التكامل المحدد

(1) $\int_{-1}^{1} 3x(x-4)\,dx = \int_{-1}^{1} 3x^2-12x\,dx$

$= \left[x^3-6x^2\right]_{-1}^{1}$

$= 1^3-6(1)^2-\left((-1)^3-6(-1)^2\right) = 2$

(2) $\int_{0}^{2} (x+1)^2\,dx = \int_{0}^{2} x^2+2x+1\,dx$

$= \left[\frac{x^3}{3}+x^2+x\right]_{0}^{2} = \frac{(2)^3}{3}+(2)^2+2-0 = \frac{26}{3}$

(3) $\int_{0}^{4} \frac{x^2-1}{x+1}\,dx = \int_{0}^{4} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}\,dx = \int_{0}^{4} x-1\,dx$

$= \left[\frac{x^2}{2}-x\right]_{0}^{4} = \frac{4^2}{2}-4-0 = 4$

(4) $\int_{0}^{\frac{\pi}{3}} \cos 3x\,dx = \left[\frac{\sin 3x}{3}\right]_{0}^{\frac{\pi}{3}}$

$= \frac{1}{3}\left(\sin 3\left(\frac{\pi}{3}\right) - \sin 3(0)\right)$

$= \frac{1}{3}(0-0) = 0$

(5) $\int_{1}^{4} \frac{8-x^4}{2x^2}\,dx = \int_{1}^{4} 4x^{-2}-\frac{1}{2}x^2\,dx$

$= \left[\frac{-4}{x}-\frac{x^3}{6}\right]_{1}^{4} = -\frac{4}{4}-\frac{(4)^3}{6}-\left(\frac{-4}{1}-\frac{1^3}{6}\right)$

$= -7.5$

(6) $\int_0^1 x\sqrt{x}\,dx = \int_0^1 x^{\frac{3}{2}}\,dx$

$= \frac{2}{5}\left[x^{\frac{5}{2}}\right]_0^1 = \frac{2}{5}(1-0) = \frac{2}{5}$

(7) $\int_1^2 \left(3e^x + \frac{5}{x}\right)dx = \left[3e^x + 5\ln|x|\right]_1^2$

$= 3e^2 + 5\ln 2 - (3e^1 + 5\ln 1) = 17.478$

(8) $\int_{-1}^3 |x-2|\,dx$

$x - 2 = 0 \Longrightarrow x = 2.$

$-(x-2)$ $x-2$

-1 $-$ 2 $+$ 3

$= \int_{-1}^2 -(x-2)\,dx + \int_2^3 x-2\,dx$

$= \left[-\left(\frac{x^2}{2} - 2x\right)\right]_{-1}^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$

$= -\left[\frac{2^2}{2} - 2(2) - \left(\frac{1^2}{2} - 2(1)\right)\right] + \frac{3^2}{2} - 2(3) - \left(\frac{2^2}{2} - 2(2)\right) = 5$

(9) $\int_{-1}^1 |x^3| = \int_{-1}^0 -x^3\,dx + \int_0^1 x^3\,dx$

$= \left[\frac{-x^4}{4}\right]_{-1}^0 + \left[\frac{x^4}{4}\right]_0^1 = 0 - \frac{-(-1)^4}{4} + \frac{1}{4} - 0 = \frac{1}{2}$

(10) $\int_{-2}^3 x|x| + 3\,dx$

$= \int_{-2}^0 -x^2+3\,dx + \int_0^3 x^2+3\,dx$

$= \left[\frac{-x^3}{3} + 3x\right]_{-2}^0 + \left[\frac{x^3}{3} + 3x\right]_0^3 =$

$= 0 - \left(\frac{-(-2)^3}{3} + 3(-2)\right) + \frac{(3)^3}{3} + 3(3) - 0 = 21.33$

دون حساب التكامل اثبت ان

(11) $\int_{-4}^{2} (x^2+2x-8)\,dx \leq 0$

$$x^2+2x-8=0$$
$$(x-2)(x+4)=0$$
$$x=2 \ , \ x=-4$$

$\therefore \ x^2+2x-8 \leq 0 \quad \forall x \in [-4,2]$

$\therefore \ \int_{-4}^{2} (x^2+2x-8)\,dx \leq 0$

(12) $\int_{-1}^{0} x^3-5x^2-6x\,dx \geq 0$

$$x^3-5x^2-6x=0$$
$$x(x^2-5x-6)=0$$
$$x(x-6)(x+1)=0$$
$$x=0 \ , \ x=-1 \ , \ x=6$$

$\therefore \ x^3-5x^2-6x \geq 0 \quad \forall x \in [-1,0]$

$\therefore \ \int_{-1}^{0} x^3-5x^2-6x\,dx \geq 0$

(13) $\int_0^1 (x^2-3x+7)\,dx \geq \int_0^1 (4x-5)\,dx$

برهن

$$(x^2-3x+7)-(4x-5) = x^2-3x+7-4x+5$$
$$= x^2-7x+12$$
$$= (x-4)(x-3)$$
$$x=4 \quad , \quad x=3$$

$+ \quad - \quad +$

$0 \quad 1 \quad 3 \quad 4$

$\therefore\ x^2-7x+12 \geq 0 \quad \forall x \in [0,1]$

$\therefore\ (x^2-3x+7)-(4x-5) \geq 0 \quad \forall x \in [0,1]$

$\therefore\ x^2-3x+7 \geq 4x-5 \quad \forall x \in [0,1]$

$\therefore\ \int_0^1 (x^2-3x+7)\,dx \geq \int_0^1 (4x-5)\,dx$

$*\ \int_2^4 x^2-3x+5\,dx \leq \int_2^4 3x-3\,dx$

$$(x^2-3x+5)-(3x-3) = x^2-3x+5-3x+3$$
$$= x^2-6x+8$$
$$= (x-4)(x-2)$$
$$x=4 \quad \text{or} \quad x=2$$

$+ \quad - \quad +$

$2 \quad 4$

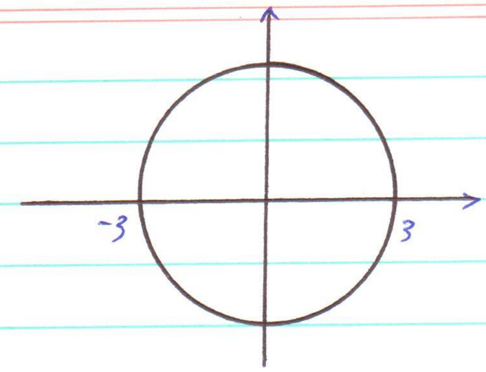
$\therefore\ x^2-6x+8 \leq 0 \quad \forall x \in [2,4]$

$\therefore\ (x^2-3x+5)-(3x-3) \leq 0 \quad \forall x \in [2,4]$

$\therefore\ x^2-3x+5 \leq 3x-3 \quad \forall x \in [2,4]$

$\therefore\ \int_2^4 x^2-3x+5\,dx \leq \int_2^4 3x-3\,dx$

(14) $\int_{-3}^{3} \sqrt{9-x^2}\, dx$



$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

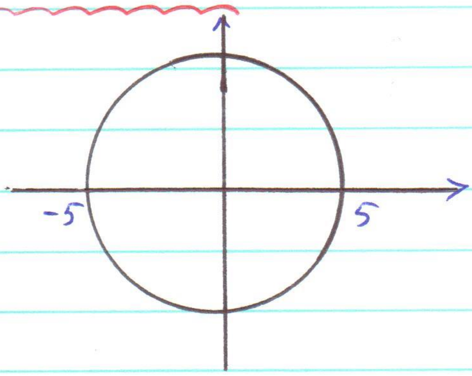
وهي معادلة دائرة مركزها نقطة الأصل وطول نصف قطرها 3 وحدات

والدالة $y = \sqrt{9-x^2}$ تمثل معادلة النصف العلوي للدائرة

$\therefore \int_{-3}^{3} \sqrt{9-x^2}\, dx =$ مساحة النصف العلوي للدائرة

$= \frac{1}{2} \pi (3)^2 = \frac{9\pi}{2}$

(15) $\int_{-5}^{0} -\sqrt{25-x^2}\, dx$



$y = -\sqrt{25-x^2}$

$y^2 = 25 - x^2$

$x^2 + y^2 = 25$

وهي معادلة دائرة مركزها نقطة الأصل وطول نصف قطرها 5 وحدات

والدالة $y = -\sqrt{25-x^2}$ تمثل معادلة النصف السفلي للدائرة

$\therefore \int_{-5}^{0} -\sqrt{25-x^2}\, dx =$ − مساحة ربع دائرة

$= -\frac{1}{4} \pi (5)^2$

$= \frac{-25\pi}{4}$

(16) $$\int_0^3 \frac{dx}{(1+x)^2} = \int_0^3 (1+x)^{-2}\,dx = \left[\frac{-1}{1+x}\right]_0^3$$

$$= \frac{-1}{1+3} - \frac{-1}{1+0} = \frac{3}{4}$$

(17) $$\int_e^6 \frac{dx}{x\ln x}$$

$u = \ln x$

$du = \frac{dx}{x}$

$x = e \Rightarrow u = \ln e = 1$

$x = 6 \Rightarrow u = \ln 6$

$$\therefore \int_e^6 \frac{dx}{x\ln x} = \int_1^{\ln 6} \frac{du}{u} = \left[\ln u\right]_1^{\ln 6} = 0.583$$

(18) $$\int_1^e \frac{\ln^6 x}{x}\,dx$$

$u = \ln x \Rightarrow du = \frac{dx}{x}$

$x = 1 \Rightarrow u = \ln 1 = 0$

$x = e \Rightarrow u = \ln e = 1$

$$\therefore \int_1^e \frac{\ln^6 x}{x}\,dx = \int_0^1 u^6\,du = \left[\frac{u^7}{7}\right]_0^1 = \frac{1}{7}$$

(19) $$\int_{-1}^3 \frac{x\,dx}{x^2+1}$$

$u = x^2+1 \Rightarrow du = 2x\,dx$

$x = -1 \Rightarrow u = (-1)^2+1 = 2$

$x = 3 \Rightarrow u = 3^2+1 = 10$

$$\therefore \int_{-1}^3 \frac{x\,dx}{x^2+1} = \int_2^{10} \frac{x}{u} \cdot \frac{du}{2x} = \frac{1}{2}\int_2^{10} \frac{du}{u}$$

$$= \frac{1}{2}\left[\ln|u|\right]_2^{10} = \frac{1}{2}(\ln 10 - \ln 2) = 0.8$$

(20) $\int_0^{\frac{\pi}{2}} x \sin x \, dx$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$= \left[-x \cos x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos x \, dx$

$= \left[-x \cos x\right]_0^{\pi/2} + \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$

(21) $\int_0^{\pi} x \cos 3x$

$u = x \qquad dv = \cos 3x \, dx$

$du = dx \qquad v = \frac{1}{3} \sin 3x$

$= \left[\frac{1}{3} x \sin 3x\right]_0^{\pi} - \int_0^{\pi} \frac{1}{3} \sin 3x \, dx$

$= \left[\frac{1}{3} x \sin 3x\right]_0^{\pi} + \left[\frac{1}{9} \cos 3x\right]_0^{\pi} = \frac{-2}{9}$

(22) $\int_1^3 x^3 \ln x \, dx =$

$u = \ln x \qquad dv = x^3 dx$

$du = \frac{dx}{x} \qquad v = \frac{1}{4} x^4$

$= \left[\frac{x^4}{4} \ln x\right]_1^3 - \int_1^3 \frac{x^4}{4} \cdot \frac{dx}{x}$

$= \left[\frac{x^4}{4} \ln x\right]_1^3 - \int_1^3 \frac{x^3}{4} dx$

$= \left[\frac{x^4}{4} \ln x\right]_1^3 - \left[\frac{x^4}{16}\right]_1^3 = \frac{27}{4} \ln 3 - \frac{41}{8}$

(23) $I = \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx$

$u = e^{2x} \qquad dv = \cos x \, dx$

$du = 2e^{2x} dx \qquad v = \sin x$

$= \left[e^{2x} \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2e^{2x} \sin x \, dx$

$= e^{\pi} - \int_0^{\frac{\pi}{2}} 2e^{2x} \sin x \, dx \quad \text{---} \ (1)$

$u = 2e^{2x} \qquad dv = \sin x \, dx$

$du = 4e^{2x} dx \qquad v = -\cos x$

$\therefore \int_0^{\frac{\pi}{2}} 2e^{2x} \sin x \, dx = \left[-2e^{2x} \cos x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-4e^{2x} \cos x) dx$

$= 2 + 4 \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx$

بالتعويض في (1)

$I = e^{\pi} - 2 - 4 \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx$

$I = e^{\pi} - 2 - 4I$

$5I = e^{\pi} - 2$

$I = \frac{e^{\pi} - 2}{5} = 4.23$

$\therefore \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = 4.23$

(24) $\int_{-1}^{1} \frac{4}{x^2-4}\,dx =$

نوجد الكسور الجزئية للدالة الحدودية النسبية

$$f(x) = \frac{4}{x^2-4} = \frac{4}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$4 = A(x+2) + B(x-2)$$

بوضع $x = 2 \Leftarrow$

$$4 = A(4) + B(0) \Rightarrow A = 1$$

بوضع $x = -2 \Leftarrow$

$$4 = A(0) + B(-2-2) \Rightarrow B = -1$$

$$\therefore f(x) = \frac{1}{x-2} + \frac{-1}{x+2}$$

$$\therefore \int_{-1}^{1} \frac{4}{x^2-4}\,dx = \int_{-1}^{1} \left(\frac{1}{x-2} - \frac{1}{x+2}\right) dx$$

$$= \int_{-1}^{1} \frac{1}{x-2}\,dx - \int_{-1}^{1} \frac{1}{x+2}\,dx$$

$$= \Big[\ln|x-2|\Big]_{-1}^{1} - \Big[\ln|x+2|\Big]_{-1}^{1}$$

$$= \ln 1 - \ln 3 - \big[\ln 3 - \ln 1\big]$$

$$= 2\ln 1 - 2\ln 3$$

(25) $\displaystyle\int_{-2}^{0} \frac{5x-1}{x^2+2x-3}\,dx$

نوجد الكسور الجزئية للدالة الحدودية النسبية

$$f(x) = \frac{5x-1}{x^2+2x-3} = \frac{5x-1}{(x+3)(x-1)}$$

$$= \frac{A}{x+3} + \frac{B}{x-1}$$

$$5x-1 = A(x-1) + B(x+3)$$

بفرض $x = 1$

$$5(1)-1 = A(0) + B(1+3) \Rightarrow 4 = 4B$$
$$\Rightarrow B = 1$$

بفرض $x = -3$

$$5(-3)-1 = A(-3-1) + B(0) \Rightarrow -16 = -4A$$
$$\Rightarrow A = 4$$

$$\therefore f(x) = \frac{4}{x+3} + \frac{1}{x-1}$$

$$\therefore \int_{-2}^{0} \frac{5x-1}{x^2+2x-3}\,dx = \int_{-2}^{0} \frac{4}{x+3}\,dx + \int_{-2}^{0} \frac{1}{x-1}\,dx$$

$$= 4\int_{-2}^{0} \frac{dx}{x+3} + \int_{-2}^{0} \frac{dx}{x-1}$$

$$= 4\Big[\ln|x+3|\Big]_{-2}^{0} + \Big[\ln|x-1|\Big]_{-2}^{0}$$

$$= 4\left[\ln 3 - \ln 1\right] + \left[\ln 1 - \ln 3\right]$$

$$= 4\ln 3 - \ln 3 = 3\ln 3$$

(26) $$\int_1^3 \frac{x^2}{(x+1)^2}\,dx = \int \frac{x^2}{x^2+2x+1}\,dx$$

$$\begin{array}{r|l} & 1 \\ \hline x^2+2x+1 & x^2 \\ & \ominus x^2 \ominus 2x \ominus 1 \\ \hline & 0 \quad -2x-1 \end{array}$$

$$\frac{x^2}{(x+1)^2} = 1 + \frac{-2x-1}{(x+1)^2} = 1 + \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$$

$$\therefore\ -2x-1 = A(x+1) + B$$

بفرض $x=-1$

$$-2(-1)-1 = A(0)+B \Rightarrow B = 1$$

بفرض $x=0$

$$-2(0)-1 = A(0+1)+1$$

$$-1 = A+1 \Rightarrow A = -2$$

$$\therefore \int_1^3 \frac{x^2}{(x+1)^2}\,dx = \int_1^3 \left(1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right)dx$$

$$= (3-1) - 2\Big[\ln|x+1|\Big]_1^3 + \left[\frac{-1}{x+1}\right]_1^3$$

$$= 2 - 2[\ln 4 - \ln 2] + \frac{-1}{4} - \frac{-1}{2}$$

$$= 2 - 2\ln\frac{4}{2} + \frac{1}{4}$$

$$= \frac{9}{2} - 2\ln\frac{1}{2}$$