



وزارة التربية والتعليم
MINISTRY OF EDUCATION



صف الثاني عشر المتقدم الفصل الأول لعام 2025-2026

احجز مكانك واستعد للامتحان بثقة كاملة

احصل على الشرح الكامل للصف من خلال:

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اضغط على الرقم: 0566991363



Mr. Aghead

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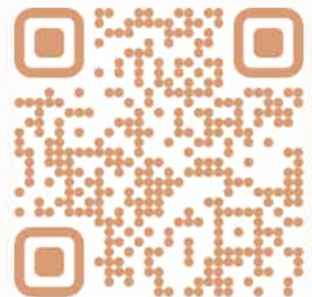


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شرح الدروس



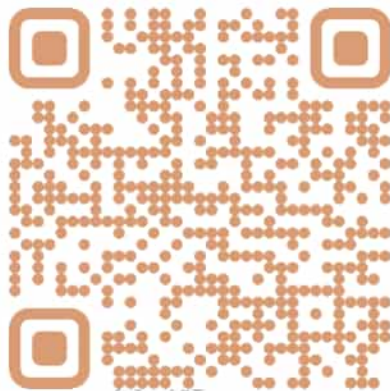
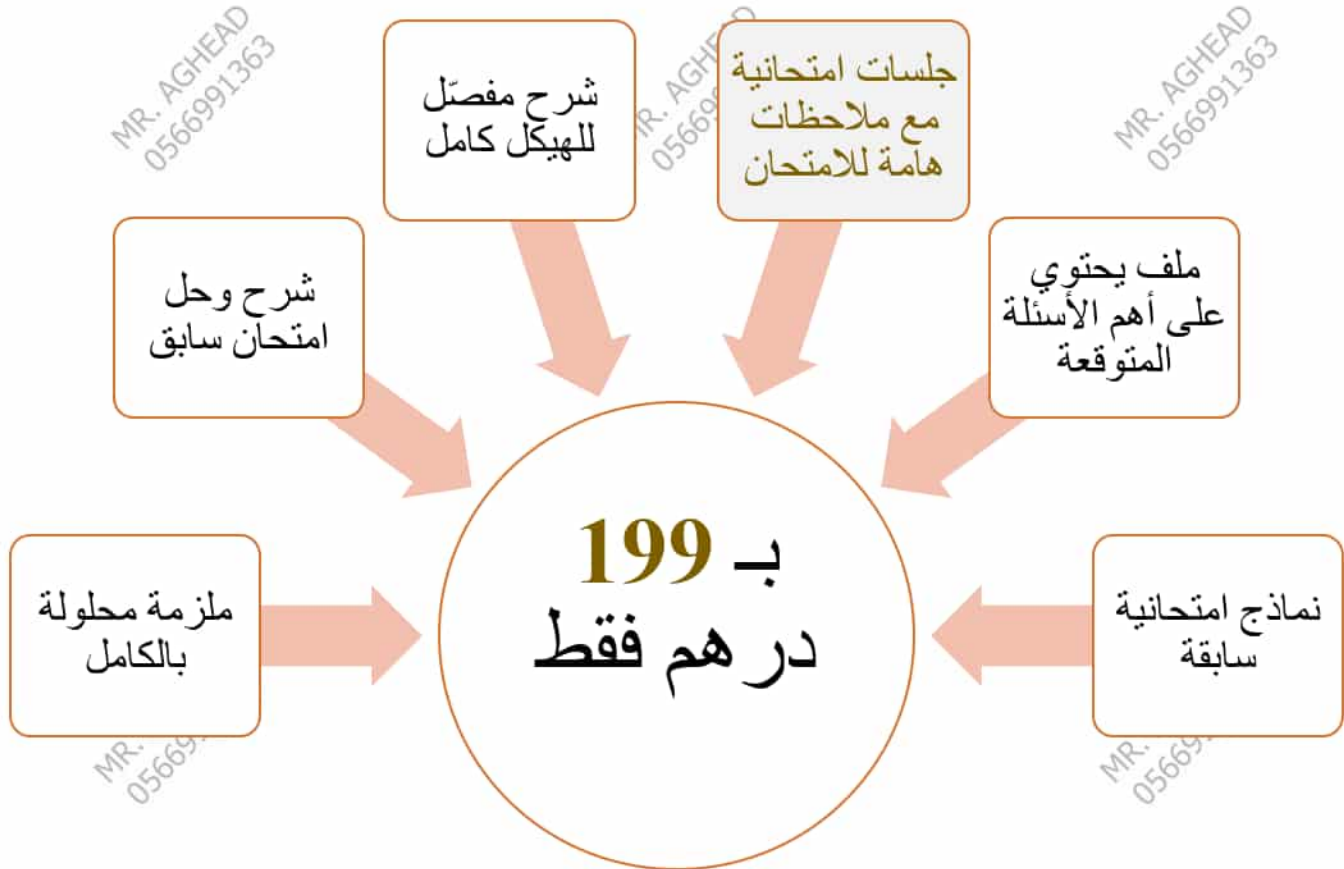
انضم للقناة



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0566991363

Mr. Aghead

يمكنكم الحصول على



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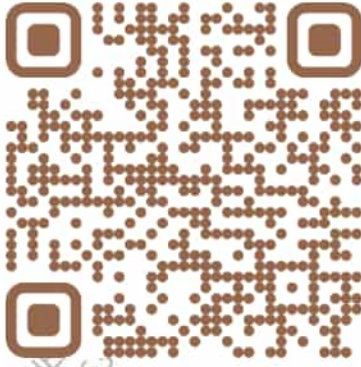
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احصل على الشرح كاملاً بـ 199 درهم فقط

Question 1: Estimate an arc length of
a given function (Page 68)

الأسئلة الموضوعية - MCQ



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Q. estimate the length of the curve $y = f(x)$ on the given interval using $n = 4$ line segments.

1. $f(x) = \cos x, 0 \leq x \leq \frac{\pi}{2}$

a) 1.906

b) 1.555

c) 0.235

d) 3.335

For the x-values of our points here we use (approximations of) $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$.

Left	Right	Length
(0, 1)	(0.393, 0.92)	0.400
(0.393, 0.92)	(0.785, 0.71)	0.449
(0.785, 0.71)	(1.18, 0.383)	0.509
(1.18, 0.383)	(1.571, 0)	0.548
	Total	1.906

2. $f(x) = \sin x, 0 \leq x \leq \frac{\pi}{2}$

a) 1.906

b) 2.8

c) 9.114

d) 0.111

For the x-values of our points here we use (approximations of) $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$.

Left	Right	Length
(0, 0)	(0.393, 0.38)	0.548
(0.393, 0.38)	(0.785, 0.71)	0.509
(0.785, 0.71)	(1.18, 0.924)	0.449
(1.18, 0.924)	(1.57, 1)	0.400
	Total	1.906

3. $f(x) = \sqrt{x+1}, 0 \leq x \leq 3$

a) 3.167

b) 1.91

c) 3.51

d) 0.6

Left	Right	Length
(0, 1)	(0.75, 1.323)	0.817
(0.75, 1.323)	(1.5, 1.581)	0.793
(1.5, 1.581)	(2.25, 1.803)	0.782
(2.25, 1.803)	(3, 2)	0.776
	Total	3.167

4. $f(x) = 1/x, 1 \leq x \leq 2$

a) 1.1310

b) 5.677

c) 9.41

d) 60.5

Left	Right	Length
(1, 1)	(1.25, 0.8)	0.3202
(1.25, 0.8)	(1.5, 0.67)	0.2833
(1.5, 0.67)	(1.75, 0.571)	0.2675
(1.75, 0.571)	(2, 0.5)	0.2600
	Total	1.1310

5. $f(x) = x^2 + 1, -2 \leq x \leq 2$

- a) 9.153 b) 3.144 c) 2.797 d) 13.157

Left	Right	Length
(-2, 5)	(-1, 2)	3.162
(-1, 2)	(0, 1)	1.414
(0, 1)	(1, 2)	1.414
(1, 2)	(2, 5)	3.162
	Total	9.153

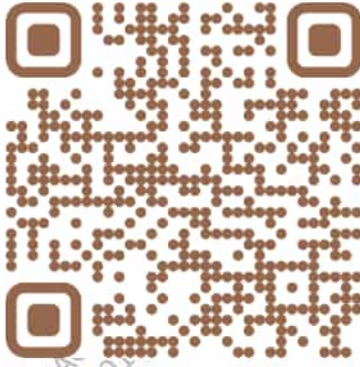
6. $f(x) = x^3 + 2, -1 \leq x \leq 1$

- a) 3.0463 b) 1.007 c) 9.42 d) 14.287

Left	Right	Length
(-1, 1)	(-0.5, 1.875)	1.0078
(-0.5, 1.875)	(0, 2)	0.5154
(0, 2)	(0.5, 2.125)	0.5154
(0.5, 2.125)	(1, 3)	1.0078
	Total	3.0463

Question 2: Find a limit algebraically
or graphically, if it exists (Page 75)

الأسئلة الموضوعية - MCQ



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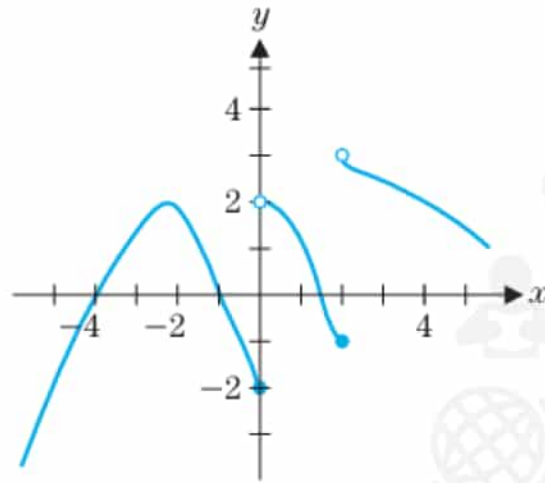
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Q. Identify each limit or state that it does not exist.



1.

a) $\lim_{x \rightarrow 0^-} f(x)$

a) -2

b) 2

c) 3

d) 4

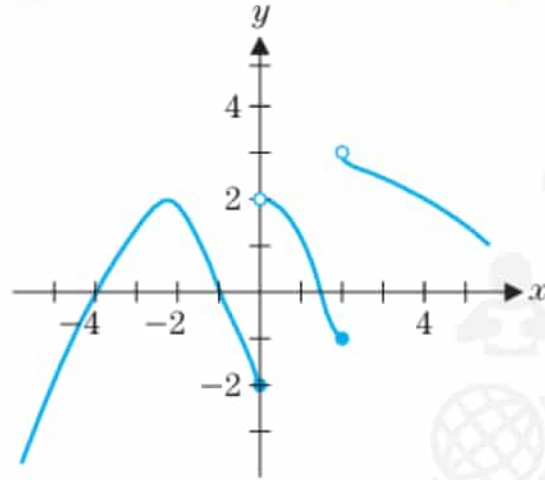
b) $\lim_{x \rightarrow 0^+} f(x)$

a) 2

b) -2

c) 0

d) 1

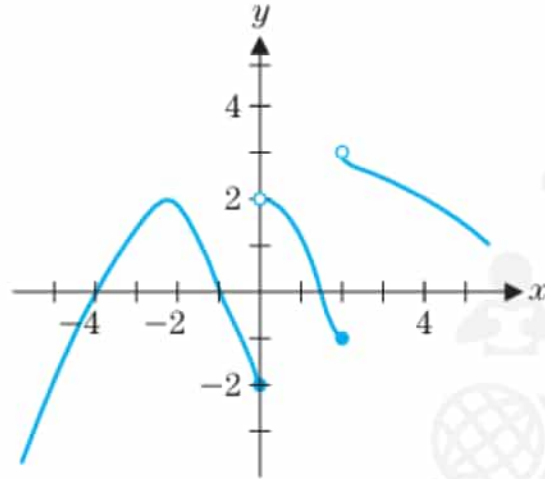


c) $\lim_{x \rightarrow 0} f(x)$

a) does not exist	b) 0	c) -1	d) 3
-------------------	------	-------	------

d) $\lim_{x \rightarrow -2^-} f(x)$

a) 2	b) -2	c) 0	d) 3
------	-------	------	------



e) $\lim_{x \rightarrow -2^+} f(x)$

a) 2

b) -3

c) 4

d) 2.5

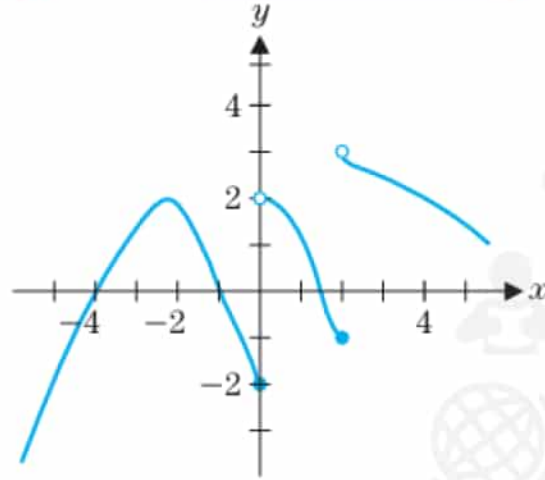
f) $\lim_{x \rightarrow -2} f(x)$

a) 2

b) 1

c) -1

d) -3



g) $\lim_{x \rightarrow -1} f(x)$

a) 0

b) 2

c) 3

d) -1

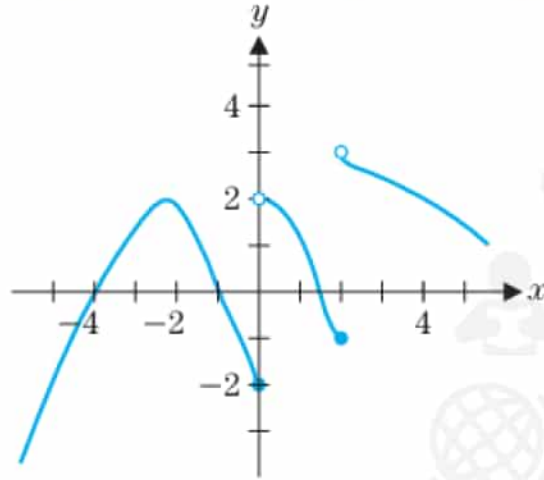
h) $\lim_{x \rightarrow -3} f(x)$

a) 1

b) -2

c) 3

d) 0



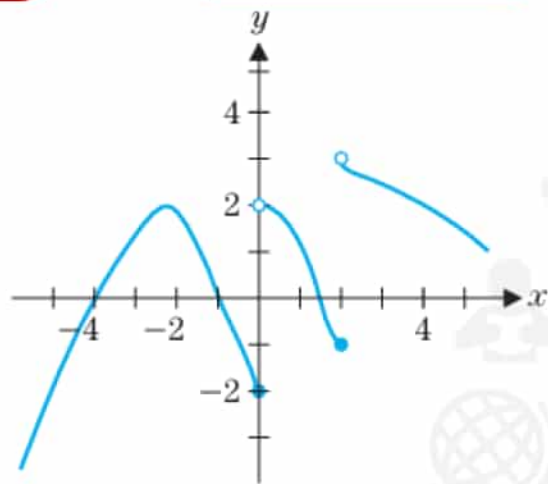
2.

a) $\lim_{x \rightarrow 1^-} f(x)$

a) 1	b) 0	c) 3	d) 5
------	------	------	------

b) $\lim_{x \rightarrow 1^+} f(x)$

a) 1	b) 5	c) 2	d) -2
------	------	------	-------

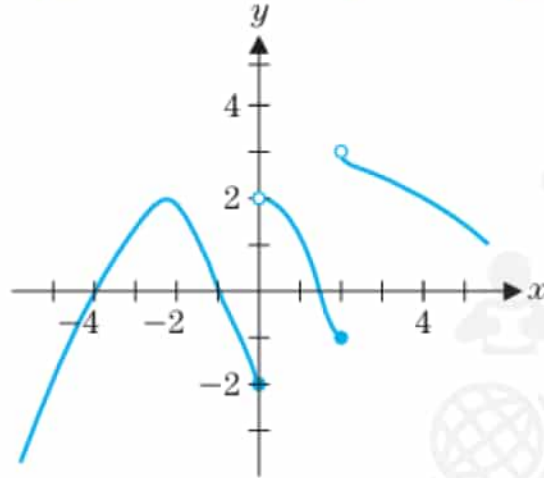


c) $\lim_{x \rightarrow 1} f(x)$

a) 1	b) 0	c) 3	d) -2
------	------	------	-------

d) $\lim_{x \rightarrow 2^-} f(x)$

a) -1	b) 1	c) 0	d) 2
-------	------	------	------



e) $\lim_{x \rightarrow 2^+} f(x)$

a) 3

b) 1

c) 0

d) -1

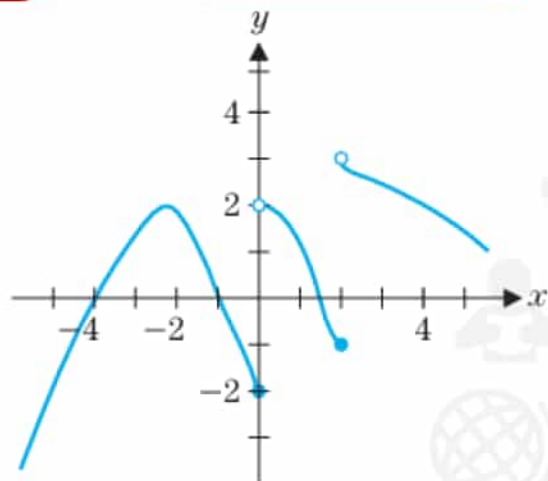
f) $\lim_{x \rightarrow 2} f(x)$

a) does not exist

b) 1

c) 0

d) -1



g) $\lim_{x \rightarrow 3^-} f(x)$

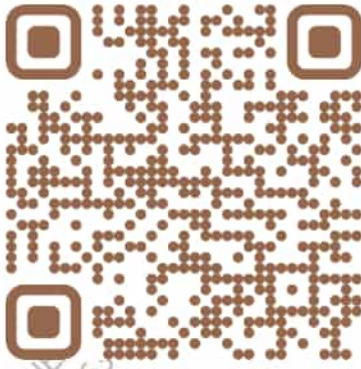
a) 2.5	b) 2	c) 3	d) -1
--------	------	------	-------

h) $\lim_{x \rightarrow -3} f(x)$

a) 1.5	b) -2	c) -3	d) 3
--------	-------	-------	------

Question 3: Find limits of polynomial, rational,
and trigonometric functions using theorems
(Page 85)

الأسئلة الموضوعية - MCQ



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Q. Evaluate the indicated limit, if it exists. Assume that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

1. $\lim_{x \rightarrow 2} f(x)$, where $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$

a) 4	b) 1	c) 2	d) 0
------	------	------	------

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} 2x = 2(2) = 4 \\ \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} x^2 = 2^2 = 4 \end{aligned}$$

2. $\lim_{x \rightarrow -1} f(x)$, where $f(x) = \begin{cases} x^2 + 1 & \text{if } x < -1 \\ 3x + 1 & \text{if } x \geq -1 \end{cases}$

a) Undefined	b) 0	c) -2	d) 2
--------------	------	-------	------

The limit from the left is 2, but the limit from the right is -2.

3. $\lim_{x \rightarrow -1} f(x)$, where $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

a) does not exist

b) 1

c) 3

d) -1

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (2x + 1) = 2(-1) + 1 = -1$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} 3 = 3$$

4. $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

a) 3

b) -1

c) 1

d) 0

$$\lim_{x \rightarrow 1^-} f(x) = 3$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2x + 1 = 3$$

5. $\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$

a) 4

b) 0

c) -4

d) 1

$$\frac{(2+h)^2 - 4}{h} = \frac{(4 + 4h + h^2) - 4}{h} = \frac{4h + h^2}{h} = 4 + h$$
$$\lim_{h \rightarrow 0} 4 + h = 4$$

6. $\lim_{h \rightarrow 0} \frac{(1+h)^3 - 1}{h}$

a) 3

b) 1

c) 2

d) 0

$$\frac{(1+h)^3 - 1}{h} = \frac{1 + 3h + 3h^2 + h^3 - 1}{h} = \frac{h(3 + 3h + h^2)}{h} = 3 + 3h + h^2$$
$$\lim_{h \rightarrow 0} 3 + 3h + h^2 = 3$$

7. $\lim_{x \rightarrow 2} \frac{\sin(x^2-4)}{x^2-4}$

a) 1

b) 0

c) 4

d) -1

Consider $f(x) = \frac{\sin x}{x}$ and a polynomial $p(x) = x^2 - 4$ such that $p(2) = 0$

Also $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Therefore, by the theorem 3.4(viii),

$$\begin{aligned} \lim_{x \rightarrow a} f(p(x)) &= L \\ \Rightarrow \lim_{x \rightarrow 2} \frac{\sin(x^2-4)}{x^2-4} &= 1 \end{aligned}$$

8. $\lim_{x \rightarrow 0} \frac{\tan x}{5x}$

a) $\frac{1}{5}$

b) 5

c) 0

d) 1

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan x}{5x} &= \lim_{x \rightarrow 0} \frac{\sin x}{5x \cos x} = \lim_{x \rightarrow 0} \left(\frac{1}{5} \frac{\sin x}{x} \frac{1}{\cos x} \right) \\ &= \frac{1}{5} \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{\cos x} \right) = \frac{1}{5} (1)(1) = \frac{1}{5} \end{aligned}$$

Question 4: Use the squeeze theorem
to find limits (Page 85)

الأسئلة الموضوعية - MCQ



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Q. Use numerical and graphical evidence to conjecture the value of $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$. Use the Squeeze Theorem to prove that you are correct: identify the functions f and h , show graphically that $f(x) \leq x^2 \sin\left(\frac{1}{x}\right) \leq h(x)$ and justify $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} h(x)$.

a) 0

b) 1

c) 3

d) 4

x^2	$x^2 \sin(1/x)$
-0.1	0.0054
-0.01	5×10^{-5}
-0.001	-8×10^{-7}
0.1	-0.005
0.01	-5×10^{-5}
0.001	8×10^{-7}

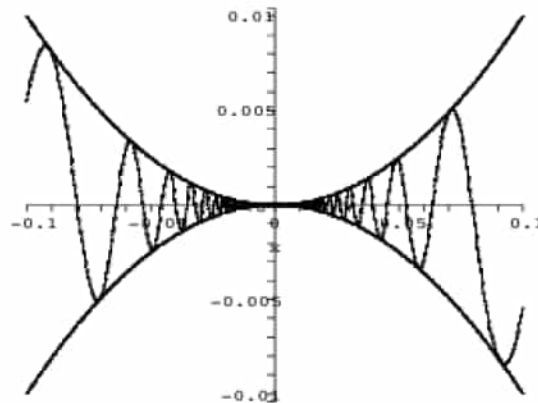
Conjecture: $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

Let $f(x) = -x^2, h(x) = x^2$.

Then $f(x) \leq x^2 \sin\left(\frac{1}{x}\right) \leq h(x)$

$\lim_{x \rightarrow 0} (-x^2) = 0, \lim_{x \rightarrow 0} (x^2) = 0$

Therefore, by the Squeeze Theorem, $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$



Q. Why can't you use the Squeeze Theorem as in the previous exercise to prove that $\lim_{x \rightarrow 0} x^2 \sec\left(\frac{1}{x}\right) = 0$? Explore this limit graphically.

You cannot use the Squeeze Theorem as in the previous exercise because the secant function is not bounded between -1 and 1 like the sine function is. This is difficult to investigate graphically because of the infinitely many vertical asymptotes as x approaches 0 .

Q. Use the Squeeze Theorem to prove that $\lim_{x \rightarrow 0^+} \left[\sqrt{x} \cos^2\left(\frac{1}{x}\right) \right] = 0$. Identify the functions f and h , show graphically that $f(x) \leq \sqrt{x} \cos^2\left(\frac{1}{x}\right) \leq h(x)$ for all $x > 0$, and justify $\lim_{x \rightarrow 0^+} f(x) = 0$ and $\lim_{x \rightarrow 0^+} h(x) = 0$.

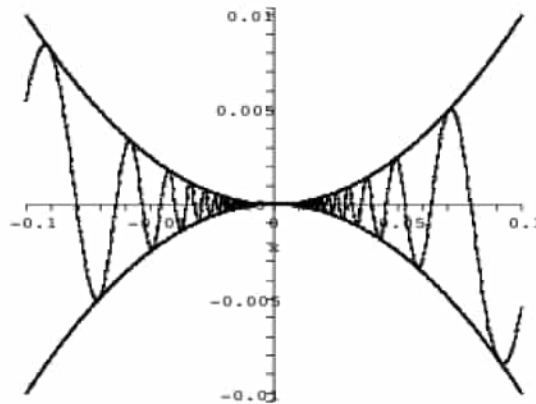
Let $f(x) = 0$, $h(x) = \sqrt{x}$. We see that:

$$f(x) \leq \sqrt{x} \cos^2\left(\frac{1}{x}\right) \leq h(x),$$

$$\lim_{x \rightarrow 0^+} 0 = 0, \quad \lim_{x \rightarrow 0^+} \sqrt{x} = 0$$

Therefore, by the Squeeze Theorem,

$$\lim_{x \rightarrow 0^+} \left[\sqrt{x} \cos^2\left(\frac{1}{x}\right) \right] = 0$$



Q. Suppose that $f(x)$ is bounded: that is, there exists a constant M such that $|f(x)| \leq M$ for all x . Use the Squeeze Theorem to prove that $\lim_{x \rightarrow 0} x^2 f(x) = 0$.

Saying that $|f(x)| \leq M$ for all x is the same as saying $-M \leq f(x) \leq M$ for all x .

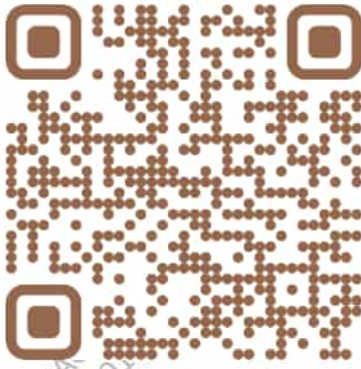
This implies that $-M \leq x^2 f(x) \leq Mx^2$.

Since $\pm Mx^2 \rightarrow 0$ as $x \rightarrow 0$, the Squeeze Theorem shows that

$$\lim_{x \rightarrow 0} x^2 f(x) = 0$$

Question 5: Use the continuity properties to study the continuity of a function or a composition of functions at a given point (Page 95)

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Mr. Aghead

Q. Determine the intervals on which f is continuous.

1. $f(x) = \sqrt{x+3}$

- | | | | |
|-------------------|--------------------|--------------------|------------------|
| a) $[-3, \infty)$ | b) $(-\infty, -2)$ | c) $(-\infty, -3)$ | d) $(2, \infty)$ |
|-------------------|--------------------|--------------------|------------------|

2. $f(x) = \sqrt{x^2 - 4}$

- | |
|--------------------------------------|
| a) $(-\infty, -2]$ and $[2, \infty)$ |
| b) $(2, \infty)$ |
| c) $(-\infty, -2)$ |
| d) $(-\infty, -4)$ and $(4, \infty)$ |

3. $f(x) = \sqrt[3]{x+2}$

- | | | | |
|------------------------|--------------------|-------------------|-------------------|
| a) $(-\infty, \infty)$ | b) $(-\infty, -2)$ | c) $(-\infty, 2)$ | d) $(-2, \infty)$ |
|------------------------|--------------------|-------------------|-------------------|

4. $f(x) = (x - 1)^{\frac{3}{2}}$

a) $[1, \infty)$	b) $(-1, \infty)$	c) $(0, \infty)$	d) $(-\infty, 1)$
------------------	-------------------	------------------	-------------------

5. $f(x) = \sin^{-1}(x + 2)$

a) $[-3, -1]$	b) $[-3, 1]$	c) $[-1, 1]$	d) $[1, 3]$
---------------	--------------	--------------	-------------

6. $f(x) = \ln(\sin x)$

a) $(2k\pi, (2k + 1)\pi)$ for all integral values of k
b) $(k\pi, (k + 1)\pi)$ for all integral values of k
c) $(0, \infty)$
d) $(-\infty, 0)$

7. $f(x) = \frac{\sqrt{x+1}+e^x}{x^2-2}$

a) $[-1, \infty)$ whenever $x \neq \sqrt{2}$

b) $[-\sqrt{2}, \infty)$

c) $(0, \infty)$

d) $(-\infty, 0)$

8. $f(x) = \frac{\ln(x^2-1)}{\sqrt{x^2-2x}}$

a) $(-\infty, -1)$ and $(2, \infty)$

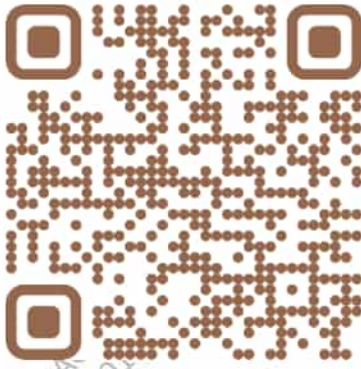
b) $(0, \infty)$

c) $(-\infty, -1)$

d) $(2, \infty)$

Question 6: Find horizontal, vertical, and slant asymptotes using limits (Page 106)

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Q. determine all horizontal and vertical asymptotes. For each side of each vertical asymptote, determine whether $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$.

1.

a) $f(x) = \frac{x}{4-x^2}$

a) vertical asymptotes at $x = \pm 2$, horizontal asymptote at $y = 0$

b) vertical asymptotes at $x = \pm 3$, horizontal asymptote at $y = 0$

c) vertical asymptotes at $x = \pm 1$, horizontal asymptote at $y = 2$

d) vertical asymptotes at $x = 0$, horizontal asymptote at $y = 0$

$4 - x^2 = 0 \Rightarrow 4 = x^2$ so we have vertical asymptotes at $x = \pm 2$.

$$f(x) \rightarrow \infty \text{ as } x \rightarrow -2^-$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -2^+$$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow 2^-$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow 2^+$$

Again we have $\lim_{x \rightarrow \pm \infty} \frac{x}{4-x^2} = \lim_{x \rightarrow \pm \infty} \frac{x}{x^2(\frac{4}{x^2}-1)} = \lim_{x \rightarrow \pm \infty} \frac{1}{x(\frac{4}{x^2}-1)} = 0$

So there is a horizontal asymptote at $y = 0$.

b) $f(x) = \frac{x^2}{4-x^2}$

a) vertical asymptotes at $x = \pm 2$, horizontal asymptote at $y = -1$

b) vertical asymptotes at $x = \pm 3$, horizontal asymptote at $y = -2$

c) vertical asymptotes at $x = \pm 2$, horizontal asymptote at $y = 1$

d) vertical asymptotes at $x = 0$, horizontal asymptote at $y = -1$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow 2^- \text{ and } x \rightarrow -2^+ \\ f(x) \rightarrow -\infty \text{ as } x \rightarrow 2^+ \text{ and } x \rightarrow -2^-$$

2.

a) $f(x) = \frac{x}{\sqrt{4+x^2}}$

a) no vertical asymptotes, horizontal asymptotes at $y = \pm 1$

b) no vertical asymptotes, horizontal asymptotes at $y = \pm 2$

c) no vertical asymptotes, horizontal asymptotes at $y = 0$

d) vertical asymptotes at $x = \pm 2$, horizontal asymptote at $y = -1$

Since $4 + x^2$ is never 0, there are no vertical asymptotes.

We have $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4+x^2}} = \lim_{x \rightarrow \infty} \frac{x}{x\sqrt{\frac{4}{x^2}+1}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{4}{x^2}+1}} = \frac{1}{\sqrt{1}} = 1$

and $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{4+x^2}} = \lim_{x \rightarrow -\infty} \frac{x}{-x\sqrt{\frac{4}{x^2}+1}} = \lim_{x \rightarrow -\infty} \frac{-1}{\sqrt{\frac{4}{x^2}+1}} = \frac{-1}{\sqrt{1}} = -1$

so there are horizontal asymptotes at $y = 1$ and $y = -1$.

b) $f(x) = \frac{x}{\sqrt{4-x^2}}$

a) vertical asymptotes at $x = \pm 2$, no horizontal asymptotes

b) vertical asymptotes at $x = \pm 1$, no horizontal asymptotes

c) vertical asymptotes at $x = \pm 5$, no horizontal asymptotes

d) no vertical asymptotes, horizontal asymptotes at $y = \pm 1$

The function is only defined in $(-2, 2)$. Two one-sided vertical asymptotes at $x = \pm 2$.

$f(x) \rightarrow \infty$ as $x \rightarrow 2^-$, and $f(x) \rightarrow -\infty$ as $x \rightarrow -2^+$.

No horizontal asymptotes.

3. $f(x) = \frac{3x^2+1}{x^2-2x-3}$

a) vertical asymptotes at $x = 3$ and $x = -1$, horizontal asymptote at $y = 3$

b) vertical asymptotes at $x = \pm 2$, no horizontal asymptotes

c) vertical asymptotes at $x = 0$ and $x = -1$, horizontal asymptote at $y = 1$

d) vertical asymptotes at $x = 2$ and $x = 1$, horizontal asymptote at $y = 3$

The denominator factors: $x^2 - 2x - 3 = (x - 3)(x + 1)$

Since neither $x = 3$ nor $x = -1$ are zeros of the numerator, we see that $f(x)$ has vertical asymptotes at $x = 3$ and $x = -1$

$f(x) \rightarrow -\infty$ as $x \rightarrow 3^-$

$f(x) \rightarrow \infty$ as $x \rightarrow 3^+$

$f(x) \rightarrow \infty$ as $x \rightarrow -1^-$

$f(x) \rightarrow -\infty$ as $x \rightarrow -1^+$

We have $\lim_{x \rightarrow \pm\infty} \frac{3x^2+1}{x^2-2x-3} = \lim_{x \rightarrow \pm\infty} \frac{3+\frac{1}{x^2}}{1-\frac{2}{x}-\frac{3}{x^2}} = 3$

So there is a horizontal asymptote at $y = 3$

4. $f(x) = \frac{1-x}{x^2+x-2}$

a) vertical asymptote at $x = -2$, horizontal asymptote at $y = 0$

b) vertical asymptote at $x = 2$, horizontal asymptote at $y = -3$

c) vertical asymptote at $x = -1$, horizontal asymptote at $y = 0$

d) vertical asymptote at $x = 0$, horizontal asymptote at $y = 3$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow -2^-$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -2^+ \text{ and } x \rightarrow -2^-$$

Again, we have $\lim_{x \rightarrow \pm\infty} \frac{1-x}{x^2+x-2} = 0$ so there is a horizontal asymptote at $y = 0$

5. $f(x) = 4 \tan^{-1} x - 1$

a) no vertical asymptotes, horizontal asymptotes at $y = 2\pi - 1$
and $y = -2\pi - 1$

b) no vertical asymptotes, horizontal asymptotes at $y = 2\pi + 1$
and $y = -2\pi - 1$

c) no vertical asymptotes, horizontal asymptotes at $y = 2\pi - 1$
and $y = 2\pi - 1$

d) no vertical asymptotes, horizontal asymptotes at $y = 2\pi - 1$
and $y = -2\pi + 1$

The function is continuous for all x , so no vertical asymptotes.

We have $\lim_{x \rightarrow \infty} 4 \tan^{-1} x - 1 = 4 \left(\lim_{x \rightarrow \infty} \tan^{-1} x \right) - 1 = 4 \left(\frac{\pi}{2} \right) - 1 = 2\pi - 1$

and $\lim_{x \rightarrow -\infty} 4 \tan^{-1} x - 1 = 4 \left(\lim_{x \rightarrow -\infty} \tan^{-1} x \right) - 1 = 4 \left(-\frac{\pi}{2} \right) - 1 = -2\pi - 1$

so there are horizontal asymptotes at $y = 2\pi - 1$ and $y = -2\pi - 1$

6. $f(x) = \ln(1 - \cos x)$

a) vertical asymptotes at $x = 2k\pi$ for any integer k
b) no vertical asymptotes
c) vertical asymptotes at $x = k\pi$ for any integer k
d) vertical asymptote at $x = 2$, horizontal asymptote at $y = -3$

The function $\ln x$ has a one-sided vertical asymptote at $x = 0$, so

$f(x) = \ln(1 - \cos x)$ will have a vertical asymptote whenever $1 - \cos x = 0$, i.e., whenever $\cos x = 1$.

This happens when $x = 2k\pi$ for any integer k .

Since $1 - \cos x \geq 0$ for all x , $f(x)$ is defined at all points except for these vertical asymptotes. Thus as $f(x)$ approaches any of these asymptotes (from either side), it behaves like $\ln x$ approaching 0 from the right, so $f(x) \rightarrow -\infty$ as x approaches any of these asymptotes from either side.

Q. Determine all vertical and slant asymptotes.

1. $y = \frac{x^3}{4-x^2}$

a) vertical asymptotes at $x = \pm 2$, the slant asymptote is $y = -x$

b) vertical asymptotes at $x = 0$, the slant asymptote is $y = x$

c) vertical asymptotes at $x = 1$, the slant asymptote is $y = -x$

d) vertical asymptotes at $x = -3$, the slant asymptote is $y = x$

2. $y = \frac{x^2+1}{x-2}$

a) vertical asymptote at $x = 2$, the slant asymptote is $y = x + 2$

b) vertical asymptote at $x = -2$, the slant asymptote is $y = x - 2$

c) vertical asymptote at $x = 3$, the slant asymptote is $y = -x + 2$

d) vertical asymptotes at $x = \pm 2$, the slant asymptote is $y = -x$

3. $y = \frac{x^3}{x^2+x-4}$

a) vertical asymptotes at $x = \frac{-1+\sqrt{17}}{2}$, the slant asymptote is $y = x - 1$

b) vertical asymptotes at $x = \frac{1+\sqrt{17}}{2}$, the slant asymptote is $y = x - 2$

c) vertical asymptotes at $x = \frac{-1-\sqrt{17}}{2}$, the slant asymptote is $y = x - 1$

d) vertical asymptotes at $x = \frac{-1+\sqrt{17}}{4}$, the slant asymptote is $y = x + 1$

4. $y = \frac{x^4}{x^3+2}$

a) vertical asymptote at $x = -\sqrt[3]{2}$, the slant asymptote is $y = x$

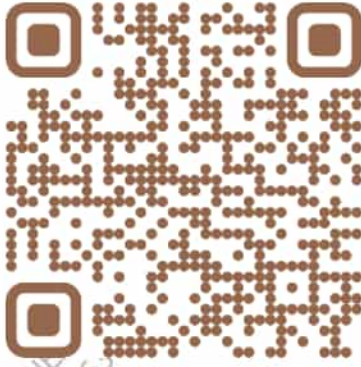
b) vertical asymptote at $x = -\sqrt[3]{3}$, the slant asymptote is $y = -x$

c) vertical asymptote at $x = \sqrt[3]{2}$, the slant asymptote is $y = 3x$

d) vertical asymptote at $x = \sqrt[3]{3}$, the slant asymptote is $y = x$

Question 7: Find limits at infinity and
limits that are infinite (Page 106)

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Q. Determine each limit (answer as appropriate, with a number, ∞ , $-\infty$ or does not exist).

1. $\lim_{x \rightarrow \infty} \frac{x^2 + 3x - 2}{3x^2 + 4x - 1}$

a) $\frac{1}{3}$	b) 1	c) 0	d) 4
------------------	------	------	------

$$\lim_{x \rightarrow \infty} \frac{x^2 + 3x - 2}{3x^2 + 4x - 1} = \lim_{x \rightarrow \infty} \frac{x^2 \left(1 + \frac{3}{x} - \frac{2}{x^2}\right)}{x^2 \left(3 + \frac{4}{x} - \frac{1}{x^2}\right)} = \lim_{x \rightarrow \infty} \frac{\left(1 + \frac{3}{x} - \frac{2}{x^2}\right)}{\left(3 + \frac{4}{x} - \frac{1}{x^2}\right)} = \frac{1}{3}$$

2. $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1}$

a) $\frac{1}{2}$	b) 2	c) 1	d) 0
------------------	------	------	------

$$\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1} = \lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1} \left(\frac{1/x^2}{1/x^2}\right) = \lim_{x \rightarrow \infty} \frac{2 - 1/x + 1/x^2}{4 - 3/x - 1/x^2} = \frac{1}{2}$$

3. $\lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{4+x^2}}$

a) 1

b) 2

c) 0

d) 3

$$\lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{4+x^2}} = \lim_{x \rightarrow -\infty} \frac{-x}{-x\sqrt{4+x^2}} = \lim_{x \rightarrow -\infty} \frac{1}{\sqrt{\frac{4}{x^2} + 1}} = \frac{1}{\sqrt{1}} = 1$$

4. $\lim_{x \rightarrow \infty} \frac{2x^2-1}{4x^3-5x-1}$

a) 0

b) 1

c) 2

d) 3

5. $\lim_{x \rightarrow \infty} \ln \left(\frac{x^2+1}{x-3} \right)$

a) ∞

b) $-\infty$

c) 0

d) 3

$$\lim_{x \rightarrow \infty} \ln \left(\frac{x^2+1}{x-3} \right) = \lim_{x \rightarrow \infty} \left[\ln \left(\frac{1 + \frac{1}{x^2}}{\frac{1}{x} - \frac{3}{x^2}} \right) \right] = \lim_{x \rightarrow \infty} [\ln x] = \infty$$

6. $\lim_{x \rightarrow 0^+} \ln(x \sin x)$

a) $-\infty$

b) 1

c) 0

d) π

$$\lim_{x \rightarrow 0^+} \ln(x \sin x) = \lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

7. $\lim_{x \rightarrow 0^+} e^{\left(-\frac{2}{x^3}\right)}$

a) 0

b) 1

c) 2

d) -1

$$\lim_{x \rightarrow 0^+} e^{\left(-\frac{2}{x^3}\right)} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$$

8. $\lim_{x \rightarrow \infty} e^{-\frac{x+1}{x^2+2}}$

a) 1

b) 0

c) 3

d) 2

$$\lim_{x \rightarrow \infty} e^{-\frac{x+1}{x^2+2}} = \lim_{x \rightarrow \infty} e^{\left(\frac{-x^2\left(\frac{1}{x} + \frac{1}{x^2}\right)}{x^2\left(1 + \frac{2}{x^2}\right)}\right)} = \lim_{x \rightarrow \infty} e^{\left[\frac{-\left(\frac{1}{x} + \frac{1}{x^2}\right)}{\left(1 + \frac{2}{x^2}\right)}\right]} = 1$$

9. $\lim_{x \rightarrow \infty} \cot^{-1} x$

a) 0

b) 1

c) 2

d) 3

10. $\lim_{x \rightarrow \infty} \sec^{-1} \frac{x^2+1}{x+1}$

a) $\frac{\pi}{2}$

b) 0

c) 1

d) 9

$$\lim_{x \rightarrow \infty} \sec^{-1} \frac{x^2+1}{x+1} = \lim_{x \rightarrow \infty} \sec^{-1} \left[\frac{x^2 \left(1 + \frac{1}{x^2}\right)}{x^2 \left(\frac{1}{x} + \frac{1}{x^2}\right)} \right] = \lim_{x \rightarrow \infty} \sec^{-1}(x) = \frac{\pi}{2}$$

11. $\lim_{x \rightarrow 0} \sin \left(e^{-\frac{1}{x^2}} \right)$

a) 0

b) 1

c) 2

d) 3

$$\lim_{x \rightarrow 0^+} \left(\sin \left(e^{-\frac{1}{x^2}} \right) \right) = \lim_{x \rightarrow -\infty} (\sin(e^x)) = \lim_{x \rightarrow 0^+} (\sin x) = 0$$

$$\lim_{x \rightarrow 0^-} \left(\sin \left(e^{-\frac{1}{x^2}} \right) \right) = \lim_{x \rightarrow -\infty} (\sin(e^x)) = \lim_{x \rightarrow 0^+} (\sin x) = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(\sin \left(e^{-\frac{1}{x^2}} \right) \right) = 0$$

12. $\lim_{x \rightarrow \infty} \sin(\tan^{-1} x)$

a) 1	b) 0	c) -1	d) 3
------	------	-------	------

$$\lim_{x \rightarrow \infty} \sin(\tan^{-1} x) = \lim_{x \rightarrow \frac{\pi}{2}} (\sin x) = 1$$

13. $\lim_{x \rightarrow \frac{\pi}{2}} e^{-\tan x}$

a) does not exist	b) 0	c) 1	d) 2
-------------------	------	------	------

$$\lim_{x \rightarrow \frac{\pi}{2}^-} e^{-\tan x} = \lim_{x \rightarrow \infty} e^{-x} = \lim_{x \rightarrow -\infty} e^x = 0$$

but

$$\lim_{x \rightarrow \frac{\pi}{2}^+} e^{-\tan x} = \lim_{x \rightarrow -\infty} e^{-x} = \lim_{x \rightarrow \infty} e^x = \infty$$

so the limit does not exist

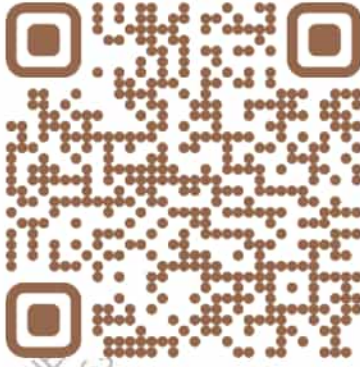
14. $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x)$

a) $-\frac{\pi}{2}$	b) $\frac{\pi}{2}$	c) 0	d) 1
---------------------	--------------------	------	------

$$\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x) = \lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

Question 8: Link and interpret the slope of a secant line and tangent line (Page 136)

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Mr. Aghead

Q. Graphically and numerically approximate the slope of the tangent line to $y = \frac{x-1}{x+1}$ at $x = 0$.

A graph of $y = \frac{x-1}{x+1}$ is shown in Figure 3.10a. We

sketch the tangent line at the point $(0, -1)$ in

Figure 3.10b, where we have zoomed in to provide better detail. To approximate the slope, we estimate

the coordinates of one point on the tangent line

other than $(0, -1)$. In Figure 3.10b, it appears that

the tangent line passes through the point $(1, 1)$. An

estimate of the slope is then $m_{tan} \approx \frac{1 - (-1)}{1 - 0} = 2$. To

approximate the slope numerically, we choose

several points near $(0, -1)$ and compute the slopes

of the secant lines. For example, rounding the

y -values to four decimal places, we have:

Second Point	m_{sec}	Second Point	m_{sec}
$(1, 0)$	$\frac{0 - (-1)}{1 - 0} = 1$	$(-0.5, -3)$	$\frac{-3 - (-1)}{-0.5 - 0} = 4.0$
$(0.1, -0.8182)$	$\frac{-0.8182 - (-1)}{0.1 - 0} = 1.818$	$(-0.1, -1.2222)$	$\frac{-1.2222 - (-1)}{-0.1 - 0} = 2.222$
$(0.01, -0.9802)$	$\frac{-0.9802 - (-1)}{0.01 - 0} = 1.98$	$(-0.01, -1.0202)$	$\frac{-1.0202 - (-1)}{-0.01 - 0} = 2.02$

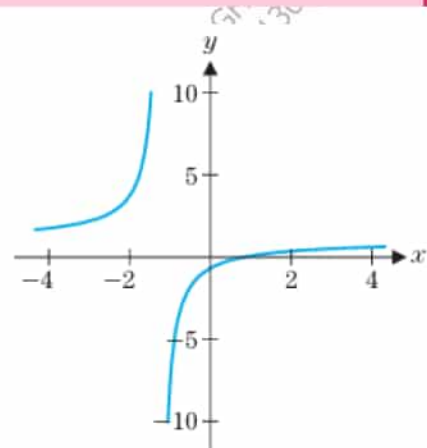


FIGURE 3.10a

$$y = \frac{x-1}{x+1}$$

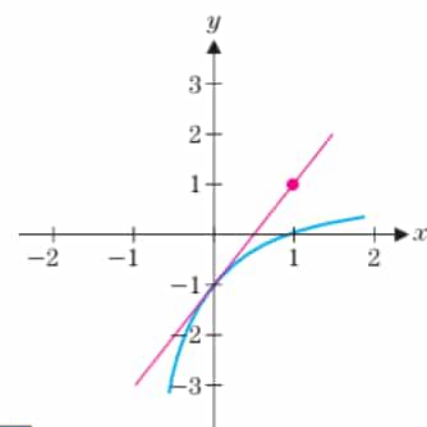


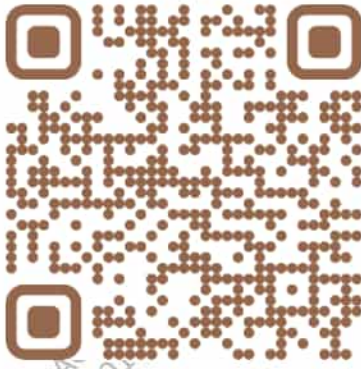
FIGURE 3.10b

Tangent line

In both columns, as the second point gets closer to $(0, -1)$, the slope of the secant line gets closer to 2. A reasonable estimate of the slope of the tangent line at the point $(0, -1)$ is then 2.

Question 9: Find the derivative of a function at a given point (Page 145)

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Q. If $f(x) = \frac{1}{x}$ ($x \neq 0$), find $f'(x)$.

a) $-x^2$

b) x^2

c) x

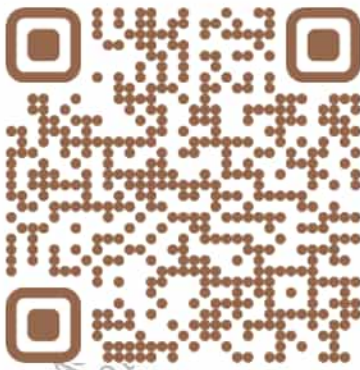
d) $2x$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left[\frac{x - (x+h)}{x(x+h)}\right]}{h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2} \end{aligned}$$

so that $f'(x) = -x^{-2}$

Question 10: Understand the relationship
between continuity and differentiability
(Page 151)

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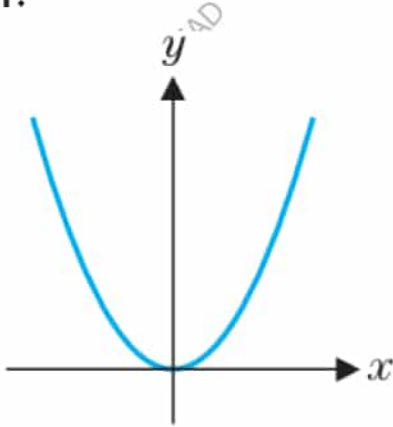
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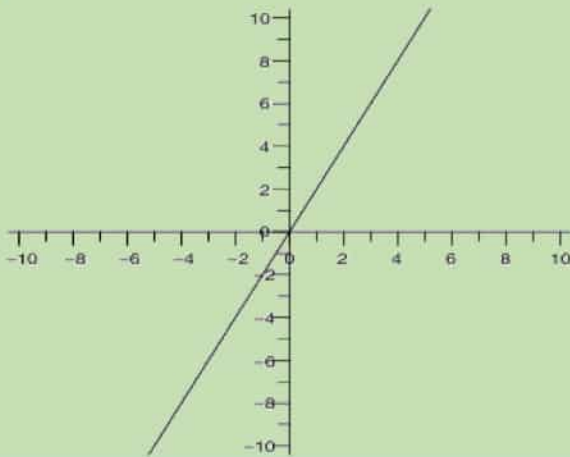
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Q. Use the graph of f to sketch a graph of f' .

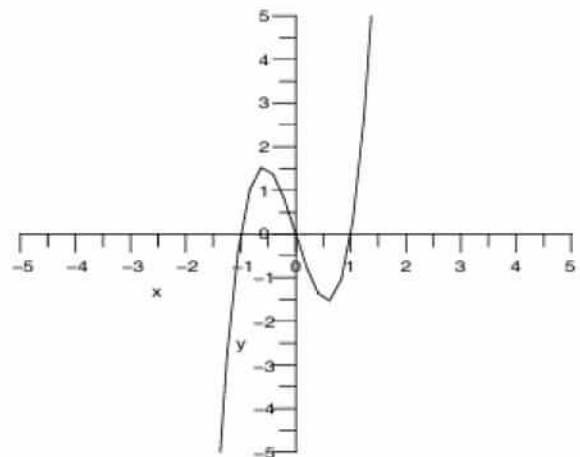
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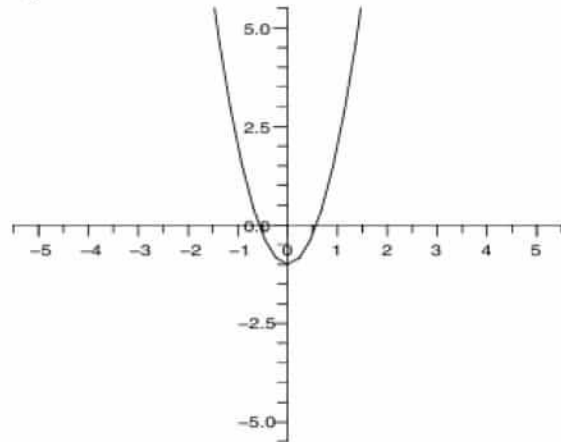
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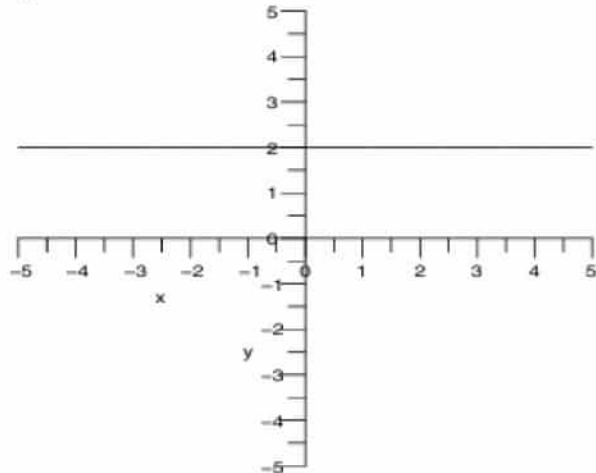
b)



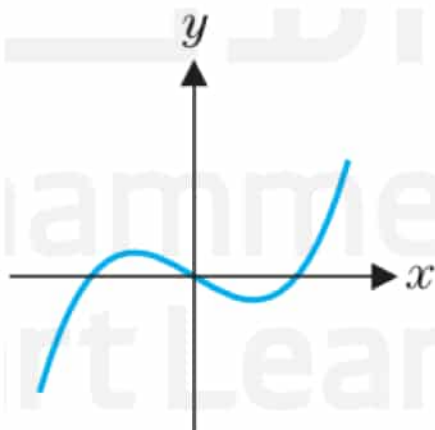
c)



d)



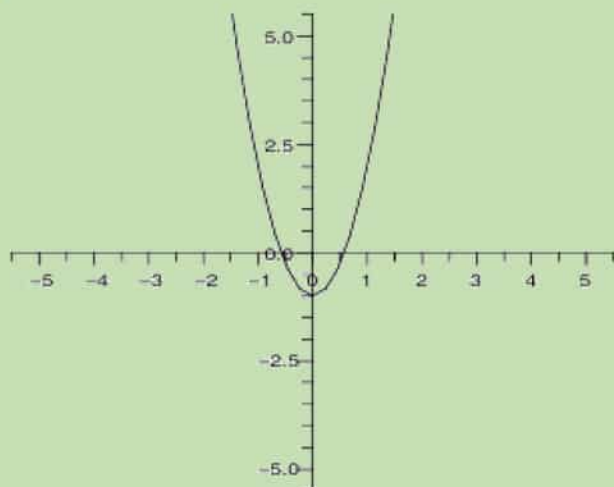
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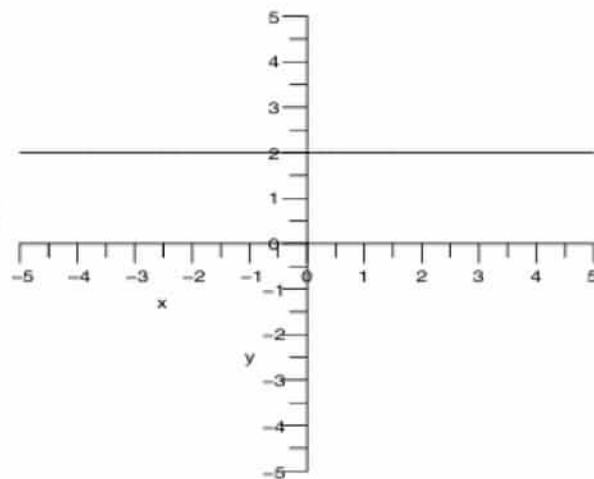
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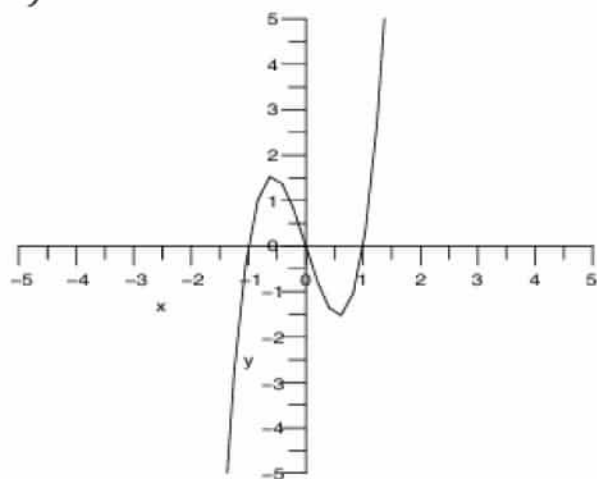
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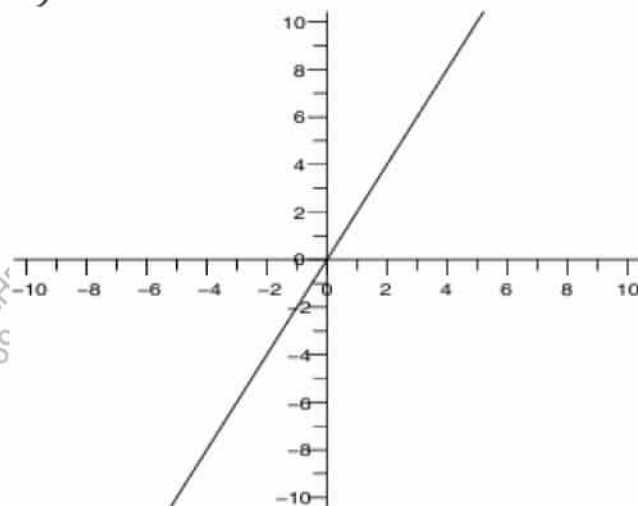
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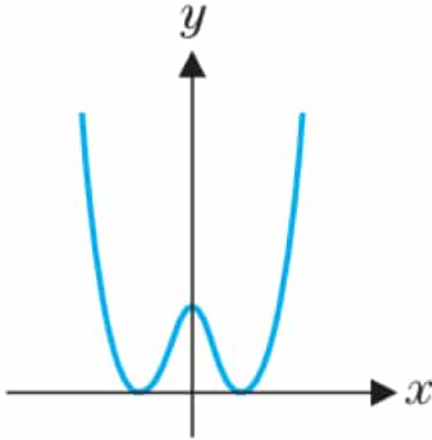
c)



d)



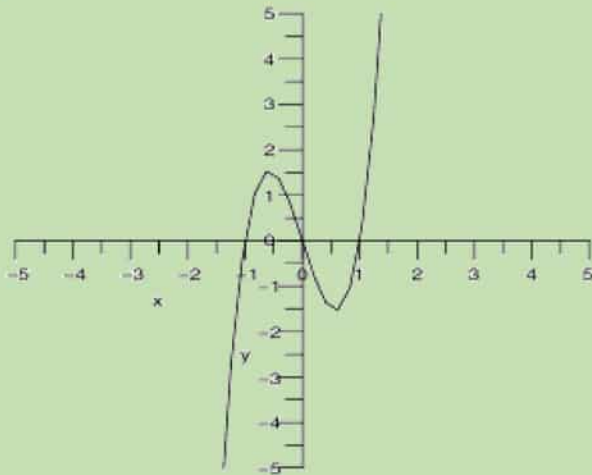
3.



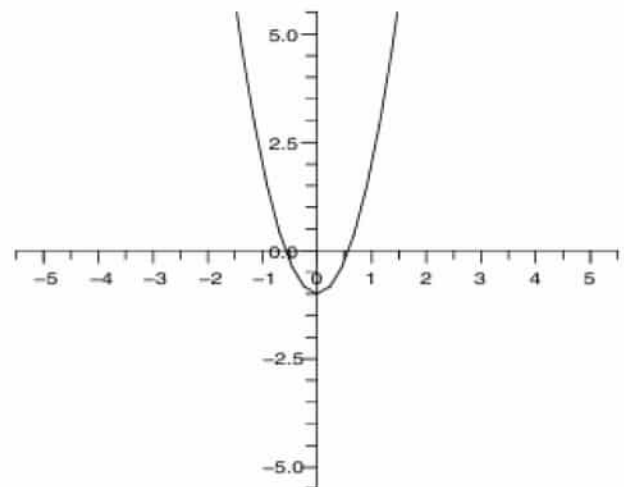
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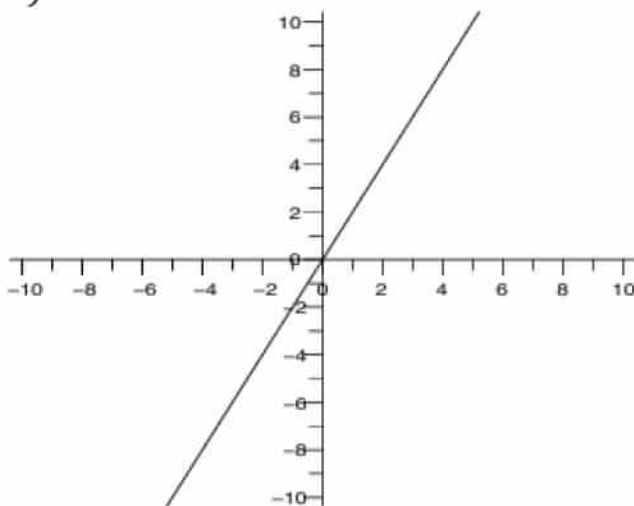
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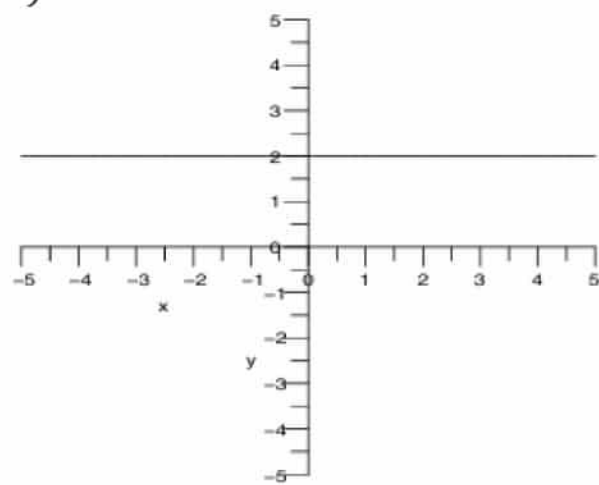
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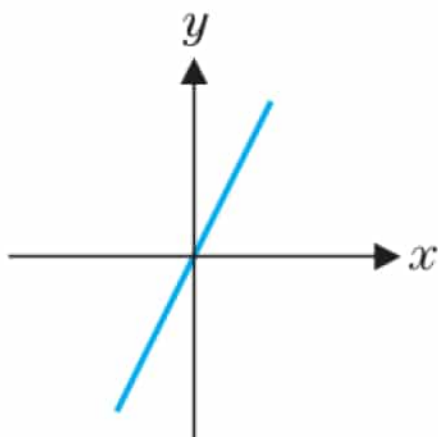
c)



d)



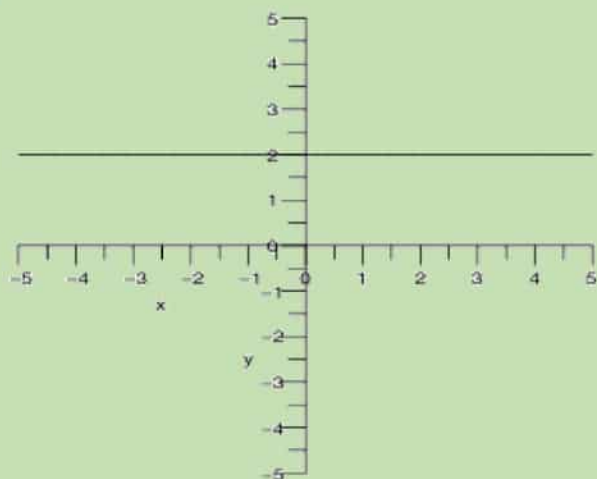
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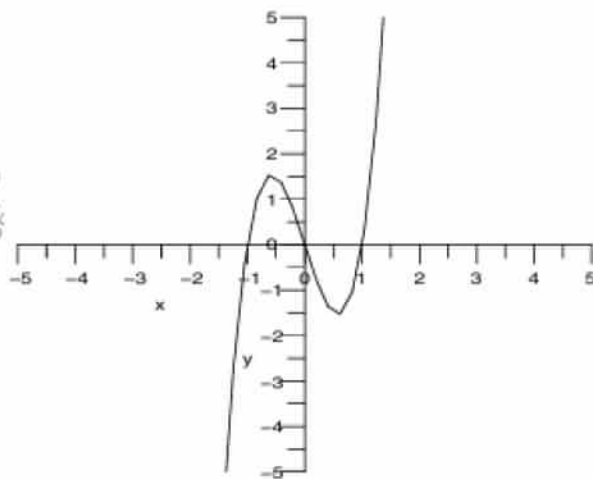
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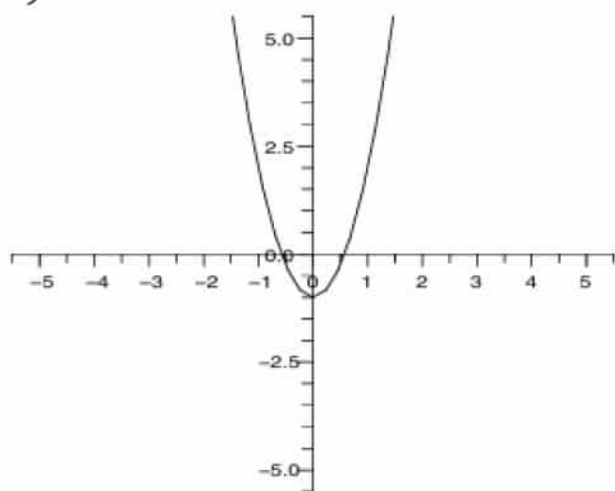
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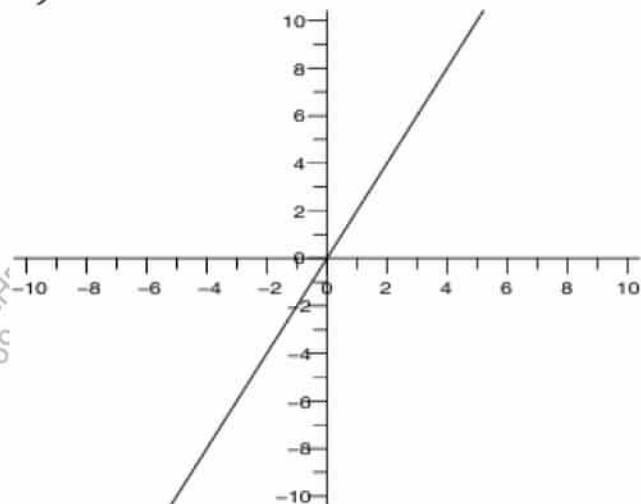
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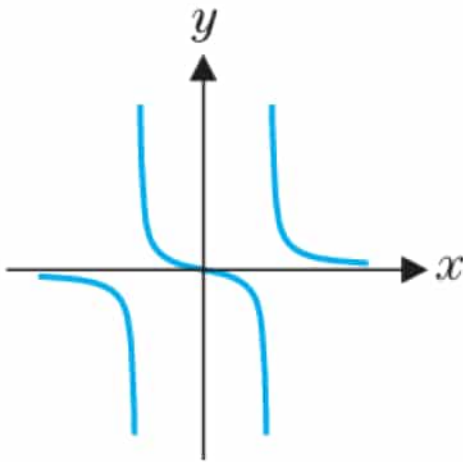
c)



d)



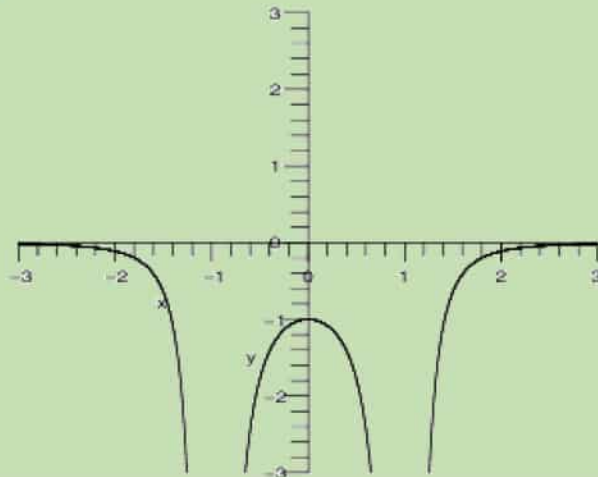
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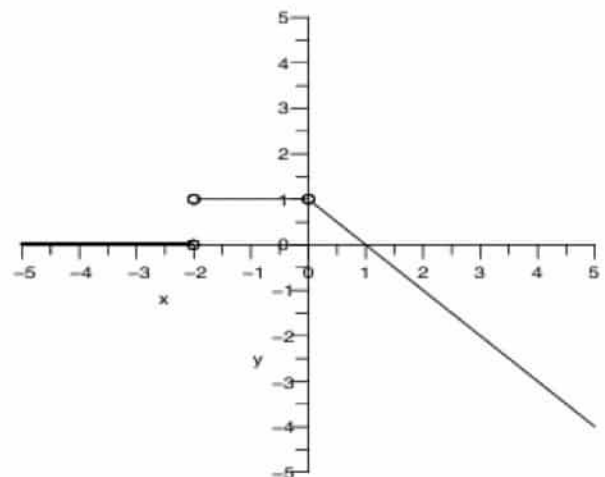
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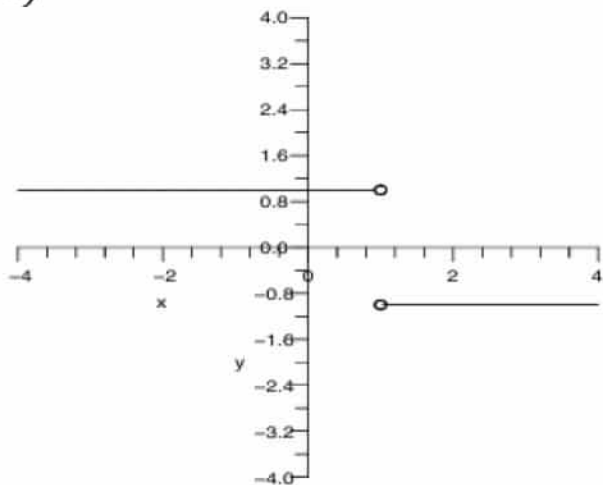
a)



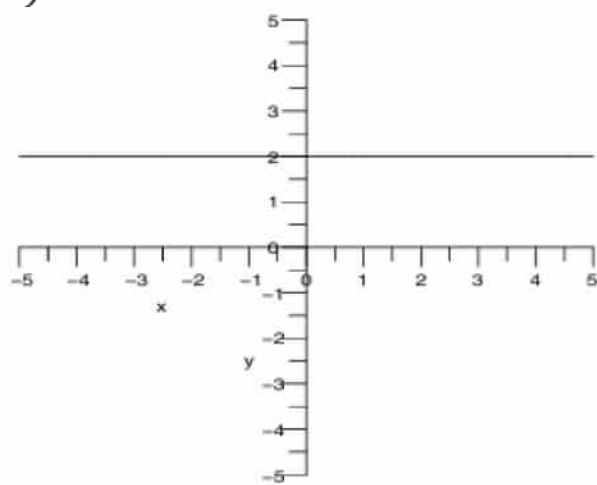
b)



c)

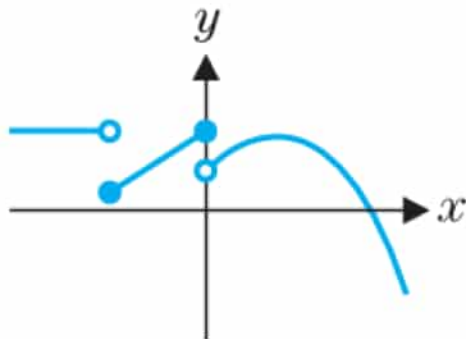


d)

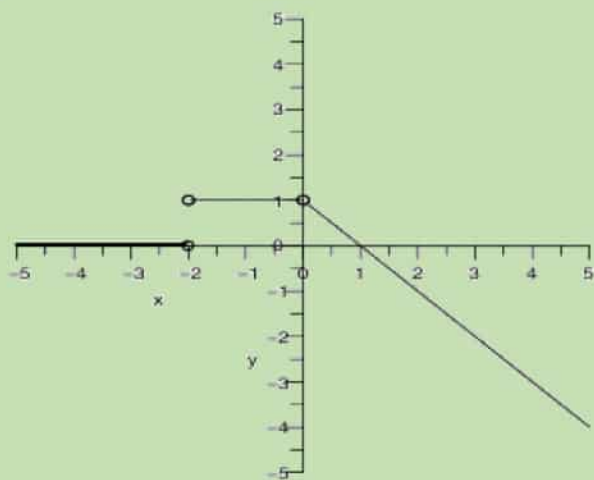


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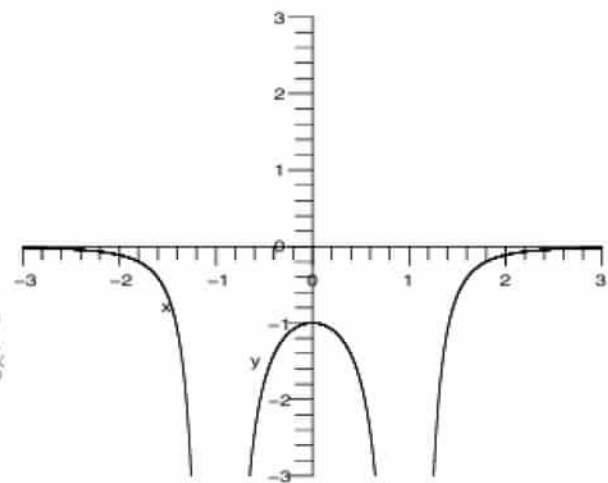
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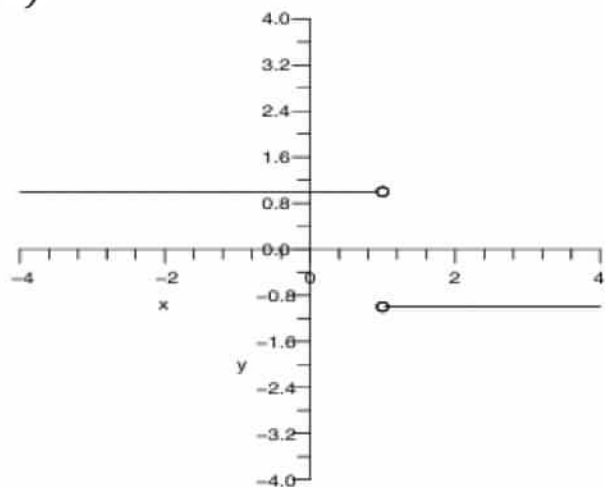
a)



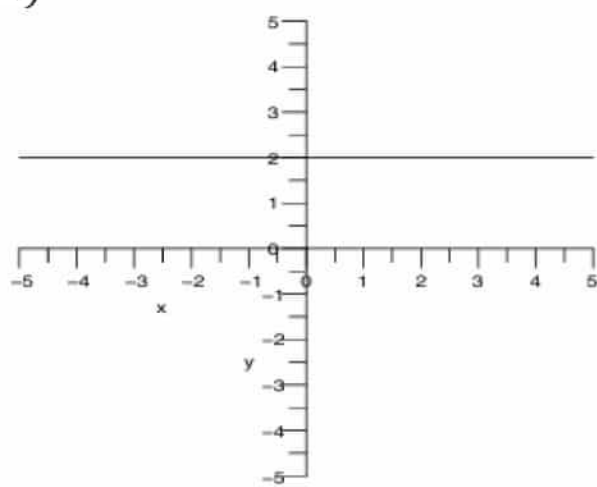
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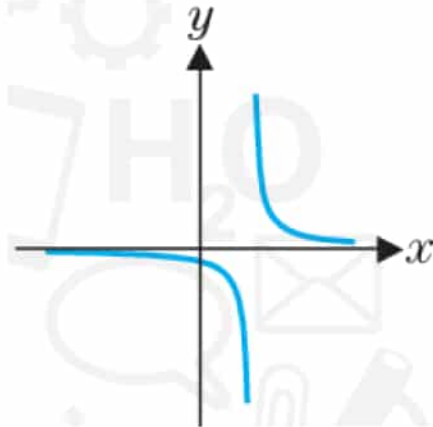
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d)



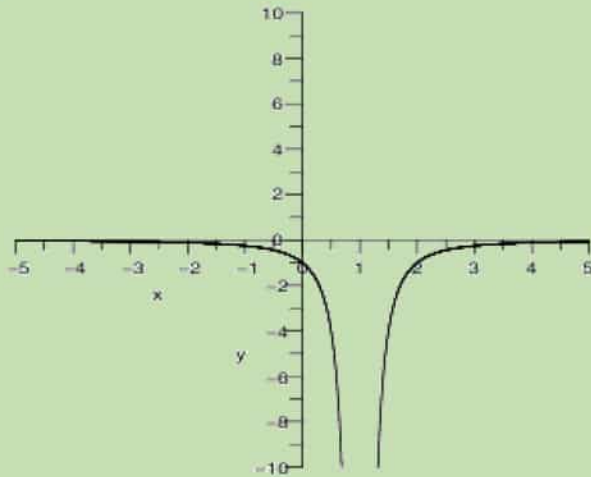
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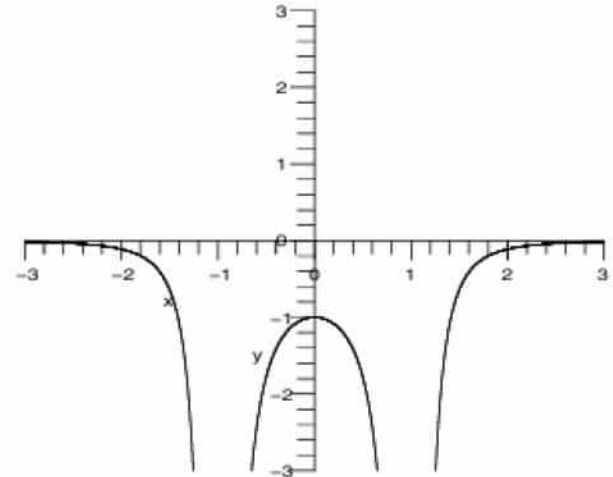
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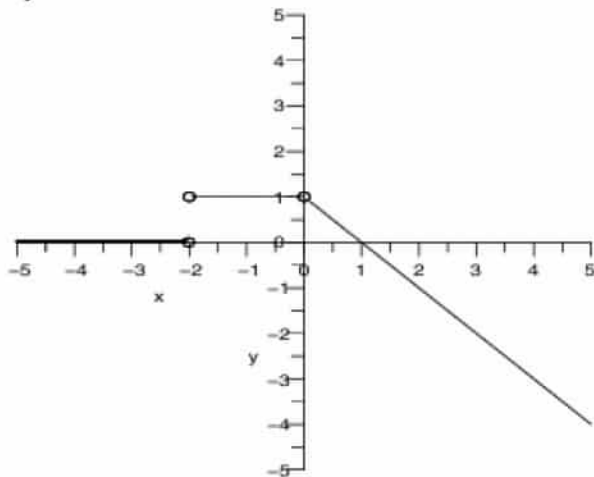
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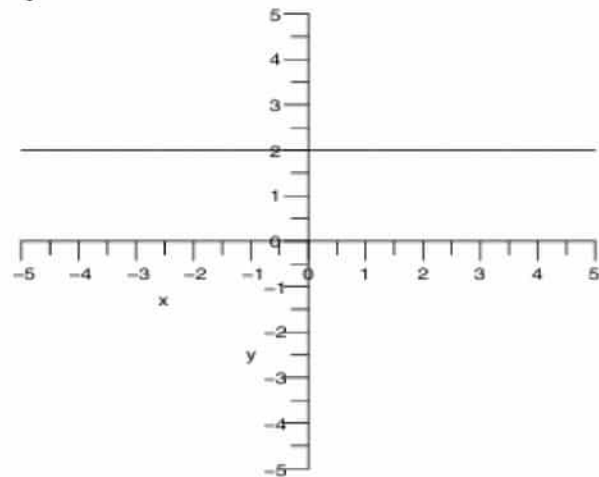
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c)

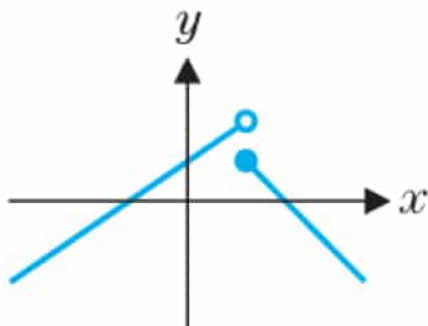


d)



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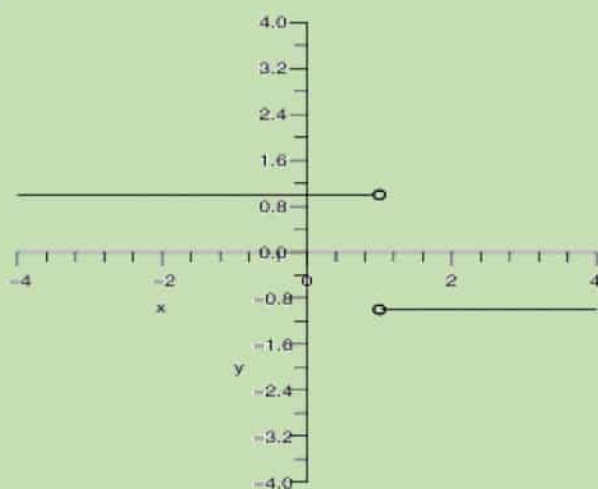
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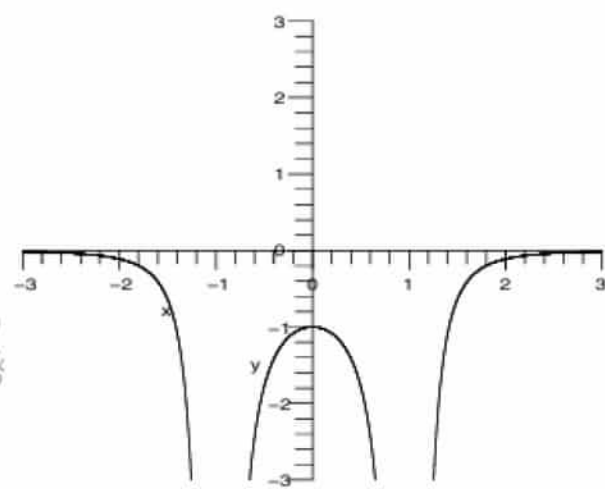
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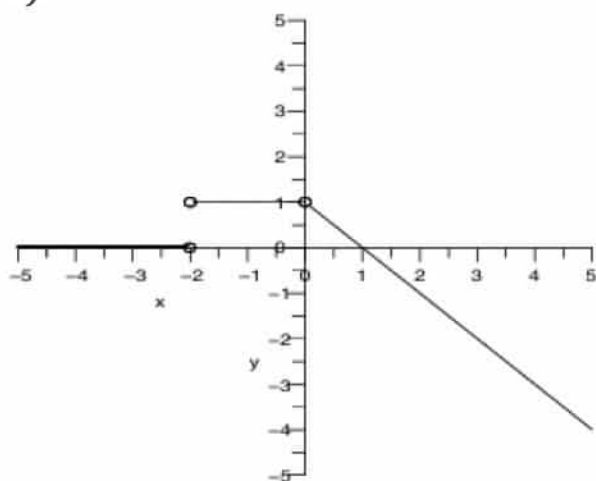
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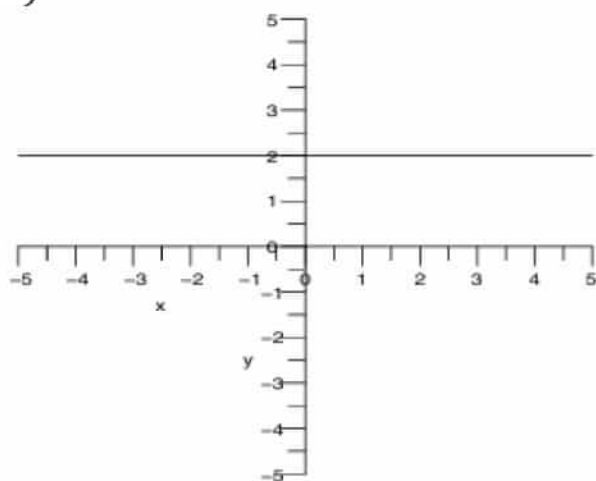
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c)

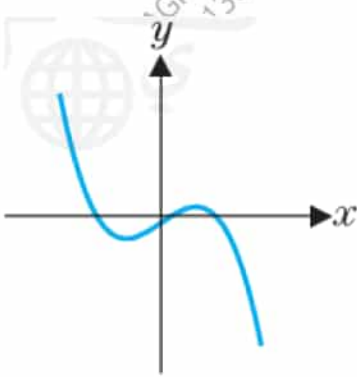


d)

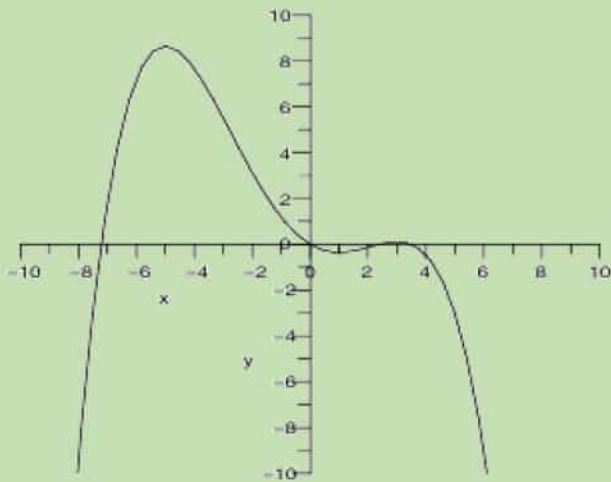


Q. Use the given graph of f' to sketch a plausible graph of a continuous function f .

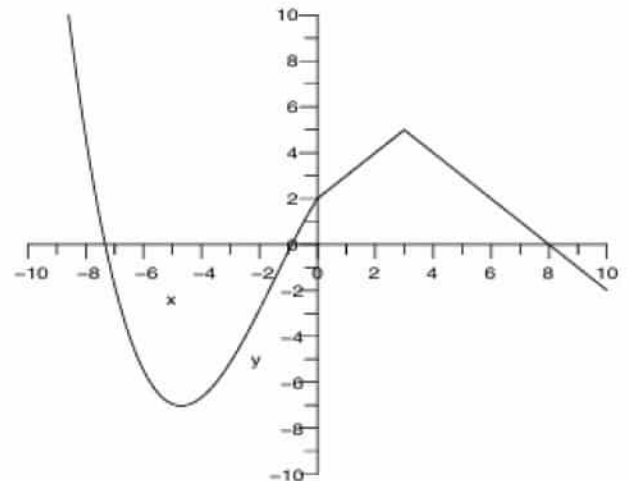
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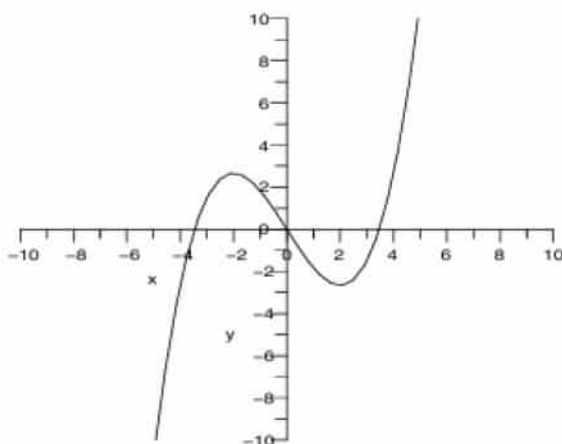
a)



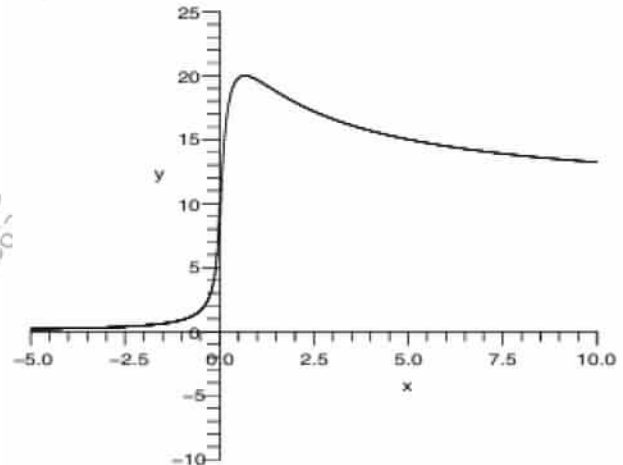
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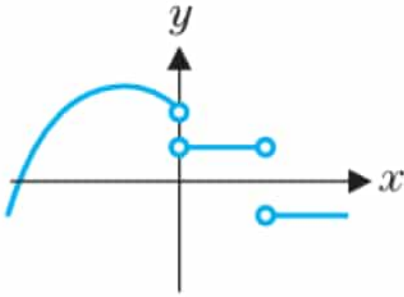
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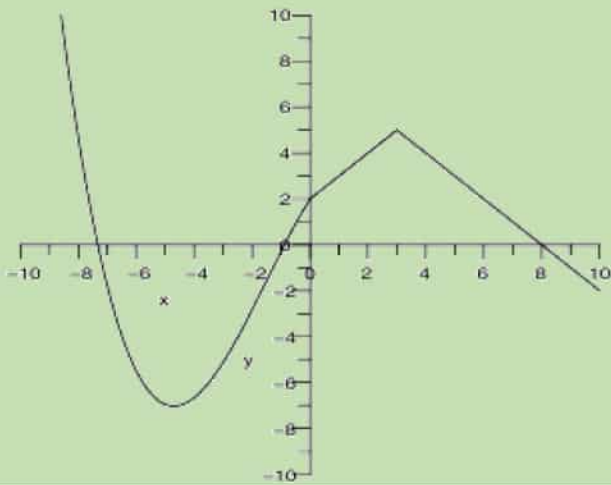
d)



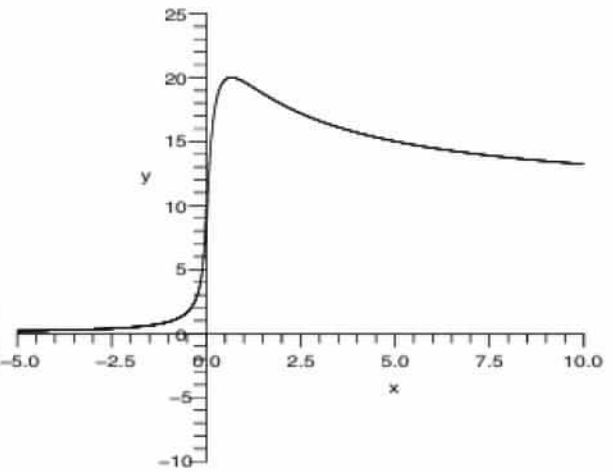
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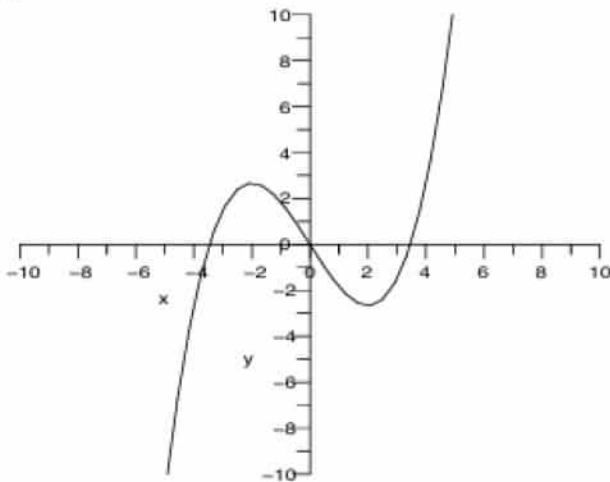
a)



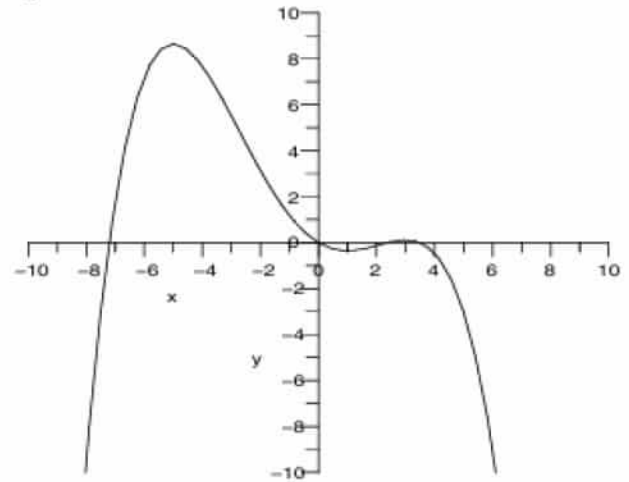
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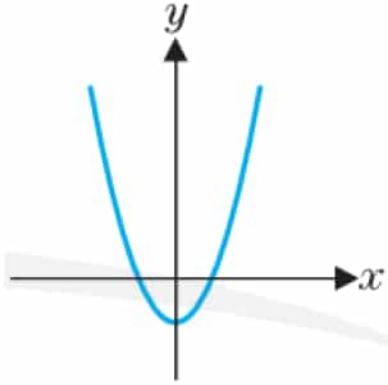
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d)



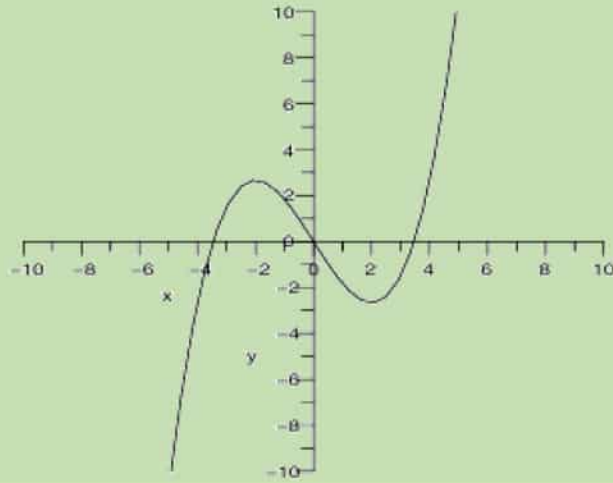
3.



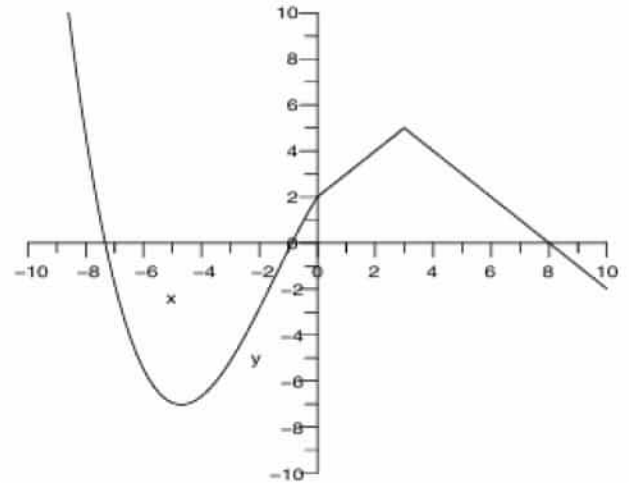
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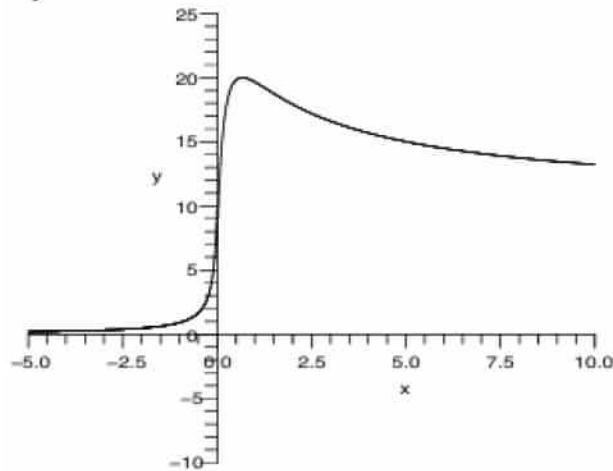
a)



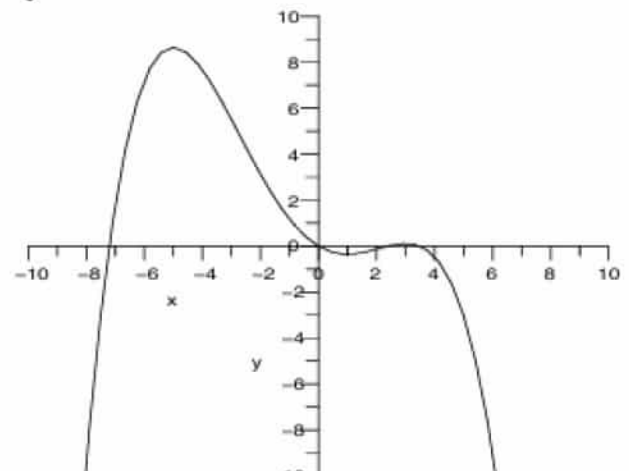
b)



c)

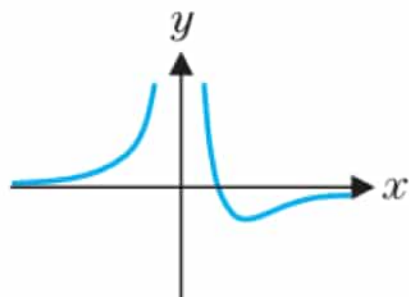


d)



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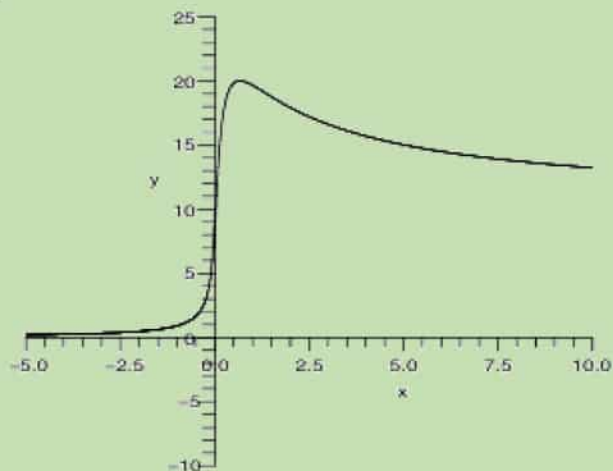
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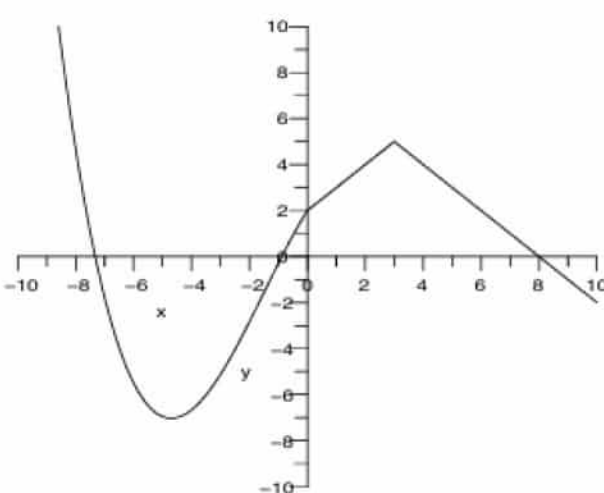
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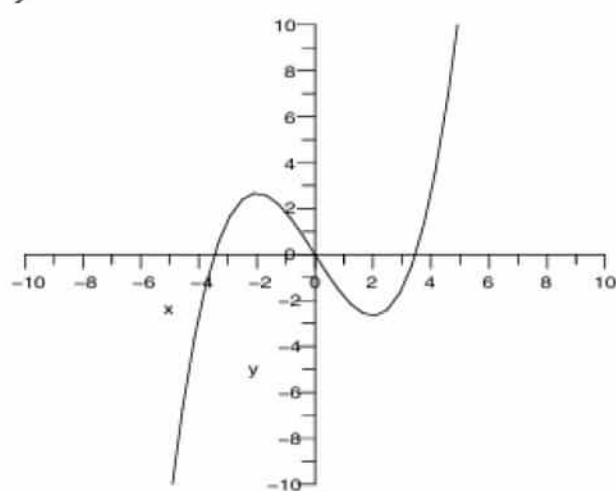
a)



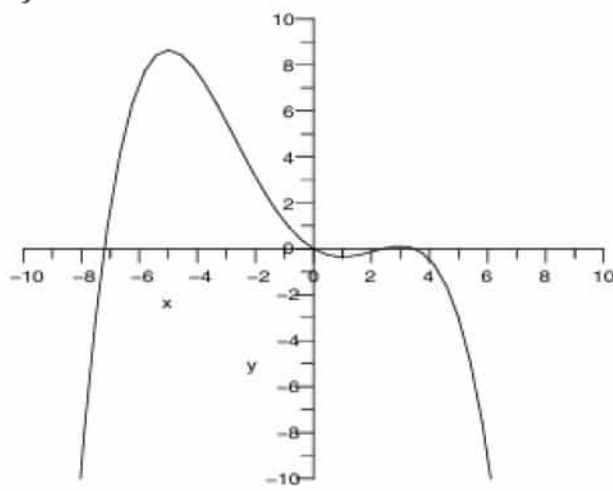
b)



c)

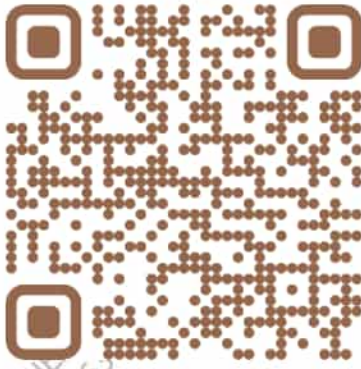


d)



Question 11: Use differentiation rules and
higher derivatives in solving real-life problems
(Page 161)

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Mr. Aghead

Q. Use the given position function to find the velocity and acceleration functions.

1. $s(t) = -16t^2 + 40t + 10$

a)	b)
$v(t) = s'(t) = -32t + 40$ $a(t) = v'(t) = s''(t) = -32$	$v(t) = s'(t) = 32t - 40$ $a(t) = v'(t) = s''(t) = -32$
c)	d)
$v(t) = s'(t) = -32t + 40$ $a(t) = v'(t) = s''(t) = 30$	$v(t) = s'(t) = -32t$ $a(t) = v'(t) = s''(t) = 0$

2. $s(t) = -4.9t^2 + 12t - 3$

a)	b)
$v(t) = s'(t) = -9.8t + 12$ $a(t) = v'(t) = s''(t) = -9.8$	$v(t) = s'(t) = +9.8t - 12$ $a(t) = v'(t) = s''(t) = 9.8$
c)	d)
$v(t) = s'(t) = \frac{1}{2}t^{-\frac{1}{2}} + 4t$ $a(t) = v'(t) = -\frac{1}{4}t^{-\frac{3}{2}} + 4$	$v(t) = s'(t) = 10t^{-2}$ $a(t) = v'(t) = s''(t) = -20t^{-3}$

3. $s(t) = \sqrt{t} + 2t^2$

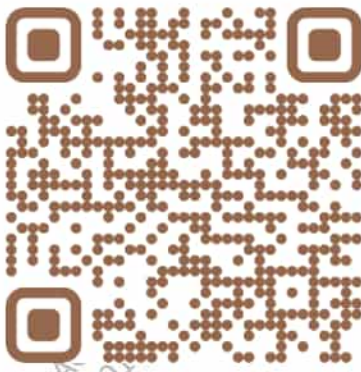
a)	b)
$v(t) = s'(t) = \frac{1}{2}t^{-\frac{1}{2}} + 4t$ $a(t) = v'(t) = -\frac{1}{4}t^{-\frac{3}{2}} + 4$	$v(t) = s'(t) = -9.8t + 12$ $a(t) = v'(t) = s''(t) = -9.8$
c)	d)
$v(t) = s'(t) = 10t^{-2}$ $a(t) = v'(t) = s''(t) = -20t^{-3}$	$v(t) = s'(t) = 9.8t + 12$ $a(t) = v'(t) = s''(t) = 9.8$

4. $s(t) = 10 - \frac{10}{t}$

a)	b)
$v(t) = s'(t) = 10t^{-2}$ $a(t) = v'(t) = s''(t) = -20t^{-3}$	$v(t) = s'(t) = -9.8t + 12$ $a(t) = v'(t) = s''(t) = -9.8$
c)	d)
$v(t) = s'(t) = \frac{1}{2}t^{-\frac{1}{2}} + 4t$ $a(t) = v'(t) = -\frac{1}{4}t^{-\frac{3}{2}} + 4$	$v(t) = s'(t) = t^{-2}$ $a(t) = v'(t) = s''(t) = 20t^{-3}$

Question 12: Use differentiation rules and
higher derivatives in solving real-life problems
(Page 161)

الأسئلة الموضوعية - MCQ



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Mr. Aghead

Q. Find all values of x for which the tangent line to $y = x^3 - 3x + 1$ is (a) at an angle of 45° with the x -axis; (b) at an angle of 30° with the x -axis, assuming that the angles are measured counterclockwise.

a)

a) $x = \frac{2}{\sqrt{3}}$ or $x = -\frac{2}{\sqrt{3}}$	b) $x = \frac{1}{\sqrt{3}}$ or $x = -\frac{1}{\sqrt{3}}$
c) $x = \left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$ or $x = -\left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$	d) $x = \left(1 - \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$ or $x = -\left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$

$$y' = 3x^2 - 3 = 3(x^2 - 1)$$

The tangent line to $y = f(x)$ intersects the x -axis at a 45° angle when $f'(x) = 1$

$$3(x^2 - 1) = 1$$

$$x^2 = 1 + \frac{1}{3}$$

$$x = \frac{2}{\sqrt{3}} \text{ or } x = -\frac{2}{\sqrt{3}}$$

b)

a) $x = \left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$ or $x = -\left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$	b) $x = \frac{2}{\sqrt{3}}$ or $x = -\frac{2}{\sqrt{3}}$
c) $x = \frac{2}{\sqrt{2}}$ or $x = -\frac{2}{\sqrt{2}}$	d) $x = \left(1 - \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$ or $x = -\left(1 - \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$

The tangent line to $y = f(x)$ intersects the x -axis at a 30° angle when

$$f'(x) = \frac{1}{\sqrt{3}}$$

$$3(x^2 - 1) = \frac{1}{\sqrt{3}}$$

$$x^2 = 1 + \frac{1}{3\sqrt{3}}$$

$$x = \left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}} \text{ or } x = -\left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{2}}$$

Q. Find all values of x for which the tangent lines to $y = x^3 + 2x + 1$ and $y = x^4 + x^3 + 3$ are (a) parallel; (b) perpendicular.

Q. Find a second-degree polynomial (of the form $ax^2 + bx + c$) such that (a) $f(0) = -2$, $f'(0) = 2$ and $f''(0) = 3$. (b) $f(0) = 0$, $f'(0) = 5$ and $f''(0) = 1$.

a) $\frac{3}{2}x^2 + 2x - 2$

b) $x^2 + 2x - 2$

c) $\frac{3}{2}x^2 - 2x - 2$

d) $\frac{3}{2}x^2 + 2x + 2$

$$f(x) = ax^2 + bx + c, \quad f(0) = c$$

$$f'(x) = 2ax + b, \quad f'(0) = b$$

$$f''(x) = 2a, \quad f''(0) = 2a$$

Given $f''(0) = 3$, we learn $2a = 3$, or $a = \frac{3}{2}$.

Given $f'(0) = 2$ we learn $b = 2$,

And given $f(0) = -2$, we learn $c = -2$.

In the end $f(x) = ax^2 + bx + c = \frac{3}{2}x^2 + 2x - 2$

Q. Find a general formula for the n th derivative $f^{(n)}(x)$ for:

a) $f(x) = \sqrt{x}$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f''(x) = \frac{1}{2} \left(-\frac{1}{2} \right) x^{-\frac{3}{2}}$$

$$f'''(x) = \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^{-\frac{5}{2}}$$

$$f^{(n)}(x) = (-1)^{n-1} \frac{1.3.5 \dots (2n-3)}{2^n} x^{-\frac{2n-1}{2}}$$

$$f^{(n)}(x) = (-1)^{n-1} \frac{1.2.3 \dots (2n-2)}{2^n \cdot 2.4 \dots (2n-2)} x^{-\frac{2n-1}{2}}$$

$$f^{(n)}(x) = (-1)^{n-1} \frac{(2n-2)!}{2^{2n-1}(n-1)!} x^{-\frac{2n-1}{2}}$$

b) $f(x) = \frac{2}{x}$

$$f'(x) = -2x^{-3}$$

$$f''(x) = 6x^{-4}$$

$$f'''(x) = -24x^{-5}$$

The pattern is:

$$f^{(n)}(x) = (-1)^n (n+1)! x^{-n-2}$$

Q. Find the area of the triangle bounded by $x = 0$, $y = 0$ and the tangent line to $y = \frac{1}{x}$ at $x = 1$. Repeat with the triangle bounded by $x = 0$, $y = 0$ and the tangent line to $y = \frac{1}{x}$ at $x = 2$. Show that you get the same area using the tangent line to $y = \frac{1}{x}$ at any $x = a > 0$.

For $y = \frac{1}{x}$, we have $y' = -\frac{1}{x^2}$. Thus, the slope of the tangent line at $x = a$ is $-\frac{1}{a^2}$.

When $a = 1$, the slope of the tangent line at $(1, 1)$ is -1 , and the equation of the tangent line is $y = -x + 2$. The tangent line intersects the axes at $(0, 2)$ and $(2, 0)$. Thus, the area of the triangle is $\frac{1}{2}(2)(2) = 2$.

When $a = 2$, the slope of the tangent line at $(2, \frac{1}{2})$ is $-\frac{1}{4}$, and the equation of the tangent line is $y = -\frac{1}{4}x + 2$. The tangent line intersects the axes at $(0, 1)$ and $(4, 0)$. Thus, the area of the triangle is $\frac{1}{2}(4)(1) = 2$.

In general, the equation of the tangent line is $y = -\left(\frac{1}{a^2}\right)x + \frac{2}{a}$. The tangent line intersects the axes at $(0, \frac{2}{a})$ and $(2a, 0)$. Thus, the area of the triangle is $\frac{1}{2}(2a)\left(\frac{2}{a}\right) = 2$.

Q. Show that the result of the previous exercise does not hold for $y = \frac{1}{x^2}$. That is, the area of the triangle bounded by $x = 0$, $y = 0$ and the tangent line to $y = \frac{1}{x^2}$ at $x = a > 0$ does depend on the value of a .

For $y = \frac{1}{x^2} = x^{-2}$, we have $f'(x) = -2x^{-3} = -\frac{2}{x^3}$.

Thus, the slope of the tangent line at $x = a$ is $-\frac{2}{a^3}$.

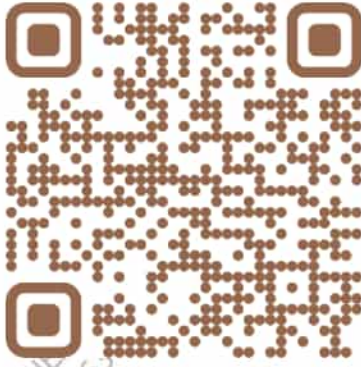
When $a = 1$, the slope of the tangent line at $(1, 1)$ is -2 , and the equation of the tangent line is $y = -2x + 3$. The tangent line intersects the axes at $(0, 3)$ and $(\frac{3}{2}, 0)$. Thus the area of the triangle is $\frac{1}{2}(3)(\frac{3}{2}) = \frac{9}{4}$.

When $a = 2$, the slope of the tangent line at $(2, \frac{1}{4})$ is $-\frac{1}{4}$, and the equation of the tangent line is $y = -\frac{1}{4}x + \frac{3}{4}$. The tangent line intersects the axes at $(0, \frac{3}{4})$ and $(3, 0)$. Thus the area of the triangle is $\frac{1}{2}(\frac{3}{4})(3) = \frac{9}{8}$.

Since $\frac{9}{4} \neq \frac{9}{8}$, we see that the result for previous exercise does not hold here.

Question 13: Apply the chain rule for
differentiation (Page 177)

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Mr. Aghead

Q. Use the relevant information to compute the derivative for $h(x) = f(g(x))$.

a. $h'(1)$, where $f(1) = 3, g(1) = 2, f'(1) = 4, f'(2) = 3, g'(1) = -2$, and $g'(3) = 5$.

a) -6	b) 3	c) 0	d) 1
-------	------	------	------

$$h'(x) = f'(g(x))g'(x)$$
$$h'(1) = f'(g(1))g'(1) = f'(2) \cdot (-2) = -6$$

b. $h'(2)$, where $f(2) = 1, g(2) = 3, f'(2) = -1, f'(3) = -3, g'(1) = 2$, and $g'(2) = 4$.

a) -12	b) 12	c) 0	d) 1
--------	-------	------	------

$$h'(x) = f'(g(x))g'(x)$$
$$h'(2) = f'(g(2))g'(2) = f'(3) \cdot (4) = -12$$

Q. A function f is an even function if $f(-x) = f(x)$ for all x and is an odd function if $f(-x) = -f(x)$ for all x . Prove that the derivative of an even function is odd and the derivative of an odd function is even.

As a temporary device given any f , set $g(x) = f(-x)$. Then by the chain rule,

$$g'(x) = f'(-x)(-1) = -f'(-x)$$

In the even case ($g = f$) this reads $f'(-x) = -f'(x)$ and shows f' is odd.

In the odd case ($g = -f$ and therefore $g' = -f'$), this reads

$-f'(x) = -f'(-x)$ or $f'(x) = f'(-x)$ and shows f' is even.

Q. In the graph of a differentiable function f is symmetric about the line $x = a$, what can you say about the symmetry of the graph of f' .

To say that $f(x)$ is symmetric about the line $x = a$ is the same as saying that $f(a+x) = f(a-x)$. Taking derivatives (using the chain rule), we have:

$$\frac{d}{dx} f(a+x) = f'(a+x)$$

$$\frac{d}{dx} f(a-x) = f'(a-x)(-1) = -f'(a-x)$$

Thus, $f'(a+x) = -f'(a-x)$ and the graph of $f'(x)$ is symmetric through the point $(a, 0)$.

Q. Find the derivative where f is an unspecified differentiable function.

1.

a) $f(x^2)$

a) $2xf'(x^2)$

b) $2f(x)f'(x)$

c) $f'(f(x))f'(x)$

d) $f'(\sqrt{x})\frac{1}{2\sqrt{x}}$

Chain rule gives:

$$\frac{d}{dx}f(x^2) = f'(x^2)\frac{d}{dx}(x^2) = f'(x^2)(2x) = 2xf'(x^2)$$

b) $[f(x)]^2$

a) $2f(x)f'(x)$

b) $2xf'(x^2)$

c) $f'(f(x))f'(x)$

d) $f'(\sqrt{x})\frac{1}{2\sqrt{x}}$

Chain rule gives:

$$\frac{d}{dx}[f(x)]^2 = 2f(x)\frac{d}{dx}f(x) = 2f(x)f'(x)$$

c) $f(f(x))$

a) $f'(f(x))f'(x)$	b) $2xf'(x^2)$	c) $2f(x)f'(x)$	d) $f'(\sqrt{x})\frac{1}{2\sqrt{x}}$
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Chain rule gives:

$$\frac{d}{dx}f(f(x)) = f'(f(x))\frac{d}{dx}f(x) = f'(f(x))f'(x)$$

2.

a) $f(\sqrt{x})$

a) $f'(\sqrt{x})\frac{1}{2\sqrt{x}}$	b) $2xf'(x^2)$	c) $2f(x)f'(x)$	d) $f'(f(x))f'(x)$
--------------------------------------	----------------	-----------------	--------------------

Chain rule gives:

$$\frac{d}{dx}f(\sqrt{x}) = f'(\sqrt{x})\frac{d}{dx}(\sqrt{x}) = f'(\sqrt{x})\frac{1}{2\sqrt{x}}$$

b) $\sqrt{f(x)}$

a) $\frac{1}{2\sqrt{f(x)}} f'(x)$	b) $-\left(\frac{1}{x^2}\right) \cdot f' \left(\frac{1}{x}\right)$	c) $\left(-\frac{1}{f(x)^2}\right) \cdot f'(x)$	d) $\frac{1}{2\sqrt{f(x)}} f'(x)$
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Chain rule gives:

$$\frac{d}{dx} (\sqrt{f(x)}) = \frac{1}{2\sqrt{f(x)}} \frac{d}{dx} f(x) = \frac{1}{2\sqrt{f(x)}} f'(x)$$

c) $f(xf(x))$

a) $f'(xf(x))(f(x) + xf'(x))$	b) $\frac{1}{2\sqrt{f(x)}} f'(x)$
c) $-\left(\frac{1}{x^2}\right) \cdot f' \left(\frac{1}{x}\right)$	d) $\left(-\frac{1}{f(x)^2}\right) \cdot f'(x)$

Chain rule gives:

$$\frac{d}{dx} [f(xf(x))] = f'(xf(x)) \frac{d}{dx} (xf(x))$$

and product rule gives:

$$= f'(xf(x)) \left(f(x) + x \frac{d}{dx} f(x) \right) = f'(xf(x))(f(x) + xf'(x))$$

3.

a) $f\left(\frac{1}{x}\right)$

a) $-\left(\frac{1}{x^2}\right) \cdot f'\left(\frac{1}{x}\right)$	b) $\frac{1}{2\sqrt{f(x)}} f'(x)$	c) $\left(-\frac{1}{f(x)^2}\right) \cdot f'(x)$	d) $\left(\frac{1}{x^2}\right) \cdot f'\left(\frac{1}{x}\right)$
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Chain rule gives:

$$\frac{d}{dx} \left[f\left(\frac{1}{x}\right) \right] = f'\left(\frac{1}{x}\right) \cdot \frac{d}{dx} \left(\frac{1}{x}\right) = f'\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) = -\left(\frac{1}{x^2}\right) \cdot f'\left(\frac{1}{x}\right)$$

b) $\frac{1}{f(x)}$

a) $\left(-\frac{1}{f(x)^2}\right) \cdot f'(x)$	b) $\frac{1}{2\sqrt{f(x)}} f'(x)$	c) $-\left(\frac{1}{x^2}\right) \cdot f'\left(\frac{1}{x}\right)$	d) $\left(\frac{1}{f(x)^2}\right) \cdot f'(x)$
---	-----------------------------------	---	--

Chain rule gives:

$$\frac{d}{dx} \left(\frac{1}{f(x)} \right) = \left(-\frac{1}{f(x)^2} \right) \cdot \frac{d}{dx} f(x) = \left(-\frac{1}{f(x)^2} \right) \cdot f'(x)$$

c) $f\left(\frac{x}{f(x)}\right)$

a) $f'\left(\frac{x}{f(x)}\right)\left(\frac{f(x)-xf'(x)}{[f(x)]^2}\right)$	b) $2xf'(x^2)$
c) $f'(x)f'(1+f(x))$	d) $xf'(x^2)$

Chain rule gives:

$$\frac{d}{dx}\left[f\left(\frac{x}{f(x)}\right)\right] = f'\left(\frac{x}{f(x)}\right)\frac{d}{dx}\left(\frac{x}{f(x)}\right)$$

and quotient rule gives

$$= f'\left(\frac{x}{f(x)}\right)\left(\frac{f(x) - xf'(x)}{[f(x)]^2}\right)$$

4.

a) $1 + f(x^2)$

a) $2xf'(x^2)$	b) $xf'(x^2)$	c) $x^2f'(x)$	d) $xf'(x^3)$
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Chain rule gives:

$$\frac{d}{dx}(1 + f(x^2)) = f'(x^2)\frac{d}{dx}(x^2) = f'(x^2)(2x) = 2xf'(x^2)$$

b) $|1 + f(x)|^2$

a) $2f'(x)[1 + f(x)]$	b) $f' \left(\frac{x}{f(x)} \right) \left(\frac{f(x) - xf'(x)}{[f(x)]^2} \right)$
c) $2xf'(x^2)$	d) $f'(x)f'(1 + f(x))$

Chain rule gives:

$$\begin{aligned} \frac{d}{dx} [1 + f(x)]^2 &= 2[1 + f(x)] \frac{d}{dx} (1 + f(x)) = 2[1 + f(x)]f'(x) \\ &= 2f'(x)[1 + f(x)] \end{aligned}$$

c) $f(1 + f(x))$

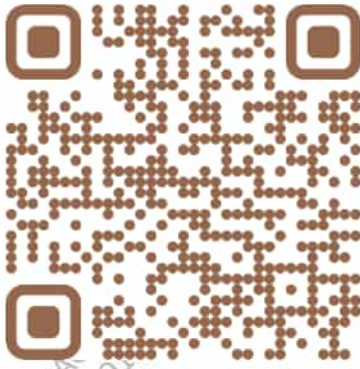
a) $f'(x)f'(1 + f(x))$	b) $f' \left(\frac{x}{f(x)} \right) \left(\frac{f(x) - xf'(x)}{[f(x)]^2} \right)$
c) $2xf'(x^2)$	d) $f'(x)f'(f(x))$

Chain rule gives:

$$\begin{aligned} \frac{d}{dx} [f(1 + f(x))] &= f'(1 + f(x)) \cdot \frac{d}{dx} (1 + f(x)) = f'(1 + f(x))f'(x) \\ &= f'(x)f'(1 + f(x)) \end{aligned}$$

Question 14: Find the derivative of an
inverse function using the Chain Rule
(Page 176)

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Mr. Aghead

Q. f has an inverse g . Use Theorem 5.2 to find $g'(a)$.

1. $f(x) = x^3 + 4x - 1$, $a = -1$

a) $\frac{1}{4}$	b) 0	c) 2	d) 1
------------------	------	------	------

$f(x) = x^3 + 4x - 1$ is a one-to-one function with $f(0) = -1$ and $f'(0) = 4$.

Therefore $g(-1) = 0$ and

$$g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(0)} = \frac{1}{4}$$

2. $f(x) = x^5 + 4x - 2$, $a = -2$

a) $\frac{1}{4}$	b) 0	c) 2	d) 1
------------------	------	------	------

$f(x) = x^5 + 4x - 2$ is a one-to-one function with $f(0) = -2$ and $f'(0) = 4$.

Therefore $g(-2) = 0$ and

$$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(0)} = \frac{1}{4}$$

3. $f(x) = x^5 + 3x^3 + x$, $a = 5$

a) $\frac{1}{15}$	b) 15	c) 2	d) 1
-------------------	-------	------	------

$f(x) = x^5 + 3x^3 + x$ is a one-to-one function with $f(1) = 5$ and

$f'(1) = 5 + 9 + 1 = 15$.

Therefore $g(5) = 1$ and

$$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(1)} = \frac{1}{15}$$

4. $f(x) = x^3 + 2x + 1$, $a = -2$

a) $\frac{1}{5}$	b) 5	c) 0	d) 1
------------------	------	------	------

$f(x) = x^3 + 2x + 1$ is a one-to-one function with $f(-1) = -2$ and

$f'(-1) = 5$. Therefore $g(-2) = -1$ and

$$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(-1)} = \frac{1}{5}$$

5. $f(x) = \sqrt{x^3 + 2x + 4}$, $A = 2$

a) 2	b) 1	c) 0	d) -2
------	------	------	-------

$f(x) = \sqrt{x^3 + 2x + 4}$ is a one-to-one function and $f(0) = 2$ so $g(2) = 0$.
Meanwhile,

$$f'(x) = \frac{1}{2\sqrt{x^3 + 2x + 4}} (3x^2 + 2)$$

$$f'(0) = \frac{1}{2}$$

$$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(0)} = 2$$

6. $f(x) = \sqrt{x^5 + 4x^3 + 3x + 1}$, $a = 3$

a) $\frac{3}{10}$	b) 10	c) 3	d) 1
-------------------	-------	------	------

$f(x) = \sqrt{x^5 + 4x^3 + 3x + 1}$ is a one-to-one function and $f(1) = 3$ so
 $g(3) = 1$. Meanwhile,

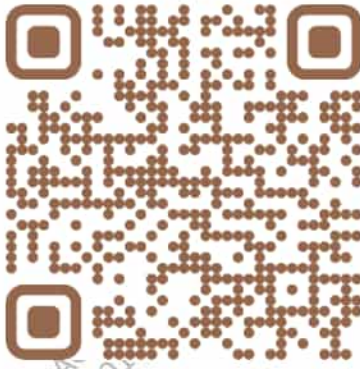
$$f'(x) = \frac{5x^4 + 12x^2 + 3}{2\sqrt{x^5 + 4x^3 + 3x + 1}}$$

$$f'(1) = \frac{20}{6} = \frac{10}{3}$$

$$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \frac{3}{10}$$

Question 15: Find the derivatives of
trigonometric functions using differentiation
rules (Page 184)

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Mr. Aghead

Q. Find the derivative of each function.

1. $f(t) = \sin 3t \sec 3t$

a) $3 \sec^2(3t)$

b) 0

c) $\frac{-4 \cos 4w}{\sin^2 4w}$

d) $6w^2 \sec^2 3w$

$$f(t) = \tan 3t$$

$$f'(t) = \frac{d}{dt} [\tan(3t)] = \sec^2(3t)(3) = 3 \sec^2(3t)$$

2. $f(t) = \sqrt{\cos 5t \sec 5t}$

a) 0

b) $3 \sec^2(3t)$

c) $\frac{-4 \cos 4w}{\sin^2 4w}$

d) $6w^2 \sec^2 3w$

$$f(t) = \sqrt{\cos 5t \cdot \frac{1}{(\cos 5t)}} = 1$$

$$f'(t) = \frac{d}{dt} (1) = 0$$

3. $f(w) = \frac{1}{\sin 4w}$

a) $\frac{-4 \cos 4w}{\sin^2 4w}$	b) $3 \sec^2(3t)$	c) 0	d) $6w^2 \sec^2 3w$
-----------------------------------	-------------------	------	---------------------

$$f'(w) = \frac{-1}{(\sin 4w)^2} \cos 4w (4) = \frac{-4 \cos 4w}{\sin^2 4w}$$

4. $f(w) = w^2 \sec^2 3w$

a) $6w^2 \sec^2 3w \tan 3w + 2w \sec^2 3w$
b) $3 \sec^2(3t)$
c) 0
d) $\frac{-4 \cos 4w}{\sin^2 4w}$

$$f'(w) = w^2 (2 \sec 3w) (\sec 3w \tan 3w) (3) + \sec^2(3w) (2w) \\ = 6w^2 \sec^2 3w \tan 3w + 2w \sec^2 3w$$

5. $f(x) = 2 \sin 2x \cos 2x$

a) $4 \cos^2(2x) - 4 \sin^2(2x)$	b) 0
c) $\sec^2 \sqrt{x^2 + 1}$	d) $8x \sin x \sec 3x$

$$f'(x) = 2\{\sin(2x)[- \sin(2x)](2) + \cos(2x)[\cos(2x)](2)\}$$

$$= -4 \sin^2(2x) + 4 \cos^2(2x) = 4 \cos^2(2x) - 4 \sin^2(2x)$$

6. $f(x) = 4 \sin^2 3x + 4 \cos^2 3x$

a) 0	b) $4 \cos^2(2x)$	c) $\sec^2 \sqrt{x^2 + 1}$	d) $8x \sin x \sec 3x$
------	-------------------	----------------------------	------------------------

$$f(x) = 4[\sin^2(3x) + \cos^2(3x)] = 4$$

$$f'(x) = \frac{d}{dx}(4) = 0$$

7. $f(x) = \tan \sqrt{x^2 + 1}$

a) $\frac{x}{\sqrt{x^2+1}} \sec^2 \sqrt{x^2 + 1}$

b) 0

c) $4 \cos^2(2x)$

d) $8x \sin x \sec 3x$

$$f'(x) = \left(\sec^2 \sqrt{x^2 + 1} \right) \cdot \left(\frac{1}{2} \right) (x^2 + 1)^{-\frac{1}{2}} (2x) = \frac{x}{\sqrt{x^2 + 1}} \sec^2 \sqrt{x^2 + 1}$$

8. $f(x) = 4x^2 \sin x \sec 3x$

a) $8x \sin x \sec 3x + 4x^2 [\cos x \sec 3x + \sin x \sec 3x \tan 3x (3)]$

b) 0

c) $4 \cos^2(2x)$

d) $\sec^2 \sqrt{x^2 + 1}$

9. $f(x) = \sin^3(\cos \sqrt{x^3 + 2x^2})$

a) $\frac{3}{2}(3x^2 + 4x)(x^3 + 2x^2)^{-\frac{1}{2}} \cdot \sin^2(\cos \sqrt{x^3 + 2x^2})$
 $\cdot \cos(\cos \sqrt{x^3 + 2x^2}) \cdot (-\sin \sqrt{x^3 + 2x^2})$

b) $4[\tan^3(\sin^2(x^3 + 2x))]. [\sec^2(\sin^2(x^3 + 2x))]. [2 \sin(x^3 + 2x)]$
 $\cdot [\cos(x^3 + 2x)]. (3x^2 + 2)$

c) $-(x^2 - x)(x^3 - 4x^2)^{-\frac{1}{2}} \cdot \sin^2(\cos \sqrt{x^3 - 2x^2})$
 $\cdot \cos(-\cos \sqrt{x^3 + 2x^2}) \cdot (\sin \sqrt{x^3 2x^2})$

d) $8[\tan^3(\sin^2(x^3 - x))]. [\sec^2(\sin^2(x^3 - x))]. [2 \sin(x^3 + 2x)]$
 $\cdot [\cos(x^3 + 2x)]. (3x^2 + 2)$

$f'(x) = 3 \sin^2(\cos \sqrt{x^3 + 2x^2}) \cdot \cos(\cos \sqrt{x^3 + 2x^2}) \cdot (-\sin \sqrt{x^3 + 2x^2})$

$\cdot \frac{1}{2}(x^3 + 2x^2)^{-\frac{1}{2}}(3x^2 + 4x)$
 $= \frac{3}{2}(3x^2 + 4x)(x^3 + 2x^2)^{-\frac{1}{2}} \cdot \sin^2(\cos \sqrt{x^3 + 2x^2})$
 $\cdot \cos(\cos \sqrt{x^3 + 2x^2}) \cdot (-\sin \sqrt{x^3 + 2x^2})$

10. $f(x) = \tan^4(\sin^2(x^3 + 2x))$

a) $4[\tan^3(\sin^2(x^3 + 2x))]. [\sec^2(\sin^2(x^3 + 2x))]. [2 \sin(x^3 + 2x)]$
 $. [\cos(x^3 + 2x)]. (3x^2 + 2)$

b) $\frac{3}{2}(3x^2 + 4x)(x^3 + 2x^2)^{-\frac{1}{2}}. \sin^2(\cos \sqrt{x^3 + 2x^2})$
 $. \cos(\cos \sqrt{x^3 + 2x^2}). (-\sin \sqrt{x^3 + 2x^2})$

c) $8[\tan^3(\sin^2(x^3 - x))]. [\sec^2(\sin^2(x^3 - x))]. [2 \sin(x^3 + 2x)]$
 $. [\cos(x^3 + 2x)]. (3x^2 + 2)$

d) $-(x^2 - x)(x^3 - 4x^2)^{-\frac{1}{2}}. \sin^2(\cos \sqrt{x^3 - 2x^2})$
 $. \cos(-\cos \sqrt{x^3 + 2x^2}). (\sin \sqrt{x^3 2x^2})$

Question 16: Find the derivatives of
natural logarithmic functions (Page 194)

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Mr. Aghead

Q. Use logarithmic differentiation to find the derivative.

1. $f(x) = x^{\sin x}$

a) $x^{\sin x} \left(\frac{x \cos x \cdot \ln x + \sin x}{x} \right)$	b) $(\sin x)^x \cdot (x \cot x + \ln(\sin x))$
c) $x^{4-x^2} \left(-2x \ln x + (4 - x^2) \frac{1}{x} \right)$	d) $(x^2)^{4x} (8 \ln x + 8)$

$$\ln f(x) = \sin x \cdot \ln x$$

$$\frac{f'(x)}{f(x)} = \frac{d}{dx} (\sin x \cdot \ln x) = \cos x \cdot \ln x + \frac{\sin x}{x}$$

$$f'(x) = x^{\sin x} \left(\frac{x \cos x \cdot \ln x + \sin x}{x} \right)$$

2. $f(x) = x^{4-x^2}$

a) $x^{4-x^2} \left(-2x \ln x + (4 - x^2) \frac{1}{x} \right)$	b) $(x^2)^{4x} (8 \ln x + 8)$
c) $(x^2)^{4x} (8 \ln x + 8)$	d) $x^{\sqrt{x}} \left(\frac{1}{2\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} \right)$

$$\ln f(x) = (4 - x^2) \ln x$$

$$\frac{f'(x)}{f(x)} = -2x \ln x + (4 - x^2) \frac{1}{x}$$

$$f'(x) = x^{4-x^2} \left(-2x \ln x + (4 - x^2) \frac{1}{x} \right)$$

3. $f(x) = (\sin x)^x$

a) $(\sin x)^x \cdot (x \cot x + \ln(\sin x))$	b) $x^{\sin x} \left(\frac{x \cos x \ln x + \sin x}{x} \right)$
c) $x^{4-x^2} \left(-2x \ln x + (4 - x^2) \frac{1}{x} \right)$	d) $(x^2)^{4x} (8 \ln x + 8)$

$$\ln f(x) = x \cdot \ln(\sin x)$$

$$\frac{f'(x)}{f(x)} = \frac{d}{dx} (x \cdot \ln(\sin x)) = \frac{x \cos x}{\sin x} + \ln(\sin x) = x \cot x + \ln(\sin x)$$

$$f'(x) = (\sin x)^x \cdot (x \cot x + \ln(\sin x))$$

4. $f(x) = (x^2)^{4x}$

a) $(x^2)^{4x} (8 \ln x + 8)$	b) $x^{4-x^2} \left(-2x \ln x + (4 - x^2) \frac{1}{x} \right)$
c) $(x)^{2x} (\ln x + 1)$	d) $x^{\sqrt{x}} \left(\frac{1}{2\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} \right)$

$$\ln f(x) = 8x \ln x$$

$$\frac{f'(x)}{f(x)} = 8 \ln x + 8x \frac{1}{x}$$

$$f'(x) = (x^2)^{4x} (8 \ln x + 8)$$

5. $f(x) = x^{\ln x}$

a) $2x^{[(\ln x - 1) \ln x]}$	b) $x^{[(\ln x + 1) \ln x]}$	c) $-x^{[(\ln x + 2) \ln x]}$	d) $x^{[(\ln x - 1) \ln x]}$
-------------------------------	------------------------------	-------------------------------	------------------------------

$$\begin{aligned}\ln f(x) &= \ln x \cdot \ln x = \ln^2 x \\ \frac{f'(x)}{f(x)} &= \frac{d}{dx} (\ln^2 x) = \frac{2 \ln x}{x} \\ f'(x) &= x^{\ln x} \left[\frac{2 \ln x}{x} \right] = 2x^{[(\ln x - 1) \ln x]}\end{aligned}$$

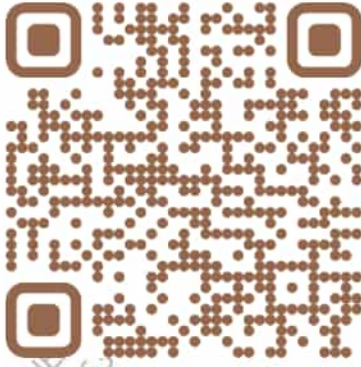
6. $f(x) = x^{\sqrt{x}}$

a) $x^{\sqrt{x}} \left(\frac{1}{2\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} \right)$	b) $2x^{[(\ln x - 1) \ln x]}$
c) $(x^2)^{4x} (8 \ln x + 8)$	d) $x^{4-x^2} \left(-2x \ln x + (4 - x^2) \frac{1}{x} \right)$

$$\begin{aligned}\ln f(x) &= \sqrt{x} \ln x \\ \frac{f'(x)}{f(x)} &= \frac{1}{2\sqrt{x}} \ln x + \sqrt{x} \frac{1}{x} \\ f'(x) &= x^{\sqrt{x}} \left(\frac{1}{2\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} \right)\end{aligned}$$

Question 17: Find the derivatives of
exponential functions (Page 193)

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Mr. Aghead

Q. Differentiate each function.

1. $f(x) = x^3 e^x$

a) $x^2 e^x (x + 3)$	b) $1 + 2^x \log 2$	c) $4^{3x} (-3x \ln 4)$	d) $8e^{4x+1}$
----------------------	---------------------	-------------------------	----------------

$$f'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = x^2 e^x (x + 3)$$

2. $f(x) = e^{2x} \cos 4x$

a) $2e^{2x} \cos 4x + e^{2x} (-\sin 4x) 4$	b) $8e^{4x+1}$
c) $1 + 2^x \log 2$	d) $x^2 e^x (x + 3)$

3. $f(t) = t + 2^t$

a) $1 + 2^t \log 2$	b) $x^2 e^t (t + 3)$	c) $-3t \ln 4$	d) $8e^{4t+1}$
---------------------	----------------------	----------------	----------------

4. $f(t) = t4^{3t}$

a) $4^{3t}(1 + 3t \ln 4)$	b) $x^2 e^t(t + 3)$
c) $1 + 2^t \log 2$	d) $8e^{4t+1}$

$$f'(t) = 4^{3t} + t4^{3t}(\ln 4)3 = 4^{3t}(1 + 3t \ln 4)$$

5. $f(x) = 2e^{4x+1}$

a) $8e^{4x+1}$	b) $x^2 e^x(x + 3)$	c) $1 + 2^t \log 2$	d) $-3t \ln 4$
----------------	---------------------	---------------------	----------------

$$f'(x) = 2e^{4x+1}(4) = 8e^{4x+1}$$

6. $f(x) = (1/e)^x$

a) $-e^{-x}$	b) $8e^{4x+1}$	c) $x^2 e^x(x + 3)$	d) $1 + 2^t \log 2$
--------------	----------------	---------------------	---------------------

7. $h(x) = (1/3)^{x^2}$

a) $-2x \cdot \ln 3 \cdot \left(\frac{1}{3}\right)^{x^2}$	b) $8e^{4x+1}$	c) $-e^{-x}$	d) $x^2 e^x (x+3)$
---	----------------	--------------	--------------------

$$h'(x) = \ln\left(\frac{1}{3}\right) \cdot 2x \cdot \left(\frac{1}{3}\right)^{x^2} = 2x \cdot \ln \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{x^2} = -2x \cdot \ln 3 \cdot \left(\frac{1}{3}\right)^{x^2}$$

8. $h(x) = 4^{-x^2}$

a) $-2x \cdot 4^{-x^2} \cdot \ln(4)$	b) $-2x \cdot \ln 3 \cdot \left(\frac{1}{3}\right)^{x^2}$	c) $2x \cdot \ln 3 \cdot \left(\frac{1}{3}\right)^{x^2}$	d) $2x \cdot 4^{-x^2} \cdot \ln(4)$
--------------------------------------	---	--	-------------------------------------

$$h'(x) = 4^{-x^2} \cdot \ln(4) \cdot (-2x) = -2x \cdot 4^{-x^2} \cdot \ln(4)$$

9. $f(u) = e^{u^2+4u}$

a) $e^{u^2+4u} \cdot (2u+4)$	b) $e^{u^2} \cdot (2u-4)$
c) $e^{u^2+4u} \cdot (u+4)$	d) $e^{u^2-4u} \cdot (-u+4)$

$$f'(u) = e^{u^2+4u} \cdot \frac{d}{du}(u^2+4u) = e^{u^2+4u} \cdot (2u+4)$$

10. $f(u) = 3e^{\tan u}$

- | | | | |
|---------------------------|--------------------------|---------------------------|---------------------------|
| a) $3e^{\tan u} \sec^2 u$ | b) $e^{\tan u} \sec^2 u$ | c) $-e^{\tan u} \sec^2 u$ | d) $2e^{\tan u} \sec^2 u$ |
|---------------------------|--------------------------|---------------------------|---------------------------|

$$f'(x) = 3e^{\tan x} \cdot \frac{d}{dx}(\tan x) = 3e^{\tan x} \sec^2 x$$

11. $f(w) = \frac{e^{4w}}{w}$

- | | | | |
|-------------------------------|--------------------------|-------------------------------|--------------------------------|
| a) $\frac{e^{4w}(4w-1)}{w^2}$ | b) $\frac{e^w(4w-1)}{w}$ | c) $\frac{e^{-w}(4w-1)}{w^2}$ | d) $\frac{-e^{4w}(4w-1)}{w^2}$ |
|-------------------------------|--------------------------|-------------------------------|--------------------------------|

$$f'(w) = \frac{w \cdot 4e^{4w} - e^{4w} \cdot 1}{w^2} = \frac{e^{4w}(4w-1)}{w^2}$$

12. $f(w) = \frac{w}{e^{6w}}$

- | | | | |
|--------------------------|--------------------------|-----------------------|-------------------------|
| a) $\frac{1-6w}{e^{6w}}$ | b) $\frac{1+6w}{e^{6w}}$ | c) $\frac{1-6w}{e^w}$ | d) $\frac{1-w}{e^{6w}}$ |
|--------------------------|--------------------------|-----------------------|-------------------------|

$$f'(w) = \frac{1 \cdot e^{6w} - w \cdot e^{6w} \cdot 6}{(e^{6w})^2} = \frac{e^{6w} - 6we^{6w}}{(e^{6w})^2} = \frac{1-6w}{e^{6w}}$$

13. $f(x) = \ln 2x$

a) $\frac{1}{x}$	b) $\frac{1}{2x}$	c) $\frac{-1}{x}$	d) $\ln x$
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$$f'(x) = \frac{1}{2x} \cdot (2) = \frac{1}{x}$$

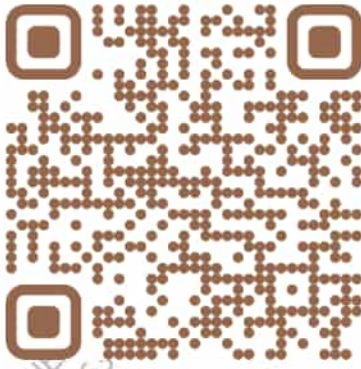
14. $f(x) = \ln \sqrt{8x}$

a) $\frac{1}{2x}$	b) $\ln x$	c) $\frac{-1}{x}$	d) $\ln \sqrt{8x}$
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$$f(x) = \frac{1}{2} \ln 8 + \frac{1}{2} \ln x$$
$$f'(x) = \frac{1}{2x}$$

Question 18: Use implicit differentiation to
find derivatives of inverse trigonometric
functions (Page 204)

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Q. Find the derivative of the given function.

1.

a) $f(x) = \sin^{-1}(x^3 + 1)$

a) $\frac{3x^2}{\sqrt{1-(x^3+1)^2}}$	b) $\frac{1}{2\sqrt{x(1-x)}}$	c) $\frac{-(2x+1)}{\sqrt{1-(x^2+x)^2}}$	d) $\frac{2}{x\sqrt{x^2-4}}$
--------------------------------------	-------------------------------	---	------------------------------

Differentiating with respect to x , $f'(x) = \frac{d}{dx} [\sin^{-1}(x^3 + 1)]$.

By the Chain rule we get,

$$f'(x) = \frac{1}{\sqrt{1-(x^3+1)^2}} \frac{d}{dx} (x^3 + 1) = \frac{1}{\sqrt{1-(x^3+1)^2}} (3x^2)$$

$$= \frac{3x^2}{\sqrt{1-(x^3+1)^2}}$$

b) $f(x) = \sin^{-1}(\sqrt{x})$

a) $\frac{1}{2\sqrt{x(1-x)}}$	b) $\frac{3x^2}{\sqrt{1-(x^3+1)^2}}$	c) $\frac{2}{x\sqrt{x^2-4}}$	d) $\frac{-(2x+1)}{\sqrt{1-(x^2+x)^2}}$
-------------------------------	--------------------------------------	------------------------------	---

Differentiating with respect to x , $f'(x) = \frac{d}{dx} [\sin^{-1}(\sqrt{x})]$.

By the chain rule, we get:

$$f'(x) = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx} (\sqrt{x}) = \frac{1}{\sqrt{1-x}} \left(\frac{1}{2\sqrt{x}} \right) = \frac{1}{2\sqrt{x(1-x)}}$$

2.

a) $f(x) = \cos^{-1}(x^2 + x)$

a) $\frac{-(2x+1)}{\sqrt{1-(x^2+x)^2}}$	b) $\frac{2}{x\sqrt{x^2-4}}$	c) $\frac{1}{2\sqrt{x(1-x)}}$	d) $\frac{3x^2}{\sqrt{1-(x^3+1)^2}}$
---	------------------------------	-------------------------------	--------------------------------------

Differentiating with respect to x , $f'(x) = \frac{d}{dx} [\cos^{-1}(x^2 + x)]$.

By using Chain rule,

$$f'(x) = \frac{-1}{\sqrt{1-(x^2+x)^2}} \frac{d}{dx} (x^2 + x) = \frac{-(2x+1)}{\sqrt{1-(x^2+x)^2}}$$

b) $f(x) = \cos^{-1}\left(\frac{2}{x}\right)$

a) $\frac{2}{x\sqrt{x^2-4}}$	b) $\frac{-(2x+1)}{\sqrt{1-(x^2+x)^2}}$	c) $\frac{1}{2\sqrt{x(1-x)}}$	d) $\frac{3x^2}{\sqrt{1-(x^3+1)^2}}$
------------------------------	---	-------------------------------	--------------------------------------

Differentiating with respect to x , $f'(x) = \frac{d}{dx} \left[\cos^{-1}\left(\frac{2}{x}\right) \right]$.

By using Chain rule,

$$f'(x) = \frac{-1}{\sqrt{1-\left(\frac{2}{x}\right)^2}} \frac{d}{dx} \left(\frac{2}{x}\right) = \frac{-1}{\sqrt{1-\left(\frac{4}{x^2}\right)}} \left(\frac{-2}{x^2}\right) = \frac{2}{x\sqrt{x^2-4}}$$

3.

a) $f(x) = \tan^{-1}(\sqrt{x})$

a) $\frac{1}{2\sqrt{x}(1+x)}$	b) $\frac{-1}{(x^2+1)}$	c) $\frac{e^{\tan^{-1} x}}{1+x^2}$	d) $\frac{1}{2(1+x^2)\sqrt{2+\tan^{-1} x}}$
-------------------------------	-------------------------	------------------------------------	---

Differentiating with respect to x , $f'(x) = \frac{d}{dx} [\tan^{-1}(\sqrt{x})]$.

By using Chain rule,

$$f'(x) = \frac{1}{1 + (\sqrt{x})^2} \frac{d}{dx} (\sqrt{x}) = \frac{1}{(1+x)} \left(\frac{1}{2\sqrt{x}} \right) = \frac{1}{2\sqrt{x}(1+x)}$$

b) $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$

a) $\frac{-1}{(x^2+1)}$	b) $\frac{1}{2\sqrt{x}(1+x)}$	c) $\frac{e^{\tan^{-1} x}}{1+x^2}$	d) $\frac{1}{2(1+x^2)\sqrt{2+\tan^{-1} x}}$
-------------------------	-------------------------------	------------------------------------	---

Differentiating with respect to x , $f'(x) = \frac{d}{dx} \left[\tan^{-1}\left(\frac{1}{x}\right) \right]$.

By using Chain rule,

$$f'(x) = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \frac{d}{dx} \left(\frac{1}{x}\right) = \frac{1}{\left(1 + \frac{1}{x^2}\right)} \left(\frac{-1}{x^2}\right) = \frac{-1}{(x^2+1)}$$

4.

a) $f(x) = \sqrt{2 + \tan^{-1} x}$

a) $\frac{1}{2(1+x^2)\sqrt{2+\tan^{-1} x}}$	b) $\frac{e^{\tan^{-1} x}}{1+x^2}$	c) $\frac{-1}{(x^2+1)}$	d) $\frac{1}{2\sqrt{x}(1+x)}$
---	------------------------------------	-------------------------	-------------------------------

Differentiating with respect to x , $f'(x) = \frac{d}{dx}(\sqrt{2 + \tan^{-1} x})$.

By using Chain rule,

$$f'(x) = \frac{1}{2\sqrt{2 + \tan^{-1} x}} \frac{d}{dx}(2 + \tan^{-1} x) = \frac{1}{2\sqrt{2 + \tan^{-1} x}} \left(\frac{1}{1+x^2} \right)$$

$$= \frac{1}{2(1+x^2)\sqrt{2 + \tan^{-1} x}}$$

b) $f(x) = e^{\tan^{-1} x}$

a) $\frac{e^{\tan^{-1} x}}{1+x^2}$	b) $\frac{1}{2(1+x^2)\sqrt{2+\tan^{-1} x}}$	c) $\frac{-1}{(x^2+1)}$	d) $\frac{1}{2\sqrt{x}(1+x)}$
------------------------------------	---	-------------------------	-------------------------------

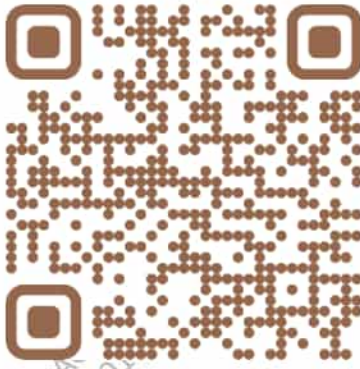
Differentiating with respect to x , $f'(x) = \frac{d}{dx}(e^{\tan^{-1} x})$.

By using Chain rule,

$$f'(x) = (e^{\tan^{-1} x}) \left(\frac{1}{1+x^2} \right) = \frac{e^{\tan^{-1} x}}{1+x^2}$$

Question 19: Learn Rolle's Theorem and use it in applications (Page 214)

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Q. Find a value of c satisfying the conclusion of Rolle's Theorem for $f(x) = x^3 - 3x^2 + 2x + 2$ on the interval $[0, 1]$.

a) $1 - \frac{1}{3}\sqrt{3}$

b) $3 + \frac{1}{3}\sqrt{3}$

c) $1 + \frac{1}{3}\sqrt{3}$

d) $1 - \frac{1}{2}\sqrt{2}$

First, we verify that the hypotheses of the theorem are satisfied: f is differentiable and continuous for all x [since $f(x)$ is a polynomial and all polynomials are continuous and differentiable everywhere]. Also, $f(0) = f(1) = 2$. We have

$$f'(x) = 3x^2 - 6x + 2$$

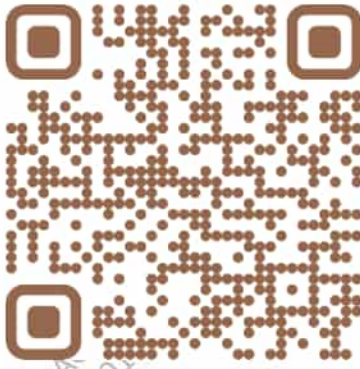
We now look for values of c such that

$$f'(c) = 3c^2 - 6c + 2 = 0$$

By the quadratic formula, we get $c = 1 + \frac{1}{3}\sqrt{3} \approx 1.5774$ [not in the interval $(0, 1)$] and $c = 1 - \frac{1}{3}\sqrt{3} \approx 0.42265 \in (0, 1)$.

Question 20: Learn the Mean Value Theorem
and use it in applications (Page 219)

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Q. Prove that $|\sin a| \leq |a|$ for all a .

First, note that $f(x) = \sin x$ is continuous and differentiable on any interval and that for any a ,

$$|\sin a| = |\sin a - \sin 0|,$$

since $\sin 0 = 0$. From the Mean Value Theorem, we have that (for $a \neq 0$)

$$\frac{\sin a - \sin 0}{a - 0} = f'(c) = \cos c,$$

for some number c between a and 0 . Notice that if we multiply both sides of by a and take absolute values, we get

$$|\sin a| = |\sin a - \sin 0| = |\cos c| |a - 0| = |\cos c| |a|$$

But, $|\cos c| \leq 1$, for all real numbers c and so, from, we have

$$|\sin a| = |\cos c| |a| \leq (1)|a| = |a|,$$

as desired.

Question 21-a: Determine the continuity
of a function at a given point (Page 96)

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Q. Determine values of a and b that make the given function continuous.

$$1. f(x) = \begin{cases} \frac{2 \sin x}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b \cos x & \text{if } x > 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{2 \sin x}{x} = 2 \lim_{x \rightarrow 0^-} \frac{\sin x}{x} = 2$$

Hence a must equal 2 if f is continuous.

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} b \cos x = b \lim_{x \rightarrow 0^+} \cos x = b$$

so b and a must equal 2 if f is continuous.

$$2. f(x) = \begin{cases} ae^x + 1 & \text{if } x < 0 \\ \sin^{-1} \frac{x}{2} & \text{if } 0 \leq x \leq 2 \\ x^2 - x + b & \text{if } x > 2 \end{cases}$$

We need $ae^0 + 1 = \sin^{-1} 0$, so $a = -1$.

We need $2^2 - 2 + b = \sin^{-1} 1$, so $b = \frac{\pi}{2} - 2$.

$$3. f(x) = \begin{cases} a(\tan^{-1} x + 2) & \text{if } x < 0 \\ 2e^{bx} + 1 & \text{if } x = 0 \\ \ln(x - 2) + x^2 & \text{if } x > 0 \end{cases}$$

First note that

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \ln(x - 2) + x^2 = \ln(3 - 2) + 3^2 = 9$$

Also $f(3) = 2e^{3b} + 1,$

so if f is continuous, $2e^{3b} + 1$ must equal 9;

that is $e^{3b} = 4$, so $b = \frac{\ln 4}{3}$. Then note that $f(0) = 2e^{(b)(0)} + 1 = 3$.

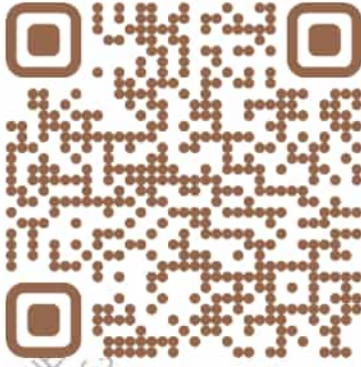
Also,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} a(\tan^{-1} x + 2) = a(\tan^{-1} 0 + 2) = a(0 + 2) = 2a,$$

so a must equal $3/2$ if f is continuous.

Question 21-b: Find limits at infinity
and limits that are infinite (Page 106)

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Q. Suppose that the size of the pupil of a certain animal is given by $f(x)$ (mm), where x is the intensity of the light on the pupil. If $f(x) = \frac{80x^{-0.3}+60}{2x^{0.3}+5}$, find the size of the pupil with no light and the size of the pupil with an infinite amount of light.

$$\lim_{x \rightarrow 0^+} \frac{80x^{-0.3} + 60}{2x^{-0.3} + 5} \left(\frac{x^{-0.3}}{x^{-0.3}} \right) = \lim_{x \rightarrow 0^+} \frac{80 + 60x^{-0.3}}{2 + 5x^{-0.3}} = \frac{80}{2} = 40 \text{ mm}$$

$$\lim_{x \rightarrow \infty} \frac{80x^{-0.3} + 60}{2x^{-0.3} + 5} = \frac{60}{5} = 12 \text{ mm}$$

Q. Repeat previous exercise with $f(x) = \frac{80x^{-0.3}+60}{8x^{-0.3}+15}$.

Re-write the function as $f(x) = \frac{80+60x^{-0.3}}{8+15x^{-0.3}}$. to see that the size with no light is $f(0) = 10 \text{ mm}$, and the size with infinite light is $\lim_{x \rightarrow \infty} f(x) = 4 \text{ mm}$

Q. Modify the function in exercise 33 to find a function f such that $\lim_{x \rightarrow 0^+} f(x) = 8$ and $\lim_{x \rightarrow \infty} f(x) = 2$.

$$f(x) = \frac{80x^{-0.3} + 60}{10x^{-0.3} + 30}$$

Q. Find a function of the form $f(x) = \frac{20x^{-0.4} + 16}{g(x)}$ such that $\lim_{x \rightarrow 0^+} f(x) = 5$ and $\lim_{x \rightarrow \infty} f(x) = 4$.

$$g(x) = 4x^{-0.4} + 4$$

$$\Rightarrow f(x) = \frac{20x^{-0.4} + 16}{4x^{-0.4} + 4}$$

Therefore, $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{(20x^{-0.4} + 16).x^{0.4}}{(4x^{-0.4} + 4).x^{0.4}} = \lim_{x \rightarrow 0^+} \frac{20 + 16x^{0.4}}{4 + 4x^{0.4}} = \frac{20}{5} = 5$ and

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{20x^{-0.4} + 16}{4x^{-0.4} + 4} = 4$$

Q. Suppose that the velocity of a skydiver t seconds after jumping is given by $v(t) = -\sqrt{\frac{32}{k} \frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}}$. Find the limiting velocity with $k = 0.00064$ and $k = 0.00128$. By what factor does a skydiver have to change the value of k to cut the limiting velocity in half?

the terminal velocity is $-\sqrt{\frac{32}{k}}$.

When $k = 0.00064$, the terminal velocity is $-\sqrt{\frac{32}{0.00064}} \approx -224$.

When $k = 0.00128$, the terminal velocity is $-\sqrt{\frac{32}{0.00128}} \approx -158$.

Solve $\sqrt{\frac{32}{ak}} = \frac{1}{2} \sqrt{\frac{32}{k}}$. Squaring both sides, $\frac{32}{ak} = \frac{1}{4} \cdot \frac{32}{k}$ so $a = 4$.

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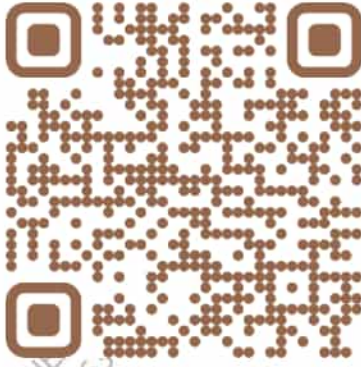
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Question 22: Apply the Product Rule on derivatives (Page 169)

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Q. Suppose that for some toy, the quantity sold $Q(t)$ at time t years decreases at a rate of 4%; explain why this translates to $Q'(t) = -0.04Q(t)$. Suppose also that the price increases at a rate of 3%; write out a similar equation for $P'(t)$ in terms of $P(t)$. The revenue for the toy is $R(t) = Q(t)P(t)$. Substituting the expressions for $Q'(t)$ and $P'(t)$ into the product rule $R'(t) = Q'(t)P(t) + Q(t)P'(t)$, show that the revenue decreases at a rate of 1%. Explain why this is "obvious."

The rate at which the quantity Q changes is Q' .

Since the amount is said to be "decreasing at a rate of 4%" we have to ask "4% of what?" The answer in this type of context is usually 4% of itself. In other words, $Q' = -0.04Q$.

As for P , the 3% rate of increase would translate as $P' = 0.03P$.

By the product rule with $R = PQ$, we have:

$$\begin{aligned} R' &= (PQ)' = P'Q + PQ' = (0.03P)Q + P(-0.04Q) \\ &= - (0.01)PQ = (-0.01)R \end{aligned}$$

In other words, revenue is decreasing at a rate of 1%.

Q. As in the previous exercise, suppose that the quantity sold decreases at a rate of 4%. By what rate must the price be increased to keep the revenue constant?

Revenue will be constant when the derivative is 0.

Substituting, $Q' = -0.04Q$ and, $P' = aP$ into the expression for R' gives,

$$R' = -0.04QP + aQP = (-0.04 + a)QP.$$

This is zero when $a = 0.04$, so price must increase by 4%.

Q. Suppose the price of an object is \$20 and 20,000 units are sold. If the price increases at a rate of \$1.25 per year and the quantity sold increases at a rate of 2000 per year, at what rate will revenue increase?

$$R' = Q'P + QP'$$

At a certain moment of time (call it t_0) we are given

$$P(t_0) = 20 \left(\frac{\$}{\text{item}} \right), \quad Q(t_0) = 20,000(\text{items})$$

$$P'(t_0) = 1.25(\$/\text{item}/\text{year})$$

$$Q'(t_0) = 2,000(\text{item}/\text{year})$$

$$R'(t_0) = 2,000(20) + (20,000)1.25$$

$$R'(t_0) = 65,000 (\$/\text{year}).$$

So, revenue is increasing by \$65,000/year at the time t_0 .

Q. Suppose the price of an object is \$14 and 12,000 units are sold. The company wants to increase the quantity sold by 1200 units per year, while increasing the revenue by \$20,000 per year. At what rate would the price have to be increased to reach these goals?

We are given $P = \$14$, $Q = 12,000$ and $Q' = 1,200$. We want $R' = \$20,000$. Substituting these values in to the expression for R' (see exercise 25) yields: $20,000 = 1200 \cdot 14 + 12,000P'$ Solve to get $P' = 0.27$ dollars per year.

Q. A baseball with mass 0.15 kg and speed 45 m/s is struck by a baseball bat of mass m kg and speed 40m/s (in the opposite direction of the ball's motion). After the collision, the ball has initial speed $u(m) = \frac{82.5m - 6.75}{m + 0.15}$ m/s. Show that $u'(m) > 0$ and interpret this in baseball terms. Compare $u'(1)$ and $u'(1.2)$.

If $u(m) = \frac{82.5m - 6.75}{m + 0.15}$ then using the quotient rule,

$$\frac{du}{dm} = \frac{(m + 0.15)(82.5) - (82.5m - 6.75)1}{(m + 0.15)^2} = \frac{19.125}{(m + 0.15)^2}$$

Which is clearly positive. It seems to be saying that initial ball speed is an increasing function of the mass of the bat. Meanwhile,

$$u'(1) = \frac{19.125}{1.15^2} \approx 14.46$$

$$u'(1.2) = \frac{19.125}{1.35^2} \approx 10.49,$$

Which suggests that the rate at which this speed is increasing is decreasing.

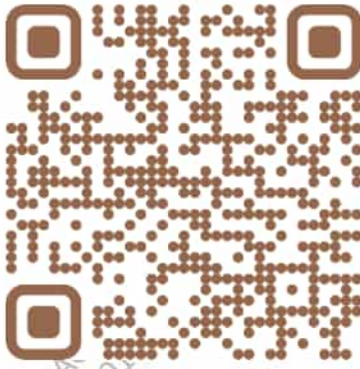
Q. In the previous exercise, if the baseball has mass M kg at speed 45 m/s and the bat has mass 1.05 kg and speed 40m/s, the ball's initial speed is $u(M) = \frac{86.625 - 45M}{M + 1.05}$ m/s. Compute $u'(M)$ and interpret its sign (positive or negative) in baseball terms.

$$\begin{aligned} u'(M) &= \frac{(M + 1.05) \frac{d}{dM} (86.625 - 45M)}{(M + 1.05)^2} \\ &= \frac{-\frac{d}{dM} (M + 1.05)(86.625 - 45M)}{(M + 1.05)^2} \\ &= \frac{(-45M - 47.25) - (86.625 - 45M)}{(M + 1.05)^2} = \frac{-133.875}{(M + 1.05)^2} \end{aligned}$$

This quantity is negative. In baseball terms, as the mass of the baseball increases, the initial velocity decreases.

Question 23: Solve real-life problems using
derivatives of exponential and logarithmic
functions (Page 194)

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Mr. Aghead

Q. find an equation of the tangent line to $y = f(x)$ at $x = 1$.

1. $f(x) = 3e^{x^2}$

$$f(1) = 3e^{1^2} = 3e$$

$$f'(x) = 3e^{x^2} 2x$$

$$f'(1) = 3e^{1^2} 2(1) = 6e$$

So, the equation of the tangent line is,

$$y = 6e(x - 1) + 3e$$

2. $f(x) = 3^{x^e}$

$$f(1) = 3^{1^e} = 3$$

$$f'(x) = 3^{x^e} \ln 3 \cdot ex^{e-1}$$

$$f'(1) = 3 \ln 3 \cdot e$$

So, the equation of the tangent line is,

$$y = 3 \ln 3 \cdot e(x - 1) + 3$$

3. $f(1) = 0$

$$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x$$

$$f'(1) = 2 \cdot 1 \ln 1 + 1 = 2 \cdot 0 + 1 = 1$$

So the equation of tangent line is

$$y = 1(x - 1) + 0 \text{ or } y = x - 1.$$

4. $f(x) = 2 \ln x^3$

$$f'(x) = \frac{2}{x^3} \cdot 3x^2 = \frac{6}{x}$$

$$\text{Slope} = f'(x) \text{ at } x = 1.$$

$$\text{Slope } m = \frac{6}{1} = 6.$$

Equation of the line passing through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$.

$$\text{At } x_1 = 1, y_1 = f(1) = 2 \cdot \ln 1^3 = 0.$$

$$\text{Therefore equation is } y - 0 = 6 \cdot (x - 1) \text{ or } y = 6x - 6.$$

Q. find all values of x for which the tangent line to $y = f(x)$ is horizontal.

1. (a) $f(x) = xe^{-2x}$

Given that, the tangent line to $f(x)$ is horizontal. Therefore slope is zero.

Hence,

$$\begin{aligned}f'(x) &= e^{-2x} - 2xe^{-2x} = 0 \\e^{-2x}(1 - 2x) &= 0 \\x &= \frac{1}{2}.\end{aligned}$$

(b) $f(x) = xe^{-3x}$

Given that, the tangent line to $f(x)$ is horizontal. Therefore slope is zero.

Hence,

$$\begin{aligned}f'(x) &= x \cdot (-3e^{-3x}) + e^{-3x} = 0. \\&\Rightarrow e^{-3x}(-3x + 1) = 0 \\&\Rightarrow 3x - 1 = 0 \\&\Rightarrow x = \frac{1}{3}\end{aligned}$$

2. (a) $f(x) = x^2 e^{-2x}$

Given that, the tangent line to $f(x)$ is horizontal. Therefore slope is zero.

Hence,

$$f'(x) = x^2 \cdot (-2e^{-2x}) + 2x \cdot e^{-2x} = 0$$

$$\Rightarrow -x + 1 = 0$$

$$\Rightarrow x = 1$$

(b) $f(x) = x^2 e^{-3x}$

Given that, the tangent line to $f(x)$ is horizontal. Therefore slope is zero.

Hence,

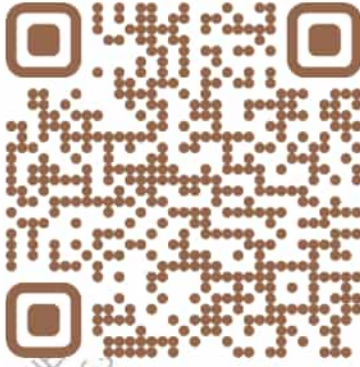
$$f'(x) = x^2 \cdot (-3e^{-3x}) + 2x \cdot e^{-3x} = 0$$

$$\Rightarrow -3x + 2 = 0$$

$$\Rightarrow x = \frac{2}{3}$$

Question 24: Find derivatives
implicitly (Page 199)

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Mr. Aghead

Q. Find $y''(x)$ implicitly for $y^2 + 2e^{-xy} = 6$. Then find the value of y'' at the point $(0, 2)$.

Solution We begin by differentiating both sides of the equation with respect to x . We have

$$\frac{d}{dx}(y^2 + 2e^{-xy}) = \frac{d}{dx}(6).$$

By the chain rule and the product rule, we have

$$2yy'(x) + 2e^{-xy}[-y - xy'(x)] = 0. \quad (8.3)$$

Notice that we don't need to solve this for $y'(x)$. Dividing out the common factor of 2 and differentiating again, we get

$$y'(x)y'(x) + yy''(x) - e^{-xy}[-y - xy'(x)][y + xy'(x)] - e^{-xy}[y'(x) + y'(x) + xy''(x)] = 0.$$

Grouping all the terms involving $y''(x)$ on one side of the equation gives us

$$yy''(x) - xe^{-xy}y''(x) = -[y'(x)]^2 - e^{-xy}[y + xy'(x)]^2 + 2e^{-xy}y'(x).$$

Factoring out the $y''(x)$ on the left, we get

$$(y - xe^{-xy})y''(x) = -[y'(x)]^2 - e^{-xy}[y + xy'(x)]^2 + 2e^{-xy}y'(x),$$

so that

$$y''(x) = \frac{-[y'(x)]^2 - e^{-xy}[y + xy'(x)]^2 + 2e^{-xy}y'(x)}{y - xe^{-xy}}. \quad (8.4)$$

Notice that (8.4) gives us a (rather messy) formula for $y''(x)$ in terms of x , y and $y'(x)$. If we need to have $y''(x)$ in terms of x and y only, we can solve (8.3) for $y'(x)$ and substitute into (8.4). However, we don't need to do this to find $y''(0)$. Instead, first substitute $x = 0$ and $y = 2$ into (8.3) to get

$$4y'(0) + 2(-2) = 0,$$

from which we conclude that $y'(0) = 1$. Then substitute $x = 0$, $y = 2$ and $y'(0) = 1$ into (8.4) to get

$$y''(0) = \frac{-1 - (2)^2 + 2}{2} = -\frac{3}{2}.$$

See Figure 3.43 for a graph of $y^2 + 2e^{-xy} = 6$ near the point $(0, 2)$.

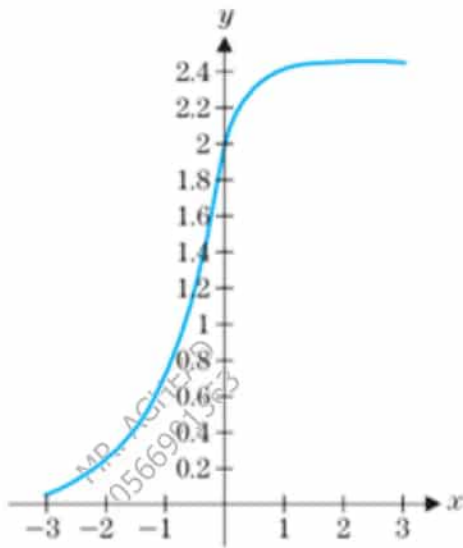
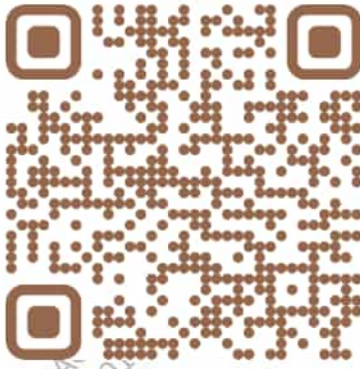


FIGURE 3.43
 $y^2 + 2e^{-xy} = 6$

Question 25: Use implicit differentiation to
find derivatives of inverse trigonometric
functions (Page 204)

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Mr. Aghead

Q. find the derivative of the given function.

1.

a) $f(x) = \sin^{-1} (x^3 + 1)$

Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} [\sin^{-1} (x^3 + 1)] .$$

By the Chain rule we get,

$$f'(x) = \frac{1}{\sqrt{1 - (x^3 + 1)^2}} \frac{d}{dx} (x^3 + 1)$$

$$= \frac{1}{\sqrt{1 - (x^3 + 1)^2}} (3x^2)$$

$$= \frac{3x^2}{\sqrt{1 - (x^3 + 1)^2}} .$$

b) $f(x) = \sin^{-1}(\sqrt{x})$

Diffrentiating with respect to x ,

$$f'(x) = \frac{d}{dx} [\sin^{-1}(\sqrt{x})].$$

By the Chain rule, we get

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \frac{d}{dx} (\sqrt{x}) \\ &= \frac{1}{\sqrt{1 - x}} \left(\frac{1}{2\sqrt{x}} \right) \\ &= \frac{1}{2\sqrt{x(1 - x)}} \end{aligned}$$

2.

a) $f(x) = \cos^{-1} (x^2 + x)$

Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} [\cos^{-1} (x^2 + x)] .$$

By using Chain rule,

$$\begin{aligned} f'(x) &= \frac{-1}{\sqrt{1 - (x^2 + x)^2}} \frac{d}{dx} (x^2 + x) \\ &= \frac{-(2x + 1)}{\sqrt{1 - (x^2 + x)^2}} \end{aligned}$$

b) $f(x) = \cos^{-1}\left(\frac{2}{x}\right)$

Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} \left[\cos^{-1} \left(\frac{2}{x} \right) \right]$$

By using Chain rule,

$$f'(x) = \frac{-1}{\sqrt{1 - \left(\frac{2}{x}\right)^2}} \frac{d}{dx} \left(\frac{2}{x} \right)$$

$$= \frac{-1}{\sqrt{1 - \left(\frac{4}{x^2}\right)}} \left(\frac{-2}{x^2} \right)$$

$$= \frac{2}{x\sqrt{x^2 - 4}}$$

3.

$$a) f(x) = \tan^{-1}(\sqrt{x})$$

Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} [\tan^{-1}(\sqrt{x})].$$

By the Chain rule,

$$f'(x) = \frac{1}{1 + (\sqrt{x})^2} \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{(1+x)} \left(\frac{1}{2\sqrt{x}} \right)$$

$$= \frac{1}{2\sqrt{x}(1+x)}$$

b) $f(x) = \tan^{-1} \left(\frac{1}{x} \right)$

Diffrentiating with respect to x ,

$$f'(x) = \frac{d}{dx} \left[\tan^{-1} \left(\frac{1}{x} \right) \right].$$

By the Chain rule,

$$f'(x) = \frac{1}{1 + \left(\frac{1}{x} \right)^2} \frac{d}{dx} \left(\frac{1}{x} \right)$$

$$= \frac{1}{\left(1 + \frac{1}{x^2} \right)} \left(\frac{-1}{x^2} \right)$$

$$= \frac{-1}{(x^2 + 1)}$$

4.

$$\text{a) } f(x) = \sqrt{2 + \tan^{-1} x}$$

Differentiating with respect to x ,

$$f'(x) = \frac{d}{dx} \left(\sqrt{2 + \tan^{-1} x} \right).$$

By the Chain rule,

$$f'(x) = \frac{1}{2\sqrt{2 + \tan^{-1} x}} \frac{d}{dx} (2 + \tan^{-1} x)$$

$$= \frac{1}{2\sqrt{2 + \tan^{-1} x}} \left(\frac{1}{1 + x^2} \right)$$

$$= \frac{1}{2(1 + x^2) \sqrt{2 + \tan^{-1} x}}$$

b) $f(x) = e^{\tan^{-1} x}$

Differentiating with respect to x ,

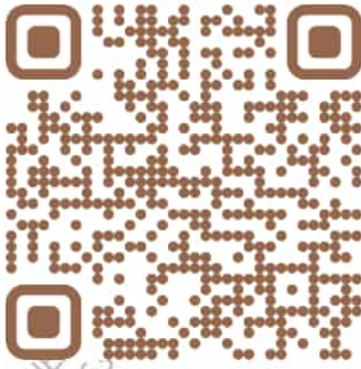
$$f'(x) = \frac{d}{dx} \left(e^{\tan^{-1} x} \right).$$

By the Chain rule,

$$\begin{aligned} f'(x) &= \left(e^{\tan^{-1} x} \right) \left(\frac{1}{1+x^2} \right) \\ &= \frac{e^{\tan^{-1} x}}{1+x^2} \end{aligned}$$

Question 26: Understand the Mean Value
Theorem and use it in applications
(Page 219)

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Mr. Aghead

Q. prove that the limit is correct using the appropriate definition (assume that k is an integer).

1. If $f'(x) < 0$ for all x , prove that $f(x)$ is a *decreasing* function: that is, if $a < b$, then $f(a) > f(b)$.

Let $a < b$. f is differentiable on (a, b) and continuous on $[a, b]$, since it is differentiable for all x . This means that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

for some $c \in (a, b)$. Therefore $f(b) - f(a) = f'(c)(b - a)$ is negative, and $f(a) > f(b)$.

2. $f(x) = x^3 + 5x + 1$

$f'(x) = 3x^2 + 5$. This is positive for all x , so $f(x)$ is increasing.

3. $f(x) = x^5 + 3x^3 - 1$

$f'(x) = 5x^4 + 9x^2 \geq 0$ for all x . $f' = 0$ only at $x = 0$, so $f(x)$ is increasing.

4. $f(x) = -x^3 - 3x + 1$

$f'(x) = -3x^2 - 3$. This is negative for all x , so $f(x)$ is decreasing.

5. $f(x) = x^4 + 2x^2 + 1$

$f'(x) = 4x^3 + 4x$ is negative for negative x , and positive for positive x , so $f(x)$ is neither an increasing function nor a decreasing function.

6. $f(x) = e^x$

$f'(x) = e^x$. This is positive for all x , so $f(x)$ is increasing.

7. $f(x) = e^{-x}$

$f'(x) = -e^{-x} < 0$ for all x , so $f(x)$ is a decreasing function.

8. $f(x) = \ln x$

$$f'(x) = \frac{1}{x}$$

$f'(x) > 0$ for $x > 0$, that is, for all x in the domain of f . So $f(x)$ is increasing.

9. $f(x) = \ln x^2$

$f'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$ is negative for negative x , and positive for positive x , so $f(x)$ is neither an increasing function nor a decreasing function.

Q. Suppose that $s(t)$ gives the position of an object at time t . If s is differentiable on the interval $[a, b]$, prove that at some time $t = c$, the instantaneous velocity at $t = c$ equals the average velocity between times $t = a$ and $t = b$.

The average velocity on $[a, b]$ is $\frac{s(b) - s(a)}{b - a}$.

By the Mean Value Theorem, there exists a $c \in (a, b)$ such that

$$s'(c) = \frac{s(b) - s(a)}{b - a}$$

Thus, the instantaneous velocity at $t = c$ is equal to the average velocity between times $t = a$ and $t = b$.

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Q. Two runners start a race at time 0. At some time $t = a$, one runner has pulled ahead, but the other runner has taken the lead by time $t = b$. Prove that at some time $t = c > 0$, the runners were going exactly the same speed.

Let $f(t)$ be the distance the first runner has gone after time t and let $g(t)$ be the distance the second runner has gone after time t . The functions $f(t)$ and $g(t)$ will be continuous and differentiable. Let $h(t) = f(t) - g(t)$.

At $t = 0$, $f(0) = 0$ and $g(0) = 0$ so $h(0) = 0$.

At $t = a$, $f(a) > g(a)$ so $h(a) > 0$. Similarly, at $t = b$, $f(b) < g(b)$ so $h(b) < 0$.

Thus, by the Intermediate Value Theorem, there is time $t = t_0$ for

$t_0 \in (a, b)$ where $h(t_0) = 0$. Rolle's Theorem then says that there is time $t = c$ where $c \in (0, t_0)$ such that $h'(c) = 0$. But $h'(t) = f'(t) - g'(t)$, so $h'(c) = f'(c) - g'(c) = 0$ implies that $f'(c) = g'(c)$, i.e., at time $t = c$ the runners are going exactly the same speed.