -1.55$$

→ Both sides are equal.

$$3) 40 \left(\frac{3}{20} + 10w \right) = -(2 - 100w)$$

Left side: $40 \left(\frac{3}{20} \right) + 40(10w) = 6 + 400w$

Right side: $-(2 - 100w) = -2 + 100w$

The equation becomes: $6 + 400w = -2 + 100w$

Subtract $100w$ from both sides: $6 + 300w = -2$

$$300w = -8 \rightarrow w = \frac{-8}{300} = -\frac{4}{150} = -\frac{2}{75}$$

Check the solution

Substitute $w = -\frac{2}{75}$.

Left side: $40 \left(\frac{3}{20} + 10w \right) = 40 \left(-\frac{7}{60} \right) = -\frac{280}{60} = -\frac{14}{3}$

Right side: $-(2 - 100w) = -\left(2 - \left(-\frac{8}{3} \right) \right) = -\left(2 + \frac{8}{3} = \frac{6}{3} + \frac{8}{3} \right) = -\frac{14}{3}$

→ Both sides are equal.

Write and Solve Linear Equations with Variables on Each Side (*No brackets*)

1. Rashid has 38 souvenir keychains

Ahmed has 52 souvenir keychains

Rashid buys 3 new keychains each week from Dubai Mall, and Ahmed buys 1 each week.

After how many weeks will Rashid and Ahmed have the same number of keychains?

Let w = number of weeks.

Rashid: $38 + 3w$

Ahmed: $52 + w$

Set them equal:

$$38 + 3w = 52 + w$$

$$3w - w = 52 - 38 = 14$$

$$2w = 14 \Rightarrow w = 7$$

After 7 weeks, each has: $38 + 3 \cdot 7 = 59$ and $52 + 7 = 59$

They will have the same number of keychains after 7 weeks.

2. Talabat charges a base delivery fee of 8 AED, plus an additional 2.5 AED per kilometre

Deliveroo charges a base delivery fee of 10 AED, plus 2 AED per kilometre.

For how many kilometers will the delivery cost be the same for both companies?

Talabat: base 8 AED, plus 2.5 AED per km $\Rightarrow C_T = 8 + 2.5k$

Deliveroo: base 10 AED, plus 2 AED per km $\Rightarrow C_D = 10 + 2k$

Let k = number of kilometres.

Set costs equal:

$$8 + 2.5k = 10 + 2k$$

$$2.5k - 2k = 10 - 8 = 2$$

$$0.5k = 2 \Rightarrow k = 4$$

The cost is the same at 4 km.

3. An e-scooter rental company on Abu Dhabi Corniche charges a 10 AED unlock fee plus 2.8 AED per minute.

Another company charges a 6 AED unlock fee plus 3.4 AED per minute.

After how many minutes will the total rental cost be the same?

Company A: 10 AED unlock fee + 2.8 AED per minute $\Rightarrow C_A = 10 + 2.8 m$

Company B: 6 AED unlock fee + 3.4 AED per minute $\Rightarrow C_B = 6 + 3.4 m$

Let m = number of minutes.

Set equal:

$$10 + 2.8m = 6 + 3.4m$$

$$10 - 6 = 3.4m - 2.8m$$

$$4 = 0.6m$$

$$m = \frac{4}{0.6} = \frac{40}{6} = \frac{20}{3} \approx 6.67$$

The costs are the same after $\frac{20}{3}$ minutes, i.e. about 6.7 minutes.

Write and solve multi-step Linear Equations (*With brackets*)

1) A right triangle has legs $9x$ and $12x$, and hypotenuse $15x$

A square has side length $7x + 1$

The triangle and the square have the same perimeter

a) Write and solve an equation to find the value of x .

b) Find the length of one side of the square.

a)

Triangle

Legs: $9x$ and $12x$

Hypotenuse: $15x$

Perimeter of triangle: $= 9x + 12x + 15x = 36x$

Square

Side length: $7x + 1$

Perimeter of square $= 4(7x + 1)$

They have the same perimeter, so: $36x = 4(7x + 1)$

Solve the equation: $36x = 4(7x + 1) \rightarrow x = \frac{1}{2}$

b) Length of one side of the square: $7x + 1 = 7\left(\frac{1}{2}\right) + 1 = \frac{7}{2} + 1 = \frac{9}{2} = 4.5$

So, one side of the square is 4.5 units.

2) A right triangle has legs $6.4x$ and $4.8x$, and a hypotenuse $8x$

A square has side length $3.2x + 0.4$

The triangle and the square have the same perimeter.

a) Write and solve an equation to find the value of x .

b) Find the length of one side of the square.

c) Find the Area of the square.

a)

Triangle

Legs: $6.4x$ and $4.8x$

Hypotenuse: $8x$

Perimeter: $6.4x + 4.8x + 8x = 19.2x$

Square:

Side: $3.2x + 0.4$

Perimeter: $4(3.2x + 0.4)$

Same perimeter: $19.2x = 4(3.2x + 0.4)$

Solve: $x = 0.25$

b) Length of one side of the square: $3.2x + 0.4 = 3.2(0.25) + 0.4 = 0.8 + 0.4 = 1.2$

One side of the square is 1.2 units.

c) Area of the square: $A = \text{side}^2 = 1.2^2 = 1.44$

So, the area is 1.44 square units.

3) Mr. Reed is drawing a blueprint of a rectangular patio.

The width of the patio is $40\frac{3}{4}$ feet shorter than twice its length.

The perimeter of the patio is $86\frac{1}{2}$ feet.

What is the length of the patio

Let L = length (ft), W = width (ft).

“The width of the patio is $40\frac{3}{4}$ feet shorter than twice its length.”

That means: $W = 2L - 40\frac{3}{4}$

Convert $40\frac{3}{4} = 40.75$: $W = 2L - 40.75$

Perimeter of a rectangle: $P = 2L + 2W$

Given $P = 86.5$ ft: $2L + 2(2L - 40.75) = 86.5$

Solve:

$$2L + 4L - 81.5 = 86.5$$

$$6L - 81.5 = 86.5$$

$$6L = 168$$

$$L = \frac{168}{6} = 28$$

So, the length is 28 feet.

4) A landscaping company in Abu Dhabi is designing a rectangular garden.

The width of the garden is $25\frac{1}{2}$ meters shorter than twice its length.

If the perimeter of the garden is 110 meters, what is the length of the garden?

Let L = length (m), W = width (m).

“The width of the garden is $25\frac{1}{2}$ meters shorter than twice its length.”

$$W = 2L - 25\frac{1}{2}$$

Convert $25\frac{1}{2} = 25.5$: $W = 2L - 25.5$

Perimeter $P = 2L + 2W$, and it's given $P = 110\text{m}$: $2L + 2(2L - 25.5) = 110$

Solve:

$$2L + 4L - 51 = 110$$

$$6L = 161$$

$$L = \frac{161}{6}$$

So, the length is $L = \frac{161}{6} \text{ m} \approx 26.83 \text{ m}$

5) Arjun purchased 5 tickets to a play, each with the same price.

He was also charged an online service fee of \$3.50 per ticket.

Emilia purchased tickets and paid twice as much for her tickets as for her tickets as Arjun.

Emilia was also charged a service fee of \$2.15 per her ticket.

If they spent the same amount, what is the cost of each of the tickets Arjun purchased?

Let x = price of one ticket Arjun purchased (in dollars).

Arjun:

5 tickets, price x each

Online fee: \$3.50 per ticket

Total cost: Arjun total = $5(x + 3.50)$

Emilia:

We'll interpret the text as:

She buys the same number of tickets (5 tickets).

Her ticket price is twice Arjun's: $2x$.

Her service fee is \$2.15 per ticket.

So, per ticket Emilia pays $2x + 2.15$, and for 5 tickets: Emilia total = $5(2x + 2.15)$

They spent the same total amount, so: $5(x + 3.50) = 5(2x + 2.15)$

Solve:

$$5(x + 3.50) = 5(2x + 2.15)$$

$$x + 3.50 = 2x + 2.15$$

$$3.50 - 2.15 = 2x - x$$

$$1.35 = x$$

So, the cost of each of Arjun's tickets is: $x = \$1.35$

6) Ahmed purchased **4** movie tickets at Vox Cinemas in Dubai, each with the same price.

He was also charged an online booking fee of **5 AED** per ticket.

Fatima purchased tickets and paid **twice as much for her tickets as Ahmed** did for his tickets.

She was also charged a booking fee of **2 AED** per ticket.

If they spent the same total amount, what is the price of each ticket Ahmed purchased?

Let x = price of one ticket Ahmed purchased (in AED).

Ahmed:

Ticket price: x

Booking fee: 5 AED per ticket

Number of tickets: 4 → Ahmed total = $4(x + 5)$

Fatima:

Ticket price is twice Ahmed's: $2x$

Booking fee: 2 AED per ticket

Number of tickets: 4 → Fatima total = $4(2x + 2)$

They spent the same total amount, so: $4(x + 5) = 4(2x + 2)$

Divide both sides by 4: $x + 5 = 2x + 2$

Solve to get $x = 3$

So, the price of each ticket Ahmed purchased is: 3 AED per ticket

7) The **Science Club** is going to a **wildlife reserve**, and each of its 10 members will pay for admission and share the cost of the guide.

The **Art Club** is going to a **museum**, and each of its 12 members will pay for admission and also purchase a souvenir.

Admission to the wildlife reserve is **1.6 times** that of the museum's admission, as in the table.

If the total cost is the same at both locations, what is the **admission per student to the museum?**

Wildlife Reserve	Museum
Admission: $\$1.6x$ per student	Admission: $\$x$ per student
Guide Fee: $\$3$ per student	Souvenir: $\$8.50$ per student

Science Club (10 students) — Wildlife Reserve

Admission per student: $1.6x$

Guide fee per student: 3

Total cost: $10(1.6x + 3)$

Art Club (12 students) — Museum

Admission per student: x

Souvenir per student: 8.50

Total cost: $12(x + 8.50)$

Both totals are equal: $10(1.6x + 3) = 12(x + 8.5)$

Solve the Equation to get $4x = 72 \rightarrow x = 18$

Thus, the admission per student to the museum is $\$18.00$

8. The **Drama Club** is visiting a **theatre**, and each of its 15 members will pay for admission and contribute to bus rental.

The **Music Club** is going to a **concert**, and each of its 10 members will pay for admission and also buy snacks.

Admission to the theatre is **1.2 times** that of the concert's admission, as shown in the table.

If the total cost is equal for both clubs, find the **admission per student to the concert**.

Theatre	Concert
Admission: $\$1.2x$ per student	Admission: $\$x$ per student
Guide Fee: $\$4$ per student	Souvenir: $\$6.50$ per student

Drama Club (15 students) — Theatre

Admission per student: $1.2x$

Guide/Bus Fee per student: 4

Total cost: $15(1.2x + 4)$

Music Club (10 students) — Concert

Admission per student: x

Souvenir per student: 6.50

Total cost: $10(x + 6.5)$

Both totals are equal: $15(1.2x + 4) = 10(x + 6.5)$

Solve the Equation to get $x = \frac{5}{8} = 0.625$

Thus, the admission per student to the concert is $\$0.625$ (62.5 cents).

9. Three friends run a **lawn mowing service**.

The table shows how many hours each friend worked.

Each friend earns **\$18.75 per hour**, and the total number of hours worked is represented by **$6h + 9$** , where **h** represents the hours worked by Omar.

What was the **total amount the friends earned**?

Name	Hours Worked
Omar	h
Sarah	$2h + 3$
Layla	$3(h - 1)$

Each friend earns \$18.75 per hour.

Hours worked:

Total hours worked is given as: $6h + 9$

We are asked to find the total amount earned.

Since each hour earns \$18.75: Total Earnings = $18.75 \times (\text{Total Hours})$

Total hours: $6h + 9$

Compute Total Earnings

Total Earnings = $18.75(6h + 9)$

Distribute: $18.75 \times 6h = 112.5h$

$18.75 \times 9 = 168.75$

So, $\text{Total Earnings} = 112.5h + 168.75$

This is the total amount of money the three friends earned.

10. Four teammates are selling **tickets for a school event**.

The table shows the number of tickets sold by each teammate.

Each ticket costs **\$12.50**, and the total number of tickets sold is represented by **$9m + 20$** , where **m** represents the tickets sold by Ahmed.

What was the **total amount collected**?

Name	Hours Worked
Ahmad	m
Fatima	$2m + 3$
Layla	$3(m - 2)$
Noor	$4m + 5$

Let **m** = tickets sold by Ahmad.

Each ticket costs **\$12.50**

The total number of tickets sold is represented by: **$9m + 20$**

Total Amount Collected: Total Money = $12.50 \times (\text{Total Tickets})$

Total Money = $12.50(9m + 20)$

Distribute

$$12.50 \times 9m = 112.5m$$

$$12.50 \times 20 = 250$$

So: **Total Money = $112.5m + 250$**

This expression represents the total amount of money collected from ticket sales.

Determine the Number of Solutions

To Determine the number of Solutions:

No Solution: The coefficients are the same, and the constants are different.

Infinite Solutions: The coefficients are the same, and the constants are the same.

One Solution: The coefficients are different.

Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*.

1) $4(x - 8) + 12 = 2(2x - 9)$

Expand both sides and simplify: $4x - 20 = 4x - 18 \rightarrow -20 = -18$ (impossible)

So, No solution

2) $3(2k - 5) = 6(k - 4) + 9$

Expand both sides and simplify: $6k - 15 = 6k - 15 \rightarrow 0 = 0$ (This is always true)

So, Infinite solutions

3) $-4y - 3 = \frac{1}{3}(12y - 9)$

Expand right side and simplify: $-4y - 3 = 4y - 3 \rightarrow 0 = 0$ (This gives $y = 0$)

So, One solution

4) $\frac{2}{3}(x + 3) = \frac{2}{3}(x - 6)$

Expand both sides and simplify: $\frac{2}{3}x + 2 = \frac{2}{3}x - 4 \rightarrow 0 = 0 \quad 2 = -4$ (impossible)

So, No solution

5) $2x + 5 = 3x + 5$

Simplify: $2x - 3x = 5 - 5 \rightarrow -x = 0 \rightarrow x = 0$

So, One solution

- What numbers would complete the equation so that it has *infinitely many solutions*?

$$1) 2x - 7(x + 10) = \boxed{-5} x - \boxed{70}$$

Explanation:

Expand and simplify Left side to get: $-5x - 70$

$$2) 12x - x + 8 + 3x = \boxed{14} x + \boxed{8}$$

Explanation:

Simplify Left side to get: $14x + 8$

- What numbers would complete the equation so that it has *no solution*?

$$1) 2x + 11 = \boxed{2} x + \boxed{\text{any number different from 11}}$$

Explanation:

When cancelling out $2x$ from both sides we get $11 = \text{Number different from 11}$, and this is impossible.

$$1) 4 - \frac{3}{5}(x + 50) = -25 + \boxed{-\frac{3}{5}} x$$

Expand and simplify Left side to get: $-26 - \frac{3}{5} x$

Module 4

4.1 Proportional Relationship and Slope

4.2 Slope of a Line

4.3 Similar triangles and slope

4.4 Direct Variation

4.5 Slope-Intercept Form

4.6 Graph Linear Equations

Hints and Notes

1. If there are two variables, x and y , related in a way that if x increases, y increases in a constant ratio, then the relation is a direct variation.

Similarly, if x decreases, y decreases in a constant ratio, then the relation is a direct variation.

2. The proportional relationship can be represented by a table, a line, or by an equation.

3. In the equation $y = mx$, the number m is called the slope, unit rate, or the constant of proportionality.

4. If a line is passing two points A (x_1, y_1) , B (x_2, y_2) , then the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

5. An equation in slope – intercept form is $y = mx + b$

m is the slope, and b is the y - intercept

6. To graph the linear equation:

a) Determine the y - intercept and plot on y – axis. This point is A

b) Choose a value for x , then find its corresponding value y . now you have a point B

c) Plot B on xy - plane

d) Use a ruler to join points A and B and extend the segment which joins them.

7. FOUR cases for lines:

1 – The equation of a **horizontal** line: $y = k$

2 – The equation of a **vertical** line: $x = k$

3 – The equation of a **rising** line: $y = mx + b$, where m is **positive**

4 – The equation of a **falling** line: $y = mx + b$, where m is **negative**

Exercises

Compare proportional relationships

1. Omar's current earnings are shown in the table.

He was offered a new job that will pay **\$9.50 per hour**.

Assume his earnings are directly proportional to the number of hours worked.

Which job offers a higher hourly rate?

Hours, x	Money Earned (\$), y
2	17.00
3	25.50
4	34.00
5	42.50

New job: \$9.50 per hour

Current job (From the table): $\frac{17}{2}$ or $\frac{25.5}{3}$ or $\frac{34}{4}$ or $\frac{42.5}{5} = \$8.50$ per hour

So, the new job offers a higher hourly rate, Since $\$9.50 > \8.50

2. Huda's current tutoring job pays as shown in the table.

Another tutoring center offers **\$8.40 per hour**.

Assume earnings are proportional to hours worked.

Which job pays more per hour?

Hours, x	Money Earned (\$), y
2	15.60
3	23.40
4	31.20
5	39.00

New job: \$8.40 per hour

Current job (From the table): $\frac{15.6}{2}$ or $\frac{23.4}{3}$ or $\frac{31.20}{4}$ or $\frac{39}{5} = \$7.80$ per hour

So, the new job pays more, Since $\$8.40 > \7.80

3. Yousef is ordering school notebooks.

For **Option A**, the price follows $y = 1.85x$.

For **Option B**, the costs are shown below.

Which option costs less for 80 notebooks? How much less?

Number of Notebooks, x	Total Cost (\$), y
40	78.00
120	234.00

Option A: The price is \$1.85 per notebook

→ For 80 notebooks: $80 \times 1.85 = \underline{\$148}$

Option B (From the table): $\frac{78}{40}$ or $\frac{234}{120} = \$1.95$ per notebook

→ For 80 notebooks: $80 \times 1.95 = \underline{\$156}$

So, **Option A** costs less for 80 notebooks, Since $\$148 < \156

Option A is less by $156 - 148 = \$8$

4. Salma is comparing the prices of two T-shirt suppliers.

For **Supplier A**, the price is given by $y = 8.75x$, where x is the number of shirts.

For **Supplier B**, the prices are shown below.

Which supplier is cheaper for 20 T-shirts? By how much?

Number of T-shirts, x	Total Cost (\$), y
10	92.50
30	277.50

Finding the slope of a line passing two points

1. Find the slope of the line passing through the points A (3, 7) and B (9, -5).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-5 - 7}{9 - 3} = \frac{-12}{6} = -2$$

2. Find the slope of the line passing through the points A (-2.5, 4.2) and B (1.5, -3.8)

$$m = \frac{-3.8 - 4.2}{1.5 - (-2.5)} = \frac{-8}{4} = -2$$

3. Find the slope of the line passing through the points A ($\frac{1}{2}$, $3\frac{3}{4}$) and B ($\frac{5}{2}$, $1\frac{1}{4}$)

$$m = \frac{\frac{11}{4} - \frac{3}{4}}{\frac{5}{2} - \frac{1}{2}} = \frac{\frac{8}{4}}{\frac{4}{2}} = \frac{2}{2} = 1$$

4. Find the slope of the line passing through the points A (-4, 6.5) and B (2, 2.5)

$$m = \frac{2.5 - 6.5}{2 - (-4)} = \frac{-4}{6} = -\frac{2}{3}$$

Finding the slope of a line given a table of points on the line

1. The points given in the table lie on a line. **Find the slope of the line.**

x	-2	1	4	7
y	6	2	-2	-6

Take any two points, e.g. (-2, 6) and (1 ,2):

$$m = \frac{2 - 6}{1 - (-2)} = \frac{-4}{3} = -\frac{4}{3}$$

2. The points given in the table lie on a line. **Find the slope of the line.**

x	0	2	4	6
y	3	1.5	0	-1.5

Using (0, 3) and (2, 1.5) :

$$m = \frac{1.5 - 3}{2 - 0} = \frac{-1.5}{2} = -0.75 = -\frac{3}{4}$$

Finding a missing value (x or y) in a table of points lying on a line

1. The points given in the table lie on a line. Find the missing value k .

x	-3	0	3	6
y	k	1	-3	-7

First, find the slope using $(3, -3)$ and $(6, -7)$:

$$m = \frac{-7 - (-3)}{6 - 3} = \frac{-4}{3} = -\frac{4}{3}$$

Now use the pair $(-3, k)$ and $(0, 1)$:

$$m = \frac{1 - k}{0 - (-3)} = \frac{1 - k}{3} = -\frac{4}{3}$$

So:

$$\frac{1 - k}{3} = -\frac{4}{3} \Rightarrow 1 - k = -4 \Rightarrow -k = -5 \Rightarrow k = 5$$

2. The points given in the table lie on a line. Find the missing value h .

x	-4.5	-1.5	h	6.5
y	9	4.5	-1.35	-7.5

First, slope using $(-4.5, 9)$ and $(-1.5, 4.5)$:

$$m = \frac{4.5 - 9}{-1.5 - (-4.5)} = \frac{-4.5}{3} = -1.5 = -\frac{3}{2}$$

Now use $(-1.5, 4.5)$ and $(h, -1.35)$:

$$m = \frac{-1.35 - 4.5}{h - (-1.5)} = -\frac{3}{2}$$

So:

$$\frac{-5.85}{h + 1.5} = -1.5 \Rightarrow -5.85 = -1.5(h + 1.5)$$

Divide both sides by -1.5:

$$h + 1.5 = \frac{-5.85}{-1.5} = 3.9 \Rightarrow h = 3.9 - 1.5 = 2.4$$

Linear equation in slope - intercept form

1. Given the linear equation $y = 3x - 4$

a) Find the slope.

3

b) Find the y-intercept.

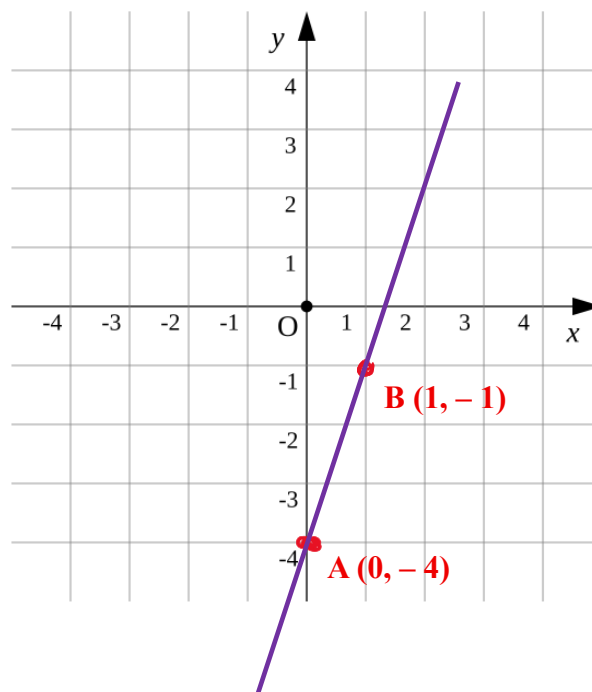
-4

c) Graph the linear equation.

To graph:

Firstly, Plot the y – intercept, Name **A (0, -4)**

Choose any point (Assume $x = 1$ for example, you will get $y = -1$), you will Get **B (1, -1)**



2. Given the linear equation $y = 1 - \frac{3}{4}x$

a) Find the slope.

$$-\frac{3}{4}$$

b) Find the y-intercept.

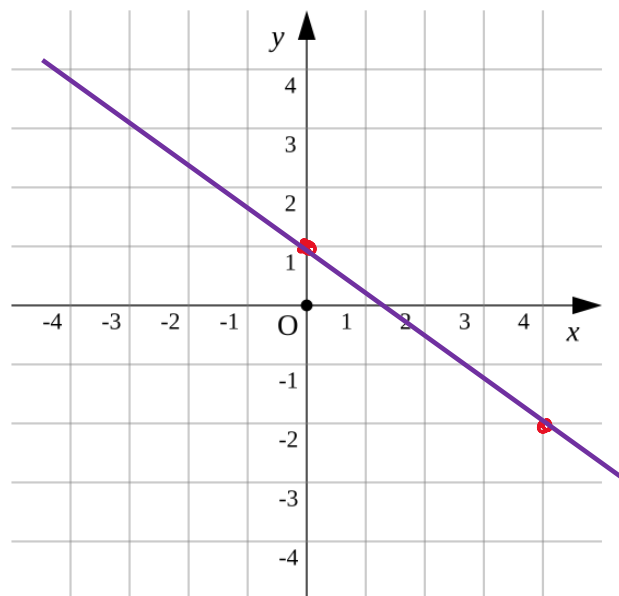
$$1$$

c) Graph the linear equation.

To graph:

Firstly, Plot the y – intercept, Name **A (0, 1)**

Choose any point (Assume $x = 4$ for example, you will get $y = -2$), you will Get **B (4, -2)**



3. Given the linear equation $y = -2 + 0.5x$

a) Find the slope.

0.5

b) Find the y-intercept.

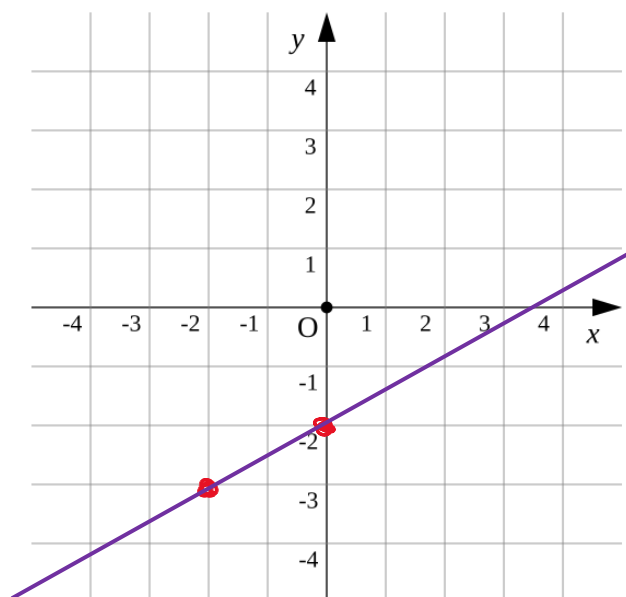
-2

c) Graph the linear equation.

To graph:

Firstly, Plot the y – intercept, Name **A (0, -2)**

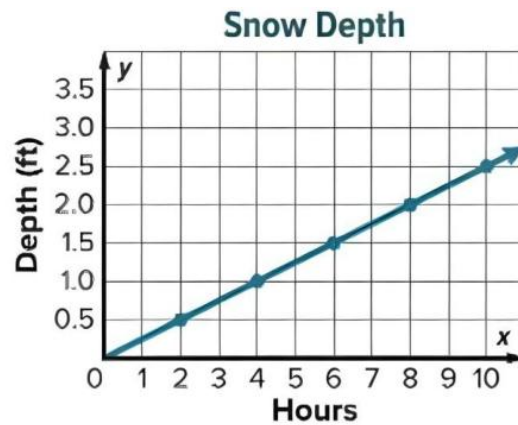
Choose any point (Assume $x = -2$ for example, you will get $y = -3$), you will Get **B (-2, -3)**



Finding the slope of a line given the graph

1. The graph shows the depth in feet of snow after each two-hour period during a storm.

Find the slope of the line.



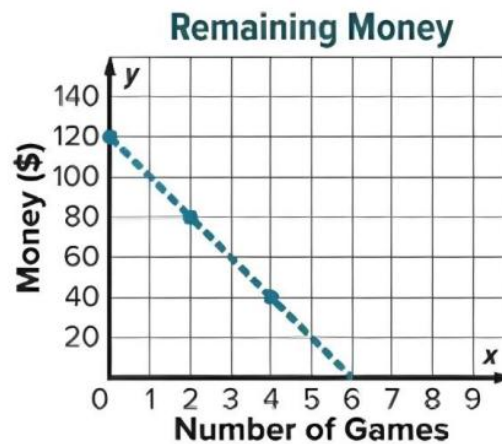
Choose any two points on the line, then apply $m = \frac{y_2 - y_1}{x_2 - x_1}$

For example, choose (4, 1), A (8, 2)

$$m = \frac{2-1}{8-4} = \frac{1}{4} = 0.25$$

2. The graph shows the amount of money left after buying video games.

Find the slope of the line.



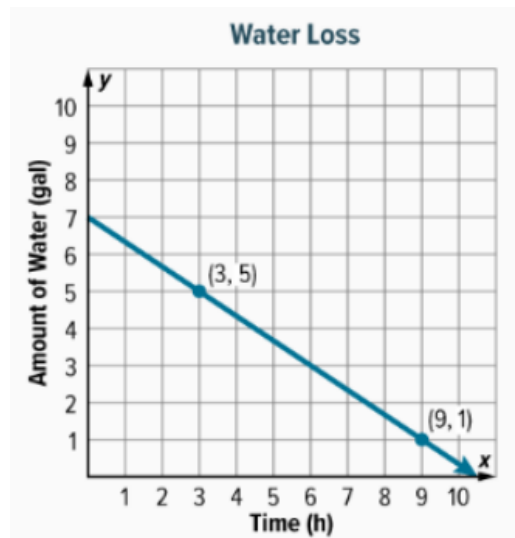
Choose any two points on the line, then apply $m = \frac{y_2 - y_1}{x_2 - x_1}$

For example, choose (4, 40), A (0, 120)

$$m = \frac{120 - 40}{0 - 4} = \frac{80}{-4} = -20$$

3. The graph shows the amount of water in a leaking bucket over time.

Find the slope of the line.



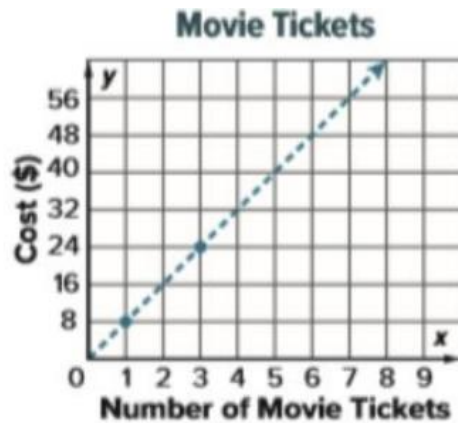
Choose any two points on the line, then apply $m = \frac{y_2 - y_1}{x_2 - x_1}$

For example, choose (9, 1), A (3, 5)

$$m = \frac{5-1}{3-9} = \frac{4}{-6} = -\frac{2}{3}$$

Use the direct variation equation to represent and solve real world problem

1. The cost y of movie tickets varies directly with the number of tickets x as shown in the graph.
 - a) Write a direct variation equation to represent this relationship.
 - b) Identify the constant of variation and interpret its meaning.



a) $y = 8x$

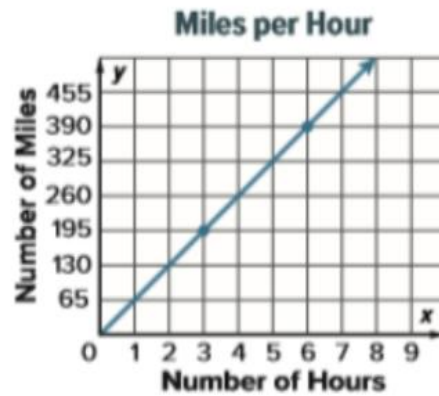
b) The constant of variation = 8

Interpretation: The cost of each movie ticket is \$8

2. The number of miles y varies directly with the number of hours x as shown in the graph.

a) Write a direct variation equation to represent this relationship.

b) Identify the constant of variation and interpret its meaning.



a) $y = 65x$

b) The constant of variation = 65

Interpretation: It needs one hour to cover 65 miles

3. The amount of flour needed for a recipe varies directly with the number of servings planned.

Three servings require 4.5 cups of flour.

a) Write a direct variation equation to represent this relationship.

b) Identify the constant of variation and interpret its meaning.

a) $y = 1.5x$

b) The constant of variation = $\frac{4.5}{3} = 1.5$

Interpretation: The number of flours needed to 1 serve is 1.5 cups

4. The cost of paper varies directly with the number of reams bought.

Suppose two reams cost \$5.10

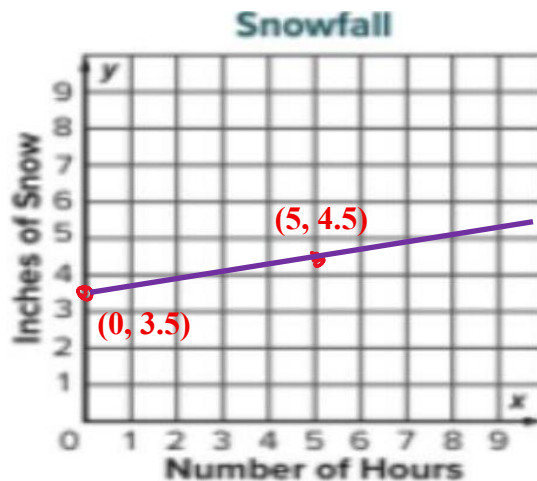
a) Write a direct variation equation to represent this relationship.

b) Identify the constant of variation and interpret its meaning.

Interpret the slope and y-intercept of a line from an equation of the form $y = mx + b$ to graph the line on the coordinate plane

1. The equation $y = \frac{1}{5}x + 3.5$ can be used to find the amount of accumulated snow y in inches x hours after 5 P.M. on a certain day.

Graph this equation.



Step 1

Choose any two x – values and then find the corresponding y – values.

Choose $x = 0$, then $y = \frac{1}{5}(0) + 3.5 = 3.5$

Choose $x = 5$, then $y = \frac{1}{5}(5) + 3.5 = 4.5$

Step 2

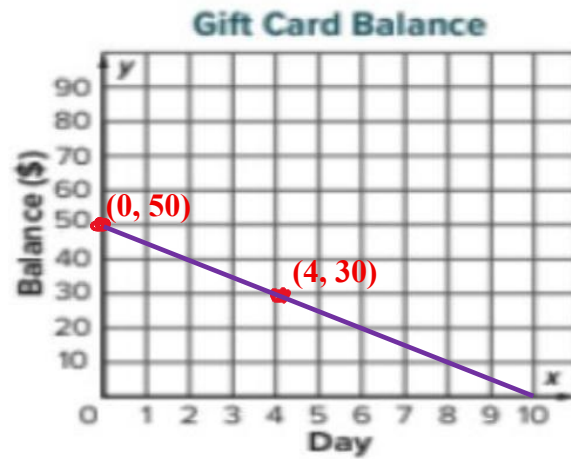
Plot the points $(0, 3.5)$, $(5, 4.5)$

Step 3

Use the ruler to join between the two points.

2. Alliyah's gift card balance can be represented by the equation $y = -5x + 50$, where y represents the gift card balance after x number of days.

Graph this equation.



Step 1

Choose any two x – values and then find the corresponding y – values.

Choose $x = 0$, then $y = -5(0) + 50 = 50$

Choose $x = 4$, then $y = -5(4) + 50 = 30$

Step 2

Plot the points $(0, 50)$, $(4, 30)$

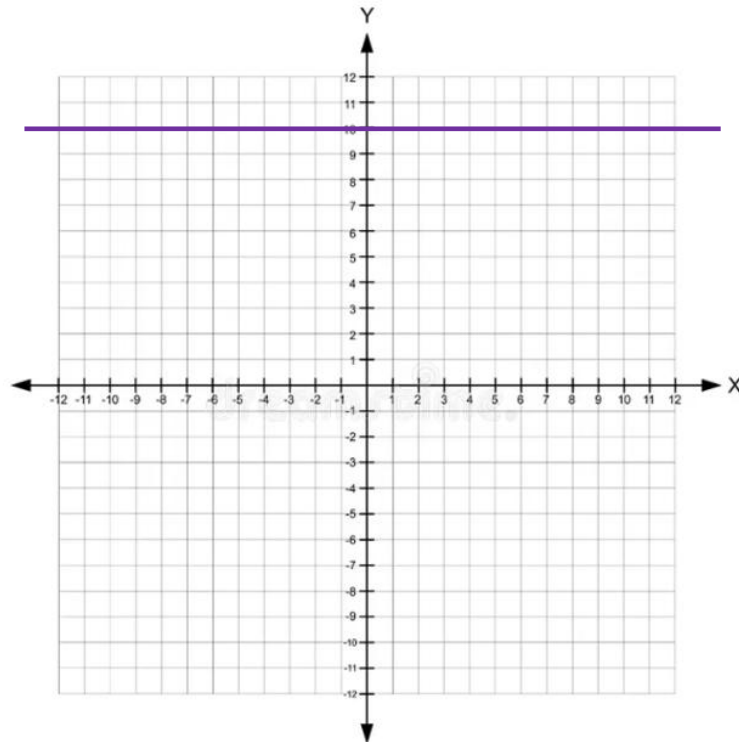
Step 3

Use the ruler to join between the two points.

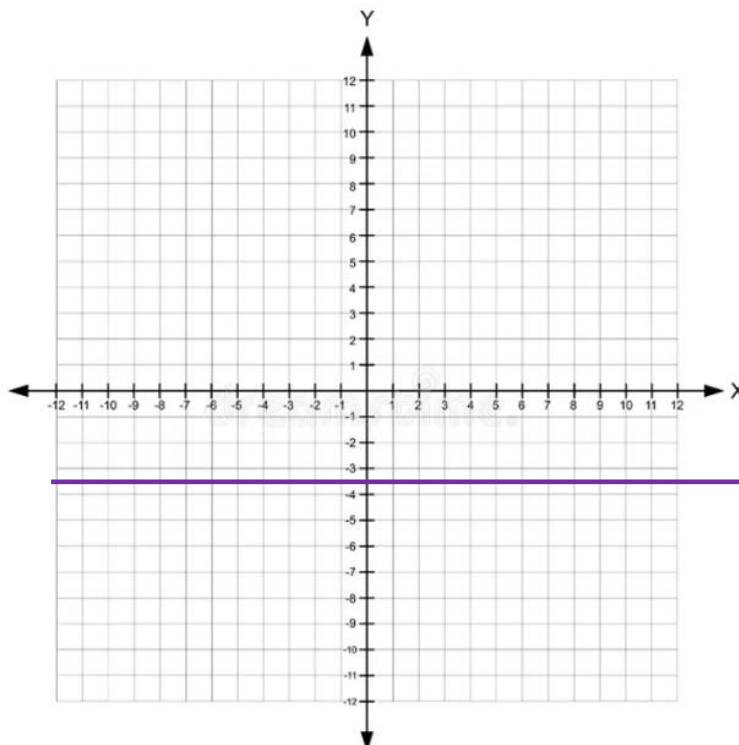
Graphing *Horizontal* lines

Graph each *horizontal* line in the xy – plane.

1. $y = 10$



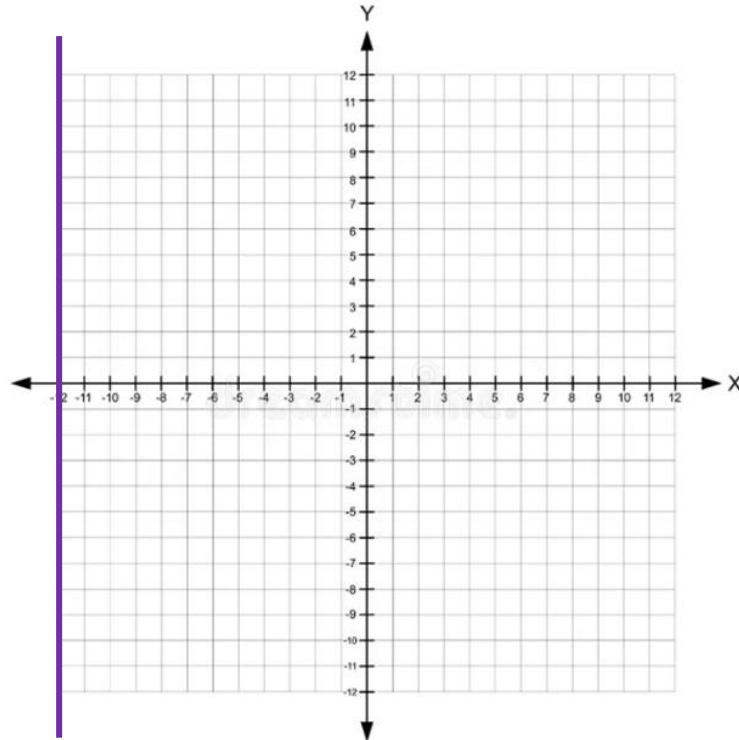
2. $y = -3.5$



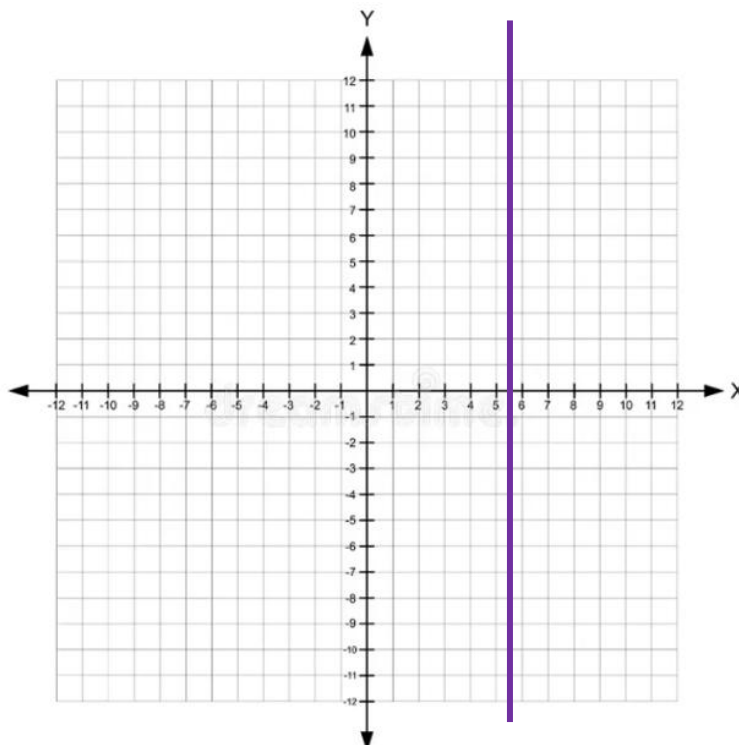
Graphing *Vertical* lines

Graph each *vertical* line in the xy - plane.

1. $x = -12$



2. $x = 5\frac{3}{4} = 5.75$



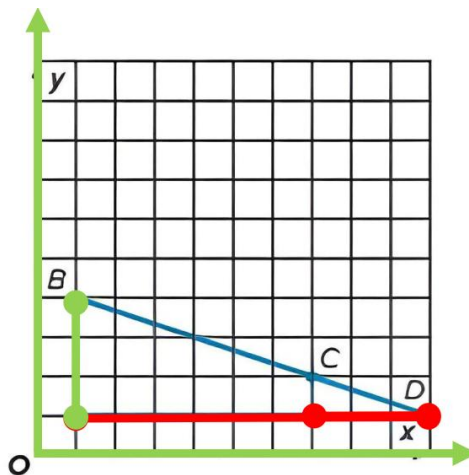
Identify similar triangles that fall on the same line in a coordinate plane and show that the slopes of the lines are equal.

1. The plans for a zipline are shown.

Use two points to determine the slope of the zipline.

Then verify that the slope is the same by choosing a different set of points.

B (1, 4), C (7, 2), D (10, 1).



Choose B (1, 4), C (7, 2)

$$m = \frac{4-2}{1-7} = \frac{2}{-6} = -\frac{1}{3}$$

Choose B (1, 4), D (10, 1)

$$m = \frac{4-1}{1-10} = \frac{3}{-9} = -\frac{1}{3}$$

Choose D (10, 1), C (7, 2)

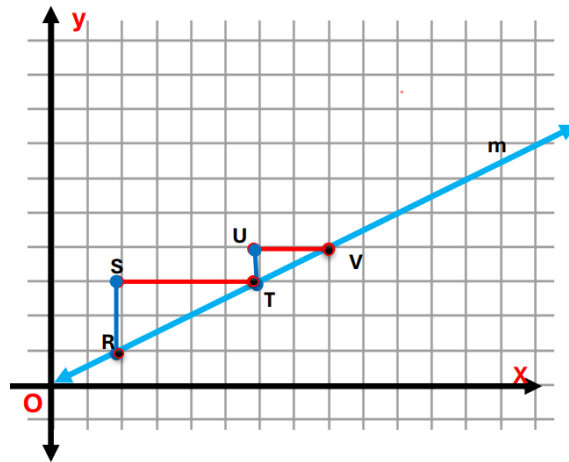
$$m = \frac{1-2}{10-7} = \frac{-1}{3} = -\frac{1}{3}$$

Conclusion:

It is verified that the slope is the same by choosing any two of the set of points.

2. The graph of line m is shown.

Use the similar slope triangles to compare the slope of segment RT and TV .



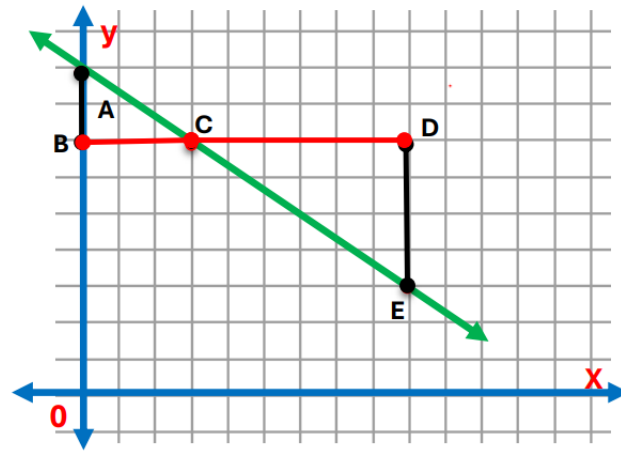
$$m_{RT} = \frac{3-1}{6-2} = \frac{2}{4} = 0.5$$

$$m_{TV} = \frac{4-3}{8-6} = \frac{1}{2} = 0.5$$

So, the Slopes are equal

3. The graph shows similar slope triangles on a line.

Select all the statements that are true.



- A) The slope of the line is negative.
- B) The slopes of each triangle are the same because they lie on the same line.
- C) Triangle CDE has a greater slope because the triangle is larger.
- D) The slope of each triangle is $\frac{2}{3}$
- E) The slope of the line is positive.

True Statements are:

Statement A (since the line falling)

Statement B (since they lie on the same line)