

1	Prove theorems about line segments by using the Segment Addition Postulate	1 to 8	743 +744
	Prove theorems about line segments by using properties of segment congruence.		

Example 1

1. **PROOF** Write the correct statements and reasons to complete the two-column proof.

Given: C is the midpoint of $\overline{AE}$,
 C is the midpoint of $\overline{BD}$.

$$\overline{AE} \cong \overline{BD}$$

Prove: $\overline{AC} \cong \overline{CD}$

Proof:



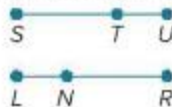
Statements	Reasons
1. _____ ? C is the midpoint of $\overline{AE}$.	1. Given
_____ ? C is the midpoint of $\overline{BD}$.	
_____ ? $\overline{AE} \cong \overline{BD}$	
2. $AC = CE$ $BC = CD$	2. _____ ? Definition of midpoint
3. $AE = BD$	3. _____ ? Definition of $\cong$ segments
4. _____ ? $AE = AC + CE$ _____ ? $BD = BC + CD$	4. Segment Addition Property
5. $AC + CE = BC + CD$	5. _____ ? Substitution Property
6. $AC + AC = CD + CD$	6. _____ ? Substitution Property
7. _____ ? $2AC = 2CD$	7. Simplify.
8. _____ ? $AC = CD$	8. Division Property
9. $\overline{AC} \cong \overline{CD}$	9. _____ ? Definition of $\cong$ segments

2. **PROOF** Write the correct statements and reasons to complete the two-column proof.

Given: $\overline{SU} \cong \overline{LR}$
 $\overline{TU} \cong \overline{LN}$

Prove: $\overline{ST} \cong \overline{NR}$

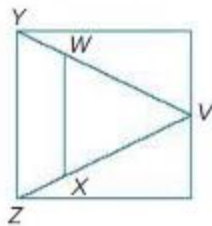
Proof:



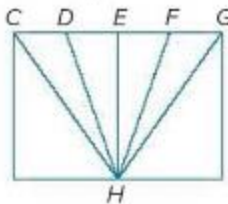
Statements	Reasons
1. $\overline{SU} \cong \overline{LR}, \overline{TU} \cong \overline{LN}$	1. _____ ? Given
2. _____ ? $SU = LR, TU = LN$	2. Definition of $\cong$ segments
3. $SU = ST + TU$ $LR = LN + NR$	3. _____ ? Segment Addition Postulate
4. $ST + TU = LN + NR$	4. _____ ? Substitution Property
5. $ST + LN = LN + NR$	5. _____ ? Substitution Property
6. $ST + LN - LN = LN + NR - LN$	6. _____ ? Subtraction Property
7. _____ ? $ST = NR$	7. Substitution Property
8. $\overline{ST} \cong \overline{NR}$	8. _____ ? Definition of $\cong$ segments

PROOF Write a two-column proof to prove each geometric relationship.

3. If $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$, then $\overline{VW} \cong \overline{VX}$.



4. If E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$, then $\overline{CE} \cong \overline{EG}$.



Example 3

5. **FAMILY** Maria is 11 inches shorter than her sister Clara. Luna is 11 inches shorter than her brother Chad. If Maria is shorter than Luna, how do the heights of Clara and Chad compare? What else can be concluded if Maria and Luna are the same height? **Clara is shorter than Chad when Maria is shorter than Luna; Clara and Chad are the same height when Maria is the same height as Luna.**
6. **LUMBER** Byron works in a lumberyard. His boss just cut a dozen planks and asked Byron to double check that they are all the same length. The planks were numbered 1 through 12. Byron took out plank number 1 and checked that the other planks are all the same length as plank 1. He concluded that they must all be the same length. Explain how you know that plank 7 and plank 10 are the same length even though they were never directly compared to each other. **Plank 7 is the same length as plank 1, and plank 1 is the same length as plank 10. By the Transitive Property, plank 7 must be the same length as plank 10.**
7. **NEIGHBORHOODS** Karla, Lola, and Mandy live in three houses that are on the same line. Lola lives between Karla and Mandy. Karla and Mandy live a mile apart. Is it possible for Lola's house to be a mile from both Karla's and Mandy's houses? **No, it's not possible. Lola's house must be less than a mile from each house because she lives between them.**

Mixed Exercises

8. **PROOF** Five lights, A, B, C, D , and E , are aligned in a row. The middle light is the midpoint of the segment between the second and fourth lights and also the midpoint of the segment between the first and last lights.

- a. Draw a figure to illustrate the situation. **See margin.**
- b. Complete this proof.

Given: C is the midpoint of $\overline{BD}$ and $\overline{AE}$.

Prove: $AB = DE$

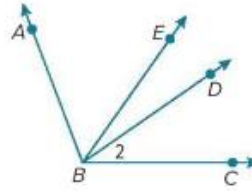
Statement	Reason
1. C is the midpoint of $\overline{BD}$ and $\overline{AE}$.	1. Given
2. $BC = CD$ and $AC = CE$	2. $?$ Def. of mdpt.
3. $AC = AB + BC$, $CE = CD + DE$	3. $?$ Seg. Add. Post.
4. $AC - BC = AB$	4. $?$ Subtr. Prop.
5. $CE - CD = AB$	5. Substitution Property
6. $CE - CD = DE$	6. $?$ Subtr. Prop.
7. $AB = CE - CD$	7. Symmetric Property of Equality

2	Prove theorems about angles by using the Angle Addition Postulate..	1 to 5	753
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Example 1

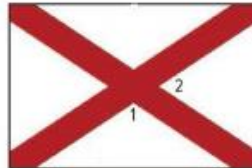
Find the measure of each angle.

- Find $m\angle ABC$ if $m\angle ABD = 70^\circ$ and $m\angle DBC = 43^\circ$. **113°**
- If $m\angle EBC = 55^\circ$ and $m\angle EBD = 20^\circ$, find $m\angle 2$. **35°**
- Find $m\angle ABD$ if $m\angle ABC = 110^\circ$ and $m\angle 2 = 36^\circ$. **74°**



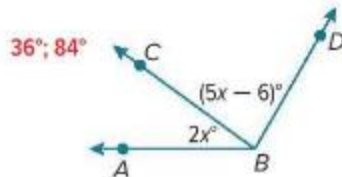
Example 2

- FLAGS** The Alabama state flag is white and has two diagonal red stripes. If the $m\angle 1 = 112^\circ$, what is $m\angle 2$? **68°**
- CONSTRUCTION** Aaron has installed a new window above the entrance of an office building. If $m\angle 2 = 44^\circ$, what is $m\angle 1$? **46°**



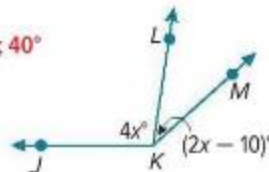
3	Prove theorems about angles by using properties and theorems of angle congruence.	11 to 18	754
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- Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120^\circ$.
- Find $m\angle JKL$ and $m\angle LKM$ if $m\angle JKM = 140^\circ$.



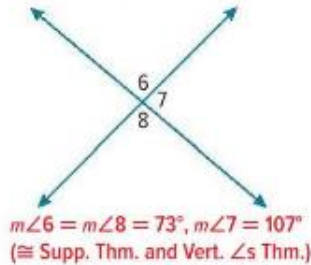
$36^\circ; 84^\circ$

$100^\circ; 40^\circ$



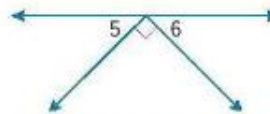
Find the measure of each numbered angle and name the theorems that you used to justify your work.

- $m\angle 6 = (2x - 21)^\circ$
 $m\angle 7 = (3x - 34)^\circ$



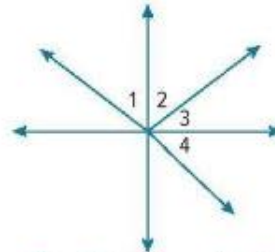
**$m\angle 6 = m\angle 8 = 73^\circ, m\angle 7 = 107^\circ$
($\cong$ Supp. Thm. and Vert. $\angle$ s Thm.)**

- $m\angle 5 = m\angle 6$



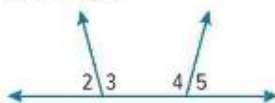
**$m\angle 5 = m\angle 6 = 45^\circ$
($\cong$ Supp. Thm.)**

- $\angle 2$ and $\angle 3$ are complementary.
 $\angle 1 \cong \angle 4$ and $m\angle 2 = 28^\circ$.



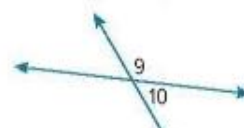
**$m\angle 3 = 62^\circ, m\angle 1 = m\angle 4 = 45^\circ$
($\cong$ Comp. and Supp. Thm.)**

- $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary.
 $m\angle 4 = 105^\circ$.



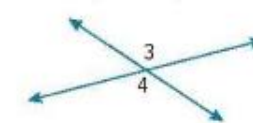
**$m\angle 2 = 75^\circ, m\angle 3 = 105^\circ, m\angle 5 = 75^\circ$
($\cong$ Supp. Thm.)**

- $m\angle 9 = (3x + 12)^\circ$
 $m\angle 10 = (x - 24)^\circ$



**$m\angle 9 = 156^\circ, m\angle 10 = 24^\circ$
($\cong$ Supp. Thm.)**

- $m\angle 3 = (2x + 23)^\circ$
 $m\angle 4 = (5x - 112)^\circ$

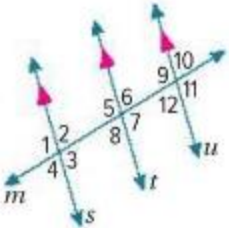


**$m\angle 3 = m\angle 4 = 113^\circ$
(Vert. $\angle$ s Thm.)**

5	Find values by applying theorems about parallel lines and transversals.	21 to 31	762
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In the figure, $m\angle 7 = 100^\circ$. Find the measure of each angle.

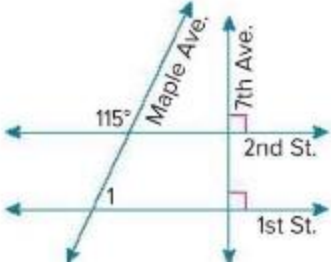
21. $\angle 9$ 100° 22. $\angle 6$ 80°
 23. $\angle 8$ 80° 24. $\angle 2$ 80°
 25. $\angle 5$ 100° 26. $\angle 11$ 100°



27. **RAMPS** A parking garage ramp rises to connect two horizontal levels of a parking lot. The ramp makes a 10° angle with the horizontal. What is the measure of angle 1 in the figure? 170°



28. **CITY ENGINEERING** Seventh Avenue runs perpendicular to 1st and 2nd Streets, which are parallel. However, Maple Avenue makes a 115° angle with 2nd Street. What is the measure of angle 1? 65°



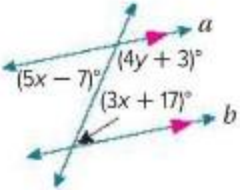
Example 5

Find the value of the variables in each figure. Explain your reasoning.

29. See margin.

30. See margin.

31. Find the value of the variables in the figure. $x = 12, y = 31$



Mixed Exercises

6	Classify lines as parallel, perpendicular, or neither by comparing the slopes of the lines.	1 to 9	771
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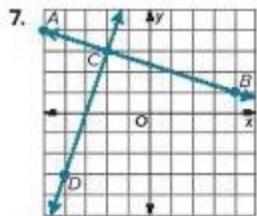
Example 1

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer. See margin for graphs.

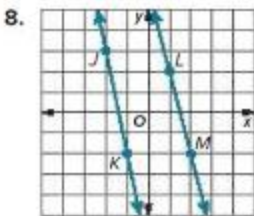
- $A(1, 5), B(4, 4), C(9, -10), D(-6, -5)$
parallel
- $A(-6, -9), B(8, 19), C(0, -4), D(2, 0)$
parallel
- $A(4, 2), B(-3, 1), C(6, 0), D(-10, 8)$
neither
- $A(8, -2), B(4, -1), C(3, 11), D(-2, -9)$
perpendicular
- $A(8, 4), B(4, 3), C(4, -9), D(2, -1)$
perpendicular
- $A(4, -2), B(-2, -8), C(4, 6), D(8, 5)$
neither

Example 2

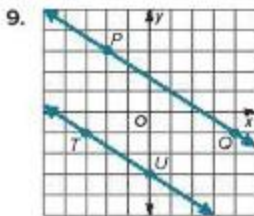
Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.



perpendicular



neither



parallel

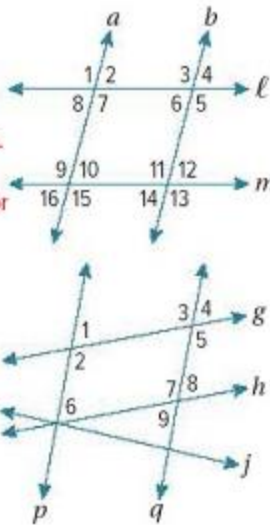
7	Apply angle relationship theorems to identify parallel lines and find missing values.	1 to 13	779
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Use the given information to determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

- $\angle 3 \cong \angle 7$
 $a \parallel b$; Alternate Interior Angles Converse
- $\angle 9 \cong \angle 11$
 $a \parallel b$; Converse of Corresponding Angles Thm.
- $\angle 2 \cong \angle 16$
 $\ell \parallel m$; Alternate Exterior Angles Converse
- $m\angle 5 + m\angle 12 = 180^\circ$
 $\ell \parallel m$; Consecutive Interior Angles Converse

Given the following information, determine which lines, if any, are parallel. State the theorem that justifies your answer.

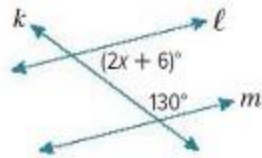
- $\angle 1 \cong \angle 6$
 $g \parallel h$; Converse of Corresponding Angles Thm.
- $m\angle 7 + m\angle 6 = 180^\circ$
 $p \parallel q$; Consecutive Interior Angles Converse



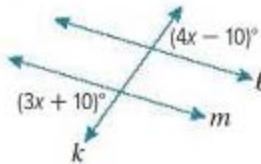
Example 3

Find the value of x so that $\ell \parallel m$.

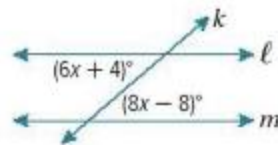
7. 22



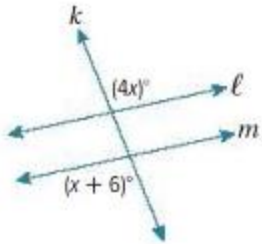
8. 20



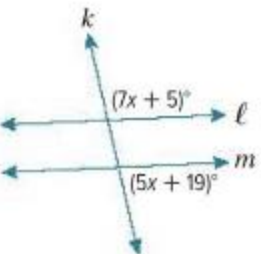
9. 6



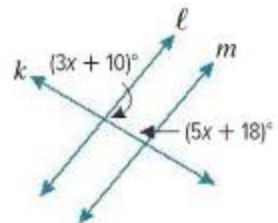
10. 2



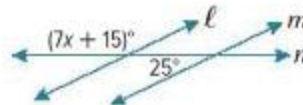
11. 13



12. 19



13. Find the value of x so that $\ell \parallel m$. 20



8	Find the distance between parallel lines by using perpendicular distance.	9 to 17	790
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Find the distance between each pair of parallel lines with the given equations.

9. $y = 7$
 $y = -1$
 8 units

10. $x = -6$
 $x = 5$
 11 units

11. $y = 3x$
 $y = 3x + 10$
 $\sqrt{10}$ or about 3.16 units

12. $y = -5x$
 $y = -5x + 26$
 $\sqrt{26}$ or about 5.10 units

13. $y = x + 9$
 $y = x + 3$
 $3\sqrt{2}$ or about 4.24 units

14. $y = -2x + 5$
 $y = -2x - 5$
 $2\sqrt{5}$ or about 4.47 units

15. $y = \frac{1}{4}x + 2$
 $4y - x = -60$
 $4\sqrt{17}$ or about 16.49 units

16. $3x + y = 3$
 $y + 17 = -3x$
 $2\sqrt{10}$ or about 6.32 units

17. $y = -\frac{5}{4}x + 3.5$
 $4y + 10.6 = -5x$
 $\sqrt{14.76}$ or about 3.84 units

9	Use symmetry to describe the reflections that carry a figure onto itself.	1 to 6	829
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Example 1
Determine whether each figure has a line of symmetry. If so, draw the lines of symmetry and state how many lines of symmetry it has. 1–2, 5–6. See margin for graphs.

1. yes; 3


2. yes; 2


3. no


4. no


5. yes; 5


6. yes; 8

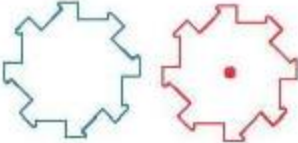


10	Use rotational symmetry to describe the rotations that carry a figure onto itself.	11 to 14	830
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Determine whether each figure has rotational symmetry. If so, locate the center of symmetry, and state the order and magnitude of symmetry.

11. yes; 3; 120°


12. yes; 8; 45°


13. yes; 2; 180°

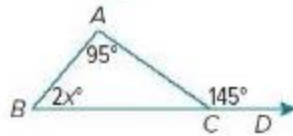

14. no



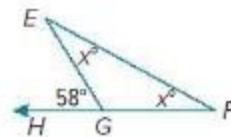
11	Prove the Exterior Angle Theorem and apply the theorem to solve problems.	5 to 8	843
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Find each measure.

5. $m\angle ABC$ **50°**



6. $m\angle F$ **29°**



7. **TOWERS** A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower. If $m\angle 1 = (7x - 7)^\circ$, $m\angle 2 = (4x + 2)^\circ$, and $m\angle 3 = (2x + 6)^\circ$, what is $m\angle 1$? **98°**



8. **GARDENING** A gardener uses a grow light to grow vegetables indoors. If $m\angle 1 = (8x)^\circ$ and $m\angle 2 = (7x - 4)^\circ$, what is $m\angle 1$? **64°**



Example 3

Find each measure.

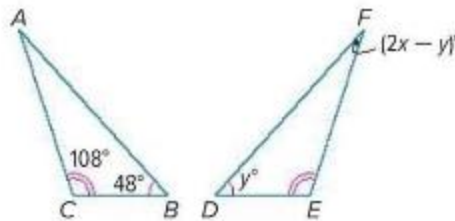
12	Use the Third Angles Theorem and the properties of triangle congruence to solve problems and to prove relationships in geometric figures.	6 to 17	851 + 852
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Example 2

In the diagram, $\triangle ABC \cong \triangle FDE$.

6. Find the value of x . **36**

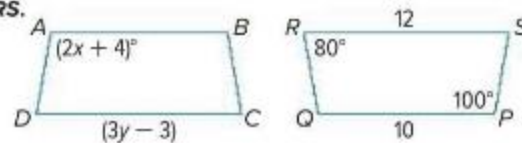
7. Find the value of y . **48**



In the diagram, polygon $ABCD \cong$ polygon $PQRS$.

8. Find the value of x . **48**

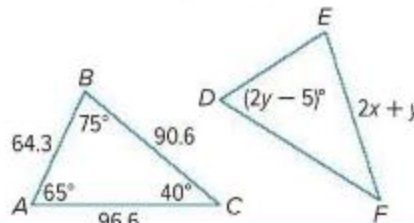
9. Find the value of y . **5**



In the diagram, $\triangle ABC \cong \triangle DEF$.

10. Find the value of x . **27.8**

11. Find the value of y . **35**



Example 3

12. **DESIGN** Camila is designing a new image for her cell phone case. If $m\angle ABC = 35^\circ$, $m\angle BAC = 29^\circ$, and $\angle ACB \cong \angle DEB$, what is $m\angle DEB$?

116°



13. **CARPENTRY** Mr. Lewis is building a rustic dining table. Instead of having four legs, the table has a set of supports at each end. If $\angle PRQ \cong \angle TVU$ and $m\angle RPQ = 49^\circ$, what is $m\angle TVU$? 41°



Example 4

PROOF For 14–16, write a two-column proof.

14. **Given:** $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, $\angle BAD \cong \angle BCD$, $\overline{BD}$ bisects $\angle ABC$.

Prove: $\triangle ABD \cong \triangle CBD$

Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{CB}$; $\overline{AD} \cong \overline{CD}$ (Given)

2. $\overline{BD} \cong \overline{BD}$ (Reflexive Prop. of Congruence)

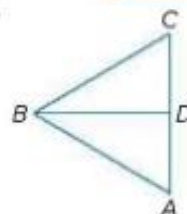
3. $\angle BAD \cong \angle BCD$ (Given)

4. $\overline{BD}$ bisects $\angle ABC$. (Given)

5. $\angle ABD \cong \angle CBD$ (Def. of angle bisector)

6. $\angle BDA \cong \angle BDC$ (Third Angles Theorem)

7. $\triangle ABD \cong \triangle CBD$ (Def. of congruent triangles)



15. **Given:** $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$, $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$

Prove: $\triangle ABD \cong \triangle CBD$

Proof:

Statements (Reasons)

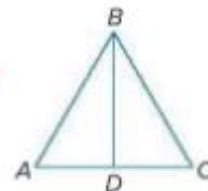
1. $\overline{AB} \cong \overline{CB}$; $\overline{AD} \cong \overline{CD}$ (Given)

2. $\overline{BD} \cong \overline{BD}$ (Reflexive Prop. of Congruence)

3. $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$ (Given)

4. $\angle A \cong \angle C$ (Third Angles Theorem)

5. $\triangle ABD \cong \triangle CBD$ (Def. of congruent triangles)



16. **Given:** $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\overline{AD} \cong \overline{CB}$, $\overline{AE} \cong \overline{CE}$, $\overline{AC}$ bisects $\overline{BD}$.

Prove: $\triangle AED \cong \triangle CEB$

Proof:

Statements (Reasons)

1. $\angle A \cong \angle C$, $\angle D \cong \angle B$ (Given)

2. $\angle AED \cong \angle CEB$ (Vertical angles are $\cong$.)

3. $\overline{AD} \cong \overline{CB}$; $\overline{AE} \cong \overline{CE}$ (Given)

4. $\overline{AC}$ bisects $\overline{BD}$. (Given)

5. $\overline{DE} \cong \overline{BE}$ (Def. of segment bisector)

6. $\triangle AED \cong \triangle CEB$ (Def. of congruent triangles)



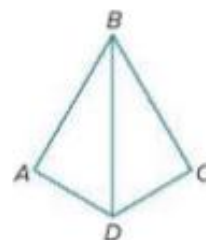
PROOF Write a paragraph proof.

17. **Given:** $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$, $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

Prove: $\triangle ABD \cong \triangle CBD$

Proof:

It is given that $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$. Therefore, $\angle ABD \cong \angle CBD$ and $\angle ADB \cong \angle CDB$ by the definition of angle bisector. By the Third Angles Theorem, $\angle A \cong \angle C$. It is given that $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$. By the Reflexive Property of Congruence, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ABD \cong \triangle CBD$ by the definition of congruent triangles.



13	Use the SSS congruence criterion for triangles to solve problems and prove relationships in geometric figures.	1 to 16	859 + 860
	Use the SAS congruence criterion for triangles to solve problems and prove relationship		

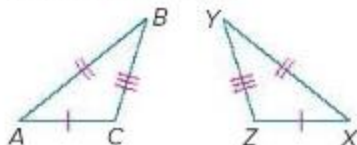
Example 1

PROOF Write the specified type of proof.

1. two-column proof

Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

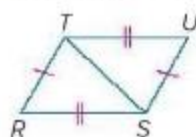
Prove: $\triangle ABC \cong \triangle XYZ$



2. flow proof

Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

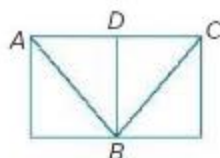
Prove: $\triangle RST \cong \triangle UTS$



3. two-column proof

Given: $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$.

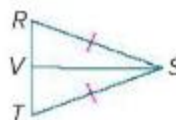
Prove: $\triangle ABD \cong \triangle CBD$



4. flow proof

Given: $\overline{RS} \cong \overline{TS}$, V is the midpoint of $\overline{RT}$.

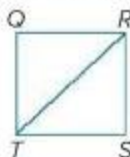
Prove: $\triangle RSV \cong \triangle TSV$



5. paragraph proof

Given: $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$

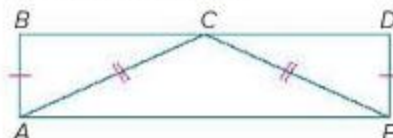
Prove: $\triangle QRT \cong \triangle SRT$



6. two-column proof

Given: $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$, $\overline{AC}$ bisects $\overline{BD}$

Prove: $\triangle ABC \cong \triangle EDC$



Example 2

REGULARITY Determine whether $\triangle DEF \cong \triangle PQR$. Explain.

7. $D(-6, 1)$, $E(1, 2)$, $F(-1, -4)$, $P(0, 5)$, $Q(7, 6)$, $R(5, 0)$ $DE = 5\sqrt{2}$, $PQ = 5\sqrt{2}$, $EF = 2\sqrt{10}$, $QR = 2\sqrt{10}$, $DF = 5\sqrt{2}$, $PR = 5\sqrt{2}$; $\triangle DEF \cong \triangle PQR$ by SSS because corresponding sides have the same measure and are congruent.
8. $D(-7, -3)$, $E(-4, -1)$, $F(-2, -5)$, $P(2, -2)$, $Q(5, -4)$, $R(0, -5)$ $DE = \sqrt{13}$, $PQ = \sqrt{13}$, $EF = 2\sqrt{5}$, $QR = \sqrt{26}$, $DF = \sqrt{29}$, $PR = \sqrt{13}$. Corresponding sides are not congruent, so $\triangle DEF$ is not congruent to $\triangle PQR$. There are no rigid motions that map one triangle onto the other.

Determine whether $\triangle ABC \cong \triangle KLM$. Explain.

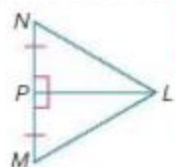
9. $A(-3, 3)$, $B(-1, 3)$, $C(-3, 1)$, $K(1, 4)$, $L(3, 4)$, $M(1, 6)$ $AB = 2$, $KL = 2$, $BC = 2\sqrt{2}$, $LM = 2\sqrt{2}$, $AC = 2$, $KM = 2$; The corresponding sides have the same measure and are congruent, so $\triangle ABC \cong \triangle KLM$ by SSS.
10. $A(-4, -2)$, $B(-4, 1)$, $C(-1, -1)$, $K(0, -2)$, $L(0, 1)$, $M(4, 1)$ $AB = 3$, $KL = 3$, $BC = \sqrt{13}$, $LM = 4$, $AC = \sqrt{10}$, $KM = 5$. The corresponding sides are not congruent, so $\triangle ABC$ is not congruent to $\triangle KLM$.

PROOF Write the specified type of proof.

11. two-column proof

Given: $NP = PM$, $\overline{NP} \perp \overline{PL}$

Prove: $\triangle NPL \cong \triangle MPL$



Proof:

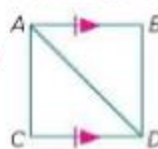
Statements (Reasons)

1. $NP = PM$, $\overline{NP} \perp \overline{PL}$ (Given)
2. $\overline{NP} \cong \overline{MP}$ (Def. of congruence)
3. $\angle MPL$ and $\angle NPL$ are rt. angles. ($\perp$ lines form rt. angles.)
4. $\angle MPL \cong \angle NPL$ (All right angles are congruent.)
5. $\overline{PL} \cong \overline{PL}$ (Reflexive Property of $\cong$)
6. $\triangle NPL \cong \triangle MPL$ (SAS)

12. two-column proof

Given: $AB = CD$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle DBA$



Proof:

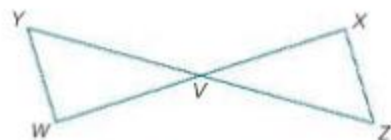
Statements (Reasons)

1. $AB = CD$, $\overline{AB} \parallel \overline{CD}$ (Given)
2. $\angle BAD \cong \angle CDA$ (Alternate Interior Angles Thm.)
3. $\overline{AD} \cong \overline{AD}$ (Reflexive Property)
4. $\overline{AB} \cong \overline{CD}$ (Def. of congruent segments)
5. $\triangle ACD \cong \triangle DBA$ (SAS)

13. paragraph proof

Given: V is the midpoint of $\overline{WX}$ and $\overline{YZ}$.

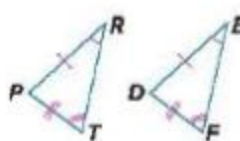
Prove: $\triangle XVZ \cong \triangle WVY$



14. flow proof

Given: $\overline{PR} \cong \overline{DE}$, $\overline{PT} \cong \overline{DF}$, $\angle R \cong \angle E$, $\angle T \cong \angle F$

Prove: $\triangle PRT \cong \triangle DEF$

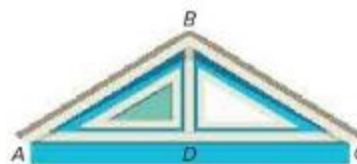


15. **GAMING** Devontae is building a house in a simulation video game. He wants the roof of the house and the main support beam to create congruent triangles. If $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ bisects $\overline{AC}$, write a two-column proof to prove $\triangle ABD \cong \triangle CBD$.

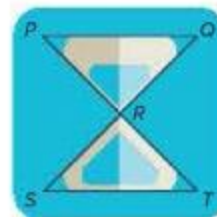
Proof:

Statements (Reasons)

1. $\overline{BD} \perp \overline{AC}$; $\overline{BD}$ bisects $\overline{AC}$. (Given)
2. $\angle BDA$ and $\angle BDC$ are rt. angles. ($\perp$ lines form rt. angles.)
3. $\angle BDA \cong \angle BDC$ (All right angles are congruent.)
4. $\overline{AD} \cong \overline{DC}$ (Def. of segment bisector)
5. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of $\cong$)
6. $\triangle ABD \cong \triangle CBD$ (SAS)



16. **TECHNOLOGY** Nevaeh has developed a new timer app. The icon for the app contains an hourglass that can be modeled by two triangles. If R is the midpoint of $\overline{QS}$ and $\overline{PT}$, write a paragraph proof to prove $\triangle PRQ \cong \triangle TRS$. See Mod. 14 Answer Appendix.

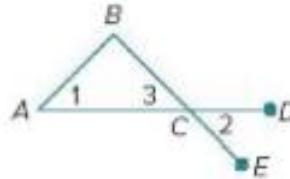


Example 1

1. **PROOF** Write a two-column proof. See Mod. 14 Answer Appendix.

Given: $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{CB}$

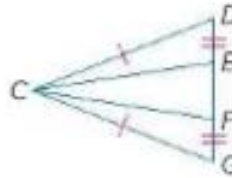


2. **PROOF** Write a two-column proof. See Mod. 14 Answer Appendix.

Given: $\overline{CD} \cong \overline{CG}$

$\overline{DE} \cong \overline{GF}$

Prove: $\overline{CE} \cong \overline{CF}$

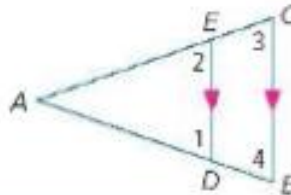


3. **PROOF** Write a two-column proof. See Mod. 14 Answer Appendix.

Given: $\overline{DE} \parallel \overline{BC}$

$\angle 1 \cong \angle 2$

Prove: $\overline{AB} \cong \overline{AC}$



4. **ROOFS** In the picture, $\overline{BD} \perp \overline{AC}$ and $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$. Write a two-column proof to prove that $\overline{BD}$ bisects the angle formed by the sloped sides of the roof, $\angle ABC$.

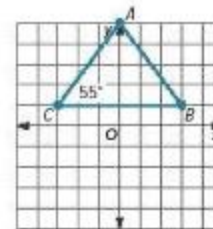
See Mod. 14 Answer Appendix.



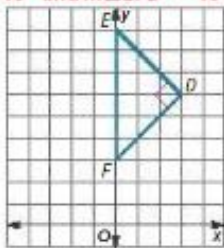
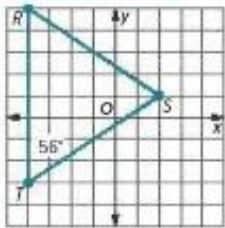
Example 2

5. Refer to the figure. See Mod. 14 Answer Appendix.

- Find the measures of the sides of $\triangle ABC$. Show your work.
- Find $m\angle A$. Show your work.

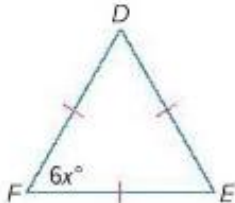
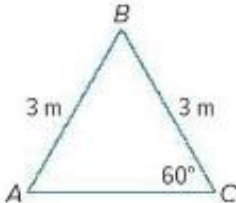
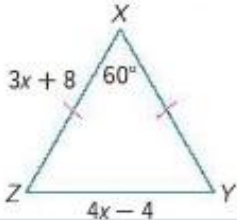


6. Find SR , ST , RT , $m\angle TRS$, and $m\angle RST$. Round to the nearest tenth, if necessary.
 $SR = 7.2$ units; $ST = 7.2$ units; $RT = 8$ units;
 $m\angle TRS = 56^\circ$; $m\angle RST = 68^\circ$
7. Find the measures of $\angle DEF$ and $\angle EFD$. Round to the nearest tenth, if necessary.
 $m\angle DEF = 45^\circ$ and $m\angle EFD = 45^\circ$

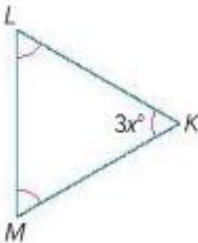
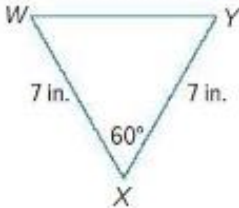


Examples 3 and 4

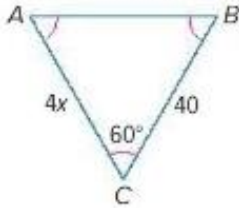
8. Find the value of x . **12**
9. Find $m\angle B$ and AC . **60° ; 3 m**
10. Find the value of x . **10**



11. Find $m\angle Y$ and WY . **60° ; 7 in.**
12. Find the value of x . **20**



13. Find the value of x . **10**

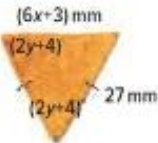


Find the value of each variable.

14. **CHIPS** Some tortilla chips can be modeled by a triangle.

a. Solve for x . **4**

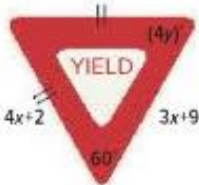
b. Solve for y . **28**



15. **SIGNS** Yield signs notify drivers to slow down and allow oncoming vehicles to proceed first.

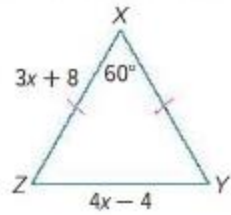
a. Solve for x . **7**

b. Solve for y . **15**

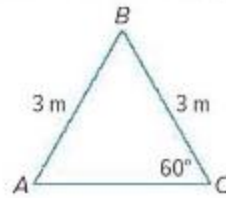


Examples 3 and 4

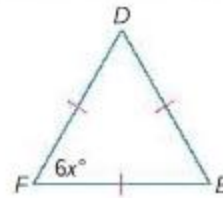
8. Find the value of x . 12



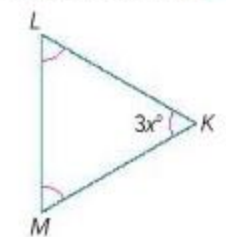
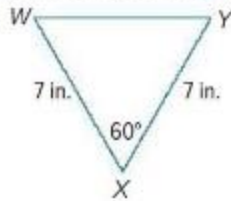
9. Find $m\angle B$ and AC . 60° ; 3 m



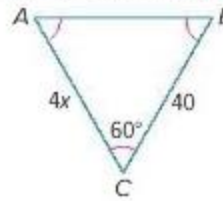
10. Find the value of x . 10



11. Find $m\angle Y$ and WY . 60° ; 7 in. 12. Find the value of x . 20



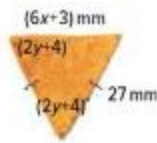
13. Find the value of x . 10



Find the value of each variable.

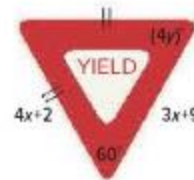
14. **CHIPS** Some tortilla chips can be modeled by a triangle.

- Solve for x . 4
- Solve for y . 28



15. **SIGNS** Yield signs notify drivers to slow down and allow oncoming vehicles to proceed first.

- Solve for x . 7
- Solve for y . 15

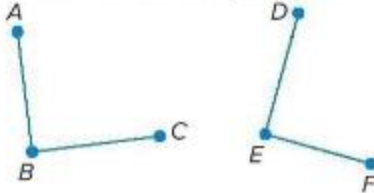


نهاية الأسئلة الموضوعية

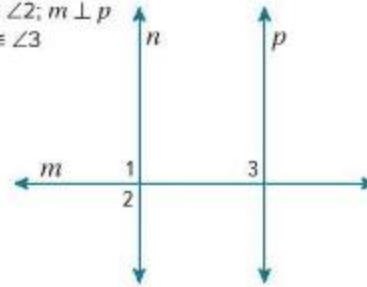
16	Prove theorems about angles by using the Angle Addition Postulate.	9 to 18	754
	Prove theorems about right angles.		

PROOF Write a two-column proof. 9–10. See margin.

9. Given: $m\angle ABC = m\angle DEF$
 $\angle ABC$ and $\angle DEF$ are supplementary.
 Prove: $\angle ABC$ and $\angle DEF$ are right angles.

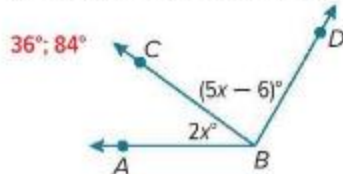


10. Given: $\angle 1 \cong \angle 2$; $m \perp p$
 Prove: $\angle 2 \cong \angle 3$



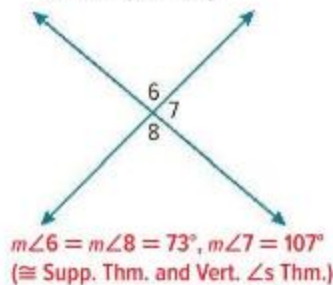
Mixed Exercises

11. Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120^\circ$. 12. Find $m\angle JKL$ and $m\angle LKM$ if $m\angle JKM = 140^\circ$.

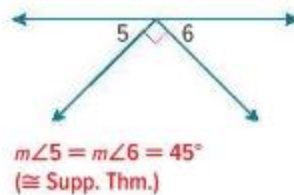


Find the measure of each numbered angle and name the theorems that you used to justify your work.

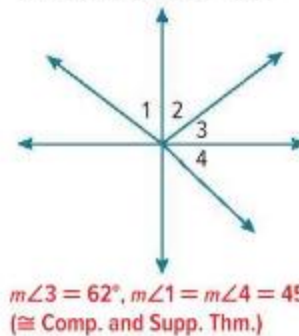
13. $m\angle 6 = (2x - 21)^\circ$
 $m\angle 7 = (3x - 34)^\circ$



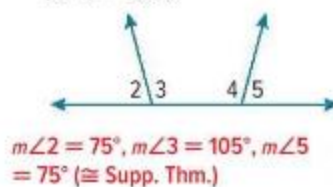
14. $m\angle 5 = m\angle 6$



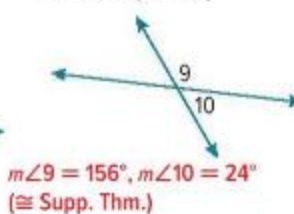
15. $\angle 2$ and $\angle 3$ are complementary.
 $\angle 1 \cong \angle 4$ and $m\angle 2 = 28^\circ$.



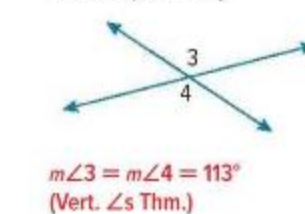
16. $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary.
 $m\angle 4 = 105^\circ$.



17. $m\angle 9 = (3x + 12)^\circ$
 $m\angle 10 = (x - 24)^\circ$

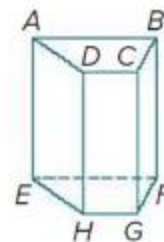


18. $m\angle 3 = (2x + 23)^\circ$
 $m\angle 4 = (5x - 112)^\circ$



Identify each of the following using the figure shown. Assume lines and planes that appear to be parallel or perpendicular are parallel or perpendicular, respectively.

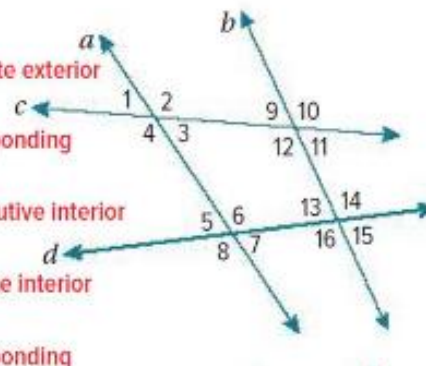
- three segments parallel to $\overline{AE}$
 $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$
- a segment skew to $\overline{AB}$
Sample answer: $\overline{EH}$
- a pair of parallel planes
 $ABCD$ and $EFGH$ or $ABFE$ and $CDHG$
- a segment parallel to $\overline{AD}$
 $\overline{EH}$
- three segments parallel to $\overline{HG}$
 $\overline{EF}$, $\overline{AB}$, and $\overline{DC}$
- five segments skew to $\overline{BC}$
 $\overline{AE}$, $\overline{EH}$, $\overline{HG}$, $\overline{EF}$, and $\overline{DH}$
- How could you characterize the relationship between faces $ABCD$ and $DCGH$? Explain. Sample answer: $ABCD$ and $DCGH$ could be characterized as perpendiculars, because $DCGH$ contains segment $\overline{CG}$, which is perpendicular to $ABCD$.



Examples 2 and 3

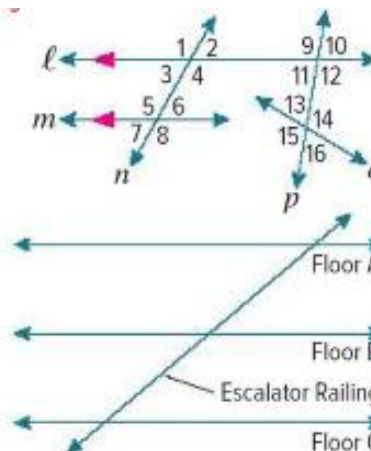
Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

- $\angle 4$ and $\angle 5$
line a ; consecutive interior
- $\angle 12$ and $\angle 14$
line b ; alternate interior
- $\angle 2$ and $\angle 12$
line c ; alternate interior
- $\angle 1$ and $\angle 9$
line c ; corresponding
- $\angle 10$ and $\angle 16$
line b ; alternate exterior
- $\angle 5$ and $\angle 15$
line d ; alternate exterior
- $\angle 7$ and $\angle 15$
line d ; corresponding
- $\angle 3$ and $\angle 6$
line a ; consecutive interior
- $\angle 3$ and $\angle 9$
line c ; alternate interior
- $\angle 5$ and $\angle 13$
line d ; corresponding



For Exercises 18 and 19, use the figure.

- What type of angles are $\angle 3$ and $\angle 10$?
alternate exterior angles
- State the transversal that connects $\angle 11$ and $\angle 13$. line p
- ESCALATORS** An escalator at a shopping mall runs up several levels. The escalator railing can be modeled by a straight line running past horizontal lines that represent the floors. Describe the relationships of these lines. The lines representing floors A, B, and C are parallel, and the escalator railing is a transversal intersecting each of the three lines.



18	Classify lines as parallel, perpendicular, or neither by comparing the equations of the lines.	10 to 20	771
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Example 3

Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.

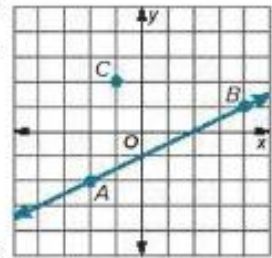
10. $y = 2x + 4$, $y = 2x - 10$
parallel
11. $y = -\frac{1}{2}x - 12$, $y - 3 = 2(x + 2)$
perpendicular
12. $y - 4 = 3(x + 5)$, $y + 3 = -\frac{1}{3}(x + 1)$
perpendicular
13. $y - 3 = 6(x + 2)$, $y + 3 = -\frac{1}{3}(x - 4)$
neither
14. $x = -2$, $y = 10$
perpendicular
15. $y = 5$, $y = -3$
parallel

Example 4

Graph the line that satisfies each condition. 16–19. See margin.

16. passes through $A(2, -5)$, parallel to $\overleftrightarrow{BC}$ with $B(1, 3)$ and $C(4, 5)$
17. passes through $X(1, -4)$, parallel to $\overleftrightarrow{YZ}$ with $Y(5, 2)$ and $Z(-3, -5)$
18. passes through $K(3, 7)$, perpendicular to $\overleftrightarrow{LM}$ with $L(-1, -2)$ and $M(-4, 8)$
19. passes through $D(-5, -6)$, perpendicular to $\overleftrightarrow{FG}$ with $F(-2, -9)$ and $G(1, -5)$

20. **SKIING** Gavin is working on an animated film about skiing. The figure shows a ski slope, represented by $\overleftrightarrow{AB}$, and one of the chairs on the chair lift, represented by point C.



- a. The chair needs to move along a straight line that is parallel to $\overleftrightarrow{AB}$. What is the equation of this line? $y = \frac{1}{2}x + \frac{5}{2}$
- b. The top of the chair lift occurs at $y = 20$. Explain how Gavin can find the coordinates of the chair when it reaches the top of the chair lift. See margin.

19	Prove the Triangle Angle-Sum Theorem and apply the theorem to solve problems.	1 to 4	843
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Example 1

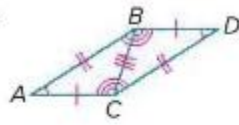
Find the measure of each numbered angle.

1. $m\angle 1 = 30^\circ$, $m\angle 2 = 60^\circ$
2. $m\angle 1 = 56^\circ$, $m\angle 2 = 56^\circ$, $m\angle 3 = 74^\circ$
3. $m\angle 1 = 109^\circ$, $m\angle 2 = 29^\circ$, $m\angle 3 = 71^\circ$
4. $m\angle 1 = m\angle 2 = 17^\circ$

Example 1

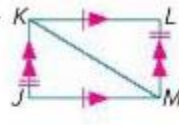
Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.

1.



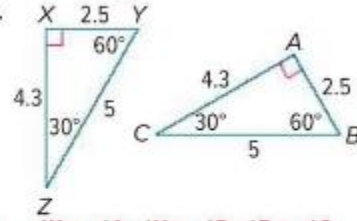
$\angle A \cong \angle D$; $\angle ABC \cong \angle DCB$;
 $\angle ACB \cong \angle DBC$; $\overline{AC} \cong \overline{DB}$;
 $\overline{AB} \cong \overline{DC}$; $\triangle ABC \cong \triangle DCB$

2.



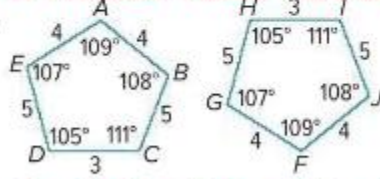
$\angle J \cong \angle L$; $\angle JKM \cong \angle LMK$;
 $\angle KML \cong \angle MJL$; $\overline{KM} \cong \overline{JL}$;
 $\overline{KL} \cong \overline{MJ}$; $\triangle JKM \cong \triangle LMK$

3.



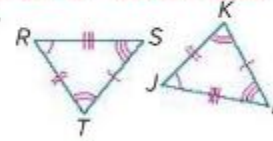
$\angle X \cong \angle A$; $\angle Y \cong \angle B$; $\angle Z \cong \angle C$;
 $\overline{XY} \cong \overline{AB}$; $\overline{XZ} \cong \overline{AC}$; $\overline{YZ} \cong \overline{BC}$;
 $\triangle XYZ \cong \triangle ABC$

4.



$\angle A \cong \angle F$; $\angle B \cong \angle J$; $\angle C \cong \angle I$; $\angle D \cong \angle H$;
 $\angle E \cong \angle G$; $\overline{AB} \cong \overline{FJ}$; $\overline{BC} \cong \overline{JI}$; $\overline{CD} \cong \overline{IH}$;
 $\overline{DE} \cong \overline{HG}$; $\overline{AE} \cong \overline{FG}$;
 polygon $ABCDE \cong$ polygon $FJIHG$

5.



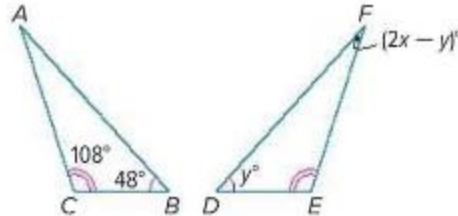
$\angle R \cong \angle J$; $\angle T \cong \angle K$; $\angle S \cong \angle L$;
 $\overline{RT} \cong \overline{JK}$; $\overline{TS} \cong \overline{KL}$; $\overline{RS} \cong \overline{JL}$; $\triangle RTS \cong \triangle JKL$

Example 2

In the diagram, $\triangle ABC \cong \triangle FDE$.

6. Find the value of x . 36

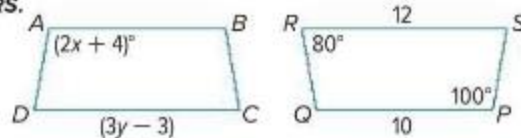
7. Find the value of y . 48



In the diagram, polygon $ABCD \cong$ polygon $PQRS$.

8. Find the value of x . 48

9. Find the value of y . 5



In the diagram, $\triangle ABC \cong \triangle DEF$.

10. Find the value of x . 27.8

11. Find the value of y . 35

