6-3

طول قوس ومعادلة منحنى دالة

(1) أوجد طول القوس من منحنى الدالة f :

$f(x) = 5 + 2\sqrt{x^3}$ في الفترة $[0 , \frac{1}{3}]$

$$f'(x) = 2 \cdot \frac{3}{2} x^{\frac{1}{2}} = 3\sqrt{x}$$

$$L = \int_0^{\frac{1}{3}} \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^{\frac{1}{3}} \sqrt{1 + 9x}\, dx = \frac{1}{9}\int_0^{\frac{1}{3}} 9(1+9x)^{\frac{1}{2}}$$

$$= \frac{1}{9} \cdot \frac{2}{3}\left[(1+9x)^{\frac{3}{2}}\right]_0^{\frac{1}{3}} = \frac{2}{27}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{14}{27} \text{ units}$$

(2) أوجد طول القوس من منحنى الدالة f :

$f(x) = \frac{1}{3}(7+4x)^{\frac{3}{2}}$ في الفترة $[1 , \frac{5}{4}]$

$$f'(x) = \frac{1}{3} \cdot \frac{3}{2}(7+4x)^{\frac{1}{2}} \cdot 4 = 2\sqrt{7+4x}$$

$$L = \int_1^{\frac{5}{4}} \sqrt{1 + 28 + 16x}\, dx$$

$$= \frac{1}{16}\int_1^{\frac{5}{4}} 16\sqrt{29 + 16x}\, dx = \frac{1}{16}\left[\frac{2}{3}(29+16x)^{\frac{3}{2}}\right]_1^{\frac{5}{4}}$$

$$= \frac{1}{24}\left[49^{\frac{3}{2}} - 45^{\frac{3}{2}}\right] \approx 1.71 \text{ units}$$

(3)

$$f(x) = \frac{1}{6}x^3 + \frac{1}{2x} \Rightarrow f'(x) = \frac{3}{6}x^2 - \frac{1}{2}x^{-2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{1}{4}x^4 - 2\left(\frac{x^2}{2}\right)\left(\frac{1}{2x^2}\right) + \frac{1}{4x^4}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{2} + \frac{1}{4}x^4 + \frac{1}{4}x^{-4}}\, dx = \int_1^2 \sqrt{\left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2}\, dx$$

$$= \int_1^2 \frac{1}{2}x^2 + \frac{1}{2x^2}\, dx = \left[\frac{1}{6}x^3 - \frac{1}{2x}\right]_1^2 = \frac{17}{12} \text{ units}$$

(4) اوجد معادلة منحنى الدالة f الذي ميله عند أي نقطة عليه (x,y) هو: $-x^2+2x-4$ ويمر بالنقطة $A(3,7)$

$$\because f'(x) = -x^2+2x-4 \Rightarrow f(x) = \int -x^2+2x-4 \, dx$$

$$\therefore f(x) = \frac{-x^3}{3} + x^2 - 4x + c$$

$$\because A(3,7) \in f \Rightarrow 7 = \frac{-27}{3} + 9 - 12 + c \Rightarrow c = 19$$

$$\therefore \quad f(x) = \frac{-x^3}{3} + x^2 - 4x + 19$$

(5) اوجد معادلة منحنى الدالة f الذي ميله عند أي نقطة عليه (x,y) هو $f'(x) = -4x^3+2x+5$ ويمر بالنقطة $A(1,3)$

$$\because f'(x) = -4x^3+2x+5 \Rightarrow f(x) = \int -4x^3+2x+5 \, dx$$

$$\therefore f(x) = -x^4 + x^2 + 5x + c$$

$$\because A(1,3) \in f \Rightarrow 3 = -1 + 1 + 5 + c \Rightarrow c = -2$$

$$\therefore f(x) = -x^4 + x^2 + 5x - 2$$

(6) $f'(x) = \cos 2x$ $\qquad A\left(\frac{-\pi}{4}, \frac{5}{2}\right)$

$$f(x) = \int \cos 2x \, dx = \frac{1}{2} \sin 2x + c$$

$$\because A\left(\frac{-\pi}{4}, \frac{5}{2}\right) \in f \Rightarrow \frac{5}{2} = \frac{1}{2} \sin\left(2\left(\frac{-\pi}{4}\right)\right) + c$$

$$\Rightarrow \frac{5}{2} = \frac{-1}{2} + c \Rightarrow c = 3$$

$$\therefore f(x) = \frac{1}{2} \sin 2x + 3$$

$A(\frac{2\pi}{9} ، \frac{7}{6})$ ، $f'(x) = \sin 3x$ (7)

$f(x) = \int \sin 3x \, dx = \frac{-1}{3} \cos 3x + C$

$\because A(\frac{2\pi}{9} ، \frac{7}{6}) \in f \Rightarrow \frac{7}{6} = \frac{-1}{3} \cos(3(\frac{2\pi}{9})) + C$

$\Rightarrow \frac{7}{6} = \frac{-1}{3} \cdot \frac{-1}{2} + C \Rightarrow C = 1$

$\therefore f(x) = \frac{-1}{3} \cos 3x + 1$

$B(-2 ، 3)$ $2x + 5$ ميل العمودي (8)

$f'(x) = \frac{-1}{2x+5} \Rightarrow f(x) = \int \frac{-1}{2x+5} \, dx$

$\therefore f(x) = \frac{-1}{2} \ln|2x+5| + C$

$\because B(-2, 3) \in f \Rightarrow 3 = \frac{-1}{2} \ln|1| + C \Rightarrow C = 3$

$\therefore f(x) = \frac{-1}{2} \ln|2x+5| + 3$

$f''(x) = 12x^2 - 24x - 1 \Rightarrow f'(x) = 4x^3 - 12x^2 - x + C$ (9)

نقطة عظمى محلية $(-\frac{1}{2} ، \frac{15}{16})$

$\therefore f'(-\frac{1}{2}) = 0 \Rightarrow 4(\frac{-1}{2})^3 - 12(\frac{-1}{2})^2 - (\frac{-1}{2}) + C = 0 \Rightarrow C = 3$

$\therefore f'(x) = 4x^3 - 12x^2 - x + 3 \Rightarrow f(x) = \int 4x^3 - 12x^2 - x + 3 \, dx$

$\therefore f(x) = x^4 - 4x^3 - \frac{x^2}{2} + 3x + C_1$

$(\frac{-1}{2} ، \frac{15}{16}) \in f \Rightarrow \frac{15}{16} = (\frac{-1}{2})^4 - 4(\frac{-1}{2})^3 - \frac{(\frac{-1}{2})^2}{2} + (\frac{-1}{2}) + C_1 \Rightarrow C_1 = 2$

$\therefore f(x) = x^4 - 4x^3 - \frac{x^2}{2} + 3x + 2$