

MATH

Done By : Mr. OBADA ALJABALI



Exam Coverage 12A Math 2024-2025 Term1



GOOD
LUCK

Grade 12 Advanced

8045 – Merbh Secondary School for Girls

Term 1 – (2024/2025)

End of Term Exam Coverage

20 Objectives

Teacher Obada Aljabali



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GRADE 12ADVANCE
EOT FIRST TERM



7. (a) For the x -values of our points here we use (approximations of) $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8},$ and $\frac{\pi}{2}$.

Left	Right	Length
(0, 1)	(0.393, 0.92)	0.400
(0.393, 0.92)	(0.785, 0.71)	0.449
(0.785, 0.71)	(1.18, 0.383)	0.509
(1.18, 0.383)	(1.571, 0)	0.548
Total		1.906

- (b) For the x -values of our points here we use (approximations of) $0, \frac{\pi}{16}, \frac{\pi}{8}, \frac{3\pi}{16}, \frac{\pi}{4}, \frac{5\pi}{16}, \frac{3\pi}{8}, \frac{7\pi}{16},$ and $\frac{\pi}{2}$.

Left	Right	Length
(0, 1)	(0.196, 0.98)	0.197
(0.196, 0.98)	(0.393, 0.92)	0.204
(0.393, 0.92)	(0.589, 0.83)	0.217
(0.589, 0.83)	(0.785, 0.71)	0.232
(0.785, 0.71)	(0.982, 0.56)	0.248
(0.982, 0.56)	(1.178, 0.38)	0.262
(1.178, 0.38)	(1.37, 0.195)	0.272
(1.37, 0.195)	(1.571, 0)	0.277
Total		1.909

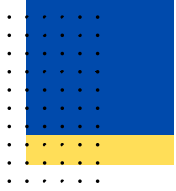
8. (a) For the x -values of our points here we use (approximations of) $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8},$ and $\frac{\pi}{2}$.

Left	Right	Length
(0, 0)	(0.393, 0.38)	0.548
(0.393, 0.38)	(0.785, 0.71)	0.509
(0.785, 0.71)	(1.18, 0.924)	0.449
(1.18, 0.924)	(1.57, 1)	0.400
Total		1.906

- (b) For the x -values of our points here we use (approximations of) $0, \frac{\pi}{16}, \frac{\pi}{8}, \frac{3\pi}{16}, \frac{\pi}{4}, \frac{5\pi}{16}, \frac{3\pi}{8}, \frac{7\pi}{16},$ and $\frac{\pi}{2}$.

Left	Right	Length
(0, 0)	(0.196, 0.2)	0.277
(0.196, 0.2)	(0.39, 0.38)	0.272
(0.39, 0.38)	(0.589, 0.56)	0.262
(0.589, 0.56)	(0.785, 0.71)	0.248
(0.785, 0.71)	(0.982, 0.83)	0.232
(0.982, 0.83)	(1.18, 0.924)	0.217
(1.18, 0.924)	(1.374, 0.98)	0.204
(1.374, 0.98)	(1.571, 1)	0.197
Total		1.909





1 Estimate an arc length of a given function.

(7-12)

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تقدير طول القوس على منحنى دالة معطاة

9. (a)

Left	Right	Length
(0, 1)	(0.75, 1.323)	0.817
(0.75, 1.323)	(1.5, 1.581)	0.793
(1.5, 1.581)	(2.25, 1.803)	0.782
(2.25, 1.803)	(3, 2)	0.776
Total		3.167

(b)

Left	Right	Length
(0, 1)	(0.375, 1.17)	0.413
(0.375, 1.17)	(0.75, 1.323)	0.404
(0.75, 1.323)	(1.125, 1.46)	0.399
(1.125, 1.46)	(1.5, 1.58)	0.395
(1.5, 1.58)	(1.88, 1.696)	0.392
(1.88, 1.696)	(2.25, 1.80)	0.390
(2.25, 1.80)	(2.63, 1.904)	0.388
(2.63, 1.904)	(3, 2)	0.387
Total		3.168

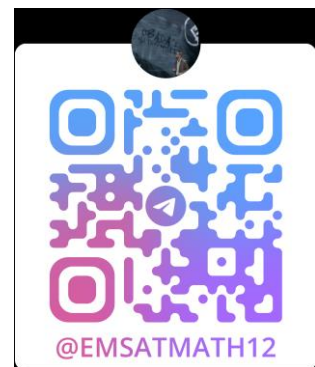
10. (a)

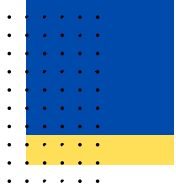
Left	Right	Length
(1, 1)	(1.25, 0.8)	0.3202
(1.25, 0.8)	(1.5, 0.67)	0.2833
(1.5, 0.67)	(1.75, 0.571)	0.2675
(1.75, 0.571)	(2, 0.5)	0.2600
Total		1.1310

(b)

Left	Right	Length
(1, 1)	(1.125, 0.89)	0.167
(1.125, 0.89)	(1.25, 0.8)	0.153
(1.25, 0.8)	(1.375, 0.73)	0.145
(1.375, 0.73)	(1.5, 0.67)	0.139
(1.5, 0.67)	(1.625, 0.62)	0.135
(1.625, 0.62)	(1.75, 0.57)	0.133
(1.75, 0.57)	(1.875, 0.53)	0.131
(1.875, 0.53)	(2, 0.5)	0.129
Total		1.132

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11. (a)

Left	Right	Length
(-2, 5)	(-1, 2)	3.162
(-1, 2)	(0, 1)	1.414
(0, 1)	(1, 2)	1.414
(1, 2)	(2, 5)	3.162
Total		9.153

(b)

Left	Right	Length
(-2, 5)	(-1.5, 3.25)	1.820
(-1.5, 3.25)	(-1, 2)	1.346
(-1, 2)	(-0.5, 1.25)	0.901
(-0.5, 1.25)	(0, 1)	0.559
(0, 1)	(0.5, 1.25)	0.559
(0.5, 1.25)	(1, 2)	0.901
(1, 2)	(1.5, 3.25)	1.346
(1.5, 3.25)	(2, 5)	1.820

12. (a)

Left	Right	Length
(-1, 1)	(-0.5, 1.875)	1.0078
(-0.5, 1.875)	(0, 2)	0.5154
(0, 2)	(0.5, 2.125)	0.5154
(0.5, 2.125)	(1, 3)	1.0078
Total		3.0463

(b)

Left	Right	Length
(-1, 1)	(-0.75, 1.58)	0.630
(-0.75, 1.58)	(-.5, 1.88)	0.388
(-.5, 1.88)	(-0.25, 1.98)	0.273
(-0.25, 1.98)	(0, 2)	0.251
(0, 2)	(0.25, 2.016)	0.251
(0.25, 2.016)	(0.5, 2.13)	0.273
(0.5, 2.13)	(0.75, 2.42)	0.388
(0.75, 2.42)	(1, 3)	0.630
Total		3.084



EXAMPLE 2.3 Determining Limits Graphically

Use the graph in Figure 2.8 to determine $\lim_{x \rightarrow 1^-} f(x)$, $\lim_{x \rightarrow 1^+} f(x)$, $\lim_{x \rightarrow 1} f(x)$ and $\lim_{x \rightarrow -1} f(x)$.

Solution For $\lim_{x \rightarrow 1^-} f(x)$, we consider the y -values as x gets closer to 1, with $x < 1$. That is, we follow the graph toward $x = 1$ *from the left* ($x < 1$). Observe that the graph dead-ends into the open circle at the point $(1, 2)$. Therefore, we say that $\lim_{x \rightarrow 1^-} f(x) = 2$. For $\lim_{x \rightarrow 1^+} f(x)$, we follow the graph toward $x = 1$ *from the right* ($x > 1$). In this case, the graph dead-ends into the solid circle located at the point $(1, -1)$. For this reason, we say that $\lim_{x \rightarrow 1^+} f(x) = -1$. Because $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$, we say that $\lim_{x \rightarrow 1} f(x)$ does not exist. Finally, we have that $\lim_{x \rightarrow -1} f(x) = 1$, since the graph approaches a y -value of 1 as x approaches -1 both from the left and from the right. ■

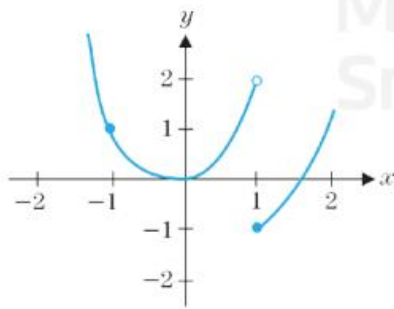
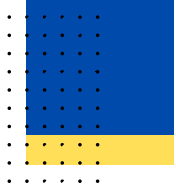


FIGURE 2.8
 $y = f(x)$





7. (a) $\lim_{x \rightarrow 0^-} f(x) = -2$

(b) $\lim_{x \rightarrow 0^+} f(x) = 2$

(c) $\lim_{x \rightarrow 0} f(x)$ Does not exist.

(d) $\lim_{x \rightarrow -2^-} f(x) = 2$

(e) $\lim_{x \rightarrow -2^+} f(x) = 2$

(f) $\lim_{x \rightarrow -2} f(x) = 2$

(g) $\lim_{x \rightarrow -1} f(x) = 0$

(h) $\lim_{x \rightarrow -3} f(x) = 1$

8. (a) $\lim_{x \rightarrow 1^-} f(x) = 1$

(b) $\lim_{x \rightarrow 1^+} f(x) = 1$

(c) $\lim_{x \rightarrow 1} f(x) = 1$

(d) $\lim_{x \rightarrow 2^-} f(x) = -1$

(e) $\lim_{x \rightarrow 2^+} f(x) = 3$

(f) $\lim_{x \rightarrow 2} f(x)$ does not exist.

(g) $\lim_{x \rightarrow 3^-} f(x) = 2.5$

(h) $\lim_{x \rightarrow -3} f(x) = 1.5$



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$$1. \lim_{x \rightarrow 0} (x^2 - 3x + 1) = 0^2 - 3(0) + 1 = 1$$

$$2. \lim_{x \rightarrow 2} \sqrt[3]{2x + 1} = \sqrt[3]{2(2) + 1} = \sqrt[3]{5}.$$

$$3. \lim_{x \rightarrow 0} \cos^{-1}(x^2) = \cos^{-1} 0 = \frac{\pi}{2}.$$

$$4. \lim_{x \rightarrow 2} \frac{x - 5}{x^2 + 4} = \frac{2 - 5}{2^2 + 4} = -\frac{3}{8}$$

$$5. \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3} \\ = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 2)}{x - 3} \\ = \lim_{x \rightarrow 3} (x + 2) = 3 + 2 = 5$$

$$6. \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 3x + 2} \\ = \lim_{x \rightarrow 1} \frac{(x - 1)(x + 2)}{(x - 1)(x - 2)} \\ = \lim_{x \rightarrow 1} \frac{(x + 2)}{(x - 2)} = \frac{3}{-1} = -3.$$

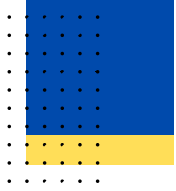
$$7. \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4} \\ = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 1)}{(x + 2)(x - 2)} \\ = \lim_{x \rightarrow 2} \frac{x + 1}{x + 2} = \frac{2 + 1}{2 + 2} = \frac{3}{4}$$

$$8. \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 2x - 3} \\ = \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{(x + 3)(x - 1)} \\ = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x + 3} = \frac{1^2 + 1 + 1}{1 + 3} = \frac{3}{4}$$

$$9. \lim_{x \rightarrow 0} \frac{\sin x}{\tan x} = \lim_{x \rightarrow 0} \frac{\sin x}{\frac{\sin x}{\cos x}} \\ = \lim_{x \rightarrow 0} \cos x = \cos 0 = 1$$

$$10. \lim_{x \rightarrow 0} \frac{\tan x}{x} \\ = \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x} \\ = \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\cos x} \right) = 1.$$





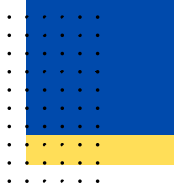
$$\begin{aligned}
 11. \lim_{x \rightarrow 0} \frac{x e^{-2x+1}}{x^2 + x} &= \lim_{x \rightarrow 0} \frac{x(e^{-2x+1})}{x(x+1)} \\
 &= \lim_{x \rightarrow 0} \frac{e^{-2x+1}}{x+1} = \frac{e^{-2(0)+1}}{0+1} = e
 \end{aligned}$$

$$\begin{aligned}
 12. \lim_{x \rightarrow 0} x^2 \csc^2 x &= \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 x} \\
 &= \left(\lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} \right) = 1
 \end{aligned}$$

$$\begin{aligned}
 13. \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \left(\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \right) \\
 &= \lim_{x \rightarrow 0} \frac{x+4-4}{x(\sqrt{x+4} + 2)} \\
 &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2} \\
 &= \frac{1}{\sqrt{4+2}} = \frac{1}{2+2} = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 14. \lim_{x \rightarrow 0} \frac{2x}{3 - \sqrt{x+9}} &= \lim_{x \rightarrow 0} \frac{2x}{(3 - \sqrt{x+9})(3 + \sqrt{x+9})} \\
 &= \lim_{x \rightarrow 0} \frac{2x(3 + \sqrt{x+9})}{-x} \\
 &= \lim_{x \rightarrow 0} -2(3 + \sqrt{x+9}) = -12
 \end{aligned}$$





$$\begin{aligned}
 15. \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} &= \lim_{x \rightarrow 1} \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}-1} \\
 &= \lim_{x \rightarrow 1} (\sqrt{x}+1) = \sqrt{1}+1 = 2
 \end{aligned}$$

$$\begin{aligned}
 16. \lim_{x \rightarrow 4} \frac{x^3-64}{x-4} &= \lim_{x \rightarrow 4} \frac{(x-4)(x^2+4x+16)}{(x-4)} \\
 &= \lim_{x \rightarrow 4} (x^2+4x+16) \\
 &= 4^2+4 \times 4+16 = 48
 \end{aligned}$$

$$\begin{aligned}
 17. \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right) &= \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)} \right) \\
 &= \lim_{x \rightarrow 1} \left(\frac{x+1}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)} \right) \\
 &= \lim_{x \rightarrow 1} \left(\frac{x-1}{(x-1)(x+1)} \right) \\
 &= \lim_{x \rightarrow 1} \left(\frac{1}{x+1} \right) = \frac{1}{2}
 \end{aligned}$$

18. Undefined. The limit from the right is 0, but the limit from the left does not exist.

$$\begin{aligned}
 19. \lim_{x \rightarrow 0} \frac{1-e^{2x}}{1-e^x} &= \lim_{x \rightarrow 0} \frac{(1-e^x)(1+e^x)}{1-e^x} \\
 &= \lim_{x \rightarrow 0} (1+e^x) = 2
 \end{aligned}$$

$$\begin{aligned}
 20. \lim_{x \rightarrow 0^+} \frac{\sin(|x|)}{x} &= \lim_{x \rightarrow 0^+} \frac{\sin(x)}{x} = 1 \\
 \lim_{x \rightarrow 0^-} \frac{\sin(|x|)}{x} &= \lim_{x \rightarrow 0^-} \frac{\sin(-x)}{x} \\
 &= \lim_{x \rightarrow 0^-} \frac{-\sin(x)}{x} = -1
 \end{aligned}$$

Since the limit from the left does not equal the limit from the right, we see that $\lim_{x \rightarrow 0} \frac{\sin(|x|)}{x}$ does not exist.

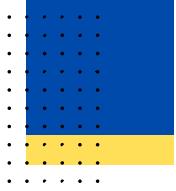
$$\begin{aligned}
 21. \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} 2x = 2(2) = 4 \\
 \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} x^2 = 2^2 = 4 \\
 \lim_{x \rightarrow 2} f(x) &= 4
 \end{aligned}$$

22. Undefined. The limit from the left is 2, but the limit from the right is -2.



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$$\begin{aligned}
 23. \quad \lim_{x \rightarrow -1^-} f(x) &= \lim_{x \rightarrow -1^-} (2x + 1) \\
 &= 2(-1) + 1 = -1 \\
 \lim_{x \rightarrow -1^+} f(x) &= \lim_{x \rightarrow -1^+} 3 = 3 \\
 \text{Therefore } \lim_{x \rightarrow -1} f(x) &\text{ does not exist.}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad \lim_{x \rightarrow 1^-} f(x) &= 3, \\
 \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^+} 2x + 1 = 3, \\
 \text{Therefore } \lim_{x \rightarrow 1} f(x) &= 3.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h} &= \lim_{h \rightarrow 0} \frac{(4 + 4h + h^2) - 4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4h + h^2}{h} = \lim_{h \rightarrow 0} 4 + h = 4
 \end{aligned}$$

$$\begin{aligned}
 26. \quad \lim_{h \rightarrow 0} \frac{(1+h)^3 - 1}{h} &= \lim_{h \rightarrow 0} \frac{1 + 3h + 3h^2 + h^3 - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(3 + 3h + h^2)}{h} \\
 &= \lim_{h \rightarrow 0} 3 + 3h + h^2 = 3
 \end{aligned}$$

$$27. \quad \text{Consider } f(x) = \frac{\sin x}{x} \text{ and a polynomial}$$

$$p(x) = x^2 - 4 \text{ such that } p(2) = 0.$$

$$\text{Also } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

Therefore by the theorem 3.4(viii),

$$\lim_{x \rightarrow a} f(p(x)) = L$$

$$\Rightarrow \lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x^2 - 4} = 1.$$

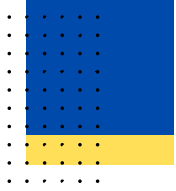
$$\begin{aligned}
 28. \quad \lim_{x \rightarrow 0} \frac{\tan x}{5x} &= \lim_{x \rightarrow 0} \frac{\sin x}{5x \cos x} \\
 &= \lim_{x \rightarrow 0} \left(\frac{1}{5} \frac{\sin x}{x} \frac{1}{\cos x} \right) \\
 &= \frac{1}{5} \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\cos x} \right) \\
 &= \frac{1}{5} (1)(1) = \frac{1}{5}
 \end{aligned}$$



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EXAMPLE 3.6 Evaluating a Limit of an Inverse Trigonometric Function

Evaluate $\lim_{x \rightarrow 0} \sin^{-1} \left(\frac{x+1}{2} \right)$.

Solution By Theorem 3.4 parts (v) and (viii), we have

$$\lim_{x \rightarrow 0} \sin^{-1} \left(\frac{x+1}{2} \right) = \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6}.$$

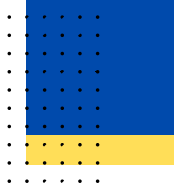
$$\begin{aligned} \lim_{x \rightarrow 0} (x \cot x) &= \lim_{x \rightarrow 0} \left(x \frac{\cos x}{\sin x} \right) = \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \cos x \right) \\ &= \left(\lim_{x \rightarrow 0} \frac{x}{\sin x} \right) \left(\lim_{x \rightarrow 0} \cos x \right) \\ &= \frac{\lim_{x \rightarrow 0} \cos x}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{1}{1} = 1, \end{aligned}$$

since $\lim_{x \rightarrow 0} \cos x = 1$ and where we have used the conjecture we made in example 2.6



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4	Find limits of polynomial, rational, and trigonometric functions using theorems. إيجاد نهاية الدوال كثيرة الحدود والنسبية والمثلثية باستخدام نظريات النهايات	Example3	81
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$$37. \lim_{x \rightarrow 0^+} \frac{\sqrt{1 - \cos x}}{x} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

$$38. \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \\ = \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) = 1.$$

$$42. \quad (a) \quad \lim_{x \rightarrow -1} (x + 1) \sin x \\ = \lim_{x \rightarrow -1} (x + 1) \lim_{x \rightarrow -1} \sin x$$

(Theorem 3.1).
Using Theorems 3.2 and 3.4 we get that
this is equal to $(-1 + 1) \sin(-1) = 0$.

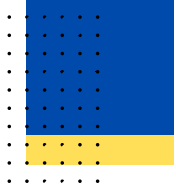
(b) By Theorem 3.1,

$$\lim_{x \rightarrow 1} \frac{x e^x}{\tan x} = \frac{(\lim_{x \rightarrow 1} x)(\lim_{x \rightarrow 1} e^x)}{\lim_{x \rightarrow 1} \tan x}.$$

Using Theorem 3.2 and Theorem 3.4 we
see that this equals $\frac{e}{\tan 1}$.

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23. (a) $4 - x^2 = 0 \Rightarrow 4 = x^2$ so we have vertical asymptotes at $x = \pm 2$.

$$f(x) \rightarrow \infty \text{ as } x \rightarrow -2^-$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -2^+$$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow 2^-$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow 2^+$$

Again, we have

$$\lim_{x \rightarrow \pm\infty} \frac{x}{4 - x^2}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{x}{x^2 \left(\frac{4}{x^2} - 1 \right)}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{1}{x \left(\frac{4}{x^2} - 1 \right)} = 0.$$

So there is a horizontal asymptote at $y = 0$.

- (b) Vertical asymptotes at $x = \pm 2$.

$$f(x) \rightarrow \infty \text{ as } x \rightarrow 2^- \text{ and } x \rightarrow -2^+.$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow 2^+ \text{ and } x \rightarrow -2^-.$$

Horizontal asymptote at $y = -1$.

24. (a) Since $4 + x^2$ is never 0, there are no vertical asymptotes. We have

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x}{\sqrt{4 + x^2}} &= \lim_{x \rightarrow \infty} \frac{x}{x\sqrt{\frac{4}{x^2} + 1}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{4}{x^2} + 1}} \\ &= \frac{1}{\sqrt{1}} = 1 \end{aligned}$$

$$\begin{aligned} \text{and} \quad \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{4 + x^2}} &= \lim_{x \rightarrow -\infty} \frac{x}{-x\sqrt{\frac{4}{x^2} + 1}} \\ &= \lim_{x \rightarrow -\infty} \frac{-1}{\sqrt{\frac{4}{x^2} + 1}} \\ &= \frac{-1}{\sqrt{1}} = -1, \end{aligned}$$

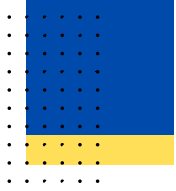
so there are horizontal asymptotes at $y = 1$ and $y = -1$.

- (b) The function is only defined in $(-2, 2)$.

Two one-sided vertical asymptotes at $x = \pm 2$. $f(x) \rightarrow \infty$ as $x \rightarrow 2^-$, and $f(x) \rightarrow -\infty$ as $x \rightarrow -2^+$.

No horizontal asymptotes.





25. The denominator factors:

$$x^2 - 2x - 3 = (x - 3)(x + 1).$$

Since neither $x = 3$ nor $x = -1$ are zeros of the numerator, we see that $f(x)$ has vertical asymptotes at $x = 3$ and $x = -1$.

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow 3^-,$$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow 3^+,$$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow -1^-, \text{ and}$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -1^+.$$

We have

$$\lim_{x \rightarrow \pm\infty} \frac{3x^2 + 1}{x^2 - 2x - 3}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{3 + 1/x^2}{1 - 2/x - 3/x^2} = 3.$$

So there is a horizontal asymptote at $y = 3$.

26. Vertical asymptote at $x = -2$.

$$f(x) \rightarrow \infty \text{ as } x \rightarrow -2^-.$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -2^+ \text{ and } x \rightarrow -2^-.$$

Again, we have

$$\lim_{x \rightarrow \pm\infty} \frac{1 - x}{x^2 + x - 2} = 0$$

So there is a horizontal asymptote at $y = 0$.

27. The function is continuous for all x , so no vertical asymptotes. We have

$$\begin{aligned} \lim_{x \rightarrow \infty} 4 \tan^{-1} x - 1 &= 4 \left(\lim_{x \rightarrow \infty} \tan^{-1} x \right) - 1 \\ &= 4(\pi/2) - 1 \\ &= 2\pi - 1 \end{aligned}$$

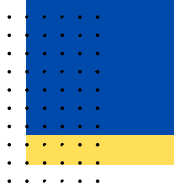
and

$$\begin{aligned} \lim_{x \rightarrow -\infty} 4 \tan^{-1} x - 1 &= 4 \left(\lim_{x \rightarrow -\infty} \tan^{-1} x \right) - 1 \\ &= 4(-\pi/2) - 1 \\ &= -2\pi - 1, \end{aligned}$$

so there are horizontal asymptotes

at $y = 2\pi - 1$ and $y = -2\pi - 1$.





28. The function $\ln x$ has a one-sided vertical asymptote at $x = 0$, so $f(x) = \ln(1 - \cos x)$ will have a vertical asymptote whenever

$1 - \cos x = 0$, i.e., whenever $\cos x = 1$.

This happens when $x = 2k\pi$ for any integer k . Since $1 - \cos x \geq 0$ for all x , $f(x)$ is defined at all points except for these vertical asymptotes. Thus as $f(x)$ approaches any of these asymptotes (from either side), it behaves like $\ln x$ approaching 0 from the right, so $f(x) \rightarrow -\infty$ as x approaches any of these asymptotes from either side.

29. Vertical asymptotes at $x = \pm 2$.

The slant asymptote is $y = -x$.

30. Vertical asymptote at $x = 2$.

The slant asymptote is $y = x + 2$.

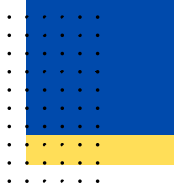
31. Vertical asymptotes at $x = \frac{-1 \pm \sqrt{17}}{2}$.

The slant asymptote is $y = x - 1$.

32. Vertical asymptote at $x = -\sqrt[3]{2}$.

The slant asymptote is $y = x$.





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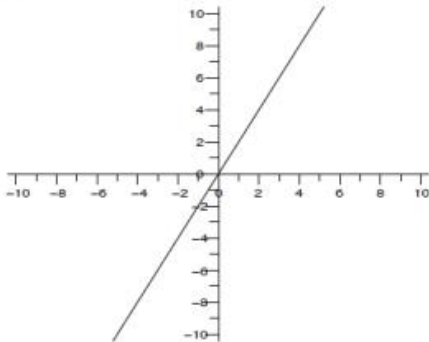
Sketch the graph of a function using the graph of its derivative.

(13-18)

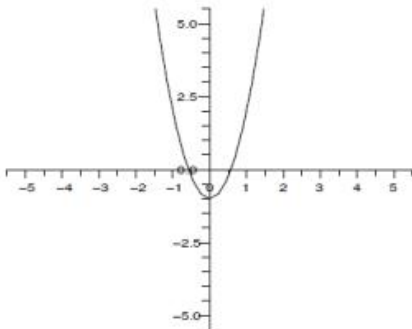
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رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها

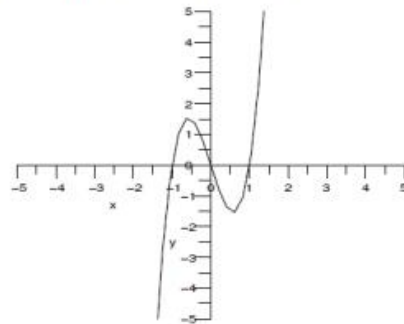
13. (a) The derivative should look like:



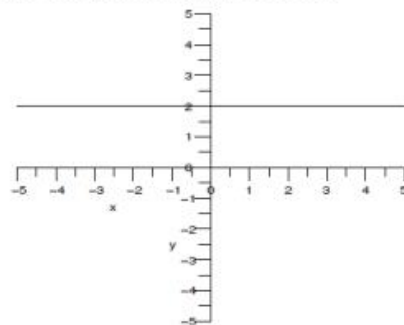
(b) The derivative should look like:



14. (a) The derivative should look like:

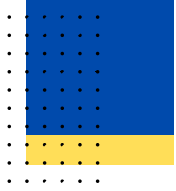


(b) The derivative should look like:



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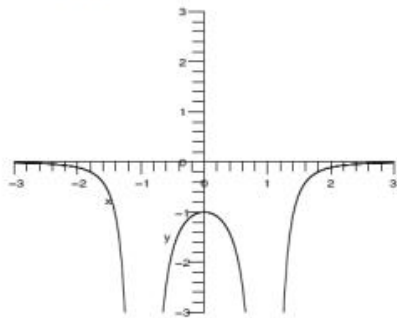
Sketch the graph of a function using the graph of its derivative.

(13-18)

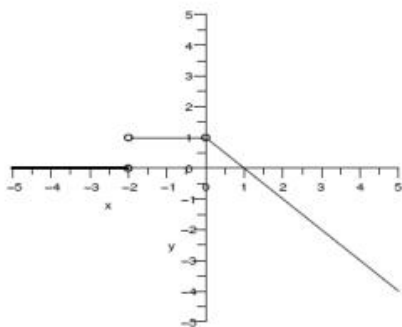
151

رسم منحنى الدالة اعتمادا على التمثيل البياني لمشتقتها

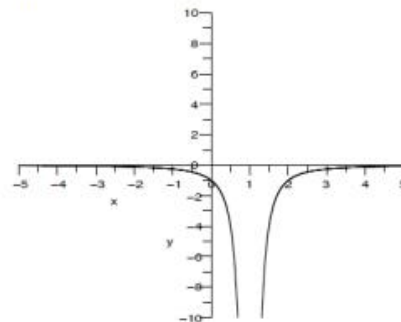
15. (a) The derivative should look like:



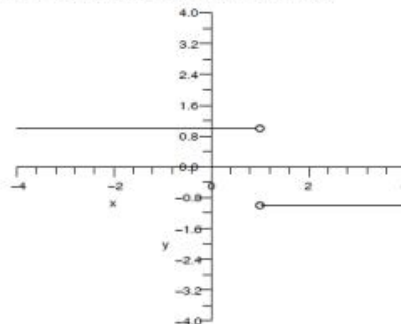
(b) The derivative should look like:



16. (a) The derivative should look like:

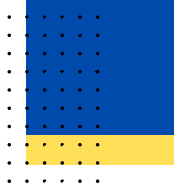


(b) The derivative should look like:

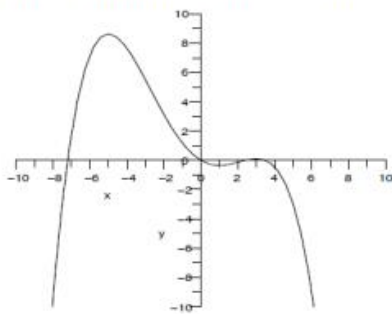


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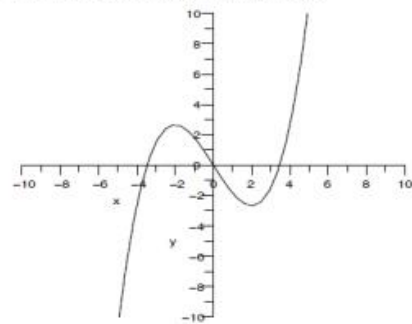




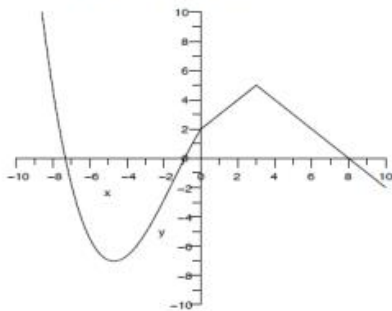
17. (a) The function should look like:



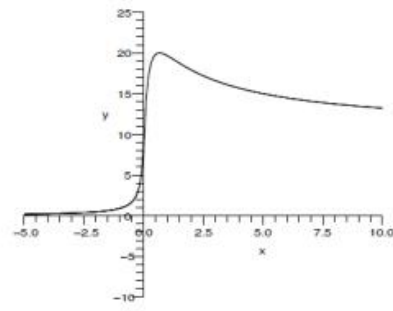
18. (a) The function should look like:

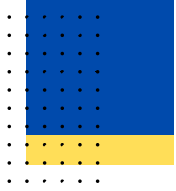


(b) The function should look like:



(b) The function should look like:





19. The left-hand derivative is

$$\begin{aligned} D_-f(0) &= \lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{2h + 1 - 1}{h} = 2 \end{aligned}$$

The right-hand derivative is

$$\begin{aligned} D_+f(0) &= \lim_{h \rightarrow 0^+} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{3h + 1 - 1}{h} = 3 \end{aligned}$$

Since the one-sided limits do not agree ($2 \neq 3$), $f'(0)$ does not exist.

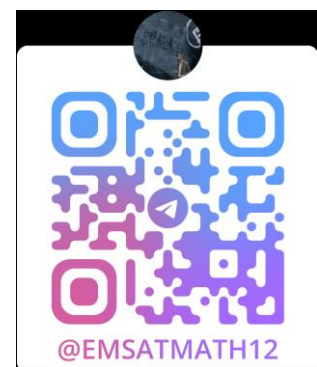
20. The left-hand derivative is

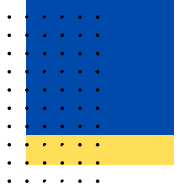
$$\begin{aligned} D_-f(0) &= \lim_{h \rightarrow 0^+} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{0 - 0}{h} = 0 \end{aligned}$$

The right-hand derivative is

$$\begin{aligned} D_+f(0) &= \lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{2h}{h} = 2 \end{aligned}$$

Since the one-sided limits do not agree ($0 \neq 2$), $f'(0)$ does not exist.





21. The left-hand derivative is

$$\begin{aligned} D_- f(0) &= \lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{h^2 - 0}{h} = 0 \end{aligned}$$

The right-hand derivative is

$$\begin{aligned} D_+ f(0) &= \lim_{h \rightarrow 0^+} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{h^3 - 0}{h} = 0 \end{aligned}$$

Since the one-sided limits are same ($0 = 0$), $f'(0)$ exist.

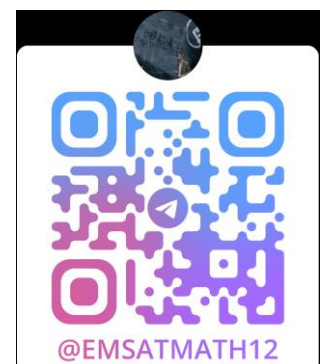
22. The left-hand derivative is

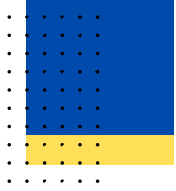
$$\begin{aligned} D_- f(0) &= \lim_{h \rightarrow 0^+} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{2h}{h} = 2 \end{aligned}$$

The right-hand derivative is

$$\begin{aligned} D_+ f(0) &= \lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{h^2 + 2h}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{h(h + 2)}{h} \\ &= \lim_{h \rightarrow 0^-} h + 2 = 2 \end{aligned}$$

Since the one-sided limits are same ($2 = 2$), $f'(0)$ exist.





21. $s(t) = -16t^2 + 40t + 10$
 $v(t) = s'(t) = -32t + 40$
 $a(t) = v'(t) = s''(t) = -32$

22. $s(t) = -4.9t^2 + 12t - 3$
 $v(t) = s'(t) = -9.8t + 12$
 $a(t) = v'(t) = s''(t) = -9.8$

23. $s(t) = \sqrt{t} + 2t^2 = t^{1/2} + 2t^2$
 $v(t) = s'(t) = \frac{1}{2}t^{-1/2} + 4t$
 $a(t) = v'(t) = s''(t) = -\frac{1}{4}t^{-3/2} + 4$

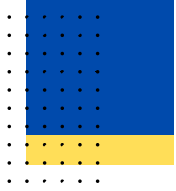
24. $s(t) = 10 - 10t^{-1}$ $v(t) = s'(t) = 10t^{-2}$
 $a(t) = s''(t) = -20t^{-3}$



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31. $h'(x) = f'(g(x))g'(x)$
 $h'(1) = f'(g(1))g'(1) = f'(2) \cdot (-2) = -6$

32. $h'(x) = f'(g(x))g'(x)$
 $h'(2) = f'(g(2))g'(2) = f'(3) \cdot (4) = -12$

33. As a temporary device given any f , set $g(x) = f(-x)$. Then by the chain rule,

$$g'(x) = f'(-x)(-1) = -f'(-x).$$

In the even case ($g = f$) this reads $f'(-x) = -f'(x)$ and shows f' is odd.

In the odd case ($g = -f$ and therefore $g' = -f'$), this reads $-f'(x) = -f'(-x)$ or $f'(x) = f'(-x)$ and shows f' is even.

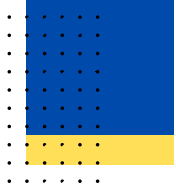
34. To say that $f(x)$ is symmetric about the line $x = a$ is the same as saying that $f(a+x) = f(a-x)$. Taking derivatives (using the chain rule), we have

$$\frac{d}{dx}f(a+x) = f'(a+x)$$

$$\frac{d}{dx}f(a-x) = f'(a-x)(-1) = -f'(a-x).$$

Thus, $f'(a+x) = -f'(a-x)$ and the graph of $f'(x)$ is symmetric through the point $(a, 0)$.





35. (a) Chain rule gives,
- $$\begin{aligned}\frac{d}{dx} f(x^2) &= f'(x^2) \frac{d}{dx} (x^2) \\ &= f'(x^2) (2x) \\ &= 2x f'(x^2).\end{aligned}$$
- (b) Chain rule gives,
- $$\begin{aligned}\frac{d}{dx} [f(x)]^2 &= 2f(x) \frac{d}{dx} f(x) \\ &= 2f(x) f'(x).\end{aligned}$$
- (c) Chain rule gives,
- $$\begin{aligned}\frac{d}{dx} f(f(x)) &= f'(f(x)) \frac{d}{dx} f(x) \\ &= f'(f(x)) f'(x).\end{aligned}$$

36. (a) Chain rule gives,
- $$\begin{aligned}\frac{d}{dx} f(\sqrt{x}) &= f'(\sqrt{x}) \frac{d}{dx} (\sqrt{x}) \\ &= f'(\sqrt{x}) \frac{1}{2\sqrt{x}}.\end{aligned}$$
- (b) Chain rule gives,
- $$\begin{aligned}\frac{d}{dx} (\sqrt{f(x)}) &= \frac{1}{2\sqrt{f(x)}} \frac{d}{dx} f(x) \\ &= \frac{1}{2\sqrt{f(x)}} f'(x).\end{aligned}$$
- (c) Chain rule gives,
- $$\begin{aligned}\frac{d}{dx} [f(xf(x))] &= f'(xf(x)) \frac{d}{dx} (xf(x)) \\ &\text{and product rule gives} \\ &= f'(xf(x)) \left(f(x) + x \frac{d}{dx} f(x) \right) \\ &= f'(xf(x)) (f(x) + x f'(x)).\end{aligned}$$



37. (a) Chain rule gives,

$$\begin{aligned}\frac{d}{dx} \left[f \left(\frac{1}{x} \right) \right] &= f' \left(\frac{1}{x} \right) \cdot \frac{d}{dx} \left(\frac{1}{x} \right) \\ &= f' \left(\frac{1}{x} \right) \cdot \left(-\frac{1}{x^2} \right) \\ &= - \left(\frac{1}{x^2} \right) \cdot f' \left(\frac{1}{x} \right).\end{aligned}$$

(b) Chain rule gives,

$$\begin{aligned}\frac{d}{dx} \left(\frac{1}{f(x)} \right) &= \left(-\frac{1}{f(x)^2} \right) \cdot \frac{d}{dx} f(x) \\ &= \left(-\frac{1}{f(x)^2} \right) \cdot f'(x).\end{aligned}$$

(c) Chain rule gives,

$$\begin{aligned}\frac{d}{dx} \left[f \left(\frac{x}{f(x)} \right) \right] &= f' \left(\frac{x}{f(x)} \right) \frac{d}{dx} \left(\frac{x}{f(x)} \right) \\ \text{and quotient rule gives,} \\ &= f' \left(\frac{x}{f(x)} \right) \left(\frac{f(x) - x f'(x)}{[f(x)]^2} \right).\end{aligned}$$

38. (a) Chain rule gives,

$$\begin{aligned}\frac{d}{dx} (1 + f(x^2)) &= f'(x^2) \frac{d}{dx} (x^2) \\ &= f'(x^2) (2x) = 2x f'(x^2).\end{aligned}$$

(b) Chain rule gives,

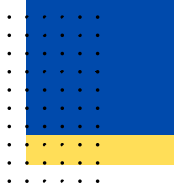
$$\begin{aligned}\frac{d}{dx} [1 + f(x)]^2 &= 2 [1 + f(x)] \frac{d}{dx} (1 + f(x)) \\ &= 2 [1 + f(x)] f'(x) = 2 f'(x) [1 + f(x)].\end{aligned}$$

(c) Chain rule gives,

$$\begin{aligned}\frac{d}{dx} [f(1 + f(x))] &= f'(1 + f(x)) \cdot \frac{d}{dx} (1 + f(x)) \\ &= f'(1 + f(x)) f'(x) \\ &= f'(x) f'(1 + f(x)).\end{aligned}$$



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17. $f(x) = x^3 + 4x - 1$ is a one-to-one function with $f(0) = -1$ and $f'(0) = 4$. Therefore $g(-1) = 0$ and

$$g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(0)} = \frac{1}{4}.$$

18. $f(x) = x^5 + 4x - 2$ is a one-to-one function with $f(0) = -2$ and $f'(0) = 4$. Therefore $g(-2) = 0$ and

$$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(0)} = \frac{1}{4}.$$

19. $f(x) = x^5 + 3x^3 + x$ is a one-to-one function with $f(1) = 5$ and $f'(1) = 5 + 9 + 1 = 15$. Therefore $g(5) = 1$ and

$$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(1)} = \frac{1}{15}.$$

20. $f(x) = x^3 + 2x + 1$ is a one-to-one function with $f(-1) = -2$ and $f'(-1) = 5$. Therefore $g(-2) = -1$ and

$$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(-1)} = \frac{1}{5}.$$

21. $f(x) = \sqrt{x^3 + 2x + 4}$ is a one-to-one function and $f(0) = 2$ so $g(2) = 0$. Meanwhile,
 $f'(x) = \frac{1}{2\sqrt{x^3 + 2x + 4}}(3x^2 + 2)$
 $f'(0) = 1/2$
 $g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(0)} = 2.$

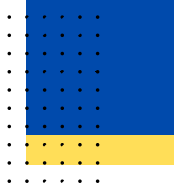
22. $f(x) = \sqrt{x^5 + 4x^3 + 3x + 1}$ is a one-to-one function and $f(1) = 3$ so $g(3) = 1$. Meanwhile,

$$f'(x) = \frac{5x^4 + 12x^2 + 3}{2\sqrt{x^5 + 4x^3 + 3x + 1}}$$

$$f'(1) = \frac{20}{6} = \frac{10}{3}$$

$$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \frac{3}{10}.$$





19. (a) $f(x) = \sin x^2$
 $f'(x) = \cos(x^2) \cdot (2x) = 2x \cos(x^2)$

(b) $f(x) = \sin^2 x$
 $f'(x) = 2 \sin x \cos x$

(c) $f(x) = \sin 2x$
 $f'(x) = \cos 2x (2) = 2 \cos 2x$

20. (a) $f(x) = \cos \sqrt{x}$
 $f'(x) = (-\sin \sqrt{x}) \cdot \frac{1}{2}(x)^{-1/2}$
 $= -\frac{1}{2}(x)^{-1/2} \sin \sqrt{x}$

(b) $f(x) = \sqrt{\cos x}$
 $f'(x) = \frac{1}{2}(\cos x)^{-1/2} \cdot (-\sin x)$
 $= -\frac{1}{2} \sin x (\cos x)^{-1/2}$

(c) $f(x) = \cos\left(\frac{1}{2}x\right)$
 $f'(x) = -\sin\left(\frac{1}{2}x\right) \cdot \left(\frac{1}{2}\right)$
 $= -\frac{1}{2} \sin\left(\frac{1}{2}x\right)$

21. (a) $f(x) = \sin x^2 \tan x$
 $f'(x) = \sin x^2 (\sec^2 x) + 2x \cos x^2 \tan x$

(b) $f(x) = \sin^2(\tan x)$
 $f'(x) = 2 \sin(\tan x) \cdot \cos(\tan x) \cdot \sec^2 x$

(c) $f(x) = \sin(\tan^2 x)$
 $f'(x) = [\cos(\tan^2 x)] (2 \tan x) (\sec^2 x)$
 $= (2 \tan x) (\sec^2 x) [\cos(\tan^2 x)]$

22. (a) $f(x) = \sec x^2 \tan x^2$
 $f'(x) = \sec^3(x^2) (2x)$
 $+ \tan^2(x^2) \sec(x^2) (2x)$
 $= 2x \sec x^2 [\sec^2 x^2 + \tan^2 x^2]$

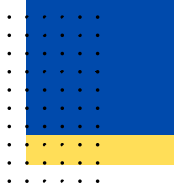
(b) $f(x) = \sec^2(\tan x)$
 $f'(x) = 2 \sec(\tan x) [\sec(\tan x)$
 $\cdot \tan(\tan x)] (\sec^2 x)$

(c) $f(x) = \sec(\tan^2 x)$
 $f'(x) = [\sec(\tan^2 x) \tan(\tan^2 x)]$
 $\cdot (2 \tan x) (\sec^2 x)$
 $= (2 \tan x \sec^2 x)$
 $\cdot [\sec(\tan^2 x) \tan(\tan^2 x)]$



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12	Find derivatives of natural logarithmic functions. إيجاد مشتقات الدوال اللوغاريتمية الطبيعية	(1-14)	192
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$$1. f'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = x^2 e^x (x + 3)$$

$$2. f'(x) = 2e^{2x} \cos 4x + e^{2x} (-\sin 4x) 4$$

$$3. f(t) = t + 2^t \\ f'(t) = 1 + 2^t \log 2$$

$$4. f(t) = t4^{3t} \\ f'(t) = 4^{3t} + t4^{3t} (\ln 4) 3 = 4^{3t} (1 + 3t \ln 4)$$

$$5. f'(x) = 2e^{4x+1} (4) = 8e^{4x+1}$$

$$6. f'(x) = e^{-x}, \text{ so } f'(x) = -e^{-x}.$$

$$7. h(x) = \left(\frac{1}{3}\right)^{x^2} \\ h'(x) = \ln\left(\frac{1}{3}\right) \cdot 2x \cdot \left(\frac{1}{3}\right)^{x^2} \\ = 2x \cdot \ln\left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right)^{x^2} \\ = -2x \cdot \ln(3) \cdot \left(\frac{1}{3}\right)^{x^2}$$

$$8. h(x) = 4^{-x^2} \\ h'(x) = 4^{-x^2} \cdot \ln(4) \cdot (-2x) \\ = -2x \cdot 4^{-x^2} \cdot \ln(4)$$

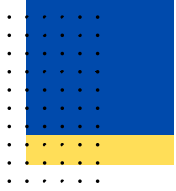
$$9. f(u) = e^{u^2+4u} \\ f'(u) = e^{u^2+4u} \cdot \frac{d}{du}(u^2 + 4u) \\ = e^{u^2+4u} \cdot (2u + 4)$$

$$10. f(x) = 3e^{\tan x} \\ f'(x) = 3e^{\tan x} \cdot \frac{d}{dx}(\tan x) \\ = 3e^{\tan x} \sec^2 x$$

$$11. f(w) = \frac{e^{4w}}{w} \\ f'(w) = \frac{w \cdot 4e^{4w} - e^{4w} \cdot 1}{w^2} \\ = \frac{e^{4w}(4w - 1)}{w^2}$$

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$$\begin{aligned} 12. \quad f(w) &= \frac{w}{e^{6w}} \\ f'(w) &= \frac{1 \cdot e^{6w} - w \cdot e^{6w} \cdot 6}{(e^{6w})^2} \\ &= \frac{e^{6w} - 6we^{6w}}{(e^{6w})^2} = \frac{(1 - 6w)}{e^{6w}} \end{aligned}$$

$$13. \quad f'(x) = \frac{1}{2x} \cdot (2) = \frac{1}{x}$$

$$\begin{aligned} 14. \quad f(x) &= \frac{1}{2} \ln 8 + \frac{1}{2} \ln x \\ f'(x) &= \frac{1}{2x} \end{aligned}$$



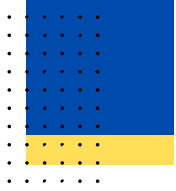
29. (a) $f(x) = \sin^{-1}(x^3 + 1)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} [\sin^{-1}(x^3 + 1)]$.
 By the Chain rule we get,
 $f'(x) = \frac{1}{\sqrt{1 - (x^3 + 1)^2}} \frac{d}{dx} (x^3 + 1)$
 $= \frac{1}{\sqrt{1 - (x^3 + 1)^2}} (3x^2)$
 $= \frac{3x^2}{\sqrt{1 - (x^3 + 1)^2}}$.

(b) $f(x) = \sin^{-1}(\sqrt{x})$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} [\sin^{-1}(\sqrt{x})]$.
 By the Chain rule, we get
 $f'(x) = \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \frac{d}{dx} (\sqrt{x})$
 $= \frac{1}{\sqrt{1 - x}} \left(\frac{1}{2\sqrt{x}} \right)$
 $= \frac{1}{2\sqrt{x}(1 - x)}$

30. (a) $f(x) = \cos^{-1}(x^2 + x)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} [\cos^{-1}(x^2 + x)]$.
 By using Chain rule,
 $f'(x) = \frac{-1}{\sqrt{1 - (x^2 + x)^2}} \frac{d}{dx} (x^2 + x)$
 $= \frac{-(2x + 1)}{\sqrt{1 - (x^2 + x)^2}}$

(b) $f(x) = \cos^{-1}\left(\frac{2}{x}\right)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} \left[\cos^{-1}\left(\frac{2}{x}\right) \right]$
 By using Chain rule,
 $f'(x) = \frac{-1}{\sqrt{1 - \left(\frac{2}{x}\right)^2}} \frac{d}{dx} \left(\frac{2}{x} \right)$
 $= \frac{-1}{\sqrt{1 - \left(\frac{4}{x^2}\right)}} \left(\frac{-2}{x^2} \right)$
 $= \frac{2}{x\sqrt{x^2 - 4}}$





31. (a) $f(x) = \tan^{-1}(\sqrt{x})$
 Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} [\tan^{-1}(\sqrt{x})].$$

 By the Chain rule,

$$f'(x) = \frac{1}{1 + (\sqrt{x})^2} \frac{d}{dx} (\sqrt{x})$$

$$= \frac{1}{(1+x)} \left(\frac{1}{2\sqrt{x}} \right)$$

$$= \frac{1}{2\sqrt{x}(1+x)}$$

(b) $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$
 Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} \left[\tan^{-1}\left(\frac{1}{x}\right) \right].$$

 By the Chain rule,

$$f'(x) = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \frac{d}{dx} \left(\frac{1}{x} \right)$$

$$= \frac{1}{\left(1 + \frac{1}{x^2}\right)} \left(\frac{-1}{x^2} \right)$$

$$= \frac{-1}{(x^2 + 1)}$$

32. (a) $f(x) = \sqrt{2 + \tan^{-1}x}$
 Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} \left(\sqrt{2 + \tan^{-1}x} \right).$$

 By the Chain rule,

$$f'(x) = \frac{1}{2\sqrt{2 + \tan^{-1}x}} \frac{d}{dx} (2 + \tan^{-1}x)$$

$$= \frac{1}{2\sqrt{2 + \tan^{-1}x}} \left(\frac{1}{1 + x^2} \right)$$

$$= \frac{1}{2(1 + x^2)\sqrt{2 + \tan^{-1}x}}$$

(b) $f(x) = e^{\tan^{-1}x}$
 Differentiating with respect to x ,

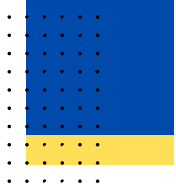
$$f'(x) = \frac{d}{dx} \left(e^{\tan^{-1}x} \right).$$

 By the Chain rule,

$$f'(x) = \left(e^{\tan^{-1}x} \right) \left(\frac{1}{1 + x^2} \right)$$

$$= \frac{e^{\tan^{-1}x}}{1 + x^2}$$





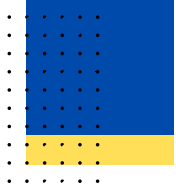
33. (a) $f(x) = 4 \sec(x^4)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} (4 \sec(x^4))$
 By Chain rule,
 $f'(x) = 4 \sec(x^4) \tan(x^4) \frac{d}{dx} (x^4)$
 $= 4 \sec(x^4) \tan(x^4) (4x^3)$
 $= 16x^3 \sec(x^4) \tan(x^4)$

(b) $f(x) = 4 \sec^{-1}(x^4)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} (4 \sec^{-1}(x^4))$
 By Chain rule,
 $f'(x) = 4 \frac{1}{x^4 \sqrt{(x^4)^2 - 1}} \frac{d}{dx} (x^4)$
 $= 4 \frac{1}{x^4 \sqrt{x^8 - 1}} (4x^3)$
 $= \frac{16}{x \sqrt{x^8 - 1}}$

34. (a) $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} \left(\sin^{-1}\left(\frac{1}{x}\right) \right)$
 By the Chain rule,
 $f'(x) = \frac{1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}} \frac{d}{dx} \left(\frac{1}{x} \right)$
 $= \frac{x}{\sqrt{x^2 - 1}} \left(\frac{-1}{x^2} \right)$
 $= -\frac{1}{x \sqrt{x^2 - 1}}$

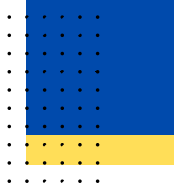
(b) $f(x) = \csc^{-1}(x)$
 Differentiating with respect to x ,
 $f'(x) = \frac{d}{dx} \csc^{-1}(x)$
 By the Chain rule,
 $f'(x) = -\frac{1}{x \sqrt{x^2 - 1}}$





1. (a) $f(x) = \sqrt{x}, x_0 = 1$
 $f(x_0) = f(1) = \sqrt{1} = 1$
 $f'(x) = \frac{1}{2}x^{-1/2}$
 $f'(x_0) = f'(1) = \frac{1}{2}$
 So,
 $L(x) = f(x_0) + f'(x_0)(x - x_0)$
 $= 1 + \frac{1}{2}(x - 1)$
 $= \frac{1}{2} + \frac{1}{2}x$
- (b) Using the approximation $L(x)$ to estimate $\sqrt{1.2}$, we get $\sqrt{1.2} = f(1.2) \approx L(1.2) = \frac{1}{2} + \frac{1}{2}(1.2) = 1.1$
2. (a) $f(x_0) = f(0) = 1$ and
 $f'(x) = \frac{1}{3}(x + 1)^{-2/3}$
 So, $f'(0) = \frac{1}{3}$
 The Linear approximation is,
 $L(x) = 1 + \frac{1}{3}(x - 0) = 1 + \frac{1}{3}x$
- (b) Using the approximation $L(x)$ to estimate $\sqrt[3]{1.2}$, we get $\sqrt[3]{1.2} = f(0.2) \approx L(0.2) = 1 + \frac{1}{3}(0.2) = 1.066$

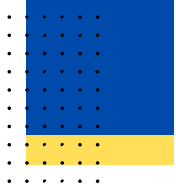




3. (a) $f(x) = \sqrt{2x+9}$, $x_0 = 0$
 $f(x_0) = f(0) = \sqrt{2 \cdot 0 + 9} = 3$
 $f'(x) = \frac{1}{2}(2x+9)^{-1/2} \cdot 2$
 $= (2x+9)^{-1/2}$
 $f'(x_0) = f'(0) = (2 \cdot 0 + 9)^{-1/2} = \frac{1}{3}$
 So,
 $L(x) = f(x_0) + f'(x_0)(x - x_0)$
 $= 3 + \frac{1}{3}(x - 0)$
 $= 3 + \frac{1}{3}x$
- (b) Using the approximation $L(x)$ to estimate $\sqrt{8.8}$, we get $\sqrt{8.8} = f(-0.1) \approx$
 $L(-0.1) = 3 + \frac{1}{3}(-0.1) = 3 - 0.033 = 2.967$

4. (a) $f(x) = \frac{2}{x}$, $x_0 = 1$
 $f(x_0) = f(1) = 2$
 $f'(x) = -\frac{2}{x^2}$ and so $f'(1) = -2$
 The linear approximation is
 $L(x) = 2 + (-2)(x - 1)$
- (b) Using the approximation $L(x)$ to estimate $\frac{2}{0.99}$, we get $\frac{2}{0.99} = f(0.99) \approx L(0.99) =$
 $2 + (-2)(0.99 - 1) = 2.02$





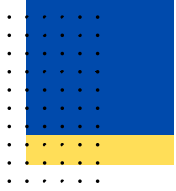
5. (a) $f(x) = \sin 3x$, $x_0 = 0$
 $f(x_0) = \sin(3 \cdot 0) = 0$
 $f'(x) = 3 \cos 3x$
 $f'(x_0) = f'(0) = 3 \cos(3 \cdot 0) = 3$
 So,
 $L(x) = f(x_0) + f'(x_0)(x - x_0)$
 $= 0 + 3(x - 0)$
 $= 3x$

(b) Using the approximation $L(x)$ to estimate $\sin(0.3)$, we get $\sin(0.3) = f(0.1) \approx L(0.1) = 3(0.1) = 0.3$

6. (a) $f(x) = \sin x$, $x_0 = \pi$
 $f(x_0) = \sin \pi = 0$
 $f'(x) = \cos x$
 $f'(x_0) = f'(\pi) = \cos \pi = -1$
 The linear approximation is,
 $L(x) = f(x_0) + f'(x_0)(x - x_0)$
 $= 0 + (-1)(x - \pi) = \pi - x$

(b) Using the approximation $L(x)$ to estimate $\sin(3.0)$, we get $\sin(3.0) = f(3.0) \approx L(3.0) = \pi - 3.0$





$$\begin{aligned}
 1. \quad \lim_{x \rightarrow -2} \frac{x+2}{x^2-4} \\
 &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x-2)} \\
 &= \lim_{x \rightarrow -2} \frac{1}{x-2} = -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \lim_{x \rightarrow 2} \frac{x^2-4}{x^2-3x+2} \\
 &= \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-2)(x-1)} \\
 &= \lim_{x \rightarrow 2} \frac{x+2}{x-1} = 4
 \end{aligned}$$

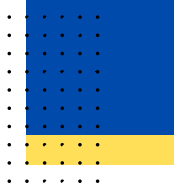
$$\begin{aligned}
 3. \quad \lim_{x \rightarrow \infty} \frac{3x^2+2}{x^2-4} \\
 &= \lim_{x \rightarrow \infty} \frac{3+\frac{2}{x^2}}{1-\frac{4}{x^2}} \\
 &= \frac{3}{1} = 3
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \lim_{x \rightarrow -\infty} \frac{x+1}{x^2+4x+3} \text{ is type } \frac{\infty}{\infty}; \\
 \text{we apply L'Hôpital's Rule to get} \\
 \lim_{x \rightarrow -\infty} \frac{1}{2x+4} = 0.
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \lim_{t \rightarrow 0} \frac{e^{2t}-1}{t} \text{ is type } \frac{0}{0}; \\
 \text{we apply L'Hôpital's Rule to get} \\
 \lim_{t \rightarrow 0} \frac{\frac{d}{dt}(e^{2t}-1)}{\frac{d}{dt}t} \\
 \lim_{t \rightarrow 0} \frac{2e^{2t}}{1} = \frac{2}{1} = 2
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \lim_{t \rightarrow 0} \frac{\sin t}{e^{3t}-1} \text{ is type } \frac{0}{0}; \\
 \text{we apply L'Hôpital's Rule to get} \\
 \lim_{t \rightarrow 0} \frac{\frac{d}{dt}(\sin t)}{\frac{d}{dt}(e^{3t}-1)} = \lim_{t \rightarrow 0} \frac{\cos t}{3e^{3t}} = \frac{1}{3}
 \end{aligned}$$





7. $\lim_{t \rightarrow 0} \frac{\tan^{-1} t}{\sin t}$ is type $\frac{0}{0}$;
we apply L'Hôpital's Rule to get
$$\lim_{t \rightarrow 0} \frac{\frac{d}{dt}(\tan^{-1} t)}{\frac{d}{dt}(\sin t)} = \lim_{t \rightarrow 0} \frac{1/(1+t^2)}{\cos t} = 1$$

8. $\lim_{t \rightarrow 0} \frac{\sin t}{\sin^{-1} t}$ is type $\frac{0}{0}$;
we apply L'Hôpital's Rule to get
$$\lim_{t \rightarrow 0} \frac{\frac{d}{dt}(\sin t)}{\frac{d}{dt}(\sin^{-1} t)} = \lim_{t \rightarrow 0} \frac{\cos t}{1/(\sqrt{1-t^2})} = 1$$

9. $\lim_{x \rightarrow \pi} \frac{\sin 2x}{\sin x}$ is type $\frac{0}{0}$;

we apply L'Hôpital's Rule to get

$$\lim_{x \rightarrow \pi} \frac{2 \cos 2x}{\cos x} = \frac{2(1)}{-1} = -2.$$

10. $\lim_{x \rightarrow -1} \frac{\cos^{-1} x}{x^2 - 1}$ is undefined (numerator goes to π , denominator goes to 0).

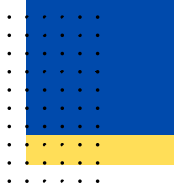
11. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$ is type $\frac{0}{0}$;
we apply L'Hôpital's Rule thrice to get
$$= \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} = \lim_{x \rightarrow 0} \frac{-\sin x}{6x}$$
$$= \lim_{x \rightarrow 0} \frac{-\cos x}{6} = -\frac{1}{6}.$$

12. $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ is type $\frac{0}{0}$;
we apply L'Hôpital's Rule to get
$$\lim_{x \rightarrow 0} \frac{\sec^2 x - 1}{3x^2}.$$

Apply L'Hôpital's Rule twice more to get

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 x \tan x}{6x}$$
$$= \lim_{x \rightarrow 0} \frac{4 \sec^2 x \tan^2 x + 2 \sec^4 x}{6} = \frac{1}{3}.$$





15	Use l'Hopital's rule to compute limits in various cases استخدم قاعدة لوبيتال لحساب النهايات في حالات مختلفة	(1-12)	247
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13. $\lim_{t \rightarrow 1} \frac{\sqrt{t} - 1}{t - 1} = \lim_{t \rightarrow 1} \frac{\sqrt{t} - 1}{t - 1} \cdot \frac{\sqrt{t} + 1}{\sqrt{t} + 1}$

$$= \lim_{t \rightarrow 1} \frac{(t - 1)}{(t - 1)\sqrt{t} + 1}$$

$$= \lim_{t \rightarrow 1} \frac{1}{\sqrt{t} + 1} = \frac{1}{2}$$

14. $\lim_{t \rightarrow 1} \frac{\ln t}{t - 1}$ is type $\frac{0}{0}$;

we apply L'Hôpital's Rule to get

$$\lim_{t \rightarrow 1} \frac{\frac{d}{dt}(\ln t)}{\frac{d}{dt}(t - 1)} = \lim_{t \rightarrow 1} \frac{\frac{1}{t}}{1} = 1$$

15. $\lim_{x \rightarrow \infty} \frac{x^3}{e^x}$ is type $\frac{\infty}{\infty}$;

we apply L'Hôpital's Rule thrice to get

$$\lim_{x \rightarrow \infty} \frac{3x^2}{e^x} = \lim_{x \rightarrow \infty} \frac{6x}{e^x}$$

$$= \lim_{x \rightarrow \infty} \frac{6}{e^x} = 0.$$

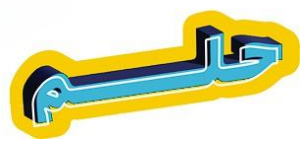
16. $\lim_{x \rightarrow \infty} \frac{e^x}{x^4}$ is type $\frac{\infty}{\infty}$;

we apply L'Hôpital's Rule four times to get

$$\lim_{x \rightarrow \infty} \frac{e^x}{4x^3} = \lim_{x \rightarrow \infty} \frac{e^x}{12x^2}$$

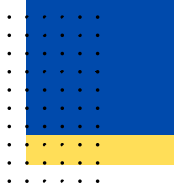
$$= \lim_{x \rightarrow \infty} \frac{e^x}{24x} = \lim_{x \rightarrow \infty} \frac{e^x}{24} = \infty.$$

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16	a) Determine the continuity of a function at a given point.	(29-32)&(39-41)	P95,96
	a) البحث في اتصال دالة عند نقطة معطاة		
	b) Find limits at infinity and limits that are infinite.	(67-71)	107
	b) إيجاد النهايات التي تؤول إلى اللانهاية والنهايات عند اللانهاية		

$$29. \lim_{x \rightarrow 10000^-} T(x) = \lim_{x \rightarrow 10000^-} 0.14x$$

$$= 0.14(10,000) = 1400$$

$$\lim_{x \rightarrow 10000^+} T(x) = \lim_{x \rightarrow 10000^+} (c + 0.21x)$$

$$= c + 0.21(10,000)$$

$$= c + 2100$$

$$c + 2100 = 1400$$

$$c = -700$$

A small change in income should not result in a big change in tax, so the tax function should be continuous.

$$30. \text{ If } \lim_{x \rightarrow 0^+} T(x) = 0, \text{ then } a = 0.$$

$$\text{If } \lim_{x \rightarrow 20,000} T(x) \text{ exists, then } b \text{ must be } 2400.$$



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FRQ Part

16

a) Determine the continuity of a function at a given point.

(29-32)&(39-41)

P95,96

a) البحث في اتصال دالة عند نقطة معطاة

b) Find limits at infinity and limits that are infinite.

(67-71)

107

b) إيجاد النهايات التي تؤول إلى اللانهاية والنهايات عند اللانهاية

31. For $T(x)$ to be continuous at $x = 141,250$ we must have

$$\lim_{x \rightarrow 141,250^-} T(x) = \lim_{x \rightarrow 141,250^+} T(x).$$

Now

$$\begin{aligned} \lim_{x \rightarrow 141,250^-} T(x) &= \lim_{x \rightarrow 141,250^-} (.30)(x)a \\ &= (.30)(141,250) - 5685 \\ &= 36690. \end{aligned}$$

On the other hand,

$$\begin{aligned} \lim_{x \rightarrow 141,250^+} T(x) &= \lim_{x \rightarrow 141,250^+} (.35)(x) - b \\ &= (.35)(141,250) - b \\ &= 49437.50 - b. \end{aligned}$$

Hence

$$b = 49437.50 - 36690 = 12,747.50.$$

For $T(x)$ to be continuous at $x = 307,050$ we must have

$$\lim_{x \rightarrow 307,050^-} T(x) = \lim_{x \rightarrow 307,050^+} T(x).$$

Now

$$\begin{aligned} \lim_{x \rightarrow 307,050^-} T(x) &= \lim_{x \rightarrow 307,050^-} (.35)(x) - b \\ &= (.35)(307,050) - 12,747.5 \\ &= 94,720. \end{aligned}$$

On the other hand,

$$\begin{aligned} \lim_{x \rightarrow 307,050^+} T(x) &= \lim_{x \rightarrow 307,050^+} (.386)(x) - c \\ &= (.386)(307,050) - c \\ &= 118,521.3 - c. \end{aligned}$$

Hence

$$c = 118,521.3 - 94,720 = 23,801.3.$$

$$32. \lim_{x \rightarrow 6,000^-} T(x) = \lim_{x \rightarrow 6,000^-} 0.10x = \$600.$$

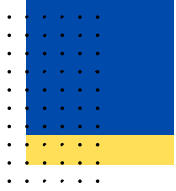
$$\lim_{x \rightarrow 6,000^+} T(x) = \lim_{x \rightarrow 6,000^+} 0.15x - 300 = \$600.$$

$$\text{So } T(6,000) = \$600 = \lim_{x \rightarrow 6,000} T(x), \text{ and}$$

$T(x)$ is continuous at $x = 6,000$.

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16	a) Determine the continuity of a function at a given point.	(29-32)&(39-41)	P95,96
	البحث في اتصال دالة عند نقطة معطاة		
	b) Find limits at infinity and limits that are infinite.	(67-71)	107
	إيجاد النهايات التي تؤول إلى اللانهاية والنهايات عند اللانهاية		

$$39. \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 2 \frac{\sin x}{x} = 2 \lim_{x \rightarrow 0^-} \frac{\sin x}{x} = 2$$

Hence a must equal 2 if f is continuous.

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} b \cos x = b \lim_{x \rightarrow 0^-} \cos x = b,$$

so b and a must equal 2 if f is continuous.

$$40. \text{ We need } ae^0 + 1 = \sin^{-1} 0, \text{ so } a = -1.$$

$$\text{We need } 2^2 - 2 + b = \sin^{-1} 1, \text{ so } b = \frac{\pi}{2} - 2.$$

41. First note that

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \ln(x-2) + x^2 = \ln(3-2) + 3^2 = 9.$$

$$\text{Also } f(3) = 2e^{3b} + 1,$$

so if f is continuous, $2e^{3b} + 1$ must equal 9;

that is $e^{3b} = 4$, so $b = \frac{\ln 4}{3}$. Then note that

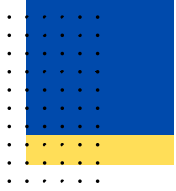
$$f(0) = 2e^{(b)(0)} + 1 = 3.$$

Also,

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} a(\tan^{-1} x + 2) \\ &= a(\tan^{-1} 0 + 2) \\ &= a(0 + 2) = 2a, \end{aligned}$$

so a must equal $3/2$ if f is continuous.





16

a) Determine the continuity of a function at a given point.

(29-32)&(39-41)

P95,96

a) البحث في اتصال دالة عند نقطة معطاة

b) Find limits at infinity and limits that are infinite.

(67-71)

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b) إيجاد النهايات التي تؤول إلى اللانهاية والنهايات عند اللانهاية

$$67. h(0) = \frac{300}{1 + 9(.8^0)} = \frac{300}{10} = 30 \text{ mm}$$

$$\lim_{t \rightarrow \infty} \frac{300}{1 + 9(.8^t)} = 300 \text{ mm}$$

68. Length at $t = 0$ is $h(0) = 20$ mm. Length eventually is $\lim_{t \rightarrow \infty} h(t) = 50$ mm.

$$69. \lim_{t \rightarrow \infty} v_N = \lim_{t \rightarrow \infty} \frac{Ft}{m} = \infty$$

$$\lim_{t \rightarrow \infty} v_E = \lim_{t \rightarrow \infty} \frac{Fct}{\sqrt{m^2c^2 + F^2t^2}}$$

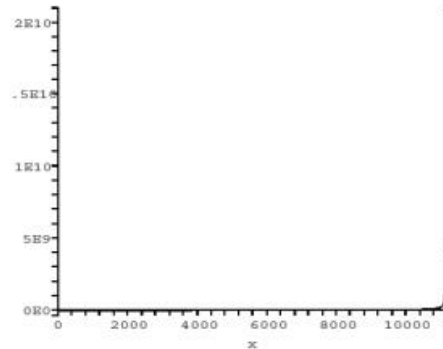
$$= \lim_{t \rightarrow \infty} \frac{Fct}{t\sqrt{\frac{m^2c^2}{t^2} + F^2}}$$

$$= \lim_{t \rightarrow \infty} \frac{Fc}{\sqrt{\frac{m^2c^2}{t^2} + F^2}}$$

$$= \frac{Fc}{\sqrt{F^2}} = c$$

70. $f(t) \rightarrow 0$ as $t \rightarrow 0$ and $t \rightarrow \infty$. This makes sense because the drug will require some time to reach the muscles, and should wear off over time.

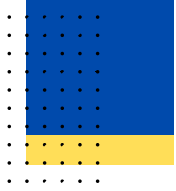
71. We must restrict the domain to $v_0 \geq 0$ because the formula makes sense only if the rocket is launched upward. To find v_e , set $19.6R - v_0^2 = 0$. Using $R \approx 6,378,000$ meters, we get $v_0 = \sqrt{19.6R} \approx 11,180$ m/s. If the rocket is launched with initial velocity $\geq v_e$, it will never return to earth; hence v_e is called the escape velocity.



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17

a) Find the derivative of a function at a given point.

ايجاد المشتقة لدالة عند نقطة ما

Write the equation of a tangent line using derivative

اكتب معادلة الخط المماس باستخدام المشتقة

(5-12)

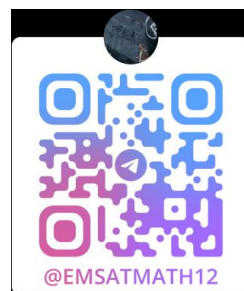
153

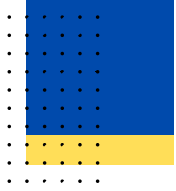
$$\begin{aligned}
 5. \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 + 1 - (3x^2 + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 1 - (3x^2 + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\
 &= \lim_{h \rightarrow 0} (6x + 3h) = 6x
 \end{aligned}$$

$$\begin{aligned}
 6. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) + 1 - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 2h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(2x + h - 2)}{h} = 2x - 2
 \end{aligned}$$

$$\begin{aligned}
 7. \lim_{b \rightarrow x} \frac{f(b) - f(x)}{b - x} \\
 &= \lim_{b \rightarrow x} \frac{b^3 + 2b - 1 - (x^3 + 2x - 1)}{b - x} \\
 &= \lim_{b \rightarrow x} \frac{(b-x)(b^2 + bx + x^2 + 2)}{b - x} \\
 &= \lim_{b \rightarrow x} \frac{b^2 + bx + x^2 + 2}{1} \\
 &= 3x^2 + 2
 \end{aligned}$$

$$\begin{aligned}
 8. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^4 - 2(x+h)^2 + 1 - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} [4x^3 + 6x^2h + 4xh^2 + h^3 - 4x - 2h] \\
 &= 4x^3 - 4x
 \end{aligned}$$





17

a) Find the derivative of a function at a given point.

ايجاد المشتقة لدالة عند نقطة ما

Write the equation of a tangent line using derivative

اكتب معادلة الخط المماس باستخدام المشتقة

(5-12)

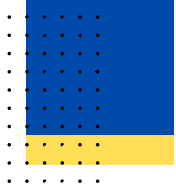
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$$\begin{aligned}
 9. \lim_{b \rightarrow x} \frac{f(b) - f(x)}{b - x} \\
 &= \lim_{b \rightarrow x} \frac{\frac{3}{b+1} - \frac{3}{x+1}}{b - x} \\
 &= \lim_{b \rightarrow x} \frac{\frac{3(x+1) - 3(b+1)}{(b+1)(x+1)}}{b - x} \\
 &= \lim_{b \rightarrow x} \frac{-3(b - x)}{(b+1)(x+1)(b - x)} \\
 &= \lim_{b \rightarrow x} \frac{-3}{(b+1)(x+1)} \\
 &= \frac{-3}{(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 10. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{2}{2(x+h)-1} - \frac{2}{2x-1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{2(2x-1) - 2(2x+2h-1)}{(2x+2h-1)(2x-1)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-4h}{(2x+2h-1)(2x-1)} \\
 &= \lim_{h \rightarrow 0} \frac{-4}{(2x+2h-1)(2x-1)} \\
 &= \frac{-4}{(2x-1)^2}
 \end{aligned}$$

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17

a) Find the derivative of a function at a given point.

ا) إيجاد المشتقة لدالة عند نقطة ما

Write the equation of a tangent line using derivative

اكتب معادلة الخط المماس باستخدام المشتقة

(5-12)

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11. $f(t) = \sqrt{3t+1}$

$$f'(t) = \lim_{b \rightarrow t} \frac{f(b) - f(t)}{(b-t)}$$

$$= \lim_{b \rightarrow t} \frac{\sqrt{3b+1} - \sqrt{3t+1}}{(b-t)}$$

Multiplying by $\frac{\sqrt{3b+1} + \sqrt{3t+1}}{\sqrt{3b+1} + \sqrt{3t+1}}$ gives

$$f'(t) = \lim_{b \rightarrow t} \frac{(3b+1) - (3t+1)}{(b-t)(\sqrt{3b+1} + \sqrt{3t+1})}$$

$$= \lim_{b \rightarrow t} \frac{3(b-t)}{(b-t)(\sqrt{3b+1} + \sqrt{3t+1})}$$

$$= \lim_{b \rightarrow t} \frac{3}{\sqrt{3b+1} + \sqrt{3t+1}}$$

$$= \frac{3}{2\sqrt{3t+1}}$$

12. $f(t) = \sqrt{2t+4}$

$$f'(t) = \lim_{b \rightarrow t} \frac{f(b) - f(t)}{(b-t)}$$

$$= \lim_{b \rightarrow t} \frac{\sqrt{2b+4} - \sqrt{2t+4}}{(b-t)}$$

Multiplying by $\frac{\sqrt{2b+4} + \sqrt{2t+4}}{\sqrt{2b+4} + \sqrt{2t+4}}$ gives

$$f'(t) = \lim_{b \rightarrow t} \frac{(2b+4) - (2t+4)}{(b-t)(\sqrt{2b+4} + \sqrt{2t+4})}$$

$$= \lim_{b \rightarrow t} \frac{2(b-t)}{(b-t)(\sqrt{2b+4} + \sqrt{2t+4})}$$

$$= \lim_{b \rightarrow t} \frac{2}{\sqrt{2b+4} + \sqrt{2t+4}}$$

$$= \frac{2}{2\sqrt{2t+4}} = \frac{1}{\sqrt{2t+4}}$$

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$$\begin{aligned}
 5. \quad \frac{d}{dx}(x^2y^2 + 3y) &= \frac{d}{dx}(4x) \\
 2xy^2 + x^2 2yy' + 3y' &= 4 \\
 y'(2x^2y + 3) &= 4 - 2xy^2 \\
 y' &= \frac{4 - 2xy^2}{2x^2y + 3}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad 3y^3 + 3x(3y^2)y' - 4 &= 20yy' \\
 (9xy^2 - 20y)y' &= 4 - 3y^3 \\
 y' &= \frac{3y^3 - 4}{20y - 9xy^2}
 \end{aligned}$$

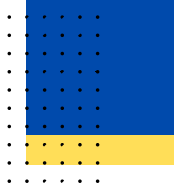
$$\begin{aligned}
 7. \quad \frac{d}{dx}(\sqrt{xy} - 4y^2) &= \frac{d}{dx}(12) \\
 \frac{1}{2\sqrt{xy}} \cdot \frac{d}{dx}(xy) - 8y \cdot y' &= 0 \\
 \frac{1}{2\sqrt{xy}} \cdot (xy' + y) - 8y \cdot y' &= 0 \\
 (xy' + y) - 16y \cdot y' \sqrt{xy} &= 0 \\
 y'(x - 16y\sqrt{xy}) &= -y \\
 y' &= \frac{-y}{(x - 16y\sqrt{xy})} = \frac{y}{16y\sqrt{xy} - x}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \cos(xy)(y + xy') &= 2x \\
 y' &= \frac{2x - y \cos(xy)}{x \cos(xy)}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad x + 3 &= 4xy + y^3 \\
 1 &= \frac{d}{dx}(4xy + y^3) = 4(xy' + y) + 3y^2y' \\
 1 - 4y &= (4x + 3y^2)y' \\
 y' &= \frac{1 - 4y}{3y^2 + 4x}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad 3x + y^3 - \frac{4y}{x+2} &= 10x^2 \\
 \text{Differentiating with respect to } x, \\
 \frac{d}{dx} \left(3x + y^3 - \frac{4y}{x+2} \right) &= \frac{d}{dx} (10x^2) \\
 \text{By the Chain rule and Product rule,} \\
 3 + 3y^2y' - \left[\frac{(x+2)4y' - 4y}{(x+2)^2} \right] &= 20x \\
 3(x+2)^2 + 3y^2y'(x+2)^2 &= 20x(x+2)^2 \\
 -4y'(x+2) + 4y &= 20x(x+2)^2 \\
 3y^2y'(x+2)^2 - 4y'(x+2) &= 20x(x+2)^2 - 3(x+2)^2 - 4y \\
 y'(x+2) [3y^2(x+2) - 4] &= (x+2)^2 (20x - 3) - 4y \\
 y' &= \frac{(x+2)^2 (20x - 3) - 4y}{(x+2) [3y^2(x+2) - 4]}
 \end{aligned}$$



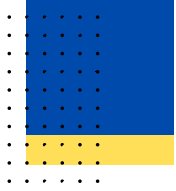


$$\begin{aligned}
 11. \quad & \frac{d}{dx}(e^{x^2y} - e^y) = \frac{d}{dx}(x) \\
 & e^{x^2y} \frac{d}{dx}(e^{x^2y}) - e^y y' = 1 \\
 & e^{x^2y}(2xy + x^2 y') - e^y y' = 1 \\
 & y'(x^2 e^{x^2y} - e^y) = 1 - 2xy e^{x^2y} \\
 & y' = \frac{1 - 2xy e^{x^2y}}{(x^2 e^{x^2y} - e^y)}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & e^y + x e^y y' - 3y' \sin x - 3y \cos x = 0 \\
 & y' = \frac{3y \cos x - e^y}{x e^y - 3 \sin x}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & y^2 \sqrt{x+y} - 4x^2 = y \\
 & \text{Differentiating with respect to } x, \\
 & \frac{d}{dx}(y^2 \sqrt{x+y} - 4x^2) = \frac{d}{dx}(y) \\
 & \text{By the Chain rule and Product rule,} \\
 & \frac{d}{dx}(y^2 \sqrt{x+y}) - 4 \frac{d}{dx}(x^2) = \frac{d}{dx}(y) \\
 & \left[y^2 \left(\frac{1}{2\sqrt{x+y}} \right) (1 + y') \right] \\
 & + 2y y' \sqrt{x+y} - 8x = y' \\
 & y^2 + y^2 y' + 4y y' (x+y) - 16x \sqrt{x+y} \\
 & = 2y' \sqrt{x+y} \\
 & y^2 y' + 4y y' (x+y) - 2y' \sqrt{x+y} \\
 & = 16x \sqrt{x+y} - y^2 \\
 & y' [y^2 + 4y(x+y) - 2\sqrt{x+y}] \\
 & = 16x \sqrt{x+y} - y^2 \\
 & y' = \frac{16x \sqrt{x+y} - y^2}{y^2 + 4y(x+y) - 2\sqrt{x+y}}
 \end{aligned}$$





14. $x \cos(x + y) - y^2 = 8$

Diffrentiating with respect to x ,

$$\frac{d}{dx} (x \cos(x + y) - y^2) = \frac{d}{dx} (8)$$

By the Chain rule and Product rule,

$$\frac{d}{dx} (x \cos(x + y)) - \frac{d}{dx} (y^2) = \frac{d}{dx} (8)$$

$$\cos(x + y) - x \sin(x + y) (1 + y') - 2yy' = 0$$

$$\cos(x + y) - x \sin(x + y) - x \sin(x + y) y' - 2yy' = 0$$

$$y' (-x \sin(x + y) - 2y)$$

$$= x \sin(x + y) - \cos(x + y)$$

$$y' = \frac{x \sin(x + y) - \cos(x + y)}{-x \sin(x + y) - 2y}$$

$$y' = \frac{\cos(x + y) - x \sin(x + y)}{x \sin(x + y) + 2y}$$

15. $e^{4y} - \ln(y^2 + 3) = 2x$

Diffrentiating with respect to x ,

$$\frac{d}{dx} (e^{4y} - \ln(y^2 + 3)) = \frac{d}{dx} (2x)$$

By the Chain rule and Product rule,

$$\frac{d}{dx} (e^{4y}) - \frac{d}{dx} (\ln(y^2 + 3)) = \frac{d}{dx} (2x)$$

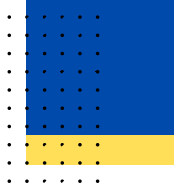
$$e^{4y} (4y') - \frac{2yy'}{y^2 + 3} = 2$$

$$4e^{4y} (y^2 + 3) y' - 2yy' = 2 (y^2 + 3)$$

$$y' (4e^{4y} (y^2 + 3) - 2y) = 2 (y^2 + 3)$$

$$y' = \frac{2 (y^2 + 3)}{4e^{4y} (y^2 + 3) - 2y}$$





16. $e^{x^2}y - 3\sqrt{y^2 + 2} = x^2 + 1$

Differentiating with respect to x ,

$$\frac{d}{dx} (e^{x^2}y - 3\sqrt{y^2 + 2}) = \frac{d}{dx} (x^2 + 1)$$

By the Chain rule and Product rule,

$$\frac{d}{dx} (e^{x^2}y) - 3 \frac{d}{dx} (\sqrt{y^2 + 2}) = 2x$$

$$e^{x^2} (2x)y + e^{x^2}y' - 3 \cdot \frac{2yy'}{2\sqrt{y^2 + 2}} = 2x$$

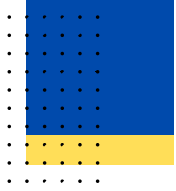
$$2xye^{x^2} + e^{x^2}y' - \frac{3yy'}{\sqrt{y^2 + 2}} = 2x$$

$$2xye^{x^2}\sqrt{y^2 + 2} + e^{x^2}y'\sqrt{y^2 + 2} - 3yy' = 2x\sqrt{y^2 + 2}$$

$$y' (e^{x^2}\sqrt{y^2 + 2} - 3y) = 2x\sqrt{y^2 + 2} - 2xye^{x^2}\sqrt{y^2 + 2}$$

$$y' = \frac{2x\sqrt{y^2 + 2} (1 - ye^{x^2})}{e^{x^2}\sqrt{y^2 + 2} - 3y}$$





30. $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\ln x} = 0$ (numerator goes to 0 and denominator goes to $-\infty$).

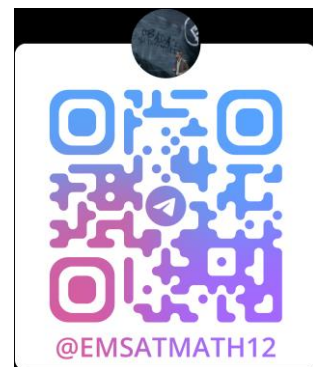
31. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$
 $= \lim_{x \rightarrow \infty} \left((\sqrt{x^2 + 1} - x) \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1} + x} \right)$
 $= \lim_{x \rightarrow \infty} \left(\frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} \right)$
 $= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2 + 1} + x} = 0$

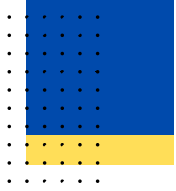
32. $\lim_{x \rightarrow \infty} \ln x - x = \lim_{x \rightarrow \infty} \frac{\frac{\ln x}{x} - 1}{\frac{1}{x}} = -\infty$ since the numerator goes to -1 and the denominator goes to 0^+ . (Recall Example 2.8 which shows $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$.)

33. Let $y = \left(1 + \frac{1}{x}\right)^x$
 $\Rightarrow \ln y = x \ln \left(1 + \frac{1}{x}\right)$. Then

$$\begin{aligned} \lim_{x \rightarrow \infty} \ln y &= \lim_{x \rightarrow \infty} x \ln \left(1 + \frac{1}{x}\right) \\ &= \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{1}{x}\right)}{\frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{1}{x}} \left(-\frac{1}{x^2}\right)}{-1/x^2} \\ &= \lim_{x \rightarrow \infty} \frac{1}{1 + \frac{1}{x}} = 1. \end{aligned}$$

Hence $\lim_{x \rightarrow \infty} y = \lim_{x \rightarrow \infty} e^{\ln y} = e$.





34. Notice that the limit in question has the indeterminate form 1^∞ . Also, note that as x gets large, $\left| \frac{x+1}{x-2} \right| = \frac{x+1}{x-2}$.

Define $y = \left(\frac{x+1}{x-2} \right)^{\sqrt{x^2-4}}$. Then

$$\ln y = \sqrt{x^2-4} \ln \left(\frac{x+1}{x-2} \right) \text{ and}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \ln y &= \lim_{x \rightarrow \infty} \left(\sqrt{x^2-4} \ln \left(\frac{x+1}{x-2} \right) \right) \\ &= \lim_{x \rightarrow \infty} \left(\frac{\ln \left(\frac{x+1}{x-2} \right)}{\frac{1}{\sqrt{x^2-4}}} \right) \end{aligned}$$

This last limit has indeterminate form $\frac{0}{0}$, so we can apply L'Hôpital's Rule and simplify to find that the above is equal to

$$\lim_{x \rightarrow \infty} \frac{-3(x^2-4)^{3/2}}{-x^3+x^2+2x} \text{ and this is equal to 3. So}$$

$$\lim_{x \rightarrow \infty} \ln y = 3.$$

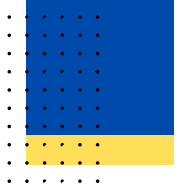
$$\text{Thus } \lim_{x \rightarrow \infty} y = \lim_{x \rightarrow \infty} e^{\ln y} = e^3 \approx 20.086.$$

$$\begin{aligned} 35. \lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{x+1}} \right) \\ &= \lim_{x \rightarrow 0^+} \left(\frac{\sqrt{x+1} - (\sqrt{x})^2}{\sqrt{x}\sqrt{x+1}} \right) \\ &= \lim_{x \rightarrow 0^+} \left(\frac{\sqrt{x+1} - x}{\sqrt{x}\sqrt{x+1}} \right) \\ &= \infty. \end{aligned}$$

$$\begin{aligned} 36. \lim_{x \rightarrow 1} \frac{\sqrt{5-x} - 2}{\sqrt{10-x} - 3} \text{ is type } \frac{0}{0} \\ \text{we apply L'Hôpital's Rule to get} \\ \lim_{x \rightarrow 1} \frac{\frac{1}{2}(5-x)^{-1/2}(-1)}{\frac{1}{2}(10-x)^{-1/2}(-1)} \\ = \lim_{x \rightarrow 1} \frac{\sqrt{10-x}}{\sqrt{5-x}} = \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} 37. \text{ Let } y = (1/x)^x. \text{ Then } \ln y = x \ln(1/x). \text{ Then} \\ \lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} x \ln(1/x) = 0, \text{ by Exercise} \\ 27. \text{ Thus } \lim_{x \rightarrow 0^+} y = \lim_{x \rightarrow 0^+} e^{\ln y} = 1. \end{aligned}$$





38. Let $y = \lim_{x \rightarrow 0^+} (\cos x)^{1/x}$. Then
- $$\ln y = \lim_{x \rightarrow 0^+} \frac{1}{x} \ln \cos x$$
- $$= \lim_{x \rightarrow 0^+} \frac{\ln(\cos x)}{x} \text{ is type } \frac{0}{0}$$
- so apply L'Hôpital's Rule to get
- $$\lim_{x \rightarrow 0^+} \frac{-\tan x}{1} = 0.$$
- Therefore the limit is $y = e^0 = 1$.

$$= \lim_{t \rightarrow \infty} \frac{\left(1 - \frac{3}{t}\right)^t}{2^t \left(1 + \frac{1/2}{t}\right)^t} = \lim_{t \rightarrow \infty} \frac{e^{-3}}{2^t e^{1/2}} = 0$$

39. $\lim_{t \rightarrow \infty} \left(\frac{t-3}{t+2}\right)^t = \lim_{t \rightarrow \infty} \frac{(t-3)^t}{(t+2)^t}$
- $$= \lim_{t \rightarrow \infty} \frac{\left(1 - \frac{3}{t}\right)^t}{\left(1 + \frac{2}{t}\right)^t} = \frac{\lim_{t \rightarrow \infty} \left(1 - \frac{3}{t}\right)^t}{\lim_{t \rightarrow \infty} \left(1 + \frac{2}{t}\right)^t}$$
- $$= \frac{\lim_{t \rightarrow \infty} \left(1 + \frac{-3}{t}\right)^t}{\lim_{t \rightarrow \infty} \left(1 + \frac{2}{t}\right)^t} = \frac{e^{-3}}{e^2} = e^{-5}$$
40. $\lim_{t \rightarrow \infty} \left(\frac{t-3}{2t+1}\right)^t = \lim_{t \rightarrow \infty} \left(\frac{1 - \frac{3}{t}}{2 + \frac{1}{t}}\right)^t$



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*Dear Students ,
Please make sure to study all
the required lessons in the Final
Exam !*

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