المساحات في المستوي

(1) اوجد مساحة المنطقة المحددة بالمنحنى $f(x) = 8x^3$ والمستقيمين $x = 1$ ، $x = 3$

$f(x) = 0 \Rightarrow 8x^3 = 0 \Rightarrow x = 0 \notin (1,3)$

$$A = \left| \int_1^3 8x^3\, dx \right| = \left| \left[2x^4\right]_1^3 \right|$$

$$= \left| 2(3)^4 - 2(1)^4 \right| = 160 \text{ unit Square}$$

(2) $f(x) = x^2 - 5x$ ، محور السينات

$f(x) = 0 \Rightarrow x^2 - 5x = 0 \Rightarrow x(x-5) = 0$

$x = 0$ or $x = 5$

$$A = \left| \int_0^5 x^2 - 5x\, dx \right| = \left| \left[\frac{x^3}{3} - \frac{5}{2}x \right]_0^5 \right|$$

$$= \left| \frac{5^3}{3} - \frac{5}{2}(5) - 0 \right| = \frac{175}{6} \text{ unit Square}$$

(3) $f(x) = 12 - x^2$ ومحور السينات

$f(x) = 0 \Rightarrow 12 - x^2 = 0 \Rightarrow x^2 = 12$

$\Rightarrow x = 2\sqrt{3}$ ، $x = -2\sqrt{3}$

$$A = \left| \int_{-2\sqrt{3}}^{2\sqrt{3}} (12 - x^2)\, dx \right| = \left| \left[12x - \frac{x^3}{3} \right]_{-2\sqrt{3}}^{2\sqrt{3}} \right|$$

$$= \left| 12(2\sqrt{3}) - \frac{(2\sqrt{3})^3}{3} - \left(12(-2\sqrt{3}) - \frac{(-2\sqrt{3})^3}{3} \right) \right|$$

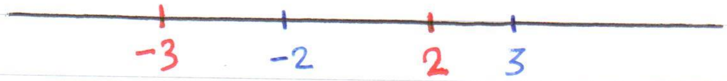
$= 55.43$ unit Square

(4) اوجد مساحة المنطقة المحدده بمنحنى الدالة f ومحور السينات في الفتره المحدده

$$f(x) = x^2 - x - 6 \quad ، \quad [-3 ، 2]$$

$$f(x) = 0 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$$

$$\Rightarrow \quad x = 3 \quad ، \quad x = -2$$



$$A = \left| \int_{-3}^{-2} x^2 - x - 6 \, dx \right| + \left| \int_{-2}^{2} x^2 - x - 6 \, dx \right|$$

$$= \left| \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_{-3}^{-2} \right| + \left| \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_{-2}^{2} \right|$$

$$= \frac{17}{6} + \frac{56}{3} = \frac{43}{2} \text{ unit Square}$$

(5)

$$f(x) = x^3 - 6x \quad ، \quad [0 ، 3]$$

$$f(x) = 0 \Rightarrow x^3 - 6x = 0 \Rightarrow x(x^2 - 6) = 0 \Rightarrow$$

$$x = 0 \quad ، \quad x = \sqrt{6} \quad ، \quad x = -\sqrt{6}$$

$$A = \left| \int_{0}^{\sqrt{6}} x^3 - 6x \, dx \right| + \left| \int_{\sqrt{6}}^{3} x^3 - 6x \, dx \right|$$

$$= \left| \left[\frac{x^4}{4} - 3x^2 \right]_{0}^{\sqrt{6}} \right| + \left| \left[\frac{x^4}{4} - 3x^2 \right]_{\sqrt{6}}^{3} \right|$$

$$= |9 - 18| + \left| \frac{-27}{4} - 9 \right| = |-9| + \left| \frac{-63}{4} \right| = \frac{99}{4} \text{ units Square}$$

$f(x) = \cos 2x$ ، $\left[\frac{-\pi}{4} ، \frac{\pi}{2}\right]$ (6)

$f(x) = 0 \Rightarrow \cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$

$$A = \left| \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \cos 2x \, dx \right| + \left| \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \, dx \right|$$

$$= \frac{1}{2} \left| \left[\sin 2x \right]_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \right| + \frac{1}{2} \left| \left[\sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \right|$$

$= 1 + \frac{1}{2} = \frac{3}{2}$ unit Square

(7) اوجد مساحة المنطقة المحدده بالمنحنيين

$f(x) = 4x - x^2$ ، $g(x) = 5 + x^2$

والمستقيمين $x=0$ ، $x=2$ علماً بأن f ، g غير متقاطعين

$$A = \left| \int_0^2 f(x) - g(x) \, dx \right|$$

$$= \left| \int_0^2 4x - x^2 - (5 + x^2) \, dx \right|$$

$$= \left| \int_0^2 -2x^2 + 4x - 5 \, dx \right|$$

$$= \left| \left[\frac{-2}{3} x^3 + 2x^2 - 5x \right]_0^2 \right|$$

$$= \left| \frac{-2}{3}(2)^3 + 2(2)^2 - 5(2) - 0 \right|$$

$= \frac{22}{3}$ unit square

(8) اوجد مساحة المنطقة المحدده بالمنحنيين :

$f(x) = x$ ، $g(x) = \sqrt[3]{x}$ والمستقيمين $x = 8$ ، $x = 1$

$$f(x) = g(x) \Rightarrow x = \sqrt[3]{x} \Rightarrow x^3 = x$$

$$\Rightarrow x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$$

$$\Rightarrow x = 0 \;,\; x = 1 \;,\; x = -1$$

وكل من هذه الأصفار لا ينتمي الى الفتره $(1, 8)$

$$\therefore A = \left| \int_1^8 x - \sqrt[3]{x} \, dx \right|$$

$$= \left| \left[\frac{x^2}{2} - \frac{3}{4} x^{\frac{4}{3}} \right]_1^8 \right| = \frac{81}{4} \text{ unit square}$$

(9) اوجد مساحة المنطقة المحدده بالمنحنيين :

$f(x) = 2x^2$ ، $g(x) = 3 - x$ ، $x = 0$ ، $x = 3$

$$f(x) = g(x) \Rightarrow 2x^2 = 3 - x \Rightarrow 2x^2 + x - 3 = 0 \Rightarrow$$

$$x = 1 \in (0, 3) \;,\; x = \frac{-3}{2} \notin (0, 3)$$

$$\therefore A = \left| \int_0^1 2x^2 - (3 - x) \, dx \right| + \left| \int_1^3 2x^2 - (3 - x) \, dx \right|$$

$$= \left| \int_0^1 2x^2 + x - 3 \, dx \right| + \left| \int_1^3 2x^2 + x - 3 \, dx \right|$$

$$= \left| \left[\frac{2}{3} x^3 - \frac{x^2}{2} - 3x \right]_0^1 \right| + \left| \left[\frac{2}{3} x^3 - \frac{x^2}{2} - 3x \right]_1^3 \right|$$

$$= \frac{17}{6} + \frac{22}{3} = \frac{61}{6} \text{ unit Square}$$

(10) اوجد مساحة المنطقة بين المنحنى $f(x)=3-x^2$

والمستقيم $g(x)=-1$

$$f(x)=g(x) \Rightarrow 3-x^2=-1 \Rightarrow x^2-4=0 \Rightarrow$$

$$x=-2 \ ، \ x=2$$

$$A=\left|\int_{-2}^{2} -1-(3-x^2)\,dx\right| = \left|\int_{-2}^{2} x^2-4\,dx\right|$$

$$=\left|\left[\frac{x^3}{3}-4x\right]_{-2}^{2}\right| = \left|\frac{(2)^3}{3}-4(2)-\left(\frac{(-2)^3}{3}-4(-2)\right)\right|$$

$$=\frac{32}{3} \text{ unit square}$$

(11) اوجد مساحة المنطقة المحدده بالمنحنيات

$$f(x)=x^2-2 \ ، \ g(x)=2$$

$$f(x)=g(x) \Rightarrow x^2-2=2 \Rightarrow x^2-4=0 \Rightarrow$$

$$x=2 \ ، \ x=-2$$

$$\therefore A=\left|\int_{-2}^{2} x^2-2-2\,dx\right| = \left|\int_{-2}^{2} x^2-4\,dx\right|$$

$$=\left|\left[\frac{x^3}{3}-4x\right]_{-2}^{2}\right|$$

$$=\left|\frac{2^3}{3}-4(2)-\left(\frac{(-2)^3}{3}-4(-2)\right)\right|$$

$$=\left|\frac{-32}{3}\right| = \frac{32}{3} \text{ Unit Square}$$

$f(x) = 2x - x^2$ و $g(x) = -2x$ (12)

$$f(x) = g(x) \Rightarrow 2x - x^2 = -2x \Rightarrow x^2 - 4x = 0$$

$$\Rightarrow x = 0 \quad ، \quad x = 4$$

$$A = \left| \int_0^4 -2x - (2x - x^2)\, dx \right| = \left| \int_0^4 x^2 - 4x\, dx \right|$$

$$= \left| \left[\frac{x^3}{3} - 2x^2 \right]_0^4 \right| = \left| \frac{(4)^3}{3} - 2(4)^2 - 0 \right|$$

$$= \left| \frac{-32}{3} \right| = \frac{32}{3} \text{ unit Square}$$

$f(x) = 7 - 2x^2$ و $g(x) = x^2 + 4$ (13)

$$f(x) = g(x) \Rightarrow 7 - 2x^2 = x^2 + 4 \Rightarrow 3x^2 - 3 = 0$$

$$\Rightarrow x = 1 \quad و \quad x = -1$$

$$A = \left| \int_{-1}^1 x^2 + 4 - (7 - 2x^2)\, dx \right|$$

$$= \left| \int_{-1}^1 3x^2 - 3\, dx \right| = \left| \left[x^3 - 3x \right]_{-1}^1 \right|$$

$$= \left| 1^3 - 3(1) - ((-1)^3 - 3(-1)) \right|$$

$$= |-4| = 4 \text{ unit Square}$$