

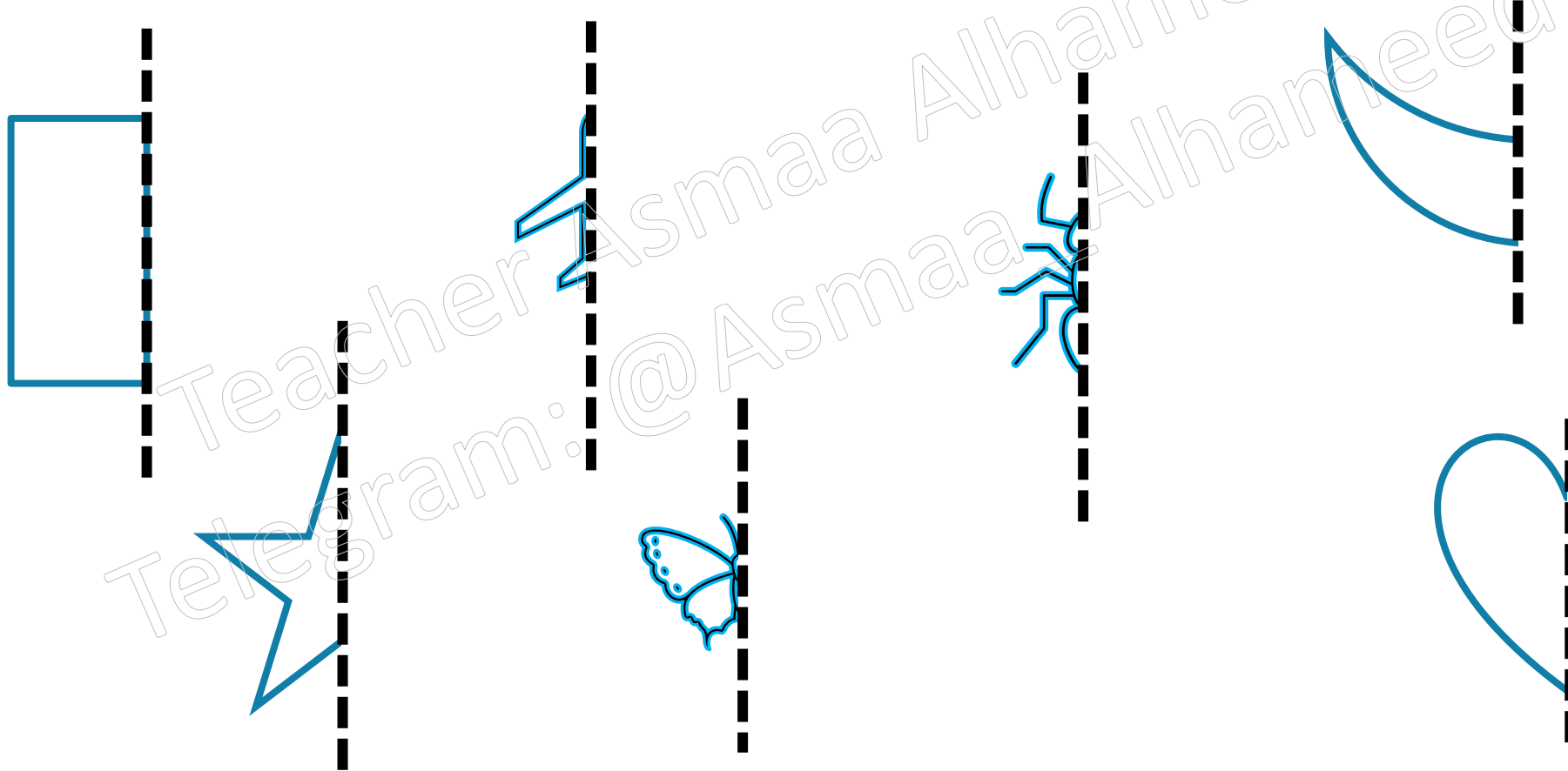
# 2.9 Special Symmetries (Part 1)

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2024-09-22

# Starter activity: complete the shape using symmetry

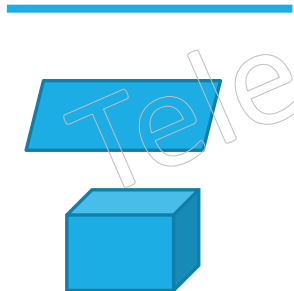
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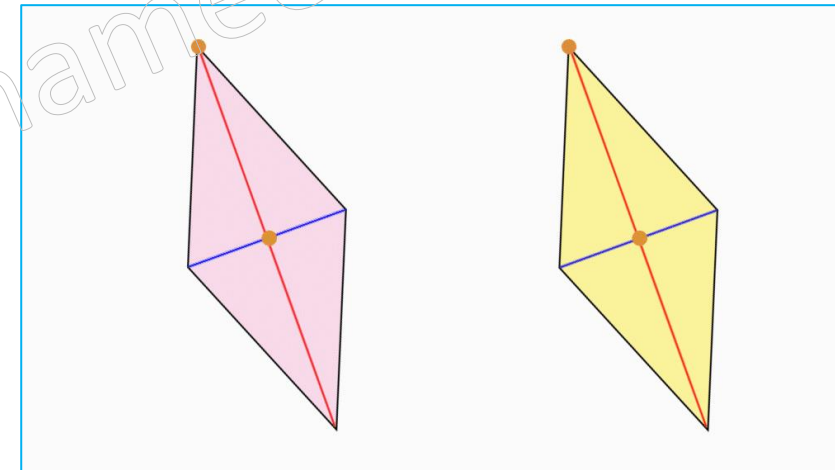
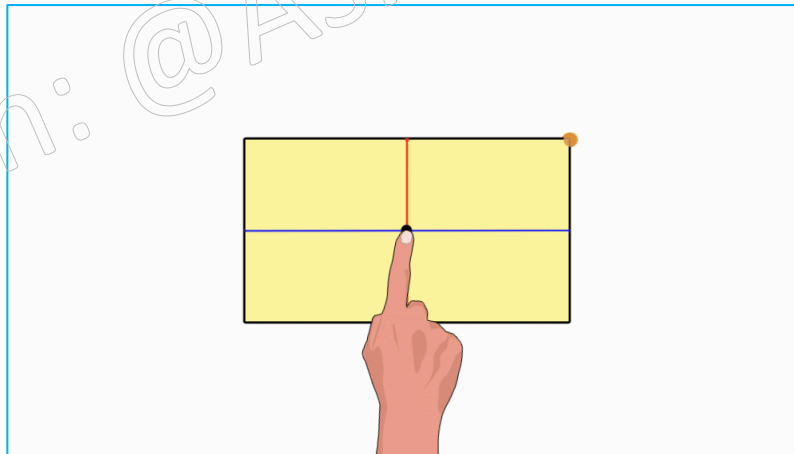
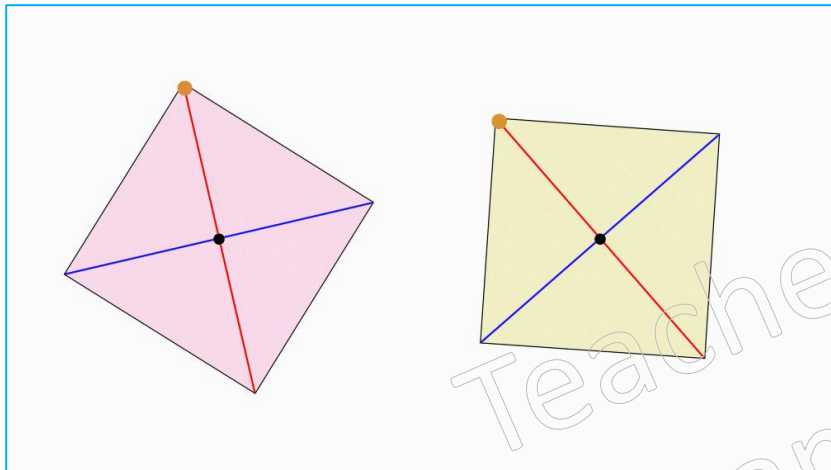
# Recap: Charge distribution

Previously we defined the charge distributions for different geometries as seen in the table below (page 46).

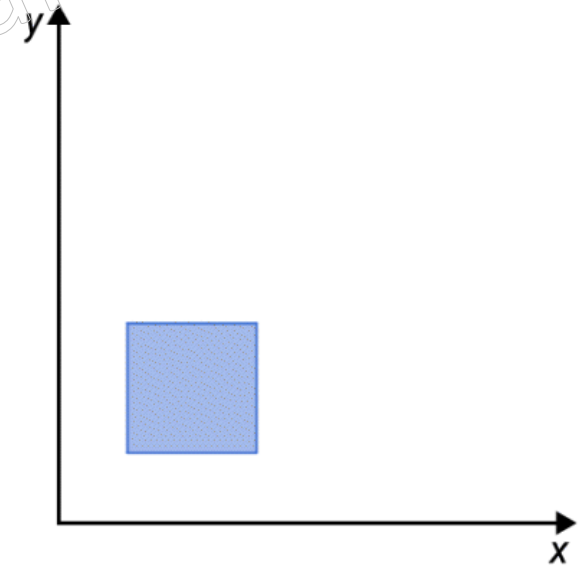
| Table 2.1 Symbols for Charge Distributions |                   |                  |
|--------------------------------------------|-------------------|------------------|
| Symbol                                     | Name              | Unit             |
| $\lambda$                                  | Charge per length | C/m              |
| $\sigma$                                   | Charge per area   | C/m <sup>2</sup> |
| $\rho$                                     | Charge per volume | C/m <sup>3</sup> |



# Rotational symmetry: examples



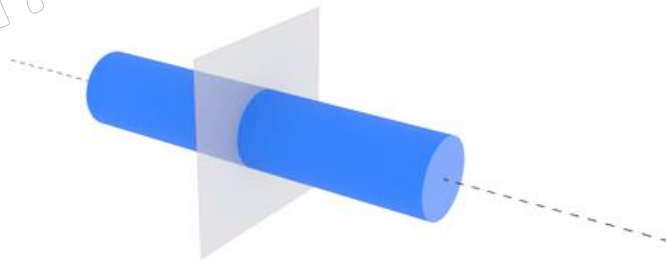
# Translational symmetry: examples



# An infinitely long cylinder:

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- The shape looks the same in the gray area as it moves through the object.
- The cylinder can also be **rotated** and still look the same. It therefore has **rotational symmetry**. In this case, the electric field doesn't depend on the rotation angle.
- The cylinder is infinitely **long**. This means the cylinder has **translational symmetry**.

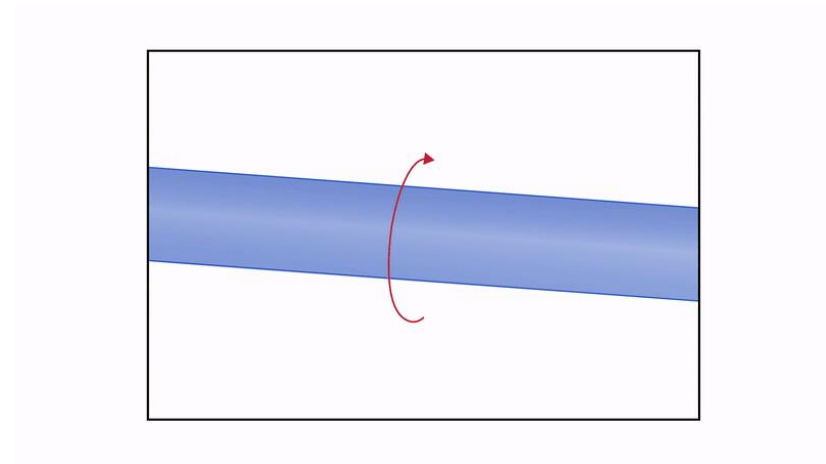
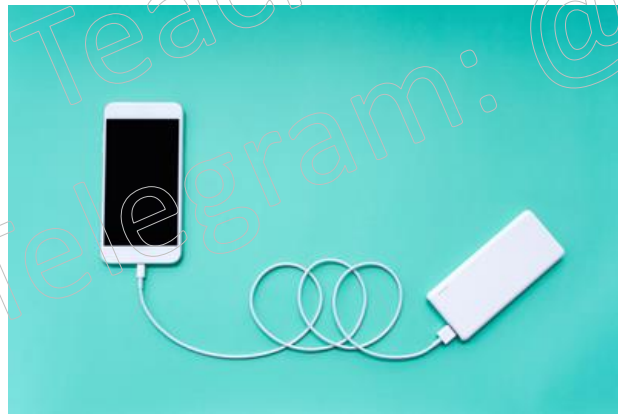


# An infinitely long cylinder:

Assume a phone charger like the one in the image below. The wire is **cylindrical** and is **long**.

A cylinder has a **constant radius**. It looks the same if rotated around its central axis. Therefore, the cylinder has **rotational symmetry**.

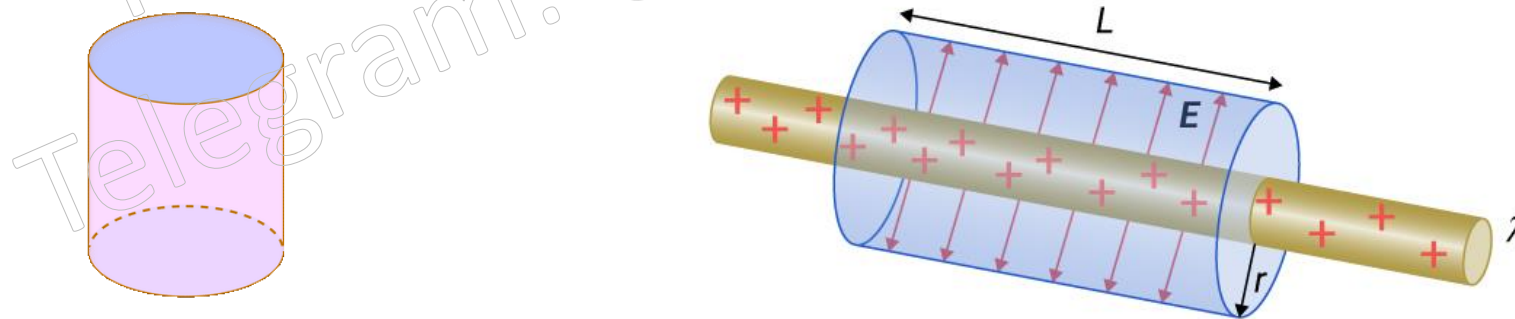
If the cylinder is infinitely long, the cylinder will look the same at any point along its length. Therefore, the cylinder also has **translational symmetry**.



# An infinitely long cylinder:

To calculate the electric field around an infinitely long 1D wire with charge density  $\lambda$ , use a cylindrical Gaussian surface.

The electric field points **radially** outwards (for positive charge). The electric field vectors point **perpendicular** to the **area vectors** of the circular end faces, so the electric flux through each circular end face of the Gaussian surface is **zero**.



# An infinitely long cylinder:

The closed loop integral of the electric field over the Gaussian surface is:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = E A_{\text{cylinder wall}}$$

$$\oiint \mathbf{E} \cdot d\mathbf{A} = E(2\pi rL)$$

Applying Gauss's law gives:

$$E(2\pi rL) = \frac{1}{\epsilon_0} q$$

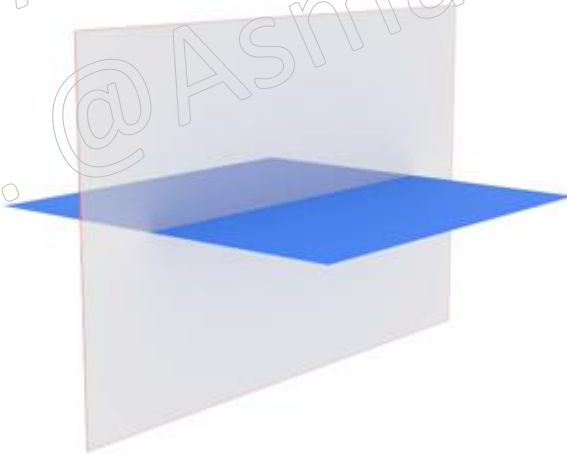
$$E = \frac{\lambda L}{2\pi\epsilon_0 r L}$$

$$E = k \frac{2\lambda}{r}$$

# An infinitely long sheet:

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- The plane can be **translated** or **rotated** and always look the same. An infinite plane has **translational** and **rotational** symmetry..



# An infinitely long sheet:

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An **infinite plane** can **rotate** around its perpendicular axis and always look the same. An infinite plane has **rotational symmetry**.

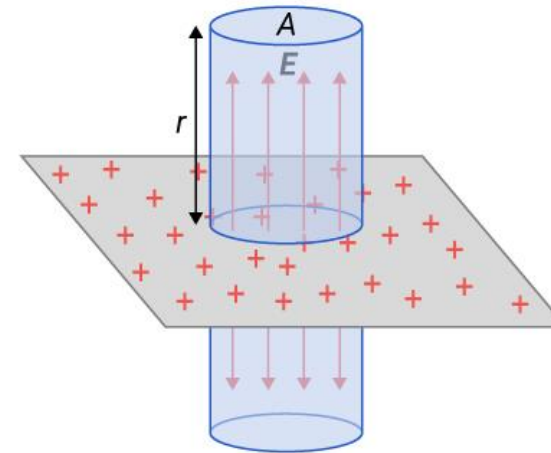
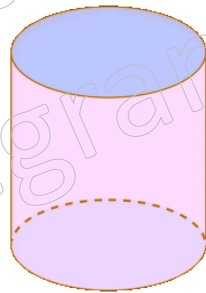
The plane can also **translate** in a parallel direction and always look the same. An infinite plane also has **translational symmetry**.

A suitable Gaussian surface for a **charged non-conducting infinite plane** is a **cylinder oriented perpendicularly to the plane**.



# An infinitely long sheet:

The **wall** of the cylinder is **parallel** to the **electric field vectors**. Therefore, the electric flux through the wall of the cylinder is **zero**, simplifying the problem.



# An infinitely long sheet:

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The closed loop integral only integrates over the area of the two end faces of the cylinder.

$$\oiint \mathbf{E} \cdot d\mathbf{A} = EA + EA$$

$$\oiint \mathbf{E} \cdot d\mathbf{A} = 2EA$$

Applying Gauss's law:

$$2EA = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{2\epsilon_0 A} q$$

$$E = \frac{\sigma}{2\epsilon_0}$$

# Activity:

$$E = k \frac{2\lambda}{r}$$

## Concept Check 2.12

A total of  $1.45 \times 10^6$  excess electrons are placed on an initially electrically neutral wire of length 1.13 m. What is the magnitude of the electric field at a point at a perpendicular distance of 0.401 m away from the center of wire? (Hint: Assume that 1.13 m is close enough to “infinitely long.”)

a)  $9.21 \times 10^{-3} \text{ N/C}$

b)  $2.92 \times 10^{-1} \text{ N/C}$

c)  $6.77 \times 10^1 \text{ N/C}$

d)  $8.12 \times 10^2 \text{ N/C}$

e)  $3.31 \times 10^3 \text{ N/C}$

1-Find  $q$  ( $q = eN_e$ )

$$q = eN_e = 1.6 \times 10^{-19} \times 1.45 \times 10^6 = 2.32 \times 10^{-13} \text{ C}$$

2-Find  $\lambda$  ( $\lambda = \frac{q}{L}$ )

$$\lambda = \frac{q}{L} = \frac{2.32 \times 10^{-13}}{1.13} = 2.05 \times 10^{-13} \frac{\text{C}}{\text{m}}$$

3-Find  $E$

$$E = \frac{2k\lambda}{r} = \frac{2 \times 8.99 \times 10^9 \times 2.05 \times 10^{-13}}{0.401} = 9.2 \times 10^{-3} \frac{\text{N}}{\text{C}}$$

# Activity:

02:00

The electric field due to a large conducting sheet with a charge density of  $\sigma = 5.0 \times 10^{-5} \text{ C/m}^2$  is  $E = 5.6 \times 10^6 \text{ N/C}$  at a point M located at a distance 5.0 cm from the sheet.

Another identical sheet is placed next to the first one as shown in the figure to the right. The electric field at M is:

$E =$  Select... ▼

$2.8 \times 10^6 \text{ N/C}$

$11.2 \times 10^6 \text{ N/C}$

$5.6 \times 10^6 \text{ N/C}$

$0 \text{ N/C}$

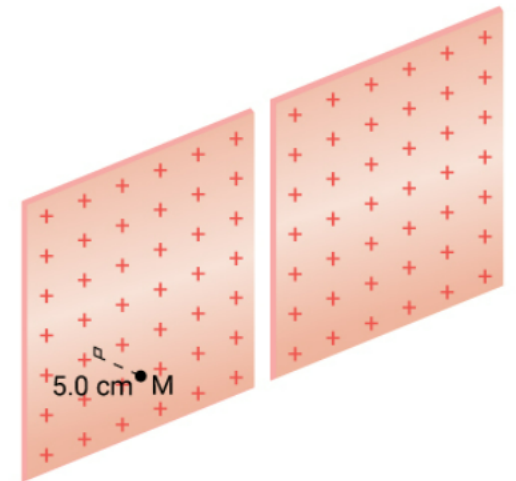
The electric field obtained by choosing a cylindrical Gaussian surface in front of either sheet is

$$E = \frac{\sigma}{\epsilon_0}$$

This expression of  $E$  shows that the charge density and not the dimension of the introduced sheets.

Thus  $E$  remains the same:

$$E = 5.6 \times 10^6 \text{ N/C}.$$

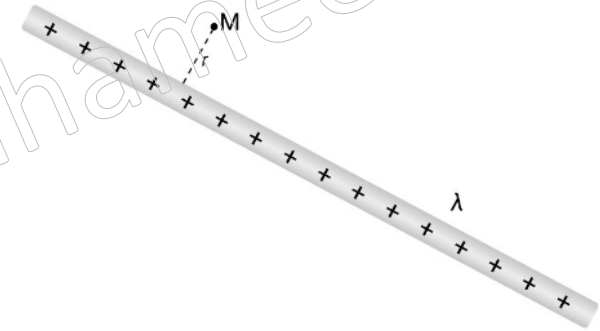


# Activity:

02:00

*Choose the correct answer.*

What is the form of the Gaussian surface needed in order to calculate the electric field,  $E$ , at a distance,  $r$ , from a very long charged wire of uniform charge distribution  $\lambda$ ?



☐ linear

☐ planar

☐ spherical

☒ cylindrical

## 2.9 Special Symmetries (Part 2)

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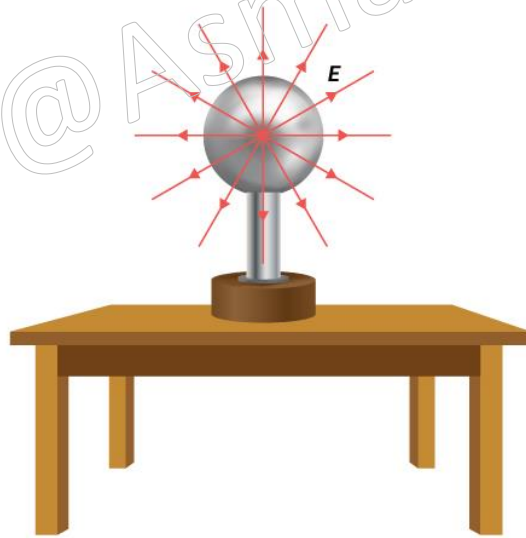
2024-09-22

# Spherical symmetry:

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A spherical conductor, like the dome of this Van de Graaff generator, will create an electric field when charged.

The dome of a Van de Graaff generator is made out of sheet **metal**, so the charge is concentrated on the surface of this sphere.



# Spherical symmetry:

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**Rotational  
symmetry?**

**yes**



**Translational  
symmetry?**

**yes**

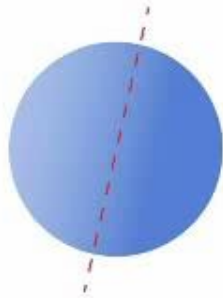
# Spherical symmetry:

---

All points on the surface of a sphere are the **same distance away from the sphere's center**.

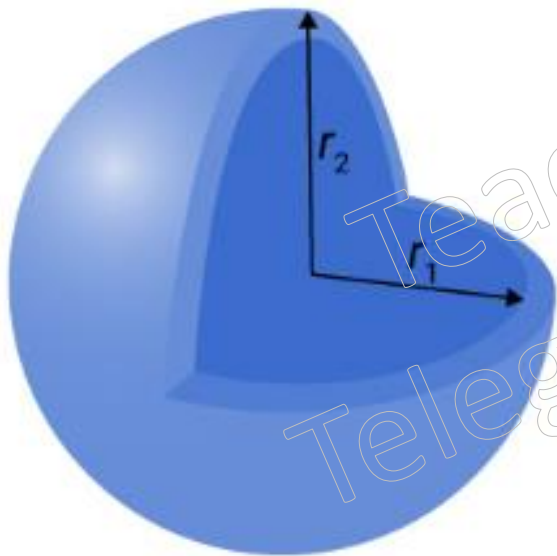
A sphere can be **rotated** by any angle or **translated** by any amount without any change to its shape.

A sphere, therefore, has **rotational symmetry**, and **translational symmetry**.



# Spherical symmetry:

Consider a thin spherical shell of charge,  $q$ . All of the sphere's charge is concentrated in the crust between  $r_1$  and  $r_2$ .



What is the electric field **inside** the hollow region in the sphere?

Use a Gaussian surface that is smaller than the inner radius of the hollow sphere.

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$
$$E dA = 0$$

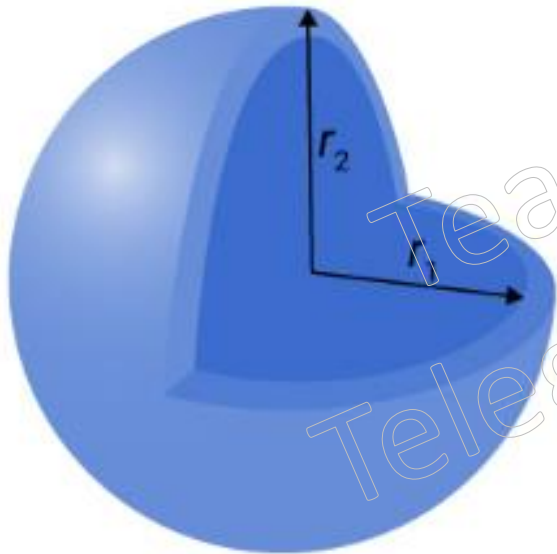
This is an example of **electrostatic shielding**.

# Spherical symmetry:

Consider a thin spherical shell of charge,  $q$ . All of the sphere's charge is concentrated in the crust between  $r_1$  and  $r_2$ .

What is the electric field **outside** of the sphere?

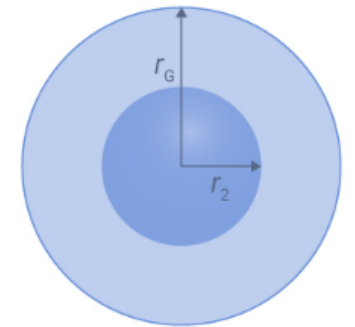
Use a **spherical Gaussian surface** surrounding the charged sphere.



$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

$$E(4\pi r_G^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r_G^2}$$



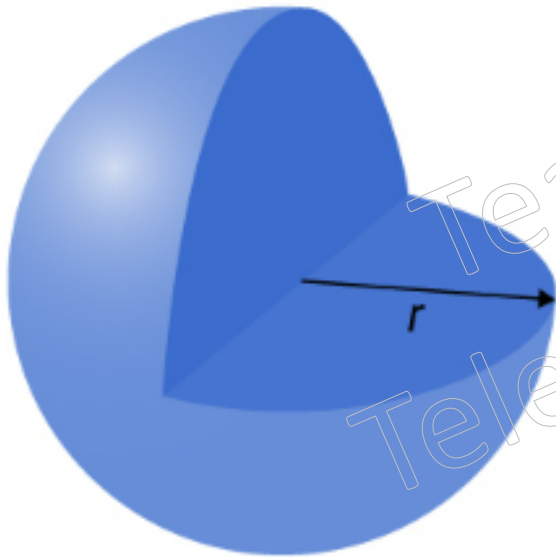
The electric field vectors and area vectors are **parallel to each other**.

# Spherical symmetry:

Consider a thin spherical shell of charge,  $q$ . All of the sphere's charge is concentrated in the crust between  $r_1$  and  $r_2$ .

**What is the electric field inside the sphere?**

Use a spherical Gaussian surface that has a smaller radius than the solid sphere.



$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

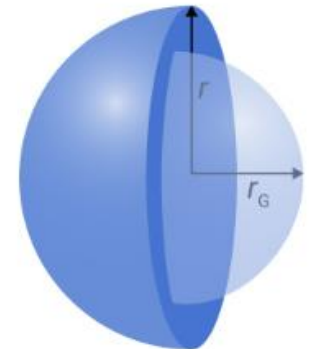
$$E A = q \frac{1}{\epsilon_0}$$

$$EA = \frac{\rho V_G}{\epsilon_0}$$

$$E(4\pi r_G^2) = \frac{\rho \left( \frac{4}{3} \pi r_G^3 \right)}{\epsilon_0}$$

$$\cancel{4} E \cancel{\pi} r_G^2 = \frac{\cancel{4} \pi}{3} \frac{\rho \cancel{r}_G^3}{\epsilon_0}$$

$$E = \frac{\rho r_G}{3\epsilon_0}$$

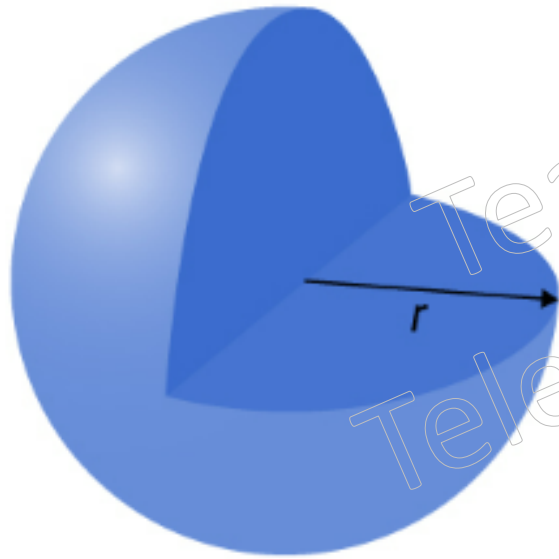


# Spherical symmetry:

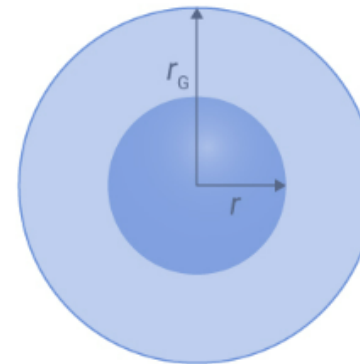
Consider a thin spherical shell of charge,  $q$ . All of the sphere's charge is concentrated in the crust between  $r_1$  and  $r_2$ .

What is the electric field **outside** of the sphere?

Use a spherical Gaussian surface that has a larger radius than the solid sphere.



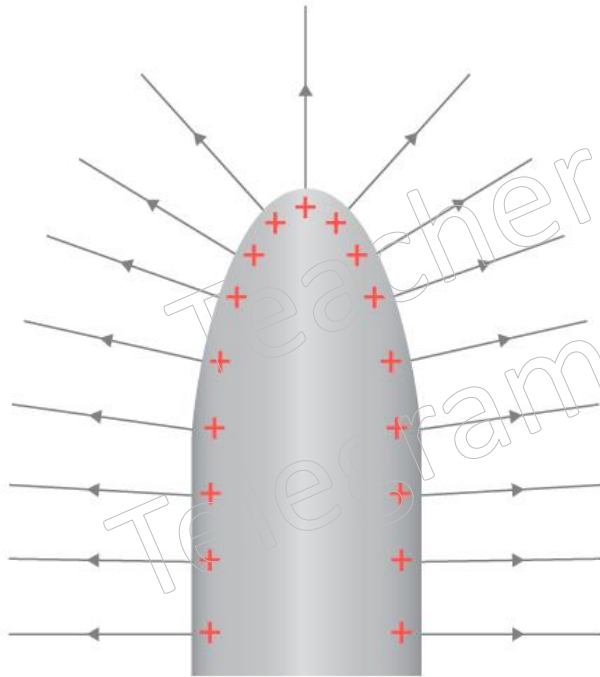
$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$
$$E(4\pi r_G^2) = \frac{q}{\epsilon_0}$$
$$E = \frac{q}{4\pi\epsilon_0 r_G^2}$$



This is the same as the case of the hollow sphere. From the outside, a uniformly charged spherical object has the same electric field as a point charge with the same magnitude positioned at the sphere's center.

# Sharp Points and Lightning Rods

An electric field vector will always point perpendicular to the surface of a charged conductor, no matter the shape of the surface.



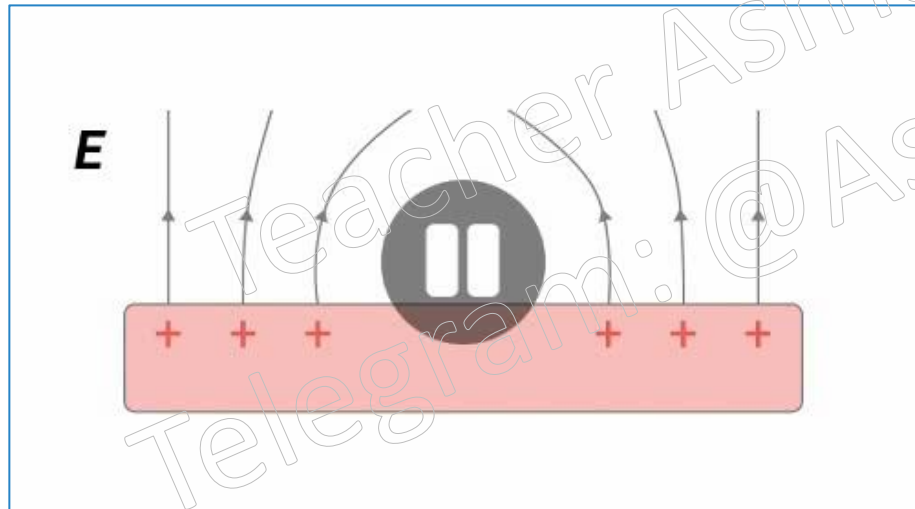
If the surface has a sharp curve, the field lines will be brought closer together. Sharp points in conductors allow for a higher density of charge

The strong electric field at a sharp point of a conductor makes it a good attractor for lightning. This device is called a **lightning rod** and they are commonly installed on the top of tall buildings.

If the point is too sharp, the electric field around the point may be too strong, and **it may create its own lightning**. Therefore, lightning rods are made using a rounder end, rather than a very sharp point.

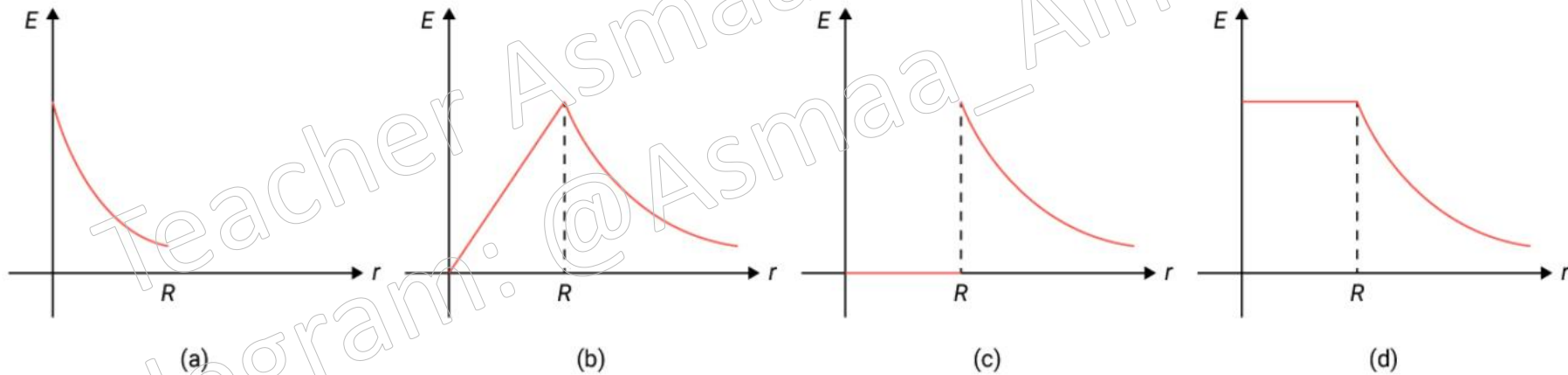
# Sharp Points and Lightning Rods

An electric field vector will always point perpendicular to the surface of a charged conductor, no matter the shape of the surface.



The region near this gap would have a weak electric field. Charges on the surface would repel, so some charges would move into this gap. All of the charges stop moving when the repulsive forces are balanced. The conductor reaches **electrostatic equilibrium**.

# Graphs:



# Activity 2:

## Concept Check 2.14

Suppose an uncharged hollow sphere made of a perfect insulator, for example a ping-pong ball, is resting on a perfect insulator. Some small amount of negative charge (say, a few hundred electrons) is placed at the north pole of the sphere. If you could check the distribution of the charge after a few seconds, what would you detect?

- a) All of the added charge has vanished, and the sphere is again electrically neutral.
- b) All of the added charge has moved to the center of the sphere.
- c) All of the added charge is distributed uniformly over the surface of the sphere.
- d) The added charge is still located at or very near the north pole of the sphere.
- e) The added charge is performing a simple harmonic oscillation on a straight line between the south and north poles of the sphere.

## Concept Check 2.13

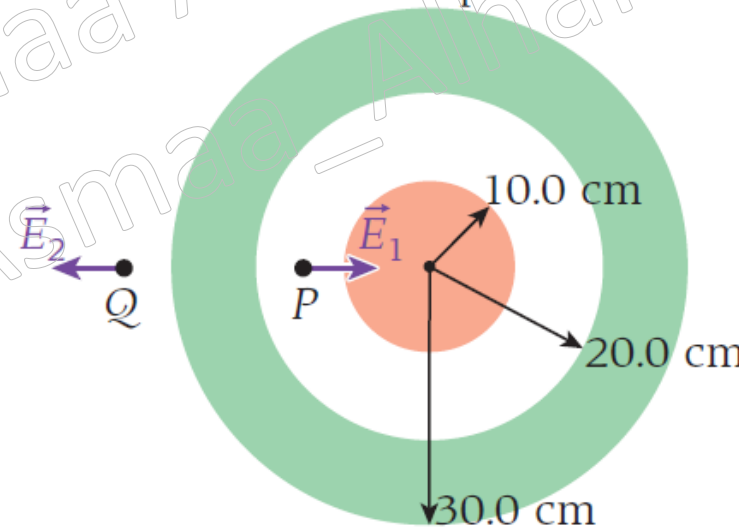
Suppose an uncharged solid steel ball, for example, one of the steel balls used in an old-fashioned pinball machine, is resting on a perfect insulator. Some small amount of negative charge (say, a few hundred electrons) is placed at the north pole of the ball. If you could check the distribution of the charge after a few seconds, what would you detect?

- a) All of the added charge has vanished, and the ball is again electrically neutral.
- b) All of the added charge has moved to the center of the ball.
- c) All of the added charge is distributed uniformly over the surface of the ball.
- d) The added charge is still located at or very near the north pole of the ball.
- e) The added charge is performing a simple harmonic oscillation on a straight line between the south and north poles of the ball.

# Activity: Bonus (page: 56)

**02:00**

**2.58** A hollow metal sphere has inner and outer radii of 20.0 cm and 30.0 cm, respectively. As shown in the figure, a solid metal sphere of radius 10.0 cm is located at the center of the hollow sphere. The electric field at a point  $P$ , a distance of 15.0 cm from the center, is found to be  $E_1 = 1.00 \times 10^4 \text{ N/C}$ , directed radially inward. At point  $Q$ , a distance of 35.0 cm from the center, the electric field is found to be  $E_2 = 1.00 \times 10^4 \text{ N/C}$ , directed radially outward. Determine the total charge on (a) the surface of the inner sphere, (b) the inner surface of the hollow sphere, and (c) the outer surface of the hollow sphere.



# Practice:

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**2.57** There is an electric field of magnitude  $150.0 \text{ N/C}$ , directed downward, near the surface of the Earth. What is the net electric charge on the Earth? You can treat the Earth as a spherical conductor of radius  $6371 \text{ km}$ .

**2.59** Two parallel, infinite, nonconducting plates are  $10.0 \text{ cm}$  apart and have charge distributions of  $+1.00 \mu\text{C/m}^2$  and  $-1.00 \mu\text{C/m}^2$ . What is the force on an electron in the space between the plates? What is the force on an electron located outside the two plates near the surface of one of the two plates?

# Practice:

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**2.60** An infinitely long charged wire produces an electric field of magnitude  $1.23 \times 10^3 \text{ N/C}$  at a distance of 50.0 cm perpendicular to the wire. The direction of the electric field is toward the wire.

- a) What is the charge distribution?
- b) How many electrons per unit length are on the wire?

# Practice:

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● **2.63** A sphere centered at the origin has a volume charge distribution of  $120. \text{ nC/cm}^3$  and a radius of  $12.0 \text{ cm}$ . The sphere is centered inside a conducting spherical shell with an inner radius of  $30.0 \text{ cm}$  and an outer radius of  $50.0 \text{ cm}$ . The charge on the spherical shell is  $-2.00 \text{ mC}$ . What are the magnitude and the direction of the electric field at each of the following distances from the origin?

a) at  $r = 10.0 \text{ cm}$

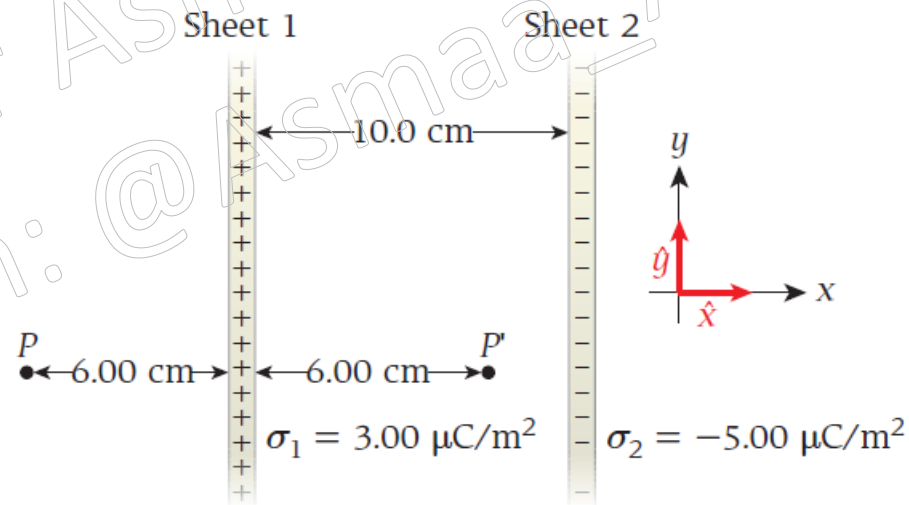
c) at  $r = 40.0 \text{ cm}$

b) at  $r = 20.0 \text{ cm}$

d) at  $r = 80.0 \text{ cm}$

# Practice:

•2.65 Two infinite sheets of charge are separated by 10.0 cm as shown in the figure. Sheet 1 has a surface charge distribution of  $\sigma_1 = 3.00 \mu\text{C}/\text{m}^2$  and sheet 2 has a surface charge distribution of  $\sigma_2 = -5.00 \mu\text{C}/\text{m}^2$ . Find the total electric field (magnitude and direction) at each of the following locations:



- a) at point P, 6.00 cm to the left of sheet 1
- b) at point P', 6.00 cm to the right of sheet 1

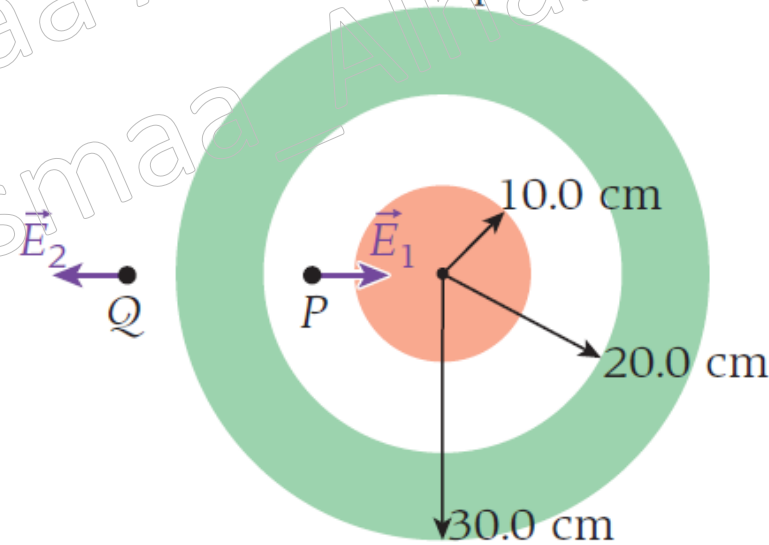
# Practice:

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- **2.66** A conducting solid sphere of radius 20.0 cm is located with its center at the origin of a three-dimensional coordinate system. A charge of 0.271 nC is placed on the sphere.
  - a) What is the magnitude of the electric field at point  $(x,y,z) = (23.1 \text{ cm}, 1.10 \text{ cm}, 0.00 \text{ cm})$ ?
  - b) What is the angle of this electric field with the x-axis at this point?
  - c) What is the magnitude of the electric field at point  $(x,y,z) = (4.10 \text{ cm}, 1.10 \text{ cm}, 0.00 \text{ cm})$ ?

# Practice:

**2.58** A hollow metal sphere has inner and outer radii of 20.0 cm and 30.0 cm, respectively. As shown in the figure, a solid metal sphere of radius 10.0 cm is located at the center of the hollow sphere. The electric field at a point  $P$ , a distance of 15.0 cm from the center, is found to be  $E_1 = 1.00 \times 10^4 \text{ N/C}$ , directed radially inward. At point  $Q$ , a distance of 35.0 cm from the center, the electric field is found to be  $E_2 = 1.00 \times 10^4 \text{ N/C}$ , directed radially outward. Determine the total charge on (a) the surface of the inner sphere, (b) the inner surface of the hollow sphere, and (c) the outer surface of the hollow sphere.



# Practice:

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**2.73** An infinitely long, solid cylinder of radius  $R = 9.00 \text{ cm}$ , with a uniform charge per unit of volume of  $\rho = 6.40 \times 10^{-8} \text{ C/m}^3$ , is centered about the  $y$ -axis. Find the magnitude of the electric field at a radius  $r = 4.00 \text{ cm}$  from the center of this cylinder.

**2.75** A solid metal sphere of radius  $8.00 \text{ cm}$ , with a total charge of  $10.0 \text{ } \mu\text{C}$ , is surrounded by a metallic shell with a radius of  $15.0 \text{ cm}$  carrying a  $-5.00 \text{ } \mu\text{C}$  charge. The sphere and the shell are both inside a larger metallic shell of inner radius  $20.0 \text{ cm}$  and outer radius  $24.0 \text{ cm}$ . The sphere and the two shells are concentric.

- What is the charge on the inner wall of the larger shell?
- If the electric field outside the larger shell is zero, what is the charge on the outer wall of the shell?