

- Solutions -

Gauss Law Practice Multiple Choice

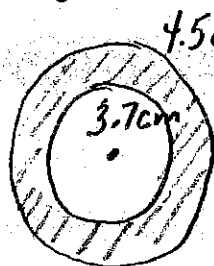
- E 1. A total charge of $6.3 \times 10^{-8} \text{ C}$ is distributed uniformly throughout a 2.7-cm radius sphere. The volume charge density is:

- A. $3.7 \times 10^{-7} \text{ C/m}^3$
 B. $6.9 \times 10^{-6} \text{ C/m}^3$
 C. $6.9 \times 10^{-6} \text{ C/m}^2$
 D. $2.5 \times 10^{-6} \text{ C/m}^3$
 E. $7.6 \times 10^{-6} \text{ C/m}^3$

$$\rho = \frac{q}{V} = \frac{6.3 \times 10^{-8} \text{ C}}{\frac{4}{3} \pi (0.027)^3} = \boxed{7.64 \times 10^{-4} \frac{\text{C}}{\text{m}^3}}$$

- A 2. A spherical shell has an inner radius of 3.7 cm and an outer radius of 4.5 cm. If charge is distributed uniformly throughout the shell with a volume density of $6.1 \times 10^{-4} \text{ C/m}^3$ the total charge is:

- A. $1.0 \times 10^{-7} \text{ C}$
 B. $1.3 \times 10^{-7} \text{ C}$
 C. $2.0 \times 10^{-7} \text{ C}$
 D. $2.3 \times 10^{-7} \text{ C}$
 E. $4.0 \times 10^{-7} \text{ C}$

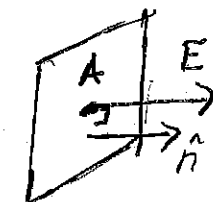


$$V_{\text{shaded}} = \frac{4}{3} \pi (0.045)^3 - \frac{4}{3} \pi (0.037)^3 = 1.695 \times 10^{-4} \text{ m}^3$$

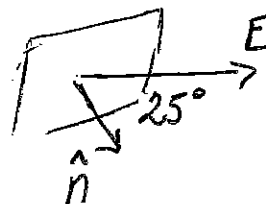
$$q = \rho V = 6.1 \times 10^{-4} \frac{\text{C}}{\text{m}^3} (1.695 \times 10^{-4} \text{ m}^3) = \boxed{1.03 \times 10^{-7} \text{ C}}$$

- D 3. When a piece of paper is held with one face perpendicular to a uniform electric field the flux through it is $25 \text{ N} \cdot \text{m}^2 / \text{C}$. When the paper is turned 25° with respect to the field the flux through it is:

- A. 0
 B. $12 \text{ N} \cdot \text{m}^2 / \text{C}$
 C. $21 \text{ N} \cdot \text{m}^2 / \text{C}$
 D. $23 \text{ N} \cdot \text{m}^2 / \text{C}$
 E. $25 \text{ N} \cdot \text{m}^2 / \text{C}$



$$\Phi_E = 25 = EA$$



$$\Phi_E = EA \cos 25^\circ$$

$$= 25 \cos 25^\circ = \boxed{22.7 \frac{\text{N} \cdot \text{m}^2}{\text{C}}}$$

- C 4. Consider Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot \hat{n} dA = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Which of the following is true?

- A. $\vec{E}$ must be the electric field due to the enclosed charge - No
 B. If $q_{\text{enc}} = 0$, then $\vec{E} = 0$ everywhere on the Gaussian surface - No
 C. If the three particles inside have charges of $+q$, $+q$, and $-2q$, then the integral is zero - Yes
 D. on the surface $\vec{E}$ is everywhere parallel to $d\vec{A}$ - No
 E. If a charge is placed outside the surface, then it cannot affect $\vec{E}$ at any point on the surface - No

E 5. Choose the INCORRECT statement:

- A. Gauss' law can be derived from Coulomb's law - Yes
- B. Gauss' law states that the net number of lines crossing any closed surface in an outward direction is proportional to the net charge enclosed within the surface - Yes
- C. Coulomb's law can be derived from Gauss' law and symmetry - Yes
- D. Gauss' law applies to a closed surface of any shape - Yes
- E. According to Gauss' law, if a closed surface encloses no charge, then the electric field must vanish everywhere on the surface - No outside charge will influence the E-field on the surface

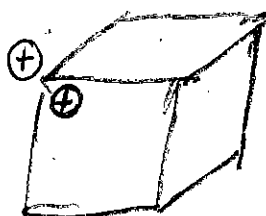
D 6. A physics instructor in an anteroom charges an electrostatic generator to $25 \mu\text{C}$, then carries it into the lecture hall. The net electric flux in $\text{N} \cdot \text{m}^2/\text{C}$ through the lecture hall walls is:

- A. 0
- B. 25×10^{-6}
- C. 2.2×10^5
- D. 2.8×10^6
- E. can not tell unless the lecture hall dimensions are given

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{25 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.8 \times 10^6 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

E 7. A particle with charge $5.0 \mu\text{C}$ is placed at the corner, and inside of, a cube. The total electric flux in $\text{N} \cdot \text{m}^2/\text{C}$ through all sides of the cube is:

- A. 0
- B. 7.1×10^4
- C. 9.4×10^4
- D. 1.4×10^5
- E. 5.6×10^5



$$\Phi_E^{\text{inside}} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{5 \times 10^{-6}}{8.85 \times 10^{-12}} = 5.6 \times 10^5 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

$$\Phi_E^{\text{outside}} = \frac{q_{\text{enc}}}{\epsilon_0} = 0 \quad \text{since } \oplus \text{ is not enclosed}$$

E 8. The table below gives the electric flux in $\text{N} \cdot \text{m}^2/\text{C}$ through the ends and round surfaces of four Gaussian surfaces in the form of cylinders. Rank the cylinders according to the charge inside, from the most negative to the most positive.

	left end	right end	rounded surface
cylinder 1:	$+2 \times 10^{-9}$	$+4 \times 10^{-9}$	-6×10^{-9}
cylinder 2:	$+3 \times 10^{-9}$	-2×10^{-9}	$+6 \times 10^{-9}$
cylinder 3:	-2×10^{-9}	-5×10^{-9}	$+3 \times 10^{-9}$
cylinder 4:	$+2 \times 10^{-9}$	-5×10^{-9}	-3×10^{-9}

Net Flux

	0
+	7
-	4
-	6

- A. 1, 2, 3, 4
- B. 4, 3, 2, 1
- C. 3, 4, 2, 1
- D. 3, 1, 4, 2
- E. 4, 3, 1, 2

- C 9. A round wastepaper basket with a 0.15-m radius opening is in a uniform electric field of 300 N/C, perpendicular to the opening. The total flux through the sides and bottom, in $\text{N} \cdot \text{m}^2/\text{C}$, is:

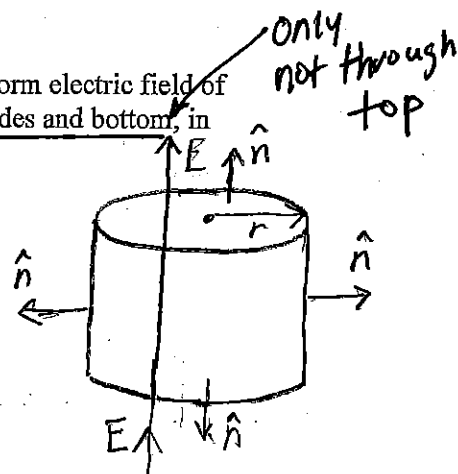
A. 0
B. 4.2
C. 21
D. 280

E. can not tell without knowing the areas of the sides and bottom

$$\Phi_E = EA \cos 0^\circ$$

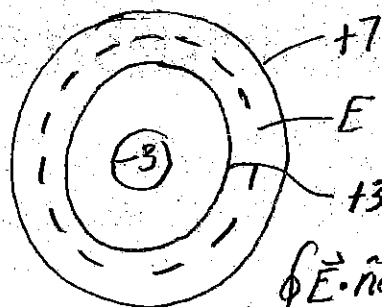
$$= 300 (\pi) (.15)^2$$

$$= 21.205$$



- E 10. 10C of charge are placed on a spherical conducting shell. A particle with a charge of -3C is placed at the center of the cavity. The net charge on the outer surface of the shell is:

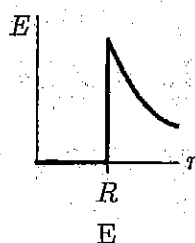
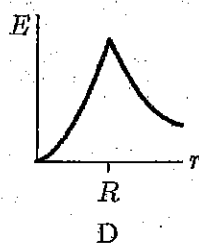
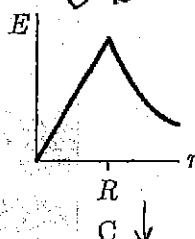
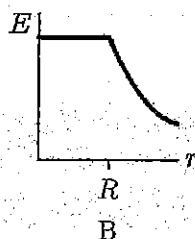
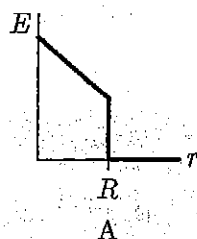
A. -7C
B. -3C
C. 0C
D. +3C
E. +7C



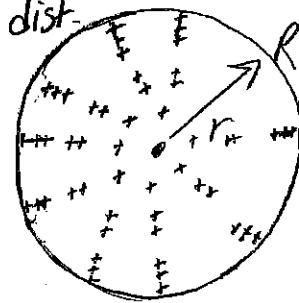
$E = 0$ Inside of a conductor and is only possible if $q_{\text{enc}} = 0 \therefore q_{\text{inner}} = +3$ and $q_{\text{outer}} = +7$

$$\oint \vec{E} \cdot \hat{n} dA = \frac{q_{\text{enc}}}{\epsilon_0}$$

- D 11. A solid insulating sphere of radius R contains positive charge that is distributed with a volume charge density that does not depend on angle but does increase with distance from the sphere center. Which of the graphs below might give the magnitude E of the electric field as a function of the distance r from the center of the sphere?



Uniform charge dist.
Insulating
Shell



$\rho = \beta r$

$$\frac{q_{\text{enc}}}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi R^3}$$

In between
C & E
Zero at center
but then grows as
charge increases.

Conducting
Shell
No charge inside



$$q_{enc} = \frac{Q}{\frac{4}{3}\pi R^3} \left(\frac{4}{3}\pi r^3\right) = \frac{Qr^3}{R^3}$$

- D 12. Charge Q is distributed uniformly throughout an insulating sphere of radius R . The magnitude of the electric field at a point $R/2$ from the center is:

- A. $Q/4\pi\epsilon_0 R^2$
 B. $Q/\pi\epsilon_0 R^2$
 C. $3Q/4\pi\epsilon_0 R^2$
 D. $Q/8\pi\epsilon_0 R^2$
 E. none of these

$$\oint \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\epsilon_0}$$

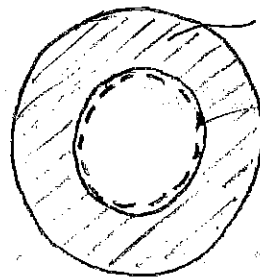
$$E(4\pi r^2) = \frac{Qr^3}{R^3\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r$$

$$\therefore E = \frac{1}{8\pi\epsilon_0} \frac{Q}{R^2}$$

- A 13. Charge Q is distributed uniformly throughout a spherical insulating shell. The net electric flux in $N \cdot m^2/C$ through the inner surface of the shell is:

- A. 0
 B. Q/ϵ_0
 C. $2Q/\epsilon_0$
 D. $Q/4\pi\epsilon_0$
 E. $Q/2\pi\epsilon_0$



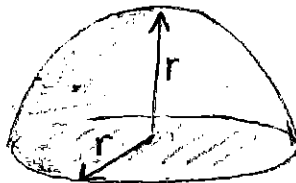
Uniform Distribution of Charge

Gaussian Surface right at but not inside shell

$$\therefore q_{enc} = 0 \text{ and } \Phi_E = 0$$

- C 14. A 3.5-cm radius hemisphere contains a total charge of $6.6 \times 10^{-7} C$. The flux through the rounded portion of the surface is $4.97 \times 10^4 N \cdot m^2/C$. The flux through the flat base is:

- A. 0
 B. $-2.49 \times 10^4 N \cdot m^2/C$
 C. $+2.49 \times 10^4 N \cdot m^2/C$
 D. $-9.94 \times 10^4 N \cdot m^2/C$
 E. $+9.94 \times 10^4 N \cdot m^2/C$



$$\Phi_{E, total} = \frac{6.6 \times 10^{-7} C}{8.85 \times 10^{-12} \frac{N \cdot C^2}{m}}$$

$$\Phi_E = 74576$$

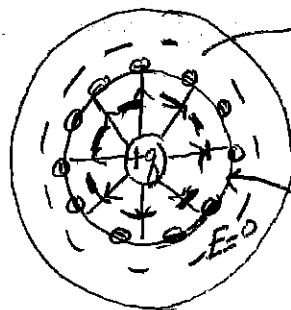
% Φ_E through the base

$$\frac{A_{base}}{A_{hemi}} = \frac{\pi r^2}{\frac{1}{2}(4\pi r^2)} = \frac{1}{2} \therefore \text{Base is } \frac{1}{2} \text{ of Total}$$

$$\frac{\Phi_E}{\text{base}} = \frac{74576}{2} = 2.49 \times 10^4$$

- C 15. Positive charge Q is placed on a conducting spherical shell with inner radius R_1 and outer radius R_2 . A particle with charge q is placed at the center of the cavity. The magnitude of the electric field at a point in the cavity, a distance r from the center, is:

- A. zero
 B. $Q/4\pi\epsilon_0 R_1^2$
 C. $q/4\pi\epsilon_0 r^2$
 D. $(q+Q)/4\pi\epsilon_0 r^2$
 E. $(q+Q)/4\pi\epsilon_0 (R_1^2 - r^2)$



$+Q$ = Total charge on shell
 Must be on surfaces
 since a conductor

$-q$ Must be on inner surface to have $E=0$ inside of conductor

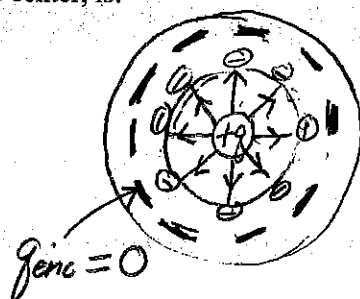
$$\oint \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{+q}{\epsilon_0}$$

A

16. Positive charge Q is placed on a conducting spherical shell with inner radius R_1 and outer radius R_2 . A point charge q is placed at the center of the cavity. The magnitude of the electric field produced by the charge on the inner surface at a point in the interior of the conductor, a distance r from the center, is:

- A. 0
B. $Q/4\pi\epsilon_0 R_1^2$
C. $Q/4\pi\epsilon_0 R_2^2$
D. $q/4\pi\epsilon_0 r^2$
E. $Q/4\pi\epsilon_0 r^2$



$-q$ on inner surface

$Q+q$ on outer surface

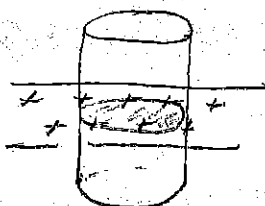
$E=0$ inside shell $\leftarrow$ Conductor

$E = \frac{1}{4\pi\epsilon_0} \frac{q_{enc}}{r^2}$ inside cavity

C

17. Charge is distributed uniformly on the surface of a large flat plate. The electric field 2 cm from the plate is 30 N/C. The electric field 4 cm from the plate is:

- A. 120 N/C
B. 80 N/C
C. 30 N/C
D. 15 N/C
E. 7.5 N/C



$$\oint \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \quad \text{Shaded Area}$$

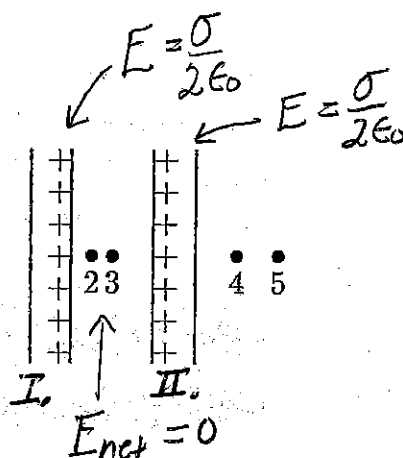
$$E(2A) = \frac{\sigma A}{\epsilon_0}$$

2 end caps

$$E = \frac{\sigma}{2\epsilon_0} = \text{const}$$

E

18. Two large parallel plates carry positive charge of equal magnitude that is distributed uniformly over their inner surfaces. Rank the points 1 through 5 according to the magnitude of the electric field at the points, least to greatest.



- A. 1, 2, 3, 4, 5
B. 5, 4, 3, 2, 1
C. 1, 4, and 5 tie, then 2 and 3 tie
D. 2 and 3 tie, then 1 and 4 tie, then 5
E. 2 and 3 tie, then 1, 4, and 5 tie

1, 4, 5 all have

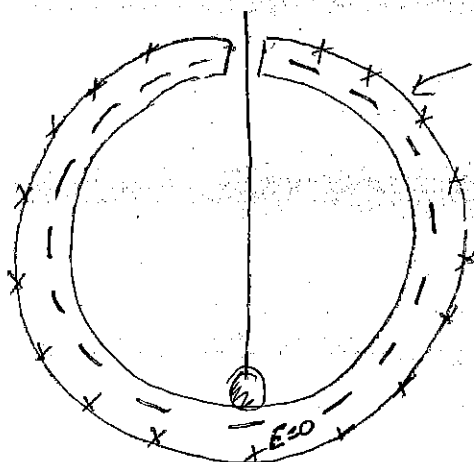
$$E_{net} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

I. + II.

C

19. A hollow conductor is positively charged. A small, uncharged metal ball is lowered by a silk thread through a small opening in the top of the conductor and allowed to touch its inner surface. After the ball is removed, it will have:

- A. a positive charge
- B. a negative charge
- C. no appreciable charge
- D. a charge whose sign depends on what part of the inner surface it touched
- E. a charge whose sign depends on where the small hole is located in the conductor



Positive charge sits on outer surface only by electrostatic repulsion

$$\oint \vec{E} \cdot \vec{n} dA = \frac{q_{enc}}{\epsilon_0} = 0$$

Since $\vec{E} = 0$ inside of conductor q_{enc} must be zero.