التكامل المحدد

P. 51 ① اوجد

$$\int_2^7 (x^3 - 2x^2 + 2)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{2}{3}x^3 + 2x\right]_2^7$$

$$= \frac{(7)^4}{4} - \frac{2}{3}(7)^3 + 2(7) - \left[\frac{(2)^4}{4} - \frac{2}{3}(2)^3 + (2(2))\right]$$

$$= \frac{4595}{12} = 382.92$$

P. 52 ② اوجد :

ⓐ $$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{1}{2}\sin 2x - \csc^2 x\right) dx$$

$$= \left[-\frac{1}{2} \times \frac{1}{2}\cos 2x + \cot x\right]_{\pi/4}^{\pi/2}$$

$$= \frac{-1}{4}\cos 2\frac{\pi}{2} + \cot\frac{\pi}{2} - \left[\frac{-1}{4}\cos 2\cdot\frac{\pi}{4} + \cot\frac{\pi}{4}\right]$$

$$= \frac{-3}{4}$$

ⓑ $$\int_2^{-3} 5\,dx = \left[5x\right]_2^{-3} = 5(-3) - 5(2) = -25$$

ⓒ $$\int_3^3 (-2x^3 + x^2)\,dx = 0$$

ⓓ $$\int_2^4 \frac{dx}{x-1} = \left[\ln|x-1|\right]_2^4$$

$$= \ln(4-1) - \ln(2-1)$$

$$= \ln 3 - \ln 1 = 1.099$$

P. 52 ③ أوجد

ⓐ $\int_{-3}^{4} |2x-4| \, dx$

$2x-4=0 \Rightarrow x=2$

$-(2x-4)$		$2x-4$
$-$	2	$+$

$= -\int_{-3}^{2} (2x-4)\,dx + \int_{2}^{4} (2x-4)\,dx$

$= -\left[x^2-4x\right]_{-3}^{2} + \left[x^2-4x\right]_{2}^{4}$

$= -\left[2^2-4(2)-((-3)^2-4(-3))\right] + \left[4^2-4(4)-2^2-4(2)\right]$

$= 29$

ⓑ $\int_{1}^{3} |x+2| \, dx$

$x+2=0$

$x=-2$

$= \int_{1}^{3} x+2 \, dx = \left[\frac{x^2}{2}+2x\right]_{1}^{3}$

$= \frac{3^2}{2}+2(3)-\left[\frac{1^2}{2}+2(1)\right] = 8$

P. 53 ④ دون حساب قيمة التكامل أثبت أن

$\int_{-1}^{0} (x^2+x)\,dx \leq 0$

بفرض $f(x)=x^2+x$

$x^2+x=0 \Rightarrow x(x+1)=0 \Rightarrow x=0$ or $x=-1$

$-\infty$				∞
$+$	-1	$-$	0	$+$

$f(x) \leq 0 \quad \forall x \in [-1,0]$

$\therefore \int_{-1}^{0} (x^2+x)\,dx \leq 0$

⑤ دون حساب قيمة التكامل أثبت أن : P.54

$$\int_{-1}^{2}(x^2+1)\,dx \geqslant \int_{-1}^{2}(x-1)\,dx$$

بفرض

$$f(x)=x^2+1 \quad , \quad g(x)=x-1$$

$$f(x)-g(x)=x^2+1-(x-1)=x^2+1-x+1$$
$$=x^2-x+2$$

$$x^2-x+2=0 \Rightarrow$$

$$\Delta=b^2-4ac$$
$$=(-1)^2-4(1)(2)=-7<0$$

∴ لا توجد للمعادلة جذور حقيقية ⟸

∴ $f(x)-g(x)$ وحيدة الإشارة .

وبأخذ قيمة إختيارية نجد أن

$$f(x)-g(x)\geqslant 0 \quad \forall x\in R$$

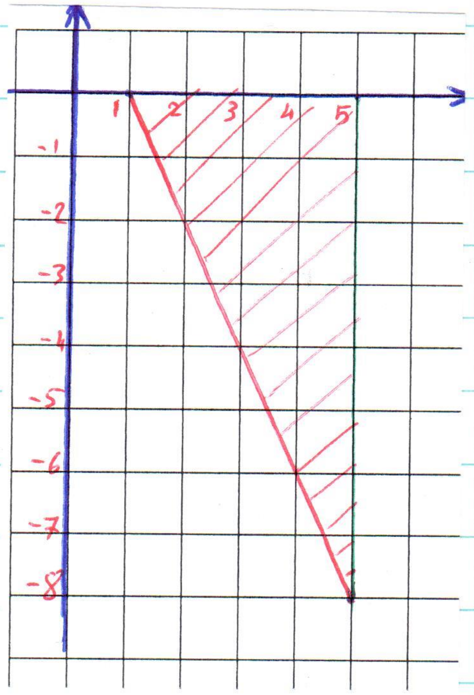
$\therefore \quad f(x)-g(x)\geqslant 0 \quad \forall x\in[-1,2]$

$\therefore \quad x^2+1-(x-1)\geqslant 0 \quad \forall x\in[-1,2]$

$\therefore \quad x^2+1\geqslant x-1 \quad \forall x\in[-1,2]$

$$\therefore \quad \int_{-1}^{2}(x^2+1)\,dx \geqslant \int_{-1}^{2}(x-1)\,dx$$

⑥ P. 55 اوجد قيمه $\int_{1}^{5}(2-2x)\,dx$ بيانياً



$f(x) = 2 - 2x$

x	1	5
$f(x)$	0	-8

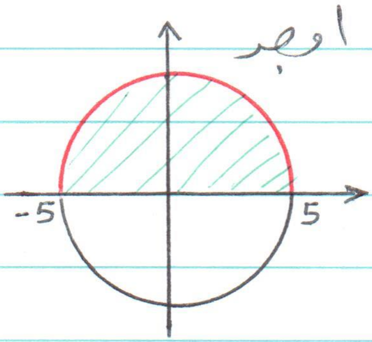
مساحة المثلث = $\frac{1}{2} \times 4 \times 8$

$= 16$ units squared

$$\int_{1}^{5}(2-2x)\,dx = \left[2x - x^2\right]_{1}^{5}$$

$$= 2(5)-(5)^2 - (2(1)-1^2) = -16$$

⑦ P.56 اوجد



ⓐ $\int_{-5}^{5}\sqrt{25-x^2}\,dx$

$$y = \sqrt{25-x^2}$$

$$y^2 = 25 - x^2$$

$$x^2 + y^2 = 25$$

هي معادلة دائرة مركزها نقطة الأصل ونصف قطرها 5 وحده

والدالة $y = \sqrt{25-x^2}$ تمثل معادلة النصف العلوي للدائرة

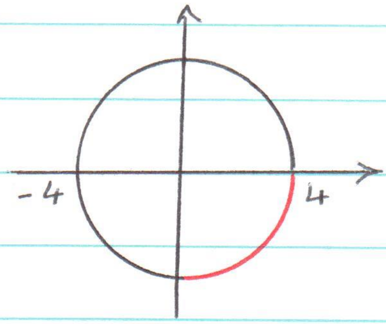
∴ مساحة المنطقة المظللة = $\int_{-5}^{5}\sqrt{25-x^2}\,dx$

$$\therefore \int_{-5}^{5}\sqrt{25-x^2} = \frac{1}{2}\pi(5)^2$$

$$= \frac{25\pi}{2}$$

P. 56 (7) أوجد

(b) $\int_0^4 -\sqrt{16-x^2}\,dx$



$$y = -\sqrt{16-x^2}$$
$$y^2 = 16 - x^2$$
$$x^2 + y^2 = 16$$

وهي معادلة دائرة مركزها النقطة الأصل ونصف قطرها 4 وحدات

والدالة $y = -\sqrt{16-x^2}$ تمثل معادلة النصف السفلي للدائرة

$$\int_0^4 -\sqrt{16-x^2}\,dx = \frac{-1}{4}\pi(4)^2 = -4\pi$$

P. 57 (8) (a) مثال 8 بطريقة ثانية

$$I = \int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx = \int_0^{\frac{\pi}{4}} \sec x \cdot \sec x \tan x\,dx$$

$u = \sec x \qquad du = \sec x \tan x\,dx$

$u = \sec 0 = 1 \Leftarrow x = 0$ عندما

$u = \sec\frac{\pi}{4} = \sqrt{2} \Leftarrow x = \frac{\pi}{4}$ عندما

$$I = \int_0^{\frac{\pi}{4}} u\,du = \left[\frac{u^2}{2}\right]_1^{\sqrt{2}} = \frac{(\sqrt{2})^2}{2} - \frac{(1)^2}{2} = 1 - \frac{1}{2} = \frac{1}{2}$$

(b) أوجد

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 2x \cos 2x\,dx = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 2x \; 2\cos 2x\,dx$$

$=$

$u = \sin 2x \qquad du = 2\cos 2x\,dx$

$u = \sin 2\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Leftarrow x = \frac{\pi}{3}$ عندما

$u = \sin 2\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \Leftarrow x = \frac{\pi}{6}$

$$I = \frac{1}{2}\int_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} u\,du = 0$$

(a) $I = \int_{-1}^{1} ((x+1)\sqrt{x^2+2x+5})\,dx$ 　　　　اوجد (9) P. 58

$= \frac{1}{2}\int_{-1}^{1} 2(x+1)\,(x^2+2x+5)^{\frac{1}{2}}\,dx$

$u = x^2+2x+5 \qquad du = (2x+2)\,dx$

$u = (-1)^2 + 2(-1) + 5 \Leftarrow x = -1$ عندما

$= 4$

$u = 1^2 + 2(1) + 5 = 8 \Leftarrow x = 1$ عندما

$I = \frac{1}{2}\int_{4}^{8} u^{\frac{1}{2}}\,du = \frac{1}{2}\,\frac{2}{3}\left[u^{\frac{3}{2}}\right]_4^8$

$= \frac{1}{3}\left(8^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) = 4.876$

(b) $I = \int_{2}^{5} x\sqrt{x-1}\,dx$

$= \int_{2}^{5} (x-1+1)(x-1)^{\frac{1}{2}}\,dx$

$= \int_{2}^{5} (x-1)^{\frac{3}{2}} + (x-1)^{\frac{1}{2}}\,dx$

$u = x-1 \qquad du = dx$

$= \int_{1}^{4} u^{\frac{3}{2}}\,du + \int_{1}^{4} u^{\frac{1}{2}}\,du$

$u = 2-1 = 1 \Leftarrow x = 2$ عندما

$u = 5-1 = 4 \Leftarrow x = 5$ عندما

$I = \left[\frac{2}{5}u^{\frac{5}{2}}\right]_1^4 + \left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4$

$= \frac{2}{5}\left(4^{\frac{5}{2}} - 1^{\frac{5}{2}}\right) + \frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$

$= 17.067$

P. 58 (10)

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$$

$u = x \qquad dv = \sec^2 x \, dx$

$du = dx \qquad v = \tan x$

$$\int u \, dv = u . v - \int v . du$$

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \Big[x \tan x\Big]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x \, dx$$

$$= \Big[x \tan x\Big]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} \, dx$$

$$= \Big[x \tan x\Big]_0^{\frac{\pi}{4}} - \Big[\ln|\cos x|\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} \tan \frac{\pi}{4} - 0 - \Big[\ln|\cos x|\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} . \tan \frac{\pi}{4} + \ln|\cos \tfrac{\pi}{4}| - \ln|\cos(0)|$$

$$= \frac{\pi}{4} - \ln\sqrt{2}$$

$$I = \int_4^7 \frac{3x^2 - 17}{x^2 - x - 6}\,dx$$ اوجد: (11) P. 59

$$\begin{array}{r|l} & \quad 3 \\ \hline x^2 - x - 6 & 3x^2 \quad\quad -17 \\ & \overset{\ominus}{3x^2} \overset{\oplus}{-} 3x \overset{\oplus}{-} 18 \\ \hline & \quad 3x + 1 \end{array}$$

$$\frac{3x^2 - 17}{x^2 - x - 6} = 3 + \frac{3x+1}{x^2 - x - 6}$$

$$= 3 + \frac{3x+1}{x^2-x-6} = 3 + \frac{A}{x-3} + \frac{B}{x+2}$$

$$\frac{3x+1}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$3x + 1 = A(x+2) + B(x-3)$$

نضع $x = 3$

$$10 = A(5) + B(0) \Rightarrow A = 2$$

نضع $x = -2$

$$-5 = A(0) + B(-5) \Rightarrow B = 1$$

$$\therefore \frac{3x+1}{x^2-x-6} = \frac{2}{x-3} + \frac{1}{x+2}$$

$$\therefore I = \int_4^7 3\,dx + \int_4^7 \frac{2}{x-3}\,dx + \int_4^7 \frac{dx}{x+2}$$

$$= [3x]_4^7 + 2\left[\ln|x-3|\right]_4^7 + \left[\ln|x+2|\right]_4^7$$

$$= 3(7-4) + 2(\ln 4 - \ln 1) + \ln 9 - \ln 6$$

$$= 12.178$$