

2-3

حساب النهاية

Computation of Limits

For any constant c and any real number a ,

لأي ثابت c وأي عدد حقيقي a ,

$$\lim_{x \rightarrow a} c = c.$$

For any real number a ,

لأي عدد حقيقي a ,

$$\lim_{x \rightarrow a} x = a.$$

THEOREM 3.1

النظرية 3.1

Suppose that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exist and let c be any constant. The following then apply:

إذا كانت $\lim_{x \rightarrow a} f(x)$ و $\lim_{x \rightarrow a} g(x)$ موجودتين وكانت c أي ثابت. فإن:

- (i) $\lim_{x \rightarrow a} [c \times f(x)] = c \times \lim_{x \rightarrow a} f(x),$
- (ii) $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x),$
- (iii) $\lim_{x \rightarrow a} [f(x) \times g(x)] = \left[\lim_{x \rightarrow a} f(x) \right] \left[\lim_{x \rightarrow a} g(x) \right]$
- (iv) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad \left(\lim_{x \rightarrow a} g(x) \neq 0 \text{ بشرط} \right).$

Apply the rules of limits to evaluate

طبّق قواعد النهايات لإيجاد

Q 1 $\lim_{x \rightarrow 2} (3x^2 - 5x + 4).$

Q 4 $\lim_{x \rightarrow 2} x$

Q 2 $\lim_{x \rightarrow 3} \frac{x^3 - 5x + 4}{x^2 - 2}.$

Q 5 $\lim_{x \rightarrow 2} 5$

Q 3 $\lim_{x \rightarrow 1} \frac{x^2 - 1}{1 - x}.$

THEOREM 3.2

إذا كانت كثيرة حدود $p(x)$ وأي عدد حقيقي a ، فإن:

For any polynomial $p(x)$ and any real number a ,

$$\lim_{x \rightarrow a} p(x) = p(a).$$

THEOREM 3.3

Suppose that $\lim_{x \rightarrow a} f(x) = L$ and n is any positive integer. Then,

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{L},$$

where for n even, we must assume that $L > 0$.

النظرية 3.3

إذا كانت $\lim_{x \rightarrow a} f(x) = L$ وأن n هو أي عدد صحيح موجب. فإن،

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{L}$$

حيث إنه لكل n زوجي، نفترض أن $L > 0$.

evaluate the indicated limit, if it exists

أوجد قيمة النهاية المشار إليها، إذا وُجدت.

Q 6 $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

Q 9 $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4}$

Q 7 $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 3x + 2}$

Q10 $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 2x - 3}$

Q 8 $\lim_{x \rightarrow 0} (x^2 - 3x + 1)$

Q11 $\lim_{x \rightarrow 2} \frac{x - 5}{x^2 + 4}$

Limit by Rationalizing

Q12 $\lim_{x \rightarrow 2} \sqrt[3]{2x + 1}$

Q15 $\lim_{x \rightarrow 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x}$

Q13 $\lim_{x \rightarrow 2} \sqrt[5]{3x^2 - 2x}$

Q16 $\lim_{x \rightarrow 0} \frac{\sqrt{x + 4} - 2}{x}$

Q14 $\lim_{x \rightarrow 2} \sqrt[3]{3x + 2}$

Q17 $\lim_{x \rightarrow 0} \frac{2x}{3 - \sqrt{x + 9}}$

Q18 $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

Q21 $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2}$

Q19 $\lim_{x \rightarrow 1} \frac{\frac{1}{x} - 1}{x - 1}$

Q22 $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

Q20 $\lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}$

Q23 $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 - x - 6}$

Q24 $\lim_{x \rightarrow 4} \frac{x-4}{x^2-16}$

Q27 $\lim_{x \rightarrow 0^+} \frac{2x}{|x|}$

Q25 $\lim_{x \rightarrow 3^+} \frac{3x-9}{|x-3|}$

Q28 $\lim_{x \rightarrow 1^+} \frac{2x-2}{|x-1|}$

Q26 $\lim_{h \rightarrow 0} \frac{\frac{1}{6+h} - \frac{1}{6}}{h}$

Q29 $\lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})}$

Q30 $\lim_{x \rightarrow 8} \frac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})}$

Q33 $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 + 4x - 5}$

Q31 $\lim_{x \rightarrow 4^+} \frac{x^2 - 2x - 8}{|-x + 4|}$

Q34 $\lim_{h \rightarrow 0} \frac{\sqrt{5-h} - \sqrt{5}}{h}$

Q32 $\lim_{x \rightarrow 0^-} \frac{|x|}{4x} =$

Q35 $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$

Q36 $\lim_{x \rightarrow 2} f(x)$, حيث $f(x) = \begin{cases} 2x & , \quad x < 2 \\ x^2 & , \quad x \geq 2 \end{cases}$

Q37 $\lim_{x \rightarrow -1} f(x)$, حيث $f(x) = \begin{cases} x^2 + 1 & , \quad x < -1 \\ 3x + 1 & , \quad x \geq -1 \end{cases}$

Q38 $\lim_{x \rightarrow -1} f(x)$, حيث $f(x) = \begin{cases} 2x + 1 & , \quad x < -1 \\ 3 & , \quad -1 \leq x < 1 \\ 2x + 1 & , \quad x > 1 \end{cases}$

THEOREM 3.4**النظرية 3.4**

For any real number a , we have

لأي عدد حقيقي a ، لدينا:

(i) $\lim_{x \rightarrow a} \sin x = \sin a,$

(v) $\lim_{x \rightarrow a} \sin^{-1} x = \sin^{-1} a, \text{ for } -1 < a < 1,$

(ii) $\lim_{x \rightarrow a} \cos x = \cos a,$

(vi) $\lim_{x \rightarrow a} \cos^{-1} x = \cos^{-1} a, \text{ for } -1 < a < 1,$

(iii) $\lim_{x \rightarrow a} e^x = e^a \text{ and}$

(vii) $\lim_{x \rightarrow a} \tan^{-1} x = \tan^{-1} a, \text{ for } -\infty < a < \infty \text{ and}$

(iv) $\lim_{x \rightarrow a} \ln x = \ln a, \text{ for } a > 0.$

(viii) if p is a polynomial and $\lim_{x \rightarrow p(a)} f(x) = L,$

then $\lim_{x \rightarrow a} f(p(x)) = L.$

Evaluate

أوجد قيمة

Q39 $\lim_{x \rightarrow 0} \sin^{-1} \left(\frac{x+1}{2} \right).$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Q40 $\lim_{x \rightarrow 0} \cos^{-1}(x^2)$

Q41 $\lim_{x \rightarrow 0} \frac{\sin x}{\tan x}$

Q42 $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

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THEOREM 3.5 (Squeeze Theorem)

Suppose that

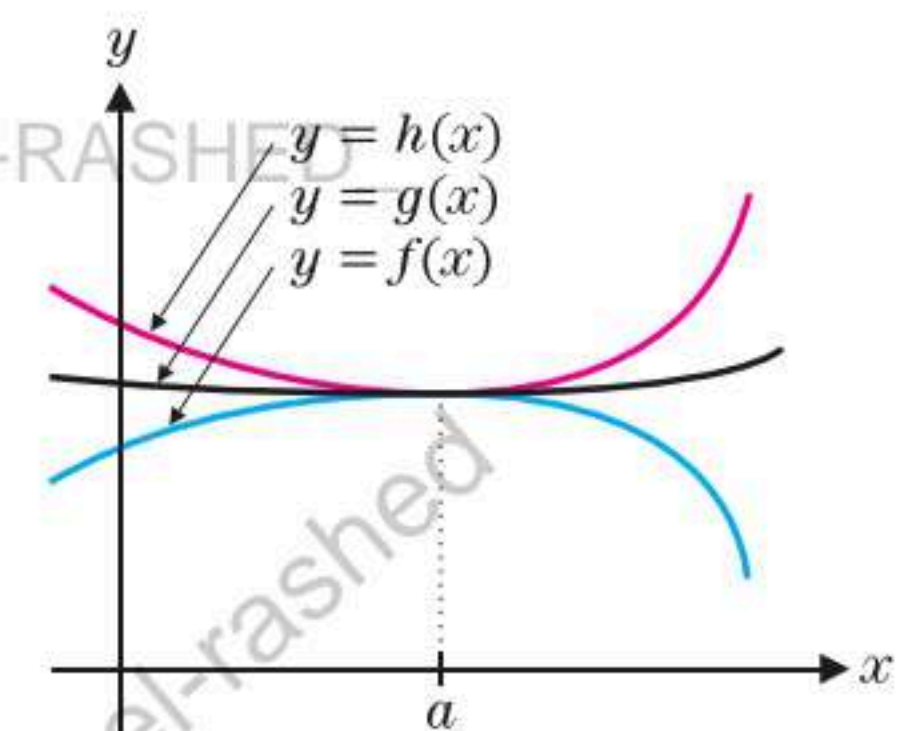
$$f(x) \leq g(x) \leq h(x)$$

for all x in some interval (c, d) , except possibly at the point $a \in (c, d)$ and that

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L,$$

for some number L . Then, it follows that

$$\lim_{x \rightarrow a} g(x) = L, \text{ also.}$$



The Squeeze Theorem

النظرية 3.5 (نظرية الشطيرة)

افترض أن

$$f(x) \leq g(x) \leq h(x)$$

لكل x في الفترة (c, d) ما عدا النقطة $a \in (c, d)$ وأن

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$$

ولعدد L . إذا، يكون:

$$\lim_{x \rightarrow a} g(x) = L \text{ كما أن}$$

REMARK 3.2

The Squeeze Theorem also applies to one-sided limits.

ملحوظة 3.2

تنطبق نظرية الشطيرة على النهايات أحادية الطرف.

Use sandwich theory to find the value of the limit

استخدم نظريه الشطيرة لايجاد قيمة النهاية

$$\text{Q43} \quad \lim_{x \rightarrow 0} \left[x^2 \cos \left(\frac{1}{x} \right) \right]$$

$$\text{Q46} \quad \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$$

$$\text{Q44} \quad \lim_{x \rightarrow 0} \left[x^2 \sin \left(\frac{1}{\sqrt{x}} \right) \right]$$

$$\text{Q47} \quad \lim_{x \rightarrow 0} x \cos x$$

$$\text{Q45} \quad \lim_{x \rightarrow \infty} \left[\frac{\cos(x)}{1 + x^2} \right]$$

Q48 $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{2x^2 - 5x - 3}.$

Q50 $\lim_{x \rightarrow 0} \left(\frac{1}{x} + \frac{5}{x(x-5)} \right).$

Q49 $\lim_{x \rightarrow 1} \frac{\frac{1}{x+1} - \frac{1}{2}}{x-1}.$

Q51 $\lim_{x \rightarrow 2^-} \frac{x-3}{x^2-2x}.$