

EOT (Maths)

Term 2 (2024-2025)

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EOT (Maths)

2nd Part

Paper based Questions

Chapter 4 and 5

Analyzing The Marginal Cost of Producing a Commercial Product

Example 9.1 Page: 307

Suppose that $C(x) = 0.02x^2 + 2x + 4000$ is the **total cost** (in AED) for a company to produce x units of a certain product. Compute the **marginal cost at $x = 100$** and compare this to the **actual cost of producing the 100th unit**.



Analyzing The Marginal Cost of Producing a Commercial Product

Example 9.1 Page: 307

Suppose that $C(x) = 0.02x^2 + 2x + 4000$ is the **total cost** (in AED) for a company to produce x units of a certain product. Compute the **marginal cost at $x = 100$** and compare this to the **actual cost of producing the 100th unit**.

Total Cost: $C(x) = 0.02x^2 + 2x + 4000$

Marginal Cost Function: $C'(x) = 0.04x + 2$

(Derivative of Total Cost)

Marginal Cost at $x = 100$: $C'(100) = 0.04(100) + 2 = 6$ AED per unit



Total Cost for Producing 100 items

Actual Cost of Producing item number 100 = $C(100) - C(99)$

Total Cost for Producing 99 items

$$= 0.02(100)^2 + 2(100) + 4000 - (0.02(99)^2 + 2(99) + 4000)$$

$$= 200 + 200 + 4000 - 196.02 - 198 - 4000$$

$$= 5.98 \text{ AED}$$

*Which is very close to marginal cost of 6 AED per unit
Marginal Cost is easier to compute.*

Minimizing The Average Cost of Producing a Commercial Product

Example 9.2 Page: 307

Suppose that $C(x) = 0.02x^2 + 2x + 4000$ is the **total cost** (in dollars) for a company to produce x **units** of a certain product. Find the **production level** x that **minimize the average cost**.

16	Solve economical and scientific problems on extrema حل مسائل اقتصادية وعلمية على القيم القصوى	Example9.1,2	307
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Minimizing The Average Cost of Producing a Commercial Product

Example 9.2 Page: 307

Suppose that $C(x) = 0.02x^2 + 2x + 4000$ is the **total cost** (in dollars) for a company to produce x **units** of a certain product. Find the **production level** x that **minimize the average cost**.

Solution

Total Cost: $C(x) = 0.02x^2 + 2x + 4000$

Average Cost: $\bar{C}(x) = \frac{C(x)}{x} \Rightarrow \bar{C}(x) = \frac{0.02x^2 + 2x + 4000}{x} = 0.02x + 2 + 4000x^{-1}$

To minimize the average cost, we will differentiate average cost function then we will find critical values

Derivative of Average Cost: $\bar{C}'(x) = 0.02 - 4000x^{-2}$

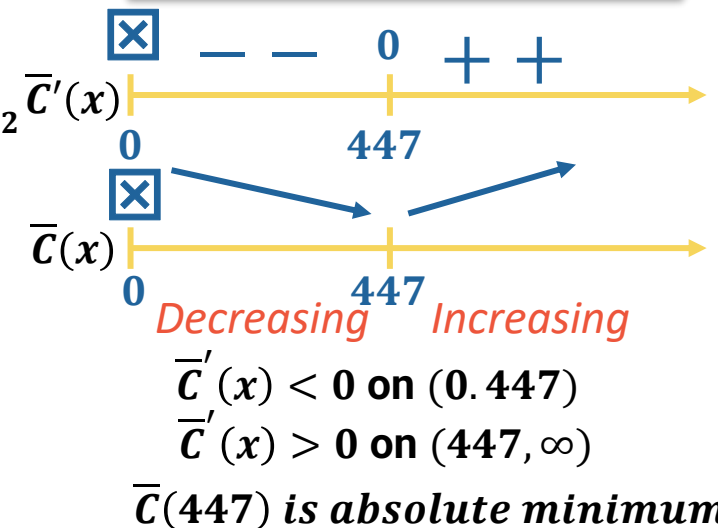
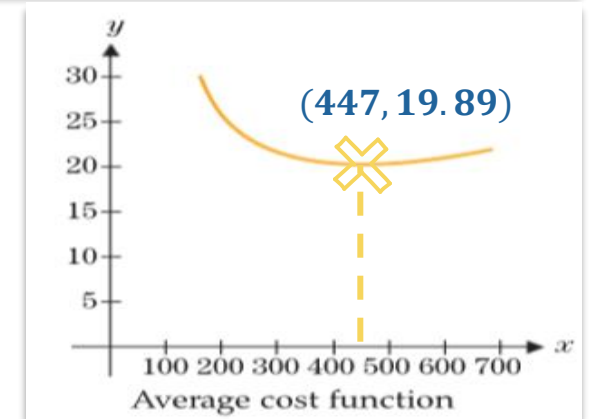
Critical Numbers: $\bar{C}'(x) = 0 \Rightarrow 0.02 - 4000x^{-2} = 0 \Rightarrow 0.02 = 4000x^{-2}$

$$\Rightarrow x^2 = \frac{4000}{0.02} \Rightarrow x^2 = 200,000$$

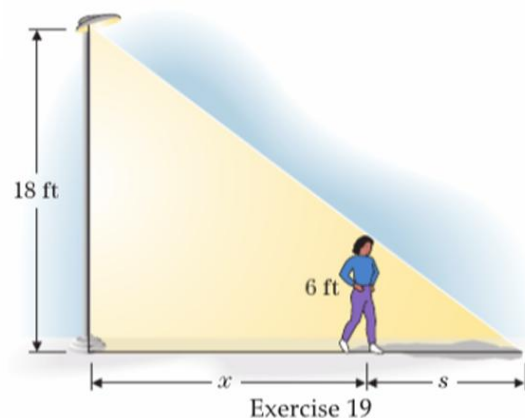
$$x \approx -447 \text{ Or } x \approx 447$$

Number of items can't be negative

Average Cost can be minimized when the level of production is $x \approx 447$ items.



19. Suppose a 6 ft-tall person is 12 ft away from an 18 ft-tall lamppost (see the figure). (a) If the person is moving away from the lamppost at a rate of 2 ft/s, at what rate is the length of the shadow changing? (Hint: Show that $\frac{x+s}{18} = \frac{s}{6}$.) (b) Repeat with the person 6 ft away from the lamppost and walking toward the lamppost at a rate of 3 ft/s.



A Related Rates Problem

Exercise Q19 Page: 304

Suppose a **6ft-tall person** is moving **away** from **18ft-tall lamppost** if the person is **moving away from lamppost** at a **rate of 2ft/sec** at what **rate is the length of the shadow changing?**

Solution

Given:

Rate of Change of x : $\frac{dx}{dt} = x'(t) = 2 \text{ ft/sec}$

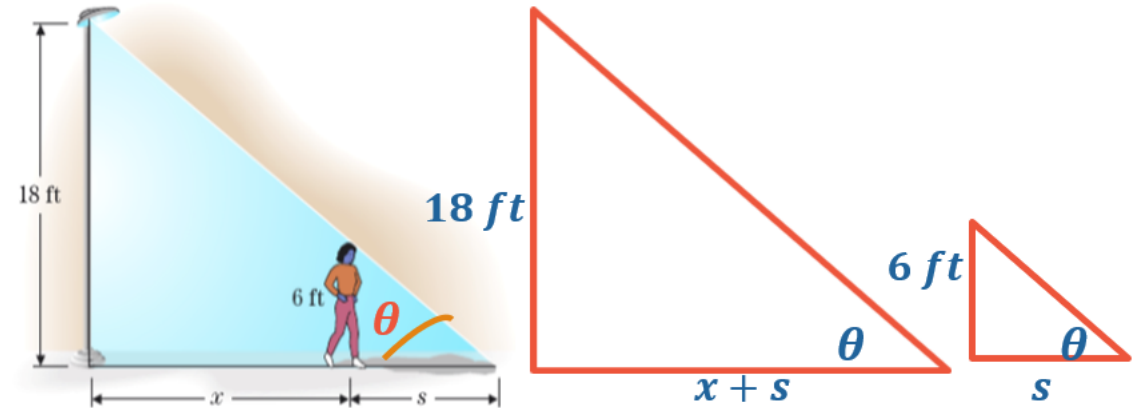
Required:

Rate of Change of shadow Length(s): $\frac{ds}{dt} = s'(t) = ?$

Equation Relating Variables:

Using similarity of triangles
Or Using tangent of angle θ

$$\begin{aligned} \tan \theta &= \frac{18}{x+s} & \tan \theta &= \frac{6}{s} \\ \Rightarrow \frac{18}{x+s} &= \frac{6}{s} & \Rightarrow \frac{x+s}{18} &= \frac{s}{6} \Rightarrow \frac{x+s}{3} = s \end{aligned}$$



$$x(t) + s(t) = 3s(t) \quad x \text{ and } s \text{ are functions of time}$$

$$x(t) = 2s(t)$$

Differentiate implicitly with respect to time:

$$x'(t) = 2s'(t)$$

$$2 = 2s'(t)$$

$$\Rightarrow s'(t) = 1 \text{ ft/sec}$$

The length of the shadow is increasing by rate of 1ft/sec as this person walking with a rate of 2 ft/sec away from lamppost.

20. Boyle's law for a gas at constant temperature is $PV = c$, where P is pressure, V is volume and c is a constant. Assume that both P and V are functions of time. (a) Show that $P'(t)/V'(t) = -c/V^2$. (b) Solve for P as a function of V . Treating V as an independent variable, compute $P'(V)$. Compare $P'(V)$ and $P'(t)/V'(t)$ from parts (a) and (b).

21. A dock is 6 ft above water. Suppose you stand on the edge of the dock and pull a rope attached to a boat at the constant rate of 2 ft/s. Assume that the boat remains at water level. At what speed is the boat approaching the dock when it is 20 feet from the dock? 10 feet from the dock? Isn't it surprising that the boat's speed is not constant?

22. Sand is poured into a conical pile with the height of the pile equalling the diameter of the pile. If the sand is poured at a constant rate of $5 \text{ m}^3/\text{s}$, at what rate is the height of the pile increasing when the height is 2 meters?

Sand is poured into a conical pile with the height of the pile equaling the diameter of the pile. If the sand is poured at a constant rate of $5\text{m}^3/\text{s}$, at what rate is the height of the pile increasing when the height is 2m ?

Solution

Given: Height : h
radius : $r = \frac{h}{2}$
Rate of Change of Volume: $\frac{dV}{dt} = 5\text{m}^3/\text{s}$

Required:
Rate of Change of Height: $\left.\frac{dh}{dt}\right|_{h=2\text{m}} = ?$

Equation Relating Variables:
Volume of Conical Pile $= \frac{1}{3}\pi r^2 h$
 $V = \frac{1}{3}\pi r^2 h$ *Substitute $r = \frac{h}{2}$*
 $V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \times h$ *Volume and height are functions of time*
 $V = \frac{1}{3}\pi \times \frac{h^2}{4} \times h \Rightarrow V(t) = \frac{1}{12}\pi h^3(t)$

Differentiate implicitly with respect to time:

$$\frac{dV}{dt} = \frac{\pi}{12} \times 3h^2(t) \frac{dh}{dt}$$
$$V'(t) = \frac{\pi}{4} h^2(t) \times h'(t)$$

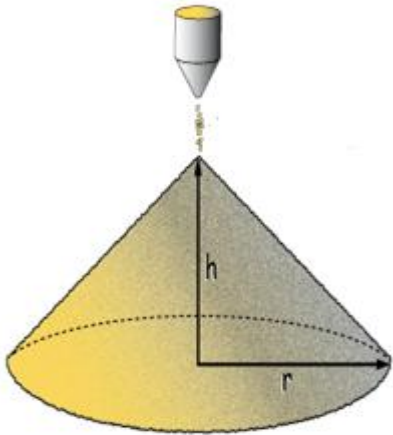
Substitute:

$$\frac{dV}{dt} = 5\text{m}^3/\text{s} , h = 2\text{m}$$
$$5 = \frac{\pi}{4} (2)^2 \times h'(t)$$

Solve for $h'(t)$:

$$5 = \pi \times h'(t)$$
$$h'(t) = \frac{5}{\pi} \approx 1.59\text{m/s}$$

The height is increasing by rate of 1.59m/s at the instant at which height is 2m



Height = Diameter
 $h = D = 2r \Rightarrow r = \frac{h}{2}$

23. The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension T to which the string is tightened, the density ρ of the string and the effective length L of the string by the equation $f = \frac{1}{2L} \sqrt{\frac{T}{\rho}}$. By running his finger along a string, a guitarist can change L by changing the distance between the bridge and his finger. Suppose that $L = \frac{1}{2}$ ft and $\sqrt{\frac{T}{\rho}} = 220$ ft/s so that the units of f are Hertz (cycles per second). If the guitarist's hand slides so that $L'(t) = -4$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double f)?

24. Suppose that you are blowing up a balloon by adding air at the rate of $1 \text{ ft}^3/\text{s}$. If the balloon maintains a spherical shape, the volume and radius are related by $V = \frac{4}{3}\pi r^3$. Compare the rate at which the radius is changing when $r = 0.01 \text{ ft}$ versus when $r = 0.1 \text{ ft}$. Discuss how this matches the experience of a person blowing up a balloon.

25. Water is being pumped into a spherical tank of radius 60 feet at the constant rate of $10 \text{ ft}^3/\text{s}$. (a) Find the rate at which the radius of the top level of water in the tank changes when the tank is half full. (b) Find the height at which the height of the water in the tank changes at the same rate as the radius.

26. Sand is dumped such that the shape of the sandpile remains a cone with height equal to twice the radius. (a) If the sand is dumped at the constant rate of $20 \text{ ft}^3/\text{s}$, find the rate at which the radius is increasing when the height reaches 6 feet.

In exercises 37 and 38, assume that $\int_1^3 f(x) \, dx = 3$ and $\int_1^3 g(x) \, dx = -2$ and find

37. (a) $\int_1^3 [f(x) + g(x)] \, dx$ (b) $\int_1^3 [2f(x) - g(x)] \, dx$

38. (a) $\int_1^3 [f(x) - g(x)] \, dx$ (b) $\int_1^3 [4g(x) - 3f(x)] \, dx$

Assume that $\int_1^3 f(x)dx = 3$ and $\int_1^3 g(x)dx = -2$

Example

Find a) $\int_1^3 [f(x) + g(x)]dx$

Find b) $\int_1^3 [2f(x) - g(x)]dx$

Solution

a $\int_1^3 [f(x) + g(x)]dx$ *Using Theorem 4.2*

$$= \int_1^3 f(x)dx + \int_1^3 g(x)dx = 3 + (-2) = 1$$

b $\int_1^3 [2f(x) - g(x)]dx$ *Using Theorem 4.2*

$$= 2 \int_1^3 f(x)dx - \int_1^3 g(x)dx = 2(3) - (-2) = 8$$

Exercise

Find a) $\int_1^3 [f(x) - g(x)]dx$

Find b) $\int_1^3 [4f(x) - 3g(x)]dx$

Solution

a $\int_1^3 [f(x) - g(x)]dx$ *Using Theorem 4.2*

$$= \int_1^3 f(x)dx - \int_1^3 g(x)dx = 3 - (-2) = 5$$

b $\int_1^3 [4f(x) - 3g(x)]dx$ *Using Theorem 4.2*

$$= 4 \int_1^3 f(x)dx - 3 \int_1^3 g(x)dx = 4(3) - 3(-2)$$

In exercises 39–42, find an equation of the tangent line at the given value of x .

39. $y = \int_0^x \sin \sqrt{t^2 + \pi^2} \, dt, x = 0$

19	Write the equation of a tangent line at a given point to a function defined as a definite integral	(39-42)	366-367
	كتابة معادلة خط المماس عند نقطة معينة لدالة معرفة كتكامل محدد		

In exercises 39–42, find an equation of the tangent line at the given value of x .

39. $y = \int_0^x \sin \sqrt{t^2 + \pi^2} dt, x = 0$

Solution

$$y = \int_0^x \sin \sqrt{t^2 + \pi^2} dt$$

Using Fundamental Theorem Part II

Find the derivative

$$\frac{dy}{dx} = \sin \sqrt{x^2 + \pi^2}$$

Slope of the tangent at $x = 0$

$$\text{Slope} = m = \left. \frac{dy}{dx} \right|_{x=0} = \sin \sqrt{(0)^2 + \pi^2}$$

$$\text{Slope} = m = \left. \frac{dy}{dx} \right|_{x=0} = \sin(\pi) = 0$$

The tangent at $x = 0$ passes through

Point

$$x = 0 \Rightarrow y = \int_0^0 \sin \sqrt{t^2 + \pi^2} dt$$

$y = 0$ *Since the upper limit equals the lower limit*

Point: $(0, 0)$

Equation of the tangent line

$$y - y_0 = m(x - x_0)$$

$$y - 0 = (0)(x - 0)$$

$$y = 0$$

In exercises 39–42, find an equation of the tangent line at the given value of x .

40. $y = \int_{-1}^x \ln(t^2 + 2t + 2) \, dt, x = -1$

19	Write the equation of a tangent line at a given point to a function defined as a definite integral	(39-42)	366-367
	كتابة معادلة خط المماس عند نقطة معينة لدالة معرفة كتكامل محدد		

In exercises 39–42, find an equation of the tangent line at the given value of x .

41. $y = \int_2^x \cos(\pi t^3) dt, x = 2$

19	Write the equation of a tangent line at a given point to a function defined as a definite integral	(39-42)	366-367
	كتابة معادلة خط المماس عند نقطة معينة لدالة معرفة كتكامل محدد		

In exercises 39–42, find an equation of the tangent line at the given value of x .

42. $y = \int_0^x e^{-t^2+1} dt, x = 0$

19	Write the equation of a tangent line at a given point to a function defined as a definite integral	(39-42)	366-367
	كتابة معادلة خط المماس عند نقطة معينة لدالة معرفة كتكامل محدد		

In exercises 39–42, find an equation of the tangent line at the given value of x .

42. $y = \int_0^x e^{-t^2+1} dt, x = 0$

Solution

$$y = \int_0^x e^{-t^2+1} dt$$

Using Fundamental Theorem Part II

Find the derivative

$$\frac{dy}{dx} = e^{-x^2+1}$$

Slope of the tangent at $x = 0$

$$\text{Slope} = m = \left. \frac{dy}{dx} \right|_{x=0} = e^{-(0)^2+1}$$

$$\text{Slope} = m = \left. \frac{dy}{dx} \right|_{x=0} = e \approx 2.718$$

The tangent at $x = 0$ passes through

Point

$$x = 0 \rightarrow y = \int_0^0 e^{-t^2+1} dt$$

$y = 0$ Since the upper limit equals the lower limit

Point: $(0, 0)$

Equation of the tangent line

$$y - y_0 = m(x - x_0)$$

$$y - 0 = (e)(x - 0)$$

$$y = ex$$

In exercises 5–30, evaluate the indicated integral.

5. $\int x^3 \sqrt{x^4 + 3} \, dx$

6. $\int \sqrt{1 + 10x} \, dx$

In exercises 5–30, evaluate the indicated integral.

7. $\int \frac{\sin x}{\sqrt{\cos x}} dx$

8. $\int \sin^3 x \cos x dx$

In exercises 5–30, evaluate the indicated integral.

9. $\int t^2 \cos t^3 dt$

10. $\int \sin t (\cos t + 3)^{3/4} dt$

In exercises 5–30, evaluate the indicated integral.

11. $\int x e^{x^2+1} dx$

12. $\int e^x \sqrt{e^x + 4} dx$

In exercises 5–30, evaluate the indicated integral.

13. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

14. $\int \frac{\cos(1/x)}{x^2} dx$

In exercises 5–30, evaluate the indicated integral.

Q13) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx$ *Make dx subject*
 $\Rightarrow dx = 2\sqrt{x} du$

$$= \int \frac{e^u}{\cancel{\sqrt{x}}} \cancel{2\sqrt{x}} du$$

$$= 2 \int e^u \cdot du$$

$$= 2e^u + c$$

Replace each occurrence of u with the corresponding expression in x .

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2e^{\sqrt{x}} + c$$

Q14) $\int \frac{\cos(1/x)}{x^2} dx$

Let $u = \frac{1}{x} \Rightarrow du = -\frac{1}{x^2} dx$ *Make dx subject*
 $\Rightarrow dx = -x^2 du$

$$= \int \frac{\cos(u)}{\cancel{x^2}} \cdot \cancel{-x^2} du$$

$$= - \int \cos u du$$

$$= -\sin u + c$$

$$\int \frac{\cos(1/x)}{x^2} dx = -\sin(1/x) + c$$

In exercises 5–30, evaluate the indicated integral.

15. $\int \frac{\sqrt{\ln x}}{x} dx$

16. $\int \sec^2 x \sqrt{\tan x} dx$

In exercises 5–30, evaluate the indicated integral.

$$15. \int \frac{\sqrt{\ln x}}{x} dx$$

Let $u = \ln x \Rightarrow du = \frac{1}{x} dx$ *Make dx subject*

$\Rightarrow dx = x du$

$$\begin{aligned} \int \frac{\sqrt{\ln x}}{x} dx &= \int \frac{\sqrt{u}}{\cancel{x}} \cdot \cancel{x} du \\ &= \int \sqrt{u} du = \int u^{\frac{1}{2}} \cdot du \\ &= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2\sqrt{u^3}}{3} + c \end{aligned}$$

$$\int \frac{\sqrt{\ln x}}{x} dx = \frac{2\sqrt{(\ln x)^3}}{3} + c$$

$$16. \int \sec^2 x \sqrt{\tan x} dx$$

In exercises 5–30, evaluate the indicated integral.

17. $\int \frac{1}{\sqrt{u}(\sqrt{u} + 1)} du$

18. $\int \frac{v}{v^2 + 4} dv$

In exercises 5–30, evaluate the indicated integral.

19. $\int \frac{4}{x(\ln x + 1)^2} dx$

20. $\int \tan 2x dx$

In exercises 5–30, evaluate the indicated integral.

21. $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

22. $\int x^2 \sec^2 x^3 dx$

In exercises 5–30, evaluate the indicated integral.

23. (a) $\int \frac{x}{\sqrt{1-x^4}} dx$

(b) $\int \frac{x^3}{\sqrt{1-x^4}} dx$

In exercises 5–30, evaluate the indicated integral. 23. (a) $\int \frac{x}{\sqrt{1-x^4}} dx$

(b) $\int \frac{x^3}{\sqrt{1-x^4}} dx$

Solution

a)

$$\int \frac{x}{\sqrt{1-x^4}} dx$$

Rewrite the integral

$$\int \frac{x}{\sqrt{1-(x^2)^2}} dx$$

Let

$$u = x^2 \Rightarrow du = 2x \cdot dx$$

$$\Rightarrow dx = \frac{du}{2x}$$

$$= \int \frac{\cancel{x}}{\sqrt{1-(u)^2}} \cdot \frac{du}{\cancel{2x}} = \frac{1}{2} \int \frac{du}{\sqrt{1-(u)^2}}$$

$$= \frac{1}{2} \sin^{-1}(u) + c$$

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{1}{2} \sin^{-1}(x^2) + c$$

Solution

b)

$$\int \frac{x^3}{\sqrt{1-x^4}} dx$$

Let

$$u = 1 - x^4 \Rightarrow du = -4x^3 \cdot dx$$

$$\Rightarrow dx = \frac{du}{-4x^3}$$

$$= \int \frac{\cancel{x^3}}{\sqrt{u}} \cdot \frac{du}{\cancel{-4x^3}} = -\frac{1}{4} \int \frac{du}{\sqrt{u}} = -\frac{1}{4} \int u^{-\frac{1}{2}} du$$

$$= -\frac{1}{4} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{1}{4} \cdot 2u^{\frac{1}{2}} + c = -\frac{1}{2} \sqrt{u} + c$$

$$\int \frac{x^3}{\sqrt{1-x^4}} dx = -\frac{1}{2} \sqrt{1-x^4} + c$$

In exercises 5–30, evaluate the indicated integral.

24. (a) $\int \frac{x^2}{1+x^6} dx$

(b) $\int \frac{x^5}{1+x^6} dx$

In exercises 5–30, evaluate the indicated integral.

24. (a) $\int \frac{x^2}{1+x^6} dx$

(b) $\int \frac{x^5}{1+x^6} dx$

Chapter 5: Lesson 6

Solution

a)

$$\int \frac{x^2}{1+x^6} dx$$

Rewrite the integral

$$\int \frac{x^2}{1+(x^2)^3} dx$$

Let $u = x^3 \Rightarrow du = 3x^2 \cdot dx$
 $\Rightarrow dx = \frac{du}{3x^2}$

$$= \int \frac{\cancel{x^2}}{1+(\cancel{u})^2} \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3} \int \frac{du}{1+(u)^2}$$

$$= \frac{1}{3} \tan^{-1}(u) + c$$

$$\int \frac{x^2}{1+x^6} dx = \frac{1}{3} \tan^{-1}(x^3) + c$$

Solution

b)

$$\int \frac{x^5}{1+x^6} dx$$

Rewrite the integral

$$= \frac{1}{6} \int \frac{6x^5 dx}{1+x^6}$$

Let

$u = 1+x^6 \Rightarrow du = 6x^5 \cdot dx$

$$= \frac{1}{6} \int \frac{\overbrace{6x^5 dx}^{du}}{\underbrace{1+x^6}_u}$$

The numerator is the derivative of denominator

$$= \frac{1}{6} \int \frac{du}{u} = \frac{1}{6} \ln|u| + c$$

$$\int \frac{x^5}{1+x^6} dx = \frac{1}{6} \ln(1+x^6) + c$$

In exercises 5–30, evaluate the indicated integral.

25. (a) $\int \frac{1+x}{1+x^2} dx$

(b) $\int \frac{1+x}{1-x^2} dx$

In exercises 5–30, evaluate the indicated integral.

25. (a) $\int \frac{1+x}{1+x^2} dx$

(b) $\int \frac{1+x}{1-x^2} dx$

Solution

a)

$$\int \frac{1+x}{1+x^2} dx$$

Rewrite the integral

$$= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$$

$$= \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{\overbrace{2x dx}^{du}}{\underbrace{1+x^2}_u}$$

Directly

The numerator is the derivative of denominator

$$u = 1+x^2 \Rightarrow du = 2x \cdot dx$$

$$= \tan^{-1}x + \frac{1}{2} \int \frac{du}{u}$$

$$= \tan^{-1}x + \frac{1}{2} \ln|u| + c$$

$$\int \frac{1+x}{1+x^2} dx = \tan^{-1}x + \frac{1}{2} \ln(1+x^2) + c$$

Solution

b)

$$\int \frac{1+x}{1-x^2} dx$$

Rewrite the integral

$$\int \frac{1+x}{(1+x)(1-x)} dx = \int \frac{1}{1-x} dx = - \int \frac{-1}{1-x} dx$$

$$u = 1-x \Rightarrow du = -dx$$

$$= - \int \frac{\overbrace{-dx}^{du}}{\underbrace{1-x}_u}$$

The numerator is the derivative of denominator

$$= - \int \frac{du}{u} = -\ln|u| + c$$

$$\int \frac{1+x}{1-x^2} dx = -\ln(1-x) + c$$

In exercises 5–30, evaluate the indicated integral.

26. (a) $\int \frac{3\sqrt{x}}{1+x^3} dx$

(b) $\int \frac{x\sqrt{x}}{1+x^5} dx$

In exercises 5–30, evaluate the indicated integral.

27. $\int \frac{2t+3}{t+7} dt$

28. $\int \frac{t^2}{\sqrt[3]{t+3}} dt$

A Substitution That Lets You Expand the Integrand

Exercise

Q27 Page: 376

Evaluate: $\int \frac{2t + 3}{t + 7} dt$

Solution

$$\int \frac{2t + 3}{t + 7} dt$$

Let

$$u = t + 7 \Rightarrow du = dt$$

$$t = u - 7$$

$$= \int \underbrace{\frac{2t + 3}{t + 7}}_u \cdot \underbrace{dt}_{du}$$

Write t in terms of u

$$t = u - 7$$

$$= \int \frac{2(u - 7) + 3}{u} \cdot du$$

$$= \int \frac{2u - 14 + 3}{u} du$$

$$= \int \frac{2u - 11}{u} du$$

$$= \int \left(2 - \frac{11}{u} \right) du$$

$$= 2u - 11 \ln|u| + c$$

$$\int \frac{2t + 3}{t + 7} dt = 2(t + 7) - 11 \ln|t + 7| + c$$

A Substitution That Lets You Expand the Integrand

Exercise

Q28 Page: 376

Evaluate: $\int \frac{t^2}{\sqrt[3]{t+3}} dt$

Solution

$$\int \frac{t^2}{\sqrt[3]{t+3}} dt$$

Let

$$u = t + 3 \Rightarrow du = dt$$

$$t = u - 3$$

Write t in terms of u

$$t = u - 3$$

$$t^2 = (u - 3)^2$$

$$t^2 = u^2 - 6u + 9$$

$$= \int \frac{\overbrace{t^2}^{t^2}}{\underbrace{\sqrt[3]{t+3}}_u} \cdot \underbrace{dt}_{du}$$

$$= \int \frac{u^2 - 6u + 9}{\sqrt[3]{u}} \cdot du$$

$$= \int (u^2 - 6u + 9) \cdot u^{-\frac{1}{3}} du$$

$$= \int (u^{\frac{5}{3}} - 6u^{\frac{2}{3}} + 9u^{-\frac{1}{3}}) du$$

$$= \frac{u^{\frac{8}{3}}}{\frac{8}{3}} - 6 \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + 9 \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + c$$

$$= 3 \frac{\sqrt[3]{u^8}}{8} - 18 \frac{\sqrt[3]{u^5}}{5} + 27 \frac{\sqrt[3]{u^2}}{2} + c$$

$$\int \frac{t^2}{\sqrt[3]{t+3}} dt = 3 \frac{\sqrt[3]{(t+3)^8}}{8} - 18 \frac{\sqrt[3]{(t+3)^5}}{5} + 27 \frac{\sqrt[3]{(t+3)^2}}{2} + c$$

In exercises 5–30, evaluate the indicated integral.

29. $\int \frac{1}{\sqrt{1 + \sqrt{x}}} dx$

30. $\int \frac{1}{x\sqrt{x^4 - 1}} dx$

In exercises 31–40, evaluate the definite integral.

31. $\int_0^2 x\sqrt{x^2 + 1} \, dx$

32. $\int_1^3 x \sin(\pi x^2) \, dx$

In exercises 31–40, evaluate the definite integral.

$$33. \int_{-1}^1 \frac{t}{(t^2 + 1)^2} dt$$

$$34. \int_0^2 t^2 e^{t^3} dt$$

Using Substitution in Definite Integral

Exercise

Q33 Page: 376

Evaluate: $\int_{-1}^1 \frac{t}{(t^2 + 1)^2} dt$

Solution

$$\int_{-1}^1 \frac{t}{(t^2 + 1)^2} dt$$

Let

$$u = t^2 + 1 \Rightarrow du = 2t dt$$

Limits of Integration

$$t = -1 \Rightarrow u = (-1)^2 + 1 = 2$$

$$t = 1 \Rightarrow u = (1)^2 + 1 = 2$$

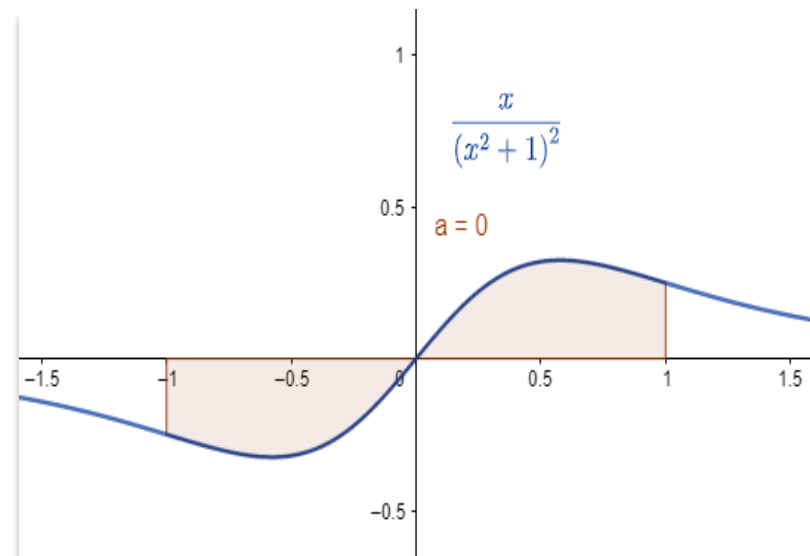
$$\int_{-1}^1 \frac{t}{(t^2 + 1)^2} dt = \frac{1}{2} \int_{-1}^1 \frac{\overbrace{2t dt}^{du}}{\underbrace{(t^2 + 1)^2}_u}$$

$$= \frac{1}{2} \int_2^2 \frac{du}{u^2}$$

$$= \frac{1}{2} \int_2^2 u^{-2} du = 0$$

The upper limit equals the lower limit

$$\boxed{\int_{-1}^1 \frac{t}{(t^2 + 1)^2} dt = 0}$$



In exercises 31–40, evaluate the definite integral.

$$35. \int_0^2 \frac{e^x}{1 + e^{2x}} dx$$

$$36. \int_0^2 \frac{e^x}{1 + e^x} dx$$

Substitution in a Definite Integral Involving an Exponential

Exercise

Q35 Page: 376

Evaluate:

$$\int_0^2 \frac{e^x}{1 + e^{2x}} dx$$

Solution

$$\int_0^2 \frac{e^x}{1 + e^{2x}} dx \quad \text{Rewrite the integral}$$

$$\int_0^2 \frac{e^x}{1 + (e^x)^2} dx$$

Let

$$u = e^x \Rightarrow du = e^x dx$$

Limits of Integration

$$x = 0 \Rightarrow u = e^{(0)} = 1$$

$$x = 2 \Rightarrow u = e^2$$

$$\int_0^2 \frac{\overbrace{e^x dx}^{du}}{1 + \underbrace{(e^x)^2}_u}$$

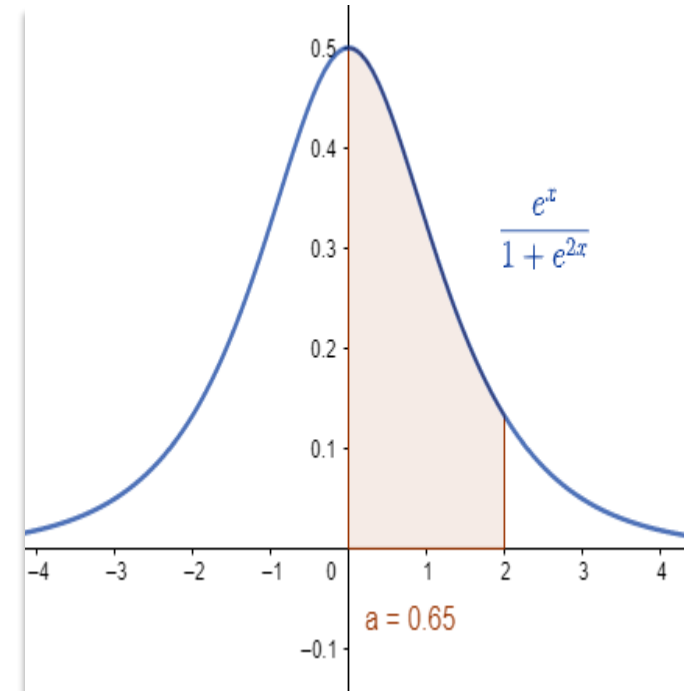
$$= \int_1^{e^2} \frac{du}{1 + (u)^2}$$

$$= \tan^{-1} u \Big|_1^{e^2}$$

$$= (\tan^{-1}(e^2)) - (\tan^{-1}(1))$$

$$= \tan^{-1} e^2 - \frac{\pi}{4} \approx 0.65$$

$$\int_0^2 \frac{e^x}{1 + e^{2x}} dx \approx 0.65$$



Substitution in a Definite Integral Involving an Exponential

Exercise

Q36 Page: 376

Evaluate:

$$\int_0^2 \frac{e^x}{1+e^x} dx$$

Solution

$$\int_0^2 \frac{e^x}{1+e^x} dx$$

Let

$$u = 1 + e^x \Rightarrow du = e^x dx$$

Limits of Integration

$$x = 0 \Rightarrow u = 1 + e^{(0)} = 2$$

$$x = 2 \Rightarrow u = 1 + e^2$$

$$\int_0^2 \frac{\overbrace{e^x dx}^{du}}{\underbrace{1 + e^x}_u}$$

$$= \int_2^{1+e^2} \frac{du}{u}$$

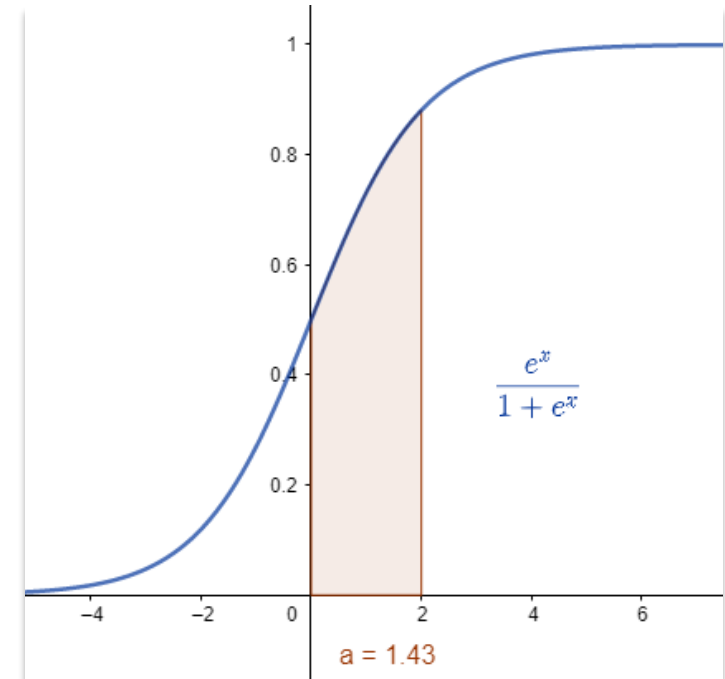
$$= \ln|u| \Big|_2^{1+e^2}$$

$$= (\ln|1 + e^2|) - (\ln|2|)$$

$$= (\ln(1 + e^2)) - (\ln(2))$$

$$= \ln\left(\frac{1 + e^2}{2}\right) \approx 1.43$$

$$\int_0^2 \frac{e^x}{1+e^x} dx \approx 1.43$$



In exercises 31–40, evaluate the definite integral.

37. $\int_{\pi/4}^{\pi/2} \cot x \, dx$

38. $\int_1^e \frac{\ln x}{x} \, dx$

In exercises 31–40, evaluate the definite integral.

$$39. \int_1^4 \frac{x-1}{\sqrt{x}} dx$$

$$40. \int_0^1 \frac{x}{\sqrt{x^2+1}} dx$$

Keep Up the
Good Work!

Ms. Duad