Physics – Grade 11 – Advanced. Academic Year: 2023 - 2024 . . . Term 2
Chapter – 5: Kinetic Energy, Work and Power.

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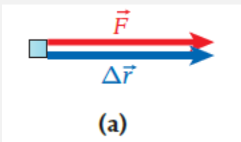
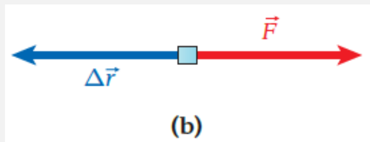
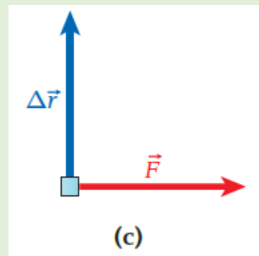
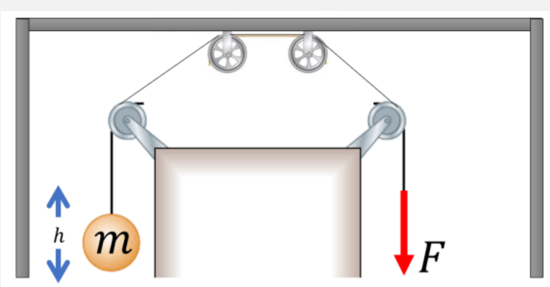
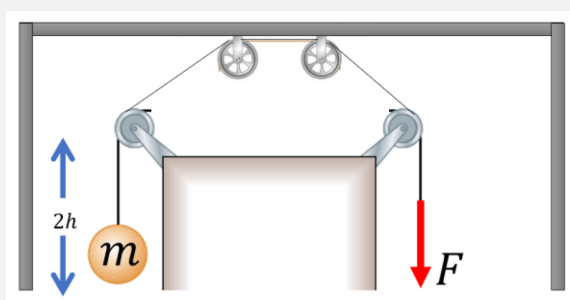
CONCEPT CHECK.

5.1: Consider an object undergoing a displacement $\Delta\vec{r}$ and experiencing a force F. In which of the three cases shown below is the work done by the force on the object zero?

		
$W = \vec{F}.\Delta\vec{r}$ $W = \lvert\vec{F}\rvert\,\lvert\Delta\vec{r}\rvert\,cos0^{\circ}$ $W = \lvert\vec{F}\rvert\,\lvert\Delta\vec{r}\rvert\,(1) \neq 0$	$W = \vec{F}.\Delta\vec{r}$ $W = \lvert\vec{F}\rvert\,\lvert\Delta\vec{r}\rvert\,cos180^{\circ}$ $W = \lvert\vec{F}\rvert\,\lvert\Delta\vec{r}\rvert\,(-1) \neq 0$	$W = \vec{F}.\Delta\vec{r}$ $W = \lvert\vec{F}\rvert\,\lvert\Delta\vec{r}\rvert\,cos90^{\circ}$ $W = \lvert\vec{F}\rvert\,\lvert\Delta\vec{r}\rvert\,(0) = 0$

5.2: If you lift an object a distance h with the aid of a rope and n pulleys and do work W, in the process, how much work will be required to lift the same object a distance 2h?

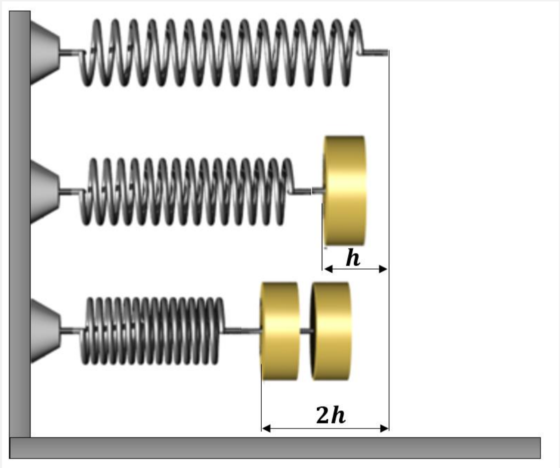
[a] W_h [b] $2W_h$ [c] $0.5W_h$ [d] nW_h [e] $\frac{2W_h}{n}$

Vertical altitude = h	Vertical altitude = 2h
	
$W = -\Delta P$ $\lvert W\rvert = \lvert-\Delta P\rvert$ $W = \Delta P$ $W = m\,g\,h$	$W = -\Delta P$ $\lvert W\rvert = \lvert-\Delta P\rvert$ $W = \Delta P$ $W = 2\,m\,g\,h$

Concept Check 5.3

If you compress a spring a distance h from its equilibrium position and do work W_h, in the process, how much work will be required to compress the same spring a distance 2h?

a) W_h
b) $2W_h$
c) $0.5W_h$
d) $4W_h$
e) $0.25W_h$

	DIAGRAM
$W = \frac{1}{2}\, k\, (\Delta x)^2$ $W_h = \frac{1}{2}\, k\, h^2$ $W_{2h} = \frac{1}{2}\, k\, (2h)^2$ $W_{2h} = 4\left(\frac{1}{2}\, k\, h^2\right)$ $W_{2h} = 4\, W_h$	

Concept Check 5.4

Is each of the following statements true or false?

a) Work cannot be done in the absence of motion.	True
b) More power is required to lift a box slowly than to lift a box quickly.	False
c) A force is required to do work.	True

[a] $W = F\, d\, cos\theta \quad \Longrightarrow \quad W\, \alpha\, d$
[b] $P = F\, v\, cos\theta \quad \Longrightarrow \quad P\, \alpha\, v$
[c] $W = F\, d\, cos\theta \quad \Longrightarrow \quad W\, \alpha\, F$

MULTIPLE-CHOICE QUESTIONS.

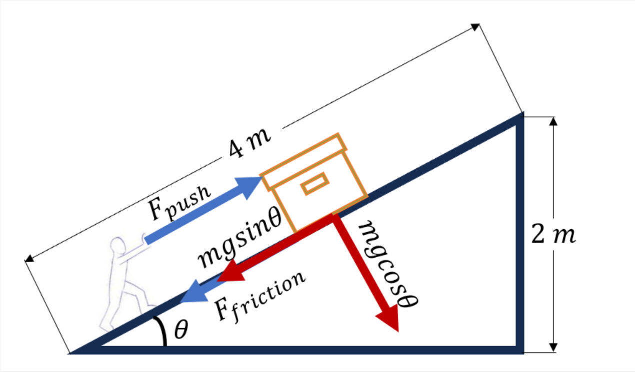
5.1 Which of the following is a correct unit of energy?

a) $kg\ m/s^2$ b) $kg\ m^2/s$ c) $kg\ m^2/s^2$ d) $kg^2\ m/s^2$ e) $kg^2\ m^2/s^2$

$W = F\, d\, cos\theta$ $[W] = N\, .m$ $J = kg\, \frac{m}{s^2}\, .m$ $J = kg\, \frac{m^2}{s^2}$	$F_{net} = m\, a$ $[F] = kg\, .\frac{m}{s^2}$ $N = kg\, .\frac{m}{s^2}$

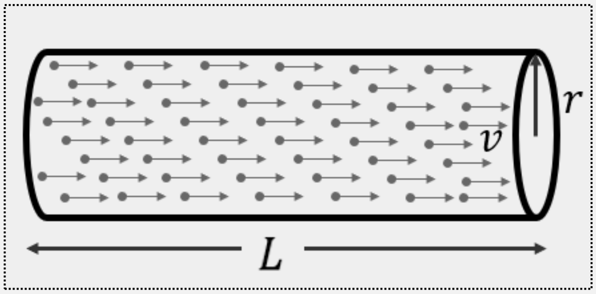
5.2 An 800-N box is pushed up an inclined plane that is 4.0 m long. It requires 3200 J of work to get the box to the top of the plane, which is 2.0 m above the base. What is the magnitude of the average friction force on the box? (Assume the box starts at rest and ends at rest.)

a) zero b) not zero but less 400 N c) greater than 400 N d) 400 N e) 800 N

SOLUTION	DIAGRAM
$v_i = v_f \equiv v \implies \Delta K.E = 0$ $W_{net} = \Delta K.E$ $F_{net}\, d\, cos\theta = 0 \implies F_{net} = 0\ N$ $W_{push} = F_{push}\, d\, cos\theta$ $3200 = (F_{push})(4.0)(cos0°)$ $F_{push} = 800\ N$ BUT $F_{push} = F_{friction} + mgsin\theta$ $800 = F_{friction} + \left(800 \times \frac{2}{4}\right)$ $F_{friction} = 400\ N$	

5.3 An engine pumps water continuously through a hose. If the speed with which the water passes through the hose nozzle is v and if k is the mass per unit length of the water jet as it leaves the nozzle, what is the power being imparted to the water?

a) $\frac{1}{2}\ k\ v^3$ b) $\frac{1}{2}\ k\ v^2$ c) $\frac{1}{2}\ k\ v$ d) $\frac{1}{2}\ \frac{v^2}{k}$ e) $\frac{1}{2}\ \frac{v^3}{k}$

SOLUTION	DIAGRAM
$P = \frac{W}{\Delta t} = \frac{\Delta K.E}{\Delta t}$ $P = \frac{\Delta(0.5mv^2)}{t} = \frac{1}{2}\, v^2 \frac{\Delta m}{\Delta t}$ $P = \frac{\Delta(0.5mv^2)}{t} = \frac{1}{2}\, v^2 \frac{\Delta kL}{\Delta t}$ $P = k\frac{1}{2}\, v^2 \frac{\Delta L}{\Delta t}$ $P = k\frac{1}{2}\, v^2 v$ $P = \frac{1}{2} k\, v^3$	 Given that: $k = \frac{m}{L} \implies\implies m = kL$

5.4 A 1500-kg car accelerates from 0 to 25 m/s in 7.0 s. What is the average power delivered by the engine (1 hp = 746 W)?

a) 60 hp b) 70 hp c) 80 hp d) 90 hp e) 180 hp

$W = \Delta K.E$ And $P = \frac{Work}{time}$

$$P = \frac{\Delta K.E}{time}$$

$$P = \frac{(0.5 \times m \times v_f^2) - (0.5 \times m \times v_i^2)}{time}$$

$$P = \frac{(0.5 \times 1500 \times 25^2) - (0.5 \times 1500 \times 0^2)}{7.0} = 66964.3\, Watts$$

$$P = 66964.3\, Watts \times \frac{1\, hp}{746\, Watts} = 90\, hp$$

5.5 Which of the following is a correct unit of power?

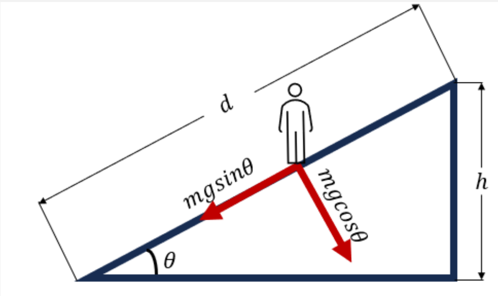
a) kg m/s² b) N c) J d) m/s² e) W

$$Power = \frac{Work}{time}$$

$$[P] = \frac{J}{s} = Watts$$

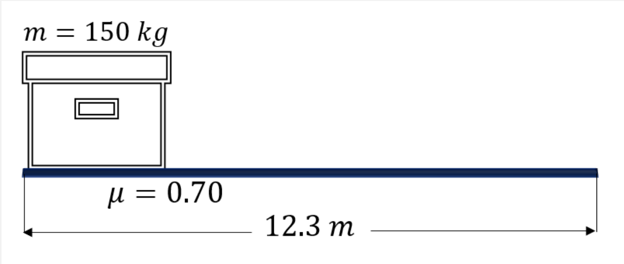
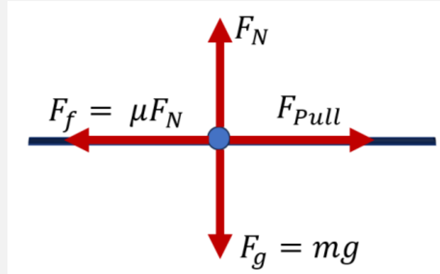
5.6 How much work is done when a 75.0-kg person climbs a flight of stairs 10.0 m high at constant speed?

a) 7.36×10^5 J b) 750 J c) 75 J d) 7500 J e) 7360 J

SOLUTION	DIAGRAM
$W = F\, d\, cos\theta$ $W = (mgsin\theta)\,(d)\,cos180°$ $W = \left(mg \times \frac{h}{d}\right)(d)\,(-1)$ $W = -mgh \Rightarrow \boldsymbol{W = -\Delta P.E}$ $W = -75.0 \times 10.0 \times 9.81 = -7357.5 \approx -7360\, J$ $Magnitude\ of\ Work = \lvert -7360 \rvert = 7360\, J$	

5.7 How much work do movers do (horizontally) in pushing a 150-kg crate 12.3 m across a floor at constant speed if the coefficient of friction is 0.70?

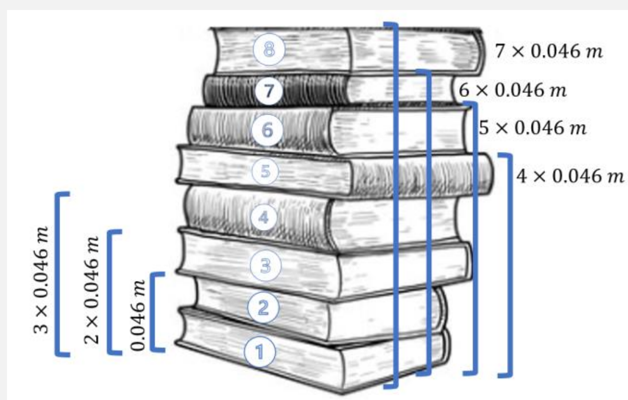
a) 1300 J b) 1845 J c) 1.3×10^4 J d) 1.8×10^4 J e) 130 J

<table>
<tr><th>Diagram</th><th>Free – Body Diagram</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Solution:
Constant speed mean that $a = 0.0\ m/s^2$
So $F_{net} = 0.0\ N$

$F_{pull} = F_f$
$F_{pull} = \mu\, F_N$
$F_{pull} = \mu\, m\, g$
$F_{pull} = (0.70)(150)(9.81)$
$F_{pull} = 1030.05\ N$</td><td>$W_{F_{pull}} = F_{pull}\, d\, cos\theta$
$W_{F_{pull}} = (1030.05)(12.3)\ cos0°$
$W_{F_{pull}} = 12669.615\ J$
$W_{F_{pull}} = 1.3 \times 10^4\ J$</td></tr>
</table>

5.8 Eight books, each 4.6 cm thick and of mass 1.8 kg, lie on a flat table. How much work is required to stack them on top of one another?

a) 141 J b) 23 J c) 230 J d) 0.81 J e) 14 J

<table>
<tr><th>SOLUTION</th><th>DIAGRAM</th></tr>
<tr><td>$$W_{total} = \sum_{x=1}^{x=8} (x-1)(mgh)$$
$$W_{total} = \sum_{x=1}^{x=8} (x-1)(0.81144)$$
$$W_{total} = 22.72032 \approx 23\ J$$</td><td>
</td></tr>
</table>

5.9 A particle moves parallel to the x-axis. The net force on the particle increases with x according to the formula $Fx = (120\ N/m)x$, where the force is in newtons when x is in meters. How much work does this force do on the particle as it moves from x = 0 to x = 0.50 m?

a) 7.5 J b) 15 J c) 30 J d) 60 J e) 120 J

Analytical Solution: $W = \int_0^{0.50} (120\ x)\ dx = 15\ J$

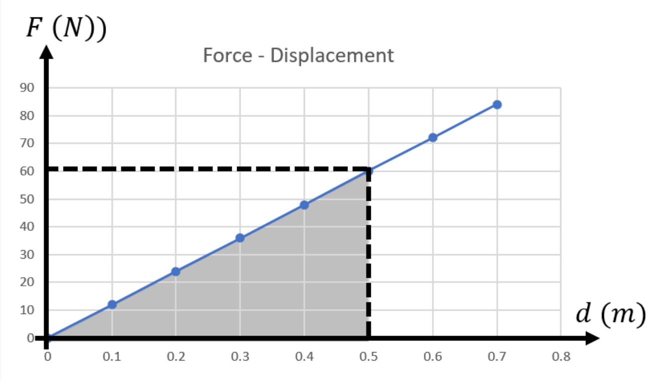
Graphical Solution:

Work = Bounded Area under F – d graph.

$W = \frac{1}{2} \times base \times height$

$W = \frac{1}{2} \times 0.50 \times 60$

$W = 15\,J$



5.10 A skydiver is subject to two forces: gravity and air resistance. Falling vertically, he reaches a constant terminal speed at some time after jumping from a plane. Since he is moving at a constant velocity from that time until his chute opens, we conclude from the work- kinetic energy theorem that, over that time interval,

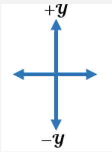
a) the work done by gravity is zero.

b) the work done by air resistance is zero.

c) the work done by gravity equals the negative of the work done by air resistance.

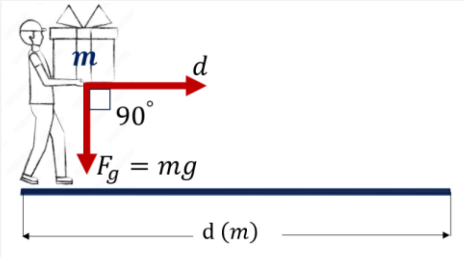
d) the work done by gravity equals the work done by air resistance.

e) his kinetic energy increases.

SOLUTION	DIAGRAM
Constant speed mean that $\vec{a} = 0.0\ m/s^2$ So $\vec{F}_{net} = 0.0\ N$ $\vec{F}_g = -\vec{F}_{AR} \Longrightarrow \lvert\vec{F}_g\rvert = \lvert -\vec{F}_{AR}\rvert \Longrightarrow \boldsymbol{F_g = F_{AR}}$	
$v_i = v_f \equiv v \Longrightarrow \boldsymbol{\Delta K.E = 0}$ $W_{total} = \Delta K.E \Longrightarrow W_{total} = 0$ $W_g + W_{AR} = 0 \Longrightarrow \boldsymbol{W_g = -W_{AR}}$	

5.11 Jack is holding a box that has a mass of m kg. He walks a distance of d m at a constant speed of v m/s. How much work, in joules, has Jack done on the box?

a) mgd b) $-mgd$ c) $\frac{1}{2}mv^2$ d) $-\frac{1}{2}mv^2$ e) zero

SOLUTION	DIAGRAM
$W = F\ d\ cos\theta$ $W = (mg)\ (d)\ cos90°$ $W = zero$	
Constant speed (v) means that $\Delta\ K.E = 0$ $W = \Delta\ K.E$ $W = zero$	

5.12 If negative work is being done by an object, which one of the following statements is true?

a) An object is moving in the negative x-direction.
b) An object has negative kinetic energy.
c) Energy is being transferred from an object.
d) Energy is being transferred to an object.

Given that: $W < 0$

$Work = -\ Change\ in\ potential\ energy$	$Work = Change\ in\ Kinetic\ energy\ energy$
$W = -\Delta P.E$ $\Delta P.E > 0$ $mg\ \Delta h > 0$ $\Delta h > 0$ $h_f > h_i$ Equivalent to lifting an object from a lower gravitational potential energy level to a higher gravitational potential level.	$W = \Delta K.E$ $\Delta K.E < 0$ $K.E_f - K.E_i < 0$ $K.E_f < K.E_i$ Kinetic energy is decreasing, so potential energy is increasing and that means energy is being transferred to the object.

5.13 The work-kinetic energy theorem is equivalent to

a) Newton's First Law.
b) Newton's Second Law.
c) Newton's Third Law.
d) Newton's Fourth Law.
e) none of Newton's laws.

$$2\,a\,\Delta d = v_f^2 - v_i^2$$

$F_{net} = m\,a$ Newtons' 2nd Law

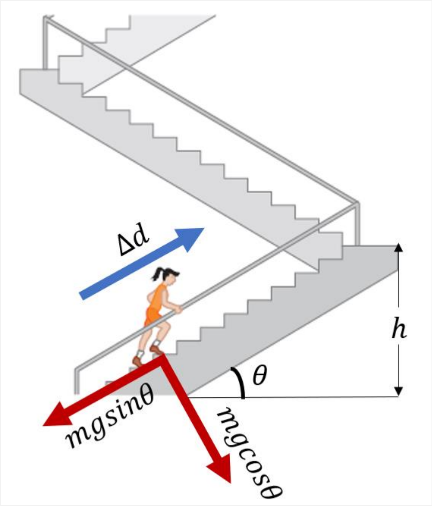
$$2\left(\frac{F_{net}}{m}\right)\Delta d = v_f^2 - v_i^2$$

$$F_{net}\,\Delta d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W = \Delta K.E$$

5.14 Shamsa climbs a flight of stairs. What can we say about the work done by gravity on her?

a) Gravity does negative work on her.
c) Gravity does no work on her.
b) Gravity does positive work on her.
d) We can't tell what work gravity does on her.

SOLUTION	DIAGRAM
$W = -\Delta P.E$ $W = -mgh$ OR $W = F\,\Delta d\,cos\theta$ $W = (mgsin\theta)\,(\Delta d)\,cos180°$ $W = -m\,g\,sin\theta\,\Delta d$ $W = -m\,g\left(\frac{h}{\Delta d}\right)\Delta d$ $W = -mgh$	

CONCEPTUAL QUESTIONS.

5.15 If the net work done on a particle is zero, what can be said about the particle's speed?

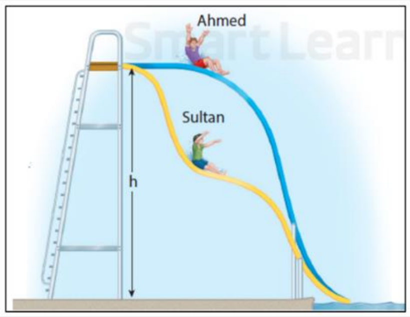
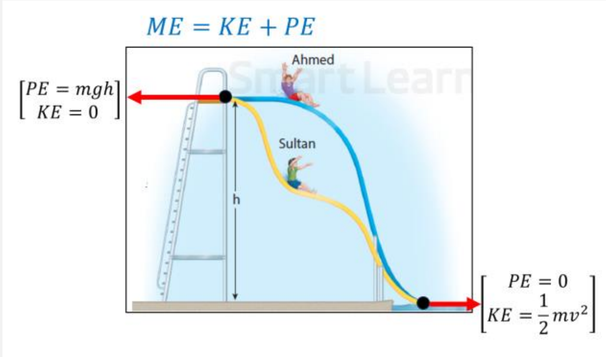
If the net work done on a particle is zero, then the net force on the particle must also be zero. If the net force is zero, then the acceleration is also zero. Hence, the particle's speed is constant.

Given that: $W = 0\,J$

$$W = \Delta K.E \implies \Delta K.E = 0 \implies v = constant.$$

5.16 Ahmed and Sultan start from rest at the same time at height h at the top of two differently configured water slides. The slides are nearly frictionless.

a) Which slider arrives first at the bottom?
b) Which slider is traveling faster at the bottom? What physical principle did you use to answer this?

<table>
<tr><td>DIAGRAM</td><td>Mechanical energy is conserved</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">

$$ME_{TOP} = ME_{BOTTOM}$$
$$mgh + 0 = 0 + \frac{1}{2}\,mv^2$$
$$mgh = \frac{1}{2}\,mv^2$$
$$v = \sqrt{2gh}$$

</td></tr>
</table>

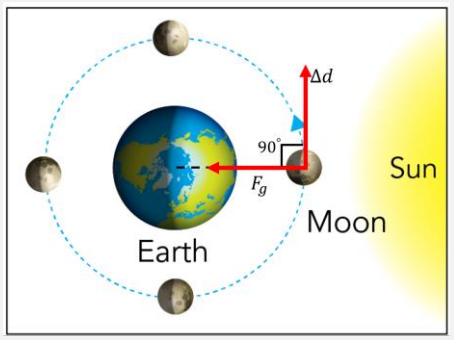
If Ahmed and Sultan both start from rest at a height h, then, by conservation of energy, they will have the same speed when they reach the bottom. That is, their initial energy is pure potential energy (mgh) and their final energy is pure kinetic energy $\left(\frac{1}{2}\,mv^2\right)$. Since energy is conserved (if we neglect friction!) then $\left(mgh = \frac{1}{2}\,mv^2\right) \Rightarrow v = \sqrt{2gh}$. Their final velocity is independent of both their mass and the shape of their respective slides!

They will in general not reach the bottom at the same time. From the figure, Sultan will likely reach the bottom first since he will accelerate faster initially and will attain a larger speed sooner. Ahmed will start off much slower and will acquire the bulk of his acceleration towards the end of his slide.

5.17 Does the Earth do any work on the Moon as the Moon moves in its orbit?

<table>
<tr><td></td><td></td></tr>
<tr><td>No. The gravitational force that the Earth exerts on the Moon is perpendicular to the Moon's displacement and so no work is done.

$$W = F\,\Delta d\,cos\theta$$
$$W = (F_g)\,(\Delta d)\,cos90°$$
$$W = 0\,J$$

</td><td>
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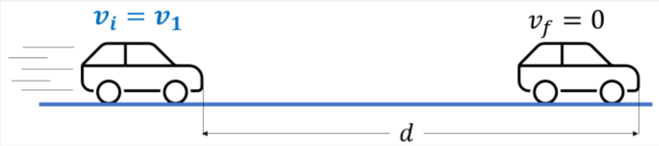
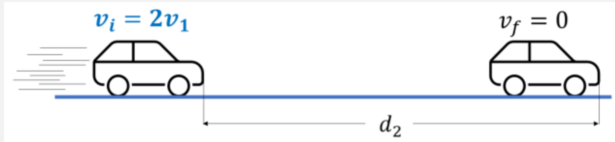
</table>

5.18 A car, of mass m, traveling at a speed v_1 can brake to a stop within a distance d. If the car speeds up by a factor of 2, so that $v_2 = 2v_1$, by what factor is its stopping distance increased, assuming that the braking force F is approximately independent of the car's speed?

First scenario $(velocity = v_1)$	Second scenario $(velocity = 2v_1)$
	
$W = \Delta K.E$ $F\, d\, cos\theta = \left(\frac{1}{2}\, m\, v_f^2\right) - \left(\frac{1}{2}\, m\, v_i^2\right)$ $F\, d\, cos180^{\circ} = (0) - \left(\frac{1}{2}\, m\, v_1^2\right)$ $F\, d\, (-1) = -\left(\frac{1}{2}\, m\, v_1^2\right)$ $Fd = \left(\frac{1}{2}\, m\, v_1^2\right) \Longrightarrow [1]$	$W = \Delta K.E$ $F\, d_2\, cos\theta = \left(\frac{1}{2}\, m\, v_f^2\right) - \left(\frac{1}{2}\, m\, v_i^2\right)$ $F\, d_2\, cos180^{\circ} = (0) - \left(\frac{1}{2}\, m\, (2v_1)^2\right)$ $F\, d_2\, (-1) = -4\left(\frac{1}{2}\, m\, v_1^2\right)$ $Fd_2 = 4\left(\frac{1}{2}\, m\, v_1^2\right) \Longrightarrow [2]$
$\frac{[2]}{[1]} \Longrightarrow\Longrightarrow \frac{Fd_2}{Fd} = \frac{4\left(\frac{1}{2}\, m\, v_1^2\right)}{\left(\frac{1}{2}\, m\, v_1^2\right)} \Longrightarrow \frac{d_2}{d} = \frac{4}{1} \Longrightarrow d_2 = 4\, d$ Doubling the velocity will quadrable the stopping distance.	

EXERCISES

Section 5.2

5.19 The damage done by a projectile on impact is correlated with its kinetic energy. Calculate and compare the kinetic energies of these three projectiles:

a) a 10.0 kg stone at 30.0 m/s

b) a 100.0 g baseball at 60.0 m/s

c) a 20.0 g bullet at 300. m/s

Solution: $K.E = 0.5\, m\, v^2$

[a]	[b]	[c]
$K.E = 0.5\, m\, v^2$ $K.E = 0.5 \times 10.0 \times 30.0^2$ $K.E = 4500\, J$ $K.E = 4.50 \times 10^3 J$	$K.E = 0.5\, m\, v^2$ $K.E = 0.5 \times 0.100 \times 60.0^2$ $K.E = 180\, J$ $K.E = 1.80 \times 10^2 J$	$K.E = 0.5\, m\, v^2$ $K.E = 0.5 \times 0.020 \times 300.0^2$ $K.E = 900\, J$ $K.E = 9.00 \times 10^2 J$

5.20 A limo is moving at a speed of 100 km/h. If the mass of the limo, including passengers, is 1900 kg, what is its kinetic energy?

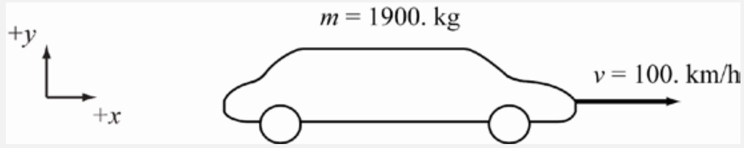
Solution:

$$K.E = 0.5\, m\, v^2$$

$$K.E = 0.5 \times 1900 \times \left(\frac{100}{3.6}\right)^2$$

$$K.E = 733024.6914\, J$$

$$K.E = 7.3 \times 10^5\, J$$



5.21 Two railroad cars, each of mass 7000 kg and traveling at 90.0 km/h, collide head on and come to rest. How much mechanical energy is lost in this collision?

$$K.E_{total} = K.E_1 + K.E_2$$

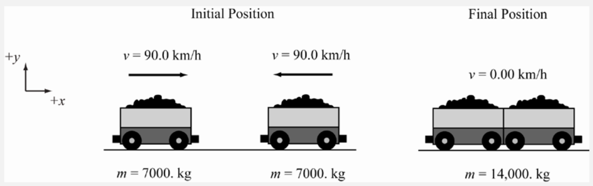
$$K.E_{total} = (0.5 \times m_1 \times v_1^2) + (0.5 \times m_2 \times v_2^2)$$

$$K.E_{total} = \left(0.5 \times 7000 \times \left(\frac{90.0}{3.6}\right)^2\right) + \left(0.5 \times 7000 \times \left(\frac{90.0}{3.6}\right)^2\right)$$

$$K.E_{total} = 4375000\, J$$

$$K.E_{total} = 4.38 \times 10^6\, J$$

$$K.E_{total} = 4.38\, MJ$$

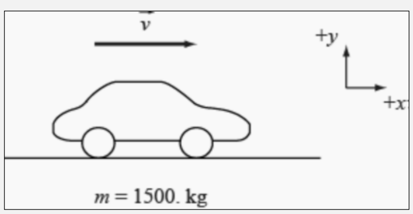


5.22 Think about the answers to these questions next time you are driving a car:
a) What is the kinetic energy of a 1500 kg car moving at 15.0 m/s?
b) If the car changed its speed to 30.0 m/s, how would the value of its kinetic energy change?

[a]	[b]
$K.E_{(v=15)} = 0.5\, m\, v^2$ $K.E_{(v=15)} = 0.5 \times 1500 \times (15)^2$ $K.E_{(v=15)} = 168750\, J$ $K.E_{(v=15)} = 1.69 \times 10^5\, J$	$K.E_{(v=30)} = 0.5\, m\, v^2$ $K.E_{(v=30)} = 0.5 \times 1500 \times (30)^2$ $K.E_{(v=30)} = 675000\, J$ $K.E_{(v=30)} = 6.75 \times 10^5\, J$

$$K.E_{(v=30)} = 4 \times K.E_{(v=15)}$$

Doubling the velocity will quadrable the kinetic energy.



5.23 A 200 kg moving tiger has a kinetic energy of 14,400 J. What is the speed of the tiger?

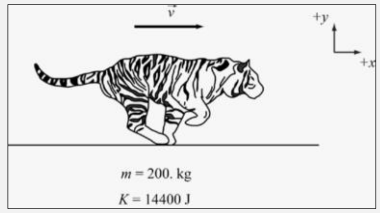
Solution

$$K.E = 0.5\, m\, v^2$$

$$14400 = 0.5 \times 200 \times v^2$$

$$v = 12\ m/s = 43.2\ km/h$$

Diagram



5.24 Two cars are moving. The first car has twice the mass of the second car but only half as much kinetic energy. When both cars increase their speed by 5.00 m/s, they then have the same kinetic energy. Calculate the original speeds of the two cars.

Solution

BEFORE.

$$(K.E)_1 = 0.5 \times (K.E)_2$$

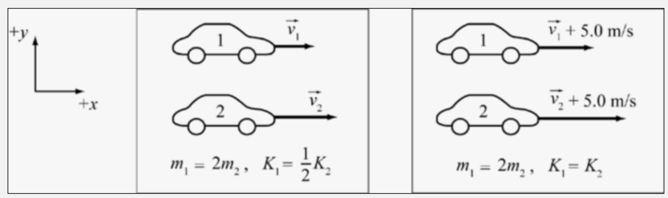
$$0.5 \times m_1 \times v_1^2 = 0.5 \times (0.5 \times m_2 \times v_2^2)$$

$$0.5 \times 2 \times m_2 \times v_1^2 = 0.5 \times (0.5 \times m_2 \times v_2^2)$$

$$v_1^2 = 0.25 \times v_2^2 \quad \Longrightarrow \quad v_1 = 0.5\, v_2$$

Diagram

Before / After



AFTER.

$$(K.E)_1 = (K.E)_2$$

$$0.5 \times m_1 \times v_1^2 = 0.5 \times m_2 \times v_2^2$$

$$0.5 \times 2 \times m_2 \times (v_1 + 5)^2 = 0.5 \times m_2 \times (v_2 + 5)^2$$

$$(v_1 + 5)^2 = 0.5 \times (v_2 + 5)^2$$

$$(0.5\, v_2 + 5)^2 = 0.5 \times (v_2 + 5)^2$$

$$0.5\, v_2 + 5 = \sqrt{0.5}(v_2 + 5) \quad \text{[Shift – Solve]}$$

$$v_2 = \sqrt{50} = 7.07\ m/s \qquad \Longrightarrow\Longrightarrow \quad v_1 = 0.5(\sqrt{50}) = 3.54\ m/s$$

5.25 What is the kinetic energy of an ideal projectile of mass 20.1 kg at the apex (highest point) of its trajectory, if it was launched with an initial speed of 27.3 m/s and at an initial angle of 46.9° with respect to the horizontal?

Solution

At maximum height: $v_y = 0\ \&\ v_x = v_o cos\theta$

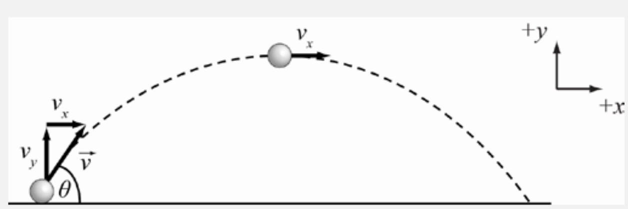
$v_x = (27.3)cos46.9^{\circ} = 18.7\ m/s$

$K.E = 0.5\, m\, v^2$

$K.E = 0.5 \times 20.1 \times (18.7)^2$

$K.E = 3496.88\, J = 3.5 \times 10^3\ J$

Diagram



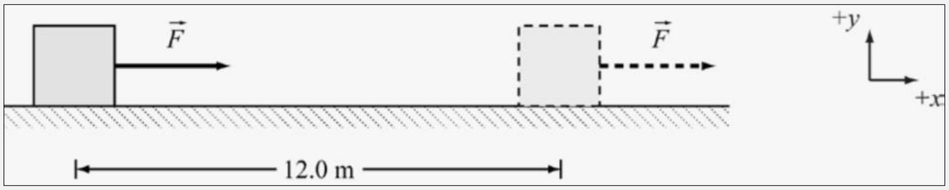
Section 5.4

5.26 A force of 5.00 N acts over a distance of 12.0 m in the direction of the force. Find the work done.

$$W = Fd\,cos\theta$$

$$W = 5.00 \times 12.0 \times cos0^{\circ}$$

$$W = 60.0\,J$$



5.27 Two baseballs are thrown off the top of a building that is 7.25 m high. Both are thrown with initial speed of 28.4 m/s. Ball 1 is thrown horizontally, and ball 2 is thrown straight down. What is the difference in the speeds of the two balls when they touch the ground? (Neglect air resistance.)

Given: $v_i = v_o \quad \& \quad \vec{a} = \vec{g} \Longrightarrow a = -g \quad \& \quad \Delta y = -h$

Ball (1)

$$\Delta y = -\frac{1}{2}\,g\,t^2 + (v_o sin\theta)t$$

$$-7.25 = -\frac{1}{2}(9.81)\,t^2 + (28.4 \times sin0^{\circ})t$$

$$7.25 = 4.905 \times t^2 \quad \Longrightarrow t^2 = \frac{2h}{g}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

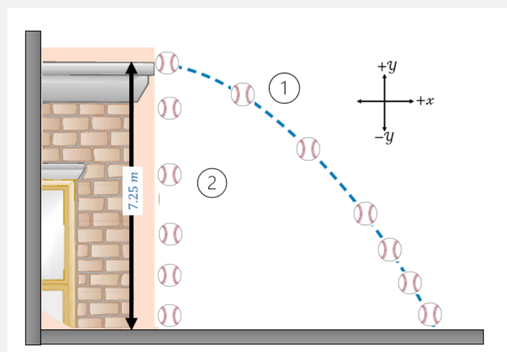
$$v = \sqrt{(v_o cos\theta)^2 + (-gt + v_o sin\theta)^2}$$

$$v = \sqrt{(v_o cos0^{\circ})^2 + (-gt + v_o sin0^{\circ})^2}$$

$$v = \sqrt{v_o^2 + g^2t^2}$$

$$v = \sqrt{v_o^2 + 2gh} = 30.8\ m/s$$

DIAGRAM



Ball (2)

$$v_f^2 - v_i^2 = 2a\Delta y$$

$$v^2 - v_o^2 = 2gh$$

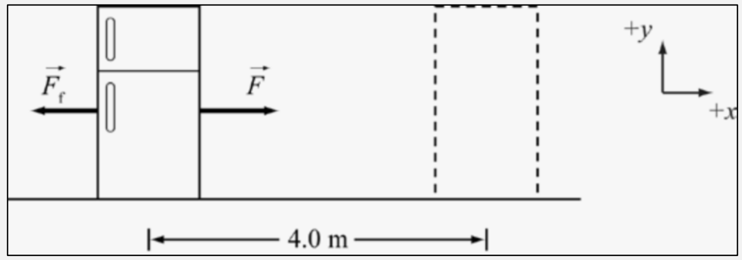
$$v^2 = v_o^2 + 2gh$$

$$v = \sqrt{v_o^2 + 2gh} = 30.8\ m/s$$

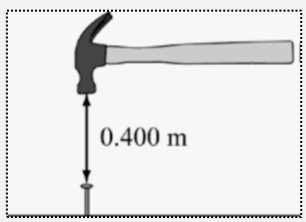
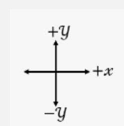
Both baseballs will arrive the ground at the same velocity.

NOTE: the gravitational field is a conservative field; the mechanical energy is conserved.

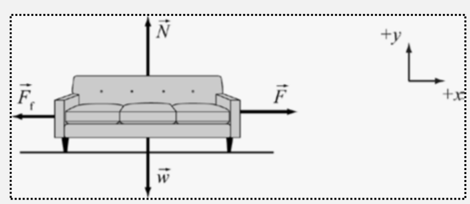
5.28 A 95.0-kg refrigerator rests on the floor. How much work is required to move it at constant speed for 4.00 m along the floor against a friction force of 180 N?

$W_F = Fd\cos\theta$ $W_F = 180 \times 4.00 \times \cos 0^{\circ}$ $W_F = 720\,J$ $W_{F_f} = Fd\cos\theta$ $W_{F_f} = 180 \times 4.00 \times \cos 180^{\circ}$ $W_{F_f} = -720\,J$	

5.29 A hammerhead of mass m = 2.00 kg is allowed to fall onto a nail from a height h = 0.400 m. Calculate the maximum amount of work it could do on the nail.

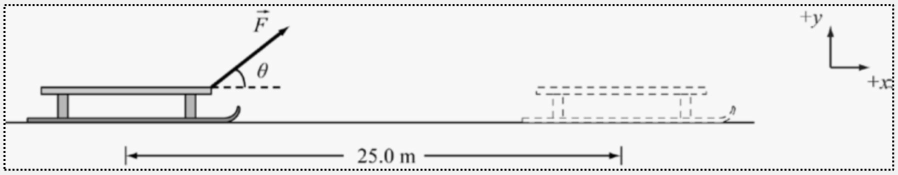
Solution	Diagram	
$W = -\Delta P.E$ $W = -m\,g\,(\Delta y)$ $W = -2.00 \times 9.81 \times (-0.400)$ $W = 7.848\,J = 7.85\,J$		

5.30 You push your couch a distance of 4.00 m across the living room floor with a horizontal force of 200.0 N. The force of friction is 150.0 N. What is the work done by you, by the friction force, by gravity, and by the net force?

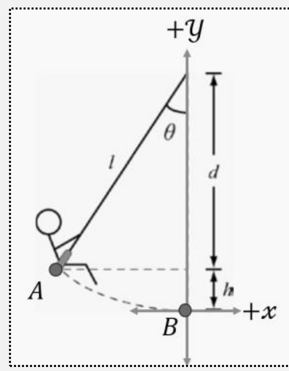
Work by (F)	Work by (F_f)	Diagram
$W_F = Fd\cos\theta$ $W_F = 200 \times 4.00 \times \cos 0^{\circ}$ $W_F = 800\,J$	$W_{F_f} = F_f d\cos\theta$ $W_{F_f} = 150 \times 4.00 \times \cos 180^{\circ}$ $W_{F_f} = -600\,J$	

Work by gravity $(w = mg)$	Work by net force
$W_w = Fd\cos\theta$ $W_w = mg \times d \times \cos 90^{\circ}$ $W_w = 0\,J$	$W_{F_{net}} = Fd\cos\theta$ $W_w = F_{net} \times d \times \cos 0^{\circ}$ $W_w = (200 - 150)(4.00)(1) = 200\,J$

5.31 Suppose you pull a sled with a rope that makes an angle of 30.0° to the horizontal. How much work do you do if you pull with 25.0 N of force and the sled moves 25.0 m?

$W = Fd\ cos\theta$ $W = 25.0 \times 25.0 \times cos30^{\circ}$ $W = 541\ J$	

5.32 A father pulls his son, whose mass is 25.0 kg and who is sitting on a swing with ropes of length 3.00 m, backward until the ropes make an angle of 33.6° with respect to the vertical. He then releases his son from rest. What is the speed of the son at the bottom of the swinging motion?

Conservation of Energy	Geometry	Diagram
Apply the principle of conservation of energy. $(M.E)_A = (M.E)_B$ $(mgh + 0.5mv^2)_A = (mgh + 0.5mv^2)_B$ $mgh = 0.5mv_B^2$ $v_B = \sqrt{2gh}$ $v_B = \sqrt{2g\ell(1 - cos\theta)}$ $v_B = \sqrt{2 \times 9.81 \times 3.00 \times (1 - cos33.6^{\circ})}$ $v_B = 3.14\ m/s$	$\ell = d + h$ $h = \ell - d$...... [1] $cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $cos\theta = \frac{d}{\ell}$ $d = \ell cos\theta$ ..... [2] $h = \ell - \ell cos\theta$ $h = \ell(1 - cos\theta)$	

5.33 A constant force, F= (4.79, -3.79, 2.09) N, acts on an object of mass 18.0 kg, causing a displacement of that object by = (4.25, 3.69, -2.45) m. What is the total work done by this force?

$W = \vec{F}.\Delta\vec{r}$

$W = (4.79, -3.79, 2.09).(4.25, 3.69, -2.45)$

$W = (4.79 \times 4.25) + (-3.79 \times 3.69) + (2.09 \times -2.45)$

$W = 1.2519\ J$

$W = 1.25\ J$

5.34 A mother pulls her daughter, whose mass is 20.0 kg and who is sitting on a swing with ropes of length 3.50 m, backward until the ropes make an angle of 35.0° with respect to the vertical. She then releases her daughter from rest. What is the speed of the daughter when the ropes make an angle of 15.0° with respect to the vertical?

Conservation of Energy

Apply the principle of conservation of energy.

$$(M.E)_A = (M.E)_B$$

$$(mgh + 0.5mv^2)_A = (mgh + 0.5mv^2)_B$$

$$mgh_1 = mgh_2 + 0.5mv_B^2$$

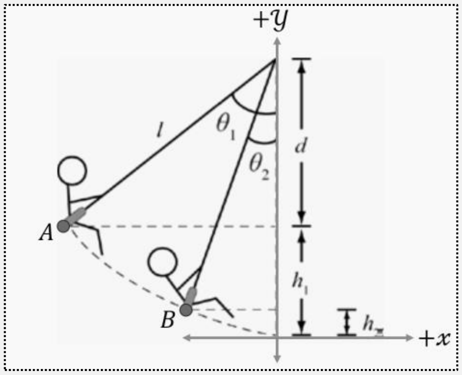
$$v_B = \sqrt{2g(h_1 - h_2)}$$

$$v_B = \sqrt{2g\ell(cos\theta_2 - cos\theta_1)}$$

$$v_B = \sqrt{2 \times 9.81 \times 3.50 \times (cos15.0^\circ - cos35.0^\circ)}$$

$$v_B = 3.17\ m/s$$

Diagram



Geometry

$$cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$Adj. = hyp. \times cos\theta$$

$$d = \ell\ cos\theta_1\ .... [1]$$

$$d + h_1 - h_2 = \ell cos\theta_2\ .... [2]$$

[2]– [1]

$$d + h_1 - h_2 - d = \ell cos\theta_2 - \ell\ cos\theta_1$$

$$h_1 - h_2 = \ell(cos\theta_2 - cos\theta_1)$$

5.35 A ski jumper glides down a 30.0° slope for 24.4 m before taking off from a negligibly short horizontal ramp. If the jumper's takeoff speed is 13.7 m/s, what is the coefficient of kinetic friction between skis and slope?

Conservation of Energy

$F_N = mgcos\theta$ & $F_f = \mu_k F_N = \mu_k mgcos\theta$

$W_{net} = \Delta K.E$ & $W_{net} = W_{F_g} + W_{F_N}$

$$\Delta K.E = W_{F_g} + W_{F_N}$$

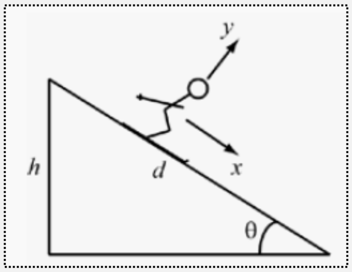
$$0.5mv_f^2 - 0.5mv_i^2 = (mgsin\theta)dcos0^\circ + (\mu_k mgcos\theta)dcos180^\circ$$

$$\mu_k = \frac{(gsin\theta)d - 0.5v_f^2}{(gcos\theta)d}$$

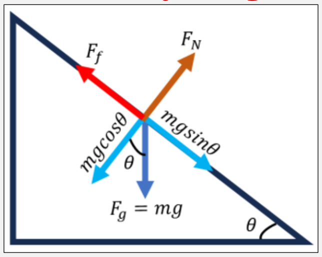
$$\mu_k = \frac{(9.81 \times sin30^\circ)(24.4) - (0.5 \times 13.7^2)}{(9.81 \times cos30^\circ)(24.4)}$$

$$\mu_k = 0.125$$

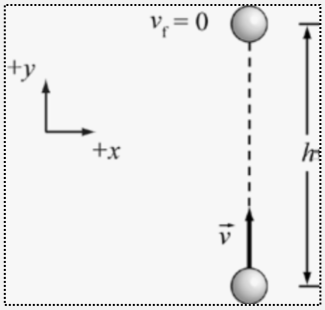
Diagram



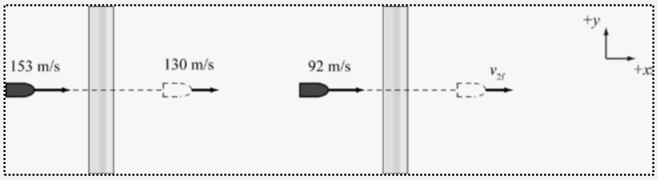
Free – Body Diagram



5.36 At sea level, a nitrogen molecule in the air has an average kinetic energy of 6.2×10^{-21}J. Its mass is 4.7×10^{-26} kg. If the molecule could shoot straight up without colliding with other molecules, how high would it rise? What percentage of the Earth's radius is this height? What is the molecule's initial speed?
(Assume that you can use g = 9.81 m/s².

Conservation of Energy	Diagram
At maximum height $v_f = 0\ m/s$ Apply the principle of conservation of energy. $(mgh + 0.5mv^2)_i = (mgh + 0.5mv^2)_f$ $0 + 0.5mv_i^2 = mgh + 0$ $h = \frac{0.5mv_i^2}{mg} = \frac{6.2 \times 10^{-21}}{4.7 \times 10^{-26} \times 9.81} = 13447\ m$ $\%\ of\ Earths'\ radius = \frac{13447\ m}{6.37 \times 10^6} = 0.0021 = 0.21\%$	 $K.E = 0.5mv^2 \implies v = \sqrt{\frac{K.E}{0.5\ m}}$ $v = \sqrt{\frac{6.2 \times 10^{-21}}{0.5 \times 4.7 \times 10^{-26}}}$ $v = 514\ m/s$

5.37 A bullet moving at a speed of 153 m/s passes through a plank of wood. After passing through the plank, its speed is 130 m/s. Another bullet, of the same mass and size but moving at 92.0 m/s, passes through an identical plank. What will this second bullet's speed be after passing through the plank? Assume that the resistance offered by the plank is independent of the speed of the bullet.

work – kinetic energy theorem.	Diagram
$W = \Delta K.E$ $F d\cos\theta = 0.5mv_f^2 - 0.5mv_i^2$ $F d\cos\theta = 0.5m(v_f^2 - v_i^2)$ $F d\cos 0^\circ = 0.5m(130^2 - 153^2)$ $Fd = 0.5m(130^2 - 153^2)$ ….. [1] $Fd = 0.5m(v_{2f}^2 - 92^2)$ ….. [2]	 $0.5m(130^2 - 153^2) = 0.5m(v_{2f}^2 - 92^2)$ $(130^2 - 153^2) = (v_{2f}^2 - 92^2)$ $v_{2f} = 44.2\ m/s$

Section 5.5

5.38 A particle of mass m is subjected to a force acting in the x-direction. $F_x = (3.00 + 0.500x)\,N$. Find the work done by the force as the particle moves from x = 0.00 to x = 4.00 m.

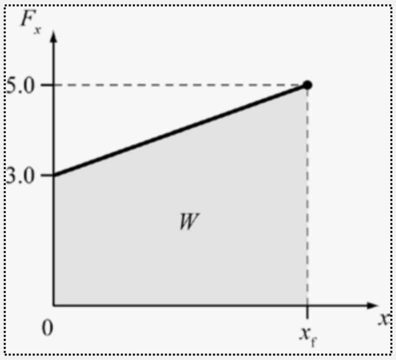
Analytical Solution: $W = \int_0^{4.00} (3.00 + 0.500\,x)\,dx = 16\,J$

Graphical Solution:
Work = Bounded Area under F – d graph.

$$W = \frac{1}{2} \times (base_1 + base_2) \times height$$

$$W = \frac{1}{2} \times (3.0 + 5.0) \times 4.0$$

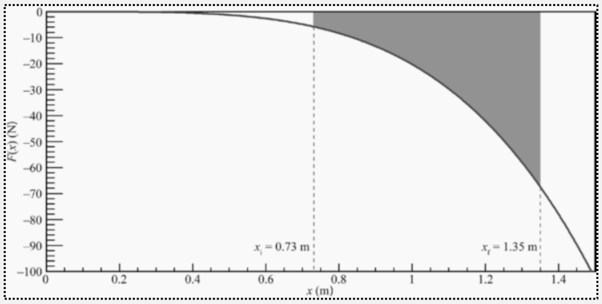
$$W = 16\,J$$



5.39 A force has the dependence $F_x(x) = -kx^4$ on the displacement x, where the constant $k = 20.3\,N/m^4$. How much work does it take to change the displacement, working against the force, from 0.730 m to 1.35 m?

Analytical Solution: $W = \int_{0.730}^{1.35} (-20.3 \times x^4)\,dx = -17.4\,J$

Graphical Solution:
Work = Bounded Area under F – d graph.



5.40 A body of mass m moves along a trajectory $\vec{r}(t)$ in three-dimensional space with constant kinetic energy. What geometric relationship has to exist between the body's velocity vector, $\vec{v}(t)$, and its acceleration vector, $\vec{a}(t)$, in order to accomplish this?

$constant\ kinetic\ energy \Longrightarrow |\vec{v}(t)| = constant$

$W = \Delta K.E$

$W = 0$

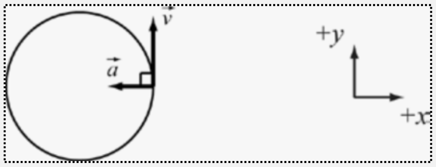
$\vec{F}.\vec{r}(t) = 0 \quad \Longrightarrow \vec{F} \perp \vec{r}(t)$

$$P = \frac{W}{t}$$

$$\vec{F}.\vec{v}(t) = \frac{\Delta K.E}{t}$$

$m\vec{a}.\vec{v}(t) = 0 \quad \Longrightarrow \vec{a} \perp \vec{v}(t)$

If a particle is moving in a circular motion at constant speed the kinetic energy is constant. The acceleration vector is always perpendicular to the velocity vector.

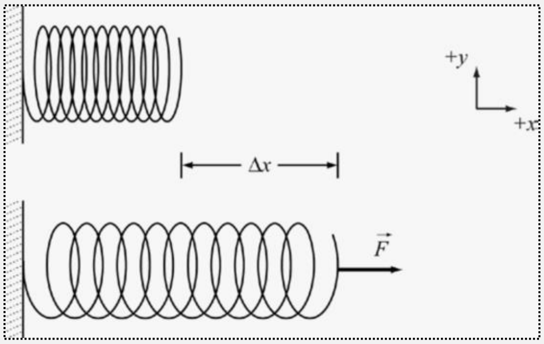


5.41 A force given by $\vec{F}(x) = 5x^3\,\hat{x}$ (in N/m^3) acts on a 1.00 kg mass moving on a frictionless surface. The mass moves from x = 2.00 m to x = 6.00 m.
a) How much work is done by the force?
b) If the mass has a speed of 2.00 m/s at x = 2.00 m, what is its speed at x = 6.00 m?

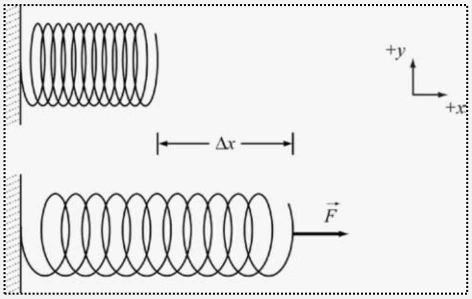
[a]	[b]
$W = \int_{2.00}^{6.00} (5x^3)\,dx = 1600\text{ J}$	$W = \Delta K.E$ $W = 0.5mv_f^2 - 0.5mv_i^2$ $1600 = 0.5(1.00)v_f^2 - 0.5(1.00)(2.00^2)$ $v_f = 56.6\ m/s$

Section 5.6

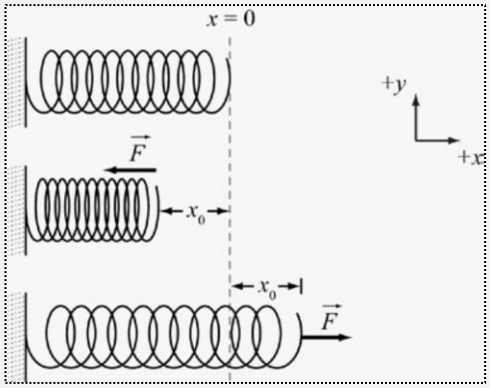
5.42 An ideal spring has the spring constant k = 440 N/m. Calculate the distance this spring must be stretched from its equilibrium position for 25.0 J of work to be done.

Elastic potential energy.	Diagram
$W_s = 0.5k(\Delta x)^2$ $25.0 = 0.5(440)(\Delta x)^2$ $\Delta x = 0.34\ m$	

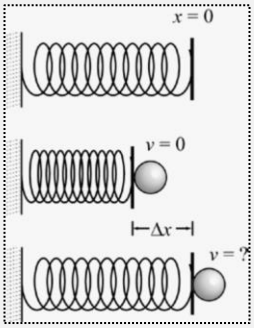
5.43 A spring is stretched 5.00 cm from its equilibrium position. If this stretching requires 30.0 J of work, what is the spring constant?

Elastic potential energy.	Diagram
$W_s = 0.5k(\Delta x)^2$ $30.0 = 0.5\,k(0.05)^2$ $k = 24000\ \ N/m$ $k = 2.40\ \times 10^4\ \ N/m$	

5.44 A spring with spring constant k is initially compressed a distance x_o from its equilibrium length. After returning to its equilibrium position, the spring is then stretched a distance x_o from that position. What is the ratio of the work that needs to be done on the spring in the stretching to the work done in the compressing?

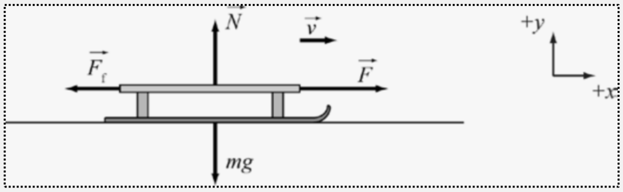
Stretching vs Compressing	Diagram
$(W_s)_{strech.} = 0.5k(\Delta x)^2$ $(W_s)_{strech.} = 0.5k(x_o - 0)^2$ $(W_s)_{strech.} = 0.5kx_o^2$ $(W_s)_{compr.} = 0.5k(\Delta x)^2$ $(W_s)_{compr.} = 0.5k(0 - x_o)^2$ $(W_s)_{compr.} = 0.5kx_o^2$ $\frac{(W_s)_{strech.}}{(W_s)_{compr.}} = \frac{0.5kx_o^2}{0.5kx_o^2} = 1 : 1$	

5.45 A spring with a spring constant of 238.5 N/m is compressed by 0.231 m. Then a steel ball bearing of mass 0.0413 kg is put against the end of the spring, and the spring is released. What is the speed of the ball bearing right after it loses contact with the spring? (The ball bearing will come off the spring exactly as the spring returns to its equilibrium position. Assume that the mass of the spring can be neglected.)

Elastic potential energy.	Work – K.E theorem.	Diagram
$W_s = 0.5k(\Delta x)^2$ $W_s = 0.5\ (238.5)(0.231)^2$ $W_s = 6.36$ J	$W = \Delta K.E$ $\text{W} = 0.5mv_f^2 - 0.5mv_i^2$ $6.36 = 0.5(0.0431)v_f^2 - 0.5(0.0431)0^2$ $v_f = 17.2\ m/s$	

Section 5.7

5.46 A horse draws a sled horizontally across a snow-covered field. The coefficient of friction between the sled and the snow is 0.195, and the mass of the sled, including the load, is 202.3 kg. If the horse moves the sled at a constant speed of 1.785 m/s, what is the power needed to accomplish this?

$Constant\ speed \Longrightarrow a = 0 \Longrightarrow \lvert\vec{F}\rvert = \lvert\vec{F}_f\rvert$ $F_f = \mu N = \mu mg$ $P = \vec{F}.\vec{v}$ $P = F\ v\ cos\theta$ $P = (\mu mg)v\ cos\theta$ $P = 0.195 \times 202.3 \times 9.81 \times 1.785 \times cos0^{\circ}$ $P = 691\ Watts$	

5.47 A horse draws a sled horizontally on snow at constant speed. The horse can produce a power of 1.060 hp. The coefficient of friction between the sled and the snow is 0.115, and the mass of the sled, including the load, is 204.7 kg. What is the speed with which the sled moves across the snow?

$Constant\ speed \implies a = 0 \implies |\vec{F}| = |\vec{F_f}|$

$F_f = \mu N = \mu mg$

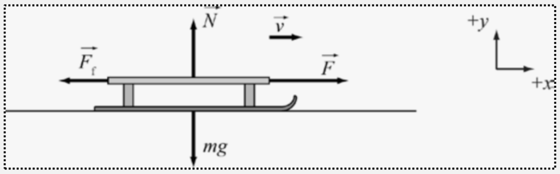
$P = \vec{F}.\vec{v}$

$P = F\ v\ cos\theta$

$P = (\mu mg) v\ cos\theta$

$1.060 \times 746 = 0.115 \times 204.7 \times 9.81 \times v \times cos0^\circ$

$v = 3.42\ m/s$



5.48 While a boat is being towed at a speed of 12.0 m/s, the tension in the towline is 6.00 kN. What is the power supplied to the boat through the towline?

$P = \vec{F}.\vec{v}$

$P = F\ v\ cos\theta$

$P = 6.00 \times 10^3 \times 12.0 \times cos0^\circ$

$P = 6.00 \times 10^3 \times 12.0 \times cos0^\circ$

$P = 7.20 \times 10^4\ Watts$

NOTE: nothing has been mentioned about the angle between F and v, so it was assumed to be zero.

5.49 A car of mass 1214.5 kg is moving at a speed of 27.9 m/s when it misses a curve in the road and hits a bridge piling. If the car comes to rest in 0.236 s, how much average power (in watts) is expended in this interval?

$W = 0.5mv_f^2 - 0.5mv_i^2$

$W = 0.5(1214.5)(0.00)^2 - 0.5(1214.5)(27.9)^2$

$W = -472689.4725\ J$

$|W| = 472689.4725\ J$

$P = \frac{W}{t}$

$P = \frac{472689.4725}{0.236} = 2.00 \times 10^6\ Watts$

5.50 An engine expends 40.0 hp in moving a car along a level track at a speed of 15.0 m/s. How large is the total force acting on the car in the direction opposite to the motion of the car?

$P = \vec{F}.\vec{v}$

$P = F\ v\ cos\theta$

$40.0\ \times 746 = F \times 15.0 \times\ cos180^{\circ}$

$F = -1.99\ \times 10^{3}\ N$

The minus sign indicates that F is opposite to the motion of the car.

5.51 A car of mass 942.4 kg accelerates from rest with a constant power output of 140.5 hp. Neglecting air resistance, what is the speed of the car after 4.55 s?

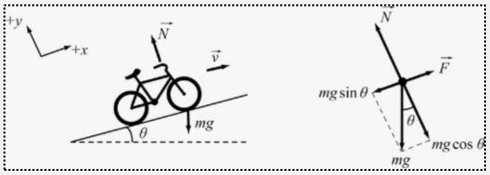
$$P = \frac{W}{t}$$

$$P = \frac{0.5mv_f^2 - 0.5mv_i^2}{t}$$

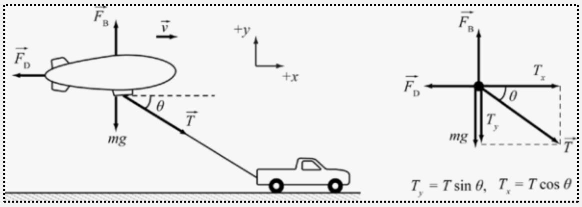
$$140.5\ \times 746 = \frac{0.5(942.4)v_f^2 - 0.5(942.4)(0.00)^2}{4.55}$$

$v_f = 31.8\ m/s$

5.52 A bicyclist coasts down a 7.0° slope at a steady speed of 5.0 m/s. Assuming a total mass of 75 kg (bicycle plus rider), what must the cyclist's power output be to pedal up the same slope at the same speed?

Total Power.	Diagram
$Steady\ speed \Longrightarrow a = 0 \Longrightarrow \lvert\vec{F}\rvert = mgsin\theta$ $\lvert\vec{F}\rvert = mgsin\theta = 89.7\ N$ $P_F = \vec{F}.\vec{v}$ $P_F = F\ v\ cos\varphi$ $P_F = 89.7 \times 5.0 \times\ cos0.0^{\circ}$ $P_F = 448\ Watts$ $P_{mgsin\theta} = -448\ Watts$ $\lvert P_{mgsin\theta}\rvert = 448Watts$	 $P_{total} = 448 + 448\ Watts$ $P_{total} = 896\ Watts$

5.53 A small blimp is used for advertising purposes at a football game. It has a mass of 93.5 kg and is attached by a towrope to a truck on the ground. The towrope makes an angle of 53.3° downward from the horizontal, and the blimp hovers at a constant height of 19.5 m above the ground. The truck moves on a straight line for 840.5 m on the level surface of the stadium parking lot at a constant velocity of 8.90 m/s. If the drag coefficient (K in $F = Kv^2$) is 0.500 kg/m, how much work is done by the truck in pulling the blimp (assuming there is no wind)?

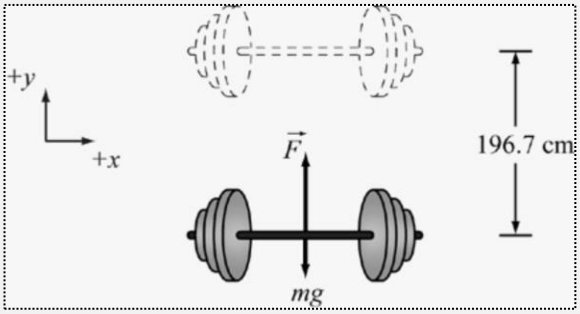
Dynamics + Energy.	Diagram
$Constant\ velocity \Longrightarrow a = 0 \Longrightarrow F_D = T\cos\theta$ $0.500v^2 = T\cos\theta$ $W = F\ d\ \cos\varphi$ $W = (T\cos\theta)\ d\ \cos\varphi$ $W = (0.500v^2)\ d\ \cos\varphi$ $W = (0.500 \times 8.90^2)\ (840.5)\ \cos 0.0^{\circ}$ $W = 33288.0025\ J = 3.33\ \times 10^4 J$	

5.54 A car of mass m accelerates from rest along a level straight track, not at a constant acceleration but with constant engine power, P. Assume that air resistance is negligible.
a) Find the car's velocity as a function of time.
b) A second car starts from rest alongside the first car on the same track but maintains a constant acceleration. Which car takes the initial lead? Does the other car overtake it? If yes, write a formula for the distance from the starting point at which this happens.
c) You are in a drag race, on a straight level track, with an opponent whose car maintains a constant acceleration of 12.0 m/s². Both cars have identical masses of 1000 kg. The cars start together from rest. Air resistance is assumed to be negligible. Calculate the minimum power your engine needs for you to win the race, assuming the power output is constant and the distance to the finish line is 0.250 mi.

[a]	[b]	[c]
$P = \dfrac{W}{t}$ $W = Pt$ $W = Pt$ $0.5mv_f^2 - 0.5mv_i^2 = P\ t$ $0.5mv_f^2 = P\ t$ $v_f = \sqrt{\dfrac{2Pt}{m}}$	$v = at + v_i$ $v = at$ $\Delta x = 0.5at^2 + v_i$ $\Delta x = 0.5at^2$	$v_1 = \sqrt{\dfrac{2Pt}{m}} = \sqrt{\dfrac{Pt}{500}}$ $v_2 = at = 12t$

ADDITIONAL EXERCISES

5.55 At the 2004 Olympic Games in Athens, Greece, athlete Hossein Rezazadeh won the super-heavyweight class gold medal in weightlifting. He lifted 472.5 kg combined in his two best lifts in the competition. Assuming that he lifted the weights a height of 196.7 cm, what work did he do?

Solution.	Diagram
$W = F\,d$ $W = mgd$ $W = 472.5 \times 9.81 \times 1.967$ $W = 9117.487575\,J$ $W = 9117.5\,J$	

5.56 How much work is done against gravity in lifting a 6.00 kg weight through a distance of 20.0 cm?

$W = F\,d$

$W = mgd$

$W = 6.00 \times 9.81 \times 0.20$

$W = 11.8\,J$

5.57 A certain tractor is capable of pulling with a steady force of 14.0 kN while moving at a speed of 3.00 m/s. How much power in kilowatts and in horsepower is the tractor delivering under these conditions?

$P = \vec{F}.\vec{v}$

$P = F\,v\,cos\theta$

$P = 14.0 \times 10^3 \times 3.00 \times cos0.0^\circ$

$P = 42000\,Watts = 42.0\,kW$

$P = \frac{42000}{746} = 56.3\,hp$

5.58 A shot-putter accelerates a 7.30 kg shot from rest to 14.0 m/s. If this motion takes 2.00 s, what average power was supplied?

$P = \frac{W}{t}$

$P = \frac{0.5mv_f^2 - 0.5mv_i^2}{t}$

$$P = \frac{0.5(7.30)(14.0)^2 - 0.5(7.30)(0.00)^2}{2.00}$$

$$P = 357.7 \approx 358\, Watts$$

5.59 An advertisement claims that a certain 1200 kg car can accelerate from rest to a speed of 25.0 m/s in 8.00 s. What average power must the motor supply in order to cause this acceleration? Ignore losses due to friction.

$$P = \frac{W}{t}$$

$$P = \frac{0.5mv_f^2 - 0.5mv_i^2}{t}$$

$$P = \frac{0.5(1200)(25.0)^2 - 0.5(1200)(0.00)^2}{8.00}$$

$$P = 46875\ Watts$$

$$P = \frac{46875}{746} = 62.8\, hp$$

5.60 A car of mass m = 1250 kg is traveling at a speed of $v_o = 105\, km/h$ (29.2 m/s). Calculate the work that must be done by the brakes to completely stop the car.

$$W = 0.5mv_f^2 - 0.5mv_i^2$$

$$W = 0.5(1250)(0.00)^2 - 0.5(1250)(29.2)^2$$

$$W = -532900\text{ J}$$

$$W \approx -533\text{ kJ}$$

5.61 An arrow of mass m = 88.0 g (0.0880 kg) is fired from a bow. The bowstring exerts an average force of F = 110 N on the arrow over a distance d = 78.0 cm (0.780 m). Calculate the speed of the arrow as it leaves the bow.

$$W = 0.5mv_f^2 - 0.5mv_i^2$$

$$Fd = 0.5mv_f^2 - 0.5mv_i^2$$

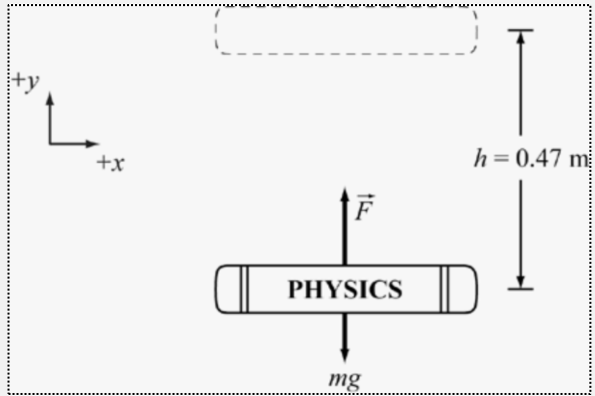
$$(110)(0.780) = 0.5(0.0880)v_f^2 - 0.5(0.0880)(0.00)^2$$

$$v_f = 44.2\ m/s$$

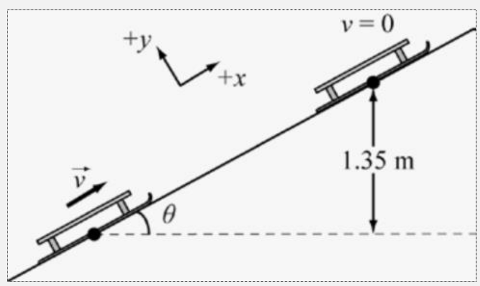
5.62 The mass of a physics textbook is 3.40 kg. You pick the book up off a table and lift it 0.470 m at a constant speed of 0.270 m/s.

a) What is the work done by gravity on the book?

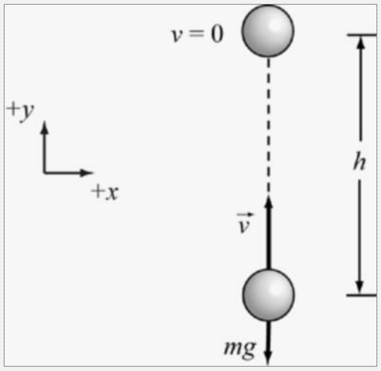
b) What is the power you supplied to accomplish this task?

Solution.	Diagram
[a] $W = F\ d\ cos\theta$ $W = -mgd$ $W = -3.40 \times 9.81 \times 0.47$ $W = -15.67638\ J$ $W = -15.7\ J$ [b] $P = F\ v\ cos\theta$ $P = (3.40 \times 9.81)\ (0.270)\ cos0^{\circ}$ $P = 9.01\ Watts$	

5.63 A sled, with mass m, is given a shove up a frictionless incline, which makes a 28.0° angle with the horizontal. Eventually, the sled comes to a stop at a height of 1.35 m above where it started. Calculate its initial speed.

Solution.	Diagram
$W = \Delta K.E \quad \& \quad W = -\Delta P.E$ $\Delta K.E = -\Delta P.E$ $0.5mv_f^2 - 0.5mv_i^2 = -mg\Delta h$ $0.5m(v_f^2 - v_i^2) = -mg\Delta h$ $0.5(v_f^2 - v_i^2) = -g\Delta h$ $0.5(0^2 - v_i^2) = (-9.8)(1.35)$ $-0.5v_i^2 = (-9.8)(1.35)$ $v_i = \sqrt{2\ g\ h}$ $v_i = 5.14\ m/s$	 Remember: The principle of conservation of energy states that: *Change in K.E = − Change in P.E*

5.64 A man throws a rock of mass m = 0.325 kg straight up into the air. In this process, his arm does a total amount of work W_{net} = 115 J on the rock. Calculate the maximum distance, h, above the man's throwing hand that the rock will travel. Neglect air resistance.

Solution.	Diagram
$W = -\Delta P.E$ $W = -mg\Delta h$ $-115 = -(0.325)(9.81)h$ $h = 36.1\ m$ The man deposited amount of K.E energy in the rock by providing it with an initial velocity, and then that K.E started to convert into P.E against the gravitational field. The gravitational field then starts doing work on the rock until the rock came to rest at its maximum height.	

5.65 A car does the work W = 7.00 × 10⁴ J in traveling a distance car x = 2.80 km at constant speed. Calculate the average force F (from all sources) acting on the car in this process.

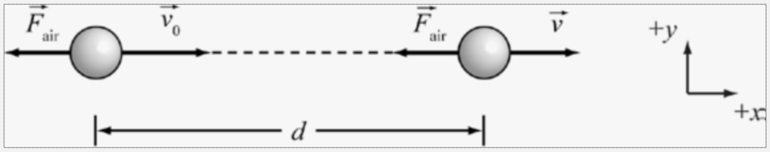
$W = F\ d\ cos\theta$

$7 \times 10^4 = F\ (2.80 \times 10^3)\ cos0^\circ$

$F = \dfrac{7 \times 10^4}{2.80 \times 10^3}$

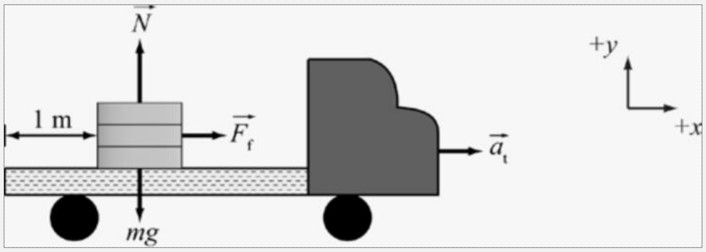
$F = 25.0\ N$

5.66 A softball, of mass m = 0.250 kg, is pitched at a speed v = 26.4 m/s. Due to air resistance, by the time it reaches home plate, it has slowed by 10.0%. The distance between the plate and the pitcher is d = 15.0 m. Calculate the average force of air resistance, F_{air} that is exerted on the ball during its movement from the pitcher to the plate.



$v_f = (90\%)v_i$ $v_f = (0.90)(26.4)$ $v_f = 23.76\ m/s$	$W_{air} = F_{air}\ d\ cos\theta$ $W_{air} = F_{air}\ d\ cos\theta$ $0.5mv_f^2 - 0.5mv_i^2 = F_{air}\ d\ cos\theta$ $F_{air} = \dfrac{0.5m(v_f^2 - v_i^2)}{d\ cos\theta}$ $F_{air} = \dfrac{0.5(0.250)(23.76^2 - 26.4^2)}{(15.0)cos180^\circ} = 1.10\ N$

5.67 A flatbed truck is loaded with a stack of sacks of cement whose combined mass is 1143.5 kg. The coefficient of static friction between the bed of the truck and the bottom sack in the stack is 0.372, and the sacks are not tied down but held in place by the force of friction between the bed and the bottom sack. The truck accelerates uniformly from rest to 56.6 mph in 22.9 s. The stack of sacks is 1 m from the end of the truck bed. Does the stack slide on the truck bed? The coefficient of kinetic friction between the bottom sack and the truck bed is 0.257. What is the work done on the stack by the force of friction between the stack and the bed of the truck?



The truck [c]	The stack [s]
$56.6\,mph = 56.6 \times \frac{1639.34\,m}{3600\,s}$ $56.6\,mph = 25.8\ m/s$ $a_t = \frac{\Delta v}{\Delta t}$ $a_t = \frac{25.8}{22.9} = 1.13\ m/s^2$	$F_f = \mu_s F_N$ $m\,a_c = \mu_s mg$ $a_c = \mu_s g$ $a_c = (0.372)(9.81)$ $a_c = 3.65\ m/s^2$

$a_t < a_c$ So, the stack will NOT slide.

NOTE: The acceleration of the stack must be the same as the acceleration of the truck.

Work done on the stack during the period of acceleration is:

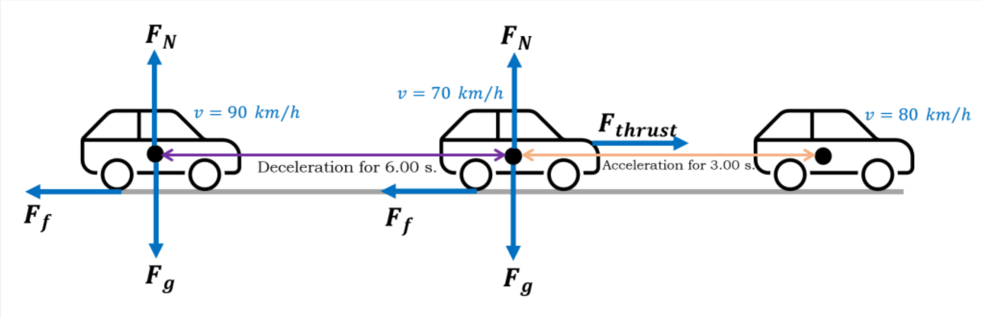
$W = \Delta K.E$

$W = 0.5m(v_f^2 - v_i^2)$

$W = 0.5(1143.5)(25.8^2 - 0^2)$

$W = 380579\,J$

5.68 A driver notices that her 1000 kg car slows from $v_o = 90.0\,km/h$ (25.0 m/s) to v = 70.0 km/h (19.4 m/s) in t = 6.00 s moving on level ground in neutral gear. Calculate the power needed to keep the car moving at a constant speed, $v_{ave} = 80.0\,km/h$ (22.2 m/s). Assume that energy is lost at a constant rate during the deceleration.



During the period of deceleration.

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{19.4 - 25.0}{6} = -0.926\ m/s^2$$

$$P = \frac{W}{t}$$

$$P = \frac{0.5m\left(v_f^2 - v_i^2\right)}{t}$$

$$P = \frac{0.5 \times 1000((19.4)^2 - (25.0)^2)}{6.00}$$

$$P = -20720\ Watts$$

$$F_{net} = m\,a$$

$$\mu_k F_N = m\,a$$

$$\mu_k mg = m\,a$$

$$\mu_k = \frac{a}{g}$$

$$\mu_k = \frac{0.926}{9.81}$$

$$\mu_k = 0.0944$$

During the period of deceleration.

$$F_{net} = m\,a$$

$$F_{thrust} - \mu_k F_N = m\,a$$

$$F_{thrust} - \mu_k mg = m\,a$$

$$F_{thrust} = m\,a + \mu_k mg$$

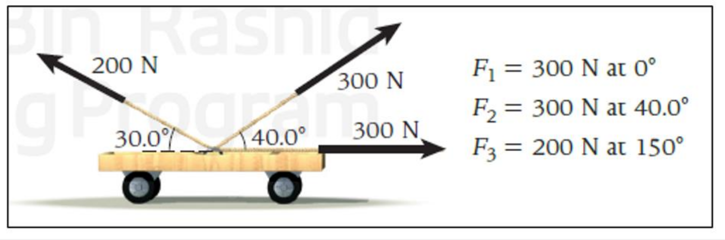
$$F_{thrust} = 1852\ N$$

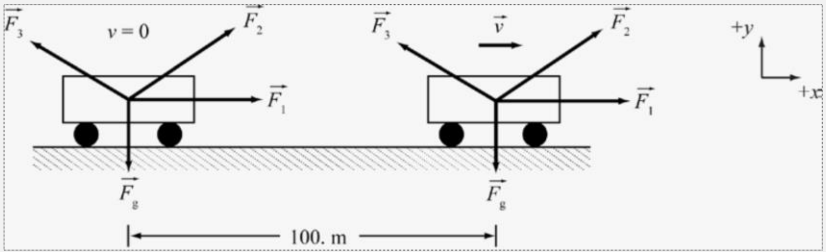
$$P = F_{thrust}\,v$$

$$P = (1852)(22.2)$$

$$P = 41114\ Watts$$

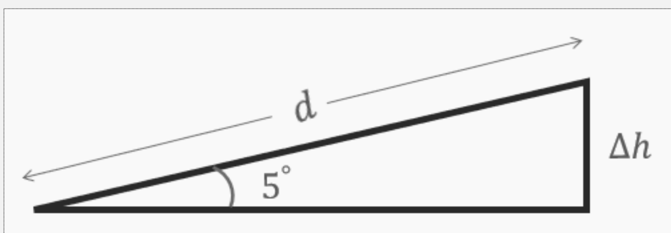
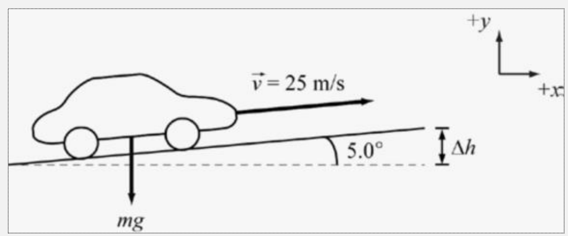
5.69 The 125-kg cart in the figure starts from rest and rolls with negligible friction. It is pulled by three ropes as shown. It moves 100 m horizontally. Find the final velocity of the cart.





$W = \Delta K.E$
$F_{net}\, d\, cos\theta = 0.5m\left(v_f^2 - v_i^2\right)$
$(300 + 300cos40^{\circ} + 200cos150^{\circ})(100)cos0^{\circ} = 0.5(125)\left(v_f^2 - 0^2\right)$
$v_f = 23.9\ m/s$

5.70 Calculate the power required to propel a 1000.0 kg car at 25.0 m/s up a straight slope inclined 5.00° above the horizontal. Neglect friction and air resistance.

Solution.	Diagram.
	

$$sin5^{\circ} = \frac{\Delta h}{d} \Longrightarrow \Delta h = d\ sin5^{\circ}$$

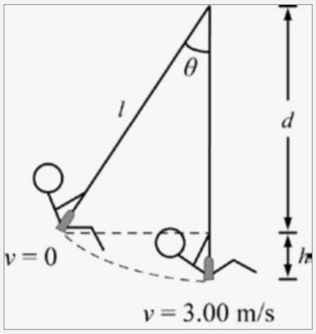
$$Power = \frac{Work}{time}$$

$$Power = \frac{mg\Delta h}{t}$$

$$Power = \frac{mgd\ sin5^{\circ}}{t} \quad BUT \quad v = \frac{d}{t}$$

$Power = mg\ v\ sin5^{\circ}$
$Power = (1000)(9.81)\ (25.0)\ sin5^{\circ}$
$Power = 21374.9\ Watts$
$Power = 21.4\ kW$

5.71 A grandfather pulls his granddaughter, whose mass is 21.0 kg and who is sitting on a swing with ropes of length 2.50 m, backward and releases her from rest. The speed of the granddaughter at the bottom of the swinging motion is 3.00 m/s. What is the angle (in degrees, measured relative to the vertical) from which she is released?

Geometry.	Diagram.
$\ell = d + h$ $h = \ell - d$......[1] $cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $cos\theta = \frac{d}{\ell}$ $d = \ell cos\theta$ .....[2] $h = \ell - \ell cos\theta$ $h = \ell(1 - cos\theta)$	

Apply the principle of conservation of energy. $\Delta K.E = -\Delta P.E$

$$0.5m(v_f^2 - v_i^2) = mgh$$

$$0.5(v_f^2 - v_i^2) = g\ell(1 - cos\theta)$$

$$\frac{0.5(v_f^2 - v_i^2)}{g\ell} = 1 - cos\theta$$

$$cos\theta = 1 - \frac{0.5(v_f^2 - v_i^2)}{g\ell}$$

$$\theta = \cos^{-1}\left(1 - \frac{0.5(v_f^2 - v_i^2)}{g\ell}\right)$$

$$\theta = \cos^{-1}\left(1 - \frac{0.5(3.00^2 - 0^2)}{(9.81)(2.50)}\right)$$

$$\theta = 35.3^{\circ}$$

5.72 A 65 kg hiker climbs to the second base camp on Nanga Parbat in Pakistan, at an altitude of 3900 m, starting from the first base camp at 2200 m. The climb is made in 5.0 h. Calculate:
(a) the work done against gravity,
(b) the average power output, and
(c) the rate of energy input required, assuming the energy conversion efficiency of the human body is 15%.

Solutions.	Diagram.

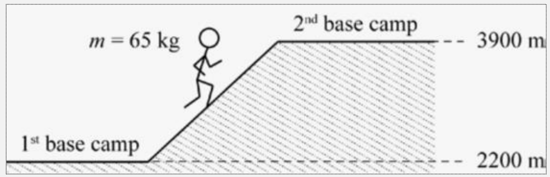
[a]

$W = -\Delta P.E$

$W = -mg\Delta h$

$W = -(65)(9.81)(3900 - 2200)$

$W = -1084005\,J$



[b]

$$P = \frac{Work}{time}$$

$$P = \frac{1084005}{5 \times 3600}$$

$P = 60.2225\,Watts$

[c]

$$Efficiency = \frac{E_{out}}{E_{in}}$$

$$0.15 = \frac{1084005}{E_{in}}$$

$E_{in} = 7226700\,J$

5.73 An x-component of a force has the dependence $F(x) = -cx^3$ on the displacement x, where the constant c = 19.1 N/m³. How much work does it take to oppose this force and change the displacement from 0.810 m to 1.39 m?

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

$$W = \int_{0.810}^{1.39} -19.1\,x^3\,dx$$

$W = -15.8\,J$

5.74 A massless spring lying on a smooth horizontal surface is compressed by a force of 63.5 N, which results in a displacement of 4.35 cm from the initial equilibrium position. How much work will it take to compress the spring from 4.35 cm to 8.15 cm?

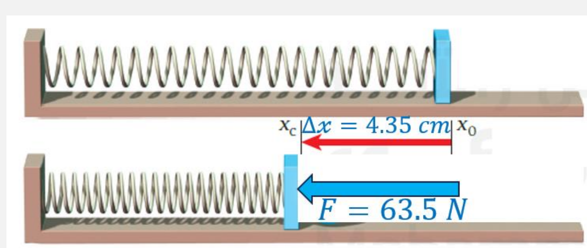
Solution.	Diagram.

$\vec{F} = -k\,\Delta\vec{x}$ ...Hook's Law

$F = k\,\Delta x$ Scalar version of Hook's Law

$63.5 = k\,(0.0435)$

$k = 1459.77\ N/m$



$W = -0.5\,k\left(x_f^2 - x_i^2\right)$

$W = -0.5\,(1459.77)\,(0.0815^2 - 0.0435^2)$

$W = -3.47\,J$

5.75 A car is traveling at a constant speed of 26.8 m/s. It has a drag coefficient $c_d =$ 0.333 and a cross-sectional area of 3.25 m². How much power is required just to overcome air resistance and keep the car traveling at this constant speed? Assume the density of air is 1.15 kg/m³.

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In general, the magnitude of the friction force due to air resistance, or drag force, can be expressed as $F_{darg} = K_o + K_1 v + K_2\, v^2 + \ldots$, with the constants K, K_1, K_2, ... determined experimentally. For the drag force on macroscopic objects moving at relatively high speeds, we can neglect the linear term in the velocity. The magnitude of the drag force is then approximately $F_{darg} = K\, v^2$. . . . . . (4.13)

To compute the terminal speed for a falling object, we need to know the value of the constant K. This constant depends on many variables, including the size of the cross-sectional area, A, exposed to the air stream. In general terms, the bigger the area, the bigger is the constant K. K also depends linearly on the air density, ρ. All other dependences on the shape of the object, on its inclination relative to the direction of motion, on air viscosity, and compressibility are usually collected in a drag coefficient, c_d.

$$K = \frac{1}{2}\, c_d\, A\, \rho$$

Drag Force.	Power
$F_{darg} = \left(\frac{1}{2}\, c_d\, A\, \rho\right) v^2$	$P = F_{drag}\, v$ $P = \left(\frac{1}{2}\, c_d\, A\, \rho\right) v^2\, v$ $P = \left(\frac{1}{2}\, c_d\, A\, \rho\right) v^3$ $P = \left(\frac{1}{2} \times 0.333 \times 3.25 \times 1.15\right) 26.8^3$ $P = 11978.4\, Watts$ $P = 16.1\, hp$

MULTI-VERSION EXERCISES

5.76 A variable force is given by $F(x) = Ax^6$, where $A = 11.45\, N/m^6$. This force acts on an object of mass 2.735 kg that moves on a frictionless surface. Starting from rest, the object moves from x = 1.093 m to x = 4.429 m. How much does the kinetic energy of the object change?

Combine: $W = \Delta K.E$ *and* $W = \int_{x_i}^{x_f} F(x)\, dx$

$$\Delta K.E = \int_{x_i}^{x_f} F(x)\, dx$$

$$\Delta K.E = \int_{1.093}^{4.429} 11.45\, x^6\, dx$$

$$\Delta K.E = 5.468 \times 10^4\, J$$

5.77 A variable force is given by $F(x) = Ax^6$, where $A = 13.75\, N/m^6$. This force acts on an object of mass 3.433 kg that moves on a frictionless surface. Starting from rest, the object moves from x = 1.105 m to a new position, x. The object gains 5.662×10^3 J of kinetic energy. What is the new position x?

$$\Delta K.E = \int_{x_i}^{x_f} F(x)\, dx$$

$$5.662 \times 10^3 = \int_{1.105}^{x_f} 13.75\, x^6\, dx$$

$$5.662 \times 10^3 = 13.75 \left(\frac{x^7}{7}\right)_{1.105}^{x_f}$$

$$5.662 \times 10^3 = \frac{13.75}{7}\left(x_f^7 - 1.105^7\right)$$

SHIFT + SOLVE:

$$x_f = 3.121\, m$$

5.78 A variable force is given by $F(x) = Ax^6$, where $A = 16.05\ N/m^6$. This force acts on an object of mass 3.127 kg that moves on a frictionless surface. Starting from rest, the object moves from a position x_o to a new position, x = 3.313 m. The object gains 1.00396×10^4 J of kinetic energy. What is the initial position x_o?

$$\Delta K.E = \int_{x_i}^{x_f} F(x)\, dx$$

$$1.00369 \times 10^4 = \int_{x_o}^{3.313} 16.05\, x^6\, dx$$

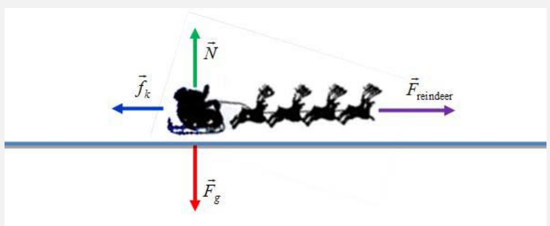
$$1.00369 \times 10^4 = 16.05 \left(\frac{x^7}{7}\right)_{x_o}^{3.313}$$

$$1.00369 \times 10^4 = \frac{16.05}{7}(3.313^7 - x_o^7)$$

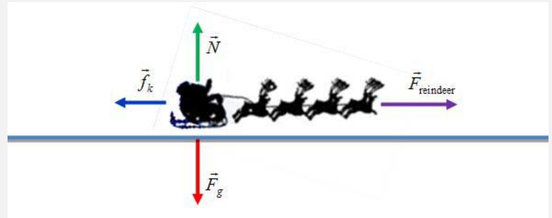
SHIFT + SOLVE:

$$x_o = 1.186\ m$$

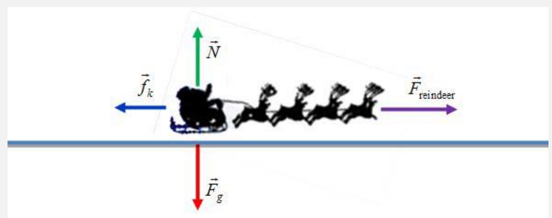
5.79 Reindeer pull a sleigh through the snow at a speed of 3.333 m/s. The mass of the sleigh, including the driver and packages, is 537.3 kg. Assuming that the coefficient of kinetic friction between the runners of the sleigh and the snow is 0.1337, what is the total power (in hp) that the reindeer are providing?

Solution.	Diagram.
$v = constant \implies a = 0\ m/s^2$ $F_{reindeer} = F_f$ $F_{reindeer} = \mu_k N$ $F_{reindeer} = \mu_k mg$ $P = F_{reindeer}\, v$ $P = \mu_k mg\, v$ $P = (0.1337)(537.3)(9.80665)(3.333)$ $P = 3.147\ hp$	

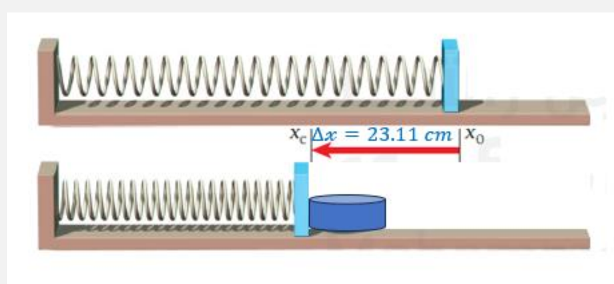
5.80 Reindeer pull a sleigh through the snow at a speed of 2.561 m/s. The mass of the sleigh, including the driver and packages, is 540.3 kg. Assuming that the reindeer can provide a total power of 2.666 hp, what is the coefficient of friction between the runners of the sleigh and the snow?

Solution.	Diagram.
$v = constant \quad \Longrightarrow \quad a = 0\ m/s^2$ $F_{reindeer} = F_f$ $F_{reindeer} = \mu_k\ N$ $F_{reindeer} = \mu_k\ mg$ $P = F_{reindeer}\ v$ $P = \mu_k\ mg\ v$ $2.666\ \times 746 = (\mu_k)(540.3)(9.806)(2.561)$ $\mu_k = 0.1466$	

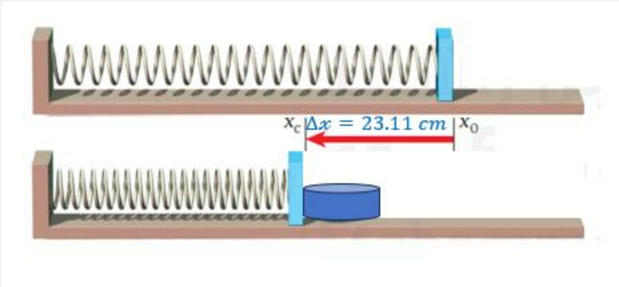
5.81 Reindeer pull a sleigh through the snow at a speed of 2.791 m/s. Assuming that the reindeer can provide a total power of 3.182 hp and the coefficient of friction between the runners of the sleigh and the snow is 0.1595, what is the mass of the sleigh, including the driver and packages?

Solution.	Diagram.
$v = constant \quad \Longrightarrow \quad a = 0\ m/s^2$ $F_{reindeer} = F_f$ $F_{reindeer} = \mu_k\ N$ $F_{reindeer} = \mu_k\ mg$ $P = F_{reindeer}\ v$ $P = \mu_k\ mg\ v$ $3.182\ \times 746 = (0.1595)(m)(9.806)(2.791)$ $m = 543.8\ kg$	

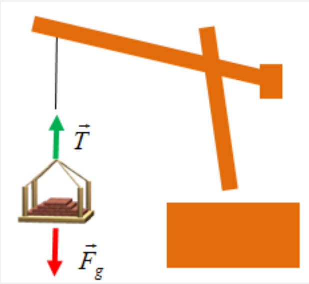
5.82 A horizontal spring with spring constant k = 15.19 N/m is compressed 23.11 cm from its equilibrium position. A hockey puck with mass m = 170.0 g is placed against the end of the spring. The spring is released, and the puck slides on horizontal ice, with a coefficient of kinetic friction of 0.02221 between the puck and the ice.
How far does the hockey puck travel on the ice after it leaves the spring?

Solution.	Diagram.
$W = P.E_s$ $F_f\ d = 0.5\ k\ (\Delta x)^2$ $\mu_k\ N\ d = 0.5\ k\ (\Delta x)^2$ $\mu_k\ mg\ d = 0.5\ k\ (\Delta x)^2$ $d = \dfrac{0.5\ k\ (\Delta x)^2}{\mu_k\ mg}$ $d = \dfrac{0.5\ (15.19)(0.2311)^2}{(0.02221)(0.170)(9.81)} = 10.95\ m$	 Travelled distance = 10.95 – 0.2311 Travelled distance = 10.72 m

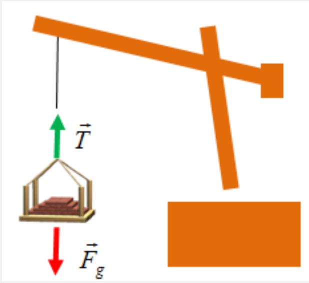
5.83 A horizontal spring with spring constant k = 17.49 N/m is compressed 23.31 cm from its equilibrium position. A hockey puck with mass m = 170.0 g is placed against the end of the spring. The spring is released, and the puck slides on horizontal ice a distance of 12.13 m after it leaves the spring. What is the coefficient of kinetic friction between the puck and the ice?

Solution.	Diagram.
$W = P.E_s$ $F_f\, d = 0.5\, k\, (\Delta x)^2$ $\mu_k\, N\, d = 0.5\, k\, (\Delta x)^2$ $\mu_k\, mg\, d = 0.5\, k\, (\Delta x)^2$ $\mu_k = \dfrac{0.5\, k\, (\Delta x)^2}{d\, mg}$ $\mu_k = \dfrac{0.5\,(17.49)(0.2331)^2}{(12.13 + 0.2311)(0.170)(9.81)} = 0.02305$	 Travelled distance = 10.95 – 0.2311 Travelled distance = 10.72 m

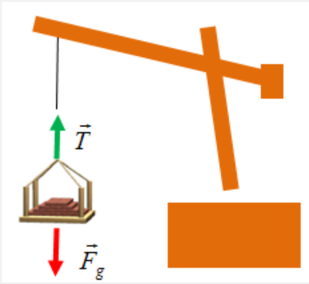
5.84 A load of bricks at a construction site has a mass of 75.0 kg. A crane raises this load from the ground to a height of 45.0 m in 52.0 s at a low constant speed. What is the average power of the crane?

Solution.	Diagram.
constant speed $\Longrightarrow a = 0 \Longrightarrow F_g = T$ $P = \dfrac{Work}{time}$ $P = \dfrac{mg\Delta h}{t}$ $P = \dfrac{75.0 \times 9.81 \times 45.0}{52.0}$ $P = 636.7\ Watts$	

5.85 A load of bricks at a construction site has a mass of 75.0 kg. A crane with 725 W of power raises this load from the ground to a height of 45.0 m at a low constant speed. How long does it take to raise the load?

Solution.	Diagram.
$constant\ speed \implies a = 0 \implies F_g = T$ $P = \frac{Work}{time}$ $P = \frac{mg\Delta h}{t}$ $725 = \frac{75.0 \times 9.81 \times 45.0}{t}$ $t = 45.7\ s$	

5.86 A load of bricks at a construction site has a mass of 75.0 kg. A crane with 815 W of power raises this load from the ground to a certain height in 52.0 s at a low constant speed. What is the final height of the load?

Solution.	Diagram.
$constant\ speed \implies a = 0 \implies F_g = T$ $P = \frac{Work}{time}$ $P = \frac{mg\Delta h}{t}$ $815 = \frac{75.0 \times 9.81 \times \Delta h}{52.0}$ $\Delta h = 57.6\ m$	

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