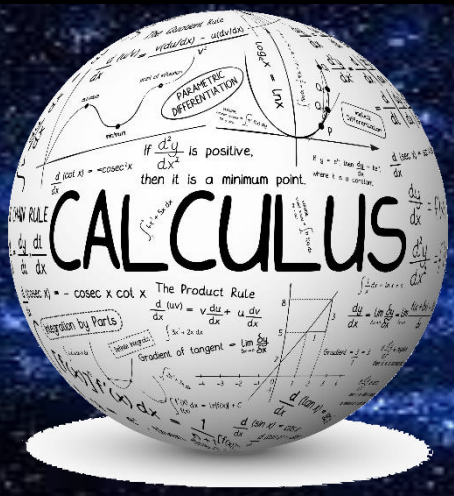
Mr. Ahmed Ata
The Featured Program

12
ADV

12 ADVACED - ENG

2023-2024

Mr. Ahmed Ata



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البرنامج المميز طريقك للتميز

6 - Applications of the Definite Integral

1. Area Between Curves
2. Volume: Slicing, Disks and Washers
3. Volumes by Cylindrical Shells
4. Arc Length and Surface Area
5. Projectile Motion
6. Applications of Integration to Physics and Engineering
7. Probability .

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6-1

Area Between Curves

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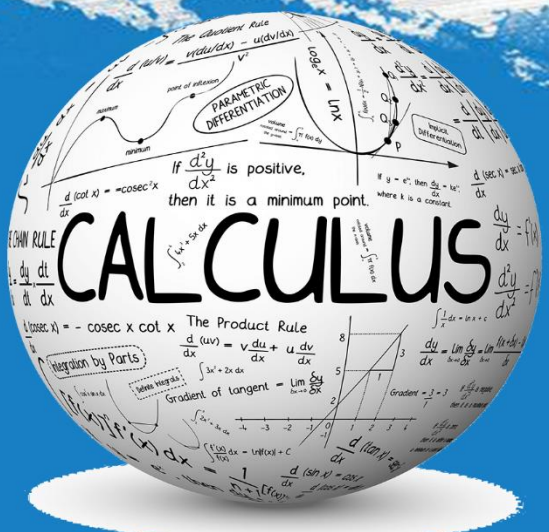

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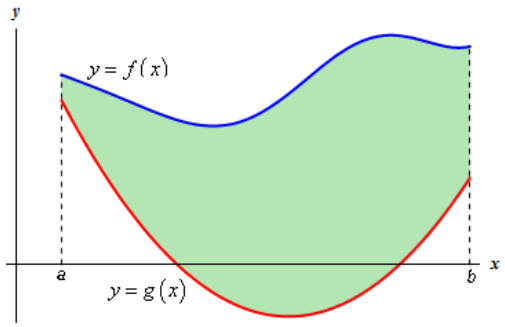
Lesson (6-1)

Area Between Curves

In this section we are going to look at finding the area between two curves. There are actually two cases that we are going to be looking at.

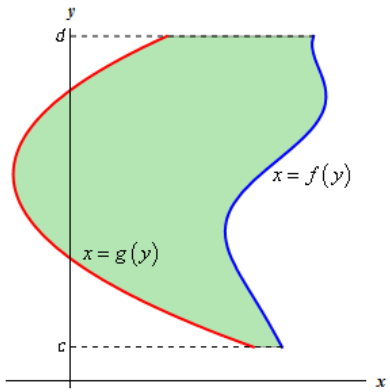
In the first case we want to determine the area between $y = f(x)$ **and** $y = g(x)$ on the interval $[a, b]$ We are also going to assume that $f(x) \geq g(x)$ Take a look at the following sketch to get an idea of what we're initially going to look at.

$$A = \int_a^b f(x) - g(x)\, dx$$



The second case is almost identical to the first case. Here we are going to determine the area between $x = f(y)$ **and** $x = g(y)$ on the interval $[c, d]$ with $f(y) \geq g(y)$

$$A = \int_c^d f(y) - g(y)\, dy$$

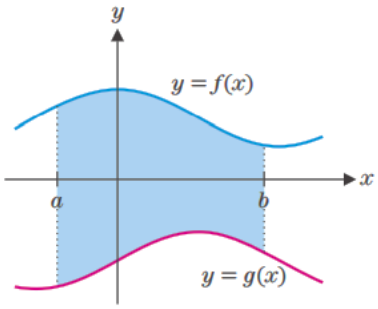


In the first case we will use,

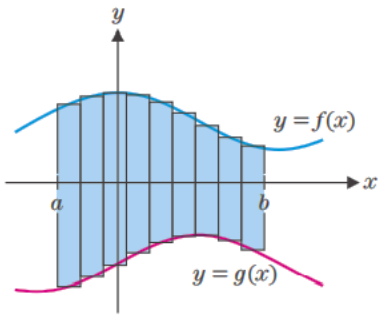
$$A = \int_a^b \left(\begin{matrix}\text{upper}\\ \text{function}\end{matrix}\right) - \left(\begin{matrix}\text{lower}\\ \text{function}\end{matrix}\right) dx, \qquad a \leq x \leq b$$

In the second case we will use,

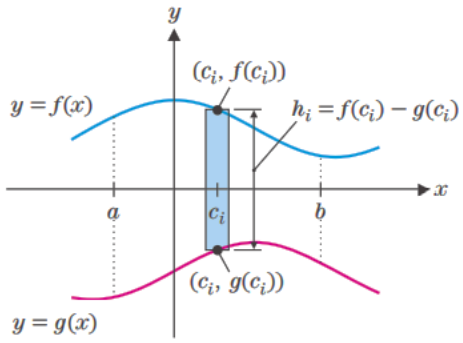
$$A = \int_c^d \left(\begin{matrix}\text{right}\\ \text{function}\end{matrix}\right) - \left(\begin{matrix}\text{left}\\ \text{function}\end{matrix}\right) dy, \qquad c \leq y \leq d$$



Area between two curves



Approximate area

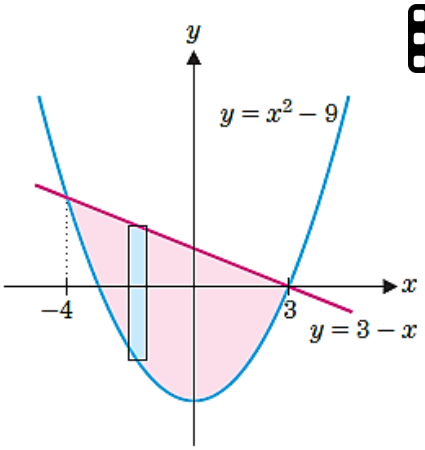


Area of *i*th rectangle

AREA BETWEEN TWO CURVES

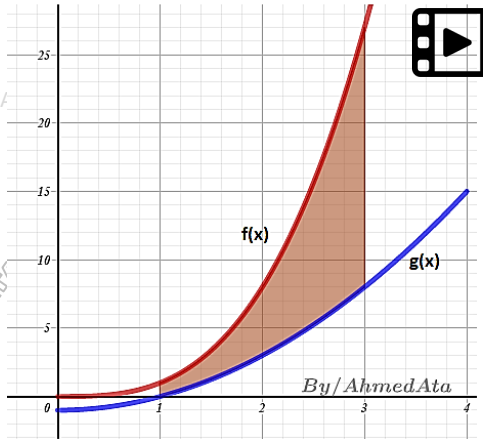
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} [f(c_i) - g(c_i)]\,\Delta x = \int_a^b [f(x) - g(x)]\,dx. \qquad (1.1)$$

1 Find the area bounded by the graphs of $y = 3 - x$ and $y = x^2 - 9$.



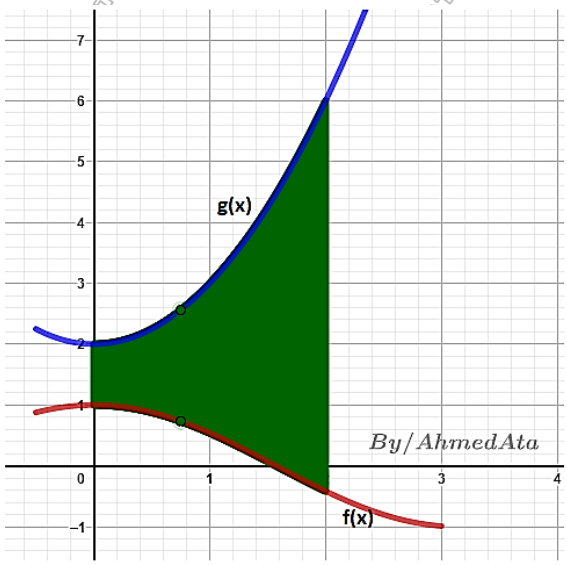
2 Find the area between the curves on the given interval.

$$f(x) = x^3 \text{ and } g(x) = x^2 - 1 \;, 1 \le x \le 3$$



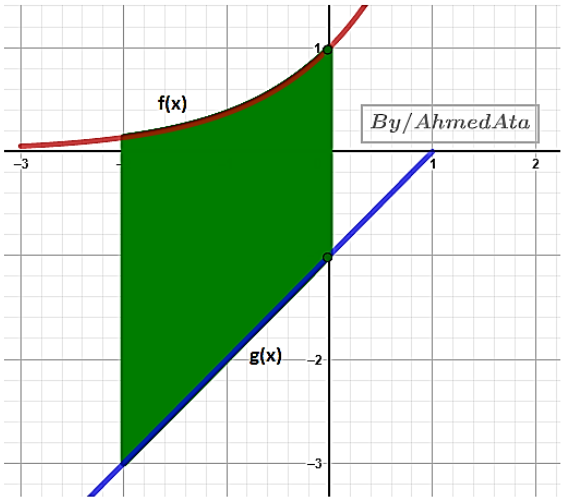
3 Find the area between the curves on the given interval.

$$f(x) = cosx \text{ and } g(x) = x^2 + 2 \quad , 0 \le x \le 2$$



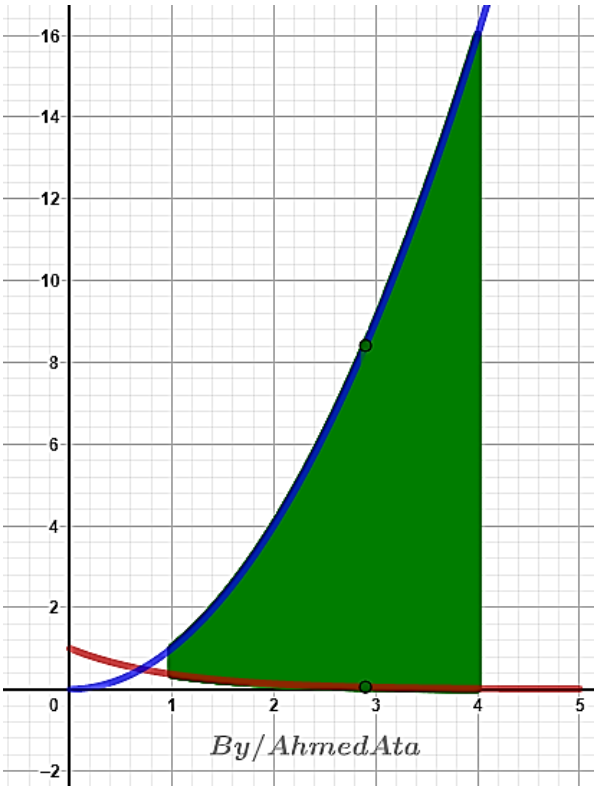
4 Find the area between the curves on the given interval.

$$f(x) = e^x \text{ and } g(x) = x - 1 \quad , -2 \le x \le 0$$



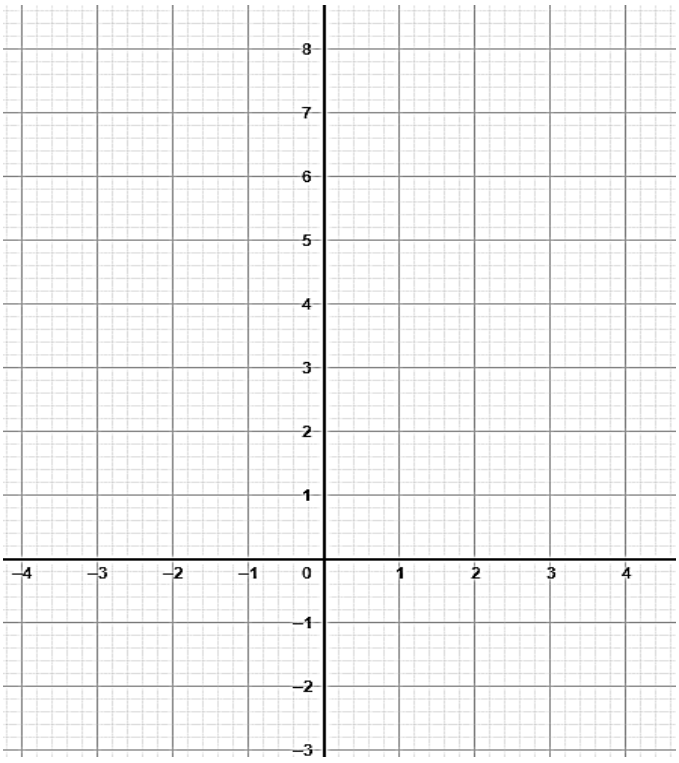
5 Find the area between the curves on the given interval.

$$f(x) = e^{-x} \text{ and } g(x) = x^2 \ , 1 \le x \le 4$$

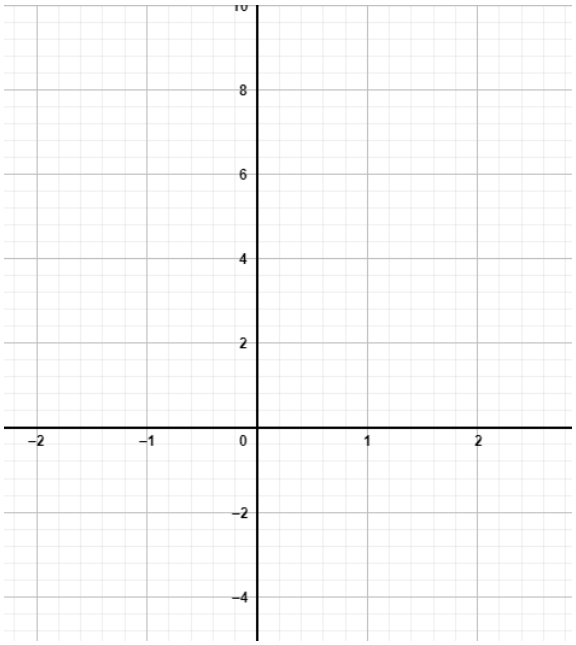


6 Sketch and find the area of the region determined by the intersections of the curves

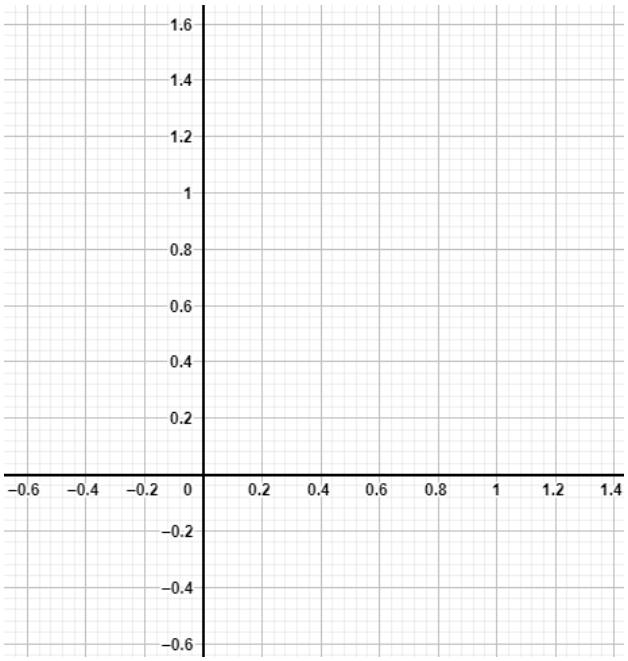
$$y = x^2 - 1, y = 7 - x^2$$



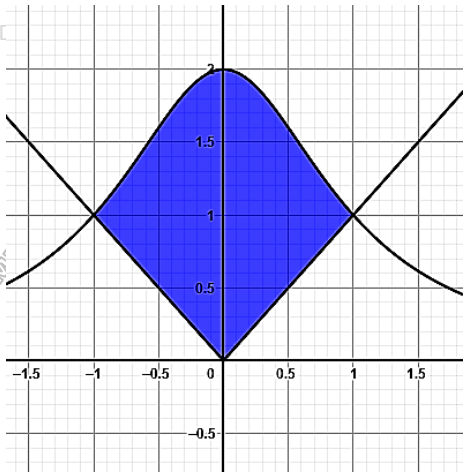
7 $y = x^3, y = 3x + 2$



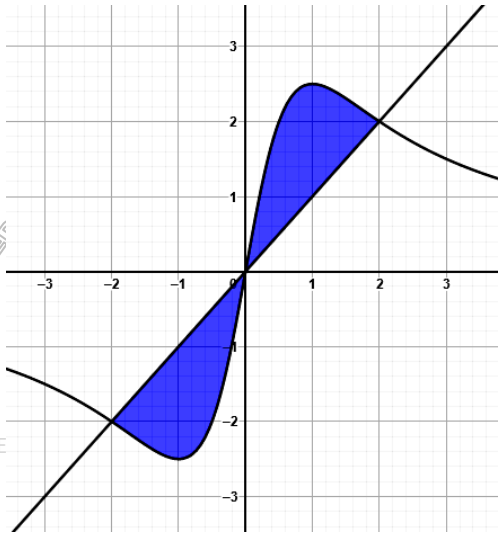
8 $y = \sqrt{x}, y = x^2$



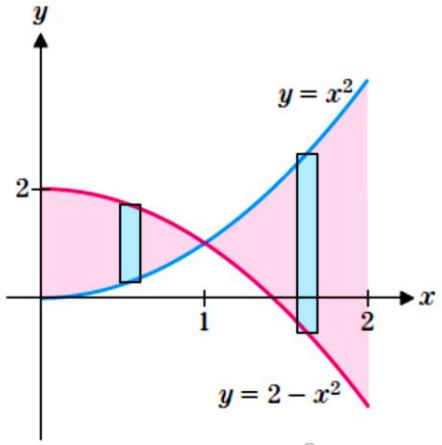
9 $y = \dfrac{2}{x^2+1}, y = |x|$



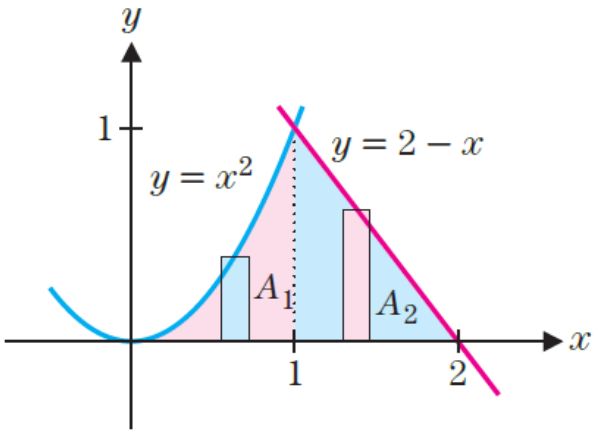
10 $y = \dfrac{5x}{x^2+1}, y = x$



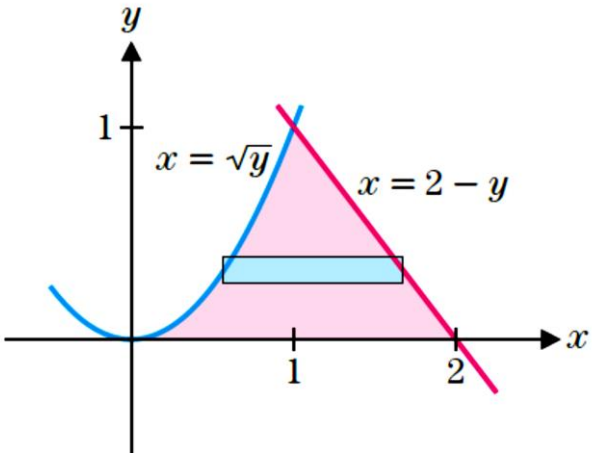
11 Find the area bounded by the graphs of $y = x^2$ and $y = 2 - x^2$ for $0 \le x \le 2$.



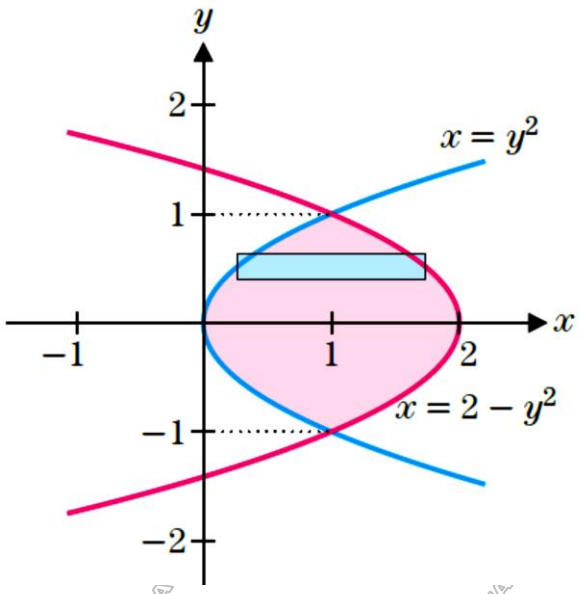
12 Find the area bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$.



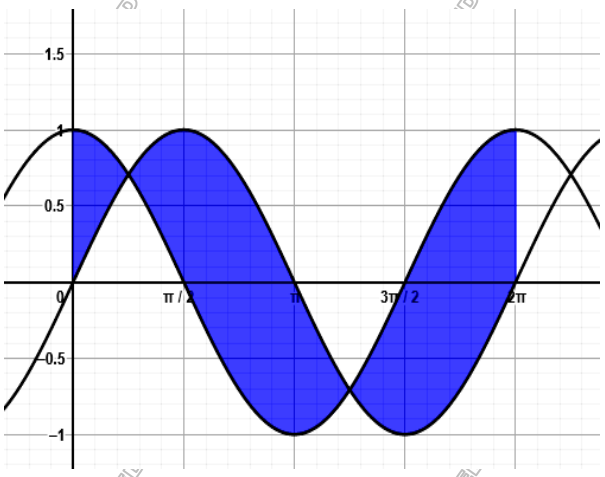
13 Find the area bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$.



14 Find the area bounded by the graphs of $x = y^2$ and $x = 2 - y^2$.

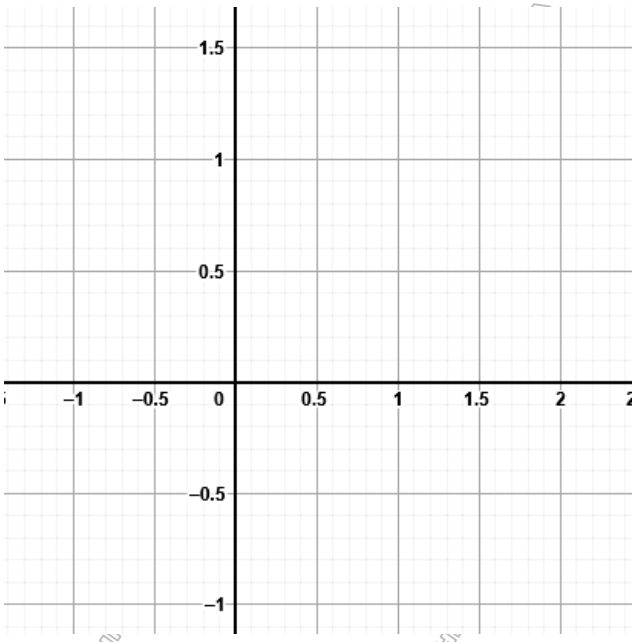


15 $y = \sin x \, (0 \le x \le 2\pi), \; y = \cos x$

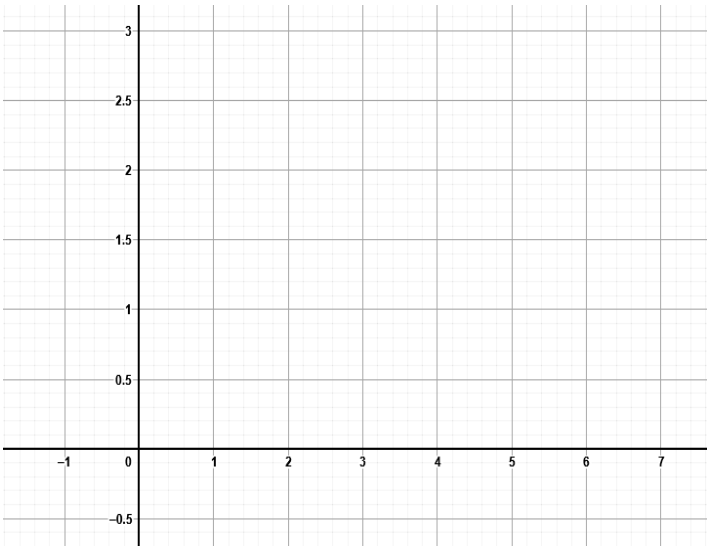


Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

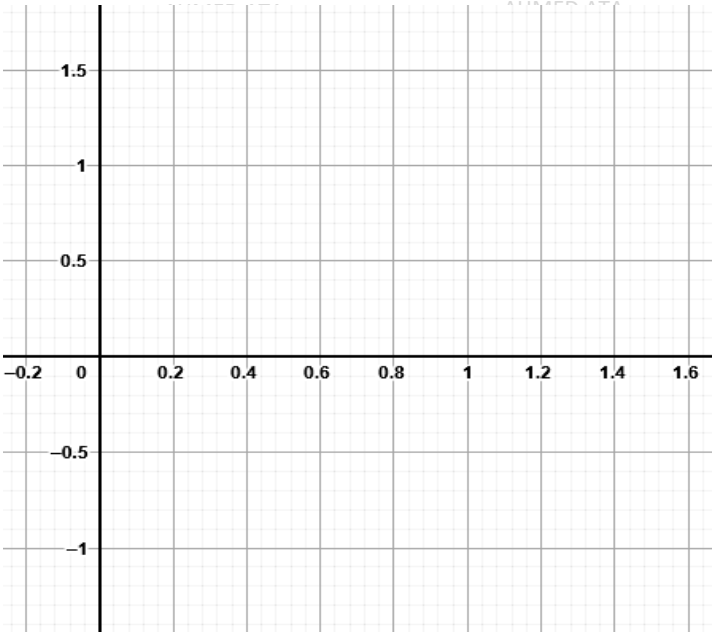
16 $y = x, y = 2 - x, y = 0$



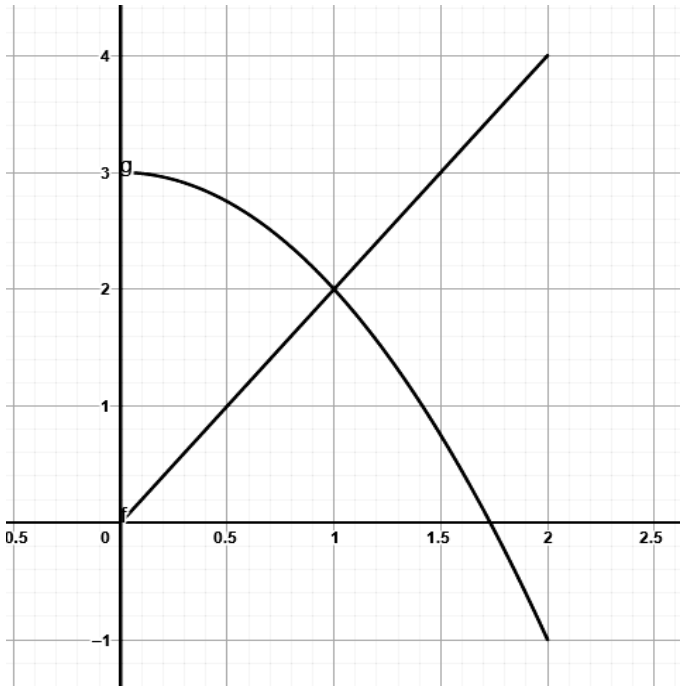
17 $y = x, y = 2, y = 6 - x, y = 0$



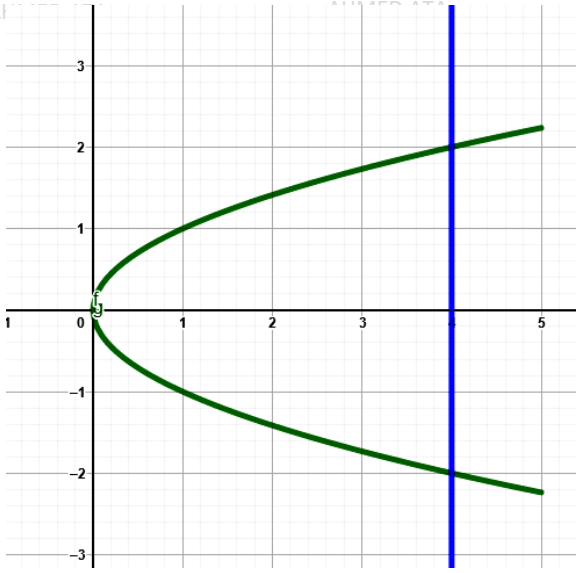
18 $x = y, x = -y, x = 1$



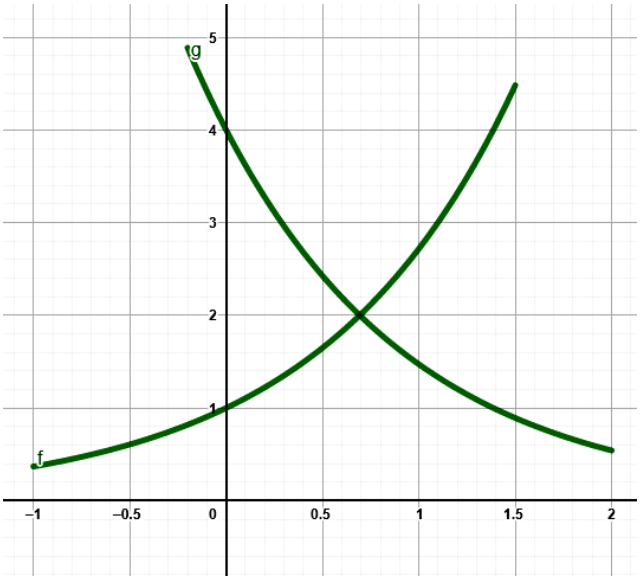
19 $y = 2x\ (x > 0), y = 3 - x^2, x = 0$



20 $x = y^2, x = 4$

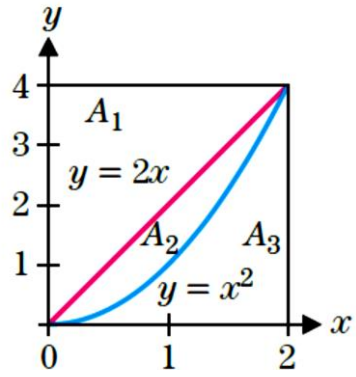


21 $y = e^x, y = 4e^{-x}, x = 0$

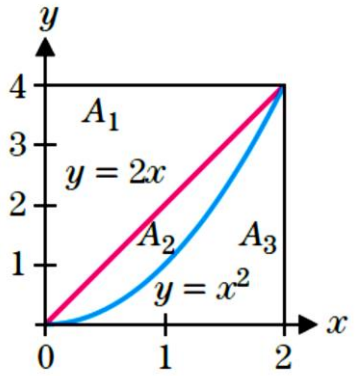


22 **In terms of A1, A2 and A3, identify the area given by each integral**

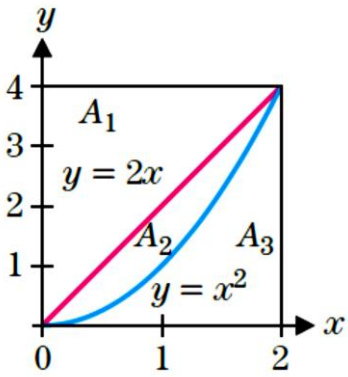
(a) $\int_0^2 (2x - x^2)\,dx$



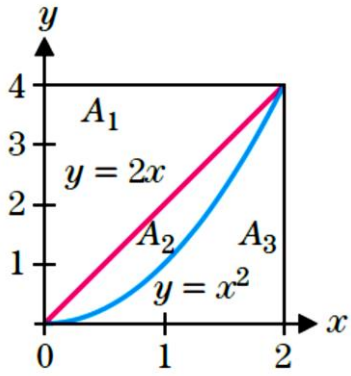
(b) $\int_0^2 (4 - x^2)\,dx$



(c) $\int_0^4 (2 - \sqrt{y})\,dy$



(d) $\int_0^4 (\sqrt{y} - \frac{y}{2})\,dy$



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Volume: Slicing, Disks and Washers

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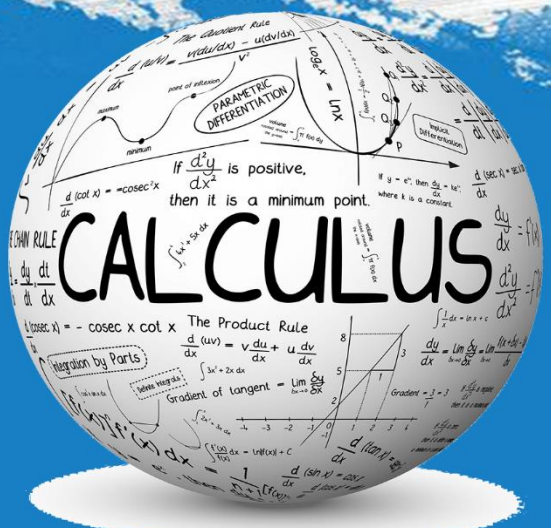

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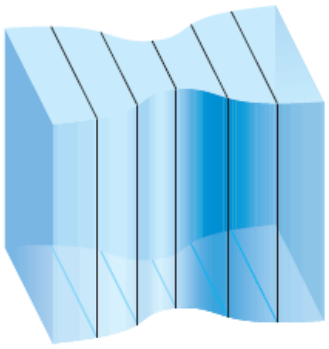
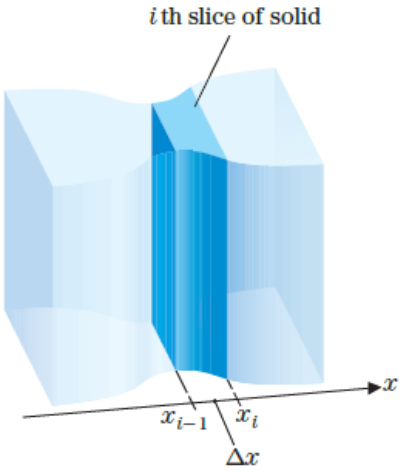
Lesson (6-2)

Volume: Slicing, Disks and Washers

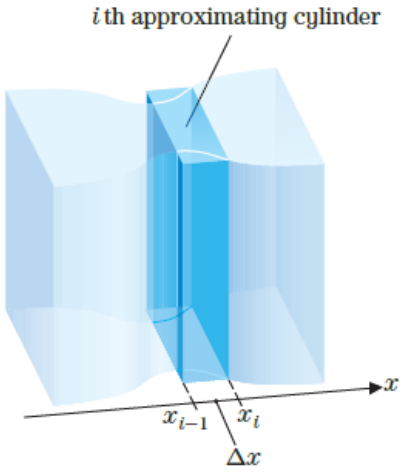
Volumes by Slicing



Sliced solid



ith slice of solid

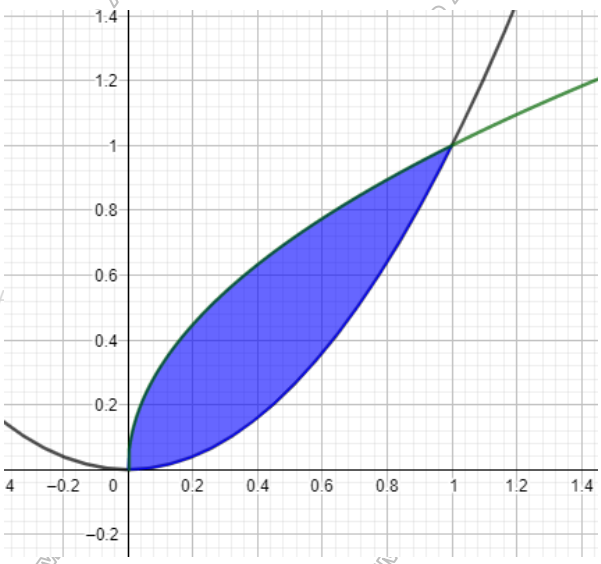


ith approximating cylinder

$$V = \int_a^b A(x)\,dx$$

1 Find the Volume of the solid whose base is bounded by $y = x^2$ *and* $y = \sqrt{x}$ with the indicated cross sections taken perpendicular

a) Square



b) Equilateral Triangle

c) Semicircle

Which one of the definite integrals gives the volume of the solid?

2 Let $f(x) = 1 - \frac{1}{4}x$ *and* $g(x) = 3cos\left(\frac{\pi x}{8}\right)$, Let R be the region enclosed by the graphs of functions f and g and the line $y - axis$

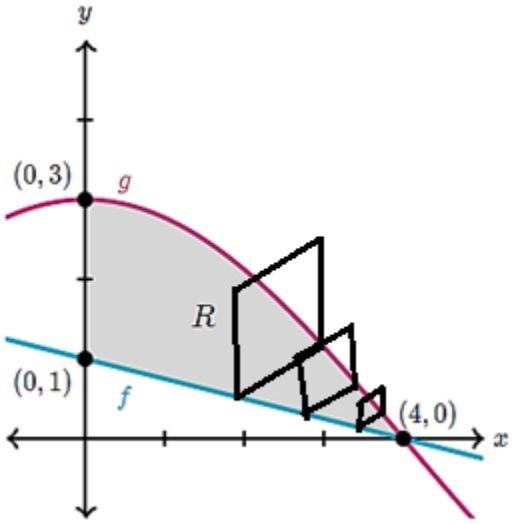
Find the volume of the solid whose cross-section perpendicular to the x-axis is a square with side lies in R

(A) $\int_1^3 \left[3 \cdot \cos\left(\frac{\pi x}{8}\right) + \frac{1}{4}x - 1\right]^2 dx$

(B) $\int_1^3 \left[3 \cdot \cos\left(\frac{\pi x}{8}\right) - \frac{1}{4}x + 1\right]^2 dx$

(C) $\int_0^4 \left[3 \cdot \cos\left(\frac{\pi x}{8}\right) + \frac{1}{4}x - 1\right]^2 dx$

(D) $\int_0^4 \left[3 \cdot \cos\left(\frac{\pi x}{8}\right) - \frac{1}{4}x + 1\right]^2 dx$



3 Let $f(x) = 2^{\frac{x}{3}}$ *and* $g(x) = 2 + \frac{1}{3}x$, Let R be the region enclosed by the graphs of functions f and g and the line $y - axis$

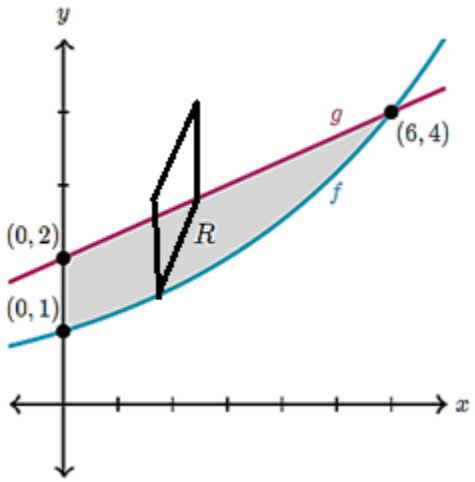
Find the volume of the solid whose cross-section perpendicular to the $x - axis$ is a square with side lies in R

(A) $\int_0^4 \left[2 + \frac{1}{3}x - 2^{\frac{x}{3}}\right]^2 dx$

(C) $\int_0^6 \left[2 + \frac{1}{3}x - 2^{\frac{x}{3}}\right]^2 dx$

(B) $\int_1^6 \left[2 + \frac{1}{3}x - 2^{\frac{x}{3}}\right]^2 dx$

(D) $\int_1^2 \left[2 + \frac{1}{3}x - 2^{\frac{x}{3}}\right]^2 dx$



4 Let R be the region enclosed by the graphs of functions f and g and the line $x = p$

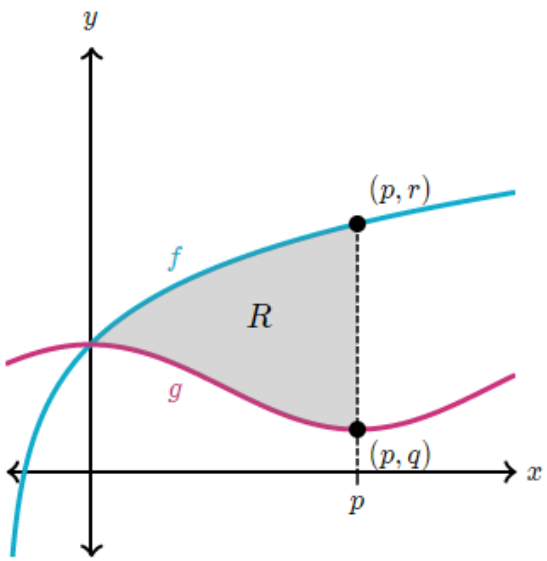
Find the volume of the solid whose cross-section perpendicular to the x-axis is a square with side lies in R

(A) $\int_q^r [f(x) - g(x)] \cdot x \, dx$

(B) $\int_q^r [f(x) - g(x)]^2 \, dx$

(C) $\int_0^p [f(x) - g(x)] \cdot x \, dx$

(D) $\int_0^p [f(x) - g(x)]^2 \, dx$



Find the volume of the solid with cross sectional area A(x).

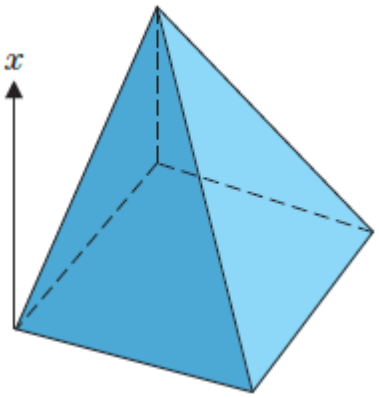
5 $A(x) = x + 2, -1 \le x \le 3$

6 $A(x) = 10e^{0.01x}, 0 \le x \le 10$

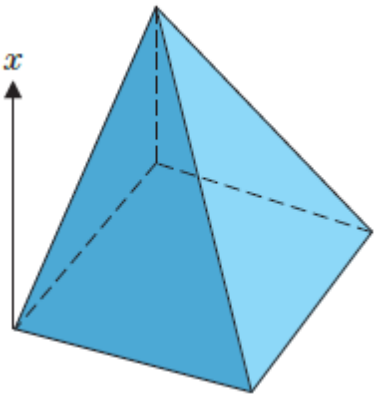
7 $A(x) = \pi(4 - x)^2, 0 \le x \le 2$

8 $A(x) = 2(x + 1)^2, 1 \le x \le 4$

9 The Pyramid Arena in Memphis has a square base of side approximately 180 meters and a height of approximately 100 meters. Find the volume of the pyramid with these measurements.



10 The great pyramid at Gizeh is 500 feet high, rising from a square base of side 750 feet. Compute its volume using integration.



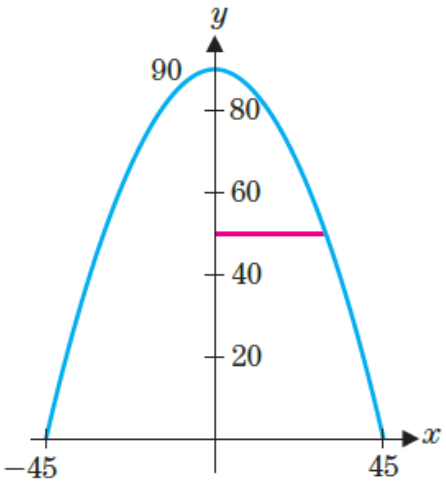
11 Suppose that instead of completing a pyramid, the builders at Gizeh had stopped at height 250 feet (with a square plateau top of side 375 feet). Compute the volume of this structure

12 Find the volume of a pyramid of height 160 feet that has a square base of side 300 feet.

13 **Suppose that a dome has circular cross sections, without line**

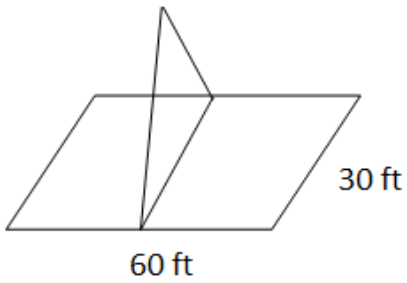
$$y = -\frac{2}{45}x^2 + 90$$

For -45≤ x ≤ 45. (In units of centimeters, this gives dimensions of a dome model similar to the Capito l Dome in Figure) Find the volume of the dome.



14 A church steeple is 30 feet tall with square cross sections. The square at the base has side 3 feet, the square at the top has side 6 inches and the side varies linearly in between. Compute the volume

15 **A house attic has rectangular cross sections parallel to the rectangle is 30 feet by 60 feet at the bottom of the attic and the triangles have base 30 feet and height 10 feet. Compute the volume of the attic.**



16 The outline of a dome is given by $y = 60 - \frac{x^2}{60}\ for\ -60 \le x \le 60$ (units of feet), with circular cross-sections perpendicular to the y-axis. Find its volume

17 A pottery jar has circular cross sections of radius

$4 + sin\ \frac{x}{2}\ inches\ for\ 0 \le x \le 2\pi$.compute its volume

18 A pottery jar has circular cross sections of radius

$4 - sin\ \frac{x}{2}\ inches\ for\ 0 \le x \le 2\pi$. compute its volume.

SIMPSON'S RULE

$$\int_a^b f(x)\,dx \approx S_n(f) = \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)].$$

19 Suppose an MRI scan indicates that cross-sectional areas of adjacent slices of a tumor are as given in the table. Use Simpson's Rule to estimate the volume

x (cm)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$A(x)$ (cm^2)	0.0	0.1	0.2	0.4	0.6	0.4	0.3	0.2	0.2	0.1	0.0

20 Suppose an MRI scan indicates that cross-sectional areas of adjacent slices of a tumor are as given in the table. Use Simpson's Rule to estimate the volume

x (cm)	0.0	0.2	0.4	0.6	0.8	1.0	1.2
$A(x)$ (cm^2)	0.0	0.2	0.3	0.2	0.4	0.2	0.0

21 **Suppose an MRI scan indicates that cross-sectional areas of adjacent slices of a tumor are as given in the table. Use Simpson's Rule to estimate the volume**

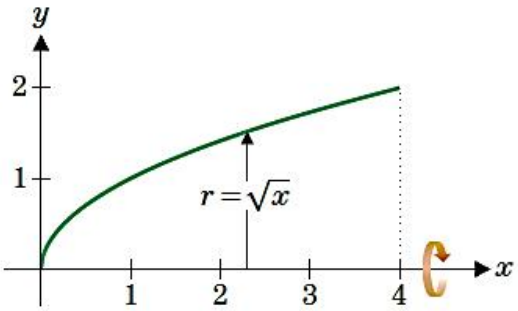
x (m)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$A(x)$ (m^2)	2.0	1.8	1.7	1.6	1.8	2.0	2.1	2.2	2.4

The Method of Disks

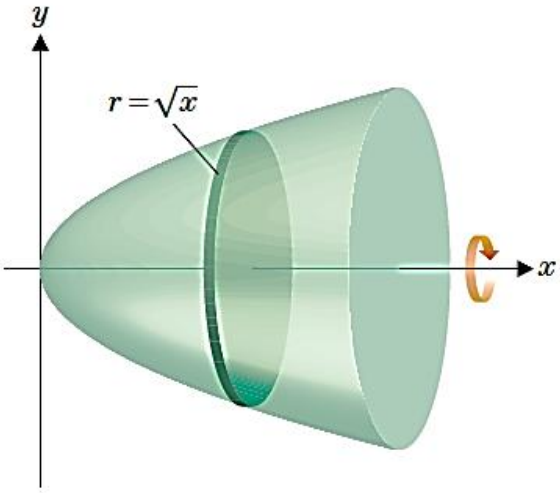
$$V = \int_a^b \underbrace{\pi[f(x)]^2}_{\text{cross-sectional area} = \pi r^2}\, dx.$$

22

Revolve the region under the curve $y = \sqrt{x}$ on the interval [0, 4] about the $x-axis$ and find the volume of the resulting solid of revolution



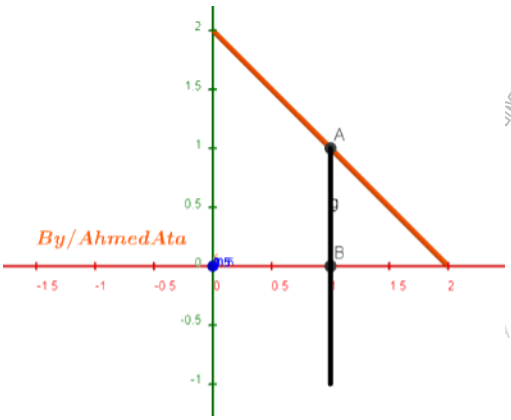
$y = \sqrt{x}$

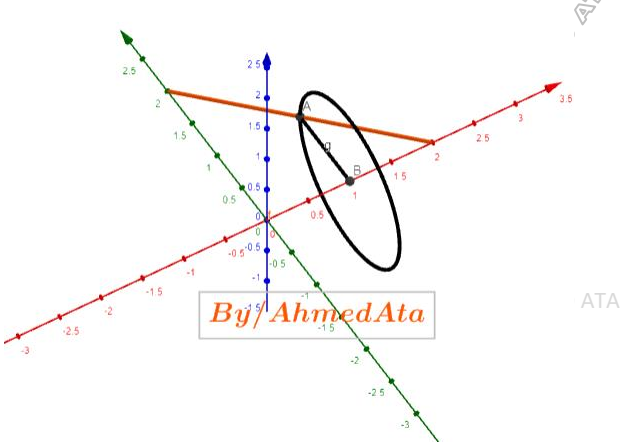


Solid of revolution

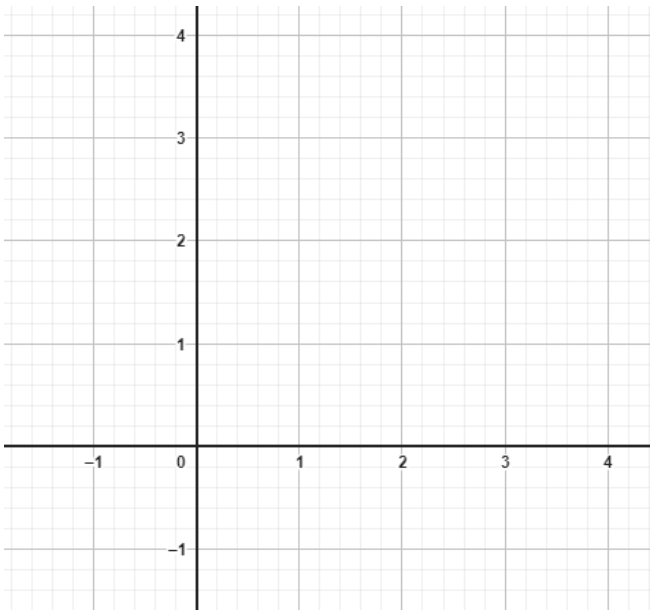
23 Region bounded by $y = 2 - x, y = 0 \text{ and } x = 0$ about

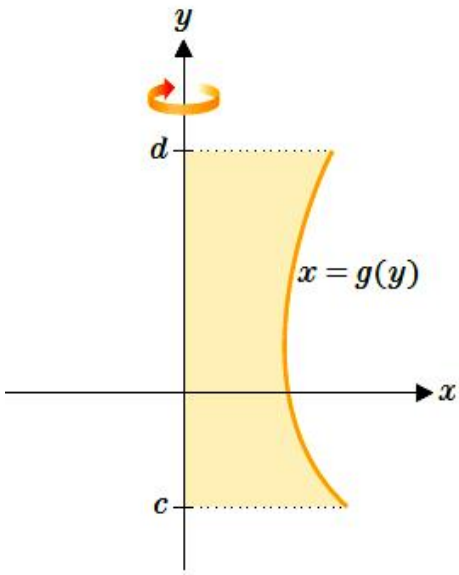
(a) $the\ x-axis$;





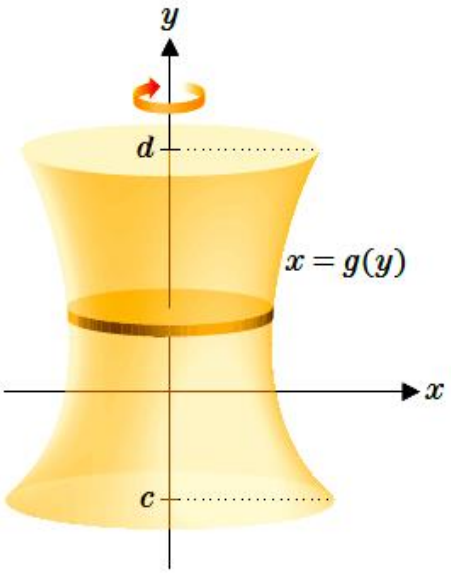
(b) $y = 3$





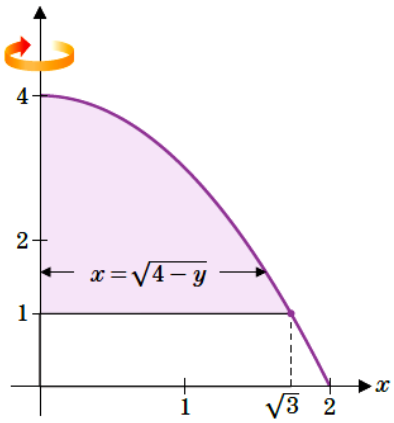
Revolve about the y-axis

$$V = \int_c^d \underbrace{\pi[g(y)]^2}_{\text{cross-sectional area} = \pi r^2}\, dy.$$

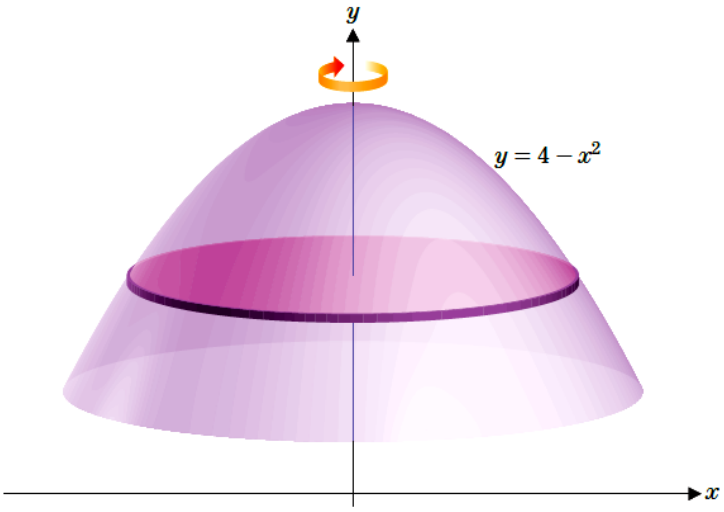


Solid of revolution

24 Find the volume of the solid resulting from revolving the region bounded by the curves $y = 4 - x^2$ *and* $y = 1$ *from* $x = 0$ *to* $x = \sqrt{3}$ *about the* $y-$*axis*



$y = 4 - x^2$

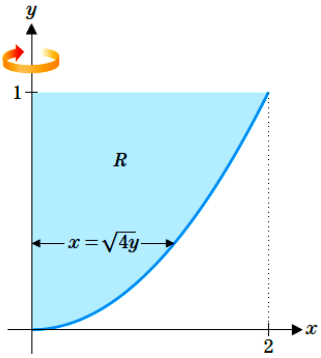


Solid of revolution

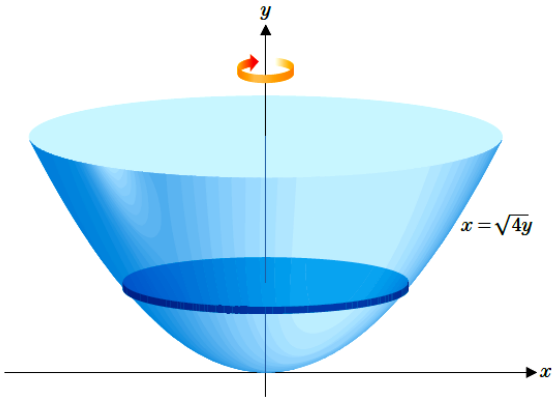
The Method of Washers

25 Let R be the region bounded by the graphs of $y = \frac{1}{4}x^2, x = 0 \text{ and } y = 1$. Computethe volume of the solid formed by revolving R about

(a) $the\ y-axis$

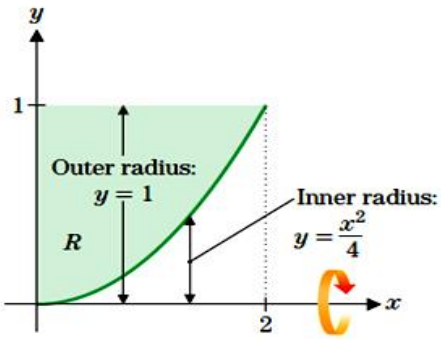


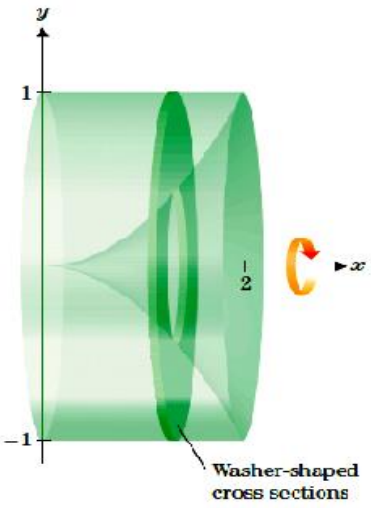
$x = \sqrt{4y}$



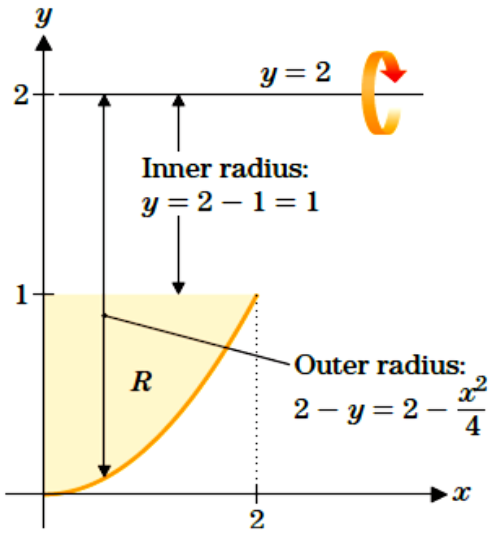
Solid of revolution

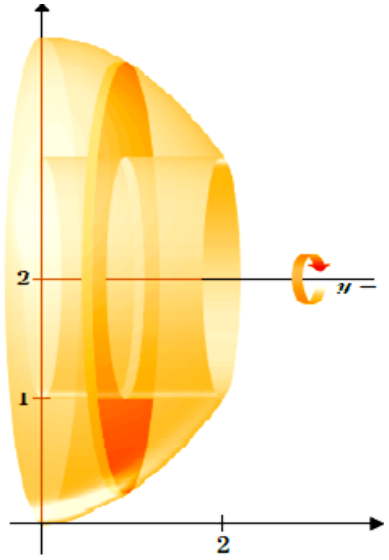
(b) $the\ x-axis$





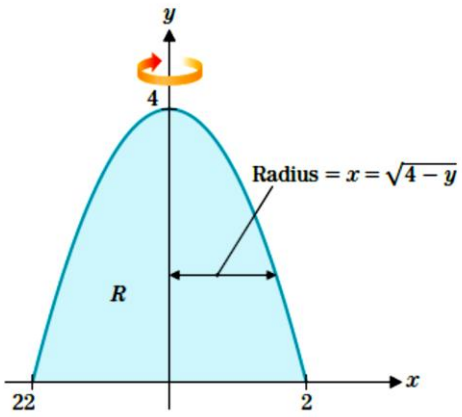
(c) *the line* $y = 2$.

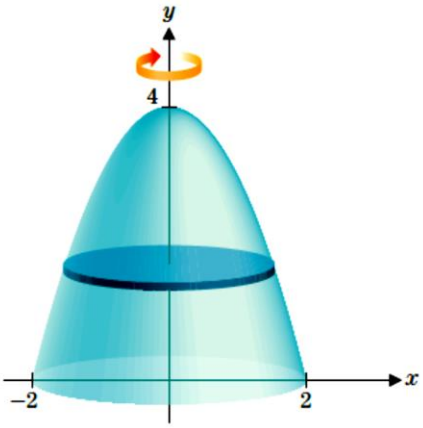




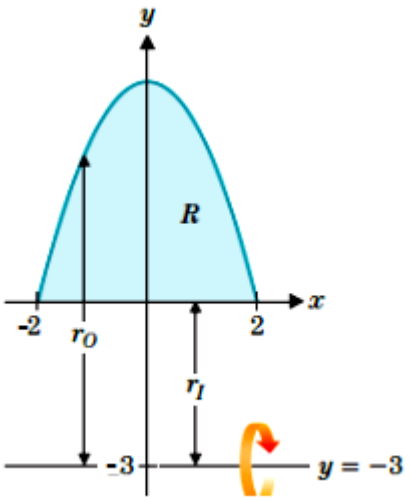
26 Let R be the region bounded by $y = 4 - x^2$ and y = 0. Find the volume of the solids obtained by revolving R about each of the following:

(a) *the* $y-$ *axis*

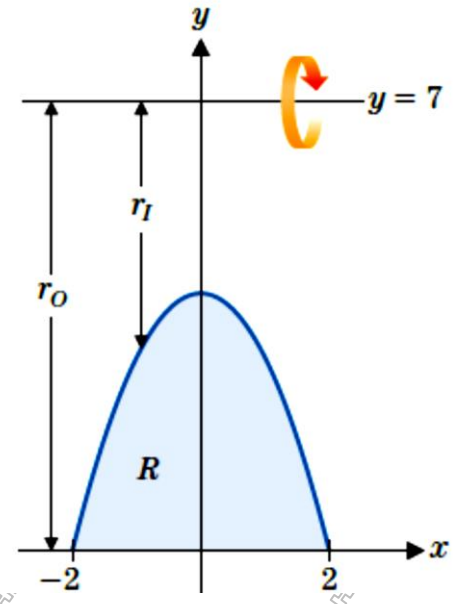




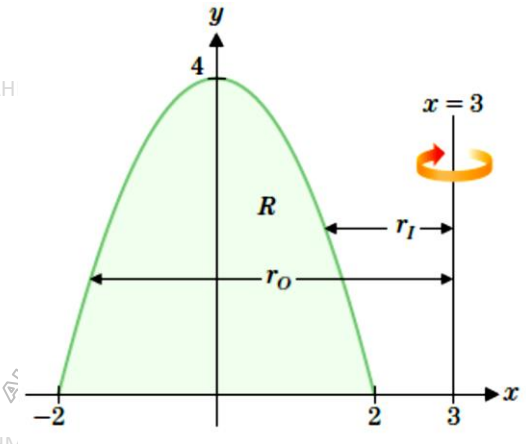
(b) *the line* $y = -3$



(c) *the line* $y = 7$

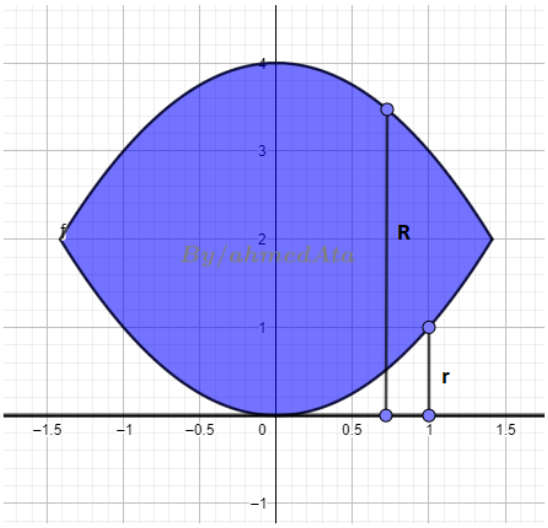


(d) *the line* $x = 3$

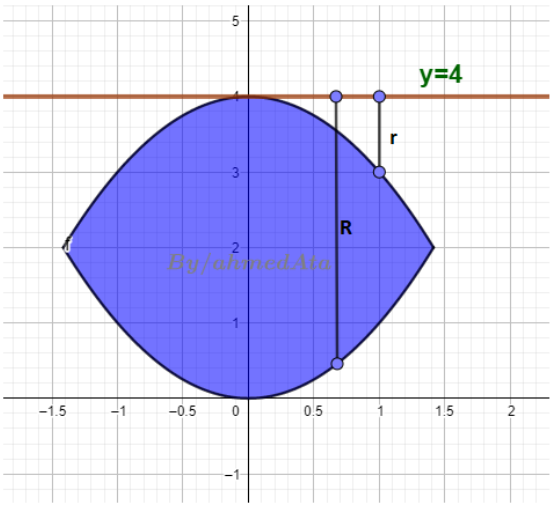


27 compute the volume of the solid formed byrevolving the given region about the given line Region bounded by $y = x^2, y = 4 - x^2$ about

(a) $the\ x - axis$

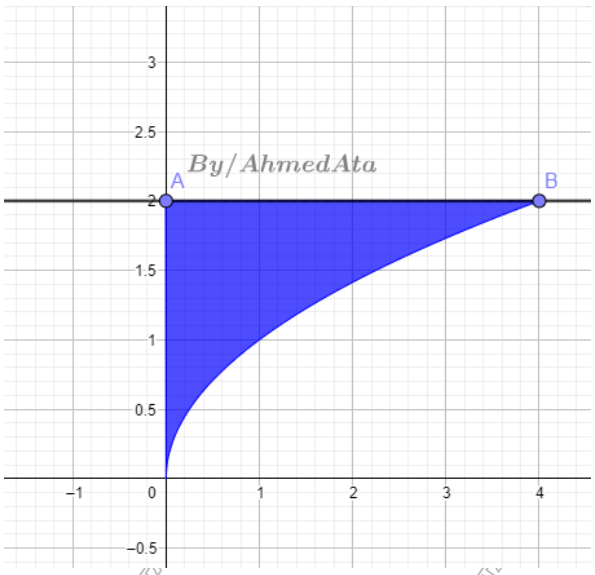


(b) y = 4

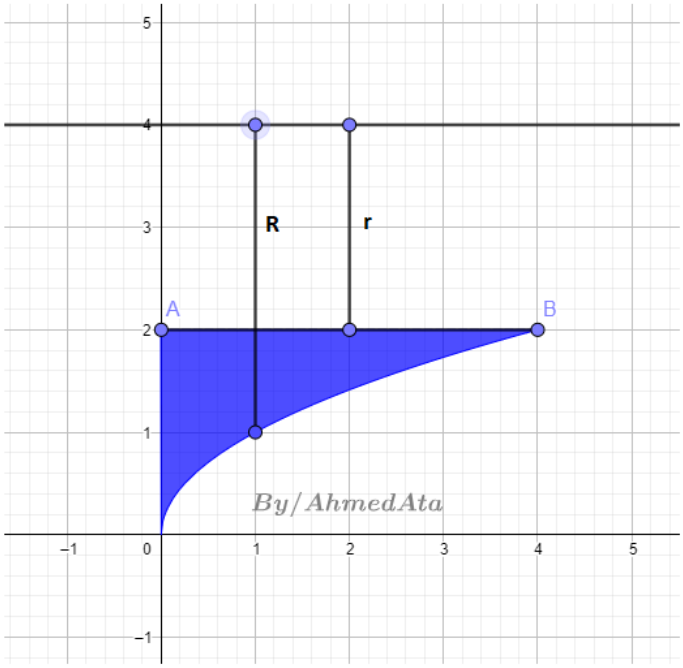


28 compute the volume of the solid formed byrevolving the given region about the given line Region bounded by $y = \sqrt{x}$ and $y = 2$ about

(a) the $y-$ axis



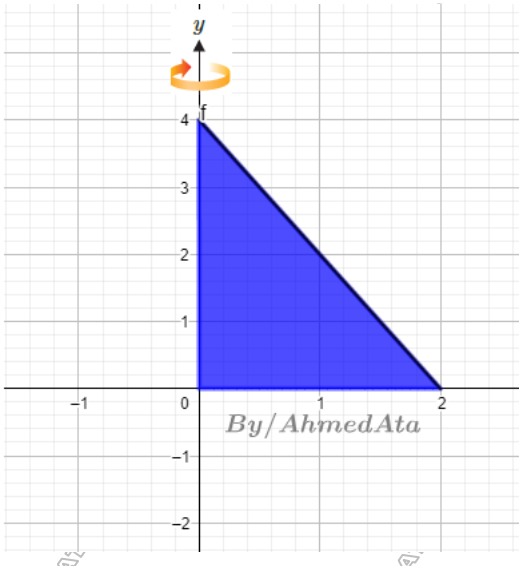
$(b)\ y = 4$



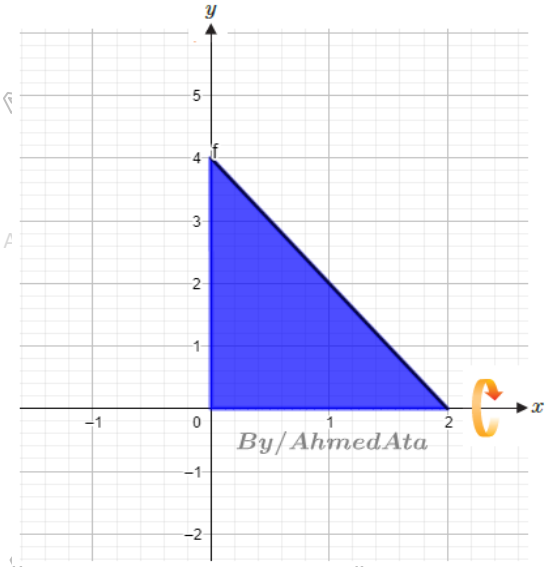
29 Let R be the region bounded by $y = 4 - 2x$, *the* $x-$*axis and the* $y-$*axis*.

Compute the volume of the solid formed by revolving R about the given line

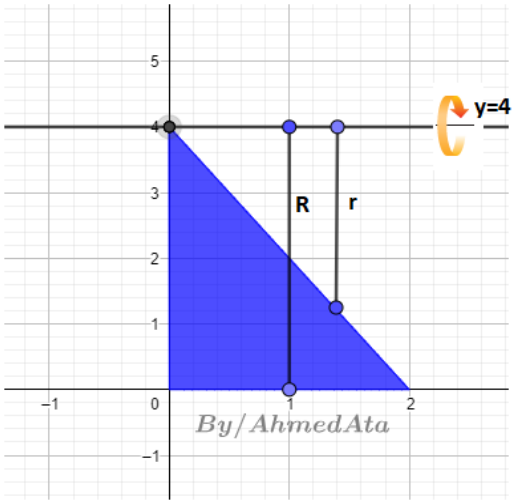
(a) *the* $y-$*axis*



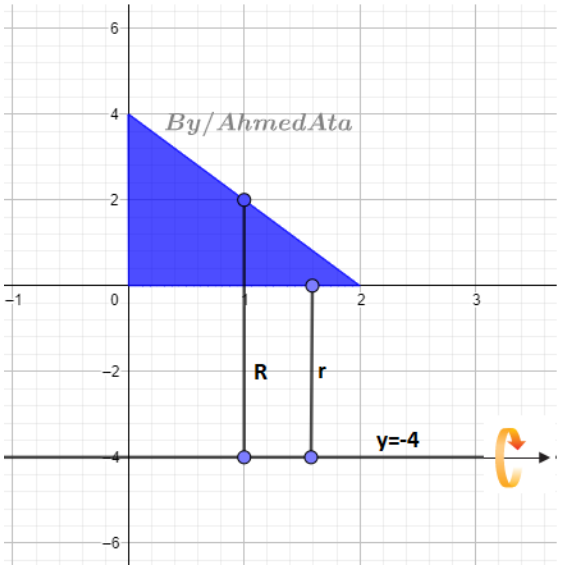
(b) *the* $x-$*axis*



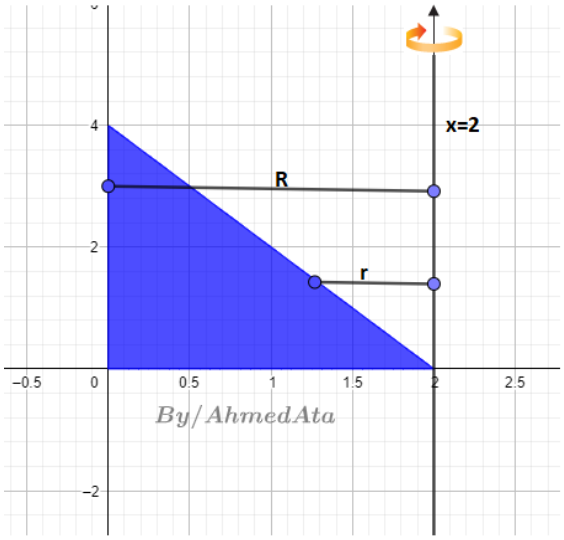
(c) $y = 4$



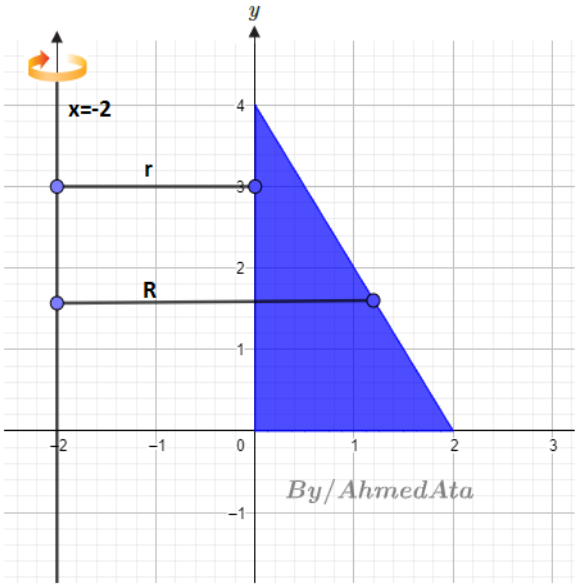
$(d)\ y = -4$



$(e)\ x = 2$



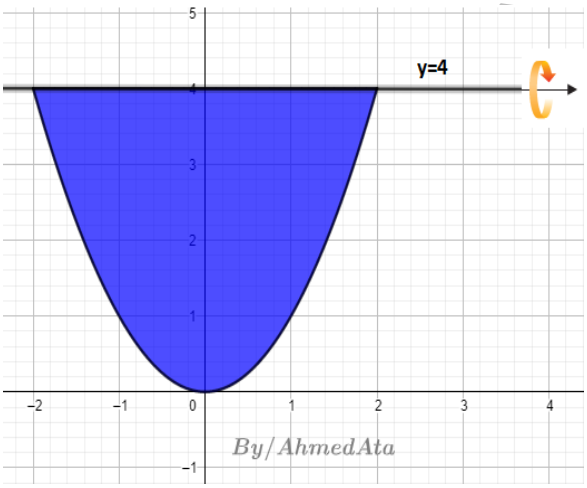
$(f)\ x = -2$



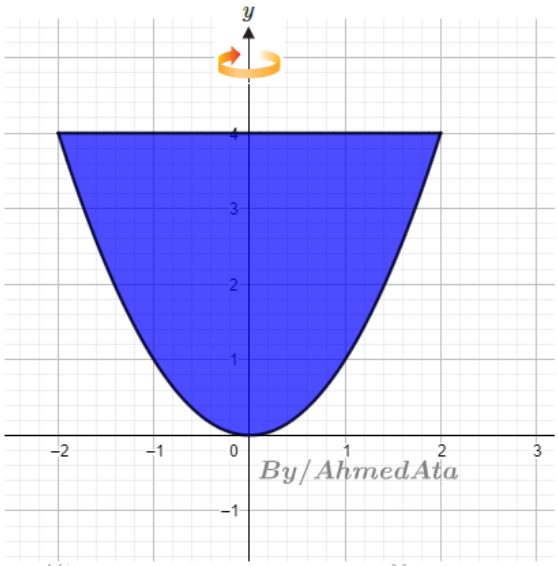
30 Let R be the region bounded by $y = x^2$, *and the* $y = 4$.

Compute the volume of the solid formed by revolving R about the given line

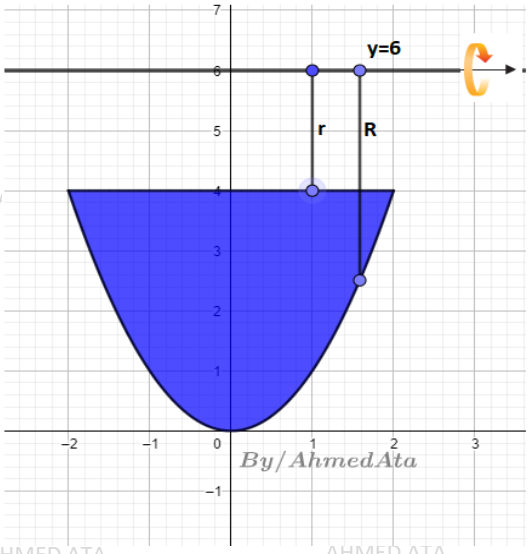
$(a)\ y = 4$



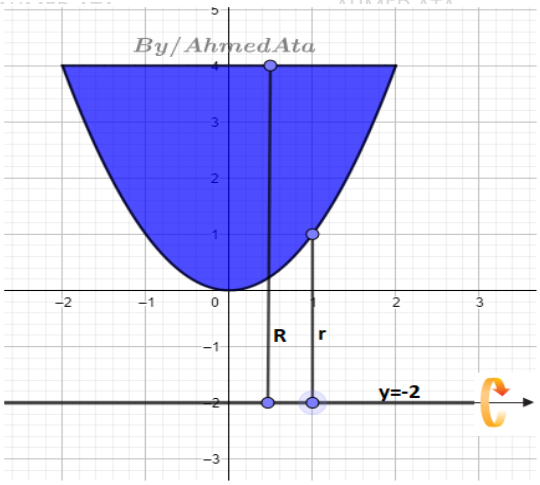
(b) *the* $y-$ *axis*



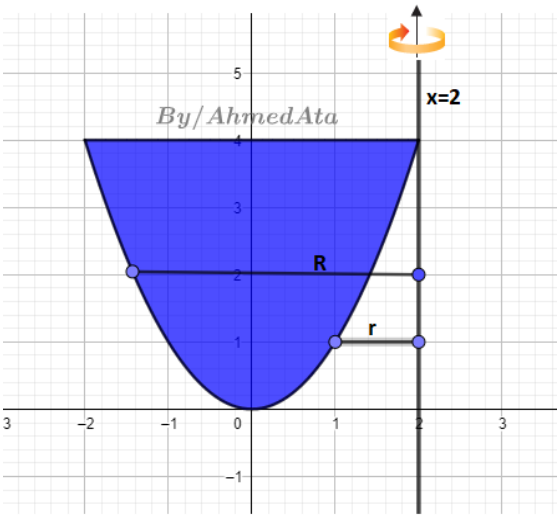
$(c) y = 6$



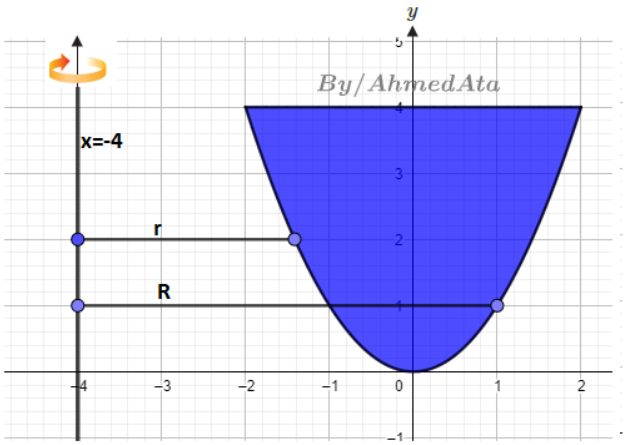
$(d)\ y = -2$



$(e)\ x = 2$



$(f)\ x = -4$



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6-4

Arc Length and Surface Area

MR.Ahmed Ata

https://t.me/ahmedatachat

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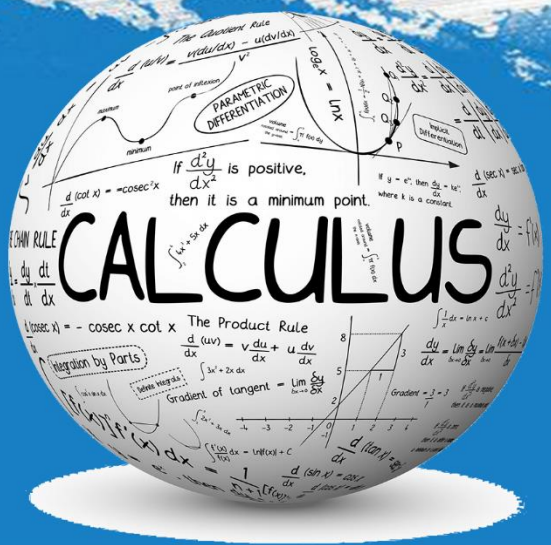
https://cutt.us/Instagramata

ata-tfp.com



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Lesson (6-4)

Arc Length and Surface Area

Arc Length

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx,$$

1 Find the arc length of the portion of the curve $y = \sin x$ with $0 \le x \le \pi$.

2 Find the arc length of the portion of the curve $y = x^2$ with $0 \le x \le 1$.

compute the arc length exactly

3 $y = 2x + 1, 0 \le x \le 2$

4 $y = 4x^{3/2} + 1, 1 \leq x \leq 2$

5 $y = \frac{1}{4}(e^{2x} + e^{-2x}), 0 \leq x \leq 1$

6 $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, 1 \leq x \leq 2$

7 $y = \frac{1}{6}x^3 + \frac{1}{2x}, 1 \le x \le 3$

8 $x = \frac{1}{8}y^4 + \frac{1}{4y^2}, -2 \le y \le -1$

9 $y = 2\ln(4 - x^2), 0 \le x \le 1$

10 $y = x^3, -1 \le x \le 1$

11 $y = \tan x, 0 \le x \le \pi/4$

12 $y = \cos x, 0 \le x \le \pi$

13 $y = \ln x, 1 \le x \le 3$

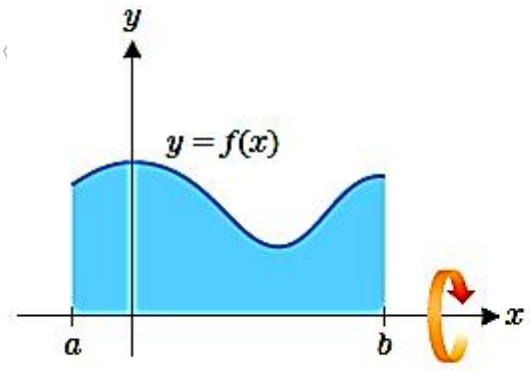
14 $y = \int_0^x u \sin u \, du, 0 \le x \le \pi$

15 $y = \int_0^x e^{-u} \sin u \, du, 0 \leq x \leq \pi$

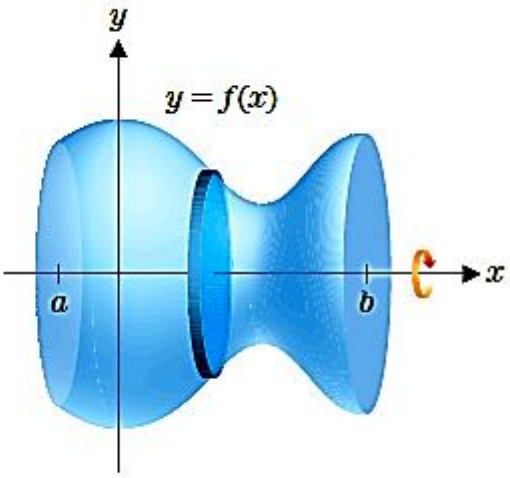
16 A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 10\left(e^{\frac{x}{20}} + e^{\frac{-x}{20}}\right), -20 \leq x \leq 20$, compute the length of the rope.

17 A rope is to be hung between two poles 40 meters apart. If the rope assumes the shape of the catenary $y = 15\left(e^{\frac{x}{30}} + e^{\frac{-x}{30}}\right), -30 \leq x \leq 30$, compute the length of the rope

Surface Area



Revolve about x-axis



Surface of revolution

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx,$$

Find the surface area of the surface generated by revolving

18 $y = x^2, 0 \le x \le 1$, revolved about the x-axis

19 $y = \sin x, 0 \le x \le \pi$, revolved about the x-axis

20 $y = 2x - x^2, 0 \le x \le 2$, revolved about the x-axis

21 $y = x^3 - 4x, -2 \le x \le 0$, revolved about the x-axis

22 $y = e^x, 0 \le x \le 1$, revolved about the x-axis

23 $y = \ln x, 1 \le x \le 2$, revolved about the x-axis

24 $y = \cos x, 0 \le x \le \pi/2$, revolved about the x-axis

25 $y = \sqrt{x}, 1 \le x \le 2$, revolved about the x-axis

26 **compute the arc length L1 of the curve and the length L2 of the secant line connecting the endpoints of the curve. Compute the ratio $\frac{L2}{L1}$**

$y = \sin x, -\frac{\pi}{6} \le x \le \frac{\pi}{6}$

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Projectile Motion

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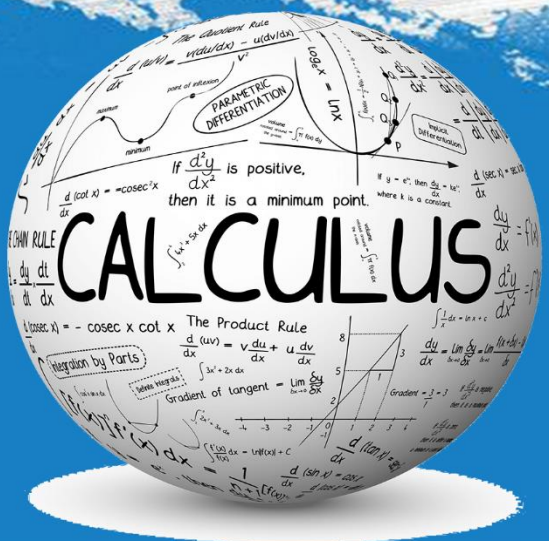

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Lesson (6-5)

Projectile Motion

Newton's second law then gives us $h''(t) = -g$

$$g = 9.8\ m/s^2 \quad or\ g = 32\ ft/\ s^2$$

$$hint: h(t) = -\frac{1}{2}gt^2 + v_0t + h_0 \qquad x(t) = v_0t + x_0$$

1 identify the initial conditions $y(0)\ and\ y'(0)$

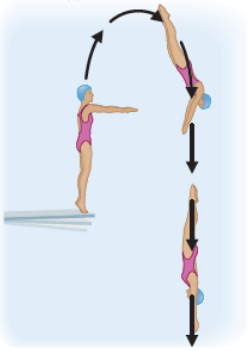
a) An object is dropped from a height of 24 meters

b) An object is dropped from a height of 30 meters

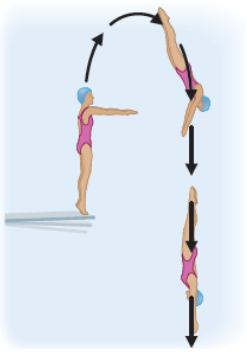
c) An object is released from a height of 18 meters with an upward velocity of 3

d) An object is released from a height of 6 meters with a downward velocity of 1.2

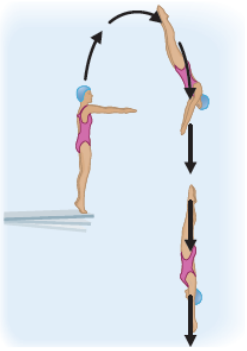
2 If a diving board is 4.5 meters above the surface of the water and a diver starts with initial velocity 2.4 m/s (in the upward direction), what is the diver's velocity at impact (assuming no air resistance)?



3 **Adiver drops from 9 meters above the water (about the height of an Olympic platform dive).What is the diver's velocity at impact**

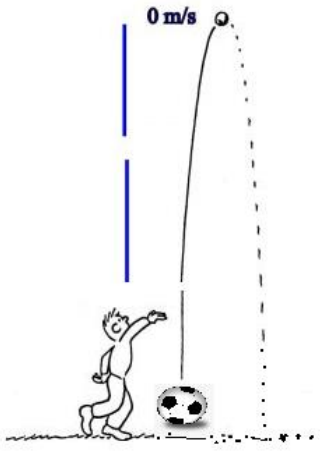


4 **A diver drops from 36 meters above the water (about the height of divers at the Acapulco Cliff Diving competition). What is the diver's velocity at impact**



5 A ball is propelled straight upward from the ground with initial velocity $19.6\ m/s$. Ignoring air resistance,

a) Find an equation for the height of the ball at any time t.
b) Determine the maximum height
c) The amount of time the ball spends in the air



6 A certain not-so-wily coyote discovers that he just stepped off the edge of a cliff. Four seconds later, he hits the ground in a puff of dust. How high in meters was the cliff?

7 The coyote's next scheme involves launching himself into the air with an Acme catapult. If the coyote is propelled vertically from the ground with initial velocity 19.6 m/s,

a) Find an equation for the height of the coyote at any time t.

b) Find his maximum height,

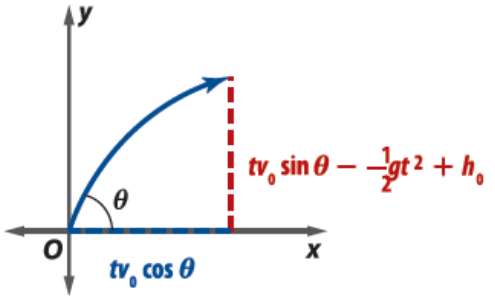
c) The amount of time spent in the air

d) The velocity when he smacks back into the catapult

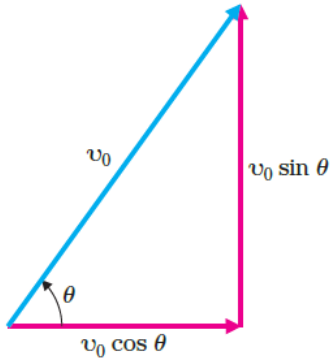
KeyConcept Projectile Motion

For an object launched at an angle θ with the horizontal at an initial velocity v_0, where g is the gravitational constant, t is time, and h_0 is the initial height:

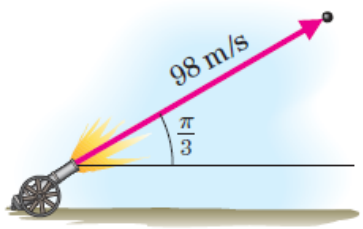
Horizontal Distance	$x = tv_0 \cos\theta$
Vertical Position	$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$



8 **An object is launched at angle $\theta = \pi / 6$ from the horizontal with initial speed $v_0 = 98\ m/s$. Determine the time of flight and the (horizontal) range of the projectile**



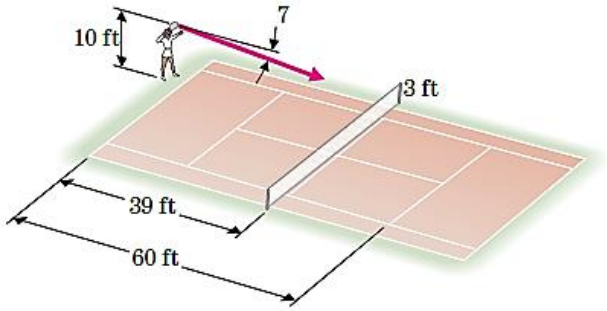
9 **An object is launched at angle $\theta = \pi / 3$ from the horizontal with initial speed $v_0 = 98\ m/s$. Determine the time of flight and the (horizontal) range of the projectile**



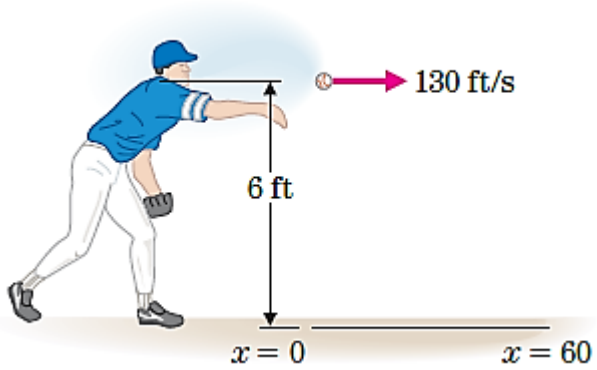
10 Find the time of flight and horizontal range of an object launched at angle 30° with initial speed 40 m/s.

11 A daredevil plans to jump over 25 cars. If the cars are all compact cars with a width of 5 feet and the ramp angle is 30°, determine the initial velocity required to complete the jump successfully

12 **Venus Williams has one of the fastest serves in women's tennis. Suppose that she hits a serve from a height of 10 feet at an initial speed of 120 mph and at an angle of 7° below the horizontal. The serve is "in" if the ball clears a 3 ft-high net that is 39 ft away and hits the ground in front of the service line 60 ft away. (We illustrate this situation in Figure) Determine whether the serve is in or out.**

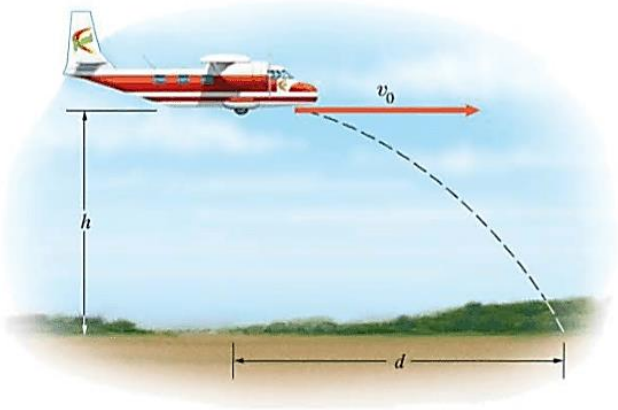


13 A baseball pitcher releases the ball horizontally from a height of 6 ft with an initial speed of 130 ft/s. Find the height of the ball when it reaches home plate 60 ft away. (Hint: Determine the time of flight from the x-equation, then use the y-equation to determine the height.)



14 A plane at an altitude of 256 feet wants to drop supplies to a specific location on the ground. If the plane has a horizontal velocity of 100 ft/s, how far away from the target should the plane release the supplies in order to hit the target location?

(Hint: Use the y-equation to determine the time of flight, then use the x-equation to determine how far the supplies will drift)



An Equation for the Motion of a Knuckleball

Newton's second law applied to the lateral motion of the knuckleball gives

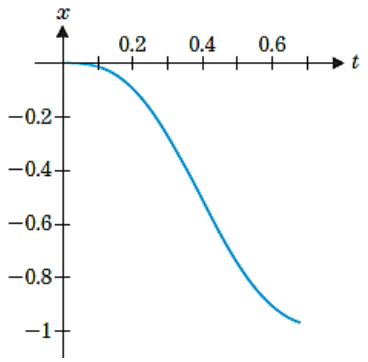
$$mx''(t) = -0.1\,sin(4\theta).$$

The mass of a baseball is about $0.098\ kg$. We now have

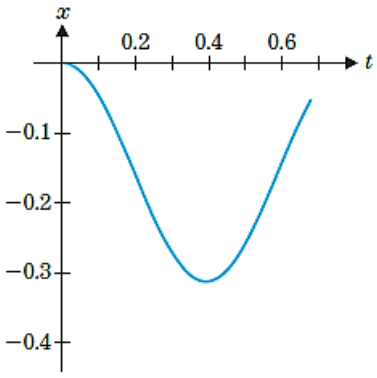
$$x''(t) = -10\,sin(4\omega t + \theta_0)$$

with initial conditions $x'(0) = 0$ and $x(0) = 0$.

15 For a spin rate of $\omega = 2$ radians per second and $\theta_0 = 0$, find an equation for the lateral motion of the knuckleball and graph it for $0 \le t \le 0.68$.



16 Repeat this for $\theta_0 = \frac{\pi}{2}$



7 - Integration Techniques and First-Order Differential Equations

1. Review of Formulas and Techniques
2. Integration by Parts
3. Trigonometric Techniques of Integration
4. Integration of Rational Functions Using Partial Fractions
5. Integration Tables and Computer Algebra Systems
6. Modeling with Differentioal Equations
7. Separable Differential Equations

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Review of Formulas and Techniques

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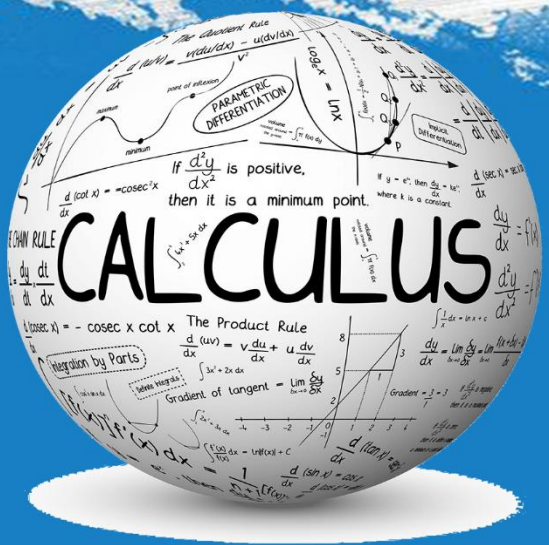

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Lesson (7-1)

Review of Formulas and Techniques

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c, \quad \text{for } r \neq -1 \text{ (power rule)} \qquad \int \frac{1}{x}\,dx = \ln|x| + c, \quad \text{for } x \neq 0$$

$$\int \sin x\,dx = -\cos x + c \qquad \int \cos x\,dx = \sin x + c$$

$$\int \sec^2 x\,dx = \tan x + c \qquad \int \sec x \tan x\,dx = \sec x + c$$

$$\int \csc^2 x\,dx = -\cot x + c \qquad \int \csc x \cot x\,dx = -\csc x + c$$

$$\int e^x\,dx = e^x + c \qquad \int e^{-x}\,dx = -e^{-x} + c$$

$$\int \tan x\,dx = -\ln|\cos x| + c \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c \qquad \int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c \qquad \int sin(ax)dx = -\frac{1}{a}cos(ax) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c \qquad \int \frac{1}{\sqrt{a^2-x^2}} = sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{f'(x)}{f(x)}dx = ln|f(x)| + c \qquad \int \frac{1}{a^2+x^2} = \frac{1}{|a|}tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int a^x dx = \frac{1}{lna}a^x + c \qquad \int \frac{1}{|x|\sqrt{x^2-a^2}} = \frac{1}{|a|}sec^{-1}\left(\frac{x}{a}\right) + c$$

Evaluate the integral

1. $\int e^{ax}\,dx, a \neq 0$
2. $\int \cos(ax)\,dx, a \neq 0$
3. $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx, a > 0$
4. $\int \frac{1}{|x|\sqrt{x^2 - a^2}}\,dx, a > 0$
5. $\int \sin 6t\,dt$
6. $\int \sec 2t \tan 2t\,dt$
7. $\int (x^2 + 4)^2\,dx$
8. $\int x(x^2 + 4)^2\,dx$

9 $\int \frac{3}{16+x^2}\,dx$

10 $\int \frac{2}{4+4x^2}\,dx$

11 $\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$

12 $\int \frac{x+1}{\sqrt{3-2x-x^2}}\,dx$

13 $\int \frac{4}{5+2x+x^2}\,dx$

14 $\int \frac{4x+4}{5+2x+x^2}\,dx$

15 $\int \frac{4t}{5+2t+t^2}\,dt$

16 $\int \frac{t+1}{t^2+2t+4}\,dt$

17 $\int e^{3-2x}\,dx$

18 $\int \frac{3}{e^{6x}}\,dx$

19 $\int \frac{4}{x^{1/3}(1+x^{2/3})}\,dx$

20 $\int \frac{2}{x^{1/4}+x}\,dx$

21 $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

22 $\int \frac{\cos(1/x)}{x^2}\,dx$

23 $\int_0^{\pi} \cos x e^{\sin x}\, dx$

24 $\int_0^{\pi/4} \sec^2 x e^{\tan x}\, dx$

25 $\int_{-\pi/4}^{0} \frac{\sin t}{\cos^2 t}\, dt$

26 $\int \frac{x^2}{1+x^6}\, dx$

27 $\int \frac{x^5}{1+x^6}\,dx$

28 $\int \frac{1}{\sqrt{4-x^2}}\,dx$

29 $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

30 $\int \frac{x}{\sqrt{1-x^4}}\,dx$

31 $\int \frac{2x^3}{\sqrt{1-x^4}}\,dx$

32 $\int \frac{1+x}{1+x^2}\,dx$

33 $\int \frac{\ln x^2}{x}\,dx$

34 $\int_3^4 x\sqrt{x-3}\,dx$

35 $\int_0^1 x(x-3)^2\,dx$

36 $\int_1^4 \frac{x^2+1}{\sqrt{x}}\,dx$

37 $\int_{-2}^{0} xe^{-x^2}\,dx$

38 $\int \frac{5}{3+x^2}$

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Integration by Parts

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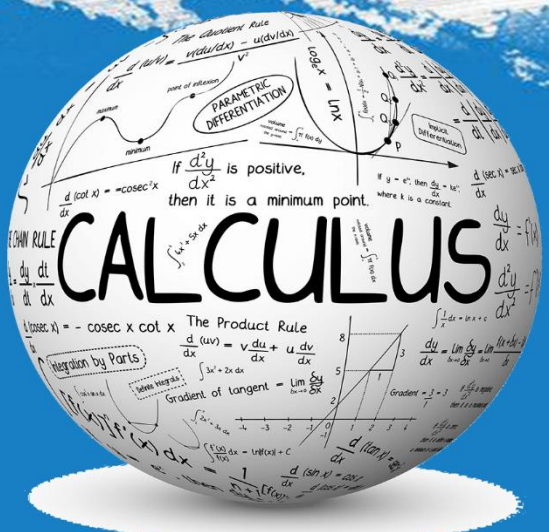

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Lesson (7-2)

Integration by Parts

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du. \qquad (2.1)$$

Sign	F(x) Differentiate	F(y) Integration
+ ⇨	F(x)	F(y)
- ⇨	First derivative of F(x)	First Integrate of F(y)
+ ⇨	Second derivative of F(x)	Second Integrate of F(y)
- ⇨	Third derivative of F(x)	Third Integrate of F(y)

Evaluate the integrals

1 $\int x\cos x\,dx$

2 $\int x \sin 4x \, dx$

3 $\int x e^{2x} dx$

4 $\int x^2 \ln x \, dx$

5 $\int x^2 e^{-3x} dx$

6 $\int x^2 e^{x^3}\, dx$

7 $\int e^x \sin 4x\, dx$

8 $\int e^{2x} \cos x\, dx$

9 $\int \cos x \cos 2x \, dx$

10 $\int x \sec^2 x \, dx$

11 $\int x^3 e^{x^2} \, dx$

12 $\int \cos x \ln(\sin x)\, dx$

13 $\int_1^2 x^3 \ln x\, dx.$

14 $\int_0^1 x \sin 2x\, dx$

15 $\int_0^1 x^2 \cos \pi x \, dx$

16 $\int_0^1 x^2 e^{3x} dx$

17 $\int_1^{10} \ln 2x \, dx$

18 $\int_1^2 x \ln x \, dx$

19 $\int e^{ax} x^2 \, dx, a \neq 0$

20 $\int x \sin(ax) \, dx, a \neq 0$

21 $\int x^4 \sin x \, dx$

22 $\int x^4 e^x \, dx$

23 $\int x^5 \cos 2x \, dx$

Evaluate the integral using integration by parts and substitution.

24 $\int \cos^{-1} x \, dx$

25 $\int \tan^{-1} x \, dx$

26 $\int \sin \sqrt{x}\, dx$

27 $\int \sin(\ln x)\, dx$

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Volume: Slicing, Disks and Washers

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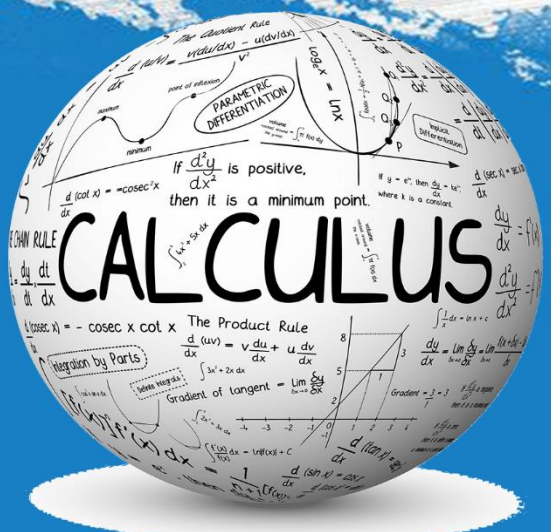

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Lesson (7-3)

Trigonometric Techniques of Integration

Case 1: m or n Is an Odd Positive Integer

$$\int sin^m x\, cos^n x\ dx$$

where m and n are positive integers.

Evaluate

1 $\int \cos^4 x \sin x\, dx.$

2 $\int \cos^4 x \sin^3 x\, dx.$

3 $\int \sqrt{\sin x}\cos^5 x\,dx.$

4 $\int \cos x \sin^4 x\,dx$

5 $\int \cos^3 x \sin^4 x\,dx$

6 $\int_0^{\pi/4} \cos 2x \sin^3 2x \, dx$

7 $\int_{\pi/4}^{\pi/3} \cos^3 3x \sin^3 3x \, dx$

8 $\int_0^{\pi/2} \cos^2 x \sin x \, dx$

Case 2: m and n Are Both Even Positive Integers

Evaluate

9 $\int \sin^2 x \, dx.$

10 $\int \cos^2 (x+1) \, dx$

11 $\int \cos^4 x \, dx.$

12 $\int \sin^4 (x - 3) \, dx$

$\int tan^m x\, sec^n x\, dx$ where m and n are integers.

Case 1: m Is an Odd Positive Integer

Evaluate

13 $\int \tan^3 x \sec^3 x\, dx.$

14 $\int \tan x \sec^3 x\, dx$

15 $\int \cot x \csc^4 x\, dx$

16 $\int \tan(2x+1)\sec^3(2x+1)\,dx$

17 $\int x\tan^3(x^2+1)\sec(x^2+1)\,dx$

Case 2: n Is an Even Positive Integer

18 $\int \tan^2 x \sec^4 x \, dx.$

19 $\int \cot^2 x \csc^4 x \, dx$

20 $\int \cot^2 x \csc^2 x \, dx$

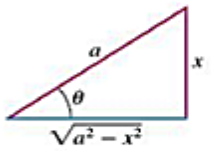
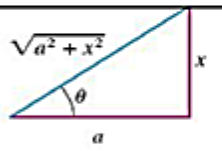
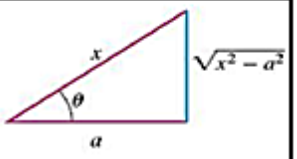
21 $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$

22 $\int \cos^2 x \sin^2 x \, dx$

23 $\int_{-\pi/3}^{0} \sqrt{\cos x} \sin^3 x \, dx$

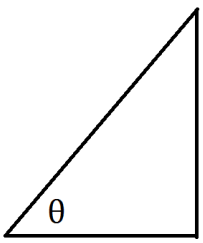
Trigonometric Substitution

TRIGONOMETRIC SUBSTITUTION

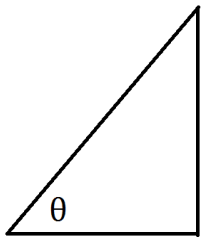
Expression	substitution	Identity	domain	triangle
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$1-\sin^2\theta=\cos^2\theta$	$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$	
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$1+\tan^2\theta=\sec^2\theta$	$-\frac{\pi}{2}<\theta<\frac{\pi}{2}$	
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$\sec^2\theta-1=\tan^2\theta$	$0\le\theta<\frac{\pi}{2}$ or $\pi\le\theta<\frac{3\pi}{2}$	

Evaluate the integral

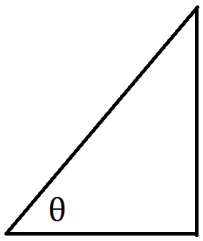
24 $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx.$



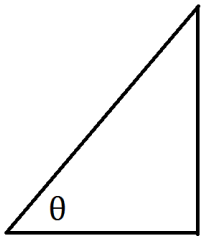
25 $\int \frac{1}{\sqrt{9+x^2}}\,dx.$



26 $\int \frac{\sqrt{x^2-25}}{x}\,dx$, for $x \geq 5$.



27 $\int \frac{x^2}{\sqrt{16-x^2}}\,dx$



28 $\int \frac{2}{\sqrt{x^2-4}}\,dx$

29 $\int \frac{1}{\sqrt{4+x^2}}\,dx$

30 $\int_0^1 x\sqrt{x^2+8}\,dx$

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Integration of Rational Functions Using Partial Fractions

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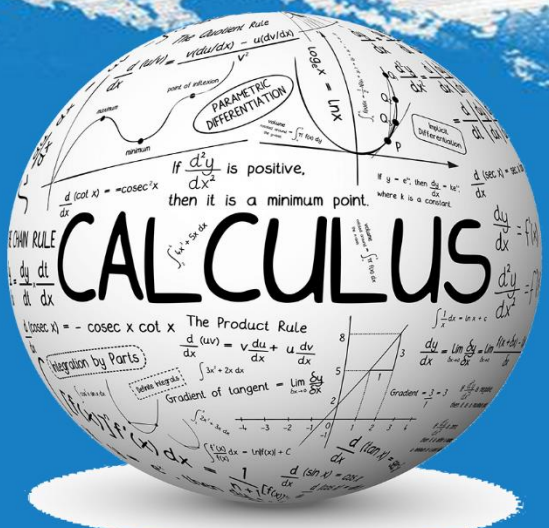

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Lesson (7-4)

Integration of Rational Functions Using Partial Fractions

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	• where $x^2 + bx + c$ cannot be factorised further	

Evaluate

1 $\int \frac{1}{x^2+x-2}\,dx.$

Find the partial fractions decomposition and an antiderivative.

2 $\dfrac{x-5}{x^2-1}$

3 $\dfrac{6x}{x^2-x-2}$

4 $\dfrac{-x+5}{x^3-x^2-2x}$

5 $\dfrac{5x-23}{6x^2-11x-7}$

6 $\dfrac{x-1}{x^3+4x^2+4x}$

7 $\dfrac{x+2}{x^3+x}$

8 $\dfrac{2x+3}{x^2+2x+1}$

9 $\dfrac{2x}{x^2-6x+9}$

10 $\dfrac{x^3+x}{x^2-1}$

Evaluate the integral

11 $\int \dfrac{x^3+x+2}{x^2+2x-8}\,dx$

12 $\int \frac{x+4}{x^3+3x^2+2x}\,dx$

13 $\int \frac{4x^3-1}{x^4-x}\,dx$

14 $\int \frac{\sin x \cos x}{\sin^2 x - 4}\,dx$

15 $\dfrac{1}{x^3 + 4x}$

16 $\displaystyle\int \frac{2e^x}{e^{3x} + e^x}\, dx$

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Modeling with Differential Equations

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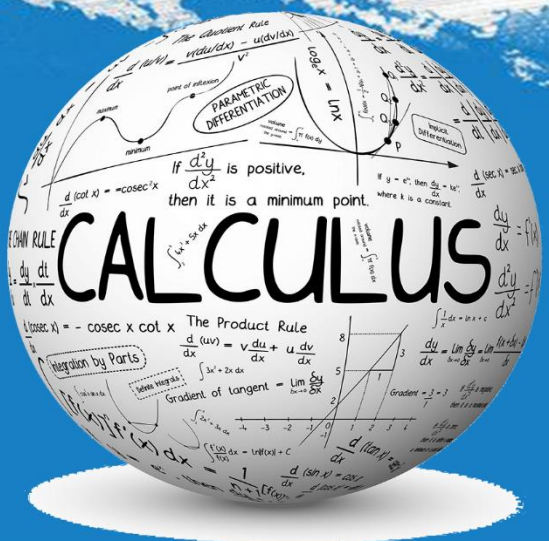

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Lesson (7-6)

Modeling with Differential Equations

Growth and Decay Problems

$y'(t) = ky(t)$ $y(t) = A(e)^{kt}$ $A = y(0)$

Find the solution of the given differential equation satisfying the indicated initial condition

1. $y' = 4y, y(0) = 2$

2. $y' = 3y, y(0) = -2$

3. $y' = -3y, y(0) = 5$

4. $y' = -y, y(1) = 2$

5 A freshly inoculated bacterial culture of Streptococcus A (a common group of microorganisms that cause strep throat) contains 100 cells. When the culture is checked 60 minutes later, it is determined that there are 450 cells present. Assuming exponential growth, determine the number of cells present at any time t (measured in minutes) and find the doubling time.

6 Suppose a bacterial culture initially has 400 cells. After 1 hour, the population has increased to 800.

(a) Quickly determine the population after 3 hours.

(b) Find an equation for the population at any time.

(c)What will the population be after 3.5 hours?

7 Suppose a bacterial culture initially has 100 cells. After 2 hours, the population has increased to 400.

(a) Quickly find the population after 6 hours.

(b) Find an equation for the population at any time.

(c) What will the population be after 7 hours?

8 Suppose a bacterial culture doubles in population every 4 hours. If the population is initially 100,

(a) quickly determine when the population will reach 400.

(b) Find an equation for the population at any time.

(c) Determine when the population will reach 6000

9 Suppose a bacterial culture triples in population every 5 hours. If the population is initially 200,

(a) quickly determine when the population will reach 5400.

(b) Find an equation for the population at any time.

(c) Determine when the population will reach 20,000.

The doubling time is $\frac{ln\,2}{r}$ or $r = \frac{ln2}{t}$

The half-life is $-\frac{ln\,2}{r}$ or $r = -\frac{ln\,2}{t}$

10 If you have 50 g of ^{14}C today, how much will be left in 100 years? Such that the half-life of carbon-14 (^{14}C) is approximately 5730 years.

11 Strontium-90 is a dangerous radioactive isotope. Because of its similarity to calcium, it is easily absorbed into human bones. The half-life of strontium-90 is 28 years. If a certain amount is absorbed into the bones due to exposure to a nuclear explosion, what percentage will remain after 84 years?

12 The half-life of uranium ^{235}U is approximately 0.7×10^9 years. If 50 grams are buried at a nuclear waste site, how much will remain after 100 years?

13 The half-life of morphine in the human bloodstream is 3 hours. If initially there is 0.4 mg of morphine in the bloodstream, Find an equation for the amount in the bloodstream at any time. When does the amount drop below 0.1 mg?

Newton's Law of Cooling

$$y'(t) = k[y(t) - T_a] \qquad y(t) = A(e)^{kt} + T_a \qquad A = y(0) - T_a$$

Find the solution of the given differential equation satisfying the indicated initial condition

14 $y' = y - 50, y(0) = 70$

15 $y' = 0.1y - 10, y(0) = 80$

16 A cup of fast-food coffee is $80^{\circ}C$ when freshly poured. After 2 minutes in a room at $20^{\circ}C$, the coffee has cooled to $75^{\circ}C$. Find the temperature at any time t and find the time at which the coffee has cooled to $50^{\circ}C$

17 A bowl of porridge at $200^{\circ}F$ (too hot) is placed in a $70^{\circ}F$ room. One minute later the porridge has cooled to $180^{\circ}F$. When will the temperature be $120^{\circ}F$ (just right)?

18 A cold drink is poured out at $50^{\circ}F$. After 2 minutes of sitting in a $70^{\circ}F$ room, its temperature has risen to $56^{\circ}F$.

(a) Find the drink's temperature at any time t.

(b) What will the temperature be after 10 minutes?

(c) When will the drink have warmed to $66^{\circ}F$?

19 If you invest AED 7000 at an annual Murabaha rate of 5.75%, compare the value of your investment after 5 years under various forms of compounding.

a) With annual compounding, the value is

b) With monthly compounding, this becomes

c) With daily compounding, these yields

d) With continuous compounding, the value is

20 **If you invest AED 1000 at an annual Murabaha rate of 8%, compare the value of the investment after 1 year under the following forms of compounding:**

e) **With annual compounding, the value is**

f) **With monthly compounding, this becomes**

g) **With daily compounding, these yields**

h) **With continuous compounding, the value is**

21 **Suppose that the value of a AED 10,000 asset decreases continuously at a constant rate of 24% per year. Find its worth**

(a) **10 years**

(b) **20 years**

22 **Suppose that the value of a AED 40,000 asset decreases at a constant percentage rate of 10%. Find its worth after**

(a) 10 years

(b) 20 years

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Separable Differential Equations

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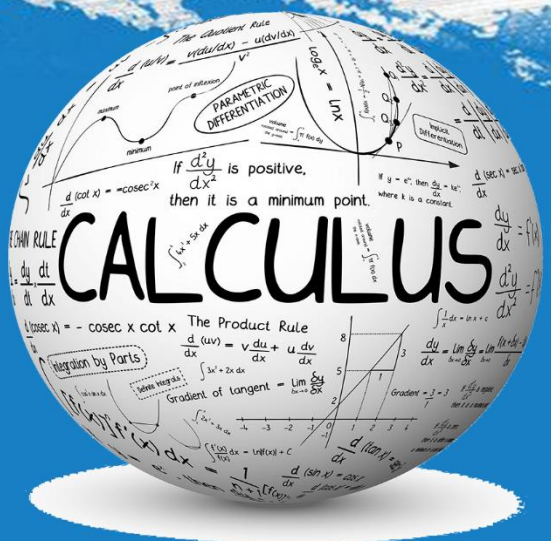

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Lesson (7-7)

Separable Differential Equations

Determine whether the differential equation is separable.

1 $y' = xy^2 - 2xy$

2 $y' = (3x + 1)\cos y$

3 $y' = (3x + y)\cos y$

4 $y' = 2x(\cos y - 1)$

5 $y' = 2x(y - x)$

6 $y' = x^2y + y\cos x$

7 $y' = x^2y - x\cos y$

8 $y' = 2x\cos y - xy^3$

Solve the differential equation

9 $y' = \dfrac{x^2 + 7x + 3}{y^2}$.

10 $y' = (x^2 + 1)y$

11 $y' = 2x^2y^2$

12 $y' = \dfrac{6x^2}{y(1+x^3)}$

13 $y' = \dfrac{2x}{y}e^{y-x}$

14 $y' = \dfrac{\cos x}{\sin y}$

15 $y' = \dfrac{xy}{1+x^2}$

16 $y' = x\cos^2 y$

Find the general solution in an explicit form and sketch several members of the family of solutions

17 $y' = -xy$

18 $y' = \frac{-x}{y}$

19 $y' = \frac{1}{y}$

Solve the IVP (Initial Value Problem) explicitly if possible

20 $y' = 3(x+1)^2 y,\ y(0) = 1$

21 $y' = \frac{x-1}{y^2}, y(0) = 2$

22 $y' = \frac{4x^2}{y}, y(0) = 2$

23 $y' = \frac{x-1}{y}, y(0) = -2$

24 $y' = \frac{4y}{x+3}, y(-2) = 1$

25 $y' = \frac{3x}{4y+1}, y(1) = 4$

26 $y' = \frac{4x}{\cos y}, y(0) = 0$

$$y'(t) = ky(M - y). \quad (7.6)$$

$$y = \frac{AMe^{kMt}}{1 + Ae^{kMt}}.$$

Use equation (7.6) to help solve the IVP.

27 $y' = 3y(2 - y), y(0) = 1$

28 $y' = y(3 - y), y(0) = 2$

29 $y' = 2y(5 - y), y(0) = 4$

30 $y' = y(2 - y), y(0) = 1$

31 **Given a maximum sustainable population of M = 1000 (this could be measured in millions or tons, etc.) and growth rate k = 0.007, find an expression for the population at any time t, given an initial population of y(0) = 350 and assuming logistic growth**