

2.8 Gauss's Law (part 1)

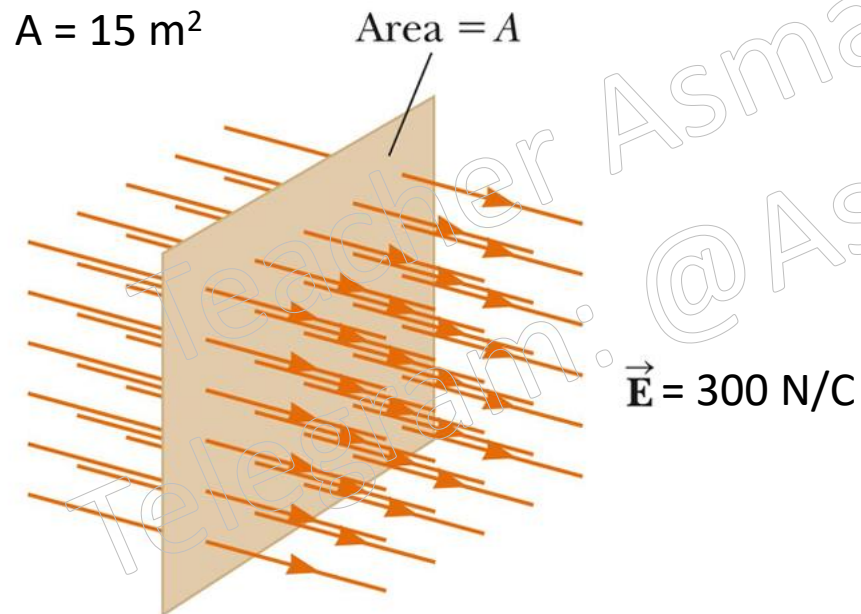
2024-09-22

Quick review:

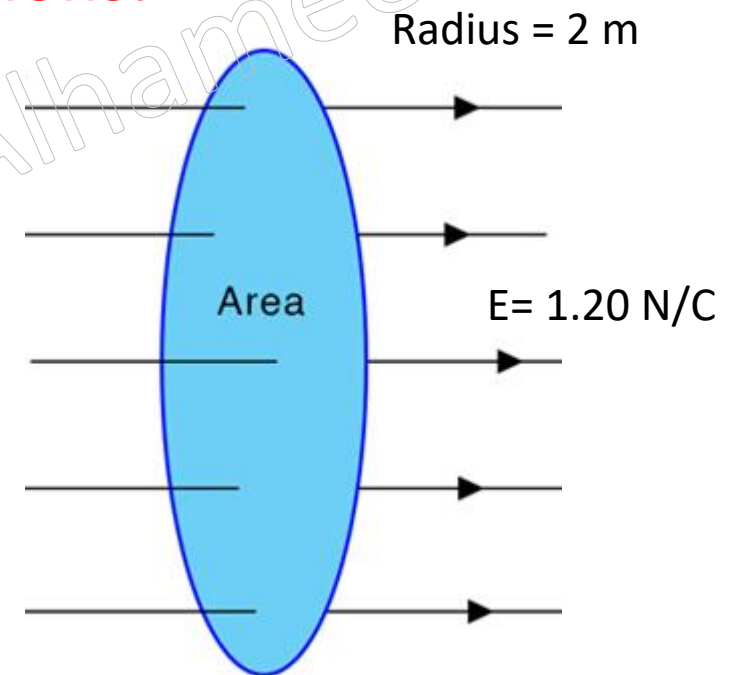
$$\phi = \vec{E} \cdot \vec{A} = EA \cos\theta$$

02:00

Find the electric flux for the following situations:



Answer: $\phi = 300 \times 15 \times \cos(0) = 4.5 \times 10^3 \frac{\text{Nm}^2}{\text{C}}$



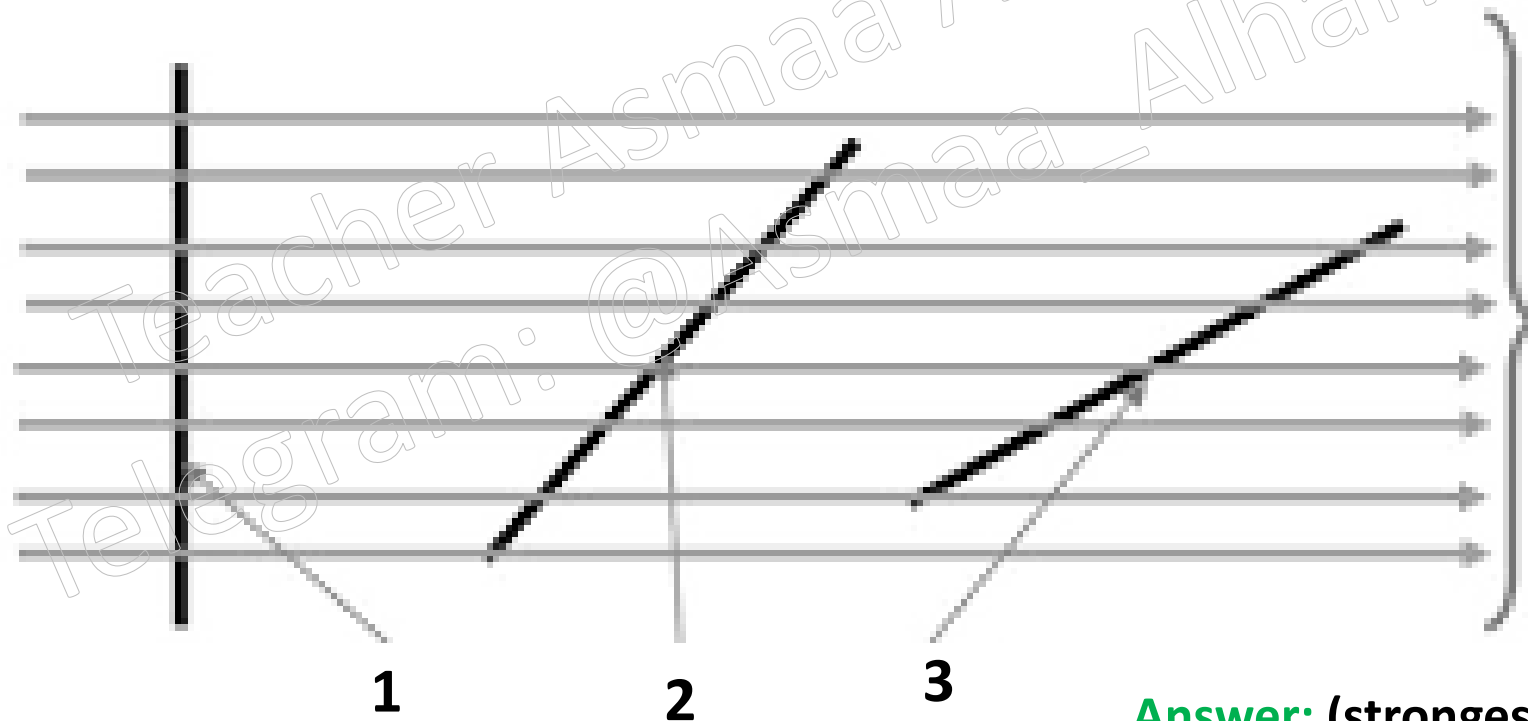
Answer: $\phi = 1.2 \times \pi(2)^2 \times \cos(0)$
 $= 15.1 \frac{\text{Nm}^2}{\text{C}}$

Quick review:

$$\phi = E \cdot A = EA \cos\theta$$

02:00

Re-order the surfaces as per the electric flux from weakest to strongest:



Answer: (strongest 1 – 2 – 3 weakest)

Quick review:

02:00

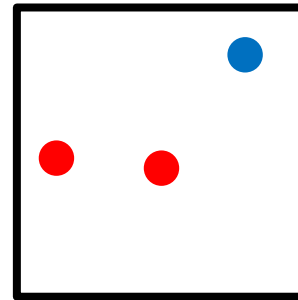
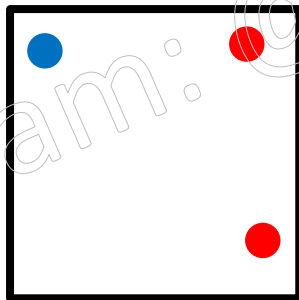
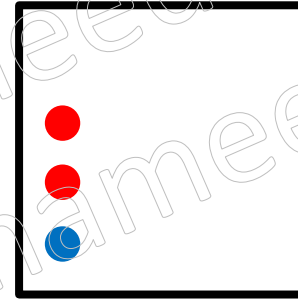
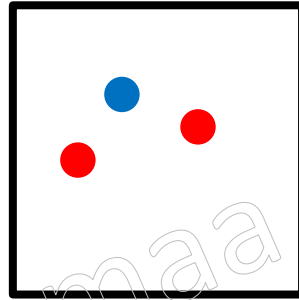
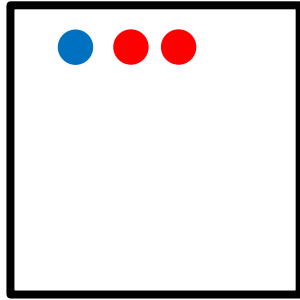
Determine the electric flux of a flat square having an area of 10m^2 is a uniform electric field of 8000 N/C passing perpendicular to it.

$$\phi = E \cdot A = EA \cos\theta$$

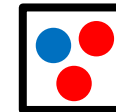
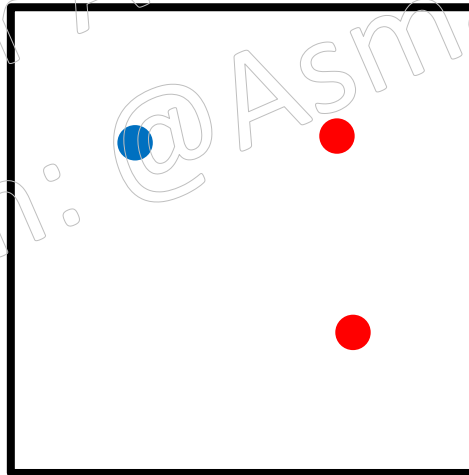
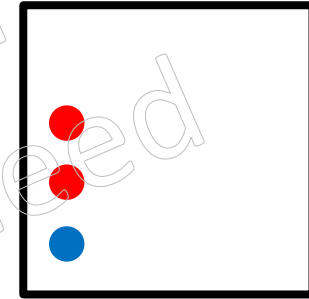
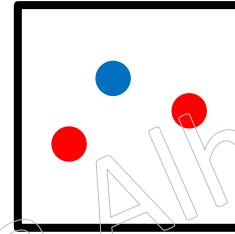
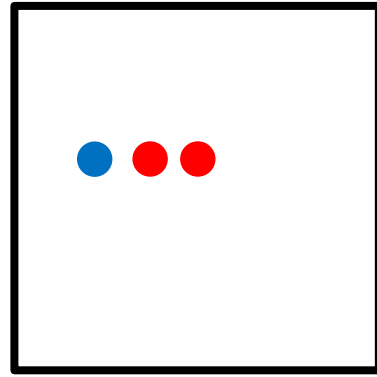
Answer:

$$\begin{aligned}\phi &= E \cdot A = EA \cos\theta \\ &= 8000 \times 10 \times \cos(0) = 8 \times 10^4 \frac{\text{Nm}^2}{\text{C}}\end{aligned}$$

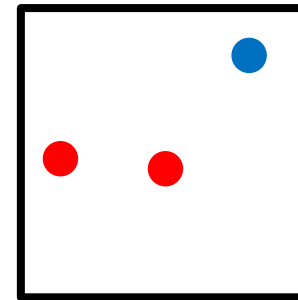
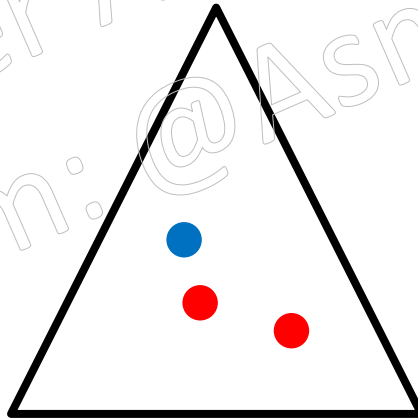
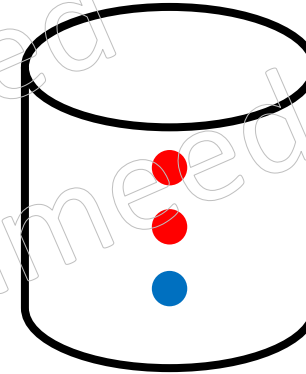
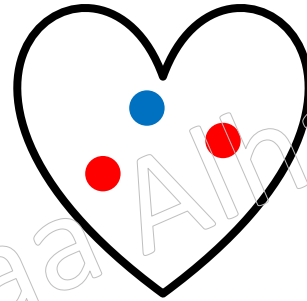
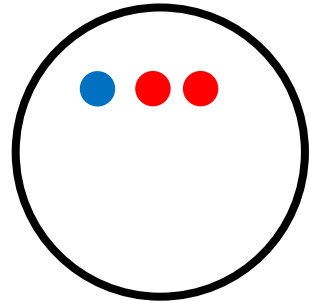
Location
of charge
inside is
not
important
!!



Size of the
shell is **not**
important
!!



Shape is **not**
important
(but you need
it for the area
or volume)

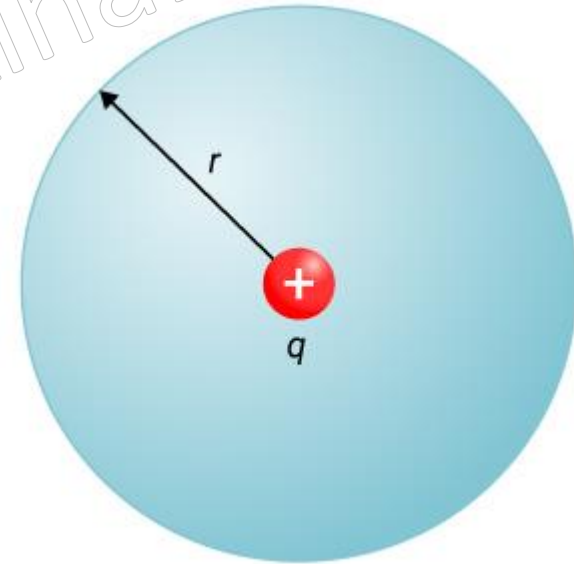


Gauss's Law:

Consider a point charge, q , positioned at the center and surrounded by a **spherical surface** of the radius, r .

What is the electric field produced by this charge at any point on this sphere?

$$E = k \frac{q}{r^2}$$



Where the charge, q , is the charge of the point located at the center and the direction of the electric field lines is away from the center.

Gauss's Law:

The electric flux through any closed surface is calculated as:

$$\phi = \frac{q}{\epsilon_0}$$

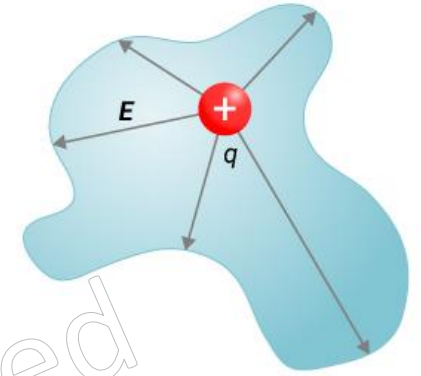
flux → charge

→ The electric flux is proportional to the net charge inside a closed surface **only**.

permittivity constant
 $= 8.85 \times 10^{-12}$

Gauss's law states that the electric flux across **any closed surface** is equal to the total charge enclosed in this surface, q , divided by the permittivity of free space, ϵ_0 .

Gauss's Law:



A general form of Gauss's law is:

Loop: double integral
(2 variables)

Electric field

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

Charge inside

Permittivity
constant

Area Element

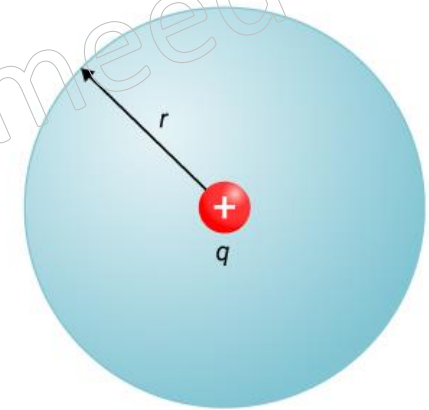
- E represents the electric field at any point
- q represents the net charge inside an enclosed surface

Example:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

A point charge, q , is surrounded by a gaussian surface of the radius, r . Click on the tabs below to see what happens to the electric flux through the surface if:

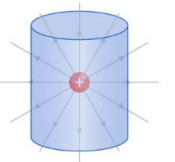
1. the charge is doubled
2. the radius of the sphere is tripled
3. the sphere is replaced with a cylinder
4. the point charge, q , is moved to another position inside the sphere



Answer (1): The electric flux is proportional to the charge inside the surface, the electric flux will double.

Answer (2): The electric flux does not depend on the radius of the surface. So, the electric flux will stay the same.

Answer (3): The electric flux stays the same because the electric field lines pass through the surface radially regardless of the surrounding shape.



Answer (4): The electric flux depends on the charge enclosed in the surface. So, wherever the point charge is inside the closed surface, the electric flux is the same.

Activity: find the electric flux:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

$$\Phi = 0$$



No net electric charge is enclosed in this surface, so the net electric flux is **zero**.

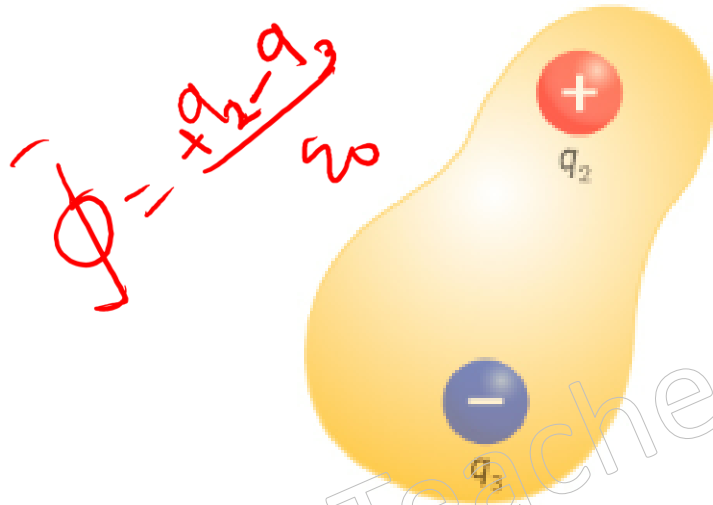


$$\Phi = \frac{q_1}{\epsilon_0}$$

This surface surrounds a charge, q_1 , so the net electric flux is:

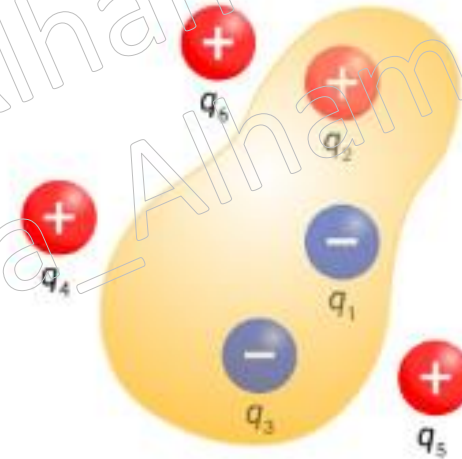
$$\phi = \frac{q_1}{\epsilon_0}$$

Activity: find the electric flux:



This surface surrounds two charges, q_2 , and q_3 . So, the net electric flux will be the sum of the two charges divided by the permittivity of free space:

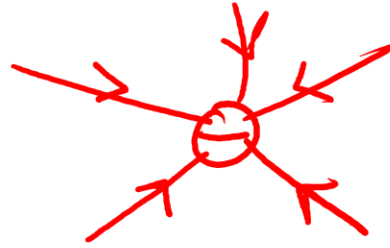
$$\phi = \frac{q_2 + q_3}{\epsilon_0}$$



So, the total electric flux in the three surfaces is:

$$\phi = \frac{q_1 + q_2 + q_3}{\epsilon_0}$$

Activity:



$$\phi = \frac{q_{\text{net}}}{\epsilon_0}$$

Three charged particles of charges -11 C , $+5.0\text{ C}$, and -8.0 C are placed in an irregular shape. Another three charged particles are placed outside the shape, two of them are positively charged and have a charge of 12 C and 15 C respectively, the other one is negatively charged, and has a charge of -4.0 C .

Answer:

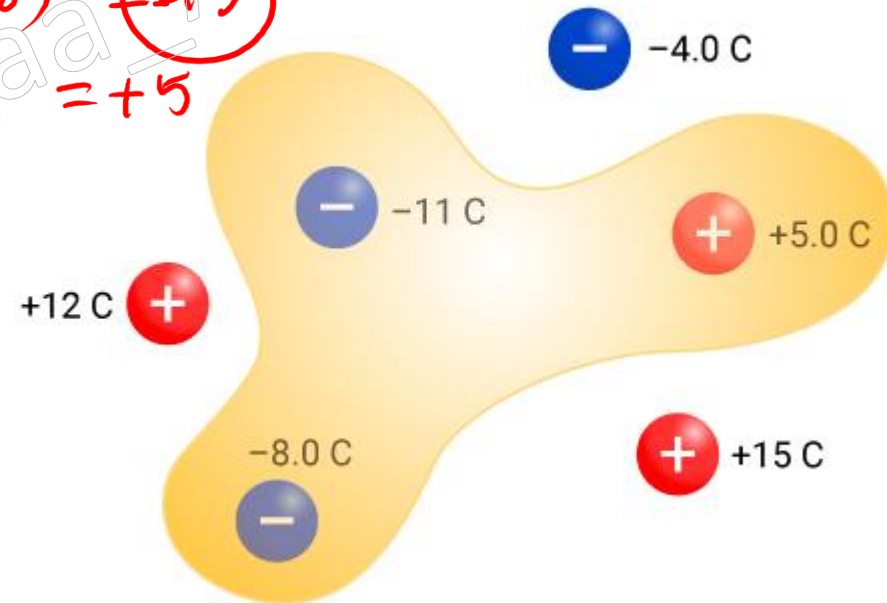
$$\phi = \frac{q_{\text{net}}}{\epsilon_0}$$

$$\phi = \frac{-8.0\text{ C} - 11\text{ C} + 5.0\text{ C}}{8.85 \times 10^{-12}\text{ C}^2/\text{N m}^2}$$

$$\phi = -1.6 \times 10^{12}\text{ N m}^2/\text{C}$$

The negative electric flux means that the electric field lines are moving toward the surface (inward)

$$\begin{aligned} &-(11 + 8) = -19 \\ &+ (5) = +5 \\ &\hline &-14 \end{aligned}$$



Activity:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

- i** An electric drill of radius 2.00 mm is used to produce a hole in a hollow non conducting sphere of radius 15.0 cm. Inside the hollow sphere, a charge of magnitude 14.5 μC is found.

The electric flux through the hole is

- Select -

44.1 $\text{N m}^2/\text{C}$

67.3 $\text{N m}^2/\text{C}$

24.9 $\text{N m}^2/\text{C}$

72.8 $\text{N m}^2/\text{C}$



Enter

0 of 2



$$\phi_{\text{hole}} = \oiint \mathbf{E} \cdot d\mathbf{A}_{\text{hole}}$$

$$\phi_{\text{hole}} = E \times A_{\text{hole}}$$

$$\phi_{\text{hole}} = k \frac{q}{r_{\text{sphere}}^2} \times \pi r_{\text{hole}}^2$$

$$\phi_{\text{hole}} = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \times \frac{14.5 \times 10^{-6} \text{ C}}{(0.150 \text{ m})^2} \times \pi (2.00 \times 10^{-3} \text{ m})^2$$

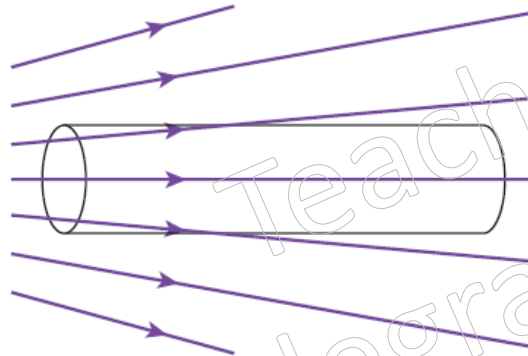
$$\phi_{\text{hole}} = 72.8 \text{ N m}^2/\text{C}$$

Activity: Page 44:

02:00

Concept Check 2.8

A cylinder made of an insulating material is placed in an electric field as shown in the figure. The net electric flux passing through the surface of the cylinder is

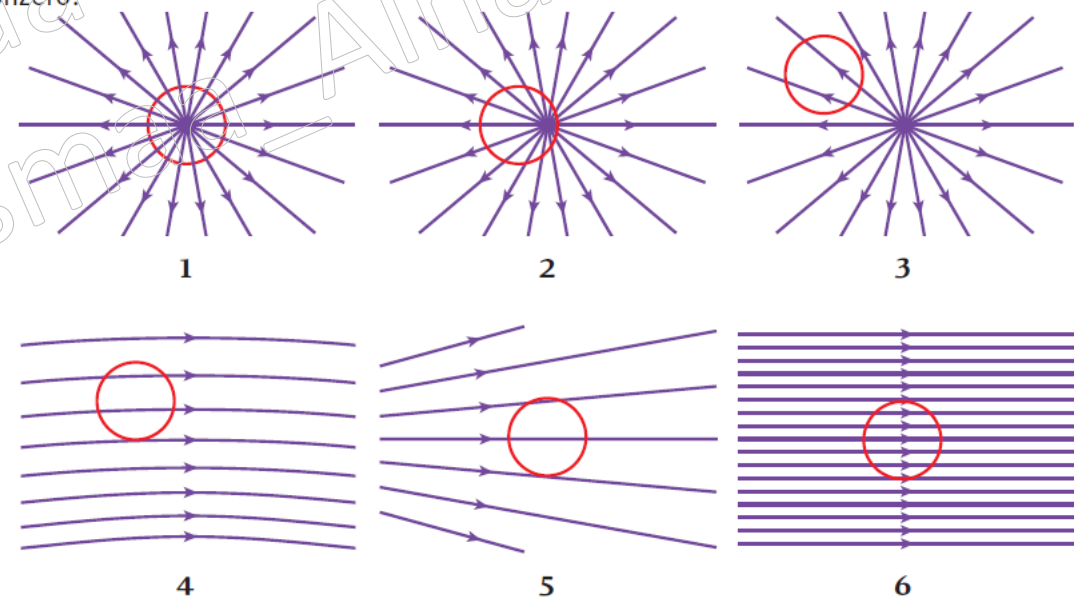


- a) positive.
- b) negative.
- ☒ c) zero.

Concept Check 2.9

The lines in the figure are electric field lines, and the circle is a Gaussian surface. For which case(s) is (are) the total electric flux nonzero?

- a) 1 only
- b) 2 only
- c) 4, 5, and 6
- d) 6 only
- ☒ e) 1 and 2



Activity:

02:00

According to Gauss's law, the net electric flux crossing an open surface, not parallel to the lines of the field, can never be .

zero

radial

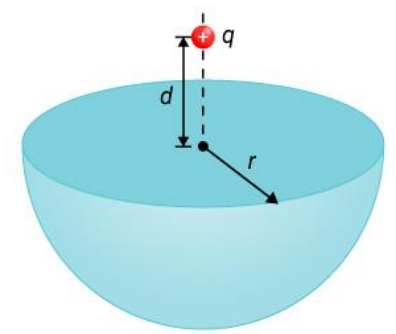
positive

negative

SUBMIT ANSWER



Since the surface is not parallel to the lines of the field, the $\cos \theta$ cannot be zero. Using the expression of the electric flux $\phi = EA \cos \theta$ then ϕ cannot be zero.



Electric Flux of a Hemisphere:

A positive charge, q , is placed a distance, d , above the center of a hemisphere that has a radius, r . If the distance, d , tends to zero, find the electric flux through the curved part of the hemisphere and the flat face.

Curved Part:

If the distance, d , is zero, all points on the hemisphere will have a distance, r , from the point charge.

The electric flux is:

$$\phi_{\text{curved}} = \int \mathbf{E} \cdot d\mathbf{A}$$

Since the electric field points in the same direction as the area vector, then:

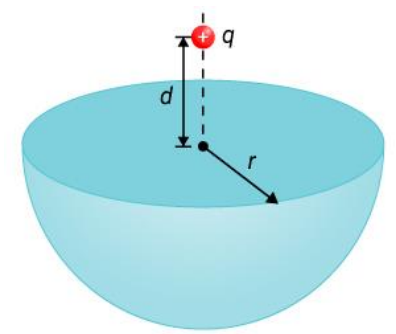
$$\phi_{\text{curved}} = EA_{\text{half hemisphere}}$$

$$\phi_{\text{curved}} = \left(k \frac{q}{r^2} \right) (2\pi r^2)$$

$$\phi_{\text{curved}} = kq2\pi$$

$$\phi_{\text{curved}} = \frac{1}{4\pi\epsilon_0} q2\pi$$

$$\phi_{\text{curved}} = \frac{q}{2\epsilon_0}$$



Electric Flux of a Hemisphere:

A positive charge, q , is placed a distance, d , above the center of a hemisphere that has a radius, r . If the distance, d , tends to zero, find the electric flux through the curved part of the hemisphere and the flat face.

Flat Part:

The closed surface in the flat case encloses zero charge, so the electric flux from Gauss's law gives:

$$\phi_{\text{curved}} + \phi_{\text{flat}} = 0$$

$$\phi_{\text{flat}} = -\phi_{\text{curved}}$$

$$\phi_{\text{flat}} = -\frac{q}{2\epsilon_0}$$

2.8 Gauss's Law (part 2)

2024-09-22

Quick review:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

02:00

$$\epsilon_0 = 8.85 \times 10^{-12}$$

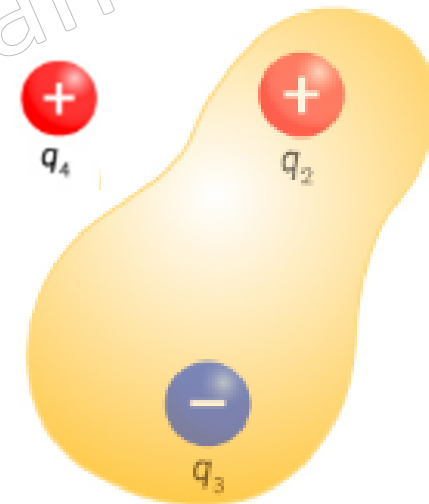
Assume the following:

$$q_2 = 4.6 \mu\text{C}$$

$$q_3 = -5.3 \mu\text{C}$$

$$q_4 = 2.2 \mu\text{C}$$

- 1) calculate the flux through this Gaussian surface.
- 2) Identify the sign of the flux and the direction of flow.
- 3) Identify if this is a sink or a source.



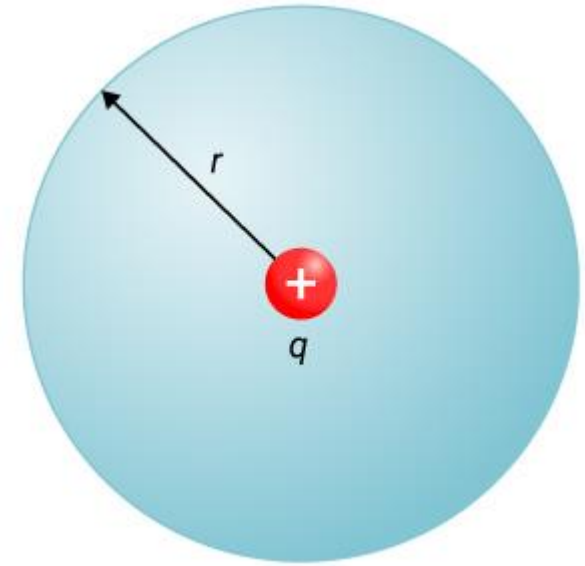
Answer: $\phi = \frac{q_2 - q_3}{\epsilon_0} = \frac{4.6 \times 10^{-6} - 5.3 \times 10^{-6}}{8.85 \times 10^{-12}} = -7.9 \times 10^4 \frac{\text{Nm}^2}{\text{C}}$

Its negative (most of the charge is negative charge) it's a sink

Recap: Gauss' Law

Gauss's law states that the electric flux across any closed surface is equal to the total charge enclosed in this surface, q , divided by the **permittivity of free space**, ϵ_0 .

$$\phi = \oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$



Deriving Gauss' law from coulomb's law:

Gauss's Law and Coulomb's Law

We can derive Gauss's Law from Coulomb's Law. To do this, we start with a positive point charge, q . The electric field due to this charge is radial and pointing outward, as we saw in Section 2.3. According to Coulomb's Law (Section 1.5), the magnitude of the electric field from this charge is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

We now find the electric flux passing through a closed surface resulting from this point charge. For the Gaussian surface, we choose a spherical surface with radius r , with the

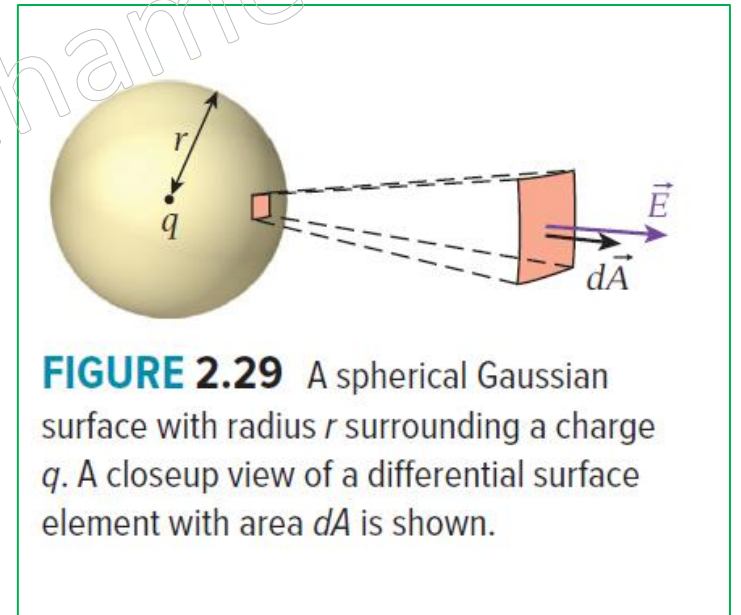
Deriving Gauss' law from coulomb's law:

charge at the center of the sphere, as shown in Figure 2.29. The electric field due to the positive point charge intersects each differential element of the surface of the Gaussian sphere perpendicularly. Therefore, at each point of this Gaussian surface, the electric field vector, $\vec{E}$ and the differential surface area vector, $d\vec{A}$ are parallel. The surface area vector will always point outward from the spherical Gaussian surface, but the electric field vector can point outward or inward depending on the sign of the charge. For a positive charge, the scalar product of the electric field and the surface area element is $\vec{E} \cdot d\vec{A} = E dA \cos 0^\circ = E dA$. The electric flux in this case, according to equation 2.14, is

$$\Phi = \oiint \vec{E} \cdot d\vec{A} = \oiint E dA$$

Because the electric field has the same magnitude anywhere in space at a distance r from the point charge q , we can take E outside the integral:

$$\Phi = \oiint E dA = E \oiint dA$$

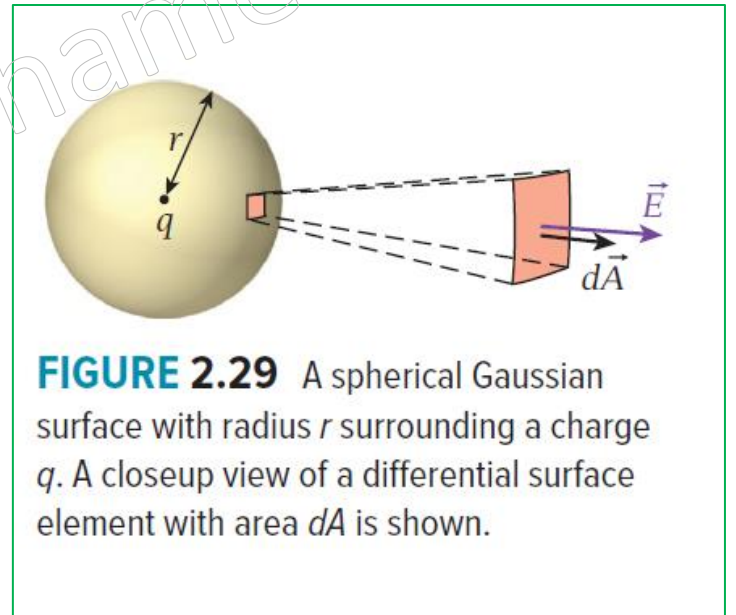


Deriving Gauss' law from coulomb's law:

Now what we have left to evaluate is the integral of the differential area over a spherical surface, which is given by $\oint dA = 4\pi r^2$. Therefore, we have found from Coulomb's Law for the case of a point charge

$$\Phi = (E) \left(\oint dA \right) = \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right) (4\pi r^2) = \frac{q}{\epsilon_0}$$

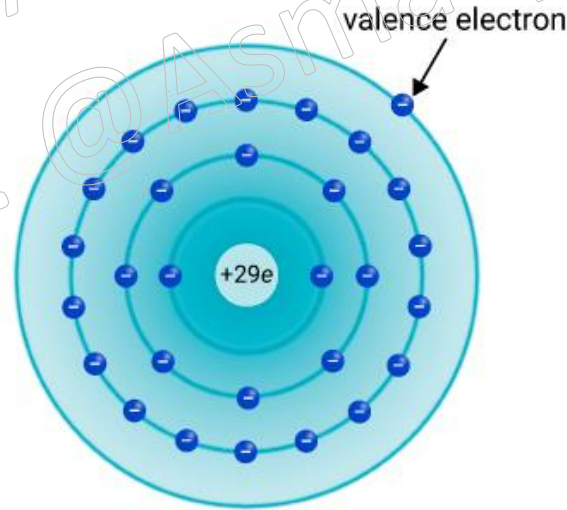
which is the same as the expression for Gauss's Law in equation 2.15. We have shown that Gauss's Law can be derived from Coulomb's Law for a positive point charge, but it can also be shown that Gauss's Law holds for any distribution of charge inside a closed surface.



Conductors:

Good conductors contain a big number of **free charges** that are able to move within the material because they are not bound to any atom.

A copper atom has 29 protons and 35 neutrons in its nucleus, and **29 electrons** orbiting around.

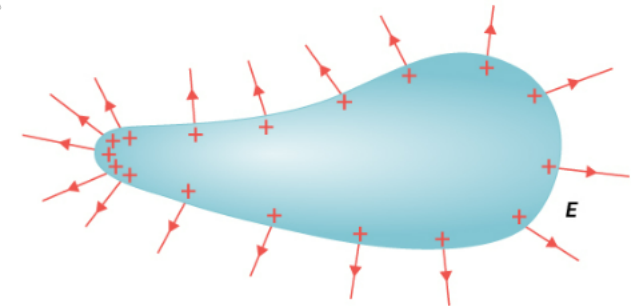


When the charges inside a conductor are not moving, the conductor is said to be in **static equilibrium**.

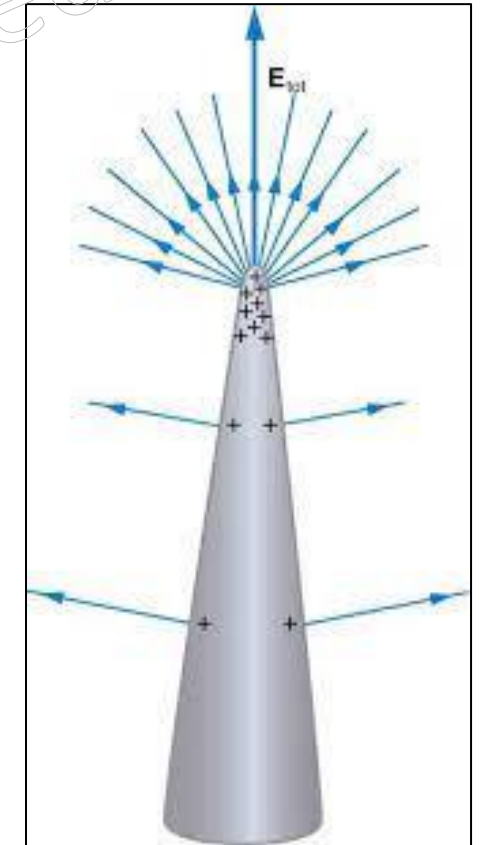
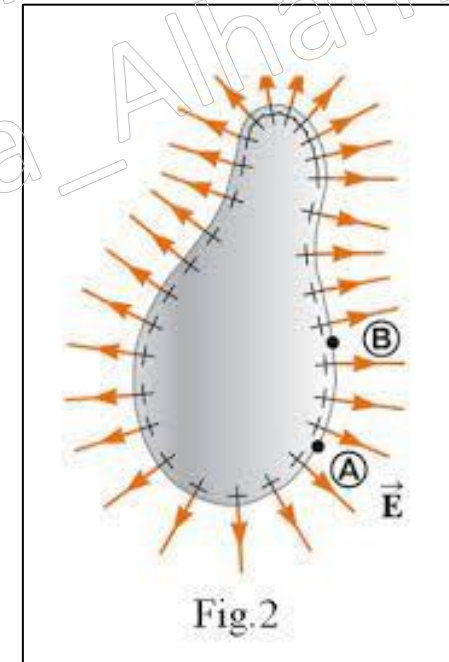
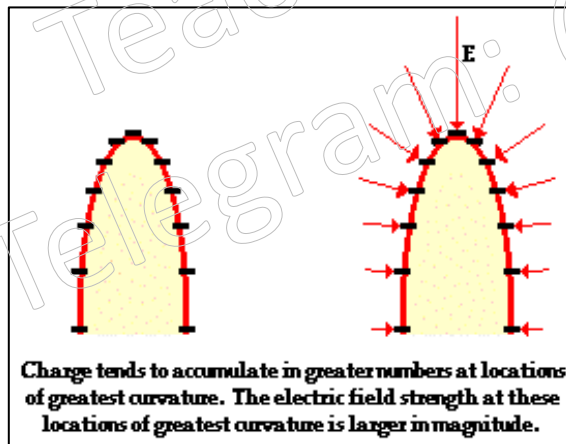
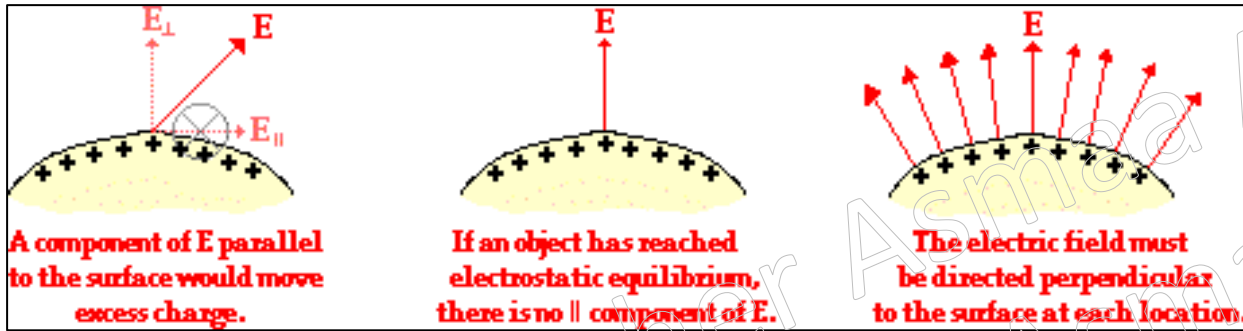
1- Conductors:

Static equilibrium of conductors

- Charges will reside on the surface of a conductor.
- The electric field at the surface of any conductor has a magnitude of $\frac{\sigma}{\epsilon_0}$.
- The surface charge density is greatest at **sharp points** in a conductor with an irregular shape.



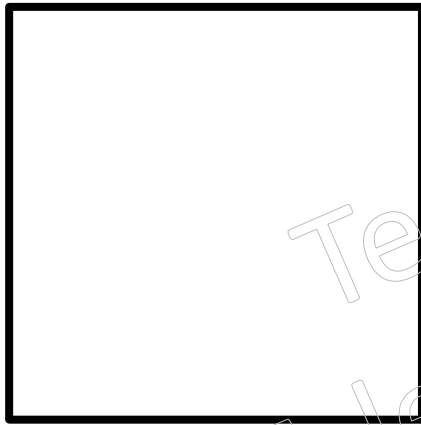
Examples of electrostatic equilibrium:



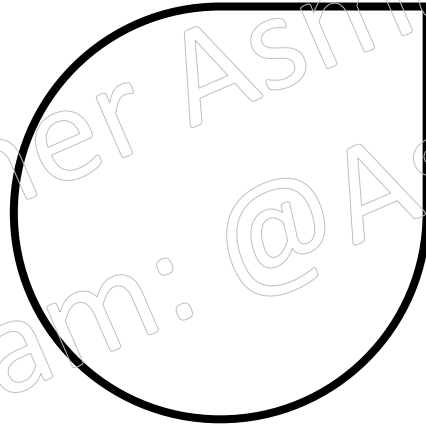
Activity:

02:00

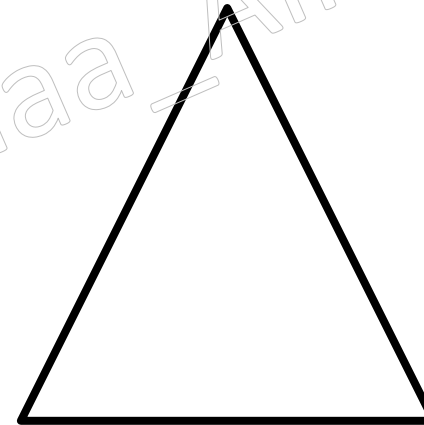
Identify the direction of the field and the concentration of charge:



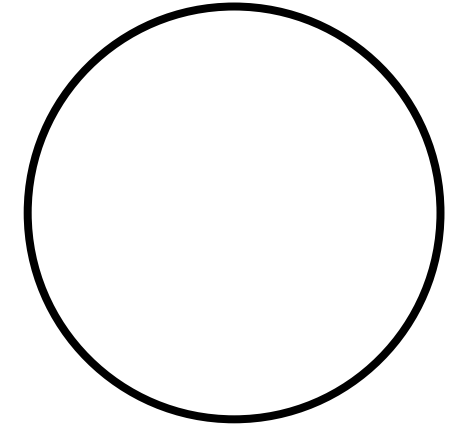
Positive charge



Negative charge



Negative charge

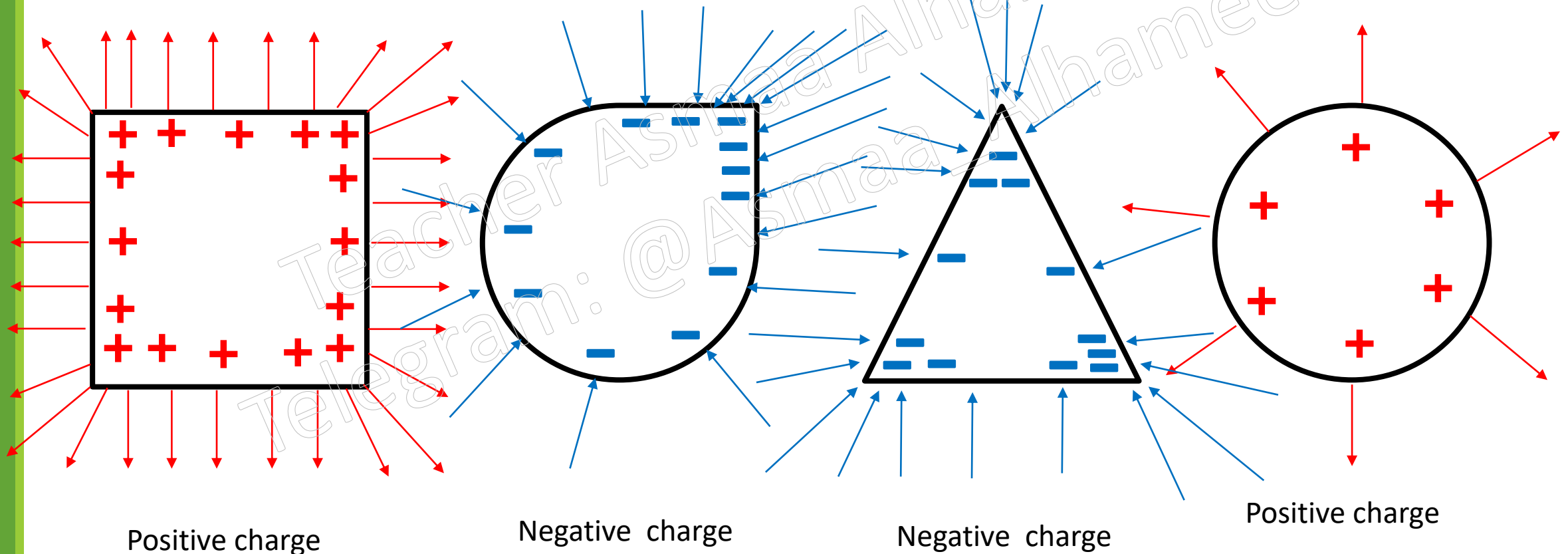


Positive charge

Activity:

02:00

Identify the direction of the field and the concentration of charge:



1- Conductors

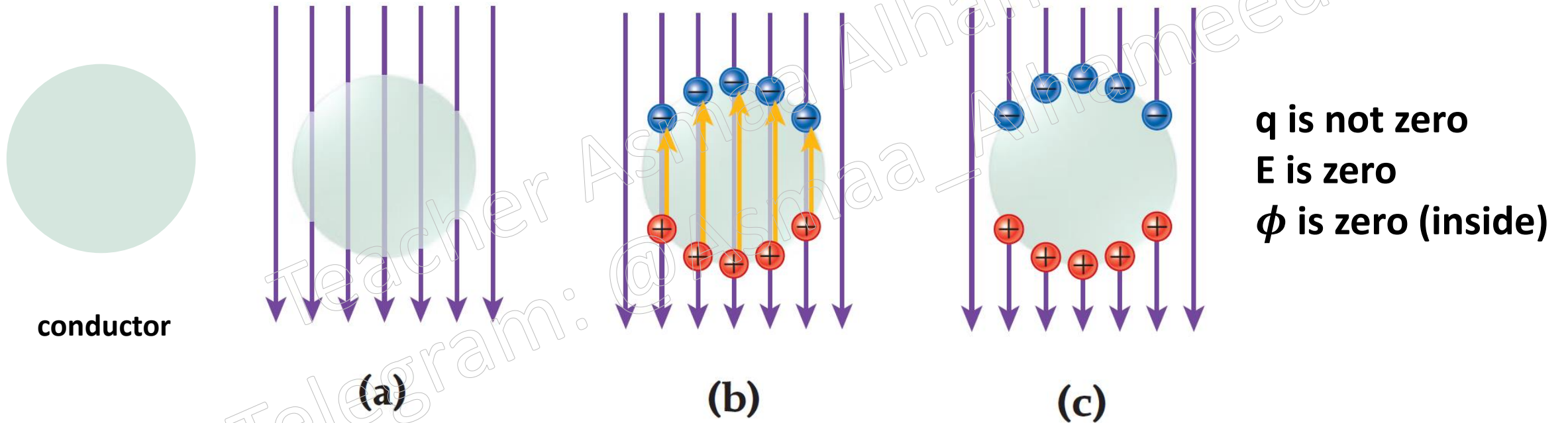
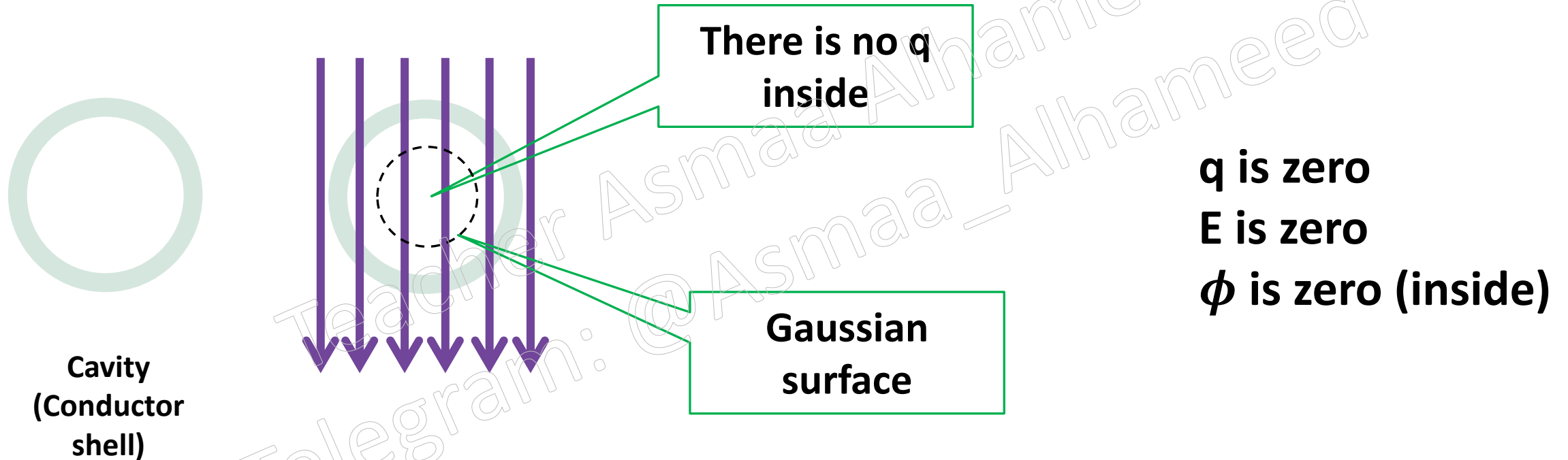


FIGURE 2.30 Shielding of an external electric field (purple vertical arrows) from the inside of a conductor.

Read the book
page 45

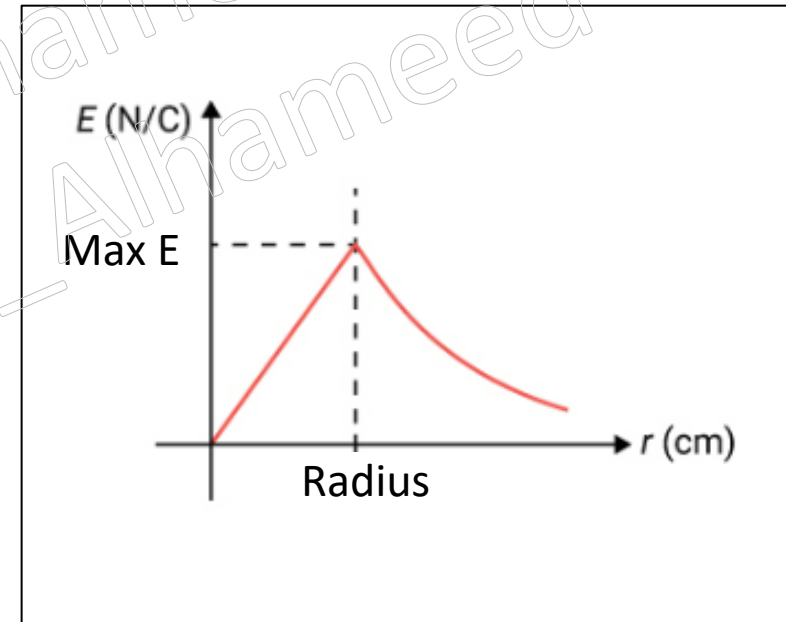
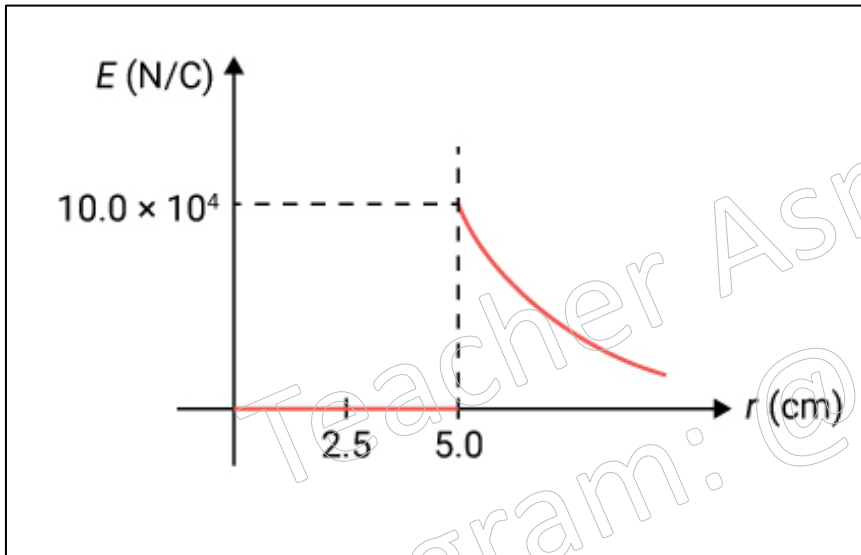
2- Cavities (Conductors shell)



Every point inside a conductor is shielded from an external electric field. This phenomena is known as **electrostatic shielding**.

**Read the book
page 45**

Conductor vs shell



Electric Field in a Conductor

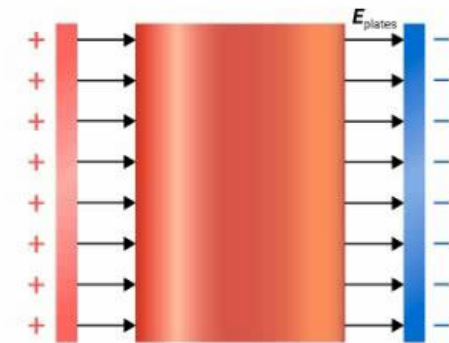
A copper block is placed between two oppositely charged parallel plates, as shown on the right.

Copper is a good conductor since it has one free valence electron.

The electric field in the copper block is zero. Watch the slideshow below to see why.



Two oppositely charged parallel plates create a uniform electric field from the positive plate to the negative plate.



Applications of shielding: (page:46)

The conductor surrounding the cavity does not have to be a solid piece of metal; even a wire mesh is sufficient to provide shielding. This can be demonstrated most impressively by seating a person inside a cage and then hitting the cage with a lightning-like electrical discharge (Figure 2.32). The person inside the cage is unhurt, even if he or she touches the metal of the cage *from the inside*. (It is important to realize that severe injuries can result if any body parts stick out of the cage, for example, if hands are wrapped around the bars of the cage!) This cage is called a *Faraday cage*, after British physicist Michael Faraday (1791–1867), who invented it.

A Faraday cage has important consequences, probably the most relevant of which is the fact that your car protects you from being hit by lightning while inside it—unless you drive a convertible. The sheet metal and steel frame that surround the passenger compartment provide the necessary shielding. (But as fiberglass, plastic, and carbon fiber begin to replace sheet metal in auto bodies, this shielding is not assured any more.)

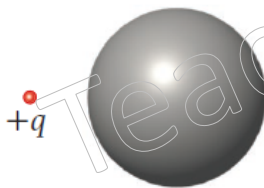
Activity:

02:00

Concept Check 2.10

A hollow, conducting sphere is initially given an evenly distributed negative charge. A positive charge $+q$ is brought near the sphere and placed at rest as shown in the figure. What is the direction of the electric field inside the hollow sphere?

- a) 
- b) 
- c) 
- d) 

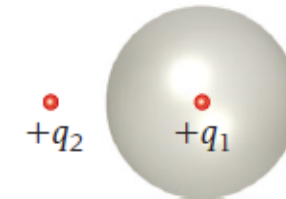


e) The field is zero.

Concept Check 2.11

A hollow, conducting sphere is initially uncharged. A positive charge, $+q_1$, is placed inside the sphere, as shown in the figure. Then, a second positive charge, $+q_2$, is placed near the sphere but outside it. Which of the following statements describes the net electric force on each charge?

- a) There is a net electric force on $+q_2$ but not on $+q_1$.
- b) There is a net electric force on $+q_1$ but not on $+q_2$.
- c) Both charges are acted on by a net electric force with the same magnitude and in the same direction.
- d) Both charges are acted on by a net electric force with the same magnitude but in opposite directions.



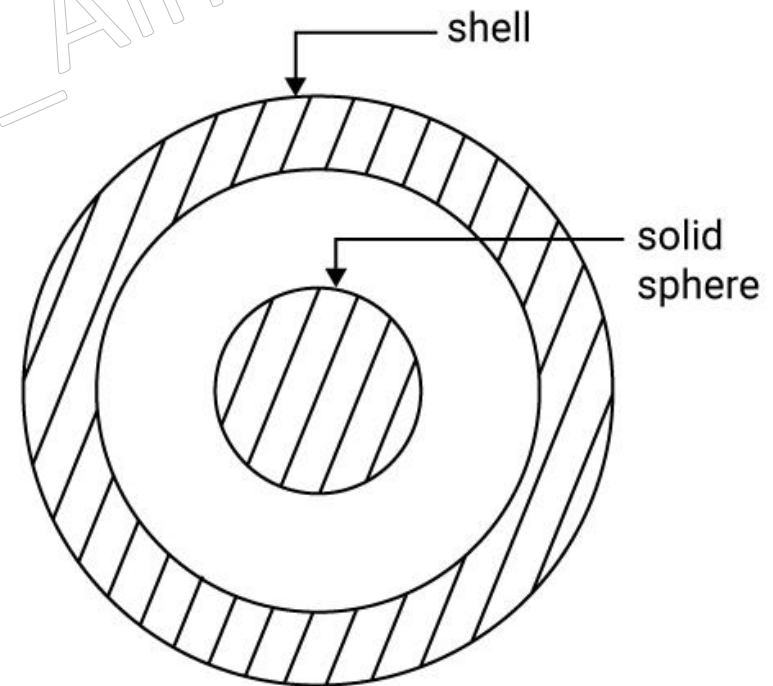
e) There is no net electric force on either charge.

Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$

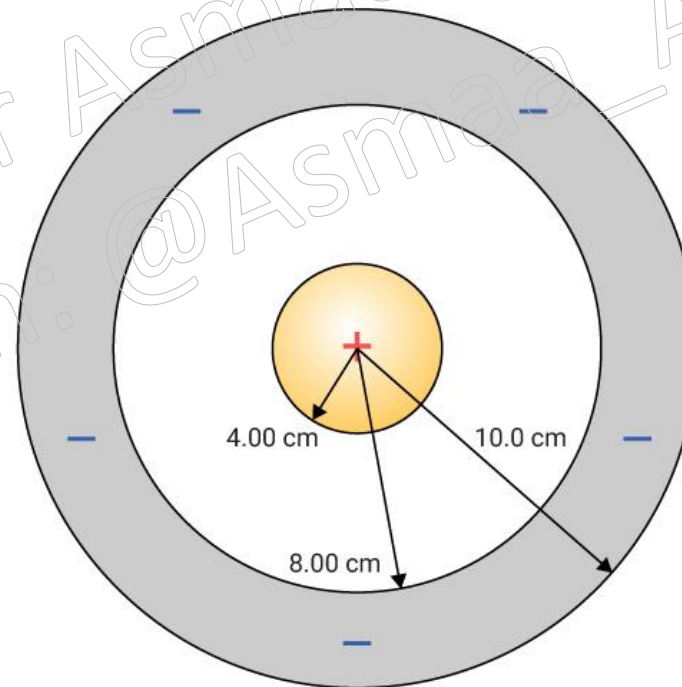
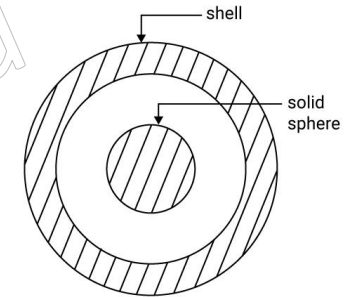


Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$



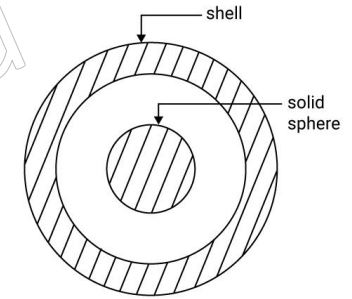
sketch

Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$



Draw a gaussian surface for every distance, r .

The electric field can be calculated as:

Research

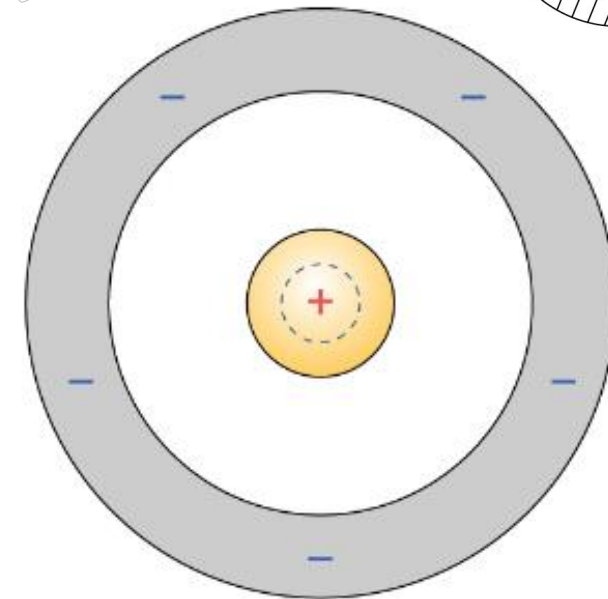
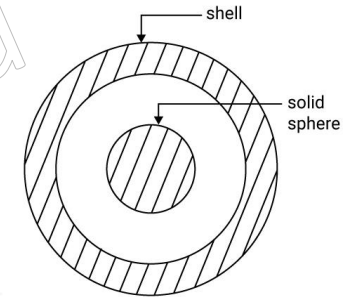
$$E = k \frac{q}{r^2}$$

Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$



Calculate
(a)

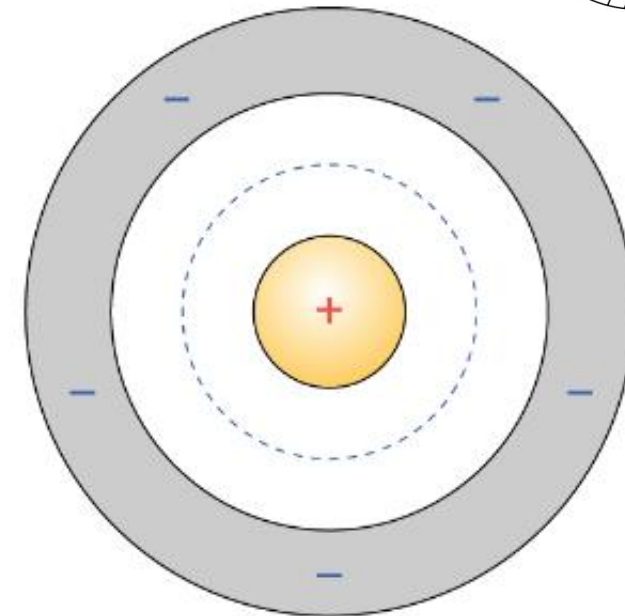
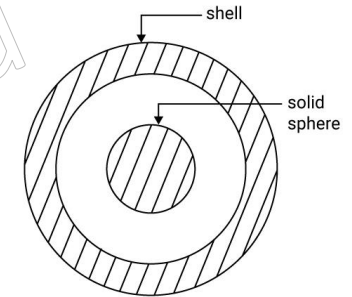
The gaussian surface is inside the solid sphere, the electric field is **zero**.

Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$



The magnitude of the electric field is flowing radially outward from the conducting sphere to the spherical shell.

$$E = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \times \frac{16.0 \times 10^{-6} \text{ C}}{(0.06 \text{ m})^2}$$
$$E = 4.00 \times 10^7 \text{ N/C}$$

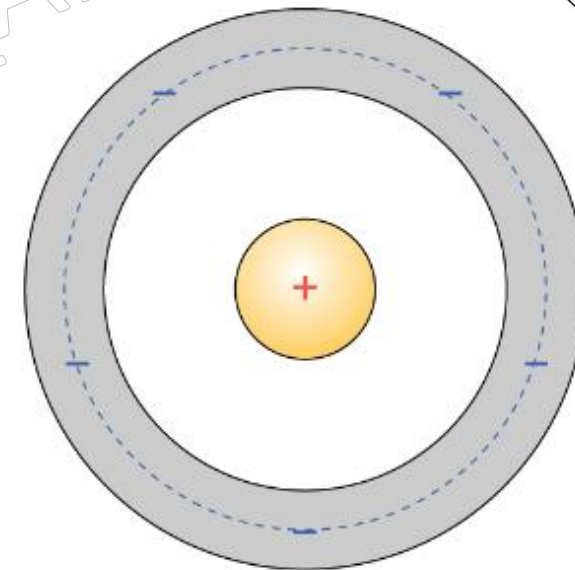
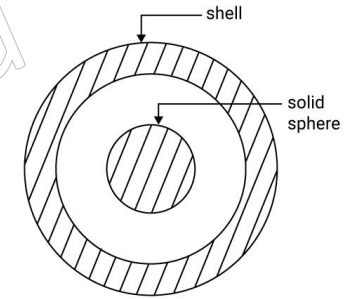
Calculate
(b)

Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$



Calculate
(c)

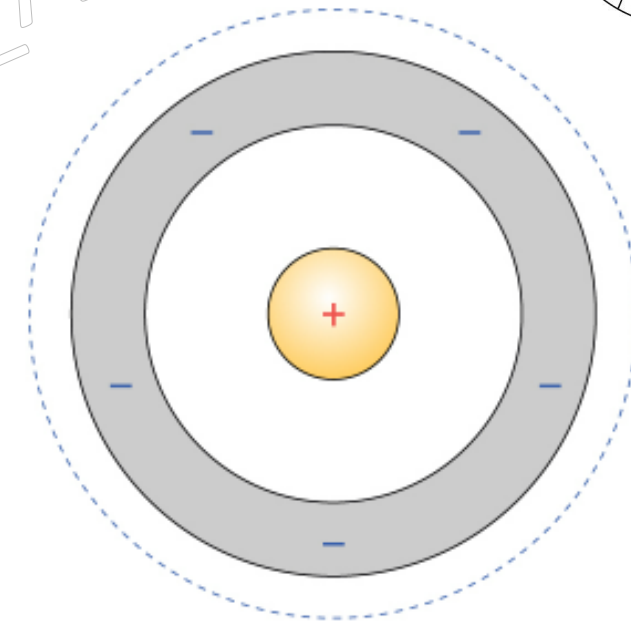
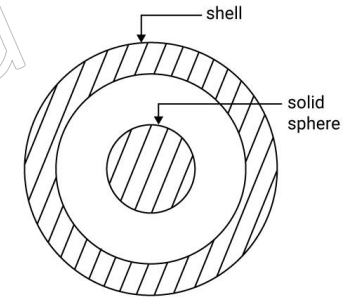
The electric field inside the spherical shell is **zero**.

Activity:

02:00

A solid conducting sphere of radius 4.00 cm has a charge of $16.0 \mu\text{C}$. A conducting spherical shell surrounds the sphere, in which the shell has a 8.00 cm inner radius, and 10.0 cm outer radius. The spherical shell has a charge of $-8.00 \mu\text{C}$. What is the electric field vector at:

- a. $r = 2.00 \text{ cm}$
- b. $r = 6.00 \text{ cm}$
- c. $r = 9.00 \text{ cm}$
- d. $r = 14.0 \text{ cm}$



The electric field is flowing radially outward, since the total charge enclosed at $r = 14.0 \text{ cm}$ is:

$$q_{\text{net}} = -8.00 \mu\text{C} + 16.0 \mu\text{C}$$

$$q_{\text{net}} = +8.00 \mu\text{C}$$

The magnitude of the electric field is:

$$E = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \times \frac{+8.00 \times 10^{-6} \text{ C}}{(0.14 \text{ m})^2}$$

$$E = 3.67 \times 10^6 \text{ N/C}$$

**Calculate
(d)**