

Chapter 1: Overview

Concept Checks

1.1. a 1.2. a) 4 b) 3 c) 5 d) 6 e) 2 1.3. a, c and e 1.4. b 1.5. e 1.6. a) 4th b) 2nd c) 3rd d) 1st

Multiple-Choice Questions

1.1. c 1.2. c 1.3. d 1.4. b 1.5. a 1.6. b 1.7. b 1.8. c 1.9. c 1.10. b 1.11. d 1.12. b 1.13. c 1.14. a 1.15. e 1.16. a

Conceptual Questions

1.17. (a) In Europe, gas consumption is in L/100 km. In the US, fuel efficiency is in miles/gallon. Let's relate these two: 1 mile = 1.609 km, 1 gal = 3.785 L.

$$\frac{1 \text{ mile}}{\text{gal}} = \frac{1.609 \text{ km}}{3.785 \text{ L}} = \frac{1.609}{3.785} \left(\frac{1}{100} \right) (100) \frac{\text{km}}{\text{L}} = (0.00425) \left(\frac{1}{\text{L}/100 \text{ km}} \right) = \frac{1}{235.24 \text{ L}/100 \text{ km}}$$

Therefore, 1 mile/gal is the reciprocal of 235.2 L/100 km.

(b) Gas consumption is $\frac{12.2 \text{ L}}{100 \text{ km}}$. Using $\frac{1 \text{ L}}{100 \text{ km}} = \frac{1}{235.24 \text{ miles/gal}}$ from part (a),

$$\frac{12.2 \text{ L}}{100 \text{ km}} = 12.2 \left(\frac{1 \text{ L}}{100 \text{ km}} \right) = 12.2 \left(\frac{1}{235.24 \text{ miles/gal}} \right) = \frac{1}{19.282 \text{ miles/gal}}.$$

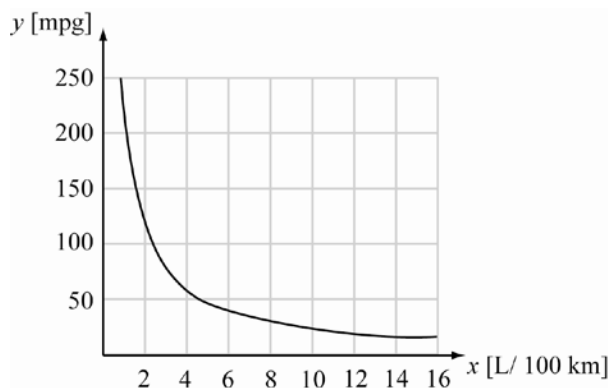
Therefore, a car that consumes 12.2 L/100 km of gasoline has a fuel efficiency of 19.3 miles/gal.

(c) If the fuel efficiency of the car is 27.4 miles per gallon, then

$$\frac{27.4 \text{ miles}}{\text{gal}} = \frac{27.4}{235.24 \text{ L}/100 \text{ km}} = \frac{1}{8.59 \text{ L}/100 \text{ km}}.$$

Therefore, 27.4 miles/gal is equivalent to 8.59 L/100 km.

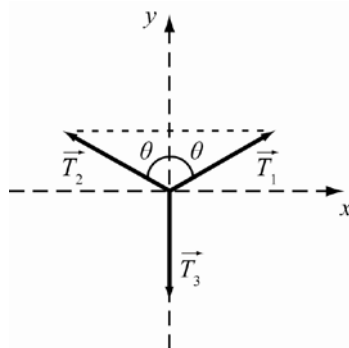
(d)



1.18. A vector is described by a set of components in a given coordinate system, where the components are the projections of the vector onto each coordinate axis. Therefore, on a two-dimensional sheet of paper there are two coordinates and thus, the vector is described by two components. In the real three-dimensional world, there are three coordinates and a vector is described by three components. A four-dimensional world would be described by four coordinates, and a vector would be described by four components.

1.19. A vector contains information about the distance between two points (the magnitude of the vector). In contrast to a scalar, it also contains information direction. In many cases knowing a direction can be as important as knowing a magnitude.

- 1.20. In order to add vectors in magnitude-direction form, each vector is expressed in terms of component vectors which lie along the coordinate axes. The corresponding components of each vector are added to obtain the components of the resultant vector. The resultant vector can then be expressed in magnitude-direction form by computing its magnitude and direction.
- 1.21. The advantage to using scientific notation is two-fold: Scientific notation is more compact (thus saving space and writing), and it also gives a more intuitive way of dealing with significant figures since you can only write the necessary significant figures and extraneous zeroes are kept in the exponent of the base.
- 1.22. The SI system of units is the preferred system of measurement due to its ease of use and clarity. The SI system is a metric system generally based on multiples of 10, and consisting of a set of standard measurement units to describe the physical world. In science, it is paramount to communicate results in the clearest and most widely understood manner. Since the SI system is internationally recognized, and its definitions are unambiguous, it is used by scientists around the world, including those in the United States.
- 1.23. It is possible to add three equal-length vectors and obtain a vector sum of zero. The vector components of the three vectors must all add to zero. Consider the following arrangement with $|\vec{T}_1| = |\vec{T}_2| = |\vec{T}_3|$:



The horizontal components of $\vec{T}_1$ and $\vec{T}_2$ cancel out, so the sum $\vec{T}_1 + \vec{T}_2$ is a vertical vector whose magnitude is $T \cos \theta + T \cos \theta = 2T \cos \theta$. The vector sum $\vec{T}_1 + \vec{T}_2 + \vec{T}_3$ is zero if

$$2T \cos \theta - T = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

Therefore it is possible for three equal-length vectors to sum to zero.

- 1.24. Mass is not a vector quantity. It is a scalar quantity since it does not make sense to associate a direction with mass.
- 1.25. The volume of a sphere is given by $V = (4/3)\pi r^3$. Doubling the volume gives $2V = 2(4/3)\pi r^3 = (4/3)\pi(2^{3/3})r^3 = (4/3)\pi(2^{1/3})r^3$. Now, since the distance between the flies is the diameter of the sphere, $d = 2r$, and doubling the volume increases the radius by a factor of $2^{1/3}$, the distance between the flies is then increased to $2(2^{1/3}r) = 2^{1/3}(2r) = 2^{1/3}d$. Therefore, the distance is increased by a factor of $2^{1/3}$.
- 1.26. The volume of a cube of side r is $V_c = r^3$, and the volume of a sphere of radius r is $V_{sp} = (4/3)\pi r^3$. The ratio of the volumes is:

$$\frac{V_c}{V_{sp}} = \frac{r^3}{\frac{4}{3}\pi r^3} = \frac{3}{4\pi}.$$

The ratio of the volumes is independent of the value of r .

- 1.27. The surface area of a sphere is given by $4\pi r^2$. A cube of side length s has a surface area of $6s^2$. To determine s set the two surface areas equal:

$$6s^2 = 4\pi r^2 \Rightarrow s = \sqrt{\frac{4\pi r^2}{6}} = r\sqrt{\frac{2\pi}{3}}.$$

- 1.28. The mass of Sun is $2 \cdot 10^{30}$ kg, the number of stars in the Milky Way is about $100 \cdot 10^9 = 10^{11}$, the number of galaxies in the Universe is about $100 \cdot 10^9 = 10^{11}$, and the mass of an H-atom is $2 \cdot 10^{-27}$ kg.

(a) The total mass of the Universe is roughly equal to the number of galaxies in the Universe multiplied by the number of stars in a galaxy and the mass of the average star:

$$M_{\text{universe}} = (10^{11})(10^{11})(2 \cdot 10^{30}) = 2 \cdot 10^{(11+11+30)} \text{ kg} = 2 \cdot 10^{52} \text{ kg}.$$

$$(b) n_{\text{hydrogen}} \approx \frac{M_{\text{universe}}}{M_{\text{hydrogen}}} = \frac{2 \cdot 10^{52} \text{ kg}}{2 \cdot 10^{-27} \text{ kg}} = 10^{79} \text{ atoms}$$

- 1.29. The volume of 1 teaspoon is about $4.93 \cdot 10^{-3}$ L, and the volume of water in the oceans is about $1.35 \cdot 10^{21}$ L.

$$\frac{1.35 \cdot 10^{21} \text{ L}}{4.93 \cdot 10^{-3} \text{ L/tsp}} = 2.74 \cdot 10^{23} \text{ tsp}$$

There are about $2.74 \cdot 10^{23}$ teaspoons of water in the Earth's oceans.

- 1.30. The average arm-span of an adult human is $d = 2$ m. Therefore, with arms fully extended, a person takes up a circular area of $\pi r^2 = \pi (d/2)^2 = \pi (1 \text{ m})^2 = \pi \text{ m}^2$. Since there are approximately $6.5 \cdot 10^9$ humans, the amount of land area required for all humans to stand without being able to touch each other is $6.5 \cdot 10^9 \text{ m}^2 (\pi) = 6.5 \cdot 10^9 \text{ m}^2 (3.14) = 2.0 \cdot 10^{10} \text{ m}^2$. The area of the United States is about $3.5 \cdot 10^6$ square miles or $9.1 \cdot 10^{12} \text{ m}^2$. In the United States there is almost five hundred times the amount of land necessary for all of the population of Earth to stand without touching each other.

- 1.31. The diameter of a gold atom is about $2.6 \cdot 10^{-10}$ m. The circumference of the neck of an adult is roughly 0.40 m. The number of gold atoms necessary to link to make a necklace is given by:

$$n = \frac{\text{circumference of neck}}{\text{diameter of atom}} = \frac{4.0 \cdot 10^{-1} \text{ m}}{2.6 \cdot 10^{-10} \text{ m/atom}} = 1.5 \cdot 10^9 \text{ atoms}.$$

The Earth has a circumference at the equator of about $4.008 \cdot 10^7$ m. The number of gold atoms necessary to link to make a chain that encircles the Earth is given by:

$$N = \frac{\text{circumference of Earth}}{\text{diameter of a gold atom}} = \frac{4.008 \cdot 10^7 \text{ m}}{2.6 \cdot 10^{-10} \text{ m}} = 1.5 \cdot 10^{17} \text{ atoms}.$$

Since one mole of substance is equivalent to about $6.022 \cdot 10^{23}$ atoms, the necklace of gold atoms has $(1.5 \cdot 10^9 \text{ atoms}) / (6.022 \cdot 10^{23} \text{ atoms/mol}) = 2.5 \cdot 10^{-15}$ moles of gold. The gold chain has $(1.5 \cdot 10^{17} \text{ atoms}) / (6.022 \cdot 10^{23} \text{ atoms/mol}) = 2.5 \cdot 10^{-7}$ moles of gold.

- 1.32. The average dairy cow has a mass of about $1.0 \cdot 10^3$ kg. Estimate the cow's average density to be that of water, $\rho = 1000. \text{ kg/m}^3$.

$$\text{volume} = \frac{\text{mass}}{\rho} = \frac{1.0 \cdot 10^3 \text{ kg}}{1000. \text{ kg/m}^3} = 1.0 \text{ m}^3$$

Relate this to the volume of a sphere to obtain the radius.

$$\text{volume} = \frac{4}{3}\pi r^3 \Rightarrow r = \left[\frac{3V}{4\pi} \right]^{1/3} = \left[\frac{3(1.0 \text{ m}^3)}{4\pi} \right]^{1/3} \approx 0.62 \text{ m}$$

A cow can be roughly approximated by a sphere with a radius of 0.62 m.

- 1.33. The mass of a head can be estimated first approximating its volume. A rough approximation to the shape of a head is a cylinder. To obtain the volume from the circumference, recall that the circumference is $C = 2\pi r$, which gives a radius of $r = C / 2\pi$. The volume is then:

$$V = (\pi r^2)h = \pi \left(\frac{C}{2\pi} \right)^2 h = \frac{C^2 h}{4\pi}.$$

The circumference of a head is about 55 cm = 0.55 m, and its height is about 20 cm = 0.20 m. These values can be used in the volume equation:

$$V = \frac{(0.55 \text{ m})^2}{4\pi} (0.20 \text{ m}) = 4.8 \cdot 10^{-3} \text{ m}^3.$$

Assuming that the density of the head is about the same as the density of water, the mass of a head can then be estimated as follows:

$$\text{mass} = \text{density} \cdot \text{volume} = (1.0 \cdot 10^3 \text{ kg/m}^3)(4.8 \cdot 10^{-3} \text{ m}^3) = 4.8 \text{ kg}.$$

- 1.34. The average adult human head is roughly a cylinder 15 cm in diameter and 20. cm in height. Assume about 1/3 of the surface area of the head is covered by hair.

$$A_{\text{hair}} = \frac{1}{3}(A_{\text{cylinder}}) = \frac{1}{3}(2\pi r^2 + 2\pi rh) = \frac{2\pi}{3}(r^2 + rh) = \frac{2\pi}{3}[(7.5 \text{ cm})^2 + (7.5 \text{ cm})(20. \text{ cm})] \\ \approx 4.32 \cdot 10^2 \text{ cm}^2$$

On average, the density of hair on the scalp is $\rho_{\text{hair}} = 2.3 \cdot 10^2 \text{ hairs/cm}^2$. Therefore, you have $A_{\text{hair}} \times \rho_{\text{hair}}$ hairs on your head.

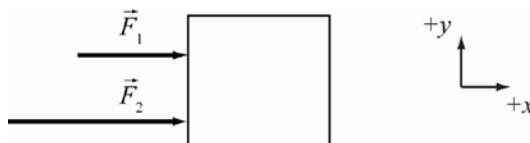
$$A_{\text{hair}} \rho_{\text{hair}} = (4.32 \cdot 10^2 \text{ cm}^2)(2.3 \cdot 10^2 \text{ hairs/cm}^2) = 9.9 \cdot 10^4 \text{ hairs}.$$

Exercises

- 1.35. (a) Three (b) Four (c) One (d) Six (e) One (f) Two (g) Three

- 1.36. **THINK:** The known quantities are: $F_1 = 2.0031 \text{ N}$ and $F_2 = 3.12 \text{ N}$. Both F_1 and F_2 are in the same direction, and act on the same object. The total force acting on the object is F_{total} .

SKETCH:



RESEARCH: Forces that act in the same direction are summed, $F_{\text{total}} = \sum F_i$.

SIMPLIFY: $F_{\text{total}} = \sum F_i = F_1 + F_2$

CALCULATE: $F_{\text{total}} = 2.0031 \text{ N} + 3.12 \text{ N} = 5.1231 \text{ N}$

ROUND: When adding (or subtracting), the precision of the result is limited by the least precise value used in the calculation. F_1 is precise to four places after the decimal and F_2 is precise to only two places after the decimal, so the result should be precise to two places after the decimal: $F_{\text{total}} = 5.12 \text{ N}$.

DOUBLE-CHECK: This result is reasonable as it is greater than each of the individual forces acting on the object.

- 1.37. The result should have the same number of decimal places as the number with the fewest of them. Therefore, the result is $2.0600 + 3.163 + 1.12 = 6.34$.
- 1.38. In a product of values, the result should have as many significant figures as the value with the smallest number of significant figures. The value for x only has two significant figures, so $w = (1.1 \cdot 10^3)(2.48 \cdot 10^{-2})(6.000) = 1.6 \cdot 10^2$.
- 1.39. Write “one ten-millionth of a centimeter” in scientific notation. One millionth is $1/10^6 = 1 \cdot 10^{-6}$. Therefore, one ten-millionth is $1/[10 \cdot 10^6] = 1/10^7 = 1 \cdot 10^{-7}$ cm.
- 1.40. $153,000,000 = 1.53 \cdot 10^8$
- 1.41. There are 12 inches in a foot and 5280 feet in a mile. Therefore there are 63,360 inch/mile. $30.7484 \text{ miles} \cdot 63,360 \text{ inch/mile} = 1948218.624 \text{ inches}$. Rounding to six significant figures and expressing the answer in scientific notation gives $1.94822 \cdot 10^6$ inches.
- 1.42. (a) kilo (b) centi (c) milli
- 1.43. $1 \text{ km} = 1 \text{ km} \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1000 \text{ mm}}{1 \text{ m}} \right) = 1,000,000 \text{ mm} = 1 \cdot 10^6 \text{ mm}$
- 1.44. 1 hectare = 100 ares, and 1 are = 100 m^2 , so:
- $$1 \text{ km}^2 = 1 \text{ km}^2 \left(\frac{(1000)^2 \text{ m}^2}{1 \text{ km}^2} \right) \left(\frac{1 \text{ are}}{100 \text{ m}^2} \right) \left(\frac{1 \text{ hectare}}{100 \text{ ares}} \right) = 100 \text{ hectares.}$$
- 1.45. 1 milliPascal
- 1.46. **THINK:** The known quantities are the masses of the four sugar cubes. Crushing the sugar cubes doesn't change the mass. Their masses, written in standard SI units, using scientific notation are $m_1 = 2.53 \cdot 10^{-2} \text{ kg}$, $m_2 = 2.47 \cdot 10^{-2} \text{ kg}$, $m_3 = 2.60 \cdot 10^{-2} \text{ kg}$ and $m_4 = 2.58 \cdot 10^{-2} \text{ kg}$.

SKETCH: A sketch is not needed to solve this problem.

RESEARCH:

(a) The total mass equals the sum of the individual masses: $M_{\text{total}} = \sum_{j=1}^4 m_j$.

(b) The average mass is the sum of the individual masses, divided by the total number of masses:

$$M_{\text{average}} = \frac{m_1 + m_2 + m_3 + m_4}{4}.$$

SIMPLIFY:

(a) $M_{\text{total}} = m_1 + m_2 + m_3 + m_4$

(b) $M_{\text{average}} = \frac{M_{\text{total}}}{4}$

CALCULATE:

(a) $M_{\text{total}} = 2.53 \cdot 10^{-2} \text{ kg} + 2.47 \cdot 10^{-2} \text{ kg} + 2.60 \cdot 10^{-2} \text{ kg} + 2.58 \cdot 10^{-2} \text{ kg}$
 $= 10.18 \cdot 10^{-2} \text{ kg}$
 $= 1.018 \cdot 10^{-1} \text{ kg}$

(b) $M_{\text{average}} = \frac{10.18 \cdot 10^{-2} \text{ kg}}{4} = 2.545 \cdot 10^{-2} \text{ kg}$

ROUND:

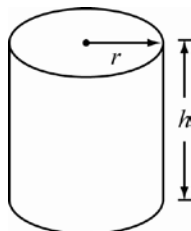
(a) Rounding to three significant figures, $M_{\text{total}} = 1.02 \cdot 10^{-1} \text{ kg}$.

(b) Rounding to three significant figures, $M_{\text{average}} = 2.55 \cdot 10^{-2} \text{ kg}$.

DOUBLE-CHECK: There are four sugar cubes weighing between $2.53 \cdot 10^{-2}$ kg and $2.60 \cdot 10^{-2}$ kg, so it is reasonable that their total mass is $M_{\text{total}} = 1.02 \cdot 10^{-1}$ kg and their average mass is $2.55 \cdot 10^{-2}$ kg.

- 1.47. **THINK:** The cylinder has height $h = 20.5$ cm and radius $r = 11.9$ cm.

SKETCH:



RESEARCH: The surface area of a cylinder is $A = 2\pi rh + 2\pi r^2$.

SIMPLIFY: $A = 2\pi r(h + r)$

CALCULATE: $A = 2\pi(11.9 \text{ cm})(20.5 \text{ cm} + 11.9 \text{ cm}) = 2422.545 \text{ cm}^2$

ROUND: Three significant figures: $A = 2.42 \cdot 10^3 \text{ cm}^2$.

DOUBLE-CHECK: The units of area are a measure of distance squared so the answer is reasonable.

- 1.48. **THINK:** When you step on the bathroom scale, your mass and gravity exert a force on the scale and the scale displays your weight. The given quantity is your mass $m_1 = 125.4$ lbs. Pounds can be converted to SI units using the conversion $1 \text{ lb} = 0.4536 \text{ kg}$. Let your mass in kilograms be m_2 .

SKETCH: A sketch is not needed to solve this problem.

RESEARCH: $m_2 = m_1 \left(\frac{0.4536 \text{ kg}}{1 \text{ lb}} \right)$

SIMPLIFY: It is not necessary to simplify.

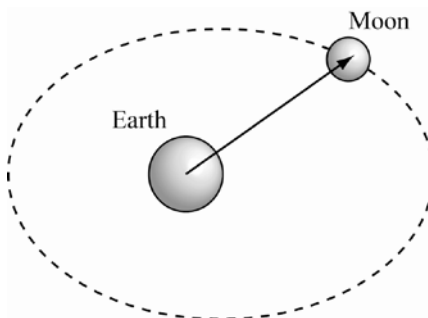
CALCULATE: $m_2 = 125.4 \text{ lbs} \left(\frac{0.4536 \text{ kg}}{1 \text{ lb}} \right) = 56.88144 \text{ kg}$

ROUND: The given quantity and conversion factor contain four significant figures, so the result must be rounded to 56.88 kg.

DOUBLE-CHECK: The SI units of mass are kg, so the units are correct.

- 1.49. **THINK:** The orbital distance from the center of the Moon to the center of the Earth ranges from 356,000 km to 407,000 km. Recall the conversion factor $1 \text{ mile} = 1.609344 \text{ kilometer}$.

SKETCH:



RESEARCH: Let d_1 be a distance in kilometers, and d_2 the equivalent distance in miles. The formula to convert from kilometers to miles is $d_2 = d_1 / 1.609344$.

SIMPLIFY: It is not necessary to simplify.

CALCULATE: $356,000 \text{ km} \left(\frac{1 \text{ mile}}{1.609344 \text{ km}} \right) = 221208.144 \text{ miles}$

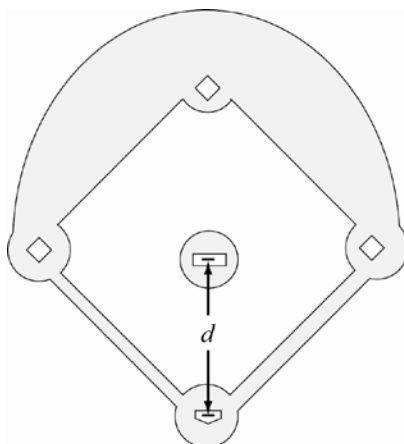
$407,000 \text{ km} \left(\frac{1 \text{ mile}}{1.609344 \text{ km}} \right) = 252898.0752 \text{ miles}$

ROUND: The given quantities have three significant figures, so the calculated values must be rounded to 221,000 miles and 253,000 miles respectively.

DOUBLE-CHECK: A kilometer is roughly 2/3 of a mile, and the answers are roughly 2/3 of the given values, so the conversions appear correct.

- 1.50. THINK:** It is a distance $d = 60$ feet, 6 inches from the pitcher's mound to home plate. Recall the conversion factors: 1 foot = 12 inches, 1 inch = 2.54 cm, 100 cm = 1 m.

SKETCH:



RESEARCH: If the distance is x in meters and y in feet, then using the conversion factor c , $x = cy$.

$$c = 1 \text{ foot} \left(\frac{12 \text{ inches}}{1 \text{ foot}} \right) \left(\frac{2.54 \text{ cm}}{1 \text{ inch}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) / \text{foot}$$

SIMPLIFY: $c = 0.3048 \text{ meters/foot}$

CALCULATE: 60 feet plus 6 inches = 60.5 feet. Then, converting 60.5 feet to meters:

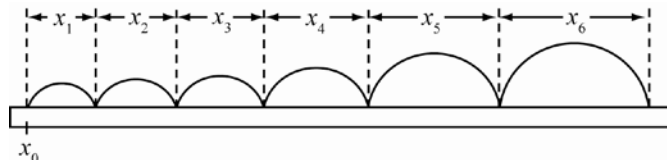
$$d = 60.5 \text{ ft} \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 18.440 \text{ m}.$$

ROUND: Rounding to three significant figures, the distance is 18.4 m.

DOUBLE-CHECK: The answer is a reasonable distance for a pitcher to throw the ball.

- 1.51. THINK:** The given quantities, written in scientific notation and in units of meters, are: the starting position, $x_0 = 7 \cdot 10^{-3} \text{ m}$ and the lengths of the flea's successive hops, $x_1 = 3.2 \cdot 10^{-2} \text{ m}$, $x_2 = 6.5 \cdot 10^{-2} \text{ m}$, $x_3 = 8.3 \cdot 10^{-2} \text{ m}$, $x_4 = 10.0 \cdot 10^{-2} \text{ m}$, $x_5 = 11.5 \cdot 10^{-2} \text{ m}$ and $x_6 = 15.5 \cdot 10^{-2} \text{ m}$. The flea makes six jumps in total.

SKETCH:



RESEARCH: The total distance jumped is $x_{\text{total}} = \sum_{n=1}^6 x_n$. The average distance covered in a single hop is:

$$x_{\text{avg}} = \frac{1}{6} \sum_{n=1}^6 x_n.$$

SIMPLIFY: $x_{\text{total}} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$, $x_{\text{avg}} = \frac{x_{\text{total}}}{6}$

CALCULATE: $x_{\text{total}} = (3.2 \text{ m} + 6.5 \text{ m} + 8.3 \text{ m} + 10.0 \text{ m} + 11.5 \text{ m} + 15.5 \text{ m}) \cdot 10^{-2} = 55.0 \cdot 10^{-2} \text{ m}$

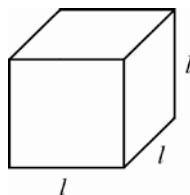
$$x_{\text{avg}} = \frac{55.0 \cdot 10^{-2} \text{ m}}{6} = 9.16666 \cdot 10^{-2} \text{ m}$$

ROUND: Each of the hopping distances is measured to 1 mm precision. Therefore the total distance should also only be quoted to 1 mm precision: $x_{\text{total}} = 55.0 \cdot 10^{-2} \text{ m}$. Rounding the average distance to the right number of significant digits, however, requires a few more words. As a general rule of thumb the average distance should be quoted to the same precision as the least precision of the individual distances, if there are only a few measurements contributing to the average. This is the case here, and so we state $x_{\text{avg}} = 9.17 \cdot 10^{-2} \text{ m}$. However, suppose we had 10,000 measurements contributing to an average. Surely we could then specify the average to a higher precision. The rule of thumb is that we can add one additional significant digit for every order of magnitude of the number of independent measurements contributing to an average. You see that the answer to this problem is yet another indication that specifying the correct number of significant figures can be complicated and sometimes outright tricky!

DOUBLE-CHECK: The flea made 6 hops, ranging from $3.2 \cdot 10^{-2} \text{ m}$ to $15.5 \cdot 10^{-2} \text{ m}$, so the total distance covered is reasonable. The average distance per hop falls in the range between $3.2 \cdot 10^{-2} \text{ m}$ and $1.55 \cdot 10^{-1} \text{ m}$, which is what is expected.

- 1.52. **THINK:** The question says that 1 cm^3 of water has a mass of 1 g, that $1 \text{ mL} = 1 \text{ cm}^3$, and that 1 metric ton is 1000 kg.

SKETCH:



RESEARCH: For the first part of the question, use the conversion equation:

$$1 \text{ L} = 1 \text{ L} \left(\frac{1000 \text{ mL}}{1 \text{ L}} \right) \left(\frac{1 \text{ cm}^3}{1 \text{ mL}} \right) \left(\frac{1 \text{ g}}{1 \text{ cm}^3} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right).$$

For the second part of the question, use:

$$1 \text{ metric ton} = 1 \text{ metric ton} \left(\frac{1000 \text{ kg}}{1 \text{ metric ton}} \right) \left(\frac{1000 \text{ g}}{1 \text{ kg}} \right) \left(\frac{1 \text{ cm}^3}{1 \text{ g}} \right).$$

For the last part, recall that the volume of a cube is $V = l^3$.

SIMPLIFY: Re-arranging the formula for the volume of the cubical tank to solve for the length gives $l = \sqrt[3]{V_c}$.

$$\text{CALCULATE: } 1 \text{ L} = 1 \text{ L} \left(\frac{1000 \text{ mL}}{1 \text{ L}} \right) \left(\frac{1 \text{ cm}^3}{1 \text{ mL}} \right) \left(\frac{1 \text{ g}}{1 \text{ cm}^3} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 1 \text{ kg}$$

$$1 \text{ metric ton} = 1 \text{ metric ton} \left(\frac{1000 \text{ kg}}{1 \text{ metric ton}} \right) \left(\frac{1000 \text{ g}}{1 \text{ kg}} \right) \left(\frac{1 \text{ cm}^3}{1 \text{ g}} \right) = 1000000 \text{ cm}^3$$

$$l = \sqrt[3]{1,000,000} = 100 \text{ cm} = 1 \text{ m}$$

ROUND: No rounding is necessary.

DOUBLE-CHECK: In each calculation the units work out correctly, so the answers are reasonable.

- 1.53. THINK:** The given quantity is the speed limit, which is 45 miles per hour. The question asks for the speed limit in millifurlongs per microfortnight. The conversions 1 furlong = 1/8 mile, and 1 fortnight = 2 weeks are given in the question.

SKETCH: A sketch is not needed.

RESEARCH:

$$\frac{1 \text{ mile}}{1 \text{ hour}} = \frac{1 \text{ mile}}{1 \text{ hour}} \left(\frac{8 \text{ furlongs}}{1 \text{ mile}} \right) \left(\frac{10^3 \text{ millifurlongs}}{1 \text{ furlong}} \right) \left(\frac{24 \text{ hours}}{1 \text{ day}} \right) \left(\frac{14 \text{ days}}{1 \text{ fortnight}} \right) \left(\frac{1 \text{ fortnight}}{10^6 \text{ microfortnights}} \right)$$

SIMPLIFY: $\frac{1 \text{ mile}}{1 \text{ hour}} = 2.688 \frac{\text{millifurlongs}}{\text{microfortnight}}$

CALCULATE: $\frac{45 \text{ miles}}{\text{hour}} = 45 \left(2.688 \frac{\text{millifurlongs}}{\text{microfortnight}} \right) = 120.96 \frac{\text{millifurlongs}}{\text{microfortnight}}$

ROUND: Because the given quantity contains two significant figures, the result must be rounded to remain consistent. A speed of 45 miles per hour is equivalent to a speed of 120 millifurlongs/microfortnight.

DOUBLE-CHECK: The conversion factor works out to be roughly 3 millifurlongs per microfortnight to each mile per hour, so the answer is reasonable.

- 1.54. THINK:** The density of water is $\rho = 1000. \text{ kg/m}^3$. Determine if a pint of water weighs a pound. Remember that 1.00 kg = 2.21 lbs and 1.00 fluid ounce = 29.6 mL.

SKETCH: A sketch is not needed.

RESEARCH: 1 pint = 16 fluid ounces, mass = density · volume

SIMPLIFY: $1 \text{ pint} = 1 \text{ pint} \left(\frac{16 \text{ fl. oz}}{1 \text{ pint}} \right) \left(\frac{29.6 \text{ mL}}{1.00 \text{ fl. oz}} \right) \left(\frac{1 \text{ cm}^3}{1 \text{ mL}} \right) \left(\frac{1 \text{ m}^3}{(100)^3 \text{ cm}^3} \right) = 4.736 \cdot 10^{-4} \text{ m}^3$

CALCULATE: $m = (1000. \text{ kg/m}^3)(4.736 \cdot 10^{-4} \text{ m}^3) = 0.4736 \text{ kg}$

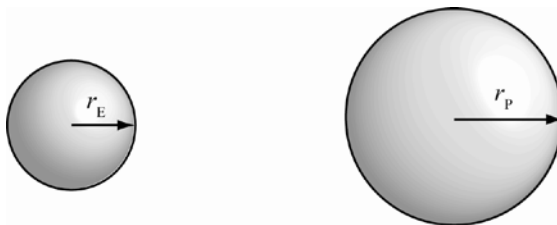
In pounds m is equal to $0.4736 \text{ kg} \left(\frac{2.21 \text{ lbs}}{1.00 \text{ kg}} \right) = 1.046656 \text{ lbs}$.

ROUND: Rounding to three significant figures, the weight is 1.05 lbs.

DOUBLE-CHECK: A pint is still a common measure for beverages, such as beer. A beer is relatively light and mainly comprised of water, so the answer is reasonable.

- 1.55. THINK:** The radius of a planet, r_p , is 8.7 times greater than the Earth's radius, r_E . Determine how many times bigger the surface area of the planet is compared to the Earth's. Assume the planets are perfect spheres.

SKETCH:



RESEARCH: The surface area of a sphere is $A = 4\pi r^2$, so $A_E = 4\pi r_E^2$, and $A_p = 4\pi r_p^2$, and $r_p = 8.7r_E$.

SIMPLIFY: $A_p = 4\pi (8.7r_E)^2$

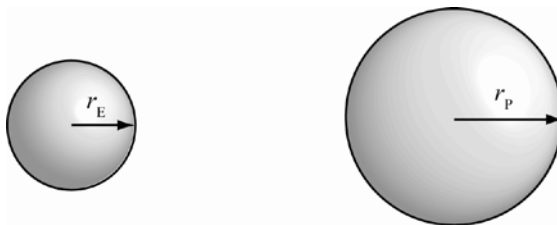
CALCULATE: $A_p = (75.69)4\pi r_E^2$, and $A_E = 4\pi r_E^2$. By comparison, $A_p = 75.69A_E$.

ROUND: Rounding to two significant figures, the surface area of the planet is 76 times the surface area of Earth.

DOUBLE-CHECK: Since the area is proportional to the radius squared, it is expected that the surface area of the planet will be much larger than the surface area of the Earth, since its radius is 8.7 times Earth's radius.

- 1.56. **THINK:** The radius of the planet r_p is 5.8 times larger than the Earth's radius r_E . Assume the planets are perfect spheres.

SKETCH:



RESEARCH: The volume of a sphere is given by $V = (4/3)\pi r^3$. The volume of the planet is $V_p = (4/3)\pi r_p^3$. The volume of the Earth is $V_E = (4/3)\pi r_E^3$. $r_p = 5.8r_E$.

SIMPLIFY: $V_p = (4/3)\pi(5.8r_E)^3$

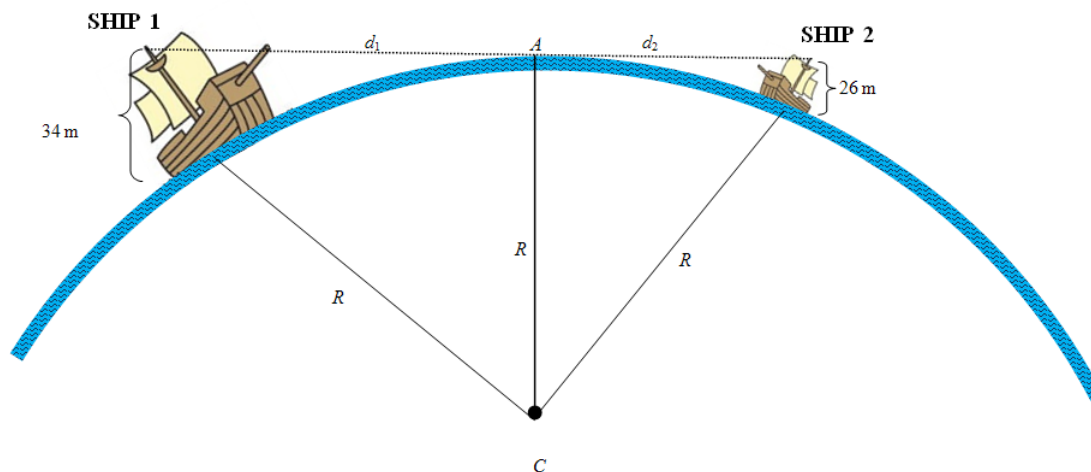
CALCULATE: $V_p = (4/3)\pi(5.8r_E)^3 = 195.112r_E^3(4/3)\pi$. Recall, $V_E = (4/3)\pi r_E^3$. Comparing the expressions, $V_p = 195.112V_E$.

ROUND: To two significant figures, so $V_p = 2.0 \cdot 10^2 V_E$.

DOUBLE-CHECK: The volume of the planet is about 200 times the volume of the Earth. The volume of a sphere is proportional to the radius cubed, it is reasonable to get a much larger volume for the planet compared to the Earth's volume.

- 1.57. **THINK:** It is necessary to take the height of both masts and the curvature of the Earth into account when calculating the distance at which they can see one another. If the ships are at the maximum distance at which the sailors can see one another, then the line between the first sailor and the second sailor will be tangent to the surface of the earth.

SKETCH: Since it is necessary to take the curvature of the earth into account when solving this problem, the sketch will not be to scale. The radius of the Earth is labeled R and the center of the earth is labeled C . The farthest point on the horizon that can be seen by both sailors, which is also the point at which the line of sight between them is tangent to the Earth, is labeled A . The distance from the first sailor to point A is d_1 and the distance from the second sailor to point A is d_2 .



RESEARCH: Because the line of sight between the sailors is tangent to the earth, it is perpendicular to the radius of the earth at point A. This means that the triangle formed by the first sailor, point A, and the center of the earth (point C) is a right triangle. The second sailor, point A, and point C also form a right triangle. Examining the figure, we can use the Pythagorean Theorem to find equations relating d_1 and d_2 to R : $R^2 + d_1^2 = (R + 34)^2$ and $R^2 + d_2^2 = (R + 26)^2$. The total distance will be the sum $d_1 + d_2$.

SIMPLIFY: First find expressions for the distances d_1 and d_2 and then use those to find the sum. The equation for d_1 gives:

$$R^2 + d_1^2 - R^2 = (R + 34)^2 - R^2 \Rightarrow$$

$$d_1^2 = (R^2 + 2 \cdot 34R + 34^2) - R^2 \Rightarrow$$

$$d_1 = \sqrt{2 \cdot 34R + 34^2}$$

Similar calculations are used to find d_2 :

$$R^2 + d_2^2 - R^2 = (R + 26)^2 - R^2 \Rightarrow$$

$$d_2^2 = (R^2 + 2 \cdot 26R + 26^2) - R^2 \Rightarrow$$

$$d_2 = \sqrt{2 \cdot 26R + 26^2}$$

Combine to get an expression for the total distance between the ships:

$$d_1 + d_2 = \sqrt{2 \cdot 34R + 34^2} + \sqrt{2 \cdot 26R + 26^2}.$$

CALCULATE: The radius of the earth is given in Solved Problem 1.2 as 6.37×10^6 m. Using this gives a final answer of:

$$\begin{aligned} d_1 + d_2 &= \sqrt{2 \cdot 34 \text{ m} \cdot (6.37 \times 10^6 \text{ m}) + (34 \text{ m})^2} + \sqrt{2 \cdot 26 \text{ m} \cdot (6.37 \times 10^6 \text{ m}) + (26 \text{ m})^2} \\ &= 39,012.54259 \text{ m.} \end{aligned}$$

ROUND: The radius of the earth used in this problem is known to three significant figures. However, the heights of the masts of the two ships are given to two significant figures. So, the final answer should have two significant figures: 3.9×10^4 m.

DOUBLE-CHECK: The maximum distance between the ships is a distance, so the units of meters seem correct. The calculated maximum distance at which the two sailors can see one another is 39 km. Calculating

$$d_1 = \sqrt{2 \cdot 26 \text{ m} \cdot (6.37 \times 10^6 \text{ m}) + (26 \text{ m})^2} = 21 \text{ km}$$

and

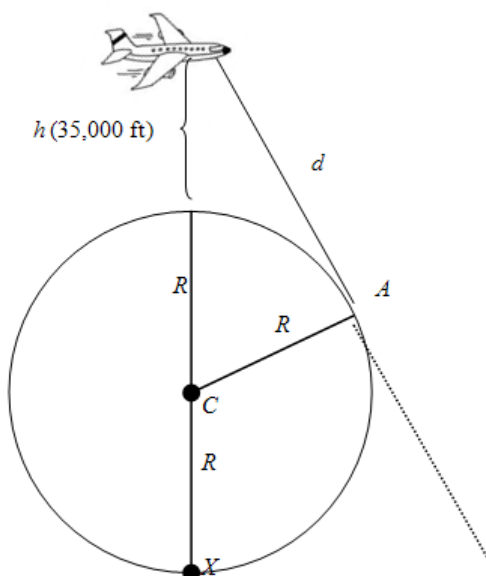
$$d_2 = \sqrt{2 \cdot 34 \text{ m} \cdot (6.37 \times 10^6 \text{ m}) + (34 \text{ m})^2} = 18 \text{ km}$$

confirms that the sailor from ship 1, sitting at the top of a slightly taller mast, can see about 3 km further than his companion. These distances seem reasonable: an average person looking out over the ocean sees about 4.7 km, and the view from 413 m atop the Willis Tower was calculated to be 72.5 km. Since the masts are significantly taller than a person but much shorter than the height of the Willis Tower, the final answer of 39 km seems reasonable.

An alternate way to calculate this would have been to use the secant-tangent theorem, which states that the square of the distance from the sailor to the horizon equals the product the height of the mast times the sum of the height of the mast and the diameter of the earth: $(d_1)^2 = (2R + 34) \cdot 34$ and $(d_2)^2 = (2R + 26) \cdot 26$. Using this formula confirms the answer:

$$\begin{aligned}
 d_1 + d_2 &= \sqrt{(2R + 34) \cdot 34} + \sqrt{(2R + 26) \cdot 26} \\
 &= \sqrt{\left(2\left(6.37 \times 10^6 \text{ m}\right) + 34 \text{ m}\right) \cdot 34 \text{ m}} + \sqrt{\left(2\left(6.37 \times 10^6 \text{ m}\right) + 26 \text{ m}\right) \cdot 26 \text{ m}} \\
 &= 39 \text{ km}
 \end{aligned}$$

- 1.58. THINK:** The altitude of the jet liner is given in feet, so it will be necessary to convert to meters before calculating the answer. The horizon is the furthest point that can be seen in perfect weather conditions. Since we don't know where and when the plane is flying, we will approximate the Earth as a perfect sphere. **SKETCH:** Since the radius of the earth is important here, the sketch will not be to scale. Point A is a furthest point on the horizon that can be seen from the plane, and C marks the center of the Earth, and R indicates the radius of the Earth. Point X is on the surface of the Earth directly opposite from where the plane is flying.



RESEARCH: The line of sight from the plane to the furthest point on the horizon (point A) is tangential to the Earth at point A. So, it must be perpendicular to the radius of the Earth at point A. This means that the plane, point A, and point C form a right triangle. The Pythagorean Theorem states that $d^2 + R^2 = (R + h)^2$. To find the distance d , it is necessary to use consistent units. Since the radius of the Earth (R) is given in meters, it is easiest to convert the height h from feet to meters using the fact that $1 \text{ m} = 3.281 \text{ ft}$.

SIMPLIFY: First convert the height of the plane from feet to meters, multiplying h by $\frac{1 \text{ m}}{3.281 \text{ ft}}$. Then,

solve the expression $d^2 + R^2 = (R + h)^2$ for d , the distance we want to find:

$$\begin{aligned}
 d^2 + R^2 - R^2 &= (R + h)^2 - R^2 \Rightarrow \\
 d^2 &= (R^2 + 2Rh + h^2) - R^2 \Rightarrow \\
 d &= \sqrt{2Rh + h^2}
 \end{aligned}$$

CALCULATE: The radius of the Earth $R = 6.37 \cdot 10^6 \text{ m}$ and the plane is flying $h = 35,000 \text{ ft} \cdot \frac{1 \text{ m}}{3.281 \text{ ft}}$ above the ground. Using these numbers, the distance to the horizon is

$$d = \sqrt{2\left(6.37 \times 10^6 \text{ m}\right)\left(35,000 \text{ ft} \frac{1 \text{ m}}{3.281 \text{ ft}}\right) + \left(35,000 \text{ ft} \frac{1 \text{ m}}{3.281 \text{ ft}}\right)^2} = 368,805.4813 \text{ m}.$$

ROUND: Though it is ambiguous, the height of the jetliner above the ground is known to at least two significant figures. The radius of the Earth is known to three significant figures and the conversion from feet to meters uses four significant figures. So, the answer is known to two significant digits. This gives a final distance of $3.7 \cdot 10^5$ m or 370 km.

DOUBLE-CHECK: The answer is given in units of meters or kilometers. Since the distance to the horizon is a length, the units are correct. 370 km is the approximate distance between Los Angeles and Las Vegas. Indeed, in an airplane at cruising altitude, it is just possible to see the Los Angeles coast as you fly over Las Vegas, so this answer seems reasonable. It is also possible to check the answer by working backwards. The secant-tangent theorem states that the square of the distance d equals the product of the height of the plane over the Earth h and the distance from the jetliner to point X on the other side of the Earth. Use this to find the height of the plane in terms of the distance to the horizon and the radius of the earth:

$$\begin{aligned} d^2 &= h(h + 2R) = h^2 + 2Rh \Rightarrow \\ 0 &= h^2 + 2Rh - d^2 \Rightarrow \\ h &= \frac{-2R \pm \sqrt{(2R)^2 + 4d^2}}{2} = -R \pm \sqrt{R^2 + d^2} \\ &= -(6.37 \times 10^6 \text{ m}) + \sqrt{(6.37 \times 10^6 \text{ m})^2 + (3.7 \times 10^5 \text{ m})^2} \\ &= 10736.63 \text{ m} \end{aligned}$$

Converting this back to feet and rounding to 2 significant figures gives confirmation that the answer was

$$\text{correct: } 10736.63459 \text{ m} \cdot \frac{3.281 \text{ ft}}{1 \text{ m}} = 35,000 \text{ ft.}$$

- 1.59. THINK:** The given quantity is 1.56 barrels of oil. Calculate how many cubic inches are in 1.56 barrels. 1 barrel of oil = 42 gallons = (42 gal)(231 cu. in./gal) = 9702 cubic inches.

SKETCH: A sketch is not needed.

RESEARCH: If a volume V_1 is given in barrels then the equivalent volume V_2 in cubic inches is given by

$$\text{the formula } V_2 = V_1 \frac{9702 \text{ cu. in.}}{1 \text{ barrel}}$$

SIMPLIFY: Not applicable.

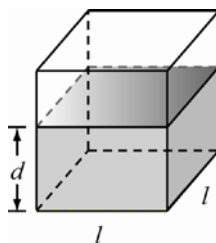
$$\text{CALCULATE: } 1.56 \text{ barrels} \left(\frac{9702 \text{ cu. in.}}{1 \text{ barrel}} \right) = 15135.12 \text{ cu. in.}$$

ROUND: The value given in the question has three significant figures, so the final answer is that 1.56 barrels is equivalent to $1.51 \cdot 10^4$ cubic inches.

DOUBLE-CHECK: Barrels are not commonly used units. However, since the proper conversion factor of 9702 cubic inches per barrel was used, the answer is accurate.

- 1.60. THINK:** The car's gas tank has the shape of a rectangular box with a square base whose sides measure $l = 62$ cm. The tank has only 1.5 L remaining. The question asks for the depth, d of the gas remaining in the tank. The car is on level ground, so that d is constant.

SKETCH:



RESEARCH: $A_{\text{tank}} = l^2$. The volume of gas remaining is $V_{\text{gas}} = A_{\text{tank}} \times d$. Convert the volume 1.5 L to 1500 cm^3 by using $1 \text{ mL} = 1 \text{ cm}^3$.

SIMPLIFY: $d = V_{\text{gas}} / A_{\text{tank}}$, but $A_{\text{tank}} = l^2$, so substitute this into the expression for d : $d = V_{\text{gas}} / l^2$.

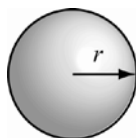
CALCULATE: $d = \frac{1500 \text{ cm}^3}{(62 \text{ cm})^2} = 0.390218 \text{ cm}$

ROUND: To two significant figures $d = 0.39 \text{ cm}$.

DOUBLE-CHECK: The car's gas tank will hold 52 L but only has 1.5 L remaining. The sides of the gas tank are 62 cm and because the gas tank is almost empty, there should be a small depth of gas in the bottom of the tank, so the answer is reasonable.

- 1.61. THINK:** The formula for the volume of a sphere is given by $V_{\text{sphere}} = (4/3)\pi r^3$. The formula for density is given by $\rho = m/V$. Refer to Appendix B in the text book and express the answers in SI units using scientific notation.

SKETCH:



RESEARCH: The radius of the Sun is $r_s = 6.96 \cdot 10^8 \text{ m}$, the mass of the Sun is $m_s = 1.99 \cdot 10^{30} \text{ kg}$, the radius of the Earth is $r_E = 6.37 \cdot 10^6 \text{ m}$, and the mass of the Earth is $m_E = 5.98 \cdot 10^{24} \text{ kg}$.

SIMPLIFY: Not applicable.

CALCULATE:

(a) $V_s = \frac{4}{3}\pi r_s^3 = \frac{4}{3}\pi(6.96 \cdot 10^8)^3 = 1.412265 \cdot 10^{27} \text{ m}^3$

(b) $V_E = \frac{4}{3}\pi r_E^3 = \frac{4}{3}\pi(6.37 \cdot 10^6)^3 = 1.082696 \cdot 10^{21} \text{ m}^3$

(c) $\rho_s = \frac{m_s}{V_s} = \frac{1.99 \cdot 10^{30}}{1.412265 \cdot 10^{27}} = 1.40908 \cdot 10^3 \text{ kg/m}^3$

(d) $\rho_E = \frac{m_E}{V_E} = \frac{5.98 \cdot 10^{24}}{1.082696 \cdot 10^{21}} = 5.523249 \cdot 10^3 \text{ kg/m}^3$

ROUND: The given values have three significant figures, so the calculated values should be rounded as:

(a) $V_s = 1.41 \cdot 10^{27} \text{ m}^3$

(b) $V_E = 1.08 \cdot 10^{21} \text{ m}^3$

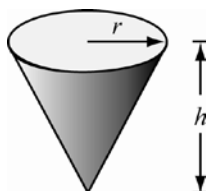
(c) $\rho_s = 1.41 \cdot 10^3 \text{ kg/m}^3$

(d) $\rho_E = 5.52 \cdot 10^3 \text{ kg/m}^3$

DOUBLE-CHECK: The radius of the Sun is two orders of magnitude larger than the radius of the Earth. Because the volume of a sphere is proportional to the radius cubed, the volume of the Sun should be $(10^2)^3$ or 10^6 larger than the volume of the Earth, so the calculated volumes are reasonable. Because density depends on mass and volume, and the Sun is roughly 10^6 times larger and more massive than the Earth, it is not surprising that the density of the Sun is on the same order of magnitude as the density of the Earth (e.g. $10^6 / 10^6 = 1$). Earth is primarily solid, but the Sun is gaseous, therefore it is reasonable that the Earth is denser than the Sun.

- 1.62. THINK:** The tank is in the shape of an inverted cone with height $h = 2.5$ m and radius $r = 0.75$ m. Water is poured into the tank at a rate of $w = 15$ L/s. Calculate how long it will take to fill the tank. Recall the conversion $1 \text{ L} = 1000 \text{ cm}^3$.

SKETCH:



RESEARCH: The volume of a cone is $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$. The rate water enters the cone is $w = \frac{V_{\text{water}}}{t}$ where t

is time. When the cone is full, $V_{\text{cone}} = V_{\text{water}}$, therefore $\frac{1}{3}\pi r^2 h = wt$.

SIMPLIFY: $t = \frac{\pi r^2 h}{3w}$

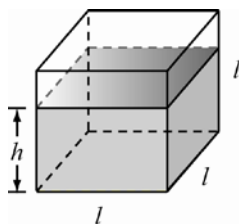
CALCULATE: $t = \frac{\pi(75 \text{ cm})^2(250 \text{ cm})}{3\left(\frac{15000 \text{ cm}^3}{\text{s}}\right)} = 98.1748 \text{ s}$

ROUND: To two significant figures, $t = 98 \text{ s}$.

DOUBLE-CHECK: The calculation resulted in the correct units, so the answer is reasonable.

- 1.63. THINK:** The rate of water flow is 15 L/s, the tank is cubical, and the top surface of the water rises by 1.5 cm/s. Let h be the height of the water in the tank.

SKETCH:



RESEARCH: The change in the volume of the water, ΔV_{water} , is $15 \text{ L/s} = 15000 \text{ cm}^3/\text{s}$. The change in the height of the water is $\Delta h = 1.5 \text{ cm/s}$. An equation to find the side length of the tank is $\Delta V_{\text{water}} = l^2 \Delta h$.

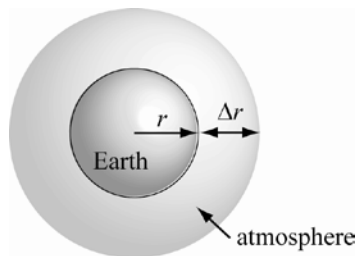
SIMPLIFY: $l = \sqrt{\frac{\Delta V_{\text{water}}}{\Delta h}}$

CALCULATE: $l = \sqrt{\left(\frac{15000 \text{ cm}^3/\text{s}}{1.5 \text{ cm/s}}\right)} = 100. \text{ cm}$

ROUND: $l = 1.0 \cdot 10^2 \text{ cm}$

DOUBLE-CHECK: The flow rate of 15 L/s is quite fast, but the level of the water is rising by only 1.5 cm/s, so it is reasonable that the tank is relatively large.

- 1.64. THINK:** The atmosphere has an effective weight of 15 pounds per square inch. By computing the surface area of the Earth, it will be easy to compute the mass of the atmosphere. Then, since the atmosphere is assumed to have a uniform density of 1.275 kg/m^3 , the mass can be converted to a volume. The volume of the atmosphere is the difference of two spheres, whose radii are the radius of the Earth, r_E , and the radius of the Earth plus the thickness of the atmosphere, Δr . The result will be a cubic equation with one real root which can be approximated to give the thickness of the atmosphere.

SKETCH:

RESEARCH: Recall the conversions $1 \text{ inch} = 0.0254 \text{ m}$ and $1 \text{ kg} = 2.205 \text{ lbs}$. The radius of the Earth is about 6378 km . The surface area of the Earth is $A_E = 4\pi r_E^2$. The mass of the atmosphere is $m_A = A_E (15 \text{ lb/sq in})$. The volume of the atmosphere can be computed using the ratio $V_A = m_A / \rho_A$, where ρ_A is the density of the atmosphere. This volume is the difference of the two spheres, as shown in the sketch. The volume of the Earth (without its atmosphere) is $V_E = (4/3)\pi r_E^3$, and the volume of the Earth and atmosphere is $V_{EA} = (4/3)\pi (r_E + \Delta r)^3$. A second method of computing the volume of the atmosphere is $V_A = V_{EA} - V_E$. Set the two values of V_A equal and solve for r .

SIMPLIFY: The first expression for the volume of the atmosphere is

$$V_A = \frac{m_A}{\rho_A} = \frac{4\pi r_E^2}{\rho_A} \left(\frac{15 \text{ lb}}{1 \text{ square inch}} \right).$$

The second expression is $V_A = (4\pi/3)((r_E + \Delta r)^3 - r_E^3)$. Setting these expressions equal to each other gives an equation to solve for Δr .

CALCULATE: $r_E = 6378 \text{ km} \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ in}}{0.0254 \text{ m}} \right) = 2.511 \cdot 10^8 \text{ in}$

$$\rho_A = 1.275 \frac{\text{kg}}{\text{m}^3} \left(\frac{0.0254 \text{ m}}{1 \text{ in}} \right)^3 = 2.089 \cdot 10^{-5} \frac{\text{kg}}{\text{in}^3}$$

Substituting into the first equation for V_A gives:

$$V_A = \frac{4\pi (2.511 \cdot 10^8 \text{ in})^2}{2.089 \cdot 10^{-5} \text{ kg/in}^3} \left(\frac{15 \text{ lb}}{1 \text{ square inch}} \right) \left(\frac{1 \text{ kg}}{2.205 \text{ lb}} \right) = 2.580 \cdot 10^{23} \text{ in}^3.$$

The second equation for V_A becomes:

$$V_A = \frac{4\pi}{3} \left((2.511 \cdot 10^8 \text{ in} + \Delta r)^3 - (2.511 \cdot 10^8 \text{ in})^3 \right) = 4.1888 \Delta r^3 + (3.155 \cdot 10^9) \Delta r^2 + (7.923 \cdot 10^{17}) \Delta r.$$

Setting the two equations for V_A equal results in the equation:

$$4.1888 \Delta r^3 + (3.155 \cdot 10^9) \Delta r^2 + (7.923 \cdot 10^{17}) \Delta r = 2.580 \cdot 10^{23},$$

a cubic equation in Δr . This equation can be solved by a number of methods. A graphical estimate is sufficient. It has one real root, and that is at approximately

$$\Delta r = 325300 \text{ in} = 325300 \text{ in} \frac{0.0254 \text{ m}}{1 \text{ in}} = 8263 \text{ m}.$$

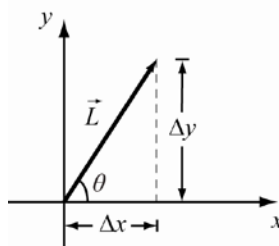
ROUND: The least precise value given in the question had two significant figures, so the answer should be rounded to 8300 m .

DOUBLE-CHECK: The result has units of distance, which is what is expected. What may not be expected is that our result is not as big as the height of the tallest mountain on Earth, Mt. Everest, which has a height of 8.8 km . We can obtain a simple approximation of our result by realizing that our calculated value for Δr is small compared to the radius of Earth. This means that the surface of a sphere with radius $R_E + \Delta r$ and

one with radius R_E are not very different, allowing us to write an approximation to our result as $\Delta r \approx V_A / (4\pi r_E^2) = (2.580 \cdot 10^{23} \text{ inch}^3) / (4\pi (2.511 \cdot 10^8 \text{ inch})^2) = 3.256 \cdot 10^5 \text{ inch} = 8.3 \text{ km}$.

- 1.65. **THINK:** Let $\vec{L}$ be the position vector. Then $|\vec{L}| = 40.0 \text{ m}$ and $\theta = 57.0^\circ$ (above x -axis).

SKETCH:



RESEARCH: From trigonometry, $\sin \theta = \Delta y / |\vec{L}|$ and $\cos \theta = \Delta x / |\vec{L}|$. The length of the vector $\vec{L}$ is given by the formula $|\vec{L}| = \sqrt{\Delta x^2 + \Delta y^2}$.

SIMPLIFY: $\Delta x = |\vec{L}| \cos \theta$, $\Delta y = |\vec{L}| \sin \theta$

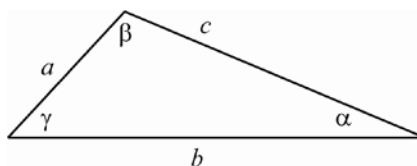
CALCULATE: $\Delta x = (40.0 \text{ m}) \cos(57.0^\circ) = 21.786 \text{ m}$, $\Delta y = (40.0 \text{ m}) \sin(57.0^\circ) = 33.547 \text{ m}$

ROUND: $\Delta x = 21.8 \text{ m}$ and $\Delta y = 33.5 \text{ m}$.

DOUBLE-CHECK: $|\vec{L}| = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(21.8 \text{ m})^2 + (33.5 \text{ m})^2} \approx 40.0 \text{ m}$, to three significant figures.

- 1.66. **THINK:** $a = 6.6 \text{ cm}$, $b = 13.7 \text{ cm}$, and $c = 9.2 \text{ cm}$ are the given quantities.

SKETCH:



RESEARCH: Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$

SIMPLIFY: $c^2 = a^2 + b^2 - 2ab \cos \gamma$

$$2ab \cos \gamma = a^2 + b^2 - c^2$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

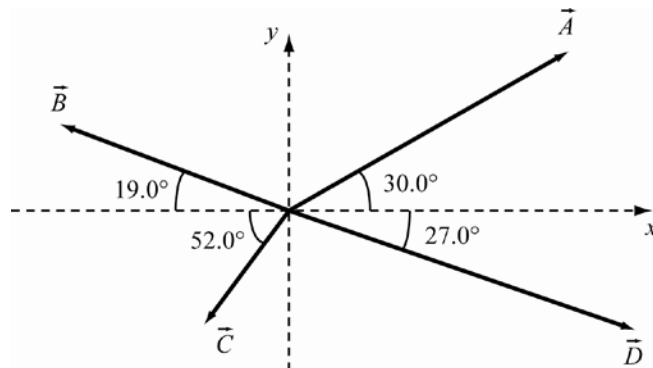
$$\gamma = \cos^{-1} \left(\frac{a^2 + b^2 - c^2}{2ab} \right)$$

CALCULATE: $\gamma = \cos^{-1} \left[\frac{(6.6 \text{ cm})^2 + (13.7 \text{ cm})^2 - (9.2 \text{ cm})^2}{2(6.6 \text{ cm})(13.7 \text{ cm})} \right] = 35.83399^\circ$

ROUND: $\gamma = 36^\circ$

DOUBLE-CHECK: The angle γ in the figure is less than 45° , so the answer is reasonable.

- 1.67. **THINK:** The lengths of the vectors are given as $|\vec{A}| = 75.0$, $|\vec{B}| = 60.0$, $|\vec{C}| = 25.0$ and $|\vec{D}| = 90.0$. The question asks for the vectors to be written in terms of unit vectors. Remember, when dealing with vectors, the x - and y -components must be treated separately.

SKETCH:**RESEARCH:** The formula for a vector in terms of unit vectors is $\vec{V} = V_x\hat{x} + V_y\hat{y}$. Since

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \text{ and } \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \sin\theta = \frac{A_y}{|\vec{A}|} \text{ and } \cos\theta = \frac{A_x}{|\vec{A}|}.$$

$$\theta_A = 30.0^\circ, \quad \theta_B = 19.0^\circ = 161.0^\circ \text{ (with respect to the positive } x\text{-axis),}$$

$$\theta_C = 52.0^\circ = 232.0^\circ \text{ (with respect to the positive } x\text{-axis),}$$

$$\theta_D = 27.0^\circ = 333.0^\circ \text{ (with respect to the positive } x\text{-axis).}$$

SIMPLIFY: $A_x = |\vec{A}|\cos\theta_A$, $A_y = |\vec{A}|\sin\theta_A$, $B_x = |\vec{B}|\cos\theta_B$, $B_y = |\vec{B}|\sin\theta_B$, $C_x = |\vec{C}|\cos\theta_C$, $C_y = |\vec{C}|\sin\theta_C$,

$$D_x = |\vec{D}|\cos\theta_D, \text{ and } D_y = |\vec{D}|\sin\theta_D.$$

$$\textbf{CALCULATE: } A_x = 75.0\cos 30.0^\circ = 64.9519\hat{x}, \quad A_y = 75.0\sin 30.0^\circ = 37.5\hat{y}$$

$$B_x = 60.0\cos 161.0^\circ = -56.73\hat{x}, \quad B_y = 60.0\sin 161.0^\circ = 19.534\hat{y}$$

$$C_x = 25.0\cos 232.0^\circ = -15.3915\hat{x}, \quad C_y = 25.0\sin 232.0^\circ = -19.70027\hat{y}$$

$$D_x = 90.0\cos 333.0^\circ = 80.19058\hat{x}, \quad D_y = 90.0\sin 333.0^\circ = -40.859144\hat{y}$$

ROUND: The given values had three significant figures so the answers must be rounded to:

$$\vec{A} = 65.0\hat{x} + 37.5\hat{y}, \quad \vec{B} = -56.7\hat{x} + 19.5\hat{y}, \quad \vec{C} = -15.4\hat{x} - 19.7\hat{y}, \quad \vec{D} = 80.2\hat{x} - 40.9\hat{y}.$$

DOUBLE-CHECK: Comparing the calculated components to the figure provided shows that this answer is reasonable.

- 1.68. THINK:** Use the components in Question 1.65 to find the sum of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$. Also, calculate the magnitude and direction of $\vec{A} - \vec{B} + \vec{D}$. Remember, when dealing with vectors the x and y components must be treated separately. Treat the values given in the question as accurate to the nearest decimal, and hence as having two significant figures.

SKETCH: Not applicable.**RESEARCH:**

(a) The resultant vector is $\vec{V} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$.

(b) The magnitude of a vector is $|\vec{V}| = \sqrt{(V_x)^2 + (V_y)^2}$. The direction of the vector $\vec{V}$ is

$$\theta_V = \tan^{-1}(V_y / V_x).$$

SIMPLIFY:

(a) $\vec{A} + \vec{B} + \vec{C} + \vec{D} = (A_x + B_x + C_x + D_x)\hat{x} + (A_y + B_y + C_y + D_y)\hat{y}$

(b) $|\vec{V}| = |\vec{A} - \vec{B} + \vec{D}| = \sqrt{(A_x - B_x + D_x)^2 + (A_y - B_y + D_y)^2}$

$$\theta_V = \tan^{-1}\left(\frac{A_y - B_y + D_y}{A_x - B_x + D_x}\right)$$

CALCULATE:

$$(a) \vec{A} + \vec{B} + \vec{C} + \vec{D} = (65.0 - 56.7 - 15.4 + 80.2)\hat{x} + (37.5 + 19.5 - 19.7 - 40.9)\hat{y} \\ = 73.1\hat{x} - 3.6\hat{y}$$

$$(b) |\vec{A} - \vec{B} + \vec{D}| = \sqrt{(65.0 - (-56.7) + 80.2)^2 + (37.5 - 19.7 - 40.9)^2} = 203.217$$

$$\theta_v = \tan^{-1} \left[\frac{37.5 - 19.7 - 40.9}{65.0 - (-56.7) + 80.2} \right] = -6.5270^\circ$$

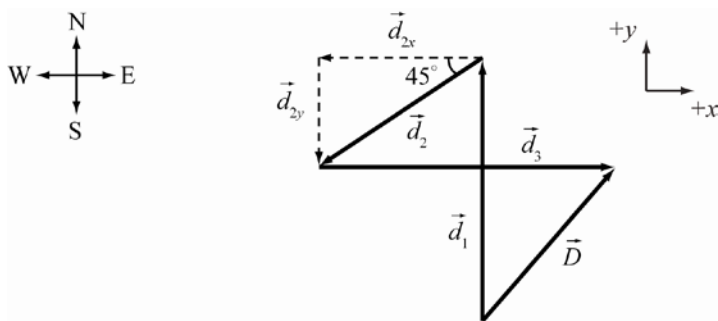
ROUND:

(a) Not necessary.

(b) The given magnitudes have three significant figures, so $|\vec{A} - \vec{B} + \vec{D}| = 203$, at -6.53° (below the x -axis).

DOUBLE-CHECK: The length of the resulting vector is less than the sum of the lengths of the component vectors. Since the vector points into the fourth quadrant, the angle made with the x -axis should be negative, as it is.

- 1.69. THINK:** The problem involves adding vectors, therefore break the vectors up into their components and add the components. SW is exactly 45° south of W. $\vec{d}_1 = 4.47$ km N, $\vec{d}_2 = 2.49$ km SW, $\vec{d}_3 = 3.59$ km E.

SKETCH:

RESEARCH: Use $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = D_x\hat{x} + D_y\hat{y}$, and recall the formula for the length of $\vec{D}$: $|\vec{D}| = \sqrt{D_x^2 + D_y^2}$. Decompose each summand vector into components $\vec{d}_i = d_{ix}\hat{x} + d_{iy}\hat{y}$, with summand vectors: $\vec{d}_1 = d_1\hat{y}$, $\vec{d}_2 = d_{2x}\hat{x} + d_{2y}\hat{y} = -d_2 \sin(45^\circ)\hat{x} - d_2 \cos(45^\circ)\hat{y}$, $\vec{d}_3 = d_3\hat{x}$.

SIMPLIFY: Therefore, $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (d_3 - d_2 \sin(45^\circ))\hat{x} + (d_1 - d_2 \cos(45^\circ))\hat{y}$ and

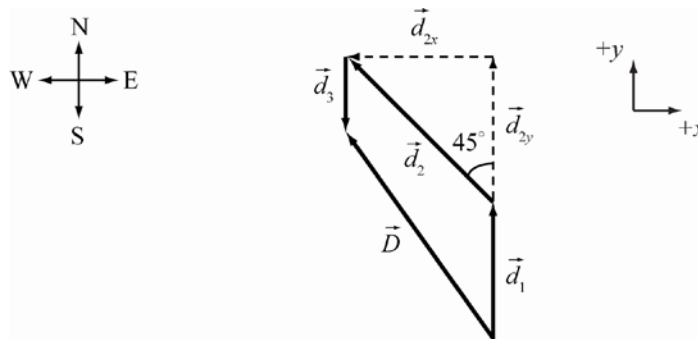
$$|\vec{D}| = \sqrt{(d_3 - d_2 \sin(45^\circ))^2 + (d_1 - d_2 \cos(45^\circ))^2}.$$

$$\text{CALCULATE: } |\vec{D}| = \sqrt{(3.59 - 2.49 \cos(45^\circ))^2 + (4.47 - 2.49 \sin(45^\circ))^2} = 3.269 \text{ km}$$

$$\text{ROUND: } |\vec{D}| = 3.27 \text{ km}$$

DOUBLE-CHECK: Given that all vectors are of the same order of magnitude, the distance from origin to final position is less than d_1 , as is evident from the sketch. This means that the calculated answer is reasonable.

- 1.70. THINK:** The problem involves adding vectors, therefore break the vectors up into their components and add the components. NW is exactly 45° north of west. $\vec{d}_1 = 20$ paces N, $\vec{d}_2 = 30$ paces NW, $\vec{d}_3 = 10$ paces S. Paces are counted to the nearest integer, so treat the number of paces as being precise.

SKETCH:

RESEARCH: Use $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = D_x \hat{x} + D_y \hat{y}$, and recall the formula for the length of $\vec{D}$: $|\vec{D}| = \sqrt{D_x^2 + D_y^2}$. Decompose each summand vector into components $\vec{d}_i = d_{ix} \hat{x} + d_{iy} \hat{y}$, with summand vectors: $\vec{d}_1 = d_1 \hat{y}$, $\vec{d}_2 = d_{2x} \hat{x} + d_{2y} \hat{y} = -d_2 \sin(45^\circ) \hat{x} - d_2 \cos(45^\circ) \hat{y}$, $\vec{d}_3 = -d_3 \hat{y}$

SIMPLIFY: $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = -d_2 \sin(45^\circ) \hat{x} + (d_1 - d_3 + d_2 \cos(45^\circ)) \hat{y}$ and

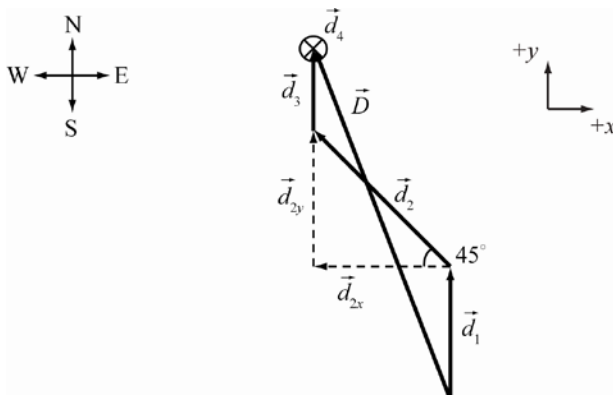
$$|\vec{D}| = \sqrt{(-d_2 \sin(45^\circ))^2 + (d_1 - d_3 + d_2 \cos(45^\circ))^2}.$$

CALCULATE: $|\vec{D}| = \sqrt{(-30 \sin(45^\circ))^2 + (20 - 10 + 30 \cos(45^\circ))^2} = 37.739 \text{ paces}$

ROUND: 38 paces

DOUBLE-CHECK: Given that $d_1 > d_3$, the calculated answer makes sense since the distance D should be greater than d_2 .

- 1.71. **THINK:** The problem involves adding vectors, therefore break the vectors up into their components and add the components. NW is 45° north of west. $\vec{d}_1 = 20 \text{ paces N}$, $\vec{d}_2 = 30 \text{ paces NW}$, $\vec{d}_3 = 12 \text{ paces N}$, $\vec{d}_4 = 3 \text{ paces into ground}$ ($\vec{d}_4$ implies 3 dimensions). Paces are counted to the nearest integer, so treat the number of paces as being precise.

SKETCH:

RESEARCH: $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4$, $\vec{d}_i = d_{ix} \hat{x} + d_{iy} \hat{y} + d_{iz} \hat{z}$, $|\vec{D}| = \sqrt{D_x^2 + D_y^2 + D_z^2}$, $\vec{d}_1 = d_1 \hat{y}$, $\vec{d}_2 = -d_{2x} \hat{x} + d_{2y} \hat{y} = -d_2 \cos(45^\circ) \hat{x} + d_2 \sin(45^\circ) \hat{y}$, $\vec{d}_3 = d_3 \hat{y}$, and $\vec{d}_4 = -d_4 \hat{z}$.

SIMPLIFY: $\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4 = -d_2 \cos(45^\circ) \hat{x} + (d_1 + d_3 + d_2 \sin(45^\circ)) \hat{y} - d_4 \hat{z}$ and

$$|\vec{D}| = \sqrt{(-d_2 \cos(45^\circ))^2 + (d_1 + d_3 + d_2 \sin(45^\circ))^2 + (-d_4)^2}.$$

CALCULATE: $\vec{D} = -30\frac{\sqrt{2}}{2}\hat{x} + \left(20 + 12 + 30\frac{\sqrt{2}}{2}\right)\hat{y} - 3\hat{z}$

$$|\vec{D}| = \sqrt{(-21.213)^2 + (53.213)^2 + (-3)^2} = 57.36 \text{ paces}$$

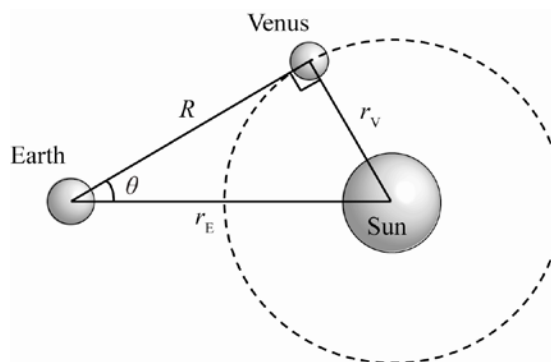
ROUND: $\vec{D} = -15\sqrt{2}\hat{x} + (32 + 15\sqrt{2})\hat{y} - 3\hat{z}$ and round the number of paces to the nearest integer:

$$|\vec{D}| = 57 \text{ paces.}$$

DOUBLE-CHECK: Distance should be less than the sum of the magnitudes of each vector, which is 65. Therefore, the calculated answer is reasonable.

- 1.72. THINK:** Consider the Sun to be the centre of a circle with the distance from the Sun to Venus, as the radius. Earth is located a distance $r_E = 1.5 \cdot 10^{11} \text{ m}$ from the Sun, so that the three bodies make a triangle and the vector from Earth to the Sun is at 0° . The vector pointing from Earth to Venus intersects Venus' orbit one or two times, depending on the angle Venus makes with the Earth. This angle is at a maximum when the vector intersects the orbit only once, while all other angles cause the vector to intersect twice. If the vector only intersects the circle once, then that vector is tangential to the circle and therefore is perpendicular to the radius vector of the orbit. This means the three bodies make a right triangle with r_E as the hypotenuse. Simple trigonometry can then be used to solve for the angle and distance.

SKETCH:



RESEARCH: $r_E^2 = r_v^2 + R^2$, $r_E \sin \theta = r_v$

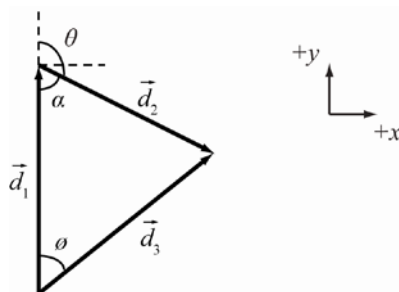
SIMPLIFY: $R = \sqrt{r_E^2 - r_v^2}$, $\theta = \sin^{-1}(r_v / r_E)$

CALCULATE: $R = \sqrt{(1.5 \cdot 10^{11})^2 - (1.1 \cdot 10^{11})^2} = 1.0198 \cdot 10^{11} \text{ m}$, $\theta = \sin^{-1}\left(\frac{1.1 \cdot 10^{11}}{1.5 \cdot 10^{11}}\right) = 47.17^\circ$

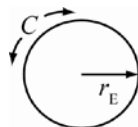
ROUND: $R = 1.0 \cdot 10^{11} \text{ m}$, $\theta = 47^\circ$

DOUBLE-CHECK: If it had been assumed that $\theta = \tan^{-1}(r_v / r_E)$ when the E-to-S-to-V angle was 90° , then $\tan \theta$ would be about 36° . Therefore the maximum angle should be greater than this, so the answer is reasonable.

- 1.73. THINK:** All angles and directions of vectors are unknown. All that is known are the distances walked, $d_1 = 550 \text{ m}$ and $d_2 = 178 \text{ m}$, and the distance $d_3 = 432 \text{ m}$ that the friend is now away from you. Since the distances are the sides of a triangle, use the cosine law to determine the internal (and then external) angles. Also, since $d_3 < d_1$, he must have turned back towards you partially, i.e. he turned more than 90° , but less than 180° .

SKETCH:**RESEARCH:** $d_2^2 = d_1^2 + d_3^2 - 2d_1d_3 \cos \phi$, $d_3^2 = d_1^2 + d_2^2 - 2d_1d_2 \cos \alpha$, $\theta + \alpha = 180^\circ$ **SIMPLIFY:** $2d_1d_3 \cos \phi = d_1^2 + d_3^2 - d_2^2 \Rightarrow \phi = \cos^{-1} \left(\frac{d_1^2 + d_3^2 - d_2^2}{2d_1d_3} \right)$ Likewise, $\alpha = \cos^{-1} \left(\frac{d_1^2 + d_2^2 - d_3^2}{2d_1d_2} \right)$.**CALCULATE:** $\phi = \cos^{-1} \left(\frac{(550 \text{ m})^2 + (432 \text{ m})^2 - (178 \text{ m})^2}{2(550 \text{ m})(432 \text{ m})} \right) = 15.714^\circ$ $\alpha = \cos^{-1} \left(\frac{(550 \text{ m})^2 + (178 \text{ m})^2 - (432 \text{ m})^2}{2(550 \text{ m})(178 \text{ m})} \right) = 41.095^\circ$, $\theta = 180^\circ - 41.095^\circ = 138.905^\circ$ **ROUND:** Since $d_1 = 550 \text{ m}$ has two significant figures (which is the fewest) the answers should be rounded to two significant figures. This means: $\phi = 16^\circ$, $\alpha = 41^\circ$ and then $\theta = 139^\circ$. The two possibilities are that the friend turned to the right or the left (a right turn is shown in the diagram). The direction the friend turned doesn't matter, he turns by the same amount regardless of which direction it was.**DOUBLE-CHECK:** The friend turned through an angle of 140 degrees. The angle between the initial departure and the final location is 16 degrees. These are both reasonable angles.

- 1.74. **THINK:** Assume that the Earth is a perfect sphere with radius, $r_E = 6378 \text{ km}$, and treat the circumference of Earth as the circumference of a circle.

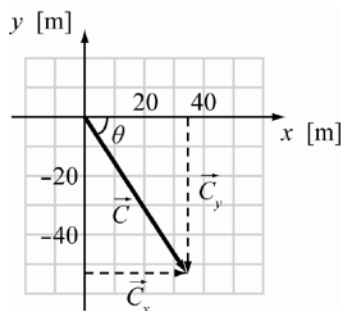
SKETCH:**RESEARCH:** The circumference of a circle is given by $C = 2\pi r$.**SIMPLIFY:** $C = 2\pi r_E$ **CALCULATE:** $C = 2\pi(6378 \text{ km}) = 40074 \text{ km}$ **ROUND:** The instructions from the question say to round to three significant figures: $C = 4.01 \cdot 10^4 \text{ km}$.**DOUBLE-CHECK:** Assuming a hot air balloon has an average velocity of 20 km/h, then it would take about 80 days to travel, hence the phrase around the world in 80 days.

1.75. $4,308,229 \approx 4 \cdot 10^6$; $44 \approx 4 \cdot 10^1$, $(4 \cdot 10^6)(4 \cdot 10^1) = 16 \cdot 10^7 = 2 \cdot 10^8$

1.76. $3\hat{x} + 6\hat{y} - 10\hat{z} + \vec{C} = -7\hat{x} + 14\hat{y}$, $\vec{C} = (-7\hat{x} - 3\hat{x}) + (14\hat{y} - 6\hat{y}) + 10\hat{z} = -10\hat{x} + 8\hat{y} + 10\hat{z}$

- 1.77. **THINK:** An angle is measured counter-clockwise from the positive x -axis (0°). $\vec{C} = (34.6 \text{ m}, -53.5 \text{ m})$. It is also possible to measure clockwise from the positive x -axis and consider the measure to be negative.

SKETCH:



RESEARCH: $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\tan \theta = \left(\frac{C_y}{C_x} \right)$

SIMPLIFY: $\theta = \tan^{-1} \left(\frac{C_y}{C_x} \right)$

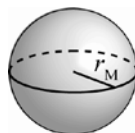
CALCULATE: $|\vec{C}| = \sqrt{(34.6 \text{ m})^2 + (-53.5 \text{ m})^2} = 63.713 \text{ m}$, $\theta = \tan^{-1} \left(\frac{-53.5 \text{ m}}{34.6 \text{ m}} \right) = -57.108^\circ$

ROUND: $\vec{C} = 63.7 \text{ m}$, $\theta = -57.1^\circ$ or 303° (equivalent angles).

DOUBLE-CHECK: The magnitude is greater than each component but less than the sum of the components and the angle is also in the correct quadrant. The answer is reasonable.

- 1.78. **THINK:** Assume Mars is a sphere whose radius is $r_M = 3.39 \cdot 10^6 \text{ m}$.

SKETCH:



RESEARCH: $C = 2\pi r$, $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$

SIMPLIFY: $C = 2\pi r_M$, $A = 4\pi r_M^2$, $V = \frac{4}{3}\pi r_M^3$

CALCULATE: $C = 2\pi(3.39 \cdot 10^6 \text{ m}) = 2.12999 \cdot 10^7 \text{ m}$

$$A = 4\pi(3.39 \cdot 10^6 \text{ m})^2 = 1.44414 \cdot 10^{14} \text{ m}^2$$

$$V = \frac{4}{3}\pi(3.39 \cdot 10^6 \text{ m})^3 = 1.63188 \cdot 10^{20} \text{ m}^3$$

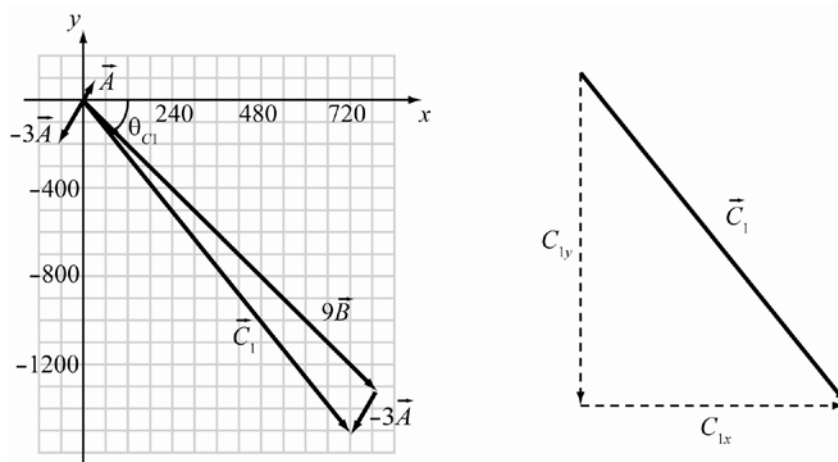
ROUND: $C = 2.13 \cdot 10^7 \text{ m}$, $A = 1.44 \cdot 10^{14} \text{ m}^2$, $V = 1.63 \cdot 10^{20} \text{ m}^3$

DOUBLE-CHECK: The units are correct and the orders of magnitude are reasonable.

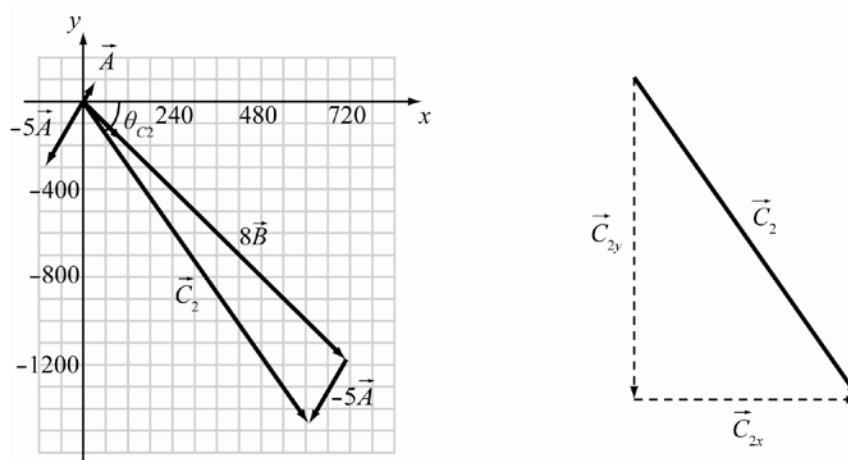
- 1.79. **THINK:** Sum the components of both vectors and find the magnitude and the angle from the positive x -axis of the resultant vector. $\vec{A} = (23.0, 59.0)$ and $\vec{B} = (90.0, -150.0)$.

SKETCH:

(a)



(b)



RESEARCH: $\vec{C} = (C_x, C_y)$, $C_i = nA_i + mB_i$, $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\tan \theta_C = \frac{C_y}{C_x}$

SIMPLIFY:

(a) Since $n = -3$ and $m = 9$, $C_x = -3A_x + 9B_x$ and $C_y = -3A_y + 9B_y$. Also, $\theta_C = \tan^{-1}(C_y / C_x)$.

(b) Since $n = -5$ and $m = 8$, $C_x = -5A_x + 8B_x$ and $C_y = -5A_y + 8B_y$. Also, $\theta_C = \tan^{-1}(C_y / C_x)$.

CALCULATE:

(a) $C_x = -3(23.0) + 9(90.0) = 741.0$, $C_y = -3(59.0) + 9(-150) = -1527.0$

$\vec{A} = (A_x, A_y) = (-30.0 \text{ m}, -50.0 \text{ m})$

(b) $C_x = -5(23.0) + 8(90.0) = 605.0$, $C_y = -5(59.0) + 8(-150) = -1495.0$

$$\theta_C = \tan^{-1}\left(\frac{-1495.0}{605.0}\right) = -67.97^\circ$$

ROUND:

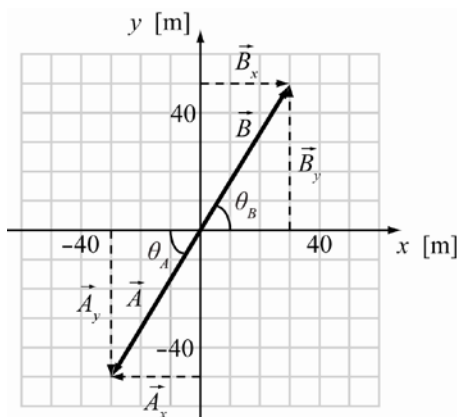
(a) $\vec{C} = 1.70 \cdot 10^3$ at -64.1° or 296°

(b) $\vec{C} = 1.61 \cdot 10^3$ at -68.0° or 292°

DOUBLE-CHECK: Each magnitude is greater than the components but less than the sum of the components and the angles place the vectors in the proper quadrants. The calculated answers are reasonable.

- 1.80. **THINK:** The vectors are $\vec{A} = (A_x, A_y) = (-30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$. Find the magnitude and angle with respect to the positive x -axis for each.

SKETCH:



RESEARCH: $\vec{C} = \sqrt{C_x^2 + C_y^2}$, $\tan \theta_C = \frac{C_y}{C_x}$

SIMPLIFY: $\vec{A} = \sqrt{A_x^2 + A_y^2}$, $\vec{B} = \sqrt{B_x^2 + B_y^2}$, $\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$, $\theta_B = \tan^{-1}\left(\frac{B_y}{B_x}\right)$

CALCULATE: $|\vec{A}| = \sqrt{(-30.0 \text{ m})^2 + (-50.0 \text{ m})^2} = 58.3095 \text{ m}$, $|\vec{B}| = \sqrt{(30.0 \text{ m})^2 + (50.0 \text{ m})^2} = 58.3095 \text{ m}$

$$\theta_A = \tan^{-1}\left(\frac{-50.0 \text{ m}}{-30.0 \text{ m}}\right) = 59.036^\circ \Rightarrow 180^\circ + 59.036^\circ = 239.036^\circ$$

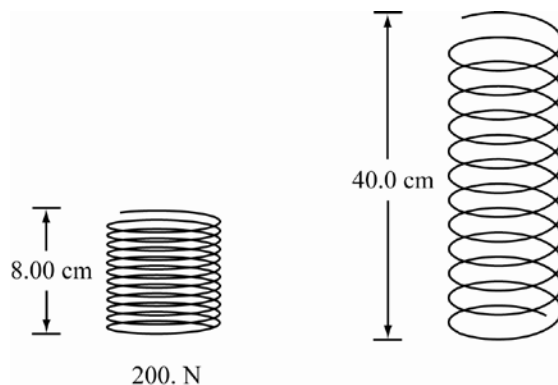
$$\theta_B = \tan^{-1}\left(\frac{50.0 \text{ m}}{30.0 \text{ m}}\right) = 59.036^\circ$$

ROUND: $\vec{A} = 58.3 \text{ m}$ at 239° or -121° , and $\vec{B} = 58.3 \text{ m}$ at 59.0° .

DOUBLE-CHECK: Each magnitude is greater than the components of the vector but less than the sum of the components and the angles place the vectors in the proper quadrants.

- 1.81. **THINK:** A variable is proportional to some other variable by a constant. This means the ratio of one variable to another is a constant. Therefore, both ratios are equal. $F_1 = 200. \text{ N}$, $x_1 = 8.00 \text{ cm}$ and $x_2 = 40.0 \text{ cm}$.

SKETCH:



RESEARCH: $\frac{F_1}{x_1} = \frac{F_2}{x_2}$

SIMPLIFY: $F_2 = \frac{F_1 x_2}{x_1}$

CALCULATE: $F_2 = \frac{(200. \text{ N})(40.0 \text{ cm})}{8.00 \text{ cm}} = 1000.0 \text{ N}$

ROUND: $F_2 = 1.00 \cdot 10^3 \text{ N}$

DOUBLE-CHECK: The ratio of distance to force remains 1:25 for the two distances. The answers are reasonable.

- 1.82. THINK:** When a variable is proportional to another, it is equal to the other variable multiplied by a constant. Call the constant “a”.

SKETCH: A sketch is not needed to solve this problem.

RESEARCH: $d = at^2$

SIMPLIFY: $d_0 = at_0^2$, $d_0' = a(3t_0)^2$

CALCULATE: $d_0' = 9at_0^2 = 9d_0$

ROUND: The distance increases by a factor of 9.

DOUBLE-CHECK: Acceleration is a quadratic relationship between distance and time. It makes sense for the amount of time to increase by a factor larger than 3.

- 1.83. THINK:** Consider the 90° turns to be precise turns at right angles.

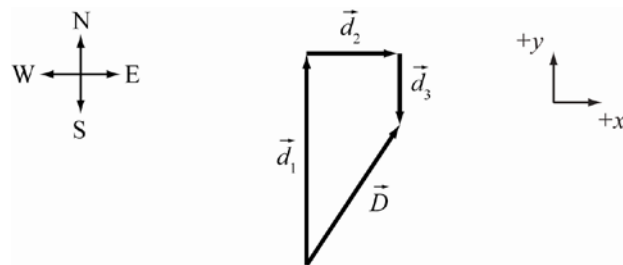
(a) The pilot initially flies N, then heads E, then finally heads S. Determine the vector $\vec{D}$ that points from the origin to the final point and find its magnitude. The vectors are $\vec{d}_1 = 155.3$ miles N, $\vec{d}_2 = 62.5$ miles E and $\vec{d}_3 = 47.5$ miles S.

(b) Now that the vector pointing to the final destination has been computed, $\vec{D} = d_2 \hat{x} + (d_1 - d_3) \hat{y} = (62.5 \text{ miles}) \hat{x} + (107.8 \text{ miles}) \hat{y}$, determine the angle the vector makes with the origin. The angle the pilot needs to travel is then 180° from this angle.

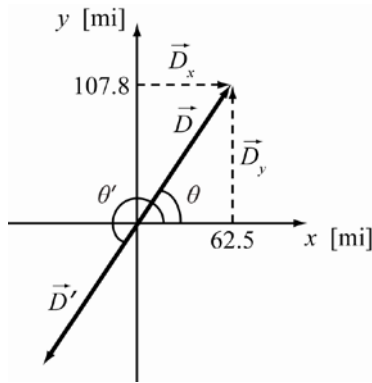
(c) Before the pilot turns S, he is farthest from the origin. This is because when he starts heading S, he is negating the distance travelled N. The only vectors of interest are $\vec{d}_1 = 155.3$ miles N and $\vec{d}_2 = 62.5$ miles E.

SKETCH:

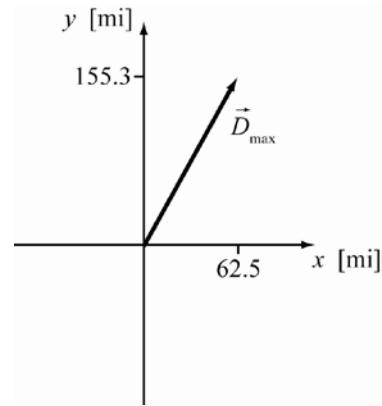
(a)



(b)



(c)

**RESEARCH:**

$$(a) \vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = D_x \hat{x} + D_y \hat{y}, \quad \vec{d}_i = d_{ix} \hat{x} + d_{iy} \hat{y}, \quad |\vec{D}| = \sqrt{D_x^2 + D_y^2}$$

$$(b) \tan \theta = \frac{D_y}{D_x}, \quad \theta' = \theta \pm 180^\circ$$

$$(c) \vec{D}_{\max} = \vec{d}_1 + \vec{d}_2, \quad \vec{d}_i = d_{ix} \hat{x} + d_{iy} \hat{y}, \quad |\vec{D}_{\max}| = \sqrt{D_x^2 + D_y^2}$$

SIMPLIFY:

$$(a) \vec{d}_1 = d_1 \hat{y}, \quad \vec{d}_2 = d_2 \hat{x}, \quad \vec{d}_3 = -d_3 \hat{y}$$

$$\text{Therefore, } \vec{D} = d_2 \hat{x} + (d_1 - d_3) \hat{y} \text{ and } |\vec{D}| = \sqrt{d_2^2 + (d_1 - d_3)^2}.$$

$$(b) \theta = \tan^{-1} \left(\frac{D_y}{D_x} \right) \text{ and } \theta' = \tan^{-1} \left(\frac{D_y}{D_x} \right) \pm 180^\circ$$

$$(c) \vec{d}_1 = d_1 \hat{y}, \quad \vec{d}_2 = d_2 \hat{x}, \quad \vec{D}_{\max} = d_2 \hat{x} + d_1 \hat{y} \Rightarrow |\vec{D}_{\max}| = \sqrt{d_2^2 + d_1^2}$$

CALCULATE:

$$(a) |\vec{D}| = \sqrt{(62.5 \text{ miles})^2 + (155.3 \text{ miles} - 47.5 \text{ miles})^2} \\ = 124.608 \text{ miles}$$

$$(b) \theta' = \tan^{-1} \left(\frac{107.8 \text{ miles}}{62.5 \text{ miles}} \right) \pm 180^\circ \\ = 59.896^\circ \pm 180^\circ = 239.896^\circ \text{ or } -120.104^\circ$$

$$(c) |\vec{D}_{\max}| = \sqrt{(62.5 \text{ miles})^2 + (155.3 \text{ miles})^2} = 167.405 \text{ miles}$$

ROUND:

$$(a) |\vec{D}| = 125 \text{ miles}$$

$$(b) \theta' = 240.^\circ \text{ or } -120.^\circ \text{ (from positive } x\text{-axis or E)}$$

$$(c) |\vec{D}_{\max}| = 167 \text{ miles}$$

DOUBLE-CHECK:

(a) The total distance is less than the distance travelled north, which is expected since the pilot eventually turns around and heads south.

(b) The pilot is clearly NE of the origin and the angle to return must be SW.

(c) This distance is greater than the distance which included the pilot travelling S, as it should be.

1.84. THINK:

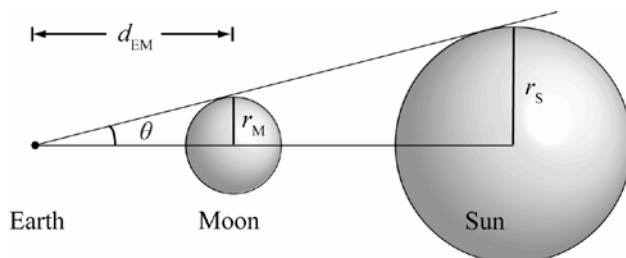
(a) If an observer sees the Moon fully cover the Sun, then light rays from the outer edge of the Sun are blocked by the outer edge of the Moon. This means a line pointing to the outer edge of the Moon also points to the outer edge of the Sun. This in turn means that the lines share a common angle. The radii of the Moon and Sun are, respectively, $r_M = 1.74 \cdot 10^6$ m and $r_S = 6.96 \cdot 10^8$ m. The distance from the Moon to the Earth is $d_{EM} = 3.84 \cdot 10^8$ m.

(b) In part (a), the origin of the light ray is assumed to be the centre of the Earth. In fact, the observer is on the surface of the Earth, $r_E = 6378$ km. This difference in observer position should then be related to the actual distance to the Moon. The observed Earth to Moon distance remains the same, $d_{EM} = 3.84 \cdot 10^8$ m, while the actual distance is the observed distance minus the radius of the Earth.

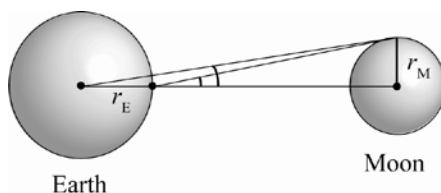
(c) Given the relative error of 1.69% between the actual and observed distance to the Moon, there should be the same relative error in the difference between the observed and actual distance to the Sun.
 $d_{ES}(\text{observed}) = 1.54 \cdot 10^{11}$ m.

SKETCH:

(a)



(b)



(c) Not applicable.

RESEARCH:

$$(a) \tan \theta = \left(\frac{\text{opposite}}{\text{adjacent}} \right)$$

$$(b) \text{ relative error} = \frac{d_{EM}(\text{observed}) - d_{EM}(\text{actual})}{d_{EM}(\text{actual})}$$

$$(c) d_{ES}(\text{actual}) = (1 - \text{relative error})d_{ES}(\text{observed})$$

SIMPLIFY:

$$(a) \tan \theta = \left(\frac{r_M}{d_{EM}} \right) = \left(\frac{r_S}{d_{ES}} \right) \Rightarrow d_{ES} = \frac{r_S d_{EM}}{r_M}$$

$$(b) \text{ relative error} = \frac{d_{EM}(\text{observed}) - (d_{EM}(\text{observed}) - r_E)}{d_{EM}(\text{observed}) - r_E} = \frac{r_E}{d_{EM}(\text{observed}) - r_E}$$

$$(c) d_{ES}(\text{actual}) = (1 - 0.0169)d_{ES}(\text{observed}) = 0.9831d_{ES}(\text{observed})$$

CALCULATE:

$$(a) d_{\text{ES}} = \frac{(6.96 \cdot 10^8 \text{ m})(3.84 \cdot 10^8 \text{ m})}{(1.74 \cdot 10^6 \text{ m})} = 1.536 \cdot 10^{11} \text{ m}$$

$$(b) \text{ relative error} = \frac{6378000 \text{ m}}{3.84 \cdot 10^8 \text{ m} - 6378000} = 0.01689$$

$$(c) d_{\text{ES}}(\text{actual}) = 0.9831(1.54 \cdot 10^{11} \text{ m}) = 1.513 \cdot 10^{11} \text{ m}$$

ROUND:

$$(a) d_{\text{ES}} = 1.54 \cdot 10^{11} \text{ m}$$

$$(b) \text{ relative error} = 1.69\%$$

$$(c) d_{\text{ES}}(\text{actual}) = 1.51 \cdot 10^{11} \text{ m}$$

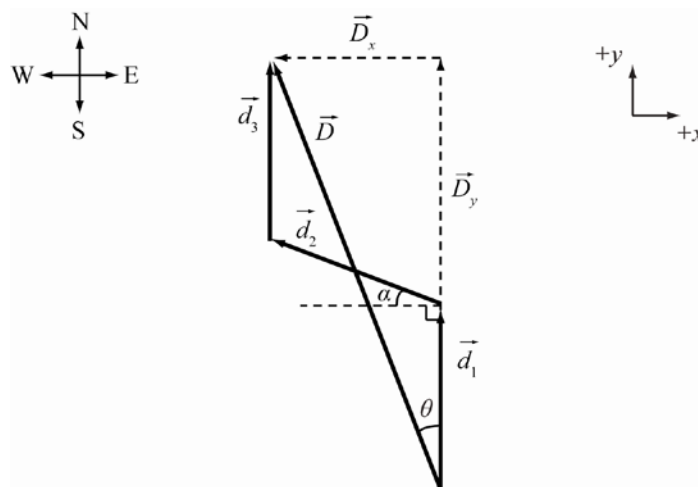
DOUBLE-CHECK:

(a) The distance from the Earth to the Sun is about 300 times the distance from the Earth to the Moon, so the answer is reasonable.

(b) The radius of Earth is small compared to the distance from the Earth to the Moon, so the error calculated is small.

(c) The relative error is small so there should be a small difference between the actual and the observed distance from the Earth to the Sun.

- 1.85. **THINK:** The problem involves adding vectors. Break the vectors into components and sum the components. The vectors are: $\vec{d}_1 = 1.50 \text{ km}$ due N, $\vec{d}_2 = 1.50 \text{ km}$ 20.0° N of W and $\vec{d}_3 = 1.50 \text{ km}$ due N. Find the length of the resultant, and the angle it makes with the vertical. Let $\alpha = 20.0^\circ$.

SKETCH:

$$\text{RESEARCH: } \vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3, \quad \vec{d}_i = d_{ix}\hat{x} + d_{iy}\hat{y}, \quad |\vec{D}| = \sqrt{D_x^2 + D_y^2}, \quad \tan\theta = \left(\frac{D_x}{D_y}\right)$$

$$\text{SIMPLIFY: } \vec{d}_1 = d_1\hat{y}, \quad \vec{d}_2 = -d_2\cos\alpha\hat{x} + d_2\sin\alpha\hat{y} = -d_2(\cos\alpha)\hat{x} + d_2(\sin\alpha)\hat{y}, \quad \vec{d}_3 = d_3\hat{y}$$

$$|\vec{D}| = \sqrt{(-d_2\cos\alpha)^2 + (d_1 + d_3 + d_2\sin\alpha)^2}$$

$$\theta = \tan^{-1}\left(\frac{D_x}{D_y}\right)$$

$$\text{CALCULATE: } |\vec{D}| = \sqrt{(-1.50\cos(20.0^\circ) \text{ km})^2 + (3.00 \text{ km} + 1.50\sin(20.0^\circ) \text{ km})^2}$$

$$= \sqrt{1.9868 \text{ km}^2 + 12.3414 \text{ km}^2} = 3.7852 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{-1.4095 \text{ km}}{3.5130 \text{ km}}\right) = -21.862^\circ$$

ROUND: $|\vec{D}| = 3.79 \text{ km}$ at 21.9° W of N

DOUBLE-CHECK: The only directions travelled were N or NW, so the final direction should be in the NW region.

- 1.86. THINK:** If the number of molecules is proportional to the volume, then the ratio of volumes should be the same as the ratio of the molecules. $1 \text{ mol} = 6.02 \cdot 10^{23}$ molecules, volume of mol = 22.4 L and the volume of one breath is 0.500 L. Only 80.0% of the volume of one breath is nitrogen.

SKETCH: Not applicable.

RESEARCH: $V_{\text{Nitrogen}} = 0.800V_{\text{breath}}$, $\frac{V_{\text{Nitrogen}}}{V_{\text{mol}}} = \frac{\# \text{ molecules in one breath}}{\# \text{ molecules in a mol}} = \frac{N_{\text{breath}}}{N_{\text{mol}}}$

SIMPLIFY: $N_{\text{breath}} = \frac{V_{\text{Nitrogen}}(N_{\text{mol}})}{V_{\text{mol}}} = \frac{0.800V_{\text{Breath}}(N_{\text{mol}})}{V_{\text{mol}}}$

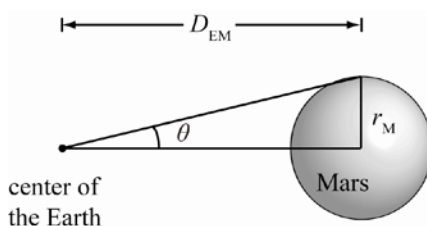
CALCULATE: $N_{\text{breath}} = \frac{0.800(0.500 \text{ L})(6.02 \cdot 10^{23} \text{ molecules})}{(22.4 \text{ L})} = 1.07500 \cdot 10^{22} \text{ molecules}$

ROUND: $N_{\text{breath}} = 1.08 \cdot 10^{22} \text{ molecules}$

DOUBLE-CHECK: Since the volume of one breath is about 50 times smaller than the volume of one mole of gas, the number of nitrogen molecules in one breath should be about 50 times smaller than the number of molecules in a mole.

- 1.87. THINK:** 24.9 seconds of arc represents the angle subtended by a circle with diameter = $2r_M$ located a distance D_{EM} from Earth. This value must be converted to radians. The diameter of Mars is $2r_M = 6784 \text{ km}$.

SKETCH:



RESEARCH: The angular size is related to the angle θ shown in the sketch by $\theta_{\text{angular size}} = 2\theta$. From the sketch, we can see that

$$\tan \theta = \frac{r_M}{D_{EM}}.$$

Because Mars is a long distance from the Earth, even at closet approach, we can make the approximation $\tan \theta \approx \theta$.

SIMPLIFY: Putting our equations together gives us $\theta_{\text{angular size}} = 2\theta = \frac{2r_M}{D_{EM}}$.

CALCULATE: We first convert the observed angular size from seconds of arc to radians

$$24.9 \text{ arc seconds} \left(\frac{1^\circ}{3600 \text{ arc seconds}} \right) \left(\frac{2\pi \text{ radians}}{360^\circ} \right) = 1.207 \cdot 10^{-4} \text{ radians}.$$

The distance is then

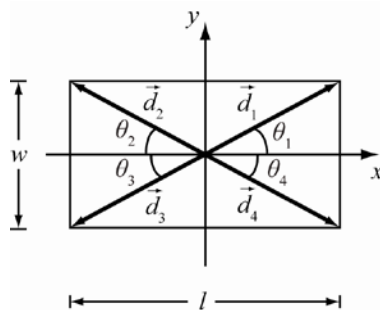
$$D_{EM} = \frac{2r_M}{\theta_{\text{angular size}}} = \frac{6784 \text{ km}}{(1.207 \cdot 10^{-4} \text{ radians})} = 5.6205 \cdot 10^7 \text{ km}.$$

ROUND: We specify our answer to three significant figures, $D_{EM} = 5.62 \cdot 10^7 \text{ km}$.

DOUBLE-CHECK: The mean distance from Earth to Mars is about $7 \cdot 10^7$ km. Because the distance calculated is for a close approach and the distance is less than the mean distance, the answer is reasonable.

- 1.88. THINK:** If the quarterback is in the exact centre of a rectangular field, then each corner should be the same distance from the centre. Only the angle changes for each corner. The width of the field is 53 1/3 yards and the length is 100. yards. Since the question states that the length is exactly 100 yards, the precision of the final answer will be limited by the width.

SKETCH:



RESEARCH: $\vec{d}_i = d_{ix}\hat{x} + d_{iy}\hat{y}$, $|\vec{d}_i| = \sqrt{d_{ix}^2 + d_{iy}^2}$, $\tan \theta_i = \frac{d_{iy}}{d_{ix}}$

SIMPLIFY: $|\vec{d}_1| = |\vec{d}_2| = |\vec{d}_3| = |\vec{d}_4| = \sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = |\vec{d}|$, $\theta_1 = \tan^{-1} \left[\frac{\left(\frac{w}{2}\right)}{\left(\frac{l}{2}\right)} \right] = \tan^{-1} \left(\frac{w}{l} \right)$

CALCULATE:

(a) $|\vec{d}| = \sqrt{\left(\frac{53 \frac{1}{3} \text{ yards}}{2}\right)^2 + \left(\frac{100 \text{ yards}}{2}\right)^2} = 56.667 \text{ yards}$, $\theta_1 = \tan^{-1} \left(\frac{53 \frac{1}{3}}{100} \right) = 28.072^\circ$

(b) $\theta_2 = 180^\circ - \theta_1 = 180^\circ - 28.072^\circ = 151.928^\circ$

$\theta_3 = 180^\circ + \theta_1 = 180^\circ + 28.072^\circ = 208.072^\circ$

$\theta_4 = 360^\circ - \theta_1 = 360^\circ - 28.072^\circ = 331.928^\circ$

ROUND:

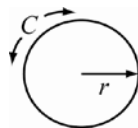
(a) $\vec{d}_1 = 56.7 \text{ yards at } 28.1^\circ$

(b) $\vec{d}_2 = 56.7 \text{ yards at } 152^\circ$, $\vec{d}_3 = 56.7 \text{ yards at } 208^\circ$, $\vec{d}_4 = 56.7 \text{ yards at } 332^\circ$

DOUBLE-CHECK: $\vec{d}_1$ & $\vec{d}_3$ and $\vec{d}_2$ & $\vec{d}_4$ are 180° apart. This is expected when throwing at opposite corners of the field. The answers are reasonable.

- 1.89. THINK:** Assume the Cornell Electron Storage Ring is a perfect circle with a circumference of $C = 768.4$ m. Recall the exact conversion $1 \text{ m} = (100 / 2.54) \text{ inches}$.

SKETCH:



RESEARCH: $C = 2\pi r$, $d = 2r$

SIMPLIFY: $d = \frac{C}{\pi} \left(\frac{100 \text{ in}}{2.54 \text{ m}} \right)$

CALCULATE: $d = \frac{(768.4 \text{ m})}{\pi} \left(\frac{100 \text{ in}}{2.54 \text{ m}} \right) = 9629.5007 \text{ inches}$

ROUND: $d = 9630$ inches

DOUBLE-CHECK: There are 12 inches in a foot and 5280 feet in a mile. Therefore there are 63,360 inch/mile. Our answer for the Cornell ring is thus about $1/6^{\text{th}}$ of a mile, which seems the right order of magnitude.

- 1.90. THINK:** 4% of the 0.5 L for each exhalation is composed of carbon dioxide. Assume 1 mole ($6.02 \cdot 10^{23}$ molecules) has a volume of 22.4 L. The particular numbers are actually not that important. The only important thing is that they have the right order of magnitude. So it also could be 0.3 or 0.6 L that we exhale in each breath, which are also numbers you can find in the literature; and some sources quote 5% CO_2 in the air that we breathe out.

SKETCH: Not applicable.

RESEARCH: How many times do we breathe per day? You can count the number of breaths you take in a minutes, and that number is around 15. This means that you breath around 800 to 1,000 times per hour and around 20,000 to 25,000 times per day.

$$V_{\text{CO}_2} = 0.04 V_{\text{breath}}, \quad \frac{V_{\text{CO}_2}}{V_{\text{mol}}} = \frac{\# \text{ molecules in one breath}}{\# \text{ molecules in a mol}} = \frac{CO_{2 \text{ breath}}}{CO_{2 \text{ mol}}}$$

$$\text{SIMPLIFY: } CO_{2 \text{ breath}} = \frac{V_{\text{CO}_2}}{V_{\text{mol}}} (CO_{2 \text{ mol}}) = \frac{0.04 V_{\text{breath}}}{V_{\text{mol}}} (CO_{2 \text{ mol}})$$

$$\text{CALCULATE: } CO_{2 \text{ breath}} = \frac{0.04(0.5 \text{ L})}{22.4 \text{ L}} (6.02 \cdot 10^{23} \text{ molecules}) = 5.375 \cdot 10^{20} \text{ molecules}$$

$$(a) \ CO_{2 \text{ day}} = \# \text{ molecules exhaled in a day}$$

$$\begin{aligned} &= (2.5 \cdot 10^4) CO_{2 \text{ breath}} \\ &= (2.5 \cdot 10^4) (5.375 \cdot 10^{20} \text{ molecules}) \\ &= 1.34375 \cdot 10^{25} \text{ molecules} \end{aligned}$$

$$(b) \ m_{\text{CO}_2} = \frac{1.34375 \cdot 10^{25} \text{ molecules}}{1 \text{ day}} \left(\frac{365 \text{ days}}{1 \text{ year}} \right) \left(\frac{1 \text{ mole}}{6.02 \cdot 10^{23} \text{ molecules}} \right) \left(\frac{44 \text{ g}}{1 \text{ mole}} \right) = 3.58482 \cdot 10^2 \text{ kg/year}$$

ROUND: In this case we only estimate order of magnitudes. And so it makes no sense to give more than one significant digit. We can therefore state our answer as

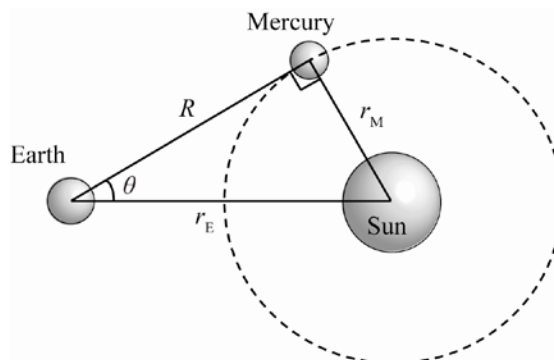
$$(a) \ CO_{2 \text{ day}} = 10^{25} \text{ molecules}$$

$$(b) \ m_{\text{CO}_2} = 300 \text{ to } 400 \text{ kg/year}$$

DOUBLE-CHECK: Does it makes sense that we breathe out around 300 to 400 kg of CO_2 in a year, which implies that we breathe out approximately 1 kg of CO_2 in a day. Where does this materials come from? The oxygen comes from the air we breathe in. So the carbon has to be part of what we eat each day. Since $\sim 1/4$ of the mass of a CO_2 molecule resides in the carbon, this means that we have to eat at least $\sim 1/2$ of a pound of carbon each day. Since carbon, hydrogen, and oxygen are the main components of our food, and since we eat several pounds of food per day, this seems in the right ballpark.

- 1.91. THINK:** Consider the Sun to be at the centre of a circle with Mercury on its circumference. This gives $r_{\text{M}} = 4.6 \cdot 10^{10} \text{ m}$ as the radius of the circle. Earth is located a distance $r_{\text{E}} = 1.5 \cdot 10^{11} \text{ m}$ from the Sun so that the three bodies form a triangle. The vector from Earth to the Sun is at 0° . The vector from Earth to Mercury intersects Mercury's orbit once when Mercury is at a maximum angular separation from the Sun in the sky. This tangential vector is perpendicular to the radius vector of Mercury's orbit. The three bodies form a right angle triangle with r_{E} as the hypotenuse. Trigonometry can be used to solve for the angle and distance.

SKETCH:



RESEARCH: $r_E^2 = r_M^2 + R^2$, $r_E \sin \theta = r_M$

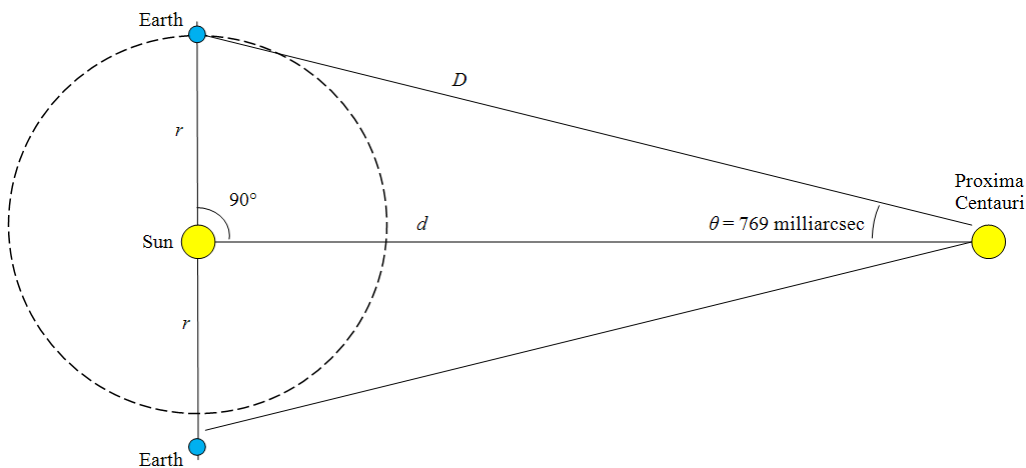
SIMPLIFY: $R = \sqrt{r_E^2 - r_M^2}$, $\theta = \sin^{-1} \left(\frac{r_M}{r_E} \right)$

CALCULATE: $R = \sqrt{(1.5 \cdot 10^{11})^2 - (4.6 \cdot 10^{10})^2} = 1.4277 \cdot 10^{11} \text{ m}$, $\theta = \sin^{-1} \left(\frac{4.6 \cdot 10^{10}}{1.5 \cdot 10^{11}} \right) = 17.858^\circ$

ROUND: $R = 1.4 \cdot 10^{11} \text{ m}$, $\theta = 18^\circ$

DOUBLE-CHECK: If it had been assumed that the maximum angular separation occurred when the Earth to Sun to Mercury angle was 90° , $\theta = \tan^{-1}(r_M / r_E)$ would be about 17° . The maximum angle should be greater than this and it is, so the answer is reasonable.

- 1.92. THINK:** This question asks about the distance to Proxima Centauri, which can be calculated using parallax. To do so, it will be necessary to know the radius of Earth's orbit. It will also be necessary to convert from milliarcseconds to degrees or radians. Then, geometry should be used to find the distance.
- SKETCH:** Because of the distances involved, the diagram will not be to scale. The earth is shown at two locations, $\frac{1}{2}$ year apart in its revolution around the Sun. The radius of Earth's orbit is labeled r and the distance to Proxima Centauri is labeled d .



RESEARCH: The goal is to find d , the distance between the Sun and Proxima Centauri. Note that the Earth at either of the positions shown, the Sun, and Proxima Centauri form right triangles. The textbook lists the mean orbital radius of Earth as $1.496 \times 10^8 \text{ km}$. The final answer needs to be in light-years, so it will be necessary to convert from km to light-years at some point using the fact that $1 \text{ light-year} = 9.461 \times 10^{12} \text{ km}$.

10^{12} km. Knowing the parallax and the radius of the Earth's orbit, it is then possible to use trigonometry to find the distance d from the Sun to Proxima Centauri: $\tan \theta = \frac{r}{d}$.

SIMPLIFY: Using algebra to find the distance d in terms of r and θ gives $d = \frac{r}{\tan \theta}$. It is more difficult to convert from milliarcseconds to a more familiar unit of angle measure, the degree. Since there are 60 arcseconds to the arcminute, and 60 arcminutes make one degree, the conversion will look like this:

$$\text{angle in milliarcseconds} \cdot \frac{10^{-3} \text{ arcseconds}}{1 \text{ milliarcsecond}} \cdot \frac{1 \text{ arcminute}}{60 \text{ arcseconds}} \cdot \frac{1 \text{ degree}}{60 \text{ arcminutes}} = \text{angle in degrees}.$$

CALCULATE: It is important to perform this calculation with the computer or calculator in degree (not radian) mode. Using the textbook value for the radius of the earth $r = 1.496 \times 10^8$ km and the given value for the parallax of 769 milliarcsec gives:

$$\begin{aligned} d &= \frac{r}{\tan \theta} \\ &= \frac{1.496 \times 10^8 \text{ km}}{\tan \left(769 \text{ milliarcsec} \cdot \frac{10^{-3} \text{ arcsec}}{1 \text{ milliarcsec}} \cdot \frac{1 \text{ arcminute}}{60 \text{ arcsec}} \cdot \frac{1 \text{ degree}}{60 \text{ arcminutes}} \right)} \cdot \frac{1 \text{ light-year}}{9.461 \times 10^{12} \text{ km}} \\ &= 4.241244841 \text{ light-years} \end{aligned}$$

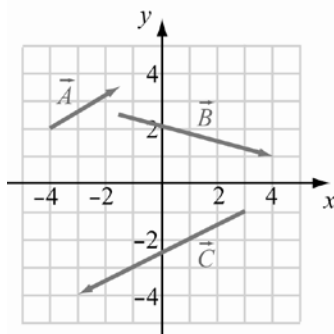
ROUND: The parallax has three significant figures. The radius of the earth is given to four, and all of the conversions are exact except light-years to km, which is given to four. So the final answer should have three figures. This gives a total distance of 4.24 light-years.

DOUBLE-CHECK: A distance to the Proxima Centauri of 4.24 light-years means that it takes light about $4\frac{1}{4}$ years to travel between the Sun and Proxima Centauri. Knowing what we do of astronomical scales, this makes sense.

Multi-Version Exercises

- 1.93. **THINK:** The lengths of the x and y components of the vectors can be read from the provided figure. Remember to decompose the vectors in terms of their x and y components.

SKETCH:



RESEARCH: A vector can be written as $\vec{V} = V_x \hat{x} + V_y \hat{y}$, where $V_x = x_f - x_i$ and $V_y = y_f - y_i$.

SIMPLIFY: Not applicable.

CALCULATE: $\vec{A} = (-1.5 - (-2))\hat{x} + (3.5 - 2)\hat{y} = 0.5\hat{x} + 1.5\hat{y}$, $\vec{B} = (3.5 - 0)\hat{x} + (2 - 2)\hat{y} = 3.5\hat{x}$, $\vec{C} = (-3 - 0)\hat{x} + (-3 - (-2))\hat{y} = -3\hat{x} - 1\hat{y}$

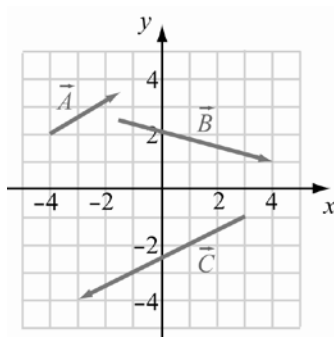
$\vec{C} = (-3 - 0)\hat{x} + (-3 - (-2))\hat{y} = -3\hat{x} - 1\hat{y}$

ROUND: Not applicable.

DOUBLE-CHECK: Comparing the signs of the x - and y -components of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ to the provided figure, the calculated components all point in the correct directions. The answer is therefore reasonable.

- 1.94. THINK:** The question asks for the length and direction of the three vectors. The x and y components of the vectors can be read from the provided figure. Remember when dealing with vectors, the components must be treated separately.

SKETCH:



RESEARCH: The length of a vector is given by the formula $\vec{L} = \sqrt{x^2 + y^2}$. The direction of a vector (with respect to the x -axis) is given by $\tan \theta = y / x$.

SIMPLIFY: $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

CALCULATE: $|\vec{A}| = \sqrt{(2.5)^2 + (1.5)^2} = 2.9$, $\theta_A = \tan^{-1}\left(\frac{1.5}{2.5}\right) = 30.9638^\circ$

$$|\vec{B}| = \sqrt{(5.5)^2 + (-1.5)^2} = 5.700877, \theta_B = \tan^{-1}\left(\frac{-1.5}{5.5}\right) = -15.2551^\circ$$

$$|\vec{C}| = \sqrt{(-6)^2 + (-3)^2} = 6.7082,$$

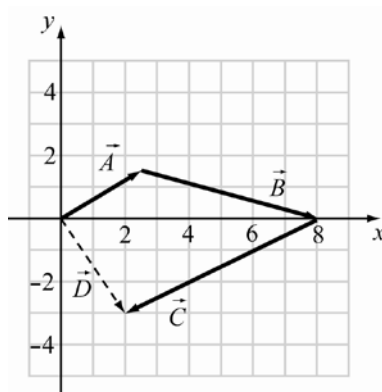
$$\theta_C = \tan^{-1}\left(\frac{-3}{-6}\right) = 26.565^\circ = 180^\circ + 26.565^\circ = 206.565^\circ$$

ROUND: The figure can reasonably be read to two significant digits, so the rounded values are $|\vec{A}| = 2.9$

$$\theta_A = 31^\circ, |\vec{B}| = 5.7, \theta_B = -15^\circ, |\vec{C}| = 6.7, \text{ and } \theta_C = 210^\circ.$$

DOUBLE-CHECK: Comparing the graphical values to the calculated values, the calculated values are reasonable.

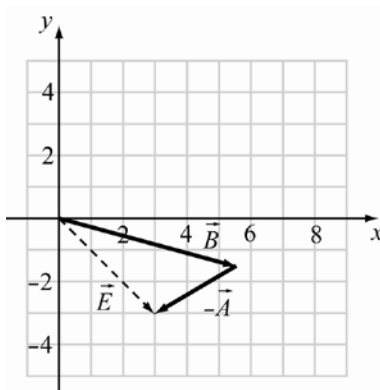
- 1.95.** Vectors add tip to tail, $\vec{A} + \vec{B} + \vec{C} = \vec{D}$.



By inspecting the image, it is clear that $\vec{D} = (2, -3)$.

- 1.96. THINK:** To subtract two vectors, reverse the direction of the vector being subtracted, and treat the operation as a sum. Denote the difference as $\vec{E} = \vec{B} - \vec{A}$.

SKETCH:



RESEARCH: $\vec{E} = \vec{B} - \vec{A} = \vec{B} + (-\vec{A})$

SIMPLIFY: No simplification is necessary.

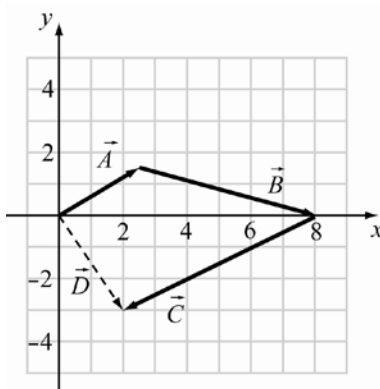
CALCULATE: By inspection, $\vec{E} = (3, -3)$.

ROUND: No rounding is necessary.

DOUBLE-CHECK: The resultant vector $\vec{E}$ points from the origin to the fourth quadrant, so its x -component should be positive and its y -component should be negative. This gives some support to the reasonableness of the answer.

- 1.97. THINK:** When adding vectors, you must add the components separately.

SKETCH:



RESEARCH: $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

SIMPLIFY: $\vec{D} = (A_x + B_x + C_x)\hat{x} + (A_y + B_y + C_y)\hat{y}$

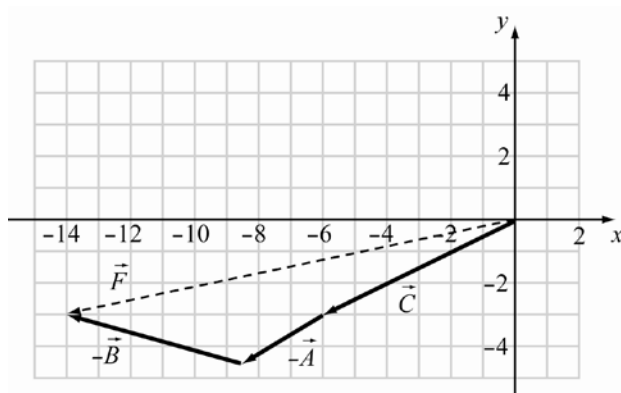
CALCULATE: $\vec{D} = (2.5 + 5.5 - 6)\hat{x} + (1.5 - 1.5 - 3)\hat{y} = 2\hat{x} - 3\hat{y}$

ROUND: The answers are precise, so no rounding is necessary.

DOUBLE-CHECK: The calculation seems consistent with the provided figure.

- 1.98. **THINK:** When subtracting vectors, you must subtract the x and y components separately.

SKETCH:



RESEARCH: $F_x = C_x - A_x - B_x$ and $F_y = C_y - A_y - B_y$. The length is computed using $|\vec{F}| = \sqrt{F_x^2 + F_y^2}$ with $F = F_x \hat{x} + F_y \hat{y}$.

SIMPLIFY: $|\vec{F}| = \sqrt{(C_x - A_x - B_x)^2 + (C_y - A_y - B_y)^2}$

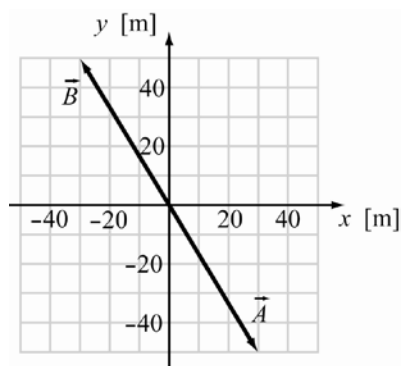
CALCULATE: $|\vec{F}| = \sqrt{((-6.0) - 2.5 - 5.5)^2 + ((-3.0) - (1.5) - (-1.5))^2}$
 $= \sqrt{205}$
 $= 14.318$

ROUND: To two significant figures, the length of $\vec{F}$ is 14.

DOUBLE-CHECK: The size of $|\vec{F}|$ is reasonable.

- 1.99. **THINK:** The two vectors are $\vec{A} = (A_x, A_y) = (30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (-30.0 \text{ m}, 50.0 \text{ m})$. Sketch and find the magnitudes.

SKETCH:



RESEARCH: The length of a vector $\vec{C} = C_x \hat{x} + C_y \hat{y}$ is $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$.

SIMPLIFY: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2}$

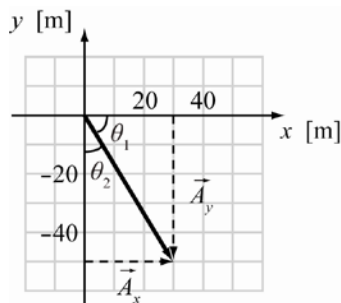
CALCULATE: $|\vec{A}| = \sqrt{(30)^2 + (-50)^2} = 58.3095 \text{ m}$, $|\vec{B}| = \sqrt{(-30)^2 + (50)^2} = 58.3095 \text{ m}$

ROUND: $|\vec{A}| = 58.3 \text{ m}$, $|\vec{B}| = 58.3 \text{ m}$

DOUBLE-CHECK: The calculated magnitudes are larger than the lengths of the component vectors, and are less than the sum of the lengths of the component vectors. Also, the vectors are opposites, so they should have the same length.

- 1.100. THINK:** Use trigonometry to find the angles as indicated in the sketch below. $\vec{A} = (A_x, A_y) = (30.0 \text{ m}, -50.0 \text{ m})$.

SKETCH:



RESEARCH: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

SIMPLIFY: $\tan \theta_1 = (A_y / A_x) \Rightarrow \theta_1 = \tan^{-1}(A_y / A_x)$, $\tan \theta_2 = (A_x / A_y) \Rightarrow \theta_2 = \tan^{-1}(A_x / A_y)$

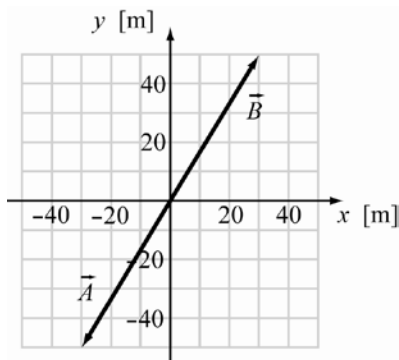
CALCULATE: $\theta_1 = \tan^{-1}(-50 / 30) = -59.036^\circ$, $\theta_2 = \tan^{-1}(30 / -50) = -30.963^\circ$

ROUND: Drop the signs of the angles and just use their size: $\theta_1 = 59.0^\circ$, $\theta_2 = 31.0^\circ$.

DOUBLE-CHECK: The two angles add up to 90° , which they should. The answers are reasonable.

- 1.101. THINK:** The two vectors are $\vec{A} = (A_x, A_y) = (-30.0 \text{ m}, -50.0 \text{ m})$ and $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$. Sketch and find the magnitudes.

SKETCH:



RESEARCH: $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$

SIMPLIFY: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2}$

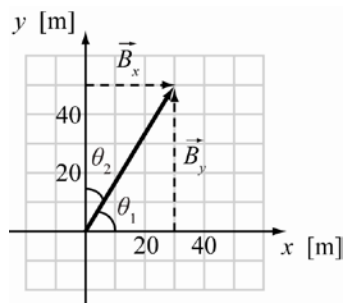
CALCULATE: $|\vec{A}| = \sqrt{(-30.0 \text{ m})^2 + (-50.0 \text{ m})^2} = 58.3095 \text{ m}$, $|\vec{B}| = \sqrt{(30.0 \text{ m})^2 + (50.0 \text{ m})^2} = 58.3095 \text{ m}$

ROUND: $|\vec{A}| = 58.3 \text{ m}$, $|\vec{B}| = 58.3 \text{ m}$

DOUBLE-CHECK: The magnitudes are bigger than individual components, but not bigger than the sum of the components. Therefore, the answers are reasonable.

- 1.102. THINK:** Using trigonometry find the angles indicated in the diagram below. The vector $\vec{B} = (B_x, B_y) = (30.0 \text{ m}, 50.0 \text{ m})$.

SKETCH:



RESEARCH: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

SIMPLIFY: $\theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$, $\theta_1 = \tan^{-1}\left(\frac{B_y}{B_x}\right)$, $\theta_2 = \tan^{-1}\left(\frac{B_x}{B_y}\right)$

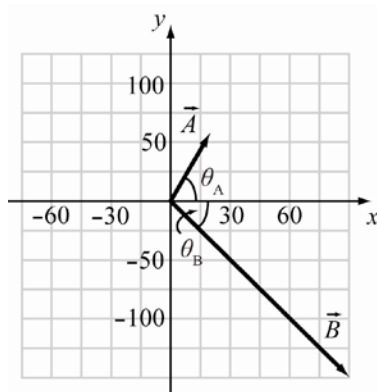
CALCULATE: $\theta_1 = \tan^{-1}\left(\frac{50.0 \text{ m}}{30.0 \text{ m}}\right) = 59.036^\circ$, $\theta_2 = \tan^{-1}\left(\frac{30.0 \text{ m}}{50.0 \text{ m}}\right) = 30.963^\circ$

ROUND: $\theta_1 = 59.0^\circ$, $\theta_2 = 31.0^\circ$

DOUBLE-CHECK: The angles sum to 90° , which is expected from the sketch. Therefore, the answers are reasonable.

- 1.103. THINK:** The two vectors are $\vec{A} = (23.0, 59.0)$ and $\vec{B} = (90.0, -150.0)$. Find the magnitude and angle with respect to the positive x -axis.

SKETCH:



RESEARCH: For any vector $\vec{C} = C_x \hat{x} + C_y \hat{y}$, the magnitude is given by the formula $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, and

the angle θ_C made with the x -axis is such that $\tan \theta_C = \frac{C_y}{C_x}$.

SIMPLIFY: $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2}$, $\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$, $\theta_B = \tan^{-1}\left(\frac{B_y}{B_x}\right)$

CALCULATE: $|\vec{A}| = \sqrt{(23.0)^2 + (59.0)^2} = 63.3246$, $|\vec{B}| = \sqrt{(90.0)^2 + (-150.0)^2} = 174.9286$

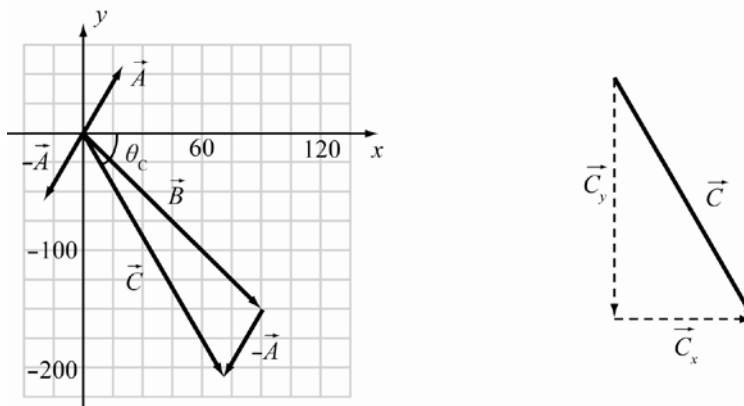
$\theta_A = \tan^{-1}\left(\frac{59.0}{23.0}\right) = 68.7026^\circ$, $\theta_B = \tan^{-1}\left(\frac{-150.0}{90.0}\right) = -59.0362^\circ$

ROUND: Three significant figures: $\vec{A} = 63.3$ at 68.7° , $\vec{B} = 175$ at -59.0° or 301.0° .

DOUBLE-CHECK: Each magnitude is greater than the components but less than the sum of the components, and the angles place the vectors in the proper quadrants.

- 1.104. THINK:** Add the components of the vectors. Find the magnitude and the angle from the positive x -axis of the resultant vector. $\vec{A} = (23.0, 59.0)$ and $\vec{B} = (90.0, -150.0)$.

SKETCH:



RESEARCH: $\vec{C} = (C_x, C_y)$, $C_i = nA_i + mB_i$ with $n = -1$ and $m = +1$, $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\tan \theta_C = \frac{C_y}{C_x}$.

SIMPLIFY: $C_x = -A_x + B_x$, $C_y = -A_y + B_y$, $\theta_C = \tan^{-1}\left(\frac{C_y}{C_x}\right)$

CALCULATE: $C_x = -23.0 + 90.0 = 67.0$, $C_y = -59.0 + (-150) = -209.0$,

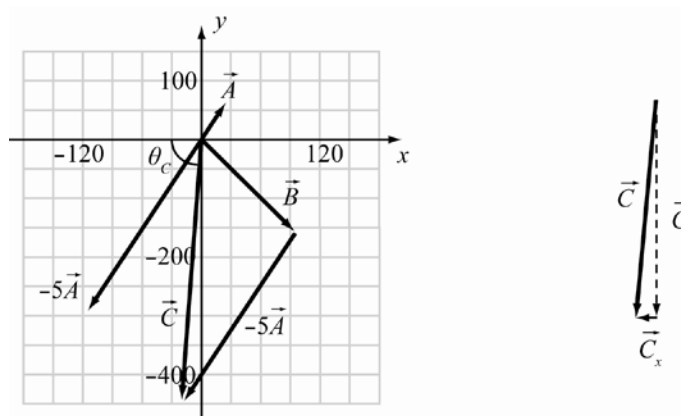
$|\vec{C}| = \sqrt{(67.0)^2 + (-209.0)^2} = 219.477$, and $\theta_C = \tan^{-1}\left(\frac{-209.0}{67.0}\right) = -72.225^\circ$.

ROUND: $\vec{C} = 219$ at -72.2° or 288°

DOUBLE-CHECK: The magnitude is greater than each component but less than the sum of the components and the angle is also in the correct quadrant. The answer is reasonable.

- 1.105. THINK:** Add the components of the vectors (with applicable multiplication of each vector). Find the magnitude and the angle from the positive x -axis of the resultant vector. $\vec{A} = (23.0, 59.0)$ and $\vec{B} = (90.0, -150.0)$.

SKETCH:



RESEARCH: $\vec{C} = (C_x, C_y)$, $C_i = nA_i + mB_i$ with $n = -5$ and $m = +1$, $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\tan \theta_C = \frac{C_y}{C_x}$.

SIMPLIFY: $C_x = -5A_x + B_x$, $C_y = -5A_y + B_y$, $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\theta_C = \tan^{-1}\left(\frac{C_y}{C_x}\right)$

CALCULATE: $C_x = -5(23.0) + 90.0 = -25.0$, $C_y = -5(59.0) + (-150) = -445.0$

$$|\vec{C}| = \sqrt{(-25.0)^2 + (-445.0)^2} = 445.702$$

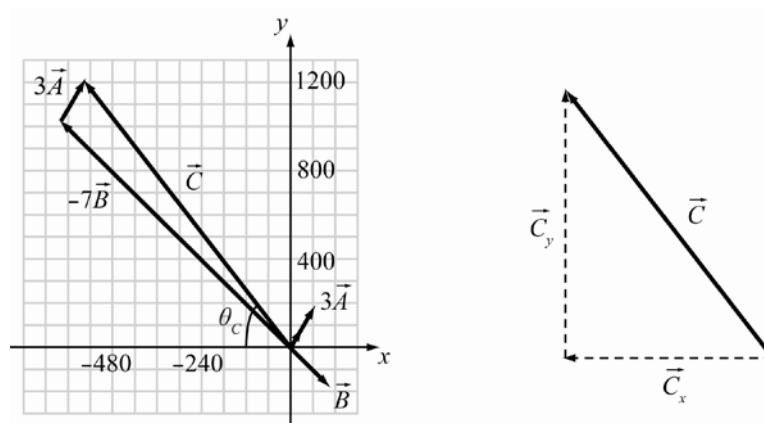
$$\theta_C = \tan^{-1}\left(\frac{-445.0}{-25.0}\right) = 86.785^\circ \Rightarrow 180^\circ + 86.785^\circ = 266.785^\circ$$

ROUND: $\vec{C} = 446$ at 267° or -93.2°

DOUBLE-CHECK: The magnitude is greater than each component but less than the sum of the components and the angle is also in the correct quadrant. The answer is reasonable.

- 1.106. THINK:** Add the components of the vectors (with applicable multiplication of each vector). Find the magnitude and the angle from the positive x -axis of the resultant vector. $\vec{A} = (23.0, 59.0)$ and $\vec{B} = (90.0, -150.0)$.

SKETCH:



RESEARCH: $\vec{C} = (C_x, C_y)$, $C_i = nA_i + mB_i$ with $n = 3$ and $m = -7$, $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$, $\tan \theta_C = \frac{C_y}{C_x}$.

SIMPLIFY: $C_x = 3A_x - 7B_x$, $C_y = 3A_y - 7B_y$, $\theta_C = \tan^{-1}\left(\frac{C_y}{C_x}\right)$

CALCULATE: $C_x = 3(23.0) - 7(90.0) = -561.0$, $C_y = 3(59.0) - 7(-150) = 1227.0$

$$|\vec{C}| = \sqrt{(-561.0)^2 + (1227.0)^2} = 1349.17$$

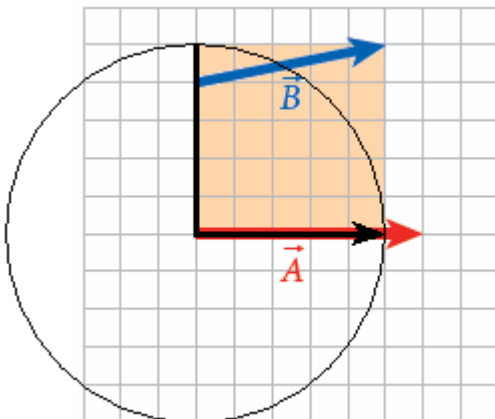
$$\theta_C = \tan^{-1}\left(\frac{1227.0}{-561.0}\right) = -65.43^\circ \Rightarrow 180^\circ - 65.43^\circ = 114.57^\circ$$

ROUND: $|\vec{C}| = 1.35 \cdot 10^3$ at 115°

DOUBLE-CHECK: The magnitude is greater than each component but less than the sum of the components and the angle is also in the correct quadrant.

- 1.107. THINK:** The scalar product of two vectors equals the length of the two vectors times the cosine of the angle between them. Geometrically, think of the absolute value of the scalar product as the length of the projection of vector $\vec{B}$ onto vector $\vec{A}$ times the length of vector $\vec{A}$, or the area of a rectangle with one side the length of vector $\vec{A}$ and the other side the length of the projection of vector $\vec{B}$ onto vector $\vec{A}$. Algebraically, use the formula $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$ to find the scalar product from the components, which can be read from the graphs.

SKETCH: Using the geometric interpretation, sketch the projection of vector $\vec{B}$ onto vector $\vec{A}$ and then draw the corresponding rectangular area, for instance for case (e):



Note, however, that this method of finding the scalar product is cumbersome and does not readily produce exact results. The algebraic approach is much more efficient.

RESEARCH: Use the formula $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$ to find the scalar product from the components, which can be read from the graphs. $(A_x, A_y) = (6, 0)$ in all cases. In (a), $(B_x, B_y) = (1, 5)$; in (b), $(B_x, B_y) = (0, 3)$; in (c), $(B_x, B_y) = (2, 2)$; in (d), $(B_x, B_y) = (-6, 0)$; in (e), $(B_x, B_y) = (5, 1)$; and in (f), $(B_x, B_y) = (1, 4)$.

SIMPLIFY: Using the formula $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$, find that in part (a), $\vec{A} \cdot \vec{B} = 6 \cdot 1 + 0 \cdot 5$. In part (b), $\vec{A} \cdot \vec{B} = 6 \cdot 0 + 0 \cdot 3$. In part (c), $\vec{A} \cdot \vec{B} = 6 \cdot 2 + 0 \cdot 2$. In part (d), $\vec{A} \cdot \vec{B} = 6 \cdot (-6) + 0 \cdot 0$. In part (e), $\vec{A} \cdot \vec{B} = 6 \cdot 5 + 0 \cdot 1$. Finally, in part (f), $\vec{A} \cdot \vec{B} = 6 \cdot 1 + 0 \cdot 4$.

CALCULATE: Performing the multiplication and addition as shown above, the scalar product in (a) is 6 units, in (b) it is 0 units, and in part (c) the scalar product is 12 units. In parts (d), (e), and (f), the scalar products are -36 units, 30 units, and 6 units, respectively. The one with the largest absolute value is case (d), $|-36| = 36$.

ROUND: Rounding is not required in this problem.

DOUBLE-CHECK: Double-check by looking at the rectangles with sides the length of vector $\vec{A}$ and the length of the projection of vector $\vec{B}$ onto vector $\vec{A}$. The results agree with what was calculated using the formula.

1.108. When the scalar products are evaluated as described in the preceding solution, the one with the smallest absolute value is case (b), where $|\vec{A} \cdot \vec{B}| = 0$. It is characteristic of the scalar product that it comes out zero for perpendicular vectors, and zero is of course the smallest possible absolute value.

1.109. The vector product of two non-parallel vectors $\vec{A}$ and $\vec{B}$ that lie in the xy -plane is a vector in the z -direction. As given by Eq. (1.32), the magnitude of that vector is $|A_x B_y - A_y B_x|$.

$$(a) |A_x B_y - A_y B_x| = |(6)(5) - (0)(1)| = 30$$

$$(b) |A_x B_y - A_y B_x| = |(6)(3) - (0)(0)| = 18$$

$$(c) |A_x B_y - A_y B_x| = |(6)(2) - (0)(2)| = 12$$

$$(d) |A_x B_y - A_y B_x| = |(6)(0) - (0)(-6)| = 0$$

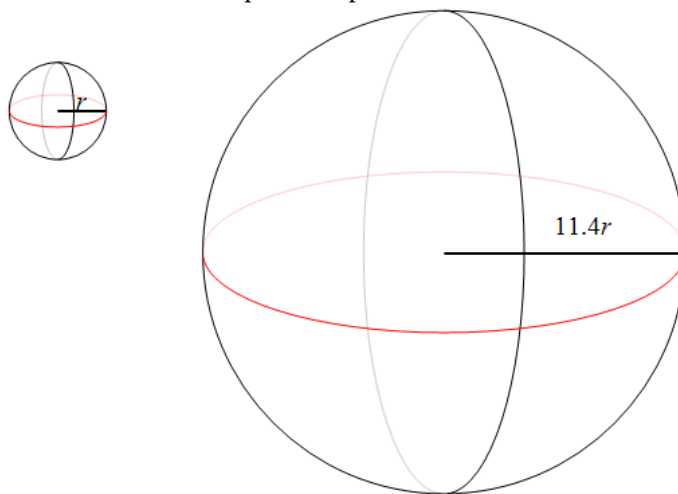
$$(e) \left| A_x B_y - A_y B_x \right| = \left| (6)(1) - (0)(5) \right| = 6$$

$$(f) \left| A_x B_y - A_y B_x \right| = \left| (6)(4) - (0)(1) \right| = 24$$

The largest absolute value of a vector product is case (a).

- 1.110.** When the vector products are evaluated as described in Solution 1.109, the one with the smallest absolute value is case (d). It is characteristic of the vector product that it comes out zero for parallel or antiparallel vectors, and zero is of course the smallest possible absolute value.
- 1.111.** Taking the absolute value and ranking in order from least to greatest, we find that $0 < 6 = 6 < 12 < 30 < 36$. This gives us the ordering from least to greatest of the absolute value of the scalar product in parts b, a = f, c, e, and d.
- 1.112.** Ranking the absolute values found in Solution 1.109 in order from least to greatest, we find that $0 < 6 < 12 < 18 < 24 < 30$. This gives us the ordering from least to greatest of the absolute value of the vector product in parts d, e, c, b, f, and a.
- 1.113. THINK:** We are given the change in the star's radius. So, if we can express the surface area, circumference, and volume in terms of the radius, we can find by what factors these change as the radius changes.

SKETCH: We can think of the star as a sphere in space with radius r .



RESEARCH: We can use the formulas for volume and surface area of a sphere given in Appendix A. We find that the volume of the sphere on the left is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$. Similarly, the volume of the sphere on the right is $\frac{4}{3}\pi(11.4r)^3$ and its surface area is $4\pi(11.4r)^2$. The circumference of a sphere is the same as the circumference of a great circle around it (shown in red in the sketch). Finding the radius of the circle will give us the radius of the sphere. Using this method, we find that the circumference of the sphere on the left is $2\pi r$, while the sphere on the right has a circumference of $2\pi(11.4r)$.

SIMPLIFY: We use algebra to find the volume, surface area, and circumference of the larger sphere in terms of the volume, surface area, and circumference of the smaller sphere. (a) We find the surface area of the sphere on the right is $4\pi(11.4r)^2 = 4\pi(11.4^2 r^2) = 11.4^2 \cdot 4\pi r^2$ (b) The circumference of the larger sphere is $2\pi(11.4r) = 11.4 \cdot (2\pi r)$. (c) The volume of the larger sphere is $\frac{4}{3}\pi(11.4r)^3 = \frac{4}{3}\pi(11.4^3 r^3) = (11.4^3) \frac{4}{3}\pi r^3$.

CALCULATE: Since we don't know the star's original radius, we take the ratio of the new value divided by the old value to get the factor by which the surface area, volume, and circumference have increased. In part (a), we find that the ratio of the new surface area to the original surface area is

$$\frac{11.4^2 \cdot 4\pi r^2}{4\pi r^2} = 11.4^2 \frac{4\pi r^2}{4\pi r^2} = 11.4^2 \cdot 1 = 129.96.$$

(b) Similarly, we can divide the new circumference by the original one to get $\frac{11.4 \cdot (2\pi r)}{2\pi r} = 11.4 \frac{2\pi r}{2\pi r} = 11.4 \cdot 1 = 11.4$. (c) The new volume divided by the original

$$\text{volume is } \frac{(11.4^3) \frac{4}{3}\pi r^3}{\frac{4}{3}\pi r^3} = (11.4^3) \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r^3} = 11.4^3 \cdot 1 = 1481.544.$$

ROUND: For all of these calculations, we round to three significant figures. This gives us that (a) the surface area has increased by a factor of 130., (b) the circumference has increased by a factor of 11.4, and (c) the volume has increased by a factor of 1.48×10^3 .

DOUBLE-CHECK: Think about what these values represent. The circumference is a one-dimensional quantity, with units such as km, which is proportional to r . The surface area is a two-dimensional quantity with units such as km^2 , and is proportional to r^2 . The volume is a three-dimensional quantity with units such as km^3 and is proportional to r^3 . So it makes sense that, when we increase the radius by a given amount (11.4 in this case), the circumference increases in proportion to that amount, while the surface area increases by that amount squared, and the volume increases by the cube of that amount.

- 1.114.** The circumference is directly proportional to the radius.
 (a) The surface area is proportional to the square of the radius and therefore to the square of the circumference. It will increase by a factor of $12.5^2 = 156$.
 (b) The radius is directly proportional to the circumference. It will increase by a factor of 12.5.
 (c) The volume is proportional to the cube of the radius and therefore to the cube of the circumference. It will increase by a factor of $12.5^3 = 1950$.
- 1.115.** The volume is proportional to the cube of the radius, so if the volume increases by a factor of 872, then the radius increases by a factor of $\sqrt[3]{872} = 9.553712362$.
 (a) The surface area is proportional to the square of the radius. It will increase by a factor of $9.553712362^2 = 91.3$.
 (b) The circumference is directly proportional to the radius. It will increase by a factor of 9.55.
 (c) The diameter is directly proportional to the radius. It will increase by a factor of 9.55.
- 1.116.** (a) The volume is proportional to the cube of the radius, so if the volume increases by a factor of 274, then the radius increases by a factor of $\sqrt[3]{274} = 16.55294536 = 16.6$.
 (b) The volume is proportional to the cube of the radius. It will increase by a factor of $16.55294536^3 = 4535.507028 = 4540$.
 (c) The density is *inversely* proportional to the volume. It will decrease by a factor of $4535.507028^{-1} = 2.20 \cdot 10^{-4}$.

Chapter 2: Motion in a Straight Line

Concept Checks

2.1. d 2.2. b 2.3. b 2.4. c 2.5. a) 3 b) 1 c) 4 d) 2 2.6. c 2.7. d 2.8. c 2.9. d

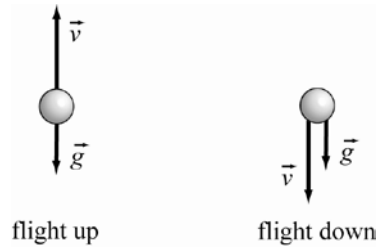
Multiple-Choice Questions

2.1. e 2.2. c 2.3. c 2.4. b 2.5. e 2.6. a 2.7. d 2.8. c 2.9. a 2.10. b 2.11. b 2.12. d 2.13. c 2.14. d 2.15. a 2.16. c

Conceptual Questions

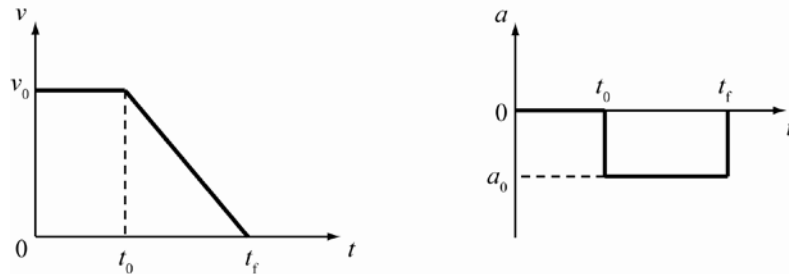
- 2.17. Velocity and speed are defined differently. The magnitude of average velocity and average speed are the same only when the direction of movement does not change. If the direction changes during movement, it is known that the net displacement is smaller than the net distance. Using the definition of average velocity and speed, it can be said that the magnitude of average velocity is less than the average speed when the direction changes during movement. Here, only Christine changes direction during her movement. Therefore, only Christine has a magnitude of average velocity which is smaller than her average speed.

- 2.18. The acceleration due to gravity is always pointing downward to the center of the Earth.



It can be seen that the direction of velocity is opposite to the direction of acceleration when the ball is in flight upward. The direction of velocity is the same as the direction of acceleration when the ball is in flight downward.

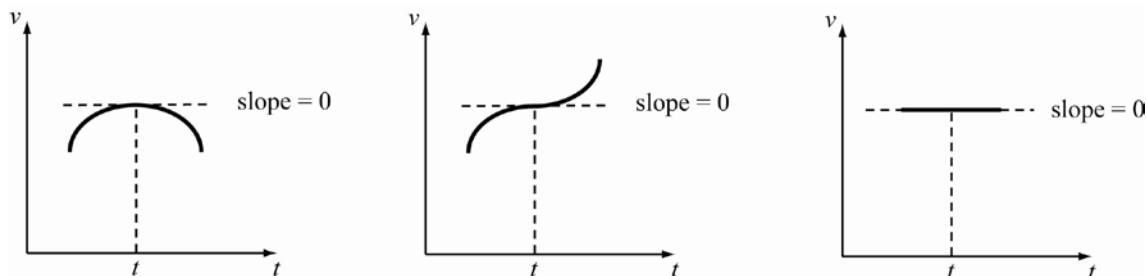
- 2.19. The car, before the brakes are applied, has a constant velocity, v_0 , and zero acceleration. After the brakes are applied, the acceleration is constant and in the direction opposite to the velocity. In velocity versus time and acceleration versus time graphs, the motion is described in the figures below.



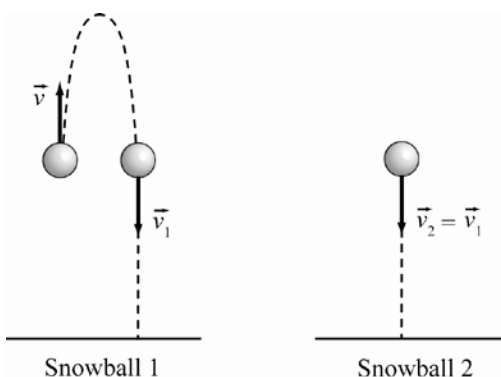
- 2.20. There are two cars, car 1 and car 2. The decelerations are $a_1 = 2a_2 = -a_0$ after applying the brakes. Before applying the brakes, the velocities of both cars are the same, $v_1 = v_2 = v_0$. When the cars have completely stopped, the final velocities are zero, $v_f = 0$. $v_f = v_0 + at = 0 \Rightarrow t = -\frac{v_0}{a}$. Therefore, the ratio of time taken

$$\text{to stop is Ratio} = \frac{\text{time of car 1}}{\text{time of car 2}} = \frac{-v_0 / -a_0}{-v_0 / \left(-\frac{1}{2}a_0\right)} = \frac{1}{2}. \text{ So the ratio is one half.}$$

- 2.21. Here a and v are instantaneous acceleration and velocity. If $a = 0$ and $v \neq 0$ at time t , then at that moment the object is moving at a constant velocity. In other words, the slope of a curve in a velocity versus time plot is zero at time t . See the plots below.

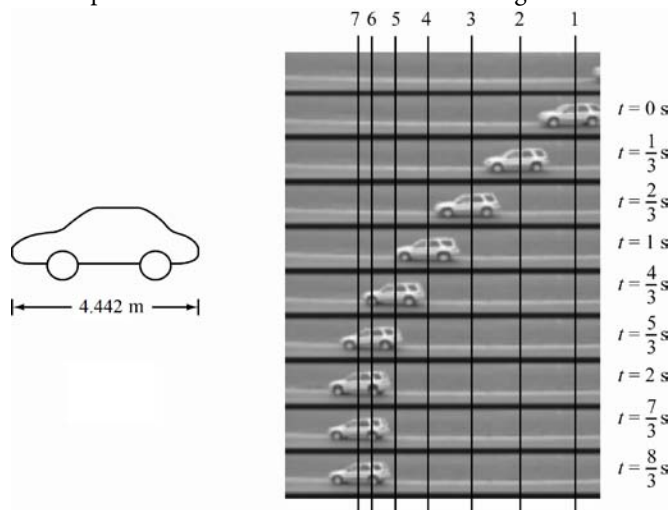


- 2.22. The direction of motion is determined by the direction of velocity. Acceleration is defined as a change in velocity per change in time. The change in velocity, Δv , can be positive or negative depending on the values of initial and final velocities, $\Delta v = v_f - v_i$. If the acceleration is in the opposite direction to the motion, it means that the magnitude of the objects velocity is decreasing. This occurs when an object is slowing down.
- 2.23. If there is no air resistance, then the acceleration does not depend on the mass of an object. Therefore, both snowballs have the same acceleration. Since initial velocities are zero, and the snowballs will cover the same distance, both snowballs will hit the ground at the same time. They will both have the same speed.
- 2.24. Acceleration is independent of the mass of an object if there is no air resistance.

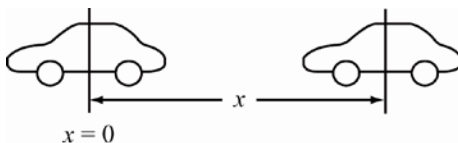


Snowball 1 will return to its original position after Δt , and then it falls in the same way as snowball 2. Therefore snowball 2 will hit the ground first since it has a shorter path. However, both snowballs have the same speed when they hit the ground.

- 2.25. Make sure the scale for the displacements of the car is correct. The length of the car is 174.9 in = 4.442 m.



Measuring the length of the car in the figure above with a ruler, the car in this scale is 0.80 ± 0.05 cm. Draw vertical lines at the center of the car as shown in the figure above. Assume line 7 is the origin ($x = 0$).



Assume a constant acceleration $a = a_0$. Use the equations $v = v_0 + at$ and $x = x_0 + v_0 t + (1/2)at^2$. When the car has completely stopped, $v = 0$ at $t = t_0$.

$$0 = v_0 + at_0 \Rightarrow v_0 = -at_0$$

Use the final stopping position as the origin, $x = 0$ at $t = t_0$.

$$0 = x_0 + v_0 t_0 + \frac{1}{2}at_0^2$$

Substituting $v_0 = -at_0$ and simplifying gives

$$x_0 - at_0^2 + \frac{1}{2}at_0^2 = 0 \Rightarrow x_0 - \frac{1}{2}at_0^2 = 0 \Rightarrow a = \frac{2x_0}{t_0^2}$$

Note that time t_0 is the time required to stop from a distance x_0 . First measure the length of the car. The length of the car is 0.80 cm. The actual length of the car is 4.442 m, therefore the scale is $\frac{4.442 \text{ m}}{0.80 \text{ cm}} = 5.5 \text{ m/cm}$. The error in measurement is $(0.05 \text{ cm}) 5.5 \text{ m/cm} \approx 0.275 \text{ m}$ (round at the end).

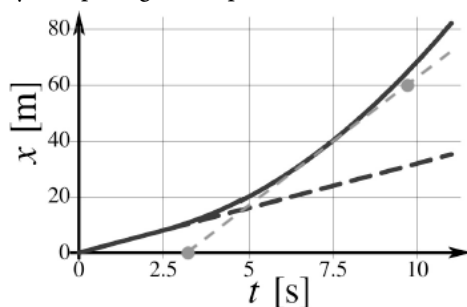
So the scale is $5.5 \pm 0.275 \text{ m/cm}$. The farthest distance of the car from the origin is $2.9 \pm 0.05 \text{ cm}$. Multiplying by the scale, 15.95 m, $t_0 = (0.333)(6 \text{ s}) = 1.998 \text{ s}$. The acceleration can be found using

$$a = 2x_0 / t_0^2: a = \frac{2(15.95 \text{ m})}{(1.998 \text{ s})^2} = 7.991 \text{ m/s}^2. \text{ Because the scale has two significant digits, round the result to}$$

two significant digits: $a = 8.0 \text{ m/s}^2$. Since the error in the measurement is $\Delta x_0 = 0.275 \text{ m}$, the error of the acceleration is

$$\Delta a = \frac{2\Delta x_0}{t_0^2} = \frac{2(0.275 \text{ m})}{(1.998 \text{ s})^2} \approx 0.1 \text{ m/s}^2.$$

- 2.26. Velocity can be estimated by computing the slope of a curve in a distance versus time plot.



Velocity is defined by $v = \Delta x / \Delta t$. If acceleration is constant, then $a = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$. (a) Estimate the slope of the dashed blue line. Pick two points: it is more accurate to pick a point that coincides with horizontal lines of the grid. Choosing points $t = 0$ s, $x = 0$ m and $t = 6.25$ s, $x = 20$ m:

$$v = \frac{20. \text{ m} - 0 \text{ m}}{6.25 \text{ s} - 0 \text{ s}} = 3.2 \text{ m/s}$$

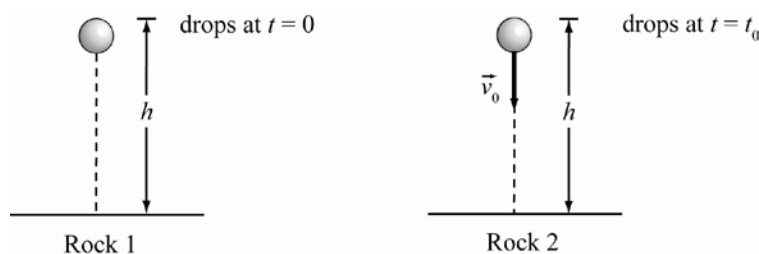
(b) Examine the sketch. There is a tangent to the curve at $t = 7.5$ s. Pick two points on the line. Choosing points: $t = 3.4$ s, $x = 0$ m and $t = 9.8$ s, $x = 60$ m:

$$v = \frac{60. \text{ m} - 0 \text{ m}}{9.8 \text{ s} - 3.4 \text{ s}} = 9.4 \text{ m/s}$$

(c) From (a), $v = 3.2$ m/s at $t = 2.5$ s and from (b), $v = 9.4$ m/s at $t = 7.5$ s. From the definition of constant acceleration,

$$a = \frac{9.4 \text{ m/s} - 3.2 \text{ m/s}}{7.5 \text{ s} - 2.5 \text{ s}} = \frac{6.2 \text{ m/s}}{5.0 \text{ s}} = 1.2 \text{ m/s}^2.$$

- 2.27. There are two rocks, rock 1 and rock 2. Both rocks are dropped from height h . Rock 1 has initial velocity $v = 0$ and rock 2 has $v = v_0$ and is thrown at $t = t_0$.



Rock 1: $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

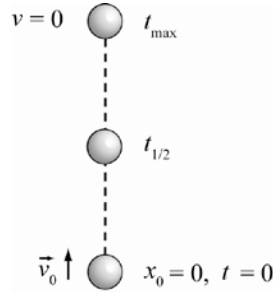
Rock 2: $h = v_0(t - t_0) + \frac{1}{2}g(t - t_0)^2 \Rightarrow \frac{1}{2}g(t - t_0)^2 + v_0(t - t_0) - h = 0$

This equation has roots $t - t_0 = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$. Choose the positive root since $(t - t_0) > 0$. Therefore

$t_0 = t + \frac{v_0 - \sqrt{v_0^2 + 2gh}}{g}$. Substituting $t = \sqrt{\frac{2h}{g}}$ gives:

$$t_0 = \sqrt{\frac{2h}{g}} + \frac{v_0}{g} - \frac{\sqrt{v_0^2 + 2gh}}{g} \text{ or } \sqrt{\frac{2h}{g}} + \frac{v_0}{g} - \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}.$$

- 2.28. I want to know when the object is at half its maximum height. The wrench is thrown upwards with an initial velocity $v(t=0) = v_0$, $x = x_0 + v_0 t - \frac{1}{2} g t^2$, $v = v_0 - g t$, and $g = 9.81 \text{ m/s}^2$.



At maximum height, $v = 0$. $v = v_0 - g t \Rightarrow 0 = v_0 - g t_{\max} \Rightarrow v_0 = g t_{\max}$. Substitute $t_{\max} = v_0 / g$ into $x = x_0 + v_0 t - (1/2) g t^2$.

$$x_{\max} = v_0 \left(\frac{v_0}{g} \right) - \frac{1}{2} g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{g} - \frac{1}{2} \left(\frac{v_0^2}{g} \right) = \frac{v_0^2}{g} \left(1 - \frac{1}{2} \right) = \frac{v_0^2}{2g}$$

Therefore, half of the maximum height is $x_{1/2} = \frac{v_0^2}{4g}$. Substitute this into the equation for x .

$$x_{1/2} = \frac{v_0^2}{4g} = v_0 t_{1/2} - \frac{1}{2} g t_{1/2}^2 \Rightarrow \frac{1}{2} g t_{1/2}^2 - v_0 t_{1/2} + \frac{v_0^2}{4g} = 0$$

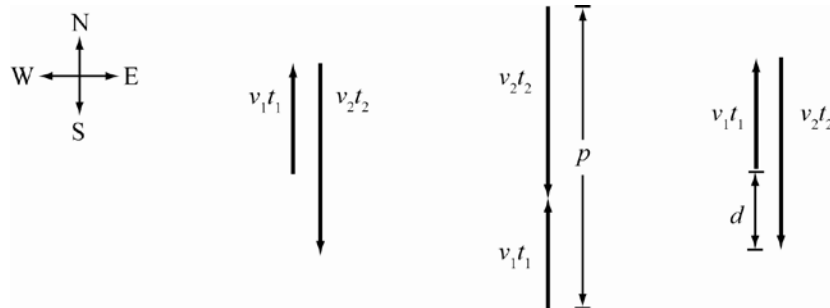
This is a quadratic equation with respect to $t_{1/2}$. The solutions to this equation are:

$$t_{1/2} = \frac{v_0 \pm \sqrt{v_0^2 - 4 \left(\frac{1}{2} g \right) \left(\frac{v_0^2}{4g} \right)}}{2 \left(\frac{1}{2} g \right)} = \frac{v_0 \pm \sqrt{v_0^2 - \frac{1}{2} v_0^2}}{g} = \frac{v_0 \pm v_0 \left(\frac{1}{\sqrt{2}} \right)}{g} = \frac{v_0}{g} \left(1 \pm \frac{1}{\sqrt{2}} \right)$$

Exercises

- 2.29. **THINK:** What is the distance traveled, p , and the displacement d if $v_1 = 30.0 \text{ m/s}$ due north for $t_1 = 10.0 \text{ min}$ and $v_2 = 40.0 \text{ m/s}$ due south for $t_2 = 20.0 \text{ min}$? Times should be in SI units: $t_1 = 10.0 \text{ min} (60 \text{ s/min}) = 6.00 \cdot 10^2 \text{ s}$, $t_2 = 20.0 \text{ min} (60 \text{ s/min}) = 1.20 \cdot 10^3 \text{ s}$.

SKETCH:



RESEARCH: The distance is equal to the product of velocity and time. The distance traveled is $p = v_1 t_1 + v_2 t_2$ and the displacement is the distance between where you start and where you finish, $d = v_1 t_1 - v_2 t_2$.

SIMPLIFY: There is no need to simplify.

CALCULATE: $p = v_1 t_1 + v_2 t_2 = (30. \text{ m/s})(6.00 \cdot 10^2 \text{ s}) + (40. \text{ m/s})(1.20 \cdot 10^3 \text{ s}) = 66,000. \text{ m}$

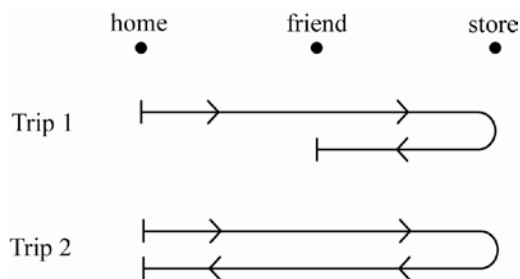
$$d = v_1 t_1 - v_2 t_2 = (30. \text{ m/s})(6.00 \cdot 10^2 \text{ s}) - (40. \text{ m/s})(1.20 \cdot 10^3 \text{ s}) = -30,000. \text{ m}$$

ROUND: The total distance traveled is 66.0 km, and the displacement is 30.0 km in southern direction.

DOUBLE-CHECK: The distance traveled is larger than the displacement as expected. Displacement is also expected to be towards the south since the second part of the trip going south is faster and has a longer duration.

- 2.30. **THINK:** I want to find the displacement and the distance traveled for a trip to the store, which is 1000. m away, and back. Let $l = 1000. \text{ m}$.

SKETCH:



RESEARCH: displacement (d) = final position – initial position
distance traveled = distance of path taken

SIMPLIFY:

(a) $d = \frac{1}{2}l - 0 = \frac{1}{2}l$

(b) $p = l + \frac{1}{2}l = \frac{3}{2}l$

(c) $d = 0 - 0 = 0$

(d) $p = l + l = 2l$

CALCULATE:

(a) $d = \frac{1}{2}l = \frac{1}{2}(1000. \text{ m}) = 500.0 \text{ m}$

(b) $p = \frac{3}{2}l = \frac{3}{2}(1000. \text{ m}) = 1500. \text{ m}$

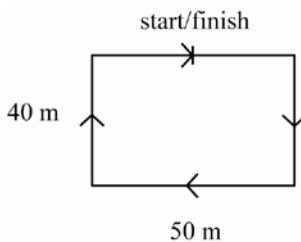
(c) $d = 0 \text{ m}$

(d) $p = 2l = 2(1000. \text{ m}) = 2000. \text{ m}$

ROUND: No rounding is necessary.

DOUBLE-CHECK: These values are reasonable: they are of the order of the distance to the store.

- 2.31. **THINK:** I want to find the average velocity when I run around a rectangular 50 m by 40 m track in 100 s.
SKETCH:



RESEARCH: average velocity = $\frac{\text{final position} - \text{initial position}}{\text{time}}$

SIMPLIFY: $\bar{v} = \frac{x_f - x_i}{t}$

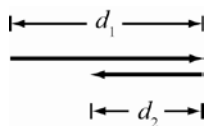
CALCULATE: $\bar{v} = \frac{0 \text{ m} - 0 \text{ m}}{100 \text{ s}} = 0 \text{ m/s}$

ROUND: Rounding is not necessary, because the result of 0 m/s is exact.

DOUBLE-CHECK: Since the final and initial positions are the same point, the average velocity will be zero. The answer may be displeasing at first since someone ran around a track and had no average velocity. Note that the speed would not be zero.

- 2.32. THINK:** I want to find the average velocity and the average speed of the electron that travels $d_1 = 2.42 \text{ m}$ in $t_1 = 2.91 \cdot 10^{-8} \text{ s}$ in the positive x -direction then $d_2 = 1.69 \text{ m}$ in $t_2 = 3.43 \cdot 10^{-8} \text{ s}$ in the opposite direction.

SKETCH:



RESEARCH:

(a) average velocity = $\frac{\text{final position} - \text{initial position}}{\text{time}}$

(b) speed = $\frac{\text{total distance traveled}}{\text{time}}$

SIMPLIFY:

(a) $\bar{v} = \frac{d_1 - d_2}{t_1 + t_2}$

(b) $s = \frac{d_1 + d_2}{t_1 + t_2}$

CALCULATE:

(a) $\bar{v} = \frac{d_1 - d_2}{t_1 + t_2} = \frac{2.42 \text{ m} - 1.69 \text{ m}}{2.91 \cdot 10^{-8} \text{ s} + 3.43 \cdot 10^{-8} \text{ s}} = 11,514,195 \text{ m/s}$

(b) $s = \frac{d_1 + d_2}{t_1 + t_2} = \frac{2.42 \text{ m} + 1.69 \text{ m}}{2.91 \cdot 10^{-8} \text{ s} + 3.43 \cdot 10^{-8} \text{ s}} = 64,826,498 \text{ m/s}$

ROUND:

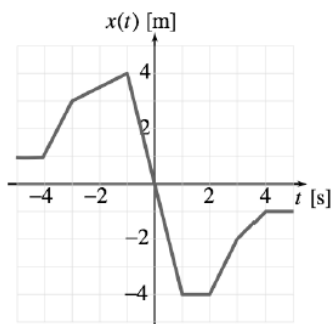
(a) $\bar{v} = 1.15 \cdot 10^7 \text{ m/s}$

(b) $s = 6.48 \cdot 10^7 \text{ m/s}$

DOUBLE-CHECK: The average velocity is less than the speed, which makes sense since the electron changes direction.

- 2.33. **THINK:** The provided graph must be used to answer several questions about the speed and velocity of a particle. Questions about velocity are equivalent to questions about the slope of the position function.

SKETCH:



RESEARCH: The velocity is given by the slope on a distance versus time graph. A steeper slope means a greater speed.

$$\text{average velocity} = \frac{\text{final position} - \text{initial position}}{\text{time}}, \quad \text{speed} = \frac{\text{total distance traveled}}{\text{time}}$$

- (a) The largest speed is where the slope is the steepest.
- (b) The average velocity is the total displacement over the time interval.
- (c) The average speed is the total distance traveled over the time interval.
- (d) The ratio of the velocities is $v_1 : v_2$.
- (e) A velocity of zero is indicated by a slope that is horizontal.

SIMPLIFY:

- (a) The largest speed is given by the steepest slope occurring between -1 s and $+1$ s.

$$s = \frac{|x(t_2) - x(t_1)|}{t_2 - t_1}, \quad \text{with } t_2 = 1 \text{ s and } t_1 = -1 \text{ s.}$$

- (b) The average velocity is given by the total displacement over the time interval.

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \quad \text{with } t_2 = 5 \text{ s and } t_1 = -5 \text{ s.}$$

- (c) In order to calculate the speed in the interval -5 s to 5 s, the path must first be determined. The path is given by starting at 1 m, going to 4 m, then turning around to move to -4 m and finishing at -1 m. So the total distance traveled is

$$\begin{aligned} p &= |(4 \text{ m} - 1 \text{ m})| + |((-4 \text{ m}) - 4 \text{ m})| + |(-1 \text{ m} - (-4 \text{ m}))| \\ &= 3 \text{ m} + 8 \text{ m} + 3 \text{ m} \\ &= 14 \text{ m} \end{aligned}$$

This path can be used to find the speed of the particle in this time interval.

$$s = \frac{p}{t_2 - t_1}, \quad \text{with } t_2 = 5 \text{ s and } t_1 = -5 \text{ s.}$$

- (d) The first velocity is given by $v_1 = \frac{x(t_3) - x(t_2)}{t_3 - t_2}$ and the second by $v_2 = \frac{x(t_4) - x(t_3)}{t_4 - t_3}$,

- (e) The velocity is zero in the regions 1 s to 2 s, -5 s to -4 s, and 4 s to 5 s.

CALCULATE:

$$(a) \quad s = \frac{|-4 \text{ m} - 4 \text{ m}|}{1 \text{ s} - (-1 \text{ s})} = 4.0 \text{ m/s}$$

$$(b) \quad \bar{v} = \frac{-1 \text{ m} - 1 \text{ m}}{5 \text{ s} - (-5 \text{ s})} = -0.20 \text{ m/s}$$

$$(c) \bar{s} = \frac{14 \text{ m}}{5 \text{ s} - (-5 \text{ s})} = 1.4 \text{ m/s}$$

$$(d) v_1 = \frac{(-2 \text{ m}) - (-4 \text{ m})}{3 \text{ s} - 2 \text{ s}} = 2.0 \text{ m/s}, v_2 = \frac{(-1 \text{ m}) - (-2 \text{ m})}{4 \text{ s} - 3 \text{ s}} = 1.0 \text{ m/s}, \quad \text{so } v_1 : v_2 = 2 : 1.$$

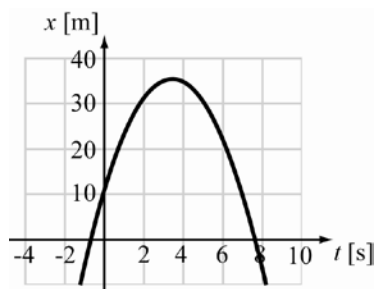
(e) There is nothing to calculate.

ROUND: Rounding is not necessary in this case, because we can read the values of the positions and times off the graph to at least 2 digit precision.

DOUBLE-CHECK: The values are reasonable for a range of positions between -4 m and 4 m with times on the order of seconds. Each calculation has the expected units.

- 2.34. **THINK:** I want to find the average velocity of a particle whose position is given by the equation $x(t) = 11 + 14t - 2.0t^2$ during the time interval $t = 1.0 \text{ s}$ to $t = 4.0 \text{ s}$.

SKETCH:



RESEARCH: The average velocity is given by the total displacement over the time interval.

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \text{ with } t_2 = 4.0 \text{ s and } t_1 = 1.0 \text{ s}.$$

$$\text{SIMPLIFY: } \bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{(11 + 14t_2 - 2.0t_2^2) - (11 + 14t_1 - 2.0t_1^2)}{t_2 - t_1} = \frac{14(t_2 - t_1) - 2.0(t_2^2 - t_1^2)}{t_2 - t_1}$$

$$\text{CALCULATE: } \bar{v} = \frac{14(4.0 \text{ s} - 1.0 \text{ s}) - 2.0((4.0 \text{ s})^2 - (1.0 \text{ s})^2)}{4.0 \text{ s} - 1.0 \text{ s}} = 4.0 \text{ m/s}$$

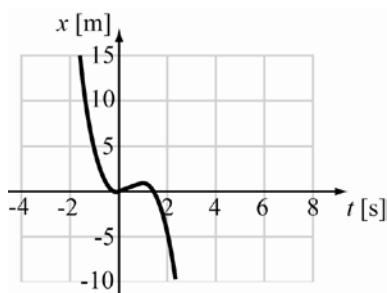
ROUND: The values given are all accurate to two significant digits, so the answer is given by two significant digits: $v = 4.0 \text{ m/s}$.

DOUBLE-CHECK: A reasonable approximation of the average velocity from $t = 1$ to $t = 4$ is to look at the instantaneous velocity at the midpoint. The instantaneous velocity is given by the derivative of the position, which is

$$v = \frac{d}{dt}(11 + 14t - 2.0t^2) = 0 + 1(14) - 2(2.0t) = 14 - 4.0t.$$

The value of the instantaneous velocity at $t = 2.5 \text{ s}$ is $14 - 4.0(2.5) = 4.0 \text{ m/s}$. The fact that the calculated average value matches the instantaneous velocity at the midpoint lends support to the answer.

- 2.35. **THINK:** I want to find the position of a particle when it reaches its maximum speed. I know the equation for the position as a function of time: $x = 3.0t^2 - 2.0t^3$. I will need to find the expression for the velocity and the acceleration to determine when the speed will be at its maximum. The maximum speed in the x -direction will occur at a point where the acceleration is zero.

SKETCH:

RESEARCH: The velocity is the derivative of the position function with respect to time. In turn, the acceleration is given by derivative of the velocity function with respect to time. The expressions can be found using the formulas:

$$v(t) = \frac{d}{dt}x(t), \quad a(t) = \frac{d}{dt}v(t).$$

Find the places where the acceleration is zero. The maximum speed will be the maximum of the speeds at the places where the acceleration is zero.

SIMPLIFY: $v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}(3.0t^2 - 2.0t^3) = 2 \cdot 3.0t^{2-1} - 3 \cdot 2.0t^{3-1} = 6.0t - 6.0t^2$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(6.0t - 6.0t^2) = 6.0t^{1-1} - 2 \cdot 6.0t^{2-1} = 6.0 - 12t$$

CALCULATE: Solving for the value of t where a is zero:

$$0 = 6.0 - 12t \Rightarrow 6.0 = 12t \Rightarrow t = 0.50 \text{ s}$$

This time can now be used to solve for the position:

$$x(0.50) = 3.0(0.50)^2 - 2.0(0.50)^3 = 0.500 \text{ m}$$

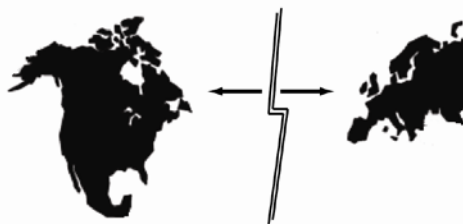
Since there is only one place where the acceleration is zero, the maximum speed in the positive x -direction must occur here.

ROUND: Since all variables and parameters are accurate to 2 significant digits, the answer should be too: $x = 0.50 \text{ m}$.

DOUBLE-CHECK: The validity of the answer can be confirmed by checking the velocity at $t = 0.50 \text{ s}$ and times around this point. At $t = 0.49 \text{ s}$, the velocity is 1.4994 m/s , and at $t = 0.51 \text{ s}$ the velocity is also 1.4994 m/s . Since these are both smaller than the velocity at 0.50 s ($v = 1.5 \text{ m/s}$), the answer is valid.

- 2.36. **THINK:** I want to find the time it took for the North American and European continents to reach a separation of 3000 mi if they are traveling at a speed of 10 mm/yr. First convert units:

$$d = (3000 \text{ mi})(1609 \text{ m/mi}) = 4827000 \text{ m}, \quad v = (10 \text{ mm/yr})(10^{-3} \text{ m/mm}) = 0.01 \text{ m/yr}.$$

SKETCH:

RESEARCH: The time can be found using the familiar equation: $d = vt$.

SIMPLIFY: The equation becomes $t = d/v$.

CALCULATE: $t = \frac{4827000 \text{ m}}{0.01 \text{ m/yr}} = 482,700,000 \text{ yr}$

ROUND: The values given in the question are given to one significant digit, thus the answer also should only have one significant digit: $t = 5 \cdot 10^8 \text{ yr}$.

DOUBLE-CHECK: The super continent Pangea existed about 250 million years ago or $2.5 \cdot 10^8$ years. Thus, this approximation is in the ballpark.

2.37.

THINK:

(a) I want to find the velocity at $t = 10.0 \text{ s}$ of a particle whose position is given by the function $x(t) = At^3 + Bt^2 + Ct + D$, where $A = 2.10 \text{ m/s}^3$, $B = 1.00 \text{ m/s}^2$, $C = -4.10 \text{ m/s}$, and $D = 3.00 \text{ m}$. I can differentiate the position function to derive the velocity function.

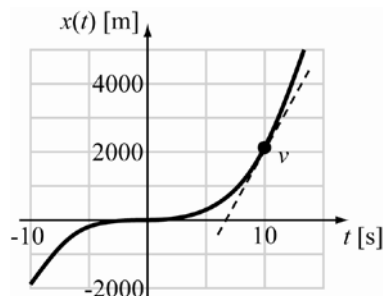
(b) I want to find the time(s) when the object is at rest. The object is at rest when the velocity is zero. I'll solve the velocity function I obtain in (a) equal to zero.

(c) I want to find the acceleration of the object at $t = 0.50 \text{ s}$. I can differentiate the velocity function found in part (a) to derive the acceleration function, and then calculate the acceleration at $t = 0.50 \text{ s}$.

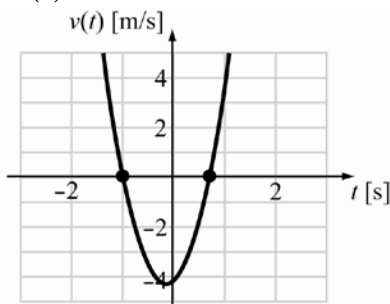
(d) I want to plot the function for the acceleration found in part (c) between the time range of -10.0 s to 10.0 s .

SKETCH:

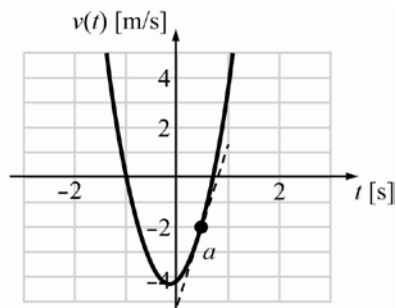
(a)



(b)



(c)



(d) The plot is part of CALCULATE.

RESEARCH:

(a) The velocity is given by the time derivative of the position function $v(t) = \frac{d}{dt}x(t)$.

(b) To find the time when the object is at rest, set the velocity to zero, and solve for t . This is a quadratic equation of the form $ax^2 + bx + c = 0$, whose solution is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(c) The acceleration is given by the time derivative of the velocity: $a(t) = \frac{d}{dt}v(t)$.

(d) The equation for acceleration found in part (c) can be used to plot the graph of the function.

SIMPLIFY:

$$(a) \quad v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}(At^3 + Bt^2 + Ct + D) = 3At^2 + 2Bt + C$$

(b) Set the velocity equal to zero and solve for t using the quadratic formula:

$$t = \frac{-2B \pm \sqrt{4B^2 - 4(3A)(C)}}{2(3A)} = \frac{-2B \pm \sqrt{4B^2 - 12AC}}{6A}$$

$$(c) \quad a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(3At^2 + 2Bt + C) = 6At + 2B$$

(d) There is no need to simplify this equation.

CALCULATE:

$$(a) \quad v(t = 10.0 \text{ s}) = 3(2.10 \text{ m/s}^3)(10.0 \text{ s})^2 + 2(1.00 \text{ m/s}^2)(10.0 \text{ s}) - 4.10 \text{ m/s} = 645.9 \text{ m/s}$$

$$(b) \quad t = \frac{-2(1.00 \text{ m/s}^2) \pm \sqrt{4(1.00 \text{ m/s}^2)^2 - 12(2.10 \text{ m/s}^3)(-4.10 \text{ m/s})}}{6(2.10 \text{ m/s}^3)}$$

$$= 0.6634553 \text{ s}, -0.9809156 \text{ s}$$

$$(c) \quad a(t = 0.50 \text{ s}) = 6(2.10 \text{ m/s}^3)(0.50 \text{ s}) + 2(1.00 \text{ m/s}^2) = 8.30 \text{ m/s}^2$$

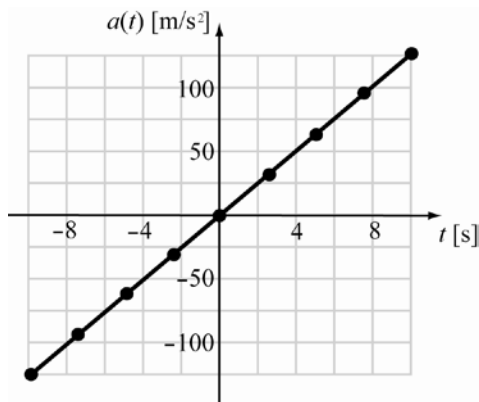
(d) The acceleration function, $a(t) = 6At + 2B$, can be used to compute the acceleration for time steps of 2.5 s. For example:

$$a(t = -2.5 \text{ s}) = 6(2.10 \text{ m/s}^3)(-2.5 \text{ s}) + 2(1.00 \text{ m/s}^2) = -29.5 \text{ m/s}^2$$

The result is given in the following table.

t [s]	-10.0	-7.5	-5.0	-2.5	0.0	2.5	5.0	7.5	10.0
a [m/s^2]	-124.0	-92.5	-61.0	-29.5	2.0	33.5	65.0	96.5	128.0

These values are used to plot the function.



ROUND:

(a) The accuracy will be determined by the factor $3(2.10 \text{ m/s}^3)(10.0 \text{ s})^2$, which only has two significant digits. Thus the velocity at 10.0 s is 646 m/s.

(b) The parameters are accurate to two significant digits, thus the solutions will also have three significant digits: $t = 0.663 \text{ s}$ and -0.981 s

(c) The accuracy is limited by the values with the smallest number of significant figures. This requires three significant figures. The acceleration is then $a = 8.30 \text{ m/s}^2$.

(d) No rounding is necessary.

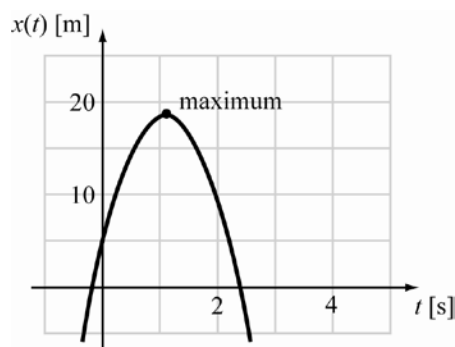
DOUBLE-CHECK:

- (a) This result is reasonable given the parameters. For example, $t^2 = (10.0 \text{ s})^2 = 100. \text{ s}$, so the velocity should be in the hundreds of meters per second.
- (b) Since the function is quadratic, there should be two solutions. The negative solution means that the object was at rest 0.98 seconds before the time designated $t = 0 \text{ s}$.
- (c) These values are consistent with the parameters.
- (d) The function for the acceleration is linear which the graph reflects.

- 2.38. THINK:** I want to determine the time when a particle will reach its maximum displacement and what the displacement will be at that time. The equation of the object's displacement is given as:

$$x(t) = 4.35 \text{ m} + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2$$

Differentiating x with respect to t gives the equation for the velocity. This is important since the time at which the velocity is zero is the moment at which the object has reached its maximum displacement.

SKETCH:

RESEARCH: The velocity is the derivative: $v = \frac{d}{dt}x(t)$. Find the value of t that makes the velocity zero.

Then, for part (b), substitute that value of t back into $x(t)$.

$$\begin{aligned} \text{SIMPLIFY: } v &= \frac{d}{dt}[4.35 \text{ m} + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2] \\ &= 25.9 \text{ m/s} - 2(11.79 \text{ m/s}^2)t \end{aligned}$$

Time for the maximum displacement is found by solving for t in the equation:
 $25.9 \text{ m/s} - 2(11.79 \text{ m/s}^2)t = 0$.

CALCULATE:

$$(a) \ t = \frac{25.9 \text{ m/s}}{2(11.79 \text{ m/s}^2)} = 1.0984 \text{ s}$$

$$\begin{aligned} (b) \ x(t) &= 4.35 \text{ m} + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2 \\ &= 4.35 \text{ m} + (25.9 \text{ m/s})(1.10 \text{ s}) - (11.79 \text{ m/s}^2)(1.10 \text{ s})^2 \\ &= 18.5741 \text{ m} \end{aligned}$$

ROUND:

- (a) The accuracy of this time is limited by the parameter 25.9 m/s, thus the time is $t = 1.10 \text{ s}$.
- (b) The least accurate term in the expression for $x(t)$ is accurate to the nearest tenth, so $x_{\text{max}} = 18.6 \text{ m}$.

DOUBLE-CHECK: Consider the positions just before and after the time $t = 1.10 \text{ s}$. $x = 18.5 \text{ m}$ for $t = 1.00 \text{ s}$, and $x = 18.5 \text{ m}$ for $t = 1.20 \text{ s}$. These values are less than the value calculated for x_{max} , which confirms the accuracy of the result.

- 2.39. THINK:** I want to calculate the average acceleration of the bank robbers getaway car. He starts with an initial speed of 45 mph and reaches a speed of 22.5 mph in the opposite direction in 12.4 s. First convert the velocities to SI units:

$$v_i = (45 \text{ mph}) \left(0.447 \frac{\text{m/s}}{\text{mph}} \right) = 20.115 \text{ m/s}$$

$$v_f = (-22.5 \text{ mph}) \left(0.447 \frac{\text{m/s}}{\text{mph}} \right) = -10.0575 \text{ m/s}$$

SKETCH:



RESEARCH: average acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$

SIMPLIFY: $\bar{a} = \frac{v_f - v_i}{t}$

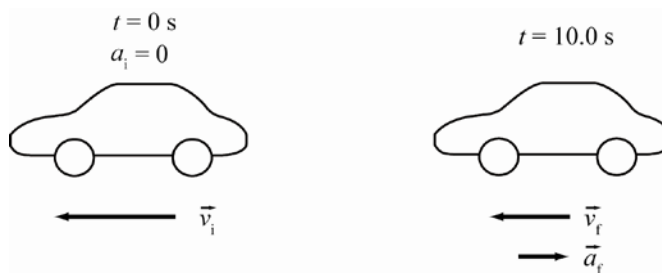
CALCULATE: $\bar{a} = \frac{(-10.0575 \text{ m/s}) - (20.115 \text{ m/s})}{12.4 \text{ s}} = -2.433 \text{ m/s}^2$

ROUND: The least precise of the velocities given in the question had two significant figures. Therefore, the final answer should also have two significant figures. The acceleration is $\bar{a} = -2.4 \text{ m/s}^2$, or 2.4 m/s^2 in the backward direction.

DOUBLE-CHECK: A top-of-the-line car can accelerate from 0 to 60 mph in 3 s. This corresponds to an acceleration of 8.94 m/s^2 . It is reasonable for a getaway car to be able to accelerate at a fraction of this value.

- 2.40. THINK:** I want to find the magnitude and direction of average acceleration of a car which goes from 22.0 m/s in the west direction to 17.0 m/s in the west direction in 10.0 s: $v_f = 17.0 \text{ m/s}$, $v_i = 22.0 \text{ m/s}$, $t = 10.0 \text{ s}$.

SKETCH:



RESEARCH: $\bar{a} = \frac{v_f - v_i}{t}$

SIMPLIFY: There is no need to simplify the above equation.

CALCULATE: $\bar{a} = \frac{17.0 \text{ m/s} - 22.0 \text{ m/s}}{10.0 \text{ s}} = -0.5000 \text{ m/s}^2$. The negative indicates the acceleration is east.

ROUND: The average acceleration is $\bar{a} = 0.500 \text{ m/s}^2$ east.

DOUBLE-CHECK: An acceleration of -0.500 m/s^2 is reasonable since a high performance car can accelerate at about 9 m/s^2 .

- 2.41. **THINK:** I want to find the magnitude of the constant acceleration of a car that goes 0.500 km in 10.0 s: $d = 0.500 \text{ km}$, $t = 10.0 \text{ s}$.

SKETCH:



RESEARCH: The position of the car under constant acceleration is given by $d = \frac{1}{2}at^2$.

SIMPLIFY: Solving for acceleration gives $a = \frac{2d}{t^2}$.

CALCULATE: $a = \frac{2(0.500 \text{ km})}{(10.0 \text{ s})^2} = 0.0100 \text{ km/s}^2$

ROUND: The values all have three significant figures. Thus, the average acceleration is $a = 0.0100 \text{ km/s}^2$, which is 10.0 m/s^2 .

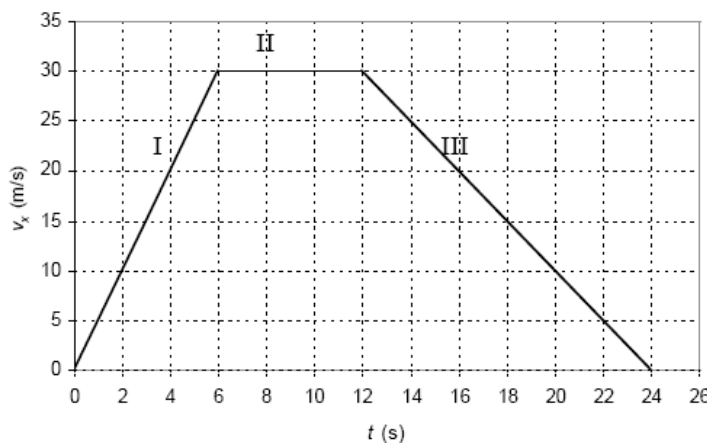
DOUBLE-CHECK: This acceleration is on the order of a high performance car which can accelerate from 0 to 60 mph in 3 seconds, or 9 m/s^2 .

- 2.42. **THINK:**

(a) I want to find the average acceleration of a car and the distance it travels by analyzing a velocity versus time graph. Each segment has a linear graph. Therefore, the acceleration is constant in each segment.

(b) The displacement is the area under the curve of a velocity versus time graph.

SKETCH:



RESEARCH:

(a) The acceleration is given by the slope of a velocity versus time graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

(b) The displacement is the sum of the areas of two triangles and one rectangle. Recall the area formulas:

$$\text{area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

$$\text{area of a rectangle} = \text{base} \times \text{height}$$

SIMPLIFY:

$$(a) \quad a_I = \frac{v_{I_2} - v_{I_1}}{t_{I_2} - t_{I_1}}, \quad a_{II} = \frac{v_{II_2} - v_{II_1}}{t_{II_2} - t_{II_1}}, \quad a_{III} = \frac{v_{III_2} - v_{III_1}}{t_{III_2} - t_{III_1}}$$

$$(b) \quad x = \frac{1}{2}v_{I_2}(t_{I_2} - t_{I_1}) + v_{II_2}(t_{II_2} - t_{II_1}) + \frac{1}{2}v_{III_2}(t_{III_2} - t_{III_1})$$

CALCULATE:

$$(a) \quad a_I = \frac{30.0 \text{ m/s} - 0 \text{ m/s}}{6.0 \text{ s} - 0 \text{ s}} = 5.0 \text{ m/s}^2, \quad a_{II} = \frac{30.0 \text{ m/s} - 30.0 \text{ m/s}}{12.0 \text{ s} - 6.0 \text{ s}} = 0.0 \text{ m/s}^2,$$

$$a_{III} = \frac{0.0 \text{ m/s} - 30.0 \text{ m/s}}{24.0 \text{ s} - 12.0 \text{ s}} = -2.50 \text{ m/s}^2$$

$$(b) \quad x = \frac{1}{2}(30.0 \text{ m/s})(6.0 \text{ s} - 0.0 \text{ s}) + (30.0 \text{ m/s})(12.0 \text{ s} - 6.0 \text{ s}) + \frac{1}{2}(30.0 \text{ m/s})(24.0 \text{ s} - 12.0 \text{ s}) = 450.0 \text{ m}$$

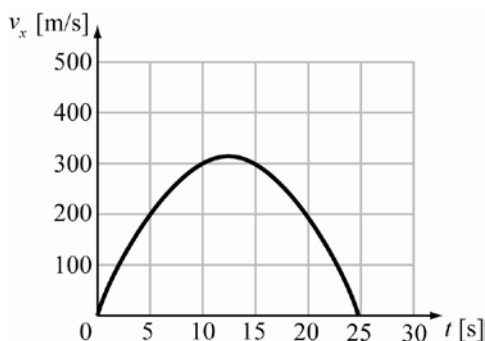
ROUND:

(a) Rounding is not necessary in this case, because the values of the velocities and times can be read off the graph to at least two digit precision.

(b) The answer is limited by the value 6.0 s, giving $x = 450 \text{ m}$.

DOUBLE-CHECK: The accelerations calculated in part (a) are similar to those of cars. The distance of 450 m is reasonable. The acceleration in I should be -2 times the acceleration in III, since the change in velocities are opposites, and the time in III for the change in velocity is twice the change in time that occurs in I.

- 2.43. **THINK:** I want to find the acceleration of a particle when it reaches its maximum displacement. The velocity of the particle is given by the equation $v_x = 50.0t - 2.0t^3$. The maximum displacement must occur when the velocity is zero. The expression for the acceleration can be found by differentiating the velocity with respect to time.

SKETCH:

RESEARCH: The acceleration is the derivative of the velocity: $a = \frac{d}{dt}v_x$. The maximum displacement will occur at a point where the velocity is zero. So, I can find the time at which the displacement is maximal by solving $v_x = 50.0t - 2.0t^3 = 0$ for t . The question says to consider after $t = 0$, so I will reject zero and negative roots. Then differentiate v with respect to t to obtain a formula for the acceleration. Evaluate the acceleration at the time where the displacement is maximized (which is when the velocity is zero).

SIMPLIFY: No simplification is required.

CALCULATE: Solving $v_x = 50.0t - 2.0t^3 = 0$ for t : $0 = 2.0t(25 - t^2)$, so $t = 0, \pm 5.0$. So, take $t = 5$. Now, differentiate v with respect to t to find the expression for the acceleration.

$$\begin{aligned} a &= \frac{d}{dt}(50.0t - 2.0t^3) \\ &= 50.0 - 6.0t^2 \end{aligned}$$

Substitute $t = 5.0$ s into the expression for acceleration:

$$a = 50.0 - 6.0t^2 = 50.0 - 6.0(5.0 \text{ s})^2 = -100 \text{ m/s}^2$$

ROUND: The solution is limited by the accuracy of $6.0t^2$, where $t = 5.0$ s, so it must be significant to two digits. This gives $50.0 - 150 = -100 \text{ m/s}^2$, which is also accurate to two significant figures. Therefore, the acceleration must be accurate to two significant figures: $a = -1.0 \cdot 10^2 \text{ m/s}^2$.

DOUBLE-CHECK: The acceleration must be negative at this point, since the displacement would continue to increase if a was positive.

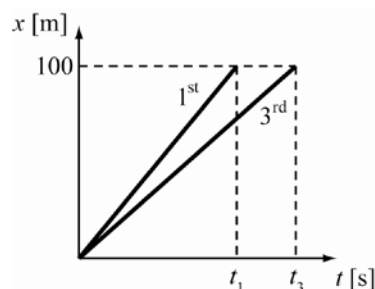
2.44. THINK:

(a) I want to know the distance between the first and third place runner when the first crosses the finish line, assuming they run at their average speeds throughout the race. The race is 100. m and the first place runner completes the race in 9.77 s while the third place runner takes 10.07 s to reach the finish line: $d = 100.$ m, $t_1 = 9.77$ s, and $t_3 = 10.07$ s.

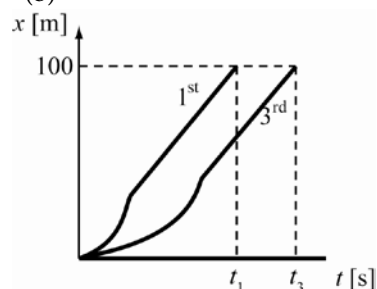
(b) I want to know the distance between the two runners when the first crosses the finish line, assuming they both accelerate to a top speed of 12 m/s: $d = 100.$ m, $t_1 = 9.77$ s, $t_3 = 10.07$ s, and $v = 12$ m/s.

SKETCH:

(a)



(b)



RESEARCH:

(a) First the average speed of each runner must be calculated: $\bar{s} = d/t$. From this the distance between the two runners can be found: $\Delta d = d_1 - d_2$, where d_1 is 100. m and d_2 is the position of the third place runner at 9.77 s.

(b) Since both runners are running at 12 m/s at the end of the race, the distance between the runners will be the distance the 3rd place runner runs after the first place runner crosses the line: $\Delta d = v\Delta t$.

SIMPLIFY:

$$(a) \Delta d = d_1 - d_2 = d_1 - \bar{s}_3 t_1 = d_1 - \left(\frac{d_1}{t_3}\right) t_1 = d_1 - d_1 \left(\frac{t_1}{t_3}\right) = d_1 \left(1 - \frac{t_1}{t_3}\right)$$

$$(b) \Delta d = v(t_3 - t_1)$$

CALCULATE:

$$(a) \Delta d = d_1 \left(1 - \frac{t_1}{t_3}\right) = (100. \text{ m}) \left(1 - \frac{9.77 \text{ s}}{10.07 \text{ s}}\right) = 2.9791 \text{ m}$$

$$(b) \Delta d = (12 \text{ m/s})(10.07 \text{ s} - 9.77 \text{ s}) = 3.6 \text{ m}$$

ROUND:

(a) The answer is limited to 3 significant figures from 9.77 s so $\Delta d = 2.98$ m.

(b) The distance then is 3.60 m between the first and third place runners.

DOUBLE-CHECK:

The two calculated distances are a small fraction (about 3%) of the race. It is reasonable for the third place runner to finish a small fraction of the track behind the first place finisher.

2.45. THINK:

(a) Since the motion is all in one direction, the average speed equals the distance covered divided by the time taken. I want to know the distance between the place where the ball was caught and midfield. I also want to know the time taken to cover this distance. The average speed will be the quotient of those two quantities.

(b) Same as in (a), but now I need to know the distance between midfield and the place where the run ended.

(c) I do not need to calculate the acceleration over each small time interval, since all that matters is the velocity at the start of the run and at the end. The average acceleration is the difference between those two quantities, divided by the time taken.

SKETCH:

In this case a sketch is not needed, since the only relevant quantities are those describing the runner at the start and end of the run, and at midfield.

RESEARCH:

The distance between two positions can be represented as $\Delta d = d_f - d_i$, where d_i is the initial position and d_f is the final one. The corresponding time difference is $\Delta t = t_f - t_i$. The average speed is $\Delta d / \Delta t$.

(a) Midfield is the 50-yard line, so $d_i = -1$ yd, $d_f = 50$ yd, $t_i = 0.00$ s, and $t_f = 5.73$ s.

(b) The end of the run is 1 yard past $d = 100$ yd, so $d_i = 50$ yd, $d_f = 101$ yd, $t_i = 5.73$ s, and $t_f = 12.01$ s.

(c) The average velocity is $\Delta v / \Delta t = (v_f - v_i) / (t_f - t_i)$. For this calculation, $t_i = 0$, $t_f = 12.01$ s, and $v_i = v_f = 0$ m/s, since the runner starts and finishes the run at a standstill.

SIMPLIFY:

$$(a), (b) \quad \frac{\Delta d}{\Delta t} = \frac{d_f - d_i}{t_f - t_i}$$

(c) No simplification needed

CALCULATE:

$$(a) \quad \frac{\Delta d}{\Delta t} = \frac{(50 \text{ yd}) - (-1 \text{ yd})}{(5.73 \text{ s}) - (0.00 \text{ s})} = 8.900522356 \text{ yd/s} \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 8.138638743 \text{ m/s}$$

$$(b) \quad \frac{\Delta d}{\Delta t} = \frac{(101 \text{ yd}) - (50 \text{ yd})}{(12.01 \text{ s}) - (5.73 \text{ s})} = 8.121019108 \text{ yd/s} \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 7.425859873 \text{ m/s}$$

$$(c) \quad \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 \text{ m/s} - 0 \text{ m/s}}{12.01 \text{ s} - 0.00 \text{ s}} = 0 \text{ m/s}^2$$

ROUND:

(a) We assume that the yard lines are exact, but the answer is limited to 3 significant figures by the time data. So the average speed is 8.14 m/s.

(b) The average speed is 7.43 m/s.

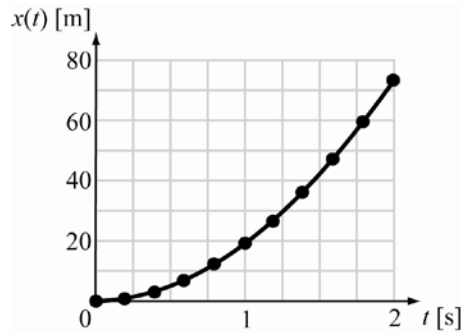
(c) The average velocity is 0 m/s².

DOUBLE-CHECK:

The average speeds in parts (a) and (b) are reasonable speeds (8.9 ft/s is about 18 mph), and it makes sense that the average speed during the second half of the run would be slightly less than during the first half, due to fatigue. In part (c) it is logical that average acceleration would be zero, since the net change in velocity is zero.

- 2.46. **THINK:** Use the difference formula to find the average velocity, and then the average acceleration of the jet given its position at several times, and determine whether the acceleration is constant.

SKETCH:



RESEARCH: The difference formula $m = \frac{\text{final point} - \text{initial point}}{\text{final time} - \text{initial time}}$.

SIMPLIFY: For velocity the difference formula is $v = (x_f - x_i) / (t_f - t_i)$ and the corresponding difference formula for the acceleration is $a = (v_f - v_i) / (t_f - t_i)$.

CALCULATE: As an example, $v = (6.6 \text{ m} - 3.0 \text{ m}) / (0.60 \text{ s} - 0.40 \text{ s}) = 18.0 \text{ m/s}$, and the acceleration is $a = (26.0 \text{ m/s} - 18.0 \text{ m/s}) / (0.80 \text{ s} - 0.60 \text{ s}) = 40.0 \text{ m/s}^2$.

t [s]	x [m]	v [m/s]	a [m/s ²]
0.00	0	0.0	
0.20	0.70	3.5	17.5
0.40	3.0	11.5	40
0.60	6.6	18	32.5
0.80	11.8	26	40
1.00	18.5	33.5	37.5
1.20	26.6	40.5	35
1.40	36.2	48	37.5
1.60	47.3	55.5	37.5
1.80	59.9	63	37.5
2.00	73.9	70	35

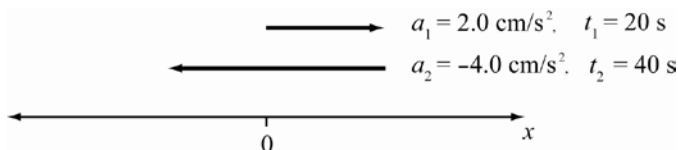
ROUND: The position measurements are given to the nearest tenth of a meter, and the time measurements are given to two significant figures. Therefore each of the stated results for velocity and acceleration should be rounded to two significant figures.

t [s]	x [m]	v [m/s]	a [m/s ²]
0.00	0.0	0.0	
0.20	0.70	3.5	18
0.40	3.0	12	40.
0.60	6.6	18	33
0.80	11.8	26	40.
1.00	18.5	34	38
1.20	26.6	41	35
1.40	36.2	48	38
1.60	47.3	56	38
1.80	59.9	63	38
2.00	73.9	70.	35

DOUBLE-CHECK: The final speed of the jet is 70. m/s, which is equivalent to 250 km/hr, the typical take-off speed of a commercial jet airliner.

- 2.47. **THINK:** I want to find the position of a particle after it accelerates from rest at $a_1 = 2.00 \text{ cm/s}^2$ for $t_1 = 20.0 \text{ s}$ then accelerates at $a_2 = -4.00 \text{ cm/s}^2$ for $t_2 = 40.0 \text{ s}$.

SKETCH:



RESEARCH: The position of a particle undergoing constant acceleration is given by the formula $x = x_0 + v_0 t + \frac{1}{2} a t^2$. The same particle's velocity is given by $v = v_0 + a t$. The final speed at the end of the first segment is the initial speed for the second segment.

SIMPLIFY: For the first 20 s the particle's position is $x_1 = \frac{1}{2} a_1 t_1^2$. This is the initial position for the second segment of the particle's trip. For the second segment, the particle is no longer at rest but has a speed of $v = a_1 t_1$.

$$x = x_1 + v_0 t_2 + \frac{1}{2} a_2 t_2^2 = \frac{1}{2} a_1 t_1^2 + a_1 t_1 t_2 + \frac{1}{2} a_2 t_2^2$$

CALCULATE:

$$x = \frac{1}{2} (2.00 \text{ cm/s}^2) (20.0 \text{ s})^2 + (2.00 \text{ cm/s}^2) (20.0 \text{ s}) (40.0 \text{ s}) + \frac{1}{2} (-4.00 \text{ cm/s}^2) (40.0 \text{ s})^2 = -1200 \text{ cm}$$

ROUND: The variables are given with three significant figures. Therefore, the particle is $-1.20 \cdot 10^3 \text{ cm}$ from its original position.

DOUBLE-CHECK: Note that the second phase of the trip has a greater magnitude of acceleration than the first part. The duration of the second phase is longer; thus the final position is expected to be negative.

- 2.48. **THINK:** The car has a velocity of +6 m/s and a position of +12 m at $t = 0$. What is its velocity at $t = 5.0$ s? The change in the velocity is given by the area under the curve in an acceleration versus time graph.

SKETCH:



RESEARCH: $v = v_0 + \text{area}$, area of triangle = $\frac{\text{base} \cdot \text{height}}{2}$

SIMPLIFY: $v = v_0 + \frac{\Delta a \Delta t}{2}$

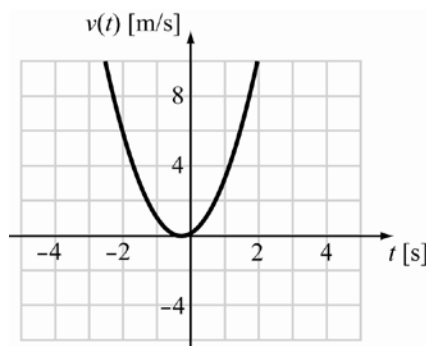
CALCULATE: $v = 6 \text{ m/s} + \frac{(4.0 \text{ m/s}^2)(5.0 \text{ s})}{2} = 16 \text{ m/s}$

ROUND: The function can only be accurate to the first digit before the decimal point. Thus $v = 16 \text{ m/s}$.

DOUBLE-CHECK: 16 m/s is approximately 58 km/h, which is a reasonable speed for a car.

- 2.49. **THINK:** I want to find the position of a car at $t_f = 3.0$ s if the velocity is given by the equation $v = At^2 + Bt$ with $A = 2.0 \text{ m/s}^3$ and $B = 1.0 \text{ m/s}^2$.

SKETCH:



RESEARCH: The position is given by the integral of the velocity function: $x = x_0 + \int_0^{t_f} v(t) dt$.

SIMPLIFY: Since the car starts at the origin, $x_0 = 0$ m.

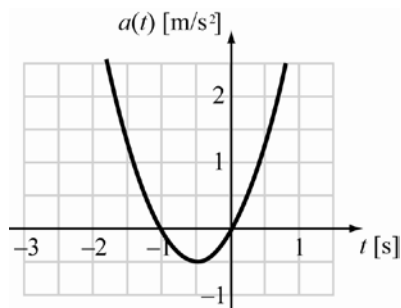
$$x = \int_0^{t_f} v(t) dt = \int_0^{t_f} (At^2 + Bt) dt = \frac{1}{3} At_f^3 + \frac{1}{2} Bt_f^2$$

CALCULATE: $x = \frac{1}{3}(2.0 \text{ m/s}^3)(3.0 \text{ s})^3 + \frac{1}{2}(1.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 22.5 \text{ m}$

ROUND: The parameters are given to two significant digits, and so the answer must also contain two significant digits: $x = 23 \text{ m}$.

DOUBLE-CHECK: This is a reasonable distance for a car to travel in 3.0 s.

- 2.50. **THINK:** An object starts at rest (so $v_0 = 0 \text{ m/s}$) and has an acceleration defined by $a(t) = Bt^2 - (1/2)Ct$, where $B = 2.0 \text{ m/s}^4$ and $C = -4.0 \text{ m/s}^3$. I want to find its velocity and distance traveled after 5.0 s. Measure the position from the starting point $x_0 = 0$ m.

SKETCH:**RESEARCH:**(a) The velocity is given by integrating the acceleration with respect to time: $v = \int a(t) dt$.(b) The position is given by integrating the velocity with respect to time: $x = \int v(t) dt$.**SIMPLIFY:**

$$v = \int a(t) dt = \int \left(Bt^2 - \frac{1}{2} Ct \right) dt = \frac{1}{3} Bt^3 - \frac{1}{4} Ct^2 + v_0, \text{ and}$$

$$x = \int v dt = \int \left(\frac{1}{3} Bt^3 - \frac{1}{4} Ct^2 + v_0 \right) dt = \frac{1}{12} Bt^4 - \frac{1}{12} Ct^3 + v_0 t + x_0$$

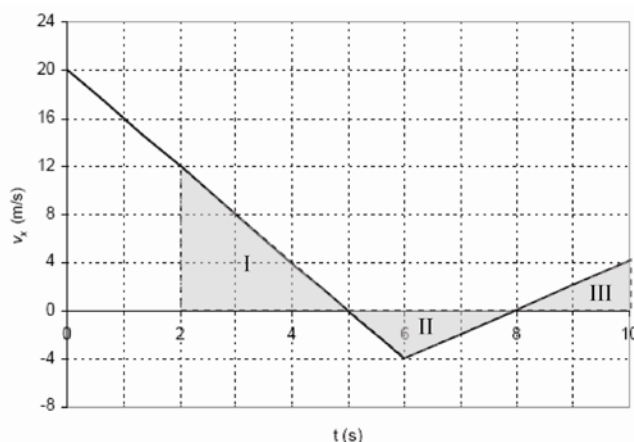
CALCULATE:

$$v = \frac{1}{3} Bt^3 - \frac{1}{4} Ct^2 + v_0 = \frac{1}{3} (2.0 \text{ m/s}^4) (5.0 \text{ s})^3 - \frac{1}{4} (-4.0 \text{ m/s}^3) (5.0 \text{ s})^2 + 0 \text{ m/s} = 108.33 \text{ m/s}$$

$$x = \frac{1}{12} (2.0 \text{ m/s}^4) (5.0 \text{ s})^4 - \frac{1}{12} (-4.0 \text{ m/s}^3) (5.0 \text{ s})^3 + (0 \text{ m/s}) (5.0 \text{ s}) + (0 \text{ m}) = 145.83 \text{ m}$$

ROUND: All parameters have two significant digits. Thus the answers should also have two significant figures: at $t = 5.0 \text{ s}$, $v = 110 \text{ m/s}$ and $x = 150 \text{ m}$.**DOUBLE-CHECK:** The distance traveled has units of meters, and the velocity has units of meters per second. These are appropriate units for a distance and velocity, respectively.

- 2.51. **THINK:** A car is accelerating as shown in the graph. At $t_0 = 2.0 \text{ s}$, its position is $x_0 = 2.0 \text{ m}$. I want to determine its position at $t = 10.0 \text{ s}$.

SKETCH:

RESEARCH: The change in position is given by the area under the curve of the velocity versus time graph plus the initial displacement: $x = x_0 + \text{area}$. Note that region II is under the t -axis will give a negative area. Let A_1 be the area of region I, let A_2 be the area of region II, and let A_3 be the area of region III.

SIMPLIFY: $x = x_0 + A_I + A_{II} + A_{III}$

CALCULATE:

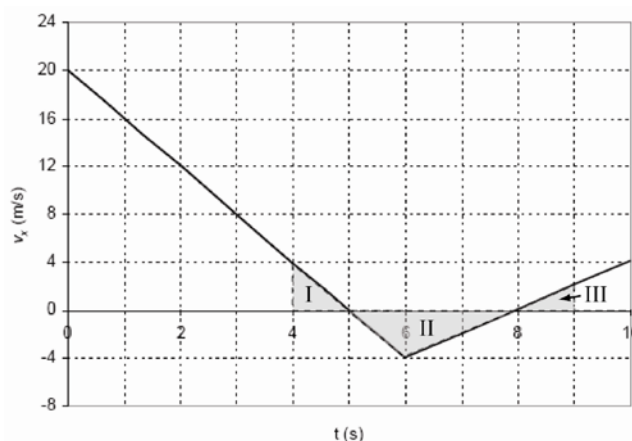
$$x = 2.0 \text{ m} + \frac{1}{2}(12.0 \text{ m/s})(5.0 \text{ s} - 2.0 \text{ s}) + \frac{1}{2}(-4.0 \text{ m/s})(8.0 \text{ s} - 5.0 \text{ s}) + \frac{1}{2}(4.0 \text{ m/s})(10.0 \text{ s} - 8.0 \text{ s}) = 18 \text{ m}$$

ROUND: The answer should be given to the least accurate calculated area. These are all accurate to the meter, thus the position is $x = 18 \text{ m}$.

DOUBLE-CHECK: The maximum velocity is 12 m/s. If this were sustained over the 8 second interval, the distance traveled would be $2.0 \text{ m} + (12 \text{ m/s})(8.0 \text{ s}) = 98 \text{ m}$. Since there was a deceleration and then an acceleration, we expect that the actual distance will be much less than the value 98 m.

- 2.52. **THINK:** A car is accelerating as shown in the graph. I want to determine its displacement between $t = 4 \text{ s}$ and $t = 9 \text{ s}$.

SKETCH:



RESEARCH: The change in position is given by the area under the curve of a velocity versus time graph. Note that it is hard to read the value of the velocity at $t = 9.0 \text{ s}$. This difficulty can be overcome by finding the slope of the line for this section. Using the slope, the velocity during this time can be determined:

$\Delta x = \text{Area}$, $m = \frac{\text{rise}}{\text{run}}$. Let A_I be the area of region I, let A_{II} be the area of region II, and let A_{III} be the area

of region III.

SIMPLIFY: $\Delta x = A_I + A_{II} + A_{III}$

CALCULATE: $m = \frac{4.0 \text{ m/s} - (-4.0 \text{ m/s})}{10.0 \text{ s} - 6.0 \text{ s}} = 2.0 \text{ m/s}^2$

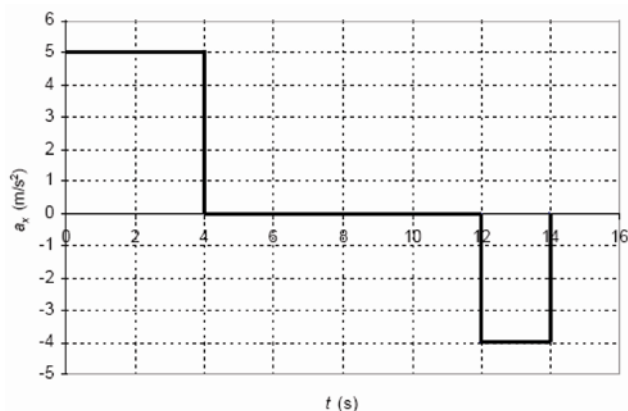
$$\Delta x = \frac{1}{2}(4.0 \text{ m/s})(5.0 \text{ s} - 4.0 \text{ s}) + \frac{1}{2}(-4.0 \text{ m/s})(8.0 \text{ s} - 5.0 \text{ s}) + \frac{1}{2}(2.0 \text{ m/s})(9.0 \text{ s} - 8.0 \text{ s}) = -3.0 \text{ m}$$

ROUND: $\Delta x = -3.0 \text{ m}$

DOUBLE-CHECK: The car will end up with a negative displacement since the area of region II is larger than the combined areas of regions I and III. The overall displacement is less than if the car had traveled constantly at its maximum velocity of 4 m/s (when the displacement would have been 20 m).

- 2.53. **THINK:** A motorcycle is accelerating at different rates as shown in the graph. I want to determine (a) its speed at $t = 4.00$ s and $t = 14.0$ s, and (b) its total displacement between $t = 0$ and $t = 14.0$ s.

SKETCH:



RESEARCH:

(a) The velocity of the motorcycle is defined by the area under the curve of the acceleration versus time graph. This area can be found by counting the blocks under the curve then multiply by the area of one block: 1 block = $(2 \text{ s}) (1 \text{ m/s}^2) = 2 \text{ m/s}$.

(b) The displacement can be found by separating the acceleration into three parts: The first phase has an acceleration of $a_1 = 5 \text{ m/s}^2$ for times between 0 to 4 seconds. The second phase has no acceleration, thus the motorcycle has a constant speed. The third phase has a constant acceleration of $a_3 = -4 \text{ m/s}^2$. Recall the position and velocity of an object under constant acceleration is $x = x_0 + v_0 t + (1/2)at^2$ and $v = v_0 + at$, respectively.

SIMPLIFY: At $t = 4.00$ s and 14.0 s, there are 10 blocks and 6 blocks respectively. Recall that blocks under the time axis are negative. In the first phase the position is given by $x = (1/2)a_1(\Delta t_1)^2$ where Δt is the duration of the phase. The velocity at the end of this phase is $v = a_1\Delta t_1$. The position and velocity of the first phase gives the initial position and velocity for the second phase.

$$x = x_0 + v_0\Delta t_2 + \frac{1}{2}a_1(\Delta t_1)^2 + a_1\Delta t_1\Delta t_2$$

Since the velocity is constant in the second phase, this value is also the initial velocity of the third phase.

$$x = x_0 + v_0\Delta t_3 + \frac{1}{2}a_3(\Delta t_3)^2 = \frac{1}{2}a_1(\Delta t_1)^2 + a_1\Delta t_1\Delta t_2 + a_1\Delta t_1\Delta t_3 + \frac{1}{2}a_3(\Delta t_3)^2$$

CALCULATE:

(a) $v(t = 4.00 \text{ s}) = 10(2.00 \text{ m/s}) = 20.0 \text{ m/s}$, $v(t = 14.0 \text{ s}) = 6(2.00 \text{ m/s}) = 12.0 \text{ m/s}$

(b)

$$\begin{aligned} x &= \frac{1}{2}(5.0 \text{ m/s}^2)(4.00 \text{ s} - 0 \text{ s})^2 + (5.0 \text{ m/s}^2)(4.00 \text{ s} - 0 \text{ s})(12.0 \text{ s} - 4.0 \text{ s}) + (5.0 \text{ m/s}^2)(4.00 \text{ s} - 0 \text{ s})(14.0 \text{ s} - 12.0 \text{ s}) \\ &\quad + \frac{1}{2}(-4.0 \text{ m/s}^2)(14.0 \text{ s} - 12.0 \text{ s})^2 \\ &= 232 \text{ m} \end{aligned}$$

ROUND:

(a) Rounding is not necessary in this case, because the values of the accelerations and times can be read off the graph to at least two digit precision.

(b) The motorcycle has traveled 232 m in 14.0 s.

DOUBLE-CHECK: The velocity of the motorcycle at $t = 14$ s is less than the speed at $t = 4$ s, which makes sense since the bike decelerated in the third phase. Since the bike was traveling at a maximum speed of 20 m/s, the most distance it could cover in 14 seconds would be 280 m. The calculated value is less than this, which makes sense since the bike decelerated in the third phase.

- 2.54. **THINK:** I want to find the time it takes the car to accelerate from rest to a speed of $v = 22.2$ m/s. I know that $v_0 = 0$ m/s, $v = 22.2$ m/s, distance = 243 m, and a is constant.

SKETCH:



RESEARCH: Recall that given constant acceleration, $d = (1/2)(v_0 + v)t$.

SIMPLIFY: $t = \frac{2d}{v_0 + v}$

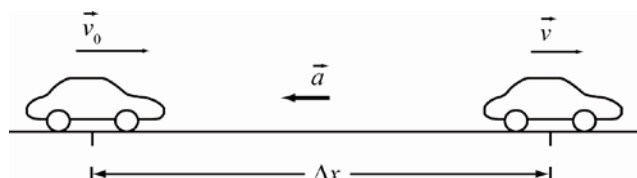
CALCULATE: $t = \frac{2(243 \text{ m})}{0.0 \text{ m/s} + 22.2 \text{ m/s}} = 21.8919 \text{ s}$

ROUND: Therefore, $t = 21.9$ s since each value used in the calculation has three significant digits.

DOUBLE-CHECK: The units of the solution are units of time, and the calculated time is a reasonable amount of time for a car to cover 243 m.

- 2.55. **THINK:** I want to determine (a) how long it takes for a car to decelerate from $v_0 = 31.0$ m/s to $v = 12.0$ m/s over a distance of 380. m and (b) the value of the acceleration.

SKETCH:



RESEARCH: Since the acceleration is constant, the time can be determined using the equation:

$$\Delta x = (1/2)(v_0 + v)t, \text{ and the acceleration can be found using } v^2 = v_0^2 + 2a\Delta x.$$

SIMPLIFY:

(a) $\Delta x = \frac{1}{2}(v_0 + v)t \Rightarrow (v_0 + v)t = 2\Delta x \Rightarrow t = \frac{2\Delta x}{v_0 + v}$

(b) $v^2 = v_0^2 + 2a\Delta x \Rightarrow 2a\Delta x = v^2 - v_0^2 \Rightarrow a = \frac{v^2 - v_0^2}{2\Delta x}$

CALCULATE:

(a) $t = \frac{2\Delta x}{v_0 + v} = \frac{2(380. \text{ m})}{(31.0 \text{ m/s} + 12.0 \text{ m/s})} = 17.674 \text{ s}$

(b) $a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(12.0 \text{ m/s})^2 - (31.0 \text{ m/s})^2}{2(380. \text{ m})} = -1.075 \text{ m/s}^2$

ROUND: Each result is limited to three significant figures as the values used in the calculations each have three significant figures.

(a) $t = 17.7 \text{ s}$

(b) $a = -1.08 \text{ m/s}^2$

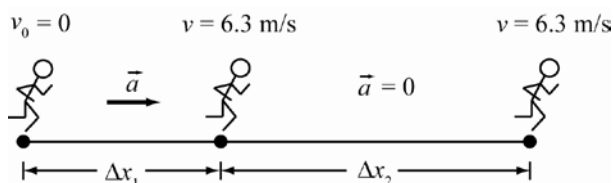
DOUBLE-CHECK:

(a) The resulting time has appropriate units and is reasonable for the car to slow down.

(b) The acceleration is negative, indicating that it opposes the initial velocity, causing the car to slow down.

- 2.56. THINK:** I want to find (a) the total distance covered in time $t = 59.7$ s, and (b) the velocity of the runner at $t = 59.7$ s. It will be useful to know the time taken to accelerate, t_1 , and the time taken to run at the achieved constant velocity, t_2 . Note that the mass of the runner is irrelevant.

SKETCH:



RESEARCH: The runner accelerates from rest to some velocity, v , then continues to run at this constant velocity. The total distance covered, Δx , will be the sum of the distance covered while accelerating, Δx_1 , and the distance covered while at constant velocity, Δx_2 : $\Delta x = \Delta x_1 + \Delta x_2$. The distance Δx_1 is determined by $v^2 = v_0^2 + 2a\Delta x_1$. The distance Δx_2 is determined by $\Delta x_2 = vt_2$. It will be necessary to know the time taken to run this distance Δx_2 . This time, t_2 , and the time taken to cover the distance Δx_1 , t_1 , must sum to the given total time of 59.7 s: $t_{\text{total}} = t_1 + t_2$. The time t_1 can be determined using the equation: $v = v_0 + at_1$.

SIMPLIFY:

$$v^2 = v_0^2 + 2a\Delta x_1 \Rightarrow \Delta x_1 = \frac{v^2 - v_0^2}{2a}, \quad v = v_0 + at_1 \Rightarrow t_1 = \frac{v - v_0}{a}$$

$$\Delta x_2 = vt_2 \Rightarrow \Delta x_2 = v(t_{\text{total}} - t_1) \Rightarrow \Delta x_2 = v\left(t_{\text{total}} - \frac{v - v_0}{a}\right)$$

$$\text{Finally, the total distance covered is } \Delta x = \Delta x_1 + \Delta x_2 = \frac{v^2 - v_0^2}{2a} + v\left(t_{\text{total}} - \frac{v - v_0}{a}\right) = \frac{v^2}{2a} + v\left(t_{\text{total}} - \frac{v}{a}\right).$$

CALCULATE:

$$\begin{aligned} \text{(a) } \Delta x &= \frac{(6.3 \text{ m/s})^2}{2(1.25 \text{ m/s}^2)} + (6.3 \text{ m/s})\left(59.7 \text{ s} - \frac{6.3 \text{ m/s}}{1.25 \text{ m/s}^2}\right) \\ &= 360.234 \text{ m} \end{aligned}$$

$$\text{(b) Since } t_1 = \frac{6.3 \text{ m/s} - 0 \text{ m/s}}{1.25 \text{ m/s}^2} = 5.0 \text{ s} \text{ is the time taken to reach the final velocity, the velocity of the runner}$$

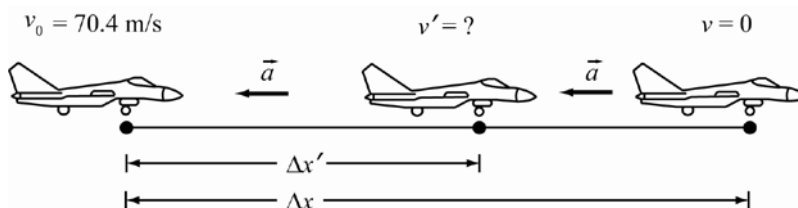
at $t_{\text{total}} = 59.7$ s is 6.3 m/s.

ROUND: Since v has only two significant digits, $\Delta x = 360$ m, or $3.6 \cdot 10^2$ m.

DOUBLE-CHECK: This seems like a reasonable distance to cover in the total time, given most of the distance is covered at the constant velocity 6.3 m/s. Since the runner stops accelerating after 5.0 s, the velocity of the runner is still 6.3 m/s at 59.7 s.

- 2.57. THINK:** I am given $v_0 = 70.4$ m/s, $v = 0$, $\Delta x = 197.4$ m, and constant acceleration. I am asked to find the velocity v' when the jet is 44.2 m from its stopping position. This means the jet has traveled $\Delta x' = 197.4 \text{ m} - 44.2 \text{ m} = 153.2$ m on the aircraft carrier.

SKETCH:



RESEARCH: The initial and final velocities are known, as is the total distance traveled. Therefore the equation $v^2 = v_0^2 + 2a\Delta x$ can be used to find the acceleration of the jet. Once the acceleration is known, the intermediate velocity v' can be determined using $(v')^2 = v_0^2 + 2a\Delta x'$.

SIMPLIFY: First find the constant acceleration using the total distance traveled, Δx , the initial velocity, v_0 , and the final velocity, v : $a = \frac{v^2 - v_0^2}{2\Delta x} = -\frac{v_0^2}{2\Delta x}$ (since $v = 0$ m/s). Next, find the requested intermediate velocity, v' :

$$(v')^2 = v_0^2 + 2a\Delta x' \Rightarrow (v')^2 = v_0^2 + 2\left(-\frac{v_0^2}{2\Delta x}\right)\Delta x' \Rightarrow v' = \sqrt{v_0^2 - \frac{v_0^2}{\Delta x}\Delta x'}$$

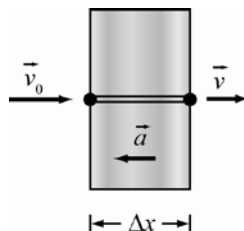
CALCULATE: $v' = \sqrt{(70.4 \text{ m/s})^2 - \frac{(70.4 \text{ m/s})^2}{(197.4 \text{ m})}(153.2 \text{ m})} = 33.313 \text{ m/s}$

ROUND: At $\Delta x' = 153.2 \text{ m}$, the velocity is $v' = 33.3 \text{ m/s}$.

DOUBLE-CHECK: This v' is less than v_0 , but greater than v , and therefore makes sense.

- 2.58. **THINK:** I want to find the acceleration of a bullet passing through a board, given that $\Delta x = 10.0 \text{ cm} = 0.100 \text{ m}$, $v_0 = 400. \text{ m/s}$, and $v = 200. \text{ m/s}$. I expect the acceleration to be negative, since the bullet is slowing down.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a\Delta x$

SIMPLIFY: $a = \frac{v^2 - v_0^2}{2\Delta x}$

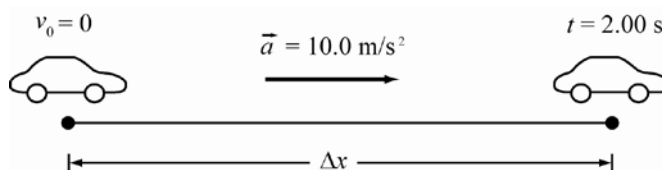
CALCULATE: $a = \frac{(200. \text{ m/s})^2 - (400. \text{ m/s})^2}{2(0.100 \text{ m})} = -600,000. \text{ m/s}^2$

ROUND: Since each velocity is given to three significant digits, $a = -6.00 \cdot 10^5 \text{ m/s}^2$.

DOUBLE-CHECK: That a is negative indicates it is in the opposite direction of the initial velocity, so the bullet slows down. The speed of the bullet decreases by 200 m/s in 0.1 m, so I am not surprised to get such a large value for the acceleration.

- 2.59. **THINK:** A car accelerates from rest with $a = 10.0 \text{ m/s}^2$. I want to know how far it travels in 2.00 s .

SKETCH:



RESEARCH: $\Delta x = v_0 t + \frac{1}{2}at^2$

SIMPLIFY: Since $v_0 = 0 \text{ m/s}$, $\Delta x = \frac{1}{2}at^2$.

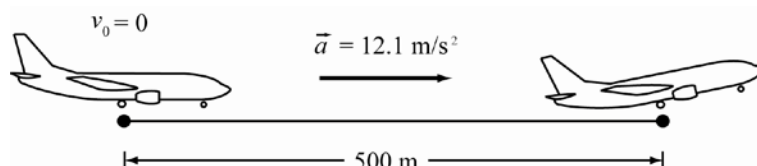
CALCULATE: $\Delta x = \frac{1}{2}(10.0 \text{ m/s}^2)(2.00 \text{ s})^2 = 20.0 \text{ m}$

ROUND: $\Delta x = 20.0 \text{ m}$

DOUBLE-CHECK: This seems like a reasonable distance to cover within 2.00 s given $a = 10.0 \text{ m/s}^2$.

2.60. THINK: A airplane accelerates from rest at $a = 12.1 \text{ m/s}^2$. I want to know its velocity at 500. m.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a\Delta x$; $v_0 = 0$, $a = 12.1 \text{ m/s}^2$, $\Delta x = 500 \text{ m}$

SIMPLIFY: $v = \sqrt{v_0^2 + 2a\Delta x}$
 $= \sqrt{2a\Delta x}$

CALCULATE: $v = \sqrt{2(12.1 \text{ m/s}^2)(500. \text{ m})} = 110.00 \text{ m/s}$

ROUND: $v = 110. \text{ m/s}$

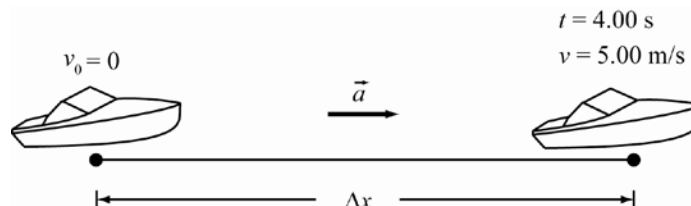
DOUBLE-CHECK: This take-off speed is about 400 kph, which is reasonable for a small plane.

2.61. THINK:

(a) I know that $v_0 = 0 \text{ m/s}$, $v = 5.00 \text{ m/s}$, and a is constant. I want to find v_{avg} .

(b) $t = 4.00 \text{ s}$ is given. I want to find Δx .

SKETCH:



RESEARCH:

(a) $v_{\text{avg}} = \frac{v_0 + v}{2}$

(b) a is unknown, so use $\Delta x = \frac{1}{2}(v_0 + v)t$

SIMPLIFY: It is not necessary to simplify the equations above.

CALCULATE:

(a) $v_{\text{avg}} = \frac{5.00 \text{ m/s} + 0 \text{ m/s}}{2} = 2.50 \text{ m/s}$

(b) $\Delta x = \frac{1}{2}(5.00 \text{ m/s} + 0 \text{ m/s})(4.00 \text{ s}) = 10.00 \text{ m}$

ROUND:

(a) v is precise to three significant digits, so $v_{\text{avg}} = 2.50 \text{ m/s}$.

(b) Each v and t have three significant digits, so $\Delta x = 10.0 \text{ m}$.

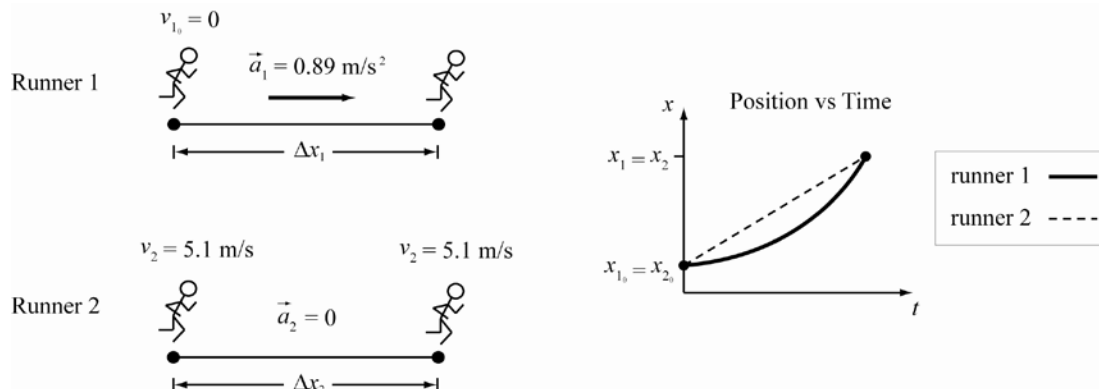
DOUBLE-CHECK:

(a) This v_{avg} is between the given v_0 and v , and therefore makes sense.

(b) This is a reasonable distance to cover in 4.00 s when $v_{\text{avg}} = 2.50 \text{ m/s}$.

- 2.62. THINK:** I have been given information on two runners. Runner 1 has an initial velocity $v_{1_0} = 0$ and an acceleration $a_1 = 0.89 \text{ m/s}^2$. Runner 2 has a constant velocity of $v_2 = 5.1 \text{ m/s}$. I want to know the distance traveled by runner 1 before he catches up to runner 2. Note that both runners cover the same distance, that is, $\Delta x_1 = \Delta x_2$, in the same time, t .

SKETCH:



RESEARCH: For runner 1, $\Delta x_1 = v_{1_0} t + (1/2) a_1 t^2$. For runner 2, $\Delta x_2 = v_2 t$.

SIMPLIFY: Since the time is not given, substitute the equation for runner 2 for the value of t : $t = \Delta x_2 / v_2$. Then for runner 1:

$$\Delta x_1 = v_{1_0} t + \frac{1}{2} a_1 t^2 \Rightarrow \Delta x_1 = \frac{1}{2} a_1 t^2 \Rightarrow \Delta x_1 = \frac{1}{2} a_1 \left(\frac{\Delta x_2}{v_2} \right)^2$$

Since $\Delta x_1 = \Delta x_2$, I write:

$$\Delta x_1 = \frac{1}{2} a_1 \left(\frac{\Delta x_1}{v_2} \right)^2 \Rightarrow \frac{1}{2} a_1 \frac{\Delta x_1^2}{v_2^2} - \Delta x_1 = 0 \Rightarrow \Delta x_1 \left(\frac{1}{2} a_1 \frac{\Delta x_1}{v_2^2} - 1 \right) = 0$$

Observe that one solution is $\Delta x_1 = 0$. This is true when runner 2 first passes runner 1. The other solution occurs when runner 1 catches up to runner 2: $\frac{1}{2} a_1 \frac{\Delta x_1}{v_2^2} - 1 = 0$. Then $\Delta x_1 = \frac{2v_2^2}{a_1}$.

CALCULATE: $\Delta x_1 = \frac{2(5.1 \text{ m/s})^2}{(0.89 \text{ m/s}^2)} = 58.449 \text{ m}$

ROUND: $\Delta x_1 = 58 \text{ m}$

DOUBLE-CHECK: A runner might catch up to another runner on a race track in 58 m.

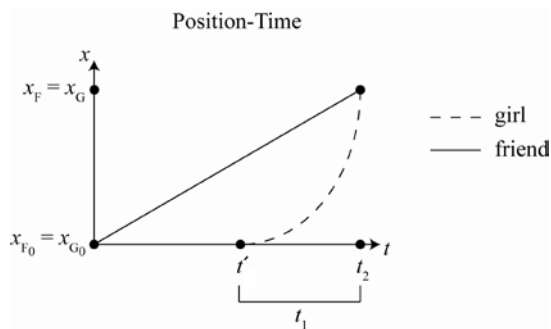
- 2.63. THINK:**

(a) The girl is initially at rest, so $v_{1_0} = 0$, and then she waits $t' = 20 \text{ s}$ before accelerating at $a_1 = 2.2 \text{ m/s}^2$. Her friend has constant velocity $v_2 = 8.0 \text{ m/s}$. I want to know the time required for the girl to catch up with her friend, t_1 . Note that both people travel the same distance: $\Delta x_1 = \Delta x_2$. The time the girl spends riding her bike is t_1 . The friend, however, has a t' head-start; the friend travels for a total time of $t_2 = t' + t_1$.

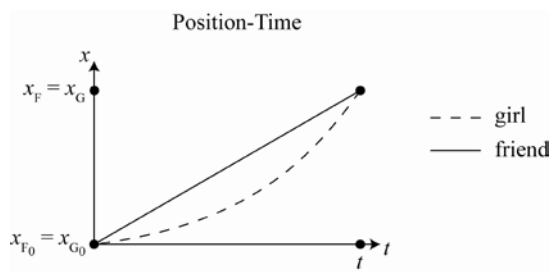
(b) The initial conditions of the girl have changed. Now $v_{1_0} = 1.2 \text{ m/s}$. The initial conditions of the friend are the same: $v_2 = 8.0 \text{ m/s}$. Now there is no time delay between when the friend passes the girl and when the girl begins to accelerate. The time taken to catch up is that found in part a), $t = 20 \text{ s}$. I will use $t = 16.2 \text{ s}$ for my calculations, keeping in mind that t has only two significant figures. I want to know the acceleration of the girl, a_1 , required to catch her friend in time t .

SKETCH:

(a)



(b)


RESEARCH:

 (a) The distance the girl travels is $\Delta x_1 = v_{i_0} t_1 + \frac{1}{2} a_1 t_1^2$. The distance her friend travels is $\Delta x_2 = v_2 t_2$.

 (b) $\Delta x_1 = v_{i_0} t + \frac{1}{2} a_1 t^2$, $\Delta x_2 = v_2 t$
SIMPLIFY:

 (a) Since $v_{i_0} = 0$, $\Delta x_1 = \frac{1}{2} a_1 t_1^2$. Also, since $t_2 = t' + t_1$, $\Delta x_2 = v_2 (t' + t_1)$. Recall that $\Delta x_1 = \Delta x_2$. This leads to $\frac{1}{2} a_1 t_1^2 = v_2 (t' + t_1)$. Now solve for t_1 : $\frac{1}{2} a_1 t_1^2 = v_2 t' + v_2 t_1 \Rightarrow \frac{1}{2} a_1 t_1^2 - v_2 t_1 - v_2 t' = 0$.

The quadratic formula gives:

$$t_1 = \frac{v_2 \pm \sqrt{v_2^2 - 4\left(\frac{1}{2}a_1\right)(-v_2 t')}}{2\left(\frac{1}{2}a_1\right)} = \frac{v_2 \pm \sqrt{v_2^2 + 2a_1 v_2 t'}}{a_1}$$

 (b) As in part (a), $\Delta x_1 = \Delta x_2$, and so $v_{i_0} t + \frac{1}{2} a_1 t^2 = v_2 t$. Solving for a_1 gives:

$$\frac{1}{2} a_1 t^2 = v_2 t - v_{i_0} t \Rightarrow a_1 = \frac{2(v_2 - v_{i_0})}{t}$$

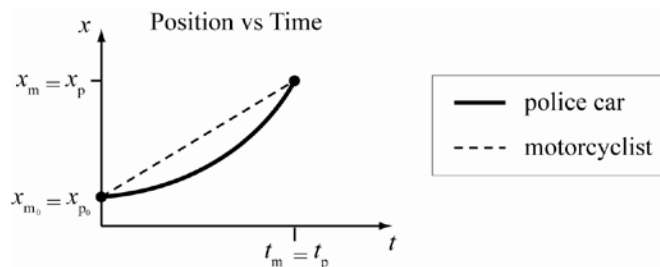
CALCULATE:

$$\begin{aligned}
 \text{(a) } t_1 &= \frac{(8.0 \text{ m/s}) \pm \sqrt{(-8.0 \text{ m/s})^2 + 2(2.2 \text{ m/s}^2)(8.0 \text{ m/s})(20 \text{ s})}}{2.2 \text{ m/s}^2} \\
 &= \frac{(8.0 \text{ m/s}) \pm \sqrt{64 \text{ m}^2/\text{s}^2 + 704 \text{ m}^2/\text{s}^2}}{2.2 \text{ m/s}^2} \\
 &= \frac{(8.0 \text{ m/s}) \pm 27.7 \text{ m/s}}{2.2 \text{ m/s}^2} \\
 &= 16.2272, -8.9545 \\
 \text{(b) } a_1 &= \frac{2(8.0 \text{ m/s} - 1.2 \text{ m/s})}{16.2 \text{ s}} = 0.840 \text{ m/s}^2
 \end{aligned}$$

ROUND:(a) Time must be positive, so take the positive solution, $t_1 = 16 \text{ s}$.(b) $a_1 = 0.84 \text{ m/s}^2$ **DOUBLE-CHECK:**(a) The units of the result are those of time. This is a reasonable amount of time to catch up to the friend who is traveling at $v_2 = 8.0 \text{ m/s}$.

(b) This acceleration is less than that in part (a). Without the 20 s head-start, the friend does not travel as far, and so the acceleration of the girl should be less in part (b) than in part a), given the same time.

2.64. THINK: The motorcyclist is moving with a constant velocity $v_m = 36.0 \text{ m/s}$. The police car has an initial velocity $v_{p0} = 0$, and acceleration $a_p = 4.0 \text{ m/s}^2$.

(a) I want to find the time required for the police car to catch up to the motorcycle. Note both the police car and the motorcycle travel for the same amount of time: $t_p = t_m$.(b) I want to find the final speed of the police car, v_p .(c) I want to find the distance traveled by the police car at the moment when it catches up to the motorcycle. Note the motorcyclist and the police car will have both traveled the same distance from the police car's initial position, once the police car catches up to the motorcycle. That is, $\Delta x_m = \Delta x_p$.**SKETCH:****RESEARCH:**(a) To find t_p , use $\Delta x_p = v_{p0} t_p + (1/2) a_p t_p^2$ for the police car and $\Delta x_m = v_m t_m$ for the motorcycle.(b) To find v_p , use $v_p = v_{p0} + a_p t_p$ for the police car.(c) Since $\Delta x_p = \Delta x_m$, $\Delta x_p = v_m t_m$ for the police car.

SIMPLIFY:(a) Since $\Delta x_p = \Delta x_m$:

$$v_{p0} t_p + \frac{1}{2} a_p t_p^2 = v_m t_m$$

$$v_{p0} t_p + \frac{1}{2} a_p t_p^2 = v_m t_p \quad \text{Since } t_m = t_p,$$

$$\frac{1}{2} a_p t_p^2 = v_m t_p \quad \text{Since } v_{p0} = 0,$$

$$\frac{1}{2} a_p t_p^2 - v_m t_p = 0$$

$$t_p \left(\frac{1}{2} a_p t_p - v_m \right) = 0$$

There are two solutions for t_p here: $t_p = 0$ or $\left(\frac{1}{2} a_p t_p - v_m \right) = 0$. The first solution corresponds to the time when the motorcycle first passes the stationary police car. The second solution gives the time when the police car catches up to the motorcycle. Rearranging gives: $t_p = 2v_m / a_p$.

(b) $v_p = v_{p0} + a_p t_p \Rightarrow v_p = a_p t_p$, since $v_{p0} = 0$. Substituting $t_p = 2v_m / a_p$ into this equation gives:

$$v_p = a_p (2v_m / a_p) = 2v_m.$$

(c) No simplification is necessary.

CALCULATE:

$$(a) \quad t_p = \frac{2(36.0 \text{ m/s})}{4.0 \text{ m/s}^2} = 18.0 \text{ s}$$

$$(b) \quad v_p = 2(36.0 \text{ m/s}) = 72.0 \text{ m/s}$$

$$(c) \quad \Delta x_p = (36.0 \text{ m/s})(18.0 \text{ s}) = 648 \text{ m}$$

ROUND:

(a) a_p has only two significant figures, so $t_p = 18 \text{ s}$.

(b) v_m has three significant digits, so $v_p = 72.0 \text{ m/s}$.

(c) a_p has only two significant digits, so $\Delta x_p = 650 \text{ m}$.

DOUBLE-CHECK:

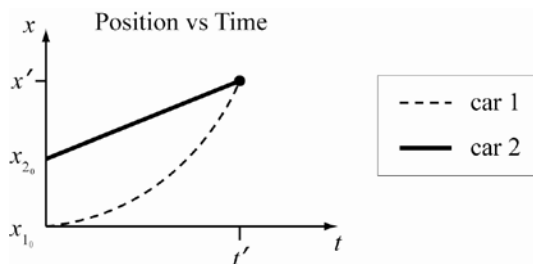
(a) The calculated time is reasonable for the police car to catch the motorcyclist.

(b) The calculated speed is fast, but it is a realistic speed for a police car to achieve while chasing a speeding vehicle.

(c) The distance is a reasonable distance to cover in 18 s given that the average speed of the police car is

$$v_{\text{avg}} = (1/2)(v_{p0} + v_p) = (1/2)v_p = 36.0 \text{ m/s}.$$

- 2.65. THINK:** Since no information is given about the direction of the second car, it is assumed that both cars travel in the same direction. The first car accelerates from rest with $a_1 = 2.00 \text{ m/s}^2$. The second car moves with constant velocity $v_2 = 4.00 \text{ m/s}$. The cars are 30.0 m apart. Take the initial position of car 1 to be $x_{10} = 0$. Then the initial position of car 2 is $x_{20} = 30.0 \text{ m}$. Both cars will have the same final position: $x_1 = x_2 = x'$. Both cars will travel for the same amount of time: $t_1 = t_2 = t'$.
- (a) I want to know the position of the collision, x' .
- (b) I want to know the time at which the collision occurs, t' .

SKETCH:

RESEARCH: car 1: $\Delta x_1 = v_{i_0} t_1 + \frac{1}{2} a_1 t_1^2$; car 2: $\Delta x_2 = v_2 t_2$

SIMPLIFY:

(a) To solve for x' , use $\Delta x_1 = v_{i_0} t_1 + (1/2) a_1 t_1^2 \Rightarrow x' - x_{i_0} = v_{i_0} t_1 + (1/2) a_1 t_1^2$. Since $x_{i_0} = 0$ and $v_{i_0} = 0$, $x' = (1/2) a_1 t_1^2$. Time t_1 is not known, but $t_1 = t_2$ and $\Delta x_2 = v_2 t_2$, therefore, $t_1 = t_2 = \Delta x_2 / v_2 = (x' - x_{2_0}) / v_2$. Inserting this t_1 into the first equation yields

$$x' = \frac{1}{2} a_1 \left(\frac{x' - x_{2_0}}{v_2} \right)^2 = \frac{a_1}{2 v_2^2} \left((x')^2 - 2 x_{2_0} x' + x_{2_0}^2 \right)$$

Rearranging gives:

$$\frac{a_1}{2 v_2^2} (x')^2 - \left(\frac{a_1 x_{2_0}}{v_2^2} + 1 \right) x' + \frac{a_1 x_{2_0}^2}{2 v_2^2} = 0 \Rightarrow (x')^2 - \left(2 x_{2_0} + \frac{2 v_2^2}{a_1} \right) x' + x_{2_0}^2 = 0.$$

This is a quadratic equation. Solving for x' :

$$x' = \frac{\left(2 x_{2_0} + \frac{2 v_2^2}{a_1} \right) \pm \sqrt{\left(2 x_{2_0} + \frac{2 v_2^2}{a_1} \right)^2 - 4 x_{2_0}^2}}{2}$$

(b) To solve for t' , use $t' = t_2 = \frac{x' - x_{2_0}}{v_2}$, from above.

CALCULATE:

$$(a) \quad x' = \frac{\left(2(30.0 \text{ m}) + \frac{2(4.00 \text{ m/s})^2}{2.00 \text{ m/s}^2} \right) \pm \sqrt{\left(2(30.0 \text{ m}) + \frac{2(4.00 \text{ m/s})^2}{2.00 \text{ m/s}^2} \right)^2 - 4(30.0 \text{ m})^2}}{2}$$

$$= 14.68 \text{ m}, 61.32 \text{ m}$$

The first solution may be disregarded; with both cars moving in the same direction, the position of the collision cannot be between their two initial positions. That is, x' cannot be between $x_{i_0} = 0$ and $x_{2_0} = 30 \text{ m}$.

$$(b) \quad t' = \frac{x' - x_{2_0}}{v_2} = \frac{61.32 \text{ m} - 30.0 \text{ m}}{4.00 \text{ m/s}} = 7.830 \text{ s}$$

ROUND:

$$(a) \quad x' = 61.3 \text{ m}$$

$$(b) \quad t' = 7.83 \text{ s}$$

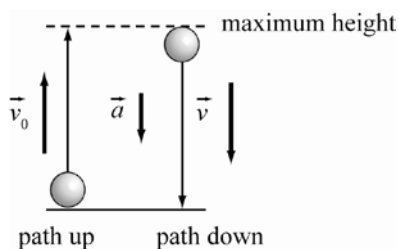
DOUBLE-CHECK:

(a) This collision position has units of distance, and is greater than the initial positions of both cars, as it should be.

(b) The time is reasonable since this is the time required for car 2 to travel $\Delta x_2 = x' - x_{2_0} = 30 \text{ m}$ at a speed of $v_2 = 4.0 \text{ m/s}$.

- 2.66. THINK:** I know that $v_0 = 26.4 \text{ m/s}$ and $a = -g = -9.81 \text{ m/s}^2$. I want to find t_{total} . Note that once the ball gets back to the starting point, $v = -26.4 \text{ m/s}$, or $v = -v_0$.

SKETCH:



RESEARCH: $v = v_0 + at$

SIMPLIFY: $t = \frac{v - v_0}{a} = \frac{-v_0 - v_0}{-g} = \frac{2v_0}{g}$

CALCULATE: $t = \frac{2(26.4 \text{ m/s})}{9.81 \text{ m/s}^2} = 5.38226 \text{ s}$

ROUND: Since all the values given have three significant digits, $t = 5.38 \text{ s}$.

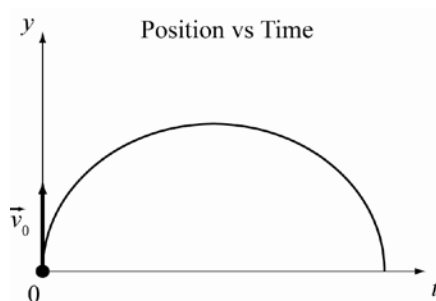
DOUBLE-CHECK: This seems like a reasonable amount of time for the ball to be up in the air.

- 2.67. THINK:** I know that $v_0 = 10.0 \text{ m/s}$, $a = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0 \text{ m}$.

(a) I want to find the velocity v at $t = 0.50 \text{ s}$.

(b) I want to find the height h of the stone at $t = 0.50 \text{ s}$.

SKETCH:



RESEARCH:

(a) $v = v_0 + at$

(b) $\Delta y = v_0 t + \frac{1}{2}at^2$ and $\Delta y = h$

SIMPLIFY:

(a) $v = v_0 - gt$

(b) $h = v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{1}{2}gt^2$

CALCULATE:

(a) $v = 10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.50 \text{ s})$
 $= 10.0 \text{ m/s} - 4.905 \text{ m/s}$
 $= 5.095 \text{ m/s}$

(b) $h = (10.0 \text{ m/s})(0.50 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.50 \text{ s})^2$
 $= 5.0 \text{ m} - 1.226 \text{ m}$
 $= 3.774 \text{ m}$

ROUND:

(a) Subtracting two numbers is precise to the least precise decimal place of the numbers. Therefore, $v = 5.1 \text{ m/s}$.

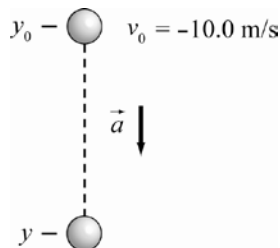
(b) $h = 3.8 \text{ m}$

DOUBLE-CHECK:

(a) $v < v_0$, and this makes sense since speed decreases as the object rises.

(b) This is a reasonable height for a ball to achieve in 0.50 s after it is thrown upward.

2.68. THINK: I know that $v_0 = -10.0 \text{ m/s}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find v at $t = 0.500 \text{ s}$.

SKETCH:

RESEARCH: $v = v_0 + at$

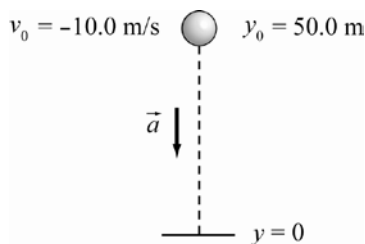
SIMPLIFY: $v = v_0 - gt$

CALCULATE: $v = -10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.500 \text{ s}) = -10.0 \text{ m/s} - 4.905 \text{ m/s} = -14.905 \text{ m/s}$

ROUND: Subtracting two numbers is precise to the least precise decimal place of the numbers. Therefore, $v = -14.9 \text{ m/s}$.

DOUBLE-CHECK: A negative v indicates that the stone is (still) falling downward. This makes sense, since the stone was thrown downward.

2.69. THINK: Take “downward” to be along the negative y -axis. I know that $v_0 = -10.0 \text{ m/s}$, $\Delta y = -50.0 \text{ m}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find t , the time when the ball reaches the ground.

SKETCH:

RESEARCH: $\Delta y = v_0 t + \frac{1}{2}at^2$

SIMPLIFY: $\frac{1}{2}at^2 + v_0 t - \Delta y = 0$. This is a quadratic equation. Solving for t :

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-\Delta y)}}{2\left(\frac{1}{2}a\right)} = \frac{-v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{-g}$$

CALCULATE: $t = \frac{-(-10.0 \text{ m/s}) \pm \sqrt{(-10.0 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(-50.0 \text{ m})}}{-9.81 \text{ m/s}^2}$
 $= -4.3709 \text{ s}, 2.3322 \text{ s}$

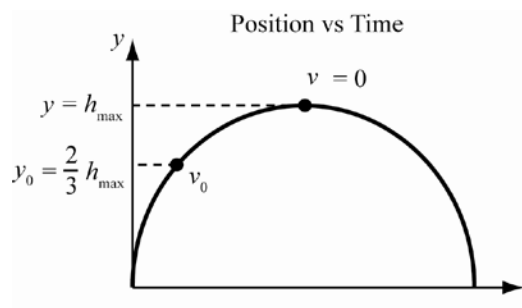
The time interval has to be positive, so $t = 2.3322 \text{ s}$.

ROUND: All original quantities are precise to three significant digits, therefore $t = 2.33 \text{ s}$.

DOUBLE-CHECK: A negative v indicates that the stone is (still) falling downward. This makes sense, since the stone was thrown downward. The velocity is even more negative after 0.500 s than it was initially, which is consistent with the downward acceleration.

- 2.70. **THINK:** I know that $v_0 = 20.0 \text{ m/s}$, $y_0 = (2/3)h_{\text{max}}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find h_{max} . Note that when $y = h_{\text{max}}$, the velocity is $v = 0$.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: $v^2 = v_0^2 - 2g\left(h_{\text{max}} - \frac{2}{3}h_{\text{max}}\right) \Rightarrow v^2 - v_0^2 = -2g\left(\frac{1}{3}h_{\text{max}}\right) \Rightarrow h_{\text{max}} = -\frac{3(v^2 - v_0^2)}{2g} \Rightarrow h_{\text{max}} = \frac{3v_0^2}{2g}$

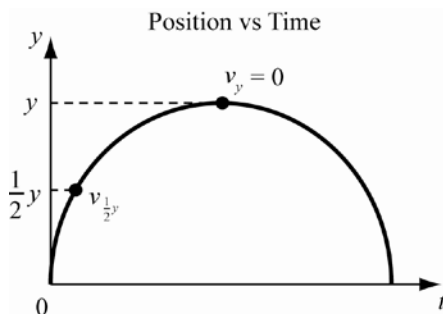
CALCULATE: $h_{\text{max}} = \frac{3(20.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 61.16 \text{ m}$

ROUND: $h_{\text{max}} = 61.2 \text{ m}$

DOUBLE-CHECK: h_{max} is positive which is consistent with the sketch. This seems like a reasonable height to achieve by throwing the ball upward.

- 2.71. **THINK:** I know the final height is y and the initial velocity is v_0 . The velocity at this height is zero: $v_y = 0$. Also, $a_y = -g$. I want to know the velocity at half of the final height, $v_{\frac{1}{2}y}$. Assume $y_0 = 0$.

SKETCH:



RESEARCH: $v_y^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: The initial velocity, v_0 , is $v_0 = \sqrt{v_y^2 - 2a(y - y_0)} = \sqrt{2gy}$. Then $v_{\frac{1}{2}y}$, in terms of the maximum height y , is

$$\left(v_{\frac{1}{2}y}\right)^2 = v_0^2 + 2a\left(\left(\frac{1}{2}y\right) - y_0\right) \Rightarrow v_{\frac{1}{2}y}^2 = (\sqrt{2gy})^2 - 2g\left(\frac{1}{2}y\right) \Rightarrow v_{\frac{1}{2}y}^2 = 2gy - gy \Rightarrow v_{\frac{1}{2}y} = \sqrt{gy}$$

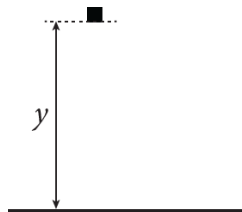
CALCULATE: This step is not necessary.

ROUND: This step is not necessary.

DOUBLE-CHECK: The units of $v_{\frac{1}{2}y}$ are: $\left[v_{\frac{1}{2}y}\right] = \sqrt{\left(\frac{\text{m}}{\text{s}^2}\right)(\text{m})} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \text{m/s}$, which is a unit of velocity.

- 2.72. **THINK:** The acceleration of an object due to gravity on the surface of the Moon is independent of the mass of the object.

SKETCH:



RESEARCH: We can use $y = \frac{1}{2}gt^2$, where y is the distance the objects fall, t is the time it takes for the objects to fall, and g is the acceleration of gravity on the Moon.

SIMPLIFY: We can solve our equation for g : $y = \frac{1}{2}gt^2 \Rightarrow g = \frac{2y}{t^2}$.

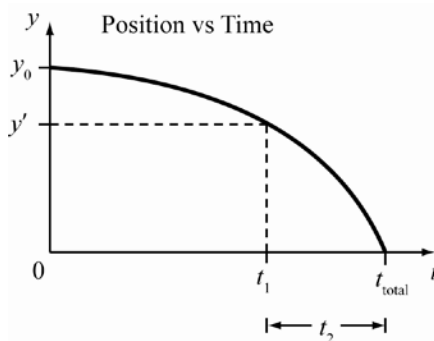
CALCULATE: $g = \frac{2y}{t^2} = \frac{2(1.6 \text{ m})}{(1.4 \text{ s})^2} = 1.6327 \text{ m/s}^2$.

ROUND: The values given are all accurate to two significant digits, so the answer is given two by two significant digits: $g = 1.6 \text{ m/s}^2$.

DOUBLE-CHECK: The Moon is smaller and less dense than the Earth, so it makes sense that the acceleration of gravity on the surface of the Moon is about 6 times less than the acceleration of gravity on the surface of the Earth.

- 2.73. **THINK:** The bowling ball is released from rest. In such a case we have already studied the relationship between vertical distance fallen and time in Example 2.5, “Reaction Time”, in the book. With this result in our arsenal, all we have to do here is to compute the time t_{total} it takes the ball to fall from Bill’s apartment down to the ground and subtract from it the time t_1 it takes the ball to fall from Bill’s apartment down to John’s apartment.

SKETCH:



RESEARCH: We will use the formula $t = \sqrt{2h/g}$ from Example 2.5. If you look at the sketch, you see that $t_{\text{total}} = \sqrt{2h_{\text{total}}/g} = \sqrt{2y_0/g}$ and that $t_1 = \sqrt{2h_1/g} = \sqrt{2(y_0 - y')/g}$.

SIMPLIFY: Solving for the time difference gives:

$$t_2 = t_{\text{total}} - t_1 = \sqrt{2y_0/g} - \sqrt{2(y_0 - y')/g}$$

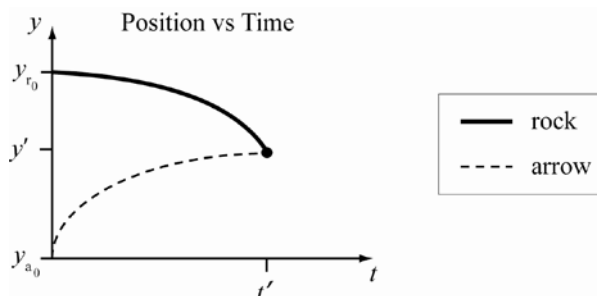
CALCULATE: $t_2 = \sqrt{2(63.17 \text{ m}) / (9.81 \text{ m/s}^2)} - \sqrt{2(63.17 \text{ m} - 40.95 \text{ m}) / (9.81 \text{ m/s}^2)}$
 $= 1.4603 \text{ s}$

ROUND: We round to $t_2 = 1.46 \text{ s}$, because g has three significant figures.

DOUBLE-CHECK: The units of the solution are those of time, which is already a good minimum requirement for a valid solution. But we can do better! If we compute the time it takes an object to fall 40.95 m from rest, we find from again using $t = \sqrt{2h/g}$ that this time is 2.89 s. In the problem here the bowling ball clearly already has a significant downward velocity as it passes the height of 40.95 m, and so we expect a time t_2 shorter than 2.89 s, which is clearly fulfilled for our solution.

- 2.74. **THINK:** The information known for the rock is the initial velocity, $v_{r_0} = 0$ and the initial height, $y_{r_0} = 18.35 \text{ m}$. The information known for the arrow is the initial velocity, $v_{a_0} = 47.4 \text{ m/s}$ and the initial height, $y_{a_0} = 0$. For both, $a = -g = -9.81 \text{ m/s}^2$. Note that both the rock and the arrow will have the same final position, y' , and both travel for the same time, t' . I want to find t' .

SKETCH:



RESEARCH: $\Delta y = v_0 t + \frac{1}{2} a t^2$

SIMPLIFY: For the rock, $y_r - y_{r_0} = v_{r_0} t' + (1/2) a (t')^2 \Rightarrow y_r = -\frac{1}{2} g (t')^2 + y_{r_0}$. For the arrow, $y_a - y_{a_0} = v_{a_0} t' + (1/2) a (t')^2 \Rightarrow y_a = v_{a_0} t' - (1/2) g (t')^2$. As the final positions for each are the same, we know $y_r = y_a \Rightarrow -\frac{1}{2} g (t')^2 + y_{r_0} = v_{a_0} t' - \frac{1}{2} g (t')^2 \Rightarrow y_{r_0} = v_{a_0} t' \Rightarrow t' = \frac{y_{r_0}}{v_{a_0}}$.

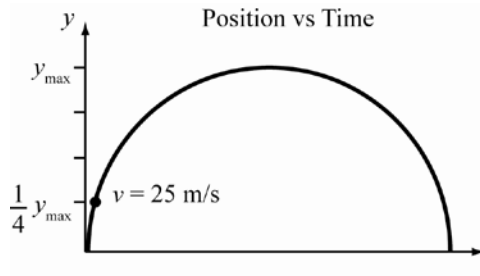
CALCULATE: $t' = \frac{18.35 \text{ m}}{47.4 \text{ m/s}} = 0.38713 \text{ s}$

ROUND: v_{a_0} is given to three significant figures, so $t' = 0.387 \text{ s}$.

DOUBLE-CHECK: This is a reasonable time for an arrow of initial velocity 47.4 m/s to rise to a height less than 18.35 m (the height from which the rock was dropped).

- 2.75. **THINK:** At $y = (1/4)y_{\max}$, $v = 25$ m/s. Also, $a = -g = -9.81$ m/s² and $y_0 = 0$. I want to find v_0 . It will be useful to know $y_{\max}$. At $y_{\max}$, $v' = 0$.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: $v^2 = v_0^2 + 2a(y - y_0) = v_0^2 - 2g\left(\frac{1}{4}y_{\max}\right)$

$$\Rightarrow v_0^2 = v^2 + 2g\left(\frac{1}{4}y_{\max}\right) = v^2 + \frac{1}{2}gy_{\max}$$

Now I must find $y_{\max}$. When $y_{\max}$ is achieved, the velocity v' is zero. Then

$$(v')^2 = v_0^2 + 2a(y_{\max} - y_0) \Rightarrow 0 = v_0^2 - 2gy_{\max} \Rightarrow y_{\max} = \frac{v_0^2}{2g}.$$

Inserting this into the equation above gives

$$v_0^2 = v^2 + \frac{1}{2}g\left(\frac{v_0^2}{2g}\right) \Rightarrow v_0^2 = v^2 + \frac{1}{4}v_0^2 \Rightarrow \frac{3}{4}v_0^2 = v^2 \Rightarrow v_0^2 = \frac{4}{3}v^2 \Rightarrow v_0 = \frac{2}{\sqrt{3}}v.$$

CALCULATE: $v_0 = \frac{2}{\sqrt{3}}(25 \text{ m/s}) = 28.87 \text{ m/s}$

ROUND: The value for v limits the calculation to two significant figures. So $v_0 = 29$ m/s.

DOUBLE-CHECK: v_0 is greater than $v = 25$ m/s, as it should be.

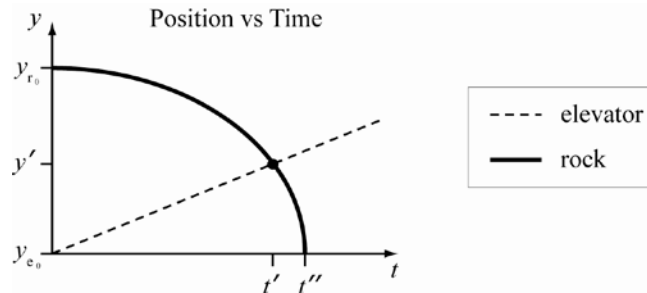
- 2.76. **THINK:** For the elevator, the velocity is $v_e = 1.75$ m/s, the acceleration is $a_e = 0$, and the initial height is $y_{e_0} = 0$. For the rock, the initial velocity is $v_{r_0} = 0$, the acceleration is $a_r = -g = -9.81$ m/s², and the initial height is $y_{r_0} = 80.0$ m.

(a) I need to find the time it takes the rock to intercept the elevator, t' .

(b) I need to find the time it takes the rock to hit the ground at $y_r = 0$, t'' .

When the rock intercepts the elevator, both are at the same position y' , and have taken the same time, t' , to arrive there.

SKETCH:



RESEARCH: The elevator position is determined from $\Delta y_e = v_e t$. For the rock, $\Delta y_r = v_{r_0} t + (1/2)a_r t^2$.

SIMPLIFY: For the elevator, $y_e - y_{e_0} = v_e t \Rightarrow y_e = v_e t$. For the rock, $y_r - y_{r_0} = v_{r_0} t + (1/2)a_r t^2 \Rightarrow y_r - y_{r_0} = -(1/2)gt^2$.

(a) Both objects take the same time to intercept each other, and both have the same position at interception:

$$y_e = y_r \Rightarrow v_e t' = y_{r_0} - \frac{1}{2}g(t')^2 \Rightarrow \frac{1}{2}g(t')^2 + v_e t' - y_{r_0} = 0.$$

Solving for t' in the quadratic equation gives:

$$t' = \frac{-v_e \pm \sqrt{v_e^2 - 4\left(\frac{1}{2}g\right)(-y_{r_0})}}{2\left(\frac{1}{2}g\right)} = \frac{-v_e \pm \sqrt{v_e^2 + 2gy_{r_0}}}{g}.$$

(b) The total fall time, t'' , for the rock is $\Delta y_r = y_r - y_{r_0} = v_{r_0} t'' + (1/2)a_r (t'')^2$. The final position is $y_r = 0$. With $v_{r_0} = 0$, $-y_{r_0} = -(1/2)g(t'')^2 \Rightarrow t'' = \sqrt{2y_{r_0}/g}$.

CALCULATE:

$$(a) \quad t' = \frac{-(1.75 \text{ m/s}) \pm \sqrt{(1.75 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(80.0 \text{ m})}}{9.81 \text{ m/s}^2} = 3.8641 \text{ s}, -4.2209 \text{ s}$$

$$(b) \quad t'' = \sqrt{\frac{2(80.0 \text{ m})}{9.81 \text{ m/s}^2}} = 4.0386 \text{ s}$$

ROUND: The values given have three significant figures, so the final answers will also have three significant figures.

(a) Taking the positive solution for time, $t' = 3.86 \text{ s}$.

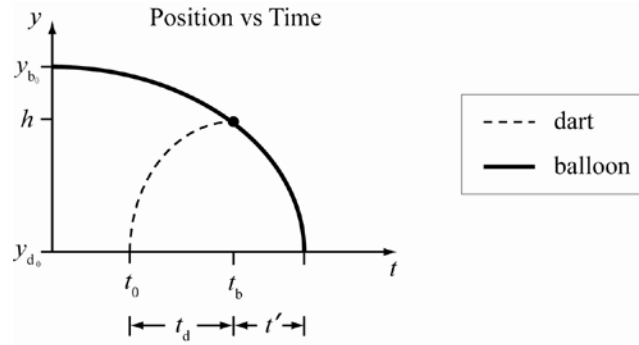
(b) $t'' = 4.04 \text{ s}$

DOUBLE-CHECK: The total time to fall is greater than the intercept time, as it should be.

2.77. THINK: It is probably a good idea to read through the solution of the “Melon Drop” problem, Solved Problem 2.5 in the textbook before getting started with the present problem. The present problem has the additional complication that the water balloon gets dropped some time before the dart get fired, whereas in the “Melon Drop” problem both projectiles get launched simultaneously. For the first 2 seconds, only our water balloon is in free fall, and we can calculate its position y_{b_0} and velocity v_{b_0} at the end of this time interval.

a) After the initial two seconds the dart also gets launched, and then both objects (water balloon and dart) are in free-fall. Their initial distance is y_{b_0} , and their relative velocity is the difference between the initial velocity of the dart and v_{b_0} . The time until the two objects meet is then simply the ratio of the initial distance and the relative velocity.

b) For this part we simply calculate the time it takes for the balloon to free-fall the entire height h and subtract our answer from part a).

SKETCH:**RESEARCH:**

(a) The position and velocity of the balloon after the time $t_0 = 2$ s are

$$y_{b_0} = h - \frac{1}{2}gt_0^2$$

$$v_{b_0} = -gt_0$$

The time it takes then for the balloon and the dart to meet is the ratio of their initial distance to their initial relative velocity:

$$t_d = y_{b_0} / (v_{d_0} - v_{b_0})$$

Our answer for part a) is the sum of the time t_0 , during which the balloon was in free-fall alone, and the time t_1 , $t_b = t_d + t_0$.

(b) The total time it takes for the balloon to fall all the way to the ground is

$$t_{\text{total}} = \sqrt{2h/g}$$

We get our answer for part b) by subtracting the result of part a) from this total time:

$$t' = t_{\text{total}} - t_d$$

SIMPLIFY:

(a) If we insert the expressions for the initial distance and relative speed into $t_d = y_{b_0} / (v_{d_0} - v_{b_0})$, we find

$t_d = y_{b_0} / (v_{d_0} - v_{b_0}) = (h - \frac{1}{2}gt_0^2) / (v_{d_0} + gt_0)$. Adding t_0 then gives us our final answer:

$$t_b = t_0 + (h - \frac{1}{2}gt_0^2) / (v_{d_0} + gt_0)$$

(b) For the time between the balloon being hit by the dart and the water reaching the ground we find by

inserting $t_{\text{total}} = \sqrt{2h/g}$ into $t' = t_{\text{total}} - t_b$:

$$t' = \sqrt{2h/g} - t_b$$

CALCULATE:

$$(a) \quad t_b = 2.00 \text{ s} + \frac{80.0 \text{ m} - \frac{1}{2}(9.81 \text{ m/s}^2)(2.00 \text{ s})^2}{20.0 \text{ m/s} + (9.81 \text{ m/s}^2)(2.00 \text{ s})} = 3.524 \text{ s}$$

$$(b) \quad t' = \sqrt{2(80.0 \text{ m}) / (9.81 \text{ m/s}^2)} - 3.524 \text{ s} = 0.515 \text{ s}.$$

ROUND:

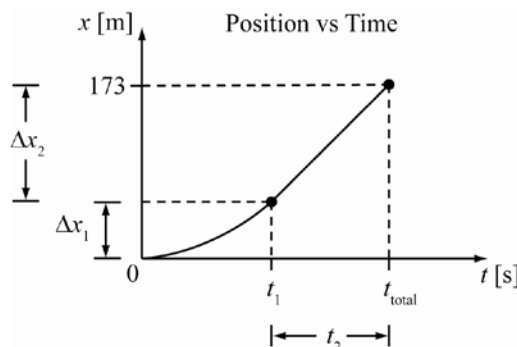
$$(a) \quad t_b = 3.52 \text{ s}$$

$$(b) \quad t' = 0.515 \text{ s}$$

DOUBLE-CHECK: The solution we showed in this problem is basically the double-check step in Solved Problem 2.5. Conversely, we can use the solution method of Solved Problem 2.5 as a double-check for what we have done here. This is left as an exercise for the reader.

- 2.78. THINK:** I know the runner's initial velocity, $v_0 = 0$, her acceleration, $a = 1.23 \text{ m/s}^2$, her final velocity, $v = 5.10 \text{ m/s}$, and the distance she traveled, $\Delta x = 173 \text{ m}$. I want to know the total time t_{total} . Note that Δx is composed of a displacement Δx_1 which occurs while accelerating and a displacement Δx_2 which occurs at a constant velocity. That is, $\Delta x = \Delta x_1 + \Delta x_2$. Mass is irrelevant.

SKETCH:



RESEARCH: The total time is the sum of the times for each displacement. Let $t_{\text{total}} = t_1 + t_2$ with t_1 the time for displacement Δx_1 and t_2 the time for displacement Δx_2 . For t_1 , use $v = v_0 + at_1$. For Δx_1 , use $\Delta x_1 = (1/2)(v + v_0)t_1$. For t_2 , use $\Delta x_2 = vt_2$.

SIMPLIFY: Note that

$$\Delta x_1 = \frac{1}{2}(v + v_0)t_1 = \frac{1}{2}vt_1 = \frac{1}{2}v \frac{v}{a} = \frac{v^2}{2a}.$$

To compute the value of t_{total} , first simplify expressions for t_1 and t_2 :

$$t_1 = \frac{v - v_0}{a} = \frac{v}{a} \quad \text{and} \quad t_2 = \frac{\Delta x_2}{v} = \frac{\Delta x - \Delta x_1}{v} = \frac{\Delta x - \frac{v^2}{2a}}{v} = \frac{\Delta x}{v} - \frac{v}{2a}.$$

Using the last two equations t_{total} can be calculated as follows:

$$t_{\text{total}} = t_1 + t_2 = \frac{v}{a} + \frac{\Delta x}{v} - \frac{v}{2a} = \frac{v}{2a} + \frac{\Delta x}{v}.$$

CALCULATE: $t_{\text{total}} = \frac{5.10 \text{ m/s}}{2(1.23 \text{ m/s}^2)} + \frac{173 \text{ m}}{5.10 \text{ m/s}} = 35.995 \text{ s}$

ROUND: Each initial value has three significant figures, so $t_{\text{total}} = 36.0 \text{ s}$

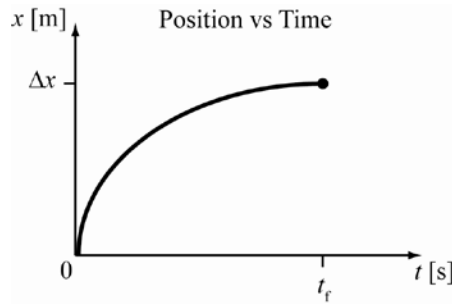
DOUBLE-CHECK: This is a reasonable amount of time required to run 173 m.

- 2.79. THINK:** Let the moment the jet touches down correspond to the time $t = 0$. The initial velocity is

$$v_0 = \left(\frac{142.4 \text{ mi}}{1 \text{ hr}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{1 \text{ mi}} \right) = 63.66 \text{ m/s}.$$

The jet comes to rest in a time of $t_f = 12.4 \text{ s}$. The jet comes to a complete stop, which makes the final velocity zero, so $v_f = 0$. I want to compute the distance the jet travels after it touches down, Δx .

SKETCH:



RESEARCH: To determine the distance traveled, the following equation can be used: $\Delta x = \frac{1}{2}(v_0 + v)t$.

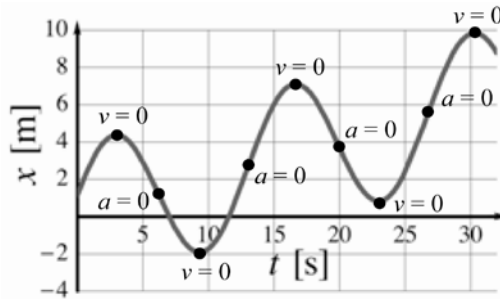
SIMPLIFY: With $v = 0$, the equation becomes $\Delta x = (v_0 t) / 2$.

CALCULATE: $\Delta x = \frac{1}{2}(63.66 \text{ m/s})(12.4 \text{ s}) = 394.7 \text{ m}$.

ROUND: Since t has three significant digits, the result should be rounded to $\Delta x = 395 \text{ m}$.

DOUBLE-CHECK: This is a reasonable distance to decelerate from 63.66 m/s in 12.4 s.

- 2.80. Velocity is the slope of the position versus time graph. Therefore, $v = 0$ at the local maxima and minima. Acceleration is the slope of the velocity versus time graph. On a position versus time graph, acceleration, a is zero at inflection points on the curve that are not maxima or minima, i.e. $a = 0$ as the slope of x vs. t approaches a constant value over some non-zero time interval, Δt :



- 2.81. **THINK:** The acceleration is the derivative of the velocity with respect to time, that is, the instantaneous change in velocity. Since the car is stopped and then accelerates to 60.0 miles per hour, we can infer that the acceleration is positive in the direction of the car's motion.

SKETCH: Sketch the motion of the car at 0 s and time 4.20 s. Since the acceleration is unknown, use the variable a to represent the size of the acceleration.

$$t = 0$$

$$|\vec{v}_x| = 0$$

$$|\vec{a}| = a$$

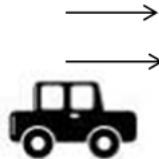


$$x_0 = 0$$

$$0 < t < 4.2$$

$$0 < |\vec{v}_x| < 60.0$$

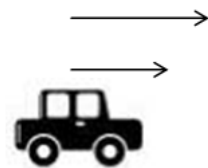
$$|\vec{a}| = a$$



$$t = 4.20$$

$$|\vec{v}_x| = 60.0$$

$$|\vec{a}| = a$$



$$x = ?$$

RESEARCH: Since this problem involves motion with constant acceleration, use equation (2.23). (a) The velocity in the positive x direction at time t is equal to the velocity in the positive x direction at time 0 plus the acceleration in the x direction multiplied by the time, $v_x = v_{x0} + a_x t$. (b) Having found the acceleration in the positive x direction, use the equation $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$ to find the position x at time $t = 4.20$ s. To make the calculations simple and straightforward, take the position of the car at time $t = 0$ s to be the zero of our coordinate system, so $x_0 = 0$ miles.

SIMPLIFY: (a) Use the velocity at times $t = 0$ and $t = 4.2$, solve the equation $v_x = v_{x0} + a_x t$ for a_x to get:

$$\begin{aligned} v_x &= v_{x0} + a_x t \\ -v_{x0} + v_x &= -v_{x0} + v_{x0} + a_x t = a_x t \\ (v_x - v_{x0})/t &= (a_x t)/t \\ \frac{v_x - v_{x0}}{t} &= a_x \end{aligned}$$

(b) Using the expression for a_x and algebra to find an expression for the total distance traveled:

$$\begin{aligned} x &= x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ &= x_0 + v_{x0}t + \frac{1}{2}\left(\frac{v_x - v_{x0}}{t}\right)t^2 \\ &= x_0 + v_{x0}t + \frac{1}{2}(v_x - v_{x0})t \end{aligned}$$

CALCULATE: (a) Since the car starts at rest, $v_{x0} = 0$ mph. Also, the velocity at time $t = 4.20$ s is $v_x = 60.0$ mph in the positive x direction. Using these values gives $a_x = \frac{v_x - v_{x0}}{t} = \frac{60.0 - 0}{4.2} = \frac{100}{7} \text{ mph} \cdot \text{s}^{-1}$.

Since time t is given in miles per hour, it is necessary to convert this to miles per second to make the units consistent. Convert this to a more convenient set of units, such as $\text{mi} \cdot \text{s}^{-2}$ to make future calculations easier:

$$\frac{100}{7} \cdot \frac{\text{mi}}{\text{hour} \cdot \text{sec}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} = \frac{1}{252} \cdot \frac{\text{mi}}{\text{sec}^2} \quad \text{(b) Plug in values to find } x, \text{ the location of the car at time } t = 4.20 \text{ s.}$$

This gives:

$$\begin{aligned} x &= x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ &= 0 + 0 + \frac{1}{2} \cdot \frac{1}{252} \cdot (4.20)^2 \\ &= \frac{7}{200} = 0.035 \text{ mi} \end{aligned}$$

ROUND: Since the measured values have 3 significant figures, the answers in both parts will have 3 significant figures. (a) For the acceleration, $\frac{1}{252} \frac{\text{mi}}{\text{sec}^2} \approx 0.00397 \frac{\text{mi}}{\text{sec}^2}$ or $3.97 \times 10^{-3} \frac{\text{mi}}{\text{sec}^2}$. (b) Using

scientific notation, $0.035 = 3.50 \times 10^{-2} \text{ mi}$. Note also that if we convert to SI units, we obtain (a) 6.39 m/s^2 for the acceleration and (b) 56.3 m for the distance.

DOUBLE-CHECK:

(a) Accelerating at a constant rate of $3.97 \times 10^{-3} \frac{\text{mi}}{\text{sec}^2}$ for 4.20 seconds from a standing start means that the car will be going $3.97 \times 10^{-3} \frac{\text{mi}}{\text{sec}^2} \cdot 4.2 \text{ sec} \cdot \frac{3600 \text{ sec}}{\text{hour}}$ or 60.0 mph after 4.20 seconds. This agrees with the question statement.

(b) Since the car is at position $x = \frac{1}{2}a_x t^2 = \frac{1}{504}t^2$ miles at time t seconds, the derivative with respect to time gives the velocity as a function of time: $\frac{dx}{dt} = \frac{d\left(\frac{1}{504}t^2\right)}{dt} = \frac{1}{504} \cdot 2t = \frac{1}{252}t$ miles per second, or $\frac{1}{252}t \frac{\text{mi}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{\text{hour}} = \frac{100}{7}t \frac{\text{mi}}{\text{hour}}$ at time t . At time $t = 4.20 \text{ s}$, this gives a velocity of $\frac{100}{7}(4.2) = 60.0 \text{ mph}$, which agrees with the setup for this problem.

2.82. THINK: It is known for a car that when the initial velocity is

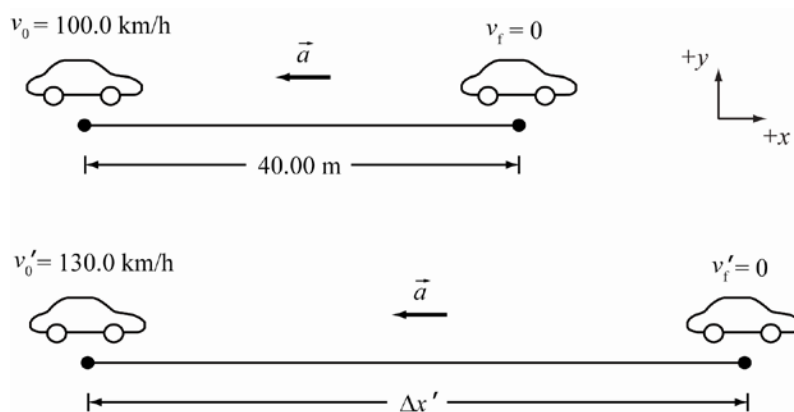
$$v_0 = 100.0 \frac{\text{km}}{\text{h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 27.7778 \text{ m/s},$$

and the final velocity is $v_f = 0$, the stopping distance is $\Delta x = 40 \text{ m}$. Determine the stopping distance, $\Delta x'$ when the initial velocity is

$$v_0' = 130.0 \frac{\text{km}}{\text{h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 36.1111 \text{ m/s},$$

and the final velocity is $v_f' = 0$. The road conditions are the same in each case, so it can be assumed that the acceleration does not change.

SKETCH:



RESEARCH: The acceleration, a can be determined from the original conditions with $v^2 = v_0^2 + 2a\Delta x$. Substitute the value of the acceleration computed from the first set of conditions as the acceleration in the second conditions to determine $\Delta x'$.

SIMPLIFY: With $v = 0$, $0 = v_0^2 + 2a\Delta x \Rightarrow a = -v_0^2 / (2\Delta x)$. Then, $v_f'^2 = v_0'^2 + 2a\Delta x'$. With $v_f' = 0$,

$$\Delta x' = -\frac{v_0'^2}{2a} = -\frac{v_0'^2}{2\left(-\frac{v_0^2}{2\Delta x}\right)} = \frac{v_0'^2}{v_0^2} \Delta x.$$

CALCULATE: $\Delta x' = \frac{(36.1111 \text{ m/s})^2}{(27.7778 \text{ m/s})^2} (40.00 \text{ m}) = 67.5999 \text{ m}$

Note that the unit conversion from km/h to m/s was not necessary as the units of velocity cancel each other in the ratio.

ROUND: $\Delta x' = 67.60 \text{ m}$

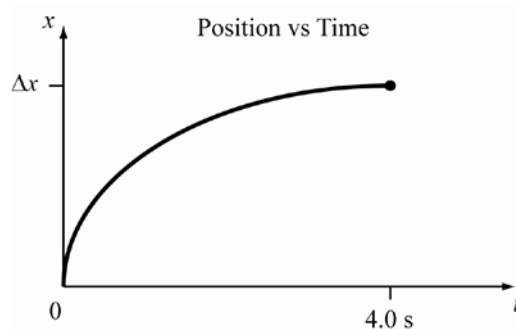
DOUBLE-CHECK: The stopping distance for the larger initial velocity is greater than the stopping distance for the small initial velocity, as it should be.

2.83. THINK: The initial velocity is

$$v_0 = 60.0 \frac{\text{km}}{\text{h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 16.67 \text{ m/s}.$$

The final velocity is $v = 0$. The stop time is $t = 4.00 \text{ s}$. The deceleration is uniform. Determine (a) the distance traveled while stopping, Δx and (b) the deceleration, a . I expect $a < 0$.

SKETCH:



RESEARCH:

(a) To determine the stopping distance, use $\Delta x = t(v_0 + v)/2$.

(b) To determine a , use $v = v_0 + at$.

SIMPLIFY:

(a) With $v = 0$, $\Delta x = v_0 t / 2$.

(b) With $v = 0$, $0 = v_0 + at \Rightarrow a = -v_0 / t$.

CALCULATE:

$$(a) \Delta x = \frac{(16.67 \text{ m/s})(4.00 \text{ s})}{2} = 33.34 \text{ m}$$

$$(b) a = -\frac{16.67 \text{ m/s}}{4.00 \text{ s}} = -4.167 \text{ m/s}^2$$

ROUND:

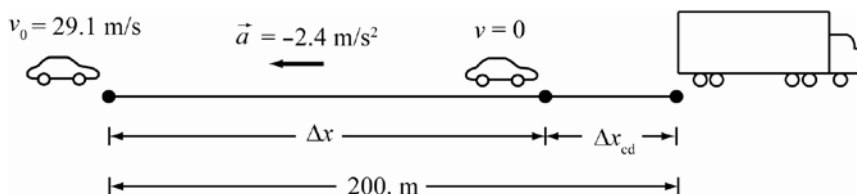
$$(a) \Delta x = 33.3 \text{ m}$$

$$(b) a = -4.17 \text{ m/s}^2$$

DOUBLE-CHECK: The distance traveled while stopping is of an appropriate order of magnitude. A car can reasonably stop from 60 km/h in a distance of about 30 m. The acceleration is negative, indicating that the car is slowing down from its initial velocity.

2.84. THINK: The car's initial velocity is $v_0 = 29.1 \text{ m/s}$. The deceleration is $a = -2.4 \text{ m/s}^2$. Assume that the final velocity is $v = 0$, that is the car does not hit the truck. The truck is a distance $d = 200.0 \text{ m}$ when the car begins to decelerate. Determine (a) the final distance between the car and the truck, Δx_{cd} and (b) the time it takes to stop, t .

SKETCH:



RESEARCH:

(a) The distance to the truck is the difference between the initial distance d and the stopping distance Δx :

$$\Delta x_{cd} = d - \Delta x. \Delta x \text{ can be determined from } v^2 = v_0^2 + 2a\Delta x.$$

(b) The stop time is determined from $v = v_0 + at$.

SIMPLIFY:

(a) With $v = 0$, $v^2 = v_0^2 + 2a\Delta x \Rightarrow 0 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -v_0^2 / 2a$. Then, $\Delta x_{cd} = d + (v_0^2 / 2a)$.

(b) With $v = 0$, $t = -v_0 / a$.

CALCULATE:

$$(a) \Delta x_{cd} = 200.0 \text{ m} + \frac{(29.1 \text{ m/s})^2}{2(-2.4 \text{ m/s}^2)} = 200.0 \text{ m} - 176.4 \text{ m} = 23.6 \text{ m}$$

$$(b) t = -\frac{(29.1 \text{ m/s})}{(-2.4 \text{ m/s}^2)} = 12.13 \text{ s}$$

ROUND:

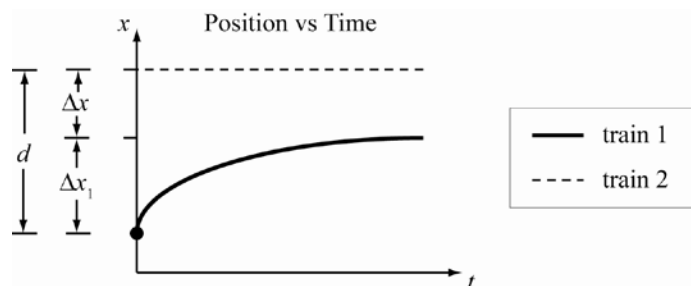
(a) Since the acceleration has two significant figures, $\Delta x_{cd} = 24 \text{ m}$

(b) Rounding to two significant figures, $t = 12 \text{ s}$.

DOUBLE-CHECK: The stopping time and distance are realistic for a car decelerating from 29.1 m/s.

- 2.85. **THINK:** For train 1, it is known that $v_{1,0} = 40.0 \text{ m/s}$, $a_1 = -6.0 \text{ m/s}^2$ and $v_1 = 0$. For train 2, it is known that $v_2 = 0$ and $a_2 = 0$. The distance between the trains is $d = 100.0 \text{ m}$. Determine the distance between the trains after train 1 stops, Δx .

SKETCH:



RESEARCH: The final distance between the trains, Δx is the difference between the initial distance, d and the stopping distance of train 1, Δx_1 : $\Delta x = d - \Delta x_1$.

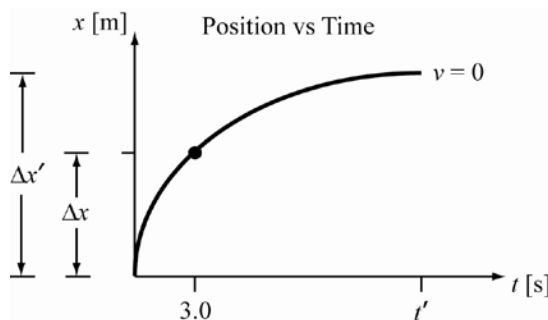
SIMPLIFY: With $v_1 = 0$, $v_1^2 = v_{1,0}^2 + 2a_1\Delta x_1 \Rightarrow 0 = v_{1,0}^2 + 2a_1\Delta x_1 \Rightarrow \Delta x_1 = -\frac{v_{1,0}^2}{2a_1}$. Then, $\Delta x = d + \frac{v_{1,0}^2}{2a_1}$.

$$\text{CALCULATE: } \Delta x = 100.0 \text{ m} + \frac{(40.0 \text{ m/s})^2}{2(-6.0 \text{ m/s}^2)} = 100.0 \text{ m} - 133.3 \text{ m} = -33.3 \text{ m}$$

ROUND: Note that Δx is determined to be a negative value. This is due to the stopping distance being greater than the initial distance between the trains. This implies that train 1 actually collides with train 2. Then the distance between the two trains is zero.

DOUBLE-CHECK: It is reasonable for train 1 to collide with train 2 given the initial separation of only 100.0 m and an initial velocity for train 1 of 40.0 m/s.

- 2.86. **THINK:** The initial velocity is $v_0 = 25.0 \text{ m/s}$. The acceleration is $a = -1.2 \text{ m/s}^2$. Determine (a) the distance Δx traveled in $t = 3.0 \text{ s}$, (b) the velocity, v after traveling this distance, (c) the stopping time, t' and (d) the stopping distance, $\Delta x'$. Note when the car is stopped, $v' = 0$.

SKETCH:**RESEARCH:**

(a) To determine Δx , use $\Delta x = v_0 t + (at^2)/2$.

(b) To determine v , use $v = v_0 + at$.

(c) To determine t' , use $v = v_0 + at$.

(c) To determine $\Delta x'$, use $v^2 = v_0^2 + 2a\Delta x$.

SIMPLIFY:

(a) It is not necessary to simplify.

(b) It is not necessary to simplify.

(c) With $v' = 0$, $v' = v_0 + at' \Rightarrow t' = -v_0 / a$.

(d) With $v' = 0$, $v'^2 = v_0^2 + 2a\Delta x' \Rightarrow \Delta x' = -v_0^2 / 2a$.

CALCULATE:

$$(a) \Delta x = (25.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-1.2 \text{ m/s}^2)(3.0 \text{ s})^2 = 69.6 \text{ m}$$

$$(b) v = 25.0 \text{ m/s} + (-1.2 \text{ m/s}^2)(3.0 \text{ s}) = 21.4 \text{ m/s}$$

$$(c) t' = -\frac{(25.0 \text{ m/s})}{(-1.2 \text{ m/s}^2)} = 20.83 \text{ s}$$

$$(d) \Delta x' = -\frac{(25.0 \text{ m/s})^2}{2(-1.2 \text{ m/s}^2)} = 260.4 \text{ m}$$

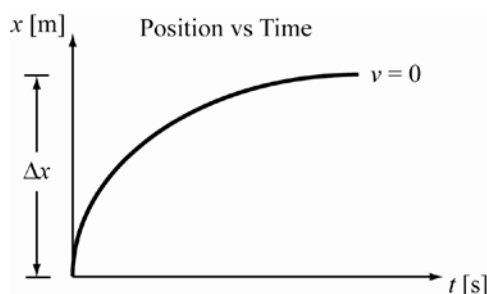
ROUND: Both the acceleration and the time have two significant figures, so the results should be rounded to $\Delta x = 70. \text{ m}$, $v = 21 \text{ m/s}$, $t' = 21 \text{ s}$ and $\Delta x' = 260 \text{ m}$.

DOUBLE-CHECK: The car travels 70 m while decelerating, which is less than the 75 m it would have traveled in the same time if it had not been decelerating. The velocity after decelerating is less than the initial velocity. The stopping distance is greater than the distance traveled in 3.0 s, and the stopping time is greater than the intermediate time of 3.0 s. All of these facts support the calculated values.

2.87. THINK: The initial velocity is

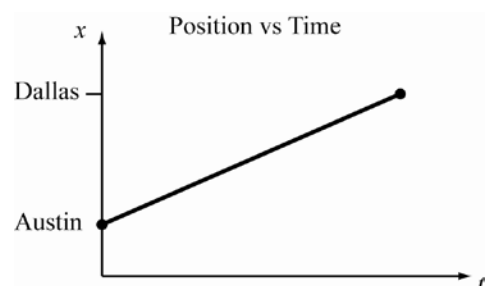
$$v_0 = 212.809 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 95.1315 \text{ m/s.}$$

The acceleration is $a = -8.0 \text{ m/s}^2$. The final speed is $v = 0$. Determine the stopping distance, Δx .

SKETCH:**RESEARCH:** Use $v^2 = v_0^2 + 2a\Delta x$.**SIMPLIFY:** With $v = 0$, $0 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -v_0^2 / 2a$.**CALCULATE:** $\Delta x = -\frac{(95.1315 \text{ m/s})^2}{2(-8.0 \text{ m/s}^2)} = 565.6 \text{ m}$ **ROUND:** The acceleration has two significant figures, so the result should be rounded to $\Delta x = 570 \text{ m}$.**DOUBLE-CHECK:** The initial velocity is large and the deceleration has a magnitude close to that of gravity. A stopping distance greater than half of a kilometer is reasonable.**2.88. THINK:** The velocity can be converted to SI units as follows:

$$v_0 = 245 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 109.5 \text{ m/s}.$$

The distance is $\Delta x = 362 \text{ km} = 3.62 \cdot 10^5 \text{ m}$. Determine the time, t to travel the distance, Δx . Note the acceleration is $a = 0$.

SKETCH:**RESEARCH:** For $a = 0$, use $\Delta x = vt$.**SIMPLIFY:** $t = \frac{\Delta x}{v}$ **CALCULATE:** $t = \frac{3.62 \cdot 10^5 \text{ m}}{109.5 \text{ m/s}} = 3306 \text{ s}$ **ROUND:** The distance Δx has three significant figures, so the result should be rounded to $t = 3310 \text{ s}$.**DOUBLE-CHECK:** The time in hours is

$$3310 \text{ s} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 0.919 \text{ h}.$$

An hour is a reasonable amount of time to fly a distance of 362 km.

2.89. The position is given by $x = at^3 + bt^2 + c$, where $a = 2.0 \text{ m/s}^3$, $b = 2.0 \text{ m/s}^2$ and $c = 3.0 \text{ m}$.(a) Determine the sled's position between $t_1 = 4.0 \text{ s}$ and $t_2 = 9.0 \text{ s}$.

$$x(4.0 \text{ s}) = (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 + (2.0 \text{ m/s}^2)(4.0 \text{ s})^2 + 3.0 \text{ m} = 163 \text{ m} \approx 160 \text{ m}$$

$$x(9.0 \text{ s}) = (2.0 \text{ m/s}^3)(9.0 \text{ s})^3 + (2.0 \text{ m/s}^2)(9.0 \text{ s})^2 + 3.0 \text{ m} = 1623 \text{ m} \approx 1600 \text{ m}$$

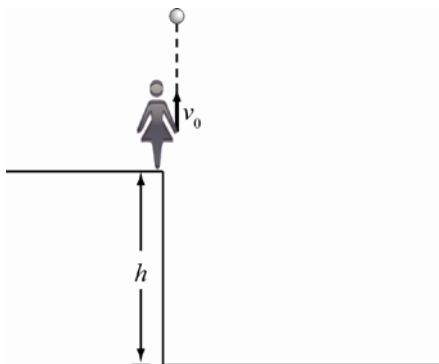
The sled is between $x = 160 \text{ m}$ and $x = 1600 \text{ m}$.

(b) Determine the sled's average speed over this interval.

$$V_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{1623 \text{ m} - 163 \text{ m}}{9.0 \text{ s} - 4.0 \text{ s}} = \frac{1460 \text{ m}}{5.0 \text{ s}} \approx 292 \text{ m/s} = 290 \text{ m/s}$$

- 2.90. THINK:** The cliff has a height of $h = 100. \text{ m}$ above the ground. The girl throws a rock straight up with a speed of $v_0 = 8.00 \text{ m/s}$. Determine how long it takes for the rock to hit the ground and find the speed, v of the rock just before it hits the ground. The acceleration due to gravity is $a = -g = -9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: The total displacement in the vertical direction is given by $\Delta y = y_f - y_i$. If the top of the cliff is taken to be the origin of the system, then $y_i = 0$ and $y_f = -h = -100. \text{ m}$. Therefore, $\Delta y = -h$.

$$(a) \Delta y = v_0 t + \frac{1}{2} a t^2$$

$$(b) v^2 = v_0^2 + 2a\Delta y$$

SIMPLIFY:

(a) The quadratic equation can be used to solve for t from the equation $gt^2/2 - v_0 t + \Delta y = 0$:

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 4\left(\frac{g}{2}\right)(-h)}}{2\left(\frac{g}{2}\right)} = \frac{v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$$

$$(b) v = \sqrt{v_0^2 + 2gh}$$

CALCULATE:

$$(a) t = \frac{8.00 \text{ m/s} \pm \sqrt{(8.00 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(100. \text{ m})}}{(9.81 \text{ m/s}^2)}$$

$$= 5.40378 \text{ s or } -3.77 \text{ s}$$

The negative time is impossible.

$$(b) v = \sqrt{(8.00 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(100. \text{ m})} = 45.011 \text{ m/s}$$

ROUND:

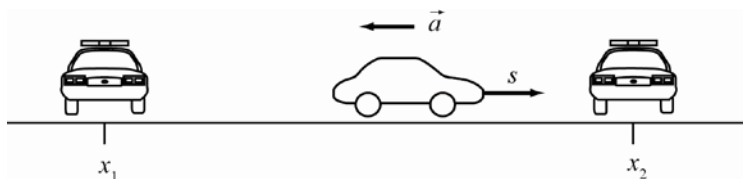
$$(a) t = 5.40 \text{ s}$$

$$(b) v = 45.0 \text{ m/s}$$

DOUBLE-CHECK: The calculated time and speed for the rock are reasonable considering the height of the cliff. Also, the units are correct units for time and speed.

- 2.91. THINK:** The police have a double speed trap set up. A sedan passes the first speed trap at a speed of $s_1 = 105.9$ mph. The sedan decelerates and after a time, $t = 7.05$ s it passes the second speed trap at a speed of $s_2 = 67.1$ mph. Determine the sedan's deceleration and the distance between the police cruisers.

SKETCH:



RESEARCH:

(a) Convert the speeds to SI units as follows:

$$s_1 = 105.9 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 47.34 \text{ m/s}$$

$$s_2 = 67.1 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 29.996 \text{ m/s}.$$

The sedan's velocity, v can be written in terms of its initial velocity, v_0 the time t , and its acceleration a :

$v = v_0 + at$. Substitute s_1 for v_0 and s_2 for v .

(b) The distance between the cruisers is given by: $\Delta x = x_2 - x_1 = v_0 t + (1/2)at^2$.

SIMPLIFY:

$$(a) \quad a = \frac{v - v_0}{t} = \frac{s_2 - s_1}{t}$$

(b) Substitute s_1 for v_0 and the expression from part (a) for a : $\Delta x = s_1 t + (1/2)at^2$

CALCULATE:

$$(a) \quad a = \frac{29.996 \text{ m/s} - 47.34 \text{ m/s}}{7.05 \text{ s}} = -2.4602 \text{ m/s}^2$$

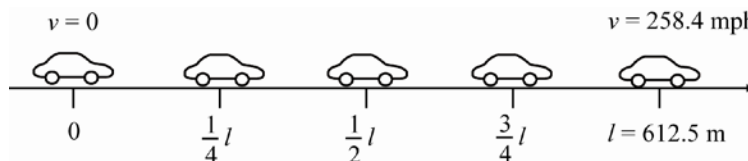
$$(b) \quad \Delta x = (47.34 \text{ m/s})(7.05 \text{ s}) + \frac{1}{2}(-2.4602 \text{ m/s}^2)(7.05 \text{ s})^2 = 272.6079 \text{ m}$$

ROUND: The least number of significant figures provided in the problem are three, so the results should be rounded to $a = -2.46 \text{ m/s}^2$ and $\Delta x = 273 \text{ m}$.

DOUBLE-CHECK: The sedan did not have its brakes applied, so the values calculated are reasonable for the situation. The acceleration would have been larger, and the distance would have been much smaller, if the brakes had been used. The results also have the proper units.

- 2.92. THINK:** The initial speed of a new racecar is $v_0 = 0$ (standing start). The car accelerates with a constant acceleration and reaches a speed of $v = 258.4$ mph at a distance of $l = 612.5$ m. Determine a relationship between the speed and distance.

SKETCH:



RESEARCH: The acceleration is constant, so there are two expressions for velocity and distance:

$$v = v_0 + at, \quad x = x_0 + v_0 t + (1/2)at^2.$$

SIMPLIFY: It is given that $v_0 = 0$ and $x_0 = 0$, so the above expressions simplify to $v = at$, $x = \frac{1}{2}at^2$.

Thus, $t = \sqrt{2x/a}$. Substituting this expression into $v = at$,

$$v = a\sqrt{\frac{2x}{a}} = \sqrt{2ax}.$$

CALCULATE: (1) The speed at a distance of $x = l/4$ is given by:

$$v_{l/4} = \sqrt{2a\frac{l}{4}} = \sqrt{\frac{1}{4}}(\sqrt{2al}) = \frac{1}{2}\sqrt{2al}.$$

Note that $v = \sqrt{2al}$, therefore, $v_{l/4} = v/2$.

$$v_{l/4} = \frac{1}{2}(258.4 \text{ mph}) = 129.2 \text{ mph}$$

(2) Similarly, substituting $x = l/2$ into $v = \sqrt{2ax}$,

$$v_{l/2} = \sqrt{\frac{1}{2}}v = \sqrt{\frac{1}{2}}(258.4 \text{ mph}) = 182.716 \text{ mph}.$$

(3) Substituting $x = 3l/4$ into $v = \sqrt{2ax}$,

$$v_{3l/4} = \sqrt{\frac{3}{4}}v = \sqrt{\frac{3}{4}}(258.4 \text{ mph}) = 223.781 \text{ mph}.$$

ROUND: Initially there are four significant figures, so the results should be rounded to $v_{l/4} = 129.2 \text{ mph}$, $v_{l/2} = 182.7 \text{ mph}$ and $v_{3l/4} = 223.8 \text{ mph}$.

DOUBLE-CHECK: Note that $v_{l/4} < v_{l/2} < v_{3l/4} < v$ as expected.

2.93. THINK: An expression of y as a function of t is given. Determine the speed and acceleration from this function, $y(t)$. The first derivative of $y(t)$ yields speed as a function of time, $v = dy/dt$, and the second derivative yields acceleration as a function of time, $a = dv/dt$.

SKETCH: A sketch is not needed to solve the problem.

RESEARCH: From a table of common derivatives: $\frac{d \sin(\alpha t + \beta)}{dt} = \alpha \cos(\alpha t + \beta)$, and

$$\frac{d \cos(\alpha t + \beta)}{dt} = -\alpha \sin(\alpha t + \beta).$$

SIMPLIFY: It is not necessary to simplify.

CALCULATE:

$$(a) \quad v = \frac{d}{dt}(3.8 \sin(0.46t/s - 0.31) \text{ m} - 0.2t \text{ m/s} + 5.0 \text{ m})$$

$$= 3.8(0.46) \cos(0.46t/s - 0.31) \text{ m/s} - 0.2 \text{ m/s}$$

$$= 1.748 \cos(0.46t/s - 0.31) \text{ m/s} - 0.2 \text{ m/s}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(1.748 \cos(0.46t/s - 0.31) \text{ m/s} - 0.2 \text{ m/s})$$

$$= -1.748(0.46) \sin(0.46t/s - 0.31) \text{ m/s}^2$$

$$= -0.80408 \sin(0.46t/s - 0.31) \text{ m/s}^2$$

(b) Set $a = 0$: $0 = -0.80408 \sin(0.46t/s - 0.31) \text{ m/s}^2 \Rightarrow \sin(0.46t/s - 0.31) = 0$. It is known that $\sin \alpha = 0$ when $\alpha = n\pi$ and n is an integer. Therefore,

$$0.46t/s - 0.31 = n\pi \Rightarrow t = \frac{n\pi + 0.31}{0.46} \text{ s}.$$

The times between 0 and 30 s that satisfy $a = 0$ are:

$$\begin{aligned} t &= 6.8295n + 0.6739 \text{ s} \\ &= 0.6739 \text{ s for } n = 0 \\ &= 7.5034 \text{ s for } n = 1 \\ &= 14.3329 \text{ s for } n = 2 \\ &= 21.1624 \text{ s for } n = 3 \\ &= 27.9919 \text{ s for } n = 4. \end{aligned}$$

ROUND: Rounding to two significant figures,

(a) $v = 1.7 \cos(0.46t / \text{s} - 0.31) \text{ m/s} - 0.2 \text{ m/s}$, $a = -0.80 \sin(0.46t / \text{s} - 0.31) \text{ m/s}^2$

(b) $t = 0.67 \text{ s}$, 7.5 s , 14 s , 21 s and 28 s .

DOUBLE-CHECK: For oscillatory motion, where the position is expressed in terms of a sinusoidal function, the velocity is always out of phase with respect to the position. Out of phase means if $x = \sin t$, then $v = \cos t = \sin(t + \pi/2)$. The acceleration is proportional to the position function. For example, if $x = A \sin t$, $a = -A \sin t$.

2.94. THINK: An expression for position as a function of time is given as $x(t) = 4t^2$.

SKETCH: A sketch is not needed to solve the problem.

RESEARCH: $(a + b)^2 = a^2 + 2ab + b^2$

SIMPLIFY: Simplification is not necessary.

CALCULATE:

(a) $x(2.00) = 4(2.00)^2 \text{ m} = 16.00 \text{ m}$

(b) $x(2.00 + \Delta t) = 4(2.00 + \Delta t)^2 \text{ m} = 4(4.00 + 4.00\Delta t + \Delta t^2) \text{ m}$
 $= (16.00 + 16.00\Delta t + 4\Delta t^2) \text{ m}$

(c) $\frac{\Delta x}{\Delta t} = \frac{x(2.00 + \Delta t) - x(2.00)}{\Delta t}$
 $= \frac{16.00 + 16.00\Delta t + 4\Delta t^2 - 16.00 \text{ m}}{\Delta t \text{ s}}$
 $= (16.00 + 4\Delta t) \text{ m/s}$

Taking the limit as $\Delta t \rightarrow 0$: $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \rightarrow 0} 16.00 + \lim_{\Delta t \rightarrow 0} 4\Delta t = 16.00 \text{ m/s}$.

ROUND: Rounding to three significant figures,

(a) $x(2.00) = 16.0 \text{ m}$

(b) $x(2.00 + \Delta t) = (16.0 + 16.0\Delta t + 4\Delta t^2) \text{ m}$

(c) $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = 16.0 \text{ m/s}$

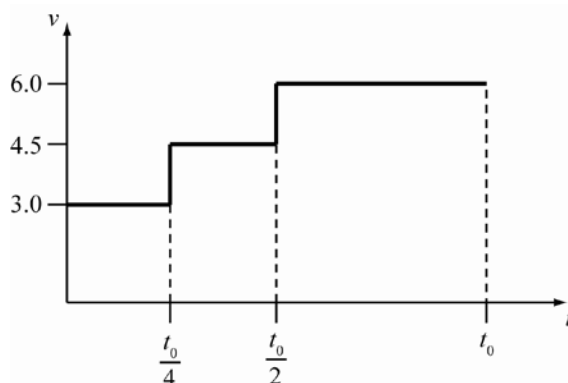
DOUBLE-CHECK: The value of the position function near $t = 2.00 \text{ s}$ coincides with its value at $t = 2.00 \text{ s}$. This should be the case, since the position function is continuous. The value of the velocity can also be

found from the derivative: $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$. Substitute $t = 2.00 \text{ s}$,

$\left. \frac{dx}{dt} \right|_{t=2.00} = 8(2.00) = 16.00 \text{ m/s}$. This value agrees with what was calculated in part (c).

- 2.95. THINK:** The distance to the destination is 199 miles or 320 km. To solve the problem it is easiest to draw a velocity versus time graph. The distance is then given by the area under the curve.

SKETCH:



RESEARCH: For a constant speed, the distance is given by $x = vt$.

SIMPLIFY: To simplify, divide the distance into three parts.

Part 1: from $t = 0$ to $t = t_0 / 4$.

Part 2: from $t = t_0 / 4$ to $t = t_0 / 2$.

Part 3: from $t = t_0 / 2$ to $t = t_0$.

CALCULATE:

(a) The distances are $x_1 = 3.0t_0 / 4$, $x_2 = 4.5t_0 / 4$ and $x_3 = 6.0t_0 / 2$. The total distance is given by

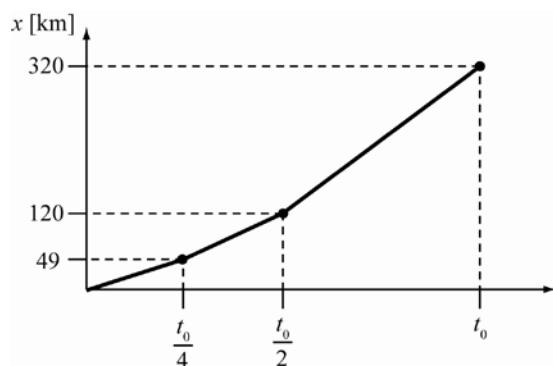
$$x = x_1 + x_2 + x_3 = \frac{(3.0 + 4.5 + 12)t_0}{4} \text{ m} = \frac{19.5t_0}{4} \text{ m} \Rightarrow t_0 = \frac{4x}{19.5} \text{ s.}$$

$$t_0 = \frac{4(320 \cdot 10^3)}{19.5} \text{ s} = 65.6410 \cdot 10^3 \text{ s} = 65641 \text{ s} \Rightarrow t_0 = 18.2336 \text{ h}$$

(b) The distances are:

$$x_1 = 3.0 \left(\frac{65641}{4} \right) \text{ m} = 49.23 \text{ km}, \quad x_2 = 4.5 \left(\frac{65641}{4} \right) \text{ m} = 73.85 \text{ km}, \quad x_3 = 6.0 \left(\frac{65641}{2} \right) \text{ m} = 196.92 \text{ km}.$$

ROUND: Since the speeds are given to two significant figures, the results should be rounded to $x_1 = 49 \text{ km}$, $x_2 = 74 \text{ km}$ and $x_3 = 2.0 \cdot 10^2 \text{ km}$. $x_1 + x_2 = 123 \text{ km} \approx 120 \text{ km}$, and then $x = x_1 + x_2 + x_3 = 323 \text{ km} \approx 320 \text{ km}$.

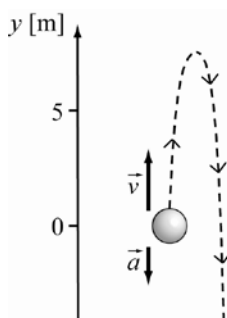


DOUBLE-CHECK: The sum of the distances x_1 , x_2 and x_3 must be equal to the total distance of 320 km:

$x_1 + x_2 + x_3 = 49.23 + 73.85 + 196.92 = 320 \text{ km}$ as expected. Also, note that $x_1 < x_2 < x_3$ since $v_1 < v_2 < v_3$.

- 2.96. THINK:** The initial speed is $v_0 = 15.0$ m/s. Assume there is no air resistance. The acceleration due to gravity is given by $g = 9.81$ m/s². t_1 is the time taken from the original position to the 5.00 m position on the way up. The time it takes from the initial position to 5.00 m on its way down is t_2 .

SKETCH:



RESEARCH: For motion with a constant acceleration, the expressions for speed and distances are

$$v = v_0 + at, \quad y = y_0 + v_0 t + \frac{1}{2}at^2. \quad \text{The acceleration due to gravity is } a = -g.$$

SIMPLIFY:

- (a) At the maximum height, the velocity is $v = 0$. Using $y_0 = 0$:

$$0 = v_0 - gt \Rightarrow t = \frac{v_0}{g},$$

$$y = y_{\max} = v_0 t - \frac{1}{2}gt^2.$$

Substituting $t = v_0 / g$,

$$y_{\max} = v_0 \left(\frac{v_0}{g} \right) - \frac{1}{2}g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{2g}.$$

- (b) If the motion of the ball starts from the maximum height, there is free fall motion with $v_0 = 0$.

$$v = -gt \Rightarrow t = -\frac{v}{g}$$

$$y = y_{\max} + v_0 t - \frac{1}{2}gt^2 = y_{\max} - \frac{1}{2}gt^2$$

Substituting $t = v/g$:

$$y = y_{\max} - \frac{v^2}{2g} \Rightarrow v = \sqrt{(y_{\max} - y)2g}.$$

CALCULATE:

$$(a) \quad y_{\max} = \frac{(15.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 11.468 \text{ m}$$

$$(b) \quad v = \sqrt{(11.468 - 5.00)(2)(9.81 \text{ m/s}^2)} = 11.265 \text{ m/s. Thus the speed at this point is 11.265 m/s.}$$

(c,d) Using $y = y_0 + v_0 t + (at^2)/2$, $y = v_0 t - (1/2)gt^2$. Using $v_0 = 15.0$ m/s, $g = 9.81$ m/s² and $y = 5.00$ m, the quadratic equation is $(1/2)(9.81 \text{ m/s}^2)t^2 - 15.0t + 5.00 \text{ m} = 0$. Solving the quadratic equation:

$$t = \frac{15.0 \pm \sqrt{(15.0)^2 - 2(9.81)5}}{9.81} \text{ s} = \frac{15.0 \pm 11.265}{9.81} \text{ s} = 1.529 \pm 1.1483 = 2.6773 \text{ s and } 0.3807 \text{ s}$$

ROUND:

- (a) Rounding to three significant figures, $y_{\max} = 11.5$ m.

(b) All the numerical values have three significant figures, so the result is rounded to $v = 11.3$ m/s. Note the speed on the way up is the same as the speed on the way down.

(c) Rounding the values to three significant figures, $t_1 = 0.381$ s.

(d) $t_2 = 2.68$ s.

DOUBLE-CHECK: The speed at $t_1 = 0.381$ s and $t_2 = 2.68$ s must be the same and it is equal to the speed determined in part (b).

$$v = v_0 - gt$$

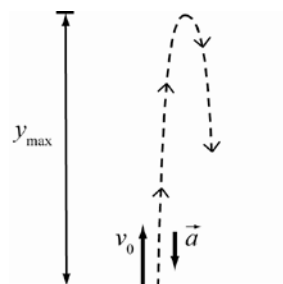
$$v_1 = 15.0 - (9.81)(0.381) = 11.2624 \text{ m/s} \approx 11.3 \text{ m/s}$$

$$v_2 = 15.0 - (9.81)(2.68) = -11.2908 \text{ m/s} \approx -11.3 \text{ m/s}$$

As can be seen, $|v_1| = |v_2|$ is equal to the result in part (b).

2.97. THINK: The maximum height is $y_{\max} = 240$ ft = 73.152 m. The acceleration due to gravity is given by $g = 9.81$ m/s².

SKETCH:



RESEARCH: To solve this constant acceleration problem, use $v = v_0 - gt$ and $y = y_0 + v_0 t - (gt^2/2)$.
 $y_0 = 0$.

SIMPLIFY:

(a) At a maximum height, the velocity v is zero.

$$v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$$

$$y_{\max} = v_0 \left(\frac{v_0}{g} \right) - \frac{1}{2} g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{2g} \Rightarrow v_0 = \sqrt{2gy_{\max}}$$

(b) If the motion is considering as starting from the maximum height $y_{\max}$, there is free fall motion with $v_0 = 0$.

$$v = -gt \Rightarrow t = \frac{v}{g}$$

$$y = y_{\max} - \frac{1}{2} gt^2 = y_{\max} - \frac{1}{2} g \left(\frac{v}{g} \right)^2 = y_{\max} - \frac{v^2}{2g} \Rightarrow v = \sqrt{(y_{\max} - y)2g}$$

(c) Note that v_0 is equal to the speed in part (b), $v_0 = -26.788$ m/s and v is equal to the original speed but in the opposite direction, $v = -37.884$ m/s.

$$t = \frac{v_0 - v}{g}$$

CALCULATE:

$$(a) \quad v_0 = \sqrt{2(9.81)(73.152)} = 37.885 \text{ m/s}$$

(b) $y = \frac{y_{\max}}{2}$, so $v = \sqrt{\left(y_{\max} - \frac{y_{\max}}{2}\right)2g} = \sqrt{gy_{\max}} = \sqrt{(9.81)73.152} = 26.788 \text{ m/s}$. Choose the positive root

because the problem asks for the speed, which is never negative.

(c) $t = \frac{37.884 \text{ m/s} - 26.788 \text{ m/s}}{(9.81 \text{ m/s}^2)} = 1.131 \text{ s}$

ROUND:

(a) Rounding to three significant figures, $v_0 = 37.9 \text{ m/s}$.

(b) Rounding to three significant figures, $v = 26.8 \text{ m/s}$.

(c) Rounding to three significant figures, $t = 1.13 \text{ s}$.

DOUBLE-CHECK: It is known that $v = \sqrt{2gy}$. This means that the ratio of two speeds is:

$$\frac{v_1}{v_2} = \frac{\sqrt{2gy_1}}{\sqrt{2gy_2}} = \sqrt{\frac{y_1}{y_2}}.$$

The result in part (b) is for $y = y_{\max}/2$, so the ratio is

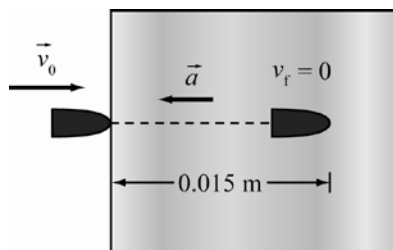
$$\frac{v_{1/2}}{v_0} = \sqrt{\frac{\frac{1}{2}y_{\max}}{y_{\max}}} = \sqrt{\frac{1}{2}} = 0.7071.$$

Using the results in parts (a) and (b):

$$\frac{v_{1/2}}{v_0} = \frac{26.8 \text{ m/s}}{37.9 \text{ m/s}} = 0.7071 \text{ as expected.}$$

2.98. THINK: The initial velocity is $v_0 = 200. \text{ m/s}$. There is constant acceleration and the maximum distance is $x_{\max} = 1.5 \text{ cm} = 0.015 \text{ m}$.

SKETCH:



RESEARCH: To solve a constant acceleration motion, use $v = v_0 + at$. There is a deceleration of a .

$$x = v_0 t + \frac{1}{2}at^2$$

SIMPLIFY: At the final position, $v = 0$.

$$v_0 - at = 0 \Rightarrow a = \frac{v_0}{t}$$

Substituting $a = v_0/t$ into $x_{\max} = v_0 t - (1/2)at^2$ gives:

$$x_{\max} = v_0 t - \frac{1}{2} \frac{v_0}{t} t^2 = \frac{1}{2} v_0 t \Rightarrow t = \frac{2x_{\max}}{v_0}$$

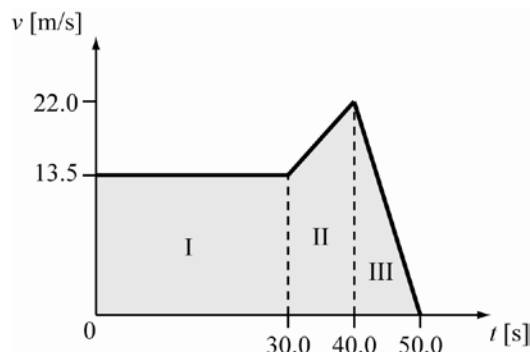
CALCULATE: $t = \frac{2(0.015 \text{ m})}{200. \text{ m/s}} = 1.5 \cdot 10^{-4} \text{ s}$

ROUND: Rounding to two significant figures yields the same result, $t = 1.5 \cdot 10^{-4} \text{ s}$

DOUBLE-CHECK: It is expected the resulting time is small for the bullet to stop at a short distance.

- 2.99. THINK:** $v_1 = 13.5 \text{ m/s}$ for $\Delta t = 30.0 \text{ s}$. $v_2 = 22.0 \text{ m/s}$ after $\Delta t = 10.0 \text{ s}$ (at $t = 40.0 \text{ s}$). $v_3 = 0$ after $\Delta t = 10.0 \text{ s}$ (at $t = 50.0 \text{ s}$). It will be easier to determine the distance from the area under the curve of the velocity versus time graph.

SKETCH:



RESEARCH: Divide and label the graph into three parts as shown above.

SIMPLIFY: The total distance, d is the sum of the areas under the graph, $d = A_1 + A_2 + A_3$.

CALCULATE:
$$d = (13.5 \text{ m/s})(30.0 \text{ s}) + \frac{1}{2}(13.5 \text{ m/s} + 22.0 \text{ m/s})(10.0 \text{ s}) + \frac{1}{2}(22.0 \text{ m/s})(10.0 \text{ s})$$

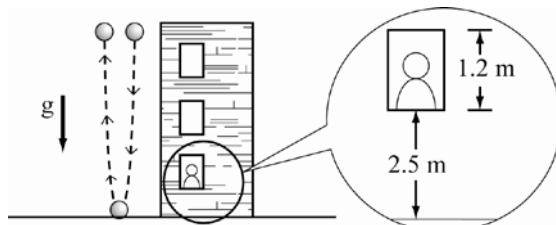
$$= 405 \text{ m} + 177.5 \text{ m} + 110 \text{ m} = 692.5 \text{ m}$$

ROUND: The speeds are given in three significant figures, so the result should be rounded to $d = 693 \text{ m}$.

DOUBLE-CHECK: From the velocity versus time plot, the distance can be estimated by assuming the speed is constant for all time, t : $d = (13.5 \text{ m/s})(50.0 \text{ s}) = 675 \text{ m}$. This estimate is in agreement with the previous result.

- 2.100. THINK:** It is given that the initial velocity is $v_0 = 0$. The time for the round trip is $t = 5.0 \text{ s}$.

SKETCH:



RESEARCH: $a = -g$. Using two expressions for velocity and distance:

(a) $v = v_0 + at$

(b) $y = y_0 + v_0 t + \frac{1}{2}at^2$

SIMPLIFY:

(a) $y_0 = y_{\max}$, $v = -gt$

(b) $y = y_{\max} - \frac{1}{2}gt^2$

(c) The distance from the top of the window to the ground is $1.2 + 2.5 = 3.7 \text{ m}$. From part (b),

$$y = y_{\max} - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(y_{\max} - y)}{g}}.$$

CALCULATE: The time taken from the roof to the ground is half the time of the round trip, $t = 5.0/2 = 2.5 \text{ s}$.

(a) The velocity before the ball hits the ground is $v = -(9.81)(2.5) = -24.525 \text{ m/s}$. So the speed is 24.525 m/s .

(b) $y = 0$ (ground), and t is the time from the roof to the ground.

$$0 = y_{\max} - \frac{1}{2}gt^2 \Rightarrow y_{\max} = \frac{1}{2}gt^2 \Rightarrow y_{\max} = \frac{1}{2}(9.81 \text{ m/s}^2)(2.5 \text{ s})^2 = 30.656 \text{ m}$$

$$(c) \quad t = \sqrt{\frac{2(30.656 - 3.7)}{(9.81)}} = 2.3443 \text{ s}$$

ROUND: Rounding to two significant figures, $|v| = 25 \text{ m/s}$, $y_{\max} = 31 \text{ m}$ and $t = 2.3 \text{ s}$.

DOUBLE-CHECK: The speed in part (a) is consistent with an object accelerating uniformly due to gravity. The distance in (b) is a reasonable height for a building. For the result of part (c), the time must be less than 2.5 s, which it is.

2.101. From a mathematical table: $\frac{d}{dt}e^{\alpha t} = \alpha e^{\alpha t}$.

$$(a) \quad x(t) = 2x_0 = \frac{1}{4}x_0 e^{3\alpha t} \Rightarrow e^{3\alpha t} = 8 \Rightarrow 3\alpha t = \ln 8 \Rightarrow t = \frac{1}{3\alpha} \ln 8$$

$$(b) \quad v(t) = \frac{dx}{dt} = \frac{3\alpha}{4}x_0 e^{3\alpha t}$$

$$(c) \quad a(t) = \frac{dv}{dt} = \frac{(3\alpha)^2}{4}x_0 e^{3\alpha t} = \frac{9\alpha^2}{4}x_0 e^{3\alpha t}$$

(d) αt must be dimensionless. Since the units of t are s, the units of α are s^{-1} .

2.102. Note that $\frac{d}{dt}(t^n) = nt^{n-1}$.

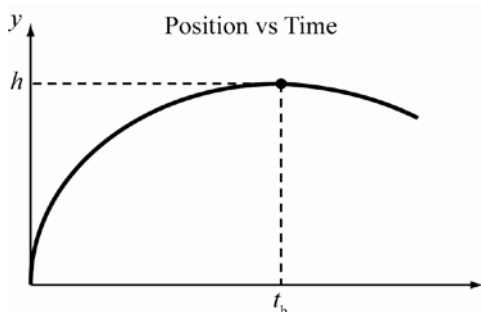
$$(a) \quad v = \frac{dx}{dt} = 4At^3 - 3Bt^2$$

$$(b) \quad a = \frac{dv}{dt} = 12At^2 - 6Bt$$

Multi-Version Exercises

2.103. THINK: The initial velocity is $v_0 = 28.0 \text{ m/s}$. The acceleration is $a = -g = -9.81 \text{ m/s}^2$. The velocity, v is zero at the maximum height. Determine the time, t to achieve the maximum height.

SKETCH:



RESEARCH: To determine the velocity use $v = v_0 + at_h$.

$$\text{SIMPLIFY: } at_h = v - v_0 \Rightarrow t_h = \frac{v - v_0}{a} = \frac{-v_0}{-g} = \frac{v_0}{g}$$

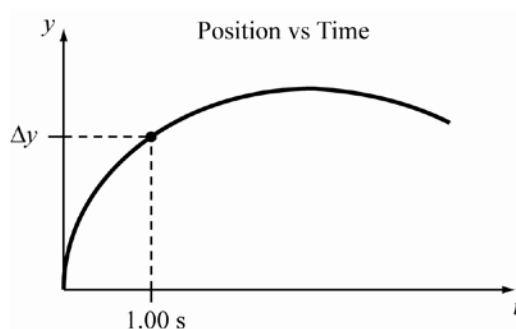
$$\text{CALCULATE: } t_h = \frac{28.0 \text{ m/s}}{9.81 \text{ m/s}^2} = 2.8542 \text{ s}$$

ROUND: The initial values have three significant figures, so the result should be rounded to $t_h = 2.85 \text{ s}$.

DOUBLE-CHECK: The initial velocity of the object is about 30 m/s, and gravity will cause the velocity to decrease about 10 m/s per second. It should take roughly three seconds for the object to reach its maximum height.

- 2.104. THINK:** The initial velocity is $v_0 = 28.0$ m/s. The time is $t = 1.00$ s. The acceleration is $a = -g = -9.81$ m/s². Determine the height above the initial position, Δy .

SKETCH:



RESEARCH: To determine the height use $\Delta y = v_0 t + (at^2)/2$.

SIMPLIFY: $\Delta y = v_0 t - \frac{1}{2}(gt^2)$

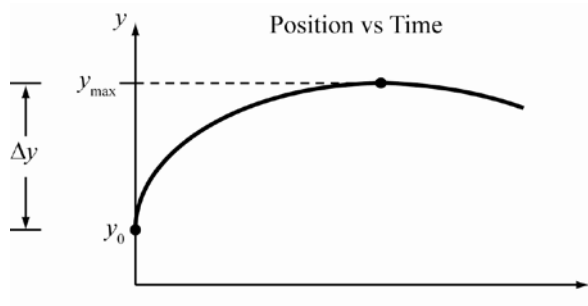
CALCULATE: $\Delta y = (28.0 \text{ m/s})(1.00 \text{ s}) - \frac{1}{2}((9.81 \text{ m/s}^2)(1.00 \text{ s})^2) = 23.095 \text{ m}$

ROUND: As all initial values have three significant figures, the result should be rounded to $\Delta y = 23.1$ m.

DOUBLE-CHECK: The displacement Δy is positive, indicating that the final position is higher than the initial position. This is consistent with the positive initial velocity.

- 2.105. THINK:** The initial velocity is $v_0 = 28.0$ m/s. The acceleration is $a = -g = -9.81$ m/s². The velocity, v is zero at the maximum height. Determine the maximum height, Δy above the projection point.

SKETCH:



RESEARCH: The maximum height can be determined from the following equation: $v^2 = v_0^2 + 2a\Delta y$.

SIMPLIFY: With $v = 0$, $0 = v_0^2 - 2g\Delta y \Rightarrow \Delta y = \frac{v_0^2}{2g}$.

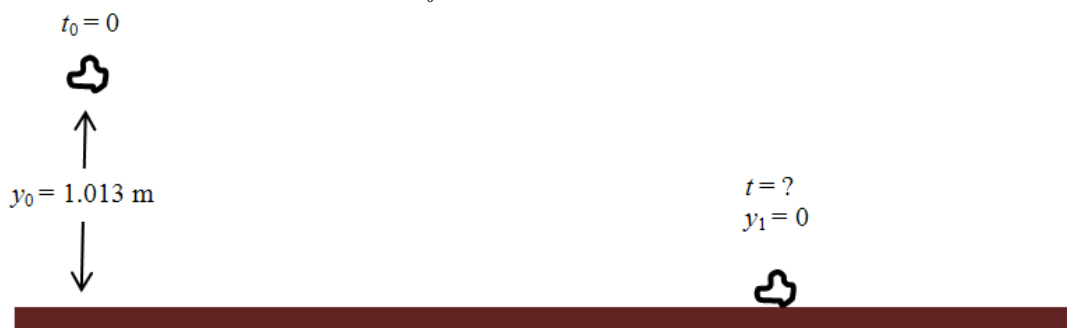
CALCULATE: $\Delta y = \frac{(28.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 39.96 \text{ m}$

ROUND: $v_0 = 28.0$ m/s has three significant figures, so the result should be rounded to $\Delta y = 40.0$ m.

DOUBLE-CHECK: The height has units of meters, which are an appropriate unit of distance. The calculated value is a reasonable maximum height for an object launched with a velocity of 28 m/s to achieve.

- 2.106. THINK:** Since the rock is dropped from a fixed height and allowed to fall to the surface of Mars, this question involves free fall. It is necessary to impose a coordinate system. Choose $y = 0$ to represent the surface of Mars and $t_0 = 0$ to be the time at which the rock is released.

SKETCH: Sketch the situation at time $t_0 = 0$ and time t , when the rock hits the surface.



RESEARCH: For objects in free fall, equations 2.25 can be used to compute velocity and position. In particular, the equation $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ can be used. In this case, $y_0 = 1.013$ m. Since the object is not thrown but dropped with no initial velocity, $v_{y0} = 0$ m/s, and $g = 3.699$ m/s² on the surface of Mars.

SIMPLIFY: The starting position and velocity ($y_0 = 1.013$ m and $v_{y0} = 0$ m/s), final position ($y = 0$ m) and gravitational acceleration are known. Using the fact that $v_{y0} = 0$ m/s and solving the equation for t gives:

$$\begin{aligned} 0 &= y_0 + 0t - \frac{1}{2}(g)t^2 = \\ \frac{g}{2}t^2 &= y_0 \Rightarrow \\ t^2 &= \frac{2 \cdot y_0}{g} \\ t &= \sqrt{\frac{2 \cdot y_0}{g}} \end{aligned}$$

CALCULATE: On Mars, the gravitational acceleration $g = 3.699$ m/s². Since the rock is dropped from a height of 1.013 m, $y_0 = 1.013$ m. Plugging these numbers into our formula gives a time $t = \sqrt{\frac{2.026}{3.699}}$ s.

ROUND: In this case, all measured values are given to four significant figures, so our final answer has four significant digits. Using the calculator to find the square root gives a time $t = 0.7401$ s.

DOUBLE-CHECK: First note that the answer seems reasonable. The rock is not dropped from an extreme height, so it makes sense that it would take less than one second to fall to the Martian surface. To check the answer by working backwards, first note that the velocity of the rock at time t is given by the equation $v_y = v_{y0} - gt = 0 - gt = -gt$ in this problem. Plug this and the value $v_{y0} = 0$ into the equation to find the average velocity $\bar{v}_y = \frac{1}{2}(v_y + 0) = \frac{1}{2}(-gt)$. Combining this with the expression for position gives:

$$\begin{aligned} y &= y_0 + \bar{v}_y t \\ &= y_0 + \left(\frac{1}{2}(-gt)\right)t \end{aligned}$$

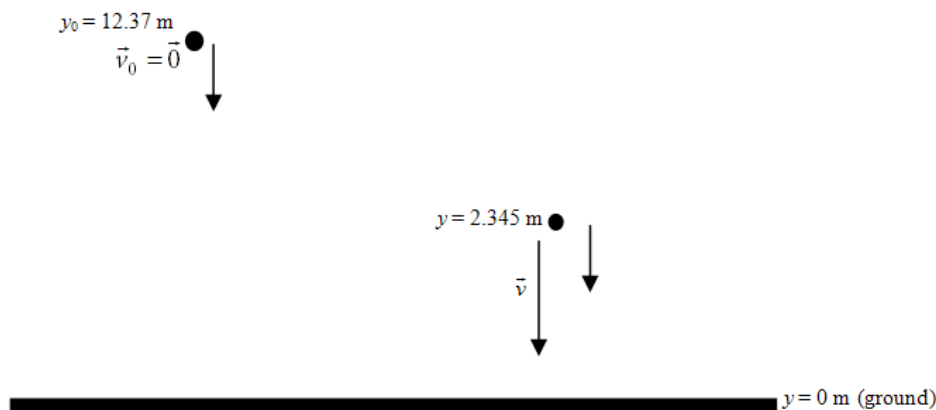
Using the fact that the rock was dropped from a height of $y_0 = 1.013$ m and that the gravitational acceleration on Mars is $g = 3.699$ m/s², it is possible to confirm that the height of the rock at time $t = 0.7401$ s is $y = 1.013 + \frac{1}{2}(-3.699)(0.7401^2) = 0$, which confirms the answer.

- 2.107.** The time that the rock takes to fall is related to the distance it falls by $y = \frac{1}{2}gt^2$.

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(3.699 \text{ m/s}^2)(0.8198 \text{ s})^2 = 1.243 \text{ m}$$

- 2.108. THINK:** Since the ball is dropped from a fixed height with no initial velocity and allowed to fall freely, this question involves free fall. It is necessary to impose a coordinate system. Choose $y = 0$ to represent the ground. Let $t_0 = 0$ be the time when the ball is released from height $y_0 = 12.37 \text{ m}$ and t_1 be the time the ball reaches height $y_1 = 2.345 \text{ m}$.

SKETCH: Sketch the ball when it is dropped and when it is at height 2.345 m.



RESEARCH: Equations (2.25) are used for objects in free fall. Since the ball is released with no initial velocity, we know that $v_{0y} = 0$. We also know that on Earth, the gravitational acceleration is 9.81 m/s^2 . In this problem, it is necessary to find the time that the ball reaches 2.345 m and find the velocity at that time. This can be done using equations (2.25) part (i) and (iii):

$$(i) \quad y = y_0 - \frac{1}{2}gt^2$$

$$(iii) \quad v_y = -gt$$

SIMPLIFY: We use algebra to find the time t_1 at which the ball will reach height $y_1 = 2.345 \text{ m}$ in terms of the initial height y_0 and gravitational acceleration g :

$$\begin{aligned} y_1 &= y_0 - \frac{1}{2}g(t_1)^2 \Rightarrow \\ \frac{1}{2}g(t_1)^2 &= y_0 - y_1 \Rightarrow \\ (t_1)^2 &= \frac{2}{g}(y_0 - y_1) \Rightarrow \\ t_1 &= \sqrt{\frac{2}{g}(y_0 - y_1)} \end{aligned}$$

Combining this with the equation for velocity gives $v_{y1} = -gt_1 = -g\sqrt{\frac{2}{g}(y_0 - y_1)}$.

CALCULATE: The ball is dropped from an initial height of 12.37 m above the ground, and we want to know the speed when it reaches 2.345 m above the ground, so the ball is dropped from an initial height of 12.37 m above the ground, and we want to know the speed when it reaches 2.345 m above the ground, so $y_0 = 12.37$ and $y_1 = 2.345 \text{ m}$. Use this to calculate $v_{y1} = -9.81\sqrt{\frac{2}{9.81}(12.37 - 2.345)} \text{ m/s}$.

ROUND: The heights above ground (12.37 and 2.345) have four significant figures, so the final answer should be rounded to four significant figures. The speed of the ball at time t_1 is then $-9.81\sqrt{\frac{2}{9.81}(12.37 - 2.345)} = -14.02 \text{ m/s}$. The velocity of the ball when it reaches a height of 2.345 m above the ground is 14.02 m/s towards the ground.

DOUBLE-CHECK: To double check that the ball is going 14.02 m/s towards the ground, we use equation (2.25) (v) to work backwards and find the ball's height when the velocity is 14.02 m/s. We know that:

$$\begin{aligned}
 v_y^2 &= v_{y0}^2 - 2g(y - y_0) \Rightarrow \\
 v_y^2 &= 0^2 - 2g(y - y_0) = -2g(y - y_0) \Rightarrow \\
 \frac{v_y^2}{-2g} &= y - y_0 \Rightarrow \\
 \frac{v_y^2}{-2g} + y_0 &= y
 \end{aligned}$$

We take the gravitational acceleration $g = 9.81 \text{ m/s}^2$ and the initial height $y_0 = 12.37 \text{ m}$, and solve for y

when $v_y = -14.02 \text{ m/s}$. Then $y = \frac{v_y^2}{-2g} + y_0 = \frac{(-14.02)^2}{-2(9.81)} + 12.37 = 2.352 \text{ m}$ above the ground. Though this

doesn't match the question exactly, it is off by less than 4 mm, so we are very close to the given value. In fact, if we keep the full accuracy of our calculation without rounding, we get that the ball reaches a velocity of $14.0246\dots \text{ m/s}$ towards the ground at a height of 2.345 m above the ground.

- 2.109.** Using the results noted in the double-check step of the preceding problem,

$$\begin{aligned}
 y_0 - y &= \frac{v^2}{2g} \\
 y &= y_0 - \frac{v^2}{2g} = 13.51 \text{ m} - \frac{(14.787 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 2.37 \text{ m}.
 \end{aligned}$$

By the rule for subtraction, the result is significant to the hundredths place.

- 2.110.** Again using the results from the double-check step of the earlier problem,

$$\begin{aligned}
 y_0 - y &= \frac{v^2}{2g} \\
 y_0 &= y + \frac{v^2}{2g} = 2.387 \text{ m} + \frac{(15.524 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 14.670 \text{ m}.
 \end{aligned}$$

Note that if the value of g is treated as exact, by the addition rule the result has five significant figures.

Chapter 3: Motion in Two and Three Dimensions

Concept Checks

3.1. b 3.2. a 3.3. b, f 3.4. d 3.5. b 3.6. a 3.7. b 3.8. b

Multiple-Choice Questions

3.1. c 3.2. d 3.3. d 3.4. d 3.5. c 3.6. d 3.7. a 3.8. c 3.9. a 3.10. c 3.11. a 3.12. d 3.13. a 3.14. b 3.15. a

Conceptual Questions

- 3.16. For ideal projectile motion, both a_x and a_y are constant and a_x is in fact zero since no horizontal force acts on the projectile. This results in v_x remaining constant since there is no horizontal acceleration. From the same logic, since a_y is a non-zero constant ($a_y = -9.81 \text{ m/s}^2$), v_y does not remain constant. Both x and y do not remain constant when the angle is between 0° and 90° .
- 3.17. If air resistance is neglected, the ball will land back in the passenger's hands since both the ball and the passenger have the same speed in the x -direction as the ball leaves the passenger's hand. If the train is accelerating, the answer does change. After the ball leaves the passenger's hand, it no longer has the same acceleration as the train and passenger in the x -direction and as result lands behind the passenger.
- 3.18. Even though the rock was thrown, this would only add initial velocity to the rock. After leaving the thrower's grasp the only force acting on the rock is due to gravity. Therefore, according to Newton's second law, the rock's acceleration is identical to the acceleration due to gravity.
- 3.19. Since the balls all start with an initial vertical velocity of zero ($v_y = 0$) and acceleration due to gravity is constant, they all take the same amount of time to reach the ground.
- 3.20. The maximum height is given by $y_{\max} = \frac{(-v_0^2 \sin^2 \theta)}{g}$. In order to maximize the height you want $\sin \theta = 1$, which occurs when $\theta = 90^\circ$.
- 3.21. (a) Neglecting air resistance, the package travels at horizontal speed v_x . Since the package and the plane travel at the same horizontal velocity, they will both have traveled the same horizontal distance when the package hits the lake. The distance then between the package and the plane is the altitude h .
 (b) When the package hits the lake, the horizontal component of the velocity vector remains v_x .
 (c) The vertical component of the package's velocity is

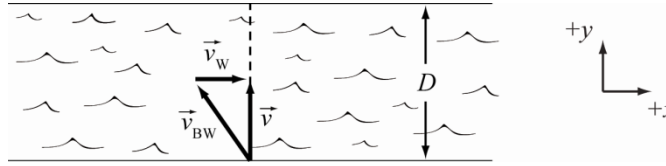
$$v_y = gt = \sqrt{2hg}, \quad \text{since } h = \frac{1}{2}gt^2, \text{ and } t = \sqrt{\frac{2h}{g}}.$$

The speed s is given by $s = \sqrt{v_x^2 + v_y^2} = \sqrt{v_x^2 + 2hg}$.

- 3.22. First the answer for vacuum: Here, it is possible to simply apply ideal trajectory considerations, which neglect air resistance. The cannonballs are launched with the same velocity at the same angle, and the equation for the range does not depend on mass; therefore the material of the cannon balls could not be distinguished based on trajectories or range. However, in the presence of air resistance one can distinguish between the lead and the wood ball. While the force of air friction is the same on both balls, the lead ball has a much greater mass and thus a much smaller acceleration due to air resistance than the wood ball. Thus the lead cannonball is the one which deviates less from the ideal projectile motion trajectory.

3.23. In order to minimize impact, speed should be minimized. The person should jump in the opposite direction of the vehicle's motion. This will reduce the magnitude of v_x and thus the impact is reduced.

3.24. (a)



The velocity of the boat will be $v = \sqrt{v_{BW}^2 - v_W^2}$ by the Pythagorean Theorem. The time is then equal to:

$$t = \frac{d}{v} = \frac{2D}{\sqrt{v_{BW}^2 - v_W^2}}.$$

(b)



For the first part of the trip the velocity is $v_1 = v_{BW} + v_W$, while on the way back the velocity

is $v_2 = v_{BW} - v_W$, so the time of the trip is $t = \frac{D}{v_1} + \frac{D}{v_2} = \frac{D}{v_{BW} + v_W} + \frac{D}{v_{BW} - v_W}$

$$= \frac{D(v_{BW} - v_W)}{(v_{BW} + v_W)(v_{BW} - v_W)} + \frac{D(v_{BW} + v_W)}{(v_{BW} + v_W)(v_{BW} - v_W)} = \frac{D(v_{BW} - v_W) + D(v_{BW} + v_W)}{v_{BW}^2 - v_W^2} = \frac{2Dv_{BW}}{v_{BW}^2 - v_W^2}.$$

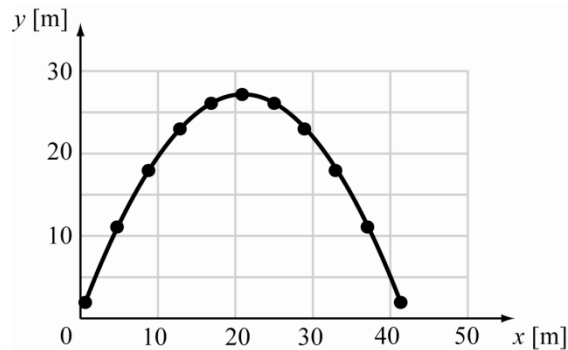
3.25. $x = x_0 + v_x t$; $y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$. a_y can be found by calculating the slope of the v_y versus time graph.

$$a_y = \frac{v_f - v_i}{t_f - t_i} = \frac{-10 \text{ m/s} - 10 \text{ m/s}}{10 \text{ s} - 0 \text{ s}} = -2 \text{ m/s}^2$$

Now, use the equations with values to get different points.

$$x = 1 \text{ m} + 4t \text{ m/s} \text{ and } y = 2 \text{ m} + 10t \text{ m/s} + \frac{1}{2}(-2 \text{ m/s}^2)t^2$$

$t(\text{s})$	$x(\text{m})$	$y(\text{m})$
0	1	2
1	5	11
2	9	18
3	13	23
4	17	26
5	21	27
6	25	26
7	29	23
8	33	18
9	37	11
10	41	2



- 3.26. The projection of the object's trajectory onto the xy -plane is uniform linear motion along the diagonal ($x = y$). The projection onto the z -axis has a motion that is parabolic as a function of time. The components of the velocity are $v_x(t) = \sqrt{2}/2$, $v_y(t) = \sqrt{2}/2$ and $v_z(t) = \sqrt{3} - 9.8t$, since the velocity is the time derivative of position. The components of the acceleration are the time derivatives of velocity: $a_x = 0$, $a_y = 0$, $a_z = -9.8$. This represents projectile motion in the vertical plane that bisects the xy -plane, with gravity acting in the $-z$ direction.

- 3.27. Differentiate the position components with respect to time to determine the velocity components.

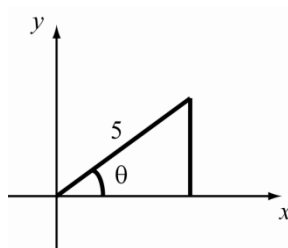
$$v_x(t) = \frac{dx(t)}{dt} = \frac{d}{dt}(4.9t^2 + 2t + 1) = 9.8t + 2, \quad v_y(t) = \frac{dy(t)}{dt} = \frac{d}{dt}(3t + 2) = 3$$

From these equations, differentiate again with respect to time to find the acceleration components.

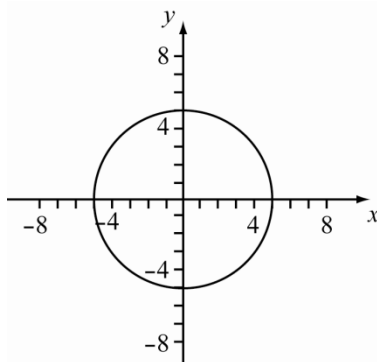
$$a_x(t) = \frac{dv_x(t)}{dt} = \frac{d}{dt}(9.8t + 2) = 9.8, \quad a_y(t) = \frac{dv_y(t)}{dt} = \frac{d}{dt}(3) = 0$$

The acceleration vector at $t = 2$ s is 9.8 m/s^2 in the x -direction.

- 3.28. (a) Observe the following sketch.



From this sketch, it can be seen that as θ is increased, a circle of radius 5 is drawn.

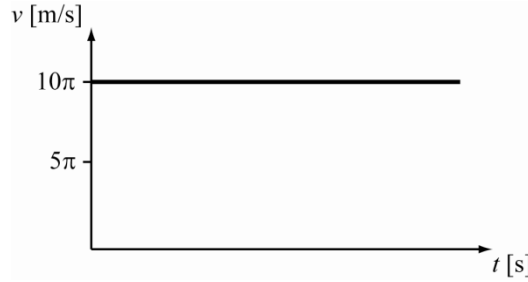


- (b) Differentiate $x(t)$ and $y(t)$ with respect to time to find the velocity components.

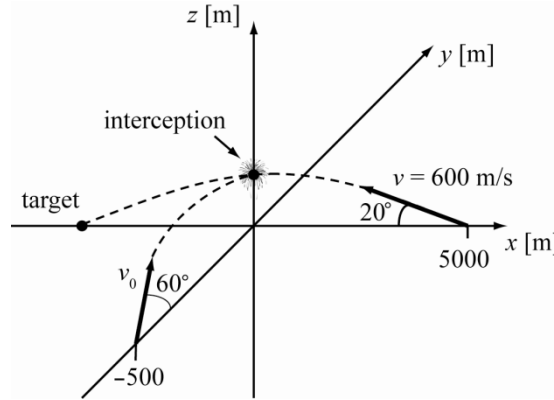
$$v_x(t) = \frac{dx(t)}{dt} = \frac{d}{dt}(5 \cos 2t) = -10 \sin 2t, \quad v_y(t) = \frac{dy(t)}{dt} = \frac{d}{dt}(5 \sin 2t) = 10 \cos 2t$$

(c) The particle's speed is given by:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10\pi)^2 \sin^2(2\pi) + (10\pi)^2 \cos^2(2\pi)} = 10\pi \sqrt{\sin^2 2\pi + \cos^2 2\pi} = 10\pi \sqrt{1} = 10\pi$$



3.29. A diagram of the situation is shown.



When the missile reaches the z -axis, it has traveled for $t = \frac{x_0}{v_0 \cos \theta_0}$. This means it has attained a height of:

$$z_m = v_0 \sin \theta_0 t + \frac{1}{2} a t^2 = \frac{v_0 \sin \theta_0 x_0}{v_0 \cos \theta_0} - \frac{g x_0^2}{2 v_0^2 \cos^2 \theta_0}$$

Inserting values, it is determined that $t = 8.868$ s and $z_m = 1434$ m. Also:

$$z_r = y_0 \tan \alpha_0 - \frac{g y_0^2}{2 v_r^2 \cos^2 \alpha_0} \Rightarrow v_r = \sqrt{\frac{-g y_0^2}{2 \cos^2 \alpha_0 (z_r - |y_0| \tan \alpha_0)}}$$

For the rocket to collide with the missile, $z_r = z_m$. This results in an imaginary number for v_r because $z_r > |y_0| \tan \alpha_r$ and $g = 9.81 \text{ m/s}^2$. This means that there is no speed that will intercept the missile at this distance and angle.

3.30. The range is given by $x = -\frac{(2v_0^2 \cos^2 \theta \sin \theta)}{g}$. The height is given by $y = -\frac{(v_0^2 \sin^2 \theta)}{2g}$. Note

that $\cos 45^\circ = \sin 45^\circ = 1/\sqrt{2}$.

$$x = -\frac{\left[2v_0^2 \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}\right)\right]}{g} = -\frac{v_0^2}{g}; y = -\frac{\frac{1}{2}v_0^2}{2g} = -\frac{v_0^2}{4g} \Rightarrow \frac{x}{y} = \frac{\left(\frac{-v_0^2}{g}\right)}{\left(\frac{-v_0^2}{4g}\right)} = 4.$$

The ratio is not dependent on the initial velocity so the answer does not change when the speed is doubled.

3.31. (a) The range is given by $x = -\frac{(2v_0^2 \cos\theta \sin\theta)}{g}$. The height is given by $y = -\frac{(v_0^2 \sin^2\theta)}{2g}$.

$$-\frac{2v_0^2}{g} \cos\theta \sin\theta = -\frac{v_0^2}{2g} \sin^2\theta \Rightarrow 4 = \frac{\sin\theta}{\cos\theta} \Rightarrow 4 = \tan\theta \Rightarrow \theta = \tan^{-1}(4) = 75.96^\circ$$

The launch angle should be 75.96° .

(b) The range is given by $x = -\frac{(2v_0^2 \cos\theta \sin\theta)}{g}$. Half this range is given by:

$$\frac{x_0}{2} = -\frac{v_0^2}{g} \cos\theta_0 \sin\theta_0 = -\frac{2v_0^2}{g} \cos\theta \sin\theta,$$

since $(1/2)\cos\theta_0 \sin\theta_0 = \cos\theta \sin\theta$. Using the trigonometric identity $\sin 2x = 2\sin x \cos x$:

$$\frac{1}{4} \sin 2\theta_0 = \frac{1}{2} \sin 2\theta$$

$$\sin 2\theta_0 = 2 \sin 2\theta$$

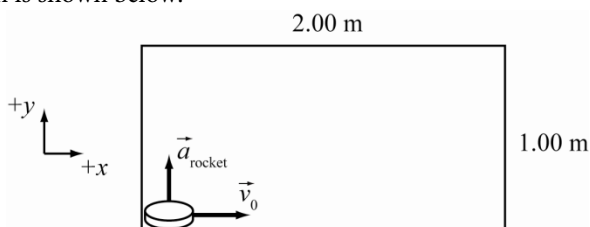
$$\theta_0 = \frac{1}{2} \sin^{-1}(2 \sin 2\theta)$$

$$\theta_0 = \frac{1}{2} \sin^{-1}[2 \sin(2 \tan^{-1} 4)]$$

$$\theta_0 = 35.13^\circ$$

For the range to be half of what it was in part (a), the angle should be 35.13° .

3.32. A diagram of the situation is shown below.



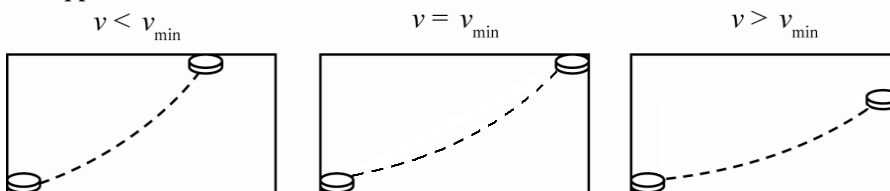
For the original velocity to be the minimum so that the puck does not hit the long side, it must reach the end of the table in the time it takes the puck to cross the short side. The x and y components can be considered separately, since the acceleration acts only in the y direction. The time it takes the rocket to push the puck across the width of the table is:

$$y_f = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2y_f}{a}}$$

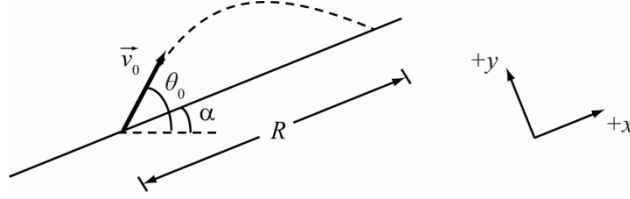
Since there is no acceleration in the x -direction,

$$x_f = v_x t \Rightarrow v_x = \frac{x_f}{t} = \frac{x_f}{\sqrt{2y_f/a}} = \frac{2.00}{\sqrt{2 \cdot 1.00/2.00}} = 2.00 \text{ m/s}$$

The trajectory is a parabola. If $v < v_{\min}$, the puck will hit the opposite long side of the table. If $v > v_{\min}$, the puck will hit the opposite short side.



3.33. A diagram of the situation is shown.



The projectile's position is given by $x = v_0 \cos \theta_0 t$ and $y = v_0 \sin \theta_0 t - \frac{1}{2}gt^2$. The possible values of x and y along the slope can be found by $x = R \cos \alpha$ and $y = R \sin \alpha$, where R is the range along the hill. From this, $R \cos \alpha = v_0 \cos \theta_0 t$ and $R \sin \alpha = v_0 \sin \theta_0 t - \frac{1}{2}gt^2$. The two unknowns in these equations are t and R . First, solve for t :

$$R \cos \alpha = v_0 \cos \theta_0 t \Rightarrow t = \frac{R \cos \alpha}{v_0 \cos \theta_0}.$$

Putting this result into the second equation:

$$\begin{aligned} R \sin \alpha &= (v_0 \sin \theta_0) \frac{R \cos \alpha}{v_0 \cos \theta_0} - \frac{1}{2}g \frac{R^2 \cos^2 \alpha}{v_0^2 \cos^2 \theta_0} \\ R \sin \alpha &= \tan \theta_0 R \cos \alpha - \frac{1}{2}g \frac{R^2 \cos^2 \alpha}{v_0^2 \cos^2 \theta_0} \\ 0 &= R \left(\tan \theta_0 \cos \alpha - \sin \alpha - \frac{g \cos^2 \alpha}{2v_0^2 \cos^2 \theta_0} R \right) \end{aligned}$$

The horizontal range should be positive, so $R > 0$. Then:

$$R = \frac{2(\sin \alpha - \tan \theta_0 \cos \alpha)v_0^2 \cos^2 \theta_0}{g \cos^2 \alpha}.$$

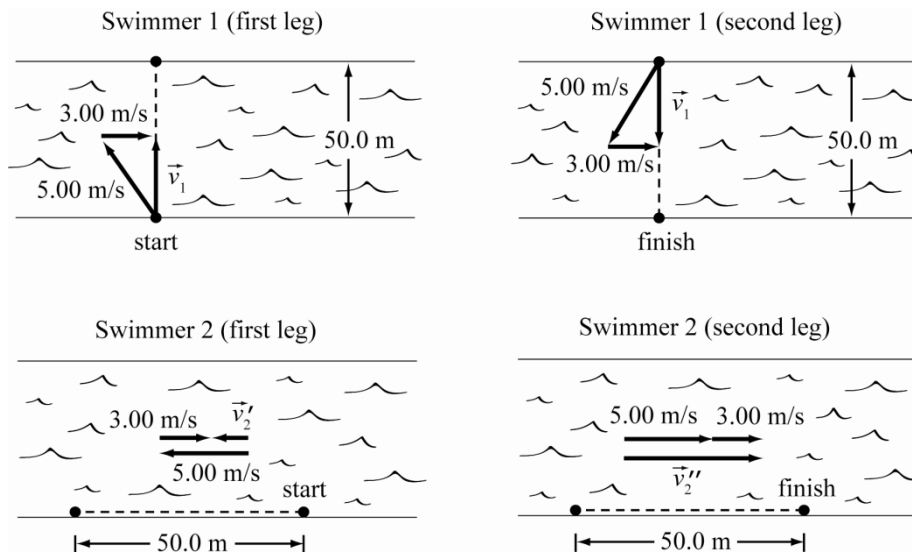
Using the horizontal range equation from the text (equation 3.25), $R' = \frac{v_0^2 \sin(2\theta_0)}{g} = \frac{2v_0^2 \cos \theta_0 \sin \theta_0}{g}$,

the objective is to find a comparative factor β so that $R = R'\beta$.

$$\begin{aligned} R &= \frac{2v_0^2 \cos \theta_0 \sin \theta_0}{g} \cdot \frac{\cos \theta_0}{\sin \theta_0} \cdot \frac{\sin \alpha - \tan \theta_0 \cos \alpha}{\cos^2 \alpha} \\ &= R' \frac{\cot \theta_0}{1} \frac{\tan \alpha - \tan \theta_0}{\cos \alpha} \\ &= R' \frac{\cot \theta_0 \tan \alpha - 1}{\cos \alpha} \end{aligned}$$

Set $\frac{\cot \theta_0 \tan \alpha - 1}{\cos \alpha} \beta = \frac{\cot \theta_0}{1} \frac{\tan \alpha - \tan \theta_0}{\cos \alpha}$ to obtain the comparative factor.

3.34. A diagram of the situation is shown.



The first swimmer crosses at a speed $v_1 = \sqrt{5.00^2 - 3.00^2} = 4.00$ m/s both ways. The total trip time for this swimmer is $t = 2d / v_1 = 2(50.0 \text{ m}) / (4.00 \text{ m/s}) = 25.0$ s. The second swimmer travels at $v_2' = 5 \text{ m/s} - 3 \text{ m/s} = 2 \text{ m/s}$ and $v_2'' = 5 \text{ m/s} + 3 \text{ m/s} = 8 \text{ m/s}$ for the first and second part of the trip, respectively. The total time for the trip is

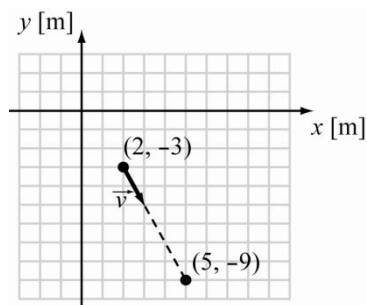
$$t = \frac{d}{v_2'} + \frac{d}{v_2''} = \frac{50.0 \text{ m}}{2.00 \text{ m/s}} + \frac{50.0 \text{ m}}{8.00 \text{ m/s}} = 31.3 \text{ s}.$$

The first swimmer gets back faster.

Exercises

3.35. **THINK:** To calculate the magnitude of the average velocity between $x = 2.0$ m, $y = -3.0$ m and $x = 5.0$ m, $y = -9.0$ m, the distance between these coordinates must be calculated, then the time interval can be used to determine the average velocity.

SKETCH:



RESEARCH: The equation to find the distance is $d = \sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}$. The average velocity is given by $v = d / t$.

SIMPLIFY: $|\vec{v}| = \frac{\sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}}{t}$

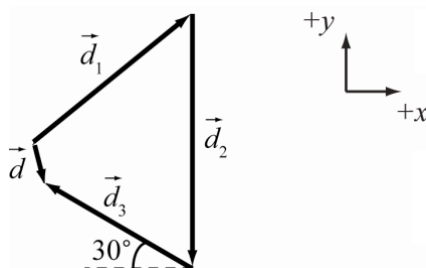
CALCULATE: $|\vec{v}| = \frac{\sqrt{(5.0 \text{ m} - 2.0 \text{ m})^2 + ((-9.0 \text{ m}) - (-3.0 \text{ m}))^2}}{2.4 \text{ s}} = 2.7951 \text{ m/s}$

ROUND: Rounding to two significant figures, $|\vec{v}| = 2.8 \text{ m/s}$.

DOUBLE-CHECK: This result is on the same order of magnitude as the distances and the time interval.

- 3.36. THINK:** The given distances must be converted to SI units; but let's save this until after the calculations. If you convert and round too early, this calculation turns into total nonsense. The magnitude and direction of the total displacement are to be determined.

SKETCH:



RESEARCH: Break the distances into north (+y) and east (+x) components. The displacement is given by $\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$. The magnitude is given by $|\vec{d}| = \sqrt{d_x^2 + d_y^2}$ and the direction is given by $\theta = \tan^{-1}(d_y / d_x)$.

SIMPLIFY: $d_x = d_{1x} + d_{2x} + d_{3x} = d_1 \cos 45^\circ - d_3 \cos 30^\circ = (1/\sqrt{2})d_1 - (\sqrt{3}/2)d_3$

$d_y = d_{1y} + d_{2y} + d_{3y} = d_1 \sin 45^\circ - d_2 + d_3 \sin 30^\circ = (1/\sqrt{2})d_1 - d_2 + (1/2)d_3$

CALCULATE: $d_x = (1/\sqrt{2})(10 \text{ mi}) - (\sqrt{3}/2)(8 \text{ mi}) = 0.14286 \text{ mi}$

$d_y = (1/\sqrt{2})(10 \text{ mi}) - 12 \text{ mi} + (1/2)(8 \text{ mi}) = -0.92893 \text{ mi}$

$|\vec{d}| = \sqrt{(0.14286 \text{ mi})^2 + (0.92893 \text{ mi})^2} = 0.93985 \text{ mi} = 1.5122 \text{ km}$

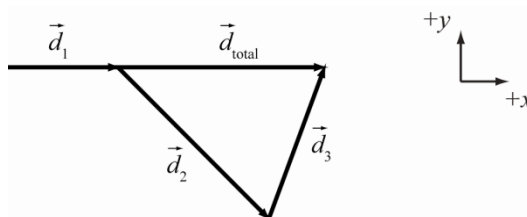
$\theta = \tan^{-1}\left(\frac{-0.92893 \text{ mi}}{0.14286 \text{ mi}}\right) = -81.257^\circ$

ROUND: If you are driving, your odometer usually does not give you the distances to a greater precision than $1/10^{\text{th}}$ of a mile. Rounding to two significant figures, the resultant displacement is 1.5 km. The direction is -81° , or 81° below the positive side of the x-axis.

DOUBLE-CHECK: Our sketch already tells us that the man will end up close to his house and almost exactly south of it, a conclusion which the sketch supports.

- 3.37. THINK:** Determine the third vector $\vec{d}_3$ for a sail boat ride that results in a displacement of $\vec{d}_{\text{total}} = 6.00 \text{ km}$ east when the first two legs of the journey are given as $\vec{d}_1 = 2.00 \text{ km}$ east and $\vec{d}_2 = 4.00 \text{ km}$ southeast.

SKETCH:



RESEARCH: The component representation of vectors can be used to determine the x and y components of each vector: $d_x \neq d \cos$ and $d_y \neq d \sin$. The other equations that can be used are: $\vec{d}_{\text{total}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$;

$$|\vec{d}| = \sqrt{d_x^2 + d_y^2}; \text{ and } \theta = \tan^{-1}(d_y / d_x).$$

SIMPLIFY: $\vec{d}_3 = \vec{d}_{\text{total}} - \vec{d}_1 - \vec{d}_2$

$$d_{3x} = d_{\text{total}x} - d_{1x} - d_{2x} = d_{\text{total}} \cos 0^\circ - d_1 \cos 0^\circ - d_2 \cos 45^\circ = d_{\text{total}} - d_1 - d_2 / \sqrt{2}$$

$$d_{3y} = d_{\text{total}y} - d_{1y} - d_{2y} = d_{\text{total}} \sin 0^\circ - d_1 \sin 0^\circ - d_2 \sin 45^\circ = 0 - 0 + d_2 / \sqrt{2} = d_2 / \sqrt{2}$$

$$|\vec{d}_3| = \sqrt{(d_{\text{total}} - d_1 - d_2 / \sqrt{2})^2 + (d_2 / \sqrt{2})^2}, \quad \theta = \tan^{-1}\left(\frac{d_2 / \sqrt{2}}{d_{\text{total}} - d_1 - d_2 / \sqrt{2}}\right)$$

CALCULATE: $|\vec{d}_3| = \sqrt{(6.00 \text{ km} - 2.00 \text{ km} - 4.00 \text{ km} / \sqrt{2})^2 + (4.00 \text{ km} / \sqrt{2})^2} = 3.0615 \text{ km}$

$$\theta = \tan^{-1}\left(\frac{4.00 \text{ km} / \sqrt{2}}{6.00 \text{ km} - 2.00 \text{ km} - 4.00 \text{ km} / \sqrt{2}}\right) = 67.500^\circ$$

ROUND: Rounding to three significant figures, $|\vec{d}_3| = 3.06 \text{ km}$ and $\theta = 67.5^\circ$. The missing part of the trip was 3.06 km 67.5° North of East.

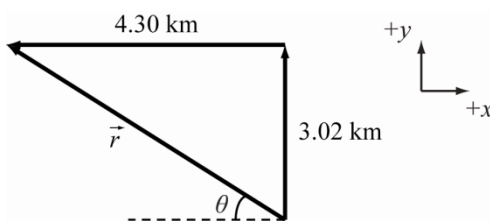
DOUBLE-CHECK: The result is on the same order of magnitude as the other parts of the trip and by looking at the sketch, the angle is reasonable.

3.38. THINK:

(a) The net displacement vector must be found for a truck that drives 3.02 km north then drives another 4.30 km west.

(b) Determine the average velocity of the trip if it takes 5.00 minutes to complete. Convert the time to seconds: $t = 5.00 \text{ min}(60 \text{ s/min}) = 300. \text{ s}$.

SKETCH:



RESEARCH:

(a) The displacement is given by $\vec{r} = (x, y)$.

(b) The magnitude of the average velocity is $|\vec{v}| = d / t$, where $d = |\vec{r}| = \sqrt{x^2 + y^2}$.

SIMPLIFY:

(a) It is not necessary to simplify.

$$(b) |\vec{v}| = \frac{d}{t} = \frac{\sqrt{x^2 + y^2}}{t}$$

CALCULATE:

$$(a) \vec{r} = (-4.30 \text{ km}, 3.02 \text{ km})$$

$$(b) |\vec{v}| = \frac{\sqrt{(-4.30)^2 + (3.02)^2}}{300. \text{ s}} = 0.017515 \text{ km/s}$$

ROUND:

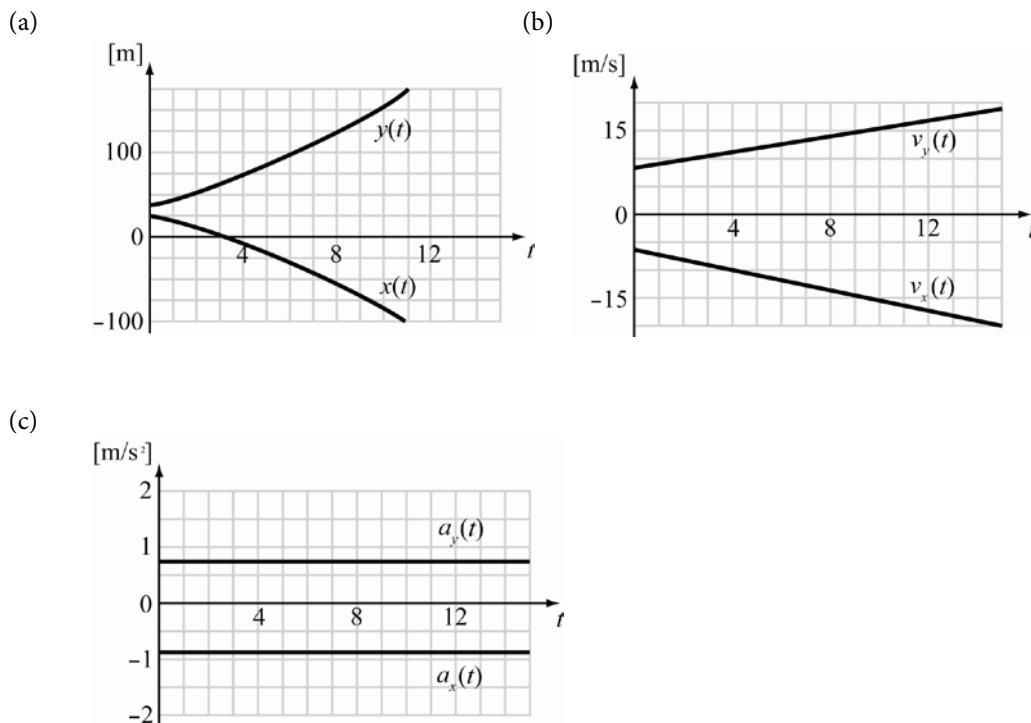
(a) It is not necessary to round since the data is simply restated.

(b) Rounding to three significant figure, $|\vec{v}| = 0.0175 \text{ km/s}$.

DOUBLE-CHECK:

- (a) The distance is reasonable for a truck to travel.
 (b) This speed corresponds to 63 km/h or 39 mph, which is reasonable for a truck.

- 3.39. **THINK:** The position components $x(t) = -0.45t^2 - 6.5t + 25$ and $y(t) = 0.35t^2 + 8.3t + 34$ can be used to find the magnitude and direction of the position at $t = 10.0$ s. The velocity and acceleration at $t = 10.0$ s must then be determined.

SKETCH:**RESEARCH:**

- (a) Insert $t = 10$ s into the given equations, then use $|\vec{r}| = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$.
 (b) Differentiate the given components with respect to time to get functions of velocity.
 (c) Differentiate the velocity functions with respect to time to get functions of acceleration.

SIMPLIFY:

- (a) It is not necessary to simplify.

$$(b) v_x(t) = \frac{dx(t)}{dt} = \frac{d(-0.45t^2 - 6.5t + 25)}{dt} = (-0.90t - 6.5) \text{ m/s}$$

$$v_y(t) = \frac{dy(t)}{dt} = \frac{d(0.35t^2 + 8.3t + 34)}{dt} = (0.70t + 8.3) \text{ m/s}$$

$$(c) a_x(t) = \frac{dv_x(t)}{dt} = \frac{d(-0.90t - 6.5)}{dt} = -0.90 \text{ m/s}^2, \quad a_y(t) = \frac{dv_y(t)}{dt} = \frac{d(0.70t + 8.3)}{dt} = 0.70 \text{ m/s}^2$$

CALCULATE:

$$(a) x(10.0) = -0.45(10.0)^2 - 6.5(10.0) + 25 = -85 \text{ m}, \quad y(10.0) = 0.35(10.0)^2 + 8.3(10.0) + 34 = 152 \text{ m}$$

Now, insert these values into the magnitude and distance equations:

$$|\vec{r}| = \sqrt{(-85 \text{ m})^2 + (152 \text{ m})^2} = 174 \text{ m}, \quad \theta = \tan^{-1}\left(\frac{152}{-85}\right) = -60.786^\circ$$

(b) $v_x(10.0) = -0.90(10.0) - 6.5 = -15.5 \text{ m/s}$, $v_y(10.0) = 0.70(10.0) + 8.3 = 15.3 \text{ m/s}$

$$|\vec{v}| = \sqrt{(-15.5)^2 + (15.3)^2} = 21.8 \text{ m/s}, \quad \theta = \tan^{-1}\left(\frac{15.3}{-15.5}\right) = -44.6^\circ$$

(c) Since there is no time dependence, the acceleration is always $\vec{a} = (a_x, a_y) = (-0.90 \text{ m/s}^2, 0.70 \text{ m/s}^2)$.

The $|\vec{a}| = \sqrt{(-0.90 \text{ m/s}^2)^2 + (0.70 \text{ m/s}^2)^2} = 1.140 \text{ m/s}^2$, $\theta = \tan^{-1}\left(\frac{0.70}{-0.90}\right) = -37.87^\circ$.

ROUND:

(a) Both distances and the magnitude are accurate to the meter, $|\vec{r}| = 174 \text{ m}$. Round the angle to three significant figures, 60.8° north of west (note: west is used because x was negative).

(b) The equation's parameters are accurate to a tenth of a meter. The rabbit's velocity is then 21.8 m/s , 44.6° north of west.

(c) It is not necessary to consider the significant figures since the original parameters of the function are used. The rabbit's velocity is 1.14 m/s^2 , 37.9° north of west.

DOUBLE-CHECK:

(a) 174 m in 10 s seems reasonable for a rabbit, considering the world record for the 100 m dash is about 10 s .

(b) The velocity of a rabbit ranges from 12 m/s to 20 m/s . This rabbit would be at the top of that range.

(c) A rabbit may accelerate at this rate but it can not sustain this acceleration for too long.

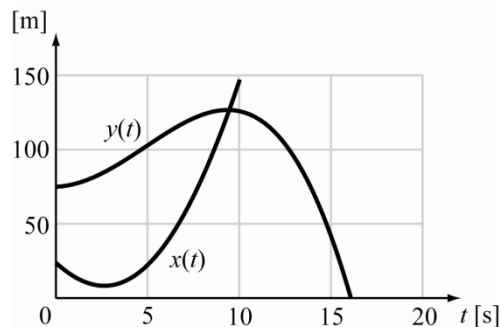
3.40. THINK: (a) The position vector is given. The distance the car is away from the origin at $t = 5.00 \text{ s}$ is to be determined.

(b) Now, the velocity vector for the car is to be determined.

(c) The speed (magnitude of the velocity) is to be determined at 5 s .

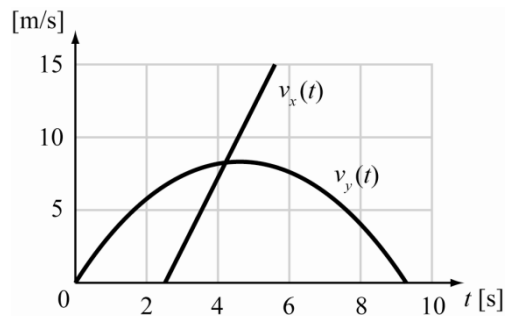
SKETCH:

(a)



(b) See sketch above.

(c)



RESEARCH:

(a) The time can be inserted into the position vector and the magnitude can then be found using $d = |\vec{r}| = \sqrt{x^2 + y^2}$.

(b) The velocity vector is given by derivative of the position vector with respect to time, $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$.

(c) To determine the speed, use the magnitude equation $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$.

SIMPLIFY:

(a) It is not necessary to simplify.

$$(b) \quad \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{d}{dt} \left[\left[24.4 - 12.3t + 2.43t^2 \right], \frac{d}{dt} \left[74.4 + 1.80t^2 - 0.130t^3 \right] \right] \\ = (-12.3 \text{ m/s} + 4.86t \text{ m/s}^2, 3.60t \text{ m/s}^2 - 0.390t^2 \text{ m/s}^3)$$

(c) It is not necessary to simplify.

CALCULATE:

$$(a) \quad \vec{r}(5) = (24.4 \text{ m}) - 5(12.3 \text{ m/s}) + (5)^2(2.43 \text{ m/s}^2), (74.4 \text{ m}) + (5)^2(1.80 \text{ m/s}^2) - (5)^3(0.130 \text{ m/s}^3) \\ = (23.65 \text{ m}, 103.150 \text{ m})$$

Now, insert the components into the equation for the magnitude:

$$d = |\vec{r}| = \sqrt{(23.65 \text{ m})^2 + (103.150 \text{ m})^2} = 105.8265 \text{ m}.$$

(b) There are no other calculations to be done.

$$(c) \quad \vec{v}(5) = (-12.3 \text{ m/s} + 4.86(5) \text{ m/s}^2, 3.60(5) \text{ m/s}^2 - 0.390(5)^2 \text{ m/s}^3) = (12.0 \text{ m/s}, 8.25 \text{ m/s})$$

$$|\vec{v}| = \sqrt{(12.0 \text{ m/s})^2 + (8.25 \text{ m/s})^2} = 14.5624 \text{ m/s}$$

ROUND:

(a) The equation is accurate to four significant figures so the distance is now $d = 105.8 \text{ m}$.

(b) The significant figures remain the same for the parameters.

(c) Rounding to the first place after the decimal, $|\vec{v}| = 14.6 \text{ m/s}$.

DOUBLE-CHECK:

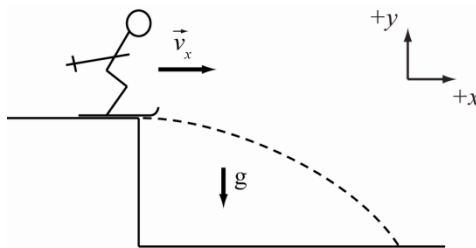
(a) The distance is reasonable for a car to travel.

(b) The derivatives were done correctly.

(c) The speed is reasonable for a car.

- 3.41. **THINK:** Ignoring air resistance, the skier's horizontal velocity will remain unchanged, while her vertical velocity is influenced solely by gravity. $v_x = 30.0 \text{ m/s}$, $g = 9.81 \text{ m/s}^2$ and $t = 2.00 \text{ s}$.

SKETCH:



RESEARCH: $v_{ix} = v_{fx}$ and $v_{fy} = v_{iy} + at$.

SIMPLIFY: $v_{fy} = 0 - gt = -gt$

CALCULATE: $v_{fx} = 30.0 \text{ m/s}$ and $v_{fy} = -(9.81 \text{ m/s}^2)(2.00 \text{ s}) = -19.62 \text{ m/s}$.

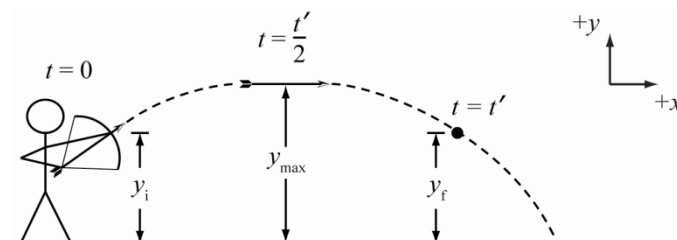
ROUND: $|v_{ix}| = 30.0 \text{ m/s}$ and $|v_{iy}| = 19.6 \text{ m/s}$.

DOUBLE-CHECK: The order of magnitude is reasonable.

- 3.42. THINK:** When the arrow is horizontal it is at its maximum height. This occurs when the vertical velocity is zero. The time to reach maximum height is half the time it takes to fall back to the same height.

$y_0 = 1.14 \text{ m}$, $v_0 = 47.5 \text{ m/s}$, $\theta = 35.2^\circ$, and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $y - y_0 = v_{y0}t + \frac{1}{2}at^2$; $v_{y0} = v_0 \sin \theta$

SIMPLIFY: For $y = y_0$, use $t = t'$:

$$0 = v_0 \sin \theta - \frac{1}{2} g \left(\frac{t'}{2} \right)^2.$$

Since $t' \neq 0$, $t' = \frac{2v_0 \sin \theta}{g}$. Therefore, at the maximum height, $t = \frac{t'}{2} = \frac{v_0 \sin \theta}{g}$.

CALCULATE: $t = \frac{(47.5 \text{ m/s}) \sin(35.2^\circ)}{9.81 \text{ m/s}^2} = 2.7911 \text{ s}$

ROUND: To three significant figures, $t = 2.79 \text{ s}$.

DOUBLE-CHECK: The arrow is horizontal when its vertical velocity is zero:

$$v_y = 0 = v_{y0} + at = v_0 \sin \theta - \frac{1}{2} g t' \Rightarrow t' = \frac{2v_0 \sin \theta}{g}$$

This is the same as the result obtained above.

- 3.43. THINK:** Assume the ball starts on the ground so that the initial and final heights are the same. The initial velocity of the ball is $v_i = 27.5 \text{ m/s}$, with $\theta = 56.7^\circ$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$ and $v_{iy} = v_i \sin \theta$.

SIMPLIFY: $0 = v_i \sin \theta - \frac{1}{2} g t^2 \Rightarrow t = \frac{2v_i \sin \theta}{g}$

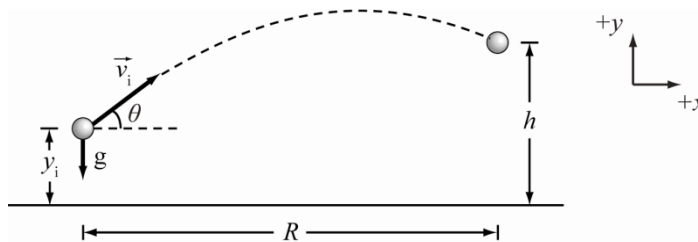
CALCULATE: $t = \frac{2(27.5 \text{ m/s}) \sin(56.7^\circ)}{9.81 \text{ m/s}^2} = 4.6860 \text{ s}$

ROUND: Rounding to three significant figures, $t = 4.69 \text{ s}$.

DOUBLE-CHECK: Given the large angle the ball was kicked, about 5 seconds is a reasonable amount of time for it to remain in the air.

- 3.44. **THINK:** Use the horizontal distance and velocity to determine the time it takes for the ball to reach the net. Then, use this time to determine the vertical height of the ball. Use this vertical height to determine if the ball clears the net and by how much it does or does not.

SKETCH:



RESEARCH: $v_{ix} = v_i \cos \theta$; $v_{iy} = v_i \sin \theta$; $R = v_{ix} t$; and $y_f - y_i = v_{iy} t + \frac{1}{2} a t^2$.

SIMPLIFY: $t = \frac{R}{v_i \cos \theta}$ and $y_f = y_i + (v_i \sin \theta) t - \frac{1}{2} g t^2$; $y_f = y_i + R \tan \theta - \frac{g R^2}{2 v_i^2 \cos^2 \theta}$. In order to compare the

height of the net, h , subtract h from the final height, y_f : $\Delta h = y_f - h = y_i - h + R \tan \theta - \frac{g R^2}{2 v_i^2 \cos^2 \theta}$.

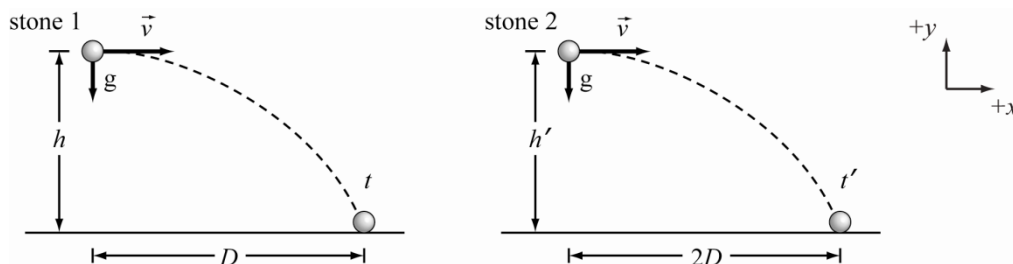
CALCULATE: $\Delta h = (1.8 \text{ m} - 1.07 \text{ m}) + (11.83 \text{ m}) \tan(7.00^\circ) - \frac{(9.81 \text{ m/s}^2)(11.83 \text{ m})^2}{2(18.0 \text{ m/s})^2 \cos^2(7.00^\circ)} = 0.031929 \text{ m}$

ROUND: Rounding the result to two significant figures, the ball clears the net by 3.2 cm.

DOUBLE-CHECK: Given the long distance to the net and the small angle that the ball is hit at, it is reasonable that the ball would clear the net by such a small distance.

- 3.45. **THINK:** Simply find a relation between the height of the building and the distance traveled. With the use of this equation, determine how the height is affected when the distance is doubled. No values are needed.

SKETCH:



RESEARCH: $x_f - x_i = vt$ and $y_f - y_i = v_{iy} t + \frac{1}{2} a t^2$.

SIMPLIFY: $D = vt \Rightarrow t = \frac{D}{v}$, $2D = vt' \Rightarrow t' = \frac{2D}{v}$, $-h = 0 - \frac{1}{2} g t^2 = -\frac{g D^2}{2 v^2}$.
 $-h' = 0 - \frac{1}{2} g (t')^2 = -\frac{4 g D^2}{2 v^2} = -\frac{2 g D^2}{v^2}$

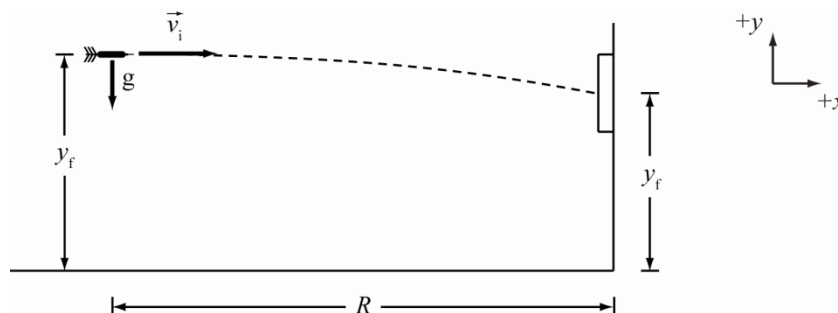
CALCULATE: $\frac{h'}{h} = \frac{(2 g D^2 / v^2)}{(g D^2 / 2 v^2)} = 4$

ROUND: The ratio of the heights of the buildings is 4 to 1.

DOUBLE-CHECK: Seeing as the paths of the stones are parabolic, the ratio of the heights is proportional to the square of the ratio of the distances, so the result makes sense.

- 3.46. THINK:** Assume there is no air resistance and the horizontal velocity remains constant. Also, assume there is no initial vertical velocity. $R = 3.0 \text{ m}$, $y_i = 2.0 \text{ m}$, $y_f = 1.65 \text{ m}$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH:

- (a) $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$ and $v_{iy} = 0$ and $a = -g$.
 (b) $v_{ix} = v_{fx}$ and $R = v_{ix}t$.
 (c) $\vec{v}_f = v_{fx}\hat{x} + v_{fy}\hat{y}$; $v_{fy} = v_{iy} + at$; and $|\vec{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2}$.

SIMPLIFY:

- (a) $y_f - y_i = 0 - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{-2(y_f - y_i)}{g}}$
 (b) $v_i = v_{ix} = \frac{R}{t}$
 (c) $v_{fy} = -gt \Rightarrow \vec{v}_f = v_i\hat{x} - gt\hat{y} \Rightarrow |\vec{v}_f| = \sqrt{v_i^2 + (-gt)^2}$

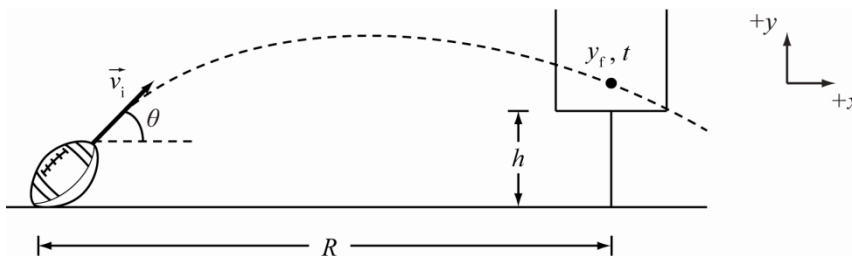
CALCULATE:

- (a) $t = \sqrt{\frac{-2(1.65 \text{ m} - 2.00 \text{ m})}{9.81 \text{ m/s}^2}} = 0.26712 \text{ s}$
 (b) $v_i = \frac{3.0 \text{ m}}{0.26712 \text{ s}} = 11.231 \text{ m/s}$
 (c) $|\vec{v}_f| = \sqrt{(11.231 \text{ m/s})^2 + ((-9.81 \text{ m/s}^2)(0.26712 \text{ s}))^2} = 11.532 \text{ m/s}$

ROUND: Rounding to two significant figures, $t = 0.27 \text{ s}$, $v_i = 11 \text{ m/s}$ and $|\vec{v}_f| = 12 \text{ m/s}$.

DOUBLE-CHECK: The initial and final velocities of the dart are reasonable for a player to achieve. At these velocities, the flight time is also reasonable.

- 3.47. THINK:** Use the horizontal distance and velocity to determine the time it takes to reach the post. Use the time to determine the height of the ball at that point. Then compare the height of the ball to the height of the goal post. Vertical velocity at this point can be determined as well: $v_i = 22.4 \text{ m/s}$, $\theta = 49.0^\circ$, $R = 39.0 \text{ m}$, $h = 3.05 \text{ m}$, $g = 9.81 \text{ m/s}^2$. Assume the ball is kicked off the ground, $y_i = 0 \text{ m}$.

SKETCH:

RESEARCH: $R = v_{ix} t$; $y_f - y_i = v_{iy} t + \frac{1}{2} a t^2$; $v_{fy} = v_{iy} + a t$; $v_{ix} \theta = v_i \cos \theta$; and $v_{iy} \theta = v_i \sin \theta$.

SIMPLIFY:

$$(a) \ t = \frac{R}{v_i \cos \theta} \text{ and } y_f \theta = (v_i \sin \theta) \left(\frac{R}{v_i \cos \theta} \right) - \frac{1}{2} g \left(\frac{R}{v_i \cos \theta} \right)^2 \Rightarrow \theta = \tan^{-1} \left(\frac{g R^2}{2 v_i^2 \cos^2 \theta} \right).$$

In order to compare the height of the ball to the height of the goal post, subtract h from both sides of the equation,

$$\Delta h = y_f \theta - h = -h + R \tan \theta - \frac{g R^2}{2 v_i^2 \cos^2 \theta}.$$

$$(b) \ v_{fy} \theta = v_i \sin \theta - \frac{g R}{v_i \cos \theta}$$

CALCULATE:

$$(a) \ \Delta h = -3.05 \text{ m} + (39.0 \text{ m}) \tan(49.0^\circ) - \frac{(9.81 \text{ m/s}^2)(39.0 \text{ m})^2}{2(22.4 \text{ m/s})^2 \cos^2(49.0^\circ)} = 7.2693 \text{ m}$$

$$(b) \ v_{fy} = (22.4 \text{ m/s}) \sin(49.0^\circ) - \frac{(9.81 \text{ m/s}^2)(39.0 \text{ m})}{(22.4 \text{ m/s}) \cos(49.0^\circ)} = -9.1286 \text{ m/s}$$

ROUND: Round to the appropriate three significant figures:

(a) The ball clears the post by 7.27 m.

(b) The ball is heading downward at 9.13 m/s.

DOUBLE-CHECK: The initial velocity certainly seems high enough to clear the goal post from about 1/3 of the field away. It also makes sense that the vertical velocity at this point is lower than the initial velocity and the ball is heading down.

- 3.48. THINK:** Since the time of the last portion of the flight and the horizontal displacement during that time are given, the x component of the initial velocity can be determined, because the horizontal velocity component remains constant throughout the flight. The initial velocity can then be determined, because we also know the initial angle of $\theta = 35.0^\circ$. The vertical displacement of the projectile during the last flight phase is also given. However, since we do not know the relative altitude of the beginning and end of the trajectory, the vertical displacement provides no useful information and is thus a distractor, which we can and should ignore.

SKETCH: A sketch is not needed in this case.

RESEARCH: $v_{ix} = v_{fx} = v_i \cos \theta$; $v_{ix} = \frac{d}{\Delta t}$

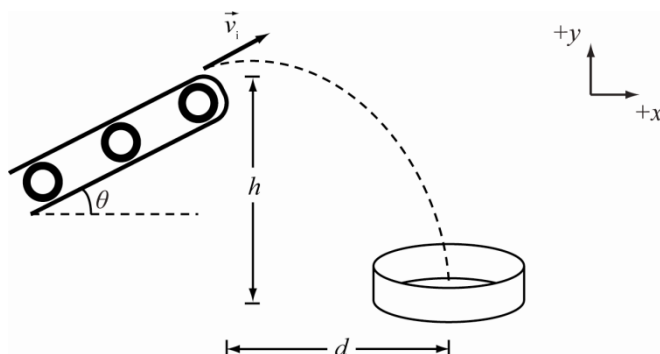
SIMPLIFY: $v_i \cos \theta = \frac{d}{\Delta t} \Rightarrow v_i = \frac{d}{\cos \theta \Delta t}$.

CALCULATE: $v_i = (10.0 \text{ m}) / [\cos(35.0^\circ)(1.50 \text{ s})] = 8.138497 \text{ m/s}$

ROUND: Rounding to three significant figures, $v_i = 8.14 \text{ m/s}$.**DOUBLE-CHECK:** This speed is reasonable.

- 3.49. THINK:** Assume the sand moves as a single object and that the sand is moving fast enough to be projected from the conveyor belt at the top. To solve for the distance, the quadratic equation must be used. $\theta = 14.0^\circ$, $h = 3.00$ m, $v_i = 7.00$ m/s and $g = 9.81$ m/s².

SKETCH:



RESEARCH: $v_{ix}t = d$; $v_{ix} = v_i \cos \theta$; $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$; and $v_{iy} = v_i \sin \theta$.

SIMPLIFY: $v_{ix}t = d \Rightarrow t = \frac{d}{v_i \cos \theta}$

$$-h - v_i \sin \theta t - \frac{1}{2}gt^2 = 0 \Rightarrow \left(\frac{gd^2}{2v_i^2 \cos^2 \theta} \right) + (d \tan \theta) - h = 0, \text{ so}$$

$$d = \frac{\tan \theta \pm \sqrt{\tan^2 \theta + 2\left(gh / (v_i^2 \cos^2 \theta)\right)}}{g / (v_i^2 \cos^2 \theta)}.$$

CALCULATE:

$$d = \frac{\tan(14.0^\circ) \pm \sqrt{\tan^2(14.0^\circ) + 2(9.81 \text{ m/s}^2)(3.00 \text{ m}) / [(7.00 \text{ m/s})^2 \cos^2(14.0^\circ)]}}{9.81 \text{ m/s}^2 [(7.00 \text{ m/s})^2 \cos^2(14.0^\circ)]}$$

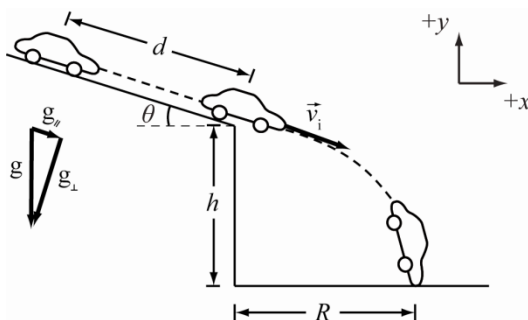
$$= 6.6122 \text{ m or } -4.2672 \text{ m}$$

ROUND: $d = 6.61$ m (the answer must be a positive value)

DOUBLE-CHECK: If launched at $\theta = 0^\circ$, the sand would travel about 6 m, so 7 m for a small angle is reasonable.

- 3.50. THINK:** Since gravity is a vector, it can be broken up into components. This means that a portion of gravity accelerates the car down the incline. Once the car reaches the edge, it is under the whole force of gravity. $g = 9.81$ m/s², $d = 29.0$ m, $\theta = 17.0^\circ$, $h = 55.0$ m. The quadratic equation can be used to solve for distance.

SKETCH:



RESEARCH: $g_{\parallel} = g \sin \theta$; $v_f^2 = v_i^2 + 2ad$; $R = v_{ix} t$; $y_f - y_i = v_{iy} t + \frac{1}{2} g t^2$; $v_{ix} = v_i \cos \theta$; and $v_{iy} = v_i \sin \theta$.

SIMPLIFY: $v_i^2 = 0 + 2g_{\parallel} d = 2(g \sin \theta) d \Rightarrow v_i = \sqrt{2gd \sin \theta}$

$$(a) \quad -h = -v_i (\sin \theta) \left(\frac{R}{v_i \cos \theta} \right) - \frac{1}{2} g \left(\frac{R}{v_i \cos \theta} \right)^2 \quad \left(\text{since } t = \frac{R}{v_i \cos \theta} \right)$$

$$-h = -R \tan \theta - \frac{g R^2}{2 v_i^2 \cos^2 \theta}$$

$$0 = \frac{g R^2}{2 \cos^2 \theta (2 g d \sin \theta)} + R \tan \theta - h \quad \left(\text{since } v_i = \sqrt{2 g d \sin \theta} \right)$$

$$0 = \left(\frac{1}{4 d \cos^2 \theta \sin \theta} \right) R^2 + R \tan \theta - h$$

Therefore, using the quadratic formula, $R = \frac{-\tan \theta \pm \sqrt{\tan^2 \theta + \frac{h}{d \cos^2 \theta \sin \theta}}}{1 / (2 d \cos^2 \theta \sin \theta)}$.

$$(b) \quad t = \frac{R}{v_i \cos \theta} = \frac{R}{\sqrt{2 g d \sin \theta} \cos \theta}$$

CALCULATE:

$$(a) \quad R = \frac{-\tan(17^\circ) \pm \sqrt{\tan^2(17^\circ) + 55.0 \text{ m} / ((29.0 \text{ m}) \cos^2(17^\circ) \sin(17^\circ))}}{1 / (2(29.0 \text{ m}) \cos^2(17^\circ) \sin(17^\circ))}$$

$$= 36.8323 \text{ m or } -46.3148 \text{ m}$$

$$(b) \quad t = \frac{36.8323 \text{ m}}{\left(\sqrt{2(9.81 \text{ m/s}^2)(29.0 \text{ m}) \sin(17^\circ)} \right) \cos(17^\circ)} = 2.9862 \text{ s}$$

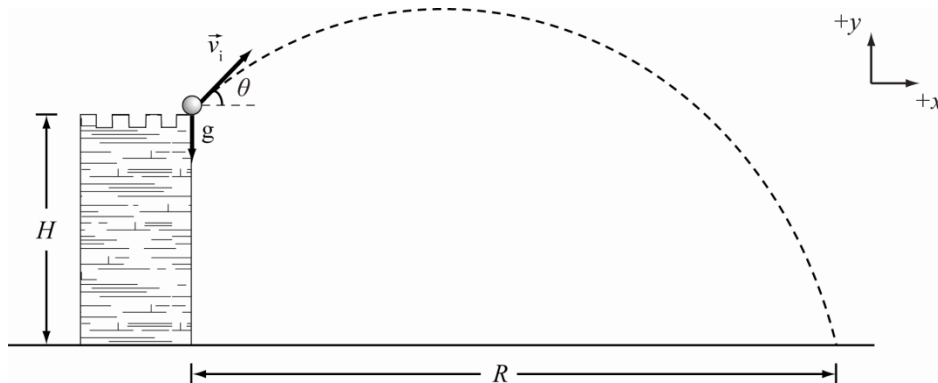
ROUND:

(a) The car falls 36.8 m from the base of the cliff.

(b) The car is in the air for 2.99 seconds.

DOUBLE-CHECK: If the car went over the cliff horizontally, it would travel about 43 m and take about 3.3 seconds to fall. The fact that the car travels a shorter distance in a shorter period of time with a small initial vertical velocity makes sense.

- 3.51. **THINK:** Though not explicitly stated, assume the launch angle is not zero. The projectile's height as a function of time is given. This function can be related to a more general function and the specifics of the motion can be determined. $g = 9.81 \text{ m/s}^2$ and $v_i = 20.0 \text{ m/s}$.

SKETCH:

RESEARCH: $y(t) = -4.90t^2 + 19.32t + 60.0$; $R = v_{ix}t$; $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$; $v_{iy} = v_i \sin \theta$; and $v_{ix} = v_i \cos \theta$.

SIMPLIFY:

(a) $y_f = y_i + v_i \sin \theta t - \frac{1}{2}gt^2$, where $(1/2)g = 4.90 \text{ m/s}^2$, $v_i \sin \theta = 19.32 \text{ m/s}$, and $y_i = 60.0 \text{ m}$.

$$(b) \sin \theta = \frac{19.32 \text{ m/s}}{v_i} \Rightarrow \theta = \sin^{-1} \left(\frac{19.32}{v_i} \right)$$

$$(c) 0 = y_i + v_i \sin \theta t - \frac{1}{2}gt^2 = y_i + R \tan \theta - \frac{gR^2}{2v_i^2 \cos^2 \theta} \quad \left(\text{since } t = \frac{R}{v_i \cos \theta} \right)$$

Therefore, $\left(\frac{g}{2v_i^2 \cos^2 \theta} \right) R^2 - (\tan \theta) R - y_i = 0$. Using the quadratic formula,

$$R = \frac{\tan \theta \pm \sqrt{\tan^2 \theta + \frac{2y_i g}{v_i^2 \cos^2 \theta}}}{g / (v_i^2 \cos^2 \theta)}.$$

CALCULATE:

(a) $y_i = H = 60.0 \text{ m}$

$$(b) \theta = \sin^{-1} \left(\frac{19.32 \text{ m/s}}{20.0 \text{ m/s}} \right) = 75.02^\circ$$

$$(c) R = \frac{\tan(75.02^\circ) \pm \sqrt{\tan^2(75.02^\circ) + \frac{2(60.0 \text{ m})(9.81 \text{ m/s}^2)}{(20.0 \text{ m/s})^2 \cos^2(75.02^\circ)}}}{\frac{9.81 \text{ m/s}^2}{(20.0 \text{ m/s}^2) \cos^2(75.02^\circ)}} = 30.9386 \text{ m or } -10.5715 \text{ m}$$

ROUND:

(a) The building height is 60.0. m.

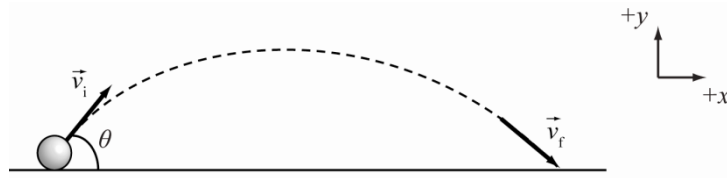
(b) The launch angle is 75.0° .

(c) The object travels 31.0 m (the positive value must be chosen).

DOUBLE-CHECK: Given the large launch angle, it makes sense that the object doesn't travel too far.

- 3.52. **THINK:** The x -component of the velocity will remain unchanged, while the y -component of the velocity will remain the same in magnitude but opposite in direction. $\Delta \vec{v} = -20 \hat{y}$ m/s and $\theta = 60^\circ$.

SKETCH:



RESEARCH: $\Delta \vec{v} = \vec{v}_f - \vec{v}_i$; $\vec{v} = v_x \hat{x} + v_y \hat{y}$; $v_{ix} = v_{fx}$; $v_{iy} = -v_{fy}$; $v_x \neq v \cos$; and $v_y \neq v \sin$.

SIMPLIFY: $\Delta \vec{v} = (v_{fx} \hat{x} + v_{fy} \hat{y}) - (v_{ix} \hat{x} + v_{iy} \hat{y}) = (v_{fx} - v_{ix}) \hat{x} + (v_{fy} - v_{iy}) \hat{y} = -2v_{iy} \hat{y} = -2v_i \sin \theta \hat{y}$

Therefore, $v_i = \frac{\Delta v}{-2 \sin \theta}$ and $v_{ix} = v_i \cos \theta = \frac{\Delta v}{-2 \tan \theta}$.

CALCULATE: $-2v_{iy} = -20$ m/s $\Rightarrow v_{iy} = 10$ m/s. Therefore, $v_{fy} = -10$ m/s and

$$v_{ix} = \frac{-20 \text{ m/s}}{-2 \tan(60^\circ)} = 5.7735 \text{ m/s}.$$

ROUND: $\vec{v}_i = (5.8 \hat{x} + 10 \hat{y})$ m/s and $\vec{v}_f = (5.8 \hat{x} - 10 \hat{y})$ m/s.

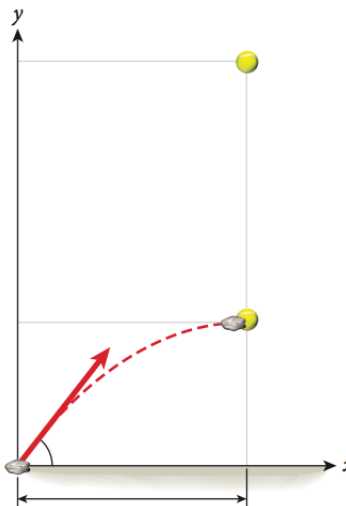
DOUBLE-CHECK: Because the projectile is shot on level ground, it makes sense that the initial and final y velocities are equal in magnitude and opposite in direction. Since the angle is greater than 45° , it also makes sense that the vertical velocity is greater than the horizontal velocity.

- 3.53. **THINK:** At its most basic this problem is just the “Shoot the Monkey” problem (Example 3.1) again. The tennis ball plays the role of the monkey, and the rock that of the tranquilizer dart. The time and the position in space where the collision occurs are given, so the information about the tennis ball is irrelevant. But note that the initial height of the golf ball ($h = 54.1$ m) and the height ($y = 10.0$ m) and time ($t = 3.00$ s) at which the collision occurs are consistent, because the tennis ball drops a distance $\frac{1}{2}gt^2 = 0.5(9.81 \text{ m/s}^2)(3 \text{ s})^2 = 44.1$ m during 3.00 s, which is exactly $h - y$.

In order to calculate the horizontal velocity component of the rock, we have to keep in mind that it has to travel a given distance x in a given time. Note that the horizontal velocity component remains unchanged during the flight of the rock.

For the vertical velocity component, we know that the rock also has to travel a given distance y in the same time, but that it is in free-fall during that time period.

SKETCH:



RESEARCH: For the initial velocity components we use $x - x_0 = v_{x0}t$ and $y - y_0 = v_{y0}t - \frac{1}{2}gt^2$ with $x_0 = y_0 = 0$. For the final velocity components at the collision moment we use $v_x = v_{x0}$ and $v_y = v_{y0} - gt$.

SIMPLIFY: We solve the first two equations for the initial velocity components and find

$$v_{x0} = x/t \quad \text{and} \quad v_{y0} = (y + \frac{1}{2}gt^2)/t = y/t + \frac{1}{2}gt$$

The two equations for the final velocity components are already in the form we can use for inserting numbers.

CALCULATE: $v_x = v_{x0} = \frac{50.0 \text{ m}}{3.00 \text{ s}} = 16.667 \text{ m/s},$

$$v_{y0} = \frac{10.0 \text{ m} + \frac{1}{2}(9.81 \text{ m/s}^2)(3.00 \text{ s})^2}{3.00 \text{ s}} = 18.0483 \text{ m/s},$$

$$|\vec{v}_0| = \sqrt{(16.667 \text{ m/s})^2 + (18.0483 \text{ m/s})^2} = 24.57 \text{ m/s}, \quad \theta_0 = \tan^{-1}\left(\frac{18.0483}{16.6667}\right) = 47.28^\circ$$

$$v_y = 18.0483 \text{ m/s} - (9.81 \text{ m/s}^2)(3.00 \text{ s}) = -11.3817 \text{ m/s}$$

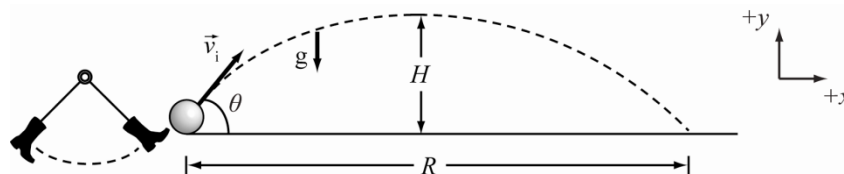
$$|\vec{v}| = \sqrt{(16.667 \text{ m/s})^2 + (-11.3817 \text{ m/s})^2} = 20.18 \text{ m/s}, \quad \theta = \tan^{-1}\left(\frac{-11.3817}{16.6667}\right) = -34.33^\circ$$

ROUND: The initial velocity is $v_0 = 24.6 \text{ m/s}$ at 47.3° above horizontal. The final velocity is $v = 20.2 \text{ m/s}$ at 34.3° below horizontal.

DOUBLE-CHECK: In Example 3.1, “Shoot the Monkey”, we learned that in order to hit the monkey during its free-fall, one has to aim directly in a straight line at it at the beginning. This means that the initial velocity vector has to point from the origin directly at the release point of the ball (x, h) . This implies that the tangent of the initial velocity vector is given by $\tan \theta_0 \equiv v_{y0}/v_{x0} = h/x$. From the values given in the problem you can see that $h/x = (54.1 \text{ m})/(50.0 \text{ m}) = 1.08$, and our numerical answer yields $v_{y0}/v_{x0} = (18.0 \text{ m/s})/(16.7 \text{ m/s}) = 1.08$.

- 3.54. **THINK:** Since the initial and final heights are the same, the range and maximum height equations can be used. Assume the only acceleration is gravity so the horizontal velocity remains unchanged. When the golf ball is at the maximum height, its vertical velocity is zero. $g = 9.81 \text{ m/s}^2$, $\theta = 31.5^\circ$ and $v_i = 11.2 \text{ m/s}$.

SKETCH:



RESEARCH: $R = \frac{v_i^2 \sin(2\theta)}{g}$; $H = y_0 + \frac{v_{iy}^2}{2g}$; $v_{ix} \neq v_{fx} = v_i \cos \theta$; and $v_{iy} \neq v_{if} = v_i \sin \theta$.

SIMPLIFY:

(a) $R = \frac{v_i^2 \sin(2\theta)}{g}$

(b) $H = 0 + \frac{v_i^2 \sin^2 \theta}{2g} = \frac{v_i^2 \sin^2 \theta}{2g}$

(c) $\vec{v} = v_{ix}\hat{x} + 0\hat{y} = v_i \cos \theta \hat{x}$

(d) $\vec{a} = 0\hat{x} - g\hat{y} = -g\hat{y}$

CALCULATE:

$$(a) R = \frac{(11.2 \text{ m/s})^2 \sin(63^\circ)}{9.81 \text{ m/s}^2} = 11.393 \text{ m}$$

$$(b) H = \frac{(11.2 \text{ m/s})^2 \sin^2(31.5^\circ)}{2(9.81 \text{ m/s}^2)} = 1.7454 \text{ m}$$

$$(c) v = (11.2 \text{ m/s}) \cos(31.5^\circ) \hat{x} = 9.5496 \hat{x} \text{ m/s}$$

$$(d) \vec{a} = -9.81 \hat{y} \text{ m/s}^2$$

ROUND:

$$(a) R = 11.4 \text{ m}$$

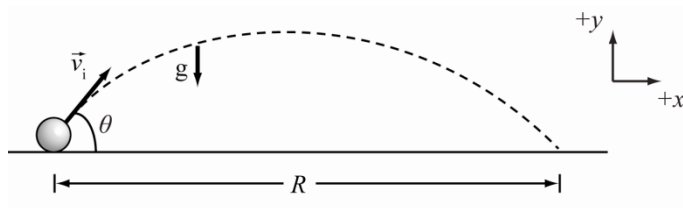
$$(b) H = 1.75 \text{ m}$$

$$(c) v = 9.55 \hat{x} \text{ m/s}$$

$$(d) \vec{a} = -9.81 \hat{y} \text{ m/s}^2$$

DOUBLE-CHECK: Given the initial velocity and angle, the height and range are reasonable.

- 3.55. **THINK:** The question does not specify a launch angle. However, for maximum distance, the launch angle is 45° . Assume the initial and final heights are the same so the range equation can be used. $R = 0.67 \text{ km}$, $g = 9.81 \text{ m/s}^2$ and $\theta = 45^\circ$.

SKETCH:

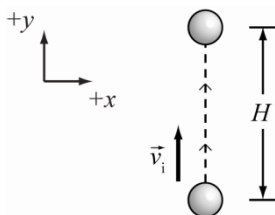
$$\text{RESEARCH: } R = \frac{v_i^2 \sin(2\theta)}{g}$$

$$\text{SIMPLIFY: } v_i = \sqrt{\frac{gR}{\sin(2\theta)}}$$

$$\text{CALCULATE: } v_i = \sqrt{\frac{(9.81 \text{ m/s}^2)(670 \text{ m})}{\sin(2 \cdot 45^\circ)}} = 81.072 \text{ m/s}$$

ROUND: Rounding to two significant figures: $v_i = 81 \text{ m/s}$.**DOUBLE-CHECK:** This speed is equivalent to about 300 km/h, which seems reasonable for a catapult.

- 3.56. **THINK:** In order to attain maximum height, the launch angle must be exactly 90° . The mass of the object is irrelevant to the kinematic equations. $v_i = 80.3 \text{ m/s}$, $\theta = 90.0^\circ$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:

$$\text{RESEARCH: } H = y_0 + \frac{v_i^2 \sin^2 \theta}{2g}$$

SIMPLIFY: $H = \frac{v_i^2 \sin^2 \theta}{2g}$

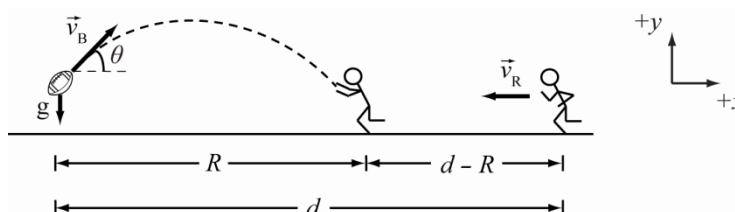
CALCULATE: $H = \frac{(80.3 \text{ m/s})^2 \sin^2(90.0^\circ)}{2(9.81 \text{ m/s}^2)} = 328.65 \text{ m}$

ROUND: Rounding to three significant figures, $H = 329 \text{ m}$.

DOUBLE-CHECK: Given the initial velocity, this height is reasonable.

- 3.57. **THINK:** Since the initial and final heights are the same, the range equation can be used to determine where the runner catches the ball. The time of flight of the ball must also be determined, thus the runner must run the remaining distance in the same amount of time. $v_B = 25.0 \text{ m/s}$, $\theta = 35.0^\circ$, $d = 70.0 \text{ m}$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $R = \frac{v_i^2 \sin 2\theta}{g}$; $y_f - y_i = v_{iy}t + \frac{1}{2}at^2$; $v_{ix} = v_{fx}$; $v_{ix} = v_i \cos \theta$; $v_{iy} = v_i \sin \theta$.

SIMPLIFY: $R = \frac{v_B^2 \sin 2\theta}{g}$ and $0 = v_B \sin \theta t - \frac{1}{2}gt^2 \Rightarrow t = \frac{2v_B \sin \theta}{g}$.

$$v_R = \frac{d - R}{t} = \frac{d - \left(\frac{v_B^2 \sin 2\theta}{g} \right)}{\left(\frac{2v_B \sin \theta}{g} \right)} = \frac{dg}{2v_B \sin \theta} - v_B \cos \theta$$

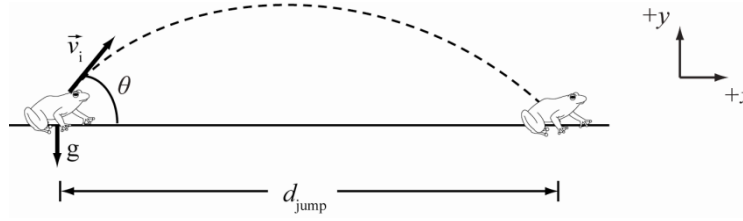
CALCULATE: $v_R = \frac{(70.0 \text{ m})(9.81 \text{ m/s}^2)}{2(25.0 \text{ m/s})\sin(35.0^\circ)} - (25.0 \text{ m/s})\cos(35.0^\circ) = 3.466 \text{ m/s}$

ROUND: Rounding to three significant figures, $v_R = 3.47 \text{ m/s}$.

DOUBLE-CHECK: The maximum speed a person can run at is around 12 m/s (see Chapter 1); so the result obtained here is possible.

- 3.58. **THINK:** If the frog is to maximize its distance, it must jump at an angle of 45° . Since the initial and final heights are the same, the range equation can be used to determine the velocity it jumps with. This velocity can then be used to determine the amount of time of one jump, and then the number of jumps in an hour. This means the total distance traveled is the number of jumps multiplied by the distance per jump. $d_{\text{jump}} = 1.3 \text{ m}$, $g = 9.81 \text{ m/s}^2$ and $\theta = 45^\circ$. The total jump time is $0.8(1 \text{ h}) = 2880 \text{ s}$.

SKETCH:



RESEARCH: $d_{\text{jump}} = R_{\text{max}} = \frac{v_i^2}{g}$ and $v_{ix} = v_i \cos 45^\circ = v_i / \sqrt{2} = \frac{d_{\text{jump}}}{t_{\text{jump}}}$.

SIMPLIFY: $v_i = \sqrt{gd_{\text{jump}}}$ and $v_{ix} = \sqrt{gd_{\text{jump}}/2} = \frac{d_{\text{jump}}}{t_{\text{jump}}}$. Therefore,

$$t_{\text{jump}} = \frac{d_{\text{jump}}}{\sqrt{gd_{\text{jump}}/2}}.$$

The number of jumps, n , is equal to $2880 \text{ s} / t_{\text{jump}}$. This implies that the total distance, D , is

$$D = n d_{\text{jump}} = \frac{2880 \text{ s}}{t_{\text{jump}}} d_{\text{jump}} = d_{\text{jump}} \frac{2880 \text{ s}}{d_{\text{jump}}} \sqrt{gd_{\text{jump}}/2} = (2880 \text{ s}) \sqrt{gd_{\text{jump}}/2}.$$

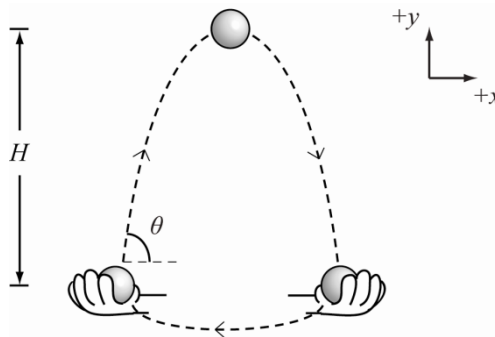
CALCULATE: $D = (2880 \text{ s}) \sqrt{(9.81 \text{ m/s}^2)(1.3 \text{ m})/2} = 7272.5 \text{ m}$

ROUND: The frog jumps a total of 7.3 km.

DOUBLE-CHECK: This result is the same distance a person walking briskly at 2 m/s would cover in an hour, so it is reasonable that a frog could cover the same distance in the same amount of time.

- 3.59. **THINK:** If the juggler has a ball in her left hand, he can also have one in his right hand, assuming her right hand is throwing the ball up. This means if the juggler has x number of balls, the minimum time between balls is 0.200 s. If the time between any two balls is less than 0.200 s, then the hands can't act fast enough and she'll drop the balls. The given information must be used to determine the time of flight, and thus the total time of one ball going around the loop. Then determine the largest integer, n , which can be multiplied by $t_{\text{pass}} = 0.200 \text{ s}$ which is still less than the total time. $H = 90.0 \text{ cm}$, $\theta = 75.0^\circ$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $H = y_i + \frac{v_i^2 \sin^2 \theta}{2g}$ and $y_f - y_i = v_{yi} t + \frac{1}{2} a t^2$.

SIMPLIFY: $H = \frac{(v_i \sin \theta)^2}{2g} \Rightarrow v_i \sin \theta = \sqrt{2gH}$. Use this when finding t_{throw} :

$$0 = v_i \sin \theta t_{\text{throw}} - \frac{1}{2} g t_{\text{throw}}^2 \Rightarrow t_{\text{throw}} = \frac{2v_i \sin \theta}{g} = \frac{2\sqrt{2gH}}{g} = \sqrt{\frac{8H}{g}}$$

Therefore, $nt_{\text{pass}} \leq (t_{\text{pass}} + t_{\text{throw}}) \Rightarrow n \leq \frac{t_{\text{pass}} + t_{\text{throw}}}{t_{\text{pass}}} \Rightarrow n \leq \frac{t_{\text{pass}} + \sqrt{\frac{8H}{g}}}{t_{\text{pass}}}$.

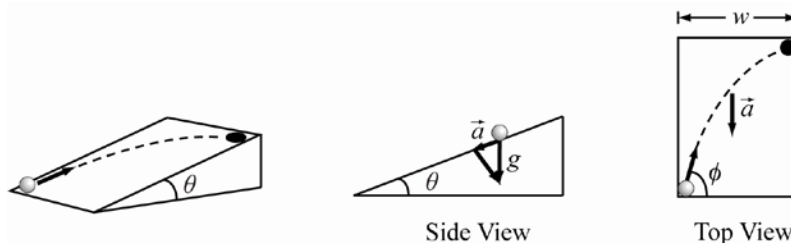
CALCULATE: $n \leq \frac{0.2 \text{ s} + \sqrt{\frac{8(0.9 \text{ m})}{9.81 \text{ m/s}^2}}}{0.2 \text{ s}} = 5.2835$

ROUND: The maximum number of balls is $n = 5$.

DOUBLE-CHECK: It is reasonable for a person to juggle five balls at a time.

- 3.60. THINK:** Since the plane is at an angle with the horizontal, it will tend to accelerate down the incline at only a fraction of gravity. Looking directly down the board, it can be considered as regular projectile motion with a vertical acceleration less than gravity. If the ball is to land in a hole on the opposite corner, the ball can't overshoot the corner, so the ball must be at its maximum height in the trajectory to make it in the hole. $\theta = 30.0^\circ$, $\phi = 45.0^\circ$, $w = 50.0 \text{ cm}$, $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $a = g \sin \theta$; $R = \frac{v_i^2 \sin(2\phi)}{g}$; and $w = \frac{R}{2}$.

SIMPLIFY: $2w = \frac{v_i^2 \sin(2\phi)}{g \sin \theta} \Rightarrow v_i = \sqrt{\frac{2wg \sin \theta}{\sin(2\phi)}}$.

CALCULATE: $v_i = \sqrt{\frac{2(0.500 \text{ m})(9.81 \text{ m/s}^2) \sin(30.0^\circ)}{\sin(90.0^\circ)}} = 2.2147 \text{ m/s}$

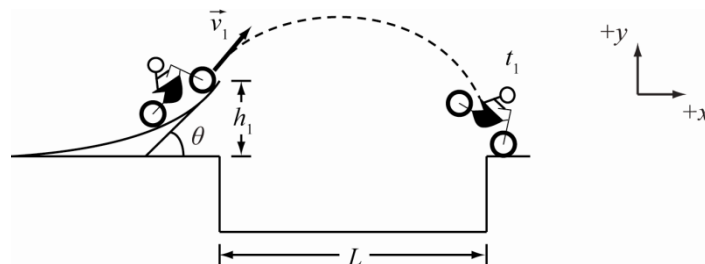
ROUND: $v_i = 2.21 \text{ m/s}$

DOUBLE-CHECK: The speed is typical for a pinball game, and so we have confidence that our solution is the right order of magnitude.

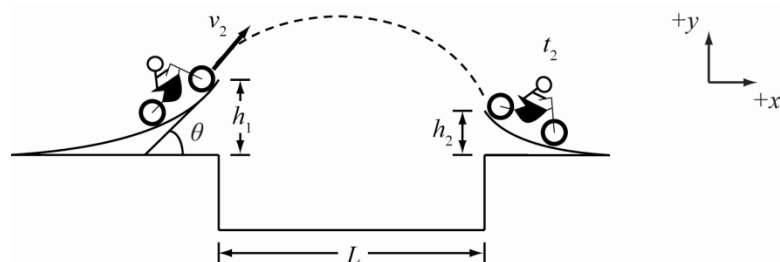
- 3.61. THINK:** Ignoring air resistance, the biker's horizontal velocity will remain unchanged. Initially the biker lands on flat land and afterwards he tries again with a landing ramp on the edge. $L = 400. \text{ m}$, $\theta = 45.0^\circ$, $h_1 = 8.00 \text{ m}$, $h_2 = 3.00 \text{ m}$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:

(a)



(b)



RESEARCH: $v_{xi}t = x_f - x_i$; $y_f - y_i = v_{yi}t + \frac{1}{2}at^2$; $v_{xi} = v_i \cos \theta$; and $v_{yi} = v_i \sin \theta$.

SIMPLIFY:

(a) $\frac{L}{v_1 \cos \theta} = t_1$ and $0 - h_1 = (v_1 \sin \theta)t_1 - \frac{1}{2}gt_1^2$. Therefore, $-h_1 = L \tan \theta - \frac{gL^2}{2v_1^2 \cos^2 \theta}$ or

$$\frac{gL^2}{2v_1^2 \cos^2 \theta} = L \tan \theta + h_1. \text{ Solving for } v_1 \text{ this gives } v_1 = \sqrt{\frac{gL^2}{2 \cos^2 \theta (L \tan \theta + h_1)}}.$$

(b) $\frac{L}{v_2 \cos \theta} = t_2$ and $h_2 - h_1 = (v_2 \sin \theta)t_2 - \frac{1}{2}gt_2^2$. So, $h_2 - h_1 = L \tan \theta - \frac{gL^2}{2v_2^2 \cos^2 \theta}$. Solving for v_2 gives

$$v_2 = \sqrt{\frac{gL^2}{2 \cos^2 \theta (L \tan \theta + h_1 - h_2)}}.$$

CALCULATE:

(a) $v_1 = \sqrt{\frac{(9.81 \text{ m/s}^2)(400. \text{ m})^2}{2 \cos^2(45.0^\circ)((400. \text{ m})\tan(45.0^\circ) + 8.00 \text{ m})}} = 62.025 \text{ m/s}$

(b) $v_2 = \sqrt{\frac{(9.81 \text{ m/s}^2)(400. \text{ m})^2}{2 \cos^2(45.0^\circ)((400. \text{ m})\tan(45.0^\circ) + 8.00 \text{ m} - 3.00 \text{ m})}} = 62.254 \text{ m/s}$

ROUND:

(a) $v_1 = 62.0 \text{ m/s}$

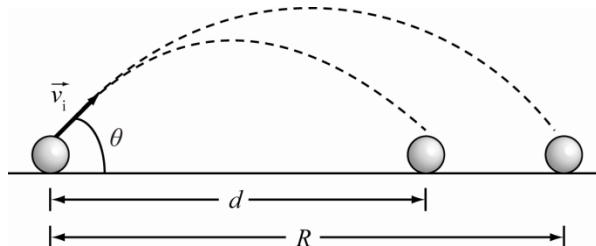
(b) $v_2 = 62.3 \text{ m/s}$

DOUBLE-CHECK: The initial speed is approximately 220 km/h (around 140 mph), which is achievable with a motorcycle. Also note that we find that the required initial speeds for parts (a) and (b) are essentially the same. The change of 3.00 m does not matter much compared to 400. m.

- 3.62. **THINK:** Determine the distance the golf ball should travel, then compare to the distance it actually travels. Assume the initial and final heights of the ball are the same, so the range equation can be used. $\theta = 35.5^\circ$, $d = 86.8 \text{ m}$ and $g = 9.81 \text{ m/s}^2$.

$$v_i = 83.3 \text{ mph} \cdot \frac{1.602 \text{ km}}{1 \text{ mile}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 37.0685 \text{ m/s}$$

SKETCH:



RESEARCH: $R = \frac{v_i^2 \sin 2\theta}{g}$ and $\Delta d = R - d$.

SIMPLIFY: It is not necessary to simplify.

CALCULATE: $R = \frac{(37.0685 \text{ m/s})^2 \sin(71.0^\circ)}{9.81 \text{ m/s}^2} = 132.44 \text{ m}$

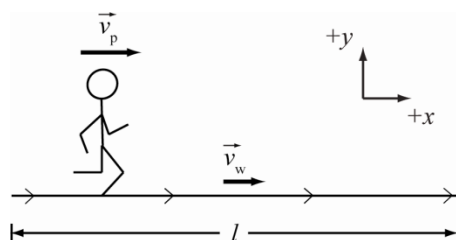
$\Delta d = 132.44 \text{ m} - 86.8 \text{ m} = 45.64 \text{ m}$

ROUND: Rounding to three significant figures, 45.6 m.

DOUBLE-CHECK: The ball lost about 1/3 of its distance to the wind. If you are a golfer or if you watch golf on TV, then you know that golf shots do not quite follow parabolic trajectories, and that the result found here is quite possible.

- 3.63. **THINK:** If an object is moving in a reference frame that is in motion, then to a stationary observer the object moves at a sum of the velocities. $l = 59.1 \text{ m}$, $v_w = 1.77 \text{ m/s}$ and $v_p = 2.35 \text{ m/s}$.

SKETCH:



RESEARCH: $x_f - x_i = v_x t$

SIMPLIFY: $t = \frac{l}{v_w + v_p}$

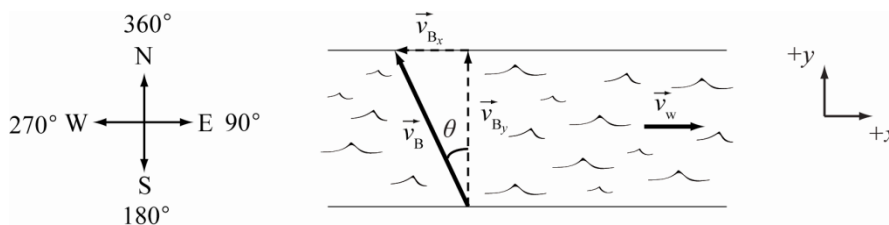
CALCULATE: $t = \frac{59.1 \text{ m}}{2.35 \text{ m/s} + 1.77 \text{ m/s}} = 14.345 \text{ s}$

ROUND: Rounding to three significant figures, $t = 14.3 \text{ s}$.

DOUBLE-CHECK: Given the long length of the walkway and the slow speed, a large time is reasonable.

- 3.64. **THINK:** If the captain wants to get directly across the river, he must angle the boat so that the component of the boat's velocity that counters the river is the same in magnitude and opposite in direction. $v_w = 1.00 \text{ m/s}$ and $v_B = 6.10 \text{ m/s}$.

SKETCH:



RESEARCH: $\vec{v} = v_x \hat{x} + v_y \hat{y}$; $|\vec{v}_{Bx}| = |\vec{v}_w|$; $v_x \theta = v \sin$; and $v_y \theta = v \cos$.

SIMPLIFY: $|\vec{v}_B| \sin \theta = |\vec{v}_w| \Rightarrow \theta = \sin^{-1} \left(\frac{v_w}{v_B} \right)$

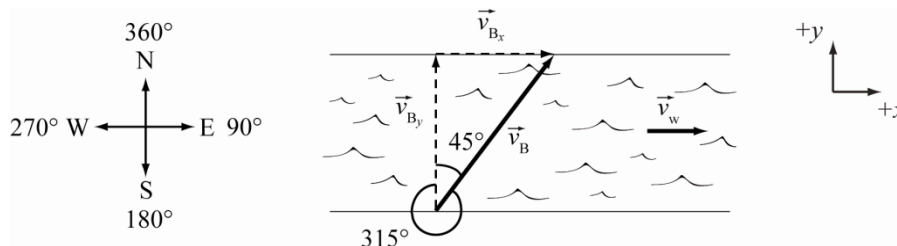
CALCULATE: $\theta = \sin^{-1} \left(\frac{1.00 \text{ m/s}}{6.10 \text{ m/s}} \right) = 9.4353^\circ$

ROUND: Round to three significant figures, as the boat must travel at 9.44° west of north.

DOUBLE-CHECK: Since the velocity of the water is small compared to the boat's velocity, a small angle is expected.

- 3.65. **THINK:** If the captain wants to get directly across the river, he must angle the boat so that the component of the boat's velocity that counters the river is the same in magnitude and opposite in direction. $\theta = 315^\circ$ and $v_B = 5.57$ m/s.

SKETCH:



RESEARCH: $\vec{v} = v_x \hat{x} + v_y \hat{y}$; $|\vec{v}_{Bx}| = |\vec{v}_w|$; $v_x \neq v \sin$; and $v_y \neq v \cos$.

SIMPLIFY: $|\vec{v}_w| = |\vec{v}_B \sin|$

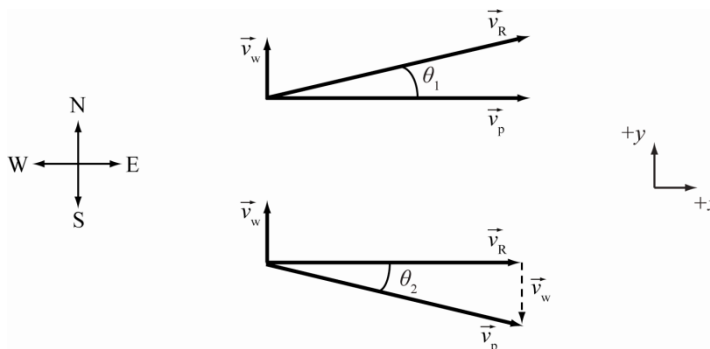
CALCULATE: $v_w = |(5.57 \text{ m/s}) \sin(315^\circ)| = 3.9386 \text{ m/s}$

ROUND: Three significant figures: The river is flowing at 3.94 m/s.

DOUBLE-CHECK: Given the large angle that the boat makes with the river, it is reasonable that the water has this large velocity.

- 3.66. **THINK:** The components of the velocity vectors of the wind and the plane can be summed to determine a resultant vector of the plane's velocity with respect to the ground. If the pilot wants to travel directly east, the plane must travel in a direction such that the component of plane's velocity in the wind's direction is equal in magnitude to the wind speed. $v_p = 350$ km/h and $v_w = 40.0$ km/h.

SKETCH:



RESEARCH: $\vec{v} = v_x \hat{x} + v_y \hat{y}$; $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$; $\tan \theta = \left(\frac{v_y}{v_x}\right)$; and $\sin \theta = \left(\frac{v_y}{v}\right)$.

SIMPLIFY: $\vec{v}_R = v_p \hat{x} + v_w \hat{y} \Rightarrow |\vec{v}| = \sqrt{v_p^2 + v_w^2}$; $\theta_1 = \tan^{-1}\left(\frac{v_w}{v_p}\right)$; and $\theta_2 = \sin^{-1}\left(\frac{v_w}{v_p}\right)$.

CALCULATE: $|\vec{v}| = \sqrt{(350 \text{ km/h})^2 + (40.0 \text{ km/h})^2} = 352.28 \text{ km/h}$

$$\theta_1 = \tan^{-1}\left(\frac{40.0 \text{ km/h}}{350 \text{ km/h}}\right) = 6.5198^\circ$$

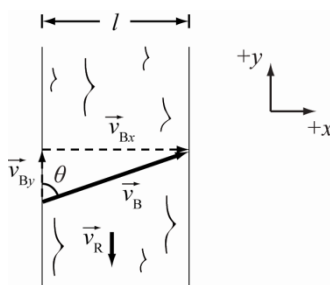
$$\theta_2 = \sin^{-1} \left(\frac{40.0 \text{ km/h}}{350. \text{ km/h}} \right) = 6.5624^\circ$$

ROUND: The plane has a velocity of 352 km/h at 6.52° north of east. To travel directly east, the plane must travel 6.56° south of east.

DOUBLE-CHECK: Given the small wind speed relative to the plane's speed, small angles are expected.

- 3.67. **THINK:** If the boaters want to travel directly over to the other side, they must angle the boat so that the component of the boat's velocity that counters the river is equal in magnitude. The time it takes to get over will then be based solely on the velocity perpendicular to the river flow. The minimum time to cross the river is when the boat is pointed exactly at the other side. Also, as long as the boat's velocity component is countering the river, any velocity in the perpendicular direction will get the boat across the river: $l = 127 \text{ m}$, $v_B = 17.5 \text{ m/s}$, and $v_R = 5.33 \text{ m/s}$.

SKETCH:



RESEARCH: $\vec{v} = v_x \hat{x} + v_y \hat{y}$; $|v_{By}| = |-v_R|$; $v_x \neq v \sin$; $v_y \neq v \cos$; $x_f - x_i = v_x \Delta t$; $\tan \theta = \left(\frac{v_x}{v_y} \right)$; and

$\cos \theta = \left(\frac{v_y}{v} \right)$. In part (e), the minimum speed, technically an infimum, will be when the angle θ is arbitrarily close to 0, and the component of the velocity directly across the stream is arbitrarily close to 0.

SIMPLIFY:

$$(a) v_{By} \neq v_R \Rightarrow v_B \cos \theta = v_R \Rightarrow \cos \theta = \left(\frac{v_R}{v_B} \right)$$

$$(b) l \neq (v_B \sin \theta) t \Rightarrow t = \frac{l}{v_B \sin \theta}$$

$$(c) \theta_{\min} = 90^\circ$$

$$(d) t_{\min} = \frac{l}{v_B \sin \theta_{\min}}$$

$$(e) \vec{v} \approx -\vec{v}_R$$

CALCULATE:

$$(a) \theta = \cos^{-1} \left(\frac{5.33 \text{ m/s}}{17.5 \text{ m/s}} \right) = 72.27^\circ$$

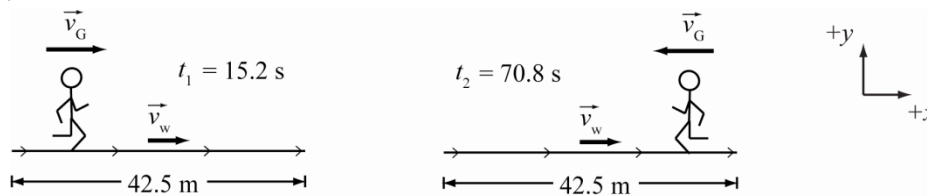
$$(b) t = \frac{127 \text{ m}}{(17.5 \text{ m/s}) \sin(72.27^\circ)} = 7.619 \text{ s}$$

$$(c) \theta = 90^\circ$$

$$(d) t_{\min} = \frac{127 \text{ m}}{17.5 \text{ m/s}} = 7.257 \text{ s}$$

(e) $v = 5.33 \text{ m/s}$ **ROUND:**(a) $\theta = 72.3^\circ$ (b) $t = 7.62 \text{ s}$ (c) $\theta = 90^\circ$ (d) $t = 7.26 \text{ s}$ (e) $v = 5.33 \text{ m/s}$ **DOUBLE-CHECK:** Given the width of the river and the velocities, these answers are reasonable.

- 3.68. THINK:** When the girl walks in the same direction as the walkway, her velocity relative to the terminal is the sum of her velocity relative to the walkway and the velocity of the walkway relative to the terminal. When she walks in the opposite direction, her velocity relative to the terminal is the difference between her velocity relative to the walkway and the velocity of the walkway relative to the terminal. $l = 42.5 \text{ m}$, $t_1 = 15.2 \text{ s}$ and $t_2 = 70.8 \text{ s}$.

SKETCH:**RESEARCH:** $x_f - x_i = v_x \Delta t$ **SIMPLIFY:** $l = v_x \Delta t \Rightarrow v_x = \frac{l}{t}$; $v_G + v_w = \frac{l}{t_1}$; and $v_G - v_w = \frac{l}{t_2}$.

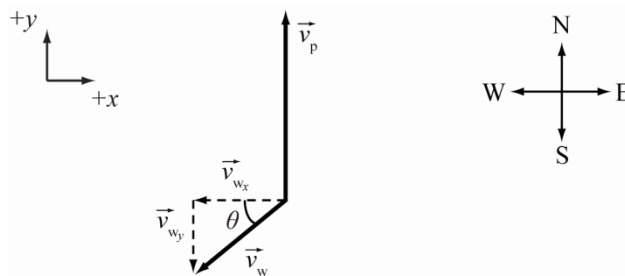
$$v_w = \frac{l}{t_1} - v_G = v_G - \frac{l}{t_2} \Rightarrow l \left(\frac{1}{t_1} + \frac{1}{t_2} \right) = 2v_G \Rightarrow v_G = \frac{l}{2} \left(\frac{1}{t_1} + \frac{1}{t_2} \right)$$

Therefore,

$$v_w = \frac{l}{t_1} - \frac{l}{2} \left(\frac{1}{t_1} + \frac{1}{t_2} \right) = \frac{l}{2} \left(\frac{1}{t_1} - \frac{1}{t_2} \right).$$

CALCULATE: $v_G = \frac{42.5 \text{ m}}{2} \left(\frac{1}{15.2 \text{ s}} + \frac{1}{70.8 \text{ s}} \right) = 1.698 \text{ m/s}$, $v_w = \frac{42.5 \text{ m}}{2} \left(\frac{1}{15.2 \text{ s}} - \frac{1}{70.8 \text{ s}} \right) = 1.0979 \text{ m/s}$ **ROUND:** Rounding to three significant figures, $v_w = 1.70 \text{ m/s}$ and $v_G = 1.10 \text{ m/s}$.**DOUBLE-CHECK:** The velocities are small, which makes sense for a walkway.

- 3.69. THINK:** Since the wind and plane velocities are vectors, simply add the components of the two vectors to determine the resultant vector. Southwest is 45° South of West. $v_p = 126.2 \text{ m/s}$, $v_w = 55.5 \text{ m/s}$ and $\theta = 45^\circ$.

SKETCH:**RESEARCH:** $\vec{v} = v_x \hat{x} + v_y \hat{y}$; $v_x = v \cos \theta$; $v_y = v \sin \theta$; and $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$.

SIMPLIFY: $v_{xw} = v_w \cos \theta$ and $v_{yw} = v_w \sin \theta$. Therefore, $\vec{v} = (v_p - v_{xw})\hat{x} + (v_{yw})\hat{y}$ and

$$|\vec{v}| = \sqrt{(v_p - v_w \cos \theta)^2 + (v_w \sin \theta)^2}.$$

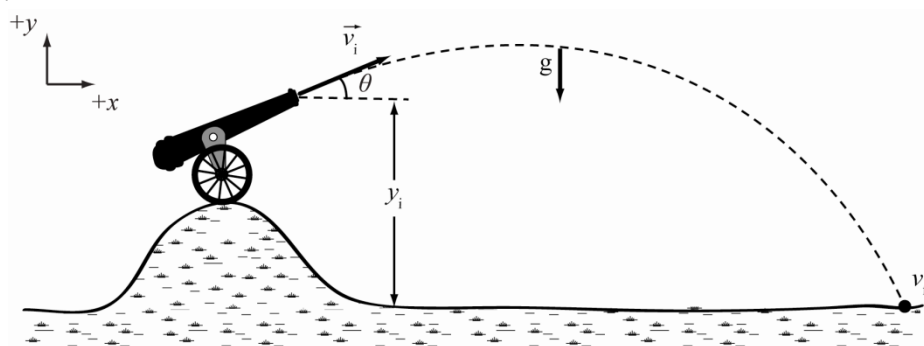
CALCULATE: $|\vec{v}| = \sqrt{(-55.5 \text{ m/s} \cos(45.0^\circ))^2 + (126.2 \text{ m/s} - 55.5 \text{ m/s} \sin(45.0^\circ))^2} = 95.401 \text{ m/s}$

ROUND: Rounding to three significant figures, $|\vec{v}| = 95.4 \text{ m/s}$.

DOUBLE-CHECK: Given that the wind is blowing against the plane, the magnitude of the resultant velocity should be less than the plane's speed.

- 3.70. THINK:** The horizontal velocity is constant. Determine the vertical velocity as it hits the ground and then determine the overall velocity. Mass is irrelevant. $y_i = 116.7 \text{ m}$, $\theta = 22.7^\circ$, $v_i = 36.1 \text{ m/s}$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $v_f^2 = v_i^2 + 2ad$; $v_x = v \cos \theta$; $v_y = v \sin \theta$; $v_{ix} = v_{fx}$; and $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$.

SIMPLIFY: $v_{fx} = v_i \cos \theta$ and $v_{fy} = (v_i \sin \theta)^2 - 2y_i$ ($v_f^2 = (v_i \sin \theta)^2 + 2y_i$). Also,

$$|\vec{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(v_i \cos \theta)^2 + (v_i \sin \theta)^2 + 2y_i} = \sqrt{v_i^2 + 2y_i}.$$

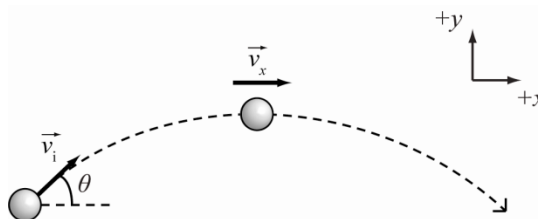
CALCULATE: $|\vec{v}_f| = \sqrt{(36.1 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(116.7 \text{ m})} = 59.941 \text{ m/s}$

ROUND: Rounding to three significant figures, $|\vec{v}_f| = 59.9 \text{ m/s}$.

DOUBLE-CHECK: The final velocity is greater than the initial velocity, which makes sense.

- 3.71. THINK:** The horizontal velocity remains unchanged at any point. $v_i = 31.1 \text{ m/s}$ and $\theta = 33.4^\circ$.

SKETCH:



RESEARCH: $v_{ix} = v_{fx} = v_i \cos \theta$

SIMPLIFY: $v_x = v_i \cos \theta$

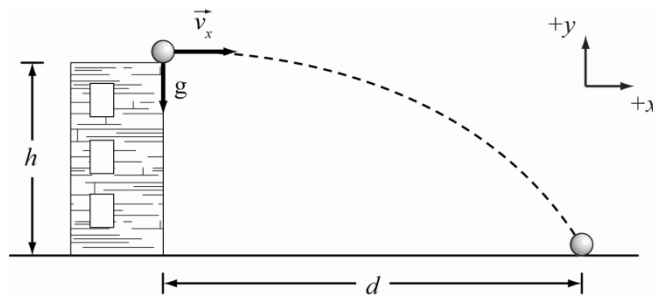
CALCULATE: $v_x = (31.1 \text{ m/s}) \cos(33.4^\circ) = 25.964 \text{ m/s}$

ROUND: Rounding to three significant figures, $v_x = 26.0 \text{ m/s}$.

DOUBLE-CHECK: The result is smaller than the initial velocity, which makes sense.

- 3.72. **THINK:** Determine an equation that relates the height of the object being launched to how far horizontally it travels. $v_x = 10.1 \text{ m/s}$, $d = 57.1 \text{ m}$ and $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $v_x t = x_f - x_i$; $y_f - y_i = v_{iy} t + \frac{1}{2} a t^2$; and $v_{iy} = v_i \sin \theta$.

SIMPLIFY: $d = v_x t \Rightarrow t = \frac{d}{v_x}$ and $-h = 0 - \frac{1}{2} g t^2 \Rightarrow h = \frac{g d^2}{2 v_x^2}$.

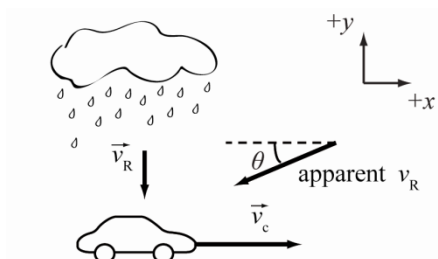
CALCULATE: $h = \frac{(9.81 \text{ m/s}^2)(57.1 \text{ m})^2}{2(10.1 \text{ m/s})^2} = 156.77 \text{ m}$

ROUND: Rounding to three significant figures, $h = 157 \text{ m}$.

DOUBLE-CHECK: Given the velocity and distance, this height is reasonable.

- 3.73. **THINK:** If rain is falling straight down, to someone moving it would appear as if the rain was heading towards said person. $v_c = 19.3 \text{ m/s}$ and $v_R = 8.9 \text{ m/s}$.

SKETCH:



RESEARCH: $\tan \theta = \left(\frac{v_y}{v_x} \right)$

SIMPLIFY: $\theta = \tan^{-1} \left(\frac{v_R}{v_c} \right)$

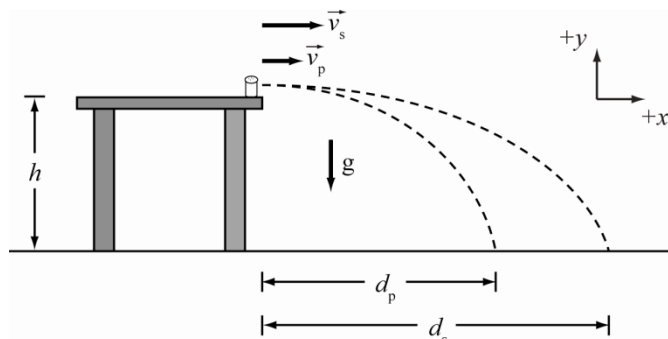
CALCULATE: $\theta = \tan^{-1} \left(\frac{8.9 \text{ m/s}}{19.3 \text{ m/s}} \right) = 24.756^\circ$

ROUND: Rounding to two significant figures for angles gives the rain an angle of 25° with the horizontal.

DOUBLE-CHECK: The car is moving faster than the rain so this angle is reasonable.

- 3.74. THINK:** Since both shakers leave the table with no initial vertical velocity, it should take both the same amount of time to hit the ground. $h = 0.85$ m, $v_p = 2.5$ m/s, $v_s = 5$ m/s and $g = 9.81$ m/s².

SKETCH:



RESEARCH: (a) The ratio of the times is 1:1 since the times are the same. (b) The ratio of the distances will be the ratio of the speeds.

SIMPLIFY:

(a) Not necessary.

(b) $d_p : d_s = v_p : v_s$

CALCULATE:

(a) 1:1

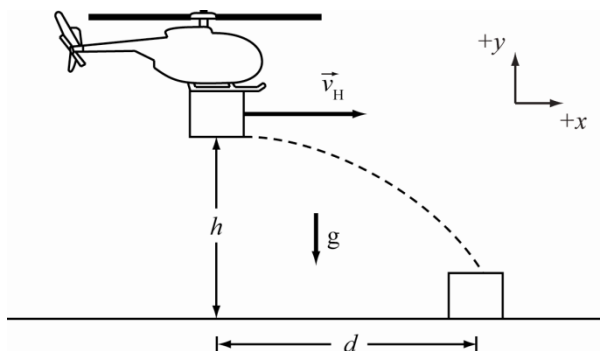
(b) $d_p : d_s = v_p : v_s = 2.5 \text{ m/s} : 5 \text{ m/s} = 1:2$

ROUND: Rounding is not necessary.

DOUBLE-CHECK: Since the horizontal component of the velocity is constant, and the initial horizontal velocity of the salt shaker is double that of the pepper shaker, it makes sense that the salt shaker travels twice as far as the pepper shaker.

- 3.75. THINK:** Assuming the box has no parachute, at the time of drop it will have the same horizontal velocity as the velocity of the helicopter, $\vec{v}_H$, and this will remain constant throughout the fall. The initial vertical velocity is zero. The vertical component of the velocity will increase due to the acceleration of gravity. The final speed of the box at impact can be found from the horizontal and final vertical velocity of the box just before impact. $h = 500$ m, $d = 150$ m, and $g = 9.81$ m/s².

SKETCH:



RESEARCH: To find the speed of the helicopter we use: $y_f - y_i = v_{yi}t + \frac{1}{2}at^2$ and $x_f - x_i = v_x t$. To find the final speed of the box use $v_{fy} = v_{iy} + at$ as the vertical component, the helicopter speed as the

horizontal component, and $|\vec{v}| = \sqrt{v_y^2 + v_x^2}$ to find the final speed of the box when it hits the ground.

SIMPLIFY: $-h = 0 - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$ and so $v_x = v_H = \frac{d}{t} = d\sqrt{\frac{g}{2h}}$.

$$v_y = -gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh} \text{ and so } |\vec{v}| = \sqrt{(-\sqrt{2gh})^2 + \left(d\sqrt{\frac{g}{2h}}\right)^2} = \sqrt{g(2h + d^2/[2h])}$$

CALCULATE: $v_H = (150. \text{ m})\sqrt{\frac{9.81 \text{ m/s}^2}{2(500. \text{ m})}} = 14.857 \text{ m/s}$

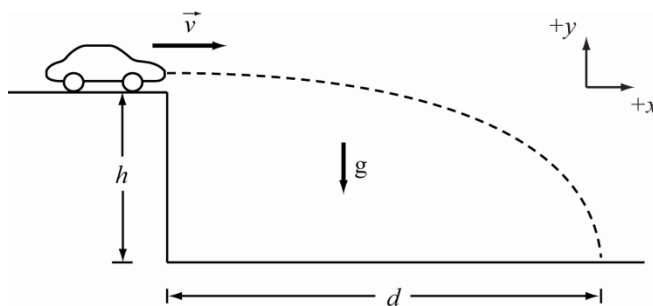
$$|\vec{v}| = \sqrt{9.81 \text{ m/s}^2 (2(500. \text{ m}) + (150. \text{ m})^2 / [2(500. \text{ m})])} = 100.15 \text{ m/s}$$

ROUND: $v_H = 14.9 \text{ m/s}$ and $|\vec{v}| = 100. \text{ m/s}$.

DOUBLE-CHECK: The helicopter velocity is equivalent to about 50 km/h and the speed that the box has when it hits the ground is equivalent to about 360 km/h. Both are reasonable values.

- 3.76. **THINK:** Ignoring air resistance, the horizontal velocity remains constant. Assume the car had no initial vertical velocity when it went over the cliff. $d = 150. \text{ m}$, $h = 60.0 \text{ m}$, $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $x_f - x_i = v_x t$ and $y_f - y_i = v_{iy} t + \frac{1}{2}at^2$.

SIMPLIFY: $d = v_x t \Rightarrow v_x = \frac{d}{t}$ and $-h = 0 - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$. Therefore, $v_x = d\sqrt{\frac{g}{2h}}$.

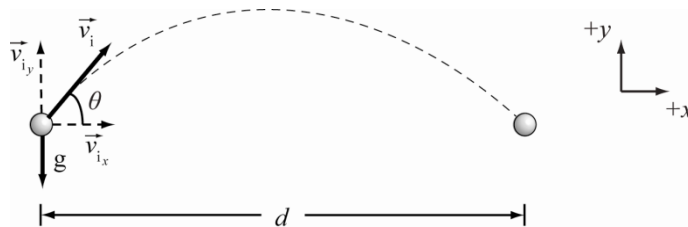
CALCULATE: $v_x = (150. \text{ m})\sqrt{\frac{9.81 \text{ m/s}^2}{2(60.0 \text{ m})}} = 42.888 \text{ m/s}$

ROUND: $v_x = 42.9 \text{ m/s}$

DOUBLE-CHECK: The vertical distance is smaller than the horizontal distance, so the car must have been going fast.

- 3.77. **THINK:** The initial horizontal velocity must be used to determine the time of flight, then this time can be used to determine the initial vertical velocity. $v_{ix} = 3.90 \text{ m/s}$, $d = 30.0 \text{ m}$, $g = 9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: $x_f - x_i = v_x t$; $y_f - y_i = v_{iy} t + \frac{1}{2} a t^2$; $v_{iy} = v_i \sin \theta$; and $v_{ix} = v_i \cos \theta$.

SIMPLIFY: $d = v_x t \Rightarrow t = \frac{d}{v_x}$ and so $0 = v_{iy} t - \frac{1}{2} g t^2 \Rightarrow v_{iy} = \frac{1}{2} g t = \frac{g d}{2 v_x}$.

Also, $\frac{v_{iy}}{v_{ix}} = \tan \theta \Rightarrow \theta = \tan^{-1} \left(\frac{v_{iy}}{v_{ix}} \right)$.

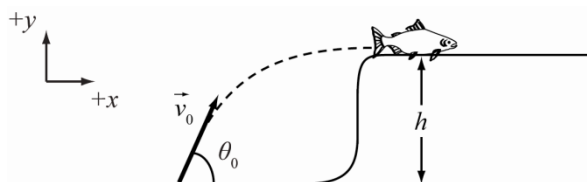
CALCULATE: $v_{iy} = \frac{(9.81 \text{ m/s}^2)(30.0 \text{ m})}{2(3.90 \text{ m/s})} = 37.73 \text{ m/s}$, $\theta = \tan^{-1} \left(\frac{37.73 \text{ m/s}}{3.90 \text{ m/s}} \right) = 84.10^\circ$

ROUND: The vertical velocity is 37.7 m/s and the launch angle is 84.1° .

DOUBLE-CHECK: The exam bundle has a small horizontal velocity but travels 30 m, so the vertical velocity and launch angle must be large to allow the bundle to remain in the air longer.

- 3.78. THINK:** Determine the initial speed of the jump v_0 . The height of the waterfall is $h = 1.05 \text{ m}$. The time taken for the jump is $t = 2.1 \text{ s}$. The initial launch angle is $\theta_0 = 35^\circ$.

SKETCH:



RESEARCH: Assuming an ideal parabolic trajectory, the kinematic equation for the vertical direction,

$\Delta y = v_{y0} t - \frac{1}{2} g t^2$ can be used to solve for v_0 .

SIMPLIFY: $\Delta y = v_{y0} t - \frac{1}{2} g t^2$ and $h = v_0 \sin \theta_0 t - \frac{1}{2} g t^2 \Rightarrow v_0 = \frac{h + \frac{1}{2} g t^2}{\sin \theta_0 t}$.

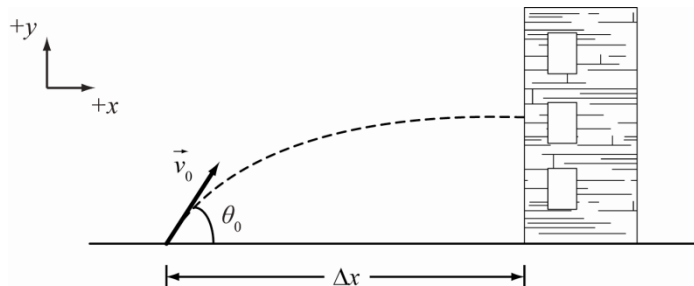
CALCULATE: $v_0 = \frac{1.05 \text{ m} + \frac{1}{2} (9.81 \text{ m/s}^2) (2.1 \text{ s})^2}{\sin(35^\circ) (2.1 \text{ s})} = \frac{1.05 \text{ m} + 21.63 \text{ m}}{1.2045 \text{ s}} = 18.83 \text{ m/s}$

ROUND: There are two significant figures in both the sum in the numerator and the product in the denominator of the equation for v_0 , so $v_0 = 19 \text{ m/s}$.

DOUBLE-CHECK: Salmon migrate great distances (hundreds of kilometers) up rivers to spawn, and must overcome large obstacles to do so; this is a fast but reasonable speed for a salmon to exert on its journey.

- 3.79. THINK:** Determine which floor of the building the water strikes (each floor is $h_f = 4.00 \text{ m}$ high). The horizontal distance between the firefighter and the building is $\Delta x = 60.0 \text{ m}$. The initial angle of the water stream is $\theta_0 = 37.0^\circ$. The initial speed is $v_0 = 40.3 \text{ m/s}$.

SKETCH:



RESEARCH: To determine which floor the water strikes, the vertical displacement of the water with respect to the ground must be determined. The trajectory equation can be used, assuming ideal parabolic motion:

$$\Delta y = (\tan \theta_0) \Delta x - \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}.$$

The floor, at which the water strikes, n , is the total vertical displacement of the water divided by the height of each floor:

$$n = \frac{\Delta y}{h_f}.$$

SIMPLIFY: $n = \frac{\Delta y}{h_f} = \frac{1}{h_f} \left[(\tan \theta_0) \Delta x - \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0} \right]$

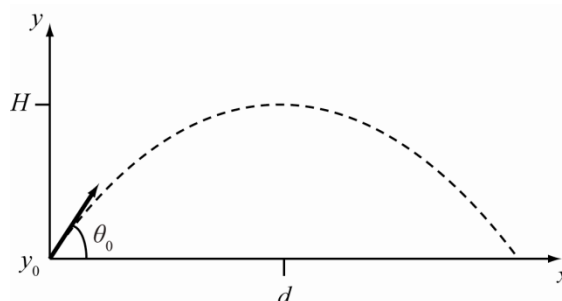
CALCULATE: $n = \frac{1}{4 \text{ m}} \left[\tan(37^\circ)(60 \text{ m}) - \frac{(9.81 \text{ m/s}^2)(60 \text{ m})^2}{2(40.3 \text{ m/s})^2 \cos^2(37^\circ)} \right] = 7.042$

ROUND: $n = 7.042$ floors above ground level (where $n = 0$) corresponds to ankle height on the 8th floor.

DOUBLE-CHECK: It is reasonable to expect the water from a high-pressure fire hose to reach the 8th floor of a building. Also, the answer is unitless, as it should be.

- 3.80. **THINK:** For a projectile, the launch angle is $\theta_0 = 68^\circ$. In time t , it achieves a maximum height $y = H$ and travels a horizontal distance $\Delta x = d$. Assume $y_0 = 0$. Determine H/d .

SKETCH:



RESEARCH: Assuming ideal parabolic motion, the maximum height equation can be used,

$$H = y_0 + \frac{v_{y_0}^2}{2g}.$$

Note that at the maximum height, the horizontal distance traveled is half of the total range. That is,

$$d = R/2 \text{ and thus, } R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

SIMPLIFY: $\frac{H}{d} = \frac{y_0 + v_{y_0}^2 / (2g)}{R/2} = \frac{v_0^2 \sin^2 \theta_0 / (2g)}{g \left(\frac{v_0^2 \sin 2\theta_0}{g} \right)} = \frac{\sin^2 \theta_0}{\sin 2\theta_0} = \frac{\sin \theta_0 \sin \theta_0}{2 \sin \theta_0 \cos \theta_0} = \frac{1}{2} \tan \theta_0$

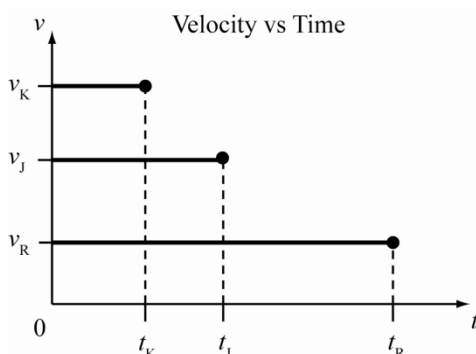
CALCULATE: $\frac{H}{d} = \frac{1}{2} \tan 68^\circ = 1.2375$

ROUND: The launch has two significant figures, so $H/d = 1.2$.

DOUBLE-CHECK: The ratio shows that the maximum height is 1.2 times the horizontal distance achieved in the same period of time. For such a steep launch angle, it is reasonable to expect $H > d$.

- 3.81. THINK:** The time it takes Robert to cover the distance of the walkway is $t_R = 30.0$ s. John's time is $t_J = 13.0$ s. Kathy walks on the walkway at the same speed as Robert, $v_K = v_R$. Determine the time, t_K , that it takes Kathy to cover the distance of the walkway. Take the distance of the walkway to be Δx .

SKETCH:



RESEARCH: Kathy's velocity relative to the airport reference frame, v_{Ka} , is the sum of the walkway velocity, v_{wa} , and her velocity relative to the walkway, v_{Kw} . v_{wa} is actually John's velocity relative to the airport reference frame, v_{Ja} , and v_{Kw} is Robert's velocity relative to the airport, v_{Ra} . So, $v_{Ka} = v_{wa} + v_{Kw}$; $v_{wa} = v_{Ja}$; and $v_{Kw} = v_{Ra}$. The constant velocity equation $v = \Delta x / t$ can be used to find t_K .

SIMPLIFY: $v_{Ka} = \frac{\Delta x}{t_K}$, or $v_{Ka} = v_{wa} + v_{Kw} = v_{Ja} + v_{Ra} = \frac{\Delta x}{t_J} + \frac{\Delta x}{t_R}$. $\frac{\Delta x}{t_K} = \frac{\Delta x}{t_J} + \frac{\Delta x}{t_R}$ and $t_K = \left(\frac{1}{t_J} + \frac{1}{t_R} \right)^{-1}$.

CALCULATE: $t_K = \left(\frac{1}{13.0 \text{ s}} + \frac{1}{30.0 \text{ s}} \right)^{-1} = 9.0698 \text{ s}$

ROUND: Each time given has three significant figures, so $t_K = 9.07$ s.

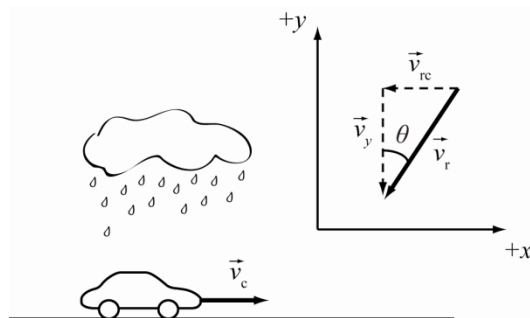
DOUBLE-CHECK: Kathy's time should be the fastest.

- 3.82. THINK:** The rain speed is $v_y = 7.00$ m/s downward. The car speed is

$$v_c = \left(\frac{60.0 \text{ km}}{1 \text{ h}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) = 16.67 \text{ m/s}.$$

Relative to the car, the rain has a horizontal speed of $v_{rc} = 16.67$ m/s directed towards the car. Determine the angle, θ , from the vertical at which the rain appears to be falling relative to the traveling car.

SKETCH: In the reference frame of the car:



RESEARCH: The x and y components of the velocity of the rain, relative to the traveling car are known. These components make a right-triangle (shown above), such that $\tan \theta = v_{rc} / v_y$.

SIMPLIFY: $\theta = \tan^{-1} \left(\frac{v_{rc}}{v_y} \right)$

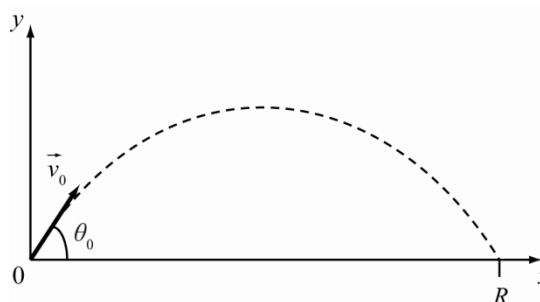
CALCULATE: $\theta = \tan^{-1} \left(\frac{16.67 \text{ m/s}}{7.00 \text{ m/s}} \right) = 67.22^\circ$

ROUND: $\theta = 67.2^\circ$ from the vertical.

DOUBLE-CHECK: This angle is less than 90° so it is reasonable that the rain is falling at such an angle.

- 3.83. **THINK:** Determine g when the range is $R = 2165 \text{ m}$, $v_0 = 50.0 \text{ m/s}$ and $\theta_0 = 30.0^\circ$.

SKETCH:



RESEARCH: Since the initial and final heights are equal, the range equation can be used:

$$R = \frac{v_0^2 \sin(2\theta_0)}{g}$$

SIMPLIFY: $g = \frac{v_0^2 \sin(2\theta_0)}{R}$

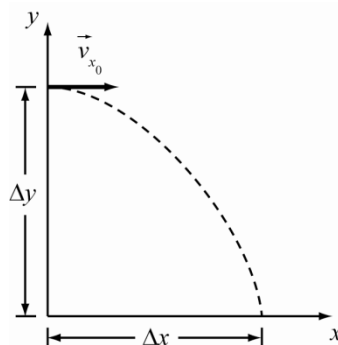
CALCULATE: $g = \frac{(50.0 \text{ m/s})^2 \sin(2(30.0^\circ))}{2165 \text{ m}} = 1.00003 \text{ m/s}^2$

ROUND: $g = 1.00 \text{ m/s}^2$

DOUBLE-CHECK: The units of the calculated g are correct.

- 3.84. **THINK:** The height is $y_0 = 40.0 \text{ m}$. The horizontal distance is $\Delta x = 7.00 \text{ m}$. Determine the minimum initial horizontal speed v_{x_0} . Assume the diver does not jump up, but rather out ($v_{y_0} = 0$).

SKETCH:



RESEARCH: If $v_{y_0} = 0$ for the diver, the initial angle is also $\theta_0 = 0$. The trajectory equation can be used,

$$y - y_0 = (\tan \theta_0) \Delta x - \frac{g(\Delta x)^2}{2v_{x_0}^2}$$

SIMPLIFY: Taking $y = 0$ (the base of the cliff),

$$0 = y_0 - \frac{g(\Delta x)^2}{2(v_{x_0})^2} \Rightarrow \frac{g(\Delta x)^2}{2v_{x_0}^2} = y_0 \Rightarrow v_{x_0} = \sqrt{\frac{g(\Delta x)^2}{2y_0}}$$

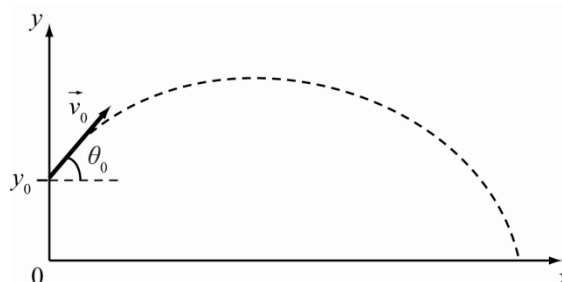
CALCULATE: $v_{x_0} = \sqrt{\frac{(9.81 \text{ m/s}^2)(7.00 \text{ m})^2}{2(40.0 \text{ m})}} = 2.451 \text{ m/s}$

ROUND: $v_{x_0} = 2.45 \text{ m/s}$

DOUBLE-CHECK: This is a reasonable velocity for a person to achieve at the start of their dive (i.e. a running start).

- 3.85. THINK:** The initial velocity is $v_0 = 32.0 \text{ m/s}$, the launch angle is $\theta_0 = 23.0^\circ$ and the initial height is $y_0 = 1.83 \text{ m}$. Determine the travel time t for the ball before it hits the ground at $y = 0$.

SKETCH:



RESEARCH: Assuming an ideal parabolic trajectory, the kinematic equation can be used for the vertical displacement, $y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$.

SIMPLIFY: $y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - (v_0 \sin \theta_0)t + (y_0 - y) = 0$

Solve for t using the quadratic formula:

$$t = \frac{v_0 \sin \theta_0 \pm \sqrt{(v_0 \sin \theta_0)^2 - 2g(y_0 - y)}}{g} = \frac{v_0 \sin \theta_0 \pm \sqrt{(v_0 \sin \theta_0)^2 - 2g(y_0 - y)}}{g}$$

CALCULATE: $t = \frac{(32 \text{ m/s})\sin(23^\circ) \pm \sqrt{((32 \text{ m/s})\sin(23^\circ))^2 + (2(9.81 \text{ m/s}^2))(1.83 \text{ m} - 0)}}{9.81 \text{ m/s}^2}$

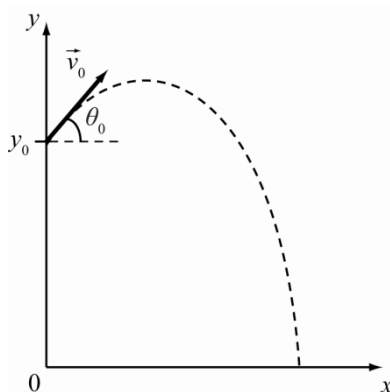
$$= \frac{12.5034 \text{ m/s} \pm \sqrt{156.33 \text{ m}^2/\text{s}^2 + 35.90 \text{ m}^2/\text{s}^2}}{9.81 \text{ m/s}^2}$$

$$= 2.6879 \text{ s or } -0.1388 \text{ s}$$

ROUND: Round to three significant figures. Then, $t = 2.69 \text{ s}$ (choosing the positive solution).

DOUBLE-CHECK: The units of the result are units of time. This seems to be a reasonable flight time for a thrown baseball.

- 3.86. THINK:** For the rock, the initial height is $y_0 = 34.9 \text{ m}$, the initial speed is $v_0 = 29.3 \text{ m/s}$ and the launch angle is $\theta_0 = 29.9^\circ$. Determine the speed of the rock, v , when it hits the ground at the bottom of the cliff, $y = 0$.

SKETCH:

RESEARCH: Assuming an ideal parabolic trajectory, when the rock descends to the height of the cliff after reaching its maximum height, it has the same vertical speed as it was launched with, only directed downward. That is, $v_y' = -v_{y_0}$, where v_y' occurs on the trajectory at the vertical position y_0 . In addition, $v_{y_0} \theta = v_0 \sin \theta_0$. Since, due to gravity, the speed changes only in the vertical direction, the constant acceleration equation, $v_y^2 = v_{y_0}^2 + 2a_y(y - y_0)$, can be used to determine the vertical speed at the bottom of the cliff. As the horizontal speed v_x remains constant ($v_x = v_{0x} = v_0 \cos \theta_0$), the speed at the bottom is $v = \sqrt{v_x^2 + v_y^2}$.

SIMPLIFY: Take the initial vertical speed to be v_y' . Then

$$v_y^2 = (v_y')^2 - 2g(-y_0) \Rightarrow v_y^2 = (-v_0 \sin \theta_0)^2 + 2g y_0$$

Also, $v_x = v_0 \cos \theta_0$. Then,

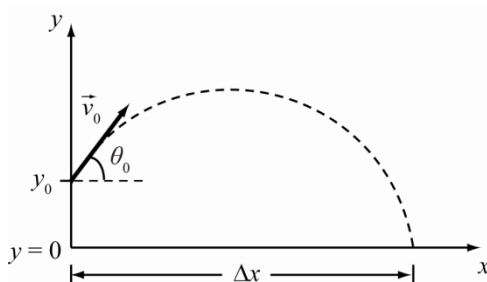
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(v_0 \cos \theta_0)^2 + (-v_0 \sin \theta_0)^2 + 2g y_0} = \sqrt{v_0^2 (\cos^2 \theta_0 + \sin^2 \theta_0) + 2g y_0} = \sqrt{v_0^2 + 2g y_0}.$$

CALCULATE: $v = \sqrt{(29.3 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)34.9 \text{ m}} = \sqrt{858.49 \text{ m}^2/\text{s}^2 + 684.74 \text{ m}^2/\text{s}^2}$
 $= \sqrt{1543.2 \text{ m}^2/\text{s}^2} = 39.284 \text{ m/s}$

ROUND: The sum in the square root is precise to the ones-place and therefore has four significant figures. Thus, $v = 39.28 \text{ m/s}$.

DOUBLE-CHECK: This velocity is greater than v_0 . It should be, given the acceleration in the vertical direction due to gravity.

- 3.87. **THINK:** For the shot-put, the initial speed is $v_0 = 13.0 \text{ m/s}$, the launch angle is $\theta_0 = 43.0^\circ$ and the initial height is $y_0 = 2.00 \text{ m}$. Determine (a) the horizontal displacement Δx and (b) the flight time t , after the shot hits the ground at $y = 0$.

SKETCH:

RESEARCH: Assuming ideal parabolic motion, find (a) Δx from the trajectory equation:

$$y = y_0 + \tan \theta_0 \Delta x - \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$$

and (b) t from the equation $\Delta x = v_0 \cos \theta_0 t$.

SIMPLIFY:

(a) With $y = 0$, $\frac{g}{2v_0^2 \cos^2 \theta_0}(\Delta x)^2 - (\tan \theta_0)\Delta x - y_0 = 0$. Solving this quadratic equation yields:

$$\Delta x = \frac{\tan \theta_0 \pm \sqrt{\tan^2 \theta_0 - 4\left(\frac{g}{2v_0^2 \cos^2 \theta_0}\right)(-y_0)}}{2\left(\frac{g}{2v_0^2 \cos^2 \theta_0}\right)} = \left(\tan \theta_0 \pm \sqrt{\tan^2 \theta_0 + \left(\frac{2gy_0}{v_0^2 \cos^2 \theta_0}\right)} \right) \left(\frac{v_0^2 \cos^2 \theta_0}{g} \right).$$

$$(b) \quad t = \frac{\Delta x}{v_0 \cos \theta_0}$$

CALCULATE:

$$\begin{aligned} (a) \quad \Delta x &= \left(\tan(43^\circ) \pm \sqrt{\tan^2(43^\circ) + \left(\frac{2(9.81 \text{ m/s}^2)(2 \text{ m})}{(13.0 \text{ m/s})^2 \cos^2(43^\circ)}\right)} \right) \left(\frac{(13.0 \text{ m/s})^2 \cos^2(43^\circ)}{9.81 \text{ m/s}^2} \right) \\ &= (0.9325 \pm \sqrt{0.8696 + 0.4341})(9.2145 \text{ m}) \\ &= (0.9325 \pm \sqrt{1.3037})(9.2145 \text{ m}) \\ &= 19.114 \text{ m or } -1.928 \text{ m} \end{aligned}$$

$$(b) \quad t = \frac{19.114 \text{ m}}{(13.0 \text{ m/s}) \cos(43^\circ)} = 2.0104 \text{ s}$$

ROUND:

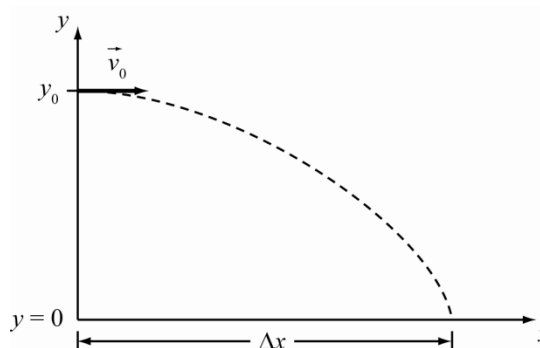
(a) The sum under the square root is precise to the tenth-place, and so has three significant figures. Then, $\Delta x = 19.1 \text{ m}$ (take the positive root).

(b) Since θ_0 and Δx have three significant figures, $t = 2.01 \text{ s}$.

DOUBLE-CHECK: For near optimal launch angle (optimal being $\theta_0 = 45^\circ$), a horizontal displacement of 19.1 m is reasonable. The flight time of 2.0 s is reasonable for this horizontal displacement.

- 3.88. **THINK:** For the phone, the initial height is $y_0 = 71.8 \text{ m}$, the launch angle is $\theta_0 = 0^\circ$ and the initial speed is $v_0 = 23.7 \text{ m/s}$. Determine (a) the horizontal distance Δx and (b) the final speed v upon hitting the water at $y = 0$. Note: $v_{y_0} = 0$.

SKETCH:



RESEARCH: Assuming ideal parabolic motion,

(a) Δx can be determined from the trajectory equation, $y = y_0 + \tan \theta_0 \Delta x - \frac{g \Delta x^2}{2v_0^2 \cos^2 \theta_0}$.

(b) $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$. v_x does not change, as there is no horizontal acceleration; $v_x = v_{x_0} = v_0 \cos \theta_0$. Since the vertical kinematics are governed by free-fall under gravity, to find v_y use $v_y^2 = v_{y_0}^2 - 2g(y - y_0)$.

SIMPLIFY:

(a) With $y = 0$, the trajectory equation becomes

$$\frac{g \Delta x^2}{2v_0^2 \cos^2 \theta_0} - \tan \theta_0 \Delta x - y_0 = 0.$$

With $\theta_0 = 0^\circ$, this equation reduces to $\frac{g \Delta x^2}{2v_0^2} - y_0 = 0$. Thus, $\Delta x = \sqrt{\frac{2v_0^2 y_0}{g}}$.

(b) $v_x = v_0$. Since $v_{y_0} = 0$ and $y = 0$, $v_y^2 = 2gy_0$. Then, $|\vec{v}| = \sqrt{v_0^2 + 2gy_0}$.

CALCULATE:

(a) $\Delta x = \sqrt{\frac{2(23.7 \text{ m/s})^2 (71.8 \text{ m})}{9.81 \text{ m/s}^2}} = 90.676 \text{ m}$

(b) $|\vec{v}| = \sqrt{(23.7 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(71.8 \text{ m})} = \sqrt{561.69 \text{ m}^2/\text{s}^2 + 1408.716 \text{ m}^2/\text{s}^2} = 44.389 \text{ m/s}$

ROUND:

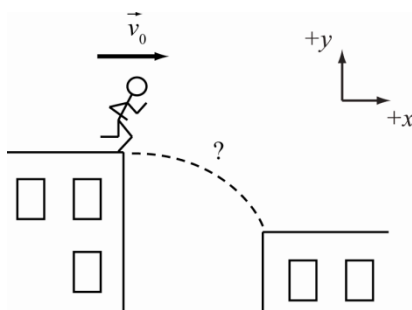
(a) All the values used in the product have three significant figures, so $\Delta x = 90.7 \text{ m}$.

(b) The sum under the square root is precise to the tens-place, and has three significant figures, so $v = 44.4 \text{ m/s}$.

DOUBLE-CHECK: The horizontal displacement is close to the height; this is reasonable for an object thrown from the given height. The final speed is greater than the initial speed, as it should be.

- 3.89. THINK:** For the burglar, the initial speed is $v_0 = 4.20 \text{ m/s}$ and the angle is $\theta_0 = 0^\circ$. Determine if the burglar can make it to the next roof, which is a horizontal distance of 5.50 m away and a vertical distance of 4.00 m lower.

SKETCH:



RESEARCH: There are two ways to approach this problem. Firstly, the vertical displacement can be determined when the horizontal displacement is $\Delta x = 5.50 \text{ m}$. If the magnitude of Δy is less than 4.0 m, the burglar will reach the next rooftop. Secondly, the horizontal displacement Δx can be determined when the vertical displacement is $\Delta y = -4.00 \text{ m}$. For this solution, Δy will be determined. The trajectory equation for ideal parabolic motion can be used:

$$y = y_0 + \tan \theta_0 \Delta x - \frac{g \Delta x^2}{2v_0^2 \cos^2 \theta_0}.$$

SIMPLIFY: With $\theta_0 = 0^\circ$, $y = -\frac{g\Delta x^2}{2v_0^2}$.

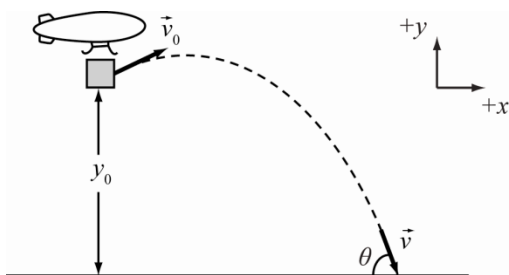
CALCULATE: $y = -\frac{(9.81 \text{ m/s}^2)(5.5 \text{ m})^2}{2(4.2 \text{ m/s})^2} = -8.4114 \text{ m}$

ROUND: The values used in the product have three significant figures, so $y = -8.41 \text{ m}$. This means that after the burglar reaches a horizontal displacement of 5.50 m, he has dropped 8.41 m from the first rooftop and cannot reach the second rooftop.

DOUBLE-CHECK: The alternate method listed above also reveals that the burglar will not reach the second rooftop.

- 3.90. THINK:** The initial vertical speed of the package is $v_{y_0} = 7.50 \text{ m/s}$. Its horizontal speed is $v_{x_0} = v_x = 4.70 \text{ m/s}$ and the initial height is $y_0 = 80.0 \text{ m}$. Determine (a) the fall time, t , to the ground, $y = 0$ and (b) the magnitude and direction of the velocity upon impact.

SKETCH:



RESEARCH:

(a) To determine t , use the equation $y - y_0 = v_{y_0}t - (gt^2)/2$.

(b) The speed v is given by $v = \sqrt{v_x^2 + v_y^2}$. v_x remains constant. To determine v_y , use the constant acceleration equation, $v_y^2 = v_{y_0}^2 + 2a\Delta y$. To determine the direction, note that $\tan\theta = v_y / v_x$, where θ is above the horizontal.

SIMPLIFY:

(a) With $y = 0$, $-y_0 = v_{y_0}t - \frac{gt^2}{2} \Rightarrow \frac{gt^2}{2} - v_{y_0}t - y_0 = 0$. Solving the quadratic gives:

$$t = \frac{v_{y_0} \pm \sqrt{v_{y_0}^2 - 4\left(\frac{g}{2}\right)(-y_0)}}{2\left(\frac{g}{2}\right)} = \frac{v_{y_0} \pm \sqrt{v_{y_0}^2 + 2gy_0}}{g}.$$

(b) $v_y^2 = v_{y_0}^2 - 2g(y - y_0) = v_{y_0}^2 + 2gy_0$. Then, $v = \sqrt{v_x^2 + v_{y_0}^2 + 2gy_0}$, and

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-\sqrt{v_{y_0}^2 + 2gy_0}}{v_x}\right),$$

where theta is measured with respect to the $+x$ -axis and therefore comes out negative.

CALCULATE:

$$(a) t = \frac{7.50 \text{ m/s} \pm \sqrt{(7.50 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(80.0 \text{ m})}}{9.81 \text{ m/s}^2} = 4.875 \text{ s} \text{ or } -3.346 \text{ s}$$

$$(b) \ v = \sqrt{(4.70 \text{ m/s})^2 + (7.50 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(80.0 \text{ m})} = 40.59 \text{ m/s}$$

$$\theta = \tan^{-1} \left(\frac{-\sqrt{(7.50 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(80.0 \text{ m})}}{(4.70 \text{ m/s})} \right) = -83.35^\circ$$

ROUND: $t = 4.88 \text{ s}$ and $\theta = -83.4^\circ$.

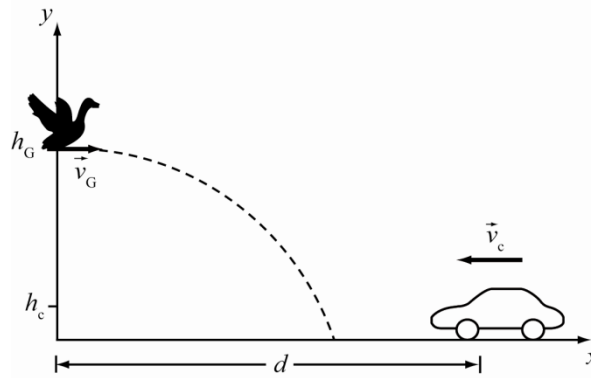
DOUBLE-CHECK: A five second fall is reasonable for such a height. The final speed is greater than the initial speed $v_0 = \sqrt{v_{x_0}^2 + v_{y_0}^2} = 8.9 \text{ m/s}$. The impact angle is almost -90° , as it should be since $|v_y|$ is much greater than $|v_x|$ after the fall.

- 3.91. THINK:** The height of the goose is $h_g = 30.0 \text{ m}$. The height of the windshield is $h_c = 1.00 \text{ m}$. The speed of the goose is $v_g = 15.0 \text{ m/s}$. The speed of the car is

$$v_c = \frac{100.0 \text{ km}}{1 \text{ hr}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 27.7778 \text{ m/s}.$$

The initial horizontal distance between the goose and the car is $d = 104.0 \text{ m}$. The goose and the car move toward each other. Determine if (a) the egg hits the windshield and (b) the relative velocity of the egg with respect to the windshield, v_g' . Let θ be the angle the egg makes with the horizontal when it impacts.

SKETCH:



RESEARCH:

(a) The egg must have a vertical displacement $\Delta y_g = h_c - h_g$ to hit the windshield. Use $\Delta y_g = (v_0 \sin \theta_0)t - \frac{1}{2}(gt^2)$ to determine the fall time t . Use $\Delta x_g = (v_0 \cos \theta_0)t$ to determine the horizontal displacement of the egg. In order for the egg to collide with the windshield, the car must travel $\Delta x_c = d - \Delta x_g$ in time t . Use $\Delta x = vt$ to determine the car's travel distance Δx_c . Note that the launch angle of the egg is $\theta_0 = 0^\circ$.

(b) The horizontal component of the egg's speed in the reference frame of the windshield will be $v_{gx}' = v_{gx} + v_c$ because the car is moving toward the egg in the horizontal direction. Because the car has no vertical speed, the vertical speed of the egg in the reference frame of the car is unchanged, $v_{gy}' = v_{gy}$. To

determine v_{gy} , use $v_y^2 = v_{y_0}^2 + 2a\Delta y$. Then $v_g' = \sqrt{(v_{gx}')^2 + (v_{gy}')^2}$. The angle of impact is implicitly given

$$\text{by } \tan \theta = \frac{v_{gy}'}{v_{gx}'}$$

SIMPLIFY:

(a) Since $\theta_0 = 0^\circ$ and $v_0 = v_g$, $v_{gy_0} = v_g \sin \theta_0 = 0$ and $v_{gx_0} = v_g \cos \theta_0 = v_g$. Then

$$\Delta y_g = -\frac{1}{2}gt^2 \Rightarrow h_c - h_g = -\frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(h_g - h_c)}{g}},$$

and also $\Delta x_g = v_g t = v_g \sqrt{\frac{2(h_g - h_c)}{g}}$. In this time, the car travels $\Delta x_c = v_c t = v_c \sqrt{\frac{2(h_g - h_c)}{g}}$.

(b) $v_{gx}' = v_{gx} + v_c = v_g + v_c$ and $v_{gy}' = v_{gy} = \sqrt{2g\Delta y_g} = \sqrt{2g(h_g - h_c)}$. Then, substituting gives:

$v_g' = \sqrt{(v_g + v_c)^2 + 2g(h_g - h_c)}$. The angle of impact relative to the car is given by the equation:

$$\theta = \arctan\left(\frac{v_{gy}'}{v_{gx}'}\right) = \arctan\left(\frac{\sqrt{2g(h_g - h_c)}}{v_g + v_c}\right).$$

CALCULATE:

$$(a) \Delta x_c = (27.778 \text{ m/s})\sqrt{\frac{2(30.0 \text{ m} - 1.00 \text{ m})}{9.81 \text{ m/s}^2}} = 67.542 \text{ m}$$

$$d - \Delta x_g = d - v_g \sqrt{\frac{2(h_g - h_c)}{g}} = (104.0 \text{ m}) - (15.0 \text{ m/s})\sqrt{\frac{2(30.0 \text{ m} - 1.00 \text{ m})}{9.81 \text{ m/s}^2}} = 67.527 \text{ m}$$

$$(b) v_g' = \sqrt{(15.0 \text{ m/s} + 27.7778 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(30.0 \text{ m} - 1.00 \text{ m})} = \sqrt{2398.94 \text{ m}^2/\text{s}^2} = 48.98 \text{ m/s}$$

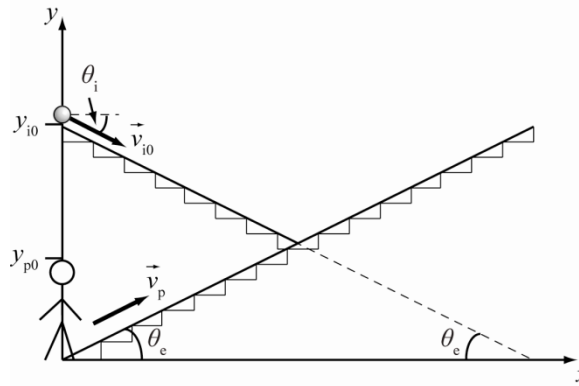
$$\theta = \arctan\left(\frac{\sqrt{2(9.81 \text{ m/s}^2)(30.0 \text{ m} - 1.00 \text{ m})}}{15.0 \text{ m/s} + 27.7778 \text{ m/s}}\right) = 29.144^\circ$$

ROUND: Rounding to three significant figures, $\Delta x_c = 67.5 \text{ m}$ and $d - \Delta x_g = 67.5 \text{ m}$. The egg hits the windshield at a speed of 49.0 m/s relative to the windshield at an angle of 29.1° above the horizontal.

DOUBLE-CHECK: The speed of the egg relative to the windshield is greater than the speed of the car and the goose.

- 3.92. THINK:** The initial speed of the ice cream is $v_{i0} = 0.400 \text{ m/s}$ and the escalator angle is $\theta_e = 40.0^\circ$. The ice cream launch angle is $\theta_i = -\theta_e$ (below the horizontal) and the initial height of the ice cream is $y_{i0} = 10.0 \text{ m}$. The initial height of the professor's head is $y_{p0} = 1.80 \text{ m}$, the professor's speed is $v_p = 0.400 \text{ m/s}$, and the professor's direction is $\theta_p = \theta_e$. If the ice cream lands on the professor's head then the kinematic equations can be used to determine the ice cream flight time t , the final vertical position of the ice cream $y_i = y_p$, and the speed of the ice cream relative to the professor's head, v_{ip} .

SKETCH:



RESEARCH: Since the professor and the ice cream have the same horizontal starting point and horizontal speed, the ice cream does land on your professor's head because they always have the same horizontal position. At the time t of the collision, the final vertical positions of the ice cream and the professor are equal, $y_i = y_p \equiv y$. To find t use $y_i - y_{i0} = v_{i0,y}t - \frac{1}{2}gt^2$ and $y_p - y_{p0} = v_{p0,y}t$ (the professor does not accelerate). With t known, the vertical height y can be found. The speed of the ice cream v_{iy} can be found from $v_{iy}^2 = v_{i0,y}^2 - 2g(y - y_{i0})$.

SIMPLIFY: At time t , $y_i = y_p$:

$$y_i = y_{i0} + v_{i0,y}t - \frac{1}{2}gt^2; \quad y_p = y_{p0} + v_{p,y}t$$

$$y_{i0} + v_{i0,y}t - \frac{1}{2}gt^2 = y_{p0} + v_{p,y}t$$

$$y_{i0} - v_{i0} \sin \theta_e y - \frac{1}{2}gt^2 = y_{p0} + v_p \sin \theta_e$$

$$\frac{1}{2}gt^2 - 2v_{i0} \sin \theta_e y + (y_{p0} - y_{i0}) = 0,$$

since $v_{i0} = v_p$. Use the quadratic formula to solve for t :

$$t = \frac{-2v_{i0} \sin \theta_e \pm \sqrt{4v_{i0}^2 \sin^2 \theta_e g + 2(y_{i0} - y_{p0})}}{g}.$$

Recall that at time t , the professor's head and the ice cream are at the same vertical height y :

$$y = v_{p0}t + v_p \sin \theta_e.$$

The speed of the ice cream relative to the professor is $v_{ip} = \sqrt{(v_{ip,x})^2 + (v_{ip,y})^2}$, where $v_{ip,x} = 0$ (they have the same horizontal speed), $v_{ip,y} = v_{iy} + v_{py}$ (they move towards each other vertically), v_{iy} can be found from $v_{iy}^2 = v_{i0,y}^2 - 2g(y - y_{i0})$, and $v_{py} = v_{p,y}$ (since the professor does not accelerate). Putting this altogether gives:

$$v_{ip} = \sqrt{(v_{i0} \sin \theta_e)^2 - 2v_{i0}(-\theta_{i0}) + v_p \sin \theta_e}.$$

CALCULATE: The time that the ice cream lands on the professor's head is:

$$t = \frac{-2(0.400 \text{ m/s}) \sin(40.0^\circ) \pm \sqrt{4(0.400 \text{ m/s})^2 \sin^2(40.0^\circ) + 2(9.81 \text{ m/s}^2)(10.0 \text{ m} - 1.8 \text{ m})}}{9.81 \text{ m/s}^2}$$

$$= 1.2416 \text{ s or } -1.3464 \text{ s}$$

The positive answer is correct. With t known, the vertical height at which the ice cream lands on the professor's head is:

$$y = (1.80 \text{ m}) + (0.400 \text{ m/s})(\sin 40.0^\circ)(1.2416 \text{ s}) = 2.1192 \text{ m}.$$

The relative speed of the ice cream with respect to the professor's head at the time of impact is:

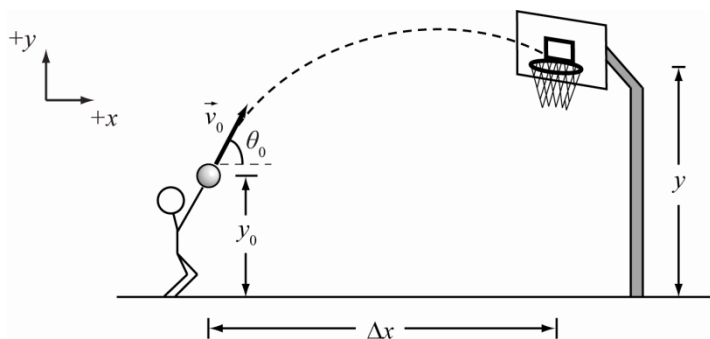
$$v_{ip} = \sqrt{(0.400 \text{ m/s})^2 \sin^2(40.0^\circ) - 2(9.81 \text{ m/s}^2)(2.1192 \text{ m} - 10.0 \text{ m}) + (0.400 \text{ m/s})\sin 40.0^\circ} = 12.694 \text{ m/s}.$$

ROUND: $t = 1.24 \text{ s}$, $y = 2.12 \text{ m}$, and $v_{ip} = 12.7 \text{ m/s}$.

DOUBLE-CHECK: Considering the slow speed of the escalator and the original vertical positions, these values are reasonable.

- 3.93. THINK:** The ball's horizontal distance from the hoop is $\Delta x = 7.50 \text{ m}$. The initial height is $y_0 = 2.00 \text{ m}$. The final height is $y = 3.05 \text{ m}$. The launch angle is $\theta_0 = 48.0^\circ$. Determine the initial speed v_0 .

SKETCH:



RESEARCH: To find v_0 , use $\Delta y = \tan \theta_0 \Delta x - \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$.

SIMPLIFY:
$$\tan \theta_0 \Delta x - \Delta y = \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$$

$$\tan \theta_0 \Delta x - (y - y_0) = \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$$

$$2v_0^2 \cos^2 \theta_0 = \frac{g(\Delta x)^2}{\tan \theta_0 \Delta x - y + y_0}$$

$$v_0 = \sqrt{\frac{g(\Delta x)^2}{2 \cos^2 \theta_0 (\tan \theta_0 \Delta x - y + y_0)}}$$

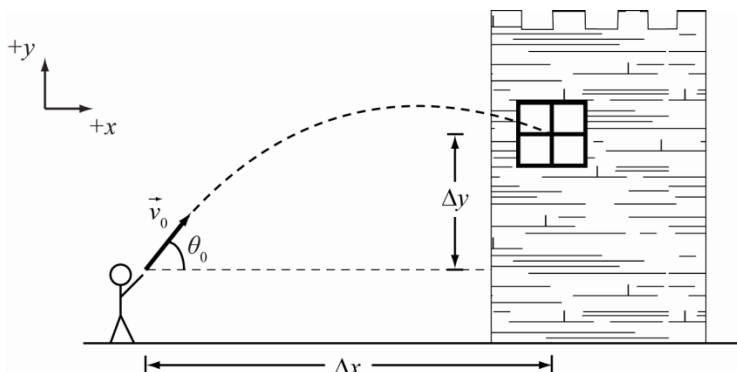
CALCULATE:
$$v_0 = \sqrt{\frac{9.81 \text{ m/s}^2 (7.50 \text{ m})^2}{2 \cos^2(48.0^\circ) ((7.50 \text{ m}) \tan(48.0^\circ) - 3.05 \text{ m} + 2.00 \text{ m})}} = 9.2006 \text{ m/s}$$

ROUND: As θ_0 has three significant figures, $v_0 = 9.20 \text{ m/s}$.

DOUBLE-CHECK: The result has units of velocity. This is a reasonable speed to throw a basketball.

- 3.94. **THINK:** The pebble's launch angle is $\theta_0 = 37^\circ$. The vertical displacement is $\Delta y = 7.0$ m. The horizontal displacement is $\Delta x = 10.0$ m. Determine the pebble's initial speed v_0 .

SKETCH:



RESEARCH: To find v_0 , use $\Delta y = \tan \theta_0 \Delta x - \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$.

SIMPLIFY: $\tan \theta_0 \Delta x - \Delta y = \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$, $v_0 = \sqrt{\frac{g(\Delta x)^2}{2 \cos^2 \theta_0 (\tan \theta_0 \Delta x - \Delta y)}}$

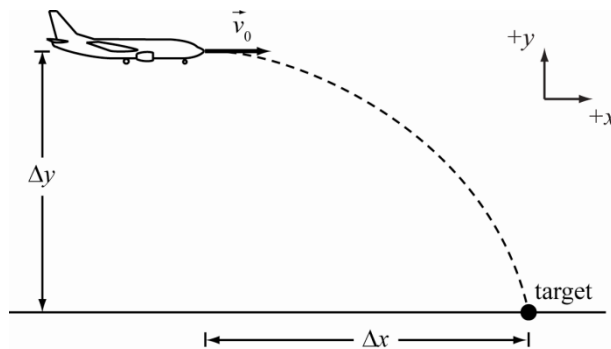
CALCULATE: $v_0 = \sqrt{\frac{9.81 \text{ m/s}^2 (10.0 \text{ m})^2}{2 \cos^2 (37^\circ) ((10.0 \text{ m}) \tan (37^\circ) - 7.0 \text{ m})}} = 37.89 \text{ m/s}$

ROUND: With two significant figures in θ_0 , $v_0 = 38 \text{ m/s}$.

DOUBLE-CHECK: This speed is fast, but reasonable if, for example, Romeo is a pitcher in the Major leagues.

- 3.95. **THINK:** The bomb's vertical displacement is $\Delta y = -5.00 \cdot 10^3$ m (falling down). The initial speed is $v_0 = 1000. \text{ km/h} \left(\frac{\text{h}}{3600 \text{ s}} \right) (1000 \text{ m/km}) = 277.8 \text{ m/s}$. The launch angle is $\theta_0 = 0^\circ$. Determine the distance from a target, Δx , and the margin of error of the time Δt if the target is $d = 50.0$ m wide.

SKETCH:



RESEARCH: To find Δx , use the trajectory equation: $\Delta y = \tan \theta_0 \Delta x - \frac{g(\Delta x)^2}{2v_0^2 \cos^2 \theta_0}$.

To find Δt , consider the time it would take the bomb to travel the horizontal distance d ; this is the margin of error in the time. The margin of error for the release time Δt can be determined from $d = v_0 \cos \theta_0 \Delta t$.

SIMPLIFY: Note, $\tan\theta_0 = 0$ and $\cos\theta_0 = 1$ for $\theta_0 = 0$.

$$\Delta y = -\frac{g(\Delta x)^2}{2v_0^2} \text{ and } \Delta x = \sqrt{\frac{-2v_0^2 \Delta y}{g}}$$

For Δt , use $d = v_0 \cos \theta_0 \Delta t \Rightarrow \Delta t = d / v_0$.

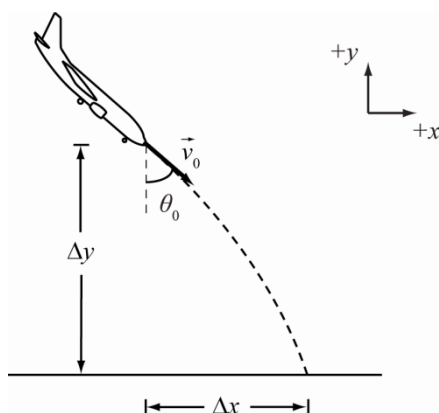
CALCULATE: $\Delta x = \sqrt{\frac{-2(277.8 \text{ m/s})^2(-5.00 \cdot 10^3 \text{ m})}{9.81 \text{ m/s}^2}} = 8869 \text{ m}, \quad \Delta t = \frac{50.0 \text{ m}}{277.8 \text{ m/s}} = 0.1800 \text{ s}$

ROUND: $\Delta x = 8.87 \text{ km}$ and $\Delta t = 0.180 \text{ s}$.

DOUBLE-CHECK: Considering the altitude and the terrific speed of the airplane, these values are reasonable.

- 3.96. THINK:** The package's launch angle is $\theta_0 = 49.0^\circ$ downward from the vertical. The vertical displacement is $\Delta y = -600. \text{ m}$. The flight time is $t = 3.50 \text{ s}$. Determine the horizontal displacement Δx .

SKETCH:



RESEARCH: Note, $v_{x_0} = v_0 \sin \theta_0$ for the given θ_0 and $v_{y_0} = -v_0 \cos \theta_0$. To determine Δx , use $\Delta x = v_{x_0} t$.

First, determine v_0 from $\Delta y = v_{y_0} t - \frac{1}{2} g t^2$.

SIMPLIFY: $\Delta y = v_{y_0} t - \frac{1}{2} g t^2 = -v_0 \cos \theta_0 t - \frac{1}{2} g t^2 \Rightarrow v_0 = -\frac{\left(\Delta y + \frac{1}{2} g t^2\right)}{\cos \theta_0 t}$

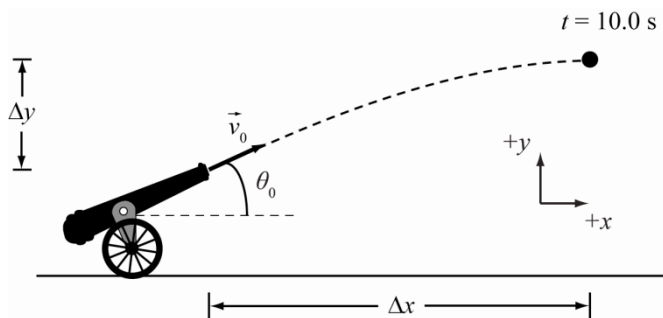
$$\Delta x = v_{x_0} t = v_0 \sin \theta_0 t = -\frac{\left(\Delta y + \frac{1}{2} g t^2\right)}{\cos \theta_0 t} \sin \theta_0 t = -\left(\Delta y + \frac{1}{2} g t^2\right) \tan \theta_0$$

CALCULATE: $\Delta x = -\left(-600. \text{ m} + \frac{1}{2}(9.81 \text{ m/s}^2)(3.50 \text{ s})^2\right) \tan(49.0^\circ) = 621.1 \text{ m}$

ROUND: $\Delta x = 621 \text{ m}$

DOUBLE-CHECK: The units for the result are units of distance.

- 3.97. THINK:** During the flight of the cannonball it moves with constant horizontal velocity component while at the same time undergoing free-fall in vertical direction.
- Since the time to hit a given point in space is given, we can easily extract the initial velocity from using our kinematic equations.
 - If we know the initial velocity, then we can simply use our equation for the maximum height of the trajectory.
 - If we know the initial velocity, then we can calculate the velocity at any given point in time, simply by applying our kinematic equations one more time.

SKETCH:**RESEARCH:**

(a) The horizontal velocity component is $v_x = v_{x0} = x/t$. We can get the vertical component of the initial velocity from $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

(b) To determine H , use $H = y_0 + \frac{v_{y0}^2}{2g}$.

(c) Since we have determined the initial velocity components in part (a), we get the velocity components at any time from $v_x = v_{x0}$; $v_y = v_{y0} - gt$.

SIMPLIFY:

(a) Since $y_0 = 0$ we get for the initial vertical velocity component $y + \frac{1}{2}gt^2 = v_{y0}t \Rightarrow v_{y0} = y/t + \frac{1}{2}gt$

Parts (b) and (c) are already in the shape we can use to put in numbers.

CALCULATE:

(a) $v_{x0} = (500. \text{ m}) / (10.0 \text{ s}) = 50.0 \text{ m/s}$; $v_{y0} = (100. \text{ m}) / (10.0 \text{ s}) + \frac{1}{2}(9.81 \text{ m/s}^2)(10.0 \text{ s}) = 59.05 \text{ m/s}$

$v_0 = \sqrt{(50.0 \text{ m/s})^2 + (59.05 \text{ m/s})^2} = 77.375 \text{ m/s}$, Angle $= \tan^{-1}((59.05 \text{ m/s}) / (50.0 \text{ m/s})) = 49.7441^\circ$

(b) $H = \frac{(59.05 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 177.72 \text{ m}$

(c) $v_x = 50.0 \text{ m/s}$; $v_y = 59.05 \text{ m/s} - (9.81 \text{ m/s}^2)(10.0 \text{ s}) = -39.05 \text{ m/s}$

$v = \sqrt{(50.0 \text{ m/s})^2 + (-39.05 \text{ m/s})^2} = 63.442 \text{ m/s}$, Angle $= \tan^{-1}((-39.05 \text{ m/s}) / (50.0 \text{ m/s})) = -37.9898^\circ$

ROUND:

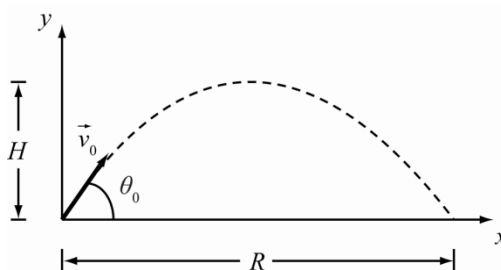
(a) $\vec{v}_0 = 77.4 \text{ m/s}$ at 49.7° above the horizontal

(b) $H = 178 \text{ m}$

(c) $v = 63.4 \text{ m/s}$ at 38.0° below the horizontal

DOUBLE-CHECK: The velocity is reasonable for a cannon. $H \geq \Delta y$ as it must be, and $v < v_0$, which it should be with $y > y_0$.

- 3.98. **THINK:** When the ball is at height $y = 12.5 \text{ m}$, its velocity is $\vec{v} = (v_x \hat{x} + v_y \hat{y}) = (5.6 \hat{x} + 4.1 \hat{y}) \text{ m/s}$. The kinematic equations can be used to determine (a) the maximum height H , (b) the range R and (c) the magnitude and direction of the velocity when the ball hits the ground.

SKETCH:**RESEARCH:**

(a) The maximum height that the soccer ball rises to is given by $H = y_0 + \frac{v_{y0}^2}{2g}$. The initial vertical velocity

v_{y0} can be determined from $v_y^2 = v_{y0}^2 - 2g(y - y_0)$.

(b) The horizontal distance that the soccer ball travels is given by $R = \frac{v_0^2}{g} \sin 2\theta_0$.

(c) For ideal parabolic motion when the initial and final height of the projectile is the same, the initial and final speeds are the same. The angle of impact is the same, but it is below the horizontal.

SIMPLIFY:

(a) Since $y_0 = 0$,

$$v_{y0} = \sqrt{v_y^2 + 2gy} \Rightarrow H = \frac{v_y^2 + 2gy}{2g} = \frac{v_y^2}{2g} + y$$

(b) The initial velocity v_0 is given by $v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{v_x^2 + v_y^2 + 2gy}$ and the launch angle θ_0 is given

$$\text{by } v_x = v_0 \cos \theta_0 \Rightarrow \theta_0 = \cos^{-1} \left(\frac{v_x}{v_0} \right).$$

(c) $v = v_0$ and $\theta = -\theta_0$.

CALCULATE:

$$(a) \quad H = \frac{(4.1 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 12.5 \text{ m} = 13.357 \text{ m}$$

$$(b) \quad v_0 = \sqrt{(5.6 \text{ m/s})^2 + (4.1 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(12.5 \text{ m})} = 17.13 \text{ m/s}, \quad \theta_0 = \cos^{-1} \left(\frac{5.6 \text{ m/s}}{17.13 \text{ m/s}} \right) = 70.92^\circ,$$

$$R = \frac{(17.13 \text{ m/s})^2}{(9.81 \text{ m/s}^2)} \sin(2(70.92^\circ)) = 18.48 \text{ m}$$

(c) $v = 17.13 \text{ m/s}$, $\theta = -70.92^\circ$

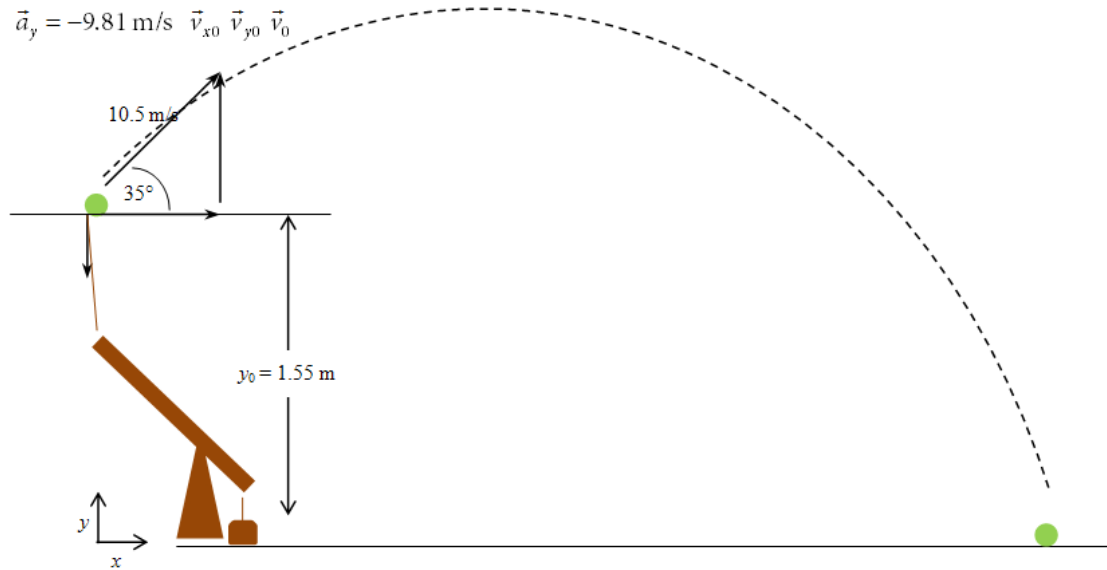
ROUND: (a) To the first decimal place, $H = 13.4 \text{ m}$. To two significant figures, (b) $R = 18 \text{ m}$ and (c) $v = 17 \text{ m/s}$, 71° below the horizontal.

DOUBLE-CHECK: With such a steep launch angle, the range is not expected to be large. Also, H is greater than $y = 12.5 \text{ m}$, which is expected.

Multi-Version Exercises

- 3.99. **THINK:** The initial height, velocity, and angle of the tennis ball are known. To find the total horizontal distance covered before the tennis ball hits the ground it makes sense to decompose the motion into horizontal and vertical components. Then, find the time at which the tennis ball hits the ground ($y = 0$) and determine the horizontal position at that time. To make the problem as simple as possible, choose $y = 0$ m to be the ground and $x = 0$ m to be the location of the trebuchet where the ball is released.

SKETCH: A sketch helps to see exactly how to decompose the initial velocity into horizontal and vertical components.



RESEARCH: This problem involves ideal projectile motion. Since there is no horizontal acceleration and the tennis ball starts at $x_0 = 0$, the equation (3.11) for the horizontal position at time t is $x = v_{x0}t$. Equation (3.13) gives the vertical position as $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$. To find a final answer, it is necessary to determine the x - and y -components of the initial velocity, given by $v_{y0} = v_0 \sin \theta$ and $v_{x0} = v_0 \cos \theta$.

SIMPLIFY: To find the time when the tennis ball hits the ground, it is necessary to find a non-negative solution to the equation $0 = y_0 + v_{y0}t - \frac{1}{2}gt^2$. The quadratic formula gives a solution of

$$t = \frac{-v_{y0} \pm \sqrt{(v_{y0})^2 - 4(-\frac{1}{2}g)(y_0)}}{2(-\frac{1}{2}g)} = \frac{v_{y0} \pm \sqrt{(v_{y0})^2 + 2gy_0}}{g}. \text{ It will be necessary to take the positive square}$$

root here: the tennis ball cannot land *before* it is released. This time can then be used with the equation for horizontal position to get the position when the tennis ball hits the ground at

$$x = v_{x0}t = v_{x0} \left(\frac{v_{y0} + \sqrt{(v_{y0})^2 + 2gy_0}}{g} \right). \text{ Combining this with the equations for the horizontal and vertical}$$

components of the initial velocity ($v_{y0} = v_0 \sin \theta$ and $v_{x0} = v_0 \cos \theta$) gives that the tennis ball lands at

$$x = (v_0 \cos \theta) \left(\frac{v_0 \sin \theta + \sqrt{(v_0 \sin \theta)^2 + 2gy_0}}{g} \right).$$

CALCULATE: The problem states that the initial height $y_0 = 1.55$ m. The initial velocity $v_0 = 10.5$ m/s at an angle of $\theta = 35^\circ$ above the horizontal. The gravitational acceleration on Earth is -9.81 m/s². Thus the tennis ball lands at

$$\begin{aligned}
 x &= (v_0 \cos \theta) \left(\frac{v_0 \sin \theta + \sqrt{(v_0 \sin \theta)^2 + 2gy_0}}{g} \right) \\
 &= [10.5 \cos(35^\circ)] \left(\frac{10.5 \sin(35^\circ) + \sqrt{(10.5 \sin(35^\circ))^2 + 2 \cdot 9.81 \cdot 1.55}}{\sqrt{9.81}} \right) \\
 &= 12.43999628
 \end{aligned}$$

ROUND: Since the measured values have 3 significant figures, the answer should also have three significant figures. Thus the tennis ball travels a horizontal distance of 12.4 m before it hits the ground.

DOUBLE-CHECK: Using equation (3.22) for the path of a projectile, it is possible to work backwards from the initial position $(x_0, y_0) = (0, 1.55)$ and the position when the tennis ball lands $(x, y) = (12.4, 0)$, and angle $\theta_0 = 35^\circ$ to find the starting velocity, which should confirm what was given originally.

$$\begin{aligned}
 y &= y_0 + (\tan \theta_0)x - \frac{g}{2v_0^2 \cos^2 \theta_0} x^2 \Rightarrow \\
 0 &= 1.55 + (\tan 35^\circ) \cdot 12.4 - \frac{9.81}{2(v_0)^2 \cos^2 35^\circ} (12.4)^2 \Rightarrow \\
 1.55 + (\tan 35^\circ) \cdot 12.4 &= \frac{9.81}{2(v_0)^2 \cos^2 35^\circ} (12.4)^2 \Rightarrow \\
 (1.55 + (\tan 35^\circ) \cdot 12.4) \left[\frac{(v_0)^2}{1.55 + (\tan 35^\circ) \cdot 12.4} \right] &= \frac{9.81 \cdot (12.4)^2}{2(v_0)^2 \cos^2 35^\circ} \left[\frac{(v_0)^2}{1.55 + (\tan 35^\circ) \cdot 12.4} \right] \Rightarrow \\
 (v_0)^2 &= \frac{9.81 \cdot (12.4)^2}{2 \cos^2 35^\circ (1.55 + (\tan 35^\circ) \cdot 12.4)} \Rightarrow \\
 v_0 &= \sqrt{\frac{9.81 \cdot (12.4)^2}{2 \cos^2 35^\circ (1.55 + (\tan 35^\circ) \cdot 12.4)}} \Rightarrow \\
 v_0 &= 10.4805539
 \end{aligned}$$

Rounded to three significant figures, this becomes $v_0 = 10.5$, which confirms the answer.

3.100. The x -component of velocity does not change during flight. It is always equal to v_{x0} .

$$v_x \neq v_{x0} = v_0 \cos \theta = (10.5 \text{ m/s}) \cos(35.0^\circ) = 8.601096465 \text{ m/s} = 8.60 \text{ m/s}.$$

- 3.101. The time of travel was previously found to be $t = \frac{v_{y0} + \sqrt{(v_{y0})^2 + 2gy_0}}{g}$, and $v_{y0} = v_0 \sin \theta$. Therefore, the y -component of velocity just before impact is

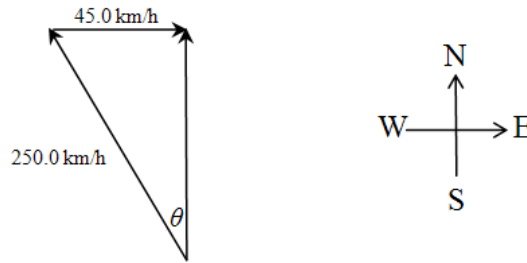
$$\begin{aligned} v_y &= v_{y0} - gt \\ &= v_{y0} - g \left(\frac{v_{y0} + \sqrt{(v_{y0})^2 + 2gy_0}}{g} \right) \\ &= -\sqrt{(v_{y0})^2 + 2gy_0} = -\sqrt{(v_0 \sin \theta)^2 + 2gy_0} \\ &= -\sqrt{[(10.5 \text{ m/s}) \sin(35.0^\circ)]^2 + 2(9.81 \text{ m/s}^2)(1.55 \text{ m})} \\ &= -8.165913274 \text{ m/s} = 8.17 \text{ m/s downward.} \end{aligned}$$

- 3.102. From the two preceding problems,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.601096465 \text{ m/s})^2 + (-8.165913274 \text{ m/s})^2} = 11.86 \text{ m/s.}$$

- 3.103. **THINK:** This question involves flying a plane through the air. The speed of the airplane with respect to the wind and the velocity of the wind with respect to the ground are both given, so this problem involves relative motion. The vector sum of the plane's velocity with respect to the air and the velocity of the air with respect to the ground must point in the direction of the pilot's destination. Since the wind is blowing from the west to the east and the pilot wants to go north, the plane should head to the northwest.

SKETCH: The sketch should show the velocity of the plane with respect to the wind ($\vec{v}_{pa}$), the velocity of the wind (which can be thought of as the velocity of the air with respect to the ground or $\vec{v}_{ag}$), and the velocity of the plane with respect to the ground ($\vec{v}_{pg}$). The distance the airplane is to travel will affect how long it takes the pilot to get to her destination, but will not effect in the $\vec{v}_{pa}$ $\vec{v}_{pg}$ $\vec{v}_{ag}$ direction she flies.



RESEARCH: To solve this problem, it is first necessary to note that the wind, blowing from West to East, is moving in a direction perpendicular to the direction the pilot wants to fly. Since these two vectors form

a right angle, it is easy to use trigonometry to find the angle θ with the equation $\sin \theta = \frac{|\vec{v}_{ag}|}{|\vec{v}_{pa}|}$. Since North is 360° and West is 270° , the final answer will be a heading of $(360 - \theta)^\circ$.

SIMPLIFY: To find the final answer, it is necessary to use the inverse sine function. The equation for the angle θ can be found using algebra and trigonometry:

$$\begin{aligned}\sin^{-1}(\sin \theta) &= \sin^{-1}\left(\frac{|\vec{v}_{\text{ag}}|}{|\vec{v}_{\text{pa}}|}\right) \\ \theta &= \sin^{-1}\left(\frac{|\vec{v}_{\text{ag}}|}{|\vec{v}_{\text{pa}}|}\right) \Rightarrow \\ 360 - \theta &= 360 - \sin^{-1}\left(\frac{|\vec{v}_{\text{ag}}|}{|\vec{v}_{\text{pa}}|}\right)\end{aligned}$$

CALCULATE: To find the final answer in degrees, it is necessary first to make sure that the calculator or computer program is in degree mode. Note that the speed of the wind $|\vec{v}_{\text{ag}}| = 45.0 \text{ km/h}$ and the speed of the plane with respect to the air $|\vec{v}_{\text{pa}}| = 250.0 \text{ km/h}$ are given in the problem. Plugging these in and solving gives a heading of:

$$\begin{aligned}360 - \sin^{-1}\left(\frac{|\vec{v}_{\text{ag}}|}{|\vec{v}_{\text{pa}}|}\right) &= 360^\circ - \sin^{-1}\left(\frac{45.0 \text{ km/h}}{250.0 \text{ km/h}}\right) \\ &= 349.6302402^\circ\end{aligned}$$

ROUND: Rounding to four significant figures, the final heading is 349.6° .

DOUBLE-CHECK: Intuitively, this answer seems correct. The pilot wants to fly North and the wind is blowing from West to East, so she should head somewhere towards the Northeast. Since the speed of the airplane with respect to the air is much greater than the speed of the air with respect to the ground (wind speed), the East-West component of the airplane's velocity with respect to the air should be less than the North-South component. Resolving the motion of the plane into horizontal (East-West) and vertical components gives that the horizontal speed of the plane with respect to the ground v_{px} is $250.0 \frac{\text{km}}{\text{h}} \cdot \cos(349.6^\circ - 270^\circ)$ or 45.1 km/h from East to West, which is within rounding the same as the known wind speed.

- 3.104.** As in the preceding problem, the velocity vectors form a right triangle, with the wind velocity as the east-west leg, the plane's ground velocity as the north-south leg, and the plane's velocity relative to the air as the hypotenuse. The plane's ground speed can therefore be found by applying the Pythagorean Theorem:

$$\sqrt{(250.0 \text{ km/h})^2 - (45.0 \text{ km/h})^2} = 245.9166525 \text{ km/h} = 245.9 \text{ km/h}.$$

- 3.105.** Using the result from the preceding problem, the time required will be

$$t = \frac{d}{v} = \frac{200.0 \text{ km}}{245.9166525 \text{ km/h}} = 0.8132836794 \text{ h} = 48.78 \text{ min}.$$

Chapter 4: Force

Concept Checks

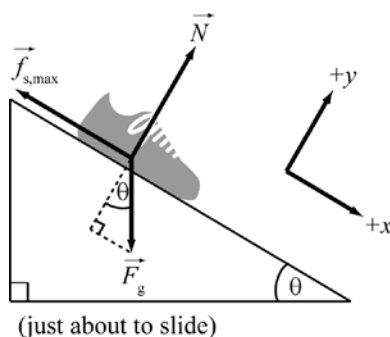
4.1. c 4.2. c 4.3. b 4.4. c 4.5. b 4.6. d

Multiple-Choice Questions

4.1. d 4.2. a 4.3. d 4.4. a 4.5. a 4.6. b and c 4.7. b 4.8. a 4.9. a 4.10. b 4.11. b 4.12. b 4.13. c and d
4.14. a, b and d 4.15. b 4.16. a

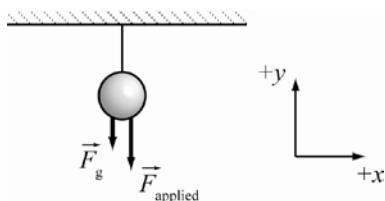
Conceptual Questions

- 4.17. Determine an expression for μ_s in terms of θ . Since the shoe just starts to slide, the maximum static friction has been achieved. Sketching shows the forces acting on the shoe:



In the y -direction: $F_{\text{net}y} = N - F_{gy} = 0 \Rightarrow N = F_{gy} = F_g \cos \theta$. In the x -direction: $F_{\text{net}x} = F_{gx} - f_{s,\text{max}} = 0$
 $\Rightarrow f_{s,\text{max}} = F_{gx} = F_g \sin \theta$. With $f_{s,\text{max}} = \mu_s N$, $\mu_s = (F_g \sin \theta) / N$. With $N = F_g \cos \theta$,
 $\mu_s = (F_g \sin \theta) / (F_g \cos \theta) = \tan \theta$.

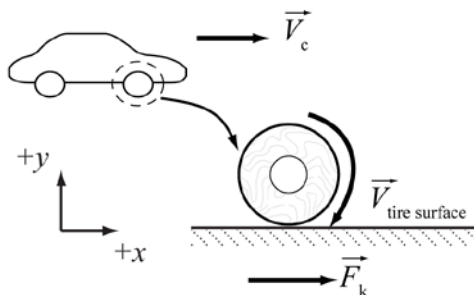
4.18.



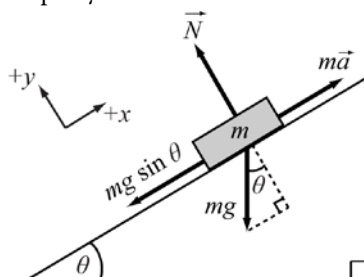
If the lower string is pulled slowly, the upper string breaks due to the greater tension in the upper string. If the lower string is pulled fast enough, the lower string breaks due to the inertia of the ball. The quick jerk increases the tension in the lower string faster than in the upper string.

- 4.19. The car and the trailer are no longer in an inertial reference frame due to their acceleration up the hill. However, the inertial forces of the car-trailer system do not change. That is, $F_t = F_c$ is still true. The sum of the internal forces of a system should always be zero.

- 4.20. It is the frictional forces against the tires that drive the car. The direction of the force of the tire against the road is actually opposite to the direction the car travels in.



- 4.21. A force external to the two interacting bodies can cause them to accelerate. For example, when a horse pulls a cart, both objects exert a force on each other equal in magnitude and opposite in direction, but the friction force acting on the horse-cart system causes it to accelerate.
- 4.22. If the textbook is initially at rest and there is no net force acting on it, then it will remain at rest. However, if the textbook is moving across a frictionless table at a constant velocity ($d\vec{v}/dt = 0$), then it will remain in motion unless acted upon by an external force. The statement is false.
- 4.23. A mass, m is sliding on a ramp that is elevated at an angle, θ with respect to the floor. The coefficient of friction between the mass and ramp is μ .



(a) The forces acting on the box are as follows:

- The force due to gravity acting down the ramp, $mg \sin \theta$.
- The force of friction, $f_k = \mu_k N$, where the subscript k denotes kinetic friction, acting opposite to the direction of motion, in this case down the ramp.

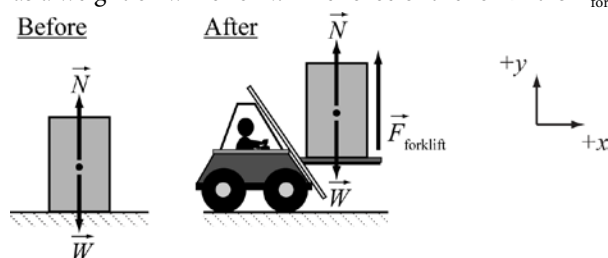
The sum of the force components along the ramp is $\sum F = ma = -mg \sin \theta - \mu_k N$. This expression can be simplified by noting that $N = mg \cos \theta$. Substituting this into the expression yields:

$$ma = -mg \sin \theta - mg \cos \theta \mu_k \Rightarrow a = -g(\sin \theta + \mu_k \cos \theta).$$

(b) If the mass is sliding down the ramp, the force of gravity remains the same, but the friction force still opposes the direction of motion and therefore now points up the ramp. Thus the expression for Newton's second law can now be written $ma = -mg \sin \theta + \mu_k N$. The equation for the acceleration is then:

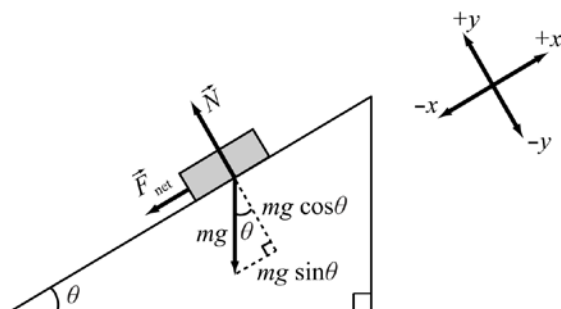
$$a = -g(\sin \theta - \mu_k \cos \theta).$$

- 4.24. The shipping crate has a weight of $w = 340 \text{ N}$. The force of the forklift is $F_{\text{forklift}} = 500 \text{ N}$.



The force due to gravity equals the weight of the crate. The mass of the crate, m is constant and so is the acceleration due to gravity, g . Therefore, the force due to gravity is $F_g = mg = w = 340 \text{ N}$.

- 4.25. The near frictionless slope is at an incline of $\theta = 30.0^\circ$. It will be useful to draw a diagram of the sliding block of mass, m , and label the forces on it. The diagram shows a block of mass, m , sliding down a (near) frictionless incline.

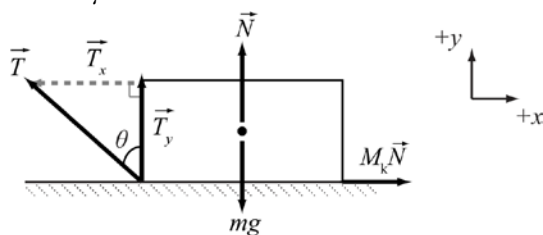


The magnitude of the net force acting on the block as it slides down the ramp is $F_{\text{net}} = mg \sin \theta$. The magnitude of the normal force is $N = mg \cos \theta$. This means that

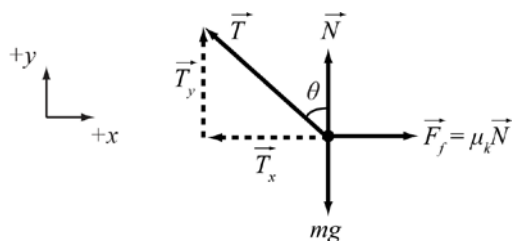
$$F_{\text{net}} / N = (mg \sin \theta) / (mg \cos \theta) = \tan \theta.$$

For an angle of 30° this means that $F_{\text{net}} / N = \tan 30^\circ \approx 0.58$. The net force is approximately 58% of the normal force.

- 4.26. The tow truck's mass is M . The mass of shipping crate is m . The angle with respect to the vertical is θ . The coefficient of kinetic friction is μ .



(a)



(b) The truck pulls the crate at a constant speed, so the net force on the crate is $\vec{F}_{\text{net}} = m\vec{a} = 0$. The components of the net force can be written as $\sum F_x = 0$; $\sum F_y = 0$. The sum of the forces in the x direction are $\sum F_x = T_x - \mu_k N = 0$.

Trigonometry can be used to obtain the expression for T_x :

$$\sin \theta = \frac{T_x}{T} \Rightarrow T_x = T \sin \theta.$$

Therefore, $\sum F_x = \mu_k N - T \sin \theta = 0$. The sum of the forces in the y direction is $\sum F_y = 0 = T_y + N - mg = 0$. To simplify this expression, $T_y = T \cos \theta$ can be used. In summary, our two equations for the x and y directions yield:

$$(1) T \cos \theta + N - mg = 0$$

$$(2) \mu_k N - T \sin \theta = 0$$

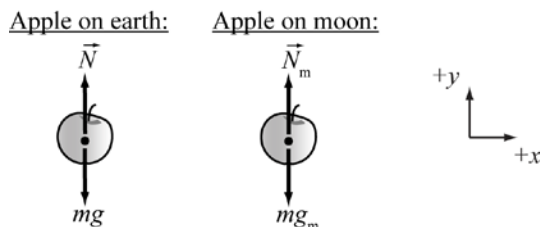
Equation (2) can be solved in terms of N to get $N = T \sin \theta / \mu_k$. Substituting this into equation (1) yields:

$$T(\cos \theta + \sin \theta / \mu_k) - mg = 0 \Rightarrow T = \frac{\mu_k mg}{\sin \theta + \mu_k \cos \theta}.$$

Exercises

- 4.27. THINK:** The gravitational constant on the Moon is $g_m = g/6$, where g is the Earth's gravitational constant. The weight of an apple on the Earth is $w = 1.00$ N.

SKETCH:



RESEARCH: The gravitational constant on the Earth is $g = 9.81 \text{ m/s}^2$. The weight of the apple, w is given by its mass times the gravitational constant, $w = mg$.

SIMPLIFY:

(a) The weight of the apple on the Moon is $w_m = mg_m$. Simplify this expression by substituting $g_m = g/6$: $w_m = m(g/6)$. Mass is constant so I can write expressions for the mass of the apple on Earth and on the Moon and then equate the expressions to solve for m . On Earth, $m = w/g$. On the Moon, $m = 6w_m/g$. Therefore, $w/g = 6w_m/g \Rightarrow w_m = w/6$.

(b) The expression for the weight of an apple on Earth can be rearranged to solve for m : $m = w/g$.

CALCULATE:

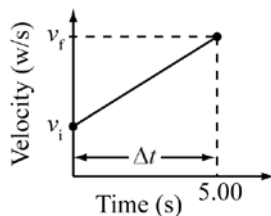
$$(a) w_m = \frac{1}{6}(1.00 \text{ N}) = 0.166667 \text{ N}$$

$$(b) m = \frac{1.00 \text{ N}}{9.81 \text{ m/s}^2} = 0.101931 \text{ kg}$$

ROUND: Rounding to three significant figures, (a) $w_m = 0.167 \text{ N}$ and (b) $m = 0.102 \text{ kg}$.

DOUBLE-CHECK: It is expected that the weight of the apple on the Moon is much less than the weight of the apple on the Earth. Also, a mass of about 100 g is reasonable for an apple.

- 4.28. THINK:** The go-cart is accelerated by having a force, $F = 423.5 \text{ N}$ applied to it. The initial velocity is given as $v_i = 10.4 \text{ m/s}$ and the final velocity is $v_f = 17.9 \text{ m/s}$. The time interval over which the change in velocity occurs is $\Delta t = 5.00 \text{ s}$. Determine the mass, m of the go-cart and the driver.

SKETCH:

RESEARCH: Because the force is constant, the acceleration is constant and the increase in velocity, v is linear. The equation for force is $F = ma$, and the expression for the acceleration is $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$.

SIMPLIFY: The expression for a can be substituted into the expression for force:

$$F = m \left(\frac{v_f - v_i}{t_f - t_i} \right) \Rightarrow m = \frac{F}{\left(\frac{v_f - v_i}{t_f - t_i} \right)}$$

CALCULATE: $m = \frac{423.5 \text{ N}}{\left(\frac{17.9 \text{ m/s} - 10.4 \text{ m/s}}{5.00 \text{ s} - 0 \text{ s}} \right)} = 282.33 \text{ kg}$

ROUND: Rounding to three significant figures, $m = 282 \text{ kg}$.

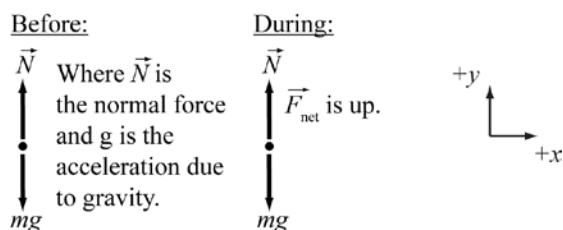
DOUBLE-CHECK: This mass is reasonable for a go-cart with a driver. The units are also correct.

- 4.29. **THINK:** The scale is in an elevator and the gym member has a mass of $m = 183.7 \text{ lb}$ when the elevator is stationary. The elevator accelerates upward with an acceleration of $a = 2.43 \text{ m/s}^2$. The mass must be converted to SI units:

$$m = 183.7 \text{ lb} \cdot \frac{1 \text{ kg}}{2.205 \text{ lb}} = 83.31 \text{ kg}.$$

Determine the weight, w measured by the scale as the elevator accelerates upward.

SKETCH: It will be useful to sketch free body diagrams for the forces acting on the scale before and during the elevator's acceleration.



RESEARCH: Before the elevator accelerates, the net force is $F_{\text{net}} = 0$. The sum of the forces at this point is $F_{\text{net},i} = 0 = N - mg$, therefore $N = mg$. Once the elevator starts to accelerate upward, there is a net force, $F_{\text{net},a} = ma = N - mg$. Since the mass of the person, m , the gravitational acceleration, g , and the acceleration, a are known, the equation can be used to determine N , which is the normal force that the scale displays.

SIMPLIFY: While the elevator is accelerating, the net force acting on the scale can be written as $ma = N - mg$. Rearranging to solve for N yields $N = m(a + g)$.

CALCULATE: $N = 83.31 \text{ kg}(2.43 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 1019.7144 \text{ N}$. However, scales are calibrated to read weight based on the assumption of $a = g = 9.81 \text{ m/s}^2$. The acceleration of the elevator “tricks” the scale. The gym member’s weight as displayed on the scale will be

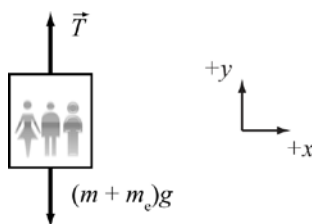
$$\frac{1019.7144 \text{ N}}{9.81 \text{ m/s}^2} \cdot \frac{2.205 \text{ lb}}{1 \text{ kg}} = 229.20 \text{ lb}.$$

ROUND: The acceleration is given with three significant figures, so the gym member’s weight is 229 lb.

DOUBLE-CHECK: Since the acceleration of the elevator is up, a person’s apparent weight should increase. The weight of the person when the elevator is at rest is 183.7 pounds and the apparent weight of the person while the elevator is accelerating is 229 pounds, so the calculation makes sense.

- 4.30. THINK:** The given quantities are the cabin mass of the elevator, $m_e = 358.1 \text{ kg}$ and the combined mass of the people in the cabin, $m = 169.2 \text{ kg}$. The elevator is undergoing a constant acceleration, $a = 4.11 \text{ m/s}^2$ due to being pulled by a cable. Determine the tension, T in the cable.

SKETCH:



RESEARCH: The sum of all the forces acting on the elevator is $F_{\text{net}} = \sum F_y = ma + m_e a = T - mg - m_e g$.

SIMPLIFY: Combine like terms: $a(m + m_e) = T - g(m + m_e)$. Rearrange the equation to solve for T : $T = (m + m_e)(a + g)$.

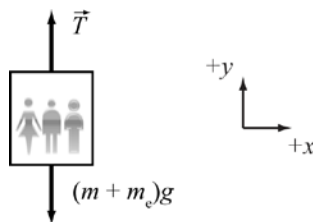
CALCULATE: $T = (169.2 \text{ kg} + 358.1 \text{ kg})(4.11 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 7340.016 \text{ N}$

ROUND: Rounding to three significant figures, $T = 7340 \text{ N}$. The force of the tension is directed upward.

DOUBLE-CHECK: The units calculated for the tension are Newtons, which are units of force. Because the elevator is accelerating upward, it is expected that the tension in the cable should be higher than if the elevator was hanging at rest. The calculated tension for the cable when the elevator was accelerating is $T = 7340 \text{ N}$. If the elevator is not accelerating, the expected tension is approximately 5173 N, so the answer is reasonable.

- 4.31. THINK:** The mass of the elevator cabin is $m_e = 363.7 \text{ kg}$ and the total mass of the people in the elevator is $m = 177.0 \text{ kg}$. The elevator is being pulled upward by a cable which has a tension of $T = 7638 \text{ N}$. The acceleration of the elevator is to be determined.

SKETCH:



RESEARCH: Force is equal to mass times acceleration, $\vec{F} = m\vec{a}$. The sum of all the forces acting on the elevator will give the net force that acts upon the elevator, $F_{\text{net},y} = \sum F_y$. In this case, $F_{\text{net},y} = ma + m_e a = T - mg - m_e g$. The gravitational acceleration is $g = 9.81 \text{ m/s}^2$.

SIMPLIFY: Group like terms, $a(m + m_e) = T - g(m + m_e)$. Rearrange to solve for a :

$$a = \frac{T - (m + m_e)g}{(m + m_e)}.$$

CALCULATE: $a = \frac{7638 \text{ N} - (177.0 \text{ kg} + 363.7 \text{ kg})9.81 \text{ m/s}^2}{(177.0 \text{ kg} + 363.7 \text{ kg})} = 4.3161 \text{ m/s}^2$

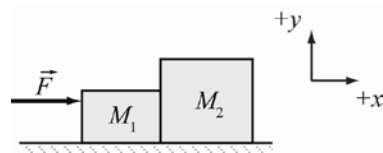
ROUND: Rounding to three significant figures because that is the precision of g , $a = 4.32 \text{ m/s}^2$.

DOUBLE-CHECK: The units of the result are correct. Also, the value determined for a is approximately 45% of the acceleration due to gravity, so the answer is reasonable.

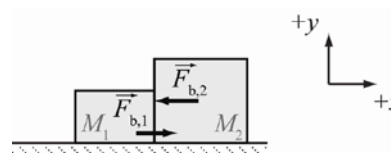
- 4.32. THINK:** Two blocks are in contact, moving across a frictionless table with a constant acceleration of $a = 2.45 \text{ m/s}^2$. The masses of the two blocks are $M_1 = 3.20 \text{ kg}$ and $M_2 = 5.70 \text{ kg}$. Determine (a) the magnitude of the applied force, F , (b) the contact force between the blocks, F_b and (c) the net force acting on block 1, $F_{\text{net},1}$.

SKETCH:

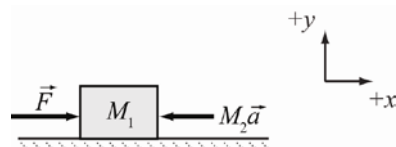
(a)



(b)



(c)



RESEARCH:

(a) Since the blocks are on a frictionless surface, there is no force due to friction. The net force is $F = M_1 a_1 + M_2 a_2$. But the blocks are in contact, so $a_1 = a_2 = a$. The equation can now be reduced to $F = (M_1 + M_2)a$.

(b) The force that block 1 feels due to block 2 is equal and opposite to the force that block 2 feels due to block 1. The contact force is $F_b = M_2 a$.

(c) The net force acting on block 1 is the sum of all the forces acting on it, $F_{\text{net},1} = F - M_2 a$.

SIMPLIFY: This step is not necessary.

CALCULATE:

(a) $F = (3.20 \text{ kg} + 5.70 \text{ kg})(2.45 \text{ m/s}^2) = 21.805 \text{ N}$

(b) $F_b = (5.70 \text{ kg})(2.45 \text{ m/s}^2) = 13.965 \text{ N}$

(c) $F_{\text{net},1} = 21.805 \text{ N} - (5.70 \text{ kg})(2.45 \text{ m/s}^2) = 7.84 \text{ N}$

ROUND: Three significant figures were provided in the question, so (a) $F = 21.8 \text{ N}$, (b) $F_b = 14.0 \text{ N}$ and

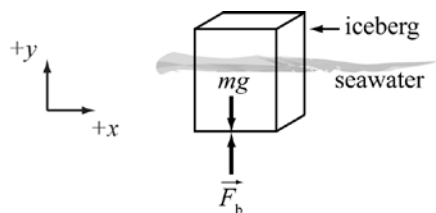
(c) $F_{\text{net},1} = 7.84 \text{ N}$.

DOUBLE-CHECK: All three results have the correct units of Newtons, which are units of force.

- 4.33. THINK:** The force that the ocean exerts on the iceberg keeps the iceberg floating. Since the iceberg floats, the net force on it must be zero. Therefore the magnitude of the force of the ocean on the iceberg must be equal to its weight.

The given quantities are the density of ice, $\rho_i = 917 \text{ kg/m}^3$ and the density of seawater, $\rho_s = 1024 \text{ kg/m}^3$. The volume of the iceberg above the sea is $v_{\text{above}} = 4205.3 \text{ m}^3$. This volume is 10.45% of the total volume of the iceberg. Determine the force, F_b that the seawater exerts on the iceberg.

SKETCH:



RESEARCH: The weight of the iceberg is $F = m_{\text{ice}}g$. The mass is the product of density and Volume, $m_{\text{ice}} = \rho_{\text{ice}} V_{\text{ice}}$. We were given the volume above the surface and were told that it represents 10.45% of the overall volume. So the total volume of the ice is: $V_{\text{ice}} = V_{\text{above}} / 0.1045$

SIMPLIFY: The weight of the iceberg is $F = m_{\text{ice}}g = \rho_{\text{ice}} V_{\text{ice}}g = \rho_{\text{ice}}gV_{\text{above}}/0.1045$

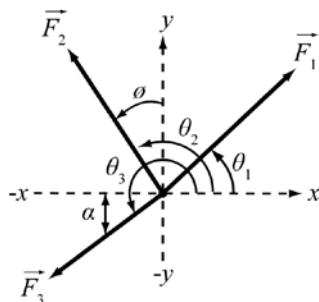
CALCULATE: $F = (917 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4205.3 \text{ m}^3)/0.1045 = 3.620087 \cdot 10^8 \text{ N}$

ROUND: There are three significant figures provided in the question, so the answer should be written as $F = 3.62 \cdot 10^8 \text{ N}$

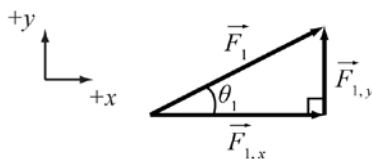
DOUBLE-CHECK: The units of Newtons that were calculated are correct units of force. Also, an iceberg is a very massive object, so it is reasonable that the seawater must exert a large force to keep it floating.

- 4.34. **THINK:** There are three massless ropes attached at one point with the following forces applied to them: $F_1 = 150. \text{ N}$ at $\theta_1 = 60.0^\circ$, $F_2 = 200. \text{ N}$ at $\theta_2 = 100.^\circ$ and $F_3 = 100. \text{ N}$ at $\theta_3 = 190.^\circ$. Determine the magnitude and direction of a fourth force that is necessary to keep the system in equilibrium.

SKETCH:



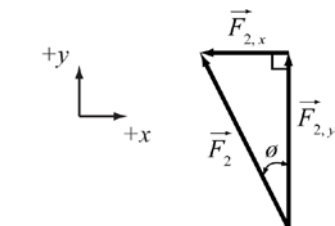
RESEARCH: For the system to remain stationary, the sum of the forces in the x and y directions must be zero, $\sum F_x = 0$ and $\sum F_y = 0$. The known forces must be broken into components to complete the calculations.



$\theta_1 = 60.0^\circ$ with respect to the x -axis.

$$F_{1,x} = F_1 \cos \theta_1$$

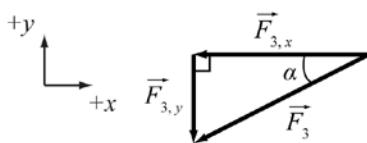
$$F_{1,y} = F_1 \sin \theta_1$$



$\phi = -10.0^\circ$ with respect to the y -axis.

$$F_{2,x} = -F_2 \sin \phi$$

$$F_{2,y} = F_2 \cos \phi$$



$\alpha = -10.0^\circ$ with respect to the negative x -axis.

$$F_{3,x} = -F_3 \cos \alpha$$

$$F_{3,y} = -F_3 \sin \alpha$$

The equations for the sum of the forces in the x and y directions are:

$$\sum F_x = 0 = F_1 \cos \theta_1 - F_2 \sin \phi - F_3 \cos \alpha + F_{4,x}, \quad \sum F_y = 0 = F_1 \sin \theta_1 + F_2 \cos \phi - F_3 \sin \alpha + F_{4,y}.$$

The angle is $\theta_4 = \tan^{-1}(F_{4,y} / F_{4,x})$. The magnitude of F_4 is given by $|F_4| = \sqrt{F_{4,x}^2 + F_{4,y}^2}$.

SIMPLIFY: The equations can be rearranged to solve for the components of the fourth force:

$$F_{4,x} = F_2 \sin \phi + F_3 \cos \alpha - F_1 \cos \theta_1 \quad \text{and} \quad F_{4,y} = F_3 \sin \alpha - F_1 \sin \theta_1 - F_2 \cos \phi.$$

CALCULATE: $F_{4,x} = (200.)\sin(10.0^\circ) \text{ N} + (100.)\cos(10.0^\circ) \text{ N} - (150.)\cos(60.0^\circ) \text{ N} = 58.2104 \text{ N}$

$$F_{4,y} = (100.)\sin(10.0^\circ) \text{ N} - (150.)\sin(60.0^\circ) \text{ N} - (200.)\cos(10.0^\circ) \text{ N} = -309.500 \text{ N}$$

$$\theta_4 = \tan^{-1}\left(\frac{-309.500 \text{ N}}{58.2104 \text{ N}}\right) = -79.3483^\circ \text{ with respect to the positive } x\text{-axis.}$$

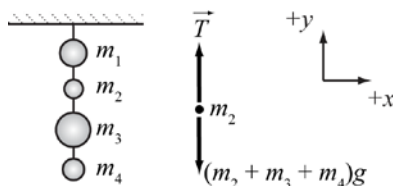
$$|F_4| = \sqrt{(58.2104 \text{ N})^2 + (-309.500 \text{ N})^2} = 314.9265 \text{ N}$$

ROUND: The given value for θ_1 has three significant figures, so the answers must be written as $|F_4| = 315 \text{ N}$ and $\theta_4 = 79.3^\circ$ below the positive x -axis.

DOUBLE-CHECK: The direction that force F_4 is applied is consistent with the diagram and the magnitude of the force is reasonable.

- 4.35. **THINK:** The given quantities are the masses of the four weights, $m_1 = 6.50 \text{ kg}$, $m_2 = 3.80 \text{ kg}$, $m_3 = 10.70 \text{ kg}$ and $m_4 = 4.20 \text{ kg}$. Determine the tension in the rope connected m_1 and m_2 .

SKETCH: Focus on an arbitrary point between m_1 and m_2 .



RESEARCH: The masses are in equilibrium, so the sum of the forces in the vertical direction is equal to zero. Therefore the tension, T in the rope between m_1 and m_2 is equal to the force exerted by gravity due to masses m_2 , m_3 and m_4 : $T - m_2 g - m_3 g - m_4 g = 0$.

SIMPLIFY: $T = (m_2 + m_3 + m_4)g$

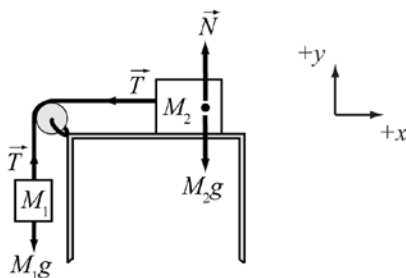
CALCULATE: $T = (3.80 \text{ kg} + 10.70 \text{ kg} + 4.20 \text{ kg})9.81 \text{ m/s}^2 = 183.447 \text{ N}$

ROUND: There are three significant figures provided in the question so the answer should be written $T = 183 \text{ N}$.

DOUBLE-CHECK: Tension is a force and the result has units of force (Newtons). The value of the tension is also reasonable considering the masses of the objects.

- 4.36. **THINK:** The value of the hanging mass is $M_1 = 0.50 \text{ kg}$ and the mass that is on the frictionless table is $M_2 = 1.50 \text{ kg}$. The masses are attached by a light string with a mass that can be neglected. Determine the magnitude of the acceleration, a of M_2 .

SKETCH:



RESEARCH: Force is a vector so the components must be considered separately. Equations can be written for the components of the force, $\vec{F}_2$ acting on mass, m_2 : $\sum F_{2,y} = 0 = N - M_2g$, $\sum F_{2,x} = M_2a = T$. The two masses are connected, so they accelerate at the same rate. Consider the components of the force, $\vec{F}_1$ acting on the mass, M_1 : $\sum F_{1,x} = 0$, $\sum F_{1,y} = -M_1a = T - M_1g$. The two expressions of interest are $T - M_2a = 0$ and $T - M_1g = -M_1a$.

SIMPLIFY: To determine a , T must be eliminated. Since the masses are rigidly connected by the string, the tension, T in the rope is constant so both equations can be solved for T , then equated to solve for a : $T = M_2a$ (1), $T = M_1g - M_1a$ (2). Therefore,

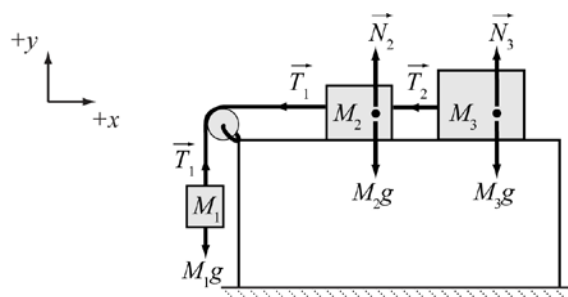
$$M_2a = M_1g - M_1a \Rightarrow a(M_2 + M_1) = M_1g \Rightarrow a = \frac{M_1g}{(M_2 + M_1)}.$$

CALCULATE: $a = \frac{0.50 \text{ kg}(9.81 \text{ m/s}^2)}{(1.50 \text{ kg} + 0.50 \text{ kg})} = 2.4525 \text{ m/s}^2$

ROUND: The least number of significant figures provided in the question is two, so the answer should be written $\vec{a} = 2.5 \text{ m/s}^2$.

DOUBLE-CHECK: The answer has the correct units for acceleration. Also, the value is approximately $\frac{1}{4}$ of the acceleration due to gravity, so it is reasonable.

- 4.37. **THINK:** The given quantities are the masses $M_1 = 0.500 \text{ kg}$, $M_2 = 1.50 \text{ kg}$ and $M_3 = 2.50 \text{ kg}$. The masses are connected by a light string with a mass that can be neglected. The string attaching M_1 is routed over a frictionless pulley. M_2 and M_3 rest on a frictionless table. Determine the magnitude of the acceleration of block 3, a , and the tension in the string between M_1 and M_2 .

SKETCH:

RESEARCH: The forces acting on each of the three masses need to be considered separately. The objects are rigidly connected so they accelerate at the same rate, $a_1 = a_2 = a_3 = a$. The forces on M_3 are $F_{3,y} = 0 = N_3 - M_3g$ (1), and $F_{3,x} = M_3a = T_2$ (2). The forces on M_2 are $F_{2,y} = 0 = N_2 - M_2g$ (3) and $F_{2,x} = M_2a = T_1 - T_2 \Rightarrow T_2 = T_1 - M_2a$ (4). The forces on M_1 are $F_{1,y} = -M_1a = T_1 - M_1g$ (5) and $F_{1,x} = 0$ (6). Substituting equation (4) into equation (2) yields $M_3a = T_1 - M_2a$ (7). This implies that $T_1 = M_3a + M_2a = (M_3 + M_2)a$.

SIMPLIFY:

$$(a) \ a = \frac{M_1g}{M_3 + M_2 + M_1}$$

(b) The tension between M_1 and M_2 is $T_1 = (M_3 + M_2)a$.

CALCULATE:

$$(a) \ a = \frac{(0.500 \text{ kg})(9.81 \text{ m/s}^2)}{(2.50 \text{ kg} + 1.50 \text{ kg} + 0.500 \text{ kg})} = 1.09 \text{ m/s}^2$$

$$(b) \ T_1 = (2.50 \text{ kg} + 1.50 \text{ kg})(1.09 \text{ m/s}^2) = 4.36 \text{ N}$$

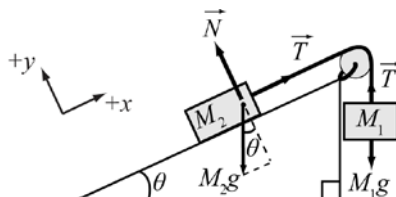
ROUND: The number of significant figures given in the question was three.

$$(a) \ a = 1.09 \text{ m/s}^2$$

$$(b) \ T_1 = 4.36 \text{ N}$$

DOUBLE-CHECK: The units calculated for the acceleration and the tension are appropriate SI units. Also, the values determined are reasonable considering the masses involved.

- 4.38. **THINK:** The given quantities are the masses, $M_1 = 0.400 \text{ kg}$ and $M_2 = 1.20 \text{ kg}$. The hanging mass, M_1 is attached by a light rope to mass, M_2 across a frictionless pulley. M_2 is initially at rest on a frictionless ramp that is elevated at an angle of $\theta = 30.0^\circ$ above the horizontal. Determine the magnitude and direction of the acceleration of M_2 .

SKETCH:

RESEARCH: The forces acting on the masses must be considered separately. The forces acting on M_2 are $F_{2,p} = 0 = N - M_2g \cos \theta$ (1) and $F_{2,R} = -M_2a = -T + M_2g \sin \theta$ (2). We have to choose a direction for acceleration so we chose for M_2 to move up the ramp – if we are wrong then the value for the acceleration will be negative. If M_2 moves up the ramp then M_1 must be descending, so the forces acting on M_1 are

$F_{1,y} = -M_1 a_1 = T - M_1 g$ (3). Since the two masses are rigidly attached, they both accelerate at the same rate, $a_1 = a_2 = a$. Also, the tension, T , in the rope between the masses is the same in equations (2) and (3).

SIMPLIFY: From equation (2), $T = M_2 g \sin \theta + M_2 a$ (4). From equation (3), $T = M_1 g - M_1 a$ (5).

Therefore $M_2 g \sin \theta + M_2 a = M_1 g - M_1 a$. Solving for a gives: $a_2 = a = \frac{g(M_1 - M_2 \sin \theta)}{M_1 + M_2}$.

CALCULATE: $a_2 = \frac{9.81 \text{ m/s}^2 (0.400 \text{ kg} - (1.20 \text{ kg}) \sin(30.0^\circ))}{(1.20 \text{ kg} + 0.400 \text{ kg})} = -1.226 \text{ m/s}^2$

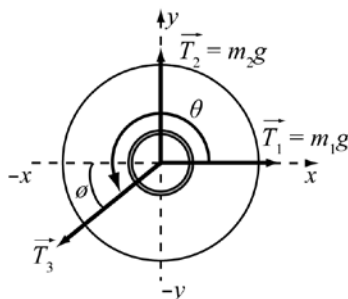
Therefore, $a_2 = 1.226 \text{ m/s}^2$ down the ramp.

ROUND: Rounding to three significant figures, the answer should be written $a_2 = 1.23 \text{ m/s}^2$ down the ramp.

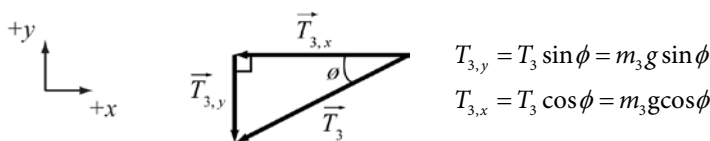
DOUBLE-CHECK: The calculated acceleration has the correct units. Also, it makes sense that a_2 is down the ramp because the force due to gravity pulling M_2 down the ramp is greater than the force exerted on M_2 up the ramp due to the force of gravity on M_1 . In addition we find the limit of the acceleration of the Atwood machine in the limit of $\theta = 90^\circ$ (See Example 4.5) and the limit of Example 4.4, Two Blocks Connected by a Rope, for $\theta = 0^\circ$ as limiting cases of our answer. This gives us additional confidence in our solution.

- 4.39. THINK:** The given quantities are the hanging masses m_1 and m_2 and the direction of the horizontal forces cause by the hanging masses on the ring. The strings that attach the hanging masses to the ring can be considered massless and the pulleys that the strings are routed through are frictionless. Determine the mass, m_3 , and the angle, θ , that will result in the ring being balanced in the middle of the table.

SKETCH: Top-down view:



RESEARCH: A sketch of the x and y components of T_3 is shown below.



The angle counterclockwise from the positive x -axis, θ is given by $\theta = 180^\circ + \phi$. For the ring to be balanced, the sum of the forces in the x and y directions must be balanced:

$$\sum F_y = 0 = T_2 - T_3 \sin \phi = m_2 g - m_3 g \sin \phi \quad (1)$$

$$\sum F_x = 0 = T_1 - T_3 \cos \phi = m_1 g - m_3 g \cos \phi \quad (2)$$

SIMPLIFY: Solve equation (1) in terms of m_3 and substitute into equation (2) to solve for ϕ . $m_3 = m_2 g / g \sin \phi = m_2 / \sin \phi$ substituted into (2) yields:

$$m_1 g = m_3 g \cos \phi \Rightarrow m_1 = \frac{m_2 \cos \phi}{\sin \phi} \Rightarrow \frac{\sin \phi}{\cos \phi} = \frac{m_2}{m_1} \Rightarrow \tan \phi = \frac{m_2}{m_1} \Rightarrow \phi = \tan^{-1} \left(\frac{m_2}{m_1} \right).$$

CALCULATE: $\phi = \tan^{-1}\left(\frac{0.0300 \text{ kg}}{0.0400 \text{ kg}}\right) = 36.8698^\circ$, $\theta = 180^\circ + 36.8698^\circ = 216.8698^\circ$,

$$m_3 = \frac{0.030 \text{ kg}}{\sin(36.8698^\circ)} = 0.05000 \text{ kg}$$

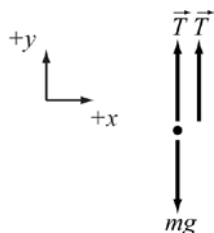
ROUND: Four significant figures are provided in the question, so the answers should be written $\theta = 216.8^\circ$ and $m_3 = 0.0500 \text{ kg}$.

DOUBLE-CHECK: By observing the sketch, it can be seen that the value of θ is reasonable to balance the forces. The mass is also a reasonable value.

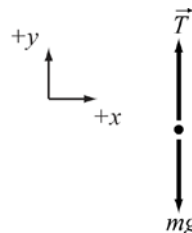
- 4.40. **THINK:** Consider the combined mass, m of the monkey and the wood plate when $m = 100. \text{ kg}$. Assume the rope's mass is negligible and the system is frictionless. In part (b), the acceleration is $a = 2.45 \text{ m/s}^2$.

SKETCH:

For monkey/plate system:



For second monkey of mass, m_2 pulling on rope,
free body diagram for m becomes:



RESEARCH:

(a) The minimum force the monkey needs to apply to lift off the ground is the force T required to balance the force of gravity due to the combined mass, m : $2T - mg = 0 \Rightarrow T = mg / 2$.

(b) $\sum F_y = ma = 2T - mg$. There is now a net upward force due to the monkey.

(c) If there is a second monkey, then in part (a), $T = mg$ and the equation in (b) becomes $T = m(a + g)$.

SIMPLIFY:

(a) It is not necessary to simplify.

$$(b) T = \frac{ma + mg}{2} = \frac{m(a + g)}{2}$$

(c) It is not necessary to simplify.

CALCULATE:

$$(a) T = \frac{(100. \text{ kg})(9.81 \text{ m/s}^2)}{2} = 490.5 \text{ N}$$

$$(b) T = \frac{(100. \text{ kg})(2.45 \text{ m/s}^2 + 9.81 \text{ m/s}^2)}{2} = 613 \text{ N}$$

$$(c) T(\text{no } a) = (100. \text{ kg})(9.81 \text{ m/s}^2) = 981.0 \text{ N}$$

$$T(a = 2.45 \text{ m/s}^2) = (100. \text{ kg})(2.45 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 1226 \text{ N}$$

ROUND: The acceleration has three significant figures, so the answers should be written:

$$(a) T = 491 \text{ N}$$

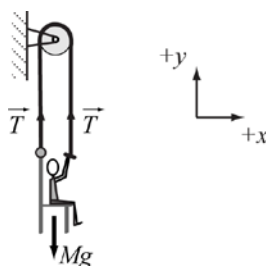
$$(b) T = 613 \text{ N}$$

$$(c) T = 981 \text{ N}, 1230 \text{ N}$$

DOUBLE-CHECK: The calculated values seem reasonable for the mass of the system of interest. It is reasonable that a larger force must be applied to give the system an upward acceleration. If a second monkey pulled the rope from the ground, the tension, T would be due to the full gravitational force of mass, m , so it is reasonable that the answers for part (c) doubled.

- 4.41. **THINK:** The rope has negligible mass and the pulley is frictionless. The chair and boatswain have a combined mass, M , of 90.0 kg. Consider two cases: (a) The magnitude of the force the boatswain must pull on the rope with to achieve constant velocity, $\vec{v}_c$ and (b) the maximum force the boatswain must pull on the rope with to achieve an acceleration of $a_{\max} = 2.00 \text{ m/s}^2$.

SKETCH:



RESEARCH:

(a) If the boatswain is moving at a constant velocity, then there is no net force on the system because $dv/dt = 0$ for a constant v . For this case, the sum of the forces is $\sum F = 0 = 2T - Mg$. The force the boatswain must pull with is $F = T$.

(b) In the case where the boatswain is accelerating, the maximum force, $F_{\max}$ can be substituted in to the sum of the forces equation, $2F_{\max} - Mg = Ma_{\max}$.

SIMPLIFY:

(a) $2T = Mg$. Substitute the force, F the boatswain must pull on the rope into the equation: $F = Mg/2$.

(b) Rearrange to solve for $F_{\max}$: $F_{\max} = \frac{M(g + a_{\max})}{2}$.

CALCULATE:

$$(a) F = \frac{(90.0 \text{ kg})(9.81 \text{ m/s}^2)}{2} = 441.45 \text{ N}$$

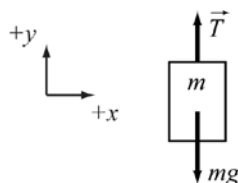
$$(b) F_{\max} = \frac{90.0 \text{ kg}(9.81 \text{ m/s}^2 + 2.00 \text{ m/s}^2)}{2} = 531.45 \text{ N}$$

ROUND: To three significant figures, the answers are (a) $F = 441 \text{ N}$ and (b) $F_{\max} = 531 \text{ N}$.

DOUBLE-CHECK: The unit of Newtons is a correct unit of force. It is reasonable that the boatswain had to pull on the rope with more force to cause a net acceleration than when the system moved at constant speed.

- 4.42. **THINK:** A granite block of mass, $m = 3311 \text{ kg}$ is suspended from a pulley system. The rope is wound around the pulley system six times. Assume the rope is massless and the pulley is frictionless. Determine the force, F , the rope would have to be pulled with to hold m in equilibrium.

SKETCH:



RESEARCH: The diagram suggests that only the rope attaches to the block, which means that for the block to be in equilibrium, $T - mg = 0$. The tension in the end of the rope that is pulled on is the same as

the tension in the rope where it attaches to the block. The tension in the rope of a pulley system is given by $T = mg / 2n$, where n is the number of loops the rope makes around the pulley system.

SIMPLIFY: The force required to hold the system in equilibrium must be equal to the tension:

$$F = T = \frac{mg}{2n}.$$

CALCULATE: $F = \frac{(3311 \text{ kg})(9.81 \text{ m/s}^2)}{(2)(6)} = 2707 \text{ N}$

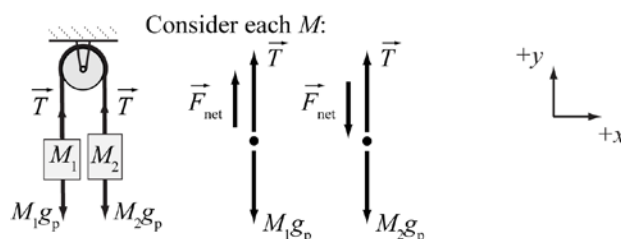
ROUND: The value of g used in the calculation has three significant figures, so the answer is rounded to $F = 2710 \text{ N}$.

DOUBLE-CHECK: The units of the result are Newtons, which are a unit of force. The calculated value is reasonable, considering the mass of the block, although it would be very difficult for one person to hold this mass in equilibrium.

- 4.43. THINK:** Two masses, $M_1 = 100.0 \text{ g}$ and $M_2 = 200.0 \text{ g}$ are placed on an Atwood device. Each mass moves a distance, $\Delta y = 1.00 \text{ m}$ in a time interval of $\Delta t = 1.52 \text{ s}$. Determine the gravitational acceleration, g_p for the planet and the tension, T in the string. The string is massless and the pulley is frictionless. M_1 and M_2 should be converted to the SI unit of kg.

$$M_1 = (100.0 \text{ g}) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 0.1000 \text{ kg}, \quad M_2 = (200.0 \text{ g}) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 0.2000 \text{ kg}$$

SKETCH:



RESEARCH:

(a) The masses are initially at rest, therefore their initial speed $v_0 = 0$. Because the masses are rigidly connected, they accelerate at the same rate, a . The net force for one mass is upward and downward for the other. The value of a can be determined using the kinematic equation $\Delta y = v_0 t + (at^2)/2$. Because the masses are initially at rest, the equation reduces to $\Delta y = (at^2)/2$ or $a = 2\Delta y / t^2$. If the forces on mass M_1 are considered, the net force equation is $F_{\text{net}} = M_1 a = T - M_1 g_p$. For mass M_2 , the net force equation is

$$F_{\text{net}} = -M_2 a = T - M_2 g_p.$$

(b) Solve for g_p and substitute into the force equation to solve for T .

SIMPLIFY:

(a) $M_1 a = T - M_1 g_p$ (1), $-M_2 a = T - M_2 g_p$ (2)

Because the tensions in the ends of the rope are the same, solve equations (1) and (2) in terms of T and equate the expressions.

$$M_2 g_p - M_2 a = M_1 g_p + M_1 a \Rightarrow a(M_1 + M_2) = g_p(M_2 - M_1) \Rightarrow g_p = a \left(\frac{M_1 + M_2}{M_2 - M_1} \right)$$

Substitute for a using $\Delta y = (at^2)/2$ to get $g_p = \frac{2\Delta y}{t^2} \left(\frac{M_1 + M_2}{M_2 - M_1} \right)$.

$$(b) T = M_2 (g_p - a) = M_2 \left(g_p - \frac{2\Delta y}{t^2} \right)$$

CALCULATE:

$$(a) g_p = \frac{2(1.00 \text{ m})}{(1.52 \text{ s})^2} \left(\frac{0.1000 \text{ kg} + 0.2000 \text{ kg}}{0.2000 \text{ kg} - 0.1000 \text{ kg}} \right) = 2.59695 \text{ m/s}^2$$

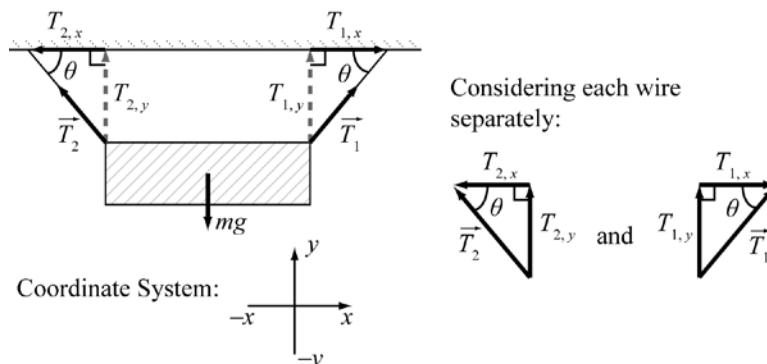
$$(b) T = 0.2000 \text{ kg} \left(2.59695 \text{ m/s}^2 - \frac{2(1.00 \text{ m})}{(1.52 \text{ s})^2} \right) = 0.346260 \text{ N}$$

ROUND: To three significant figures, the answers should be (a) $g_p = 2.60 \text{ m/s}^2$ and (b) $T = 0.346 \text{ N}$.

DOUBLE-CHECK: The units for the calculated answers were the correct units of acceleration and force. The small tension calculated is reasonable, considering the small masses.

- 4.44. THINK:** The mass of the sign is given as $m = 4.25 \text{ kg}$. The sign is hung by 2 wires that each makes an angle of $\theta = 42.4^\circ$ with the ceiling. Determine the tension in each wire.

SKETCH:



RESEARCH: Because the sign is in equilibrium, the sum of the forces in the x - and y -directions must equal zero. The sum of the forces in the y -direction is $\sum F_y = 0 = T_{1,y} + T_{2,y} - mg$, where $T_{1,y} = T_1 \sin \theta$ and $T_{2,y} = T_2 \sin \theta$. Inserting, the expression can be written $T_1 \sin \theta + T_2 \sin \theta = mg$. The sum of the forces in the x -direction can be written $\sum F_x = 0 = T_{1,x} - T_{2,x}$, where $T_{1,x} = T_1 \cos \theta$ and $T_{2,x} = T_2 \cos \theta$. Therefore, $T_1 \cos \theta = T_2 \cos \theta$ or $T_1 = T_2$.

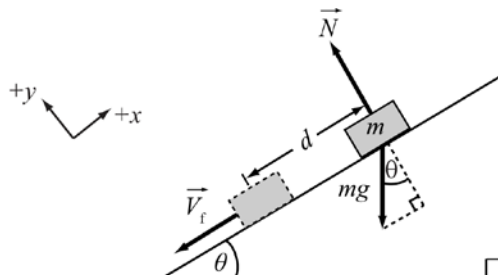
SIMPLIFY: Because $T_1 = T_2$, the tension can simply be called T . The forces in the y -direction can then be simplified, $2T \sin \theta = mg \Rightarrow T = mg / (2 \sin \theta)$.

$$\text{CALCULATE: } T = \frac{(4.25 \text{ kg})(9.81 \text{ m/s}^2)}{2 \sin(42.4^\circ)} = 30.9153 \text{ N}$$

ROUND: Rounding to three significant figures, $T = 30.9 \text{ N}$.

DOUBLE-CHECK: The calculated tension has Newtons as the unit, which is the correct SI unit for force. The calculated value is reasonable considering the given mass and angle. Note that in the limit of $\theta \rightarrow 0$ we recover $T = mg / 2$, i.e. the tension in each wire is equal to half of the weight of the sign, as expected. For all values $\theta > 0$ the tensions in the wires are larger than this value at $\theta \rightarrow 0$, which is also comforting. Finally, as $\theta \rightarrow 90^\circ$ the tensions in the wires become infinitely large, which is also expected.

- 4.45. THINK:** A crate of oranges with mass m slides down a frictionless incline. The crate has an initial velocity $v_i = 0$ and a final velocity $v_f = 5.832 \text{ m/s}$ after sliding a distance, $d = 2.29 \text{ m}$. Determine the angle of inclination, θ with respect to the horizontal.

SKETCH:

RESEARCH: The net acceleration, a , of m down the ramp can be determined from the equation $v_f^2 = v_i^2 + 2ad$. Sum all of the forces acting on the crate down the ramp: $\sum F_{\text{ramp}} = ma = mg \sin \theta$.

SIMPLIFY: $v_i = 0$, therefore $a = v_f^2 / 2d$. Substitute this relation into the equation for the forces acting down the ramp:

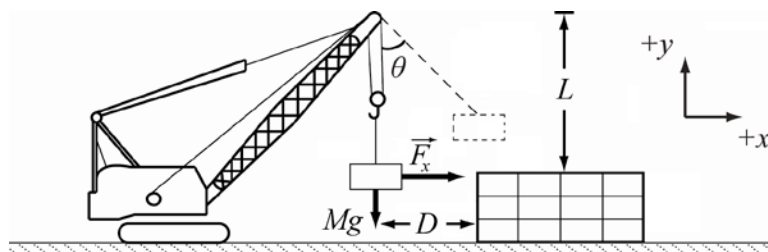
$$\frac{mv_f^2}{2d} = mg \sin \theta \Rightarrow \sin \theta = \frac{v_f^2}{2dg} \Rightarrow \theta = \sin^{-1} \left(\frac{v_f^2}{2dg} \right).$$

CALCULATE: $\theta = \sin^{-1} \left(\frac{(5.832 \text{ m/s})^2}{2(2.29 \text{ m})(9.81 \text{ m/s}^2)} \right) = 49.20114669^\circ$

ROUND: Rounding to three significant figures, $\theta = 49.2^\circ$.

DOUBLE-CHECK: The inclination of the plane is reasonable.

- 4.46. THINK:** The bricks have a mass $M = 200.0 \text{ kg}$ and are attached to a crane by a cable of negligible mass and length, $L = 3.00 \text{ m}$. In the initial vertical position, the bricks are a distance $D = 1.50 \text{ m}$ from the wall. Determine the magnitude of the horizontal force, F_x , that must be applied to the bricks to position them directly above the wall.

SKETCH:

RESEARCH: The given lengths can be used to solve for the angle, θ , that the bricks move through: $\tan \theta = D / L$ (1). By similar reasoning, the angle in the force vector diagram is given by:

$$\tan \theta = F_x / (Mg) \quad (2).$$

SIMPLIFY: Because θ is the same angle in both cases, equations (1) and (2) can be equated:

$$\frac{D}{L} = \frac{F_x}{Mg} \Rightarrow F_x = \frac{MgD}{L}.$$

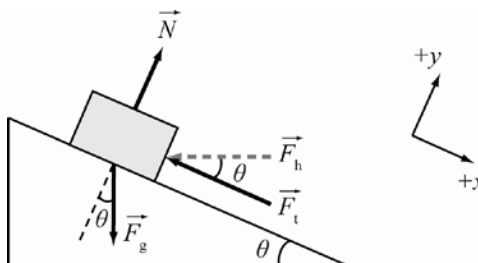
CALCULATE: $F_x = \frac{(200.0 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m})}{3.00 \text{ m}} = 981 \text{ N}$

ROUND: To three significant figures $F_x = 981 \text{ N}$.

DOUBLE-CHECK: The unit of the calculated value is Newtons, which is a SI unit for force. The result is reasonable considering the mass of the bricks.

- 4.47. THINK:** The problem asks for the force needed to hold the block in place. This means that the net force on the block has to be zero in each case, $F_{\text{net } x} = F_{\text{net } y} = 0$. The only forces to consider are the force of gravity, which act straight downward, the normal force from the plane, which is perpendicular to the plane, and the third external force we are asked to apply in parts (a) and (b). We do not need to consider friction forces, because the problem stipulates a “frictionless ramp”.

SKETCH:



RESEARCH:

- (a) To find F_t , the forces acting in the x direction on the block must be balanced.
 (b) Note that now F_t and F_h are related by $F_t = F_h \cos \theta$.

SIMPLIFY:

- (a) $F_{\text{net } x} = -F_t + F_{gx} = 0$, $F_t = F_{gx} = mg \sin \theta$
 (b) $F_h = F_t / \cos \theta = mg \sin \theta / \cos \theta = mg \tan \theta$

CALCULATE:

- (a) $F_t = (80.0 \text{ kg})(9.81 \text{ m/s}^2) \sin(36.9^\circ) = 471.2 \text{ N}$
 (b) $F_h = (80.0 \text{ kg})(9.81 \text{ m/s}^2) \tan(36.9^\circ) = 589.2 \text{ N}$

ROUND: To three significant figures,

- (a) $F_t = 471 \text{ N}$ and
 (b) $F_h = 589 \text{ N}$.

DOUBLE-CHECK: With almost all problems involving inclined planes, such as this one, one can obtain great insight and perform easy checks of the algebra by considering the limiting cases of the angle θ of the plane approaching 0 and 90 degrees.

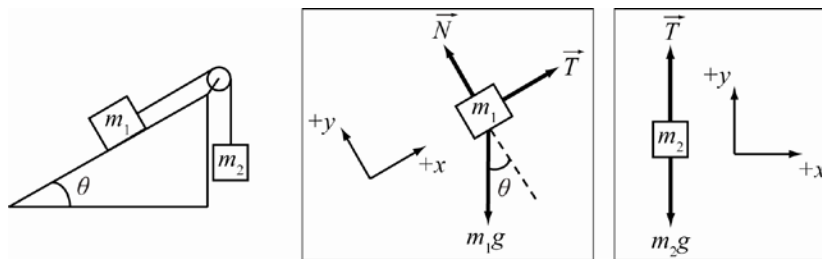
In the case of $\theta \rightarrow 0^\circ$ the block will simply sit on a horizontal surface, and no external force should be required to hold it in that position. Our calculations are compatible with this, because $\sin 0^\circ = \tan 0^\circ = 0$.

In the case of $\theta \rightarrow 90^\circ$ our results for parts (a) and (b) should be very different. In part (a) the force acts long the plane and so will be straight up in this limit, thus balancing the weight of the block all by itself.

Therefore, as $\theta \rightarrow 90^\circ$, we expect our force to approach $F_t(\theta \rightarrow 90^\circ) = mg$. This is satisfied in our solution because $\sin 90^\circ = 1$. In part (b) the external force will act perpendicular to the plane in the limit of $\theta \rightarrow 90^\circ$. Thus almost no part of it will be available to balance the weight of the block, and consequently an infinitely big force magnitude should be required. This is also born out by our analytic result for part (b), because $\tan 90^\circ \rightarrow \infty$.

- 4.48. THINK:** The mass, $m_1 = 20.0$ kg. The ramp angle is $\theta = 30.0^\circ$. The acceleration of the masses is $a_1 = a_2 = 0$.

SKETCH:



RESEARCH: For m_2 , $F_{\text{net},y} = T - m_2g = 0$. Determine T from the sum of forces on m_1 .

SIMPLIFY: For m_1 , $F_{\text{net},x} = T - F_{g1,x} = 0$, $T - m_1g \sin \theta = 0$, and $T = m_1g \sin \theta$. Then for m_2 , $T = m_2g \Rightarrow m_2 = T / g = m_1 \sin \theta$.

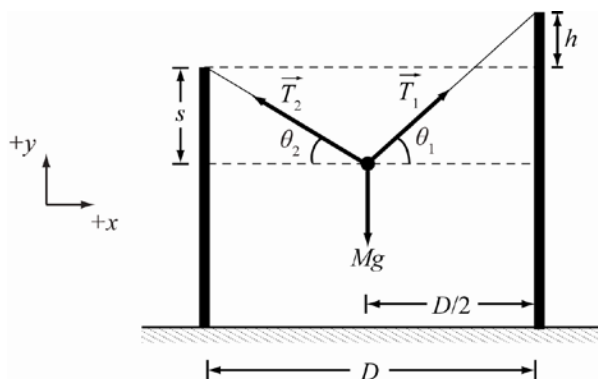
CALCULATE: $m_2 = (20.0 \text{ kg}) \sin(30.0^\circ) = 10.0 \text{ kg}$

ROUND: $m_2 = 10.0 \text{ kg}$.

DOUBLE-CHECK: m_2 is the same order of magnitude as m_1 .

- 4.49. THINK:** The piñata's mass is $M = 8.00$ kg. The distance between the poles is $D = 2.00$ m. The difference in pole heights is $h = 0.500$ m. The vertical distance between the shorter pole and the piñata is $s = 1.00$ m. Horizontally, the piñata is $D/2$ from each pole. Determine the tension in each part of the rope, T_1 and T_2 . Note, $F_{\text{net},x} = F_{\text{net},y} = 0$.

SKETCH:



RESEARCH: To find T_1 and T_2 , balance the forces on the piñata in each direction. θ_1 and θ_2 can be determined from trigonometry.

SIMPLIFY: Find θ_1 : $\tan \theta_1 = \frac{h+s}{D/2} \Rightarrow \theta_1 = \tan^{-1} \left(\frac{2(h+s)}{D} \right)$. Similarly, $\theta_2 = \tan^{-1} \left(\frac{2s}{D} \right)$.

$$F_{\text{net},x} = T_{1x} - T_{2x} = 0 \Rightarrow T_1 \cos \theta_1 - T_2 \cos \theta_2 = 0 \quad (1)$$

$$F_{\text{net},y} = T_{1y} + T_{2y} - mg = 0 \Rightarrow T_1 \sin \theta_1 + T_2 \sin \theta_2 - mg = 0 \quad (2)$$

Solve for T_1 in terms of T_2 in (1) and substitute into (2):

$$\left(T_2 \frac{\cos \theta_2}{\cos \theta_1} \right) \sin \theta_1 + T_2 \sin \theta_2 - mg = 0 \Rightarrow T_2 (\cos \theta_2 \tan \theta_1 + \sin \theta_2) = mg \Rightarrow T_2 = \frac{mg}{(\cos \theta_2 \tan \theta_1 + \sin \theta_2)}.$$

CALCULATE: $\theta_1 = \tan^{-1}\left(\frac{2(0.500 \text{ m} + 1.00 \text{ m})}{2.00 \text{ m}}\right) = 56.31^\circ$, $\theta_2 = \tan^{-1}\left(\frac{2.00 \text{ m}}{2.00 \text{ m}}\right) = 45.0^\circ$

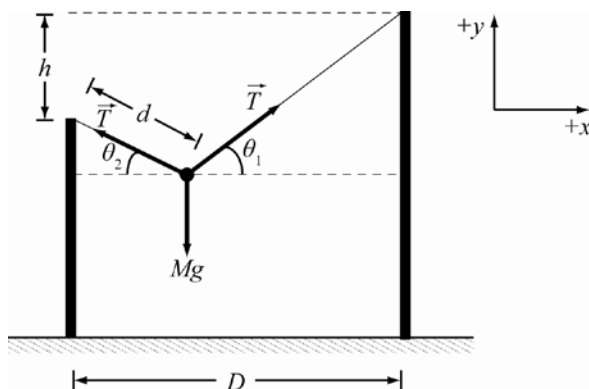
$$T_2 = \frac{(8.00 \text{ kg})(9.81 \text{ m/s}^2)}{(\cos(45.0^\circ)\tan(56.31^\circ) + \sin(45.0^\circ))} = 44.39 \text{ N}, \quad T_1 = (44.39 \text{ N}) \frac{\cos(45.0^\circ)}{\cos(56.31^\circ)} = 56.59 \text{ N}$$

ROUND: With all the given values containing three significant figures, $T_1 = 56.6 \text{ N}$ and $T_2 = 44.4 \text{ N}$.

DOUBLE-CHECK: Both T_1 and T_2 are less than the weight of the piñata and are reasonable values.

- 4.50. THINK:** The piñata's mass is $M = 12 \text{ kg}$. The distance between the poles is $D = 2.0 \text{ m}$. The difference in pole height is $h = 0.50 \text{ m}$. The rope length is $L = 3.0 \text{ m}$. The piñata is in equilibrium, so $F_{\text{net},x} = F_{\text{net},y} = 0$. Determine (a) the distance from the top of the lower pole to the ring, d and (b) the tension in the rope, T .

SKETCH:



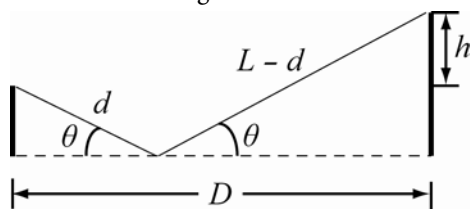
RESEARCH:

(a) Trigonometry can be used to find d . Also, a relation between the angles θ_1 and θ_2 can be established by balancing the horizontal forces on the ring: $F_{\text{net},x} = \sum F_x = 0$.

(b) To find T , the horizontal and vertical forces on the ring must be balanced. The magnitude of T is the same on each side of the ring because it is the same rope.

SIMPLIFY:

(a) First, determine how θ_1 and θ_2 relate: $F_{\text{net},x} = T \cos \theta_1 - T \cos \theta_2 = 0 \Rightarrow T \cos \theta_1 = T \cos \theta_2$. Then, $\theta_1 = \theta_2 = \theta$. To find d , consider the sketch again:



$$d \cos \theta + (L - d) \cos \theta = D \Rightarrow L \cos \theta = D \Rightarrow \theta = \cos^{-1}(D/L)$$

$$(L - d) \sin \theta - d \sin \theta = h \Rightarrow L \sin \theta - 2d \sin \theta = h \Rightarrow d = \frac{L \sin \theta - h}{2 \sin \theta}$$

(b) From (a), $\theta = 48.19^\circ$. To determine T , consider: $F_{\text{net},y} = 2T \sin \theta - Mg = 0$. Then, $T = \frac{Mg}{2 \sin \theta}$.

CALCULATE:

(a) $\theta = \cos^{-1}\left(\frac{2.0 \text{ m}}{3.0 \text{ m}}\right) = 48.19^\circ$, $d = \frac{(3.0 \text{ m}) \sin(48.19^\circ) - 0.50 \text{ m}}{2 \sin(48.19^\circ)} = \frac{1.736 \text{ m}}{1.4907} = 1.1646 \text{ m}$

$$(b) T = \frac{(12 \text{ kg})(9.81 \text{ m/s}^2)}{2 \sin(48.19^\circ)} = 78.97 \text{ N}$$

ROUND:

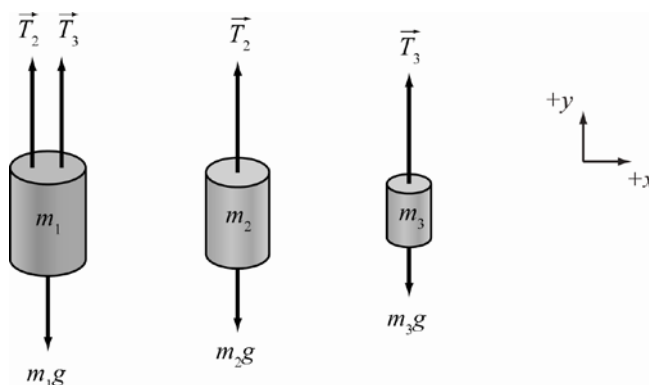
(a) Rounding to two significant figures, $d = 1.2 \text{ m}$.

(b) Rounding to two significant figures, $T = 79 \text{ N}$.

DOUBLE-CHECK: As they should be, d is less than D and T is less than Mg .

- 4.51. **THINK:** The masses are given as $m_1 = 36.5 \text{ kg}$, $m_2 = 19.2 \text{ kg}$ and $m_3 = 12.5 \text{ kg}$. Determine the acceleration of m_1 , a_1 . As there are no forces in the x -direction, only the y -direction needs to be considered.

SKETCH:



RESEARCH: To determine a_1 , use $F_{\text{net},y} = \sum F_{1,y}$. Determine T_2 and T_3 by summing the forces on m_2 and m_3 . A key idea is that $a_1 = -a_2 = -a_3$, as all the masses are connected (and ignoring any tipping of m_1). With $m_1 > m_2 + m_3$, it can be seen that m_1 moves downward while m_2 and m_3 move upward.

SIMPLIFY: $m_2: F_{2,\text{net}} = T_2 - m_2g \Rightarrow m_2a_2 = T_2 - m_2g \Rightarrow T_2 = m_2(a_2 + g)$.

$m_3: F_3 = T_3 - m_3g \Rightarrow m_3a_3 = T_3 - m_3g \Rightarrow T_3 = m_3(a_3 + g)$

$m_1: F_{1,\text{net}} = T_2 + T_3 - m_1g \Rightarrow m_1a_1 = m_2(a_2 + g) + m_3(a_3 + g) - m_1g$

With $a_1 = -a_2 = -a_3$,

$$\begin{aligned} m_1a_1 &= m_2(-a_1 + g) + m_3(-a_1 + g) - m_1g \\ m_1a_1 + m_2a_1 + m_3a_1 &= m_2g + m_3g - m_1g \\ a_1(m_1 + m_2 + m_3) &= g(m_2 + m_3 - m_1) \\ a_1 &= \frac{g(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} \end{aligned}$$

CALCULATE: $a_1 = \frac{9.81 \text{ m/s}^2 (19.2 \text{ kg} + 12.5 \text{ kg} - 36.5 \text{ kg})}{(36.5 \text{ kg} + 19.2 \text{ kg} + 12.5 \text{ kg})} = -0.69044 \text{ m/s}^2$

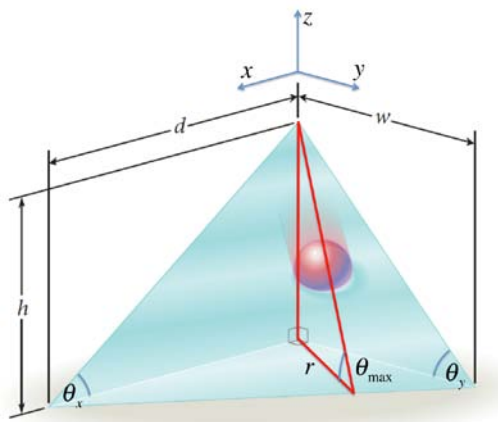
ROUND: There are two significant figures in the sum in the numerator, so the answer should be written, $a_1 = 0.69 \text{ m/s}^2$ downward.

DOUBLE-CHECK: a_1 is less than g in magnitude, which it should be for this system.

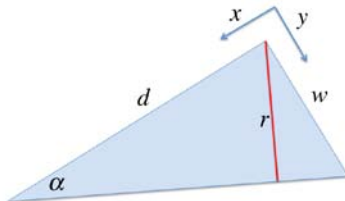
- 4.52. **THINK:** The block's dimensions are $w = 1.165 \text{ m}$, $d = 1.648 \text{ m}$ and $h = 1.051 \text{ m}$. The paperweight's mass is $m = 16.93 \text{ kg}$. Determine the paperweight's acceleration, a , down the incline. We first need to realize that the paperweight will slide down the plane in the direction of the steepest slope, i.e. the direction that has the smallest angle with the vertical. (This is the same as for a golf ball rolling down the side of a hill on a putting green.) The most difficult part of this problem is to find which way the steepest slope points. At

the edges of our slope the angles are $\theta_x = \tan^{-1}(h/d)$ and $\theta_y = \tan^{-1}(h/w)$. But the question is if there is a steeper angle somewhere in between the x - and y -directions. One may be tempted to think that this leads to a minimization problem involving some sort of derivative. However, there is a shortcut, if one realizes that the angle of steepest descent, $\theta_{\max}$, is the one for which the bottom of the slope has the shortest distance, r , to the corner of the block.

SKETCH: We use the figure in the problem and indicate our coordinate system at the top. The red triangle containing the angle $\theta_{\max}$ is a right triangle with side lengths h along the z -direction and r in the xy -plane. The hypotenuse of this triangle is the path along which the paperweight slides down.



It is also instructive to draw a top view of the bottom triangle in the xy -plane, because it helps us to determine the length of the distance r . From this drawing we see that the direction of r has to be such that it forms the height of the right triangle in the xy -plane.



RESEARCH: From trigonometry we know that the angle $\alpha = \tan^{-1}(w/d)$ and that the length of r is then given by $r = d \sin \alpha$. Once we have r , we can compute the angle $\theta_{\max} = \tan^{-1}(h/r)$. The magnitude of the acceleration is then calculated as $a = g \sin \theta_{\max}$, which is universally the case for inclined plane problems.

SIMPLIFY: Inserting all intermediate results, we find

$$\begin{aligned} a &= g \sin \theta_{\max} = g \sin \left(\tan^{-1} \left[\frac{h}{r} \right] \right) \\ &= g \sin \left(\tan^{-1} \left[\frac{h}{d \sin \alpha} \right] \right) \\ &= g \sin \left(\tan^{-1} \left[\frac{h}{d \sin(\tan^{-1}(w/d))} \right] \right) \end{aligned}$$

CALCULATE: Inserting our given numbers results in

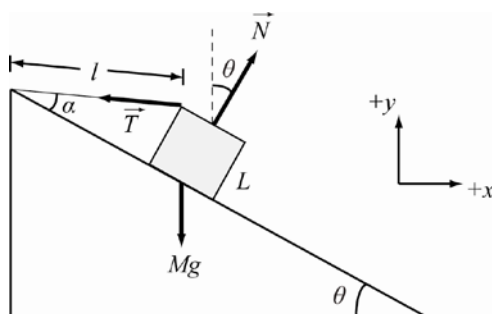
$$a = (9.81 \text{ m/s}^2) \sin \left(\tan^{-1} \left[\frac{105.1}{164.8 \sin(\tan^{-1}(116.5/164.8))} \right] \right) = 7.27309 \text{ m/s}^2$$

ROUND: Since the gravitational acceleration is only specified to three significant digits, we also round our result to $a = 7.27 \text{ m/s}^2$.

DOUBLE-CHECK: Our result is smaller than the free-fall value of the gravitational acceleration, which is comforting. What about limits? As $h \rightarrow 0$ we find $a \rightarrow 0$, as expected. Also, we find that in either limit, $d \rightarrow 0$ or $w \rightarrow 0$, that our acceleration value approaches $a \rightarrow g$, which is also expected.

- 4.53. **THINK:** The only forces that act on the block are the force of gravity, the normal force from the ramp, and the tension force from the rope. Gravity, as always, points straight down. The normal force, also as always, is perpendicular to the plane. And the tension force points in the direction of the rope. No friction is present. The block is held in position, which implies force equilibrium. The only direction that motion could occur in is along the plane, and so we should try to compute the force components along the direction of the plane and make sure they add up to 0, as required by Newton's first law.

SKETCH:



RESEARCH: Note that we have drawn the angle α , where $\alpha = \sin^{-1}(L/l)$. It is the angle between the rope and the plane. The component of the tension force along the plane is $T \cos \alpha$, where we have defined the direction up the plane as positive. As always in inclined plane problems, the force component of gravity along the plane is $Mg \sin \theta$. Since these are the only two force components along the plane, we find:

$$T \cos \alpha - Mg \sin \theta = 0$$

SIMPLIFY: It is now fairly straightforward to solve for the magnitude of the tension in the string:

$$T = \frac{Mg \sin \theta}{\cos \alpha} = \frac{Mg \sin \theta}{\cos(\sin^{-1}(L/l))}$$

We can use the trigonometric identity $\cos \phi = \sqrt{1 - \sin^2 \phi}$ (which is valid for any angle) and then find finally

$$T = \frac{Mg \sin \theta}{\cos \alpha} = \frac{Mg \sin \theta}{\sqrt{1 - (L/l)^2}}$$

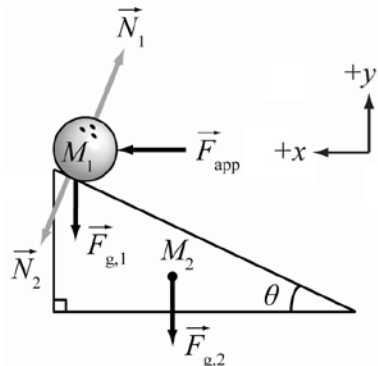
CALCULATE: $T = \frac{64.0 \text{ kg}(9.81 \text{ m/s}^2) \sin(26.0^\circ)}{\sqrt{1 - (0.400/1.60)^2}} = 284.2532 \text{ N}$

ROUND: There are three significant figures in all given values, so the answer should be written as $T = 284 \text{ N}$.

DOUBLE-CHECK: There are two limiting cases that we can study to see if our solution makes sense. First we can see what happens in the limit that L approaches 0. We can see that our solution then reduces to $T = Mg \sin \theta$, which is as expected, because then the rope is along the plane and the tension is simply equal to the component of gravity along the plane. For any value of $L > 0$, the tension has to increase. In the limiting case that the side length of the cube approaches the length of the rope, the tension in the rope has to become infinitely large, which is also born out by our analytic result. This gives us added confidence that we have solved the problem properly.

- 4.54. **THINK:** The bowling ball's mass is $M_1 = 6.00$ kg. The initial velocity is $v_{b0} = 0$. The wedge's mass is $M_2 = 9.00$ kg and it sits on a frictionless surface. The wedge's angle is $\theta = 36.9^\circ$. Determine (a) the force on the ball, F_{app} , to maintain the ball's vertical position, and (b) the magnitude of the wedge's acceleration, a_2 , if $F_{\text{app}} = 0$.

SKETCH:



RESEARCH:

(a) $F_{1\text{net},y} = 0$ in order to keep the ball in the same vertical position. Also, the ball and wedge must have the same velocity in the x direction at all times in order to prevent the ball from rolling down (or up) the wedge $\Rightarrow a_{1,x} = a_{2,x} = a_x$. To determine F_{app} , sum the forces in the x direction acting on M_2 : $F_{2\text{net},x} = \sum F_{2,x}$. The normal force of the ball on the wedge N_2 must be determined. By Newton's third law, $\vec{N}_2 = -\vec{N}_1$. $\vec{N}_1$ can be determined by summing the forces on M_1 .

(b) The wedge's acceleration, a_2 , in the absence of any external force can be found by writing one equation relating accelerations in the x -direction and three equations relating force to acceleration in the x - and y -directions. The resulting system of four equations can be solved for a_2 .

SIMPLIFY:

(a) First, consider M_1 : In the y direction, $F_{1\text{net},y} = 0$ to keep the ball in the same vertical position. Then,

$$F_{1\text{net},y} = N_{1,y} - F_{g,1} = 0 \Rightarrow N_1 \cos \theta = M_1 g \Rightarrow N_1 = \frac{M_1 g}{\cos \theta}.$$

In the x -direction, $F_{1\text{net},x} = F_{\text{app}} - N_{1,x} = F_{\text{app}} - N_1 \sin \theta = F_{\text{app}} - \frac{M_1 g}{\cos \theta} \sin \theta = F_{\text{app}} - M_1 g \tan \theta$. Now

consider M_2 : The net force in the x -direction is $F_{2\text{net},x} = N_{2,x} = N_2 \sin \theta = M_1 g \tan \theta$, where we have used $|\vec{N}_2| = |\vec{N}_1|$ from Newton's third law. To keep the ball stationary with respect to the wedge,

$$a_{1,x} = a_{2,x} \Rightarrow \frac{F_{1\text{net},x}}{M_1} = \frac{F_{2\text{net},x}}{M_2} \Rightarrow \frac{F_{\text{app}} - M_1 g \tan \theta}{M_1} = \frac{M_1 g \tan \theta}{M_2}$$

Solving for F_{app} yields $F_{\text{app}} = M_1 \left(\frac{M_1 g \tan \theta}{M_1} + \frac{M_1 g \tan \theta}{M_2} \right) = M_1 \left(1 + \frac{M_1}{M_2} \right) g \tan \theta$.

(b) To determine a_2 when $F_{\text{app}} = 0$, note that $v_2 - v_{1,x}$ is the relative velocity at which the wedge is moving horizontally out from under the ball, and therefore $(v_2 - v_{1,x}) \tan \theta$ is the rate at which the surface of the wedge drops downward beneath the ball. Since the ball drops, too, maintaining contact with the wedge, this is also the vertical speed of the ball, $|v_{1,y}|$. Using the sign convention in the figure to write the appropriate equation, and then *taking the time derivative of all each side*, we obtain

$$\begin{aligned}
(v_2 - v_{1,x}) \tan \theta &= -v_{1,y} \\
(a_2 - a_{1,x}) \tan \theta &= -a_{1,y} \\
a_{1,y} &= (a_{1,x} - a_2) \tan \theta
\end{aligned} \tag{1}$$

Now, knowing that $|N_1| = |N_2| = N$, we relate force to acceleration in the x -direction for the wedge, and in both the x - and y -directions for the ball:

$$N \sin \theta = M_2 a_2 \tag{2}$$

$$N \sin \theta = -M_1 a_{1,x} \tag{3}$$

$$N \cos \theta - M_1 g = M_1 a_{1,y} \tag{4}$$

We eliminate N using equations (2) and (3):

$$\begin{aligned}
M_2 a_2 &= -M_1 a_{1,x} \\
a_{1,x} &= -\frac{M_2}{M_1} a_2
\end{aligned} \tag{5}$$

We also eliminate N using equations (2) and (4):

$$\begin{aligned}
\frac{M_2 a_2}{\sin \theta} \cos \theta - M_1 g &= M_1 a_{1,y} \\
M_2 a_2 \cot \theta - M_1 g &= M_1 a_{1,y} \\
a_{1,y} &= \frac{M_2}{M_1} a_2 \cot \theta - g
\end{aligned} \tag{6}$$

Finally, we eliminate $a_{1,y}$ using equations (1) and (6), substitute for $a_{1,x}$ using (5), and solve for a_2 .

$$\begin{aligned}
(a_{1,x} - a_2) \tan \theta &= \frac{M_2}{M_1} a_2 \cot \theta - g \\
\left(-\frac{M_2}{M_1} a_2 - a_2 \right) \tan \theta &= \frac{M_2}{M_1} a_2 \cot \theta - g \\
-a_2 \left(\frac{M_2}{M_1} + 1 \right) \tan \theta - \frac{M_2}{M_1} a_2 \cot \theta &= -g \\
a_2 &= \frac{g}{\left(\frac{M_2}{M_1} + 1 \right) \tan \theta + \frac{M_2}{M_1} \cot \theta} \\
a_2 &= \frac{M_1 g}{(M_1 + M_2) \tan \theta + M_2 \cot \theta}
\end{aligned}$$

Note that $a_2 = a_{2,x}$, as the wedge does not accelerate in the y direction.

CALCULATE:

$$(a) \ F_{\text{app}} = (6.00 \text{ kg}) \left(1 + \frac{6.00 \text{ kg}}{9.00 \text{ kg}} \right) (9.81 \text{ m/s}^2) \tan(36.9^\circ) = 73.66 \text{ N}$$

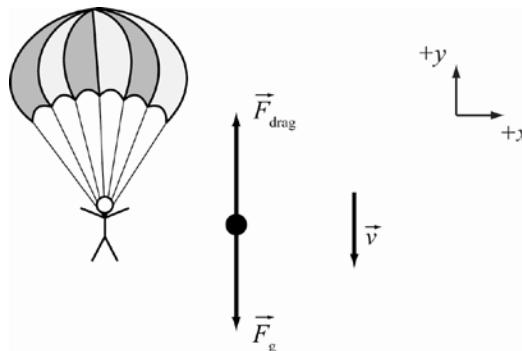
$$(b) \ a_w = \frac{(6.00 \text{ kg})(9.81 \text{ m/s}^2)}{(6.00 \text{ kg} + 9.00 \text{ kg}) \tan(36.9^\circ) + (9.00 \text{ kg}) \cot(36.9^\circ)} = 2.532 \text{ m/s}^2$$

ROUND: There are three significant figures in both masses, so the results should be written as $F_{\text{app}} = 73.7 \text{ N}$ and $a_w = 2.53 \text{ m/s}^2$.

DOUBLE-CHECK: The applied force seems reasonable for the bowling ball on an inclined wedge. A positive a_2 implies that the wedge accelerates to the left in the sketch, as it should.

- 4.55. **THINK:** The skydiver's total mass is $m = 82.3$ kg. The drag coefficient is $c_d = 0.533$. The parachute area is $A = 20.11$ m². The density of air is $\rho = 1.14$ kg/m³. The skydiver has reached terminal velocity ($a_{\text{net}} = 0$). Determine the drag force of the air, F_{drag} .

SKETCH:



RESEARCH: The skydiver has achieved terminal velocity, that is $F_{\text{net},y} = 0$. By balancing the forces in y , F_{drag} can be determined.

SIMPLIFY: $F_{\text{net},y} = F_{\text{drag}} - F_g = 0 \Rightarrow F_{\text{drag}} = F_g = mg$

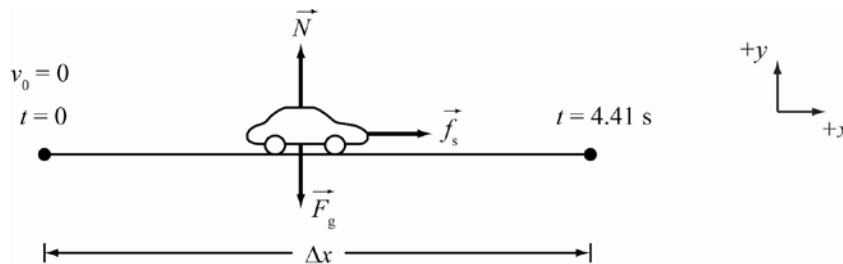
CALCULATE: $F_{\text{drag}} = (82.3 \text{ kg})(9.81 \text{ m/s}^2) = 807.36 \text{ N}$

ROUND: Since the mass has three significant figures, $F_{\text{drag}} = 807 \text{ N}$.

DOUBLE-CHECK: Since the skydiver has reached terminal velocity, the air's drag force should be equal to her weight, mg .

- 4.56. **THINK:** The dragster's initial speed is $v_0 = 0$. The distance traveled is $\Delta x = 402$ m in time, $t = 4.441$ s. Determine the coefficient of static friction, μ_s necessary to achieve this result.

SKETCH:



RESEARCH: $f_s = \mu_s N$. To determine f_s and N , the forces acting along the x and y directions must be balanced: $F_{\text{net},x} = \sum F_x$ and $F_{\text{net},y} = \sum F_y$. Note, the vertical acceleration is zero, so $F_{\text{net},y} = 0$. $F_{\text{net},x} = ma_{\text{net},x}$. Determine $a_{\text{net},x}$ by assuming a constant acceleration and using the equation $\Delta x = v_0 t + (at^2)/2$.

SIMPLIFY: To determine $a_{\text{net},x}$: $\Delta x = v_0 t + (a_{\text{net},x} t^2)/2 = (a_{\text{net},x} t^2)/2$ with $v_0 = 0$. Then, $a_{\text{net},x} = 2\Delta x / t^2$. Sum the forces in the vertical direction on the dragster: $F_{\text{net},y} = N - F_g = 0 \Rightarrow N = F_g = mg$. Sum the forces in the horizontal direction: $F_{\text{net},x} = f_s \Rightarrow ma_{\text{net},x} = \mu_s N \Rightarrow ma_{\text{net},x} = \mu_s mg$. So,

$$\mu_s = \frac{a_{\text{net},x}}{g} = \frac{2\Delta x}{gt^2}.$$

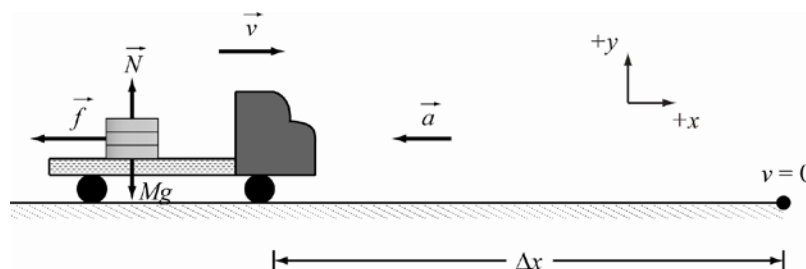
CALCULATE: $\mu_s = \frac{2(402 \text{ m})}{(9.81 \text{ m/s}^2)(4.441 \text{ s})^2} = 4.1555$

ROUND: There are three significant figures in the distance as given, so the result should be written $\mu_s = 4.16$.

DOUBLE-CHECK: The coefficient of friction that we have calculated would be extremely high for regular tires. If you ever walked around a car racetrack, you might have noticed that the road surface feels sticky. Why? The answer is that racing tires are very sticky, with the resulting higher coefficients of friction. However, the value we calculated here is too high, even for real racing tires, because we assumed that the only contribution to the normal force is the weight of the dragster. In real race situations, however, spoilers, wings, and other aerodynamics adjustments can convert some of the wind resistance into a downward force, which adds to the normal force. In addition, top fuel dragsters point their exhaust pipes almost straight up. The superchargers in the dragsters' engine expel the exhaust with very large velocities, and pointing the exhaust pipes up generates more downward force on the car. The net effect of all of these corrections is that real-life coefficients of friction do not have to be as high as our calculated result in order to achieve the accelerations reached.

- 4.57. **THINK:** The initial speed of the truck is $v = 30.0 \text{ m/s}$. The final speed of the truck is $v = 0$. The mass of the block is M . The coefficient of static friction is $\mu_s = 0.540$. Determine the minimum distance, Δx the truck can travel while stopping without causing the block to slide.

SKETCH:



RESEARCH: The minimum stopping distance occurs at the maximum acceleration the truck can undergo without causing the block to slide. Use the equation $v^2 = v_0^2 + 2a\Delta x$ to determine Δx . The acceleration is found from balancing the forces in the horizontal direction acting on the block.

SIMPLIFY: For the block, when it is just about to slide, $F_{\text{net},x} = -f_{s,\text{max}}$. Then, $Ma_{\text{net},x} = -\mu_s N = -\mu_s Mg \Rightarrow a_{\text{net},x} = -\mu_s g$. Since the block and the truck remain in contact, they form a single system with the same acceleration. With $v = 0$,

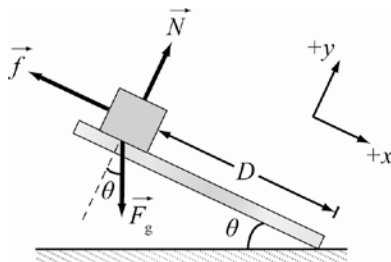
$$0 = v_0^2 + 2a_{\text{net},x} \Delta x \Rightarrow \Delta x = \frac{-v_0^2}{2a_{\text{net},x}} = \frac{-v_0^2}{2(-\mu_s g)} = \frac{v_0^2}{2\mu_s g}.$$

CALCULATE: $\Delta x = \frac{(30.0 \text{ m/s})^2}{2(0.540)(9.81 \text{ m/s}^2)} = 84.95 \text{ m}$

ROUND: To three significant figures, the result should be written $\Delta x = 84.9 \text{ m}$.

DOUBLE-CHECK: The displacement is positive, which is consistent with how the sketch is set up. This is a reasonable stopping distance.

- 4.58. **THINK:** The box's distance from the end of the board is $D = 0.540 \text{ m}$. The coefficients of friction are $\mu_s = 0.320$ and $\mu_k = 0.250$. Determine the speed of the box after it reaches the end of the board, v . It is useful to know the angle of the board with respect to the horizontal, θ .

SKETCH:

RESEARCH: The final speed, v can be found from the equation $v^2 = v_0^2 + 2a\Delta x$. Note, $v_0 = 0$ as the box starts from rest. The acceleration a is the net acceleration in the x direction. This is found by balancing the forces in the horizontal direction when the book is sliding. N is determined by balancing the forces in the vertical direction. The angle, θ can be determined from the equation $f_{s,\max} = \mu_s N$, just before the box begins to slide. Note that when the box is stationary, $F_{\text{net},x} = F_{\text{net},y} = 0$.

SIMPLIFY: First, determine θ . When the box is at rest, just about to slide, $F_{\text{net},x} = F_{g,x} - f_{s,\max} = 0 \Rightarrow F_{g,x} = f_{s,\max} \Rightarrow F_g \sin \theta = \mu_s N$. Since F_g is unknown, use the equation:

$$F_{\text{net},y} = N - F_{g,y} = 0 \Rightarrow N = F_g \cos \theta \Rightarrow F_g = \frac{N}{\cos \theta}.$$

Then, $(N / \cos \theta) \sin \theta = \mu_s N$. Rearranging, $\mu_s = \tan \theta \Rightarrow \theta = \tan^{-1}(\mu_s)$. Once the box is sliding, there is kinetic friction and a net acceleration in the horizontal direction. Determine $a_{\text{net},x}$:

$$F_{\text{net},x} = ma_{\text{net},x} = F_{g,x} - f_k = mg \sin \theta - \mu_k N.$$

From above, $N = F_g \cos \theta = mg \cos \theta$. Then, $ma_{\text{net},x} = mg \sin \theta - \mu_k mg \cos \theta$. This can be reduced:

$$a_{\text{net},x} = g(\sin \theta - \mu_k \cos \theta) \Rightarrow a_{\text{net},x} = g(\sin(\tan^{-1}(\mu_s)) - \mu_k \cos(\tan^{-1}(\mu_s))).$$

With $a_{\text{net},x}$ known, $v^2 = v_0^2 + 2a\Delta x \Rightarrow v = \sqrt{2a_{\text{net},x}D}$.

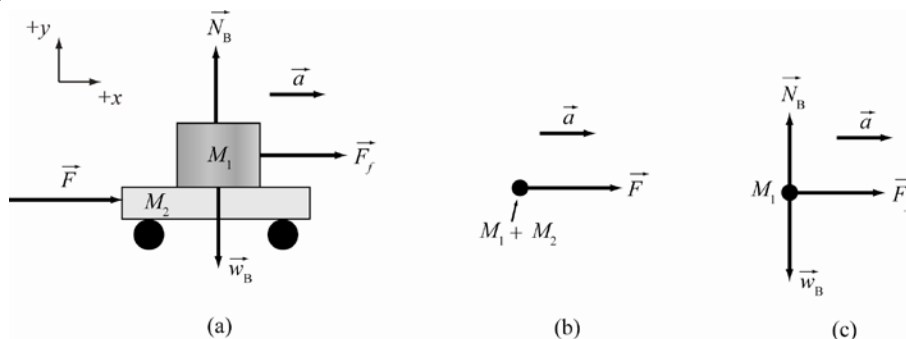
CALCULATE: $a_{\text{net},x} = (9.81 \text{ m/s}^2)(\sin(\tan^{-1}(0.320)) - (0.250)\cos(\tan^{-1}(0.320))) = 0.6540 \text{ m/s}^2$,

$$v = \sqrt{2(0.654 \text{ m/s}^2)(0.540 \text{ m})} = 0.8404 \text{ m/s}$$

ROUND: Due to the difference in values that appear in the equation for $a_{\text{net},x}$, there are two significant figures. The result should be written as $v = 0.84 \text{ m/s}$.

DOUBLE-CHECK: This is a reasonable speed to achieve after the box slides down the incline.

- 4.59. **THINK:** It is given that there is a block of mass, $M_1 = 0.640 \text{ kg}$ at rest on a cart of mass, $M_2 = 0.320 \text{ kg}$. The coefficient of static friction between the block and the cart is $\mu_s = 0.620$. Determine the maximum force on the cart and block such that the block does not slip.

SKETCH:

RESEARCH: Use Newton's second law: $\sum F_x = ma_x$, $\sum F_y = ma_y$. The force of friction is given by $F_f = \mu_s N$. First, consider a composite body (block and cart system, free-body diagram (b)). Applying Newton's second law, $\sum F_x = ma \Rightarrow F = (M_1 + M_2)a$. Note that both the block and the cart accelerate at the same rate. Second, consider only the block, applying Newton's second law in the horizontal and vertical directions: $\sum F_{B,y} = ma_y \Rightarrow N - w_B = 0 \Rightarrow N = w_B = M_1 g$ ($a_y = 0$), $\sum F_{B,x} = ma_x \Rightarrow F_f = M_1 a$.

SIMPLIFY: $F = (M_1 + M_2)a$, $F_f = M_1 a$, $N = M_1 g$. The maximum magnitude of F is when the acceleration is at a maximum. This means also that the force of friction is maximum which is equal to $F_f = \mu_s N = \mu_s M_1 g$. Note that when $F_f > \mu_s N$, the block starts to slip. $F_f = \mu_s M_1 g = M_1 a_{\max} \Rightarrow a_{\max} = \mu_s g$. Therefore, $F_{\max} = (M_1 + M_2)a_{\max} \Rightarrow F_{\max} = (M_1 + M_2)\mu_s g$.

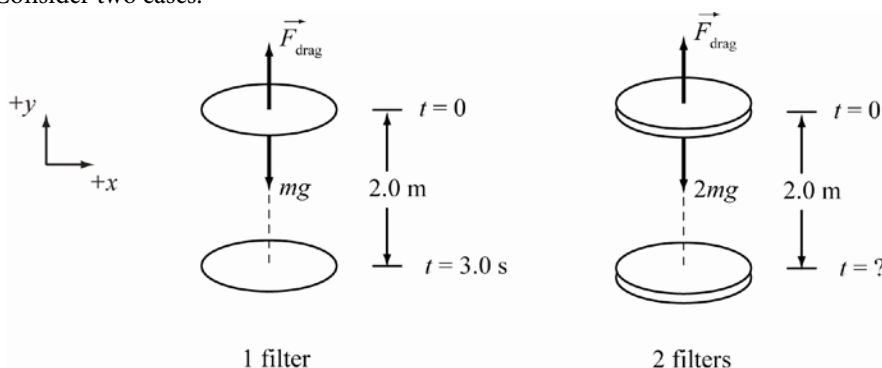
CALCULATE: $F_{\max} = (0.640 \text{ kg} + 0.320 \text{ kg})(0.620)(9.81 \text{ m/s}^2) = 5.83891 \text{ N}$

ROUND: There are three significant figures initially, so the result should be $F_{\max} = 5.84 \text{ N}$.

DOUBLE-CHECK: By checking the masses given and the coefficient of static friction, it can be determined that the result should be the same order of magnitude as gravity. This is indeed the case.

- 4.60. **THINK:** A coffee filter is dropped from a height of 2.0 m. The coffee filter reaches the ground after 3.0 s. What happens when there are two coffee filters? A drag force is $F_{\text{drag}} = Kv^2$. The drag constant, K does not change if there are one or two filters. Since the cross-sectional areas are the same for the filters, the drag force for two filters remains $F_{\text{drag}} = Kv^2$.

SKETCH: Consider two cases:



RESEARCH: Use Newton's second law to determine the acceleration of the system:

$$\sum F_y = ma \Rightarrow F_{\text{drag}} - mg = ma.$$

However, the brief period when the filters are accelerating has been neglected. This means consider only when the filters reach terminal velocity. This occurs when $a = 0$.

$$F_{\text{drag}} = Kv^2 = mg \Rightarrow v^2 = \frac{mg}{K} \Rightarrow v = \sqrt{\frac{mg}{K}}$$

Because of constant speed, use $y = y_0 - vt$ ($y = 0$ is the ground).

SIMPLIFY: $y_0 = vt \Rightarrow t = y_0 / v = y_0 / \sqrt{mg/K}$. For one filter, $m = m_0$: $t_1 = \frac{y_0}{\sqrt{\frac{m_0 g}{K}}}$.

For two filters, $m = 2m_0$:

$$t_2 = \frac{y_0}{\sqrt{\frac{2m_0 g}{K}}} = \frac{y_0}{\sqrt{2} \sqrt{\frac{m_0 g}{K}}} = \frac{t_1}{\sqrt{2}}.$$

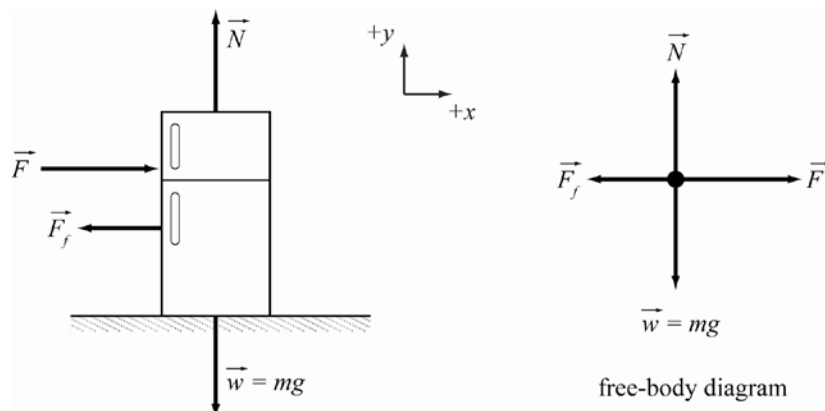
CALCULATE: $t_2 = \frac{3.0 \text{ s}}{\sqrt{2}} = 2.1213 \text{ s}$

ROUND: The first time has two significant figures, so the result should be rounded to $t_2 = 2.1 \text{ s}$.

DOUBLE-CHECK: It can be seen that t is inversely proportional to $\sqrt{m}$. This implies that if the mass of the object is increased, the time will be decreased. t_2 is less than t_1 as expected.

- 4.61. THINK:** The refrigerator has a mass of $m = 112.2 \text{ kg}$. The coefficients of static and kinetic friction are $\mu_s = 0.460$ and $\mu_k = 0.370$. Determine the force of friction if the refrigerator is pushed with a force F .

SKETCH:



RESEARCH: Use Newton's second law: $\sum F_x = ma_x \Rightarrow F - F_f = ma_x$, $\sum F_y = ma_y$ ($a_y = 0$), $N - w = 0 \Rightarrow N = mg$. To move the refrigerator, the maximum static friction needs to be overcome. The maximum static friction is given by $F_f = f_{s,\max} = \mu_s N = \mu_s mg$. After the refrigerator has moved, the force applied needs to be larger than the force of kinetic friction in order to keep the refrigerator moving. The force of kinetic friction is given by $f_k = \mu_k N = \mu_k mg$.

SIMPLIFY: $f_{s,\max} = \mu_s mg$ and $f_k = \mu_k mg$.

CALCULATE: $f_{s,\max} = 0.460(112.2 \text{ kg})(9.81 \text{ m/s}^2) = 506.31 \text{ N}$,

$f_k = 0.370(112.2 \text{ kg})(9.81 \text{ m/s}^2) = 407.25 \text{ N}$

ROUND: Rounding to three significant figures, since gravity has three significant figures, the results are $f_{s,\max} = 506 \text{ N}$ and $f_k = 407 \text{ N}$.

(a) Here, F is less than $f_s = 506 \text{ N}$. This means that the force of friction balances the force F . Therefore, the force of friction is 300 N .

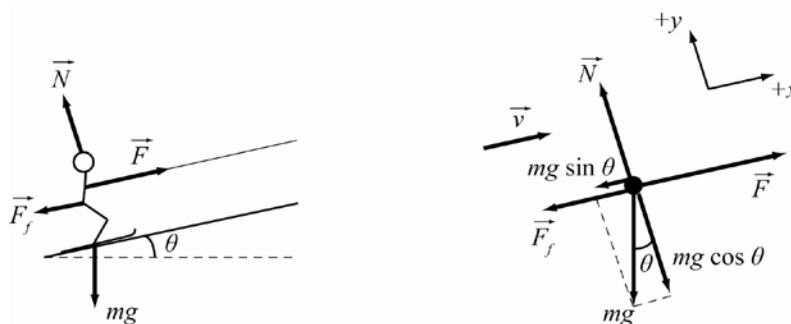
(b) F is still less than $f_s = 506 \text{ N}$. The force of friction is 500 N .

(c) F is larger than $f_s = 506 \text{ N}$. This means that initially the force of friction is 506 N , but after the refrigerator is in motion, the force of friction is the force of kinetic friction, $F_f = f_k = 407 \text{ N}$.

DOUBLE-CHECK: The force of friction must be equal or less than the force acting on an object, $F_f \leq F$. Also, the maximum static friction is always larger than kinetic friction, $f_{s,\max} > f_k$.

- 4.62. THINK:** A towrope pulls the skiers with a constant speed of 1.74 m/s . The slope of the hill is 12.4° . A child with a mass of 62.4 kg is pulled up the hill. The coefficients of static and kinetic friction are 0.152 and 0.104 , respectively. What is the force of the towrope acting on the child? Constant speed means zero acceleration, $a = 0$.

SKETCH:



free-body diagram

RESEARCH: Use Newton's second law: $\sum F_x = ma_x$, $\sum F_y = ma_y$. The maximum force of static friction is given by $f_{s,\max} = \mu_s N$ and the force of kinetic friction is given by $f_k = \mu_k N$. Initially, the force of the towrope must overcome the maximum static friction in order to move the child. $\sum F_y = ma_y = 0$ since $a_y = 0$. So $N - mg \cos \theta = 0 \Rightarrow N = mg \cos \theta$. $\sum F_x = ma_x = 0$ since $a_x = 0$. So $F - F_f - mg \sin \theta = 0 \Rightarrow F = F_f + mg \sin \theta$. Also, $F_f = f_{s,\max} = \mu_s N = \mu_s mg \cos \theta$.

SIMPLIFY: $N = mg \cos \theta$, $F = f_{s,\max} + mg \sin \theta = \mu_s mg \cos \theta + mg \sin \theta = mg(\mu_s \cos \theta + \sin \theta)$. After the child is in motion with a speed of 1.74 m/s, μ_s above is replaced by μ_k . Therefore, $F = mg(\mu_k \cos \theta + \sin \theta)$.

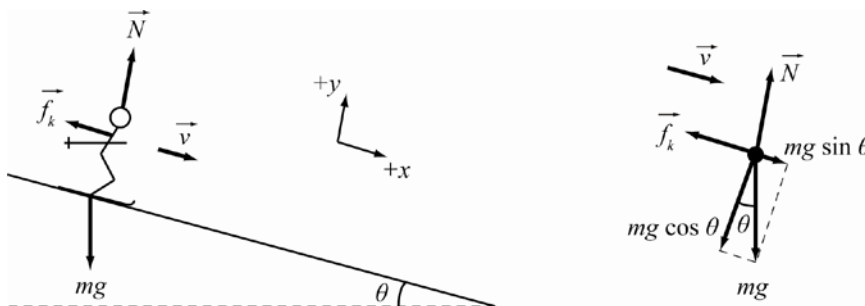
CALCULATE: $F = 62.4 \text{ kg}(9.81 \text{ m/s}^2)(0.152 \cos(12.4^\circ) + \sin(12.4^\circ)) = 222.324 \text{ N}$. After the child is in motion, $F = 62.4 \text{ kg}(9.81 \text{ m/s}^2)(0.104 \cos(12.4^\circ) + \sin(12.4^\circ)) = 193.63 \text{ N}$.

ROUND: Rounding to three significant figures, before movement, $F = 222 \text{ N}$ and after movement begins, $F = 194 \text{ N}$.

DOUBLE-CHECK: The initial force $F = 222 \text{ N}$ must be larger than the force after the child is in motion $F = 194 \text{ N}$.

- 4.63. THINK:** A skier moves down a slope with an angle of 15.0° . The initial speed is 2.00 m/s. The coefficient of kinetic friction is 0.100. Determine the speed after 10.0 s. First, the acceleration of the skier must be determined.

SKETCH:



RESEARCH: Assume the direction of motion is the positive direction of the x axis. The force of kinetic friction is given by $f_k = \mu_k N$. Use Newton's second law to determine the acceleration of the skier:

$$\begin{aligned}\sum F_x = ma_x &\Rightarrow mg \sin \theta - f_k = ma_x \Rightarrow ma_x = mg \sin \theta - \mu_k N \\ \sum F_y = ma_y \quad (a_y = 0) &\Rightarrow N - mg \cos \theta = 0 \Rightarrow N = mg \cos \theta\end{aligned}$$

SIMPLIFY: $ma_x = mg \sin \theta - \mu_k mg \cos \theta \Rightarrow a_x = g(\sin \theta - \mu_k \cos \theta)$. The speed after the time interval Δt is: $v = v_0 + a_x \Delta t = v_0 + g(\sin \theta - \mu_k \cos \theta) \Delta t$.

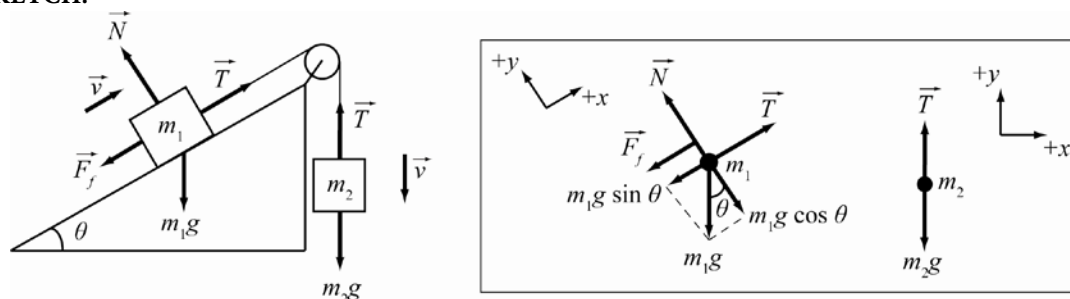
CALCULATE: $v = 2.00 \text{ m/s} + (9.81 \text{ m/s}^2)(\sin 15.0^\circ - 0.100 \cos 15.0^\circ)(10.0 \text{ s}) = 17.91 \text{ m/s}$

ROUND: Since v_0 has three significant figures, round the result to $v = 17.9 \text{ m/s}$.

DOUBLE-CHECK: 17.9 m/s is equivalent to about 64.4 km/h, which is a reasonable speed.

- 4.64. **THINK:** A block of mass, $m_1 = 21.9 \text{ kg}$ is connected to another block of mass, $m_2 = 25.1 \text{ kg}$ on an inclined plane with $\theta = 30.0^\circ$. The coefficients of friction are $\mu_s = 0.109$ and $\mu_k = 0.086$. Determine the displacement of block 2 after 1.51 s. Because block 2 is heavier than block 1, the displacement of block 2 is either downward or zero.

SKETCH:



The force of friction always opposes a motion. This means its direction is opposite to the direction of motion of the system.

RESEARCH: Use Newton's second law: $\sum F_x = ma_x$, $\sum F_y = ma_y$.

Block 2: $\sum F_y = m_2 a \Rightarrow T - m_2 g = m_2 a$.

Block 1: $\sum F_y = 0$ ($a_y = 0$) $\Rightarrow N - m_1 g \cos \theta = 0 \Rightarrow N = m_1 g \cos \theta$

$\sum F_x = m_1 a \Rightarrow m_1 g \sin \theta - T + F_f = m_1 a$.

SIMPLIFY: $T - m_2 g = m_2 a \Rightarrow T = m_2 g + m_2 a$, $F_f = \mu N = \mu m_1 g \cos \theta$ ($\mu = \mu_s$ if the blocks are at rest and $\mu = \mu_k$ if the blocks are in motion). So,

$$\begin{aligned} m_1 g \sin \theta - T + F_f &= m_1 a \\ m_1 g \sin \theta - m_2 g - m_2 a + F_f &= m_1 a \\ m_1 g \sin \theta - m_2 g + F_f &= (m_1 + m_2) a \\ m_1 g \sin \theta - m_2 g + \mu m_1 g \cos \theta &= (m_1 + m_2) a \\ a &= \frac{m_1 g \sin \theta - m_2 g + \mu m_1 g \cos \theta}{(m_1 + m_2)} \end{aligned}$$

Before a is calculated, it must be determined if the net force (excluding friction) is larger than the maximum force of static friction.

$$m_2 g - m_1 g \sin \theta > \mu_s m_1 g \cos \theta \Rightarrow m_2 > m_1 \sin \theta + \mu_s m_1 \cos \theta \Rightarrow m_2 > m_1 (\sin \theta + \mu_s \cos \theta)$$

Because $(\sin \theta + \mu_s \cos \theta) < 1$ and $m_2 > m_1$, the above condition is satisfied. So the above equation for the acceleration can be used. Displacement after t is $y = y_0 + v_0 t + (1/2)at^2$. $y_0 = 0$ and $v_0 = 0$.

$$y = \frac{1}{2}at^2 = \frac{1}{2}t^2 \left(\frac{m_1 g \sin \theta - m_2 g + \mu_k m_1 g \cos \theta}{(m_1 + m_2)} \right)$$

CALCULATE: Displacement is

$$y = \frac{1}{2}(1.51 \text{ s})^2 \left(\frac{21.9 \text{ kg}(9.81 \text{ m/s}^2)\sin(30^\circ) - 25.1 \text{ kg}(9.81 \text{ m/s}^2) + (0.086)21.9 \text{ kg}(9.81 \text{ m/s}^2)\cos(30^\circ)}{(21.9 \text{ kg} + 25.1 \text{ kg})} \right)$$

$$= -2.9789 \text{ m}$$

ROUND: Since the coefficient of kinetic friction has two significant figures, round the result to $y = -3.0 \text{ m}$.

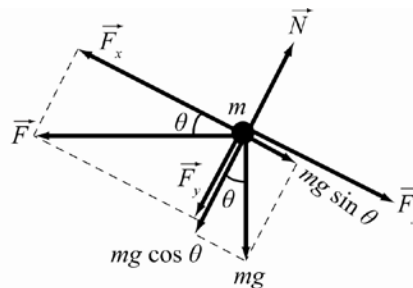
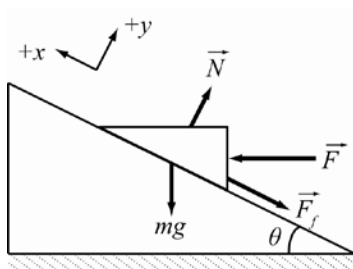
DOUBLE-CHECK: It is understandable that the displacement is negative, since m_2 is larger than m_1 . If it assumed the acceleration is equal to gravity, then

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.81 \text{ m/s}^2)(1.51)^2 = -11.2 \text{ m}.$$

The result is much smaller than -11.2 m , since the acceleration must be smaller than the acceleration due to gravity.

- 4.65. **THINK:** A wedge has a mass, $m = 36.1 \text{ kg}$. The angle is $\theta = 21.3^\circ$ and the force is $F = 302.3 \text{ N}$. The coefficient of kinetic friction is $\mu_k = 0.159$. Determine the acceleration.

SKETCH:



RESEARCH: Use Newton's second law: $\sum F_x = ma_x$ and $\sum F_y = ma_y$. $F_x = F \cos \theta$, $F_y = F \sin \theta$, and $F_f = \mu_k N$.

$$\sum F_y = ma_y = 0, \text{ since } a_y = 0 \Rightarrow N - F_y - mg \cos \theta = 0 \Rightarrow N = F_y + mg \cos \theta = F \sin \theta + mg \cos \theta$$

$$\sum F_x = ma_x \Rightarrow F_x - F_f - mg \sin \theta = ma \Rightarrow ma = F \cos \theta - \mu_k N - mg \sin \theta$$

$$\text{SIMPLIFY: } a = \frac{F \cos \theta - \mu_k (F \sin \theta + mg \cos \theta) - mg \sin \theta}{m} = \frac{F \cos \theta - \mu_k F \sin \theta - \mu_k mg \cos \theta - mg \sin \theta}{m}$$

$$= \frac{F}{m} (\cos \theta - \mu_k \sin \theta) - g (\mu_k \cos \theta + \sin \theta)$$

$$\text{CALCULATE: } a = \frac{302.3 \text{ N}}{36.1 \text{ kg}} (\cos(21.3^\circ) - 0.159 \sin(21.3^\circ)) - (9.81 \text{ m/s}^2) (0.159 \cos(21.3^\circ) + \sin(21.3^\circ))$$

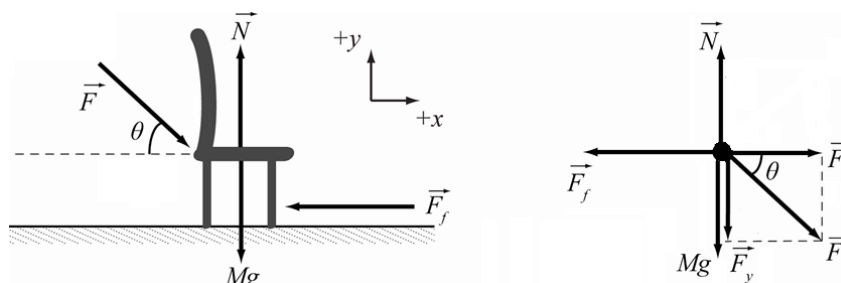
$$= 2.3015 \text{ m/s}^2$$

ROUND: Rounding to three significant figures, $a = 2.30 \text{ m/s}^2$.

DOUBLE-CHECK: If it is assumed that $\theta = 0$, then the acceleration is $a = F/m$. $a = \frac{302.2}{36.1} = 8.37 \text{ m/s}^2$

Therefore the result is reasonably less than this value.

- 4.66. **THINK:** The chair has a mass of M . The coefficient of static friction is $\mu_s = 0.560$. A force F is acting at an angle θ to the horizontal. Determine the range of θ so that the chair does not move. The condition requires that the horizontal component of the force F be equal to or less than the maximum force of static friction, $f_{s,\max} = \mu_s N$.

SKETCH:**RESEARCH:** Use Newton's second law:

$$\sum F_y = ma_y = 0 \Rightarrow N - F_y - Mg = 0 \quad \text{and} \quad \sum F_x = ma_x \Rightarrow F_x - F_f = ma$$

SIMPLIFY: $N = Mg + F \sin \theta$ and $F \cos \theta - F_f = ma$. The chair not moving means that $a = 0$, so $\sum F_x = 0$.

At the minimum angle,

$$\begin{aligned} F_x &= f_{s,\max} \\ F \cos \theta &= F_{f,\max} \\ F \cos \theta &= \mu_s N \\ F \cos \theta &= \mu_s (Mg + F \sin \theta) \\ F \cos \theta &= \mu_s Mg + \mu_s F \sin \theta \\ F(\cos \theta - \mu_s \sin \theta) &= \mu_s Mg \\ \cos \theta - \mu_s \sin \theta &= \frac{\mu_s Mg}{F} \end{aligned}$$

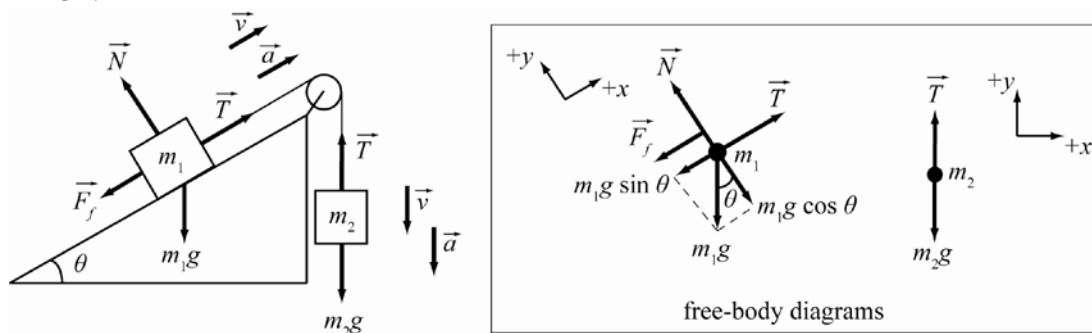
Since $\mu_s Mg / F$ is greater than or equal to zero, the critical value occurs when $\cos \theta - \mu_s \sin \theta = 0$. Solving

$$\text{for } \theta, \cos \theta = \mu_s \sin \theta \Rightarrow \frac{1}{\mu_s} = \frac{\sin \theta}{\cos \theta} = \tan \theta \Rightarrow \tan^{-1} \left(\frac{1}{\mu_s} \right) = \theta.$$

$$\text{CALCULATE: } \theta = \tan^{-1} \left(\frac{1}{0.560} \right) = 60.751^\circ$$

ROUND: Rounding to three significant figures, $\theta_c = 60.8^\circ$. Thus, the minimum angle at which any force can be applied and the chair will not move across the floor is 60.8° .**DOUBLE-CHECK:** If $\theta = 90^\circ$, the chair is pushed straight down and there are no horizontal forces, therefore the chair does not move.

- 4.67. **THINK:** The two blocks have masses $m_1 = 0.2500$ kg and $m_2 = 0.5000$ kg. The coefficients of static and kinetic friction are 0.250 and 0.123. The angle of the incline is $\theta = 30.0^\circ$. The blocks are initially at rest.

SKETCH:

RESEARCH:

(a) If there is no friction, $F_f = 0$, it is given that $m_2 > m_1$. This would cause block 1 to move up and block 2 to move down. The same motion occurs when there is friction, only the acceleration is less when there is friction.

(b) Use Newton's second law to determine the acceleration:

Body 1: $\sum F_y = 0$ and $a_y = 0$, so $N - m_1 g \cos \theta = 0 \Rightarrow N = m_1 g \cos \theta$.

Also, $\sum F_x = m_1 a$ so $T - m_1 g \sin \theta - F_f = m_1 a$.

Body 2: $\sum F_y = m_2 a$ so $m_2 g - T = m_2 a \Rightarrow T = m_2 g - m_2 a$.

SIMPLIFY: (b) $T - m_1 g \sin \theta - F_f = m_1 a$ and $F_f = \mu_k N$, so

$$m_2 g - m_2 a - m_1 g \sin \theta - \mu_k N = m_1 a$$

$$m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta = (m_1 + m_2) a$$

$$a = \frac{m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta}{(m_1 + m_2)}$$

$$a = g \frac{(m_2 - m_1 (\sin \theta + \mu_k \cos \theta))}{(m_1 + m_2)}$$

CALCULATE: (b) $a = (9.81 \text{ m/s}^2) \frac{(0.5000 \text{ kg} - 0.2500 \text{ kg} (\sin(30.0^\circ) + 0.123 \cos(30.0^\circ)))}{(0.5000 \text{ kg} + 0.2500 \text{ kg})} = 4.5567 \text{ m/s}^2$

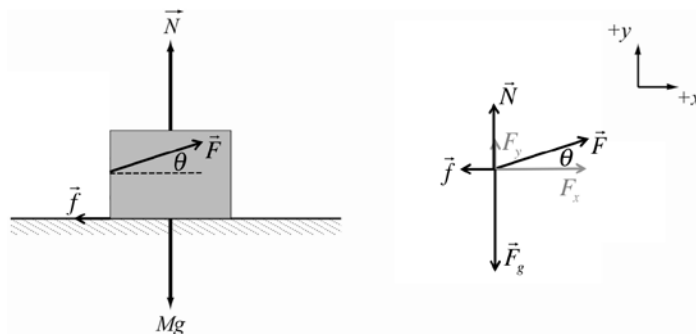
ROUND:

(b) Rounding to three significant figures,

$$a = 4.56 \text{ m/s}^2.$$

DOUBLE-CHECK: The result is reasonable since it is less than the acceleration due to gravity. In addition we find the limit of the acceleration of the Atwood machine in the limit of $\theta = 90^\circ$ (See Example 4.4) and the limit of Example 4.8, Two Blocks Connected by a Rope – with Friction, for $\theta = 0^\circ$ as limiting cases of our answer. This gives us additional confidence in our solution.

- 4.68. THINK:** Let's first consider the case where there is no friction; then the force F obviously has to act horizontally direction, because that is the direction of the intended acceleration. What, then, changes with the presence of friction? The friction force is always in the opposite direction of the motion of the block, and it is proportional to the magnitude of the normal force acting on the block from the supporting surface. Thus, if we direct the force downward, with an angle below the horizontal ($\theta < 0$), then our vertical force component adds to the normal force, which causes a large friction force ... which is bad. However, if we direct the force upward, then we *reduce* the normal force and thus reduce the friction force, and we have a chance to have a higher acceleration of the block than what we would have if the external force acted horizontally.

SKETCH:

RESEARCH: Use Newton's second law:

$$y\text{-direction: } N - Mg + F_y = 0 \Rightarrow N = Mg - F_y = Mg - F \sin \theta$$

$$x\text{-direction: } F_x - f = Ma \text{ and } f = \mu_k N, \text{ so } F_x - \mu_k N = Ma \Rightarrow F \cos \theta - \mu_k N = Ma.$$

SIMPLIFY:

$$F \cos \theta - \mu_k (Mg - F \sin \theta) = Ma \Rightarrow a = \frac{F \cos \theta - \mu_k Mg + \mu_k F \sin \theta}{M} = \frac{F}{M} (\cos \theta + \mu_k \sin \theta) - \mu_k g$$

The acceleration is maximized when $da/d\theta = 0$.

$$\frac{da}{d\theta} = \frac{F}{M} (-\sin \theta + \mu_k \cos \theta) = 0 \Rightarrow$$

$$-\sin \theta + \mu_k \cos \theta = 0$$

$$\tan \theta = \mu_k$$

$$\theta = \tan^{-1}(\mu_k)$$

CALCULATE:

$$(a) \theta = \tan^{-1}(0.41) = 22.2936^\circ$$

$$(b) a = \frac{10.0 \text{ N}}{0.5000 \text{ kg}} (\cos(22.29^\circ) + (0.41)\sin(22.29^\circ)) - (0.41)(9.81 \text{ m/s}^2) = 17.5936 \text{ m/s}^2$$

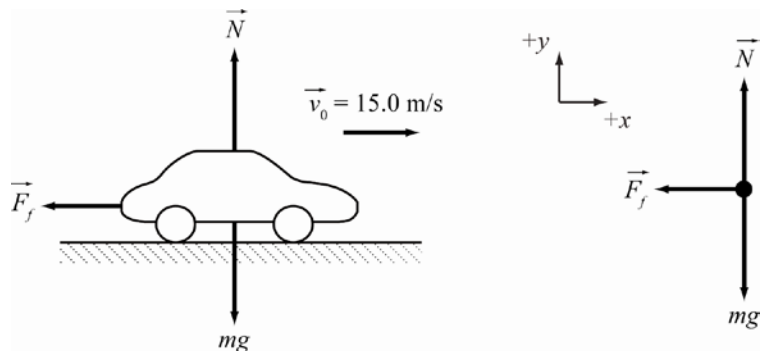
ROUND: Rounding to three significant figures, $\theta = 22.3^\circ$ and $a = 17.6 \text{ m/s}^2$.

DOUBLE-CHECK: The fact that the force is directed upward makes sense because the y -component of this applied force will reduce the normal force, acting to reduce the force of friction. Assuming no friction, the maximum acceleration is when $\theta = 0$. This means the acceleration is $a = F/M = 10.0 \text{ N}/0.500 \text{ kg} = 20.0 \text{ m/s}^2$. If there is friction and $\theta = 0$, the acceleration is

$$a = \frac{(F - \mu_k Mg)}{M} = \frac{F}{M} - \mu_k g = 20.0 \text{ m/s}^2 - 0.41(9.81 \text{ m/s}^2) = 16.4 \text{ m/s}^2.$$

The part (b) result is between 16.4 m/s^2 and 20.0 m/s^2 , as it should be.

- 4.69. THINK:** A car is initially moving at a speed of 15.0 m/s then hits the brakes to make a sudden stop. The coefficients of static and kinetic friction are 0.550 and 0.430 . Determine the acceleration and the distance traveled before the car stops.

SKETCH:

RESEARCH: Using Newton's second law: $\sum F_x = ma_x \Rightarrow F_f = ma \Rightarrow \mu_k N = ma$ and $\sum F_y = ma_y = 0 \Rightarrow N - mg \Rightarrow N = mg$. Also, $v^2 = v_0^2 - 2ax$.

SIMPLIFY:

$$(a) \mu_k mg = ma \Rightarrow a = \mu_k g$$

$$(b) v^2 = v_0^2 - 2ax \Rightarrow x = \frac{v_0^2 - v^2}{2a}$$

CALCULATE:

$$(a) \ a = 0.430(9.81 \text{ m/s}^2) = 4.2183 \text{ m/s}^2$$

$$(b) \ x = \frac{(15.0 \text{ m/s})^2 - 0^2}{2(4.2183 \text{ m/s}^2)} = 26.6695 \text{ m}$$

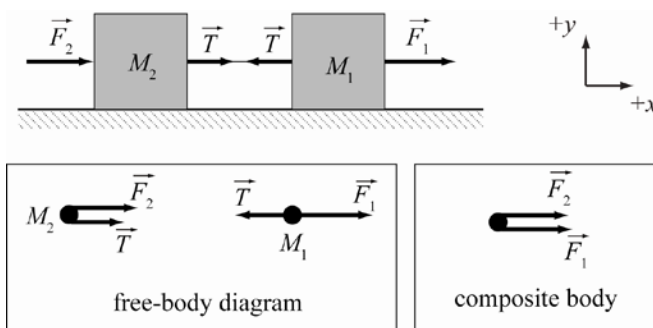
ROUND: Rounding to three significant figures,

$$(a) \ a = 4.22 \text{ m/s}^2 \text{ and}$$

$$(b) \ x = 26.7 \text{ m.}$$

DOUBLE-CHECK: The acceleration is less than the acceleration due to gravity, as expected.

- 4.70. **THINK:** There are two blocks with masses, $M_1 = 2.00 \text{ kg}$ and $M_2 = 6.00 \text{ kg}$. There are two forces, $F_1 = 10.0 \text{ N}$ and $F_2 = 5.00 \text{ N}$.

SKETCH:

RESEARCH: Using Newton's second law: $\sum F_x = ma_x$. Consider the composite body ($M_1 + M_2$):

$$F_1 + F_2 = (M_1 + M_2)a$$

SIMPLIFY:

$$(a) \ a = \frac{(F_1 + F_2)}{(M_1 + M_2)}; \text{ Consider } M_1: \sum F_x = ma, \ F_1 - T = m_1 a. \text{ Consider } M_2: \sum F_x = ma, \ F_2 + T = m_2 a.$$

$$(b) \ T = F_1 - m_1 a, \ T = m_2 a - F_2$$

$$(c) \ \text{The net force acting on } M_1 \text{ is } \sum F_x = F_1 - T = m_1 a.$$

CALCULATE:

$$(a) \ a = \frac{(10.0 \text{ N} + 5.00 \text{ N})}{(2.00 \text{ kg} + 6.00 \text{ kg})} = 1.875 \text{ m/s}^2$$

$$(b) \ T = 10.0 \text{ N} - (2.00 \text{ kg})(1.875 \text{ m/s}^2) = 6.25 \text{ N}$$

$$(c) \ \sum F = \sum F_x = (2.00 \text{ kg})(1.875 \text{ m/s}^2) = 3.75 \text{ N}$$

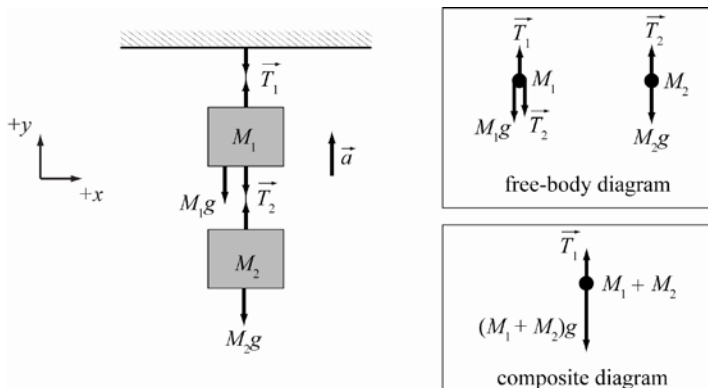
ROUND: To three significant figures, $a = 1.88 \text{ m/s}^2$, and $T = 6.25 \text{ N}$. The sum of the forces acting on M_1 is $\sum F = 3.75 \text{ N}$.

DOUBLE-CHECK: The tension T can also be calculated by $T = m_2 a - F_2$.

$$T = (6.0 \text{ kg})(1.875 \text{ m/s}^2) - 5.0 \text{ N} = 6.25 \text{ N}, \text{ which agrees with the previous result.}$$

- 4.71. **THINK:** There are two masses, $M_1 = 2.00$ kg and $M_2 = 4.00$ kg. For part (a): a constant velocity means zero acceleration.

SKETCH:



RESEARCH: Using Newton's second law:

$$\text{Mass 1: } \sum F_y = m_1 a \Rightarrow T_1 - T_2 - m_1 g = m_1 a$$

$$\text{Mass 2: } \sum F_y = m_2 a \Rightarrow T_2 - m_2 g = m_2 a$$

SIMPLIFY: $T_2 = m_2(a + g)$. Substitute into the following equation:

$$T_1 = T_2 + m_1 a + m_1 g \Rightarrow T_1 = m_2(a + g) + m_1(a + g) = (m_1 + m_2)(a + g)$$

$$\text{The composite mass } (m_1 + m_2): \sum F_y = ma \Rightarrow T_1 - (m_1 + m_2)g = (m_1 + m_2)a \Rightarrow T_1 = (m_1 + m_2)(a + g).$$

CALCULATE:

$$(a) \ a = 0, \text{ so } T_1 = (2.00 \text{ kg} + 4.00 \text{ kg})(9.81 \text{ m/s}^2) = 58.86 \text{ N}$$

$$(b) \ a = 3.00 \text{ m/s}^2, \text{ so } T_1 = (2.00 \text{ kg} + 4.00 \text{ kg})(3.00 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 76.86 \text{ N}$$

ROUND: Since the masses have three significant figures, the results should be rounded to:

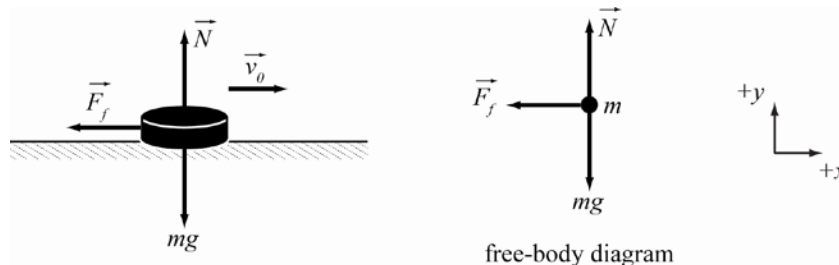
$$(a) \ T_1 = 58.9 \text{ N}$$

$$(b) \ T_1 = 76.9 \text{ N}$$

DOUBLE-CHECK: The tension increases as acceleration increases (assuming the acceleration is upward). As a check, the tension in part (a) is less than the tension in part (b).

- 4.72. **THINK:** The initial speed of a hockey puck is $v_0 = 12.5$ m/s. The puck stops after sliding a distance of 60.5 m. Determine the acceleration and then the coefficient of kinetic friction.

SKETCH:



RESEARCH: Using Newton's second law and the relation $v^2 = v_0^2 - 2ax$:

$$\sum F_x = ma_x \Rightarrow F_f = ma \Rightarrow F_f = \mu_k N$$

$$\sum F_y = ma_y = 0 \text{ (since } a_y = 0) \Rightarrow N - mg = 0 \Rightarrow N = mg$$

SIMPLIFY: $ma = \mu_k N = \mu_k mg \Rightarrow a = \mu_k g \Rightarrow \mu_k = a / g$. The final speed is zero since the puck has stopped; $v = 0$.

$$2ax = v_0^2 \Rightarrow a = \frac{v_0^2}{2x} \Rightarrow \mu_k = \frac{v_0^2}{2gx}$$

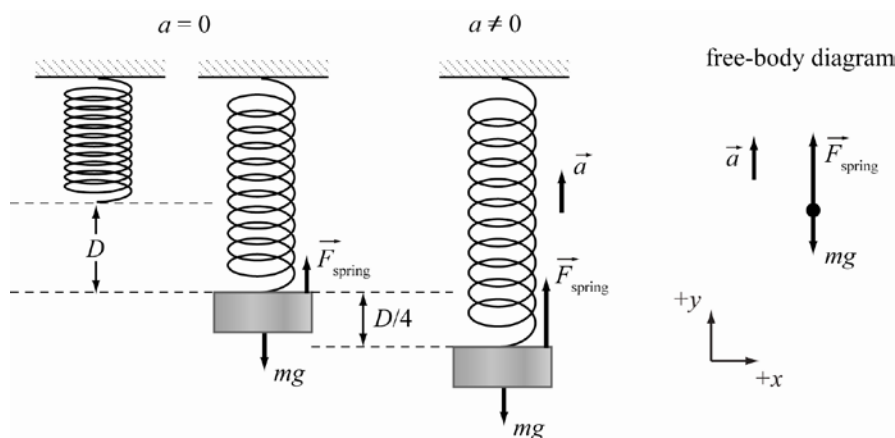
CALCULATE: $\mu_k = \frac{(12.5 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)60.5 \text{ m}} = 0.13163$

ROUND: Rounding to three significant figures, $\mu_k = 0.132$.

DOUBLE-CHECK: If the puck is on ice, it is expected that the coefficient of kinetic friction is small since the ice surface is smooth and slippery.

- 4.73. **THINK:** A mass M is attached to a massless spring. The spring stretches a distance D after the mass is attached. It stretches an additional $D/4$ after the elevator accelerates. Assume $F_{\text{spring}} = k\Delta x$.

SKETCH:



RESEARCH: Using Newton's second law and the equation for the force of the spring:

$$\sum F_y = ma \Rightarrow F_{\text{spring}} - mg = ma \Rightarrow F_{\text{spring}} = m(a + g) = k\Delta x.$$

SIMPLIFY: When $a = 0$, $\Delta x = D$. When $a \neq 0$, $\Delta x = D + D/4 = 5D/4$.

$$a = 0 \Rightarrow kD = mg \Rightarrow k = mg / D$$

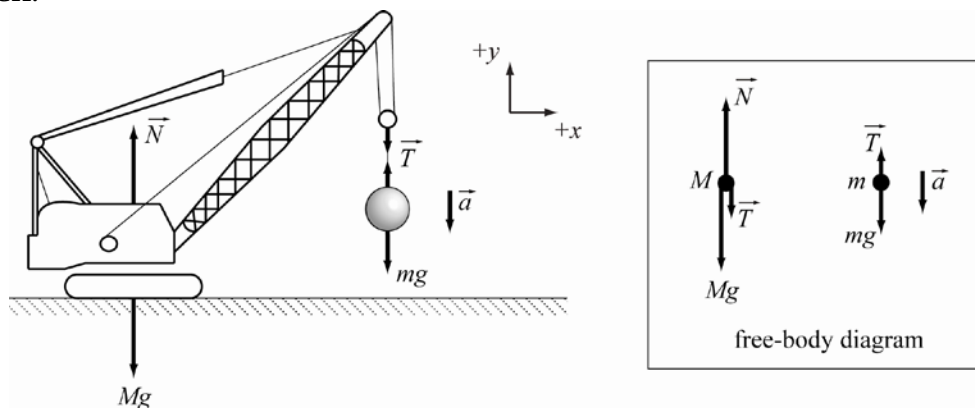
$$a \neq 0 \Rightarrow k \frac{5D}{4} = m(a + g) \Rightarrow \left(\frac{mg}{D} \right) \frac{5D}{4} = m(a + g) \Rightarrow \frac{5mg}{4} = m(a + g) \Rightarrow \frac{5g}{4} = a + g \Rightarrow a = \frac{g}{4}$$

CALCULATE: $a = \frac{(9.81 \text{ m/s}^2)}{4} = 2.4525 \text{ m/s}^2$

ROUND: $a = 2.45 \text{ m/s}^2$

DOUBLE-CHECK: It makes sense that $a = g/4$, since it produces $D/4$ displacement and D is proportional to g .

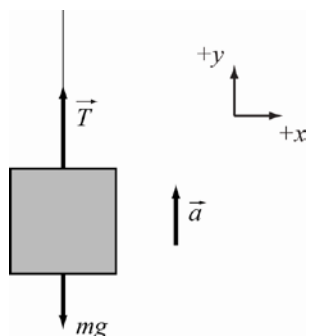
- 4.74. **THINK:** The mass of a crane is $M = 1.00 \cdot 10^4$ and the ball has a mass, $m = 1200$. kg. Determine the normal force exerted on the crane by the ground when (a) the acceleration is zero and (b) the acceleration is not zero.

SKETCH:**RESEARCH:** Use Newton's second law and the equation $v^2 = v_0^2 - 2ax$.For the crane: $\sum F_y = ma_y$, $a_y = 0$ because the crane does not move. $N - T - Mg = 0 \Rightarrow N = T + Mg$.For the ball: $\sum F_y = ma_y$, $a_y = -a$ (deceleration). $T - mg = -ma \Rightarrow T = m(g - a)$ **SIMPLIFY:** $N = m(g - a) + Mg \Rightarrow N = (m + M)g - ma$. $v = 0$ (the ball has stopped), so $a = v_0^2 / (2x)$.

$$N = (m + M)g - m \frac{v_0^2}{2x}$$

CALCULATE:(a) Constant speed means $a = 0$. $N = (1.00 \cdot 10^4 \text{ kg} + 1200. \text{ kg})(9.81 \text{ m/s}^2) = 109872 \text{ N}$.(b) $N = 109872 \text{ N} - (1200. \text{ kg}) \frac{(1.00 \text{ m/s})^2}{2(0.250 \text{ m})} = 107472 \text{ N}$ **ROUND:** Rounding to three significant figures, (a) $N = 1.10 \cdot 10^5 \text{ N}$ and (b) $N = 1.07 \cdot 10^5 \text{ N}$.**DOUBLE-CHECK:** Because the ball is decelerating, it is understandable that the normal force in part (a) is larger than the normal force in part (b). This is a similar situation to measuring weight in an elevator.

- 4.75. **THINK:** A block of mass $m = 20.0 \text{ kg}$ is initially at rest and then pulled upward with a constant acceleration, $a = 2.32 \text{ m/s}^2$.

SKETCH:**RESEARCH:** Using Newton's second law and the equation $v^2 = v_0^2 - 2ax$: $v_0 = 0 \Rightarrow v^2 = 2ax$. $\sum F_y = ma_y \Rightarrow T - mg = ma \Rightarrow T = m(a + g)$.**SIMPLIFY:** $v^2 = 2ax \Rightarrow v = \sqrt{2ax}$ **CALCULATE:**(a) $T = (20.0 \text{ kg})(2.32 \text{ m/s}^2 + 9.81 \text{ m/s}^2) = 242.6 \text{ N}$

$$(b) \sum F_y = (20.0 \text{ kg})(2.32 \text{ m/s}^2) = 46.4 \text{ N}$$

$$(c) v = \sqrt{2(2.32 \text{ m/s}^2)(2.00 \text{ m})} = 3.04631 \text{ m/s}$$

ROUND: Rounding to three significant figures, the results are

$$(a) T = 243 \text{ N},$$

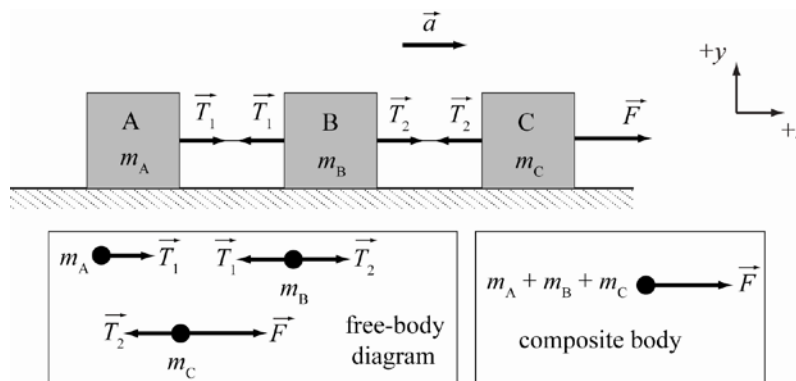
$$(b) \sum F_y = 46.4 \text{ N} \text{ and}$$

$$(c) v = 3.05 \text{ m/s}.$$

DOUBLE-CHECK: Since a is about $g/4$, the net force must be about $T/5$.

- 4.76. **THINK:** There are three blocks A, B and C. A force $F = 12 \text{ N}$ is pulling block C. Determine the tension in the string between blocks B and C.

SKETCH:



RESEARCH: First, consider the three blocks as a composite block. Using Newton's second law:

$$\sum F_x = ma_x \Rightarrow F - T_2 = m_c a \Rightarrow T_2 = F - m_c a.$$

$$\text{SIMPLIFY: } T_2 = F - m \left(\frac{F}{3m} \right) = F - \frac{F}{3} = \frac{2F}{3}$$

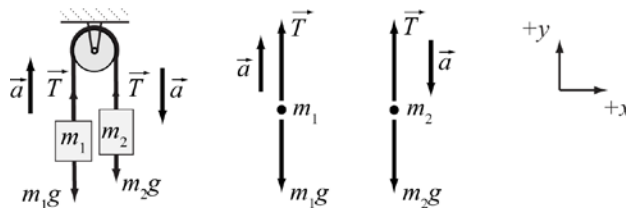
$$\text{CALCULATE: } T_2 = \frac{2(12 \text{ N})}{3} = 8.0 \text{ N}$$

ROUND: It is not necessary to round. $T_2 = 8.0 \text{ N}$

DOUBLE-CHECK: It is reasonable that $T_2 = 2F/3$ since T_2 pulls two blocks.

- 4.77. **THINK:** There are two masses, $m_1 = 3.00 \text{ kg}$ and $m_2 = 4.00 \text{ kg}$ arranged as an Atwood machine. Determine the acceleration of the blocks.

SKETCH:



RESEARCH: Using Newton's second law, block 1: $\sum F_y = ma_y$, $a_y = a$, and $T - m_1 g = m_1 a$. Block 2:

$$\sum F_y = ma_y, a_y = -a, \text{ and } T - m_2 g = -m_2 a.$$

SIMPLIFY:

$$\begin{aligned}
 T &= m_1(a + g) \\
 T - m_2g &= -m_2a \\
 m_1a + m_1g - m_2g + m_2a &= 0 \\
 (m_1 + m_2)a &= (m_2 - m_1)g \\
 a &= \frac{(m_2 - m_1)}{(m_1 + m_2)}g
 \end{aligned}$$

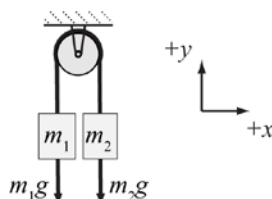
CALCULATE: $a = \frac{(4.00 \text{ kg} - 3.00 \text{ kg})}{(4.00 \text{ kg} + 3.00 \text{ kg})}(9.81 \text{ m/s}^2) = 1.4014 \text{ m/s}^2$

ROUND: Rounding to three significant figures, $a = 1.40 \text{ m/s}^2$.**DOUBLE-CHECK:** If the sum of the forces along the string are considered,

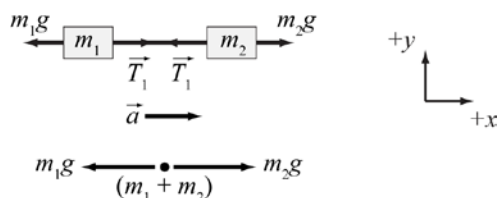
$$\sum F = (m_1 + m_2)a = m_2g - T + T - m_1g \Rightarrow a = \frac{(m_2 - m_1)}{(m_1 + m_2)}g.$$

This is the same as above.

- 4.78. THINK:** There are two blocks of masses, $m_1 = 3.50 \text{ kg}$ and m_2 arranged as an Atwood's machine. The acceleration of the blocks is $a_0 = 0.400g$. Determine the mass, m_2 .

SKETCH:

If the axis along the string is considered, the problem can be redrawn.

**RESEARCH:** The magnitude of the acceleration is given but not the direction of motion. Consider the two values of acceleration, $a = \pm a_0 = \pm 0.4g$. Using Newton's second law:

$$\sum F = ma \Rightarrow m_2g - m_1g = (m_1 + m_2)a \Rightarrow m_2(g - a) = m_1(g + a) \Rightarrow m_2 = m_1 \frac{(g + a)}{(g - a)}.$$

SIMPLIFY: If $a = +a_0$, $m_2 = m_1 \frac{(g + a_0)}{(g - a_0)}$. If $a = -a_0$, $m_2 = m_1 \frac{(g - a_0)}{(g + a_0)}$.

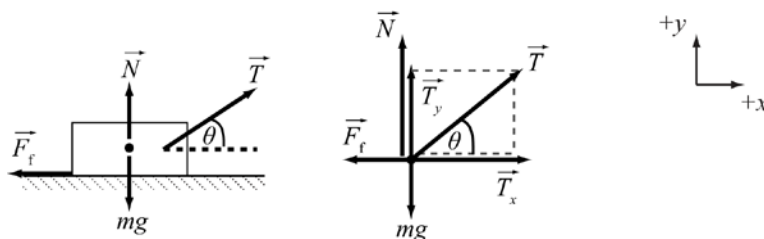
CALCULATE: $m_2 = (3.50 \text{ kg}) \frac{(9.81 \text{ m/s}^2 + 0.400g)}{(9.81 \text{ m/s}^2 - 0.400g)} = 8.1667 \text{ kg},$

or, $m_2 = (3.50 \text{ kg}) \frac{(9.81 \text{ m/s}^2 - 0.400g)}{(9.81 \text{ m/s}^2 + 0.400g)} = 1.5 \text{ kg}$

ROUND: Keeping three significant figures, the mass is either $m_2 = 8.17 \text{ kg}$ or $m_2 = 1.50 \text{ kg}$.**DOUBLE-CHECK:** There are two masses calculated for m_2 and one of them must be larger than m_1 and the other must be smaller than m_1 . This is what we have found, so the answers are reasonable.

- 4.79. THINK:** The sled has a mass, $M = 1000.$ kg. The coefficient of kinetic friction is $\mu_k = 0.600$. The sled is pulled at an angle $\theta = 30.0^\circ$ above the horizontal. Determine magnitude of the tension in the rope when the acceleration is $a = 2.00$ m/s².

SKETCH:



RESEARCH: $T_y = T \sin \theta$, $T_x = T \cos \theta$ and $F_f = \mu_k N$. Using Newton's second law: $\sum F_y = ma_y$ and $a_y = 0$, so $N + T_y - mg = 0 \Rightarrow N = mg - T_y = mg - T \sin \theta$. Also, $\sum F_x = ma \Rightarrow T_x - F_f = ma \Rightarrow T_x = F_f + ma$.

SIMPLIFY:

$$T \cos \theta = \mu_k N + ma$$

$$T \cos \theta = \mu_k (mg - T \sin \theta) + ma$$

$$T \cos \theta + \mu_k T \sin \theta = \mu_k mg + ma$$

$$T (\cos \theta + \mu_k \sin \theta) = \mu_k mg + ma$$

$$T = \frac{m(\mu_k g + a)}{(\cos \theta + \mu_k \sin \theta)}$$

CALCULATE: $T = \frac{(1000. \text{ kg})(0.600(9.81 \text{ m/s}^2) + 2.00 \text{ m/s}^2)}{(\cos(30.0^\circ) + 0.600 \sin(30.0^\circ))} = 6763.15 \text{ N}$

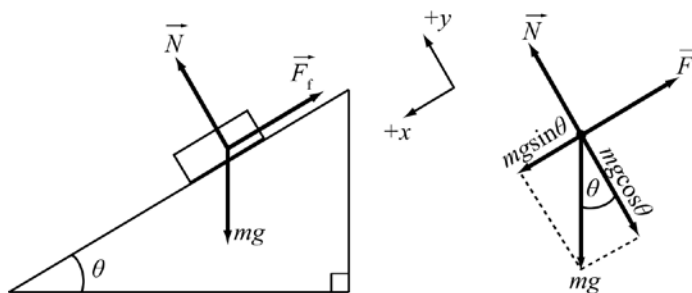
ROUND: Rounding to three significant figures, $T = 6760 \text{ N}$.

DOUBLE-CHECK: If there is no friction, $T \cos \theta = ma \Rightarrow T = \frac{ma}{\cos \theta} = \frac{(1000. \text{ kg})(2.00 \text{ m/s}^2)}{\cos 30.0^\circ} = 2309 \text{ N}$.

Since friction was considered previously, the result was larger.

- 4.80. THINK:** A block with a mass of $m = 2.00$ kg is on an inclined plane with an angle $\theta = 20.0^\circ$. The coefficient of static friction is $\mu_s = 0.60$.

SKETCH:



RESEARCH:

(a) The three forces are the normal, frictional and the gravitational forces.

(b) The maximum force of friction is given by $F_{f,\max} = f_s = \mu_s N$. To determine the normal force, use Newton's second law: $\sum F_y = ma_y = 0$ and $a_y = 0$, so $N - mg \cos \theta = 0 \Rightarrow N = mg \cos \theta$. Also, $\sum F_x = ma_x \Rightarrow mg \sin \theta - F_f = ma$.

SIMPLIFY:

(c) The block is moving if $mg \sin \theta > F_f = \mu_s N$ or $mg \sin \theta - \mu_s N > 0$.

$$\sum F_x = mg \sin \theta - \mu_s mg \cos \theta = mg (\sin \theta - \mu_s \cos \theta)$$

CALCULATE:

(b) $N = 2.00 \text{ kg} (9.81 \text{ m/s}^2) \cos 20.0^\circ = 18.437 \text{ N}$

(c) $\sum F_x > 0$ if $\sin \theta - \mu_s \cos \theta > 0$.

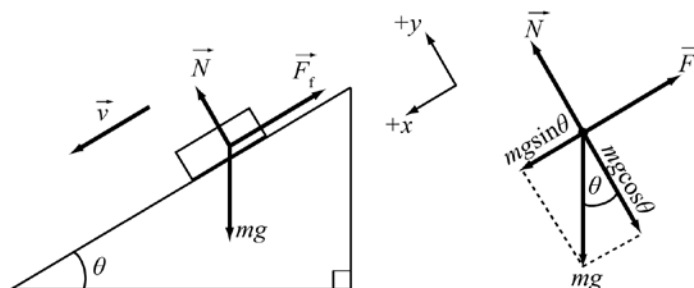
$$\sin \theta - \mu_s \cos \theta = \sin(20.0^\circ) - (0.60) \cos(20.0^\circ) = -0.2218 < 0$$

Therefore, $mg \sin \theta < \mu_s N$. This means the block does not move.

ROUND: (b) $N = 18.437 \text{ N}$. Rounding to two significant figures, $N = 18 \text{ N}$.

DOUBLE-CHECK: The critical angle of the inclined plane such that the blocks starts to move is $\tan \theta_c = \mu_s \Rightarrow \theta_c = \tan^{-1}(\mu_s)$. Here, $\theta_c = \tan^{-1}(0.60) = 30.96^\circ$. Since $\theta < \theta_c$, the block is not moving.

- 4.81. THINK:** A block of mass, $m = 5.00 \text{ kg}$ is sliding down an inclined plane of angle $\theta = 37.0^\circ$ at a constant velocity ($a = 0$). Determine the frictional force and the coefficient of kinetic friction.

SKETCH:

RESEARCH: There is no acceleration in any direction, $a_x = a_y = 0$. The force of friction is given by $F_f = \mu_k N$. Using Newton's second law: $\sum F_y = ma_y = 0$ and $a_y = 0$, so $N - mg \cos \theta = 0 \Rightarrow N = mg \cos \theta$. Also, $\sum F_x = ma_x = 0$ and $a_x = 0$, so $mg \sin \theta - F_f = 0 \Rightarrow F_f = mg \sin \theta$.

SIMPLIFY: $\mu_k N = mg \sin \theta \Rightarrow \mu_k mg \cos \theta = mg \sin \theta \Rightarrow \mu_k = \frac{\sin \theta}{\cos \theta} = \tan \theta$

CALCULATE:

(a) $F_f = (5.00 \text{ kg})(9.81 \text{ m/s}^2) \sin 37.0^\circ = 29.519 \text{ N}$

(b) $\mu_k = \tan(37.0^\circ) = 0.75355$

ROUND: Rounding to three significant figures,

(a) $F_f = 29.5 \text{ N}$ and

(b) $\mu_k = 0.754$.

DOUBLE-CHECK: μ_k is less than 1 and does not depend on the mass of the block.

- 4.82. THINK:** The mass of the skydiver, $m = 83.7 \text{ kg}$, is given as well as her drag coefficient, $c_d = 0.587$, and her surface area, $A = 1.035 \text{ m}^2$. We need to determine the terminal velocity then the time to reach a distance 296.7 m . Air density is $\rho = 1.14 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$.

SKETCH: Not needed.

RESEARCH: Terminal speed is $v_t = \sqrt{2mg / c_d \rho A}$, and for constant velocity $v = x/t$, where x is the distance traveled.

SIMPLIFY: $t = \frac{x}{v_t} = x \sqrt{\frac{c_d \rho A}{2mg}}$

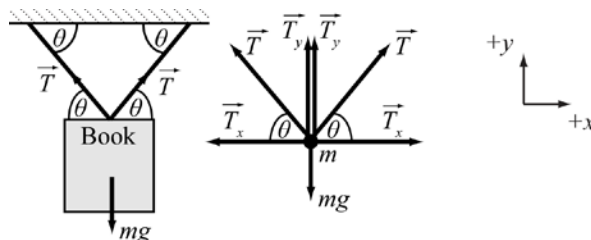
CALCULATE: $t = (296.7 \text{ m}) \sqrt{\frac{0.587(1.14 \text{ kg/m}^3)(1.035 \text{ m}^2)}{2(83.7 \text{ kg})(9.81 \text{ m/s}^2)}} = 6.09322 \text{ s}$

ROUND: Keeping only three significant digits, $t \approx 6.09 \text{ s}$.

DOUBLE-CHECK: The typical speed of a sky diver is 60 m/s. $t = (296.7 \text{ m}) / (60 \text{ m/s}) = 4.945 \text{ s}$. The result is comparable to this value.

- 4.83. THINK:** A book has a mass of $m = 0.500 \text{ kg}$. The tension on each wire is $T = 15.4 \text{ N}$. Determine the angle of the wires with the horizontal.

SKETCH:



RESEARCH: There is no acceleration in any direction, $a_x = a_y = 0$. Using Newton's second law:

$$\sum F_y = ma_y = 0 \Rightarrow 2T_y - mg = 0 \text{ and } \sum F_x = ma_x = 0 \Rightarrow T_x - T_x = 0.$$

SIMPLIFY: $T_y = T \sin \theta$, so $2T \sin \theta = mg \Rightarrow \sin \theta = \frac{mg}{2T} \Rightarrow \theta = \sin^{-1}\left(\frac{mg}{2T}\right)$.

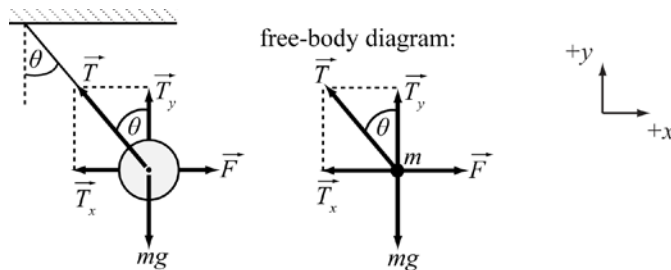
CALCULATE: $\theta = \sin^{-1}\left(\frac{0.500 \text{ kg}(9.81 \text{ m/s}^2)}{2(15.4 \text{ N})}\right) = 9.1635^\circ$

ROUND: Rounding to three significant figures, $\theta = 9.16^\circ$.

DOUBLE-CHECK: If the angle is $\theta = 90^\circ$, the tension required is $T = mg / 2 = 0.500(9.81) / 2 = 2.45 \text{ N}$. It is reasonable that a smaller angle requires more tension.

- 4.84. THINK:** A bob has a mass of $m = 0.500 \text{ kg}$. The angle is $\theta = 30.0^\circ$.

SKETCH:



RESEARCH: There is no acceleration in any direction, so $a_x = a_y = 0$. Using Newton's second law:

$$\sum F_x = 0 \Rightarrow F - T_x = 0 \text{ and } \sum F_y = 0 \Rightarrow T_y - mg = 0.$$

SIMPLIFY: $T_x = T \sin \theta$ and $T_y = T \cos \theta$. $F = T_x = T \sin \theta$ and $T \cos \theta = mg \Rightarrow T = mg / \cos \theta$. So,

$$F = \left(\frac{mg}{\cos \theta}\right) \sin \theta = mg \tan \theta$$

CALCULATE:

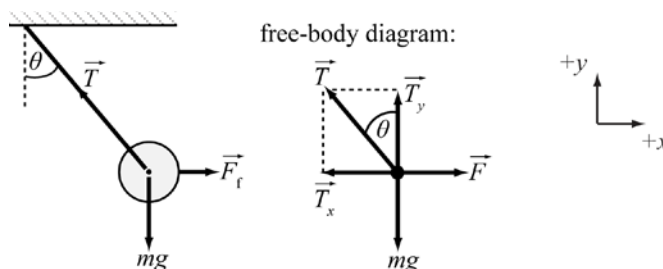
$$(a) F = (0.500 \text{ kg})(9.81 \text{ m/s}^2) \tan(30.0^\circ) = 2.8319 \text{ N}$$

$$(b) T = \frac{0.500 \text{ kg}(9.81 \text{ m/s}^2)}{\cos(30.0^\circ)} = 5.66381 \text{ N}$$

ROUND: Rounding to three significant figures, $F = 2.83 \text{ N}$ and $T = 5.66 \text{ N}$.

DOUBLE-CHECK: The ratio F/T is equal to $\sin 30.0^\circ = 0.5$, which is correct.

- 4.85. **THINK:** A ping-pong ball with a mass, $m = 2.70 \cdot 10^{-3} \text{ kg}$ is suspended by a string at an angle of $\theta = 15.0^\circ$ with the vertical. The force of friction is proportional to the square of the speed of the air stream, $v = 20.5 \text{ m/s}$.

SKETCH:

RESEARCH: Use the equation $F_f = cv^2$, where c is a constant. There is no acceleration in any direction, so $a_x = a_y = 0$. $T_y = T \cos \theta$, $T_x = T \sin \theta$. Using Newton's second law: $\sum F_x = ma_x = 0 \Rightarrow F_f - T_x = 0$ and $\sum F_y = ma_y = 0 \Rightarrow T_y - mg = 0$.

SIMPLIFY: $F_f = T \sin \theta$, and $T \cos \theta = mg \Rightarrow T = \frac{mg}{\cos \theta}$. And so,

$$F_f = cv^2 = \left(\frac{mg}{\cos \theta} \right) \sin \theta \Rightarrow c = \frac{mg}{v^2} \left(\frac{\sin \theta}{\cos \theta} \right) = \frac{mg}{v^2} \tan \theta$$

CALCULATE:

$$(a) c = \frac{2.70 \cdot 10^{-3} \text{ kg}(9.81 \text{ m/s}^2) \tan(15.0^\circ)}{(20.5 \text{ m/s})^2} = 1.688 \cdot 10^{-5} \text{ kg/m}$$

$$(b) T = \frac{2.70 \cdot 10^{-3} \text{ kg}(9.81 \text{ m/s}^2)}{\cos(15.0^\circ)} = 0.027421 \text{ N}$$

ROUND: Rounding to three significant figures, the results should be

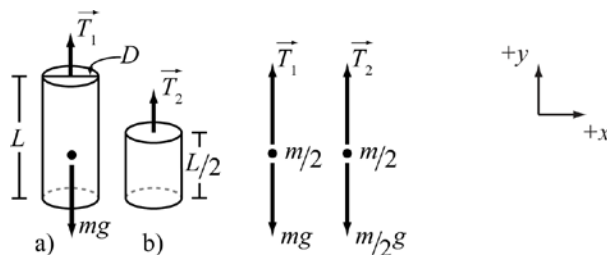
$$(a) c = 1.69 \cdot 10^{-5} \text{ kg/m} \text{ and}$$

$$(b) T = 0.0274 \text{ N.}$$

DOUBLE-CHECK: Because the mass of the ping pong ball is small, a small value for the tension is expected. Also, the cross-sectional area of the ball is small so the coefficient of friction is expected to be small.

- 4.86. THINK:** A silicon nanowire has a length of 100.0 nm and a diameter of 5.0 nm. The density of silicon is $\rho = 2.33 \text{ g/cm}^3$.

SKETCH:



RESEARCH: Using Newton's second law:

$$(a) \sum F_y = 0 \Rightarrow T_1 - mg = 0 \Rightarrow T_1 = mg$$

$$(b) \sum F_y = 0 \Rightarrow T_2 - \frac{m}{2}g = 0 \Rightarrow T_2 = \frac{mg}{2} = \frac{T_1}{2}. \text{ The mass of the nanowire is } m = \rho V = \rho(\pi R^2 L).$$

SIMPLIFY: $m = \rho \pi \left(\frac{D}{2}\right)^2 L = \rho \frac{\pi}{4} D^2 L$

CALCULATE: $\rho = 2.33 \text{ g/cm}^3 = \frac{2.33 \cdot 10^{-3} \text{ kg}}{(10^{-2} \text{ m})^3} = 2.33 \cdot 10^3 \text{ kg/m}^3, \quad D = 5.0 \text{ nm} = 5 \cdot 10^{-9} \text{ m},$

$$L = 100.0 \text{ nm} = 1.00 \cdot 10^{-7} \text{ m}, \quad m = (2.33 \cdot 10^3 \text{ kg/m}^3) \frac{\pi}{4} (5.0 \cdot 10^{-9} \text{ m})^2 (1.00 \cdot 10^{-7} \text{ m}) = 4.575 \cdot 10^{-21} \text{ kg}$$

$$(a) T_1 = 4.575 \cdot 10^{-21} \text{ kg} (9.81 \text{ m/s}^2) = 4.488 \cdot 10^{-20} \text{ N}$$

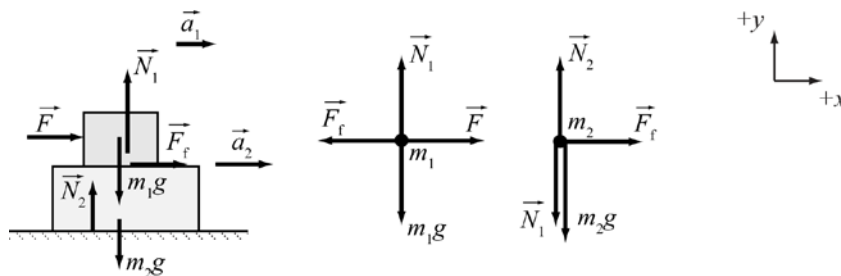
$$(b) T_2 = \frac{4.488 \cdot 10^{-20} \text{ N}}{2} = 2.244 \cdot 10^{-20} \text{ N}$$

ROUND: Rounding to two significant figures, $T_1 = 4.5 \cdot 10^{-20} \text{ N}$ and $T_2 = 2.2 \cdot 10^{-20} \text{ N}$.

DOUBLE-CHECK: Since the volume of a nanowire is very small, it is expected to get a very small tension.

- 4.87. THINK:** Two blocks have masses of $m_1 = 2.50 \text{ kg}$ and $m_2 = 3.75 \text{ kg}$. The coefficients of static and kinetic friction between the two blocks are 0.456 and 0.380. A force, F is applied horizontally on m_1 . Determine the maximum force, F , such that m_1 does not slide, and also the acceleration of m_1 and m_2 when $F = 24.5 \text{ N}$.

SKETCH:



RESEARCH: The force of friction is given by $F_f = \mu_s N_1$. First, consider m_1 . Using Newton's second law:

$$\sum F_x = ma_x \Rightarrow F - F_f = m_1 a_1 \Rightarrow F = F_f + m_1 a_1 \quad \text{and} \quad \sum F_y = ma_y = 0 \Rightarrow N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g.$$

Then, consider m_2 : $\sum F_x = ma_x$, $F_f = m_2 a_2$, and $N_2 - N_1 - m_2 g = 0$.

(a) The force is maximum when $F_f = \mu_s N_1$ and $a_1 = a_2 = a$.

(b) If $F = 24.5 \text{ N}$ is larger than F_{max} , then m_1 slides on m_2 .

SIMPLIFY:

$$(a) F_f = \mu_s N_1 = m_2 a_2 \Rightarrow \mu_s m_1 g = m_2 a_2 \Rightarrow a = \mu_s (m_1 / m_2) g$$

$$F_{\max} = F_f + m_1 a = \mu_s m_1 g + m_1 \mu_s (m_1 / m_2) g = \mu_s m_1 g (1 + m_1 / m_2)$$

(b) The force of friction is given by $F_f = \mu_k N_1$. Using the equations, $F = F_f + m_1 a_1$ and $F_f = m_2 a_2$:

$$a_2 = \frac{F_f}{m_2} = \frac{\mu_k m_1 g}{m_2} \text{ and } a_1 = \frac{F - F_f}{m_1} = \frac{F - \mu_k m_1 g}{m_1} = \frac{F}{m_1} - \mu_k g.$$

CALCULATE:

$$(a) F_{\max} = 0.456(2.50 \text{ kg})(9.81 \text{ m/s}^2) \left(1 + \frac{2.50 \text{ kg}}{3.75 \text{ kg}} \right) = 18.639 \text{ N}$$

$$(b) a_1 = \frac{24.5 \text{ N}}{2.50 \text{ kg}} - 0.380(9.81 \text{ m/s}^2) = 6.0722 \text{ m/s}^2 \text{ and } a_2 = \frac{(0.380)(2.50 \text{ kg})(9.81 \text{ m/s}^2)}{3.75 \text{ kg}} = 2.4852 \text{ m/s}^2$$

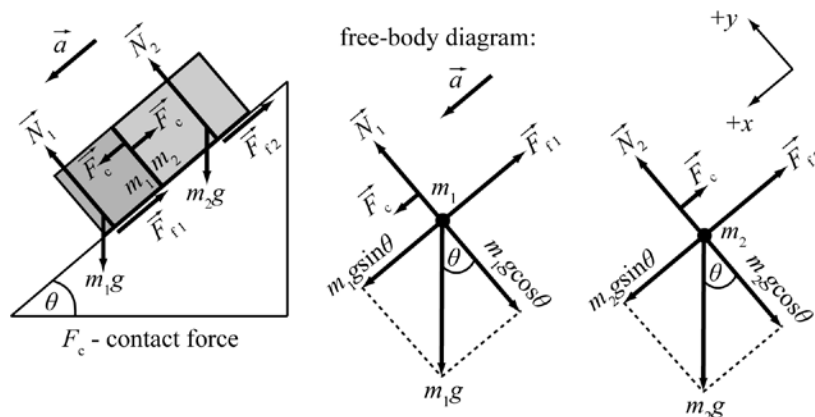
ROUND: Rounding to three significant figures,

$$(a) F_{\max} = 18.6 \text{ N},$$

$$(b) a_1 = 6.07 \text{ m/s}^2 \text{ and } a_2 = 2.49 \text{ m/s}^2.$$

DOUBLE-CHECK: For part (b), it is known that m_1 slides on m_2 . This means that a_1 is larger than a_2 .

- 4.88. **THINK:** Two blocks with masses of $m_1 = 1.23 \text{ kg}$ and $m_2 = 2.46 \text{ kg}$ are glued together, moving on an inclined plane of angle $\theta = 40.0^\circ$. The coefficients of kinetic friction are $\mu_{1k} = 0.23$ and $\mu_{2k} = 0.35$. Determine the acceleration of the blocks.

SKETCH:

RESEARCH: The forces of friction are given by $F_{f1} = \mu_{1k} N_1$ and $F_{f2} = \mu_{2k} N_2$. First, consider m_1 . Using Newton's second law:

$$\sum F_x = m a_x \Rightarrow m_1 g \sin \theta + F_c - F_{f1} = m_1 a \Rightarrow m_1 g \sin \theta + F_c - \mu_{1k} N_1 = m_1 a$$

$$\sum F_y = m a_y = 0 \Rightarrow N_1 - m_1 g \cos \theta = 0 \Rightarrow N_1 = m_1 g \cos \theta$$

Consider m_2 :

$$\sum F_x = m a_x \Rightarrow m_2 g \sin \theta - F_c - F_{f2} = m_2 a \Rightarrow m_2 g \sin \theta - F_c - \mu_{2k} N_2 = m_2 a$$

$$\sum F_y = m a_y = 0 \Rightarrow N_2 - m_2 g \cos \theta = 0 \Rightarrow N_2 = m_2 g \cos \theta$$

$$\begin{aligned} \text{SIMPLIFY: } & m_1 g \sin \theta + F_c - \mu_{1k} N_1 = m_1 a \\ & + m_2 g \sin \theta - F_c - \mu_{2k} N_2 = m_2 a \\ & \hline & (m_1 + m_2) g \sin \theta - \mu_{1k} N_1 - \mu_{2k} N_2 = (m_1 + m_2) a \end{aligned}$$

This implies $a = g \sin \theta - \frac{\mu_{1k} m_1 g \cos \theta + \mu_{2k} m_2 g \cos \theta}{(m_1 + m_2)} = g \sin \theta - g \cos \theta \left[\frac{\mu_{1k} m_1 + \mu_{2k} m_2}{(m_1 + m_2)} \right]$.

CALCULATE: $a = (9.81 \text{ m/s}^2) \sin(40.0^\circ) - (9.81 \text{ m/s}^2) \cos(40.0^\circ) \left[\frac{0.23(1.23 \text{ kg}) + 0.35(2.46 \text{ kg})}{(1.23 \text{ kg} + 2.46 \text{ kg})} \right]$
 $= 3.976 \text{ m/s}^2$

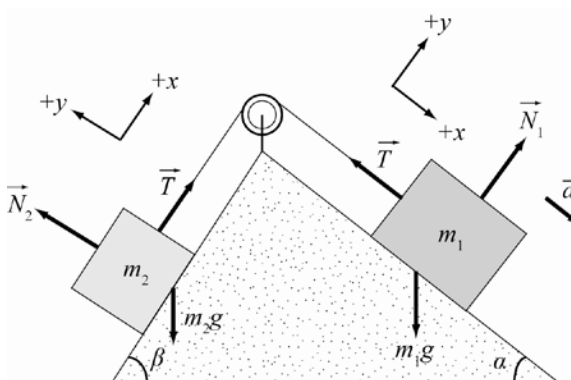
ROUND: Rounding to two significant figures, $a = 4.0 \text{ m/s}^2$.

DOUBLE-CHECK: For a one block system, the acceleration is given by $a = g \sin \theta - \mu_k g \cos \theta$.

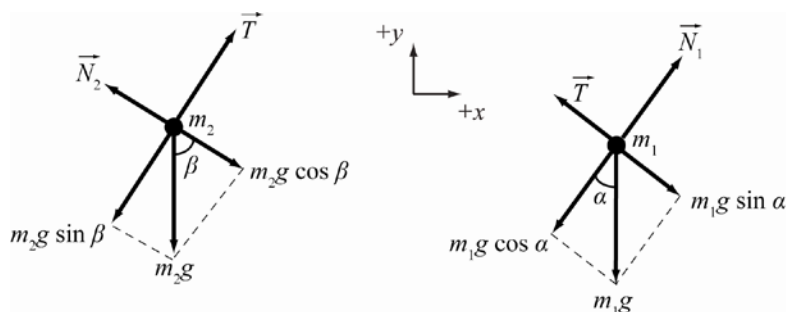
$\frac{\mu_{1k} m_1 + \mu_{2k} m_2}{(m_1 + m_2)}$ is an effective coefficient for two blocks.

- 4.89. THINK:** Two blocks with masses, $m_1 = 567.1 \text{ kg}$ and $m_2 = 266.4 \text{ kg}$ are on inclined planes with angles $\alpha = 39.3^\circ$ and $\beta = 53.2^\circ$. Assume there is no friction. Determine the acceleration of the marble block, m_1 . The marble block, m_1 , is heavier than the granite block, m_2 , but angle β is larger than angle α . It is impossible to guess which block will move up the incline, and which block will move down. The assumption will be made that the marble block accelerates down the incline, and choose that to be the positive direction. If the assumption is correct, the acceleration will be positive, and if it is incorrect, the acceleration will be negative.

SKETCH:



Assume the motion is in the positive x direction.



RESEARCH: First, consider m_1 . Using Newton's second law: $\sum F_y = ma_y = 0$, $N_1 - m_1 g \cos \alpha = 0$, $\sum F_x = ma_x$, and $m_1 g \sin \alpha - T = m_1 a$. Then, consider m_2 : $\sum F_y = ma_y = 0$, $N_2 - m_2 g \cos \beta = 0$, $\sum F_x = ma_x$, and $T - m_2 g \sin \beta = m_2 a$.

SIMPLIFY: $T = m_2 g \sin \beta + m_2 a$, and so:

$$\begin{aligned} m_1 g \sin \alpha - T &= m_1 a \\ m_1 g \sin \alpha - m_2 g \sin \beta - m_2 a &= m_1 a \\ m_1 g \sin \alpha - m_2 g \sin \beta &= (m_1 + m_2) a \\ a &= g \frac{m_1 \sin \alpha - m_2 \sin \beta}{(m_1 + m_2)} \end{aligned}$$

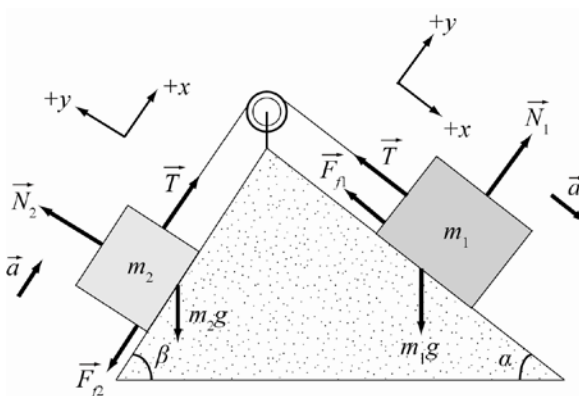
CALCULATE: $a = (9.81 \text{ m/s}^2) \frac{(567.1 \text{ kg}) \sin(39.3^\circ) - (266.4 \text{ kg}) \sin(53.2^\circ)}{(567.1 \text{ kg} + 266.4 \text{ kg})} = 1.7169 \text{ m/s}^2$

ROUND: Rounding to three significant figures, $a = 1.72 \text{ m/s}^2$.

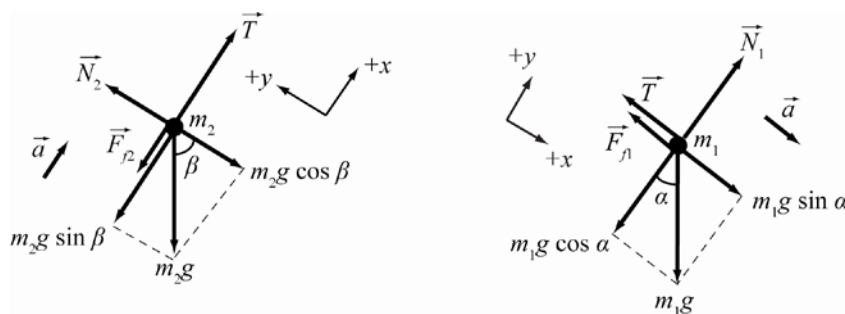
DOUBLE-CHECK: As we expect, the acceleration must be less than $g = 9.81 \text{ m/s}^2$.

- 4.90. THINK:** There are two blocks with masses, $m_1 = 559.1 \text{ kg}$ and $m_2 = 128.4 \text{ kg}$. The angles are $\alpha = 38.3^\circ$ and $\beta = 57.2^\circ$. The coefficients of friction are $\mu_1 = 0.13$ and $\mu_2 = 0.31$. Determine the acceleration of the blocks.

SKETCH:



Assume motion is in the positive x -direction.



RESEARCH: The forces of friction are given by $F_{f1} = \mu_1 N_1$ and $F_{f2} = \mu_2 N_2$. First, consider m_1 . Using Newton's second law:

$$\sum F_x = ma_x \text{ and } m_1 g \sin \alpha - T - F_{f1} = m_1 a \Rightarrow m_1 g \sin \alpha - T - \mu_1 N_1 = m_1 a$$

$$\sum F_y = ma_y = 0, \quad a_y = 0, \text{ and } N_1 - m_1 g \cos \alpha = 0 \Rightarrow N_1 = m_1 g \cos \alpha$$

Then, consider m_2 :

$$\sum F_x = ma_x \text{ and } T - m_2 g \sin \beta - F_{f2} = m_2 a \Rightarrow T - m_2 g \sin \beta - \mu_2 N_2 = m_2 a$$

$$\sum F_y = ma_y = 0 \text{ and } N_2 - m_2 g \cos \beta = 0 \Rightarrow N_2 = m_2 g \cos \beta$$

SIMPLIFY: $m_1 g \sin \alpha - T - \mu_1 m_1 g \cos \alpha = m_1 a$ and $T - m_2 g \sin \beta - \mu_2 m_2 g \cos \beta = m_2 a$, which implies $T = m_2 a + m_2 g \sin \beta + \mu_2 m_2 g \cos \beta$. Eliminate T :

$$m_1 g \sin \alpha - m_2 a - m_2 g \sin \beta - \mu_2 m_2 g \cos \beta = \mu_1 m_1 g \cos \alpha + m_1 a$$

$$m_1 g \sin \alpha - m_2 g \sin \beta - \mu_2 m_2 g \cos \beta - \mu_1 m_1 g \cos \alpha = (m_1 + m_2) a$$

$$a = g \frac{m_1 \sin \alpha - m_2 \sin \beta - \mu_2 m_2 \cos \beta - \mu_1 m_1 \cos \alpha}{(m_1 + m_2)}$$

$$a = g \left(\frac{m_1 (\sin \alpha - \mu_1 \cos \alpha)}{m_1 + m_2} - \frac{m_2 (\sin \beta + \mu_2 \cos \beta)}{m_1 + m_2} \right)$$

CALCULATE:

$$a = 9.81 \text{ m/s}^2 \left(\frac{(559.1 \text{ kg})(\sin 38.3^\circ - 0.13 \cos 38.3^\circ)}{(559.1 \text{ kg} + 128.4 \text{ kg})} - \frac{(128.4 \text{ kg})(\sin 57.2^\circ + 0.31 \cos 57.2^\circ)}{(559.1 \text{ kg} + 128.4 \text{ kg})} \right)$$

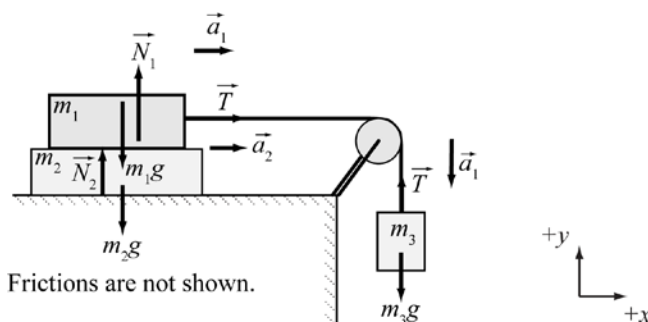
$$= 2.283 \text{ m/s}^2$$

ROUND: Rounding to two significant figures, $a = 2.3 \text{ m/s}^2$.

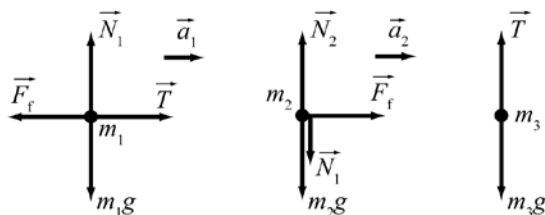
DOUBLE-CHECK: The acceleration must be less than $g = 9.81 \text{ m/s}^2$.

- 4.91. THINK:** Three blocks have masses, $m_1 = 3.50 \text{ kg}$, $m_2 = 5.00 \text{ kg}$ and $m_3 = 7.60 \text{ kg}$. The coefficients of static and kinetic friction between m_1 and m_2 are 0.600 and 0.500. Determine the accelerations of m_1 and m_2 , and tension of the string. If m_1 does not slip on m_2 , then the accelerations of both blocks will be the same. First, make the assumption that the blocks do not slide. Then, it must be determined whether the acting force of friction, F_f , is less than or greater than the maximum force of friction.

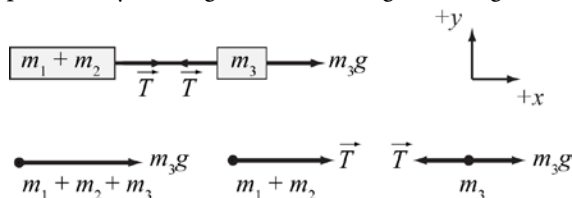
SKETCH:



free-body diagram:



RESEARCH: Simplify the problem by looking at the axis along the string.



Using Newton's second law: $\sum F = ma \Rightarrow m_3 g = (m_1 + m_2 + m_3)a$ and $\sum F = ma \Rightarrow T = (m_1 + m_2)a$.

SIMPLIFY:

$$(a) \ a = \frac{m_3 g}{(m_1 + m_2 + m_3)}$$

$$(b) \ T = \frac{(m_1 + m_2)m_3 g}{(m_1 + m_2 + m_3)}$$

CALCULATE:

$$(a) \ a = \frac{7.60 \text{ kg}(9.81 \text{ m/s}^2)}{(3.50 \text{ kg} + 5.00 \text{ kg} + 7.60 \text{ kg})} = 4.631 \text{ m/s}^2$$

$$(b) \ T = (3.50 \text{ kg} + 5.00 \text{ kg})4.631 \text{ m/s}^2 = 39.362 \text{ N}$$

Now, it must be determined if the force of friction, F_f is less than the maximum force of static friction.

From the free-body diagram of m_2 : $\sum F_x = m_2 a \Rightarrow F_f = m_2 a = 5.00(4.631 \text{ m/s}^2) = 23.15 \text{ N}$. The

maximum force of static friction is $f_{s,\max} = \mu_s N_1$ where $N_1 = m_1 g$. $f_{s,\max} = (0.600)(3.50)(9.81) = 20.60 \text{ N}$. $F_f > f_{s,\max}$, so block 1 slips on block 2. Some parts of the question must be reconsidered.

RESEARCH: The force of friction is given by $F_f = \mu_k N_1$. First, consider m_1 . Using Newton's second law:

$$\sum F_x = m a_x \Rightarrow T - F_f = m_1 a_1 \Rightarrow T - \mu_k N_1 = m_1 a_1 \Rightarrow T - \mu_k m_1 g = m_1 a_1$$

$$\sum F_y = m a_y = 0 \Rightarrow N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g$$

Now, consider m_2 : $\sum F_x = m a_x \Rightarrow F_f = m_2 a_2 \Rightarrow a_2 = \frac{\mu_k N_1}{m_2} = \mu_k \frac{m_1 g}{m_2}$. Finally, consider m_3 :

$$\sum F_y = m a_y \Rightarrow m_3 g - T = m_3 a_1.$$

SIMPLIFY: $T - \mu_k m_1 g = m_1 a_1$ and $m_3 g - T = m_3 a_1$ can be used to eliminate T :

$$m_3 g - m_1 a_1 - \mu_k m_1 g = m_3 a_1 \Rightarrow m_3 g - \mu_k m_1 g = (m_1 + m_3) a_1 \Rightarrow a_1 = g \frac{(m_3 - \mu_k m_1)}{(m_1 + m_3)}$$

Also, $a_2 = \frac{\mu_k m_1}{m_2} g$ and $T = m_1(a_1 + \mu_k g) = m_3(g - a_1)$.

CALCULATE:

$$(a) \ a_1 = (9.81 \text{ m/s}^2) \frac{(7.60 \text{ kg} - 0.500(3.50 \text{ kg}))}{(7.60 \text{ kg} + 3.50 \text{ kg})} = 5.1701 \text{ m/s}^2,$$

$$a_2 = \frac{0.500(3.50 \text{ kg})}{5.00 \text{ kg}}(9.81 \text{ m/s}^2) = 3.4335 \text{ m/s}^2$$

$$(b) \ T = 7.60 \text{ kg}(9.81 \text{ m/s}^2 - 5.1701 \text{ m/s}^2) = 35.26 \text{ N}$$

ROUND: Rounding to three significant figures,

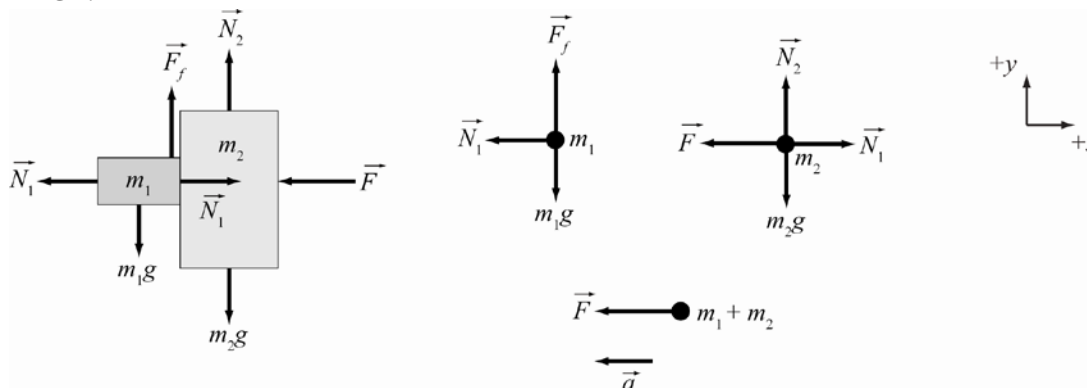
$$(a) \ a_1 = 5.17 \text{ m/s}^2, \ a_2 = 3.43 \text{ m/s}^2 \text{ and}$$

$$(b) \ T = 35.3 \text{ N}.$$

DOUBLE-CHECK: Because block 1 slips on block 2, a_1 is larger than a_2 .

- 4.92. THINK:** There are two blocks with masses, $m_1 = 2.30 \text{ kg}$ and $m_2 = 5.20 \text{ kg}$. The coefficient of static friction between the blocks is 0.65.

SKETCH:



RESEARCH:

(a) From the above diagram, there are three forces acting on block 1; the normal, frictional and gravitational forces.

(b) Consider block 1. Using Newton's second law:

$$\sum F_x = ma_x \Rightarrow N_1 = m_1 a \quad \text{and} \quad \sum F_y = ma_y = 0 \Rightarrow F_f - m_1 g = 0 \Rightarrow F_f = m_1 g$$

There is a minimum force F , when there is a minimum acceleration and $F_f = \mu_s N_1$.

SIMPLIFY: $F_f = \mu_s N_1 = m_1 g \Rightarrow N_1 = \frac{m_1 g}{\mu_s} \Rightarrow \mu_s m_1 a = m_1 g \Rightarrow a = \frac{g}{\mu_s}$. Consider a block composed of

block 1 and block 2: $\sum F_x = ma_x \Rightarrow F = (m_1 + m_2)a \Rightarrow F = \frac{(m_1 + m_2)g}{\mu_s}$.

(c) $N_1 = \frac{m_1 g}{\mu_s}$

(d) $\sum F_x = m_2 a = \frac{m_2 g}{\mu_s}$

CALCULATE:

(b) $F = \frac{(2.30 \text{ kg} + 5.20 \text{ kg})(9.81 \text{ m/s}^2)}{0.65} = 113.19 \text{ N}$

(c) $N_1 = \frac{2.30 \text{ kg}(9.81 \text{ m/s}^2)}{0.65} = 34.71 \text{ N}$

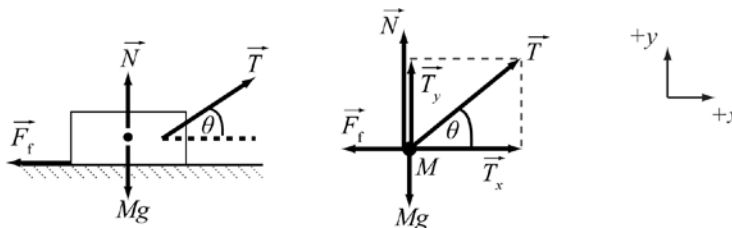
(d) $\sum F_x = \frac{5.20 \text{ kg}(9.81 \text{ m/s}^2)}{0.65} = 78.48 \text{ N}$

ROUND: Rounding to two significant figures, $F = 110 \text{ N}$, $N_1 = 35 \text{ N}$ and $\sum F_x = 78 \text{ N}$.

DOUBLE-CHECK: The net force on block 2 added to the contact force N_1 should equal F . $F = 78 \text{ N} + 35 \text{ N} = 113 \text{ N}$, as expected.

- 4.93. THINK:** The suitcase has a weight of $M_g = 450$ N. The coefficient of kinetic friction is $\mu_k = 0.640$. Determine the angle that minimizes the force to pull the suitcase.

SKETCH:



RESEARCH: The force of friction is given by $F_f = \mu_k N$. Also, $T_x = T \cos \theta$ and $T_y = T \sin \theta$. Using Newton's second law:

$$\sum F_x = ma_x, \text{ and } a_x = 0, \text{ so } T_x - F_f = 0 \Rightarrow T \cos \theta = F_f = \mu_k N$$

$$\sum F_y = ma_y = 0 \Rightarrow T_y + N - Mg = 0 \Rightarrow T \sin \theta + N - Mg = 0 \Rightarrow N = Mg - T \sin \theta$$

SIMPLIFY: $T \cos \theta = \mu_k (Mg - T \sin \theta) = \mu_k Mg - \mu_k T \sin \theta$

$$T(\cos \theta + \mu_k \sin \theta) = \mu_k Mg$$

$$T = \mu_k Mg (\cos \theta + \mu_k \sin \theta)^{-1}$$

Differentiate T with respect to θ ; $\frac{dT}{d\theta} = -\mu_k Mg (-\sin \theta + \mu_k \cos \theta) (\cos \theta + \mu_k \sin \theta)^{-2}$. The minimum

tension is when $dT/d\theta = 0$: $-\mu_k Mg (-\sin \theta + \mu_k \cos \theta) (\cos \theta + \mu_k \sin \theta)^{-2} = 0$, which simplifies to $-\sin \theta + \mu_k \cos \theta = 0$, or $\tan \theta = \mu_k$. Thus, $\theta = \tan^{-1}(\mu_k)$.

CALCULATE:

(a) $\theta = \tan^{-1}(0.640) = 32.6192^\circ$

(b) $T = \frac{0.640(450 \text{ N})}{(\cos(32.6^\circ) + 0.640 \sin(32.6^\circ))} = 242.5 \text{ N}$

ROUND: Rounding to three significant figures,

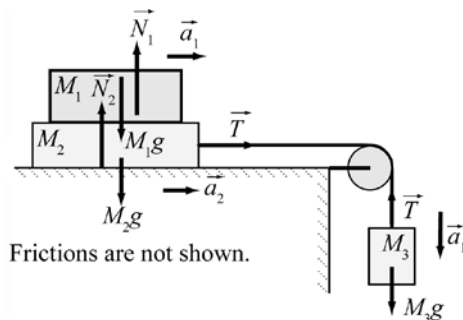
(a) $\theta = 32.6^\circ$ and

(b) $T = 243 \text{ N}$.

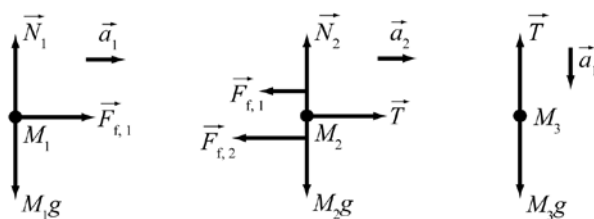
DOUBLE-CHECK: As a comparison, if $\theta = 0^\circ$, $T = \mu_k M_g = 0.640(450 \text{ N}) = 288 \text{ N}$. The minimum tension calculated is less than this value.

- 4.94. THINK:** There are three blocks with masses, $M_1 = 0.450 \text{ kg}$, $M_2 = 0.820 \text{ kg}$ and M_3 . The coefficients of static and kinetic friction are $\mu_s = 0.560$ and $\mu_k = 0.340$.

SKETCH:



free-body diagram:



RESEARCH: The force of friction is given by $F_f = \mu N$. It is given that block 1 does not slide on M_2 , which means $a_1 = a_2 = a$. First, consider M_1 . Using Newton's second law:

$$\sum F_x = ma_x \Rightarrow F_{f,1} = M_1 a \quad \text{and} \quad \sum F_y = ma_y = 0 \Rightarrow N_1 - M_1 g = 0 \Rightarrow N_1 = M_1 g$$

Then, consider M_2 : $\sum F_x = ma_x \Rightarrow T - F_{f,1} - F_{f,2} = M_2 a$

$$\text{and } \sum F_y = ma_y = 0 \Rightarrow N_2 - N_1 - M_2 g = 0 \Rightarrow N_2 = (M_1 + M_2)g$$

Finally, consider M_3 : $\sum F_y = ma_y \Rightarrow M_3 g - T = M_3 a$.

SIMPLIFY: The maximum force of friction before M_1 will slip is given by $F_{f,1} = f_{s,\max} = \mu_s N_1 = \mu_s M_1 g$.

$$F_{f,1} = \mu_s M_1 g = M_1 a \Rightarrow a = \mu_s g \quad (1)$$

Mass 2 is slipping along the table so $F_{f,2} = f_k = \mu_k N_2 = \mu_k (M_1 + M_2)g$; therefore, the equation for the x -direction yields $T - \mu_s M_1 g - \mu_k (M_1 + M_2)g = M_2 a \Rightarrow T = M_2 a + \mu_s M_1 g + \mu_k (M_1 + M_2)g \quad (2)$.

Substituting (1) in (2) yields $T = M_2 \mu_s g + \mu_s M_1 g + \mu_k (M_1 + M_2)g = \mu_s (M_1 + M_2)g + \mu_k (M_1 + M_2)g \quad (3)$.

Solving the equation x -direction equation for M_3 yields, $M_3 = \frac{T}{g - a}$. Substituting (1) and (3) for a and T ,

$$\text{respectively, yields } M_3 = \frac{\mu_s (M_1 + M_2)g + \mu_k (M_1 + M_2)g}{g - \mu_s g} = \frac{(\mu_s + \mu_k)(M_1 + M_2)}{1 - \mu_s}.$$

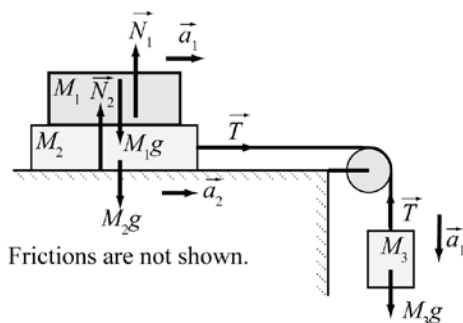
$$\text{CALCULATE: } M_3 = \frac{(0.560 + 0.340)(0.450 \text{ kg} + 0.820 \text{ kg})}{1 - 0.560} = 2.5977 \text{ kg}$$

ROUND: Rounding to three significant figures, $M_3 = 2.60 \text{ kg}$.

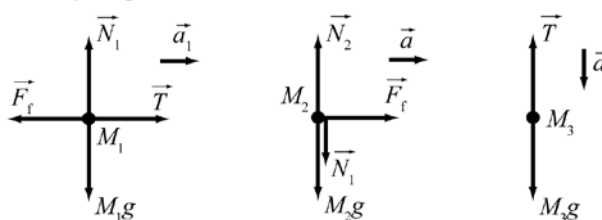
DOUBLE-CHECK: The order of magnitude of M_3 is reasonable in comparison to M_1 and M_2 .

4.95. THINK: The three blocks have masses, $M_1 = 0.250 \text{ kg}$, $M_2 = 0.420 \text{ kg}$ and $M_3 = 1.80 \text{ kg}$. The coefficient of kinetic friction is $\mu_k = 0.340$.

SKETCH:



free-body diagram:



RESEARCH: The force of friction is given by $F_f = \mu_k N$. First, consider M_1 . Using Newton's second law:

$$\sum F_x = ma_x \Rightarrow F_{f,1} = M_1 a_1 \text{ and } \sum F_y = ma_y = 0 \Rightarrow N_1 - M_1 g = 0 \Rightarrow N_1 = M_1 g$$

Then, consider M_2 : $\sum F_x = ma_x \Rightarrow T - F_{f,1} - F_{f,2} = M_2 a$ and

$$\sum F_y = ma_y = 0 \Rightarrow N_2 - N_1 - M_2 g = 0 \Rightarrow N_2 = (M_1 + M_2)g$$

Finally, consider M_3 : $\sum F_y = ma_y \Rightarrow M_3 g - T = M_3 a$.

SIMPLIFY:

(a) $F_{f,1} = \mu_k N_1 = \mu_k M_1 g = M_1 a_1 \Rightarrow a_1 = \mu_k g$, which is the acceleration of the block.

(b) For the slab (M_2) the friction with the table is $F_{f,2} = f_k = \mu_k N_2 = \mu_k (M_1 + M_2)g$; therefore, the equation for the x direction yields $T - \mu_k M_1 g - \mu_k (M_1 + M_2)g = M_2 a$. From the equation for M_3 , $T = M_3 (g - a)$. Substituting this for T yields the following.

$$\begin{aligned} M_3 (g - a) - \mu_k M_1 g - \mu_k (M_1 + M_2)g &= M_2 a \Rightarrow (M_2 + M_3)a = M_3 g - \mu_k (2M_1 + M_2)g \\ \Rightarrow a &= \frac{[M_3 - \mu_k (2M_1 + M_2)]g}{M_2 + M_3} \end{aligned}$$

CALCULATE:

$$(a) \ a_1 = (0.340)(9.81 \text{ m/s}^2) = 3.335 \text{ m/s}^2$$

$$(b) \ a = \frac{[1.80 \text{ kg} - (0.340)(2 \cdot 0.250 \text{ kg} + 0.420 \text{ kg})](9.81 \text{ m/s}^2)}{0.420 \text{ kg} + 1.80 \text{ kg}} = 6.572 \text{ m/s}^2$$

ROUND: Rounding to three significant figures,

$$(a) \ a_1 = 3.34 \text{ m/s}^2 \text{ and}$$

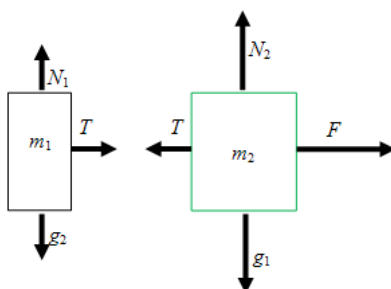
$$(b) \ a = 6.57 \text{ m/s}^2.$$

DOUBLE-CHECK: Because M_1 slides on M_2 , it is expected that a_1 is less than a . Both a_1 and a must be less than g .

Multi-Version Exercises

- 4.96. THINK:** This problem involves two blocks sliding along a frictionless surface. For these types of problems, use Newton's laws. Also note that the tension force from block 1 must be exactly equal and opposite from the force on block 2.

SKETCH: Start with the image from the text to create free body diagrams for each block.



RESEARCH: Since the blocks are lying on a frictionless surface, there is no friction force acting on the blocks. It is necessary to find the tension T on the rope in terms of the force F acting on the second block and the masses m_1 and m_2 . Since force F is acting on the blocks and rope as a single system, Newton's Second Law gives that $F = (m_1 + m_2)a$, where a is the acceleration of the blocks. Looking at the horizontal forces on block 1 gives $T = m_1 a$.

SIMPLIFY: It is necessary to find an expression for the tension in terms of the outside force F acting on block 2 and the masses of the two blocks. Rewriting $T = m_1 a$ as $a = \frac{T}{m_1}$ and combining with

$F = (m_1 + m_2)a$ gives $F = (m_1 + m_2) \frac{T}{m_1}$. This expression can be re-written to give the tension in terms of

known quantities: $T = F \frac{m_1}{m_1 + m_2}$.

CALCULATE: Using the masses and force given in the question statement gives a tension force of:

$$T = 12.61 \text{ N} \left(\frac{1.267 \text{ kg}}{1.267 \text{ kg} + 3.557 \text{ kg}} \right) \\ = 3.311954809 \text{ N}$$

ROUND: The masses of the blocks are given to four significant figures, and their sum also has four significant figures. The only other measured quantity is the external force acting on the second block, which also has four significant figures. This means that the final answer should be rounded to four significant figures, giving a total tension of $T = 3.312 \text{ N}$.

DOUBLE-CHECK: Think of the tension on the rope transmitting the force from block 2 to block 1. Since block 2 is much more massive than block 1, block 1 represents about one fourth of the total mass of the system. So, it makes sense that only about one fourth of the force will be transmitted along the string to the second block. About $12.61/4$ or 3.153 N will be transmitted, which is pretty close to our calculated value of 3.312 N .

4.97. $F = (m_1 + m_2) \frac{T}{m_1}$

$$m_1 F = m_1 T + m_2 T$$

$$m_1 (F - T) = m_2 T$$

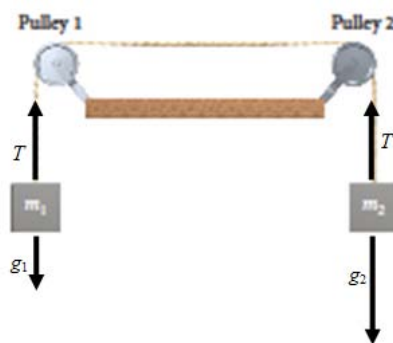
$$m_1 = m_2 \frac{T}{F - T} = (3.577 \text{ kg}) \frac{4.094 \text{ N}}{13.89 \text{ N} - 4.094 \text{ N}} = 1.495 \text{ kg}$$

$$\begin{aligned}
 4.98. \quad F &= (m_1 + m_2) \frac{T}{m_1} \\
 m_1 F &= m_1 T + m_2 T \\
 m_2 &= \frac{m_1 F - m_1 T}{T} = m_1 \frac{F - T}{T} = (1.725 \text{ kg}) \frac{15.17 \text{ N} - 4.915 \text{ N}}{4.915 \text{ N}} = 3.599 \text{ kg}
 \end{aligned}$$

$$4.99. \quad F = (m_1 + m_2) \frac{T}{m_1} = T \frac{m_1 + m_2}{m_1} = (5.777 \text{ N}) \frac{1.955 \text{ kg} + 3.619 \text{ kg}}{1.955 \text{ kg}} = 16.47 \text{ N}$$

4.100. THINK: This problem involves two hanging masses and two frictionless pulleys. The rope is massless, so it is not necessary to include the gravitational force on the spring. The forces on this system will be the gravitational force and the tension in the string. To solve this problem, it will be necessary to use Newton's laws.

SKETCH: Begin with the sketch from the text. Then, draw free body diagrams for both masses, keeping in mind that the forces exerted by the rope on the masses must be equal.



RESEARCH: First note that the two blocks will accelerate at the same rate, but in opposite directions. First note that the gravitational force on each block can be given in terms of the mass and gravitational acceleration: $g_1 = gm_1$ and $g_2 = gm_2$. It is possible to use Newton's Second law on each block individually to get two equations relating the tension T , the masses of the blocks m_1 and m_2 , and the acceleration a . For the first block we have that $(T - m_1 g) = m_1 a$. Likewise, $(T - m_2 g) = m_2 \cdot (-a)$ because block 2 is accelerating in the opposite direction from block 1. With these two equations, we should be able to solve for either of the unknown quantities a or T .

SIMPLIFY: Since the problem asks for acceleration a , first find an equation for the tension T in terms of the other quantities. $T - m_1 g = m_1 a$ means that $T = m_1 a + m_1 g$. Substitute this expression for T into the equation $(T - m_2 g) = m_2 \cdot (-a)$ to get:

$$\begin{aligned}
 -m_2 a &= (m_1 a + m_1 g) - m_2 g \Rightarrow \\
 -m_1 a - m_2 a &= -m_1 a + m_1 g - m_2 g \Rightarrow \\
 (-m_1 - m_2) a &= m_1 g - m_2 g \Rightarrow \\
 a &= \frac{m_1 g - m_2 g}{-m_1 - m_2}
 \end{aligned}$$

CALCULATE: The masses m_1 and m_2 are given in the problem. The gravitational acceleration g is about $9.81 \text{ m} \cdot \text{s}^{-2}$. Using these values gives the acceleration a :

$$\begin{aligned}
 a &= \frac{m_1 g - m_2 g}{-m_1 - m_2} \\
 &= \frac{1.183 \text{ kg} \cdot 9.81 \text{ m} \cdot \text{s}^{-2} - 3.639 \text{ kg} \cdot 9.81 \text{ m} \cdot \text{s}^{-2}}{-1.183 \text{ kg} - 3.639 \text{ kg}} = 4.99654915 \text{ m} \cdot \text{s}^{-2}
 \end{aligned}$$

ROUND: The masses of the blocks are given to four significant figures, and their sum also has four significant figures. Since they are the only measured quantities in this problem, the final answer should have four significant figures, giving a final answer of 4.997 m/s^2 .

DOUBLE-CHECK: In this situation, it is intuitively obvious that the heavier mass will fall towards the ground less quickly than if it were in free fall. So it is reasonable that our calculated value of 4.997 m/s^2 is less than the 9.81 m/s^2 , which is the rate at which objects on the Earth's surface generally accelerate towards the ground.

4.101. $m_1 g + m_1 a = m_2 g - m_2 a$

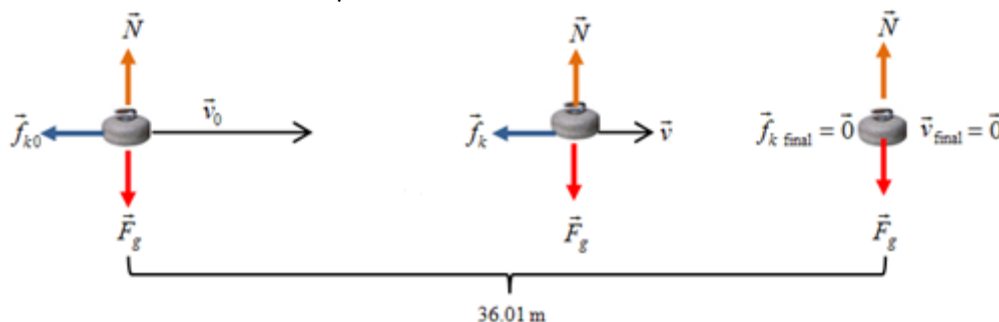
$$m_2 = m_1 \frac{g + a}{g - a} = (1.411 \text{ kg}) \frac{9.81 \text{ m/s}^2 + 4.352 \text{ m/s}^2}{9.81 \text{ m/s}^2 - 4.352 \text{ m/s}^2} = 3.661 \text{ kg}$$

4.102. $m_1 g + m_1 a = m_2 g - m_2 a$

$$m_1 = m_2 \frac{g - a}{g + a} = (3.681 \text{ kg}) \frac{9.81 \text{ m/s}^2 - 3.760 \text{ m/s}^2}{9.81 \text{ m/s}^2 + 3.760 \text{ m/s}^2} = 1.641 \text{ kg}$$

4.103. **THINK:** This problem involves friction, so the only forces acting on the curling stone are gravity, the normal force, and the frictional force. Since gravity and the normal force act in the vertical direction, the only force slowing the horizontal movement of the curling stone is the frictional force. It is necessary to come up with a way to relate the initial velocity to the mass, coefficient of friction, and total distance traveled by the stone. Since the curling stone is slowing to a stop, there is a net external force, so it will be necessary to use Newton's Second law.

SKETCH: It is helpful to draw the free body diagrams for the curling stone at three different times: the moment the curling stone is released, part of the way along its path, and after it has stopped. When the stone is at rest, there is no velocity and no kinetic friction (there is, however, static friction).



RESEARCH: The only unknown force acting on the curling stone is the kinetic friction force. The magnitude is given by $f_k = \mu_k N$. But the normal force is equal and opposite to the gravitational force ($\vec{N} = -\vec{F}_g = -(mg)$), so the magnitude of the normal force is $N = mg$. The initial kinetic energy of the stone is $K = \frac{1}{2}mv_0^2$ and the kinetic energy of the stone at rest is zero. So, all of the kinetic energy has been dissipated by friction. The energy dissipated by friction is equal to the magnitude of the force times the distance traveled, $f_k d = K$. This will allow us to find the initial velocity in terms of known quantities.

SIMPLIFY: First, it is necessary to combine $f_k d = K$ with the equations for the kinetic energy, normal force, and frictional force. So:

$$\begin{aligned} f_k d &= K \Rightarrow \\ (\mu_k N) d &= \frac{1}{2}mv_0^2 \Rightarrow \\ \mu_k (mg) d &= \frac{1}{2}mv_0^2 \end{aligned}$$

Use algebra to find an expression for the initial velocity in terms of known quantities:

$$\begin{aligned}\mu_k(mg)d &= \frac{1}{2}mv_0^2 \Rightarrow \\ \frac{2}{m}\mu_k mgd &= \frac{2}{m}\frac{1}{2}mv_0^2 \Rightarrow \\ 2\mu_k gd &= v_0^2 \Rightarrow \\ \sqrt{2\mu_k gd} &= v_0\end{aligned}$$

CALCULATE: The mass of the curling stone $m = 19.00$ kg, the coefficient of kinetic friction between the stone and the ice $\mu_k = 0.01869$ and the total distance traveled $d = 36.01$ m. The acceleration due to gravity on the surface of the Earth is not given in the problem, but it is $g = 9.81$ m/s². Using these values,

$$\begin{aligned}v_0 &= \sqrt{2\mu_k gd} \\ &= \sqrt{2 \cdot 0.01869 \cdot 9.81 \text{ m/s}^2 \cdot 36.01 \text{ m}} = 3.633839261 \text{ m/s}\end{aligned}$$

ROUND: The mass of the curling stone, distance traveled, and coefficient of friction between the ice and the stone are all given to four significant figures. These are the only measured values in the problem, so the answer should also be given to four significant figures. The initial velocity was 3.634 m/s.

DOUBLE-CHECK: Since the gravitational and normal forces are perpendicular to the direction of the motion and cancel one another exactly, they will not affect the velocity of the curling stone. Between the time the stone is released and the moment it stops, the frictional force acts in the opposite direction of the velocity and is proportional to the normal force, so this is one-dimensional motion with constant acceleration. It is possible to check this problem by working backward from the initial velocity and force to find an expression for velocity as a function of time, and then use that to find the total distance traveled. Newton's Second law and the equation for the frictional force can be combined to find the acceleration of the curling stone: $f_k = -\mu_k mg = ma_x \Rightarrow a_x = -\mu_k g$. If the spot where the curling stone was released is $x_0 = 0$, then the equations for motion in one dimension with constant acceleration become:

$$\begin{aligned}v &= v_{x0} + at & \text{and} & & d &= x_0 + v_{x0}t + \frac{1}{2}at^2 \\ &= 3.634 - 0.01869 \cdot 9.81t & & & &= 0 + 3.634t - \frac{1}{2}0.01869 \cdot 9.81t^2\end{aligned}$$

Solving the first equation for $v = 0$ gives $0 = 3.634 - 0.01869 \cdot 9.81t \Rightarrow t = \frac{3.634}{0.01869 \cdot 9.81} = 19.82$ sec. (The

stone was in motion 19.82 seconds, which is reasonable for those who are familiar with the sport of curling.) This value can be used to compute the distance traveled by the stone at the moment it stops as

$$\begin{aligned}d &= 0 + 3.634t - \frac{1}{2}0.01869 \cdot 9.81t^2 \\ &= 0 + 3.634(19.82) - \frac{1}{2}0.01869 \cdot 9.81(19.82)^2 = 36.01 \text{ m}\end{aligned}$$

This confirms the calculated result.

4.104. $(\mu_k mg)d = \frac{1}{2}mv_0^2$

$$d = \frac{mv_0^2}{2(\mu_k mg)} = \frac{v_0^2}{2\mu_k g} = \frac{(2.788 \text{ m/s})^2}{2(0.01097)(9.81 \text{ m/s}^2)} = 36.11 \text{ m}$$

4.105. $(\mu_k mg)d = \frac{1}{2}mv_0^2$

$$\mu_k = \frac{v_0^2}{2gd} = \frac{(3.070 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)(36.21 \text{ m})} = 0.01327$$