

Reveal MATH[®]

Integrated III
UAE Edition
Grade 11 General
Student Edition

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Academic Year	2025/2026
العام الدراسي	
Term	1
الفصل	
Subject	Mathematics/(Bridge.Reveal)
المادة	الرياضيات/(إبريدج و ريفيل)
Grade	12
الصف	
Stream	General
المسار	العام
Number of MCQ عدد الأسئلة الموضوعية	20
Marks of MCQ درجة الأسئلة الموضوعية	(2-4)
Number of FRQ عدد الأسئلة المقالية	5
Marks per FRQ الدرجات للأسئلة المقالية	(7-9)
Type of All Questions نوع كافة الأسئلة	الأسئلة الموضوعية / MCQ الأسئلة المقالية / FRQ
Maximum Overall Grade الدرجة القصوى الممكنة	100
Exam Duration - مدة الامتحان	150 minutes
Mode of Implementation - طريقة التطبيق	SwiftAssess & Paper-Based
Calculator	Allowed
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الأسئلة المقالية - FRQ	20	إيجاد المساحة الواقعة تحت منحنى التوزيع الطبيعي		
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الاسئلة الالكترونية لامتحان



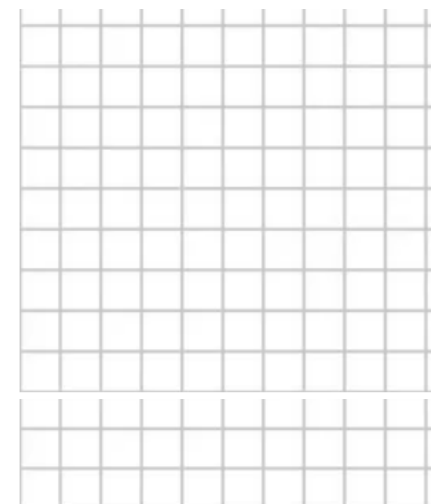
1	تمثيل الدوال التربيعية بيانياً		
	Graph quadratic functions.	Example 1 & (1-4)	4 & 9

Example 1 Graph a Quadratic Function by Using a Table

Graph $f(x) = x^2 + 2x - 3$. State the domain and range.

$$1. f(x) = x^2 + 6x + 8$$

$$2. f(x) = -x^2 - 2x + 2$$



1	تمثيل الدوال التربيعية بيانياً		
	Graph quadratic functions.	Example 1 & (1-4)	4 & 9

$$3. f(x) = 2x^2 - 4x + 3$$

$$4. f(x) = -2x^2$$



1	تمثيل الدوال التربيعية بيانياً Graph quadratic functions.	Example 1 & (1-4)	4 & 9
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solution method

Learn Graphing Quadratic Functions

A **quadratic function** has an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$. A quadratic function has a graph that is a parabola.

$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0$$



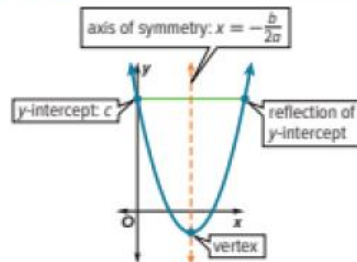
Key Concept • Graph of a Quadratic Function

$$y = ax^2 + bx + c, \text{ where } a \neq 0$$

The y -intercept is c .

The **axis of symmetry** is the line about which a parabola is symmetric. Its equation is $x = -\frac{b}{2a}$.

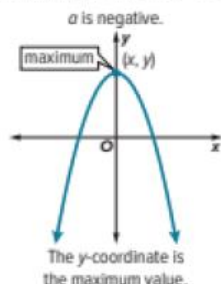
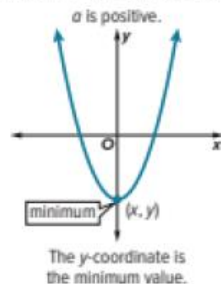
The axis of symmetry intersects the parabola at its **vertex**. Its x -coordinate is $-\frac{b}{2a}$.



You will derive the equation for the axis of symmetry in Lesson 1-5.

Key Concept • Maximum or Minimum Values of Quadratic Graphs

The vertex of a quadratic function is either a **maximum** or **minimum**. The graph of $y = ax^2 + bx + c$, where $a \neq 0$, opens up and has a minimum when $a > 0$, and opens down and has a maximum when $a < 0$.



Graph each function. Then state the domain and range.

1. $f(x) = x^2 + 6x + 8$

SOLUTION:

For $f(x) = x^2 + 6x + 8$, $a = 1$, $b = 6$, and $c = 8$. c is the y -intercept, so the y -intercept is 8.

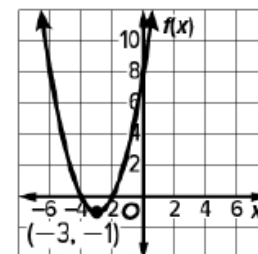
Find the axis of symmetry.

$$\begin{aligned} x &= -\frac{b}{2a} && \text{Equation of the axis of symmetry.} \\ &= -\frac{6}{2(1)} && a = 1, b = 6 \\ &= -3 && \text{Simplify.} \end{aligned}$$

The equation of the axis of symmetry is $x = -3$, so the x -coordinate of the vertex is -3 . Because $a > 0$, the vertex is a **minimum**.

Graph the function.

x	$x^2 + 6x + 8$	$(x, f(x))$
-5	$(-5)^2 + 6(-5) + 8$	3
-4	$(-4)^2 + 6(-4) + 8$	0
-3	$(-3)^2 + 6(-3) + 8$	-1
-2	$(-2)^2 + 6(-2) + 8$	0
-1	$(-1)^2 + 6(-1) + 8$	3



Analyze the graph.

The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq -1\}$.

Example 1 Graph a Quadratic Function by Using a Table

2. $f(x) = -x^2 - 2x + 2$

SOLUTION:

For $f(x) = -x^2 - 2x + 2$, $a = -1$, $b = -2$, and $c = 2$. c is the y -intercept, so the y -intercept is 2.

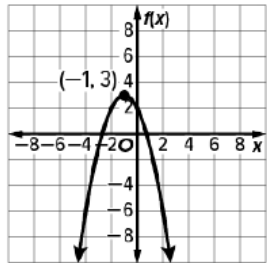
Find the axis of symmetry.

$$\begin{aligned}x &= -\frac{b}{2a} && \text{Equation of the axis of symmetry.} \\&= -\frac{-2}{2(-1)} && a = -1, b = -2 \\&= -1 && \text{Simplify.}\end{aligned}$$

The equation of the axis of symmetry is $x = -1$, so the x -coordinate of the vertex is -1 . Because $a < 0$, the vertex is a maximum.

Graph the function.

x	$-x^2 - 2x + 2$	$(x, f(x))$
-3	$-(-3)^2 - 2(-3) + 2$	-1
-2	$-(-2)^2 - 2(-2) + 2$	2
-1	$-(-1)^2 - 2(-1) + 2$	3
0	$-(0)^2 - 2(0) + 2$	2
1	$-(1)^2 - 2(1) + 2$	-1



The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \leq 3\}$.

3. $f(x) = 2x^2 - 4x + 3$

SOLUTION:

For $f(x) = 2x^2 - 4x + 3$, $a = 2$, $b = -4$, and $c = 3$. c is the y -intercept, so the y -intercept is 3.

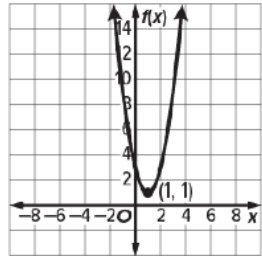
Find the axis of symmetry.

$$\begin{aligned}x &= -\frac{b}{2a} && \text{Equation of the axis of symmetry.} \\&= -\frac{-4}{2(2)} && a = 2, b = -4 \\&= 1 && \text{Simplify.}\end{aligned}$$

The equation of the axis of symmetry is $x = 1$, so the x -coordinate of the vertex is 1. Because $a > 0$, the vertex is a minimum.

Graph the function.

x	$2x^2 - 4x + 3$	$(x, f(x))$
-1	$2(-1)^2 - 4(-1) + 3$	9
0	$2(0)^2 - 4(0) + 3$	3
1	$2(1)^2 - 4(1) + 3$	1
2	$2(2)^2 - 4(2) + 3$	3
3	$2(3)^2 - 4(3) + 3$	9



Analyze the graph.

The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq 1\}$.

4. $f(x) = -2x^2$

SOLUTION:

For $f(x) = -2x^2$, $a = -2$, $b = 0$, and $c = 0$. c is the y -intercept, so the y -intercept is 0.

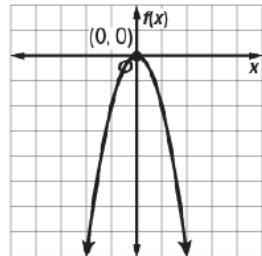
Find the axis of symmetry.

$$\begin{aligned}x &= -\frac{b}{2a} && \text{Equation of the axis of symmetry.} \\&= -\frac{0}{2(-2)} && a = -2, b = 0 \\&= 0 && \text{Simplify.}\end{aligned}$$

The equation of the axis of symmetry is $x = 0$, so the x -coordinate of the vertex is 0. Because $a < 0$, the vertex is a maximum.

Graph the function.

x	$-2x^2$	$(x, f(x))$
-2	$-2(-2)^2$	-8
-1	$-2(-1)^2$	-2
0	$-2(0)^2$	0
1	$-2(1)^2$	-2
2	$-2(2)^2$	-8



Analyze the graph.

The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \leq 0\}$.

Solution

2	حل المعادلات التربيعية باستخدام التمثيل البياني		
	Solve quadratic equations by graphing.	(4-12)	17

Solve each equation by graphing.

4. $x^2 - 10x + 21 = 0$

5. $4x^2 + 4x + 1 = 0$

6. $x^2 + x - 6 = 0$



2	حل المعادلات التربيعية باستخدام التمثيل البياني		
	Solve quadratic equations by graphing.	(4-12)	17

7. $x^2 + 2x - 3 = 0$



8. $-x^2 - 6x - 9 = 0$



9. $x^2 - 6x + 5 = 0$



2	حل المعادلات التربيعية باستخدام التمثيل البياني		
	Solve quadratic equations by graphing.	(4-12)	17

10. $x^2 + 2x + 3 = 0$

11. $x^2 - 3x - 10 = 0$

12. $-x^2 - 8x - 16 = 0$



solution method

2

حل المعادلات التربيعية باستخدام التمثيل البياني

Solve quadratic equations by graphing.

(4-12)

17

Solution

Learn Solving Quadratic Equations by Graphing

A **quadratic equation** is an equation that includes a quadratic expression.

Key Concept • Standard Form of a Quadratic Equation

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$ and a , b , and c are integers.

One method for finding the roots of a quadratic equation is to find the zeros of a related quadratic function. You can identify the solutions or roots of an equation by finding the x -intercepts of the graph of a related function. Often, exact roots cannot be found by graphing. You can estimate the solutions by finding the integers between which the zeros are located on the graph of the related function.

Example 1 One Real Solution

Solve each equation by graphing.

4. $x^2 - 10x + 21 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

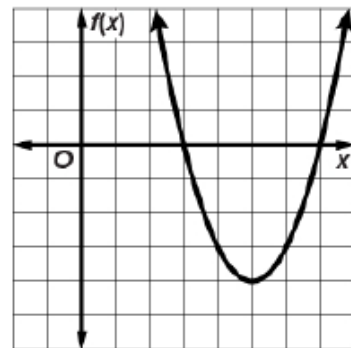
$$x = -\frac{-10}{2(1)} \quad a = 1, b = -10$$

$$x = 5 \quad \text{Simplify.}$$

Make a table of values.

x	y
3	0
4	-3
5	-4
6	-3
7	0

Plot the points and connect them with a curve.



The zeros of the function are 3 and 7.

Therefore, the solutions of the equation are $x = 3$ or $x = 7$.

5. $4x^2 + 4x + 1 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

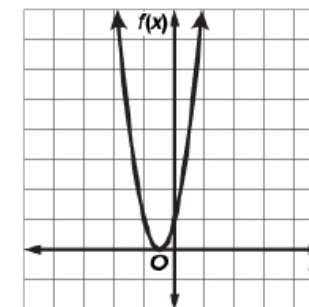
$$x = -\frac{4}{2(4)} \quad a = 4, b = 4$$

$$x = -\frac{1}{2} \quad \text{Simplify.}$$

Make a table of values.

x	y
$-1\frac{1}{2}$	4
-1	1
$-\frac{1}{2}$	0
0	1
$\frac{1}{2}$	4

Plot the points and connect them with a curve.



The zero of the function is $-\frac{1}{2}$.

Therefore, the solution of the equation is $x = -\frac{1}{2}$.

6. $x^2 + x - 6 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

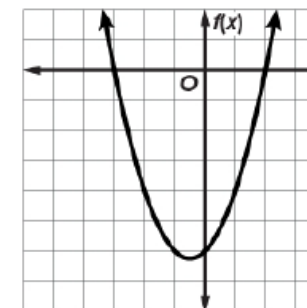
$$x = -\frac{1}{2(1)} \quad a = 1, b = 1$$

$$x = -\frac{1}{2} \quad \text{Simplify.}$$

Make a table of values.

x	y
-3	0
-2	-4
-1	-6
$-\frac{1}{2}$	-6.25
0	-6
1	-4
2	0

Plot the points and connect them with a curve.



The zeros of the function are -3 and 2.

Therefore, the solutions of the equation are $x = -3$ or $x = 2$.

7. $x^2 + 2x - 3 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

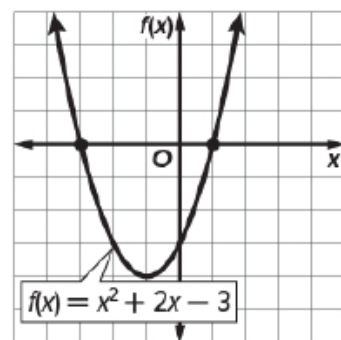
$$x = -\frac{2}{2(1)} \quad a = 1, b = 2$$

$$x = -1 \quad \text{Simplify.}$$

Make a table of values.

x	y
-3	0
-2	-3
-1	-4
0	-3
1	0

Plot the points and connect them with a curve.



The zeros of the function are -3 and 1.

Therefore, the solutions of the equation are $x = -3$ or $x = 1$.

8. $-x^2 - 6x - 9 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

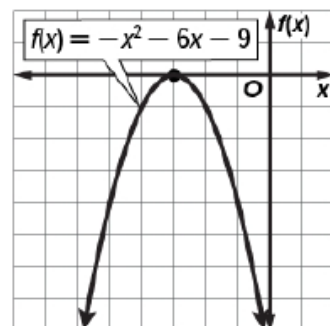
$$x = -\frac{-6}{2(-1)} \quad a = -1, b = -6$$

$$x = -3 \quad \text{Simplify.}$$

Make a table of values.

x	y
-5	-4
-4	-1
-3	0
-2	-1
-1	-4

Plot the points and connect them with a curve.



The zero of the function is -3.

Therefore, the solution of the equation is $x = -3$.

9. $x^2 - 6x + 5 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

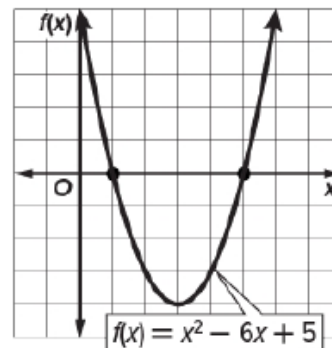
$$x = -\frac{-6}{2(1)} \quad a = 1, b = -6$$

$$x = 3 \quad \text{Simplify.}$$

Make a table of values.

x	y
1	0
2	-3
3	-4
4	-3
5	0

Plot the points and connect them with a curve.



The zeros of the function are 1 and 5.

Therefore, the solutions of the equation are $x = 1$ or $x = 5$.

10. $x^2 + 2x + 3 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

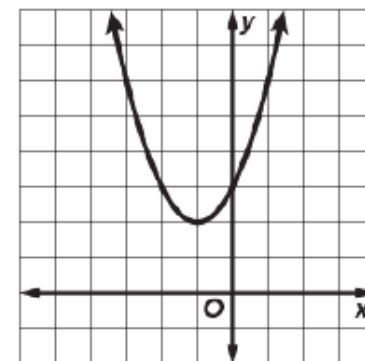
$$x = -\frac{2}{2(1)} \quad a = 1, b = 2$$

$$x = -1 \quad \text{Simplify.}$$

Make a table of values.

x	y
-3	6
-2	3
-1	2
0	3
1	6

Plot the points and connect them with a curve.



There are no zeros of the function.

Therefore, there is no real solution of the equation.

11. $x^2 - 3x - 10 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

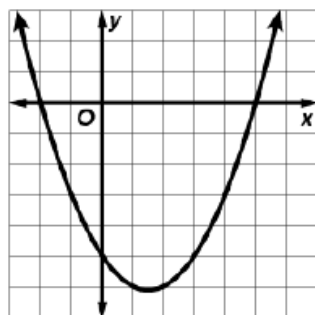
$$x = -\frac{-3}{2(1)} \quad a = 1, b = -3$$

$$x = 1.5 \quad \text{Simplify.}$$

Make a table of values.

x	y
-2	0
0	-10
1.5	-12.25
3	-10
5	0

Plot the points and connect them with a curve.



The zeros of the function are -2 and 5.

Therefore, the solutions of the equation are $x = -2$ or $x = 5$.

12. $-x^2 - 8x - 16 = 0$

SOLUTION:

Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

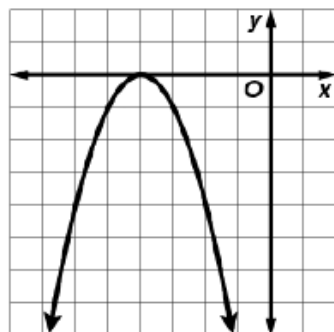
$$x = -\frac{-8}{2(-1)} \quad a = -1, b = -8$$

$$x = -4 \quad \text{Simplify.}$$

Make a table of values.

x	y
-6	-4
-5	-1
-4	0
-3	-1
-2	-4

Plot the points and connect them with a curve.



The zero of the function is -4.

Therefore, the solution of the equation is $x = -4$.

3	حل المعادلات التربيعية باستخدام التحليل إلى العوامل		
	Solve Quadratic Equations by Factoring.	Example 1 & Example 2 & Example 4	27 & 28 & 29

Example 1 Factor by Using the Distributive Property

Solve $12x^2 - 2x = x$ by factoring. Check your solution.

Example 2 Factor a Trinomial

Solve $x^2 - 6x - 9 = 18$ by factoring. Check your solution.

Example 4 Factor a Trinomial Where a is Not 1

Solve $3x^2 + 5x + 15 = 17$ by factoring. Check your solution.

3	حل المعادلات التربيعية باستخدام التحليل إلى العوامل		
	Solve Quadratic Equations by Factoring.	Example 1 & Example 2 & Example 4	27 & 28 & 29

solution method

Learn Solving Quadratic Equations by Factoring

The **factored form** of a quadratic equation is $0 = a(x - p)(x - q)$, where $a \neq 0$. In this equation, p and q represent the x -intercepts of the graph of the related function. For example, $0 = x^2 - 2x - 3$ can be written in the factored form $0 = (x - 3)(x + 1)$ and its related graph has x -intercepts of -1 and 3 .

Key Concepts • Factoring	
Using the Distributive Property	$ax + bx = x(a + b)$
Factoring Trinomials	$x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$

Key Concept • Zero Product Property
Words: For any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and $b = 0$.
Example: If $(x - 2)(x + 4) = 0$, then $x - 2 = 0$, $x + 4 = 0$, or both $x - 2 = 0$ and $x + 4 = 0$.

To solve a quadratic equation by factoring, first make sure that one side of the equation is 0, and factor the trinomial. Use the Zero Product Property to write separate equations. Then use the properties of equality to isolate the variable.

Example 1 Factor by Using the Distributive Property

Example 2 Factor a Trinomial

Example 4 Factor a Trinomial Where a is Not 1

Example 1 Factor by Using the Distributive Property

Solve $12x^2 - 2x = x$ by factoring. Check your solution.

$12x^2 - 2x = x$	Original equation
$12x^2 - 3x = 0$	Subtract x from each side.
$3x(4x) - 3x(1) = 0$	Factor the GCF.
$3x(4x - 1) = 0$	Distributive Property
$3x = 0$ or $4x - 1 = 0$	Zero Product Property
$x = 0$ $x = \frac{1}{4}$	Solve.

Example 2 Factor a Trinomial

Solve $x^2 - 6x - 9 = 18$ by factoring. Check your solution.

$x^2 - 6x - 9 = 18$	Original equation
$x^2 - 6x - 27 = 0$	Subtract 18 from each side.
$(x + 3)(x - 9) = 0$	Factor the trinomial.
$x + 3 = 0$ or $x - 9 = 0$	Zero Product Property
$x = -3$ $x = 9$	Solve.

Example 4 Factor a Trinomial Where a is Not 1

Solve $3x^2 + 5x + 15 = 17$ by factoring. Check your solution.

$3x^2 + 5x + 15 = 17$	Original equation
$3x^2 + 5x - 2 = 0$	Subtract 17 from each side.
$(3x - 1)(x + 2) = 0$	Factor the trinomial.
$3x - 1 = 0$ or $x + 2 = 0$	Zero Product Property
$x = \frac{1}{3}$ $x = -2$	Solve.

4	إكمال المربع في حالة ثلاثي حدود ليس مربعاً كاملاً	Example 4	35
	Complete the square in quadratic expressions to solve quadratic equations.		

Example 4 Complete the Square

Find the value of c that makes $x^2 - 7x + c$ a perfect square. Then write the expression as a perfect square trinomial.

Learn Solving Quadratic Equations by Completing the Square

All quadratic equations can be solved by using the properties of equality to manipulate the equation until one side is a perfect square. This process is called **completing the square**.

Key Concept • Completing the Square

Words: To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b , the coefficient of x .

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

Symbols: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

To solve an equation of the form $x^2 + bx + c = 0$ by completing the square, first subtract c from each side of the equation. Then add $\left(\frac{b}{2}\right)^2$ to each side of the equation and solve for x .

Example 4 Complete the Square**Example 4** Complete the Square

Find the value of c that makes $x^2 - 7x + c$ a perfect square. Then write the expression as a perfect square trinomial.

Step 1 Find one half of -7 . $\frac{-7}{2} = -3.5$

Step 2 Square the result from Step 1. $(-3.5)^2 = 12.25$

Step 3 Add the result from Step 2 to $x^2 - 7x$. $x^2 - 7x + 12.25$

The expression $x^2 - 7x + 12.25$ can be written as $(x - 3.5)^2$.

5	حل المعادلات التربيعية بإكمال المربع	Example 5	35
	Complete the square in quadratic expressions to solve quadratic equations		

Example 5 Solve by Completing the Square

Solve $x^2 + 18x - 4 = 0$ by completing the square.

5	حل المعادلات التربيعية بإكمال المربع	Example 5	35
	Complete the square in quadratic expressions to solve quadratic equations		

solution method

Learn Solving Quadratic Equations by Completing the Square

All quadratic equations can be solved by using the properties of equality to manipulate the equation until one side is a perfect square. This process is called **completing the square**.

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Symbols: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

To solve an equation of the form $x^2 + bx + c = 0$ by completing the square, first subtract c from each side of the equation. Then add $\left(\frac{b}{2}\right)^2$ to each side of the equation and solve for x .

Example 5 Solve by Completing the Square

Example 5 Solve by Completing the Square

Solve $x^2 + 18x - 4 = 0$ by completing the square.

$$x^2 + 18x - 4 = 0$$

Original equation

$$x^2 + 18x = 4$$

Add 4 to each side.

$$x^2 + 18x + 81 = 4 + 81$$

Add $\left(\frac{b}{2}\right)^2$ to each side.

$$(x + 9)^2 = 85$$

Factor.

$$x + 9 = \pm\sqrt{85}$$

Square Root Property

$$x = -9 \pm \sqrt{85}$$

Subtract 9 from each side.

$$x = -9 + \sqrt{85} \text{ or}$$

Write as two equations.

$$x = -9 - \sqrt{85}$$

$$x \approx 0.22, \text{ or } -18.22$$

Simplify.

The solution set is $\{x \mid x = -9 - \sqrt{85}, -9 + \sqrt{85}\}$.

6	استخدام المميز لتحديد عدد ونوع جذور معادلة تربيعية		
	Determine the number and type of roots of a quadratic equation.	Key Concept • Discriminant	45

Examples 4 and 5

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

24. $x^2 - 8x + 16 = 0$

25. $x^2 - 11x - 26 = 0$

26. $3x^2 - 2x = 0$

27. $20x^2 + 7x - 3 = 0$

28. $5x^2 - 6 = 0$

29. $x^2 - 6 = 0$

30. $x^2 + 8x + 13 = 0$

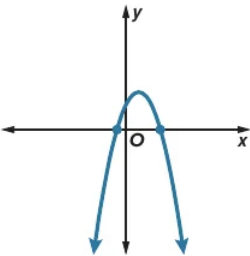
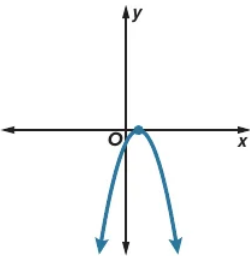
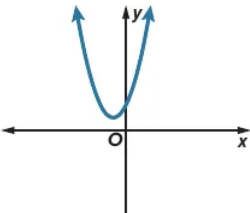
solution method

Learn Using the Discriminant

In the Quadratic Formula, the **discriminant** is the expression under the radical sign, $b^2 - 4ac$. The value of the discriminant can be used to determine the number and type of roots of a quadratic equation.

Key Concept • Discriminant

Consider $ax^2 + bx + c = 0$, where a , b , and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is not a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

Example 4 The Discriminant, Real Roots

Example 5 The Discriminant, Complex Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

24. $x^2 - 8x + 16 = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (-8)^2 - 4(1)(16) & a = 1, b = -8, c = 16 \\ &= 64 - 64 \\ &= 0 \end{aligned}$$

The discriminant is zero, so there is 1 real rational root.

25. $x^2 - 11x - 26 = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (-11)^2 - 4(1)(-26) & a = 1, b = -11, c = -26 \\ &= 121 + 104 \\ &= 225 \end{aligned}$$

The discriminant is nonzero, so there are 2 roots. The discriminant is positive and a perfect square, so the roots are rational.

26. $3x^2 - 2x = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (-2)^2 - 4(3)(0) & a = 3, b = -2, c = 0 \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

The discriminant is nonzero, so there are 2 roots. The discriminant is positive and a perfect square, so the roots are rational.

27. $20x^2 + 7x - 3 = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (7)^2 - 4(20)(-3) \quad a = 20, b = 7, c = -3 \\ &= 49 + 240 \\ &= 289 \end{aligned}$$

The discriminant is nonzero, so there are 2 roots. The discriminant is positive and a perfect square, so the roots are rational.

28. $5x^2 - 6 = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (0)^2 - 4(5)(-6) \quad a = 5, b = 0, c = -6 \\ &= 0 + 120 \\ &= 120 \end{aligned}$$

The discriminant is nonzero, so there are 2 roots. The discriminant is positive and not a perfect square, so the roots are irrational.

29. $x^2 - 6 = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (0)^2 - 4(1)(-6) \quad a = 1, b = 0, c = -6 \\ &= 0 + 24 \\ &= 24 \end{aligned}$$

The discriminant is nonzero, so there are 2 roots. The discriminant is positive and not a perfect square, so the roots are irrational.

30. $x^2 + 8x + 13 = 0$

SOLUTION:

$$\begin{aligned} b^2 - 4ac &= (8)^2 - 4(1)(13) \quad a = 1, b = 8, c = 13 \\ &= 64 - 52 \\ &= 12 \end{aligned}$$

The discriminant is nonzero, so there are 2 roots. The discriminant is positive and not a perfect square, so the roots are irrational.

7	تمثيل المتباينات التربيعية بمتغيرين اثنين بيانياً		
	Graph quadratic inequalities in two variables.	(4-9) & (37-39)	55 & 56

Examples 1 and 2

Graph each inequality.

4. $y \leq x^2 + 4$
5. $y < 2x^2 - 4x - 2$
6. $-x^2 + 12x - 36 > y$



7	تمثيل المتباينات التربيعية بمتغيرين اثنين بيانياً		
	Graph quadratic inequalities in two variables.	(4-9) & (37-39)	55 & 56

7. $y > x^2 + 6x + 7$

8. $y > x^2 - 8x + 17$

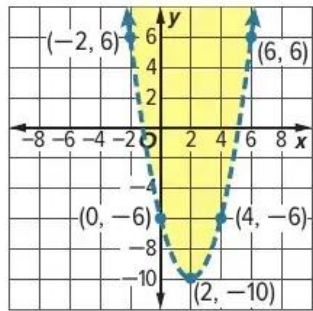
9. $y \geq x^2 + 2x + 2$



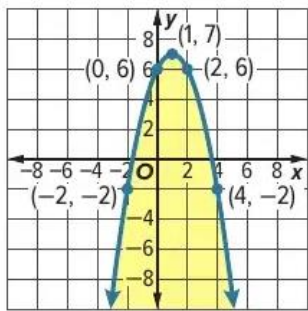
7	تمثيل المتباينات التربيعية بمتغيرين اثنين بيانياً		
	Graph quadratic inequalities in two variables.	(4-9) & (37-39)	55 & 56

Write a quadratic inequality for each graph.

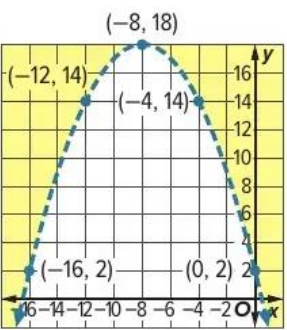
37.



38.



39.



solution method

Learn Graphing Quadratic Inequalities

You can graph quadratic inequalities in two variables by using the same techniques used to graph linear inequalities in two variables. A **quadratic inequality** is an inequality of the form $y > ax^2 + bx + c$, $y \geq ax^2 + bx + c$, $y < ax^2 + bx + c$, or $y \leq ax^2 + bx + c$.

Key Concept • Graphing Quadratic Inequalities

Step 1 Graph the related function.

Step 2 Test a point not on the parabola.

Step 3 Shade accordingly.

Example 1 Graph a Quadratic Inequality ($<$ or $\leq$)

Example 2 Graph a Quadratic Inequality ($>$ or $\geq$)

$$4. y \leq x^2 + 4$$

SOLUTION:

Step 1 Graph the related function. Because the inequality is less than or equal to, the parabola should be solid.

Step 2 Test a point not on the parabola.

$$y \leq x^2 + 4 \quad \text{Original inequality}$$

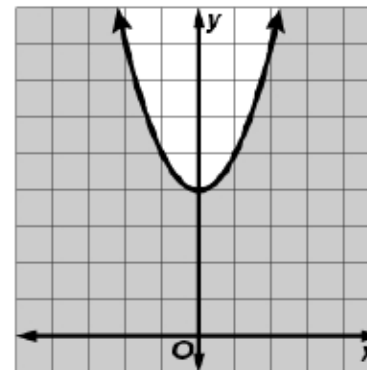
?

$$0 \leq (0)^2 + 4 \quad (x, y) = (0, 0)$$

$$0 \leq 4 \quad \text{True}$$

Shade the region that contains the point.

Step 3 Shade accordingly.



$$5. y < 2x^2 - 4x - 2$$

SOLUTION:

Step 1 Graph the related function. Because the inequality is less than, the parabola should be dashed.

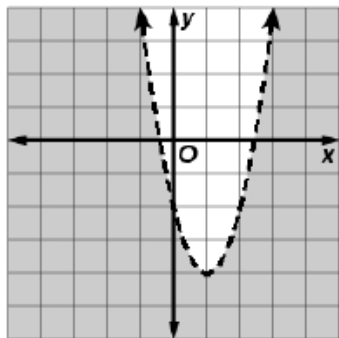
Step 2 Test a point not on the parabola.

$$\begin{array}{ll} y < 2x^2 - 4x - 2 & \text{Original inequality} \\ ? & \\ 0 < 2(0)^2 - 4(0) - 2 & (x, y) = (0, 0) \\ 0 < -2 & \text{False} \end{array}$$

So, $(0, 0)$ is not a solution of the inequality.

Step 3 Shade accordingly.

Because $(0, 0)$ is not a solution of the inequality, shade the region that does not contain the point.



$$6. -x^2 + 12x - 36 > y$$

SOLUTION:

Step 1 Graph the related function. Because the inequality is greater than, the parabola should be dashed.

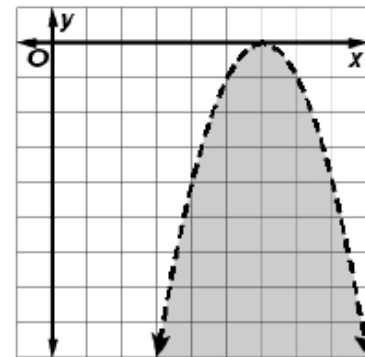
Step 2 Test a point not on the parabola.

$$\begin{array}{ll} -x^2 + 12x - 36 > y & \text{Original inequality} \\ ? & \\ -(0)^2 + 12(0) - 36 > 0 & (x, y) = (0, 0) \\ -36 > 0 & \text{False} \end{array}$$

So, $(0, 0)$ is not a solution of the inequality.

Step 3 Shade accordingly.

Because $(0, 0)$ is not a solution of the inequality, shade the region that does not contain the point.



$$7. y > x^2 + 6x + 7$$

SOLUTION:

Step 1 Graph the related function. Because the inequality is greater than, the parabola should be dashed.

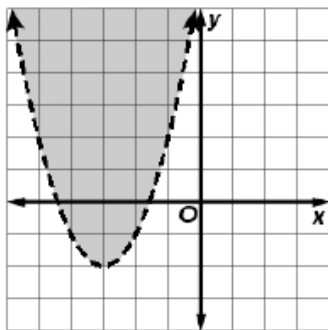
Step 2 Test a point not on the parabola.

$$\begin{array}{ll} y > x^2 + 6x + 7 & \text{Original inequality} \\ ? & \\ 0 > (0)^2 + 6(0) + 7 & (x, y) = (0, 0) \\ 0 > 7 & \text{False} \end{array}$$

So, $(0, 0)$ is not a solution of the inequality.

Step 3 Shade accordingly.

Because $(0, 0)$ is not a solution of the inequality, shade the region that does not contain the point.



$$8. y > x^2 - 8x + 17$$

SOLUTION:

Step 1 Graph the related function. Because the inequality is greater than, the parabola should be dashed.

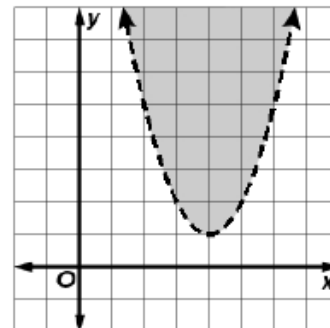
Step 2 Test a point not on the parabola.

$$\begin{array}{ll} y > x^2 - 8x + 17 & \text{Original inequality} \\ ? & \\ 0 > (0)^2 - 8(0) + 17 & (x, y) = (0, 0) \\ 0 > 17 & \text{False} \end{array}$$

So, $(0, 0)$ is not a solution of the inequality.

Step 3 Shade accordingly.

Because $(0, 0)$ is not a solution of the inequality, shade the region that does not contain the point.



$$9. y \geq x^2 + 2x + 2$$

SOLUTION:

Step 1 Graph the related function. Because the inequality is greater than or equal to, the parabola should be solid.

Step 2 Test a point not on the parabola.

$$y \geq x^2 + 2x + 2 \quad \text{Original inequality}$$

?

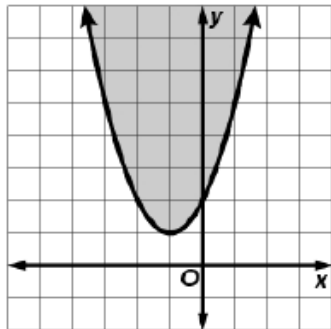
$$0 \geq (0)^2 + 2(0) + 2 \quad (x, y) = (0, 0)$$

$$0 \geq 2 \quad \text{False}$$

So, $(0, 0)$ is not a solution of the inequality.

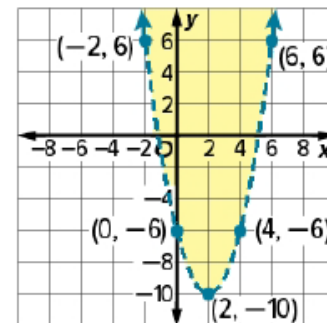
Step 3 Shade accordingly.

Because $(0, 0)$ is not a solution of the inequality, shade the region that does not contain the point.



Write a quadratic inequality for each graph.

37.



SOLUTION:

Find the value of a in the vertex form of a parabola.

$$y = a(x - h)^2 + k \quad \text{Vertex form of a parabola}$$

$$-6 = a(0 - 2)^2 - 10 \quad (h, k) = (2, -10) \text{ and } (x, y) = (0, -6)$$

$$-6 = 4a - 10 \quad \text{Simplify.}$$

$$4 = 4a \quad \text{Add 10 to each side.}$$

$$1 = a \quad \text{Divide each side by 4.}$$

Write the related equation.

$$y = a(x - h)^2 + k \quad \text{Vertex form of a parabola}$$

$$y = 1(x - 2)^2 - 10 \quad (h, k) = (2, -10) \text{ and } a = 1$$

$$y = x^2 - 4x - 6 \quad \text{Simplify.}$$

The graph shows a dashed line, so the inequality symbol is $>$ or $<$. Test the symbol $>$ with a point in the solution region, such as $(0, 0)$.

$$y = x^2 - 4x - 6 \quad \text{Related equation}$$

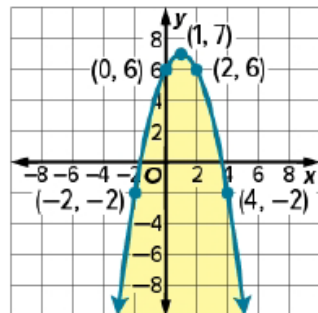
$$?$$

$$0 > (0)^2 - 4(0) - 6 \quad (x, y) = (0, 0)$$

$$0 > -6 \quad \text{True}$$

The symbol $>$ makes the statement true, so the graph of the inequality is $y > x^2 - 4x - 6$.

38.



SOLUTION:

Find the value of a in the vertex form of a parabola.

$$y = a(x - h)^2 + k \quad \text{Vertex form of a parabola}$$

$$6 = a(0 - 1)^2 + 7 \quad (h, k) = (1, 7) \text{ and } (x, y) = (0, 6)$$

$$6 = a + 7 \quad \text{Simplify.}$$

$$-1 = a \quad \text{Subtract 7 from each side.}$$

Write the related equation.

$$y = a(x - h)^2 + k \quad \text{Vertex form of a parabola}$$

$$y = -1(x - 1)^2 + 7 \quad (h, k) = (1, 7) \text{ and } a = -1$$

$$y = -x^2 + 2x + 6 \quad \text{Simplify.}$$

The graph shows a solid line, so the inequality symbol is $\geq$ or $\leq$. Test the symbol $\geq$ with a point in the solution region, such as $(0, 0)$.

$$y = -x^2 + 2x + 6 \quad \text{Related equation}$$

$$0 \geq -(0)^2 + 2(0) + 6 \quad (x, y) = (0, 0)$$

$$0 \geq 6 \quad \text{False}$$

Test the symbol $\leq$ with a point in the solution region, such as $(0, 0)$.

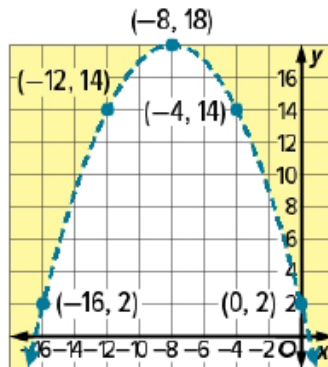
$$y = -x^2 + 2x + 6 \quad \text{Related equation}$$

$$0 \leq -(0)^2 + 2(0) + 6 \quad (x, y) = (0, 0)$$

$$0 \leq 6 \quad \text{True}$$

The symbol $\leq$ makes the statement true, so the graph of the inequality is $y \leq -x^2 + 2x + 6$.

39.



SOLUTION:

Find the value of a in the vertex form of a parabola.

$$y = a(x - h)^2 + k \quad \text{Vertex form of a parabola}$$

$$2 = a(0 - (-8))^2 + 18 \quad (h, k) = (-8, 18) \text{ and } (x, y) = (0, 2)$$

$$2 = 64a + 18 \quad \text{Simplify.}$$

$$-16 = 64a \quad \text{Subtract 18 from each side.}$$

$$-0.25 = a \quad \text{Divide each side by 64.}$$

Write the related equation.

$$y = a(x - h)^2 + k \quad \text{Vertex form of a parabola}$$

$$y = -0.25(x - (-8))^2 + 18 \quad (h, k) = (-8, 18) \text{ and } a = -0.25$$

$$y = -0.25x^2 - 4x + 2 \quad \text{Simplify.}$$

The graph shows a dashed line, so the inequality symbol is $>$ or $<$. Test the symbol $>$ with a point in the solution region, such as $(0, 4)$.

$$y = -0.25x^2 - 4x + 2 \quad \text{Related equation}$$

$$4 > -0.25(0)^2 - 4(0) + 2 \quad (x, y) = (0, 4)$$

$$4 > 2 \quad \text{True}$$

The symbol $>$ makes the statement true, so the graph of the inequality is $y > -0.25x^2 - 4x + 2$.

8	إيجاد قيمة الدوال كثيرة الحدود	Example 3	75
	Graph and analyze polynomial functions.		

Example 3 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $2x^4 - 3x^3 - 4x^2 - 5x + 6a$

b. $7x^3 - 2$

c. $4x^2 - 2xy + 8y^2$

d. $x^5 + 12x^4 - 3x^3 + 2x^2 + 8x + 4$

8	إيجاد قيمة الدوال كثيرة الحدود	Example 3	75
	Graph and analyze polynomial functions.		

solution method

Learn Graphing Polynomial Functions

A **polynomial in one variable** is an expression of the form $a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$, where $a_n \neq 0$, a_{n-1} , a_1 , and a_0 are real numbers, and n is a nonnegative integer. Because the terms are written in order from greatest to least degree, this polynomial is written in **standard form**. The **degree of a polynomial** is n and the leading coefficient is a_n .

A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable. You have learned about constant, linear, quadratic, and cubic functions. A **quartic function** is a fourth-degree function. A **quintic function** is a fifth-degree function. The degree tells you the maximum number of times that the graph of a polynomial function intersects the x -axis.

Example 3 Degrees and Leading Coefficients

Example 3 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- a. $2x^4 - 3x^3 - 4x^2 - 5x + 6a$

degree: 4 leading coefficient: 2
- b. $7x^3 - 2$

degree: 3 leading coefficient: 7
- c. $4x^2 - 2xy + 8y^2$

This is not a polynomial in one variable. There are two variables, x and y .
- d. $x^5 + 12x^4 - 3x^3 + 2x^2 + 8x + 4$

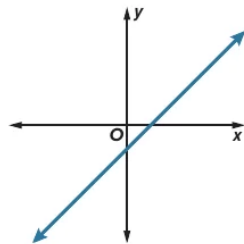
degree: 5 leading coefficient: 1

9	تحديد الأشكال العامة للتمثيلات البيانية للدوال كثيرة الحدود		
	Graph and analyze polynomial functions.	(15-20)	80

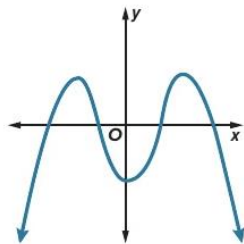
Example 5

Use the graph to state the number of real zeros of the function.

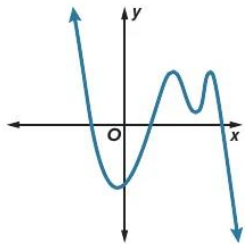
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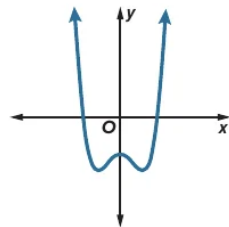
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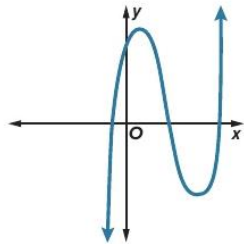
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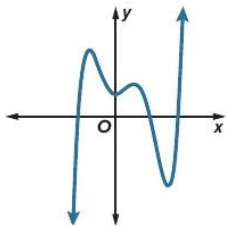
18.



19.



20.



9	تحديد الأشكال العامة للتمثيلات البيانية للدوال كثيرة الحدود		
	Graph and analyze polynomial functions.	(15-20)	80

solution method

Learn Graphing Polynomial Functions

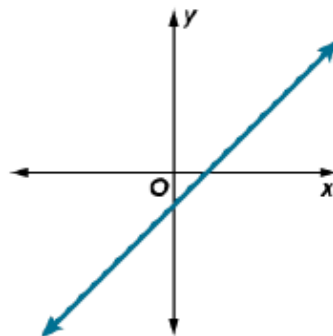
A **polynomial in one variable** is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$, where $a_n \neq 0$, a_{n-1} , a_1 , and a_0 are real numbers, and n is a nonnegative integer. Because the terms are written in order from greatest to least degree, this polynomial is written in **standard form**. The **degree of a polynomial** is n and the leading coefficient is a_n .

A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable. You have learned about constant, linear, quadratic, and cubic functions. A **quartic function** is a fourth-degree function. A **quintic function** is a fifth-degree function. The degree tells you the maximum number of times that the graph of a polynomial function intersects the x -axis.

Example 5 Zeros of a Polynomial Function

Use the graph to state the number of real zeros of the function.

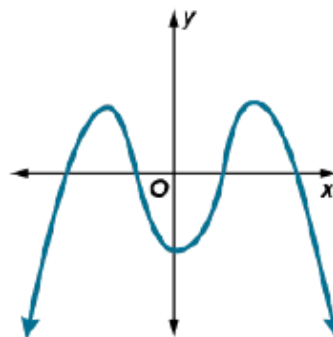
15.



SOLUTION:

The graph crosses the x -axis one time, so there is 1 real zero.

16.

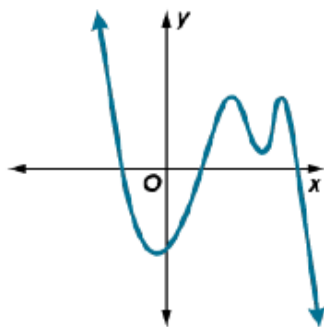


SOLUTION:

The graph crosses the x -axis four times, so there are 4 real zeroes.

9	تحديد الأشكال العامة لتمثيلات البيانات للدوال كثيرة الحدود Graph and analyze polynomial functions.	(15-20)	80
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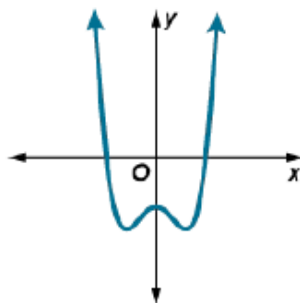
17.



SOLUTION:

The graph crosses the x -axis three times, so there are 3 real zeroes.

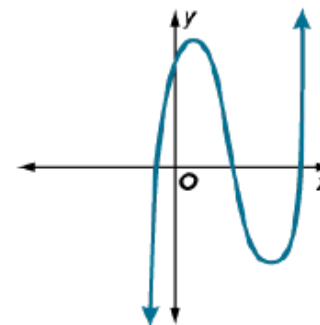
18.



SOLUTION:

The graph crosses the x -axis two times, so there are 2 real zeroes.

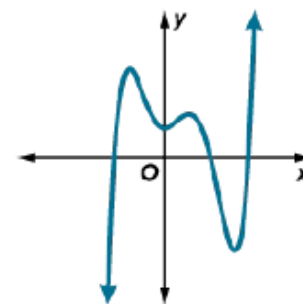
19.



SOLUTION:

The graph crosses the x -axis three times, so there are 3 real zeroes.

20.



SOLUTION:

The graph crosses the x -axis three times, so there are 3 real zeroes.

21. Examine $f(x) = x^3 - 2x^2 - 4x + 1$ and $g(x)$ shown in the graph.

10	تمثيل الدوال كثيرة الحدود بيانياً وتحديد أماكن الأصفار بها		
	Approximate zeros by graphing polynomial functions.	Example1	83

Example 1 Locate Zeros of a Function

Determine the consecutive integer values of x between which each real zero of $f(x) = x^4 - 2x^3 - x^2 + 1$ is located. Then draw the graph.

10	تمثيل الدوال كثيرة الحدود بيانياً وتحديد أماكن الأصفار بها		
	Approximate zeros by graphing polynomial functions.	Example1	83

solution method

Learn The Location Principle

If the value of a polynomial function $f(x)$ changes signs from one value of x to the next, then there is a zero between those two x -values. This is called the Location Principle.

Key Concept • Location Principle

Suppose $y = f(x)$ represents a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b .

Example 1 Locate Zeros of a Function

Example 1 Locate Zeros of a Function

Determine the consecutive integer values of x between which each real zero of $f(x) = x^4 - 2x^3 - x^2 + 1$ is located. Then draw the graph.

Step 1 Make a table.

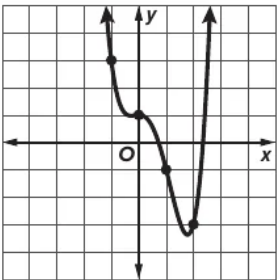
Because $f(x)$ is a fourth-degree polynomial, it will have as many as 4 real zeros or none at all.

x	−2	−1	0	1	2	3	4
$f(x)$	29	3	1	−1	−3	19	113

Using the Location Principle, there are zeros between $x = 0$ and $x = 1$ and between $x = 2$ and $x = 3$.

Step 2 Sketch the graph.

Use the table to sketch the graph and find the locations of the zeros.



Check

Use technology to check the location of the zeros.

Input the function into a graphing calculator to confirm that the function crosses the x -axis between $x = 0$ and $x = 1$ and between $x = 2$ and $x = 3$.

You can find more accurate values of the zeros by using the **zero** feature in the CALC menu to find $x \approx 0.7213$ and $x \approx 2.3486$, which confirms the estimates.

11	إيجاد القيم العظمى والصغرى للدوال كثيرة الحدود		
	Find extrema of polynomial functions.	Example2	84

Example 2 Identify Extrema

Use a table to graph $f(x) = x^3 + x^2 - 5x - 2$. Estimate the x -coordinates at which the relative maxima and relative minima occur.

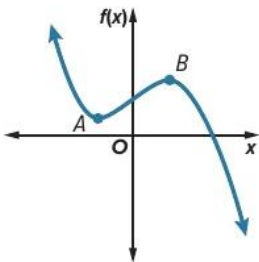
11	إيجاد القيم العظمى والصغرى للدوال كثيرة الحدود		
	Find extrema of polynomial functions.	Example2	84

solution method

Learn Extrema of Polynomials

Extrema occur at relative maxima or minima of the function.

Point *A* is a relative minimum, and point *B* is a relative maximum. Both points *A* and *B* are extrema. The graph of a polynomial of degree *n* has at most *n* – 1 extrema.



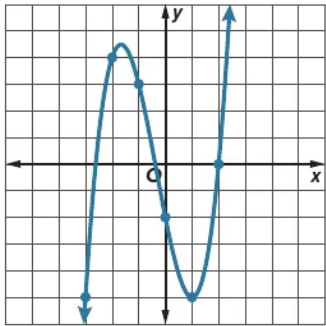
Example 2 Identify Extrema

Example 2 Identify Extrema

Use a table to graph $f(x) = x^3 + x^2 - 5x - 2$. Estimate the *x*-coordinates at which the relative maxima and relative minima occur.

Step 1 Make a table of values and graph the function.

<i>x</i>	<i>f</i> (<i>x</i>)
–4	–30
–3	–5
–2	4
–1	3
0	–2
1	–5
2	0
3	19



Step 2 Estimate the locations of the extrema.

The value of *f*(*x*) at *x* = –2 is greater than the surrounding points indicating a maximum *near* *x* = –2.

The value of *f*(*x*) at *x* = 1 is less than the surrounding points indicating a minimum *near* *x* = 1.

You can use a graphing calculator to find the extrema of a function and confirm your estimates.

12	جمع وطرح كثيرات الحدود		
	Add and subtract polynomials.	Example 2 & Example 3	94

Example 2 Add Polynomials

Find $(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1)$.

Example 3 Subtract Polynomials

Find $(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4)$.

12	جميع وطرح كثيرات الحدود		
	Add and subtract polynomials.	Example 2 & Example 3	94

solution method

Learn Adding and Subtracting Polynomials

A polynomial is a monomial or the sum of two or more monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. The degree of a polynomial is the greatest degree of any term in the polynomial.

Polynomials can be added or subtracted by performing the operations indicated and combining like terms. You can subtract a polynomial by adding its additive inverse.

The sum or difference of polynomials will have the same variables and exponents as the original polynomials, but possibly different coefficients. Thus, the sum or difference of two polynomials is also a polynomial.

A set is **closed** if and only if an operation on any two elements of the set produces another element of the same set. Because adding or subtracting polynomials results in a polynomial, the set of polynomials is closed under the operations of addition and subtraction.

Example 2 Add Polynomials

Example 3 Subtract Polynomials

Example 2 Add Polynomials

Find $(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1)$.

Method 1 Add horizontally.

Group and combine like terms.

$$\begin{aligned} &(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1) \\ &= (6x^3 + x^3) + (7x^2 - 4x^2) + (-2x - 8x) + (5 + 1) \\ &= 7x^3 + 3x^2 - 10x + 6 \end{aligned}$$

Group like terms.
Combine like terms.

Method 2 Add vertically.

Align like terms vertically and add.

$$\begin{array}{r} 6x^3 + 7x^2 - 2x + 5 \\ (+) x^3 - 4x^2 - 8x + 1 \\ \hline 7x^3 + 3x^2 - 10x + 6 \end{array}$$

Align like terms.
Combine like terms.

Check

Find $(2x^3 + 9x^2 + 6x - 3) + (4x^3 - 7x^2 + 5x)$.

Example 3 Subtract Polynomials

Find $(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4)$.

Method 1 Subtract horizontally.

Group and combine like terms.

$$\begin{aligned} &(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4) \\ &= 2x^5 + 11x^4 + 7x - 8 - 5x^4 - 9x^3 + 3x - 4 \\ &= 2x^5 + (11x^4 - 5x^4) + (-9x^3) + (7x + 3x) + (-8 - 4) \\ &= 2x^5 + 6x^4 - 9x^3 + 10x - 12 \end{aligned}$$

Original equation
Distribute -1 .
Group like terms.
Combine like terms.

Method 2 Subtract vertically.

Align like terms vertically and subtract.

$$\begin{array}{r} 2x^5 + 11x^4 + 0x^3 + 7x - 8 \\ (-) 0x^5 + 5x^4 + 9x^3 - 3x + 4 \\ \hline 2x^5 + 6x^4 - 9x^3 + 10x - 12 \end{array}$$

13	ضرب كثيرات الحدود		
	Multiply polynomials.	(16-26)	97

Examples 4, 5 and 6

Multiply.

16. $x^2(2x + 9)$

17. $(a - 5)^2$

18. $(2x - 3)(3x - 5)$

13	ضرب كثيرات الحدود		
	Multiply polynomials.	(16-26)	97

Examples 4, 5 and 6

Multiply.

19. $(x - y)(x^2 + 2xy + y^2)$

20. $(a + b)(a^3 - 3ab - b^2)$

21. $(x - y)(x + y)(2x + y)$

13	ضرب كثيرات الحدود		
	Multiply polynomials.	(16-26)	97

Examples 4, 5 and 6

Multiply.

22. $(a + b)(2a + 3b)(2x - y)$

23. $(r - 2t)(r + 2t)$

24. $(3y + 4)(2y - 3)$

13	ضرب كثيرات الحدود		
	Multiply polynomials.	(16-26)	97

Examples 4, 5 and 6

Multiply.

25. $(x^3 - 3x^2 + 1)(2x^2 - x + 2)$

26. $(4x^5 + x^3 - 7x^2 + 2)(3x - 1)$

Learn Multiplying Polynomials

Polynomials can be multiplied by using the Distributive Property to multiply each term in one polynomial by each term in the other. When polynomials are multiplied, the product is also a polynomial. Therefore, the set of polynomials is closed under the multiplication. This is similar to the system of integers, which is also closed under multiplication. To multiply two binomials, you can use a shortcut called the **FOIL method**.

Key Concept • FOIL Method

Words: Find the sum of the products of **F** the *First* terms, **O** the *Outer* terms, **I** the *Inner* terms, and **L** the *Last* terms.

Symbols:

$$\begin{aligned}
 (2x + 4)(x - 3) &= (2x)(x) + (2x)(-3) + (4)(x) + (4)(-3) \\
 &= 2x^2 - 6x + 4x - 12 \\
 &= 2x^2 - 2x - 12
 \end{aligned}$$

Example 4 Simplify by Using the Distributive Property

Example 5 Multiply Binomials

$$16. x^2(2x + 9)$$

SOLUTION:

$$\begin{aligned}
 x^2(2x + 9) &= x^2(2x) + x^2(9) && \text{Distributive Property} \\
 &= 2x^3 + 9x^2 && \text{Multiply.}
 \end{aligned}$$

$$17. (a - 5)^2$$

SOLUTION:

$$\begin{aligned}
 (a - 5)^2 &= (a - 5)(a - 5) && \text{Rewrite as two binomials.} \\
 &= a(a) + a(-5) + (-5)(a) + (-5)(-5) && \text{FOIL Method} \\
 &= a^2 - 5a - 5a + 25 && \text{Multiply.} \\
 &= a^2 - 10a + 25 && \text{Combine like terms.}
 \end{aligned}$$

$$18. (2x - 3)(3x - 5)$$

SOLUTION:

$$\begin{aligned}
 (2x - 3)(3x - 5) &= 2x(3x) + 2x(-5) + (-3)(3x) + (-3)(-5) && \text{FOIL Method} \\
 &= 6x^2 - 10x - 9x + 15 && \text{Multiply.} \\
 &= 6x^2 - 19x + 15 && \text{Combine like terms.}
 \end{aligned}$$

$$19. (x - y)(x^2 + 2xy + y^2)$$

SOLUTION:

$$\begin{aligned}
 (x - y)(x^2 + 2xy + y^2) &= x(x^2) + x(2xy) + x(y^2) + (-y)(x^2) + (-y)(2xy) + (-y)(y^2) && \text{Distributive Property} \\
 &= x^3 + 2x^2y + xy^2 - x^2y - 2xy^2 - y^3 && \text{Multiply.} \\
 &= x^3 + (2x^2y - x^2y) + (xy^2 - 2xy^2) - y^3 && \text{Group like terms.} \\
 &= x^3 + x^2y - xy^2 - y^3 && \text{Combine like terms.}
 \end{aligned}$$

13	ضرب كثيرات الحدود		
	Multiply polynomials.	(16-26)	97

20. $(a + b)(a^3 - 3ab - b^2)$

SOLUTION:

$$\begin{aligned}
 (a + b)(a^3 - 3ab - b^2) &= a(a^3) + a(-3ab) + a(-b^2) + b(a^3) + b(-3ab) + b(-b^2) && \text{Distributive Property} \\
 &= a^4 - 3a^2b - ab^2 + a^3b - 3ab^2 - b^3 && \text{Multiply.} \\
 &= a^4 + a^3b - 3a^2b + (-ab^2 - 3ab^2) - b^3 && \text{Group like terms.} \\
 &= a^4 + a^3b - 3a^2b - 4ab^2 - b^3 && \text{Combine like terms.}
 \end{aligned}$$

21. $(x - y)(x + y)(2x + y)$

SOLUTION:

$$\begin{aligned}
 (x - y)(x + y) &= x(x) + x(y) + (-y)(x) + (-y)(y) && \text{Distributive Property} \\
 &= x^2 + xy - xy - y^2 && \text{Multiply.} \\
 &= x^2 - y^2 && \text{Combine like terms.} \\
 (x^2 - y^2)(2x + y) &= x^2(2x) + x^2(y) + (-y^2)(2x) + (-y^2)(y) && \text{Distributive Property} \\
 &= 2x^3 + x^2y - 2xy^2 - y^3 && \text{Multiply.}
 \end{aligned}$$

22. $(a + b)(2a + 3b)(2x - y)$

SOLUTION:

$$\begin{aligned}
 (a + b)(2a + 3b) &= a(2a) + a(3b) + b(2a) + b(3b) && \text{Distributive Property} \\
 &= 2a^2 + 3ab + 2ab + 3b^2 && \text{Multiply.} \\
 &= 2a^2 + 5ab + 3b^2 && \text{Combine like terms.} \\
 (2a^2 + 5ab + 3b^2)(2x - y) &= 2a^2(2x) + 2a^2(-y) + 5ab(2x) + 5ab(-y) + 3b^2(2x) + 3b^2(-y) && \text{Distributive Property} \\
 &= 4a^2x - 2a^2y + 10abx - 5aby + 6b^2x - 3b^2y && \text{Multiply.}
 \end{aligned}$$

23. $(r - 2t)(r + 2t)$

SOLUTION:

$$\begin{aligned}
 (r - 2t)(r + 2t) &= r(r) + r(2t) + (-2t)(r) + (-2t)(2t) && \text{FOIL Method} \\
 &= r^2 + 2rt - 2rt - 4t^2 && \text{Multiply.} \\
 &= r^2 - 4t^2 && \text{Combine like terms.}
 \end{aligned}$$

24. $(3y + 4)(2y - 3)$

SOLUTION:

$$\begin{aligned}
 (3y + 4)(2y - 3) &= 3y(2y) + 3y(-3) + 4(2y) + 4(-3) && \text{FOIL Method} \\
 &= 6y^2 - 9y + 8y - 12 && \text{Multiply.} \\
 &= 6y^2 - y - 12 && \text{Combine like terms.}
 \end{aligned}$$

25. $(x^3 - 3x^2 + 1)(2x^2 - x + 2)$

SOLUTION:

$$\begin{aligned}
 (x^3 - 3x^2 + 1)(2x^2 - x + 2) &= x^3(2x^2) + x^3(-x) + x^3(2) + (-3x^2)(2x^2) + (-3x^2)(-x) + (-3x^2)(2) + 1(2x^2) + 1(-x) + 1(2) && \text{Distributive Property} \\
 &= 2x^5 - x^4 + 2x^3 - 6x^4 + 3x^3 - 6x^2 + 2x^2 - x + 2 && \text{Multiply.} \\
 &= 2x^5 + (-x^4 - 6x^4) + (2x^3 + 3x^3) + (-6x^2 + 2x^2) - x + 2 && \text{Group like terms.} \\
 &= 2x^5 - 7x^4 + 5x^3 - 4x^2 - x + 2 && \text{Combine like terms.}
 \end{aligned}$$

26. $(4x^5 + x^3 - 7x^2 + 2)(3x - 1)$

SOLUTION:

$$\begin{aligned}
 (4x^5 + x^3 - 7x^2 + 2)(3x - 1) &= 4x^5(3x) + 4x^5(-1) + x^3(3x) + x^3(-1) + (-7x^2)(3x) + (-7x^2)(-1) + 2(3x) + 2(-1) && \text{Distributive Property} \\
 &= 12x^6 - 4x^5 + 3x^4 - x^3 - 21x^3 + 7x^2 + 6x - 2 && \text{Multiply.} \\
 &= 12x^6 - 4x^5 + 3x^4 + (-x^3 - 21x^3) + 7x^2 + 6x - 2 && \text{Group like terms.} \\
 &= 12x^6 - 4x^5 + 3x^4 - 22x^3 + 7x^2 + 6x - 2 && \text{Combine like terms.}
 \end{aligned}$$

14	قسمة كثيرات الحدود باستخدام القسمة المطولة		
	Divide polynomials by using long division.	Example 1 & Example 2 & Example 3	102

Example 1 Divide a Polynomial by a Monomial

Find $(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$.

Example 2 Divide a Polynomial by a Binomial

Find $(x^2 - 5x - 36) \div (x + 4)$.

Example 3 Find a Quotient with a Remainder

Find $\frac{3z^3 - 14z^2 - 7z + 3}{z - 5}$.

14	قسمة كثيرات الحدود باستخدام القسمة المطولة
	Divide polynomials by using long division.

Example 1 & Example 2 & Example 3	102
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Learn Dividing Polynomials by Using Long Division

To divide a polynomial by a monomial, find the quotient of each term of the polynomial and the monomial.

$$\frac{6x^2 - 15x}{3x} = \frac{6x^2}{3x} - \frac{15x}{3x} = 2x - 5$$

You can divide a polynomial by a polynomial with more than one term by using a process similar to long division of real numbers. This process is known as the Division Algorithm. The resulting quotient may be a polynomial or a polynomial with a remainder.

Key Concept • Division Algorithm

Words: If $f(x)$ and $g(x)$ are two polynomials in which $g(x) \neq 0$ and the degree of $g(x)$ is less than the degree of $f(x)$, then there exists a unique quotient $q(x)$ and a unique remainder $r(x)$ such that $f(x) = q(x)g(x) + r(x)$, where the remainder is zero or the degree of $r(x)$ is less than the degree of $g(x)$.

Symbols: $\frac{f(x)}{g(x)} = q(x) \rightarrow f(x) = q(x)g(x)$

$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \rightarrow f(x) = q(x)g(x) + r(x)$

Example: $\frac{2x^2 - 5x - 3}{x - 3} = 2x + 1$

$$2x^2 - 5x - 3 = (x - 3)(2x + 1)$$
$$= 2x^2 - 5x - 3$$

$$\frac{x^2 - 4x - 1}{x + 1} = x - 5 + \frac{4}{x + 1}$$
$$x^2 - 4x - 1 = (x + 1)(x - 5) + 4$$
$$= x^2 - 4x - 5 + 4$$
$$= x^2 - 4x - 1$$

Example 1 Divide a Polynomial by a Monomial

Example 2 Divide a Polynomial by a Binomial

Example 3 Find a Quotient with a Remainder

Example 1 Divide a Polynomial by a Monomial

Find $(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$.

$$(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$$
$$= \frac{24a^4b^3 + 18a^2b^2 - 30ab^3}{6ab}$$
 Write a fraction.

$$= \frac{24a^4b^3}{6ab} + \frac{18a^2b^2}{6ab} - \frac{30ab^3}{6ab}$$
 Sum of quotients

$$= \frac{24}{6}a^{4-1}b^{3-1} + \frac{18}{6}a^{2-1}b^{2-1} - \frac{30}{6}a^{1-1}b^{3-1}$$
 Divide.

$$= 4a^3b^2 + 3ab - 5b^2$$
 Simplify.

Check

Find $(9x^3y^5 + 21x^4y^4 - 12x^3y^2) \div (3x^2y^2)$.

Example 2 Divide a Polynomial by a Binomial

Find $(x^2 - 5x - 36) \div (x + 4)$.

$$\begin{array}{r} x - 9 \\ x + 4 \overline{) x^2 - 5x - 36} \\ \underline{(-) x^2 + 4x} \\ -9x - 36 \\ \underline{(-) -9x - 36} \\ 0 \end{array}$$
 The quotient is $x - 9$ and the remainder is 0.

Check

Find $\frac{x^2 + 6x - 112}{x - 8}$.

Example 3 Find a Quotient with a Remainder

Find $\frac{3z^3 - 14z^2 - 7z + 3}{z - 5}$.

$$\begin{array}{r} 3z^2 + z - 2 \\ z - 5 \overline{) 3z^3 - 14z^2 - 7z + 3} \\ \underline{(-) 3z^3 - 15z^2} \\ z^2 - 7z \\ \underline{(-) z^2 - 5z} \\ -2z + 3 \\ \underline{(-) -2z + 10} \\ -7 \end{array}$$

15	حل معادلات كثيرات الحدود باستخدام التحليل إلى العوامل		
	Solve polynomial equations by writing them in quadratic form and factoring.	Example 6 , Example 7 & (16-27)	126 & 128

Example 6 Write Expressions in Quadratic Form

Write each expression in quadratic form, if possible.

- a. $4x^{20} + 6x^{10} + 15$
- b. $18x^4 + 180x^8 - 28$
- c. $9x^6 - 4x^2 - 12$

Example 7 Solve Equations in Quadratic Form

Solve $8x^4 + 10x^2 - 12 = 0$.

15	حل معادلات كثيرات الحدود باستخدام التحليل إلى العوامل		
	Solve polynomial equations by writing them in quadratic form and factoring.	Example 6 , Example 7 & (16-27)	126 & 128

Example 6

Write each expression in quadratic form, if possible.

16. $x^4 + 12x^2 - 8$

17. $-15x^4 + 18x^2 - 4$

18. $8x^6 + 6x^3 + 7$

19. $5x^6 - 2x^2 + 8$
20. $9x^8 - 21x^4 + 12$

21. $16x^{10} + 2x^5 + 6$

15	حل معادلات كثيرات الحدود باستخدام التحليل إلى العوامل		
	Solve polynomial equations by writing them in quadratic form and factoring.	Example 6 , Example 7 & (16-27)	126 & 128

Example 7

Solve each equation.

22. $x^4 + 6x^2 + 5 = 0$

23. $x^4 - 3x^2 - 10 = 0$

24. $4x^4 - 14x^2 + 12 = 0$

15	حل معادلات كثيرات الحدود باستخدام التحليل إلى العوامل		
	Solve polynomial equations by writing them in quadratic form and factoring.	Example 6 , Example 7 & (16-27)	126 & 128

25. $9x^4 - 27x^2 + 20 = 0$

26. $4x^4 - 5x^2 - 6 = 0$

27. $24x^4 + 14x^2 - 3 = 0$

15	حل معادلات كثيرات الحدود باستخدام التحليل إلى العوامل		
	Solve polynomial equations by writing them in quadratic form and factoring.	Example 6 , Example 7 & (16-27)	126 & 128

solution method

Learn Solving Polynomial Equations in Quadratic Form

Some polynomials in x can be rewritten in **quadratic form**.

Key Concept • Quadratic Form

An expression in quadratic form can be written as $au^2 + bu + c$ for any numbers a , b , and c , $a \neq 0$, where u is some expression in x . The expression $au^2 + bu + c$ is called the quadratic form of the original expression.

Example 6 Write Expressions in Quadratic Form

Example 7 Solve Equations in Quadratic Form

Example 6 Write Expressions in Quadratic Form

Write each expression in quadratic form, if possible.

a. $4x^{20} + 6x^{10} + 15$

Examine the terms with variables to choose the expression equal to u .

$$4x^{20} + 6x^{10} + 15 = (2x^{10})^2 + 3(2x^{10}) + 15 \qquad (2x^{10})^2 = 4x^{20}$$

b. $18x^4 + 180x^8 - 28$

If the polynomial is not already in standard form, rewrite it. Then examine the terms with variables to choose the expression equal to u .

$$\begin{aligned} 18x^4 + 180x^8 - 28 &= 180x^8 + 18x^4 - 28 && \text{Standard form} \\ &= 5(6x^4)^2 + 3(6x^4) - 28 && \begin{array}{l} \text{of a polynomial} \\ (6x^4)^2 = 36x^8 \end{array} \end{aligned}$$

c. $9x^6 - 4x^2 - 12$

Because $x^6 \neq (x^2)^2$, the expression cannot be written in quadratic form.

Check

What is the quadratic form of $10x^4 + 100x^8 - 9$?

Example 7 Solve Equations in Quadratic Form

Solve $8x^4 + 10x^2 - 12 = 0$.

$$8x^4 + 10x^2 - 12 = 0 \qquad \text{Original equation}$$

$$2(2x^2)^2 + 5(2x^2) - 12 = 0 \qquad 2(2x^2)^2 = 8x^4$$

$$2u^2 + 5u - 12 = 0 \qquad \text{Let } u = 2x^2.$$

$$(2u - 3)(u + 4) = 0 \qquad \text{Factor.}$$

$$u = \frac{3}{2} \text{ or } u = -4 \qquad \text{Zero Product Property}$$

$$2x^2 = \frac{3}{2} \qquad 2x^2 = -4 \qquad \text{Replace } u \text{ with } 2x^2.$$

$$x^2 = \frac{3}{4} \qquad x^2 = -2 \qquad \text{Divide each side by 2.}$$

$$x = \pm \frac{\sqrt{3}}{2} \qquad x = \pm i\sqrt{2} \qquad \text{Take the square root of each side.}$$

The solutions are $\frac{\sqrt{3}}{2}$, $-\frac{\sqrt{3}}{2}$, $i\sqrt{2}$, and $-i\sqrt{2}$.

Write each expression in quadratic form, if possible.

16. $x^4 + 12x^2 - 8$

SOLUTION:

Examine the terms with variables to choose the expression equal to u .

$$x^4 + 12x^2 - 8 = (x^2)^2 + 12(x^2) - 8 \quad (x^2)^2 = x^4$$

17. $-15x^4 + 18x^2 - 4$

SOLUTION:

Examine the terms with variables to choose the expression equal to u .

$$-15x^4 + 18x^2 - 4 = -15(x^2)^2 + 18(x^2) - 4 \quad (x^2)^2 = x^4$$

18. $8x^6 + 6x^3 + 7$

SOLUTION:

Examine the terms with variables to choose the expression equal to u .

$$8x^6 + 6x^3 + 7 = 2(2x^3)^2 + 3(2x^3) + 7 \quad (2x^3)^2 = 4x^6$$

19. $5x^6 - 2x^2 + 8$

SOLUTION:

Because $x^6 \neq (x^2)^2$, the expression cannot be written in quadratic form.

20. $9x^8 - 21x^4 + 12$

SOLUTION:

Examine the terms with variables to choose the expression equal to u .

$$9x^8 - 21x^4 + 12 = (3x^4)^2 - 7(3x^4) + 12 \quad (3x^4)^2 = 9x^8$$

21. $16x^{10} + 2x^5 + 6$

SOLUTION:

Examine the terms with variables to choose the expression equal to u .

$$16x^{10} + 2x^5 + 6 = 4(2x^5)^2 + 1(2x^5) + 6 \quad (2x^5)^2 = 4x^{10}$$

Solve each equation.

22. $x^4 + 6x^2 + 5 = 0$

SOLUTION:

$$x^4 + 6x^2 + 5 = 0 \quad \text{Original equation}$$

$$(x^2)^2 + 6(x^2) + 5 = 0 \quad (x^2)^2 = x^4$$

$$u^2 + 6u + 5 = 0 \quad \text{Let } u = x^2.$$

$$(u + 5)(u + 1) = 0 \quad \text{Factor.}$$

$$u = -5 \text{ or } u = -1 \quad \text{Zero Product Property}$$

$$x^2 = -5 \text{ or } x^2 = -1 \quad \text{Replace } u \text{ with } x^2.$$

$$x = \pm i\sqrt{5} \text{ or } x = \pm i \quad \text{Take the square root of each side.}$$

The solutions are $i\sqrt{5}$, $-i\sqrt{5}$, i , and $-i$.

23. $x^4 - 3x^2 - 10 = 0$

SOLUTION:

$$x^4 - 3x^2 - 10 = 0 \quad \text{Original equation}$$

$$(x^2)^2 - 3(x^2) - 10 = 0 \quad (x^2)^2 = x^4$$

$$u^2 - 3u - 10 = 0 \quad \text{Let } u = x^2.$$

$$(u - 5)(u + 2) = 0 \quad \text{Factor.}$$

$$u = 5 \text{ or } u = -2 \quad \text{Zero Product Property}$$

$$x^2 = 5 \text{ or } x^2 = -2 \quad \text{Replace } u \text{ with } x^2.$$

$$x = \pm\sqrt{5} \text{ or } x = \pm i\sqrt{2} \quad \text{Take the square root of each side.}$$

The solutions are $\sqrt{5}$, $-\sqrt{5}$, $i\sqrt{2}$, and $-i\sqrt{2}$.

24. $4x^4 - 14x^2 + 12 = 0$

SOLUTION:

$$4x^4 - 14x^2 + 12 = 0 \quad \text{Original equation}$$

$$(2x^2)^2 - 7(2x^2) + 12 = 0 \quad (2x^2)^2 = 4x^4$$

$$u^2 - 7u + 12 = 0 \quad \text{Let } u = 2x^2.$$

$$(u - 4)(u - 3) = 0 \quad \text{Factor.}$$

$$u = 4 \text{ or } u = 3 \quad \text{Zero Product Property}$$

$$2x^2 = 4 \text{ or } 2x^2 = 3 \quad \text{Replace } u \text{ with } 2x^2.$$

$$x^2 = 2 \text{ or } x^2 = \frac{3}{2} \quad \text{Divide each side by 2.}$$

$$x = \pm\sqrt{2} \text{ or } x = \pm\frac{\sqrt{3}}{2} \quad \text{Take the square root of each side.}$$

The solutions are $\sqrt{2}$, $-\sqrt{2}$, $\frac{\sqrt{3}}{2}$, and $-\frac{\sqrt{3}}{2}$.

25. $9x^4 - 27x^2 + 20 = 0$

SOLUTION:

$$9x^4 - 27x^2 + 20 = 0 \quad \text{Original equation}$$

$$(3x^2)^2 - 9(3x^2) + 20 = 0 \quad (3x^2)^2 = 9x^4$$

$$u^2 - 9u + 20 = 0 \quad \text{Let } u = 3x^2.$$

$$(u - 4)(u - 5) = 0 \quad \text{Factor.}$$

$$u = 4 \text{ or } u = 5 \quad \text{Zero Product Property}$$

$$3x^2 = 4 \text{ or } 3x^2 = 5 \quad \text{Replace } u \text{ with } 3x^2.$$

$$x^2 = \frac{4}{3} \text{ or } x^2 = \frac{5}{3} \quad \text{Divide each side by 3.}$$

$$x = \pm\frac{2\sqrt{3}}{3} \text{ or } x = \pm\frac{\sqrt{15}}{3} \quad \text{Take the square root of each side.}$$

The solutions are $\frac{2\sqrt{3}}{3}$, $-\frac{2\sqrt{3}}{3}$, $\frac{\sqrt{15}}{3}$, and $-\frac{\sqrt{15}}{3}$.

15	حل معادلات كثيرات الحدود باستخدام التحليل إلى العوامل		
	Solve polynomial equations by writing them in quadratic form and factoring.	Example 6 , Example 7 & (16-27)	126 & 128

26. $4x^4 - 5x^2 - 6 = 0$

SOLUTION:

$$\begin{array}{ll}
 4x^4 - 5x^2 - 6 = 0 & \text{Original equation} \\
 4(x^2)^2 - 5(x^2) - 6 = 0 & (x^2)^2 = x^4 \\
 4u^2 - 5u - 6 = 0 & \text{Let } u = x^2. \\
 (u - 2)(4u + 3) = 0 & \text{Factor.} \\
 u = 2 \text{ or } u = -\frac{3}{4} & \text{Zero Product Property} \\
 x^2 = 2 \text{ or } x^2 = -\frac{3}{4} & \text{Replace } u \text{ with } x^2. \\
 x = \pm\sqrt{2} \text{ or } x = \pm i\frac{\sqrt{3}}{2} & \text{Take the square root of each side.}
 \end{array}$$

The solutions are $\sqrt{2}$, $-\sqrt{2}$, $i\frac{\sqrt{3}}{2}$, and $-i\frac{\sqrt{3}}{2}$.

27. $24x^4 + 14x^2 - 3 = 0$

SOLUTION:

$$\begin{array}{ll}
 24x^4 + 14x^2 - 3 = 0 & \text{Original equation} \\
 24(x^2)^2 + 14(x^2) - 3 = 0 & (x^2)^2 = x^4 \\
 24u^2 + 14u - 3 = 0 & \text{Let } u = x^2. \\
 (6u - 1)(4u + 3) = 0 & \text{Factor.} \\
 u = \frac{1}{6} \text{ or } u = -\frac{3}{4} & \text{Zero Product Property} \\
 x^2 = \frac{1}{6} \text{ or } x^2 = -\frac{3}{4} & \text{Replace } u \text{ with } x^2. \\
 x = \pm\frac{\sqrt{6}}{6} \text{ or } x = \pm i\frac{\sqrt{3}}{2} & \text{Take the square root of each side.}
 \end{array}$$

The solutions are $\frac{\sqrt{6}}{6}$, $-\frac{\sqrt{6}}{6}$, $i\frac{\sqrt{3}}{2}$, and $-i\frac{\sqrt{3}}{2}$.

16	تمثيل دوال النمو الأسّي بيانيًا		
	Graph exponential growth functions.	Example 1 & Example 2	217 & 218

Example 1 Graph Exponential Growth Functions

Graph $f(x) = 2^x$. Find the domain, range, y -intercept, asymptote, and end behavior.



Example 2 Graph Transformations of Exponential Growth Functions

Graph $g(x) = -\frac{1}{2} \cdot 3^{x+4} + 1$.



16	تمثيل دوال النمو الأسّي بيانياً	Example 1 & Example 2	217 & 218
	Graph exponential growth functions.		

solution method

Learn Graphing Exponential Growth Functions

In an **exponential function**, the independent variable is an exponent. An exponential function has the form $f(x) = b^x$, where the base b is a constant and the independent variable x is the exponent. For an exponential growth function, $b > 1$. **Exponential growth** occurs when an initial amount increases by the same percent over a given period of time.

Graphs of exponential functions have asymptotes. An **asymptote** is a line that a graph approaches.

Example 1 Graph Exponential Growth Functions

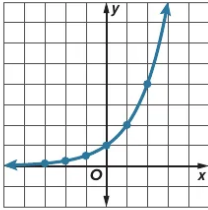
Example 2 Graph Transformations of Exponential Growth Functions

Example 1 Graph Exponential Growth Functions

Graph $f(x) = 2^x$. Find the domain, range, y -intercept, asymptote, and end behavior.

Make a table of values. Then plot the points and sketch the graph.

x	$f(x) = 2^x$
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8



domain: all real numbers

range: all positive real numbers

y -intercept: $(0, 1)$

asymptote: $y = 0$

end behavior: As $x \rightarrow -\infty$, $f(x) \rightarrow 0$ and as $x \rightarrow \infty$, $f(x) \rightarrow \infty$.

Example 2 Graph Transformations of Exponential Growth Functions

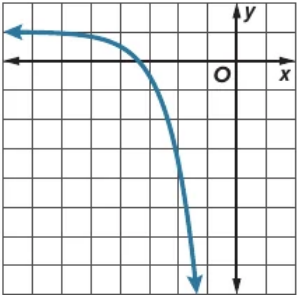
Graph $g(x) = -\frac{1}{2} \cdot 3^{x+4} + 1$.

Transform the graph of $g(x) = 3^x$.

$a = -\frac{1}{2}$; Reflect in the x -axis and compress vertically.

$h = -4$; Translate 4 units left.

$k = 1$; Translate 1 unit up.



17	تمثيل دوال التضاؤل الأسّي بيانيًا		
	Graph exponential decay functions.	Example 6 & Example 7	219 & 220

Example 6 Graph Exponential Decay Functions

Graph $f(x) = \left(\frac{1}{2}\right)^x$. Find the domain, range, y -intercept, asymptote, and end behavior.



Example 7 Graph Transformations of Exponential Decay Functions

Graph $g(x) = -2\left(\frac{1}{4}\right)^{x-4} + 3$.



17	تمثيل دوال التضاؤل الأسي بيانياً		
	Graph exponential decay functions.	Example 6 & Example 7	219 & 220

solution method

Learn Graphing Exponential Decay Functions

Exponential decay occurs when an initial amount decreases by the same percent over a given period of time. So, for an exponential function of the form $f(x) = b^x$, exponential decay occurs when b is between 0 and 1.

Like exponential growth, exponential decay can be modeled by $A(t) = a(1 - r)^t$, where r is the percent of decrease per time period. The **decay factor** is $1 - r$.

Example 6 Graph Exponential Decay Functions

Example 7 Graph Transformations of Exponential Decay Functions

Example 6 Graph Exponential Decay Functions

Graph $f(x) = \left(\frac{1}{2}\right)^x$. Find the domain, range, y-intercept, asymptote, and end behavior.

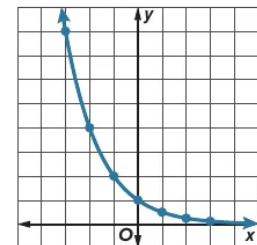
Make a table of values. Then plot the points and sketch the graph.

domain: all real numbers

range: all positive real numbers

y-intercept: (0, 1) asymptote: $y = 0$

end behavior: as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$ and as $x \rightarrow \infty$, $f(x) \rightarrow 0$



Example 7 Graph Transformations of Exponential Decay Functions

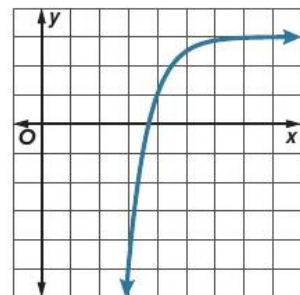
Graph $g(x) = -2\left(\frac{1}{4}\right)^{x-4} + 3$.

$g(x)$ is a transformation of $f(x) = \left(\frac{1}{4}\right)^x$.

$a = -2$; Reflect in the x-axis and stretch vertically.

$h = 4$; Translate 4 units right.

$k = 3$; Translate 3 units up.



18	تصنيف وتحديد أنواع الدراسات		
	Distinguish among sample surveys, experiments, and observational studies.	Example 4 & (11-14)	373 & 376

 Example 4 Classify Study Types

UNIFORMS A research team wants to test new football uniform designs and their appeal to young adults. They randomly select 100 young adults to view the different uniforms. The research team observes and records the reactions to the uniforms.

Example 4

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

11. An Internet service provider conducts an online study in which customers are randomly selected and asked to provide feedback on their customer service.

12. A research group randomly selects 100 business owners, half of whom started their own businesses, and compares their success.

18	تصنيف وتحديد أنواع الدراسات		
	Distinguish among sample surveys, experiments, and observational studies.	Example 4 & (11-14)	373 & 376

13. A research group randomly chooses 50 people to participate in a study to determine whether exercising regularly reduces the risk of diabetes in adults.
14. An online video streaming service mails a questionnaire to randomly selected people across the country to determine whether they prefer streaming movies or sports.

18	تصنيف وتحديد أنواع الدراسات		
	Distinguish among sample surveys, experiments, and observational studies.	Example 4 & (11-14)	373 & 376

solution method

Learn Types of Studies

In a **survey**, data are collected from responses given by members of a group regarding their characteristics, behaviors, or opinions.

In an **experiment**, the sample is divided into two randomly selected groups, the *experimental group* and the *control group*. The effect on the experimental group is then compared to the control group.

In an **observational study**, members of a sample are measured or observed without being affected by the study.

To determine when to use each type of study, think about how the data will be obtained and how the study will affect the participants. Each method of study relies on random selection of a sample to ensure that it accurately represents the population.

Example 4 Classify Study Types

Example 4 Classify Study Types

UNIFORMS A research team wants to test new football uniform designs and their appeal to young adults. They randomly select 100 young adults to view the different uniforms. The research team observes and records the reactions to the uniforms.

- Step 1 What is the purpose of the study?
- The purpose is to determine if the new uniforms will be appealing to young adults.
- Step 2 Does this situation represent a *survey*, an *experiment*, or an *observational study*?
- This is a(n) observational study because the participants are observed without being affected by the study.
- Step 3 Identify the sample and population.
- The sample is the 100 young adults involved in the study. The population is all young adults.

- Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.
11. An Internet service provider conducts an online study in which customers are randomly selected and asked to provide feedback on their customer service.

SOLUTION:

The purpose of the study is to get feedback on the internet service providers customer service. This is a survey because data are collected from responses given by members of a group regarding their characteristics, behaviors, or opinions. The sample is customers that take the online survey. The population is all customers of the internet service provider.

18	تصنيف وتحديد أنواع الدراسات		
	Distinguish among sample surveys, experiments, and observational studies.	Example 4 & (11-14)	373 & 376

12. A research group randomly selects 100 business owners, half of whom started their own businesses, and compares their success.

SOLUTION:

The purpose of the study is to compare the success of business owners. This is an observational study because the participants are observed without being affected by the study. The sample is the 100 business owners selected for the study. The population is all business owners.

13. A research group randomly chooses 50 people to participate in a study to determine whether exercising regularly reduces the risk of diabetes in adults.

SOLUTION:

The purpose of this study is to determine whether regular exercise reduces the risk of diabetes in adults. This is an experiment because the sample can be divided into two randomly selected groups, the experimental group and the control group. The effect on the experimental group can be compared to the control group. The sample is the 50 adults participating in the study. The population is all adults.

14. An online video streaming service mails a questionnaire to randomly selected people across the country to determine whether they prefer streaming movies or sports.

SOLUTION:

The purpose of this study is to determine whether viewers prefer streaming movies or sports. This is a survey because data are collected from responses given by members of a group regarding their characteristics, behaviors, or opinions. The sample is people that receive the questionnaire. The population is all online video streaming viewers.

19	إعداد دراسة إحصائية		
	Classify sampling methods and identify bias in samples and survey questions.	Example3 & (7-10)	372 &375

Example 3 Identify Biased Questions

FOOD A restaurant owner wants to determine which kinds of meals she should add to her menu: vegetarian options or more traditional meat-based dishes. She releases a survey to her customers asking the following question: *Do you prefer a plain salad or a delicious steak?* Identify any bias in the question.

Examples 2 and 3

Identify each sample or question as *biased* or *unbiased*. Explain your reasoning.

7. Do you think that the workout facility needs a new treadmill and racquetball court?

8. Which is your favorite type of music, pop, or country?

9. Are you a member of any after-school clubs?

10. Don't you agree that employees should pack their lunch?

19	إعداد دراسة إحصائية		
	Classify sampling methods and identify bias in samples and survey questions.	Example3 & (7-10)	372 &375

solution method

Learn Randomness and Bias

Statistics is an area of mathematics that deals with collecting, analyzing, and interpreting data.

A characteristic of a population is called a **parameter**. Because it is not always practical to collect data from a large population, it may be collected from a representative sample. A characteristic of a sample is called a **statistic**.

A **population** consists of all the members of a group of interest. A **bias** is an error that results in a misrepresentation of a population.

Key Concept • Types of Samples

simple random	Each member has an equal chance of being selected.
systematic	Members are selected according to a specified interval.
self-selected	Members volunteer to be included in the sample.
convenience	Members that are readily available are selected.
stratified	Members are selected from similar, nonoverlapping groups.

Example 3 Identify Biased Questions

Example 3 Identify Biased Questions

FOOD A restaurant owner wants to determine which kinds of meals she should add to her menu: vegetarian options or more traditional meat-based dishes. She releases a survey to her customers asking the following question: *Do you prefer a plain salad or a delicious steak?* Identify any bias in the question.

Step 1 Identify the purpose of the question. The purpose is to determine which dishes are most popular with customers.

Step 2 Identify the bias in the sample. The question provides descriptions of each kind of food, and the description for the steak favors it over the salad.

19	إعداد دراسة إحصائية		
	Classify sampling methods and identify bias in samples and survey questions.	Example3 & (7-10)	372 &375

7. Do you think that the workout facility needs a new treadmill and racquetball court?

SOLUTION:

biased; Sample answer: The question is asking about two issues: whether the workout facility needs a new treadmill and whether the workout facility needs a new racquetball court.

8. Which is your favorite type of music, pop, or country?

SOLUTION:

biased; Sample answer: The question only gives two options, and thus encourages a certain response.

9. Are you a member of any after-school clubs?

SOLUTION:

unbiased; Sample answer: The question does not influence participants.

10. Don't you agree that employees should pack their lunch?

SOLUTION:

biased; Sample answer: The question encourages a certain response. The phrase "don't you agree" suggests that the people surveyed should agree.

الاسئلة الكتابيه لامتحان



الأسئلة المقالية - FRQ	20	إيجاد المساحة الواقعة تحت منحنى التوزيع الطبيعي Analyze normally distributed variables by using the Empirical Rule.	Key Concept • The Empirical Rule	396
	21	حل المعادلات التربيعية باستخدام القانون العام Solve quadratic equations by using the Quadratic Formula.	(1-6,12,13)	47
	22	قسمة كثيرات الحدود باستخدام القسمة التركيبية Divide polynomials by using synthetic division.	Example 4 & Example5	103 & 104
	23	تحليل كثيرات الحدود إلى العوامل Solve polynomial equations by factoring.	(1-10)	127
	24	حل المعادلات الأسية Solve exponential equations in one variable.	(1-6)	229
	25	(سوف يتم إدراج الجدول ضمن السؤال) إيجاد احتمالات التوزيعات الطبيعية، وإيجاد قيم البيانات عند إعطاء الاحتمالات (a,b) (a,b) Analyze standardized data and distributions by using z-values. (The table will be provided in the question)	Example 8 & 14	400 & 402

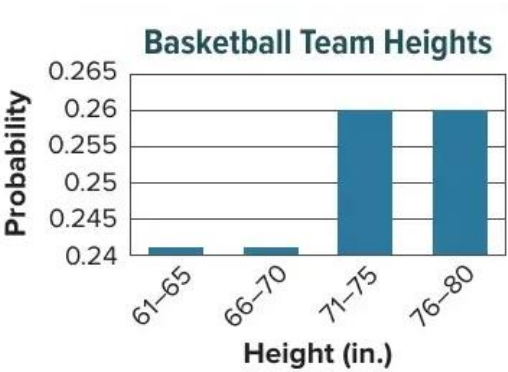
20	إيجاد المساحة الواقعة تحت منحني التوزيع الطبيعي		
	Analyze normally distributed variables by using the Empirical Rule.	Key Concept • The Empirical Rule	396

Examples 2-4

4. **FUNDRAISING** At a fundraising dinner, the underside of 200 plates were randomly tagged with a sticker to indicate winning a cash prize. The frequency table shows the number of winning plates for each prize. Construct a relative frequency table, and graph the probability distribution.

Prize, (X)	Frequency
\$5	150
\$50	40
\$100	9
\$1000	1

5. **BASKETBALL** An athletic director made a probability distribution of the heights of her team’s basketball players, and distributed a flyer that claimed that the majority of the players on the basketball team are 71 inches or taller. Identify any flaws in the representation of the probability distribution.



20	إيجاد المساحة الواقعة تحت منحنى التوزيع الطبيعي		
	Analyze normally distributed variables by using the Empirical Rule.	Key Concept • The Empirical Rule	396

6. **TRACK** The preliminary times for a 110-meter hurdles race are shown. Create a histogram of the set of data. Determine whether the data can be approximated with a normal distribution.

Times (seconds)
14.75, 14.77, 14.31, 14.83, 14.84, 14.35, 14.69, 14.63, 14.74, 14.82, 14.25, 14.93

Learn Normal Distributions

The **normal distribution** is the most common continuous probability distribution of a random variable. A normal distribution occurs as a sample size increases due to the Law of Large Numbers.

Key Concept • Law of Large Numbers

As the number of trials increases, the experimental probability approaches the theoretical probability.

Key Concept • The Normal Distribution

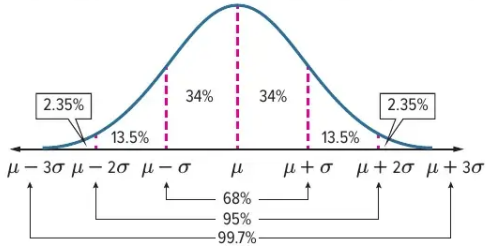
- The graph of a normal distribution is continuous, bell-shaped, and symmetric with respect to the mean.
- The mean, median, and mode are equal and located at the center.
- The curve approaches, but never touches, the x-axis.
- The total area under the curve is equal to 1, or 100%.

The area under the normal curve represents the amount of data within a certain interval, which can be used to determine the probability that a random data value falls within that interval. This area is defined in relation to the mean and standard deviation of the data.

When a set of data is normally distributed, or approximately normal, the Empirical Rule can be used.

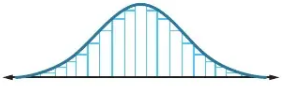
Key Concept • The Empirical Rule

In a normal distribution with mean μ and standard deviation σ ,

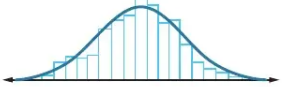


- approximately 68% of the data fall within 1σ of the mean,
- approximately 95% of the data fall within 2σ of the mean, and
- approximately 99.7% of the data fall within 3σ of the mean.

When a set of data is *not* approximately normal, it cannot be represented by the Empirical Rule. Skewed data is one example of a set of data that is not approximately normal.



The data are normally distributed, symmetric about the mean, and bell-shaped.



The data are approximately normally distributed. The data can be modeled by the normal distribution.



The data are skewed to the left. A normal curve would not be the best curve to model the distribution.

20	إيجاد المساحة الواقعة تحت منحنى التوزيع الطبيعي		
	Analyze normally distributed variables by using the Empirical Rule.	Key Concept • The Empirical Rule	396

solution method

4. **FUNDRAISING** At a fundraising dinner, the underside of 200 plates were randomly tagged with a sticker to indicate winning a cash prize. The frequency table shows the number of winning plates for each prize. Construct a relative frequency table, and graph the probability distribution.

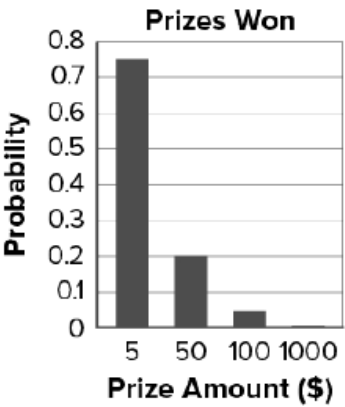
Prize, (X)	Frequency
\$5	150
\$50	40
\$100	9
\$1000	1

SOLUTION:

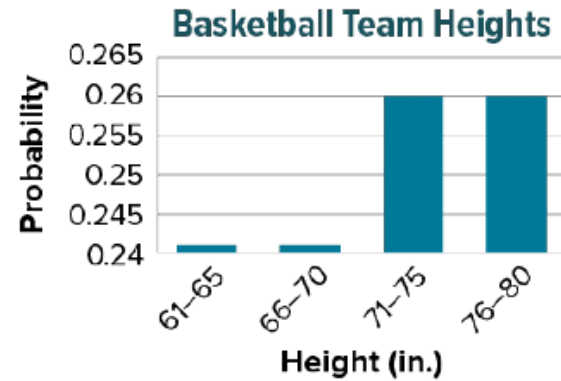
The relative frequency table converts the frequencies to probabilities. Divide each frequency by the total number of

Prize (X)	Frequency	Relative Frequency
\$5	150	$150 \div 200 = 0.75$
\$50	40	$40 \div 200 = 0.2$
\$100	9	$9 \div 200 = 0.045$
\$1000	1	$1 \div 200 = 0.005$

The bars are separated on the graph because the distribution is discrete. Each unique outcome is indicated on the horizontal axis, and the probability of each outcome occurring is indicated on the vertical axis.



5. **BASKETBALL** An athletic director made a probability distribution of the heights of her team's basketball players taller. Identify any flaws in the representation of the probability distribution.



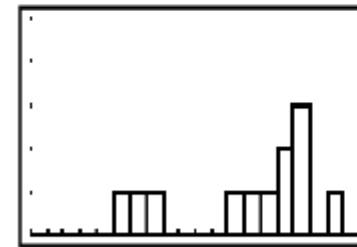
SOLUTION:

The scale of the y-axis misleadingly shows the differences in probabilities. The scale makes it appear that many mor

6. **TRACK** The preliminary times for a 110-meter hurdles race are shown. Create a histogram of the set of data. Determine whether the data can be approximated with a normal distribution.

Times (seconds)
14.75, 14.77, 14.31, 14.83, 14.84, 14.35, 14.69, 14.63, 14.74, 14.82, 14.25, 14.93

SOLUTION:



[14, 15] scl: 0.05 by [0, 5] scl: 1

The data set is skewed. Thus, the data cannot be approximated with a normal distribution.

21	حل المعادلات التربيعية باستخدام القانون العام		
	Solve quadratic equations by using the Quadratic Formula.	(1-6,12,13)	47

Example 1

Solve each equation by using the Quadratic Formula.

1. $x^2 + 8x + 15 = 0$

2. $x^2 - 18x + 72 = 0$

3. $12x^2 - 22x + 6 = 0$

4. $4x^2 - 6x = -2$

5. $x^2 + 8x + 5 = 0$

6. $-8x^2 + 4x = -5$

21	حل المعادلات التربيعية باستخدام القانون العام		
	Solve quadratic equations by using the Quadratic Formula.	(1-6,12,13)	47

12. $x^2 + x - 8 = 0$

13. $8x^2 + 5x - 1 = 0$

Learn Using the Quadratic Formula

To solve any quadratic equation, you can use the Quadratic Formula.

Key Concept • Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

 **Go Online** You can see how the Quadratic Formula is derived.

Example 1 Real Roots, c Is Positive

CONTEST At the World Championship Punkin Chunkin contest in Bridgeville, Delaware, pumpkins are launched hundreds of yards. The path of a pumpkin can be modeled by $h = -4.9t^2 + 11.7t + 42$, where h is the height and t is the number of seconds after launch.

Part A Use the Quadratic Formula to solve $0 = -4.9t^2 + 11.7t + 42$.

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\ &= \frac{-11.7 \pm \sqrt{(11.7)^2 - 4(-4.9)(42)}}{2(-4.9)} && a = -4.9, b = 11.7, c = 42 \\ &= \frac{-11.7 \pm \sqrt{136.89 + 823.2}}{-9.8} && \text{Square and multiply.} \\ &= \frac{-11.7 \pm \sqrt{960.09}}{-9.8} && \text{Add.} \\ &= \frac{11.7 + \sqrt{960.09}}{9.8} \text{ or } \frac{11.7 - \sqrt{960.09}}{9.8} && \text{Multiply by } \frac{-1}{-1}. \end{aligned}$$

The approximate solutions are 4.4 seconds and -2.0 seconds.

Part B Interpret the roots.

The negative root does not make sense in this context because the pumpkin launches at 0 seconds. The pumpkin lands after 4.4 seconds.

 **Example 1** Real Roots, c Is Positive

Example 2 Real Roots, c Is Negative

Example 3 Complex Roots

Solve each equation by using the Quadratic Formula.

1. $x^2 + 8x + 15 = 0$

SOLUTION:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\ &= \frac{-8 \pm \sqrt{(8)^2 - 4(1)(15)}}{2(1)} && a = 1, b = 8, c = 15 \\ &= \frac{-8 \pm \sqrt{64 - 60}}{2} && \text{Square and multiply.} \\ &= \frac{-8 \pm \sqrt{4}}{2} && \text{Subtract.} \\ &= \frac{-8 \pm 2}{2} && \sqrt{4} = 2 \\ &= -3 \text{ or } -5 && \text{Simplify.} \end{aligned}$$

2. $x^2 - 18x + 72 = 0$

SOLUTION:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\ &= \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(72)}}{2(1)} && a = 1, b = -18, c = 72 \\ &= \frac{18 \pm \sqrt{324 - 288}}{2} && \text{Square and multiply.} \\ &= \frac{18 \pm \sqrt{36}}{2} && \text{Subtract.} \\ &= \frac{18 \pm 6}{2} && \sqrt{36} = 6 \\ &= 12 \text{ or } 6 && \text{Simplify.} \end{aligned}$$

3. $12x^2 - 22x + 6 = 0$

SOLUTION:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\
 &= \frac{-(-22) \pm \sqrt{(-22)^2 - 4(12)(6)}}{2(12)} && a = 12, b = -22, c = 6 \\
 &= \frac{22 \pm \sqrt{484 - 288}}{24} && \text{Square and multiply.} \\
 &= \frac{22 \pm \sqrt{196}}{24} && \text{Subtract.} \\
 &= \frac{22 \pm 14}{24} && \sqrt{196} = 14 \\
 &= \frac{3}{2} \text{ or } \frac{1}{3} && \text{Simplify.}
 \end{aligned}$$

4. $4x^2 - 6x = -2$

SOLUTION:

Set the equation equal to 0.

$$\begin{aligned}
 4x^2 - 6x &= -2 && \text{Original equation} \\
 4x^2 - 6x + 2 &= 0 && \text{Add 2 to each side.}
 \end{aligned}$$

Use the Quadratic Formula.

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\
 &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(2)}}{2(4)} && a = 4, b = -6, c = 2 \\
 &= \frac{6 \pm \sqrt{36 - 32}}{8} && \text{Square and multiply.} \\
 &= \frac{6 \pm \sqrt{4}}{8} && \text{Subtract.} \\
 &= \frac{6 \pm 2}{8} && \sqrt{4} = 2 \\
 &= 1 \text{ or } \frac{1}{2} && \text{Simplify.}
 \end{aligned}$$

5. $x^2 + 8x + 5 = 0$

SOLUTION:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\
 &= \frac{-8 \pm \sqrt{(8)^2 - 4(1)(5)}}{2(1)} && a = 1, b = 8, c = 5 \\
 &= \frac{-8 \pm \sqrt{64 - 20}}{2} && \text{Square and multiply.} \\
 &= \frac{-8 \pm \sqrt{44}}{2} && \text{Subtract.} \\
 &= \frac{-8 \pm \sqrt{4} \cdot \sqrt{11}}{2} && \text{Product Property of Square Roots} \\
 &= \frac{-8 \pm 2\sqrt{11}}{2} && \sqrt{4} = 2 \\
 &= -4 \pm \sqrt{11} && \text{Divide the numerator and denominator by 2.}
 \end{aligned}$$

6. $-8x^2 + 4x = -5$

SOLUTION:

Set the equation equal to 0.

$$\begin{aligned}
 -8x^2 + 4x &= -5 && \text{Original equation} \\
 -8x^2 + 4x + 5 &= 0 && \text{Add 5 to each side.}
 \end{aligned}$$

Use the Quadratic Formula.

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\
 &= \frac{-4 \pm \sqrt{(4)^2 - 4(-8)(5)}}{2(-8)} && a = -8, b = 4, c = 5 \\
 &= \frac{-4 \pm \sqrt{16 + 160}}{-16} && \text{Square and multiply.} \\
 &= \frac{-4 \pm \sqrt{176}}{-16} && \text{Add.} \\
 &= \frac{-4 \pm \sqrt{16} \cdot \sqrt{11}}{-16} && \text{Product Property of Square Roots} \\
 &= \frac{-4 \pm 4\sqrt{11}}{-16} && \sqrt{16} = 4 \\
 &= \frac{1 \pm \sqrt{11}}{4} && \text{Divide the numerator and denominator by } -4.
 \end{aligned}$$

21	حل المعادلات التربيعية باستخدام القانون العام		
	Solve quadratic equations by using the Quadratic Formula.	(1-6,12,13)	47

12. $x^2 + x - 8 = 0$

SOLUTION:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\
 &= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-8)}}{2(1)} && a = 1, b = 1, c = -8 \\
 &= \frac{-1 \pm \sqrt{33}}{2} && \text{Simplify.}
 \end{aligned}$$

13. $8x^2 + 5x - 1 = 0$

SOLUTION:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\
 &= \frac{-5 \pm \sqrt{(5)^2 - 4(8)(-1)}}{2(8)} && a = 8, b = 5, c = -1 \\
 &= \frac{-1 \pm \sqrt{57}}{16} && \text{Simplify.}
 \end{aligned}$$

22	قسمة كثيرات الحدود باستخدام القسمة التركيبية		
	Divide polynomials by using synthetic division.	Example 4 & Example5	103 & 104

Example 4 Use Synthetic Division

Find $(3x^3 - 2x^2 - 53x - 60) \div (x + 3)$.

Example 5 Divisor with a Coefficient Other Than 1

Find $\frac{4x^4 - 37x^2 + 4x + 9}{2x - 1}$.

solution method

Learn Dividing Polynomials by Using Synthetic Division

Synthetic division is an alternate method used to divide a polynomial by a binomial of degree 1. You may find this to be a quicker, simpler method.

Key Concept • Synthetic Division

Step 1 After writing a polynomial in standard form, write the coefficients of the dividend. If the dividend is missing a term, use 0 as a placeholder. Write the constant a of the divisor $x - a$ in the box. Bring the first coefficient down.

Step 2 Multiply the number just written in the bottom row by a , and write the product under the next coefficient.

Step 3 Add the product and the coefficient above it.

Step 4 Repeat **Steps 2** and **3** until you reach a sum in the last column.

Step 5 Write the quotient. The numbers along the bottom row are the coefficients of the quotient. The power of the first term is one less than the degree of the dividend. The final number is the remainder.

Example 4 Use Synthetic Division**Example 5** Divisor with a Coefficient Other Than 1**Example 4** Use Synthetic Division

Find $(3x^3 - 2x^2 - 53x - 60) \div (x + 3)$.

Step 1 Write the coefficients of the dividend and write the constant a in the box. Because $x + 3 = x - (-3)$, $a = -3$. Then bring the first coefficient down.

$$\begin{array}{r|rrrrr} -3 & 3 & -2 & -53 & -60 & \\ & \downarrow & & & & \\ & 3 & & & & \end{array}$$

Step 2 Multiply by a and write the product. The product of the coefficient and a is $3(-3) = -9$.

$$\begin{array}{r|rrrrr} -3 & 3 & -2 & -53 & -60 & \\ & & -9 & & & \\ & 3 & & & & \end{array}$$

Step 3 Add the product and the coefficient.

$$\begin{array}{r|rrrrr} -3 & 3 & -2 & -53 & -60 & \\ & & -9 & & & \\ & 3 & -11 & & & \end{array}$$

Step 4 Repeat **Steps 2** and **3** until you reach a sum in the last column.

$$\begin{array}{r|rrrrr} -3 & 3 & -2 & -53 & -60 & \\ & & -9 & 33 & 60 & \\ & 3 & -11 & -20 & 0 & \end{array}$$

Step 5 Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2. The final sum in the synthetic division is 0, so the remainder is 0.

The quotient is $3x^2 - 11x - 20$.

Example 5 Divisor with a Coefficient Other Than 1

Find $\frac{4x^4 - 37x^2 + 4x + 9}{2x - 1}$.

To use synthetic division, the lead coefficient of the divisor must be 1.

$$\frac{(4x^4 - 37x^2 + 4x + 9) \div 2}{(2x - 1) \div 2}$$

Divide the numerator and denominator by 2.

$$= \frac{2x^4 - \frac{37}{2}x^2 + 2x + \frac{9}{2}}{x - \frac{1}{2}}$$

Simplify the numerator and denominator.

$$x - a = x - \frac{1}{2}, \text{ so } a = \frac{1}{2}.$$

Complete the synthetic division.

$\frac{1}{2}$	2	0	$-\frac{37}{2}$	2	$\frac{9}{2}$
		1	$\frac{1}{2}$	-9	$-\frac{7}{2}$
	2	1	-18	-7	1

The resulting expression is $2x^3 + x^2 - 18x - 7 + \frac{1}{x - \frac{1}{2}}$. Now simplify the fraction.

$$\frac{1}{x - \frac{1}{2}} = \frac{(1)^2}{\left(x - \frac{1}{2}\right) \cdot 2}$$

Multiply the numerator and denominator by 2.

$$= \frac{2}{2x - 1}$$

Simplify.

The solution is $2x^3 + x^2 - 18x - 7 + \frac{2}{2x - 1}$.

You can check your answer by using long division.

23	تحليل كثيرات الحدود إلى العوامل		
	Solve polynomial equations by factoring.	(1-10)	127

Examples 1-3

Factor completely. If the polynomial is not factorable, write *prime*.

1. $8c^3 - 27d^3$

2. $64x^4 + xy^3$

3. $a^8 - a^2b^6$
4. $x^6y^3 + y^9$

5. $18x^6 + 5y^6$

6. $w^3 - 2y^3$

23	تحليل كثيرات الحدود إلى العوامل		
	Solve polynomial equations by factoring.	(1-10)	127
24	حل المعادلات الأسية		
	Solve exponential equations in one variable.	(1-10)	127

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$

8. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

9. $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

solution method

23	تحليل كثيرات الحدود إلى العوامل		
	Solve polynomial equations by factoring.	(1-10)	127
24	حل المعادلات الأسية		
	Solve exponential equations in one variable.	(1-10)	127

Learn Solving Polynomial Equations by Factoring

Like quadratics, some polynomials of higher degrees can be factored. A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**. Like a prime real number, the only factors of a prime polynomial are 1 and itself.

Similar to quadratics, some cubic polynomials can be factored by using polynomial identities.

Key Concept • Sum and Difference of Cubes

Factoring Technique	General Case
Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Polynomials can be factored by using a variety of methods, the most common of which are summarized in the table below. When factoring a polynomial, always look for a common factor first to simplify the expression. Then, determine whether the resulting polynomial factors can be factored using one or more methods.

Concept Summary • Factoring Techniques

Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$2a^4b^3 + 6ab = 2ab(a^3b^2 + 6)$
two	Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
	Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
	Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd$ $= (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Example 1 Factor Sums and Differences of Cubes

Example 2 Factor by Grouping

Example 3 Combine Cubes and Squares

Factor completely. If the polynomial is not factorable, write *prime*.

1. $8c^3 - 27d^3$

SOLUTION:

$$8c^3 - 27d^3$$
$$= (2c)^3 - (3d)^3$$
$$= (2c - 3d)[(2c)^2 + 2c(3d) + (3d)^2]$$
$$= (2c - 3d)(4c^2 + 6cd + 9d^2)$$

Original expression

$(2c)^3 = 8c^3$; $(3d)^3 = 27d^3$

Difference of two cubes

Simplify.

2. $64x^4 + xy^3$

SOLUTION:

$$64x^4 + xy^3$$
$$= x(64x^3 + y^3)$$
$$= x[(4x)^3 + (y)^3]$$
$$= x(4x + y)[(4x)^2 - 4x(y) + (y)^2]$$
$$= x(4x + y)(16x^2 - 4xy + y^2)$$

Original expression

Factor out the GCF.

$(4x)^3 = 64x^3$; $(y)^3 = y^3$

Sum of two cubes

Simplify.

3. $a^8 - a^2b^6$

SOLUTION:

$$a^8 - a^2b^6$$
$$= a^2(a^6 - b^6)$$
$$= a^2[(a^3)^2 - (b^3)^2]$$
$$= a^2(a^3 - b^3)(a^3 + b^3)$$
$$= a^2[(a)^3 - (b)^3][(a)^3 + (b)^3]$$
$$= a^2(a - b)[(a)^2 + a(b) + (b)^2](a + b)[(a)^2 - a(b) + (b)^2]$$
$$= a^2(a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)$$

Original expression

Factor out the GCF.

$(a^3)^2 = a^6$; $(b^3)^2 = b^6$

Difference of two squares

$(a)^3 = a^3$; $(b)^3 = b^3$

Sum and difference of two cubes

Simplify.

23	تحليل كثيرات الحدود إلى العوامل		
	Solve polynomial equations by factoring.	(1-10)	127

4. $x^6y^3 + y^9$

SOLUTION:

$x^6y^3 + y^9$	Original expression
$= y^3(x^6 + y^6)$	Factor out the GCF.
$= y^3[(x^2)^3 + (y^2)^3]$	$(x^2)^3 = x^6$; $(y^2)^3 = y^6$
$= y^3(x^2 + y^2)[(x^2)^2 - x^2(y^2) + (y^2)^2]$	Sum of two cubes
$= y^3(x^2 + y^2)(x^4 - x^2y^2 + y^4)$	Simplify.

5. $18x^6 + 5y^6$

SOLUTION:

There is not an integer, that when raised to the third power is equal to 18. There is not an integer, that when raised to the third power is equal to 5. Therefore, the polynomial $18x^6 + 5y^6$ is prime.

6. $w^3 - 2y^3$

SOLUTION:

There is not an integer, that when raised to the third power is equal to 2. Therefore, the polynomial $w^3 - 2y^3$ is prime.

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$

SOLUTION:

$gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$	Original expression
$= (6fx^2 + gx^2 - 3hx^2) + (-6fy^2 - gy^2 + 3hy^2)$	Group to find a GCF.
$= x^2(6f + g - 3h) - y^2(6f + g - 3h)$	Factor out the GCF.
$= (x^2 - y^2)(6f + g - 3h)$	Distributive Property
$= [(x)^2 - (y)^2](6f + g - 3h)$	$(x)^2 = x^2$; $(y)^2 = y^2$
$= (x + y)(x - y)(6f + g - 3h)$	Difference of two squares

8. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

SOLUTION:

$12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$	Original expression
$= (12ax^2 - 18bx^2 + 24cx^2) + (-10ay^2 + 15by^2 - 20cy^2)$	Group to find a GCF.
$= 6x^2(2a - 3b + 4c) - 5y^2(2a - 3b + 4c)$	Factor out the GCF.
$= (6x^2 - 5y^2)(2a - 3b + 4c)$	Distributive Property

9. $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

SOLUTION:

$a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$	Original expression
$= (a^3x^2 - 16a^3x + 64a^3) + (-b^3x^2 + 16b^3x - 64b^3)$	Group to find a GCF.
$= a^3(x^2 - 16x + 64) - b^3(x^2 - 16x + 64)$	Factor out the GCF.
$= (a^3 - b^3)(x^2 - 16x + 64)$	Distributive Property
$= [(a)^3 - (b)^3](x^2 - 16x + 64)$	$(x)^3 = x^3$; $(y)^3 = y^3$
$= (a - b)[(a)^2 + a(b) + (b)^2](x^2 - 16x + 64)$	Difference of two cubes
$= (a - b)(a^2 + ab + b^2)(x^2 - 16x + 64)$	Simplify.
$= (a - b)(a^2 + ab + b^2)(x - 8)^2$	Perfect square trinomial

10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

SOLUTION:

$8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$	Original expression
$= (8x^5 + 80x^4 + 200x^3) + (-x^2y^3 - 10xy^3 - 25y^3)$	Group to find a GCF.
$= 8x^3(x^2 + 10x + 25) - y^3(x^2 + 10x + 25)$	Factor out the GCF.
$= (8x^3 - y^3)(x^2 + 10x + 25)$	Distributive Property
$= [(2x)^3 - (y)^3](x^2 + 10x + 25)$	$(2x)^3 = 8x^3$; $(y)^3 = y^3$
$= (2x - y)[(2x)^2 + 2x(y) + (y)^2](x^2 + 10x + 25)$	Difference of two cubes
$= (2x - y)(4x^2 + 2xy + y^2)(x^2 + 10x + 25)$	Simplify.
$= (2x - y)(4x^2 + 2xy + y^2)(x + 5)^2$	Perfect square trinomial

25	(a,b) إيجاد احتمالات التوزيعات الطبيعية، وإيجاد قيم البيانات عند إعطاء الاحتمالات (سوف يتم إدراج الجدول ضمن السؤال)		
	(a,b) Analyze standardized data and distributions by using z-values. (The table will be provided in the question)	Example 8 & 14	400 & 402

Example 8 Find Area Under the Standard Normal Curve by Using a Calculator

INTERNET TRAFFIC The number of daily hits to a local news Web site is normally distributed with $\mu = 98,452$ hits and $\sigma = 10,325$ hits. Find the probability that the Web site will get at least 100,000 hits on a given day, $P(X > 100,000)$.



25	(a,b) إيجاد احتمالات التوزيعات الطبيعية، وإيجاد قيم البيانات عند إعطاء الاحتمالات (a,b) (سوف يتم إدراج الجدول ضمن السؤال)		
	(a,b) Analyze standardized data and distributions by using z-values. (The table will be provided in the question)	Example 8 & 14	400 & 402

Example 8

14. TESTING The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.

a. What percent of the scores are between 70 and 80?

b. What percent of the scores are over 115?

c. If 75 people take the test, how many would you expect to score lower than 75?

solution method

Learn The Standard Normal Distribution

The Empirical Rule is only useful for evaluating specific values. Once the data set is *standardized*, any value can be evaluated. Data are standardized by converting them to z-values. The **z-value** of a specific data value is a measure of position, and it represents the number of standard deviations that a given data value is from the mean.

The z-value for a data value X in a set of normally distributed data is $z = \frac{X - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation.

If the distributions are standardized by using z-values, any distribution can be compared to any other distribution.

The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1. You can take any normal distribution and convert it to the standard normal distribution by standardizing it.

The standard normal distribution allows you to determine how much of the data falls between any two z-values, or to the left or right of any z-value. The area under the normal curve between two points corresponds to the proportion of the data that falls in a given interval.

Because the area under the curve is 1, the area under the curve between two points represents the probability that a data value falls in that interval.

The proportion of values within each interval, or the associated probability of a data point falling within that interval, can also be found by using probability tables. For these values, round to the hundredths place.

To use a Standard Normal Distribution Table to find the probability of data being in the interval $[-1.09, 0.69]$, complete the following steps.

Example 8 Find Area Under the Standard Normal Curve by Using a Calculator

Solution

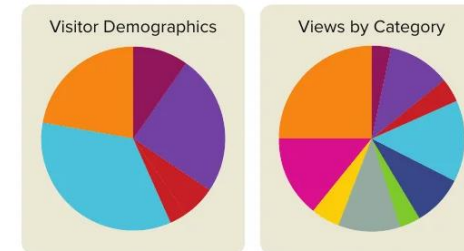
Example 8 Find Area Under the Standard Normal Curve by Using a Calculator

INTERNET TRAFFIC The number of daily hits to a local news Web site is normally distributed with $\mu = 98,452$ hits and $\sigma = 10,325$ hits. Find the probability that the Web site will get at least 100,000 hits on a given day, $P(X > 100,000)$.

Visits Summary



Visits Summary - Last 30 Days



Step 1 Find the corresponding z-value for $X = 100,000$.

$$z = \frac{X - \mu}{\sigma}$$

Formula for z-value

$$z = \frac{100,000 - 98,452}{10,325}$$

$X = 100,000$, $\mu = 98,452$, and $\sigma = 10,325$

$$z \approx 0.150$$

Simplify.

Step 2 Find the probability.

The area under the curve when $z > 4$ is negligible. So $z = 4$ can be used as an upper bound for finding area. You can use a graphing calculator to find the area between $z = 0.150$ and $z = 4$.

Press **2nd** **[DISTR]** and select **normalcdf**. Enter the interval and press **enter** to display the area.

The area is 0.44. Therefore, the probability of the Web site getting at least 100,000 hits in one day is 44.0%.

```
normalcdf(0.150,
4)
.4403506018
```

25	(a,b) إيجاد احتمالات التوزيعات الطبيعية، وإيجاد قيم البيانات عند إعطاء الاحتمالات (سوف يتم إدراج الجدول ضمن السؤال)		
	(a,b) Analyze standardized data and distributions by using z-values. (The table will be provided in the question)	Example 8 & 14	400 & 402

Solution

14. **TESTING** The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.

- What percent of the scores are between 70 and 80?
- What percent of the scores are over 115?
- If 75 people take the test, how many would you expect to score lower than 75?

SOLUTION:

- Find the corresponding z-values for $X = 70$ and $X = 80$.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values.}$$

$$z = \frac{70 - 100}{12.3} \quad X = 70, \mu = 100, \sigma = 12.3$$

$$z = \frac{-30}{12.3} \quad \text{Subtract.}$$

$$z \approx -2.439 \quad \text{Divide.}$$

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values.}$$

$$z = \frac{80 - 100}{12.3} \quad X = 80, \mu = 100, \sigma = 12.3$$

$$z = \frac{-20}{12.3} \quad \text{Subtract.}$$

$$z \approx -1.626 \quad \text{Divide.}$$

You can use a graphing calculator to find the area between $z = -2.44$ and $z = -1.63$. Press **2nd** [DISTR] and select **normalcdf**. Enter the interval and press **ENTER** to display the area. The area is 0.0446. Therefore, the probability of the scores being between 70 and 80 is 4.5%.

- The area under the curve when $z > 4$ is negligible so this can be used as the upper bound. Find the corresponding z-value for $X = 115$.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values.}$$

$$z = \frac{115 - 100}{12.3} \quad X = 115, \mu = 100, \sigma = 12.3$$

$$z = \frac{15}{12.3} \quad \text{Subtract.}$$

$$z \approx 1.220 \quad \text{Divide.}$$

You can use a graphing calculator to find the area between $z = 1.220$ and $z = 4$. Press **2nd** [DISTR] and select **normalcdf**. Enter the interval and press **ENTER** to display the area. The area is 0.1112. Therefore, the probability of the scores being over 115 is 11.1%.

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	(a,b) Analyze standardized data and distributions by using z-values. (The table will be provided in the question)	Example 8 & 14	400 & 402

c. Find the corresponding z-values for $X = 0$ and $X = 75$.

Solution

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values.}$$

$$z = \frac{0 - 100}{12.3} \quad X = 0, \mu = 100, \sigma = 12.3$$

$$z = \frac{-100}{12.3} \quad \text{Subtract.}$$

$$z \approx -8.130 \quad \text{Divide.}$$

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for } z\text{-values.}$$

$$z = \frac{75 - 100}{12.3} \quad X = 75, \mu = 100, \sigma = 12.3$$

$$z = \frac{-25}{12.3} \quad \text{Subtract.}$$

$$z \approx -2.033 \quad \text{Divide.}$$

You can use a graphing calculator to find the area between $z = -8.130$ and $z = -2.033$. Press **2nd [DISTR]** and select **normalcdf**. Enter the interval and press **ENTER** to display the area.

The area is 0.0210. Therefore, the probability of the scores being between 0 and 75 is 2.1%.

Since 75 people took the test, multiply by the probability to predict the number of people that would score lower than 75.

$$75 \times 2.1\% = 1.575 \text{ or } 2 \text{ people.}$$