

5-4

الدوال الأسية واللوغاريتمية

$$\frac{d}{dx} a^x = a^x \ln a \quad , \quad \frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx} \quad , \quad \frac{d}{dx} e^u = e^u \frac{du}{dx} = e^u u'$$

$$\frac{d}{dx} (\ln x) = \frac{1}{x}$$

$$\frac{d}{dx} (\ln u) = \frac{1}{u} \frac{du}{dx} = \frac{u'}{u}$$

$$\int e^x dx = e^x + C$$

$$\int e^u u' dx = e^u + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{u'}{u} dx = \ln|u| + C$$

5-4

$$\textcircled{1} \quad y = 7^x \Rightarrow \frac{dy}{dx} = 7^x \ln 7$$

$$\textcircled{2} \quad y = 5^{\sqrt{x+1}} \Rightarrow \frac{dy}{dx} = 5^{\sqrt{x+1}} \cdot \ln 5 \cdot \frac{1}{2\sqrt{x+1}}$$

$$\textcircled{3} \quad y = 8^{\tan x} \Rightarrow \frac{dy}{dx} = 8^{\tan x} \cdot \ln 8 \cdot \sec^2 x$$

$$\textcircled{4} \quad y = 2e^x \Rightarrow \frac{dy}{dx} = 2e^x$$

$$\textcircled{5} \quad y = e^{-x} \Rightarrow \frac{dy}{dx} = -e^{-x}$$

$$\textcircled{6} \quad y = 3e^{\frac{x}{5}} \Rightarrow \frac{dy}{dx} = 3e^{\frac{x}{5}} \cdot \frac{1}{5} = \frac{3}{5}e^{\frac{x}{5}}$$

$$\textcircled{7} \quad y = e^{x^2-x+1} \Rightarrow \frac{dy}{dx} = e^{x^2-x+1} \cdot (2x-1)$$

$$\textcircled{8} \quad y = e^{2\sqrt{x}+3} \Rightarrow \frac{dy}{dx} = e^{2\sqrt{x}+3} \cdot \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}} e^{2\sqrt{x}+3}$$

$$\textcircled{9} \quad y = e^{\csc x} \Rightarrow \frac{dy}{dx} = -\csc x \cot x \cdot e^{\csc x}$$

$$\textcircled{10} \quad y = e^{x^4-5} \Rightarrow \frac{dy}{dx} = 4x^3 e^{x^4-5}$$

$$\textcircled{11} \quad y = \ln(x^3) \Rightarrow \frac{dy}{dx} = \frac{3x^2}{x^3} = \frac{3}{x}$$

$$\textcircled{12} \quad y = \ln\left(\frac{1}{x^2}\right) \Rightarrow \frac{dy}{dx} = \frac{\frac{-2}{x^3}}{\frac{1}{x^2}} = \frac{-2}{x}$$

$$\textcircled{13} \quad y = \ln(x+2) \Rightarrow \frac{dy}{dx} = \frac{1}{x+2}$$

$$\textcircled{14} \quad y = \ln(2 - \cos x) \Rightarrow \frac{dy}{dx} = \frac{\sin x}{2 - \cos x}$$

$$\textcircled{15} \quad y = \ln(\ln x) \Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x \ln x}$$

---

$$\textcircled{16} \quad \int e^{0.1x} dx = 10e^{0.1x} + c$$

$$\textcircled{17} \quad \int \frac{-1}{x^2} e^{\frac{1}{x}} dx \qquad u = \frac{1}{x} \rightarrow u' = \frac{-1}{x^2}$$
$$= -e^{\frac{1}{x}} + c$$

$$18 \quad \int (2x+1) e^{x^2+x+4} dx \qquad u = x^2+x+4$$
$$\qquad \qquad \qquad u' = 2x+1$$
$$= e^{x^2+x+4} + c$$

$$(19) \int (x^2 - 2) e^{x^3 - 6x} dx \quad \begin{array}{l} u = x^3 - 6x \\ u' = 3x^2 - 6 \end{array}$$

$$\frac{1}{3} \int 3(x^2 - 2) e^{x^3 - 6x} dx = \frac{1}{3} e^{x^3 - 6x} + C$$

$$(20) \int (e^{0.5x} + \frac{0.5}{x}) dx$$

$$= \int e^{0.5x} dx + \int \frac{0.5}{x} dx$$

$$= 2 e^{0.5x} + 0.5 \ln|x| + C$$

$$(21) \int \frac{e^x}{e^x + 1} dx \quad \begin{array}{l} u = e^x + 1 \\ u' = e^x \end{array}$$

$$= \int \frac{u'}{u} dx = \ln|u| + C = \ln|e^x + 1| + C$$

$$(22) \int \frac{x+1}{x^2+2x+5} dx \quad \begin{array}{l} u = x^2 + 2x + 5 \\ u' = 2x + 2 \end{array}$$

$$\frac{1}{2} \int \frac{2(x+1)}{x^2+2x+5} dx = \frac{1}{2} \int \frac{u'}{u} dx$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2 + 2x + 5| + C$$

$$(23) \int \frac{x^3 - x}{x^4 - 2x^2} dx$$

$$u = x^4 - 2x^2$$

$$u' = 4x^3 - 4x$$

$$\frac{1}{4} \int \frac{4(x^3 - x)}{x^4 - 2x^2} dx = \frac{1}{4} \int \frac{u'}{u} dx$$

$$= \frac{1}{4} \ln|u| + c = \frac{1}{4} \ln|x^4 - 2x^2| + c$$

$$(24) \int \frac{x^2 + 1}{x} dx = \int \frac{x^2}{x} + \frac{1}{x} dx$$

$$= \int x + \frac{1}{x} dx = \frac{x^2}{2} + \ln|x| + c$$

$$(25) \int \frac{2}{3x+1} dx$$

$$u = 3x + 1$$

$$u' = 3$$

$$= \frac{2}{3} \int \frac{3}{3x+1} dx = \frac{2}{3} \ln|3x+1| + c$$

$$(26) \int (2 \tan x - \csc^2 x) dx$$

$$= -2 \int \frac{-\sin x}{\cos x} dx - \int \csc^2 x dx$$

$$= -2 \int \frac{u'}{u} dx + \cot x + c \quad ; \quad u = \cos x$$

$$u' = -\sin x$$

$$= -2 \ln|u| + \cot x + c$$

$$= -2 \ln|\cos x| + \cot x + c$$

$$(27) \int (\cot x + x^2) dx$$

$$= \int \frac{\cos x}{\sin x} dx + \int x^2 dx$$

$$u = \sin x \rightarrow u' = \cos x$$

$$= \int \frac{u'}{u} dx + \int x^2 dx$$

$$= \ln|u| + \frac{x^3}{3} + C$$

$$= \ln|\sin x| + \frac{x^3}{3} + C$$