

حجوم الأجسام الدورانية

① P. 77

اوجد حجم الجسم الناتج من دوران المنطقة المستوية دره كامله حول محور السينات والمحوره بخص الداله f :

$$f(x) = \sqrt{x-1} \quad \text{ومحور السينات في الفترة } [1, 5]$$

$$V = \pi \int_1^5 (f(x))^2 dx = \pi \int_1^5 (\sqrt{x-1})^2 dx$$

$$= \pi \int_1^5 (x-1) dx = \pi \left[\frac{x^2}{2} - x \right]_1^5$$

$$= \pi \left[\left(\frac{25}{2} - 5 \right) - \left(\frac{1}{2} - 1 \right) \right] = 8\pi \text{ units cube}$$

② P. 78

$$f(x) = r \quad \text{في الفترة } [0, h] \quad ; r \neq 0$$

$$V = \pi \int_0^h (f(x))^2 dx = \pi \int_0^h r^2 dx \quad ; \text{ ثابت } r$$

$$= \pi r^2 (h - 0) = \pi r^2 h$$

$$f(x) = \frac{x^2}{2} + 1 \quad \text{و} \quad g(x) = \frac{x}{2} + 2 \quad \text{③ P. 79}$$

$$f(x) = g(x) \Rightarrow \frac{x^2}{2} + 1 = \frac{x}{2} + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow$$

$$\Rightarrow x = -1 \quad \text{و} \quad x = 2$$

نأخذ قيمه اقصاويه في الفترة $(-1, 2)$ ونلاحظ $x=0$

$$\Rightarrow f(0) = 1 \quad , \quad g(0) = 2 \Rightarrow g(x) \geq f(x) \geq 0 \quad \forall x \in [-1, 2]$$

$$V = \int_{-1}^2 \pi [(g(x))^2 - (f(x))^2] dx = \pi \int_{-1}^2 \left[\left(\frac{x}{2} + 2 \right)^2 - \left(\frac{x^2}{2} + 1 \right)^2 \right] dx$$

$$= \pi \int_{-1}^2 \left[\left(\frac{x^2}{4} + 2x + 4 \right) - \left(\frac{x^4}{4} + x^2 + 1 \right) \right] dx = \pi \int_{-1}^2 \left[-\frac{1}{4}x^4 - \frac{3}{4}x^2 + 2x + 3 \right] dx$$

$$= \pi \left[-\frac{1}{20}x^5 - \frac{1}{4}x^3 + x^2 + 3x \right]_{-1}^2 = \frac{81}{10} \pi$$

$$y_1 = x + 3 \quad \text{and} \quad y_2 = x^2 + 1$$

④ p. 79

$$y_1 = y_2 \Rightarrow x + 3 = x^2 + 1 \Rightarrow x^2 - x - 2 = 0 \Rightarrow$$

$$\Rightarrow x = -1 \quad \text{and} \quad x = 2$$

$$\text{So } x = 0 \in (-1, 2) \Rightarrow y_1 = 3 \quad \text{and} \quad y_2 = 1$$

$$\therefore y_1 \geq y_2 \geq 0 \quad \forall x \in [-1, 2]$$

$$V = \pi \int_{-1}^2 [y_1^2 - y_2^2] dx = \pi \int_{-1}^2 [(x+3)^2 - (x^2+1)^2] dx$$

$$= \pi \int_{-1}^2 [(x^2 + 6x + 9) - (x^4 + 2x^2 + 1)] dx$$

$$= \pi \int_{-1}^2 (-x^4 - x^2 + 6x + 8) dx$$

$$= \pi \left[-\frac{1}{5}x^5 - \frac{1}{3}x^3 + 3x^2 + 8x \right]_{-1}^2$$

$$= \pi \left[\left(-\frac{(2)^5}{5} - \frac{(2)^3}{3} + 3(2)^2 + 8(2) \right) - \left(-\frac{(-1)^5}{5} - \frac{(-1)^3}{3} + 3(-1)^2 + 8(-1) \right) \right]$$

$$= \frac{117}{5} \pi \text{ units cube}$$