Chapter 29: Electromagnetic Induction

Concept Checks

29.1. c **29.2.** a **29.3.** c **29.4.** c **29.5.** a **29.6.** a **29.7.** a **29.8.** e

Multiple-Choice Questions

29.1. d **29.2.** c **29.3.** a **29.4.** a **29.5.** a **29.6.** c **29.7.** d **29.8.** b **29.9.** a **29.10.** d **29.11.** c **29.12.** d **29.13.** e **29.14.** a

Conceptual Questions

29.15. A refrigerator's electrical circuit contains a motor with a large number of winding coils, making it highly inductive. The electromagnetic induction due to the coil can create a large voltage, on the order of kV between the prongs. This voltage is great enough to ionize the air and the process of ionization produces light, creating a visible spark.

29.16. Large machinery and motors often convert electrical energy to mechanical energy or vice-versa to complete a task. The conversion from electrical energy to mechanical energy requires the creation of magnetic fluxes. Changes in the magnetic flux reaching a pacemaker, due to movements of the machine or the person, will create currents in the circuitry of the pacemaker, changing its behavior; this can be dangerous.

29.17. As the metal moves through the non-uniform magnetic field, it experiences a changing magnetic flux. The flux induces an emf in the metal, if it is a conductor, and produces eddy currents. Lenz's law states that the induced currents create a force to oppose the movement of the metal through the field. This action is analogous to the drag force or force of friction used to create the damping of a harmonic oscillator.

29.18. Lenz's law requires that as the magnet moves down the cylinder, a current is produced in the aluminum cylinder, which in turn creates a magnetic field that opposes the magnet's motion. The force of the currents on the magnet is proportional to the velocity of the magnet. Thus, the magnet will continue to accelerate until it reaches a terminal speed that creates a force equal and opposite to the force of gravity.

29.19. (a) The currents produced in the aluminum, by induction, create a force that opposes the motion of the magnet. The magnet falling in the glass tube does not create a current since glass is an insulator. Thus, the magnet in the glass tube falls faster since there is no magnetic field produced to oppose the force of gravity.
(b) Because the glass has nearly infinite resistance, no eddy currents are created as the magnet passes through it. The aluminum being a good conductor does produce eddy currents as the magnet falls through it. Thus, the aluminum tube has a larger eddy current.

29.20. (a) The *B* field inside the solenoid is uniform and equal to $B_{\text{i}} = \mu_0 ni.$ Outside the solenoid, the field is zero, $B_{\text{o}} = 0.$ The *B* field through the ring is only that of the field inside the solenoid of radius, *a*. The flux is then $\Phi = BA = \mu_0 ni\pi a^2 = \mu_0 n\pi a^2 Ct^2.$ Thus, the emf is $\left|\Delta V_{\text{ind}}\right| = \frac{d\Phi}{dt} = 2\mu_0 n\pi a^2 Ct.$

(b) The magnitude of the electric field is then $2\pi rE = \Delta V = 2\mu_0 n\pi a^2 Ct$ or $E = \frac{\mu_0 na^2 Ct}{r}.$

(c) The ring is not necessary for the induced electric field to exist. The solenoid will produce a magnetic field from the current being passed through the wire inducing an electric field on each concurrent loop of wire.

29.21. Lenz's law requires that the induced current opposes the change in the magnetic field. Therefore, the *B* field created by the induced current is downward. To produce a magnetic field in this direction, the current must flow clockwise as seen from above.

29.22. The area of the loop perpendicular to the field is given by $A = L^2\cos(\omega t)$. The potential difference is:

$$\Delta V_{\text{in,L}} = -\frac{d\Phi}{dt} = -\frac{d(AB)}{dt} = -B\frac{dA}{dt} = -B\frac{d}{dt}\left(L^2\cos(\omega t)\right) = -BL^2\left(-\omega\sin(\omega t)\right) = BL^2\omega\sin(\omega t).$$

29.23. The emf produced by a loop is given by $\Delta V_{\text{ind}} = vBL,$ where L is the length of the moving conductor. By taking a differentially small element of the disk, we convert L into the differential, dr, and integrate from the center of the disk to the edge for the emf of the disk: $\Delta V_{\text{ind}} = \int_0^R vBdr.$ The velocity of an element, dr, is given by $v = r\omega.$ The emf is then:

$$\Delta V_{\text{ind}} = \int_0^R r\omega Bdr = \frac{1}{2}\omega R^2 B.$$

29.24. Separation of charge due to the magnetic force, $q\vec{v}\times\vec{B},$ engenders a compensating electric field of magnitude $E = \vec{v}\times\vec{B} = vB.$ The corresponding potential difference across height, l, is:
$V = lE = lvB = (1.80\text{ m})(2.00\text{ m/s})(25.0\text{ T}) = 90.0\text{ V}.$ In equilibrium this drives no current. However, such a large magnetic field offers further hazards due to any metal objects about the man's body and to stress on blood vessels, which are carrying conducting fluids in motion like iron.

29.25. The flux through the inside copper cylinder is constant during the process, so:

$$\Phi_{\text{i}} = \Phi_{\text{f}} \Rightarrow B_{\text{i}}A_{\text{i}} = B_{\text{f}}A_{\text{f}} \Rightarrow B_{\text{i}}\pi r_{\text{i}}^2 = B_{\text{f}}\pi r_{\text{f}}^2.$$

The final magnetic field is given by:

$$B_{\text{f}} = \left(\frac{r_{\text{i}}}{r_{\text{f}}}\right)^2 B_{\text{i}}.$$

If the initial B field is 1.0 T and the radius compresses by a factor of 14, then final field is given by:

$$B_{\text{f}} = \left(\frac{r_{\text{i}}}{r_{\text{i}}/14}\right)^2 B_{\text{i}} = (14)^2 B_{\text{i}} = (14)^2(1.0\text{ T}) = 2.0\cdot 10^2\text{ T}.$$

Experimental magnetic fields are typically lower than 10 T. This is a huge magnetic field.

29.26. Lenz's law requires that the induced current opposes the change in the magnetic field. Therefore, the B field created by the induced current is downward. To produce a magnetic field in this direction, the current must flow clockwise as seen from above.

29.27. The inductance of a solenoid is given by $L = \mu_0 n^2 lA.$ Let d denote the length of the wire. The number of turns in each case is $N = d/2\pi r.$ The inductance is then:

$$L = \mu_0 n^2 lA = \mu_0 n(nl)A = \mu_0 nNA = \mu_0 n\left(\frac{d}{2\pi r}\right)\pi r^2 = \frac{1}{2}\mu_0 ndr.$$

For both solenoids, the number of turns per unit length is equal, and the distance of the wire is the same. Therefore, the ratio of the inductances is:

$$\frac{L_1}{L_2} = \frac{\mu_0 ndr/2}{\mu nd2r/2} = \frac{1}{2}.$$

Thus, the inductance of the second solenoid is twice that of the first solenoid.

Exercises

29.28. The magnetic flux through the coil is given by:

$$\Phi = NBA\cos\theta = 20(5.00\text{ T})\pi(0.400\text{ m})^2\cos(90° - 25.8°) = 21.9\text{ T m}^2$$

29.29. The potential difference around the loop is:

$$V_{\text{emf}} = -\frac{d\Phi}{dt} \approx -\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta(AB)}{\Delta t} = -A\frac{\Delta B}{\Delta t} = -\pi r^2\frac{\Delta B}{\Delta t} = -\pi(0.0100\text{ m})^2\left(\frac{0\text{ T}-1.20\text{ T}}{20.0\text{ s}}\right) = 1.89\cdot 10^{-5}\text{ V}.$$

Note that the area of the ring is perpendicular to the field. Thus, the normal of the area is parallel to the field and $\cos\theta = 1$.

29.30. If the angle between the B-field and the plane of the loop is $40°$, then the angle between the B-field and the normal to the loop is $90° - 40° = 50°$, and so the voltage across the loop is given by:

$$V_{\text{ind}} = -\frac{d\Phi_{\text{B}}}{dt} = -\frac{d}{dt}(NAB\cos\theta) = -NA\cos\theta\frac{dB}{dt} = -NA\cos\theta\frac{d(1.50t^3)}{dt} = -NL^2(\cos 50.0°)4.50t^2.$$

The current induced if the loop has a resistance of $R = 3.00\ \Omega$ is:

$$i = \frac{|V_{\text{ind}}|}{R} = \frac{NL^2(\cos 50.0°)4.50t^2}{R} = \frac{(8)(0.200\text{ m})^2(\cos 50.0°)4.50(2.00\text{ s})^2}{(3.00\ \Omega)} = 1.23\text{ A}.$$

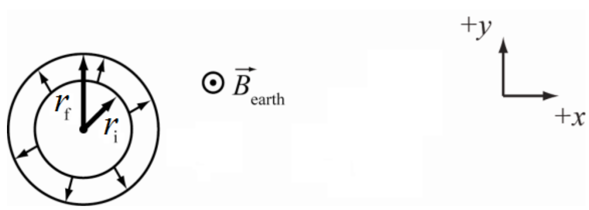
29.31. Because the magnetic field is perpendicular to the normal of the loop, there is no flux through the loop:

$$\Phi = AB\cos 90° = 0.$$

Since there is no flux through the loop, there is no induced voltage: $V = -\frac{d\Phi}{dt} = -\frac{d(0)}{dt} = 0.$

29.32. **THINK:** The change in the area of the loop creates a change in the magnetic flux through the loop. The change in flux produces a current. The loop has a resistance of $R = 30.0\ \Omega$ and a radius which changes from $r_{\text{i}} = 20.0$ cm to $r_{\text{f}} = 25.0$ cm in 1.00 s. The magnetic field of the Earth is about $4.26\cdot 10^{-5}$ T.

SKETCH:



RESEARCH: The flux through the loop is $\Phi_{\text{B}} = AB\cos\theta$ or $\Phi_{\text{B}} = AB$, since the B field is perpendicular to the surface of the loop. The induced potential difference is given by $V_{\text{ind}} = -d\Phi_{\text{B}}/dt$. This potential must also satisfy $V = iR$.

SIMPLIFY: The induced current in the loop is:

$$i = \frac{V_{\text{ind}}}{R} = \frac{1}{R}\left(-\frac{d\Phi_{\text{B}}}{dt}\right) = -\frac{1}{R}\left(\frac{dAB}{dt}\right) = -\frac{B}{R}\left(\frac{dA}{dt}\right) = -\frac{B}{R}\frac{d\pi r^2}{dt} = -\frac{B\pi}{R}\left(\frac{dr^2}{dt}\right) \approx -\frac{B\pi}{R}\left(\frac{r_{\text{f}}^2 - r_{\text{i}}^2}{\Delta t}\right).$$

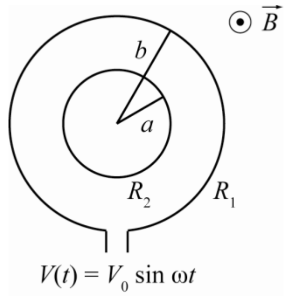
CALCULATE: $i = -\frac{(4.26\cdot 10^{-5}\text{ T})\pi}{30.0\ \Omega}\left(\frac{(0.250\text{ m})^2 - (0.200\text{ m})^2}{1.00\text{ s}}\right) = -1.00374\cdot 10^{-7}\text{ A}$

ROUND: The induced current in the loop is $i = -1.00\cdot 10^{-7}$ A.

DOUBLE-CHECK: This current is very small, as one would expect. The negative sign indicates that the direction of the induced current is such that the magnetic field due to the induced current opposes the change in magnetic flux that induces the current.

29.33. **THINK:** The current in the outer loop generates a magnetic field. Because the magnitude of the current in the outer loop changes with time, the magnetic field it generates also changes. The changing magnetic field, in turn, induces a potential difference and thus a current in the inner loop. Let I be the current in the outer loop and i be the induced current in the inner loop.

SKETCH:



RESEARCH: The current through the large loop is $I = \dfrac{V_0 \sin \omega t}{R_1}$. This creates a magnetic field at the center of the loop of:

$$B_1 = \frac{\mu_0 I}{2b}$$

which is derived from the Biot-Savart Law. Since the radius of the inner loop is much smaller than the radius of the outer loop, the magnetic field through the inner loop is $B_1 = \mu_0 V_0 \sin \omega t / 2bR_1$. This magnetic field creates a flux of:

$$\Phi_B = B_1 A = B_1 \pi a^2 = \frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t.$$

The induced potential across the inner loop is then:

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right).$$

This voltage corresponds to a current in the inner loop of:

$$i = \frac{\Delta V_{\text{ind}}}{R_2} = -\frac{1}{R_2}\frac{d}{dt}\left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right).$$

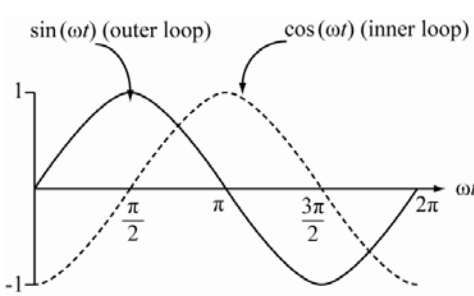
SIMPLIFY: The potential difference induced in the inner loop is:

$$\Delta V_{\text{ind}} = -\frac{d}{dt}\left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right) = -\frac{\mu_0 \pi a^2 V_0}{2bR_1}\frac{d}{dt}\left(\sin \omega t \right) = -\frac{\mu_0 \pi a^2 V_0 \omega}{2bR_1} \cos \omega t,$$

and the induced current in the inner loop is:

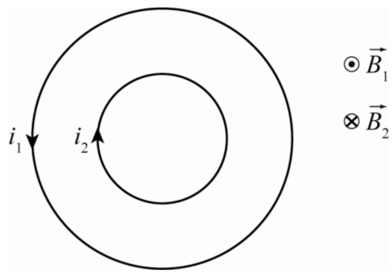
$$i = \frac{\Delta V_{\text{ind}}}{R_2} = -\frac{1}{R_2}\frac{d}{dt}\left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right) = -\frac{\mu_0 \pi a^2 V_0}{2bR_1 R_2}\frac{d}{dt}\left(\sin \omega t \right) = -\frac{\mu_0 \pi a^2 V_0 \omega}{2bR_1 R_2} \cos \omega t.$$

CALCULATE: Not applicable.
ROUND: Not applicable.
DOUBLE CHECK: The time dependence on the current for the outer loop and inner loop is shown in the plot below. For example, for $\omega t < \pi / 2$ (taking positive values to be the counterclockwise direction) if the current in the outer loop is moving counterclockwise and increasing then the current in the inner loop is increasing in the clockwise direction. This is consistent with Lenz's Law.



29.34. **THINK:** The varying current, i_1, through the outer solenoid creates a varying magnetic field, B_1, within the coil. This varying B field creates a flux in the inner solenoid, which in turn creates an induced *emf.*

SKETCH:



RESEARCH: The magnetic field generated by the outer solenoid is given by $B_1 = \mu_0 n i_1 = \mu_0 n i_0 \cos\omega t$. The flux generated in the inner solenoid is given by $\Phi = A_2 B_1$. The induced emf in the inner solenoid is given by $\Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = -A_2 \frac{dB_1}{dt}$.

SIMPLIFY: $\Delta V_{\text{ind}} = -A_2 \frac{dB_1}{dt} = -A_2 \frac{d}{dt}\left(\mu_0 n i_0 \cos\omega t\right) = -A_2 \mu_0 n i_0 \frac{d}{dt}\left(\cos\omega t\right) = A_2 \mu_0 n i_0 \omega \sin\omega t.$ This corresponds to a current of $i_2 = \frac{\Delta V}{R} = \frac{A_2 \mu_0 n i_0 \omega \sin(\omega t)}{R}$, in the inner solenoid. The current of the inner solenoid induces a B field of:

$$B_2 = \mu_0 n i_2 = \frac{\mu_0 n\left(A_2 \mu_0 n i_0 \omega \sin\omega t\right)}{R} = \frac{\mu_0^{\,2} n^2 A_2 i_0 \omega \sin\omega t}{R}.$$

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE CHECK: The induced magnetic field of the inner solenoid must oppose the change in flux of the outer solenoid. It can be seen from the expressions for B_2 and B_1 the two fields will always have opposite directions, satisfying this requirement.

29.35. (a) The decreasing B field creates a changing flux through the loop, confined to the area of the dotted circle of radius, r = 3.00 cm. The varying flux creates an emf of:

$$\Delta V_{\text{ind}} = -\frac{d\Phi_{\text{B}}}{dt} = -\frac{d(AB)}{dt} = -\frac{d\left(\pi r^2 B\right)}{dt} = -\pi r^2 \frac{dB}{dt} \approx -\pi r^2 \frac{\Delta B}{\Delta t} = -\pi r^2 \left(\frac{B_{\text{f}} - B_{\text{i}}}{\Delta t}\right).$$

This corresponds to a current of:

$$i = \frac{V}{R} = -\frac{\pi r^2}{R}\left(\frac{B_{\text{f}} - B_{\text{i}}}{\Delta t}\right) = -\frac{\pi (0.0300 \text{ m})^2}{0.200 \ \Omega}\left(\frac{1.00 \text{ T} - 2.00 \text{ T}}{2.00 \text{ s}}\right) = 0.00707 \text{ A} = 7.07 \text{ mA}$$

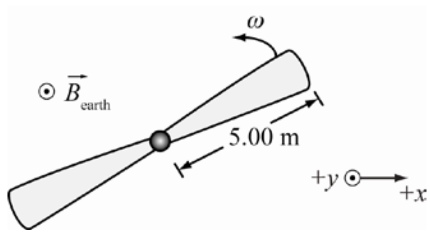
(b) The B field points into the page, thus a decrease in the B field will induce a current corresponding to a B field which points into the page. By the right-hand rule, the induced current flows clockwise.

29.36. The airplane's wings are approximated by a straight wire. The voltage across a wire moving in a B field is:

$$V = vLB = 3v_{\text{mach}}LB = 3(340. \text{ m/s})(10.0 \text{ m})\left(0.500 \cdot 10^{-4} \text{ T}\right) = 0.510 \text{ V}.$$

29.37. **THINK:** As a conductor travels through a magnetic field, perpendicular to the ground, of intensity $B = 0.426$ G, it creates a voltage difference between its ends. The length of metal of interest is L = 5.00 m and rotates at $1.00 \cdot 10^4$ rpm.

SKETCH:



RESEARCH: The potential difference across a wire moving in a magnetic field is $\Delta V_{\text{ind}} = vLB.$ Each element of the blade travels at a different speed, $v = r\omega.$ To calculate the potential difference, the length must be divided into pieces of length, *dl*, which travel at $v = l\omega.$ The value should be integrated over the total length, from 0 to *L*.

SIMPLIFY: $\int \Delta V = \int_0^L vBdl = \int_0^L l\omega Bdl = \frac{1}{2}\omega BL^2$

In terms of the blade's rpm, the potential difference is $V = \frac{1}{2}\left(\frac{2\pi(\text{rpm})}{60\text{ s}}\right)BL^2.$

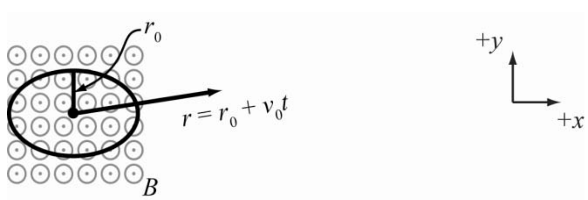
CALCULATE: $V = \left(\frac{\pi\left(1.00\cdot 10^4\text{ rpm}\right)}{60.0\text{ s/rpm}}\right)\left(0.426\cdot 10^{-4}\text{ T}\right)\left(5.00\text{ m}\right)^2 = 0.557633\text{ V} \approx 0.558\text{ V}$

ROUND: The potential difference from the hub of the helicopter's blade to its far end is $\Delta V_{\text{ind}} = 0.558$ V.

DOUBLE-CHECK: We can double-check this result by assuming that the blade moves with a constant speed equal to the speed of the middle of the blade, $v = \left(\frac{L}{2}\right)\omega = \left(\frac{L}{2}\right)2\pi f = 2\pi Lf.$ The induced potential difference would be $\Delta V = vB\frac{L}{2} = \left(2\pi Lf\right)B\frac{L}{2} = \pi fBL^2,$ which is the same answer we got by integrating over the length of the blade.

29.38. **THINK:** The expanding loop creates a changing flux through the loop. Lenz's law implies that the changing flux induces a current in the loop. This is similar to increasing the magnetic field within the loop. To counteract the increase in flux, the current must create a magnetic field opposite to the *B* field. By the right-hand rule, the current must flow clockwise. The radius of the loop expands by $r = r_0 + vt,$ where $r_0 = 0.100$ m and $v = 0.0150$ m/s. The resistance of the wire is $R = 12.0\ \Omega.$ The *B* field has a uniform value of $B_0 = 0.750$ T upward. The problem asks for the induced current at the time, $t = 5.00$ s.

SKETCH:



RESEARCH: The flux through the loop is $\Phi_B = AB = \pi r^2 B.$ The induced current of the loop is $i = V/R,$ where the voltage is given by $V = -d\Phi_B/dt.$

SIMPLIFY: The induced current in the wire is:

$$i = \frac{V}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt} = -\frac{1}{R}\frac{d}{dt}\left(r^2\right) = -\left(\frac{\pi B}{R}\right)\frac{d}{dt}\left(r_0 + vt\right)^2 = -\frac{\pi B2\left(r_0 + vt\right)v}{R} = -\frac{2\pi B}{R}v\left(r_0 + vt\right).$$

CALCULATE: The magnitude of the induced current at $t = 5.00$ s is:

$$i = -\frac{2\pi\left(0.750\text{ T}\right)}{12.0\ \Omega}\left(0.0150\text{ m/s}\right)\left[0.100\text{ m} + \left(0.0150\text{ m/s}\right)\left(5.00\text{ s}\right)\right] = 0.0010308\text{ A}.$$

ROUND: $i = 1.03$ mA at 5.00 s, travelling clockwise through the loop.

DOUBLE-CHECK: $[i] = \frac{[\text{T}]}{[\Omega]}[\text{m/s}]([\text{m}] + [\text{m/s}][\text{s}]) = \frac{[\text{T}][\text{m}^2]}{[\Omega][\text{s}]} = \frac{[\text{V}][\text{s}][\text{m}^2][\text{A}]}{[\text{m}^2][\text{V}][\text{s}]} = [\text{A}]$

29.39. **THINK:** Terminal velocity will be reached when the force due to the changing magnetic flux cancels the weight of the bar.

SKETCH: A sketch is not necessary.

RESEARCH: $\Delta V_{\text{ind}} = -\frac{d\Phi_{\text{B}}}{dt} = -B\frac{dA}{dt} == -Bw\frac{dy}{dt} = Bwv_{\text{term}}$

$i = \frac{\Delta V_{\text{ind}}}{R}$, $F_{\text{B}} = iLB = iwB$, $F_{\text{B}} = F_{\text{gravity}} = mg$.

SIMPLIFY: $iBw = mg \Rightarrow \frac{\Delta V_{\text{ind}}}{R}Bw = mg \Rightarrow \frac{Bwv_{\text{term}}}{R}Bw = mg \Rightarrow v_{\text{term}} = \frac{mgR}{w^2B^2}$.

CALCULATE: No calculations are necessary.

ROUND: Rounding is not necessary.

DOUBLE-CHECK: It makes sense the larger m is, the higher v_{term} has to be to compensate for the greater gravitational force.

29.40. **THINK:**

(a) The change in area causes an induced voltage.

(b) After finding the induced voltage, the induced current can be determined.

(c) The induced current will cause a force opposite to the direction of motion (from Lenz's law) which requires F_{ext} compensating for it.

(d) Determine W_{ext} and P_{ext} from F_{ext}.

SKETCH: Provided with the question.

RESEARCH:

(a) $|\Delta V_{\text{ind}}| = \left|-\frac{d\Phi_{\text{B}}}{dt}\right| = B\frac{dA}{dt} = BvL$

(b) $i_{\text{ind}} = \frac{\Delta V}{R}$, in the clockwise direction.

(c) $F_{\text{B}} = i_{\text{ind}}LB = F_{\text{ext}}$

(d) $W_{\text{ext}} = F_{\text{ext}}\Delta y$, $P_{\text{ext}} = Fv$

(e) $P_{\text{ext}} = P_{\text{R}} = i_{\text{ind}}^2R$

SIMPLIFY:

(a) $|\Delta V_{\text{ind}}| = BvL$

(b) $i_{\text{ind}} = \frac{BvL}{R}$

(c) $|F_{\text{B}}| = |F_{\text{ext}}| = \frac{L^2B^2v}{R}$

(d) $W_{\text{ext}} = \frac{L^2B^2v}{R}\Delta y$, $P_{\text{ext}} = \frac{L^2B^2v^2}{R}$

(e) $P_{\text{R}} = \frac{L^2B^2v^2}{R}$

CALCULATE: Not necessary.

ROUND: Not necessary.

DOUBLE-CHECK:
(e) This is due to the law of conservation of energy. The work done has to go somewhere, and in this case is dissipated by the resistor as heat.

29.41. **THINK:** The current in the wire will cause a magnetic field. The changing current will cause a changing flux through the loop, inducing a potential.
SKETCH: Provided with question.
RESEARCH: For a wire: $B=\frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)$. $\Delta V_{\text{ind}}=\frac{d\Phi_{\text{B}}}{dt}$, $i=2.00\text{ A}+(0.300\text{ A/s})t$, A = 7.00 m by 5.00 m, $\Phi_{\text{B}}=\oiint \vec{B}\bullet d\vec{A}$.

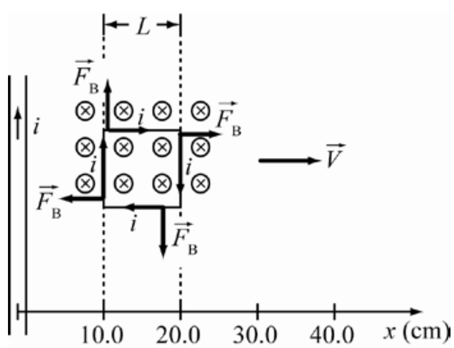
SIMPLIFY: $\Phi_{\text{B}}=(5.00\text{ m})\int_{1\text{ m}}^{8\text{ m}}\frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)dr=(5.00\text{ m})\left(\frac{\mu_0 i}{2\pi}\right)\ln\left(\frac{8.00\text{ m}}{1.00\text{ m}}\right)=(5.00\text{ m})\left(\frac{\mu_0 i}{2\pi}\right)\ln 8.00$

$$\Delta V_{\text{ind}}=\frac{d\Phi_{\text{B}}}{dt}=(5.00\text{ m})\left(\frac{\mu_0}{2\pi}\right)(\ln 8.00)\left(\frac{di}{dt}\right)=(5.00\text{ m})\left(\frac{\mu_0}{2\pi}\right)(\ln 8.00)(0.300\text{ A/s})$$

CALCULATE: $\Delta V_{\text{ind}}=(5.00\text{ m})\left(\frac{4\pi\cdot 10^{-7}\text{ H/m}}{2\pi}\right)(\ln 8.00)(0.300\text{ A/s})=6.238\cdot 10^{-7}\text{ V}$

ROUND: $\Delta V_{\text{ind}}=6.24\cdot 10^{-7}\text{ V}$
DOUBLE-CHECK: It makes sense that the larger the rate of change of the current, the larger the induced voltage.

29.42. **THINK:**
(a) By the right-hand rule, the flux is into the page. Since the square is moving away from the wire, the flux is decreasing. Lenz's law states that the current is moving clockwise.
(c) The top and bottom parts have the same contributions and cancel each other.
SKETCH:
(b)



RESEARCH: Use $x_2=20.0\text{ cm}$ and $x_1=10.0\text{ cm}$ as the end points. $B=\frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)$, $\Phi_{\text{B}}=\oiint \vec{B}\bullet d\vec{A}$, $\Delta V_{\text{ind}}=-\frac{d\Phi}{dt}$, $i_{\text{ind}}=\frac{\Delta V_{\text{ind}}}{R}$, r = 10.0 cm, i = 1.00 A, v = 10.0 cm/s, $R=0.0200\ \Omega$, L = 10.0 cm, $F_{\text{left}}=i_{\text{ind}}LBx_1$, $F_{\text{right}}=i_{\text{ind}}LBx_2$, and $F_{\text{net}}=F_{\text{right}}-F_{\text{left}}$.

SIMPLIFY: $\Phi_B=L\int_{x_1+vt}^{x_2+vt}\frac{\mu_0}{4\pi}\frac{2i}{r}dr=L\frac{\mu_0}{2\pi}i\left[\ln(x_2+vt)-\ln(x_1+vt)\right]$

$$\Delta V_{\text{ind}}=-\frac{d\Phi_{\text{B}}}{dt}=-\frac{Li\mu_0}{2\pi}\left(\frac{v}{x_2+vt}-\frac{v}{x_1+vt}\right)$$

$$F_{\text{net}} = i_{\text{ind}}LBx_2 - i_{\text{ind}}LBx_1 = i_{\text{ind}}LB(x_2 - x_1) = \frac{\Delta V_{\text{ind}}}{R}LB(x_2 - x_1)$$
$$= \left(-\frac{Li\mu_0}{2\pi R}v\left(\frac{1}{x_2+vt} - \frac{1}{x_1+vt}\right)\right)L\left(\frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)\right)(x_2 - x_1)$$
$$= -\frac{L^2i^2\mu_0 v}{2\pi R}\left(\frac{1}{x_2+vt} - \frac{1}{x_1+vt}\right)\left(\frac{\mu_0}{2\pi}\right)\left(\frac{x_2 - x_1}{r}\right)$$
$$= -\frac{L^2i^2\mu_0^2 v}{(2\pi)^2 R}\left(\frac{1}{x_2+vt} - \frac{1}{x_1+vt}\right)\left(\frac{x_2 - x_1}{r}\right)$$

CALCULATE: At time $t = 0$:

$$F_{\text{net}} = -\frac{(0.100\text{ m})^2(1.00\text{ A})^2(4\pi\cdot 10^{-7}\text{ H/m})^2(0.100\text{ m/s})}{(2\pi)^2\, 0.0200\ \Omega}\left(\frac{1}{20.0\text{ cm}} - \frac{1}{10.0\text{ cm}}\right)\left(\frac{20.0\text{ cm} - 10.0\text{ cm}}{10.0\text{ cm}}\right)$$
$$= 1.00\cdot 10^{-16}\text{ N}$$

ROUND: $F_{\text{net}} = 1.00\cdot 10^{-16}$ N

DOUBLE-CHECK: It makes sense that for larger velocities and currents through the wire, the induced force is larger. This is in some ways analogous to how a car traveling faster than another has a larger drag force.

29.43. $\Phi(t) = BA\cos(2\pi ft)$, $\Delta V_{\text{ind}} = \frac{d\Phi}{dt} = -2\pi fBA\sin(2\pi ft)$. The maximum occurs when $\left|\sin(2\pi ft)\right| = 1$.

$\Delta V_{\text{ind,max}} = 2\pi fBA = 110.$ V. Substitute the values to obtain: $f = \frac{110.\text{ V}}{2\pi BA} = \frac{110.\text{ V}}{2\pi(1.00\text{ T})(1.00\text{ m}^2)} = 17.5$ Hz.

29.44. **THINK:** First relate the magnetic flux to the angular speed and then determine the maximum angular speed. Use the values $B = 0.87$ T, $A = 0.0300\text{ m}^2$.

SKETCH: A sketch is not necessary.

RESEARCH: For a single loop: $\Phi(t) = BA\cos(\omega t)$. $\Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = \omega BA\sin(\omega t)$

$\Delta V_{\text{ind,max}} = 170\text{ V} = \omega BA$, since the maximum occurs when $\left|\sin(\omega t)\right| = 1$.

SIMPLIFY: $\omega = \frac{\Delta V_{\text{ind,max}}}{BA}$

CALCULATE: $\omega = \frac{170\text{ V}}{0.87\text{ T}(0.0300\text{ m}^2)} = 6513$ Hz

ROUND: $\omega = 6500$ Hz

DOUBLE-CHECK: It is reasonable that the higher the applied voltage, the higher the angular speed.

29.45. **THINK:** First determine an expression for the magnetic flux, and then use Faraday's law to determine the induced voltage.

SKETCH: A sketch is not necessary.

RESEARCH: $B_{\text{Earth}} = 0.300\text{ G} = 0.300\cdot 10^{-4}$ T, $\Phi_{\text{B}} = NBA\cos(\omega t)$, $A = \pi r^2$, $r = 0.250$ m, $N = 1.00\cdot 10^5$,

$\omega = 2\pi(150.\text{ Hz})$, $i_{\text{ind}} = \frac{\Delta V_{\text{ind}}}{R} = -\left(\frac{1}{R}\right)\frac{d\Phi_{\text{B}}}{dt}$, $i_{\text{ind,peak}} = -\left(\frac{1}{R}\right)\frac{d\Phi_{\text{B}}}{dt}_{\text{peak}}$, $R = 1500.\ \Omega$

SIMPLIFY:

(a) $i_{\text{ind}} = -\left(\frac{1}{R}\right)\left(-NBA\omega\sin(\omega t)\right) = \frac{NBA\omega}{R}\sin(\omega t)$; The peak occurs at $\left|\sin(\omega t)\right| = 1$: $i_{\text{ind,peak}} = \frac{NBA\omega}{R}$.

(b) $i_{avg} = 0.7071\left(i_{ind,peak}\right),\;\; P_{avg} = i_{avg}^2 R$

CALCULATE:

(a) $i_{ind,peak} = \dfrac{\left(1.00\cdot 10^5\right)\left(0.300\cdot 10^{-4}\text{ T}\right)\left(0.250\text{ m}\right)^2 2\pi^2\left(150.\text{ Hz}\right)}{\left(1500.\;\Omega\right)} = 0.3701\text{ A}$

(b) $i_{avg} = 0.7071\left(0.3701\text{ A}\right) = 0.2617\text{ A},\;\; P_{avg} = \left(0.2617\text{ A}\right)^2\left(1500.\;\Omega\right) = 102.7\text{ W}$

ROUND:

(a) $i_{ind,peak} = 0.370\text{ A}$

(b) $i_{avg} = 0.262\text{ A},\;\; P_{avg} = 103\text{ W}$

DOUBLE-CHECK: The answer seems reasonable since there are a very large number of turns for the generator turning at a very fast rate.

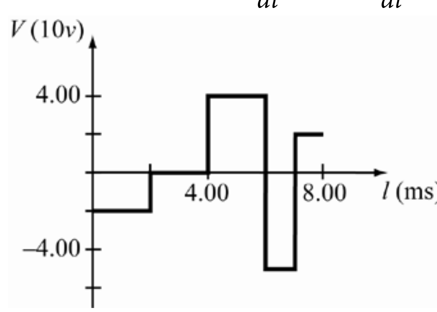
29.46. First solve for n: $B = \mu_0 ni \;\Rightarrow\; n = \dfrac{B}{\mu_0 i} = \dfrac{0.025\text{ T}}{\left(1.2566\cdot 10^{-6}\text{ m kg s}^{-2}\text{ A}^{-2}\right)\left(0.60\text{ A}\right)} = 33158.$

$$M = N_1\pi\mu_0 n_2 r_1^2 = 200\pi\left(1.2566\cdot 10^{-6}\text{ m kg s}^{-2}\text{ A}^{-2}\right)\left(33158\right)\left(0.034\text{ m}\right)^2 = 0.0302\text{ H},\;\; i(t) = i_0\left(1+\left(2.4\text{ s}^{-2}\right)t^2\right)$$

$$V = -M\frac{di}{dt} = -\left(0.0302\text{ H}\right)\left(2\right)\left(0.60\text{ A}\right)\left(2.4\text{ s}^{-2}\right)\left(2.0\text{ s}\right) = -0.17\text{ V}$$

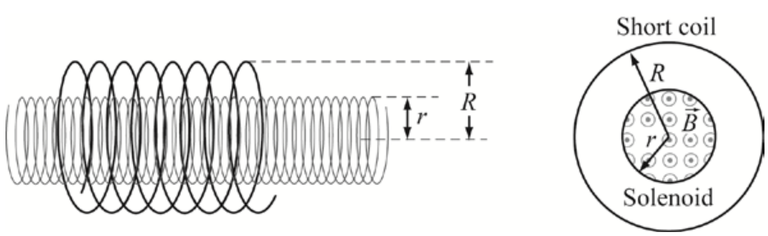
The results match those of the example.

29.47. The potential across an inductor is given by: $\Delta V_{ind} = -L\dfrac{di}{dt}$, where $\dfrac{di}{dt}$ is the slope.



29.48. **THINK:** The potential difference induced in the solenoid is due to the changing current in the coil. Using the mutual inductance of the solenoid due to the coil, the potential difference induced in the solenoid can be calculated. Assume the magnetic field of the short coil is uniform. This is not strictly accurate, but necessary to answer the question and will give a reasonable approximation.

SKETCH:



RESEARCH: The mutual inductance between the coil and the solenoid is

$$M = \frac{N_s \Phi_{c\to s}}{i_c},$$

where N_s is the number of turns in the solenoid, $\Phi_{c\to s}$ is the flux in the solenoid resulting from the magnetic field through the coil, and i_c is the current in the coil. The flux is given by

$$\Phi_{c\to s} = in\pi\mu_0 r^2.$$

$\Delta V_{ind} = M\frac{di}{dt}$, $N_s = 30$, $n = 60/\text{cm} = 6000/\text{m}$, $r = 0.0800$ m, $\frac{di}{dt} = \frac{2.00\text{ A}}{12.0\text{ s}}$.

SIMPLIFY: $\Delta V_{ind} = N_s n\pi\mu_0 r^2 \frac{di}{dt}$

CALCULATE: $\Delta V_{ind} = (30)(6000/m)\pi(4\pi\cdot 10^{-7}\text{ H/m})(0.0800\text{ m})^2\left(\frac{2.00\text{ A}}{12.0\text{ s}}\right) = 7.57986\cdot 10^{-4}\text{ V}$

ROUND: $\Delta V_{ind} = 7.58\cdot 10^{-4}$ V

DOUBLE-CHECK: It makes sense that for larger changes in current, larger potential differences are induced.

29.49. (a) $\tau_L = \frac{L}{R} = \frac{1.00\text{ H}}{1.00\text{ M}\Omega} = 1.00\ \mu\text{s}$

(b) $i(t) = \frac{V_{emf}}{R}\left(1 - e^{-t/\tau_L}\right)$. At $t = 0$, $i(t) = 0$. At $t = 2.00\ \mu$s, $i(t) = \frac{10.0\text{ V}}{1.00\text{ M}\Omega}\left(1 - e^{-(2.00\ \mu\text{s})/(1.00\ \mu\text{s})}\right) = 8.65$ μA.

At steady state. $t\to\infty$: $i(\infty) = \frac{V_{emf}}{R} = 10.0\ \mu A$.

29.50. For an *RL* circuit: $i(t) = \frac{V_{emf}}{R}\left(1 - e^{-t/\tau}\right)$, where $\tau = \frac{L}{R} = 0.0250$ s.

$$\frac{i(t)R}{V_{emf}} = 1 - e^{-t/\tau} \Rightarrow -\tau\ln\left(1 - \frac{i(t)R}{V_{emf}}\right) = t \Rightarrow t = -(0.0250\text{ s})\ln\left(1 - \frac{(0.300\text{ A})(120.\ \Omega)}{40.0\text{ V}}\right) = 0.0576\text{ s}$$

29.51. The potential drop is the sum of the potential drop across the resistor and the inductor:

$$\Delta V = iR + L\frac{di}{dt} = (3.0\text{ A})(3.25\ \Omega) + (0.440)(3.6\text{ A/s}) = 11 \approx 11.3\text{ V}.$$

29.52. **THINK:** In a circuit containing only a resistor, the current would be established almost instantaneously. However, with the RL circuit, the current must increase exponentially from zero to the steady state. $V_{emf} = 18$ V, $R_1 = R_2 = 6.0\ \Omega$, $L = 5.0$ H.

SKETCH: Provided with question.

RESEARCH:

(a) The inductor functions as an open-circuit, so $i = V_{emf}/R_2 = 18\text{ V}/6.0\ \Omega = 3.0$ A.

(b) The inductor acts as an open-circuit, so there is no current across it and hence no current across R_1.

(c) The current across R_2 is given by Ohm's Law, $i = V_{emf}/R$.

(d) The potential difference across a resistor is also given by Ohm's Law, $\Delta V = iR$.

(e) Same as (d).

(f) The sum of the voltages around any loop is zero.

(g) The rate of current change across R_1 is the same as that of L.

SIMPLIFY:

(a) $i = V/R_2$

(b) Not applicable.

(c) $i_{R_2} = V_{emf}/R_2$

(d) $\Delta V_{R_1} = i_{R_1}R_1$

(e) $\Delta V_{R_2} = i_{R_2}R_2$

(f) $V_{emf} - V_L - V_{R_1} = 0 \Rightarrow V_L = V_{emf} - V_{R_1}$

(g) $V_{\text{L}} = L\dfrac{di}{dt} \Rightarrow \dfrac{di}{dt} = \dfrac{V_{\text{L}}}{L}$

CALCULATE:

(a) $i = 18\text{ V}/6.0\ \Omega = 3.0\text{ A}$

(b) $i_{\text{R}_1} = 0$

(c) $i_{\text{R}_2} = 18\text{ V}/6.0\ \Omega = 3.0\text{ A}$

(d) $\Delta V_{\text{R}_1} = (0\text{ A})(6.0\ \Omega) = 0$

(e) $\Delta V_{\text{R}_2} = (3.0\text{ A})(6.0\ \Omega) = 18\text{ V}$

(f) $V_{\text{L}} = 18\text{ V} - 0 = 18\text{ V}$

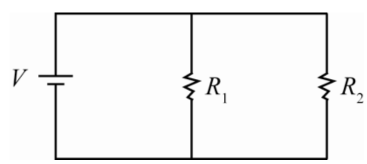
(g) $\dfrac{di}{dt} = \dfrac{18\text{ V}}{5.0\text{ H}} = 3.6\text{ A/s}$

ROUND: Not necessary. The values are already to the correct number of significant figures.

DOUBLE CHECK: The branch of the circuit which contains only a resistor and a source of emf behaves as a simple resistor circuit, with the current being established almost instantaneously. For the branch of the circuit which contains a resistor and an inductor, equation 29.29 states $i(t) = \dfrac{V_{\text{emf}}}{R}\left[1 - e^{-t/(L/R)}\right]$. When $t = 0$, $i(t) = 0$, as found above.

29.53. **THINK:** After a long time, the inductor acts like a short-circuit. The circuit is in steady state, so the current is no longer changing. $V_{\text{emf}} = 18\text{ V}$, $R_1 = R_2 = 6.0\ \Omega$, $L = 5.0\text{ H}$.

SKETCH: An equivalent sketch when the circuit is in steady-state is as follows.



RESEARCH: The current from the battery is given by $i_{\text{tot}} = \dfrac{V_{\text{emf}}}{R_{\text{net}}}$, where $R_{\text{net}} = \left(\dfrac{1}{R_2} + \dfrac{1}{R_1}\right)^{-1}$. The current through each resistor is given by Ohm's Law, $i = V/R$. The sum of the potentials around any loop must be zero: $V_{\text{emf}} + V_{\text{R}_1} = 0$, $V_{\text{emf}} + V_{\text{R}_2} + V_{\text{L}} = 0$.

SIMPLIFY:

(a) $i_{\text{tot}} = \dfrac{V_{\text{emf}}}{R_1 R_2}(R_1 + R_2)$

(b) $i_{\text{R}_1} = \dfrac{V_{\text{R}_1}}{R_1}$

(c) $i_{\text{R}_2} = \dfrac{V_{\text{R}_2}}{R_2}$

(d) $V_{\text{emf}} + V_{\text{R}_1} = 0 \Rightarrow V_{\text{R}_1} = -V_{\text{emf}}$

(e) $V_{\text{emf}} + V_{\text{R}_2} + V_{\text{L}} = 0 \Rightarrow V_{\text{R}_2} = -V_{\text{emf}} - V_{\text{L}} = -V_{\text{emf}} - L\dfrac{di}{dt}$

(f) $V_{\text{L}} = L\dfrac{di}{dt}$

(g) $\dfrac{di_{\text{R}_1}}{dt} = \dfrac{di_{\text{L}}}{dt} = \dfrac{V_{\text{L}}}{L}$

CALCULATE:

(a) $i_{tot} = \dfrac{18 \text{ V}}{(6.0 \ \Omega)(6.0 \ \Omega)}(6.0 \ \Omega + 6.0 \ \Omega) = 6.0 \text{ A}$

(b) $i_{R_1} = \dfrac{18 \text{ V}}{6.0 \ \Omega} = 3.0 \text{ A}$

(c) $i_{R_2} = \dfrac{18 \text{ V}}{6.0 \ \Omega} = 3.0 \text{ A}$

(d) $V_{R_1} = -18 \text{ V}$

(e) $V_{R_2} = -18 \text{ V} - (5.0 \text{ H})(0) = -18 \text{ V}$

(f) $V_L = 5.0 \text{ H}(0) = 0$

(g) $\dfrac{di_{R_1}}{dt} = \dfrac{0}{5.0 \text{ H}} = 0$

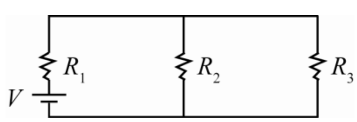
ROUND: Not necessary.

DOUBLE CHECK: Evaluating the loop containing the inductor using equation 29.29 shows that after a long time, $i_2(t) = \dfrac{V_{emf}}{R_2}\left(1 - e^{-t/(L/R)}\right) = \dfrac{V_{emf}}{R_2}$, as found above. Kirchoff's rules can be used to show that $i_{R_1} = i_{R_2}$, also as found above.

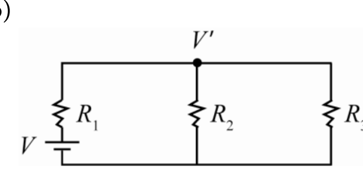
29.54. **THINK:** As the current begins to flow through the circuit, the self induced potential difference in the inductor opposes the change in current. As the change in current decreases, the self induced potential difference also decreases until the current reaches the steady state given by Ohm's Law, $i = V_{emf} / R$. When the switch is opened, the current will continue to flow, at a decreasing rate, through the loop composed of R_3, L, and R_2 until the energy which has been stored in the inductor is dissipated.

SKETCH:

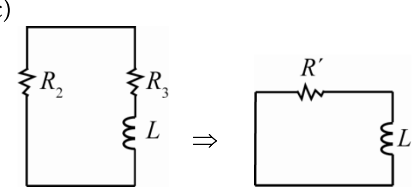
(a)



(b)



(c)



RESEARCH:

(a) Immediately after the switch is closed, the inductor is like an open-circuit. Clearly, $i_{R_3} = 0$, and $V + i_{R_1}R_1 + i_{R_2}R_2 + i_{R_3}R_3 = 0.$ so $i_{R_2} = i_{R_1} = \dfrac{V}{R_1 + R_2}.$

(b) After a long time, the inductor acts like a short-circuit. $R_{tot} = R_1 + \dfrac{R_2 R_3}{R_2 + R_3}$, $\quad V' = V - R_1\left(\dfrac{V}{R_{tot}}\right)$

(c) When the switch is opened, $i_L = i_{R_2} = i_{R_3}$. In fact, the equivalent resistance of this circuit is $R' = R_2 + R_3$ and the circuit can be redrawn accordingly. Since the current in an inductor cannot change instantaneously, from part (b): $i_{\text{initial}} = \dfrac{V'}{R_3} = \dfrac{V - R_1\left(V/R_{\text{tot}}\right)}{R_3}$, $R_{\text{tot}} = R_1 + \dfrac{R_2R_3}{R_2+R_3}$ and $\tau = \dfrac{L}{R'}$. The current for an *RL* circuit is $i(t) = i_{\text{initial}}\left(e^{-t/\tau}\right)$. Immediately after opening the switch, $t \approx 0$ and $i(t_0) = i_{\text{initial}}\left(e^0\right) = i_{\text{initial}}$.

SIMPLIFY:

(a) $V + i_{R_1}R_1 + i_{R_2}R_2 + 0 = 0 \Rightarrow i_{R_2} = i_{R_1} = \dfrac{V}{R_1+R_2}$, $i_{R_3} = 0$

(b) $i_{R_1} = \dfrac{V}{R_{\text{tot}}} = \dfrac{V}{R_1 + \dfrac{R_2R_3}{R_2+R_3}} = \dfrac{V(R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3}$

$$i_{R_2} = \frac{V'}{R_2} = \frac{V}{R_2} - \frac{R_1}{R_2}\left(\frac{V(R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3}\right) = \frac{V(R_1R_2+R_1R_3+R_2R_3) - V(R_1R_2+R_1R_3)}{R_2(R_1R_2+R_1R_2+R_2R_3)} = \frac{V(R_3)}{(R_1R_2+R_1R_2+R_2R_3)}$$

$$i_{R_3} = \frac{V'}{R_3} = \frac{V}{R_3} - \frac{R_1}{R_3}\left(\frac{V(R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3}\right) = \frac{V(R_1R_2+R_1R_3+R_2R_3) - V(R_1R_2+R_1R_3)}{R_3(R_1R_2+R_1R_2+R_2R_3)} = \frac{V(R_2)}{R_1R_2+R_1R_2+R_2R_3}$$

(c) $i(t) = i_{\text{initial}}\left(e^{-t/\tau}\right)$, where $i_{R_1} = 0,\ i_{R_3} = -i_{R_2} = i(t), i_{\text{initial}} = \dfrac{V - R_1\left(V/R_{\text{tot}}\right)}{R_3} = \dfrac{V(R_2)}{R_1R_2+R_1R_3+R_2R_3}$.

CALCULATE: Not applicable.
ROUND: Not applicable.
DOUBLE CHECK: When the switch is closed and the current is increasing according to $i(t) = \dfrac{V_{\text{emf}}}{R}\left(1-e^{-t/\tau}\right)$, as $t \to \infty$, $i(t) = \dfrac{V_{\text{emf}}}{R}$, which agrees with the result.

29.55. The energy density is given by $u_B = \dfrac{B^2}{2\mu_0}$. Determine the volume that gives $Vu_B = 1$ J:

$$V = \frac{2\mu_0(1\text{ J})}{B^2} = \frac{2\left(4\pi\cdot 10^{-7}\text{ H}\cdot\text{m}^{-1}\right)(1\text{ J})}{\left(5.0\cdot 10^{-5}\text{ T}\right)^2} = 1.01\cdot 10^3\text{ m}^3.$$

This volume is equivalent to a 10 m by 10 m by 10 m cube. This is a fraction of the size of a house.

29.56. (a) The magnetic energy density is given by: $u_B = \dfrac{1}{2\mu_0}B^2 = \dfrac{1}{2\left(4\pi\cdot 10^{-7}\text{ H}\cdot\text{m}^{-1}\right)}(3.00\text{ T})^2 = 3.58\cdot 10^6\text{ J/m}^3$.

(b) The total energy is given by $U_B = Vu_B$. $V = \pi R^2 L$, $R = 0.500$ m, $L = 1.50$ m.

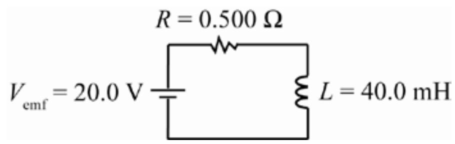
$$\Rightarrow U_B = \pi(0.500\text{ m})^2(1.50\text{ m})\left(3.58\cdot 10^6\text{ J/m}^3\right) = 4.22\cdot 10^6\text{ J}$$

29.57. (a) $u_B = \dfrac{1}{2\mu_0}B^2 = \dfrac{1}{2\left(4\pi\cdot 10^{-7}\text{ H}\cdot\text{m}^{-1}\right)}\left(4.00\cdot 10^{10}\text{ T}\right)^2 = 6.366\cdot 10^{26}\text{ J/m}^3$

(b) The associated mass density is then: $\dfrac{u_B}{c^2} = \rho_{\text{rest}} = \dfrac{6.366\cdot 10^{26}\text{ J/m}^3}{(3.00\cdot 10^8\text{ m/s}^2)^2} = 7.07\cdot 10^9\text{ kg/m}^3$

29.58. **THINK:** The emf potential and the resistance can be used to find the maximum current. Then the energy stored in the magnetic field of the inductor at one fourth of this current can be found. The equation for the rise in current as a function of time can be used to find the time for the circuit to reach a current of one fourth of its maximum value. The inductance of the inductor is $L = 40.0$ mH, the resistance of the resistor is $R = 0.500\ \Omega$, and the emf potential is $V_{\text{emf}} = 20.0$ V.

SKETCH:



RESEARCH:
(a) At steady-state,

$$i_{max} = \frac{V_{emf}}{R}.$$

The time of interest is when $i = i_{max}/4$. Use the equation $U_B = \frac{1}{2}Li^2$.

(b) $i(t) = i_{max}\left(1 - e^{-t/\tau_{RL}}\right) = \frac{1}{4}i_{max}$, $\tau_{RL} = \frac{L}{R}$

SIMPLIFY:

(a) $U_B = \frac{1}{2}L\left(\frac{1}{4}\frac{V_{emf}}{R}\right)^2 = \left(\frac{1}{32}\right)\frac{LV_{emf}^{\,2}}{R^2}$

(b) $\frac{1}{4}i_{max} = i_{max}\left(1 - e^{-t/\tau_{RL}}\right) \Rightarrow \ln\left(\frac{3}{4}\right) = -\frac{t}{\tau_{RL}} \Rightarrow t = -\tau_{RL}\ln\left(\frac{3}{4}\right) = -\frac{L}{R}\ln\left(\frac{3}{4}\right)$

CALCULATE:

(a) $U = \left(\frac{1}{32}\right)\frac{(0.0400 \text{ H})(20.0 \text{ V})^2}{(0.500\ \Omega)^2} = 2.00 \text{ J}$

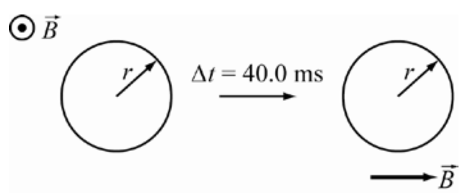
(b) $t = -\left(\frac{0.0400 \text{ H}}{0.500\ \Omega}\right)\ln\left(\frac{3}{4}\right) = 0.0230 \text{ s}$

ROUND:
(a) $U = 2.00$ J
(b) $t = 0.0230$ s

DOUBLE-CHECK: It makes sense that the time it takes to reach one fourth of the maximum value is comparable to the time constant, τ_{RL}.

29.59. **THINK:** The equation for the rate of energy production due to a potential across a resistance can be used to determine the heat generated. The induced potential can be found by using Faraday's Law. Then the rise in temperature due to this heat can be found for the ring of mass $m = 0.0150$ kg and specific heat capacity $c = 129$ J/kg °C. The strength of the magnetic field is $B = 0.0800$ T, the radius of the ring is $r = 0.00750$ m, the time change between maximum and zero magnetic flux is $\Delta t = 0.0400$ s, and the resistance of the ring is $R = 61.9 \cdot 10^{-6}\ \Omega$.

SKETCH:



RESEARCH: The induced potential in the ring is given by: $\Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = -\frac{BA}{\Delta t}$. The rate of energy production as heat is given by

$$P = \frac{(\Delta V_{\text{ind}})^2}{R}.$$

The power produced multiplied by the time difference is equal to the heat generated:

$$P\Delta t = Q = mc\Delta T.$$

SIMPLIFY: The temperature rise is

$$\Delta T = \frac{(\Delta V_{\text{ind}})^2 \Delta t}{mcR} = \frac{\Delta t}{mcR}\left(-\frac{BA}{\Delta t}\right)^2 = \frac{(\pi B r^2)^2}{mcR\Delta t}$$

CALCULATE: $\Delta T = \frac{\left(\pi(0.0800 \text{ T})(0.00750 \text{ m})^2\right)^2}{(0.0150 \text{ kg})(129 \text{ J/kg °C})(61.9\cdot 10^{-6} \ \Omega)(0.0400 \text{ s})} = 4.1715\cdot 10^{-5} \text{ °C}$

ROUND: To three significant figures, the temperature rise is $\Delta T = 4.17\cdot 10^{-5}$ °C.

DOUBLE-CHECK: It makes sense that for larger fields, ΔT is larger, and for larger masses, ΔT is smaller since it would take more work to heat up the ring. As expected, the temperature increase is quite small.

29.60. **THINK:** Consider the energy of the dipole before and after the flip and relate this to the work done.
SKETCH: A sketch is not necessary.
RESEARCH: When the dipole is in alignment: $U = -NiAB$. When the dipole is anti-parallel to the field: $U = NiAB$.
SIMPLIFY: The work done must therefore be $W = \Delta U = 2NiAB$.
CALCULATE: No calculations are necessary.
ROUND: Rounding is not necessary.
DOUBLE-CHECK: Larger fluxes (larger NAB) yield more work for the power supply.

29.61. **THINK:** Determine the energy density of the electric field and the magnetic field separately.
SKETCH: A sketch is not necessary.

RESEARCH: $u_{\text{B}} = \frac{1}{2\mu_0}|B^2|$, $u_{\text{E}} = \frac{1}{2}\varepsilon_0 |E^2|$, $|\vec{B}_0| = \left|\frac{k \times \vec{E}_0}{\omega}\right|$, $\omega = \frac{|\vec{k}|}{\sqrt{\mu_0 \varepsilon_0}}$, $\vec{E}(\vec{x},t) = \vec{E}_0 \cos(\vec{k}\bullet\vec{x} - \omega t)$, $\vec{B}(\vec{x},t) = \vec{B}_0 \cos(\vec{k}\bullet\vec{x} - \omega t)$.

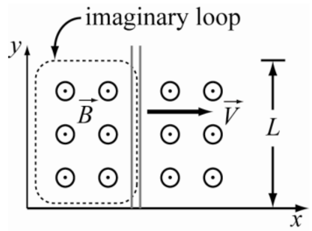
SIMPLIFY:

$$\frac{u_{\text{B}}}{u_{\text{E}}} = \frac{|B^2|}{2\mu_0}\left(\frac{1}{2}\varepsilon_0 |E^2|\right)^{-1} = \frac{1}{\mu_0\varepsilon_0}\frac{|B^2|}{|E^2|} = \frac{1}{\mu_0\varepsilon_0}\left[\frac{|\vec{B}_0|^2 \cos^2(\vec{k}\bullet\vec{x} - \omega t)}{|\vec{E}_0|^2 \cos^2(\vec{k}\bullet\vec{x} - \omega t)}\right] = \frac{1}{\mu_0\varepsilon_0}\left(\frac{1}{\omega^2}\right)\frac{|k \times \vec{E}_0|^2}{|\vec{E}_0|^2} = \frac{|k \times \vec{E}_0|^2}{|\vec{k}|^2 |\vec{E}_0|^2}$$

Note that $\vec{k}$ is perpendicular to $\vec{E}_0$ so $|\vec{k} \times \vec{E}_0|^2 = |\vec{k}|^2 |\vec{E}_0|^2$, so the above expression becomes $\frac{u_{\text{B}}}{u_{\text{E}}} = 1$.

CALCULATE: No calculations are necessary.
ROUND: Rounding is not necessary.
DOUBLE-CHECK: This result shows that the energy in this type of wave is partitioned equally between the electric and magnetic fields.

29.62.



The induced voltage is given by:

$$\left|\Delta V_{\text{ind}}\right| = \left|-\frac{d\Phi}{dt}\right| = vLB = 2.00 \text{ V} \Rightarrow v = \frac{\Delta V}{BL} = \frac{2.00 \text{ V}}{(0.100 \text{ m})(1.00 \text{ T})} = 20.0 \text{ m/s}.$$

29.63. The potential difference is given by Faraday's law:

$$\left|\Delta V_{\text{ind}}\right| = \left|-\frac{d\Phi}{dt}\right| = A\frac{dB}{dt} = \pi R^2 \frac{dB}{dt} = \pi (0.0400 \text{ m})^2 (1.50 \text{ T/s}) = 7.54 \cdot 10^{-3} \text{ V}$$

Note that the radius of the coil is irrelevant.

29.64. The inductor cannot have the current jump instantaneously. From Kirchoff's loop law:

$$V_{\text{emf}} - L\frac{di}{dt} = 0 \Rightarrow \frac{V_{\text{emf}}}{L} = \frac{di}{dt}.$$

$$\frac{V_{\text{emf}}}{L} = \frac{di}{dt} \Rightarrow di = \frac{V_{\text{emf}}}{L}dt \Rightarrow \int_0^i di = \int_0^t \frac{V_{\text{emf}}}{L}dt \Rightarrow i(t) = \frac{V_{\text{emf}}}{L}t + C$$

Since $i(0) = 0$, $C = 0$. The expression is then $i(t) = \frac{V_{\text{emf}}}{L}t$.

29.65. The energy stored in a solenoid is given by $U_{\text{B}} = Li^2/2$. The energy is dependent only on the magnitude, not the direction of the current. Therefore the energy stored in the magnetic field does not change.

29.66. Use the formulas: $u_{\text{B}} = \frac{1}{2\mu_0}B^2$, $u_{\text{E}} = \frac{1}{2}\varepsilon_0 E^2$ and $\frac{u_{\text{B}}}{u_{\text{E}}} = \frac{1}{\mu_0\varepsilon_0}\frac{B^2}{E^2}$. In particular, the values of the energy densities are:

$$u_{\text{B}} = \frac{1}{2\left(1.257 \cdot 10^{-6} \ \frac{\text{m kg}}{\text{s}^2 \ \text{A}^2}\right)}(50.0 \ \mu\text{T})^2 = 9.94 \cdot 10^{-4} \text{ J/m}^3,$$

and

$$u_{\text{E}} = \frac{1}{2}\left(8.842 \cdot 10^{-12} \ \frac{\text{s}^4 \ \text{A}^2}{\text{m}^3 \ \text{kg}}\right)(150. \text{ N/C})^2 = 9.95 \cdot 10^{-8} \text{ J/m}^3.$$

To compute the ratios, it is useful to remember that $1/\mu_0\varepsilon_0 = c^2$. This is a result from light being an electromagnetic wave where *c* is the speed of light in a vacuum.

$$\frac{u_{\text{B}}}{u_{\text{E}}} = c^2\frac{B^2}{E^2} = (3.00 \cdot 10^8 \text{ m/s})^2\left(\frac{50.0 \cdot 10^{-6} \text{ T}}{150. \text{ N/C}}\right)^2 = 1.00 \cdot 10^5$$

The energy density of the magnetic field is much larger than that of the electric field.

29.67. The current of an RL circuit is given by: $i(t) = \frac{V_{\text{emf}}}{R}\left(1 - e^{-t/\tau}\right)$, where $\tau = L/R$. For $t = 20.0 \ \mu\text{s}$:

$$\frac{1}{2}\left(\frac{V_{\text{emf}}}{R}\right) = \frac{V_{\text{emf}}}{R}\left(1 - e^{-t/\tau}\right) \Rightarrow 1 - \frac{1}{2} = e^{-t/\tau} \Rightarrow \tau \ln\left(\frac{1}{2}\right) = -t \Rightarrow \frac{L}{R}\ln\left(\frac{1}{2}\right) = -t \Rightarrow L = -\frac{Rt}{\ln(1/2)}$$

$$\Rightarrow\ L = -\frac{\left(3.00\cdot 10^{3}\ \Omega\right)\left(20.0\cdot 10^{-6}\ \text{s}\right)}{\ln\left(1/2\right)} = 0.0866\ \text{H}.$$

29.68. The current of an RL circuit is given by $i(t) = i_{\text{max}}\left(1-e^{-t/\tau}\right)$, where $\tau = L/R$.

$$0.995 i_{\text{max}} = i_{\text{max}}\left(1-e^{-t/\tau}\right)\ \Rightarrow\ e^{-t/\tau} = 0.00500\ \Rightarrow$$

$$t = -\frac{L}{R}\ln\left(0.00500\right) = -\frac{0.200\cdot 10^{-6}\ \text{H}}{500.\ \Omega}\ln\left(0.00500\right) = 2.12\ \text{ns}$$

It is interesting to note that the voltage of the battery is irrelevant to the result of the problem.

29.69. For a single loop of wire (N = 1), the induced potential difference is:

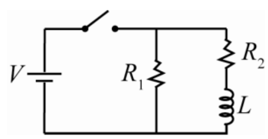
$$\Delta V_{\text{ind}} = -\frac{d\Phi_{\text{B}}}{dt} = -\frac{d}{dt}\left(BA\cos\theta\right).$$

Since the normal vector of the loop and the magnetic field is parallel, $\cos\theta = 1.$ The negative sign can be dropped and ΔV_{ind} becomes:

$$\Delta V_{\text{ind}} = A\frac{dB}{dt} = A\frac{d}{dt}\left(3.00\ \text{T} + 2.00t\ \text{T/s}\right) = A\left(2.00\ \text{T/s}\right) = \left(5.00\ \text{m}^2\right)\left(2.00\ \text{T/s}\right) = 10.0\ \text{V}.$$

Note the magnetic field, $\vec{B}$, is increasing, and it is directed into the page. By Lenz's law, the induced magnetic field, $\vec{B}_{\text{i}}$, opposes the change in magnetic flux, Φ_{B}. In this case, $\vec{B}_{\text{i}}$ is directed out of the page to oppose the increasing field, $\vec{B}$, directed into the page. The induced current is therefore counterclockwise.

29.70. The following circuit has values: V = 9.00 V, $R_1 = R_2 = 100.\ \Omega$, and L = 3.00 H.



(a) When the switch is closed at t = 0 s, the current through R_1 is: $i_1 = \dfrac{V}{R_1} = \dfrac{9.00\ \text{V}}{100.\ \Omega} = 0.0900\ \text{A}.$ The current through R_2 is $i_2(t) = \dfrac{V}{R_2}\left[1-e^{-t/(l/R)}\right]\ \Rightarrow\ i_2(0) = 0.$

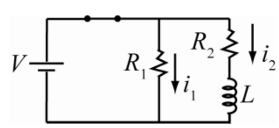
(b) At t = 50.0 ms = 0.0500 s, i_1 is still 0.0900 A, while i_2 is:

$$i_2\left(0.0500\ \text{s}\right) = \frac{9.00\ \text{V}}{100.\ \Omega}\left[1-\exp\left\{\left(-0.0500\ \text{s}\right)\left(\frac{100.\ \Omega}{3.00\ \text{H}}\right)\right\}\right] = 0.0900\left(1-0.189\right)\ \text{A} = 0.0730\ \text{A}$$

(c) At t = 500. ms = 0.500 s, i_1 is still 0.0900 A, and i_2 is:

$$i_2\left(0.500\ \text{s}\right) = \frac{9.00\ \text{V}}{100.\ \Omega}\left[1-\exp\left\{\left(-0.500\ \text{s}\right)\left(\frac{100.\ \Omega}{3.00\ \text{H}}\right)\right\}\right] = 0.0900\left(1-5.78\cdot 10^{-8}\right)\ \text{A} = 0.0900\ \text{A}.$$

(d) After 10.0 s, the equilibrium current of 0.0900 A has long since been reached. Before the switch is opened, the currents i_1 and i_2 oppose each other in the right-most loop, as shown below.



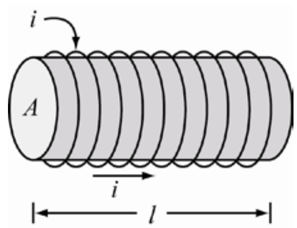
When the switch is opened (after achieving an equilibrium current in the circuit), $i_1 = -i_2$. After opening the switch, Kirchhoff's loop rule becomes $L\,di/dt + iR_1 + iR_2 = 0.$ With $R_1 = R_2 = R,$ this expression becomes $Ldi/dt + i(2R) = 0$. Solving for i yields: $i(t) = i_0 e^{-t/\tau_L}$, $\tau_L = L/2R$, and i_0 is the achieved equilibrium current, i = 0.0900 A. At t = 0 s, $-i_1(0) = i_2(0) = i_0 e^{-0/\tau_L} = i_0 = 0.0900$ A.

(e) At t = 50.0 ms = 0.0500 s, $i_1(0.0500 \text{ s}) = -i_2(0.0500 \text{ s}) = (0.0900 \text{ A})e^{-2(100. \ \Omega)(0.0500 \text{ s})/3.00 \text{ H}} = 0.00321$ A.

(f) At t = 500. ms = 0.500 s, $i_1(0.500 \text{ s}) = -i_2(0.500 \text{ s}) = (0.0900 \text{ A})e^{-2(100. \ \Omega)(0.500 \text{ s})/3.00 \text{ H}} \approx 0$ A.

29.71. **THINK:** A solenoid of length, l = 3.0 m, and n = 290 turns/m has a current of i = 3.0 A, and stores an energy of $U_B = 2.8$ J. Find the cross-sectional area, A, of the solenoid.

SKETCH:



RESEARCH: The energy stored in the magnetic field of an ideal solenoid is $U_B = \mu_0 n^2 l A i^2 / 2$.

SIMPLIFY: Solving for A yields: $A = \dfrac{2U_B}{\mu_0 n^2 l i^2}$.

CALCULATE:

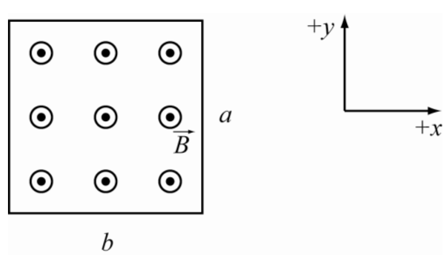
$$A = \frac{2(2.80 \text{ J})}{(4\pi \cdot 10^{-7} \text{ T m/A})(290 \text{ m}^{-1})^2 (3.00 \text{ m})(3.00 \text{ A})^2} = 1.9625 \text{ J/T A} = 1.9625 \frac{\text{N m}}{(\text{V s/m}^2)(\text{J/V s})} = 1.9625 \text{ m}^2$$

ROUND: Rounding to three significant figures, $A = 1.96 \text{ m}^2$.

DOUBLE-CHECK: Considering the length, l, of the solenoid, this is a reasonable cross-sectional area. The units of the result are also correct.

29.72. **THINK:** The rectangular loop has dimensions a by b and resistance R. It is placed on the xy-plane. The magnetic field direction points out of the page and varies in time according to $B = B_0(l + c_1 t^3)$. Determine the direction of the current induced in the loop, i_{ind}, and its value at t = 1 s (in terms of a, b, R, B_0 and c_1).

SKETCH:



RESEARCH: Since the magnetic field is increasing as it comes out of the page, the induced magnetic field, B_i, points into the page according to Lenz's law. The induced current flows clockwise. The current is found from $V_{\text{ind}} = i_{\text{ind}} R$, where $V_{\text{ind}} = -d\Phi_B / dt = -dBA\cos\theta / dt$.

SIMPLIFY: With $\cos\theta = \cos(0°) = 1$, and A constant:

$$V_{\text{ind}} = -A\frac{dB}{dt} = -A\frac{d}{dt}\left[B_0(l + c_1 t^3)\right] = -3AB_0 c_1 t^2. \text{ Then, } i_{\text{ind}} = \frac{|V_{\text{ind}}|}{R} = \frac{3AB_0 c_1 t^2}{R} = \frac{3abB_0 c_1 t^2}{R}.$$

CALCULATE: At $t = 1$ s, $i_{\text{ind}} = \dfrac{3abB_0c_1}{R}$, clockwise

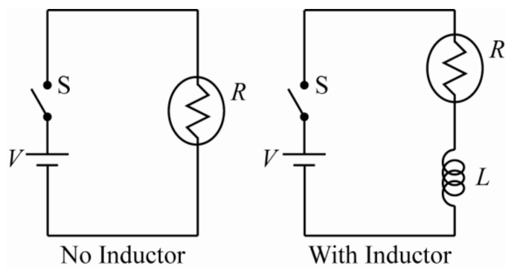
ROUND: Not applicable.

DOUBLE-CHECK: By dimensional analysis, the result has units of current:

$$\left[\frac{abB_0c_1t^2}{R}\right]=\left[\frac{\text{m}^2\ \text{T s}^2}{\text{s}^3\ \Omega}\right]=\left[\frac{\text{m}^2\ \text{V s/m}^2\ \text{s}^2}{\text{s}^3\ \text{V/A}}\right]=\left[\text{A}\right].$$

29.73. **THINK:** The battery with $V_{\text{emf}} = 12.0$ V, is connected in series with a switch and a light-bulb. When the light-bulb draws a current of $i = 0.100$ A, its glow becomes visible. This bulb draws $P = 2.00$ W when it has been connected and when the switch has been closed for a long time. When an inductor is put in series with the bulb and the rest of the circuit, the light-bulb begins to glow $t = 3.50$ ms after the switch is closed. Find the size of the inductor, L.

SKETCH:



RESEARCH: The resistance of the light-bulb can be determined from $P = V^2 / R$. When the inductor is attached, the current is given by $i(t) = i_0\left(1-\text{e}^{-tR/L}\right) = \dfrac{V_{\text{emf}}}{R}\left(1-\text{e}^{-tR/L}\right)$.

SIMPLIFY: $R = \dfrac{V^2}{P} = \dfrac{V_{\text{emf}}^2}{P}$. Substitute this expression into the equation for the current to get:

$$i(t) = \frac{V_{\text{emf}}}{R}\left(1-\text{e}^{-tR/L}\right) = \frac{V_{\text{emf}}}{V_{\text{emf}}^2/P}\left(1-\text{e}^{-tR/L}\right) = \frac{P}{V_{\text{emf}}}\left(1-\text{e}^{-tR/L}\right) \Rightarrow \text{e}^{-tR/L} = 1-\frac{V_{\text{emf}}i(t)}{P}$$

$$\Rightarrow -tR/L = \ln\left[1-\frac{V_{\text{emf}}i(t)}{P}\right] \Rightarrow L = -\frac{tR}{\ln\left[1-V_{\text{emf}}i(t)/P\right]}.$$

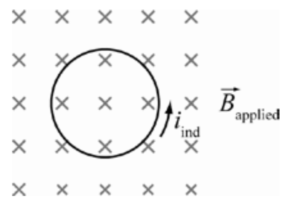
CALCULATE: $R = \dfrac{(12.0\text{ V})^2}{2.00\text{ W}} = 72.0\ \Omega,\ \ L = -\dfrac{(0.00350\text{ s})(72.0\ \Omega)}{\ln\left\{1-\left[(12.0\text{ V})(0.100\text{ A})/2.00\text{ W}\right]\right\}} = 0.27502\text{ H}$

ROUND: $L = 0.275$ H.

DOUBLE-CHECK: An inductor of this capacity in this circuit is capable of storing energy $U_{\text{B}} = \frac{1}{2}Li^2 = \frac{1}{2}(0.300\text{ H})(0.100\text{ A})^2 = 1.50$ mJ. This is sufficient energy to power a 2.00 W light bulb for 0.750 ms. This is a reasonable value for L in this light-bulb circuit.

29.74. **THINK:** A circular loop of cross-section A is placed perpendicular to a time-varying magnetic field of $B(t) = B_0 + at + bt^2$, where B_0, a, and b are constants. For purposes of making a sketch, view the loop so that the field points into the plane of the page. Determine (a) the magnetic flux, Φ_{B}, through the loop at $t = 0$, (b) an equation for the induced potential difference, V_{ind}, in the loop as a function of time, and (c) the magnitude and direction of the induced current if the resistance of the loop is R.

SKETCH:



RESEARCH:

(a) Since the loop is perpendicular to the field, the magnetic flux is given by $\Phi_B = BA$.

(b) From Faraday's law, $V_{\text{ind}} = -d\Phi_B / dt$. Since A is constant while B varies with time, this expression becomes $V_{\text{ind}} = -A(dB/dt)$.

(c) The magnitude of the induced current is found from $V = iR$. With the applied magnetic field directed into the page and increasing in time, the induced magnetic field will point out of the page to oppose the change in magnetic flux. The induced current flows counterclockwise.

SIMPLIFY:

(a) $\Phi_B(t) = BA = \left(B_0 + at + bt^2\right)A$. At $t = 0$, $\Phi_B(0) = B_0 A$.

(b) $V_{\text{ind}}(t) = -A\dfrac{d}{dt}\left(B_0 + at + bt^2\right) = -A(a + 2bt)$

(c) The magnitude of i_{ind} is given by: $i_{\text{ind}} = \left|\dfrac{V_{\text{ind}}}{R}\right| = \dfrac{A(a+2bt)}{R}$, counterclockwise.

CALCULATE: Not applicable.

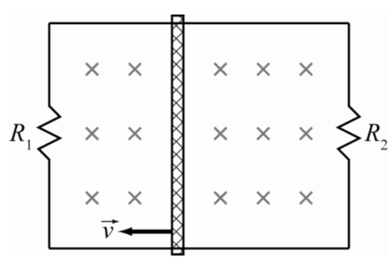
ROUND: Not applicable.

DOUBLE-CHECK: By dimensional analysis, the units are correct:

$[\Phi] = [BA] \Rightarrow \text{Wb} = \text{T m}^2$; $[V] = [AB/t] \Rightarrow \text{V} = \text{m}^2\ \text{T/s}$; $[i] = [V/R] \Rightarrow \Omega\ [\text{A}] = [\text{V}/\ ]$

29.75. **THINK:** A conducting rod of length, L = 0.500 m, slides over a frame of two metal bars placed in a magnetic field of strength, B = 1000. gauss = 0.1000 T. The ends of the rods are connected by two resistors, $R_1 = 100.\ \Omega$ and $R_2 = 200.\ \Omega$. The conducting rod moves with a constant velocity of v = 8.00 m/s. Determine (a) the current flowing through the two resistors, i_1 and i_2, (b) the power, P, delivered to the resistors, and (c) the force, F, needed for the motion of the rod with constant velocity.

SKETCH:



RESEARCH:

(a) The induced potential difference across the resistors is $V_{\text{ind}} = -d\Phi_B / dt$. Since B is constant while A varies in time at a velocity of v, this expression becomes $V_{\text{ind}} = -B(dA/dt) = -BLv$. The current in each resistor can be determined from $V_{\text{ind}} = i_{\text{ind}}R$.

(b) The power delivered to the resistors is $P = i_1^2 R_1 + i_2^2 R_2$.

(c) The force needed to move the rod with a constant velocity is obtained by calculating the total force acting on the rod. The magnetic force on the rod, F_{mag}, is given by $F_{\text{mag}} = BiL = B\left(V/R_{\text{eq}}\right)L$, where R_{eq} is the equivalent resistance.

Note for n resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_n}.$$

SIMPLIFY:

(a) $V_{\text{ind}}=-BLv,\quad i_1=\frac{|V_{\text{ind}}|}{R_1},\quad i_2=\frac{|V_{\text{ind}}|}{R_2}$

(b) $P=i_1^2R_1+i_2^2R_2$

(c) $F_{\text{mag}}=BV_{\text{ind}}L\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=B^2L^2v\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

CALCULATE:

(a) $V_{\text{ind}}=-(0.100\text{ T})(0.500\text{ m})(8.00\text{ m/s})=-0.400\text{ V},\quad i_1=\frac{|-0.400\text{ V}|}{100.\ \Omega}=0.00400\text{ A},$

$i_2=\frac{|-0.400\text{ V}|}{200.\ \Omega}=0.00200\text{ A}$

(b) $P=(0.00400\text{ A})^2(100.\ \Omega)+(0.00200\text{ A})^2(200.\ \Omega)=0.00240\text{ W}$

(c) $F_{\text{mag}}=(0.100\text{ T})^2(0.500\text{ m})^2(8.00\text{ m/s})\left(\frac{1}{100.\ \Omega}+\frac{1}{200.\ \Omega}\right)=0.000300\text{ N}$

ROUND:

(a) $i_1=4.00\text{ mA}$, $i_2=2.00\text{ mA}$

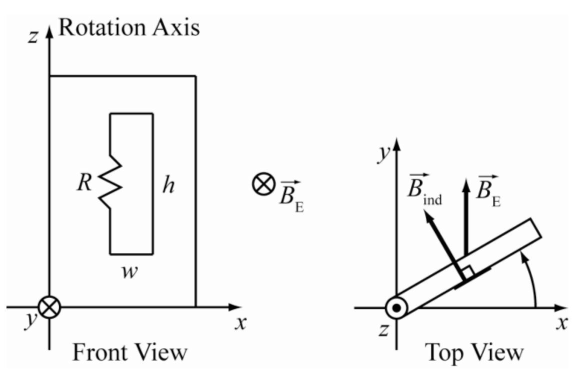
(b) $P=2.40\text{ mW}$

(c) $F_{\text{mag}}=0.300\text{ mN}$

DOUBLE-CHECK: The calculated values are consistent with the given values. Dimensional analysis confirms all the units are correct.

29.76. **THINK:** The loop on the door has dimensions h = 0.150 m, w = 0.0800 m and resistance, $R=5.00\ \Omega$. When the door is closed, it is perpendicular $(\theta=0°)$ to the Earth's uniform magnetic field, $B_{\text{E}}=2.6\cdot10^{-5}$ T. At time, t = 0 s, the door is opened (right edge moving into the page in the figure below) at a constant rate, with an opening angle of $\theta(t)=\omega t$, where $\omega=3.5$ rad/s. Determine the direction and magnitude of the induced current, $i(t)$, at t = 0.200 s.

SKETCH:



RESEARCH: The induced current, i, is found from $i=V_{\text{ind}}/R$, where V_{ind} is given by $V_{\text{ind}}=-d\Phi_{\text{B}}/dt$, and $\Phi_{\text{B}}=BA\cos\theta$. As the door opens, the B field through the loop decreases; by Lenz's law the induced B field points into the page, at an angle of $\theta(t)$ from the plane of the page. The induced current flows clockwise.

SIMPLIFY: $\Phi_B = BA\cos(\theta(t)) = whB_E\cos\omega t,\quad V_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(whB_E\cos\omega t) = whB_E\omega\sin\omega t$

The magnitude of i is $i(t) = \frac{|V_{ind}|}{R} = \frac{whB_E\omega\sin\omega t}{R}.$

CALCULATE: At $t = 0.200$ s:

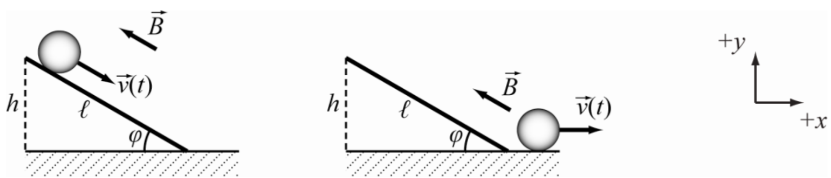
$$i(0.200\text{ s}) = \frac{(0.0800\text{ m})(0.150\text{ m})(2.6\cdot10^{-5}\text{ T})(3.5\text{ rad/s})\sin\left[(3.5\text{ rad/s})(0.200\text{ s})\right]}{5.00\ \Omega} = 1.407\cdot10^{-7}\text{ A.}$$

ROUND: Rounding to two significant figures, $i(0.200\text{ s}) = 140$ nA clockwise.

DOUBLE-CHECK: This induced current is reasonable for a loop with such a small cross-sectional area in the Earth's magnetic field.

29.77. **THINK:** The steel cylinder has radius r = 2.5 cm = 0.025 m and length L = 10.0 cm = 0.100 m. The ramp is inclined at $\phi = 15.0°$ and has a length l = 3.00 m. Determine the induced potential difference, V_{ind} between the ends at the bottom of the ramp if the ramp points in the direction of magnetic North. Use $0.426\cdot10^{-4}$ T as the magnetic field of the Earth.

SKETCH:



RESEARCH: The magnetic field of the Earth lies at an angle to the surface of the Earth (i.e. it is usually not parallel or perpendicular to the Earth's surface). Generally, as a charge q moves with velocity through a magnetic field, the magnetic force acting on the electrons in the conductor is $F_{mag} = q|\vec{v}\times\vec{B}| = qvB\sin\theta,$ where θ is the angle between the velocity $\vec{v}$ of the charge and the magnetic field vector $\vec{B}$. When the electric force, $F_E = qE,$ and the magnetic force on the electrons are in equilibrium, then $E = vB\sin\theta$. This means that the induced potential difference, V_{ind}, between the ends of the conductor is given by $V_{ind} = EL = vBL\sin\theta$. As the cylinder rolls down the ramp (in the direction of the Earth's magnetic field, B), the angle between the cylinder's velocity vector and the Earth's magnetic field vector is zero, so the induced voltage between the ends of the cylinder is zero. At the bottom of the ramp, the cylinder changes direction, and the induced potential difference between the ends is $V_{ind} = vBL\sin\theta$. To determine the speed, v, of the cylinder, recall that the cylinder rolls without slipping so the change in potential energy for the cylinder is equal in magnitude to the change in its kinetic energy: $\Delta K = -\Delta U \Rightarrow mv^2/2 + I\omega^2/2 = mgh$. Here I is the moment of inertia for the cylinder: $I = mr^2/2$, and $\omega = v/r$.

SIMPLIFY: Determine v:

$$\Delta K = -\Delta U \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{v^2}{r^2}$$

$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{4}v^2 = gh \Rightarrow v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}gl\sin\varphi}$$

At the bottom of the ramp, the angle between $\vec{B}$ and $\vec{v}$ is $\theta = \varphi$.

$$V_{ind} = vBL\sin\theta = \sqrt{\frac{4}{3}gl\sin\varphi}\left[BL\sin(\varphi)\right]$$

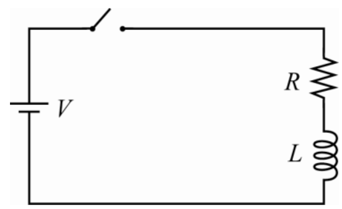
CALCULATE: $V_{ind} = \sqrt{\frac{4}{3}(9.81\text{ m/s}^2)(3.00\text{ m})\sin15.0°}\left[(0.426\cdot10^{-4}\text{ T})(0.100\text{ m})\sin(15°)\right] = 3.514\cdot10^{-6}\text{ V}$

ROUND: $V_{\text{ind}} = 3.51\ \mu\text{V}.$

DOUBLE-CHECK: Considering the given values for this problem, this result is reasonable and also has the correct units: $V_{\text{ind}} = \sqrt{[\text{m/s}^2][\text{m}]}[\text{T}][\text{m}] \Rightarrow \sqrt{\frac{[\text{m}][\text{m}]}{[\text{s}^2]}} \cdot \frac{[\text{V}][\text{s}][\text{m}]}{[\text{m}^2]} \Rightarrow [\text{V}].$

29.78. **THINK:** The battery is connected to a resistor and an inductor in series. Determine (a) the current, $i(t)$, across the circuit after the switch is closed, (b) the total energy, U, provided by the battery from time, $t = 0$ to $t = L/R$, (c) the total energy, U_{R}, dissipated from the resistor, R, for the same time period, and (d) discuss the conservation of energy in this circuit.

SKETCH:



RESEARCH:

(a) In this *RL* circuit, current at any given time, *t*, is given by equation 29.29 in the text, namely $i = i_0\left(1 - e^{-t/\tau}\right)$, where $i_0 = V/R$, and the time constant is $\tau = L/R$.

(b) The power provided by the battery is $P = Vi$. In the given time period, the total energy provided by the battery is $U = \int Vi dt$.

(c) The power dissipated in the resistor is $P = i^2 R$. In the given time period, the total energy dissipated in the resistor is $U_{\text{R}} = \int i^2 R dt$.

(d) Any discrepancy between the energy provided by the battery and the energy dissipated in the resistor is due to the fact that there is energy stored in the inductor, $U_{\text{L}} = Li^2/2$.

SIMPLIFY:

(a) $i(t) = \frac{V}{R}\left(1 - e^{-tR/L}\right)$

(b) $U = \int_{t=0}^{t=\tau} Vi(t)dt = \int_0^\tau \frac{V^2}{R}\left(1 - e^{-tR/L}\right)dt = \frac{V^2}{R}[t]_0^\tau + \frac{V^2}{R}\left(\frac{L}{R}\right)\left[e^{-tR/L}\right]_0^\tau = \frac{V^2}{R}\tau + \frac{V^2 L}{R^2}e^{-\tau R/L} - \frac{V^2 L}{R^2} = \frac{V^2 L}{R^2}e^{-1}$

$= \frac{V^2 L}{R^2}(0.368)$

(c) $U_{\text{R}} = \int_{t=0}^{t=\tau} i^2 R dt = \int_0^\tau \frac{V^2}{R}\left(1 - e^{-tR/L}\right)^2 dt = -\frac{V^2 L}{2R^2}\left[\left(e^{-tR/L}\right)^2\right]_0^\tau + \left[\frac{2V^2 L e^{-tR/L}}{R^2}\right]_0^\tau - \frac{V^2 L}{R^2}\left[\ln\left(e^{-tR/L}\right)\right]_0^\tau$

$= \frac{V^2 L}{R^2}\left(-\frac{e^{-2}}{2} + \frac{1}{2} + 2e^{-1} - 2 + 1 + 0\right) = 0.168\frac{V^2 L}{R^2}$

(d) At time, $t = \tau = L/R$: $U_{\text{L}} = \frac{1}{2}Li^2\tau = \frac{LV^2}{2R^2}\left(1 - e^{-1}\right)^2 = (0.200)\frac{V^2 L}{R^2}.$

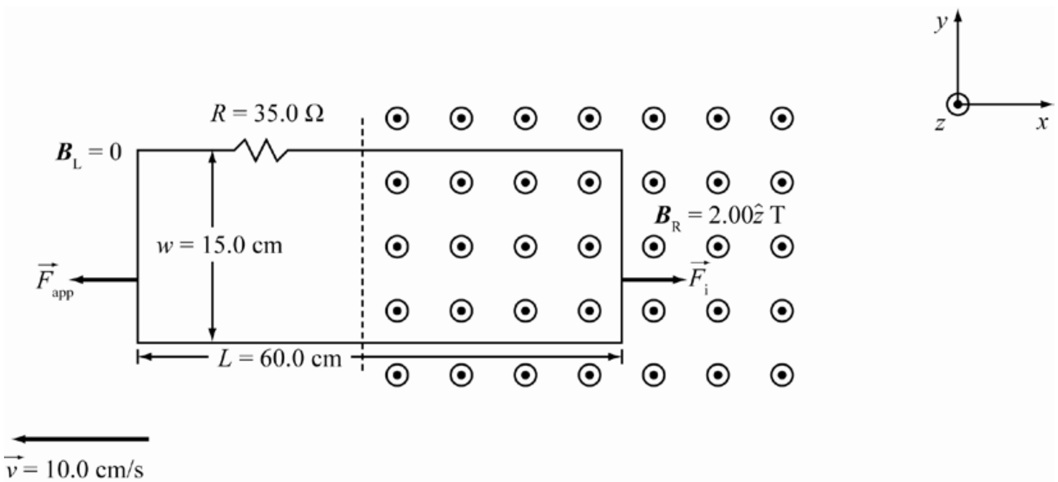
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The total energy of the battery is the sum of the energy dissipated in the resistor and the energy stored in the inductor; energy is conserved.

29.79. **THINK:** The rectangular circuit loop has length, L = 0.600 m, and width, w = 0.150 m, with resistance, $R = 35.0\ \Omega$. It is held parallel to the xy-plane with one end inside a uniform magnetic field as shown in the figure. The magnetic field is $\vec{B}_{\text{R}} = 2.00\hat{z}\text{ T}$ along the positive z-axis to the right of the dotted line; $\vec{B}_{\text{L}} = 0\text{ T}$ to the left of the dotted line. Determine the magnitude of the force, F_{app}, required to move the loop to the left at a constant speed of v = 0.100 m/s, while the right end of the loop is still in the magnetic field. Determine the power, P, used by an agent to pull the loop out of the magnetic field at this speed, and the power, P_{R}, dissipated by the resistor.

SKETCH:



RESEARCH: The magnitude of the force required to move the loop will be equal to the magnitude of the force, F_{i}, on the current induced in the segment of the loop that lies along the y-axis in the magnetic field. That is, $F_{\text{app}} = F_2 = iwB\sin\theta$. Since the angle, θ, between the loop segment of length, w, and the magnetic field is $90°$: $\sin\theta = 1$. The induced current, i, is $i = V_{\text{ind}} / R$, where $V_{\text{ind}} = vwB$ (see equation 29.15). The power, P, used by an agent to pull the loop out of the magnetic field is given by $P = F_{\text{app}}v$. The power dissipated by the resistor is given by $P_{\text{R}} = i^2R$.

SIMPLIFY: The current is $i = \dfrac{V_{\text{ind}}}{R} = \dfrac{vwB}{R}$.

(a) $F_{\text{app}} = iwB$

(b) $P = F_{\text{app}}v$

(c) $P_{\text{R}} = i^2R$

CALCULATE:

(a) $i = \dfrac{(0.100\text{ m/s})(0.150\text{ m})(2.00\text{ T})}{35.0\ \Omega} = 0.85714\text{ mA}$

$F_{\text{app}} = (0.85714\text{ mA})(0.150\text{ m})(2.00\text{ T}) = 0.25714\text{ mN}$

(b) $P = (0.25714\text{ m N})(0.100\text{ m/s}) = 25.714\ \mu\text{W}$

(c) $P_{\text{R}} = (0.85714\text{ mA})^2(35.0\ \Omega) = 25.714\ \mu\text{W}$

ROUND:

(a) $F_{\text{app}} = 0.257\text{ mN}$

(b) $P = 25.8\ \mu\text{W}$

(c) $P_{\text{R}} = 25.7\ \mu\text{W}$

DOUBLE-CHECK: All the power used to move the loop while in the magnetic field is dissipated in the resistor: $P = P_R$.

Multi-Version Exercises

29.80. $i(t) = i_{\max}\left(1 - e^{-t/\tau}\right), \quad \tau = L/R$

$$\frac{3}{4}i_{\max} = i_{\max}\left(1 - e^{-t/\tau}\right) \Rightarrow -\frac{t}{\tau} = -\frac{tR}{L} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow R = -\frac{L}{t}\ln\left(\frac{1}{4}\right) = \frac{L}{t}\ln(4) = \frac{33.03 \cdot 10^{-3}\text{ H}}{3.35 \cdot 10^{-3}\text{ s}}\ln(4) = 13.7\ \Omega$$

29.81. $i(t) = i_{\max}\left(1 - e^{-t/\tau}\right), \quad \tau = L/R$

$$\frac{3}{4}i_{\max} = i_{\max}\left(1 - e^{-t/\tau}\right) \Rightarrow -\frac{t}{\tau} = -\frac{tR}{L} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow L = -tR/\ln\left(\frac{1}{4}\right) = tR/\ln(4) = (3.45 \cdot 10^{-3}\text{ s})(17.88\ \Omega)/\ln(4) = 44.5\text{ mH}$$

29.82. $i(t) = i_{\max}\left(1 - e^{-t/\tau}\right), \quad \tau = L/R$

$$\frac{3}{4}i_{\max} = i_{\max}\left(1 - e^{-t/\tau}\right) \Rightarrow -\frac{t}{\tau} = -\frac{tR}{L} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow t = -\frac{L}{R}\ln\left(\frac{1}{4}\right) = \frac{L}{R}\ln(4) = \frac{55.93 \cdot 10^{-3}\text{ H}}{21.84\ \Omega}\ln(4) = 3.55\text{ ms}$$

29.83. $\Phi_B = NAB\cos(2\pi ft)$, which means that $V_{ind} = -d\Phi_B/dt = NAB(2\pi f)\sin(2\pi ft)$.

The maximum occurs when $\sin(2\pi ft) = 1$. $N = 1$, so

$$V_{\max} = AB(2\pi f) = (0.25\pi d^2)B(2\pi f) = 0.5\pi^2\left(0.0195\text{ m}\right)^2\left(4.77 \cdot 10^{-5}\text{ T}\right)\left(13.3\text{ s}^{-1}\right) = 1.19 \cdot 10^{-6}\text{ V}.$$

29.84. $\Phi_B = NAB\cos(2\pi ft)$, which means that $V_{ind} = -d\Phi_B/dt = NAB(2\pi f)\sin(2\pi ft)$.

The maximum occurs when $\sin(2\pi ft) = 1$. $N = 1$, so

$$V_{\max} = AB(2\pi f) = (0.25\pi d^2)B(2\pi f)$$

$$\Rightarrow d = \sqrt{2V_{\max}/Bf\pi^2} = \sqrt{2\left(1.446 \cdot 10^{-6}\text{ V}\right)/\left(4.97 \cdot 10^{-5}\text{ T}\right)\left(13.5\text{ s}^{-1}\right)\pi^2} = 2.09\text{ cm}.$$

29.85. $\Phi_B = NAB\cos(2\pi ft)$, which means that $V_{ind} = -d\Phi_B/dt = NAB(2\pi f)\sin(2\pi ft)$.

The maximum occurs when $\sin(2\pi ft) = 1$. $N = 1$, so

$$V_{\max} = AB(2\pi f) = (0.25\pi d^2)B(2\pi f)$$

$$\Rightarrow B = 2V_{\max}/(f\pi^2 d^2) = 2\left(6.556 \cdot 10^{-7}\text{ V}\right)/\left((13.7\text{ s}^{-1})\pi^2(1.63\text{ cm})^2\right) = 3.65 \cdot 10^{-5}\text{ T}.$$