كراس التمارين P.15

① حل كلاً من المعادلات التالية

(a) $3\sqrt{x}+3=15 \Rightarrow 3\sqrt{x}=15-3 \Rightarrow$

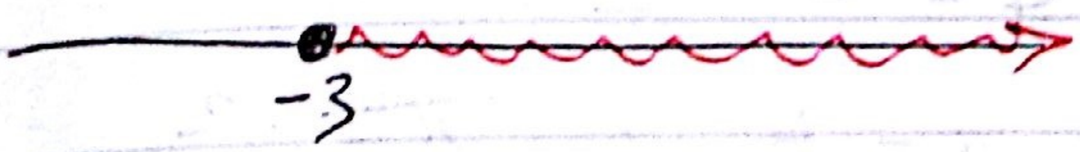
شرط الحل $x \geq 0$

$3\sqrt{x}=12 \Rightarrow \sqrt{x}=4 \Rightarrow x=16 \geq 0$

$\{16\} = ح.ص$

(b) $\sqrt{x+3}=5$

شرط الحل: $x+3 \geq 0 \Leftarrow x \geq -3$



$x \in [-3, \infty)$

$(\sqrt{x+3})^2 = (5)^2 \Rightarrow x+3=25 \Rightarrow x=22$

$22 \in [-3, \infty)$

$\{22\} = ح.ح$

(c) $(x+5)^{\frac{2}{3}} = 4$

$\left[(x+5)^{\frac{2}{3}}\right]^{\frac{3}{2}} = \left[4\right]^{\frac{3}{2}} \Rightarrow x+5 = (\sqrt{4})^3 \Rightarrow$

$\Rightarrow x+5=8 \Rightarrow x=3$

$\{3\} = ح.ص$

(d) $(x+1)^{\frac{3}{2}} - 2 = 2$

$(x+1)^{\frac{3}{2}} = 4$

شرط الحل: $4>0$ ، $x+1 \geq 0$

$x \geq -1$

$x \in [-1, \infty)$

$\left[(x+1)^{\frac{3}{2}}\right]^{\frac{2}{3}} = \left[4\right]^{\frac{2}{3}} \Rightarrow x+1 = 2.52$

$x = 2.52 - 1$

$x = 1.52 \in [-1, \infty)$

$\{1.52\} = ح.ص$

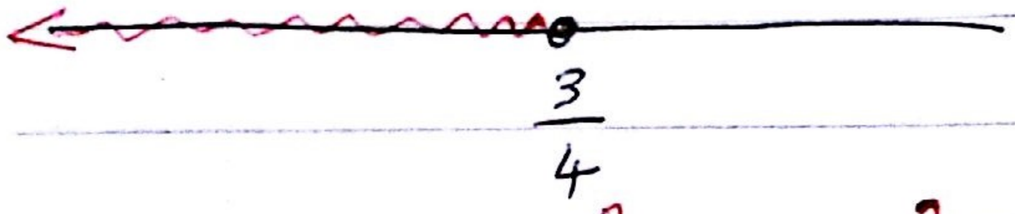
(e) $\sqrt{3-4x} - 2 = 0$

$\sqrt{3-4x} = 2$

شرط الجذر: $3-4x \geq 0 \Leftarrow 3 \geq 4x$

$4x \leq 3 \Rightarrow x \leq \frac{3}{4}$

$x \in (-\infty, \frac{3}{4}]$



$\left[\sqrt{3-4x}\right]^2 = [2]^2$

$3-4x = 4 \Rightarrow -4x = 4-3$

$-4x = +1 \Rightarrow x = \frac{1}{-4} \in (-\infty, \frac{3}{4}]$

$\left\{\frac{-1}{4}\right\} = ق.ح$

(f) $\frac{2(2x+4)^{\frac{3}{4}}}{2} = \frac{16}{2}$

$(2x+4)^{\frac{3}{4}} = 8$

شرط الجذر: $2x+4 \geq 0$

$2x \geq -4$

$x \geq -2$

$x \in [-2, \infty)$

$\left[(2x+4)^{\frac{3}{4}}\right]^{\frac{4}{3}} = [8]^{\frac{4}{3}}$

$2x+4 = 16 \Rightarrow 2x = 16-4 \Rightarrow \frac{2x}{2} = \frac{12}{2}$

$\Rightarrow x = 6 \in [-2, \infty)$

$\{6\} = ق.ح$

(g) $(5-3x)^{\frac{3}{2}} + 4 = 3$

$(5-3x)^{\frac{3}{2}} = 3-4$

$(5-3x)^{\frac{3}{2}} = -1$

$\not\exists = ق.ح$

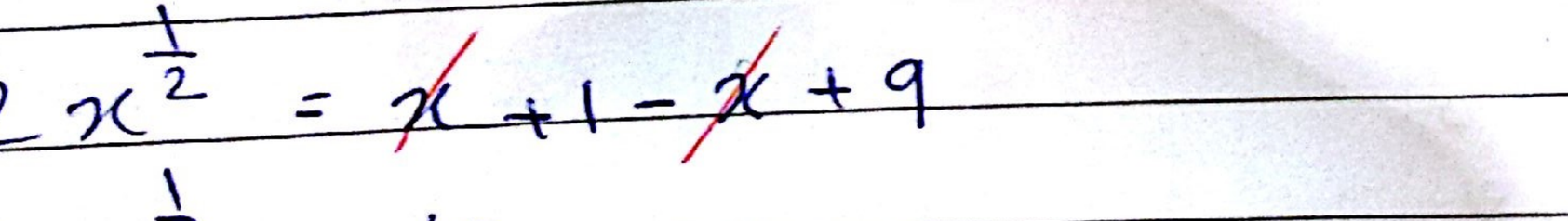
$$(x-9)^{\frac{1}{2}} + 1 = x^{\frac{1}{2}}$$

$$(x-9)^{\frac{1}{2}} = x^{\frac{1}{2}} - 1$$

$x-9 \geq 0$ and $x \geq 0$ and $x^{\frac{1}{2}} - 1 \geq 0$

$x \geq 9$ and $x \geq 0$ and $x^{\frac{1}{2}} \geq 1$

$x \geq 1$



$x \in [9, \infty)$ المجال

$$\left[(x-9)^{\frac{1}{2}}\right]^2 = \left[x^{\frac{1}{2}} - 1\right]^2$$

$$x - 9 = x - 2x^{\frac{1}{2}} + 1$$

$$2x^{\frac{1}{2}} = x + 1 - x + 9$$

$$2x^{\frac{1}{2}} = 10$$

$$x^{\frac{1}{2}} = 5$$

$$\left[x^{\frac{1}{2}}\right]^2 = [5]^2$$ بتربيع الطرفين

$x = 25 \in [9, \infty)$

$\{25\}$ = ح.ص

(3) حل كلاً من المعادلات الآتية

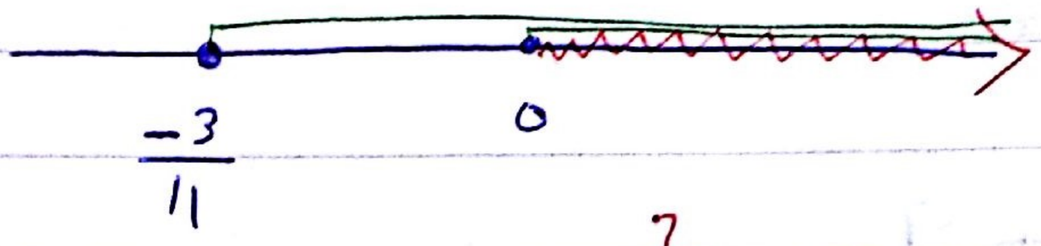
(a) $\sqrt{11x+3} - 2x = 0$

$\sqrt{11x+3} = 2x$

شرط الجذر: $11x+3 \geq 0$ ، $x \geq 0$

$11x \geq -3$

$x \geq \frac{-3}{11}$ ، $x \geq 0$



$x \in [0, \infty)$ المجال

$\left[\sqrt{11x+3}\right]^2 = [2x]^2$

$11x+3 = 4x^2 \Rightarrow 4x^2 - 11x - 3 = 0$

$x = 3 \in$ ، $x = \frac{-1}{4} \notin [0, \infty)$

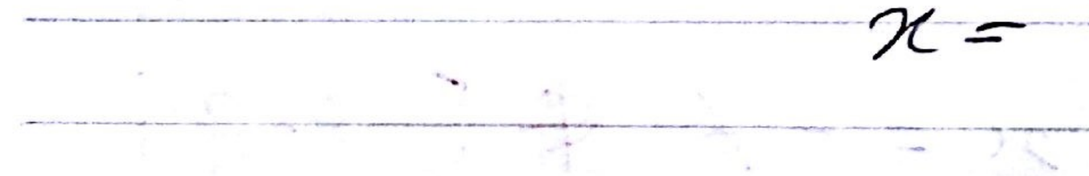
$\{3\} = ح.ص$

(b) $\sqrt{3x+13} - 5 = x$

$\sqrt{3x+13} = x+5$

$3x+13 \geq 0$ ، $x+5 \geq 0$ الشرط

$x \geq \frac{-13}{3}$ ، $x \geq -5$



$x \in [\frac{-13}{3}, \infty)$ المجال

$\left[\sqrt{3x+13}\right]^2 = [x+5]^2 \Rightarrow 3x+13 = x^2+10x+25 \Rightarrow$

$\Rightarrow x^2+7x+12 = 0 \Rightarrow (x+3)(x+4) = 0$

$x = -3 \in$ ، $x = -4 \in$

$\{-3, -4\} = ح.ص$

ⓒ $\sqrt{-3x-5} = x+3$

شرط الجذر:

$-3x-5 \geq 0$ ، $x+3 \geq 0$

$-3x \geq 5$ ، $x \geq -3$

$x \leq \frac{5}{-3}$

شرط الجذر: $x \in [-3, \frac{5}{-3}]$

$\left[\sqrt{-3x-5}\right]^2 = [x+3]^2$

$-3x-5 = x^2+6x+9$

$x^2+6x+3x+9+5=0$

$x^2+9x+14=0$

$(x+2)(x+7)=0$

$x=-2 \in [-3, \frac{5}{-3}]$ ، $x=-7 \notin [-3, \frac{5}{-3}]$

$\{-2\}$ = مجموعة الحل

ⓓ $(x+3)^{\frac{1}{2}} - 1 = x$

$(x+3)^{\frac{1}{2}} = x+1$

شرط الجذر: $x+3 \geq 0$ ، $x+1 \geq 0$

$x \geq -3$ ، $x \geq -1$

$x \in [-1, \infty)$

$\left[(x+3)^{\frac{1}{2}}\right]^2 = (x+1)^2$

$x+3 = x^2+2x+1$

$x^2+2x+1-x-3=0$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \notin$ ، $x=1 \in [-1, \infty)$

$\{1\}$ = مجموعة الحل

ⓔ $x+8=(x^2+16)^{\frac{1}{2}}$

شرط الحل

* $x^2+16>0$ محققة دوماً
* $x+8\geq 0 \Rightarrow x\geq -8 \Rightarrow$

$x\in[-8,+\infty)$

$$[x+8]^2=\left[(x^2+16)^{\frac{1}{2}}\right]^2$$ بالتربيع

$$x^2+16x+64=x^2+16$$
$$x^2-x^2+16x+64-16=0$$
$$16x+48=0$$
$$16x=-48$$
$$x=\frac{-48}{16} \Rightarrow x=-3\in[-8,\infty)$$

$\{-3\}$ = ح.ك

ⓕ $\sqrt{10x}-2\sqrt{5x-25}=0$

$$\sqrt{10x}=2\sqrt{5x-25}$$

شرط الحل:

$10x\geq 0$ و $5x-25\geq 0$

$x\geq 0$ $\quad 5x\geq 25$

$x\geq 5$

$x\in[5,\infty)$

$$[\sqrt{10x}]^2=[2\sqrt{5x-25}]^2$$

$$10x=4(5x-25)$$
$$10x=20x-100$$
$$10x-20x=-100$$
$$-10x=-100 \Rightarrow x=\frac{-100}{-10}=10\in[5,\infty)$$

$\{10\}$ = مجموعة الحل

(g) $(3x+2)^{\frac{1}{2}} - (2x+7)^{\frac{1}{2}} = 0$

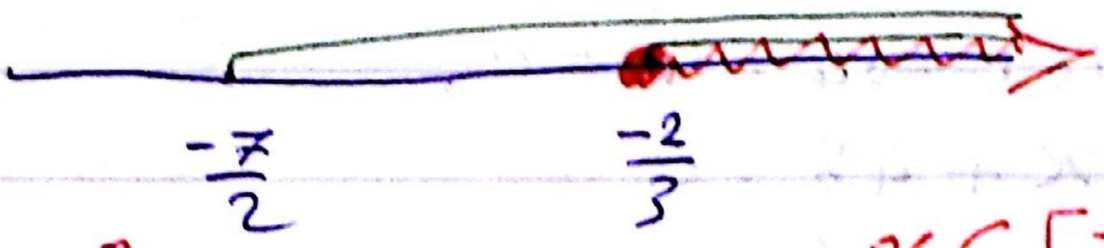
$\sqrt{3x+2} = \sqrt{2x+7}$

شرط الجذر:

$3x+2 \geq 0$ و $2x+7 \geq 0$

$3x \geq -2$ $\quad$ $2x \geq -7$

$x \geq \frac{-2}{3}$ $\quad$ $x \geq \frac{-7}{2}$



$x \in [\frac{-2}{3}, \infty)$

$\left[\sqrt{3x+2}\right]^2 = \left[\sqrt{2x+7}\right]^2$

$3x+2 = 2x+7$

$3x-2x = 7-2$

$x = 5 \in [\frac{-2}{3}, \infty)$

$\{5\} = ق.ح$

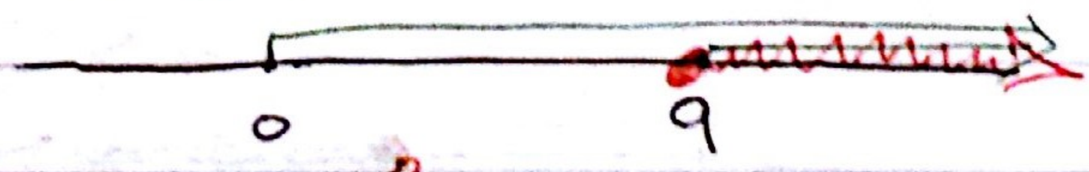
(h) $\underbrace{(x-9)^{\frac{1}{2}}}_{+} + 1 = \underbrace{x^{\frac{1}{2}}}_{+}$

شرط الجذر:

$x-9 \geq 0$ $\quad$ $x \geq 0$

$x \geq 9$

$x \in [9, \infty)$



$\left[\sqrt{x-9}+1\right]^2 = \left[\sqrt{x}\right]^2$

$x-9+2\sqrt{x-9}+1 = x$

$2\sqrt{x-9} = x-x+9-1$

$2\sqrt{x-9} = 8$

$\sqrt{x-9} = 4$

$x-9 = 16$ بالتربيع

$x = 16+9$

$x = 25 \in [9, \infty)$

$\{25\} = ق.ح$

① $(2x+3)^{\frac{3}{4}} - 3 = 5$

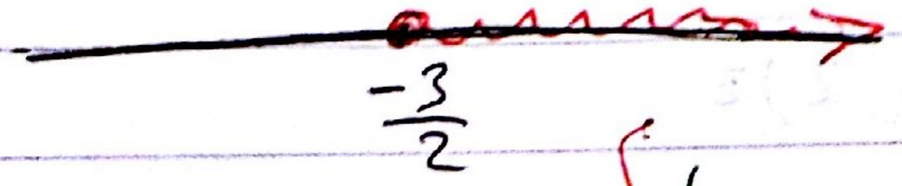
$(2x+3)^{\frac{3}{4}} = 5+3$

$(2x+3)^{\frac{3}{4}} = 8$

شرط الجذر

$2x+3 \geq 0 \Rightarrow 2x \geq -3$

$\Rightarrow x \geq \frac{-3}{2} \Rightarrow x \in [\frac{-3}{2}, \infty)$



$\left[(2x+3)^{\frac{3}{4}}\right]^{\frac{4}{3}} = \left[8\right]^{\frac{4}{3}}$

$2x+3 = 16$

$2x = 16-3 \Rightarrow 2x = 13 \Rightarrow x = \frac{13}{2} \in$

$x = \frac{13}{2} \in [\frac{-3}{2}, \infty) \Rightarrow \{\frac{13}{2}\} = S.S$

② $2(x-1)^{\frac{4}{3}} + 4 = 36$

$2(x-1)^{\frac{4}{3}} = 36-4 \Rightarrow \frac{2(x-1)^{\frac{4}{3}}}{2} = \frac{32}{2}$.

$\Rightarrow (x-1)^{\frac{4}{3}} = 16$

$\left[(x-1)^{\frac{4}{3}}\right]^{\frac{3}{4}} = \left[16\right]^{\frac{3}{4}}$

شرط الجذر $x \in \mathbb{R}$

$x-1 = 8 \Rightarrow x = 8+1 \Rightarrow x = 9$

$\{9\} = S.S$

(K) $(3x+2)^{\frac{1}{2}} = 8\,(3x+2)^{-\frac{1}{2}}$

طريقة الحل:

$3x+2 \geq 0$

$3x \geq -2$

$x \geq \frac{-2}{3}$

$x \in [\frac{-2}{3}, \infty)$

$$(3x+2)^{\frac{1}{2}} = \frac{8}{(3x+2)^{\frac{1}{2}}}$$

$$(3x+2)^{\frac{1}{2}} \cdot (3x+2)^{\frac{1}{2}} = 8$$

$$(3x+2)^{\frac{1}{2}+\frac{1}{2}} = 8$$

$$3x+2 = 8 \Longrightarrow 3x = 8-2$$

$$3x = 6$$

$$x = 2 \in [\tfrac{-2}{3}, \infty)$$

$\{2\} = $ مجموعة الحل

(L) $(2x+1)^{\frac{1}{3}} = (3x+2)^{\frac{1}{3}}$

مجموعة الحل $= x \in R$

$$\left[(2x+1)^{\frac{1}{3}}\right]^3 = \left[(3x+2)^{\frac{1}{3}}\right]^3$$

$$2x+1 = 3x+2$$

$$2x-3x = 2-1$$

$$-x = 1 \Longrightarrow x = -1$$

$\{-1\} = $ مجموعة الحل

(m) $(2x-1)^{\frac{1}{3}} = (x+1)^{\frac{1}{6}}$ ← زوجي

شرط الجذر

$2x-1 \geq 0 \rightarrow x+1 \geq 0$

$2x \geq 1 \qquad x \geq -1$

$x \geq \frac{1}{2}$

$x \in [\frac{1}{2}, \infty)$

$\left[(2x-1)^{\frac{1}{3}}\right]^6 = \left[(x+1)^{\frac{1}{6}}\right]^6$

$(2x-1)^2 = x+1$

$4x^2 - 4x + 1 = x + 1$

$4x^2 - 4x - x + 1 - 1 = 0 \Rightarrow 4x^2 - 5x = 0 \Rightarrow$

$x(4x-5) = 0$

$x = 0 \quad \text{or} \quad 4x - 5 = 0$

$x = 0 \notin [\frac{1}{2}, \infty) \qquad 4x = 5 \Rightarrow x = \frac{5}{4} \in [\frac{1}{2}, \infty)$

$\{\frac{5}{4}\} = S.S$

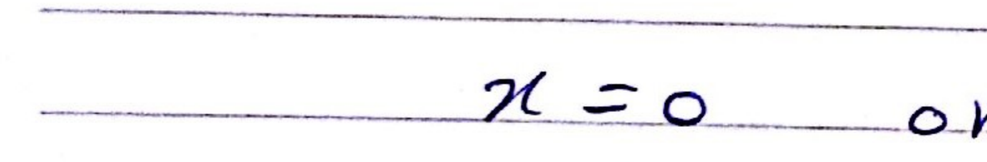
(n) $(x+5)^{\frac{1}{2}} - (5-2x)^{\frac{1}{4}} = 0 \Rightarrow (x+5)^{\frac{1}{2}} = (5-2x)^{\frac{1}{4}}$

شرط الجذر

$x+5 \geq 0 \rightarrow 5-2x \geq 0$

$x \geq -5 \qquad -2x \geq -5$

$x \leq \frac{+5}{-2}$



$x \in [-5, \frac{5}{2}]$

$\left[(x+5)^{\frac{1}{2}}\right]^4 = \left[(5-2x)^{\frac{1}{4}}\right]^4 \Rightarrow (x+5)^2 = 5-2x \Rightarrow$

$\Rightarrow x^2 + 10x + 25 + 2x - 5 = 0 \Rightarrow x^2 + 12x + 20 = 0 \Rightarrow$

$(x+2)(x+10) = 0$

$x = -2 \in \qquad x = -10 \notin [-5, \frac{5}{2}]$

$\{-2\} = S.S$