مدرسة باحثة البادية – الحلقة الثالثة بنات
Bahethat AL-Badeyah Girl's School – Cycle 3

نموذج اختبار

وفقاً لهيكلة اختبار

نهاية الفصل الدراسي الأول

2025-2026

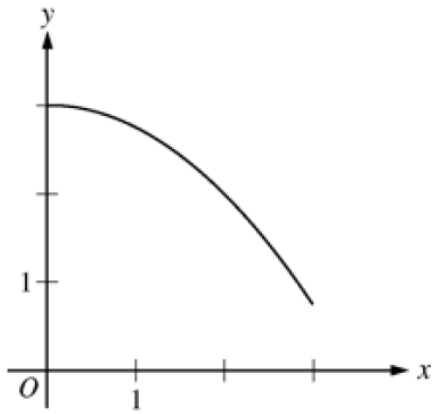
إعداد المعلم	احمد راتب ظاهر
المادة الدراسية	MATHEMATICS
الصف	12 ELITE

مديرة المدرسة .. هيام محمد عامر الحمادي

إعداد ومتابعة لجنة جودة الشؤون الأكاديمية

مبادرة .. أعلم و اتعلم

1.



Graph of f

The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

2.

Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}dt$. Which of the following must be true on the interval $0 < x < 2$?

(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

3. Which of the following is equal to $\int_0^{\pi} \sin x \, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$

(B) $\int_0^{\pi} \cos x \, dx$

(C) $\int_{-\pi}^{0} \sin x \, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$

(E) $\int_{\pi}^{2\pi} \sin x \, dx$

4. Let f and g be continuous functions such that $\int_0^6 f(x)dx = 9$, $\int_3^6 f(x)dx = 5$, and $\int_3^0 g(x)dx = -7$. What is the value of $\int_0^3 \left(\frac{1}{2}f(x) - 3g(x)\right) dx$?

(A) -23

(B) -19

(C) $-\frac{17}{2}$

(D) 19

(E) 23

5.

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

The table above gives values of a function f and its derivative at selected values of x . If f' is continuous on the interval $[-4, -1]$ what is the value of $\int_{-4}^{-1} f'(x)dx$?

(A) -4.5

(B) -2.25

(C) 0

(D) 2.25

(E) 4.5

6. $\int \frac{1}{3x+12}dx =$

(A) $-3\ln|x+4|+C$

(B) $\frac{1}{3}\ln|x+4|+C$

(C) $\ln|x+4|+C$

(D) $3\ln|x+4|+C$

7. $\int \left(x^3+2\right)^2 dx =$

(A) $\frac{\left(x^3+2\right)^3}{9x^2}+C$

(B) $\frac{1}{3}\left(x^3+2\right)^3+C$

(C) $6x^2\left(x^3+2\right)+C$

(D) $\frac{1}{7}x^7+x^4+4x+C$

8. $\int \frac{3}{x(x+1)}dx =$

(A) $3\tan^{-1}x+C$

(B) $3\ln\left|x^2+x\right|+C$

(C) $3\ln\left|\frac{x}{x+1}\right|+C$

(D) $\frac{3}{2x+1}\ln\left|x^2+x\right|+C$

(E) $3\ln\left|\frac{x+1}{x}\right|+C$

9. Students are asked to memorize a list of 100 words. The students are given periodic quizzes to see how many words they have memorized. The function L gives the number of words memorized at time t. The rate of change of the number of words memorized is proportional to the number of words left to be memorized. Which of the following differential equations could be used to model this situation, where k is a positive constant?

(A) $\frac{dL}{dt}=kL$

(B) $\frac{dL}{dt}=100-kL$

(C) $\frac{dL}{dt}=k(100-L)$

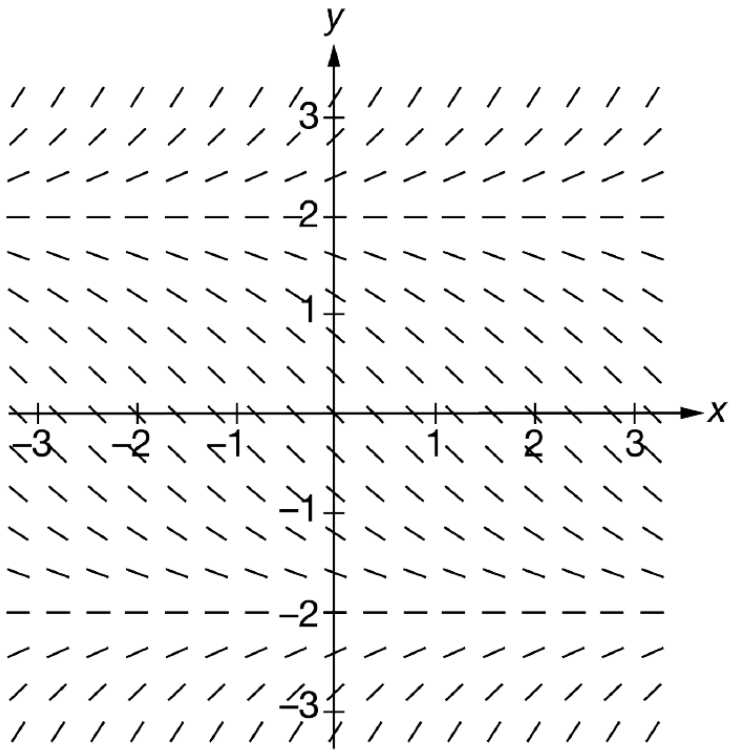
(D) $\frac{dL}{dt}=kL-100$

10. Of the following, which are solutions to the differential equation $y'' - 5y' + 6y = 0$?

I. $y = e^{2t}$
II. $y = 3e^{3t}$
III. $y = 3\sin(2t)$

(A) I only
(B) II only
(C) III only
(D) I and II only

11.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{y-2}{2}$
(B) $\frac{dy}{dx} = \frac{y^2-4}{4}$
(C) $\frac{dy}{dx} = \frac{x-2}{2}$
(D) $\frac{dy}{dx} = \frac{x^2-4}{4}$

12. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(2) = 8$. What is the approximation for $f(3)$ obtained by using Euler's method with two steps of equal length, starting at $x = 2$?

(A) 2
(B) $\frac{5}{2}$
(C) $\frac{15}{4}$
(D) $\frac{61}{4}$

13. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{2xy}{x^2+1}$ whose graph contains the points (0,1)?

(A) $y = e^{x^2}$
(B) $y = x^2 + 1$
(C) $y = \ln(x^2+1)$
(D) $y = 1 + \ln(x^2+1)$
(E) $y = \sqrt{1 + 2\ln(x^2+1)}$

14. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\int_{-2}^{4} \frac{f(x)}{8}\,dx$?

(A) $\frac{3}{2}$
(B) 3
(C) 9
(D) 72

15. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2 \sin t$ for $0 \le t \le 2$. What is the total distance traveled by the particle between t = 0 and t = 2?

(A) -3.637
(B) -1.973
(C) 1.531
(D) 2.539
(E) 3.637

16. The furnace in a home consumes heating oil during a particular month at a rate modeled by the function F given by $F(t) = 0.3 + 0.1t - 0.85\cos\left(\frac{2\pi}{15}(t+5)\right)$, where $F(t)$ is measured in gallons per day and t is the number of days since the start of the month. How many gallons of oil does the furnace consume during the first 14 days of the month (from $t = 0$ to $t = 14$) ?

(A) 10.150
(B) 13.739
(C) 17.597
(D) 25.044

17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

(A) $\frac{7}{3}$
(B) $\frac{16}{3}$
(C) $\frac{20}{3}$
(D) 9
(E) 36

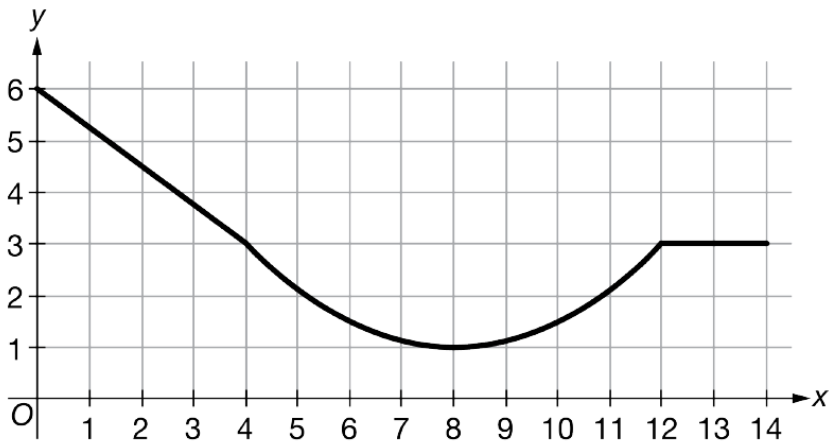
18. Let R be the region bounded by the graph of $x = e^y$, the vertical line $x = 10$, and the horizontal lines $y = 1$ and $y = 2$. Which of the following gives the area of R ?

(A) $\int_1^2 e^y dy$
(B) $\int_e^{e^2} \ln x dx$
(C) $\int_1^2 (10 - e^y) dy$
(D) $\int_e^{10} (\ln x - 1) dx$

19. Let R be the region in the first quadrant bounded by the graph of $y = \tan x$, the x-axis, and the vertical line $x = 1$. Which of the following gives the volume of the solid generated when region R is revolved about the vertical line $x = 1$?

(A) $\pi \int_0^{\tan 1} (1 - \arctan y)^2 dy$
(B) $\pi \int_0^1 (1 - \arctan y)^2 dy$
(C) $\pi \int_0^{\tan 1} (1 - \arctan y) dy$
(D) $\pi \int_0^1 \left(1^2 - (\arctan y)^2\right) dy$

20.



Graph of f

$$f(x) = \begin{cases} 6 - \frac{3}{4}x & \text{for } 0 \le x < 4 \\ 1 + \frac{1}{8}(x-8)^2 & \text{for } 4 \le x \le 12 \\ 3 & \text{for } 12 < x \le 14 \end{cases}$$

A skateboard track consists of a straight ramp followed by a curved section and a horizontal ledge. The track is modeled by the piecewise-defined function f above, and the graph of f is shown in the figure above. Which of the following expressions gives the total length of the track from $x = 0$ to $x = 14$?

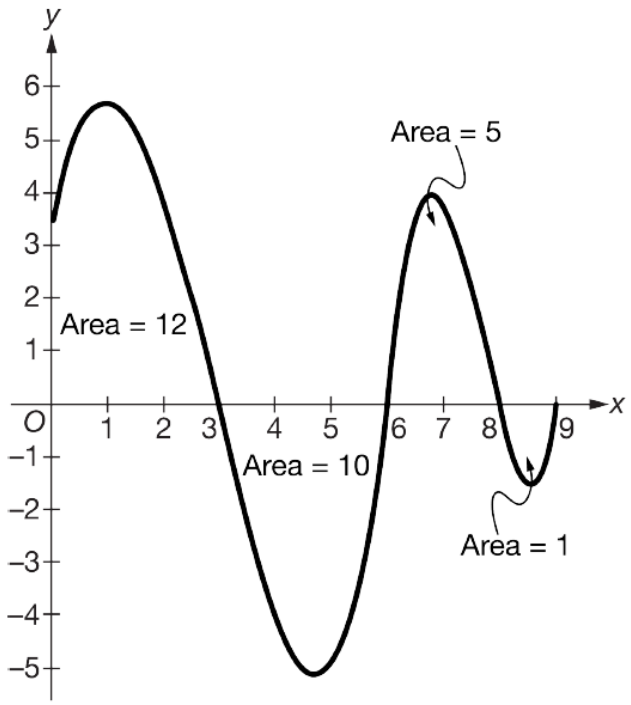
(A) $2 + \int_0^{12} \sqrt{1 + \left(-\frac{3}{4} + \frac{1}{4}(x-8)\right)^2}\, dx$

(B) $2 + \int_0^{12} \left(\sqrt{1 + \left(-\frac{3}{4}\right)^2} + \sqrt{1 + \frac{1}{16}(x-8)^2} \right) dx$

(C) $7 + \int_4^{12} \sqrt{1 + \left(1 + \frac{1}{8}(x-8)^2\right)^2}\, dx$

(D) $7 + \int_4^{12} \sqrt{1 + \frac{1}{16}(x-8)^2 dx}$

1.



Graph of f'

The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 9$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(6) = 7$.

Let g be the function defined by $g(x) = x^2 - 1$.

(a) Find the value of $\int_0^9 f'(x)\,dx$.

(b) Given that $f(6) = 7$, write an expression for $f(x)$ that involves an integral. Use this expression to find the absolute minimum value of f and the absolute maximum value of f on the closed interval $0 \le x \le 9$. Justify your answers.

(c) Find $\int g(x)\,dx$.

(d) Find the value of $\int_2^3 x\, f'(g(x))\,dx$.

2. Let f be the function defined by $f(x) = \frac{4}{x^2}$.

(a) Approximate the value of $\int_1^{12} f(x)dx$ using a left Riemann sum with the subintervals $[1, 4]$, $[4, 8]$, and $[8, 12]$.

(b) Find the value of $\int_1^{12} f(x)dx$. Show the work that leads to your answer.

(c) Find the value of $\int_1^{\infty} f(x)dx$ or explain why it does not exist.

(d) Find the value of $\int_1^{12} f(x) \cdot \ln x dx$. Show the work that leads to your answer.

3. A large vat is initially filled with a saltwater solution. A solution with a higher concentration of salt flows into the vat, and solution flows out of the vat at the same rate. The number of pounds of salt in the vat at time t minutes is modeled by the function A that satisfies the differential equation $\frac{dA}{dt} = 6 - 0.02A$. At time $t = 10$ minutes, the vat contains 50 pounds of salt.

(a) Write an equation for the line tangent to the graph of A at $t = 10$. Use the tangent line to approximate the number of pounds of salt in the vat at time $t = 12$ minutes.

(b) Show that $A(t) = 300 - 250e^{0.2-0.02t}$ satisfies the differential equation $\frac{dA}{dt} = 6 - 0.02A$ with initial condition $A(10) = 50$.

(c) The flow of solution into the vat is stopped, and the solution is drained. The depth of solution in the vat is modeled by the function h that satisfies the differential equation $\frac{dh}{dt} = -k\sqrt{h}$, where $h(t)$ is measured in meters, t is the number of minutes since draining began, and k is a constant. If the depth of the solution is 16 meters at time $t = 0$ minutes and 4 meters at time $t = 30$ minutes, what is $h(t)$ in terms of t ?

4.

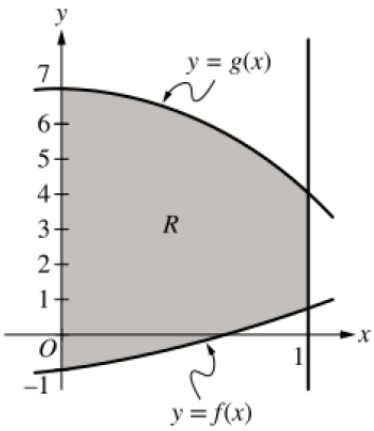


Figure 1

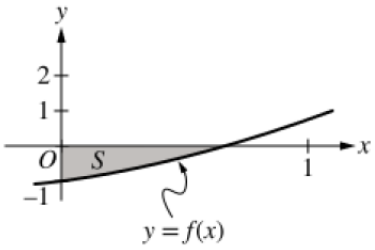


Figure 2

Let f and g be the functions given by $f(x) = e^x - 2$ and $g(x) = -3x^2 + 7$. Let R be the shaded region bounded by the graphs of f and g, the y-axis, and the vertical line $x = 1$, as shown in Figure 1. Let S be the shaded region in Quadrant IV bounded by the graph of f, the x-axis, and the y-axis, as shown in Figure 2.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 8$.

(c) The region S is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a rectangle with height 5 and base in region S. Write, but do not evaluate, an integral expression that gives the volume of this solid.

5. Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.