

Math lessons Simplification

اللهم إني أسألك فهمَ النَّبِيِّينَ وَحِفْظَ الْمُرْسَلِينَ، وَالْمَلَائِكَةَ الْمُقَرَّبِينَ، اللهم
اجعل لِسَانِي عَامِرًا بِذِكْرِكَ، وَقَلْبِي بِحُشْيَتِكَ، وَسِرِّي بِطَاعَتِكَ، إِنَّكَ عَلَى كُلِّ
شَيْءٍ قَدِيرٌ، وَحَسْبِيَ اللَّهُ وَنِعْمَ الْوَكِيلُ

Some Rules from Term 1 You Should Know::

Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

Function	
Polynomials	Continuous Everywhere
$\sin x, \cos x, \tan^{-1} x$	Continuous Everywhere
$\sqrt[n]{x}$	Continuous Everywhere when n is <u>odd</u>
$\sqrt[n]{x}$	Continuous for $x > 0$ when n is <u>even</u>
$\ln x$	Continuous for $x > 0$
$\sin^{-1} x, \cos^{-1} x$	Continuous for $-1 < x < 1$

Chpt 4: Applications of differentiation

Lesson 4.3: Maximum and Minimum Values

DEFINITION 3.3

A number c in the domain of a function f is called a **critical number** of f if $f'(c) = 0$ or $f'(c)$ is undefined.

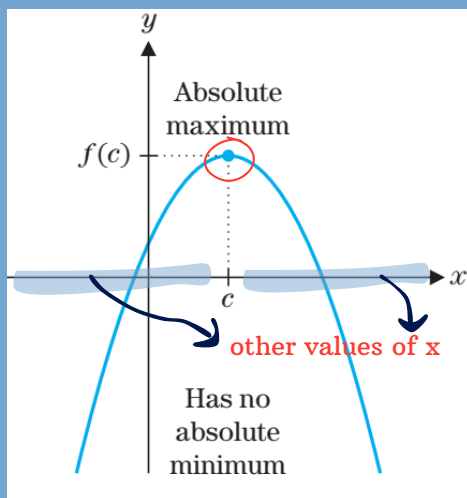
DEFINITION 3.1

For a function f defined on a set S of real numbers and a number $c \in S$,

- (i) $f(c)$ is the **absolute maximum** of f on S if $f(c) \geq f(x)$ for all $x \in S$ and
- (ii) $f(c)$ is the **absolute minimum** of f on S if $f(c) \leq f(x)$ for all $x \in S$.

An absolute maximum or an absolute minimum is referred to as an **absolute extremum**. (The plural form of extremum is **extrema**.)

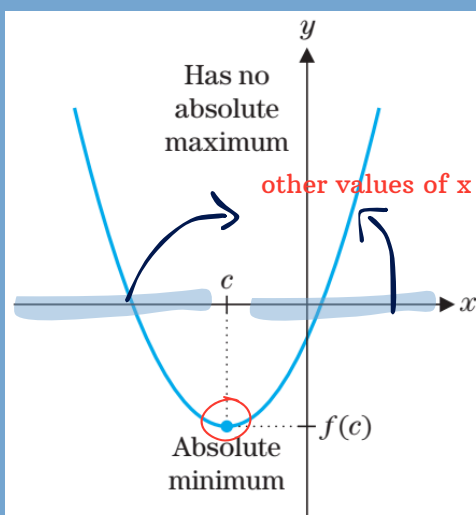
Absolute maximum and absolute minimum are also called absolute extremum



- (i) $f(c)$ is the **absolute maximum** of f on S if $f(c) \geq f(x)$ for all $x \in S$ and

this means $x=c$ will consider an absolute maximum if we substitute the value of c in the original function and then we get a number **greater than or equal** to the other values of x only.

$$f(c) \geq f(x) \text{ ---> absolute max}$$



- (ii) $f(c)$ is the **absolute minimum** of f on S if $f(c) \leq f(x)$ for all $x \in S$.

this means $x=c$ will consider an absolute minimum if we substitute the value of c in the original function and then we get a number **less than or equal** to the other values of x only. $f(c) \leq f(x) \text{ ---> absolute min.}$

THEOREM 3.1 (Extreme Value Theorem)

A continuous function f defined on a *closed, bounded* interval $[a, b]$ attains both an absolute maximum and an absolute minimum on that interval.

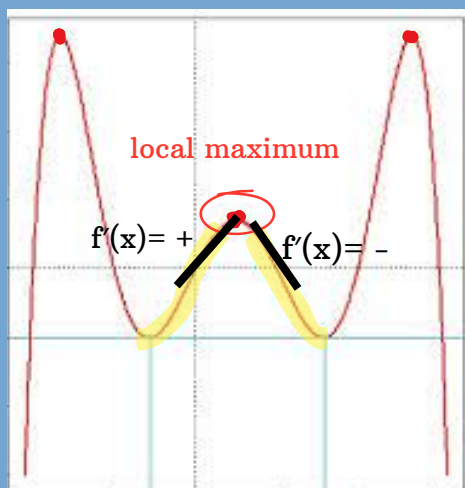
*here it means if we have a continuous function and it is defined on a closed, bounded interval $[a, b]$ it must have both absolute max. and min.

**If we have an absolute max. or min. at $x=c$, then c must be a critical number

DEFINITION 3.2

- (i) $f(c)$ is a **local maximum** of f if $f(c) \geq f(x)$ for all x in some *open* interval containing c .
- (ii) $f(c)$ is a **local minimum** of f if $f(c) \leq f(x)$ for all x in some *open* interval containing c .

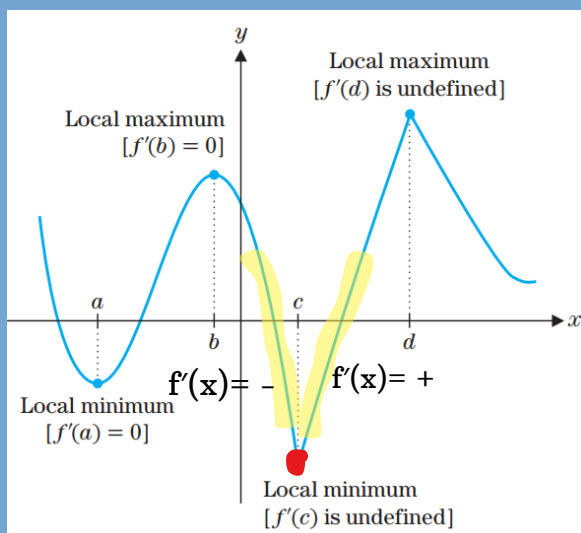
In either case, we call $f(c)$ a **local extremum** of f .



There are points higher than this point, but this point is higher than the values on both the right and left sides, so it is considered a LOCAL MAXIMUM.

**All absolute maximum is considered a local maximum BUT, local maximum is not always an absolute maximum.

same for local minimum>>



There are points lower than this point, but this point is lower than the values on both the right and left sides, so it is considered a LOCAL MINIMUM.

In summary, local max. or min. doesn't mean it has to be the highest or lowest point, but the point should be higher or lower than the values on both the right and left sides.

LET'S APPLY WHAT WE'VE LEARNED

Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[-2, 4]$.

step 1: Determine the domain of the function.

since it's a polynomial function in this qn the domain will be $(-\infty, \infty)$

step 2: Find the derivative of the original function.

$$f'(x) = 6x^2 - 6x - 12$$

step 3: equate the resulting function to 0 and find the value of x.

$$f'(x) = 0 \rightarrow 6x^2 - 6x - 12 = 0$$
$$x = 2, x = -1$$

Now see.. are the values of x within the interval or not, and then cancel the numbers if they are outside the interval.

step 4: Find both absolute max. and absolute min.

*substitute the given interval in the qn and both critical numbers we found in the original function.. $[-2, 4]$ $x = 2, x = -1$

$$f(-2) = 2x^3 - 3x^2 - 12x + 5 = 1$$

$$f(4) = 2x^3 - 3x^2 - 12x + 5 = 37 \rightarrow \text{Absolute maximum}$$

$$f(2) = 2x^3 - 3x^2 - 12x + 5 = -15 \rightarrow \text{Absolute minimum}$$

$$f(-1) = 2x^3 - 3x^2 - 12x + 5 = 12$$

Summary of the steps::

step 1: Determine the domain of the function.

step 2: Find the derivative of the original function.

step 3: equate the resulting function to 0 and find the value of x. (here x is considered a critical number (c)).

**DON'T forget to check if the resulting values of x are within the interval or not. if not cancel them.

step 4: Find both absolute max. and absolute min.

READ THE STEPS CAREFULLY AND ATTENTIVELY. ONCE YOU ARE SURE YOU UNDERSTAND, GO TO THE NEXT SLIDE TO SOLVE ANOTHER QUESTION ON YOUR OWN.

By: Fa*

CHECK YOUR UNDERSTANDING

Find the absolute extrema of $f(x) = 4x^{5/4} - 8x^{1/4}$ on the interval $[0, 4]$.

step 1: Determine the domain of the function.

$f(x) = 4\sqrt[4]{x^5} - 8\sqrt[4]{x}$ → for any even root the domain must be only positive numbers, $[0, \infty)$

step 2: Find the derivative of the original function.

$$f'(x) = 5x^{1/4} - 2x^{-3/4} = \frac{x^{3/4} \cdot 5x^{1/4}}{x^{3/4}} - \frac{2}{x^{3/4}} \rightarrow = \frac{5x^{1/4+3/4} - 2}{x^{3/4}} = \frac{5x - 2}{x^{3/4}}$$

Multiply the numerator and denominator to unify the denominators

step 3: equate the resulting function to 0 and find the value of x.

$$\frac{5x - 2}{x^{3/4}} = 0 \quad \begin{matrix} 5x - 2 = 0 \\ x = 0.4 \end{matrix}$$

Now see.. is the values of x are within the interval or not, and then cancel the numbers if they are outside the interval.

step 4: Find both absolute max. and absolute min.

*substitute the given interval in the fn and the critical numbers we found in the original function..

$$[0, 4]$$

$$x = 0.4$$

$$f(0) = 4x^{5/4} - 8x^{1/4} = 0$$

$$f(4) = 4x^{5/4} - 8x^{1/4} = 11.3 \rightarrow \text{Absolute maximum}$$

$$f(0.4) = 4x^{5/4} - 8x^{1/4} = -5.1 \rightarrow \text{Absolute minimum}$$

LAST QN

Find the absolute extrema (absolute minimum and absolute maximum) of $f(x) = x^3 - 3x + 1$ on the interval $[0, 2]$.

step 1: Determine the domain of the function.

since it's a polynomial function in this qn the domain will be $(-\infty, \infty)$

step 2: Find the derivative of the original function.

$$f'(x) = 3x^2 - 3$$

step 3: equate the resulting function to 0 and find the value of x.

$$f'(x) = 0 \implies 3x^2 - 3 = 0$$

$$x=1, x=-1$$

Now see.. are the values of x are within the interval or not, and then cancel the numbers if they are outside the interval.

we will cancel $x=-1$ because it's outside the given interval in the qn, which is $[0, 2]$

step 4: Find both absolute max. and absolute min.

*substitute the given interval in the qn and the critical number we found in the original function..

$$[0, 2]$$

$$x=1$$

$$f(0) = x^3 - 3x + 1 = 1$$

$$f(2) = x^3 - 3x + 1 = 3 \implies \text{Absolute maximum}$$

$$f(1) = x^3 - 3x + 1 = -1 \implies \text{Absolute minimum}$$

Done with lesson 4.3..
Hope you understand!

اللهم إني أَسْتودِعُكَ ما قرأتُ وما حفظتُ وما تعلمتُ، فردّه إلَيَّ عند حاجتي
إليه، إنك على كل شيء قدير.
اللهم ذكرني منه ما نسيْتُ، وعلمني منه ما جهلتُ