5 - ١ التكامل غير المحدد

(١) أثبت أن $F(x)=(3x+2)^5+7$ هي مشتقة عكسية للدالة

$$f(x)=15(3x+2)^4$$

$$F'(x)=5(3x+2)^4(3)=15(3x+2)^4=f(x)$$

∴ F دالة مشتقة عكسية للدالة f

(4) $\int(x^5-6x+3)\,dx=\frac{x^6}{6}-\frac{6x^2}{2}+3x+c$

(5) $\int(3-6x^2)\,dx=3x-2x^3+c$

(6) $\int\frac{1}{3}x^{-\frac{2}{3}}\,dx=\frac{1}{3}\cdot\frac{3}{1}x^{\frac{1}{3}}+c=\sqrt[3]{x}+c$

(7) $\int(x^3-\frac{1}{x^3})\,dx=\int(x^3-x^{-3})\,dx=\frac{x^4}{4}-\frac{x^{-2}}{-2}+c$

(8) $\int\frac{x^4-27x}{x^2-3x}\,dx=\int\frac{\cancel{x}(x^3-27)}{\cancel{x}(x-3)}\,dx$

$=\int\frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}}\,dx=\int x^2+3x+9\,dx$

$=\frac{x^3}{3}+\frac{3}{2}x^2+9x+c$

(9) $\int(x-2)(2x+3)\,dx=\int 2x^2-x-6\,dx$

$=\frac{2}{3}x^3-\frac{x^2}{2}-6x+c$

(10) $\int \frac{x-1}{\sqrt{x}+1}\,dx = \int \frac{(\sqrt{x}-1)\cancel{(\sqrt{x}+1)}}{\cancel{(\sqrt{x}+1)}}\,dx$

$= \int \sqrt{x}-1\,dx = \frac{2}{3}x^{\frac{3}{2}} - x + c$

(11) $\int \frac{x-\sqrt{x}}{x}\,dx = \int 1 - x^{-\frac{1}{2}}\,dx$

$= x - 2x^{\frac{1}{2}} + c = x - 2\sqrt{x} + c$

(12) $\int \frac{5+2x}{\sqrt{x}}\,dx = \int 5x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}\,dx$

$= 10x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

(13) $\int (x+\frac{1}{x})^2\,dx = \int x^2 + 2 + x^{-2}\,dx$

$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c$

$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$

(14) $\int (\sqrt[3]{x^2} + \sqrt[4]{x^3})\,dx = \int x^{\frac{2}{3}} + x^{\frac{3}{4}}\,dx$

$= \frac{3}{5}x^{\frac{5}{3}} + \frac{4}{7}x^{\frac{7}{4}} + c$

(15) $F(x) = \int (3x^2 - 5)\,dx = x^3 - 5x + c$

$F(2) = 3 \Rightarrow (2)^3 - 5(2) + c = 3 \Rightarrow c = 5$

$\therefore F(x) = x^3 - 5x + 5$